Forecasting Stock-Market Tail Risk and Connectedness in Advanced Economies Over a Century: The Role of Gold-to-Silver and Gold-to-Platinum Price Ratios [1]

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Abstract

We examine the predictive value of risk perceptions as measured in terms of the gold-to-silver and gold-to-platinum price ratios for stock-market tail risks and their connectedness in eight major industrialized economies using monthly data for the period 1916:02-2020:10 and 1968:01-2020:10, where we use four variants of the popular Conditional Autoregressive Value at Risk (CAViaR) framework to estimate the tail risks for both 1% and 5% VaRs. Our findings for the short sample period show that the gold-to-silver price ratio resembles the gold-to-platinum price ratios in that it is a useful proxy for global risk. Our findings for the long sample period show, despite some heterogeneity across economies, that the gold-to-silver price ratio often helps to out-of-sample forecast for both 1% and 5% stock market tail risks, particularly when a forecaster suffers a higher loss from underestimation of tail risks than from a corresponding overestimation of the same absolute size. We also find that using the gold-to-silver price ratio for forecasting the total connectedness of stock markets is beneficial for an investor who suffers a higher loss from an underestimation of total connectedness (i.e., an investor who otherwise would overestimate the benefits from portfolio diversification) than from a comparable overestimation.

JEL classification: C22, C53, G15

Keywords: Stock markets, Tail risks, Connectedness, Gold-to-silver price ratio, Gold-to-platinum price ratio, Forecasting, Asymmetric loss

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1 Introduction

Definitionally, tail risk is the additional risk which, commonly observed, fat-tailed asset return distributions have relative to normal distributions (Li and Rose, 2009). The issue of tail risks has emerged as an important research question following multiple recent episodes of financial markets-related distress such as the Lehman default, the “Great Recession", and the Global Financial Crisis (GFC), followed by the European sovereign debt crisis and the Chinese stock market crash, and, of course, the financial market jitters due to the currently ongoing COVID-19 pandemic (Adrian et al., 2019; Baker et al., 2020), over and above the important earlier ones involving Black Monday, the Asian Financial Crisis, and the bursting of the dot-com bubble. Tail risks have been shown to predict not only stock returns, but also real economic variables, such as employment, investment and output, as well as oil returns and its volatility and tail risks (Kelly and Jiang, 2014; Almeida et al., 2017; Chevapatrakul et al., 2019; Hollstein et al., 2019; Salisu et al., 2021, 2022a, 2022b, 2022c, forthcoming). Naturally, determining which factors drive the evolution of future tail risks is an important question for both investors and policymakers.

In this regard, in a recent study, Huang and Kilic (2019) *inter alia*[2] provide evidence of (positive) in-sample predictability of (option-implied) tail-risk measures of the United States (US) based on the ratio of the gold-to-platinum prices. [3] The reason behind this, as outlined by the authors, is as follows: Gold serves the dual two roles of a consumption good as jewelry (Bouri et al., 2021), and is also regarded by investors as a well-established “safe haven", i.e., investors consider it valuable in times of severe financial turmoil, as shown by Boubaker et al., (2020) based on the longest possible data available on gold prices from 1257. In contrast, platinum is a precious metal with similar uses as gold in consumption, and which also has several important industrial usages (such as, laboratory equipment, electrical contacts and electrodes, platinum resistance

[2] In addition, Huang and Kilic (2019) show that the gold-to-platinum price ratio (1) is countercyclical and increases in times of economic distress, (2) positively predicts future stock market excess returns, (3) is negatively priced in the cross-section, and, (4) is high when the default spread is high (that is, when firms with low credit ratings have a high probability of default).

[3] Some in-sample evidence of the predictability for both advanced and emerging stock-market tail risks based on macroeconomic and financial surprises, as well as oil shocks, have been provided by the published works of Gkillas et al., (2020), and Gupta et al., (2021a).

thermometers, dentistry equipment (McDonald and Hunt, 1982)). Hence, the ratio of gold-to-platinum prices should be largely unaffected by consumption (jewelry demand) shocks, and in the process reveal variation in global risk, implying that the ratio should serve as a proxy for an important economic state variable.

Against this backdrop, and given the importance of tail risks, the objective of our study is to extend this line of research by analyzing the role of gold-to-platinum price ratio in forecasting tail risks of not only the US, but also Canada, France, Germany, Italy, Japan, Switzerland, and the United Kingdom (UK). Our decision to analyze the stock markets of these eight advanced economies, besides data availability, is primarily motivated by their importance in the global economy, with these countries representing nearly two-third of global net wealth, and nearly half of world output (Das et al., 2019; Salisu et al., 2021, 2022a, 2022b, 2022c, forthcoming). Besides analyzing multiple countries, our analysis differs from the tail-risks component of the work of Huang and Kilic (2019) in multiple other dimensions. First, instead of in-sample tests of predictability, we conduct a full-fledged out-of-sample forecasting exercise of tail risks, since the later is deemed as a more robust test of predictability compared to an in-sample analysis in terms of the predictors and econometric model specifications (Campbell, 2008). Second, because silver too has similar consumption-based properties like platinum, the gold-to-silver price ratio, given gold's dual role as a consumption and investment good, would also provide a measure of worldwide financial risk, just like gold-to-platinum price ratio (Gupta et al., 2021b). Thus, in this research, we also consider the role of the ratio of gold-to-silver prices as a metric of world risk, over and above gold-to-platinum prices, in forecasting the tail risks of the stock markets of the eight advanced economies. This not only serves as a robustness check of the results associated with the gold-to-platinum price ratio over the monthly period from 1968:01 to 2020:10, but also renders it possible to study more than a century of data, i.e., from 1916:02 to 2020:10. In the process, we are able to study the evolution of tail risks over other important historical crises like the Spanish Flu, the two World Wars, the "Great Depression", besides the relatively recent crises mentioned above. The fact that we are dealing with such a long time span is a further motivation to look at the G7 countries and Switzerland, for which data on stock markets are available over this long sample period. Importantly, upon studying data for such long sample period, we avoid the issue of a possible sample-selection bias.[4] Third, we also differ in our econometric approach of computing the tail risks. Instead of option-implied measures, we use a framework based on the underlying returns data, which is understandable due to the unavailability of such long-spans of historical data on options.[5] Specifically, we estimate tail risk using the popular Value at Risk (VaR) metric at 1% and 5% by employing the conditional autoregressive quantile specification as proposed by Engle and Manganelli (2004), which, in turn, is called the CAViAR model. In this context, the models considered are: (i) the adaptive model; (ii) the symmetric slope model, (iii) the asymmetric slope model, and; (iv) the indirect generalized autoregressive conditional heteroscedasticity (GARCH) model with an autoregressive mean. In our predictive analysis, we forecast stock-market tail risk using both the gold-to-platinum and the gold-to-silver price ratios. We start our predictive analysis using a standard symmetric forecaster loss function. We then extend our out-of-sample forecasting experiment using the type of asymmetric loss functions that have been widely studied in recent research (see, for example, Elliott et al., 2005, 2008). Using a standard autoregressive model as our benchmark model, our analysis of asymmetric loss functions shows that mainly a forecaster who suffers a higher loss from underestimating rather than overestimating tail risk benefits from using the gold-to-silver price ratio to forecast stock-market tail risk. Finally, we not only forecast the individual tail risks of the eight advanced stock markets, but also their time-varying connectedness (derived based on the work of Antonakakis et al., (2020)), with the underlying idea being that if the comovement of the extreme negative returns of these markets are also forecastable by the gold-to-silver and gold-to-platinum price ratios, then following a shock to these proxies of global risk, investors might not be able to diversify across these stock markets, hence making our findings of tremendous relevance to portfolio managers. We find that those forecasters benefit most from using the gold-to-silver price and gold-to-platinum price ratios for forecasting the total connectedness of the stock markets in our sample who otherwise would overestimate the gains from diversifying their portfolio investments across the eight stock markets in our sample. The reason for this finding is that underestimation

[4]We include Switzerland in our sample of countries because it is a classic example of a safe-haven economy, hence it is interesting to study whether its stock market tail risks are sensitive to global risk and, thereby, to the gold-to-silver and/or gold-to-platinum price ratio (or both).

[5]The tail-risks data of Huang and Kilic (2019) cover the period of time from 1996:01 to 2013:08.

of total connectedness of stock markets means that the benefits from portfolio diversification are overestimated.

Having outlined the objectives of our research, we can now discuss the manifold “applied" contributions of our study. First, this is the first study to compute tail risks of eight developed stock markets spanning over a century of data. Second, given the importance of tail risks in predicting the future path of movements in financial and commodity markets, and the macroeconomy as a whole, we are the first to forecast these tail risks based on the information content of two proxies of global risks, i.e., gold-to-silver and gold-to-platinum price ratios. Third, we compute the time-varying historical connectedness of the tail risks of these eight markets, which, in turn, has not been done before. Finally, realizing the importance of the comovement of stock markets in affecting the portfolio-allocation decisions of investors, we analyze, for the first time in the literature on financial market connectedness, the role of gold-to-silver and gold-to-platinum price ratios for forecasting the time-varying connectedness of stock markets. In sum, to the best of our knowledge, this is the first study to compute and forecast the individual tail risks of the eight advanced stock markets and their connectedness, based on monthly data that runs over a century, by accounting for the role of gold-to-silver and gold-to-platinum price ratios.[6] Our findings should be of importance to not only investors, but also policymakers, and academics in understanding what drives tail risks, which seems to be the governing feature of stock-market movements these days since the GFC.

At this stage, it is important to highlight that the introduction of a silver Exchange-Traded Fund (ETF) in April 2006, and somewhat later in 2010 of platinum ETFs, by reducing costs, have made these two white precious metals more attractive as investment assets (Vigne et al., 2017).

[6]The only somewhat related papers are that of De Nicolò and Lucchetta (2017) and Salisu et al., (2022c). In the former paper, the authors present a set of multi-period forecasts of indicators of real (industrial production and employment growth) and financial (distance to insolvency measures of corporate and banking sectors) tail risks derived based on a large database of monthly US data for the sample period 1972:01−2014:12. The main finding of De Nicolò and Lucchetta (2017) is that the forecasts they compute by means of autoregressive (AR) and factor-augmented vector autoregressive (FAVAR) models significantly underestimate tail risks, while they obtain accurate forecasts and early-warning signals for real and financial tail risks (up to a 1-year horizon) when they use quantile projections. The latter study highlights the role of oil tail risks, as well as spillovers of tail risks from stock markets of other advanced countries, in forecasting the tail risks for the stock markets of Canada and the US over the monthly period from 1916:02 to 2020:10.

However, gold still tends to dominate in the category of investment assets, and this seems to be confirmed by Figure 1 (see Section 2.1), which tends to show a general increase in the gold-to-silver and gold-to-platinum price ratios during various episodes of historical and current crises as discussed above. In other words, during periods of financial market turbulence, even though silver and platinum have gained importance as investment assets, the price of gold tends to increase relatively more, given its continued importance as perhaps the most well-established safe haven (for a survey of the literature, see O'Connor et al., 2015).

We organize the remainder of our paper as follows. In Section 2, we describe the data and methodologies we use in our empirical analysis. In Section 3, we discuss and our empirical results. In Section 4, we conclude.

2 Data and Methodologies

2.1 Data

The data we use in our empirical research are monthly stock price indexes of eight industrialized economies: Canada (S&P TSX 300 Composite Index), France (CAC All-Tradable Index), Germany (CDAX Composite Index), Italy (Banca Commerciale Italiana Index), Japan (Nikkei 225 Index), Switzerland (All Share Stock Index), the UK (FTSE All Share Index), and the US (S&P500 Index). The indexes are derived from Global Financial Data[7], and covers the period from 1916:02 to 2020:10. We convert the indexes into log-returns in percentages, i.e., the first-difference of the natural logarithm of the indexes multiplied by 100. The nominal gold and silver prices are derived from Macrotrends[8], while the nominal platinum price is obtained from Kitco[9].

The nominal gold and silver prices data actually starts from 1915:01, but platinum data is available only from 1968:01. However, based on availability of data at the time of writing this paper,

[7] https://globalfinancialdata.com/.

[8] https://www.macrotrends.net/.

[9] https://www.kitco.com/.

our data period covers 1916:02 to 2020:10, with the start date governed by the availability of the stock price-index of Switzerland (i.e., 1916:01, with us losing one observation due to the computation of log-returns).

– Figure 1 about here. –

Figure 1 plots the gold-to-silver (GS) and the gold-to-platinum (GP) price ratios that we use in our empirical analysis. It must be realized that the movements in gold and silver prices were at times restricted due to price-fixation (for historical accounts of the gold and silver markets, see O'Connor et al., 2015; Corbet and O'Connor, 2020), and hence the jagged nature of the movement of the GS. Inspite of that, the fact that it continued to capture global risks is indicated by increases in ths ratio during the periods involving the Spanish Flu, the two World Wars, the "Great Depression". The GS ratio is understandably quite static following World War II in the wake of the postwar economic boom or the "Golden Age of Capitalism" (Marglin and Schor, 1992), until the 1973−1975 recession resulting from stagflation. Post 1968, of course, the GS ratio, just like the GP ratio, tends to fluctuate relatively more, which is obviously due to the lack of price controls, and also tend to increase during the Black Monday, the Asian Financial Crisis, the bursting of the dot-com bubble, the Lehman default, the "Great Recession", and the GFC, the European sovereign debt crisis, the Chinese stock market crash, and the ongoing COVID-19 pandemic.

2.2 Methodologies

2.2.1 Estimating Tail Risk

We employ the Engle and Manganelli (2004) approach, which involves a conditional autoregressive quantile specification of VaR (CAViaR), to estimate the tail risks of stock markets, $f_t(\beta)$. This approach concentrates on the asymptotic form of the tail, rather than the whole distribution.

We begin by specifying a generic representation for the CAViaR[10]:

$$f_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\beta) + \sum_{j=1}^{r} \beta_j l(x_{t-j}) \tag{1}$$

where $f_t(\beta)$, which can be equivalently expressed as $f_t(x_{t-1}, \beta_\theta)$, represents the period t θ-quantile of the distribution of stock returns at $t-1$, wherein the subscript θ is supressed from β_θ for notational convenience. Further, the dimension of β is given as $p = q + r + 1$, and l is a function of a finite number of lagged values of observables (x_{t-j}) that links to $f_t(\beta)$ to x_{t-j} belonging to the information set. In order to ensure that the quantile changes “smoothly" over time, we include the autoregressive terms $\beta_i f_{t-i}(\beta)$, $i = 1, ..., q$. Specifically, we estimate four variants of the CAViaR described as Adaptive, Symmetric absolute value, Asymmetric slope, and Indirect GARCH. The four variants are specified as follows:[11]

Adaptive:

$$f_t(\beta_1) = f_{t-1}(\beta_1) + \beta_1 [1 + exp(G[y_{t-1} - f_{t-1}(\beta_1)])]^{-1} - \theta. \tag{2}$$

Symmetric absolute value:

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3 |y_{t-1}|. \tag{3}$$

Asymmetric slope:

$$f_t(\boldsymbol{\beta}) = \beta_1 + \beta_2 f_{t-1}(\boldsymbol{\beta}) + \beta_3 (y_{t-1})^+ + \beta_4 (y_{t-1})^-. \tag{4}$$

Indirect GARCH (1,1):

$$f_t(\boldsymbol{\beta}) = (\beta_1 + \beta_2 f_{t-1}^2(\boldsymbol{\beta}) + \beta_3 y_{t-1}^2)^{1/2}. \tag{5}$$

[10]We refer technically-minded readers to Engle and Manganelli (2004) for technical details on the CAViaR methodology.

[11]Recently, a number of studies have used the CAVaiR models to measure tail risk such as the oil tail risk (see Salisu et al., 2021b, 2021c, 2021d) and have established its predictive value for out-of-sample predictability of oil returns (Salisu et al., 2021b) and stock returns (Salisu et al., 2021c).

In Equation (2), a smoothed version of a step function of the model is achieved with some positive finite number, G, while the last term in the equation converges almost surely to $\beta_1[I(y_{t-1} \leq f_{t-1}(\beta_1)) - \theta]$ if $G \rightarrow \infty$, with $I(\cdot)$ representing the indicator function. It is not difficult to show that the Symmetric absolute value and Indirect GARCH specifications in Equations (3) and (5) are symmetric, while the structure in Equation (4) is asymmetric. Thus, the response of positive and negative returns is identical for the former, but differs for the latter. Also note that while the coefficient on the lagged VaR of the adaptive model is constrained to be 1, the other three are not, and, therefore, are considered to be mean reverting.

For each returns series, we produce results for both 1% and 5% VaRs across the four variants of the CAViaR specifications, and the "best" CAViaR specification under each VaR is determined by the Dynamic Quantile test (DQ) test and %Hits in line with Engle and Manganelli (2004).[12]

2.2.2 Estimating Connectedness

Furthermore, we compute the dynamic total connectedness index (TCI) using the time-varying parameter vector autoregressive (TVP-VAR) based connectedness approach of Antonakakis, et al. (2020). Thus, we first estimate the following TVP-VAR model with a lag length of one as suggested by the Bayesian information criterion. The resulting TVP-VAR(1) model can be outlined as follows[13],

$$\boldsymbol{z}_t = \boldsymbol{B}_t \boldsymbol{z}_{t-1} + \boldsymbol{u}_t \qquad \boldsymbol{u}_t \sim N(\boldsymbol{0}, \boldsymbol{S}_t), \tag{6}$$

$$vec(\boldsymbol{B}_t) = vec(\boldsymbol{B}_{t-1}) + \boldsymbol{v}_t \qquad \boldsymbol{v}_t \sim N(\boldsymbol{0}, \boldsymbol{R}_t), \tag{7}$$

where $\boldsymbol{z}_t$, $\boldsymbol{z}_{t-1}$ and $\boldsymbol{u}_t$ are $k \times 1$ dimensional vectors, denoting the tail risk series in t, $t-1$, and the corresponding error term, respectively. $\boldsymbol{B}_t$ and $\boldsymbol{S}_t$ are $k \times k$ dimensional matrices illustrating the time-varying VAR coefficients and the time-varying variance-covariances while $vec(\boldsymbol{B}_t)$ and $\boldsymbol{v}_t$ are $k^2 \times 1$ dimensional vectors and $\boldsymbol{R}_t$ denotes a $k^2 \times k^2$ dimensional matrix.

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[12] These are standard test statistics for evaluating the relative performance of the alternative specifications of CAViaR test.

[13] We refer technically-minded readers to Antonakakis et al. (2020) for technical details on the computation of the dynamic total connected index.

As the Generalised Forecast Error Variance Decomposition (GFEVD) of Koop et al. (1996) and Pesaran and Shin (1998) rests on the vector moving average (VMA) coefficients, we apply the Wold representation theorem which transforms the TVP-VAR to its TVP-VMA process by the following equality: $\boldsymbol{z}_t = \sum_{i=1}^{p} \boldsymbol{B}_{it}\boldsymbol{z}_{t-i} + \boldsymbol{u}_t = \sum_{j=0}^{\infty} \boldsymbol{A}_{jt}\boldsymbol{u}_{t-j}$. The GFEVD, $\tilde{\psi}^g_{ij,t}(H)$, stands for the influence series j has on series i in terms of its forecast error variance share and is computed by

$$\psi^g_{ij,t}(H) = \frac{S^{-1}_{ii,t}\sum_{t=1}^{H-1}(\boldsymbol{\iota}'_i\boldsymbol{A}_t\boldsymbol{S}_t\boldsymbol{\iota}_j)^2}{\sum_{j=1}^{k}\sum_{t=1}^{H-1}(\boldsymbol{\iota}_i\boldsymbol{A}_t\boldsymbol{S}_t\boldsymbol{A}'_t\boldsymbol{\iota}_i)} \qquad \tilde{\psi}^g_{ij,t}(H) = \frac{\psi^g_{ij,t}(H)}{\sum_{j=1}^{k}\phi^g_{ij,t}(H)},$$

with $\sum_{j=1}^{k}\tilde{\psi}^g_{ij,t}(H) = 1$, $\sum_{i,j=1}^{k}\tilde{\psi}^g_{ij,t}(H) = k$, where H stands for the forecast horizon, and $\boldsymbol{\iota}_i$ for a zero vector with unity on the ith position.

Subsequently, the (corrected) TCI of Chatziantoniou and Gabauer (2021) can be constructed. This connectedness measure indicates the degree of network interconnectedness and, hence, the inherent market risk:

$$TCI = \frac{k\cdot\sum_{i,j=1,i\neq j}^{k}\tilde{\psi}^g_{ij,t}(H)}{(k-1)\cdot\sum_{i,j=1}^{k}\tilde{\psi}^g_{ij,t}(H)} \qquad 0 \leq C^g_t(H) \leq 1. \tag{8}$$

Diebold and Yilmaz (2009, 2012, 2014) point out the link between network interconnectedness and market risk by highlighting the association of TCI with several economic and financial events. Hence, a high TCI value is associated with high market risk, while a low TCI value indicates low market risk.

2.2.3 Forecasting Model

Equipped with estimates of stock market tail risks and their connectedness, we conduct an out-of-sample forecasting experiment. To this end, we consider the following simple forecasting model:

$$TR_{t+h} = c + \theta TR_t + \gamma Ratio_t + \eta_{t+h}, \tag{9}$$

where c and θ are coefficients to be estimated, TR_t is stock-market tail risk at time t, TR_{t+h} is the average value of TR_t over the forecasting horizon, where $h = 1, 3, 6, 12, 24$ months ahead are the five forecasting horizons we consider in our empirical research, and η_{t+h} denotes a disturbance term. Moreover, $Ratio_t$ denotes either the gold-to-silver price ratio (GS_t) or gold-to-platinum price ratio (GP_t) in period of time t, and γ denotes the corresponding coefficient to be estimated.[14] As our benchmark model, we use a simple autoregressive model of order one, which we obtain by letting $\gamma = 0$. Given that our long sample periods cover many important historical events, we estimate Equation (9) by means of a rolling-window estimation approach.[15] In order to rule out that our empirical results hinge on the choice of a specific rolling-estimation window, we use in Section 3 rolling-estimation windows of lengths 60, 80, 100, and 120 months. Note that, while dealing with the forecastability of connectedness, we replace TR with TCI.

