UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Teachers' questions matter: exploring the attributes of mathematics teachers' questions within the Lesson Study context

by

Lesego Brenda Mokotedi

Submitted in fulfilment of the requirements for the degree

PHILOSOPHIAE DOCTOR

in the Faculty of Education

at the

UNIVERSITY OF PRETORIA

Supervisor: Dr. RD Sekao

Co-supervisor: Dr. JJ Botha

APRIL 2023

Declaration

I declare that the thesis, which I hereby submit for the degree Philosophiae Doctor (PhD) at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

Mokotedi L.B

....

29 November 2022

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Ethical Clearance Certificate

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE	CLEARANCE NUMBER: **EDU065/19**
DEGREE AND PROJECT	PhD Teachers' questions matter: exploring the attributes of mathematics teachers' questions within the Lesson Study context
INVESTIGATOR	Ms Lesego Brenda Mokotedi
DEPARTMENT	Science Mathematics and Technology Education
APPROVAL TO COMMENCE STUDY	16 January 2020
DATE OF CLEARANCE CERTIFICATE	09 September 2022
CHAIRPERSON OF ETHICS COMMITTEE:	Prof Funke Omidire

CC	Mr Simon Jiane Dr R.D. Sekao Dr J.J. Botha

This Ethics Clearance Certificate should be read in conjunction with the Integrated Declaration Form (D08) which specifies details regarding:

- Compliance with approved research protocol,
- No significant changes,
- Informed consent/assent,
- Adverse experience or undue risk,
- Registered title, and
- Data storage requirements.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Ethics statement

The author, whose name appears on the title page of this dissertation, has obtained, for research described in this work, the applicable research ethics approval. The author declares that she has observed the ethical standards required in terms of the University of Pretoria's Code of Ethics for Researchers and the Policy guidelines for responsible research.

Signature:

Students name: Lesego Brenda MOKOTEDI

Month Year: November 2022

Dedication

I dedicate this research to my parents Ruth and Gabaake Moremedi who were my first teachers. They have taught me patience, instilled in me their wisdom and love. I thank them for their unwavering support.

Acknowledgements

Several people contributed to the success of this study in various ways. Without their help, interest, cooperation and encouragement, this study could not have been completed. Therefore, I extend my sincere thanks to all of them.

- My Heavenly Father, who provided me the strength and wisdom to complete this study;
- I forward my special thanks to Dr. R.D. Sekao (Supervisor) and Dr. J.J. Botha (Co-supervisor) for having confidence in me and accepting the responsibility of being my supervisors. I feel grateful and privileged for their contributions, their words of encouragement, invaluable advice, guidance and motivation during difficult times. They literally took my hand and walked with me throughout this journey.
- I gratefully acknowledge research participants who took time out of their hectic schedules to participate in this study. This research could have not been possible without their willingness to take part.
- Special tribute goes to my husband Reginald Mokotedi and our three beautiful daughtres Resegofetse, Resolofetse and Ogalaletseng.
- Special tribute to my family, my siblings for being so understanding when I could not attend many family activities due to my studies.
- I thank the North West Department of Education for providing me with the financial support to persue this study.
- Finally, a special thanks to Mr Solly Makgamathe who always called and checked on my progress and for encouraging me to hang in there despite the difficulties.
- My language editor, Angela Bryan for the excellent editing and making this piece of work a resounding success.

Abstract

Questions and questioning remain central to classroom conversations. Classroom interactions are mostly dominated by teachers' questions and responses offered by learners. If used properly, questions can become tools that inspire deep intellectual thought in mathematics. Quality questions cultivate the habit of reflective inquiry and tend to transform learners into active participants during teaching and learning. Important as they are, questions and questioning appear to be neglected narratives within Lesson Study contexts. Against this background, this study was conducted to explore the attributes of questions developed and used by teachers as they engage in Lesson Study activities. In this study, I used Lesson Study as a context to learn how mathematics teachers incorporate questions during the three of the five stages of LS, i.e., collaborative lesson planning, lesson presentation and observation, and post-lesson reflection. The conceptual framework which guided the study was an amalgam of LS (the context), Variation Theory (theoretical lens) and Emanuelsson's categories of classroom interactions. Extensive review of literature has shown that there is very little research conducted to explore how LS communities incorporate oral questions (questions they intend to use to facilitate learning) in their plans and how such questions filter into the lesson presentation and observation, and how they are eventually reflected upon during the post-lesson reflection stage. This study is an interpretive qualitative case study which involved five participants from four different schools and a single class of Grade 9 learners. Data were generated through observation, document analysis and informal conversational interviews. The findings show that although teachers were able to give consideration to questions they intend to use to facilitate learning, dominant questions were those that mainly stimulated interactions in the topical zone. Evidence reported in this study also revealed that there is a policy implementation gap regarding the guidance teachers need when planning questions for informal assessment. This study has also contributed to extending the body of knowledge on how an LS team plan, implement and reflect on the questions.

Key words: Lesson Study, Variation Theory, question, questioning, mathematics

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Language editor

Angela Bryan & Associates

6 Martin Crescent
Westville

Date: 13 November 2022

To whom it may concern

This is to certify that the Doctoral Thesis: Teachers' Questions Matter: Exploring the Attributes of Mathematics Teachers' Questions Within the Lesson Study Context written by Lesego Brenda Mokotedi has been edited by me for language.

Please contact me should you require any further information.

Kind Regards

Angela Bryan

angelakirbybryan@gmail.com
0832983312

List of abbreviations

CAPS	Curriculum and Assessment Policy Statement
CAQDAS	Computer-Assisted Qualitative Data Analysis Software
DBE	Department of Basic Education
GET	General Education and Training
LS	Lesson Study
NWDE	North West Department of Education
PCK	Pedagogical Content Knowledge
PLC	Proffesional Learning Community
SASA	South Afican School Act
SMT	School Management Team

TABLE OF CONTENTS

List of Figures

List of Tables

CHAPTER ONE: GENERAL ORIENTATION

1.1. INTRODUCTION

Why do people ask questions? A common response to this question would be: because they do not know the answers. However, in the classroom setting, the teacher already knows the answers to questions they ask their learners. Aziza (2018) has argued that questions have a significant role in the teaching and learning activities used in a mathematics classroom. Tienken et al. (2009) stated that teachers ask questions in order to stimulate critical thinking and to help learners develop deep conceptual understanding. Among many other aspects, questions stimulate learners' curiosity, promotes their thinking, and eventually affects their conceptual understanding. Although answers to questions posed are important in the mathematics teaching and learning, high premium needs to be placed on well-constructed and purposeful questions to drive the teaching and learning process.

Recent research (Aziza, 2018; Dong et al., 2015; Larson & Lovelace, 2013; Shahrill & Clarke, 2014) on classroom questioning continues to stimulate researchers' interests. While researchers continue to demonstrate interest in classroom questioning, there appears to be silence in the literature regarding efforts to explore teachers' questioning practices in LS contexts. LS initiatives throughout the world have one common purpose i.e. to engage in the critical examination of practice with the intention of improving practice (Amirullah, 2018; Lewis, Perry & Hurd, 2009; Lewis, Perry & Hurd, 2004).

There is research evidence that LS presents opportunities for teachers to improve their professional development (Lewis, 2016). But, what do we know of questioning practices of mathematics teachers who participate in LS? Can discussions and research-generated insights linked to classroom questioning be extended to LS contexts? This study takes up these questions by investigating the attributes of questions planned and posed by teachers during mathematics lessons in the context of LS. Of particular interest to this research is how questions are used to make it possible for learners to appropriate the object of teaching (Lo, 2012).

1.2. PROBLEM STATEMENT

The South African mathematics curriculum, viz. Curriculum and Assessment Policy Statement, [CAPS] (DBE, 2011), provides a well detailed guide for the development of questions for formal assessment tasks. The mathematics CAPS is very explicit regarding how questions should be conceptualised for formal assessment i.e., tests and examinations. This is evidenced by a well detailed cognitive level framework intentionally designed to provide guidance to teachers for the development of quality formal assessment. It is also clearly articulated within the policy that, "The questions should be carefully spread to cater for different cognitive levels of learners" (DBE, 2011, p.155). While the significance of formal assessment tasks is heightened in the curriculum policy, little is mentioned about how teachers should prepare informal assessment, i.e. assessment that is used to facilitate the teaching and learning process, therefore not used for promotion purposes as is the case with formal assessment. Predominantly, informal assessment includes oral and written questions used to inform teaching and learning, hence formative assessment. Given the significance of formative assessment during classroom interactions, and how it appeared to have been sidelined by the CAPS, this study became more appropriate.

There is also a dearth of literature focussing on examining the attributes of questions and questioning practices of mathematics teachers in LS contexts (Ong et al., 2010) globally, including in South Africa. The available literature is still inadequate to properly inform policy and guide research efforts particularly in LS. Aizikovitsh-Udi (2011) has long expressed concern about the need to consider important questions regarding how teachers implement question-asking. In this study, I sought to learn how instrumental the LS can be in co-creating knowledge about qestions and questioning with specific focus on questions planned during the collaborative lesson planning stage, how such questions permeate lesson presentation and observation stage, and how they are eventually reflected upon.

1.3. PURPOSE AND RATIONALE OF THE STUDY

The issue that led to this study was that mathematics teachers in the district where I work as a curriculum specialist for mathematics, were expected to implement LS as a form of professional development. As a curriculum specialist who had the responsibility to provide support to those teachers, it occurred to me that formulation of quality questions that needed to be used to facilitate learning during a live research lesson, formed part of a plethora of issues that surround teacher quality. Successful implementation of LS requires that teachers be conversant with content knowledge and knowledge for teaching. These key aspects cannot be detached from questions as important pedagogic tools. It was this factor that motivated me to pursue this study and make a positive contribution towards the development of mathematics teachers in the Senior Phase.

The primary purpose of this study, therefore, was to explore the attributes of questions generated within the LS context, thereby generating knowledge on how mathematics teachers grappled with the process of planning questions they intended to use to facilitate learning during a live research lesson. The purpose was achieved by exploring four key areas: (a) the attributes of questions emanating from the collaborative lesson planning, (b) the use of these questions (i.e. questioning) during teaching and learning, (c) learners' responses/experiences to the questions posed by teachers, and (d) the emerging critical features of questions from post-lesson reflection. The questions I explored involve oral and written formative assessment questions used to facilitate the mathematics lesson(s).

Research on classroom questioning in the LS context has never been conducted in South Africa. The scarcity of literature on the attributes of mathematics teachers' questions in the context of LS was a driving purpose for this study. Research on questioning (Aziza, 2017; Nappi, 2017; Ong et al., 2010; Watson, 2018) has paid little attention to how mathematics teachers who implement LS as a professional development model use questions and questioning to strengthen their practice. The significance of questioning in LS contexts has been flagged in the literature e.g. (Shahrill, 2013). In the LS, when teachers plan a research lesson they predict learners' responses and thinking (Fuji, 2019). Anticipating learners thinking cannot be done without thinking about the questions that will be asked during the lessons.

1.4. RESEARCH QUESTIONS UNDER INVESTIGATION

The purpose of this study was to explore the attributes of questions (oral and written) planned and posed by teachers during mathematics lessons in the context of LS. The study also sought to explore how the subject of questions and questioning find expression during the post-lesson reflection stage of the LS. The study was guided by the following primary research question: What are the attributes of mathematics teachers' questions in the context of LS? The secondary research questions, their purpose(s), as well as the data collection instrument used to collect data to respond to them, are presented in Table 1.1

Table 1.1: Linking the research questions with purpose and data collection instruments

	Question	Purpose	Data collection instrument
a)	What attributes characterise questions planned during collaborative lesson planning?	• To establish whether questions are planned for mathematics lessons • To explore the underlying attributes of questions. • To establish whether questions planned are purposefully aligned to the intended learning	• Observation protocol • Document analysis • Unstructured interviews
b)	How do questions planned during the collaboration planning stage permeate lesson presentation?	• To establish how the planned questions were enacted during the lesson presentation and observation stage • To establish the purposefulness of questions posed during the lesson • To explore the underlying attributes of questions posed by teachers during the lesson.	• Observation protocol • Unstructured interviews
c)	How do learners experience teachers' questions during teaching?	To explore the ideas and thoughts that learners construct when they respond to teachers' questions	Observation protocol
d)	What critical features on questioning emerge during post-lesson reflection?	• To explore the nature of contributions made in relation to questions • To establish whether, as they reflect on the lesson taught, the LS team considers the need to raise issues about questions posed during the lesson presentation.	• Observation protocol • Informal conversational interviews

1.5. CLARIFICATION OF CONCEPTS

In this section I have provided a list of concepts/terms that are susceptible to confusion due to, for instance, being homonymous or having different contextual meanings. The idea was to convey the meanings of these terms/concepts I adopted in my study.

