# Anomalous Reflection From a Phase Gradient Metasurface With Arbitrary Incident Angle 

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#### Abstract

A planar phase gradient metasurface (PGM) with phase gradients in two in-plane directions will introduce two additional wave vectors to the reflected wave vector. For small incident angles, close to the normal vector, the magnitude of the reflected wave vector is smaller than that of the incident wave vector and the direction of the anomalous reflected wave can be determined from the incident wave vector components and additional phase gradient components. The expanded generalized Snell's law, which includes diffraction order modes, is combined with array theory to accurately predict the directions of the reflected waves from a planar PGM for incident angles where the magnitude of the reflected wave vector is larger than that of the incident wave vector. The predicted directions of the reflected waves from a planar PGM are compared with simulated RCS results obtained with CST Studio Suite and measured results obtained in a compact range.


INDEX TERMS Bistatic radar, electromagnetic metamaterials, electromagnetic reflection, radar cross-sections.

## I. INTRODUCTION

Phase gradient metasurfaces (PGMs) are used to manipulate the radar cross section (RCS) of planar structures for monostatic and bistatic RCS applications [1], [2], [3]. PGMs can be designed to reduce or enhance the RCS of structures for specific scenarios if it is possible to predict the directions of the reflected waves from the PGM for arbitrary incident angles. The direction of the anomalous reflected wave is determined from the incident wave vector components and additional phase gradient components of the PGM. Predicting the directions of the scattered waves from PGMs with dual gradients are currently restricted to perpendicular incidence [4] or small incident angles close to the normal vector [5], [6]. These restrictions are limiting the application of PGMs for RCS control to scenarios where the incident angle is smaller than the critical value [7], i.e. when the summation of the reflected wave vector, due to the incident angle, and the phase gradient vector do not exceed the magnitude of the initial incident wave vector. In [8] a PGM was used to control the scattering

[^0]direction of optical waves. In [9] a general method is proposed to control the diffraction pattern both in angle and energy ratio between the scattered beams. This proposed method only consider scattered beams with incident angles smaller than the critical value. In [4], [7], and [8] it is claimed that incident angles smaller than the critical value cause anomalous reflected waves and incident angles larger than the critical value cause non-radiating surface waves.

In [10] it was shown that incident angles larger than the critical value are converted to evanescent surface waves. The phenomenon of negative reflection was also introduced in [10] for acoustic waves. A generalized Snell's law of reflection was formulated that depends on the incident and reflected wave vectors. This formulation makes it possible to predict the directions of the reflected waves from a PGM for scenarios where the incident wave angle is larger than the critical value. Subsequently the bistatic RCS of a PGM with a one dimensional phase gradient was considered and multiple directions for reflected modes were estimated in [11]. However, full wave scattering simulations of PGMs using CST Studio Suite [12] show additional reflected energy, close to
the plane of the PGM at $\theta_{r}=90^{\circ}$, not predicted by any of the valid diffraction order modes.

In [13] the angular directions of the scattered waves for a checkerboard metasurface were determined using array theory with inspiration from [14].

Destructive interference was utilized in [15] to design a surface, utilizing artificial magnetic conductor (AMC) elements, which reflect the incident wave simultaneously in phase and out of phase. The idea behind this concept is explained using array theory by modelling the unit cell as a $2 \times 2$ antenna array consisting of 4 antenna elements (representing the two different AMC elements) and assuming (as a first order approximation) that the four antennas all radiate the same amount of power.

In this paper the generalized Snell's law of reflection from [10] and [11] is extended to account for PGMs with different phase gradients in two orthogonal in-plane directions. The extended formulation is combined with antenna array theory concepts [13], [15] to estimate the finite number of scattering modes and accurately estimate the directions of the reflected waves. The formulation also takes into account the scattering beamwidth of the reflected waves from a finite PGM to estimate the directions of reflected waves close to the plane of the PGM. The estimated directions of the reflected waves from a planar PGM are compared with simulated RCS results obtained with CST Studio Suite, and measured results obtained in a compact range.