2.2.4 Evaluation of Forecasting Performance

We evaluate the forecasting performance of the forecasting model given in Equation (9) in various ways. First, we use the conventional root-mean-squared-forecast-error (RMSFE) and mean-absolute-forecast-error (MAFE) statistics to evaluate the accuracy of forecasts. These two statistics rest on the assumption that a forecaster has a loss function that is symmetric in the squared (RMSFE) and absolute (MAFE) forecast error, with the MAFE statistic putting less weight on very large forecast errors. Second, we evaluate forecasts assuming that forecasters have an asymmetric loss function. The asymmetry of the loss function accounts for the possibility that a forecaster may incur a higher (or lower) loss from an underestimation of tail risk than from an overestimation of the same absolute size. In order to model the asymmetry of the loss function, we use the asymmetric loss function studied by Elliott et al. (2005, 2008). They consider the following loss function: $\mathscr{L} = [\alpha + (1-2\alpha)\mathbf{1}(TR - \hat{TR} < 0)]|TR - \hat{TR}|^p$, where $\hat{TR}$ denotes the forecast of stock-market tail risk (we have dropped the time index for simplicity). The parameter,

[14] In addition, we consider a model that features both ratios as predictors. In this case, the model given in Equation (9) is modified to $TR_{t+h} = c + \theta TR_t + \gamma_1 Ratio_{t,1} + \gamma_2 Ratio_{t,2} + \eta_{t+h}$.

[15] This approach works as follows: 1) We start by choosing a subset of the data at the beginning of the sample period to compute initial estimates of our forecasting models. 2) We then update the predictors by one period to compute out-of-sample forecasts. 3) We then re-estimate the forecasting models, where we add one observation of the data at the end of the initial estimation period and, to hold the length of the estimation window constant, delete the first observation. 4) We next roll the estimation window over the sample of data in this way (adding one observation to the end of the estimation window, and deleting the first observation of the estimation window) until we reach the end of the sample period. 5) While rolling the estimation window over the data, we compute out-of-sample forecasts.

p, governs the functional form of the loss function. Setting $p = 1$ results in a so-called lin-lin loss function, while setting $p = 2$ gives a so-called quad-quad loss function. The parameter, $\alpha \in (0,1)$, determines the asymmetry of the loss function. For $\alpha = 0.5$, we obtain a symmetric loss function. As a result, if we set, in addition, $p = 2$ then we obtain the standard RMSFE loss function, and for $p = 1$, we obtain the MAFE loss function. For $\alpha \neq 0.5$, in turn, the loss function becomes asymmetric in the forecast error. Specifically, for $\alpha > 0.5$ ($\alpha < 0.5$), the loss from an underestimation (overestimation) of stock-market tail risk exceeds the loss from an overestimation (underestimation) of the same (absolute) size. Figure 2 plots examples of the lin-lin and quad-quad loss function.

– Figure 2 about here. –

Equipped with the loss function, we use the out-of-sample relative-loss criterion studied by Pierdzioch et al. (2014, 2016) to evaluate the forecasts. The out-of-sample relative-loss criterion, $\mathscr{R}_o$, is defined as follows:

$$\mathscr{R}_{o|\alpha,p} = 1 - \sum_{t=\tau}^{T} \mathscr{L}_{t,rival|\alpha,p} / \sum_{t=\tau}^{T} \mathscr{L}_{t,benchmark|\alpha,p}, \tag{10}$$

where τ (T) is the first (last) period for which a forecast is being made. Given the functional form (p) and the asymmetry (α) of the loss function, the rival model (the model that features the gold-to-silver and/or the gold-to-platinum price ratio as a predictor) performs better (worse) then the autoregressive benchmark model according to the out-of-sample relative-loss criterion when we observe $\mathscr{R}_{o|\alpha,p} > 0$ ($\mathscr{R}_{o|\alpha,p} < 0$).

Finally, in order to shed light on the statistical significance of our results, we use the well-known modified version of the Diebold and Mariano (1995) test proposed by Harvey et al. (1997).

3 Empirical Results

3.1 Results for Tail Risk

Before we move to the formal forecasting analyses, we compute the 1% and 5% stock-market tail risks using the four CAViaR specifications (Symmetric Absolute Value, Asymmetric Slope, Indirect GARCH and Adaptive) described in Section 2.2, where several diagnostics are computed to enable us determine the "best" CAViaR specification for each returns series. We report the results as Supplementary Material (Tables S1 to S8). In terms of the DQ test and %Hits,[16] for the 1% tail risks of the stock markets, the Asymmetric Slope model is the best fitting specification for France, Italy, Japan, Switzerland, the UK, and the US, with the Indirect GARCH model being the best framework for Canada and Germany. As far as the 5% tail risks are concerned, based again on the DQ test and %Hits, the Asymmetric Slope model produces the best fit for Canada, Germany, Italy, Japan, Switzerland, and the US, while the Indirect GARCH model is the "optimal" specification for France and the UK. Interestingly, the Symmetric Absolute Value and the Adaptive specifications are not optimal for any of the returns series. We use the tail risks computed by means of the "optimal" specifications in our forecasting analysis based on the information content of GS_t and GP_t.

Figure 3 plots the estimated 1% and 5% tail risks. The figure shows that, as expected, the estimated tail risks display substantial variation over time. For example, the estimates show that there is one large outburst of tail risk during the Second World War in the case of Germany. A natural concern is to which extent this outlier and, more generally, the interwar period affects our results. Similarly, also in the case of the other countries, important historical events and changes in policy regimes may have had an impact on the link between stock-market tail risk and the gold-to-silver and the gold-to-platinum price ratios. In order to control for such changes, we use a rolling-estimation-window approach to estimate Equation (9). In line with the discussion in the introduction, common peaks of tail risks across the countries are observed during the periods associated with the "Great Depression", the Black Monday, the Asian Financial Crisis, the bursting of the dot-com bubble, the Global Financial Crisis, the European sovereign debt crisis and the Coronavirus outbreak.

– Figure 3 about here. –

[16]We expect the %Hits to be relatively 1% for 1% VaR and 5% for 5% VaR, while the DQ test statistic is not expected to be significant. When more than one tail risk is statistically insignificant in terms of the DQ test, we consider the tail risk with the closest value to the expected value for the %Hits. In the same vein, when all the tail risks are statistically significant, the %Hits become a major criterion except when some distinctions can still be made with the significant DQ test statistics.

3.2 Results for Forecasting Performance

Table 1 depicts for TR_1, and for forecast horizons ranging from one month to two years, results for the ratio of root-mean-squared-forecasting error (RMSFE) for TR_1. The RMSFE ratio is computed as the ratio of the RMSFE of the autoregressive benchmark model and a rival model that includes the variables displayed in the first column as additional predictors. We consider three alternative rival models: one model that only includes the gold-to-silver price ratio, one model that only includes the gold-to-platinum price ratio, and one model that includes both price ratios as predictors. Because the choice of a rolling-estimation window is always to some extent arbitrary, we compute the ratios as averages across four rolling-estimation windows of length 60, 80, 100, and 120 months. A ratio that is larger than unity shows that the rival model has a better forecasting performance in terms of the RMSFE than the benchmark model. The results are based on data starting in 1968 because gold-to-platinum data are available only for that shorter sample period. The results demonstrate that accounting for the gold-to-silver and/or the gold-to-platinum price ratio improves forecasting performance in terms of the RMSFE (that is, under a loss function that is symmetric in the squared forecast error) for the intermediate and the long forecast horizons (starting with $h = 6$ or $h = 9$, depending on the country being studied). Moreover, the results for the gold-to-silver ratio closely resemble the results for the gold-to-platinum ratio and, for some countries, even are slightly better than the results for the latter.

– Tables 1 and 2 about here. –

Because Figure 3 witnesses that the measures of tail risk that we study in our empirical research displayed sharp fluctuations during our sample period, we summarize in Table 2 for TR_1 the ratio of the mean-absolute-forecasting error (MAFE), again averaged across the four rolling-estimation windows of length 60, 80, 100, and 120 months. The simple autoregressive model again is our benchmark model. The results corroborate the results for the RMSFE ratios given in Table 1. The MAFE ratios increase in the forecast horizon, and they are similar across the gold-to-silver and the gold-to-platinum ratios.

– Table 3 about here. –

We report in Table 3 results for the RMSFE ratios that we obtain when we use TR_5 rather than TR_1 as our metric of tail risk. As in the case of TR_1, we observe that the RMSFE ratios increase in the forecast horizon and that the results for the gold-to-silver price ratio are qualitatively similar to the results for the gold-to-platinum price ratio. We conclude that, irrespective of whether we study the RMSFE or MAFE ratio, or a loss function that is symmetric in the squared or the absolute forecast error, the results for the gold-to-silver price ratio resemble the results for the gold-to-platinum price ratio. Both ratios have forecasting power for the subsequent stock-market tail risk in the eight economies included in our sample, mainly at the intermediate and the long forecast horizons.

As we already pointed out, the data for the gold-to-platinum price ratio start at the beginning of the year 1968 and are, thus, available for a relatively short sample period only. The data for the gold-to-silver price ratio, in contrast, date back to 1916 and, thus, are available for a much longer sample period. The results for the shorter sample period show that the gold-to-silver price ratio, similar to the gold-to-platinum price ratio studied in recent research by Huang and Kilic (2009) and others, captures the variation in global risk as a predictor of stock-market tail risk. The results for the shorter sample period, thereby, motivate our choice to use the gold-to-silver price ratio as a proxy of global risk also for the longer sample period dating back to 1916.

– Table 4 about here. –

We present in Table 4 results for the RMSFE ratios as computed using the data for the long sample period. The results for the long sample period corroborate the results for the short sample period in that the RMSFE ratios increase in the forecast horizon. Starting at forecast horizons of $h = 6$ or $h = 9$, the RMSFE ratios start to exceed unity, demonstrating that the model that includes the gold-to-silver price ratio as a predictor outperforms the benchmark model in terms of the in terms of the RMSFE statistic. We observe this result for both measures of stock-market tail risk, TR_1 and TR_5. Hence, when we study the long sample period, the gold-to-silver price ratio, interpreted as a proxy that measures the variation in global risk, helps to predict stock-market tail risk, especially at the long forecast horizons, in the same way as the gold-to-platinum price ratio does in the short sample period. Taken together, the results we report in Tables 1 to 4, thus, lend support to the hypothesis that, because silver has similar consumption-based properties like platinum, a simple metric of the the variation in global risk can be constructed for a long sample period dating back more than a century based on the gold-to-silver price ratio. Our forecasting results for the gold-to-silver price ratio, thereby, complement the results documented in much significant recent research for the gold-to-platinum ratio as a measure of global risk.

– Figure 4 about here. –

Figure 4 plots, for the quad-quad and the lin-lin loss function, on the vertical axis the out-of-sample relative loss criterion as a function of the asymmetry parameter of the loss function on the horizontal axis for the long sample period.[17] As in the case of the RMSFE and MAFE ratios, we average the out-of-sample relative loss criterion across the four different rolling-estimation windows. The model that includes the gold-to-silver price ratio as a predictor of stock-market tail risk performs better than the benchmark model when the out-of-sample relative loss criterion takes on a positive value. The results show that, assuming a quad-quad loss function, the out-of-sample relative loss criterion increases in the asymmetry parameter for all countries except Germany (for which Figure 3 shows the presence of a large outlier during the interwar period), and crosses the zero line from below at around $\alpha = 0.5$. The functions representing the out-of-sample relative loss criterion tend to become steeper and they tend to shift upward when we increase the forecast horizon. Hence, corroborating the results for the RMSFE and MAFE ratios, the contribution of the gold-to-silver price ratio to the performance of the forecasting model tends to increase as the forecast horizons become longer. Importantly, the results demonstrate that for some countries (e.g., Japan) and forecast horizons (i.e., mainly the intermediate and long forecast horizons), a forecaster with a symmetric quadratic loss function benefits from using the

[17] Figure S1 (Supplementary Material) shows the out-of-sample relative-loss statistic for the shorter sample period beginning in 1968. The figure demonstrates that the general message that the out-of-sample relative-loss statistic increases the parameter α (displayed on the horizontal axis) is the same in the short as in the long sample period (starting in 1916).

gold-to-silver price ratio to forecast stock-market tail risk. The benefits, however, are clearly larger for a forecaster with an asymmetric loss function that is characterized by an asymmetry parameter $\alpha > 0.5$. In other words, a forecaster who suffers more from an underestimation of stock-market tail risk than from an overestimation of the same size clearly benefits from using the gold-to-silver price ratio as a predictor of subsequent stock-market tail risk.

The results for the lin-lin loss function resemble the results for the quad-quad loss function. In most cases and for both TR_1 and TR_5, the function that represents the out-of-sample relative loss criterion is upward sloping and takes on positive values for values of the asymmetry parameter in the approximate range $\alpha > 0.5$. For TR_1, the out-of-sample relative loss criterion decreases in the asymmetry parameter for Canada and Italy for the long forecast horizons, and for Germany, the shape of the function that represents the out-of-sample relative loss criterion is rather different than the shape of the function for the other countries. For TR_5, the results for Germany are more in line with the results for the other countries: the for the out-of-sample relative loss criterion increases in the asymmetry parameter for intermediate and long forecast horizons and even reaches positive values for a sufficiently large asymmetry parameter. Hence, the overall picture that emerges is that the gold-to-silver price ratio improves forecast accuracy mainly at the intermediate and long forecast horizons and when a forecaster is concerned primarily with an underestimation of future stock-market tail risk.

– Figures 5 and 6 about here. –

Figures 5 and 6 shed light on the statistical significance of our results. The figures plot the p-values (based on robust standard errors) of the Diebold and Mariano (1995) test that compares the accuracy of forecasts. We present results for the modified version of the test due to Harvey et al. (1997). We plot the p-values for the four different rolling-estimation windows as a function of the asymmetry parameter, α, and, to save journal space, focus on the case of the lin-lin loss function. In the case of TR_1, the p-values decrease in the asymmetry parameter and the test results become significant (depending on the length of the rolling-estimation window being studied) for a sufficiently large asymmetry parameter, when we study stock-market tail risks for France, Japan, Switzerland, the United Kingdom, and the United States. We observe a similar pattern for Germany, except when we study the short rolling-estimation window. For TR_1, the results are qualitatively similar to the results for TR_1, where we now observe a general tendency of the p-values to decrease in the asymmetry parameter also for Canada.

– Figures 7 and 8 about here. –

A natural question, especially from the viewpoint of an investor, is whether our results also hold when we study daily data.[18] Daily data, of course, are not available for the extended period dating back to 1916. Figures S2 to S9 report results (for the gold-to-silver and gold-to-platinum ratios) for the Diebold-Mariano test based on daily data for the sample period from 1/03/1973 to 04/08/2022, with data on stock indexes derived from Datastream, and the metal prices from the website of the London Bullion Market Association (LBMA)[19] and Macrotrends. The optimal CAViaR models for the daily data are as follows: Canada – asymmetric slope, France, Germany, Italy, Japan, UK, AND US –adaptive, Switzerland – symmetric absolute value (TR_1) and adaptive (TR_5).[20] The results for the quad-quad loss function, and the lin-lin loss function combined with our estimates of TR_5, are stronger than the results we obtain when we combine the lin-lin loss function and our estimates of TR_1, but the general message to take home from the figures is that our result that the p-values of the Diebold-Mariano tests are decreasing in the asymmetry parameter also holds for the shorter sample period of daily data starting in 1973.

[18]It should be noted that there are at least two problems with the usage of daily data. First, while we can match the opening and closing times of the metals market with the corresponding time of operation of the stock market of the UK, due to time-differences with the other economies in the sample, we cannot do the same. Second, the connectedness analysis cannot be performed because the stock markets have different opening and closing times being in separate time zones and, hence, we cannot put them in the TVP-VAR model all together to obtain the time-varying TCI. Ignoring these concerns, however, we report the predictability of the tail risks, and not the TCI (monthly results of which are discussed in the sub-section that follows), which in any event would be an infeasible exercise, given the computational time of estimating a TVP-VAR model comprising eight variables involving over 12000 observations.

[19]See: `https://www.lbma.org.uk/prices-and-data/precious-metal-prices#/`.

[20]Detailed estimation results for the CAViaR models are not reported but are available from the authors upon request.

3.3 Results for Connectedness

We next study the implications of our results with regard to total connectedness of stock-market tail risks. To this end, we plot in Figure 7 the estimated total connectedness index (TCI) for both TR_1 and TR_5. The results illustrate that total connectedness reached a first peak in the interwar period, then declined and hovered around a modest level until around the 1970s, and from then on started increasing again, where we observe that this trend increase did not occur gradually but rather was characterized four step jumps, with the latest one occurring at the very end of the sample period during the recent COVID-19 pandemic. While the others are associated with decline in stock returns in 1948 following the strong performances of the stock markets of the advanced countries that was witnessed during World War II (Ferguson, 2008), the Black Monday in 1987, and of course during the Global Financial Crisis of 2007 to 2009.

Equipped with the TCI estimates, we plot in Figure 8 the out-of-sample relative loss as a function of the asymmetry parameter for our measure of total connectedness of stock-market tail risk.[21] The figure shows that a forecaster who incurs higher costs from underestimating total connectedness than from overestimating it benefits from using the gold-to-silver price ratio as a predictor in our forecasting model. In other words, those forecasters benefit most from using the gold-to-silver price ratio for forecasting who otherwise would overestimate the gains from diversifying their portfolio investments across the eight stock markets in our sample (because underestimation of total connectedness of stock markets means that the benefits from portfolio diversification are overestimated).

4 Concluding Remarks

Stock-market tail risks have been shown to predict not only stock returns, but also real economic activity and oil market movements. Hence, determining the drivers and the forecastability of

[21] Figures S10 and S11 (Supplementary Material) depict the corresponding results for the gold-to-silver price and the gold-to-platinum price ratios for the shorter sample period (starting in 1968).

the tail risks due to these predictors, should be of immense value to investors and policymakers. Given this, the objective of our research is to forecast, for the first time, the tail risks of the advanced stock markets of Canada, France, Germany, Italy, Japan, Switzerland, the UK, and the US over the monthly period from 1916:02 to 2020:10 and 1968:01 to 2020:10, based on the ratios of gold-to-silver prices and gold-to-platinum prices, which, in turn, are proxies for global risk. In order to capture tail risks, we have employed four variants of the CAViaR framework, with the "best" tail-risk model obtained using relevant diagnostics. Consequently, we forecast the optimal tail risk estimates based on the predictive prowess of the two global risk predictors. Following this, we not only forecast the individual tail risks, but also their underlying time-varying connectedness, given the importance of comovements of stock markets, especially in the lower tails, for asset-allocation decisions of portfolio managers and stock market traders.

Using data for the shorter sample period starting in 1968, we have found that the gold-to-silver price ratio is a useful proxy for global risk similar to the conventionally used gold-to-platinum price ratio. Given this finding, we then have extended our empirical analysis to the long sample period. For the long sample period, we have found that, despite some heterogeneity across economies, the gold-to-silver price ratio often is useful for improving the accuracy of forecasts of both 1% and 5% stock-market tail risks, especially when a forecaster has an asymmetric loss function that attaches a larger weight to underestimations of tail risk than to overestimations of the same absolute seize. Similarly, such a forecaster should find the gold-to-silver price ratio beneficial for forecasting the total connectedness index that we constructed by means of a TVP-VAR model. Hence, our results offer important insights for international investors who seek to assess (and forecast) the benefits of international portfolio diversification in the wake of global risk shocks. From a policy perspective, our results imply that using the predicted path of tail risks based on the two ratios may help policymakers to design appropriate monetary and fiscal policies to insulate the effects on economic activity from global risk shocks.

As part of future research, it is interesting to extend our analysis to stock markets of emerging-market economies as well, and if, possible, from an historical perspective too, which would, however, be contingent on data availability. Given the financialization of the commodity market, such an exercise can also be implemented for commodities. The results we have documented in this research should be particularly useful for such analyses because our results suggest that, while the gold-to-silver price ratio has similar forecasting properties than the gold-to-platinum price ratio, the former is available for a much longer sample period than the latter. Hence, upon using the the gold-to-platinum price ratio, researchers can study over a much longer sample period than is possible by using the gold-to-platinum price ratio, how a variation of global risk affects subsequent stock-market returns in emerging-market economies, and the comovement of these stock markets with the stock markets of advanced economies and commodity markets.

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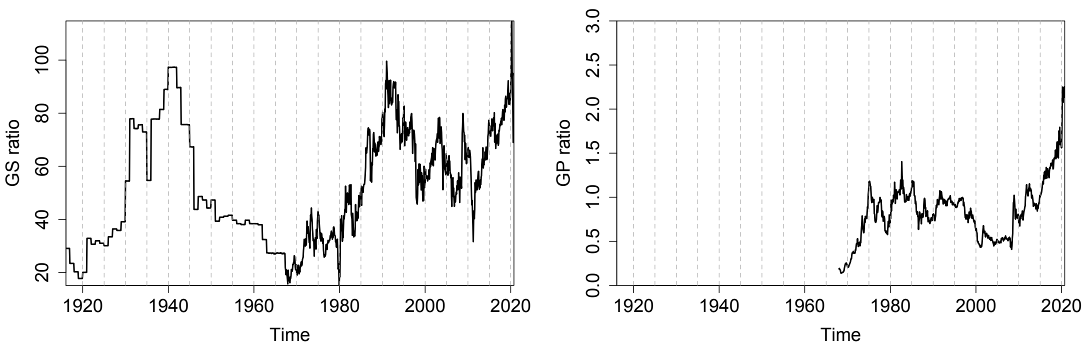
Table 1: Results for the Root-Mean-Squared-Forecasting Error (TR_1, Sample Starts 1968M01)

Model / window	h=1	h=3	h=6	h=9	h=12	h=24
CAN - GS ratio	0.9692	0.9865	1.0080	1.0226	1.0368	1.0613
CAN - GP ratio	0.9689	0.9799	1.0028	1.0088	1.0200	1.0749
CAN - both	0.9504	0.9651	0.9938	1.0111	1.0360	1.1472
FRA - GS ratio	0.9815	0.9905	0.9968	1.0025	1.0084	1.0265
FRA - GP ratio	0.9699	0.9707	0.9777	0.9867	0.9951	1.0507
FRA - both	0.9604	0.9645	0.9728	0.9828	0.9961	1.0737
GER - GS ratio	0.9840	0.9972	1.0004	1.0138	1.0270	1.0753
GER - GP ratio	0.9684	0.9778	0.9923	1.0075	1.0110	1.0679
GER - both	0.9645	0.9823	0.9947	1.0019	1.0130	1.1229
ITA - GS ratio	0.9797	0.9815	0.9834	0.9837	0.9839	0.9958
ITA - GP ratio	0.9789	0.9836	0.9894	0.9943	1.0008	1.0572
ITA - both	0.9662	0.9722	0.9817	0.9876	0.9940	1.0569
JAP - GS ratio	0.9838	0.9911	1.0061	1.0241	1.0466	1.1192
JAP - GP ratio	0.9601	0.9660	0.9763	0.9852	0.9974	1.0617
JAP - both	0.9554	0.9686	0.9881	1.0054	1.0317	1.1669
SWI - GS ratio	0.9738	0.9784	0.9884	1.0024	1.0145	1.0278
SWI - GP ratio	0.9664	0.9664	0.9828	1.0060	1.0253	1.0946
SWI - both	0.9450	0.9444	0.9614	0.9884	1.0207	1.1556
UK - GS ratio	0.9804	0.9902	0.9943	0.9959	0.9997	0.9942
UK - GP ratio	0.9754	0.9894	1.0058	1.0132	1.0156	1.0527
UK - both	0.9625	0.9751	0.9868	0.9926	1.0049	1.0869
US - GS ratio	0.9796	0.9899	1.0006	1.0035	1.0150	1.0422
US - GP ratio	0.9601	0.9730	0.9954	0.9982	1.0060	1.0491
US - both	0.9445	0.9540	0.9711	0.9769	0.9987	1.1078

Note: This table depicts the ratio of the root-mean-squared-forecasting errors (RMSFEs) as computed as the ratio of the RMSFE of the autoregressive benchmark model and a rival model that includes the variables displayed in the first column as additional predictors. The ratios are averages computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. A ratio that larger than unity indicates that the rival model has a better forecasting performance than the benchmark model.