A question:

"is an expression of inquiry that invites or calls for a reply" (Aizikovitch-Udi et al., 2013, p. 1). In the context of my study, mathematical questions, therefore, could include a problem, example, exercise or even an instruction (Smith & Julie, 2014) requiring a response from learners.

Attributes of questions:

Is a phrase that portrays the character/characteristics of mathematics questions. These characteristics include existence or lack of desirable qualities of a question such as purposefulness, well prepared in advance, its ability to stimulate mathematical thinking, ignite interest and creativity and have wait time period (Shahrill, 2013).

Purposeful question:

A question planned and posed/asked with a clear objective and definite aim to stimulate discourse and advance the attainment of intended lesson objective(s)

Formal assessment:

Formal assessment is a type of assessment carried out after a topic or topics have been completed. This assessment is used for promotional purposes. It is for this reason that it is referred to as assessment *of* learning because the results are mainly used for promotional purposes.

Informal assessment:

Informal assessment also known as formative assessment is assessment that is used to facilitate the teaching and learning process. Formative assessment is characterised by constant feedback to learners, short classwork and verbal questioning during the lesson. All these form an integral part of learning. Formative assessment is meant to assist the teaching and learning process hence assessment *for* learning.

Questioning:

The act of asking questions

Written questions:

Questions planned and written on a lesson plan, and on the chalkboard for use during a mathematics lesson presentation. These do not include questions that are used in assessment after the lesson presentation such as class work, homework or test.

Learner:

According to the South African Schools Act [SASA] (1996) learner refers to any person receiving education or obliged to receive education in schools in South Africa and it is synonymous to the term pupil(s) or student(s) as generally used in other contexts globally.

Senior Phase:

Refers to Grades 7, 8 and 9 within the General Education and Training [GET] Band in South Africa; however in this study only two grades of the Senior Phase (Grades 8 and 9) were involved.

1.6. VALUE OF THE RESEARCH

The main goal of educational research is to increase the body of knowledge by offering solutions to various pedagogical issues while enhancing teaching and learning practices. The contributions the study made are discussed in 1.6.1 to 1.6.4.

1.6.1. Collaborative lesson planning

Thinking about questions prior to the implementation of a research lesson helped teachers to think ahead of critical moments during the lesson. Although in some instances teachers were unable to focus their attention to desirable quality questions, it was found that participation in this study stimulated teachers' awareness of the subject of questions and questioning. LS is regarded as a venue for professional development of in-service teachers. In this study, the challenges that surround classroom questioning were elevated to an environment that fosters collective responsibility. This study contributes to knowledge and understanding of how secondary school teachers incorporate questions in their research lessons.

When mathematics teachers prepare formal tasks (tests/examination) they refer to the curriculum policy for guidance on how to structure and categorise questions. This is so because there is a framework in the curriculum policy dedicated for providing guidance to teachers for the development of questions for formal assessment tasks. During the collaborative lesson planning, it occurred to me that teachers did not have an explicit guide to refer to when they plan questions for formative assessment. This study revealed this policy implementation gap for consideration by curriculum policy-makers.

1.6.2. Lesson presentation and observation

There were significant pointers throughout the research processes which indicated that participation in the LS have contributed to teachers' development. This research study has carefully inspected how an LS team intentionally and unintentionally incorporated questions in what they intended for their learners and how such questions filtered into the lessons. This was done by collecting a rich set of data obtained through observation, documents analysis (lesson plans) and conversational interviews. This study therefore contributes to a body of knowledge on how LS communities can begin to think deeply about how informal assessment can be considered for each LS cycle. The study shed light on how the character of questions posed by teachers during instruction can either make or break the lesson.

1.6.3. Post-lesson reflection

LS enhances mathematics teachers' reflection on their teaching practice generally and their questioning behaviour specifically. Although teachers did not pay attention to other attributes of questions that deserved interrogation during the post-lesson reflection, it was evident that participation in LS stimulated discussions around the subject of questions and questioning to some extent.

1.6.4. The conceptual framework

The conceptual framework employed to guide the entire research process was considred a novel approach. The theoretical lens employed in this study causes us to look at the phenomena of questions and questioning in a different light.

1.7. RESEARCH DESIGN AND APPROACH

In order to explore the practices and culture of questioning in mathematics classrooms occurring in LS contexts, qualitative research was adopted as an approach for the study. Qualitative design was preferred for its special attribute of enabling the researcher to explore a holistic picture and to gain a deeper understanding, which quantitative methods may not afford (Cohen et al., 2018; Creswell & Poth, 2018). Creswell and Poth (2018) describe research design as, "A plan and procedure for research that span the decisions from broad assumptions to detailed methods of data collection and analysis" (p. 4). The plan involved a number of crucial decisions which were informed by the paradigmatic assumptions I brought to the study. In this study, a case study design was chosen because it enabled me to respond to the broader research question as truthfully and as deeply as possible (Cohen et al., 2018).

1.8. TARGET POPULATION AND SAMPLING

The broader population from which the sample was drawn was a total of six LS clusters which are formed by teachers from nineteen schools in the entire Sub-District. Five mathematics teachers from four secondary schools within the Sub District constituted a LS team that participated in the study. These teachers were purposively sampled because they had an idea of what LS is since they had been implementing it as a teacher professional development model (Cohen et al., 2018). This is consistent with the views expressed by Cohen et al. (2018) that, “Researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality or possession of the particular characteristic (s) being sort” (p. 218). For the purpose of this study, a school where all activities related to the research study were conducted was purposively selected (Cohen et al., 2018). The school was chosen because the school management team (SMT) and learners were familiar with LS since they had previously hosted LS activities for the Sub-district. The school was also convenient to the other teachers from three other schools since it was central and accessible to them.

A single Grade 9 class was sampled to participate in the study. This grade is an exit grade in the Senior Phase. Ary et al. (2006) and Cohen et al. (2018) caution qualitative researchers that purposive sampling is prone to bias in that the researcher may have pre-judged the participants and mistakenly assumed that they are all knowledgeable about the phenomenon being explored. In this study, it was necessary to ensure that all participants were familiar with the context (LS) in which the study was conducted. I acknowledged that the newly appointed teacher, who has just joined the school was not familiar with LS processes. Taking this limitation into account, I then requested participants to explain to her what LS is and how it works prior to the commencement of the study.

1.9. DATA COLLECTION

1.9.1. Data

Three strategies were used for data collection, namely: observations, informal conversational interviews and document analysis. Data were collected through an observation of teachers’ collaborative lesson planning processes, lesson presentation and observations and post-lesson reflections of all four research lessons.

1.9.2. Observation protocol

Observation is worthwhile if the researcher is intending to obtain an intimate perspective of an area of interest (Cohen et al., 2018). In this study, observations were conducted during three of five stages of LS, namely: collaborative lesson planning, lesson presentation and observation and the post-lesson reflection stage. Annexure A and

Annexure B were used to collect observational data from lesson planning and lesson presentation and observation stage respectively, while Annexure C was used to guide the collection of data from the post-lesson reflection stage. Video-recordings were used to support the observation processes (Cohen et al., 2018).

1.9.3. Interviews

Informal conversational interviews were conducted where clarity was needed (Cohen et al., 2018). In an informal conversational interview, questions emerge from an observation are posed when the need arises (Cohen et al., 2018). In this study, I interjected without pausing a video while teachers were planning the lesson by asking questions to clarify or corroborate what I was observing. This type of interview increased the relevance of questions I asked during observation. Although there was no specific schedule to guide questioning during the interviews, I was guided by the conceptual framework in chapter 4, Figure 4.1 and the research questions.

1.9.4. Personal experience

As a researcher and an experienced mathematics teacher, I learnt much during this research study. My understanding of how teachers' questions shape classroom conversations extended to how supervisors' questions helped me to tap into my inner resources and reflect deeply about the role of questions throughout my PhD journey.

1.10. DATA ANALYSIS

Data analysis in qualitative research is the process of consistently searching and arranging transcripts and observation notes that the researcher accumulated to help answer the research questions in relation to the phenomenon being studied.

Organisation – The data was organised into word files to create a computer data base.

Perusal - The data collected were read and re-read to get a sense of what was emerging.

Classification – Data were grouped into categories or themes. Questions (oral and written) and learners' responses to questions were then categorised according to Emanuelsson's (2001) categories of classroom interactions.

Synthesis – Information from multiple sources used to gather data i.e., observation, informal conversational interviews and document analysis (lesson plans) were intergrated and analysed using literature.

Induction - By reading and re-reading, reflecting and making sense of data and what they implied, I was able to systematically derive concepts and themes, through interpretations.

Deduction – By exploring all the facts (data from the field and literature) I was able to respond to all the four research questions which the study was set to answer. The outcomes made it possible for me to make findings and reccomendations and outline the contribution the study made to the body of knowledge. The conceptual framework designed to guide the research process was instrumental.

1.11. QUALITY ASSURANCE CRITERIA

There are a number of pathways which researchers can consider to ensure quality in qualitative studies. They include credibility, transferability, dependability and confirmability (Cohen et al., 2018). While these quality assurance criteria are explained in section 1.11.1 to 1.11.4 more details are presented in chapter 5.

1.11.1. Credibility

Credibility in qualitative research concerns the truthfulness of the research findings (Ary et al., 2006). In this study, triangulation was employed to ensure that findings are credible. Multiple methods of data collection such as observations, conversational interviews, and document analysis were used to triangulate data. Another method that was employed to ensure credibility is a method described by Johnson and Christensen (2017) as participant feedback or member checking. This technique involves availing and discussing the researcher's interpretations and conclusions to the study's participants so that they can establish if they agree with what is said about them.

1.11.2. Transferability

Transferability in qualitative research is achieved by demonstrating to the readers that the research study's conclusions might be applied to different contexts, situations and times (Cohen et al., 2018). In this study, I provided the data base that makes transferability judgements possible to potential appliers. The issue of transferability was addressed by providing thorough description of the context, data collection processes, assumptions that were applicable to the research and working conditions of the participants. The 'rich and thick' descriptions provided will enable readers to make their own judgements about how well this fitted in with their situations.

1.11.3. Dependability

Dependability concerns the extent to which the findings of the study are believable. Dependability in this study was achieved through keeping an audit trail of the whole research process, all transcripts, field notes, videos, initial and final drafts of typed lesson plans were recorded (Cohen et al., 2018; Johnson & Christensen, 2017).

1.11.4. Confirmability

Confirmability has to do with the degree of confidence that the research study's conclusions are based on the participants' narratives and words rather than any researcher biases (Creswell & Poth, 2018; Johnson & Christensen, 2017). In this study I employed reflexivity, during data collection and analysis (Cohen et al., 2018). I kept a reflexive journal throughout the research process where I reflected on what was happening in the research process. I was careful that my knowledge of the subject (mathematics) and LS do not influence the research process. This was necessary to ensure that findings are shaped by participants more than they were shaped by me (the researcher).

1.12. ETHICAL CONSIDERATIONS

In this study, I observed all the necessary ethical procedures during each step of the planning and implementation of this research study. The first step involved obtaining permission from all concerned (Appendices D,E,F,G,H,I). Ethics approval to collect data and ethical clearance to declare adherence to ethical practices after data collection, were requested from the Ethics Committee of the University of Pretoria. This research study was conducted in accordance with the University of Pretoria’s Ethics Guidelines. The ethical clearance and research approval were first obtained from the Faculty of Education Ethics Committee in accordance with University’s regulations. Approval to conduct research in public school (chosen research site) as well as teachers was obtained from the North-West Department of Education. In order to provide a fair explanation of the procedures and processes of a research, participants were invited to a meeting where information regarding the study, its purpose, the role of participants, obligations and responsibilities were clarified (Cohen et al., 2018; Creswell & Poth, 2018). The purpose of the initial meeting which was held with participants was to inform them about their right to freely decide whether or not they want to take part in the study and that they can withdraw their consent to participate at any point during the research process (Cohen et al., 2018; Creswell & Poth, 2018). Participants were not coerced to take part in the research. All participants were assured that their anonymity will be protected and confidentiality of data will be kept during data analysis and after dissemination of results. This study also involved learners whose permission to participate was first obtained from their parents since they are minors. Cohen et al. (2018) emphasise that seeking informed consent with regard to minors is a two-stage process.

Table 1.2: Ethical considerations

Category	Researcher's responsibility
Anonymity	Potential risk 1: Video recording has the potential to expose facial identities of learners. To mitigate this, maximum caution was exercised to avoid capturing learners' faces in the video recording.
Consent	Learners whose parents may opt not to return the consent forms and/or learners who may choose not to grant assent to participate in the study may miss out on the lessons presented. It turned out that all the parents of targeted learners returned the consent forms and learners also assented, therefore there was no need for mitigation.
Health risks	Potential risk 3: The uncertainties that were brought about by the COVID 19 pandemic were prevalent during the period of data collection. To mitigate challenges of potential health risks, all the observers, the teacher teaching the lesson, the researcher and learners wore face masks for the duration of the lesson and during post-lesson reflections

1.13. RESEARCH STRUCTURE

To assure a well-structured research report in which the content flows in a logical manner and in which the research intentions and questions are addressed, the chapters were outlined as follows:

Chapter 1: Introduction and background

This chapter introduces the study, providing the related background, context, rationale, aim and research questions. The summary of methodological considerations, contributions, quality assurance crieteria and ethical considerations to the study are outlined.