## II. PROBLEM FORMULATION

## A. GENERALISED SNELL'S LAW OF REFLECTION

The directions of the reflected waves from a PGM with phase gradients in two orthogonal in-plane directions are illustrated in Fig. 1. The relationship between the incident wave direction and reflected wave directions for incident angles smaller than the critical value is described by Snell's law of reflection [4], [5], [6], [7], [8]. The direction of the specular reflected wave, $\theta_{r}$ and $\varphi_{r}$, is expressed in [6] as

$$
\begin{align*}
& \theta_{r}=\arcsin \left(\frac{\sqrt{\left(\nabla \phi_{x}+k_{i x}\right)^{2}+\left(\nabla \phi_{y}+k_{i y}\right)^{2}}}{k_{i}}\right) \\
& \varphi_{r}=\arctan \left(\frac{k_{i y}+\nabla \phi_{y}}{k_{i x}+\nabla \phi_{x}}\right) \tag{1}
\end{align*}
$$

with $k_{i x}$ and $k_{i y}$ the magnitudes of the in-plane vector components of the incident wave vector, $\mathbf{k}_{\mathbf{i}} . \nabla \phi_{x}$ and $\nabla \phi_{y}$ are the orthogonal phase gradients of the PGM along the $x$ - and $y$-direction, respectively. If the incident angle is larger than the critical value, the numerator in (1) exceeds the denominator, and $\theta_{r}$ results in a complex value, and according to [6] the incident wave is coupled into a surface wave.

In [11] a PGM with a one dimensional phase gradient is considered and Snell's law is extended to incident angles larger than the critical value using a diffraction order element,

$$
\begin{equation*}
\sin \theta_{r}-\sin \theta_{i}=\frac{1}{\beta} \sigma \nabla \phi\left(1+\eta_{G}\right) \tag{2}
\end{equation*}
$$



FIGURE 1. A reflective PGM with phase gradients in two orthogonal in-plane directions, illustrating the incident and reflected wave vectors as well as phase gradient vectors. The dashed square indicates a sub cell consisting of $4 \times 4$ AMC elements.
where $\beta$ is the wavenumber of the incident wave, $\nabla \phi$ is the phase gradient of the PGM, $\eta_{G}$ is the diffraction order, and $\sigma=1$ or $\sigma=-1$ a parameter indicating the direction of the phase gradient.

For a PGM with phase gradients in two orthogonal in-plane directions and an incident wave from an arbitrary direction, $\left(\theta_{i}, \varphi_{i}\right)$, (2) can be rewritten as

$$
\begin{align*}
\beta \sin \theta_{r} \cos \varphi_{r} & =\beta \sin \theta_{i} \cos \left(\varphi_{i}-\pi\right)+\sigma \nabla \phi_{x}\left(1+\eta_{G)}\right) \\
\beta \sin \theta_{r} \sin \varphi_{r} & =\beta \sin \theta_{i} \sin \left(\varphi_{i}-\pi\right)+\sigma \nabla \phi_{y}\left(1+\eta_{G}\right) \tag{3}
\end{align*}
$$

$\nabla \phi_{x}$ and $\nabla \phi_{y}$ are the phase gradients of the PGM in the $x$ - and $y$-direction, respectively. The directions of reflected waves, $\theta_{r}, \varphi_{r}$, are determined by simultaneously solving (3) for different diffraction order values, $\eta_{G}$. Valid diffraction order modes will result in real values for the directions of the reflected waves [11]. However, full wave CST Studio Suite scattering simulations of PGMs show additional reflected energy, close to the plane of the PGM that is not predicted by any of the valid diffraction order modes [16].

## B. ANTENNA ARRAY THEORY AND SCATTERING FROM A PGM

Snell's law describes the relationship between the angle of incidence and reflection for a wave incident on an infinite surface. Practical PGMs are finite in size and will produce a reflected wave with a finite beamwidth, rather than a plane wave in a single direction [17].

A PGM is designed using AMC elements, which provide a uniform amplitude reflection and varied phase reflection. This phase reflection is controlled by varying certain parameters of the AMC elements [4]. Fig. 1 shows a reflective PGM consisting of $3 \times 3$ sub cells and $4 \times 4$ AMC elements per sub cell, realizing phase gradients in two orthogonal in-plane directions.