Figure 1: Gold-to-Silver and Gold-to-Platinum Price Ratios



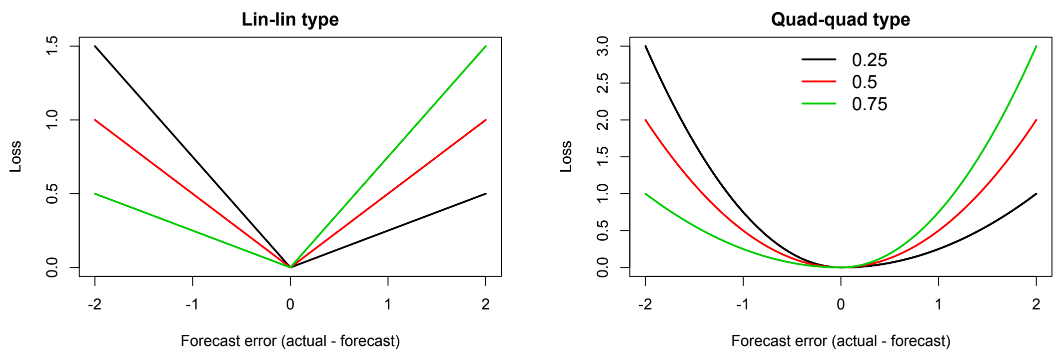
Note: This vertical dashed lines are added every five years (at 1920/01, 1925/01, 1930/01,.., 2015/01, 2020/01) to make it easier to read and interpret this figure.

Table 2: Results for the Mean-Absolute-Forecasting Error (TR_1, Sample Starts 1968M01)

Model / window	h=1	h=3	h=6	h=9	h=12	h=24
CAN - GS ratio	0.9544	0.9506	0.9928	1.0182	1.0328	1.0746
CAN - GP ratio	0.8965	0.9236	0.9761	0.9877	0.9983	1.0856
CAN - both	0.8752	0.8884	0.9613	1.0013	1.0371	1.1774
FRA - GS ratio	0.9640	0.9774	0.9838	0.9913	0.9848	1.0354
FRA - GP ratio	0.9495	0.9500	0.9684	0.9840	0.9845	1.0572
FRA - both	0.9371	0.9388	0.9483	0.9646	0.9673	1.1002
GER - GS ratio	0.9482	0.9688	0.9912	1.0245	1.0492	1.1171
GER - GP ratio	0.9206	0.9308	0.9703	1.0228	1.0519	1.1363
GER - both	0.8960	0.9259	0.9746	1.0203	1.0581	1.2182
ITA - GS ratio	0.9575	0.9677	0.9651	0.9490	0.9477	1.0004
ITA - GP ratio	0.9611	0.9799	0.9894	0.9936	1.0022	1.0945
ITA - both	0.9451	0.9655	0.9729	0.9655	0.9738	1.1020
JAP - GS ratio	0.9832	0.9829	1.0001	1.0160	1.0468	1.0787
JAP - GP ratio	0.9488	0.9591	0.9696	0.9942	1.0196	1.0771
JAP - both	0.9388	0.9514	0.9811	1.0180	1.0656	1.1582
SWI - GS ratio	0.9323	0.9373	0.9486	0.9676	0.9873	1.0198
SWI - GP ratio	0.9351	0.9521	0.9659	1.0023	1.0421	1.1767
SWI - both	0.9042	0.9240	0.9467	0.9858	1.0357	1.2123
UK - GS ratio	0.9600	0.9708	0.9821	0.9863	0.9835	0.9818
UK - GP ratio	0.9594	0.9864	1.0118	1.0195	1.0214	1.1337
UK - both	0.9316	0.9550	0.9832	0.9883	0.9994	1.1543
US - GS ratio	0.9479	0.9580	0.9783	0.9906	1.0034	1.0146
US - GP ratio	0.9276	0.9480	0.9886	1.0068	1.0210	1.0984
US - both	0.8903	0.9086	0.9631	0.9951	1.0231	1.1499

Note: This table depicts the ratio of the mean-absolute-forecasting errors (MAFEs) as computed as the ratio of the MAFE of the autoregressive benchmark model and a rival model that includes the variables displayed in the first column as additional predictors. The ratios are averages computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. A ratio that larger than unity indicates that the rival model has a better forecasting performance than the benchmark model.

Figure 2: Examples of the Loss Function



Note: Examples of the loss function for $\alpha \in 0.25, 0.5, 0.75$.

Table 3: Results for the Root-Mean-Squared-Forecasting Error (TR_5, Sample Starts 1968M01)

Model / window	h=1	h=3	h=6	h=9	h=12	h=24
CAN - GS ratio	0.9818	0.9859	0.9958	1.0073	1.0254	1.0702
CAN - GP ratio	0.9717	0.9690	0.9797	0.9876	1.0010	1.0637
CAN - both	0.9576	0.9526	0.9628	0.9725	0.9971	1.1236
FRA - GS ratio	0.9841	1.0029	1.0093	1.0124	1.0223	1.0501
FRA - GP ratio	0.9749	0.9845	0.9894	0.9923	0.9949	1.0389
FRA - both	0.9673	0.9809	0.9863	0.9850	0.9967	1.0859
GER - GS ratio	0.9839	0.9890	0.9907	1.0020	1.0140	1.0574
GER - GP ratio	0.9686	0.9741	0.9927	1.0123	1.0194	1.0758
GER - both	0.9646	0.9737	0.9864	0.9977	1.0119	1.1213
ITA - GS ratio	0.9742	0.9779	0.9803	0.9848	0.9861	0.9933
ITA - GP ratio	0.9738	0.9758	0.9804	0.9816	0.9843	1.0376
ITA - both	0.9580	0.9598	0.9634	0.9651	0.9677	1.0238
JAP - GS ratio	0.9890	0.9975	1.0105	1.0287	1.0516	1.1100
JAP - GP ratio	0.9613	0.9664	0.9780	0.9885	1.0027	1.0591
JAP - both	0.9611	0.9739	0.9915	1.0097	1.0418	1.1706
SWI - GS ratio	0.9723	0.9763	0.9865	1.0023	1.0152	1.0302
SWI - GP ratio	0.9657	0.9645	0.9795	1.0024	1.0219	1.0900
SWI - both	0.9443	0.9423	0.9574	0.9837	1.0161	1.1524
UK - GS ratio	0.9725	0.9858	0.9982	0.9942	0.9997	0.9904
UK - GP ratio	0.9657	0.9852	1.0126	1.0185	1.0215	1.0338
UK - both	0.9497	0.9673	0.9950	1.0017	1.0128	1.0431
US - GS ratio	0.9852	0.9974	1.0053	1.0110	1.0265	1.0633
US - GP ratio	0.9612	0.9761	0.9965	1.0015	1.0118	1.0785
US - both	0.9515	0.9633	0.9781	0.9875	1.0148	1.1531

Note: This table depicts the ratio of the root-mean-squared-forecasting errors (RMSFEs) as computed as the ratio of the RMSFE of the autoregressive benchmark model and a rival model that includes the variables displayed in the first column as additional predictors. The ratios are averages computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. A ratio that larger than unity indicates that the rival model has a better forecasting performance than the benchmark model.

Table 4: Results for the Root-Mean-Squared-Forecasting Error (Sample Starts 1916M02)

Panel A: TR_1

Model / window	h=1	h=3	h=6	h=9	h=12	h=24
CAN - GS ratio	0.9740	0.9827	0.9946	1.0006	1.0080	1.0199
FRA - GS ratio	0.9838	0.9898	0.9963	1.0056	1.0162	1.0502
GER - GS ratio	1.1345	1.3654	1.5375	1.6444	1.7121	1.0656
ITA - GS ratio	0.9863	0.9869	0.9856	0.9861	0.9871	1.0182
JAP - GS ratio	0.9895	0.9978	1.0140	1.0376	1.0648	1.1689
SWI - GS ratio	0.9716	0.9810	0.9996	1.0199	1.0418	1.0797
UK - GS ratio	0.9852	0.9918	1.0001	1.0087	1.0195	1.0547
US - GS ratio	0.9820	0.9854	0.9925	0.9964	1.0056	1.0340

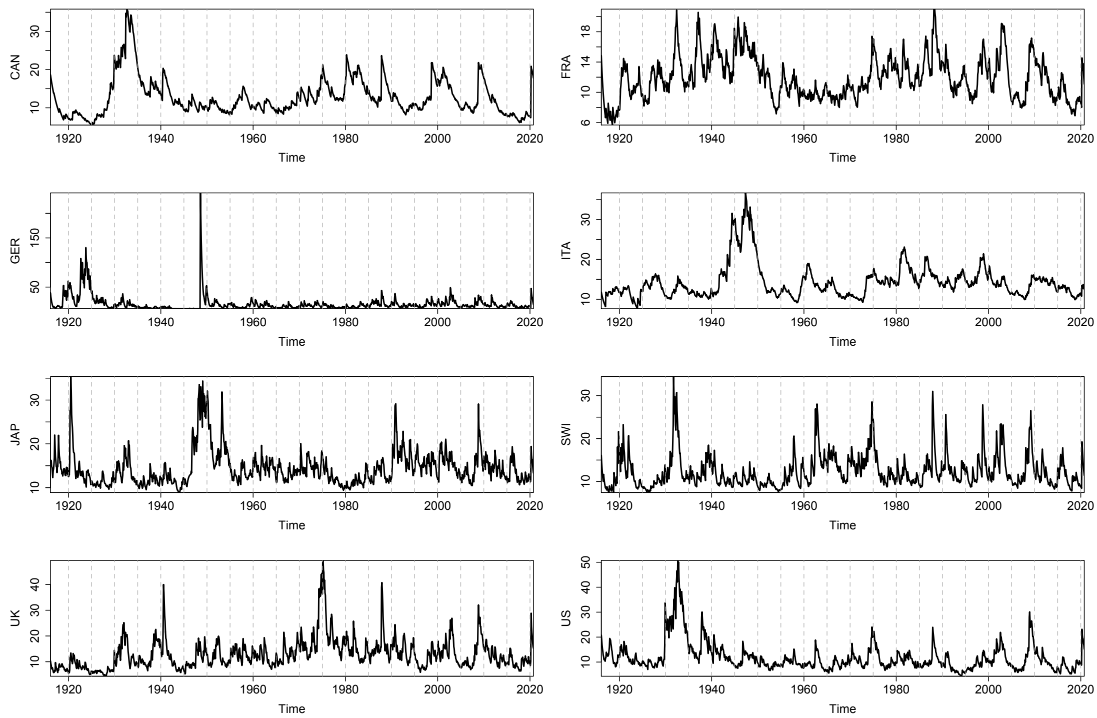
Panel A: TR_5

Model / window	h=1	h=3	h=6	h=9	h=12	h=24
CAN - GS ratio	0.9773	0.9770	0.9826	0.9873	0.9983	1.0278
FRA - GS ratio	0.9860	0.9986	1.0057	1.0131	1.0243	1.0539
GER - GS ratio	0.9615	1.0580	1.1501	1.2234	1.2921	1.1494
ITA - GS ratio	0.9829	0.9848	0.9836	0.9848	0.9855	1.0155
JAP - GS ratio	0.9962	1.0141	1.0364	1.0649	1.0940	1.1868
SWI - GS ratio	0.9718	0.9807	0.9996	1.0206	1.0430	1.0781
UK - GS ratio	0.9835	0.9981	1.0156	1.0227	1.0340	1.0760
US - GS ratio	0.9740	0.9826	0.9908	0.9976	1.0109	1.0410

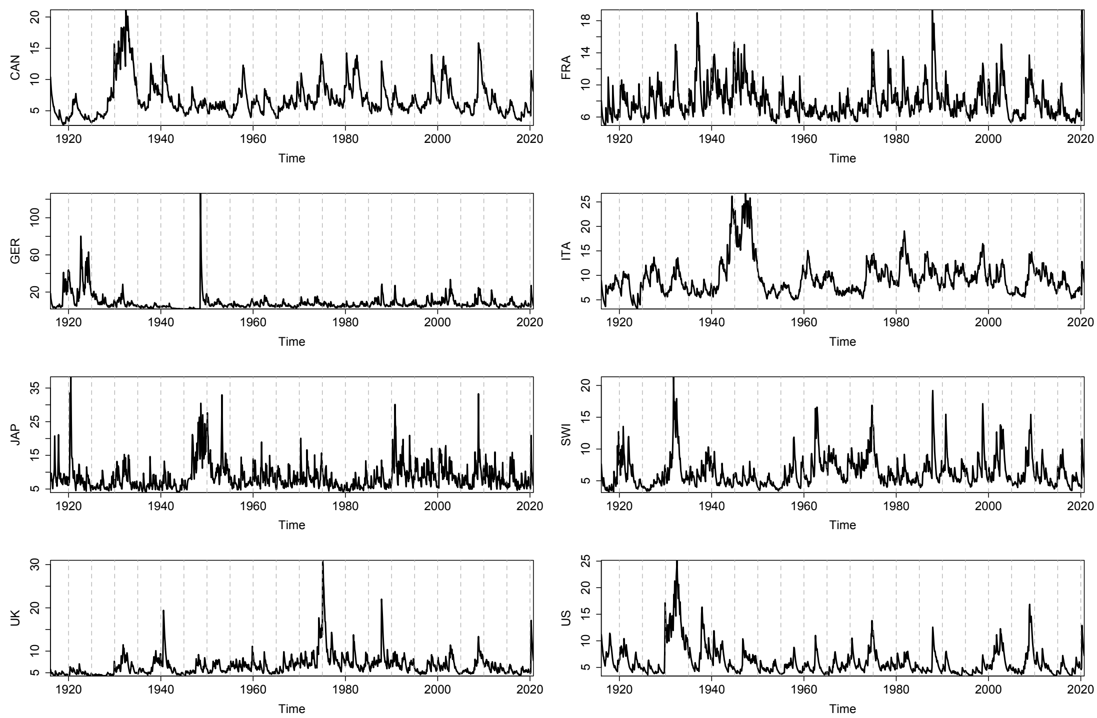
Note: This table depicts the ratio of the root-mean-squared-forecasting errors (RMSFEs) as computed as the ratio of the RMSFE of the autoregressive benchmark model and a rival model that includes the gold-to-silver price ratio as an additional predictor. The ratios are averages computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. A ratio that larger than unity indicates that the rival model has a better forecasting performance than the benchmark model.

Figure 3: Tail Risks (TR_1)

Panel A: TR_1

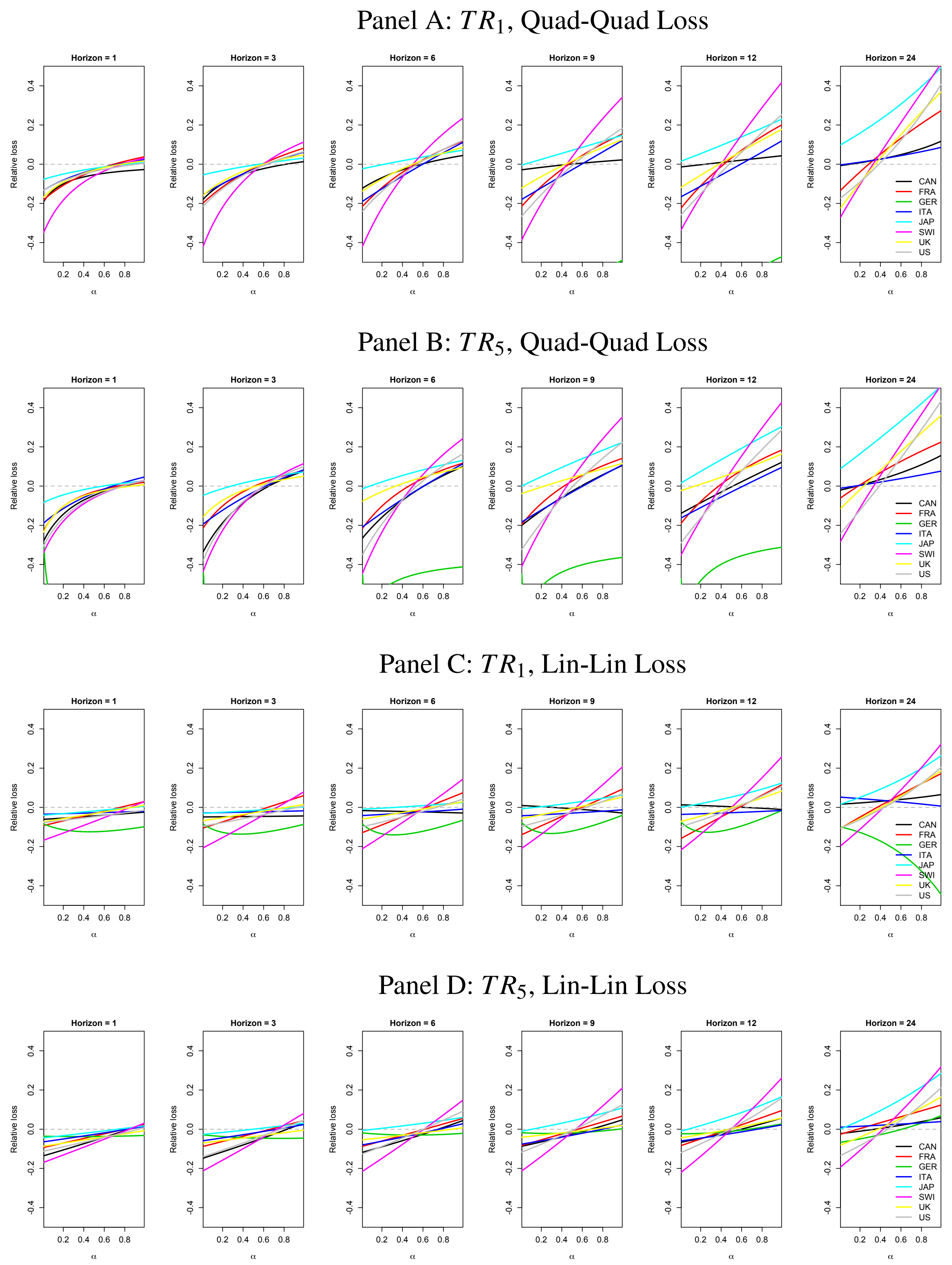


Panel A: TR_5



Note: This vertical dashed lines are added every five years (at 1920/01, 1925/01, 1930/01,.., 2015/01, 2020/01) to make it easier to read and interpret this figure.

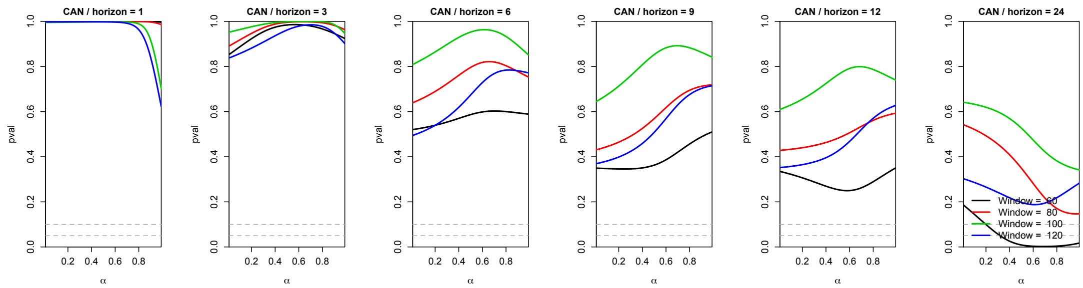
Figure 4: Out-of-Sample Relative-Loss Statistic (Sample Starts 1916M02)



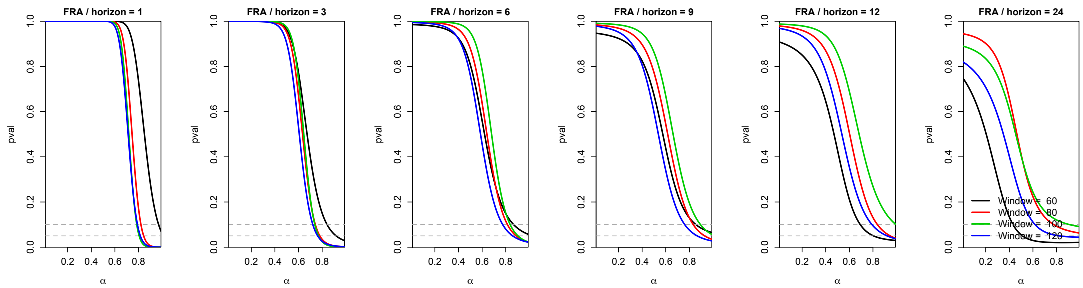
Note: This figure shows the out-of-sample relative loss criterion as a function of the asymmetry parameter and a quad-quad loss function. The relative loss criterion is an average computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure 5: Diebold-Mariano Test Under Asymmetric Loss (TR_1, Lin-Lin Loss, Sample Starts 1916M02)

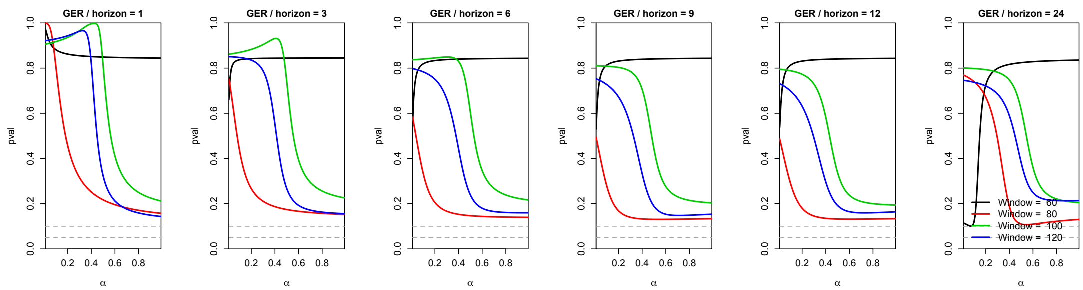
Panel A: Canada



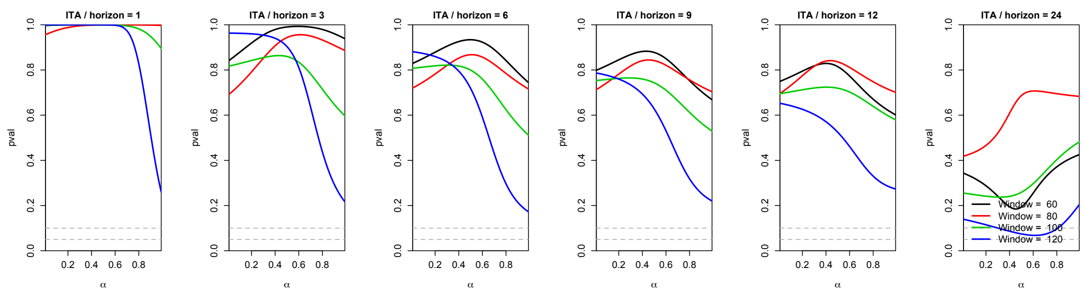
Panel B: France



Panel C: Germany



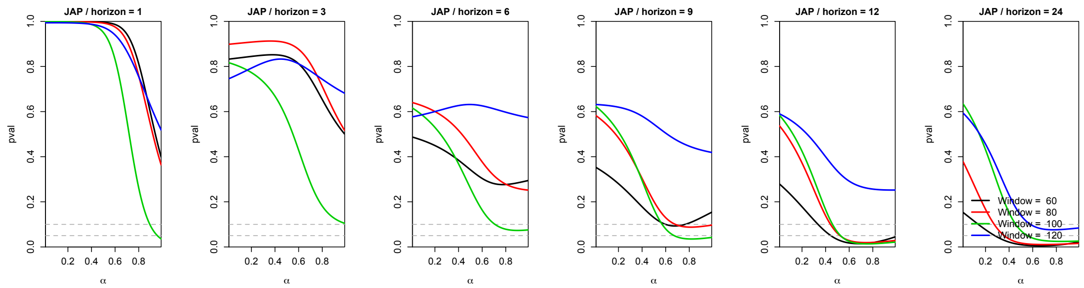
Panel D: Italy



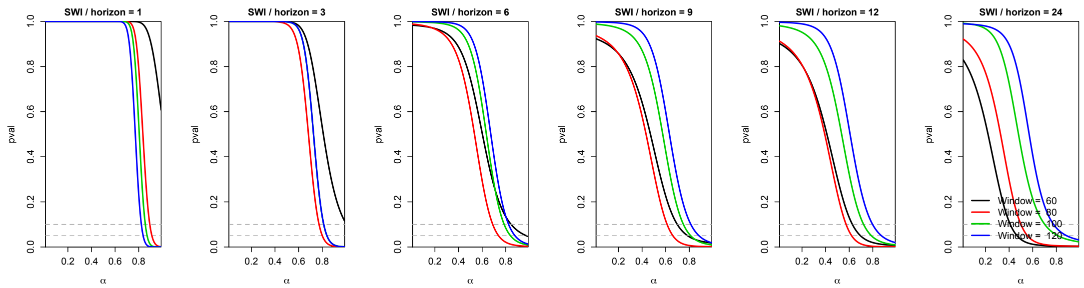
To be continued.