Chapter 2: Review of literature

This chapter gives critical synthesis of literature on questions and questioning. The chapter focuses on the attributes of mathematics teachers' questions as broadly presented in the literature. The chapter outlines questioning practices outside and within LS contexts reflecting international debates while also including insights into areas that are less known of the discussion. Significantly, the theoretical framework guided the conceptualisation of the aspects characterising the review of literature.

Chapter 3: The evolution of LS

Chapter 3 is a chapter specially dedicated to a review of literature on LS. The chapter details historical origins of LS and mechanisms by which this teacher-led professional development model improves instructional practice. The chapter further outlines the variations of LS across different cultures including South Africa.

Chapter 4: Theoretical orientations

Chapter 4 provides details of the theoretical focus of the research. The chapter chronicles the origin and background of the Variation Theory - the theory that underpins the study. The chapter introduces the reader to the key constructs of Variation Theory and broadly explores the intersections of Variation Theory (Marton, 2015) and a phenomenon of questioning as well as questions categories espoused by (Emmanuelson,2001). The chapter culminates in a detailed conceptual framework which brings together all three dimensions that constitute the conceptual framework, i.e. LS as the context, Variation Theory and Emanuelson's categories of questions.

Chapter 5: Research methods and methodology

This chapter firstly describes the philosophical foundation of the study. The chapter sheds light on how the study was conducted in relation to the research design, data collection techniques, and data analysis strategies. The chapter also details the research paradigm, the lens through which I viewed the research world. Quality assurance criteria and ethical considerations are also discussed comprehensively in this chapter.

Chapter 6: Presentation of the findings

This chapter informs the reader of the discoveries that emanated from the data collected. The findings of the study are organised in terms of the conceptual framework that guides the study.

Chapter 7: Discussions, recommendations and conclusions

This chapter presents discussions of findings in relation to the research questions presented in chapter 6. The discussions of the findings, and the implications thereof, are supported by the reviewed literature and most importantly the conceptual framework chapter provides a summary of discussions supported by literature. I also responded to each one of the four secondary research questions in this chapter.

CHAPTER TWO: REVIEW OF LITERATURE

2.1. INTRODUCTION

The aim of this chapter is to provide a critical synthesis of literature on questions and questioning. The chapter commences with the definition of key constructs ‘question’ and ‘questioning’. Attention is paid to the attributes of teachers’ questions to portray the defining characteristics of mathematics questions as broadly presented in the literature. The role of teachers as questioners and the role of questions in stimulating classroom conversations and guiding thought during mathematics lessons is brought to the fore. The chapter outlines questioning practices outside and within LS contexts reflecting international debates while also including insights into areas that are less known of the discussion. The reader is invited into larger issues related to considerations for why teachers’ questions matter. Discussions on this subject are situated alongside the South African mathematics curriculum – Curriculum and Assessment Policy Statement (CAPS) and the South African Framework for Mathematics Teaching. Lastly the affordances and challenges surrounding classroom questioning are foregrounded.

2.2. DEMYSTIFYING THE CONCEPTS QUESTION(S), QUESTION ATTRIBUTES AND QUESTIONING

What is a question? Or what counts as a question? Aizikovitch-Udi et al. (2013) define a question as, “An expression of inquiry that invites or calls for a reply” (p. 1). This definition coincides with that of Dahal et al. (2019) when they define a question as, “A statement for which a reply is expected” (p.121). We may infer from these definitions that naturally questioners await or expect responses in the form of answers. While this view is a natural occurrence, Warshauer (2015) seems to have a different view, suggesting that questions could be answered with questions. Watson (2018) defines questions as, “Information-eliciting acts” (p.358). Judging from these definitions, researchers seem to agree on what characterises a question and the expectation that comes with a question i.e., a reply is anticipated. I too concur with Warshauer (2015) that it is possible to respond to learners’ questions with questions. What this essentially means is that one possible way of heightening classroom conversations is to ask questions about questions learners pose in order to assist them to access answers to their own questions. By responding to a question with a question the teacher not only assists learners to realise that they have an answer to the question themselves, but also creates a climate that values co-construction of knowledge. While the literature provides us with a description of a question in general, there is a need to clarify what counts as a mathematical question. The description of what counts as a mathematical question from Smith and Julie's (2014) perspective is considered. According to Smith and Julie (2014) a mathematical question can be an example, a problem, an instruction or even an exercise. This description of a mathematical question is consistent with how in practice textbooks perceive a mathematical question i.e., a task, a problem, examples and even instructions such as simplify, solve, sketch etc.

Although there may be validity to the definition presented by Smith and Julie (2014) of what qualifies as a question, this description may somehow be misleading. This is so because, for example: solve for x in the equation $x^2 - 4 = 0$ is a clear instruction that needs to be responded to. A learner who engages with the task of solving for x is simply responding to an instruction. The point I am making here is, our engagements with issues that surround questions and questioning, should begin with appropriate descriptions of what qualifies it as a mathematics question. A question should remain a question i.e., an expression of inquiry with a question mark. Distinctions should be made between a question and instructions or all other prompts.

Questioning is defined by Mason (2020) as, "The use of questions and other prompts offered to students to help them get unstuck or to direct their attention in a potentially useful way so that they make mathematical progress" (p. 705). The definitions of question and questioning provided here remind us that asking questions during classroom interactions not only promotes critical thinking, but can also be used to provide learners with the responsibility to make decisions i.e. when a teacher asks questions in relation to learners' questions. Research (Aizikovitsh-Udi & Cheng 2015; Watson, 2019; Shahrill, 2013) has already pointed out that oral questioning in mathematics classrooms (which is the focus in this study) is a key component of regular classroom interactions.

2.2.1. The attributes of teachers' questions

Attributes of questions is a phrase that portrays the characteristics of mathematics questions. These characteristics include existence or lack of desirable qualities of a question such as purposefulness, relevance, its ability to stimulate mathematical thinking and to ignite interest and creativity (Shahrill, 2013). Sorto et al. (2009) have focussed on the level (cognitive domain), type (e.g., open vs closed, genuine, provoking, empowering, etc) and patterns (funnelling, focussing, probing, orienting) of individual questions as a way of characterising questions posed by teachers in a mathematics classroom. Aziza (2017) uses the phrase ‘kinds of questions’ to refer to closed and open-ended questions. Judging from the different conceptualisations of question characteristics/attributes there appears to be no consensus amongst researchers on what precisely counts as a characteristic or attribute of questions.

In this study, the phrase ‘attributes of questions’ is used to refer to the different categories of questions posed by teachers during classroom interactions such as closed and open-ended questions. The study however gravitates towards the characterisation of questions as described by Emmanuelson (2001) which I will outline later. The attributes of questions from Emanuelsson’s (2001) perspective are of particular interest to this study. There are however distinctive features of classroom interactions particularly in relation to how questions are entangled in teacher talk. The attributes of such questions were also interesting to analyse. In the literature the classification of mathematics questions according to distinct attributes are often cast in contrast: convergent versus divergent (Dahal et al., 2019; Tofade et al., 2013); closed versus open-ended questions (Aizikovitch-Udi et al., 2013; Aziza, 2018; Houen et al., 2016; Smith & Julie, 2014); reproductive versus productive questions, (Tienken, 2009); topical versus conceptual (Emanuelsson, 2001); and finally, explicit versus implicit questioning (Parks, 2010).

Several points can be mentioned about the classification of questions mentioned above, for instance the characterisation of questions is conceived as dichotomous. These “…specific predetermined classification system” (Purdum-Cassidy et al., 2015) are what characterise the questions teachers ask in mathematics classrooms (p. 88). The classifications presented here are useful categorical tools for researchers and act as lenses to view classroom discourses while paying attention to the choice of questions preferred by teachers and how those questions are asked.

Another important observation is that the first phrases in each category (e.g. convergent in the convergent-divergent dichotomy) are different labels that are basically synonymous and share similar attributes. This applies to second phrases in each category. Lower-order questions are synonymous to closed ended, convergent, factual, and reproductive questions. These types of questions require learners to think convergently (by focusing on a single aspect), prompt learners' procedural and factual knowledge, have one correct answer, make minimal demands on learners' thinking and they are intended to establish retention of previously learned knowledge (Aizikovitsh-Udi, & Cheng, 2015; Azizaa, 2017; Houen et al., 2016; Purdum-Cassidy et al., 2015; Tofade et al., 2013; Watson, 2018). Lower-order questions are framed such that several, if not all, learners arrive at the same limited number of responses (Purdum-Cassidy et al., 2015). The true character of a reproductive question is captured in Tienken et al.'s. (2009) comment, "Reproductive questions prompt students to imitate, recall, or apply knowledge and information taught by the teacher, through a mimicked process" (p. 40). It is precisely this attribute of lower-order questions that makes it less attractive for mathematics education.

Higher-order questions, on the other hand, are synonymous to the second phrases in each category (e.g. divergent in the convergent-divergent dichotomy); therefore, includes divergent, open-ended, productive and conceptual questions. These types of questions are generally intended to invoke higher order thinking, heuristic, solicit long responses, provoke critical and creative thinking, support conceptual development and sharpen problem solving skills (Aizikovitsh-Udi & Cheng, 2015; Aziza., 2017; Houen et al., 2016; Purdum-Cassidy et al., 2015; Tienken, et al., 2009; Tofade et al., 2013; Watson, 2018). They are responded to with insight as opposed to reproductive questions that require previously learned information to respond to.

Fatah et al. (2016) further characterise open-ended questions by describing and classifying the openness into three types, "The process is open, end products are open, and ways to develop are open" (p.13). The openness should be embraced and planned for by teachers because it holds promise to the fertile and rich unpredictable contributions from the learners (Ulleberg & Solem, 2018). Productive questions are capable of giving sustained attention of learners (Watson, 2018). Attentiveness to productive questions and questioning is strongly advocated for in the literature (Aizikovitsh-Udi & Cheng, 2015; Aziza, 2017; Ulleberg & Solem, 2018). The reasons for their preference abound literature and they are connected to the central goal of why we teach mathematics.

The attributes of questions described here do have a place in mathematics education, and serve different purposes at different points during the lesson (Houen, et al., 2016; Koizumi, 2013; Nappi, 2017; Pianta et al., 2012; Tofade et al., 2013; Shahrill & Clarke 2014; Warshauer, 2015). For instance, Nappi (2017) argues that reproductive questions set the stage for learning and lay a foundation for higher level cognition and as such can be used at the beginning of the lesson to establish learners' previous knowledge. Koizumi (2013) made a similar observation regarding the relevance of reproductive questions when new mathematical content is introduced. Such questions were considered useful as they required learners to recall previously learned material.

Pianta et al. (2012) espoused that teachers' questions do not have to be entirely higher-level questions rather a good balance of different questions should be considered appropriate. Tofade et al. (2013) support this view indicating that it does not mean that reproductive questions should not be considered for teaching. However, researchers (Dahal et al., 2019; Houen, et al., 2016; Nappi, 2017; Pianta et al., 2012; Tofade et al., 2013; Shahrill & Clarke, 2014; Ulleberg & Solem, 2018; Warshauer, 2015) expressed concern about the dominance of lower-order questions in mathematics classrooms. These studies reported that teachers overwhelmingly ask more questions that fail to turn on learners' intellectual engines. Although the reasons for teachers' preferences to these types of questions have been outlined towards the end of this chapter, one reason is that teacher preparation programmes do not incorporate questioning in their training (Dahal et al., 2016). The inability of teacher training programmes to develop questioning skills has direct implications for teaching practice when these teachers become qualified to teach. These teachers become incompetent questioners who in turn are unable to cultivate the habit of inquiry in their learners. It should therefore not surprise us that mathematics teachers fail to plan questions they will use to facilitate learning and promote productive exchanges during classroom conversations.

There are several viewpoints that may propel the reader to agree that the concerns raised by researchers in relation to the over-reliance on reproductive questions are genuine. Specific types of questions (and their associated attributes) attend to different cognitive needs of learners and serve different educational functions during teaching and learning (Koizumi, 2013; Desli & Galanopoulou, 2015). Hokanson (2015) on the other hand is of the view that the purpose of instruction in any mathematics classroom is to encourage learners to think deeper. Watson (2018) supports this view arguing that teachers should teach for good questioning because it contributes to the formation and cultivation of an intellectually virtuous character. The phrase ‘teach for good questioning’ implies using quality questions in the classroom to stimulate productive conversations. Following these discussions, it becomes clear that the concerns expressed in literature about the dominance of lower-order questions in classrooms are worth noting. But why is the quality of questions and questioning a cause for concern to researchers? The quality of ideas learners construct is largely influenced by the quality of questions teachers pose during instruction (Purdum-Cassidy et al., 2015; Viseu & Oliveira 2017). ‘Good questions’ and ‘good teacher questioning practice’ (Aizikovitch-Udi, et al., 2013, p. 1) lie at the centre of productive exchanges that occur between teachers and learners during teachable moments.