Following [1], [13], [14], [15] the scattering from the PGM can be analyzed as an equally spaced uniformly excited planar array. The normalized array factor for uniformly excited and equally spaced planar arrays [17], adapted to the scattering


FIGURE 2. Normalized array factor, $f_{x y}\left(\psi_{x}, \psi_{y}\right)$ with visible space regions indicated by various ellipses.
from a PGM for an incident wave from an arbitrary direction $\left(\theta_{i}, \varphi_{i}\right)$ is

$$
\begin{equation*}
f_{x y}(\theta, \varphi)=\left|\frac{\sin \left(\frac{N \psi_{x}(\theta, \varphi)}{2}\right)}{N \sin \left(\frac{\psi_{x}(\theta, \varphi)}{2}\right)} \times \frac{\sin \left(\frac{N \psi_{y}(\theta, \varphi)}{2}\right)}{N \sin \left(\frac{\psi_{y}(\theta, \varphi)}{2}\right)}\right| \tag{4}
\end{equation*}
$$

with

$$
\begin{align*}
\psi_{x}(\theta, \varphi)= & \beta d_{x}\left(\sin \theta \cos \varphi-\sin \theta_{i} \cos \left(\varphi_{i}-\pi\right)\right) \\
& +\sigma \nabla \phi_{x} d_{x}\left(1+\eta_{G}\right) \\
\psi_{y}(\theta, \varphi)= & \beta d_{y}\left(\sin \theta \sin \varphi-\sin \theta_{i} \sin \left(\varphi_{i}-\pi\right)\right) \\
& +\sigma \nabla \phi_{y} d_{y}\left(1+\eta_{G}\right) \tag{5}
\end{align*}
$$

where $N \times N$ represents the number of AMC elements in one sub cell; $\beta$ is the wavenumber of the incident wave and ( $d_{x}, d_{y}$ ) the spacing between the center points of the AMC elements in the $x$ - and $y$-direction, respectively.

The normalized array factor, $f_{x y}\left(\psi_{x}, \psi_{y}\right)$, for a sub cell with $4 \times 4$ AMC elements is shown in Fig. 2. The visible space regions [17] are given for different diffraction orders, $\eta_{G}$ by

$$
\begin{equation*}
\frac{\left(\psi_{x}-\psi_{x}(0,0)\right)^{2}}{\left(\beta d_{x}\right)^{2}}-\frac{\left(\psi_{y}-\psi_{y}(0,0)\right)^{2}}{\left(\beta d_{y}\right)^{2}}=1 \tag{6}
\end{equation*}
$$

and correspond with ellipses $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D for diffraction order values, $\eta_{\mathrm{G}}=0,-1,-2$, and -3 , respectively. The peak in the visible space regions $B$ and $C$ corresponds to valid diffraction order modes due to higher order diffraction values, $\eta_{\mathrm{G}}=-1$ and $\eta_{\mathrm{G}}=-2$, respectively. The directions of these two reflected waves, determined using (3) will result in real values for the directions of the reflected waves.

The visible space regions A and D contain no peaks and correspond with non-valid diffraction order values; (3) will return imaginary values for $\theta_{r}$ and no reflected waves are expected. Although there is no peak in ellipse D , a significant portion of the ellipse intersects with at least half of the peak


FIGURE 3. Geometry of the AMC element. (a) Top view. (b) Side view.

TABLE 1. Parameters of simulated AMC elements.

| Symbol | Description | Value (mm) |
| :---: | :--- | :---: |
|  | width of AMC element |  |
| $w_{1}$ | width of cross | 0.4 |
| $w_{2}$ | width inner square ring | 1.1 |
| $w_{3}$ | width outer square ring | 0.8 |
| $g$ | gap between the rings | 0.5 |
| $l$ | length of cross (for the 4 elements, | $3.4,4.2,4.7,5.6$ |
| $h_{1}$ | respectively) | 3 |
| $h_{2}$ | height of substrate | plane |

at the center of the normalized array factor and will result in reflected energy close to the plane of the PGM. The analogy of this reflected wave to array theory is that at least -3 dB of the grating lobe appears in the visible space [17].