Continued.

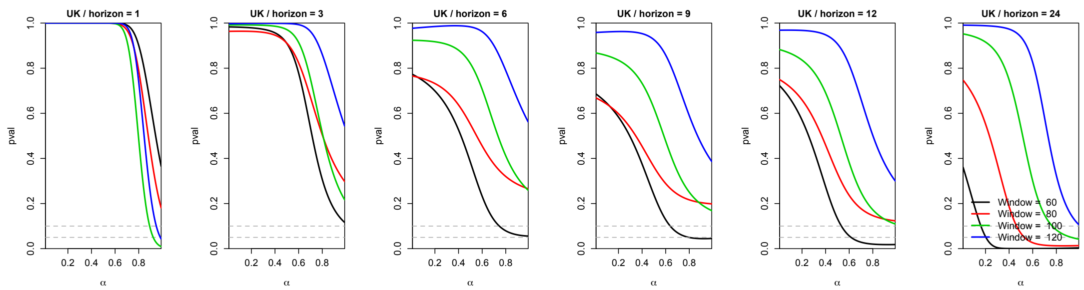
Panel E: Japan



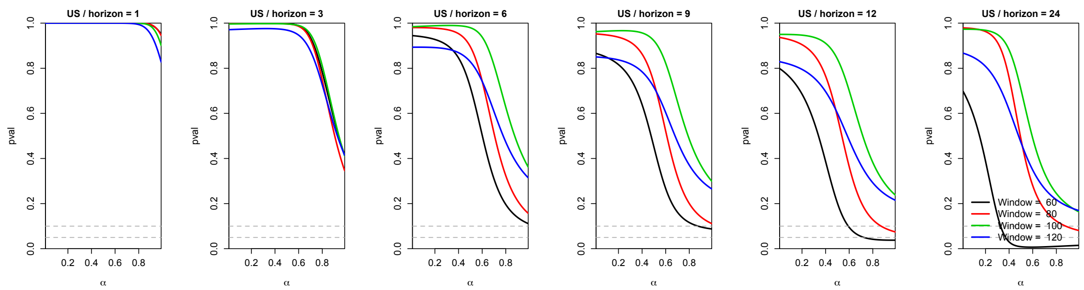
Panel F: Switzerland



Panel G: United Kingdom



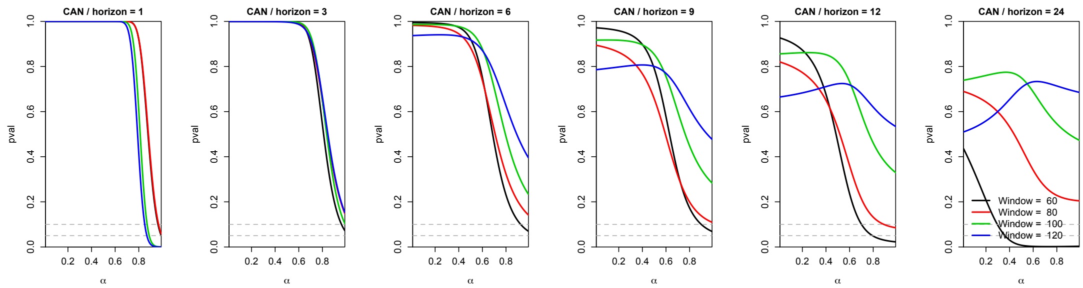
Panel H: United States



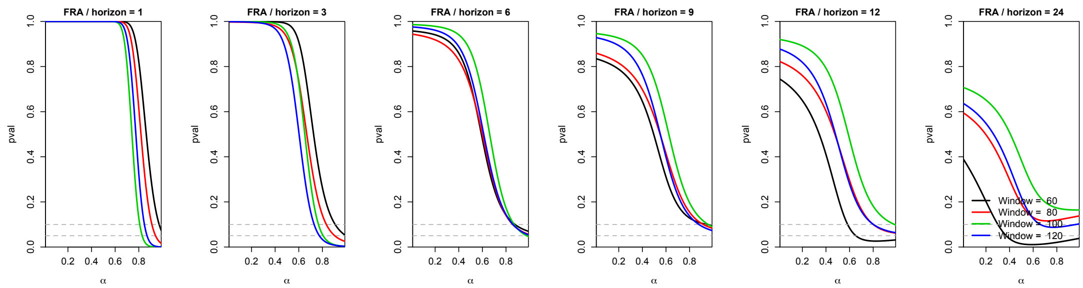
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure 6: Diebold-Mariano Test Under Asymmetric Loss (TR_5, Lin-Lin Loss, Sample Starts 1916M02)

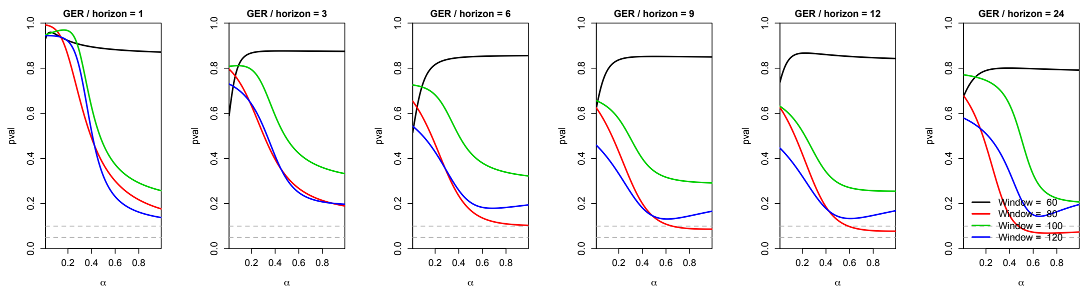
Panel A: Canada



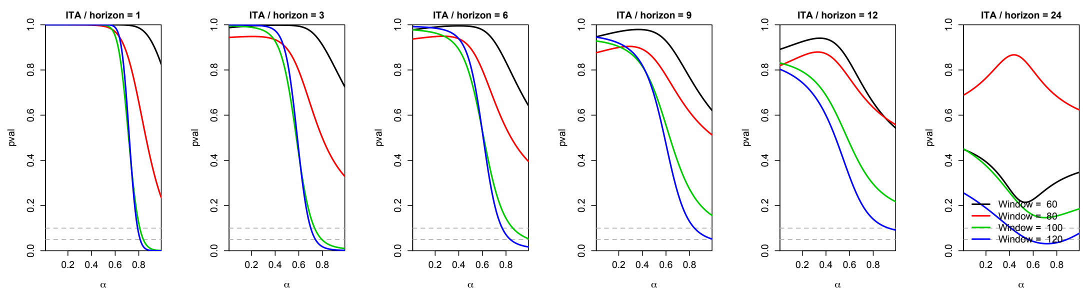
Panel B: France



Panel C: Germany



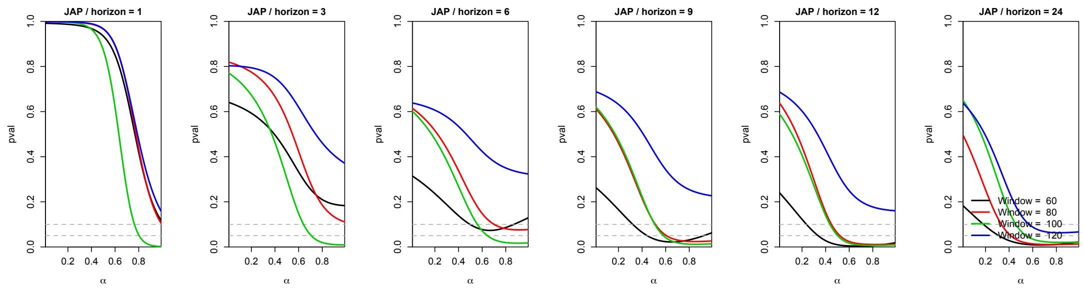
Panel D: Italy



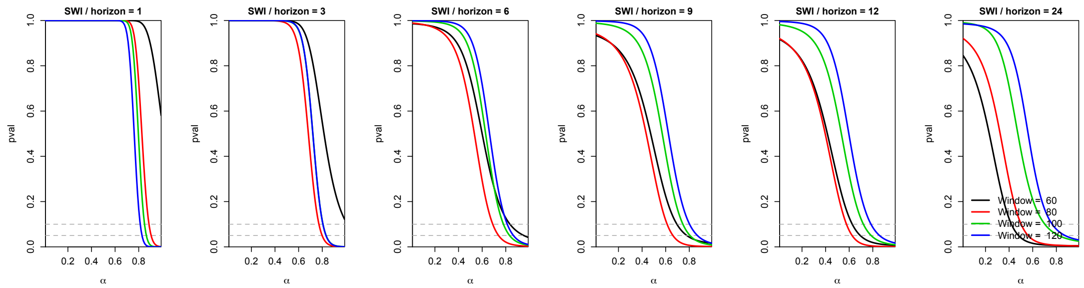
To be continued.

Continued.

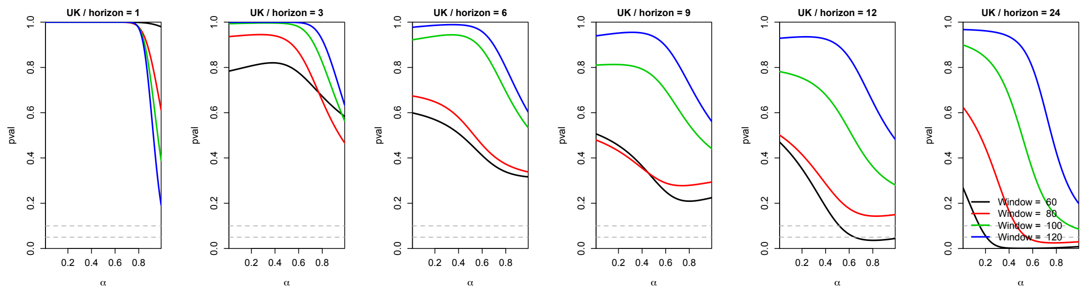
Panel E: Japan



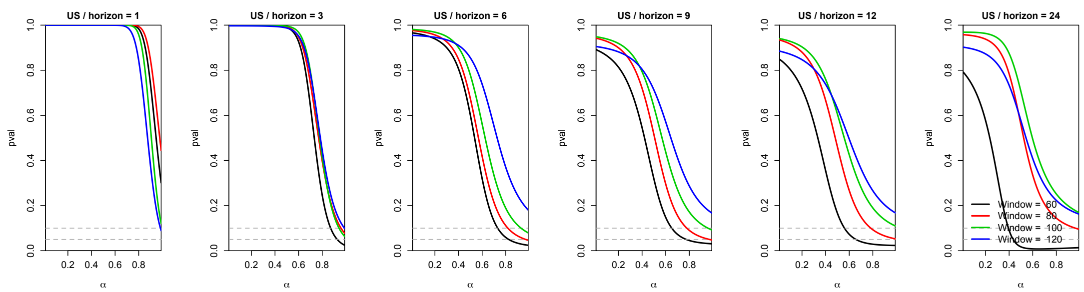
Panel F: Switzerland



Panel G: United Kingdom

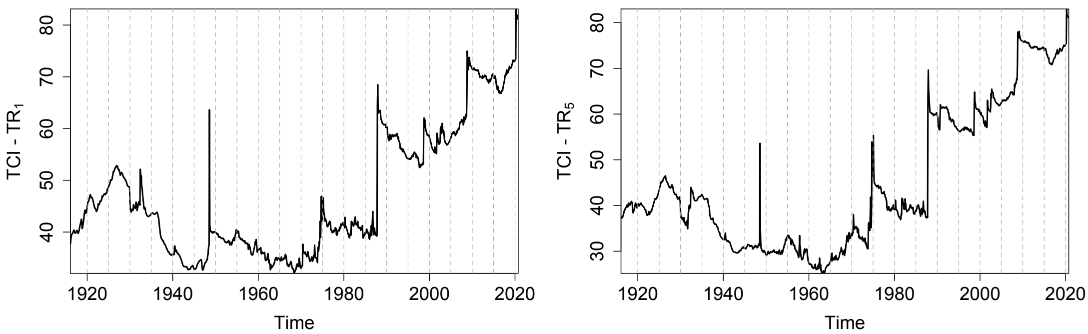


Panel H: United States



Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

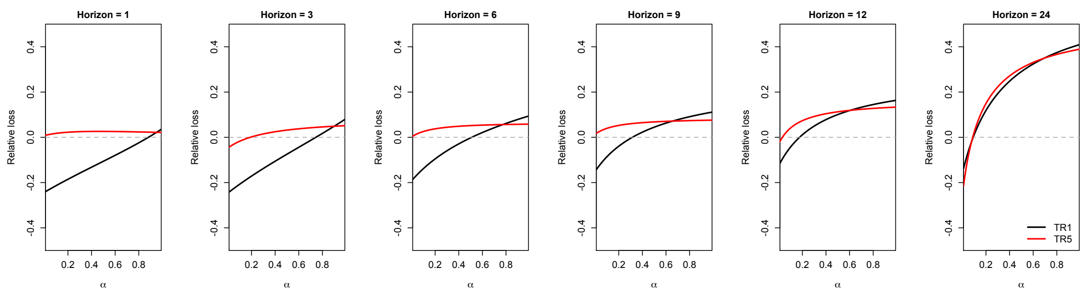
Figure 7: Total Connectedness



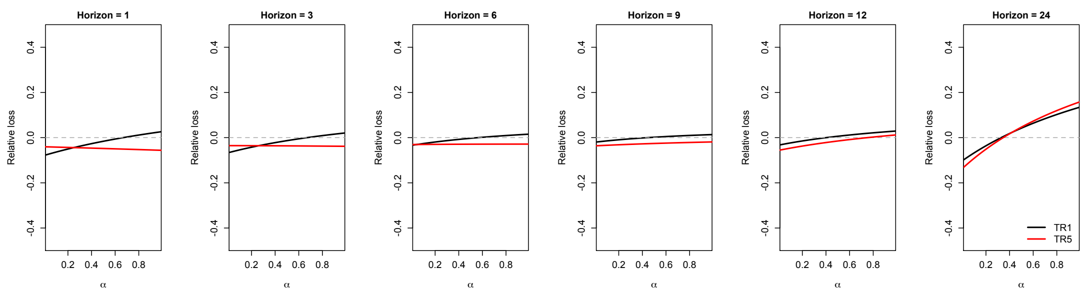
Note: This vertical dashed lines are added every five years (at 1920/01, 1925/01, 1930/01,.., 2015/01, 2020/01) to make it easier to read and interpret this figure.

Figure 8: Out-of-Sample Relative-Loss Criterion for Total Connectedness (Sample Starts 1916M02)

Panel A: Quad-Quad Loss



Panel B: Lin-Lin Loss



Note: This figure shows the out-of-sample relative loss criterion as a function of the asymmetry parameter and a quad-quad loss function. The relative loss criterion is an average computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Supplementary Material

Table S1: Estimates and relevant statistics for the countryspecific CAViaR specification [Symmetric Absolute Value]

1% VaR	Canada	France	Germany	Italy	Japan	Switzerland	UK	US
β_1	0.172	0.425	1.770	0.342	2.490	1.200	4.510	-0.004
Standard errors	0.295	0.363	1.270	0.232	1.050	1.530	1.470	0.167
P values	0.279	0.121	0.082	0.070	0.009	0.216	0.001	0.490
β_2	0.924	0.896	0.649	0.928	0.705	0.836	0.386	0.923
Standard errors	0.039	0.045	0.123	0.042	0.093	0.190	0.125	0.036
P values	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
β_3	0.284	0.216	1.010	0.148	0.492	0.327	1.020	0.365
Standard errors	0.088	0.064	0.148	0.078	0.143	0.319	0.329	0.183
P values	0.001	0.000	0.000	0.028	0.000	0.152	0.001	0.023
RQ	176.000	167.000	342.000	178.000	185.000	172.000	166.000	162.000
Hits in-sample (%)	0.993	0.993	1.090	0.894	0.993	0.993	1.090	1.090
Hits out-of-sample (%)	1.200	3.200	2.000	3.600	1.600	0.400	1.200	2.000
DQ in-sample (P-values)	0.189	0.083	0.809	0.994	0.080	0.181	0.030	0.985
DQ out-of-sample (P-values)	0.000	0.000	0.540	0.000	0.000	0.967	0.980	0.653

Note: Bolded figures indicate a country's tail risk that best "fits" the return series. The criteria used are the DQ test and %Hits (out-of-sample). For the "best" tail risk specification, we expect the %Hits to be 1% for 1% VaR (and 5% for 5% VaR). while the DQ test statistic is not expected to be significant. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, then, we consider the tail risk with the closest value to the expected value for the % Hits. In the same vein, when all the tail risks are statistically significant, then, the % Hits becomes a major criterion except when some distinctions can still be made with the significant DQ test statistics.

Table S2: Estimates and Relevant statistics for the country-specific CAViaR specification [Asymmetric Slope]

1% VaR	Canada	**France**	Germany	**Italy**	**Japan**	**Switzerland**	**UK**	**US**
β_1	0.048	**0.355**	1.520	**0.389**	**1.890**	**1.230**	**0.577**	**0.220**
Standard errors	0.283	**0.374**	0.663	**0.356**	**1.400**	**1.630**	**0.746**	**0.470**
P values	0.433	**0.171**	0.011	**0.137**	**0.089**	**0.224**	**0.220**	**0.320**
β_2	0.921	**0.893**	0.742	**0.927**	**0.765**	**0.815**	**0.802**	**0.862**
Standard errors	0.042	**0.050**	0.079	**0.054**	**0.125**	**0.176**	**0.106**	**0.093**
P values	0.000	**0.000**	0.000	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
β_3	0.273	**0.200**	0.280	**0.159**	**0.280**	**0.129**	**0.451**	**0.349**
Standard errors	0.074	**0.064**	0.124	**0.106**	**0.131**	**0.152**	**0.333**	**0.381**
P values	0.000	**0.001**	0.012	**0.066**	**0.017**	**0.199**	**0.088**	**0.180**
β_4	0.307	**0.250**	0.824	**0.102**	**0.443**	**0.619**	**0.832**	**0.619**
Standard errors	0.261	**0.100**	0.146	**0.103**	**0.100**	**0.466**	**0.371**	**0.536**
P values	0.119	**0.006**	0.000	**0.161**	**0.000**	**0.092**	**0.012**	**0.124**
RQ	176.000	**166.000**	330.000	**177.000**	**184.000**	**164.000**	**166.000**	**158.000**
Hits in-sample (%)	1.090	**0.894**	0.993	**0.993**	**1.090**	**0.993**	**0.894**	**0.993**
Hits out-of-sample (%)	1.200	**2.800**	1.600	**4.000**	**1.600**	**0.400**	**0.400**	**1.200**
DQ in-sample (P-values)	0.243	**0.993**	0.068	**0.991**	**0.118**	**0.984**	**0.932**	**0.992**
DQ out-of-sample (P-values)	0.000	**0.003**	0.964	**0.000**	**0.000**	**0.968**	**0.986**	**0.997**

Note: See notes to Table A1.

Table S3: Estimates and Relevant statistics for the country-specific CAViaR specification [Indirect GARCH]

1% VaR	**Canada**	France	**Germany**	Italy	Japan	Switzerland	UK	US
β_1	**0.766**	3.310	**13.800**	4.870	44.700	3.740	7.200	0.920
Standard errors	**3.600**	3.600	**14.000**	3.410	22.900	4.820	8.740	2.450
P values	**0.416**	0.179	**0.161**	0.077	0.026	0.219	0.205	0.354
β_2	**0.925**	0.923	**0.602**	0.922	0.663	0.862	0.758	0.817
Standard errors	**0.036**	0.031	**0.126**	0.017	0.112	0.053	0.107	0.030
P values	**0.000**	0.000	**0.000**	0.000	0.000	0.000	0.000	0.000
β_3	**0.644**	0.283	**2.740**	0.241	0.810	0.984	1.820	1.680
Standard errors	**0.649**	0.449	**1.560**	0.627	0.694	3.070	0.888	2.560
P values	**0.160**	0.264	**0.039**	0.350	0.121	0.374	0.021	0.256
RQ	**180.000**	170.000	**343.000**	178.000	184.000	168.000	170.000	161.000
Hits in-sample (%)	**1.090**	0.993	**0.993**	0.993	0.993	1.090	0.894	0.993
Hits out-of-sample (%)	**1.200**	2.800	**2.000**	3.600	1.600	0.800	0.800	1.600
DQ in-sample (P-values)	**0.978**	0.080	**0.995**	0.988	0.057	0.118	0.799	0.988
DQ out-of-sample (P-values)	**0.000**	0.002	**0.673**	0.000	0.000	0.935	1.000	0.959

Note: See notes to Table A1.