Teachers are expected to ask good questions so that learners can copy the habit of asking good questions from them. Mason (2020) argues that, “Learning and independence can really only be said to have been achieved when students spontaneously question themselves and each other” (p. 710). I can draw from this narrative that, a true measure of learning is not just about learners demonstrating competencies in standardized or systemic tests, rather it is evidenced by deep reflections that call for self-questioning. Thompson and Mackiewicz (2014) support this view when they indicate that teachers’ questions can turn out to be models for self-questioning for their learners.

2.2.2. Questioning skills

Having explored the different categories of questions as outlined in the literature, the next step is to discuss the technical know-how to implement them in practice. There is generally consensus amongst researchers (Shahrill & Clarke, 2014; McCarthy et al., 2016; Shahrill, 2013; Watson, 2018) that questioning has to be done appropriately and effectively if it is to maximise learning gains in mathematics classrooms. These sentiments are echoed in Aizikovitch-Udi et al. (2013) who maintain that the tools of questioning practice may not be useful if not properly implemented in an instructional practice. This section discusses the techniques or skills of asking questions.

- *Knowing when to ask which questions:* Viseu and Oliveira (2017) acknowledge the importance of asking the right questions (with specific attributes) at the right time. This implies that teachers should be able to make an accurate judgement of when to pose which question to elicit particular information.

- *Planning questions in advance:* Chong and Shahrill (2014) encourage teachers to consider discussing and preparing possible oral questions related to the lesson that addresses different levels of cognitive demand. In the context of this study, LS provides a fertile ground for mathematics teachers to implement this questioning technique because the lesson, including key questions, is collaboratively prepared. This implies that teachers can incorporate key productive questions which they may find difficult to think of during the lesson when the need arises.
- *Phrase questions clearly*: Shahrill and Clarke, (2014) emphasise that questions should be phrased clearly to avoid confusion. This is very important because ambiguous questions have the potential to create confusion.
- *Wait time (*sometimes referred to as *think time):* Aziza (2018) describes this technique as the interval between teachers' question and learners' answers. Etemadzadeh et al. (2013) recommend that a teacher waits for some time after posing a question to give learners the opportunity to think. Wait time is even more crucial when high order questions are asked.
- *Probing and Follow-Up:* McCarthy et al. (2016) describe probing as a teacher's way of engaging with responses of learners. The teacher may probe to follow up on learners' response to questions. Probing and follow up stimulate learners' confidence and make them feel that their contributions are valued.

Being able to distinguish between the different types of questions that can be used to navigate a classroom discourse is not sufficient. Teachers are expected to develop questioning techniques or skills to succeed in developing learning journeys for their learners.

2.2.3. Emanuelsson's categories of classroom interactions

In earlier discussions in paragraph 3.2.1, topical and conceptual questions, referred to as zones in Emanuelsson's (2001) work, were highlighted as some of the attributes of mathematics questions. There is a need for these categories of questions to be made more explicit since they are the preferred categorical lenses for this study. This preference is since these attributes referred to as zones (Emanuelsson, 2001) are grounded within Variation Theory (Mason, 2020) which guides this study. In his work Emanuelsson (2001) describes three qualitatively different ways of framing classroom interactions referred to as zones, i.e., topical, conceptual and procedural zones. From this perspective questions in the topical zone are characterised by recall of facts, they are closed and solicit short responses and focus on the correctness of answers. Contrary questions in the conceptual zone are productive, open-ended and solicit long responses and focus more on learners' thinking. In this zone teachers' questions present opportunities for learners to learn the intended mathematical content.

The two zones (topical and conceptual zones) are the only zones in which questions frame discussions. In the third zone (procedural zone) comments will be made in relation to how learners react to the teachers' questions. Interactions in the procedural zone are more about learners' contributions to the discussions, responses to questions asked by a teacher and the form of presentations and argumentations for their solutions. Teachers can learn more about how their own questions lead to classroom interactions and how interactions can lead to questions.

2.3. MATHEMATICS QUESTIONING PRACTICES

Questioning occupies centre stage in classroom conversations and is one of the most frequently used instructional techniques for mathematics teachers (Thompson & Mackiewicz, 2014). Researchers such as Aziza (2018); Chikiwa & Schafer (2018); Dahal et al, (2019) agree that questioning is one of the most commonly used teaching strategies in mathematics classrooms. They stated that questions posed by teachers are intended to guide learning towards the intended goal and to strengthen learners' understanding of mathematical concepts.

According to Shahrill (2013), "Good teachers are good questioners" (p. 226). This statement resonates well with McCarthy et al's, (2016) view that, the way teachers teach is a revelation of how they question (McCarthy et al. 2016). Asking good questions maximizes opportunities for learning mathematical concepts (Shahrill, 2013), promotes critical thinking and deepens conceptual understanding, (Chikiwa, and Schäfer, 2018); directs the focus of learners, to structure their attention to the learning process, (Mason, 2020); assists in determining the progress learners are making in the lesson as well as how learners understand the content taught (McCarthy et al., 2016), Classroom conversations are mainly dominated by questions posed by teachers and responses from learners. The nature and quality of questions teachers ask in mathematics learning is likely to create impressions about what mathematics enterprise is about (Mason, 2020). This view is endorsed by Thompson, and Mackiewicz, (2014) who argue that the quality of ideas learners construct is primarily guided by the quality of questions teachers ask. In general, researchers (Dahal et al., 2019; Mason, 2020; McCarthy et al., 2016; Shahrill 2013; Watson, 2018) seem to agree that questioning is undeniably a powerful teaching tool if used properly and if mathematics teachers are aware of its intricacies.

Chen et al. (2017) conceptualized four critical roles of teacher questioning which are, “Dispenser, moderator, coach, and participant in light of the ownership of ideas and activities” (p. 373). Chen et al. (2017) revealed that there are two significant changes in the ways teachers implement question-asking and the relationship between teachers’ question-asking and learners’ cognitive responses. Teachers became motivated to adopt multiple roles of dispensing, moderating and coaching in creating productive classroom conversations. Some researchers (Aizikovitch-Udi, 2013; Almeida, 2012; Chirinda & Barmby, 2017; McCarthy et al., 2016) point to the usefulness of providing guidance to both pre-service and in-service teachers on how they can successfully implement question asking in mathematics classrooms. Aizikovitch-Udi et al. (2013) explored the questioning practices of two experienced mathematics teachers who participated in a week-long professional development initiative which focused on the use of supplemental question-asking curriculum materials. The professional development was also intended to assist teachers to implement particular questioning techniques. The questioning practices of a teacher whose teaching was characterised by traditional methods of instruction did not change i.e., the professional development did not contribute new knowledge on questioning. It was observed that the training programme constituted an extension of a good practice which was already in-existence in a teacher who embraced contemporary teaching techniques. Materials were provided to these teachers to act as a catalyst in the effort to promote question-asking methodology, but it did not help to reconstruct question-asking practice of a teacher whose teaching was more traditional.

In a study conducted by Almeida (2012), a two-hour workshop was designed and conducted to promote teachers’ awareness of classroom questioning. Teachers were asked to reflect on the discourse pattern found in the transcripts of classroom interactions. Specific attention was paid to the kinds of questions posed by the teacher using the category closed or open questions. The study revealed that teachers are not aware of the high number of questions they pose during classroom interactions. Their reluctance to prepare oral questions for facilitating learning was also observed. Professional development initiatives that focus on creating questioning awareness were therefore recommended for further research in other contexts.

In their work, Chirinda and Barmby (2017) guided mathematics teachers by suggesting specific questions which they were to use to stimulate and guide classroom discussions when learners engage in problem-solving activities. These teachers participated in a six-month long professional development programme in which the focus was on mathematics problem-solving pedagogy. Teachers were supported on how to use Polya's problem solving process Because Polya's steps, as outlined by Nurkaeti (2018), require particular kinds of questions and questioning to guide classroom conversations, teachers were given a clue as to which questions to ask to guide classroom talk. It was found that when teachers are supported on how they can implement questioning, their questioning skills improve, and they develop different dispositions. McCarthy et al. (2016) concur and further mention that if teachers are guided through analysis of questions they ask during teaching and the responses they get from learners, they can distinguish between effective and ineffective questioning practices. This will encourage teachers to reflect on their own habits of questioning and further enable them to see how the character of the questions influence classroom conversations.

In my view and in the context of this study, professional development initiatives that recognize the need to create awareness about questioning in mathematics education and that are aimed at sharpening mathematics teachers' question-asking skills, are commendable and advocated for. However, there is something worth noting about the exposure of teachers to support programmes that were intended to transform teachers' questioning practices in these studies (Almeida, 2012; Chirinda & Barmby, 2017; Aizikovitch-Udi et al., 2013). Teachers' exposure to these interventions was brief. For desirable changes to occur in mathematics teachers' teaching methodologies, researchers (Vangrieken et al., 2017) advocate for professional development that occurs within a particular context and has to be situated in classroom practice (Feldman, 2020) so that a sustained improvement in teaching and learning can be analysed and comprehended. Professional Learning Communities (PLCs) in this regard hold promise in areas where transformation of teachers' teaching practices is much needed. In the previous chapter, a much more detailed systematic review of a specific form of PLC referred to as LS was provided. This teacher development model is considered relevant for this study because it affords teachers the opportunity to discuss issues that concern their professional development.

In the literature, decisions to consider LS as an effective model for professional development have frequently factored changes in how mathematics is taught in order to optimise learning opportunities (Amirullah, 2018; Lewis, 2016; Takahashi, 2014; Takahashi & McDougal, 2016). These studies have reported that mathematics teachers develop different dispositions and their competencies improve after participating in LS. The question now becomes: what do we know of the questioning practices of mathematics teachers who participate in LS?

In their work, Ong et al. (2010) examined the changes in experienced and novice mathematics teachers' questioning techniques. Mathematics teachers from two schools participated in the LS process for fifteen months. Their study revealed that experienced mathematics teachers developed different dispositions. This was observed in the way they moved from asking reproductive questions which required short correct answers. Experienced teachers began to focus more on questions that stimulate learners' thinking. These desirable changes were attributed to participation in LS. Changes were however not observed in novice teachers. Based on Amirullah's (2018) observations, mathematics teachers who participated in a LS in Malaysia enriched their teaching approach and their questioning techniques improved on a small scale.

There appears to be little research about questions and questioning in LS contexts and little if no focus on what precisely characterise the questions teachers ask when facilitating lessons. The scarcity of literature is of concern considering the role of questions and questioning in mathematics classrooms. By focussing attention on what characterises the oral questions mathematics teachers prepare for a research lesson, how such questions eventually filter into the lesson, how learners experience those questions, and how they are reflected upon during post-lesson reflection stage, it is hoped that this study will contribute to the conversations on classroom questioning within LS contexts.

2.4. THE ROLE OF TEACHERS AS QUESTIONERS

Teachers remain central interactional agents in navigating complex social systems known as classrooms (Pianta et al., 2012). Their role in maximizing learning opportunities includes amongst others, management of classroom discourses, encouraging learners to take risks in solving problems (Sharma, 2015), managing social, as well as emotional and organizational barriers that may impede learning (Ottmar et al., 2015). Teachers also have a huge responsibility of ensuring that what is intended for learning (the object of learning) is well thought out and learners are presented with a carefully structured variation for development of mathematical ideas (Kullberg et al., 2017). This section foregrounds the role of mathematics teachers as questioners. More precisely the section focusses on how mathematics teachers use questions to assist learners to discern the intended ideas.

2.4.1. Questioning and productive struggle

It is the contention of this study that the attributes and role of questions in enhancing mathematics learning cannot be meaningfully considered independent of the role of a teacher as a questioner. Mathematics teachers questioning practices have been explored from many directions by several researchers (Livy et al., 2018; Warshauer, 2015; Webb et al., 2019; Sayster & Mhakure, 2020). These include explicit attention to:

- how productive questioning can lead to productive learner participation and the practices of the teacher in promoting interactions (Webb et al., 2019);
- typical challenging tasks designed to provide opportunities for learners of different abilities to experience productive struggle while persisting through the task and learners' responses to the activities (Livy, et al., 2018);
- the nature and kinds of learners' struggle that emanate in classrooms when cognitively demanding questions or tasks are posed and how teachers become responsive to learners' struggles (Warshauer, 2015).