## III. SIMULATED RESULTS

The estimated directions of the reflected waves from a planar PGM are compared with simulated RCS results obtained with CST Studio Suite. Detailed design information for PGMs can be found in [4] and [18]. The AMC element from [4] was implemented on Rogers 5880 substrate to realize a sub cell consisting of $4 \times 4$ AMC elements. The parameters of the AMC elements are defined in Fig. 3 and Table 1.

The parameter $l$ is used to control the phase reflection of the unit cell. The phase differences between adjacent AMC elements, were designed as $\delta \phi_{x}=\pi / 2 \mathrm{rad}$ and $\delta \phi_{y}=\pi / 2$ rad at 10 GHz . The phase gradient of the PGM is the change in phase over the width of the AMC element,

$$
\begin{equation*}
\nabla \phi_{x, y}=\frac{\delta \phi_{x, y}}{a} \tag{7}
\end{equation*}
$$

In [4] a detailed PGM design procedure is provided which was utilized to design the simulated and manufactured PGMs in this paper. The layout of the AMC elements realizing a


FIGURE 4. A 3-D scattering pattern for incident plane wave at $\theta_{i}=60^{\circ}, \varphi_{i}=20^{\circ}$.


FIGURE 5. Bistatic scattering from PGM, illustrating the directions of reflected waves.
$3 \times 3$ PGM and simulated bistatic RCS results for an incident plane wave at $\theta_{i}=60^{\circ}, \varphi_{i}=20^{\circ}$ are shown in Figs. 4 and 5. There are three distinct scattering directions at approximately $\left(\theta_{1}=60^{\circ}, \varphi_{1}=200^{\circ}\right),\left(\theta_{2}=22^{\circ}, \varphi_{2}=115^{\circ}\right)$ and $\left(\theta_{3}=80^{\circ}, \varphi_{3}=65^{\circ}\right)$.

The predicted directions of the reflected waves using (3) for an incident plane wave from $\theta_{i}=60^{\circ}, \varphi_{i}=20^{\circ}$ are given in Table 2. Two valid diffraction orders produce real directions for reflected waves viz. $\eta_{G}=-1$ and $\eta_{G}=-2$ corresponding to directions of reflected waves

TABLE 2. Calculated results for various diffraction orders.

| Diffraction Order, $\eta_{G}$ | $\theta_{r}\left({ }^{\circ}\right)$ | $\varphi_{r}\left({ }^{\circ}\right)$ |
| :---: | :---: | :---: |
|  | $90-\mathrm{j} 109$ | 216.6 |
| 0 | $90-\mathrm{j} 64.7$ | 212.6 |
| -1 | 60 | 200 |
| -2 | 22.3 | 119.9 |
| -3 | $90-\mathrm{j} 17.8$ | 65.3 |

TABLE 3. Parameters of measured AMC elements.

| Symbol | Description | Value (mm) |
| :---: | :--- | :---: |
|  | width of AMC element | 24 |
| $w_{1}$ | width of cross | 0.8 |
| $w_{2}$ | width inner square ring | 1.1 |
| $w_{3}$ | width outer square ring | 1.6 |
| $g$ | gap between the rings | 1.0 |
| $l$ | length of cross (for the 4 elements, | $6.0,7.6,8.1,9.2$ |
| $h_{1}$ | respectively) |  |
| $h_{2}$ | height of substrate | 6.4 |
| plane | 0.0 |  |

at $\left(\theta_{1}=60^{\circ}, \varphi_{1}=200^{\circ}\right)$ and $\left(\theta_{2}=22.3^{\circ}, \varphi_{2}=119.9^{\circ}\right)$, respectively.

The visible space regions $\mathrm{A},\left(\eta_{G}=0\right)$ and $\mathrm{D},\left(\eta_{G}=-3\right)$ in Fig. 2 contain no peaks and correspond with non-valid diffraction order values; (3) returned complex values for $\theta_{r}$ and no reflected waves are expected. A significant portion of ellipse D intersects with at least half of the peak at the center of the normalized array factor and resulted in a predicted reflected wave close to the plane of the PGM $\left(\theta_{3}=90^{\circ}\right)$ at $\varphi_{3}=65.3^{\circ}$. The predicted directions for the reflected waves correspond very closely to that observed in the CST simulation, as seen in Fig. 5. The slight difference in predicted value for $\theta_{3}$ can be attributed to edge diffraction due to the finite size of the simulated PGM.