Table S4: Estimates and Relevant statistics for the country-specific CAViaR specification [Adaptive]

1% VaR	Canada	France	Germany	Italy	Japan	Switzerland	UK	US
β_1	1.040	-0.108	14.000	0.732	-1.950	0.618	1.080	3.400
Standard errors	0.602	0.330	0.000	0.730	0.015	0.796	0.433	0.551
P values	0.042	0.371	0.000	0.158	0.000	0.219	0.006	0.000
RQ	214.000	180.000	393.000	226.000	215.000	184.000	183.000	193.000
Hits in-sample (%)	0.596	0.794	0.794	1.490	0.993	0.695	1.390	0.695
Hits out-of-sample (%)	1.200	1.600	1.200	1.200	0.800	0.400	0.800	1.600
DQ in-sample (P-values)	1.000	0.020	0.996	0.041	0.003	1.000	0.008	0.000
DQ out-of-sample (P-values)	0.000	0.044	0.757	0.243	0.638	0.905	0.639	0.040

Note: See notes to Table A1.

Table S5: Estimates and Relevant statistics for the country-specific CAViaR specification [Symmetric Absolute Value]

5% VaR	Canada	France	Germany	Italy	Japan	Switzerland	UK	US	**Oil**
β_1	0.062	1.040	0.867	0.458	0.961	0.439	0.730	0.458	**0.000**
Standard errors	0.154	0.761	0.267	0.244	0.406	0.508	0.309	0.244	**0.000**
P values	0.344	0.085	0.001	0.030	0.009	0.194	0.009	0.030	**0.351**
β_2	0.923	0.752	0.601	0.874	0.765	0.826	0.777	0.874	**0.890**
Standard errors	0.048	0.125	0.102	0.051	0.076	0.127	0.080	0.051	**0.000**
P values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	**0.000**
β_3	0.182	0.259	0.680	0.169	0.283	0.286	0.279	0.169	**0.263**
Standard errors	0.109	0.090	0.191	0.060	0.113	0.122	0.105	0.060	**0.000**
P values	0.048	0.002	0.000	0.003	0.006	0.010	0.004	0.003	**0.000**
RQ	573.000	580.000	828.000	655.000	651.000	540.000	530.000	655.000	**517.000**
Hits in-sample (%)	4.970	5.060	4.970	4.970	4.970	5.060	5.060	4.970	**5.060**
Hits out-of-sample (%)	3.600	6.000	8.000	6.400	7.600	6.400	6.000	6.400	**8.800**
DQ in-sample (P-values)	0.063	0.762	0.147	0.054	0.337	0.087	0.309	0.054	**0.000**
DQ out-of-sample (P-values)	0.604	0.004	0.016	0.646	0.095	0.050	0.379	0.646	**0.000**

Note: See notes to Table A1.

Table S6: Estimates and Relevant statistics for the country-specific CAViaR specification [Asymmetric Slope]

5% VaR	**Canada**	France	**Germany**	**Italy**	**Japan**	**Switzerland**	UK	**US**
β_1	**0.231**	1.040	**0.653**	**0.373**	**1.790**	**0.609**	0.649	**0.373**
Standard errors	**0.381**	0.772	**0.821**	**0.190**	**0.845**	**0.547**	0.323	**0.190**
P values	**0.272**	0.089	**0.213**	**0.025**	**0.017**	**0.133**	0.022	**0.025**
β_2	**0.874**	0.719	**0.629**	**0.853**	**0.503**	**0.782**	0.805	**0.853**
Standard errors	**0.076**	0.118	**0.235**	**0.041**	**0.133**	**0.098**	0.099	**0.041**
P values	**0.000**	0.000	**0.004**	**0.000**	**0.000**	**0.000**	0.000	**0.000**
β_3	**0.108**	0.249	**0.344**	**0.188**	**0.386**	**0.108**	0.080	**0.188**
Standard errors	**0.147**	0.117	**0.248**	**0.059**	**0.087**	**0.101**	0.087	**0.059**
P values	**0.230**	0.017	**0.083**	**0.001**	**0.000**	**0.142**	0.177	**0.001**
β_4	**0.308**	0.319	**0.853**	**0.238**	**0.823**	**0.439**	0.344	**0.238**
Standard errors	**0.156**	0.109	**0.594**	**0.080**	**0.171**	**0.082**	0.185	**0.080**
P values	**0.024**	0.002	**0.076**	**0.001**	**0.000**	**0.000**	0.031	**0.001**
RQ	**568.000**	578.000	**797.000**	**652.000**	**648.000**	**526.000**	520.000	**652.000**
Hits in-sample (%)	**4.870**	4.870	**4.970**	**4.970**	**4.770**	**4.970**	5.060	**4.970**
Hits out-of-sample (%)	**4.800**	7.200	**8.800**	**6.400**	**7.600**	**5.200**	7.600	**6.400**
DQ in-sample (P-values)	**0.386**	0.831	**0.986**	**0.126**	**0.963**	**0.601**	0.424	**0.126**
DQ out-of-sample (P-values)	**0.886**	0.018	**0.050**	**0.701**	**0.121**	**0.877**	0.536	**0.701**

Note: See notes to Table A1.

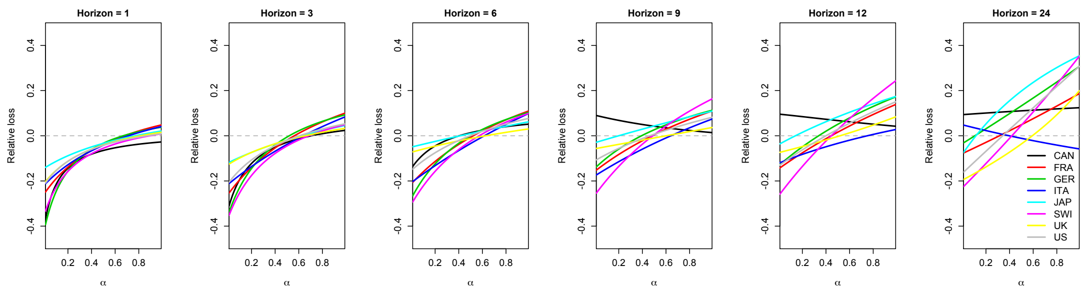
Table S7: Estimates and Relevant statistics for the country-specific CAViaR specification [Indirect GARCH]

5% VaR	Canada	**France**	Germany	Italy	Japan	Switzerland	**UK**	US
β_1	-0.384	**7.000**	4.590	3.320	7.340	1.510	**5.070**	3.320
Standard errors	0.378	**6.720**	3.460	1.820	4.000	2.380	**2.950**	1.820
P values	0.155	**0.149**	0.092	0.034	0.033	0.263	**0.043**	0.034
β_2	0.930	**0.704**	0.456	0.878	0.771	0.804	**0.709**	0.878
Standard errors	0.016	**0.132**	0.134	0.028	0.057	0.076	**0.074**	0.028
P values	0.000	**0.000**	0.000	0.000	0.000	0.000	**0.000**	0.000
β_3	0.225	**0.435**	1.200	0.181	0.308	0.502	**0.458**	0.181
Standard errors	0.132	**0.232**	0.727	0.290	0.154	0.280	**0.472**	0.290
P values	0.044	**0.030**	0.050	0.267	0.023	0.036	**0.166**	0.267
RQ	575.000	**580.000**	834.000	662.000	654.000	538.000	**533.000**	662.000
Hits in-sample (%)	5.060	**5.060**	5.060	5.060	5.060	5.060	**5.060**	5.060
Hits out-of-sample (%)	6.000	**6.800**	9.200	6.800	8.000	6.000	**7.600**	6.800
DQ in-sample (P-values)	0.084	**0.883**	0.897	0.013	0.462	0.377	**0.443**	0.013
DQ out-of-sample (P-values)	0.137	**0.012**	0.001	0.566	0.024	0.036	**0.517**	0.566

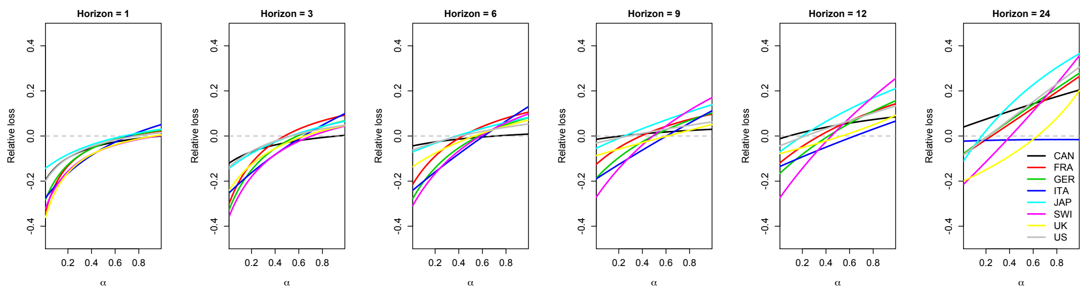
Note: See notes to Table A1.

Figure S1: Out-of-Sample Relative-Loss Statistic (Sample Starts 1968M01)

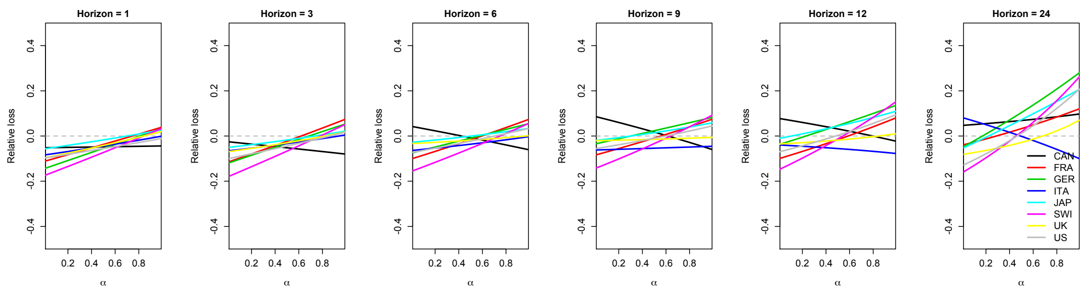
Panel A: TR_1, Quad-Quad Loss



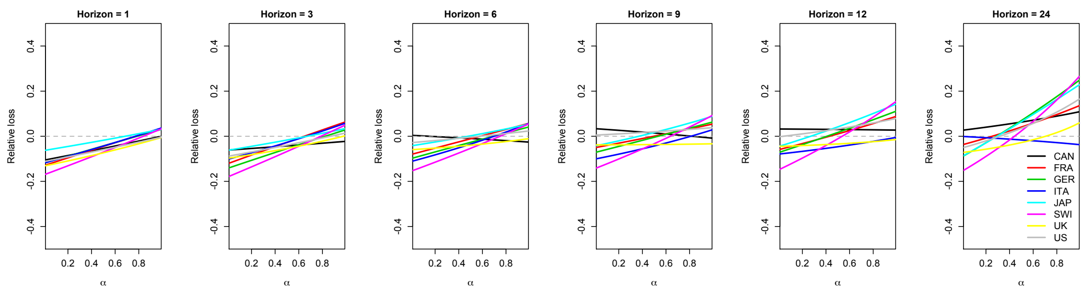
Panel B: TR_5, Quad-Quad Loss



Panel C: TR_1, Lin-Lin Loss



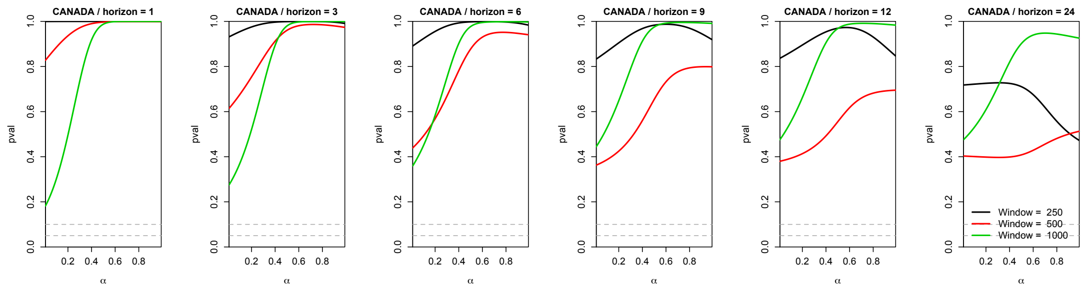
Panel D: TR_5, Lin-Lin Loss



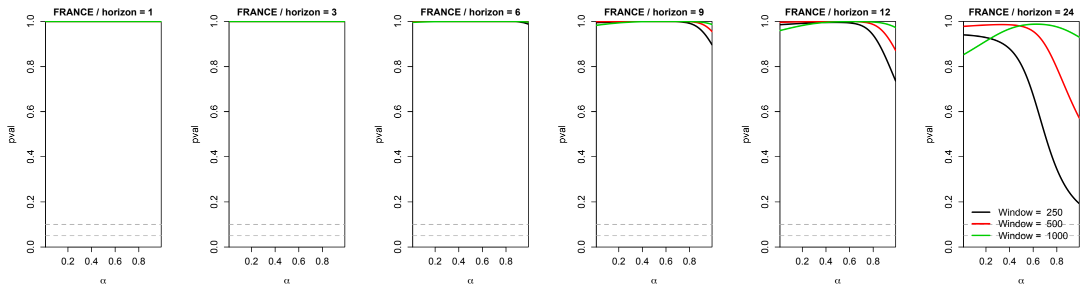
Note: This figure shows the out-of-sample relative loss criterion as a function of the asymmetry parameter and a quad-quad loss function. The relative loss criterion is an average computed across four rolling-estimation windows of length 60, 80, 100, and 120 months. The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure S2: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Silver Price Ratio, TR_1, Lin-Lin Loss, Sample Period 1/03/1973−04/08/2022)

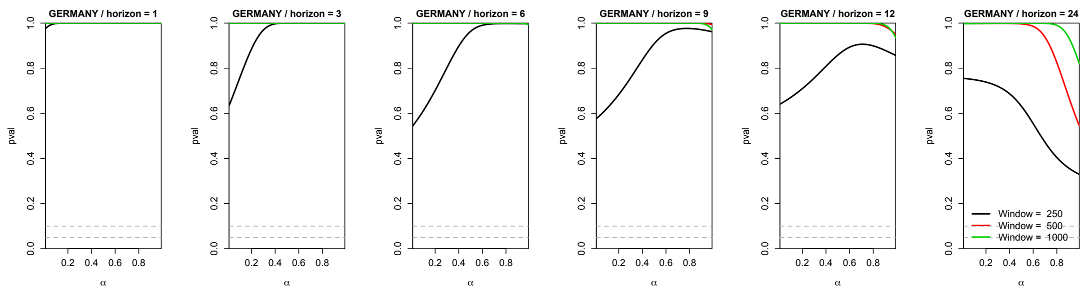
Panel A: Canada



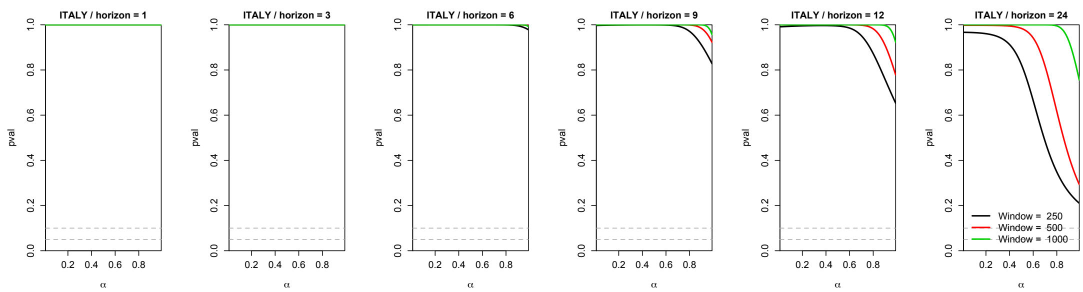
Panel B: France



Panel C: Germany



Panel D: Italy



To be continued.

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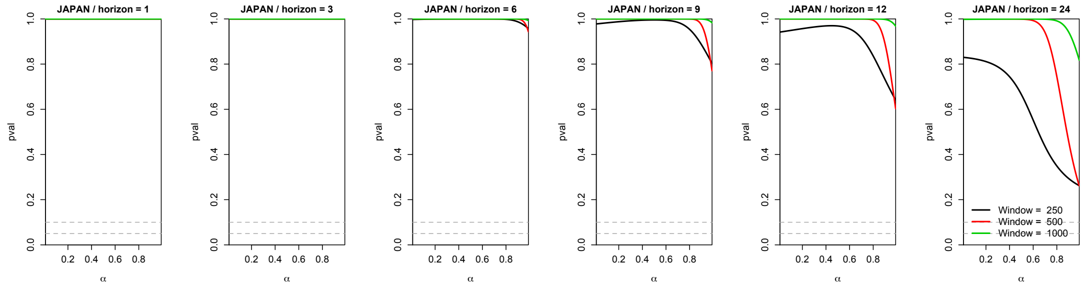
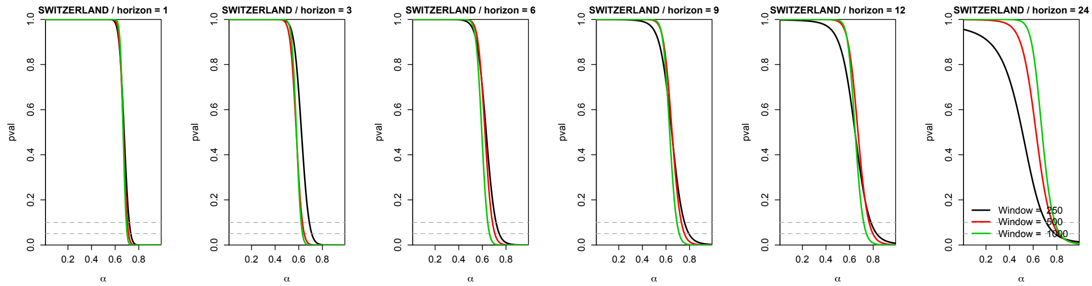
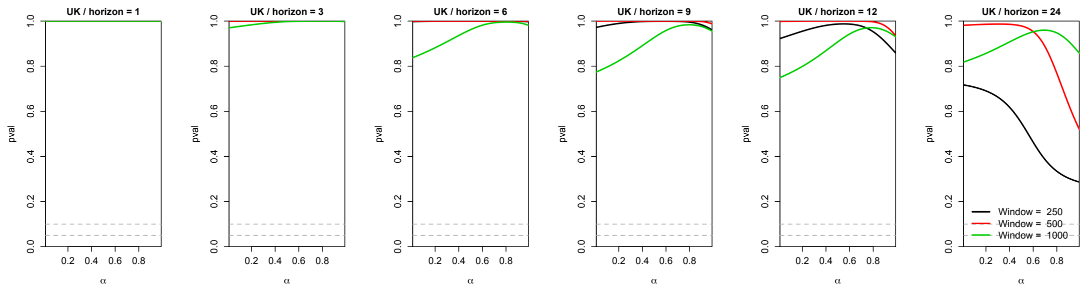
Panel E: Japan



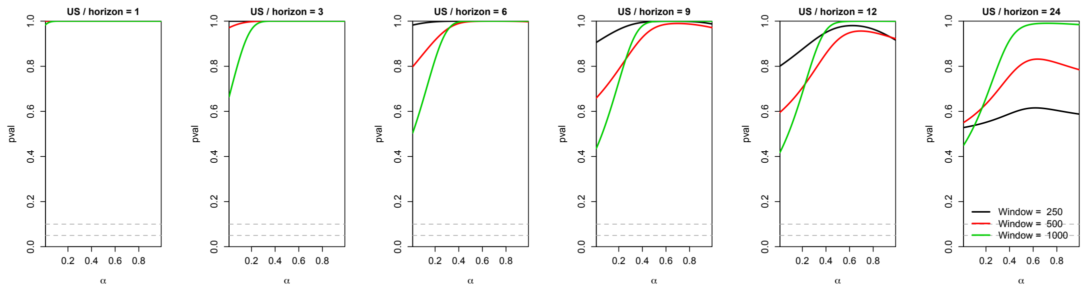
Panel F: Switzerland



Panel G: United Kingdom



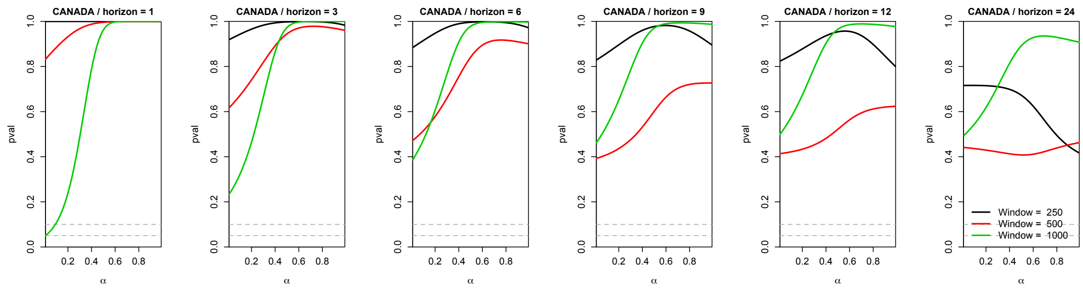
Panel H: United States



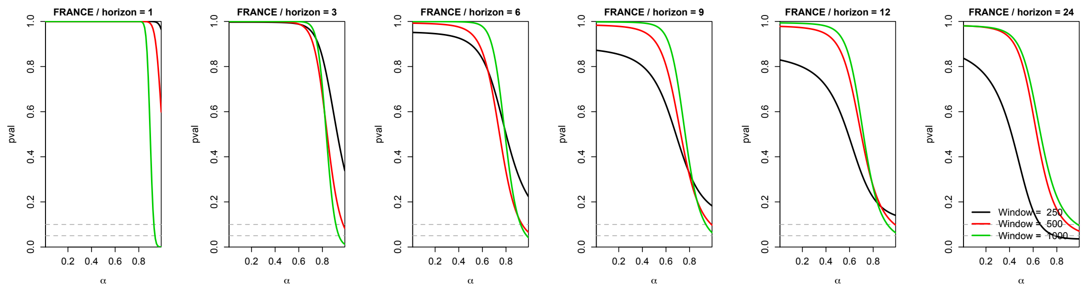
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure S3: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Silver Price Ratio, TR_5, Lin-Lin Loss, Sample Period 1/03/1973−04/08/2022)

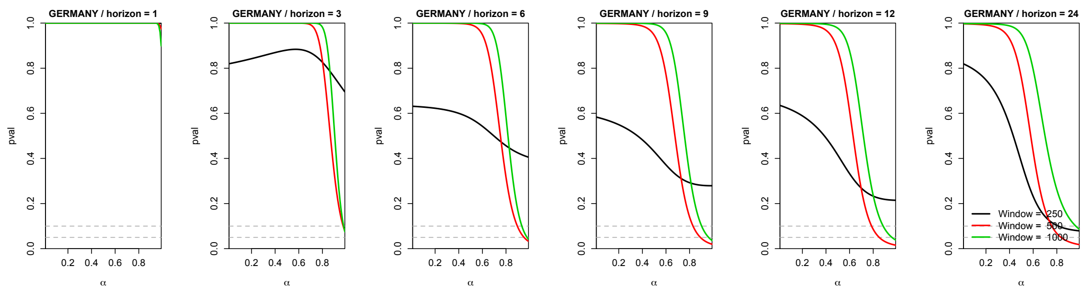
Panel A: Canada



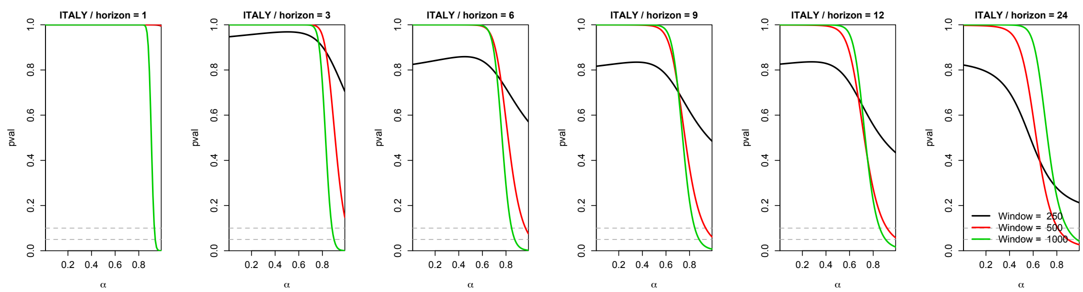
Panel B: France



Panel C: Germany



Panel D: Italy



To be continued.