Sayster and Mhakure (2020) extended the work of productive struggle arguing that questioning and noticing are inseparable constructs in the process of exploring and responding to learners' productive struggles during teaching and learning. I use Warshauer's (2015) definition of productive struggle, "A student's effort to make sense of mathematics, to figure something out that is not immediately apparent'' (p. 376). In the context of this study, learners' struggle in attending to mathematical tasks in which the solution is not immediately visible, is anticipated because of the nature of mathematical tasks that appeal to LS teams (Fuji, 2018). For instance, the South African version of LS espoused by Sekao and Engelbrecht (2021) show that, it is topics that teachers find difficult to teach or areas where learners frequently demonstrate deficiencies that influence their choice of what should be collectively researched and taught. In Japan, mathematics is taught through problem solving (Fuji, 2018; Stigler & Hiebert, 2016; da Ponte et al., 2018). By their nature, solutions to mathematics tasks administered in LS are not immediately visible to learners and so they engage in a struggle which is expected to be productive or come to fruition. How teachers question, notice and act in response to learners' struggles constitutes support structure in a classroom discourse. This support comes in the form of feedback and follow up questions to direct and scaffold thinking (Sayster & Mhakure, 2020). Warshauer (2015) concurs by stating that:

> Responding to student struggles by asking questions serves various purposes. As part of a discourse interaction between teacher and students, questions can give direction to students' thinking and opportunities for students to organize ideas as they engage with a task (Warshauer, 2015, p. 380).

The views expressed above resonate well with Mason's (2020) observations arguing that the questions teachers ask even when a learner is submerged in being stuck and unable to think, serve as intervention and interrupt a learner's state. While this may sound incongruous, the way teachers respond to learners' struggles holds significant consequences for a flow of conversations, participation and productivity. In most of the studies that view classroom discourse through a lens of 'productive struggle', the role of the teacher in asking questions that guide learners' thinking and the guidance and support learners need to persist throughout struggle is heightened. By their nature, productive questions are intellectually demanding and the higher demand on cognition lead to the struggle (Livy et al., 2018; Warshauer, 2015; Webb et al., 2019). Teachers' roles become even more intricate in ensuring that learners' struggles come to fruition. Teachers must communicate well with learners, assure them that the struggle is important and should be embraced because it is part of what mathematicians do (Livy, et al., 2018; Warshauer, 2015) and most importantly affords them opportunity to grapple with the given questions or task (Chirinda & Barmby, 2017). The role of the teacher in supporting productive struggle includes a careful selection of tasks and questions that promote productive engagement and the use of questions that guide thinking (Livy, 2018). Good questions are essential in the process because they make learners thinking visible and bring to the surface errors and misconceptions (Warshauer, 2015). Warshauer (2015) further cautions that teachers should make efforts to provide support to learners without depriving them of the opportunity to think independently. If learners are not supported to translate their efforts into success they can easily give up or stop attempting the question and their struggle will become unproductive.

2.4.2. Questioning and listening

How can mathematics teachers use their ears more intentionally when reacting to learners' responses to questions they ask? The intersections of questioning and listening have been explored by several researchers (Mason, 2020; McCarthy et al., 2016; Thompson & Mackiewicz, 2014; Warshauer, 2015) arguing that listening is an important part of effective communication and a vital component of classroom conversations. Productive questions aim to elicit long responses and teachers must listen to all parts of learners' extensive replies so that productive exchanges in mathematical discourse can be elevated (Mason, 2020). Teachers must listen attentively and show interest in what learners are saying because their responses will provide guidance on how to probe them further and to decide on which follow up questions to use (Dahale et al., 2019; Tofade et al., 2013; Viseu & Oliveira, 2017). Warshauer (2015) supports this view indicating that when teachers question and listen carefully to details of learners' struggles, suitable responses that build upon learners' thoughts and ideas can be made. Mason (2020) draws from the work of Davis (1997) on the critical role of listening in mathematics discourse and particularly how teachers can teach through listening. In this regard a distinction is made between listening *for* an anticipated response and listening *to* what learners are saying and carefully observing what they are doing.

From this perspective, when teachers listen *for* an expected response, classroom conversations become unproductive because of the teachers' inclinations to judge the correctness of the learners' contribution against what is already known. Listening *to* on the other hand is much advocated for because the teacher in this situation is listening to learners' thinking, more interested in understanding the sense learners are making and the ideas they are trying to construct (Mason, 2020). The notion of questioning and listening is further espoused by McCarthy et al, (2016) indicating that when posing questions teachers are expected to listen more while learners are expected to reason more. The demands that come with successful productive questioning require a teacher to transform their own identities and practices (Aizikovitch-Udi et al., 2013) and ultimately embrace a paradigm shift (McCarthy et al., 2016). A way of questioning that embraces the insights of contemporary teaching where a teacher sees herself as a participant rather than a provider of information is advocated for. It is clear from these conversations that the role of teachers as questioners demand them to become more than just listeners but inquirers.

2.4.3. Questioning and intellectual humility

Haggard et al. (2018) conceptualised intellectual humility as a mindset that attends to our intellectual behaviour, more precisely the existence of a specific ability, namely the capacity to acknowledge and take responsibility for one's intellectual limits. Intellectual humility entails taking ownership of one’s intellectual limitations in the process of digging deeper for knowledge and understanding (Haggard et al., 2018). Intellectual humility has received much praise from several researchers as one of the intellectual virtues (Church, 2016; Haggard et al.,2018; Watson, 2018; Whitcomb et al.,2017). The close connection between intellectual humility and good questions can be observed in Watson’s account:

> The good questioner must recognise and ‘own’ (to use the limits-owning terminology) an important gap in her knowledge or understanding. Doing so will often, perhaps typically, amount to intellectual humility. This is especially apparent given that, for the most part, we are operating under a system that values knowing things highly (Watson 2018, p. 365).

This means that teachers seeking to engage their learners in productive discussions where pedagogical choices involve good questions and good questioning, should recognise that they may have knowledge gaps. Mathematics teachers are then required to acknowledge that the information that is missing is worthwhile and act competently to access it (Church, 2016). This is especially clear given that, for the most part, the system under which we operate places a high value on knowledge. Watson (2018) emphasises that good questioning constitutes intellectual humility.

What implications do the discussions around the subject of intellectual humility have for this study generally and for the characterisation of mathematics teachers' questions in particular? The idea of intellectual humility challenges mathematics teachers' questioning actions (verbal and written) and encourages them to deeply reflect on their own beliefs. These discussions generally invite teachers to embrace openness and to recognise connections that exist between good questioning and intellectual humility.

But what pointers does intellectual humility make to the characterisation of mathematics questions? Discussions pertaining to the subject of intellectual humility makes a crucial pointer on what questions should be selected to guide investigation of mathematical rules and how such questions should be posed to learners. Secondly, considering that LS is a venue for professional development, naturally one of the first questions one may ask when observing an LS team planning a lesson is: do teachers acknowledge the limit of their knowledge in the process of planning a research lesson? Watson (2018) lamented that, "The skill of good questioning extends beyond the mechanism of constructing an interrogative sentence that expresses a desire for this or that piece of information" (p. 356). The phrase 'interrogative sentence' reveals a form of characterisation of a question not just a sentence but an information seeking question.
Intellectual humility and good questioning are inextricably linked (Watson, 2018). Key to the notion of intellectual humility is that we need to approach conversations with open-mindedness, willing to admit that our knowledge may be limited and willing to reflect on our thinking (Watson, 2018). By adopting such an attitude and willingness to be wrong one stands a better position to grow and learn. A general assumption teachers have about questioning is that correct answers imply proficiency in the subject matter being taught whereas incorrect responses are interpreted to mean learning deficiencies (Mason, 2015). For this reason, Watson (2018) emphasises that to develop into proficient questioners, teachers should be able to determine what to ask, when and how to ask, and decide on the potential respondent. The role of the teacher as a questioner then is to encourage learners to: articulate what they know and to modify their conjectures without ridiculing them (Mason, 2020), and recognize and accept that incompleteness in their knowledge is not ignorance (Watson, 2018), rather it is a step towards development of what Pohl (2016) refers to as a growth mindset where learners confront their own intellectual limitations and view them as opportunities and challenges instead of failure.

Mason (2020) cautions that specific things that a teacher wants to hear in learners' responses may reduce their awareness which may have adverse effects on their learning. Teachers must demonstrate faith in learners' capabilities and acknowledge their limitations. To realise this goal, teachers need to establish classroom cultures in which risk-taking dispositions are encouraged and developed (Sharma ,2015); mistakes and errors are not viewed as failures rather as the opportunity for the brain to grow and develop, (Pohl, 2016) and learners are encouraged to exercise their intellectual autonomies and acknowledge their own intellectual limitations (Watson, 2018).

2.5. CONSIDERATIONS FOR WHY TEACHERS' QUESTIONS MATTER

This section examines reasons why mathematics teachers' questions are worth paying attention to from the lens of the South African curriculum, (Curriculum and Assessment Policy Statement [CAPS], 2011) and Framework for mathematics teaching (Department of Basic Education [DBE], 2018). The questioning (framework) in the CAPS that is supposed to guide teachers to navigate classroom discourse appears to have been overshadowed by questioning guidelines for formal or summative assessment.

2.5.1. Why do teachers' questions matter?

Why do teachers' questions in mathematics classrooms matter? An answer to this question is significant because it has the potential to contribute to an understanding of why teachers' questions in the classroom are important in optimizing learners' learning and thereby inform instructional practices. I situate discussions around this question alongside the Mathematics Curriculum and Assessment Policy Statement [CAPS] of the Department of Education (DBE 2011) and the Mathematics Teaching and Learning Framework for South Africa, (DBE, 2018). One of the principles of mathematics learning as pronounced in the curriculum (DBE, 2011) is, "Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths" (p. 4). We may infer from this statement that the curriculum subscribes to the view that learners are not kept inside classrooms to obey mathematical knowledge already established by others, but rather they should interrogate this knowledge.

But how does this view of learning find resonance with discussions surrounding classroom questioning in mathematics? What does active and critical learning have to do with classroom questioning? Jamaludin and Osman (2014) indicated that active and critical learning can be promoted through what Smart and Marshall (2013) refer to as cognitive engagement. Smart and Marshall (2013) define cognitive engagement as, “The teacher’s skill in questioning and the students’ elaboration of an idea as an answer” (p. 265). This view of the curriculum discourages the teaching where truths are accepted without being questioned. Aizikovitsh-Udi and Cheng (2015) discourage the “uncritical learning of given truths” when they indicate that critical thinking has to be developed in learners so that they can learn to question the world rather than just accepting it. These sentiments are echoed in Watson’s (2018) compelling narrative about “educating for good questioning” for the formation of intellectual character. Watson (2018) is of the view that teachers must model good questioning because it can advance virtual intellectual inquiry and help to establish an intellectually virtuous character.

Research has shown that the questions posed by teachers during learning have a direct influence on the knowledge that learners construct and communicate (Aziza, 2018; McCarthy, Sithole, McCarthy and Gyan, 2016). The curriculum further communicates certain expectations explicitly expressed as follows, “The teaching and learning of mathematics aims to develop: a spirit of curiosity and love for mathematics, recognition that mathematics is a creative part of human activity and deep conceptual understandings to make sense of mathematics” (DBE, 2011, p. 8). The key aspects notable in this pronouncement are curiosity, love for mathematics, creativity and deep conceptual understanding. Curiosity as Abramovich et al. (2019) maintain, is the beginning of inspiration to learn while creative thinking has been made the centre of the curriculum by researchers (Daher et al., 2017; Fatah et al., 2016). Teachers can use questions to stimulate curiosity (Dahal et al., 2019). A similar view of questions being a stimulant of curiosity is put forth by (Watson, 2018). Watson maintains that inquisitiveness which is synonymous with curiosity has the strongest connection to the analytical ability of effective questioning. According to Watson (2018), good questioning is a distinguishing characteristic of virtuous inquisitiveness. From this perspective it is impossible to be virtuously inquisitive without asking good questions.

D'Ambrosio and D'Ambrosio’s (2013) question is significant, “How different is mathematical creativity from other forms of creativity?” (p.20). Being creative in mathematical ways is a very important intellectual posture. The enjoyment that comes with expressing mental creativity while engaging in productive struggle is what best describes mathematical creativity (Warshauer, 2015). In other words, mathematical creativity is characterised by the ability to choose among the combinations which are meaningful and meaningless.

Daher et al. (2017) have argued that the mathematical experiences which teachers make available to their learners can open opportunities for learners to become creative. Creativity in this way cannot only be measured by what learners do, but also by what teachers do. As observed by Daher et al. (2017), the way mathematics is handled in several classrooms can either lift or kill the spirit of curiosity and cause learners to either love or hate the subject respectively. It is therefore not surprising to learn that the curriculum foregrounds these components of 'love, creativity and curiosity for mathematics' and charge teachers with the responsibility of ensuring that learners experience this subject joyfully (Abramovich et al., 2019).