## IV. MEASURED RESULTS

Monostatic RCS measurements were performed in the compact range at the University of Pretoria and compared to simulated RCS results using CST Studio Suite. The PGM used in these measurements was designed with CST Studio Suite to operate at 5 GHz , using FR-4 substrate with no air gap between the substrate and ground plane. Four AMC elements were designed with a phase difference of $\delta \phi_{x}=\pi / 2 \mathrm{rad}$ and $\delta \phi_{y}=0 \mathrm{rad}$, the parameters are defined in Table 3.

The manufactured PGM is shown in Fig. 6. The monostatic RCS measurement setup in the compact range is shown in Fig. 7. The PGM is mounted on a polystyrene column allowing azimuth rotation in the quiet zone of the compact range and the offset parabolic reflector is used for transmit and receive. The azimuth angle of the PGM relative to the parabolic reflector corresponds to the incident angle, $\theta_{i}$, and for monostatic RCS also to the reflected angle, $\theta_{r}$. Because the incident angle is equal to the reflected angle for


FIGURE 6. PGM prototype measured in the compact range.


FIGURE 7. Setup for monostatic RCS measurement in the compact range.

TABLE 4. Directions of reflected waves for monostatic RCS results.

| Diffraction Order, $\eta_{G}$ | Measured <br> $\left(\theta_{r}, \varphi_{r}\right)\left({ }^{\circ}\right)$ | Calculated <br> $\left(\theta_{r}, \varphi_{r}\right)\left({ }^{\circ}\right)$ |
| :---: | :---: | :---: |
| -2 | $(18,0)$ | $(18.4,0)$ |
| 1 | $(38,180)$ | $(37.9,180)$ |
| 2 | $(69,180)$ | $(70.3,180)$ |

monostatic RCS measurements, only reflected waves with reflection angles equal to incident angles will be observed in the RCS measurement of the PGM.

The results of the monostatic RCS measurement and CST simulation are shown in Fig 8 and summarized in Table 4. The slight difference between the measured and simulated RCS values is probably due to alignment of the PGM in the compact range. Three distinct peaks are observed at approximately $\theta_{r 1}=18^{\circ}, \theta_{r 2}=-38^{\circ}$ and $\theta_{r 3}=-69^{\circ}$ in the


FIGURE 8. Monostatic RCS measured and simulated results.
measured and simulated results and represent the directions of the reflected waves. Using the formulation in Section II and taking into account that for monostatic RCS measurements the incident angle is equal to the reflected angle, the direction of the first reflected wave was estimated as $\theta_{r 1}=18.4^{\circ}$ for a diffraction order $\eta_{G}=-2$. The direction of the second reflected wave was estimated as $\theta_{r 2}=-37.9^{\circ}$, for a diffraction order, $\eta_{G}=1$, and the direction of the third reflected wave was estimated as $\theta_{r 3}=-70.3^{\circ}$, corresponding to a diffraction order $\eta_{G}=2$.

## V. CONCLUSION

A planar PGM with phase gradients in two in-plane directions will introduce two additional wave vectors to the reflected wave vector. For small incident angles, close to the normal vector, the magnitude of the reflected wave vector is smaller than that of the incident wave vector and the direction of the anomalous reflected wave can be determined from the incident wave vector components and additional phase gradient components.

The generalized Snell's law of reflection presented in [10], made it possible to predict the directions of reflected waves from PGMs for incident angles larger than the critical value. In this paper, the generalized Snell's law of reflection was extended to account for PGMs with different orthogonal phase gradients in the two in-plane directions. The extended formulation is used to predict all the scattering modes and to accurately estimate the directions of the reflected waves. The extended formulation is combined with antenna array theory concepts to account for the scattering beamwidth of reflected waves close to the plane of a finite PGM.

The estimated directions of the reflected waves from a planar PGM were compared with simulated RCS results obtained with CST Studio Suite and measured results.

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