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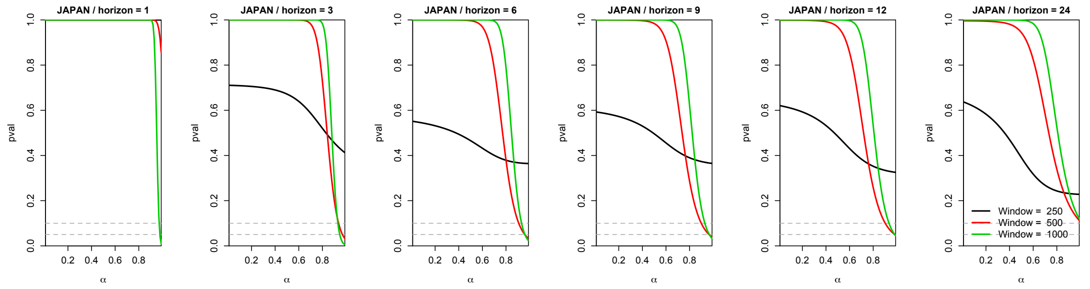
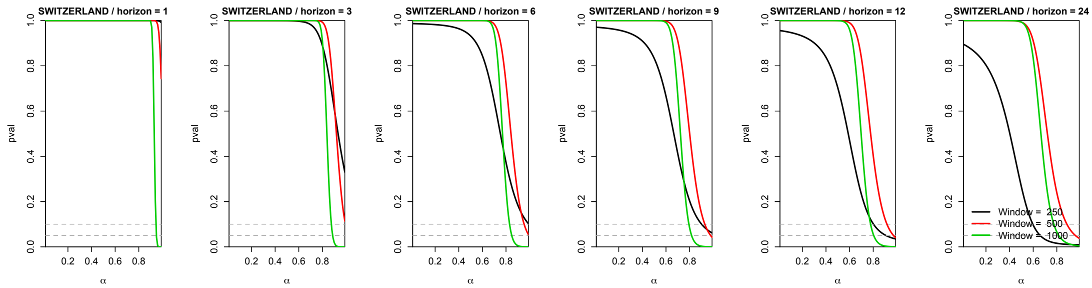
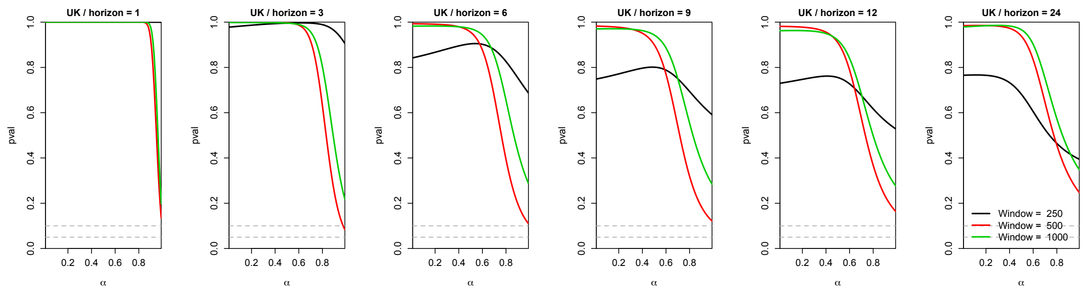
Panel E: Japan



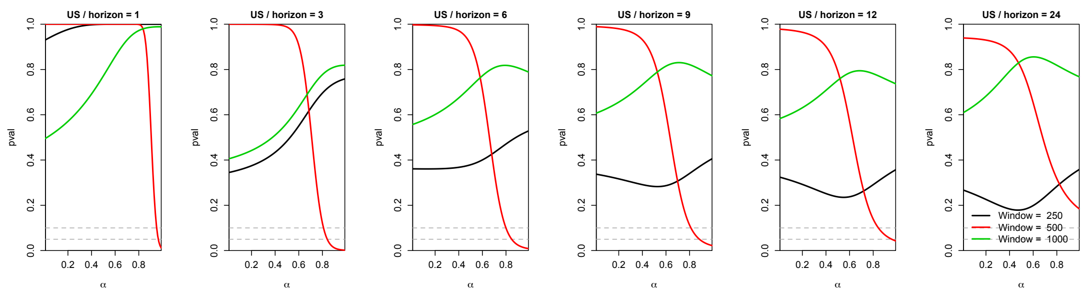
Panel F: Switzerland



Panel G: United Kingdom



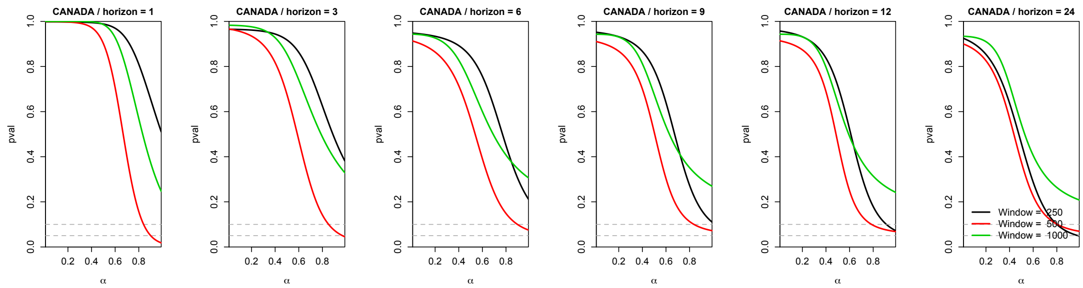
Panel H: United States



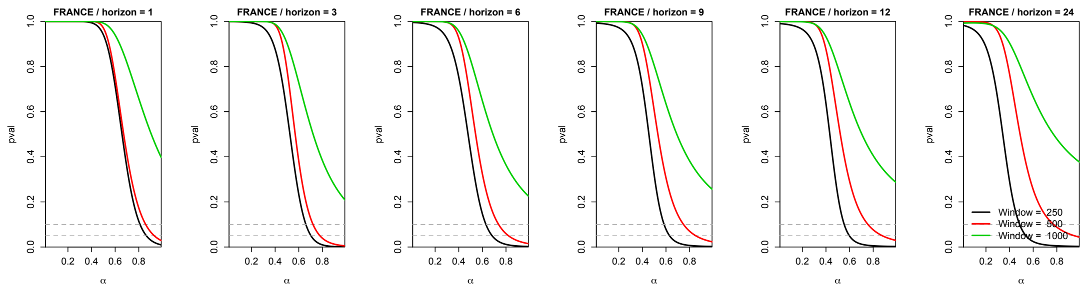
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure S4: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Silver Price Ratio, TR_1, Quad-Quad Loss, Sample Period 1/03/1973−04/08/2022)

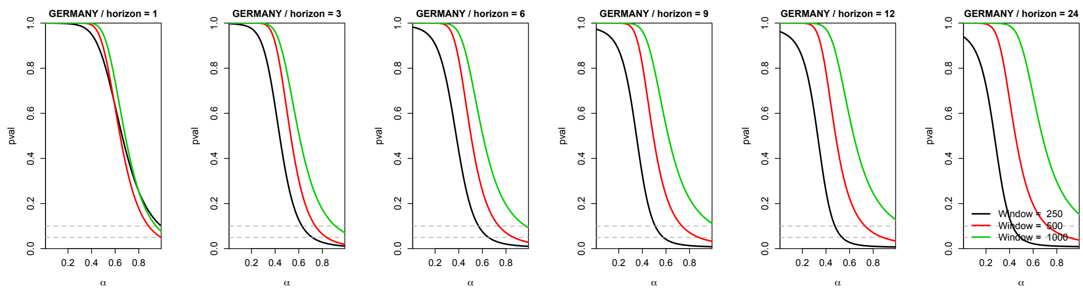
Panel A: Canada



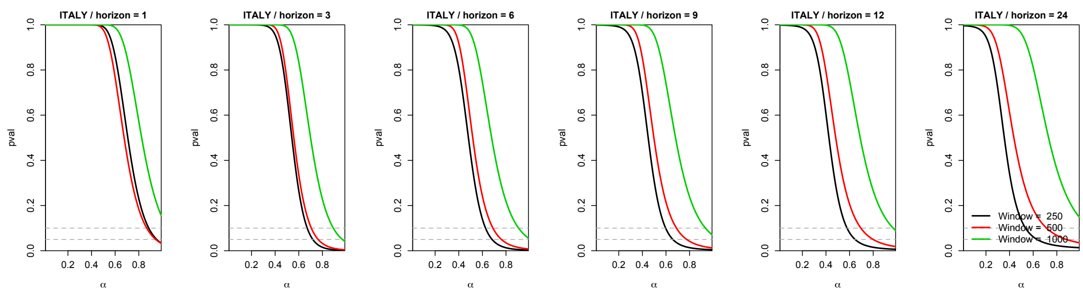
Panel B: France



Panel C: Germany



Panel D: Italy



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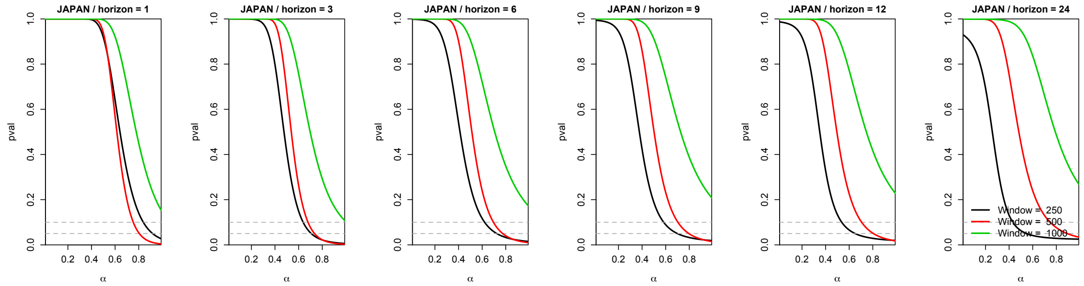
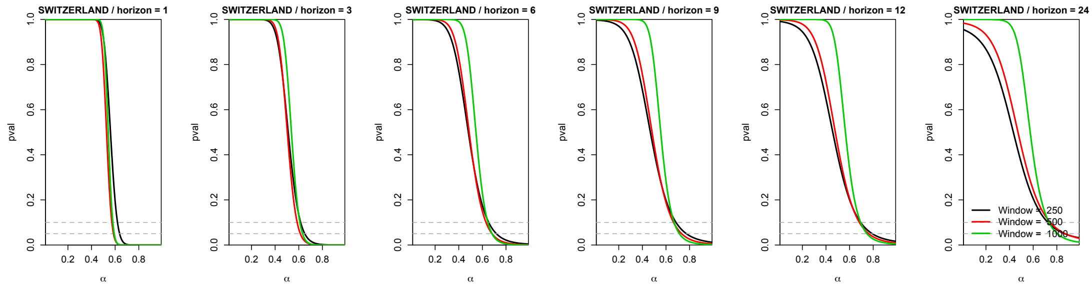
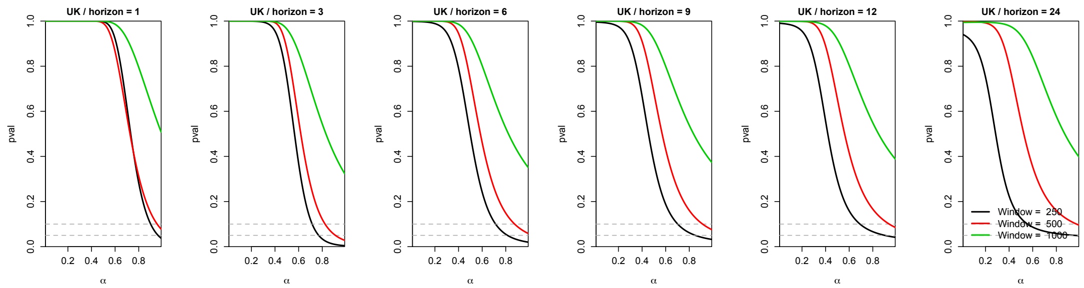
Panel E: Japan



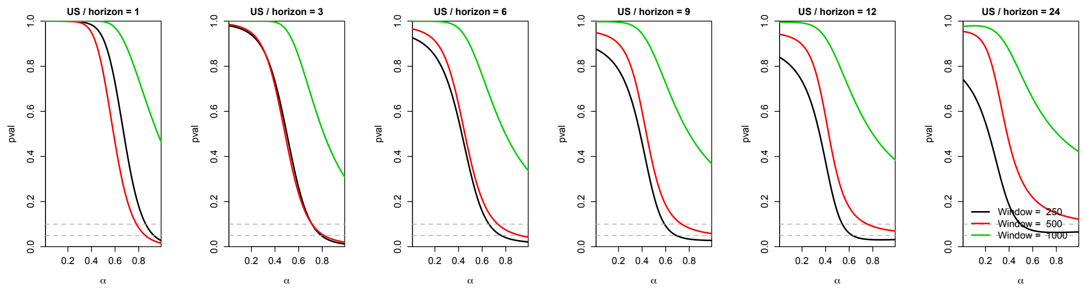
Panel F: Switzerland



Panel G: United Kingdom



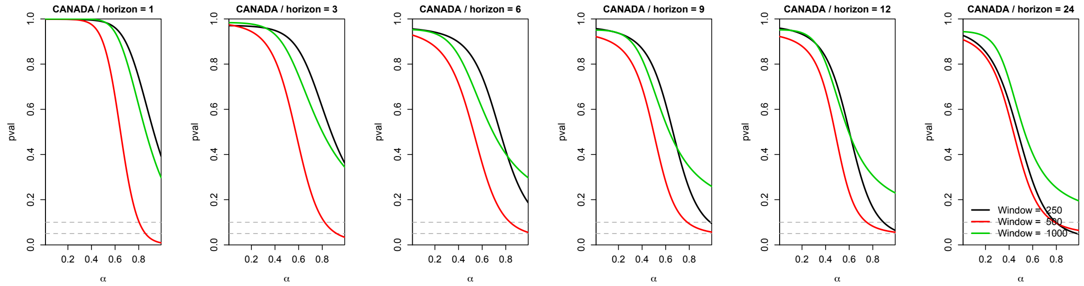
Panel H: United States



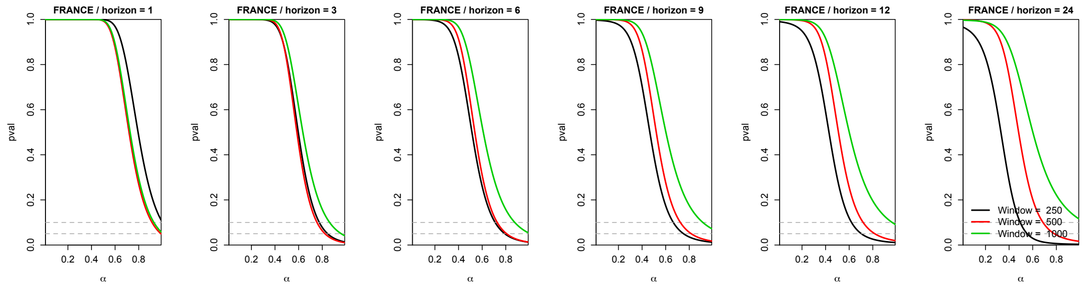
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure S5: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Silver Price Ratio, TR_5, Quad-Quad Loss, Sample Period 1/03/1973−04/08/2022)

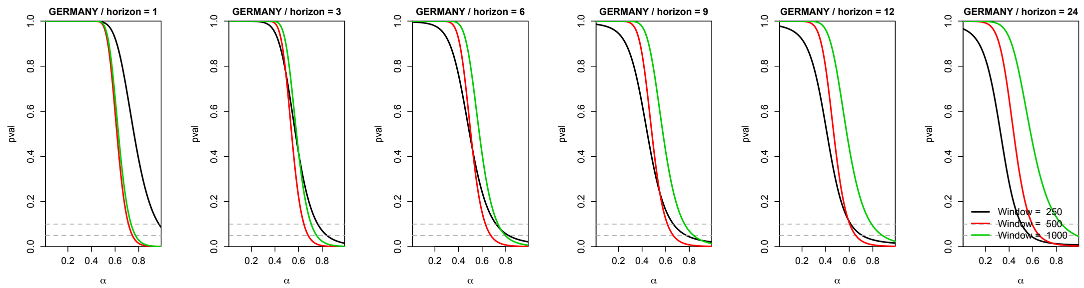
Panel A: Canada



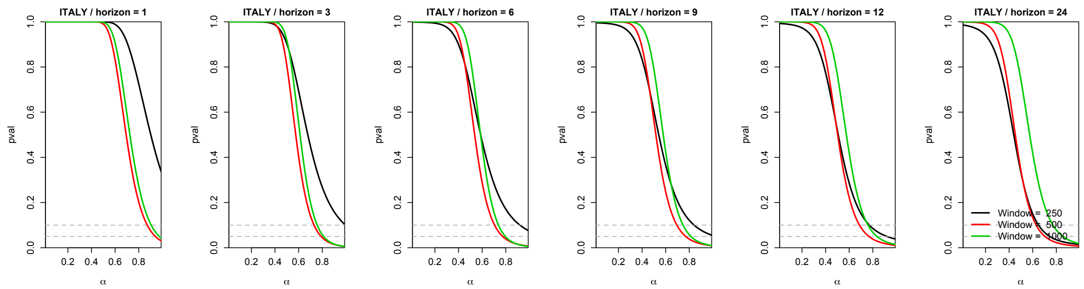
Panel B: France



Panel C: Germany



Panel D: Italy



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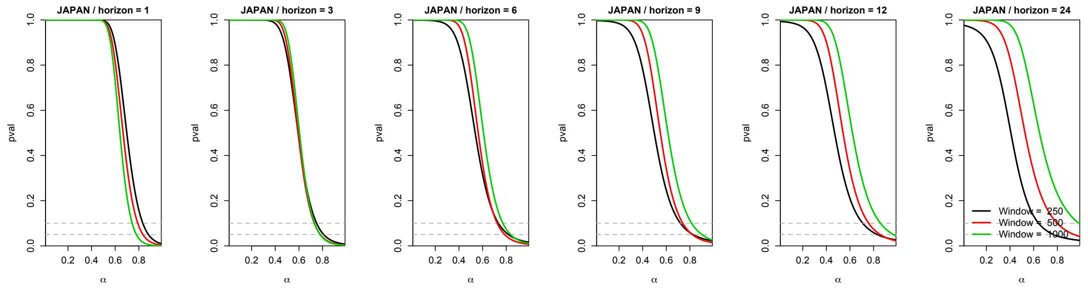
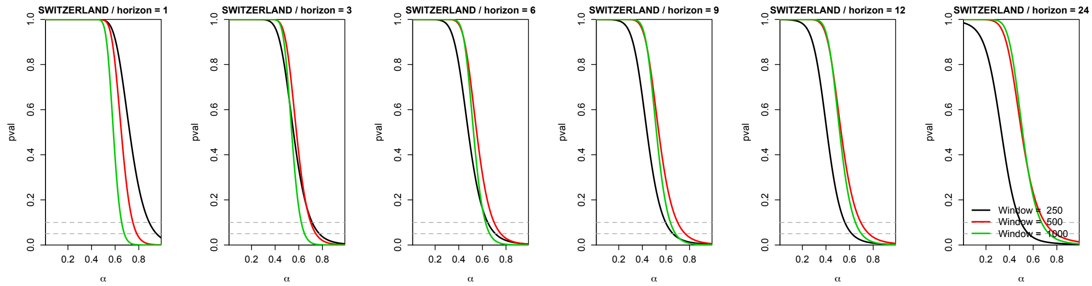
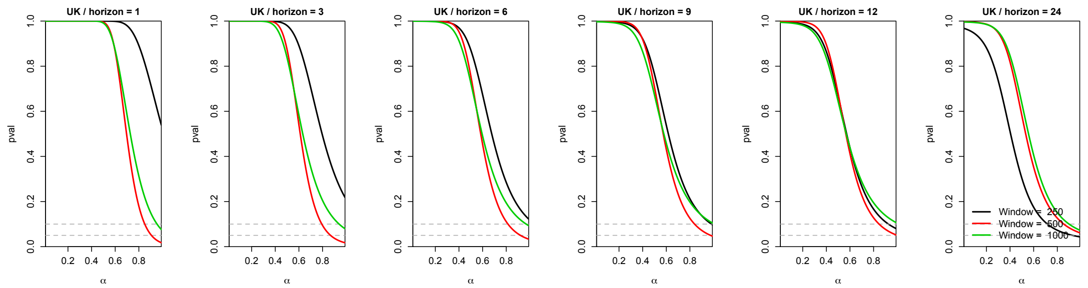
Panel E: Japan