Radford (2015) reminds us that emotions are implicated in mathematical thinking. For instance, achievement, excitement, and satisfaction can evoke positive emotions while on the other hand frustration, failure leading to disappointment evokes negative emotions in the learning process. It is for this reason that the CAPS document advocates for the teaching of mathematics where the love and curiosity for mathematics is developed (DBE, 2011). Against this background, one of the key considerations to address these curricular expectations is through the creation of a classroom climate where teachers pose the right questions that ignite interest in mathematics lessons in an appropriate manner. Questions not only stimulate curiosity and taps learners' interest, but they also create wonder and creativity in the classroom.

The notion of teaching mathematics for deep conceptual understanding is also flagged in the Mathematics Teaching and Learning Framework for South Africa (DBE, 2018). The key tenet of the Mathematics Teaching and Learning Framework is the promotion of 'teaching mathematics for conceptual understanding'. The framework model draws from Kilpatrick et al.'s (2001) strands of mathematical proficiency. The dimensions characterised in the framework include: conceptual understanding, procedural fluency, strategic competence, reasoning and learning-centred classroom. The framework identifies reasoning as a fundamental competency expected from learners across the grades. Accordingly, questioning in the formative assessment should aim at encouraging learners to reason rather than focusing on correct answers (DBE, 2018).

The framework also recognizes the role of questions in the development and support of learners' logical and adaptive reasoning as follows:

> To delve deeper in the minds of your learners, the teacher will have to be prepared to ask learners to explain their understanding. The more teachers do this, the better they will be able to prepare mathematics lessons that address all learners' potential misconceptions. (DBE, 2018, p. 49).

The phrase 'delve deeper in the minds of learners' is a metaphor implying that teachers have to ask questions that dig deeper into the cognitive processes of learners such that the mathematical depth that is potentially present can be unearthed. This expectation has implications for teachers' questioning practices in a sense that for them to exercise the components of effective questioning expressed by (Steyn & Adendorff, 2020) teachers must possess the skill of questioning because good questions and productive dialogue do not just happen (Purdum-Cassidy et al., 2015). Rather, teachers must acknowledge that verbal questioning is a skill, and like any other skill it must be developed before it is mastered (Purdum-Cassidy et al., 2015).

On the other hand, the level of preparedness and competency required from teachers to successfully navigate this role of asking questions is what researchers have noted with dismay (Abramovich et al., 2019; Aizikovitsh-Udi & Cheng, 2015; McCarthy et al., 2016). Literature has provided several reasons why teachers find it difficult to succeed in orchestrating productive discussions through questioning. These researchers have made explicit factors that limit teachers engaging learners during learning so that they can get the most out of their mathematics lessons. The aims of mathematics teaching presented in the CAPS together with the recommendations for implementations presented in the framework for mathematics teaching (DBE, 2018), call for a multi-pronged approach to transform the teaching of mathematics in South African schools. Classroom questioning is not a stand-alone indicator of effective teaching in mathematics, but it clearly underlies big issues. This is evidenced by the heightened emphasis expressed in the literature and curriculum policy intentions such as, 'teaching mathematics for understanding' (DBE, 2011); development of skills for 21st century citizenship (Abramovich et al., 2019; Ayllón et al., 2016; Daher et al., 2017) and the preparation of learners for the demands of the 4th Industrial Revolution (DBE, 2018). These expectations clearly point to the need for mathematics teachers to re-think their teaching. It further suggests a need for teachers to engage in a professional journey in which they can develop practical knowledge to address curriculum policy intentions. In this study, my interest in elevating discussions and research-generated insights in relation to classroom questioning to LS contexts meets the call of the South African curriculum and the Mathematics Teaching and Learning Framework, (DBE (2018). I do this as a way of sensitizing mathematics teachers about questioning as a matter that concerns their practice and as an item that deserves attention in on-going professional development. I align myself with the views expressed by researchers that the constraints of strategic and productive questioning are due to the fact that the skill of questioning is not developed during teacher training programmes.

2.5.2. Limitations of CAPS and the Framework for mathematics teaching

While CAPS, (DBE, 2011) shed light on what the teaching of mathematics in South African schools is aimed at, (e.g. enhance logical and critical thinking, encouraging learners to interrogate given truths, developing the spirit of curiosity and love for mathematics), there appears to be limitations on the policy in terms of the guidance teachers need to successfully orchestrate productive classroom discussions through questioning. This guidance (if provided) will make it practical for mathematics teachers to work towards achieving the aims of the curriculum. The importance of informal assessment is emphasized in the following quotation from the curriculum policy.

> Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-teacher conferences, informal classroom interactions, etc. Informal assessment may be as simple as stopping during the lesson to observe learners or to discuss with learners how learning is progressing. Informal assessment should be used to provide feedback to learners and to inform planning for teaching but need not be recorded. It should not be separate from the learning activities taking place in the classroom (DBE, 2011, p. 155).

The perspectives shared in the preceding quotation not only position assessment as a process, but it also highlights the notion of a classroom as a collaborative space, that is, mechanisms (observations, discussions, practical demonstrations, learner-teacher conferences, informal classroom interactions) through which informal assessment should be conducted to exhibit the key features of classroom interactions. On the surface these comments look like a comprehensive guide for teachers to navigate a collaborative space. However, on a closer examination these mechanisms for formative assessment are vaguely expressed without specific guidance on how discussions can be made productive during classroom interactions.

On the contrary, CAPS (DBE, 2011) provides a clear-cut guide for the development of formal assessment tasks i.e., tests and examinations. The policy provides a detailed framework showing examples of questions with descriptors that characterise the skills to be demonstrated for each cognitive level. The framework also indicates the proportion of marks to be considered for each cognitive level. The cognitive level framework is purposefully crafted to guide teachers in the development of well-balanced quality formal written tests and examinations (DBE, 2011). It is further stated in the policy that, “The questions should be carefully spread to cater for different cognitive levels of learners” (DBE, 2011, p.155).

Judging from this explicit guide for the development of questions for formal assessment tasks, it appears that questions that matter most from the perspective of the curriculum policy are those developed for the assessment *of* learning (tests and examinations) as opposed to questions that are developed for the assessment *for* learning (informal activities and informal classroom interactions). The guideline for the development of questions that can be used during learning is open-ended and left for interpretation by the teacher. The question is: If such a well-structured guide can be provided for formal assessment, then why can it not be provided for formative assessment? Why it is that guidance on verbal questioning strategies is not embedded in the curriculum policy to support and guide teachers on how to navigate classroom discourse? This observation raises a question: how do mathematics teachers grapple with the process of preparing questions to facilitate productive classroom discussions? And how can such questions be characterised? The argument I put forth is if assessment for learning is as crucial as articulated in the policy, then it does not receive the attention it deserves. As teachers plan for teaching they should equally plan for questioning (written and oral questions and or tasks) which they will use to facilitate learning.

The significance of formative assessment in mathematics is well substantiated in the literature. More than 80% of daily classroom interaction is spent working on activities, solving unfamiliar tasks, asking questions and solving problems (Rakoczy et al., 2019). The question of what role formative assessment plays in day to day teaching in mathematics classrooms is clearly addressed by Rakoczy et al, (2019) in the five key strategies, “(1) to clarify and share learning intentions and criteria for success in order to determine the direction in which learners are heading; (2) to elicit evidence of students' understanding (assessment) in order to determine the areas in which they are reaching their learning goals; (3) to provide feedback that pushes learners forward; (4) to encourage students to be instructional resources for one another; and (5) to motivate students to take responsibility for their learning” (p.155). Summative assessment on the other hand is conducted with the purpose of measuring what a learner has learnt at the end of the learning process (Bjørkli, 2014). Given the purposes of these different forms of assessments and the conditions under which they are administered, it becomes evident that summative assessment does not help much in shaping the learning process. The importance of formative assessment over summative (formal) assessment is more pronounced in the literature. The CAPS limitation of not giving a well-deserved focus on formative assessment is noted with dismay.

2.6. AFFORDANCES AND CHALLENGES OF CLASSROOM QUESTIONING

2.6.1. Affordances of classroom questioning

An extensive review of literature demonstrated that teacher questioning can either enhance or inhibit mathematics learning depending on the level of teacher preparedness to implement this important teaching skill. But what can serve as an enablement for effective questioning? Researchers propose solutions to resolve these challenges. Several researchers (Chikiwa, & Schäfer, 2018; Di Teodoro et al., 2011; Etemadzadeh et al., 2013; Nappi, 2017; Shahrill, 2013) are of the view that, teachers have to think deeply and plan questions for formative assessment ahead of the lesson. Huinker and Freckmann (2004) cited in Chikiwa and Schäfer (2018) stated that, “By choosing our words carefully and using intentionally designed questions, we can engage and transform another person’s thinking and perspective” (p.16). We can draw from this quotation that a well-constructed and carefully thought-out question is important in steering productive conversations in mathematics classrooms when the object of learning is handled.

Ulleberg and Solem (2018) designed a questioning model with four distinct areas in which particular question types may be asked, and which links these question types to the question categories of the researchers. The model as seen in Figure 2.1 was intended to serve as a guide to teachers so that they can successfully orchestrate classroom talk through questions.

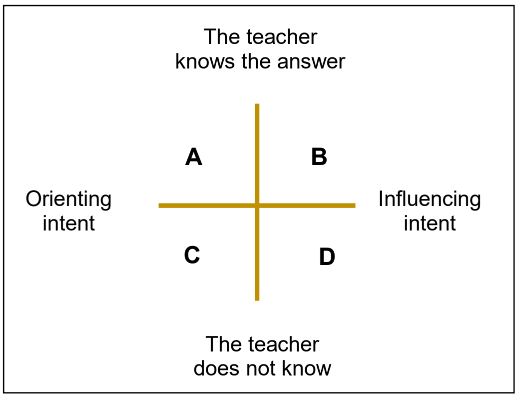


Figure 2.1: Ulleberg and Solem's (2018) questioning model

The focus of the model is on the teacher and the teacher's intentions with the questions and position about the possible answers. Ulleberg and Solem (2018) relate the axis with the dichotomies closed and open-ended questions and acknowledge fluid transitions between the teacher's expectations to the answer. Some researchers (Aizikovitch-Udi et al., 2013; Chikiwa & Schäfer, 2018; Dahal et al., 2016; Pant, 2019) have accentuated the need to consider questioning as an item for the professional development of teachers. This recommendation is necessary to dispel the myth expressed by Purdum-Cassidy et al. (2015) that teachers think that they can naturally excel in oral questioning without practice. While these recommendations are commendable, it is very important to note that the design features, the length of a professional development programme and other factors go a long way in determining success in what the professional development initiative is intended to accomplish. In a study conducted by Aizikovitch-Udi et al. (2013) a teacher who attended a week long 'question asking methodology' short course did not benefit from the professional development initiative because her teaching was primarily rooted in traditional teaching practice. This study shares the view expressed by other scholars that questioning should be an item for the ongoing professional development programmes to afford teachers the opportunity to hone their questioning skills.

2.6.2. Challenges of classroom questioning

Although there is agreement on the significance of questioning, some researchers (Aizikovitch-Udi, et al., 2013; McCarthy, et al., 2016) have indicated that mathematics teachers are generally not good at questioning and at asking productive questions. The question now becomes: what are the constraining factors that impede productive questioning in mathematics classrooms? Before I respond to this question, I share insights on what good questioning is and how it can be characterised by responding to Watson's (2018) compelling question, "What makes questioning good or bad"? (p. 357). Good or bad questioning can be characterised by three closely related elements which Watson (2018) describes as the questioners' competency, the worthiness of the question and the context in which the question is asked. A competent questioner is proficient in implementing questioning strategies in an instructional practice (Aizikovitch-Udi et al., 2013). Good questioning requires the questioner to make accurate judgment about how to obtain the information that is required. Bad questioning, in contrast, stifles learning by creating confusion, intimidates learners and limits creative thinking (Tofade et al., 2013).

Having explored the attributes of 'good questioning' and what makes it different from 'bad questioning' I attempt to share insights into the question: what are the constraining factors that impede good and productive questioning in mathematics classrooms? An examination of factors that constrain teachers to ask productive questions in mathematics classrooms is necessary here. Several researchers (Aizikovitch-Udi, et al., 2013; Dong et al., 2015; Tofade et al., 2013) have provided different reasons to explain why good questioning is a difficult teaching technique for mathematics teachers. For instance, Aizikovitch-Udi, et al. (2013) contended that, "It is not only good questions that are essential for good teaching, rather, it is good questioning" (p.1). In order to explore this phenomenon, they studied two experienced mathematics teachers who embrace different pedagogical teaching practices (traditional and reform-based teaching).

Their study revealed that the questioning practices and the character of classroom interactions demonstrated by these teachers were tied to their pedagogical practices. It appeared that the question asking methodology did not have an effect on one of the teacher's questioning behaviour. This was evidenced by commitment to detailed explanations, non-negotiability, proving formulae and answering her own questions. For the other teacher whose teaching was characterised by "contemporary reform based" practice (encouraged classroom discussions, re-phrased questions, gave rules and laws limitedly) the primary goal of asking questions was to understand learners' thinking as opposed to targeting correct answers. Aizikovitch-Udi, et al. (2013) draw from this observation that mathematics teachers' pedagogical practices mirror their questioning practice in that a change in teachers' questioning may require a change in their beliefs. The focus of this study is not about beliefs of teachers. Dong et al. (2015) are also aware of the pedagogical tensions that arise when a teacher attempts to implement a questioning strategy to create opportunities for learners to express mathematical ideas while simultaneously ensuring that lesson goals are accomplished. Dong et al. (2015) made a similar observation arguing that a departure from traditional forms of teaching and ultimately questioning to reform-based mathematics curriculum, requires teachers to be prepared to create opportunities for discussions about mathematical ideas. This however is a role which is challenging for teachers to successfully navigate since there is an element of inhibition of learning when the teacher attempts to accomplish the lesson goals.

Questioning has been perceived by some researchers as a form of art (Di Teodoro et al., 2011; Dong et al., 2015; Shahrill, 2013; Tofade et al., 2013). Art is synonymous with creativity and ability. These researchers are of the view that if this art is appropriately mastered by teachers, it has the potential to stimulate learners' thinking. To succeed as a questioner, one must master the art of questioning. Watson (2018) put it this way, "Good questioning is a complex intellectual skill in the sense that it will frequently involve the exercise of prudential, and moral judgments, alongside intellectual ones" (p.356). Put differently, effective questioning is a difficult intellectual ability in that sensible and moral judgments will often be practiced alongside academic judgments. Desli and Galanopoulou (2015) revealed that quality questions are difficult to use, particularly if teachers are not aware of their learners' mathematical knowledge. Nappi (2017) has argued that low order questions are easily accessible and do not pose difficulties for teachers to produce because of their nature. This view is consistent with that of Tienken et al. (2009) when they indicate that, "Productive questions are more difficult to generate in the heat of the moment when teaching" (p.42). Put differently, mathematics teachers lack the ability to pose quality questions (questions that foster understanding) on the spot when the need arises. Nappi (2017) and Purdum-Cassidy et al. (2015) share the view that important as it is, questioning is frequently overlooked in teacher training programmes. Purdum-Cassidy et al. (2015) indicate that failure to pay attention to questioning in teacher training is promoted by a false belief that questioning is a "natural teaching behavior that requires little to no practice or planning" (p.84).

On the other hand, McCarthy et al. (2016) attributed this inability to question well to teachers' lack of pedagogical content knowledge and an understanding of how children think. Teachers play a critical role in ensuring that learners receive quality instruction and must know and understand in detail the mathematics they teach and be able to draw on this knowledge in their teaching tasks with versatility (Ottmar et al., 2015). However, Venkant and Spaull's (2015) literature remind us that most teachers in South Africa are graduates of the education system that they are expected to improve. Despite a revised curriculum that emphasises a need to embrace new approaches to teaching, traditional teaching still dominates many South African classrooms (Chirinda & Barmby, 2017).

Choy and Dindyal (2018) mentioned that teachers who find themselves in examination driven education systems may align their questioning practices to testing as an attempt to prepare their learners for examinations instead of establishing problem solving lessons. It follows that classroom practices can be constrained by an obsession to prepare for examinations. Desli and Ganalapoulou (2015) on the other hand, noticed that some teachers have difficulty implementing, interpreting and responding to learners' responses to open-ended questions because they could not anticipate their responses. This makes it difficult for teachers to react to learners unexpected answers and are therefore reluctant to prepare quality questions that provoke deep thinking.

In their study, Chikiwa and Schäfer (2018) found that teachers developed questions using Home Language phrases with which learners were familiar. While code-switching is generally recommended by research, to aid understanding, it was found to have constrained quality questioning in that it reduced the cognitive level of questions to a lower level (Arias de Sanchez et al., 2018; Chikiwa, & Schäfer, 2018; King & Chetty, 2014). The study recommended that teachers should plan questions in advance and think carefully about ways of including Home Language mathematical terms in their questioning. It can be inferred from the study that code-switching, if not properly done, may have adverse effects on attainments of learning goals.

2.7. DISCUSSIONS AND IMPLICATIONS

Having explored factors that constrain quality questioning as an important strategy for mathematics teaching, do we still wonder why teachers struggle to ask questions that are “mathematically fruitful and pedagogically effective”? (Mason, 2020). Are our visions blurred to see why many classrooms produce learners who are unable to exercise intellectual autonomy because their brains were never sufficiently stimulated? (Watson, 2018). Indeed, the demands that come with productive questioning are quite confronting and need attention. Literature has demonstrated that teacher questioning practices are nested within a wide-ranging network of issues that embody teacher quality such as pedagogical content knowledge Venkat and Spaull, (2015) and McCarthy et al. (2016); understanding learner thinking McCarthy, et al. (2016); understanding learners’ mathematical knowledge Desli, and Galanopoulou (2015), and pedagogical practices (traditional and reform-based teaching) Aizikovitch-Udi, et al., (2013). Classroom questioning is clearly intended to stimulate productive classroom talk, but may either constrain or enable learning opportunities, depending on how the teacher is grounded in all the other elements that characterise teacher quality

2.8. CHAPTER SUMMARY

The literature reviewed provided relevant information on questions and questioning in mathematics classrooms. The constructs, question, questioning and question attributes were interrogated, and explicit attention was paid to their role in stimulating productive discussions and development of deep understanding. Central to the conversations around the subject of classroom questioning is assessment *of* learning as opposed to assessment *for* learning. The focus of curriculum policy seems to be that of assessment for learning since it is made much more explicit in the CAPS. It is therefore crucial that teachers’ pedagogical content knowledge be developed during teacher training and continuously be included in on-going professional development programmes. Discussions throughout the chapter have demonstrated that the attributes of mathematics teachers’ questions cannot be meaningfully considered without the role of the teacher as a questioner. Just as questions and questioning have an important role to play in cultivating the habits of inquiry in mathematics classrooms, the role played by the users of these important pedagogic tools should be clearly defined and understood. More importantly, how these questions can be characterised, is an important consideration in this study. These discussions contributed towards answering the key question that guides the study.

From the perspective of the South African mathematics policy and the framework, questions are tools that can be used to advance logical and critical thinking, develop conceptual understanding, and prepare an individual for critical citizenship. In the preceding discussions, attempts were made to respond to the question: Why do teachers' questions in mathematics classrooms matter? The question was responded to from the point of view of the curriculum CAPS, (2017) and the framework for Mathematics Teaching, (DBE, 2018). Good questioning is a characteristic of good teaching Nappi (2017) which is heightened in the curriculum and the framework for mathematics teaching.

CHAPTER THREE: THE EVOLUTION OF LESSON STUDY

3.1. INTRODUCTION

This chapter presents a review of literature on LS. LS is a context within which this study is located. For this reason, this chapter is specially dedicated to a review of literature on LS to give a reader an in-depth understanding of what it is and how it works. The chapter outlines historical origins of LS, mechanisms by which this teacher-led professional development model improves instructional practice. The chapter further explores the variations of LS across different cultures and it then goes on to provide research done in South Africa in relation to this teacher professional development model. The cultural assumptions that underpin the implementation of LS are well examined. Finally, an overall summary of the chapter is provided.

3.2. THE ORIGIN AND BACKGROUND OF LS IN JAPAN

The education system in Japan survived through political circumstances which were unwelcomed by the society during the Meiji era (Cheng, 2018; Kuno, 2017). Meiji is the era that was characterized by rapid modernization of Japan. The idea of state control was applied across all societal spheres i.e., politics, and economics, to society and education (Kuno, 2017). Educational reforms implemented during the Meiji era were intended to westernize Japan (Arani et al., 2010; Ishii, 2017; Fernandez & Yoshida, 2012). Elementary schools became oriented towards European and American models because of these reforms. Another goal of the state was to achieve economic power and as such, all the features of education i.e. its aims, content and methods were designed to achieve that end (Kuno, 2017). It can be inferred that the education system was also a means to reinforce the political agenda of the state.

3.3. LS DURING THE MEIJI ERA

LS emerged during the Meiji era of Japan (1868–1912) as a practice to promote the professional development of teachers (Cheng, 2018; Fernandez & Yoshida 2012; Maeda & Ono, 2019; Pjanić, 2014). LS in Japan was initiated by bureaucrats whose main objectives were to modernize Japan and bring it out of its long period of isolation (Fernandez & Yoshida, 2012). Tokyo Normal School University saw its emergence during the 1870s (Makinae, 2010; Pjanić, 2014). During the Meiji era, the national government developed a national curriculum which was to be implemented by all the schools in Japan. By implication, local administrators and teachers were required to accept the political and ideological values of the Meiji government as a point of reference for the modernization of Japan (Arani et al., 2010; Cheng, 2018). The primary concern for the state was for schools to implement the curriculum that was bureaucratically imposed (Cheng, 2018) to advance its political agendas (Kuno, 2017). The concern for teachers however was professional accountability, professional autonomy and most importantly to create a climate where professional dialogue about issues that surround their practice could be established (Cheng, 2018; Fernandez & Yoshida, 2012). Cheng (2018) describes these opposing views between teachers and the bureaucrats as a pendulum swing between professional accountability and state accountability in developing curriculum at a micro-level, i.e., school-based curriculum and the national curriculum. The limitations of the bureaucratically imposed curriculum of Meiji education methodologies which led to teachers' resistance are captured in the following words:

> Early Meiji educational methodology was concerned above all with the process of teaching lessons in the classroom. The teacher stands up straight in front of the wall map or blackboard, with pupils seated in an orderly fashion before him. Pupils respond to instructions and questions that are repeatedly posed by teachers. Western, particularly American, textbooks were translated for use in the classroom, and classes were conducted in a uniform manner regardless of the subjects being taught (Arani et al., 2010, p. 173).

The views expressed here clearly demonstrate a typical behaviourist approach to learning (Penazzi et al., 2022) characterised by control to learning by the teacher, leaving little or no space for a learner to construct meaning (Skovsmose, 2018). This lecture form of instruction was introduced by Scott, an American educator, with the intention to introduce and propagate the western-influenced whole-class method of lecturing in elementary schools (Ishii, 2017). This instructional technique was to be infused into the LS practice despite the fact that it had adverse effects. Teachers were mandated to attend the organized training courses which were conducted outside the school premises (Cheng, 2018). Teachers were left without much choice but to attend training despite the fact that in their view they were not promising to develop them professionally. This training was not holistic in that it focussed only on developing their abilities as individual teachers rather than on examining many other factors that could help the teaching process (Yamasaki & Kuno, 2017).

To exemplify how controlling and bureaucratic the Meiji government was, guidelines for lesson preparations were issued by The Ministry of Education for Elementary School Lessons (*Shoggakou kyousoku taiko*) in 1891 (Fernandez and Yoshida, 2012). These guidelines instructed school principals to prescribe a lesson design (*kyoju saimoku*) which teachers were supposed to implement unquestioningly when preparing their lesson plans. The methodological approach which was prescribed for controlling the teacher's didactic approach comprised five steps namely: preparation, presentation, comparison, integration and application. Fernandez and Yoshida (2012) noted that this rigid and formal approach did not appeal to teachers not only because it limited their flexibility and creativity but also because it did not afford them the opportunity to stimulate learners' intellectual abilities. Schools were now formalized as educational institutions where teachers were able to receive training in-school during the Meiji period. More focus was placed on prior preparation with post-lesson discussions focusing primarily on critiquing teaching skills. In contrast, the primary focus on LS was on how children react to instruction and instructional material and most importantly on how they learn.

However, schools increased in number and this made it difficult for authorities to centralize control of the education system. Power was then decentralized to schools and more authority was delegated to school principals (Cheng, 2018; Fernandez & Yoshida, 2012). According to Kuno (2017) schools were now able to develop their own mechanisms to develop teacher professional learning practice through teacher meetings. Meetings conducted by teachers to discuss professional matters and pedagogical issues, became the earliest form of LS. Schools became formal institutions where LS (which teachers embraced) became a platform for professional dialogue. The government remained uncomfortable with the idea of centralizing power to school principals (Arani et al., 2010; Cheng, 2018; Fernandez & Yoshida, 2012). An accountability system was introduced because of this discomfort. The bureaucrats, who were the school inspectors, from the Ministry of Education monitored compliance and supervised the practices of local school districts and oversaw the implementation of the prescribed Herbatian[1] teaching methodologies (Fernandez & Yoshida, 2012). Schools

[1] A teaching approach named after Herbat used for controlling teachers' pedagogic approach with formalized sequence of steps.

saw the opportunity to extend LS practice by introducing open class lesson observation as a way of responding to the new inspectorate system introduced by the government. This implied that inspectors would visit schools to monitor compliance in each subject. The scope of LS broadened to integrate the development of teaching materials (*kuozayi kenkyu*), critical discussions of lesson implementation, classroom visits, and the interrogation of subject knowledge (Cheng, 2018). A live research lesson which utilized the live classroom experience of teachers and learners became a centerpiece of LS (Arani et al., 2010; Lewis, 2011). The uniqueness of this educational practice enabled teachers to discover themselves as practitioners who perceived themselves as researchers with educational missions (Pjanić, 2014) and an underlying endeavour to improve learners' outcomes (Doig & Groves, 2011).