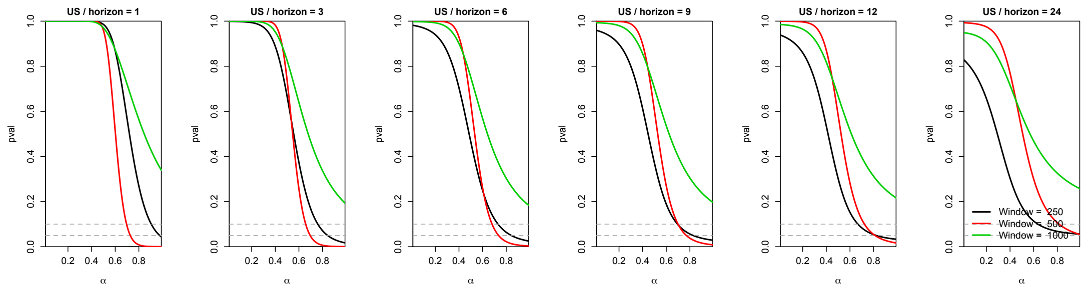
Panel F: Switzerland



Panel G: United Kingdom



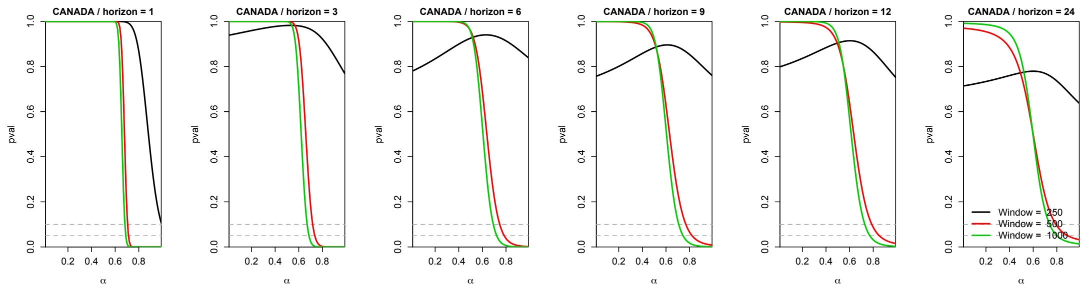
Panel H: United States



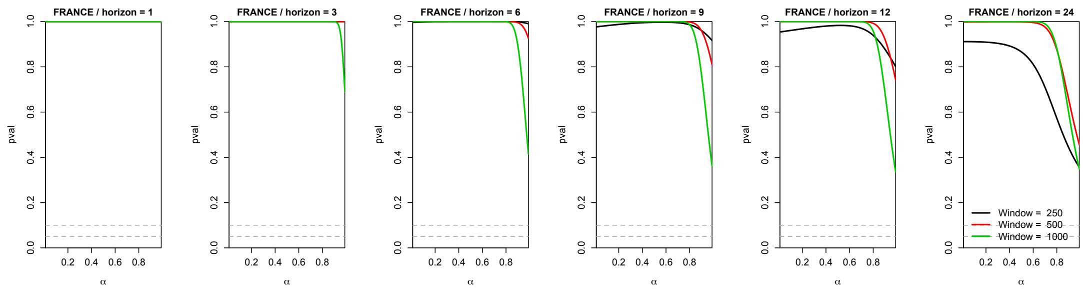
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-silver price ratio as an additional predictor

Figure S6: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Platinum Price Ratio, TR_1, Lin-Lin Loss, Sample Period 1/03/1973−04/08/2022)

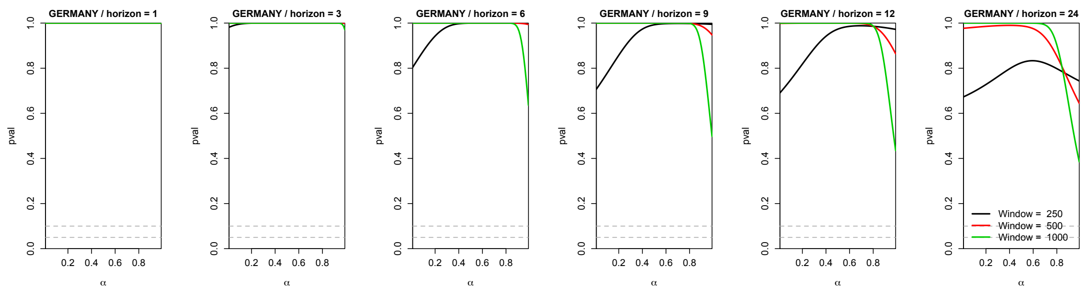
Panel A: Canada



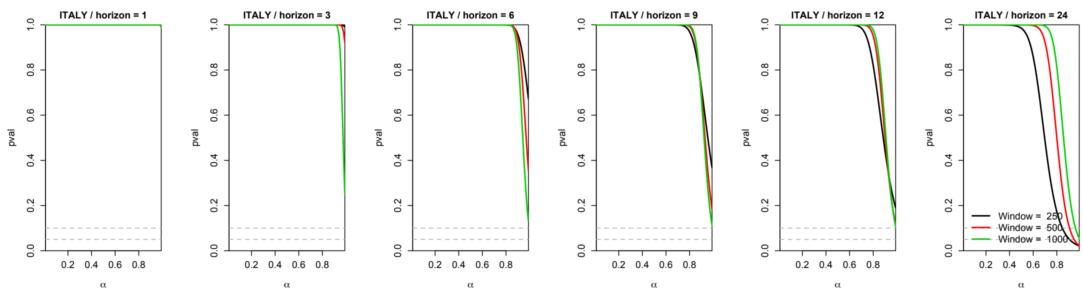
Panel B: France



Panel C: Germany



Panel D: Italy



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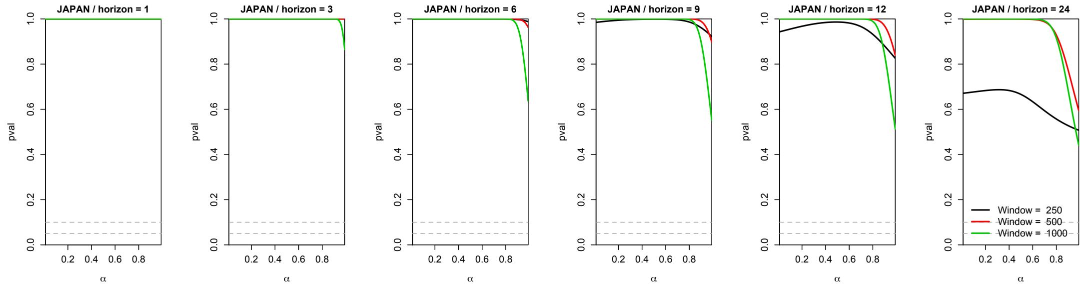
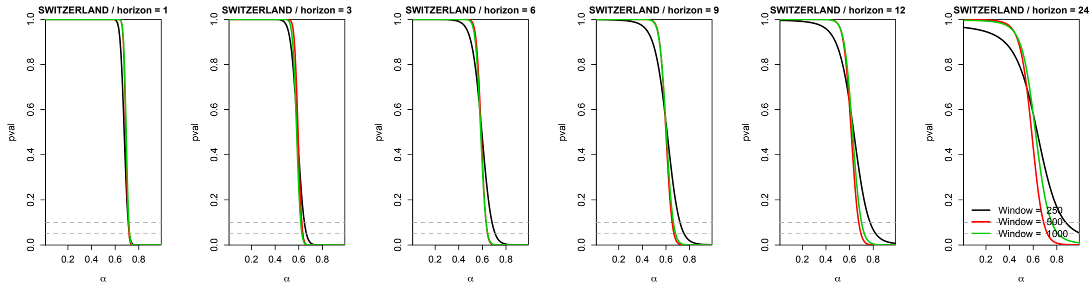
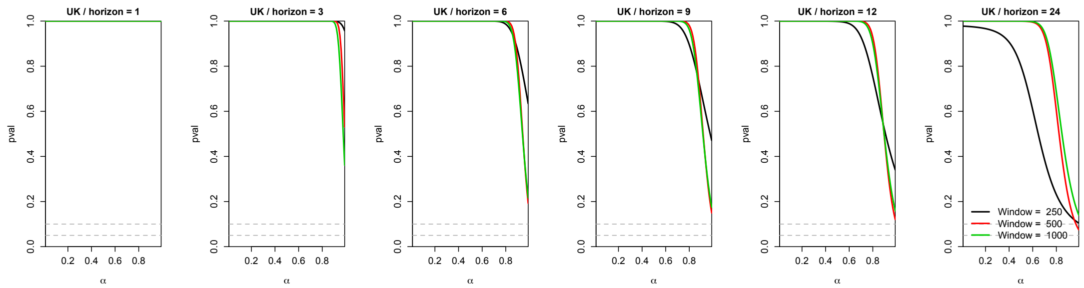
Panel E: Japan



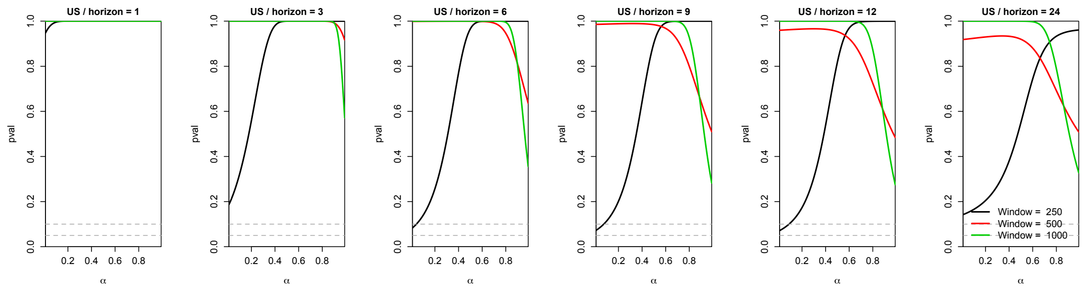
Panel F: Switzerland



Panel G: United Kingdom



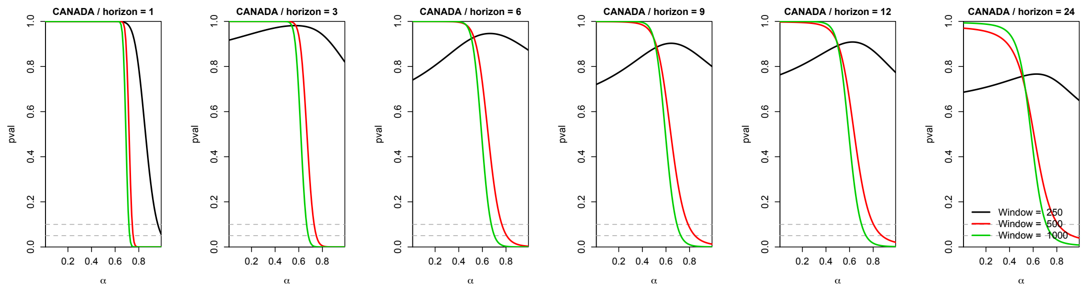
Panel H: United States



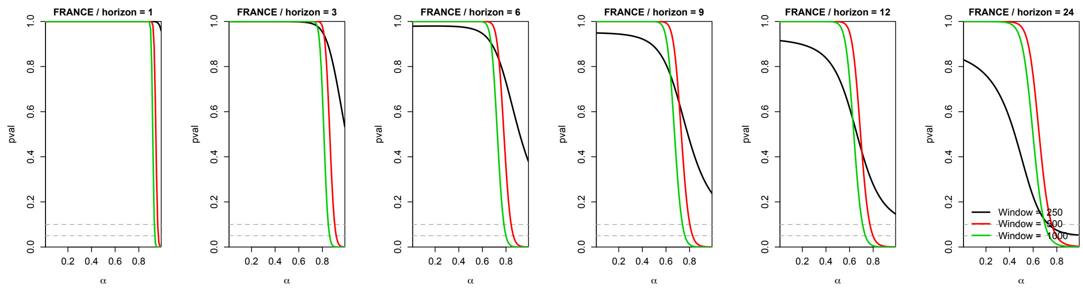
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-platinum price ratio as an additional predictor

Figure S7: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Platinum Price Ratio, TR_5, Lin-Lin Loss, Sample Period 1/03/1973–04/08/2022)

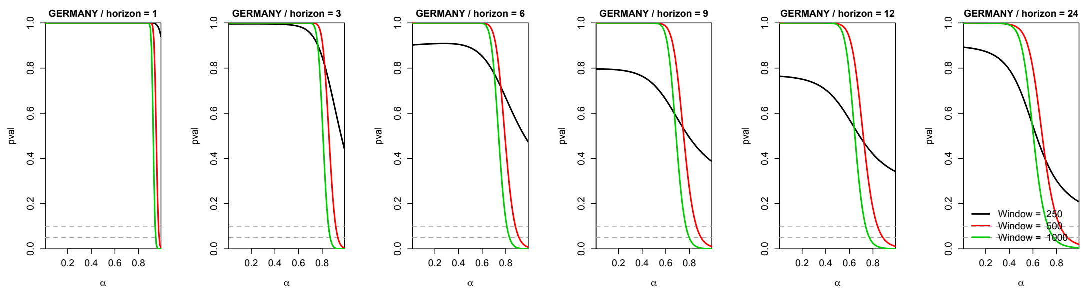
Panel A: Canada



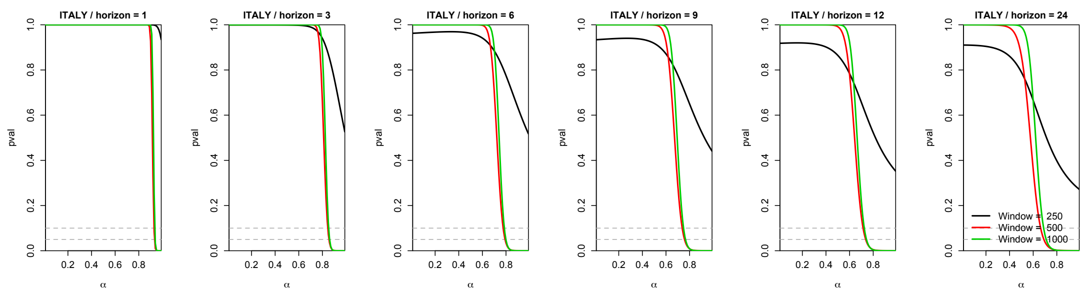
Panel B: France



Panel C: Germany



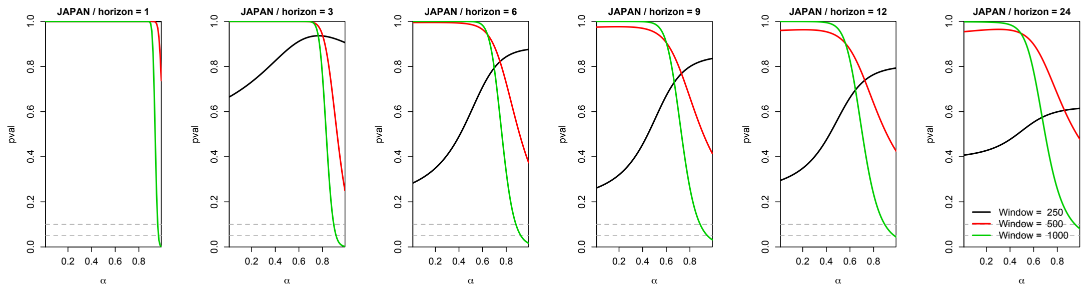
Panel D: Italy



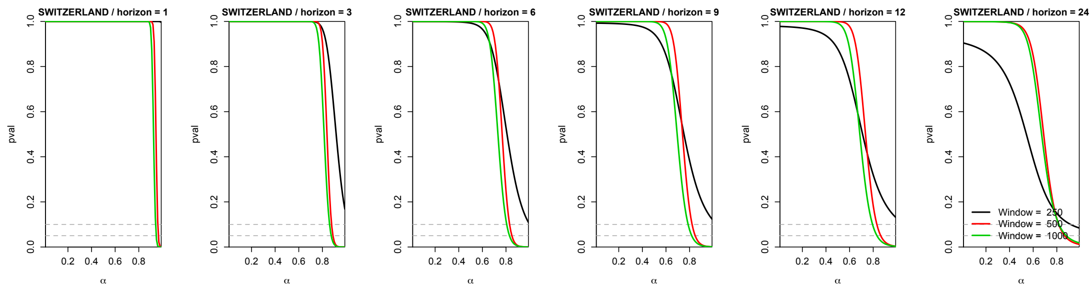
To be continued.

Continued.

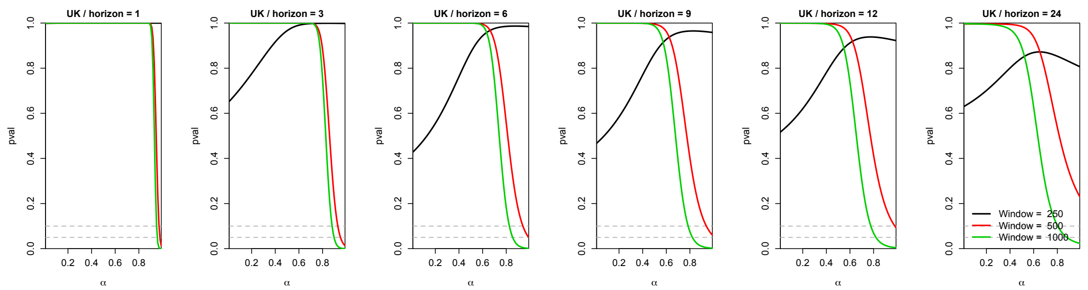
Panel E: Japan



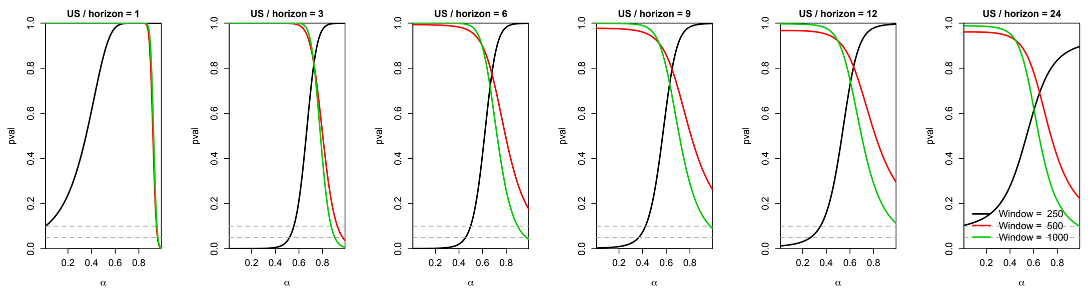
Panel F: Switzerland



Panel G: United Kingdom



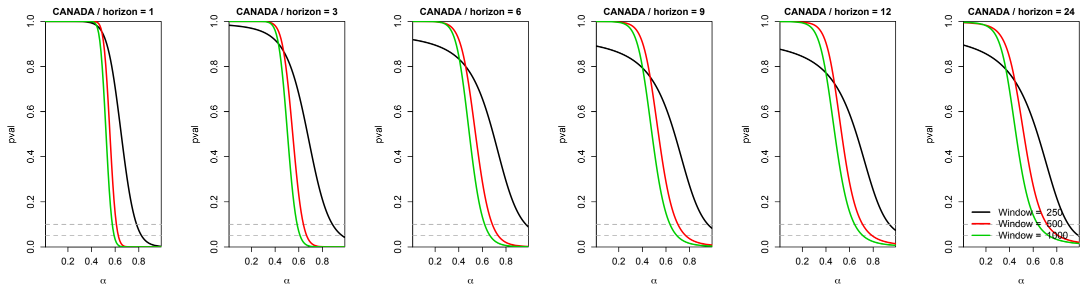
Panel H: United States



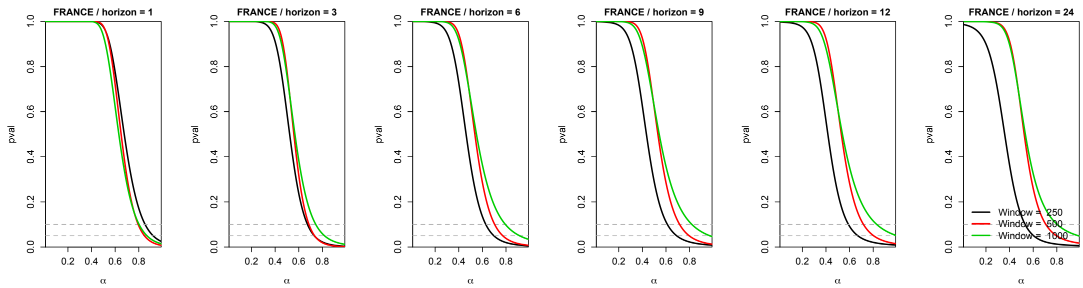
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-platinum price ratio as an additional predictor

Figure S8: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Platinum Price Ratio, TR_1, Quad-Quad Loss, Sample Period 1/03/1973−04/08/2022)

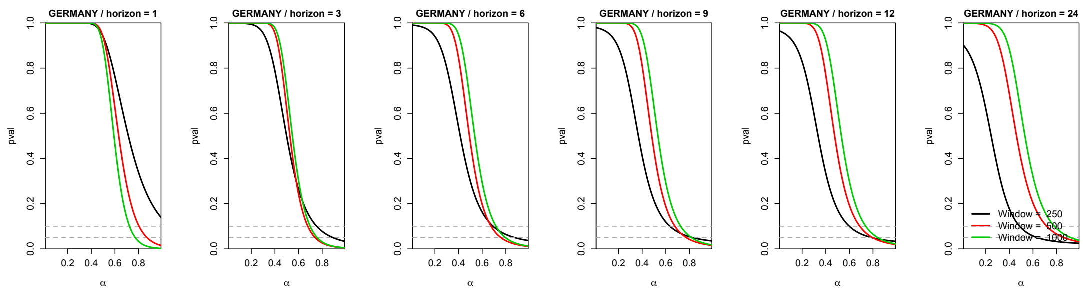
Panel A: Canada



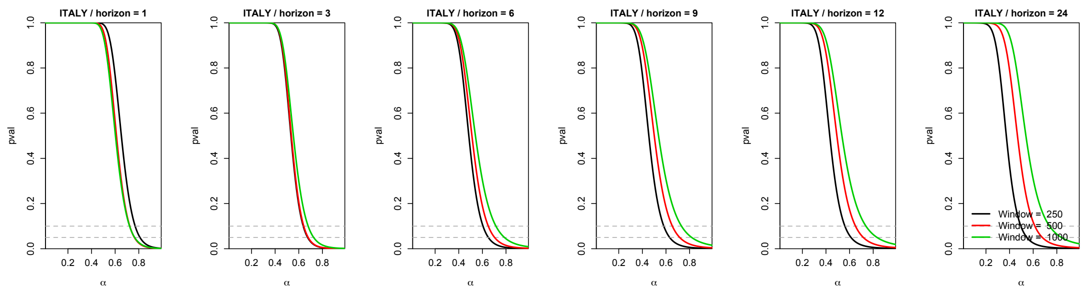
Panel B: France



Panel C: Germany



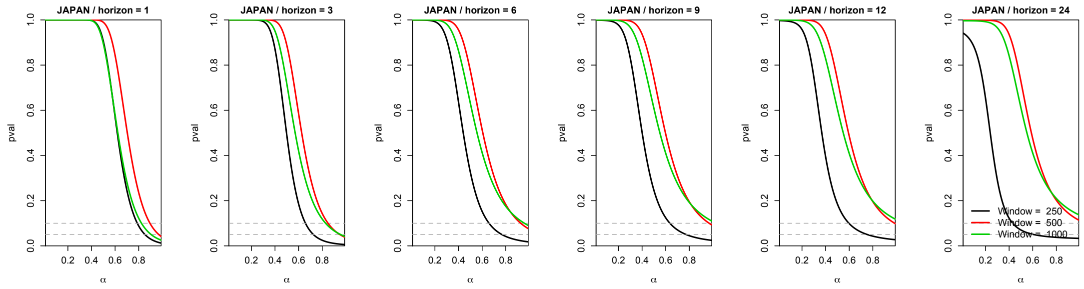
Panel D: Italy



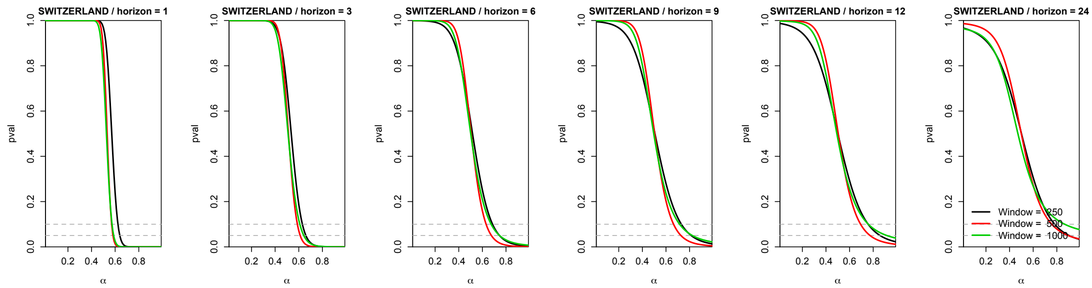
To be continued.

Continued.

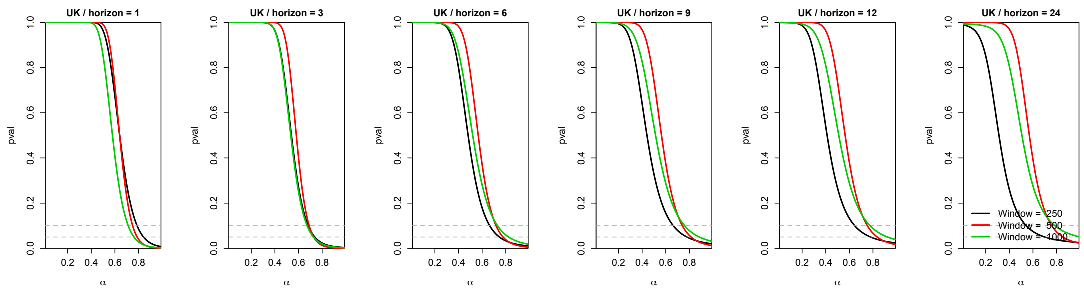
Panel E: Japan



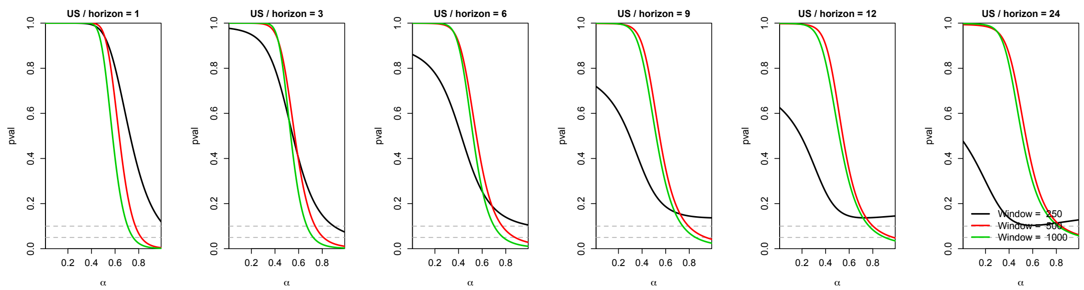
Panel F: Switzerland



Panel G: United Kingdom



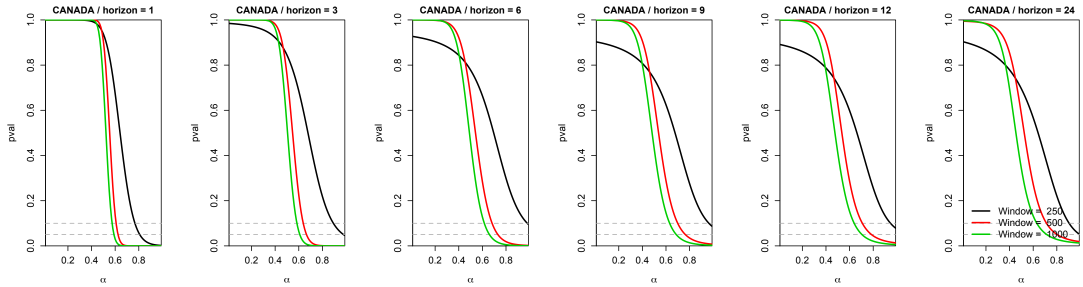
Panel H: United States



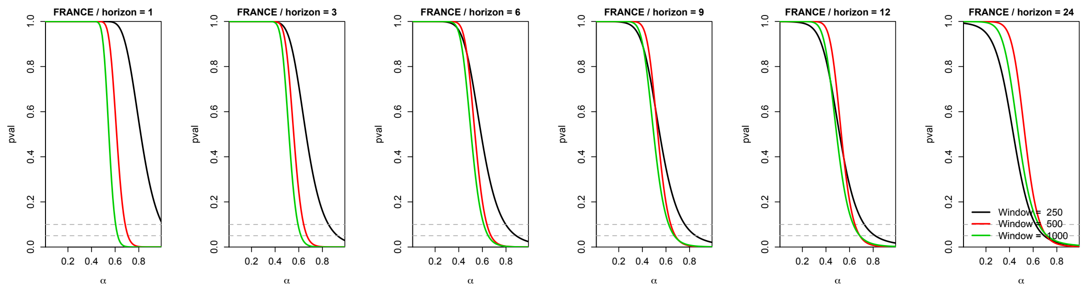
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-platinum price ratio as an additional predictor

Figure S9: Diebold-Mariano Test Under Asymmetric Loss (Daily Data, Gold-to-Platinum Price Ratio, TR_5, Quad-Quad Loss, Sample Period 1/03/1973−04/08/2022)

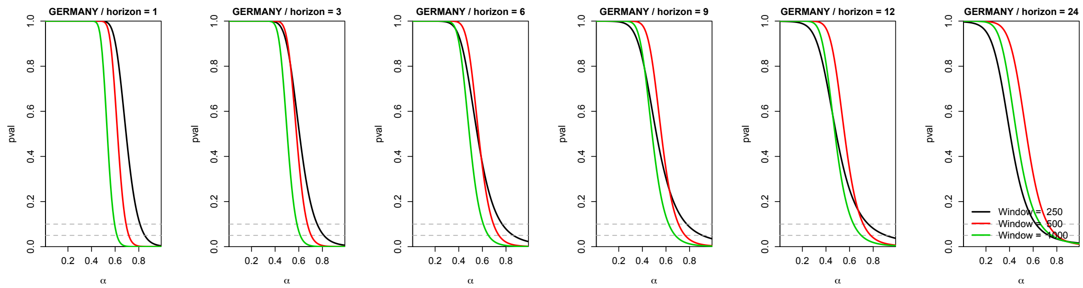
Panel A: Canada



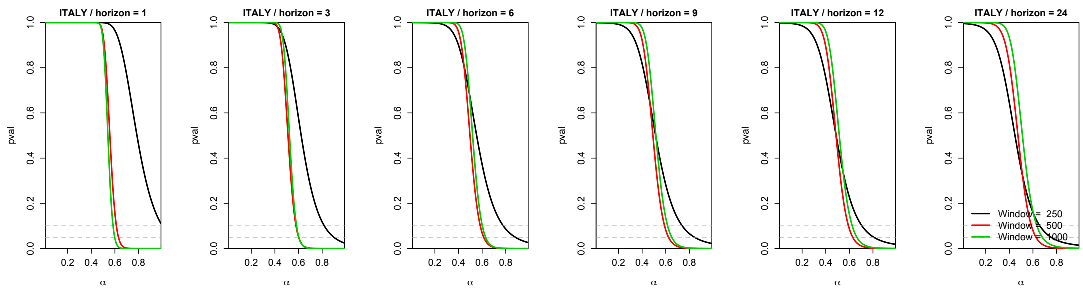
Panel B: France



Panel C: Germany



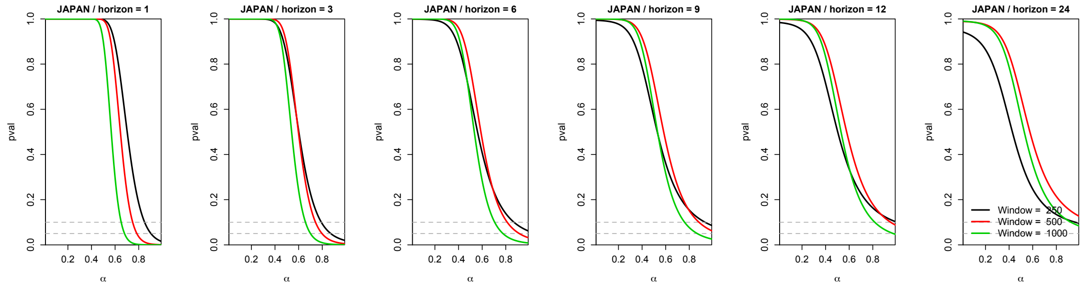
Panel D: Italy



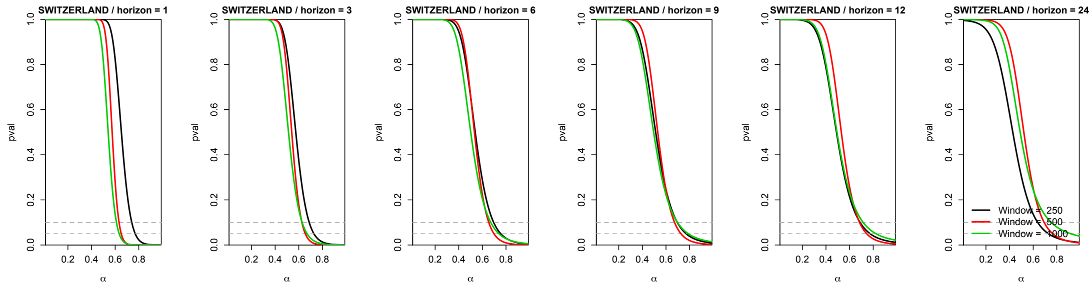
To be continued.

Continued.

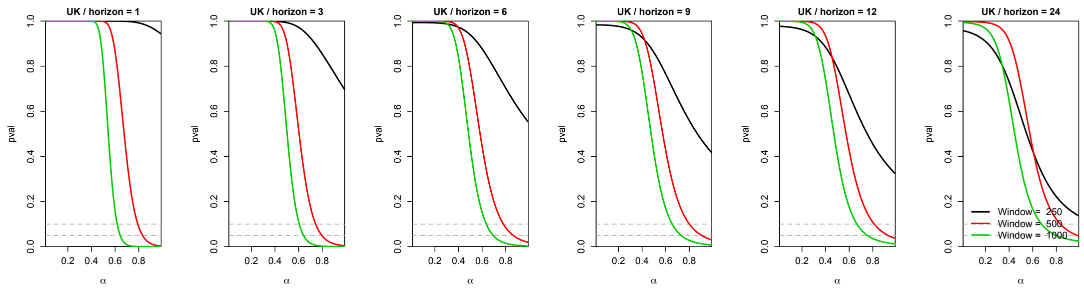
Panel E: Japan



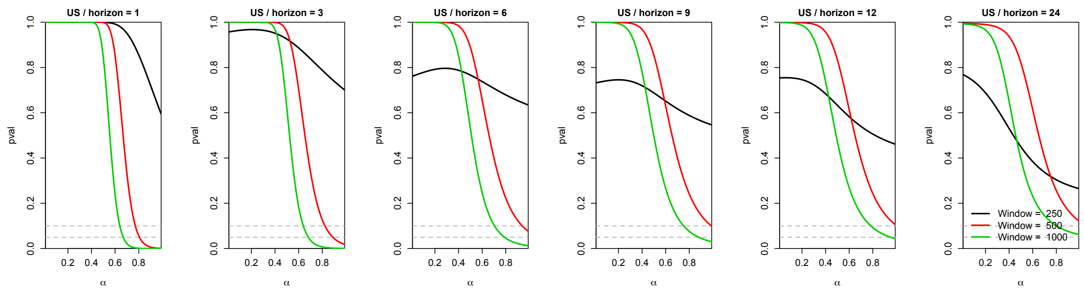
Panel F: Switzerland



Panel G: United Kingdom

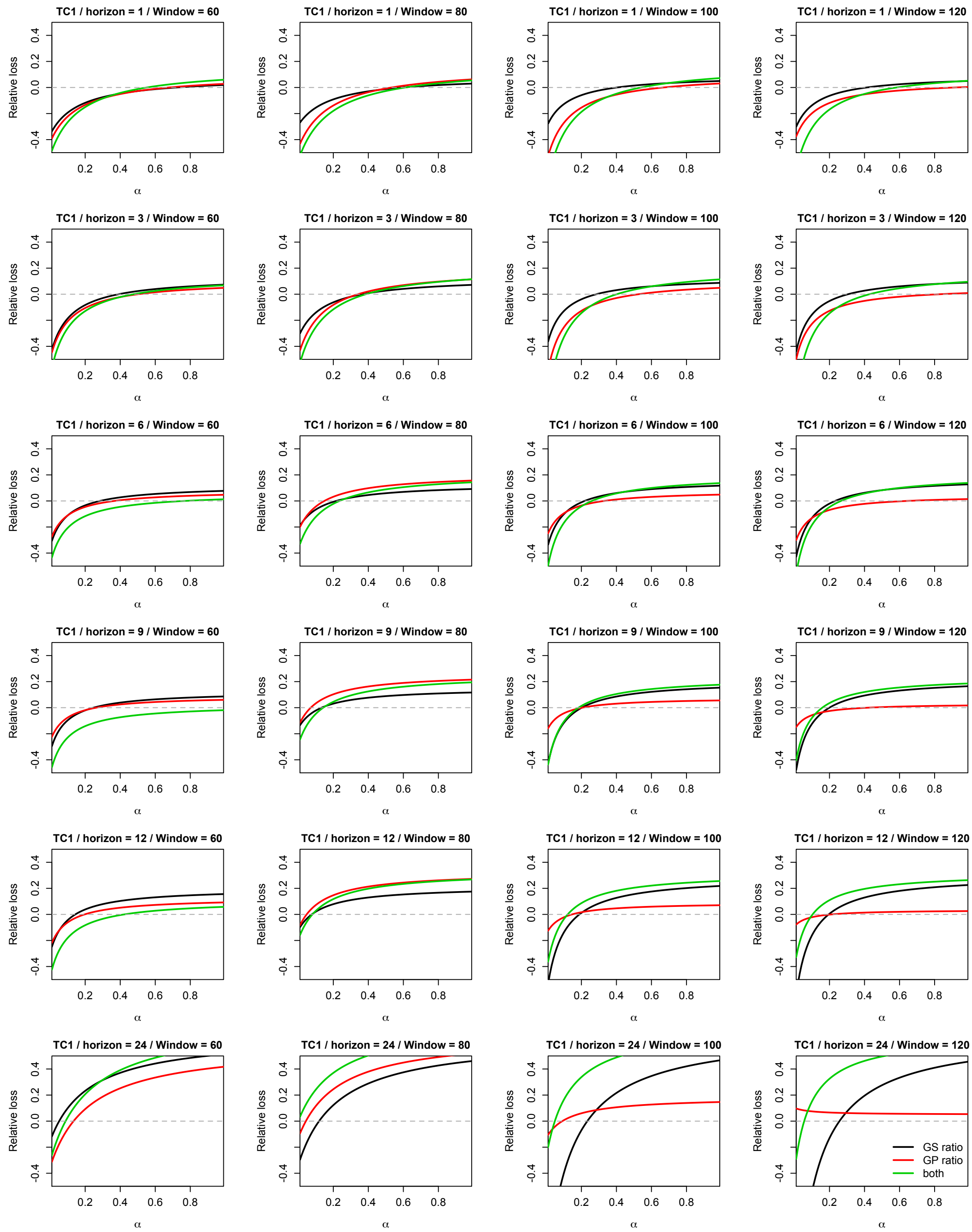


Panel H: United States



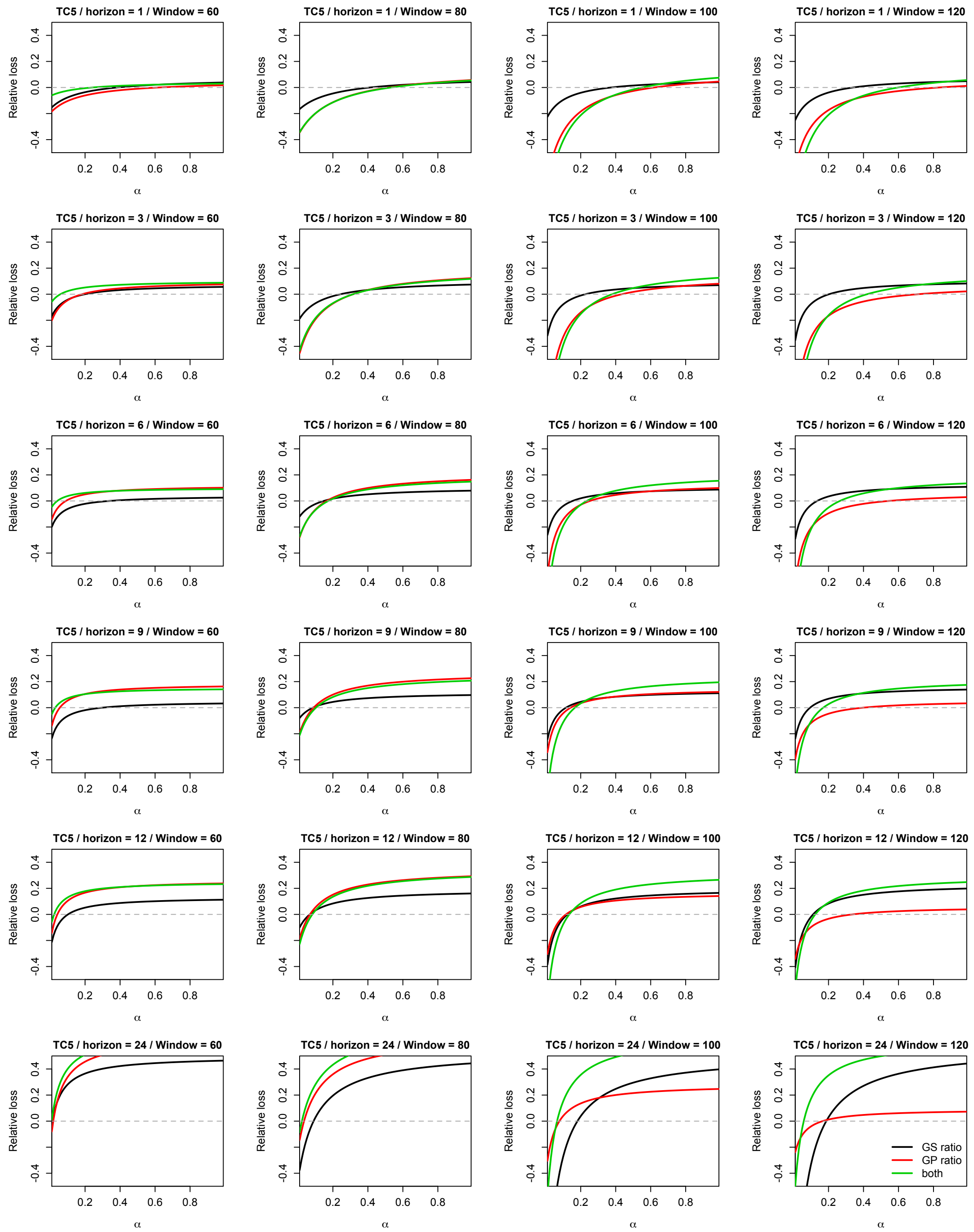
Note: This table depicts the results (p-values) of the (modified) Diebold-Mariano test computed under an asymmetric absolute-error loss function, where the asymmetry parameter is plotted on the horizontal axis. The alternative hypothesis is that the forecasts from the rival model are more accurate than the forecasts from the benchmark model (one-sided test). The autoregressive model is the benchmark model and the rival model includes the gold-to-platinum price ratio as an additional predictor

Figure S10: Out-of-Sample Relative-Loss Criterion for Total Connectedness (Quad-Quad Loss, TR_1, Sample Starts 1968M01)



Note: This figure shows the out-of-sample relative loss criterion as a function of the asymmetry parameter and a quad-quad loss function. The autoregressive model is the benchmark model and the rival model includes the predictors given in the legend of this figure.

Figure S11: Out-of-Sample Relative-Loss Criterion for Total Connectedness (Quad-Quad Loss, TR_5, Sample Starts 1968M01)



Note: This figure shows the out-of-sample relative loss criterion as a function of the asymmetry parameter and a quad-quad loss function. The autoregressive model is the benchmark model and the rival model includes the predictors given in the legend of this figure.

Table S8: Estimates and Relevant statistics for the country-specific CAViaR specification [Adaptive]

5% VaR	Canada	France	Germany	Italy	Japan	Switzerland	UK	US
β_1	0.687	-0.174	3.550	1.040	1.360	-0.002	1.060	1.040
Standard errors	0.258	0.098	0.000	0.158	0.043	0.120	0.157	0.158
P values	0.004	0.038	0.000	0.000	0.000	0.493	0.000	0.000
RQ	622.000	604.000	905.000	736.000	711.000	582.000	560.000	736.000
Hits in-sample (%)	4.370	4.470	4.670	5.060	5.160	4.270	5.060	5.060
Hits out-of-sample (%)	4.000	6.000	4.800	4.800	4.400	4.400	4.400	4.800
DQ in-sample (P-values)	0.000	0.011	0.003	0.029	0.032	0.000	0.001	0.029
DQ out-of-sample (P-values)	0.010	0.002	0.703	0.226	0.313	0.004	0.225	0.226

Note: See notes to Table A1.