3.4. LS DURING THE TAISHO ERA

Taisho era (1912-1926) marked a paradigm shift from the Meiji period which was associated with bureaucratic control of the state (Cheng, 2018; Fernandez & Yoshida, 2012). There were drastic cultural and educational changes as a result of "Taisho Freedom Movement". This era was generally associated with freedom, new ways of thinking and strengthening of social movements, hence the appearance of the Free Education Movement. To the Japanese teachers, the Taisho period presented the opportunity to re-think and reconstruct the LS model by focusing on the development and dissemination of lesson methods (Cheng, 2018; Fernandez & Yoshida, 2012). The revised LS model of the Taishō period emphasized critical feedback on lessons, assessment of lesson plans, and reflective practice (Fernandez & Yoshida, 2012).

The idea of promoting a child-centered 'liberal' education emerged in the Taisho Period (Cheng, 2018; Fernandez & Yoshida, 2012). The method of teacher training was also reviewed to improve teaching. The student-centred movement was put in place in laboratory schools i.e. schools attached to teacher training colleges. The prescribed Herbastian didactic methods which were imposed during the Meiji era to control the process of instruction were abandoned by teachers (Fernandez & Yoshida, 2012). The records of educational practices were replaced by the live classroom experiences of teachers and learners.There were positive changes as a result of the Taisho freedom movement (Cheng, 2018). Teachers gradually began to form their own culture and curriculum that was different from the national curriculum. Most importantly LS constantly featured in their curriculum. Records of lesson plans were highly detailed so that others could easily review their classroom practices. Generally, teachers began to exercise autonomy in the way they managed the curriculum and how they collaboratively created practical pedagogical knowledge.

At the beginning of 1920, LS became popular and active in schools and good teaching practices were reported and documented in lesson demonstrations (Cheng, 2018). LS was used by teachers to determine how they could incorporate the advantages of demonstration lessons into knowledge transfer within their schools. The notion of setting objectives, analyzing the teaching and learning process or developing theory from findings to improve teaching further was rarely given consideration by the teachers. The Ministry of Education reversed previous policies in 1958, returning to the spoon-feeding approach to fill up children with knowledge without interrogating it (Cheng, 2018). These attempts were rejected by The Japan Teachers' Union with a growing determination to develop its own curriculum and to develop professional practice to counterbalance bureaucratic control. The soul and spirit of LS prevailed in the way teachers exercised their professional autonomy to compensate for the bureaucratic control of the national curriculum guide (Cheng, 2018).

It is interesting to note how Cheng (2018) invokes the metaphor the 'soul of LS'. This phrase captures the essence of what made LS survive in Japan despite bureaucratic controls. Just as the soul and spirit sustain life in humans, Japanese teachers' commitment, passion and their quest to attain professional autonomy acted as sustenance mechanisms for LS to continue to exist. These qualities fueled teachers' desire to remain self-directing about matters that concerned their practice. They tirelessly engaged in discussions and debates in which their conceptions of teaching and learning were taking shape. Cheng (2018) refers to this engagement and passion as the soul of LS. These sentiments are echoed in Fujii's (2018) statement, "For Japanese educators, LS is like the air they breathe" (p.1). These comments exemplify how LS became and is still part of everyday and professional life of Japanese teachers. Despite the strong bureaucratic pressures to enforce the national curriculum, teachers did have the opportunity to exercise professional autonomy to review that curriculum (Cheng, 2018). Research activities were initiated by the newly set non-government professional education bodies in different fields. Teachers voluntarily participated in LS meetings sponsored by these bodies and discussed professional issues with other teachers during summer vacations (Cheng, 2018; Fernandez & Yoshida, 2012; Kuno, 2017). The concept 'LS' (jugyō kenkyū in Japanese) was officially accepted in the 1960s, the time at which school curriculum guidance developed through the training and research activities was widely adopted by schools in Japan (Cheng, 2018). Today LS is associated with teacher professional autonomy because of the efforts taken by Japanese teachers in shaping it to 'match' and 'fit' their vision of developing their abilities and qualities as teachers. Cheng (2018) explains that it is the reason why the professional accountability of Japanese teachers is more significant than the state accountability of Japan's Ministry of Education in ensuring quality education.

LS was not abandoned in the 1970s when the country experienced a decline in academic achievement and an increase in juvenile delinquency (Cheng, 2018). This signaled a need to reflect and review its role. A guide for streamlining LS practices was developed and published because of that review. However, the guidelines were lacking in terms of what the teachers deemed necessary for a coherent guide. The guidelines did not incorporate empirical or theoretical support and were rather too procedural. The pendulum once again swung between the two poles of professional autonomy and bureaucratic control: LS was systematized and theorized after the 1980s (Cheng, 2018). Efforts to rejuvenate LS were intended to theorize it as education science so it could be used to reorganize the school system. LS was eventually more acceptable to all the stake holders including teachers, school administrators, researchers and policy makers. Cheng (2018) clarifies the two paths which were advocated to achieving the reconstruction that resulted. First, teaching practice was viewed through a research lens by teachers - the central focus of LS. The second was to capacitate teachers (through training) on how to implement this research-based LS.
LS became a common school practice in Japan in which teachers participated to improve classroom practice. It involved formulating broader objectives of a teaching and learning plan, implementation of the plan and checking with the external experts to reflect on the plan i.e. to see whether the plan has solved problems.

3.5. DISCUSSIONS AND IMPLICATIONS

If compelled to characterize in a single phrase the journey traveled by Japanese teachers to attain professional autonomy, I will describe it as a movement from epistemological concerns (i.e. issues of knowing and the creation of knowledge) to broader ontological considerations (i.e. issues that concern the emergence of individual and collective identities). Japanese teachers' desire to develop pedagogical knowledge and practical wisdom and the pursuit to establish their own professional identities without obeying technical prescriptions of teaching by the bureaucrats, was a driving force in their journey. LS in Japan endured social and political forces that threatened teachers' efforts to acquire professional autonomy before and after the Meiji era (Cheng, 2018; Fernandez & Yoshida, 2012). These forces were overcome by teachers' strong sense of community (Cheng, 2018) coupled with the underlying endeavour to improve learning outcomes of their students (Doig & Groves, 2011). Their unshakable conviction in professional accountability represented stances and positions on the value they attached to a professional learning community in which their identities evolved to fit their visions of developing their abilities and qualities as professional teachers.

Indeed, Cheng (2018) was correct to say, "The effectiveness of this transplantation of LS depends on the 'soul' as identified in Japan: the level of professional autonomy teachers were willing to fight for, and the practical wisdom exercised by teachers in the interests of students" (p.12). This quotation triggers several significant questions which may be directed to LS communities elsewhere. For instance, can the LS efforts in other cultures demonstrate this strong sense of voluntary commitment to improve their own abilities and quality as professionals? Is the 'soul' and 'spirit' of LS prevailing in cultures where the seed of LS is already planted? To what extent can teachers in other LS communities assume full responsibility of their own professional learning?

I encourage teachers preparing to enter this journey of professional learning and willing to transpose this model to their own cultural contexts, to be prepared to wrestle with these questions. Those who may have already begun the journey and are willing to pause and reflect introspectively may have to consider these questions as a point of reference. LS teams anywhere in the world and in South Africa in particular, have to respond to the questions I raised with honesty and conviction if they are to make significant strides in the successful implementation of this teacher-led professional development model. Indeed, the implementation of LS by adoptive cultures should be done with the focus on its core vision and principles and not "without the 'soul' of its originating country" (Cheng, 2018, p. 24).

LS in Japan evolved through many lifetimes expanding and progressing, experiencing the twists and turns as the pendulum swung between professional and state accountability (Cheng, 2018). But in all these times, the Japanese teachers' vision of developing their qualities and abilities as professionals was never obscured by a bureaucratic control system to obey a national curriculum guide (Cheng, 2018; Fernandez & Yoshida, 2012). If we follow the Japanese teachers' example, it seems reasonable to say, although professional development is achieved through collaboration, it remains to be a personal matter. It requires individual and personal commitment to the course. Professional development cannot be done for teachers, rather it is a personal journey of self-discovery and self-actualization driven by the obligation and quest to improve one's own quality and identity as a professional.

A century-long journey of perfecting LS in Japan should be perceived by adoptive cultures as a classroom encounter because this rich history has important implications for those who are willing to enter this journey of professional learning. Kuno (2017) expressed it elegantly, "…an ambivalent passage between freedom and control is not limited to history, but it is equally possible in our time" (p.12). Implied here is that communities that are willing to transpose this model into their own contexts should develop mechanisms to withstand challenges that may arise and threaten LS efforts to improve teaching practice.

3.6. LS AS A TOOL FOR IMPROVING THE QUALITY OF INSTRUCTION IN MATHEMATICS

The results of teachers' actions and prolonged engagements in the LS culminates into what Pjanić (2014) refers to as the products of LS. One of these products that emerged from LS is a theory of mathematics teaching referred to as Problem Solving Approach (Fuji, 2018; Pjanić, 2014). This approach is a theory of teaching about learning commonly used in Japan. The intended goal of this approach is the ability of learners to learn mathematics independently.

LS became a tool for problem solving in Japan (Coenders & Verhoef, 2019; Fuji, 2018; Pjanić, 2014; Takahashi et al., 2013;). Mathematics research lessons in Japan follow a certain path, referred to by (Fuji, 2015) as structured problem solving. This path focusses on a single task which unfolds in four stages i.e. presentation of an identified problem for the day (5 to 10 minutes), problem solving by the learners which takes (10 to 20 minutes), followed by comparing and discussion stage (neriage) which lasts for (10 to 20 minutes). Lastly there is a summary of a lesson by the teacher (*matome*) which lasts for 5 minutes. Fuji (2018) uses the phrase, “LS and problem solving: the two wheels of a cart” to exemplify the strong connections that exist between LS and the Mathematics Problem Solving Approach in the Japanese context (p.1). This description uniquely captures the inseparable connections between LS and the Problem-Solving Approach in Japan.

Teaching mathematics through the Problem Solving Approach is achieved through a collaborative process whereby teachers collectively engage in the preparation of a mathematics problem solving lesson, sequence of lessons or a selected topic (Fuji, 2018; Lewis, 2016). This planning is characterised by exceptionally well-thought-out tasks (Lewis, 2016), coupled with prediction of learners’ possible responses and reactions to a problem-solving task (Huang & Shimizu, 2016). The planned lesson is then presented while other teachers who co-planned the lesson observe and collect information about their general impressions and how the lesson impacted the learners. The presentation of a lesson is followed by a post-lesson review to engage in discussions about what transpired, how the lesson impacted learners and to note areas that need improvement (Doig, 2013; Fuji, 2015; Lewis, 2016). Eventually teachers produce a written report of the valuable lessons they have gathered from a research lesson in relation to their research question which served as a guide for what they intend for their learners.

In Japan, “presenting the problem” means helping learners understand the context of the task and what it will mean to solve the task, but it specifically excludes any exposition by the teacher about how to solve the task. Instead, learners are expected to work independently on the task for 10 to 20 minutes, during which time at least learners solve it. The Japanese have special pedagogical terms which describe teachers’ key roles such as *hatsumon, kikan-shido, neriage, bansho* and *matome* (Pjanić, 2014). At a particular point during a lesson a teacher poses a well-crafted thought-provoking question or problem with which learners are expected to grapple. This question is referred to as hatsumon (Doig & Groves, 2011; Fujii, 2019; Pjanić, 2014). A careful selection of *hatsumon* is intentionally done to initiate a mathematically productive activity (Doig, 2013). Presentation of *hatsumon* or problem is followed by individual or groups solving the problem while the teacher engages in *kikan-shido* which Pjanić

(2014) defines as, “An instruction at students’ desk, the one-to-one discussion” (p.89). It is however important to note that there is little instruction done by the teacher while doing *kikan-shido* as he/she walks around the classroom since the main goal is to carefully observe how learners are responding to the task, take note of responses that were anticipated and provide support where necessary (Doig, 2013; Doig & Groves, 2011). The teacher may or may not talk to learners while moving around. The teacher also observes different strategies from different learners.

The third phase, neriage, assumes that students will arrive at different solution methods, which are then compared and discussed for the purpose of helping all students learn new mathematics and ways of thinking. Thus, the task should be understandable by the students with minimal teacher intervention; it should be solvable by at least some students (but not too quickly), and it should lend itself to multiple strategies. In the fourth phase, matome, the teacher may say something about which strategy may be the most sophisticated and why, but it should go beyond that to include comments by the teacher concerning the mathematical and educational values of the task and lesson (Fujii, 2018).

3.7. VARIATIONS OF LS MODELS IN SELECTED COUNTRIES

If I can reiterate Ebaeguin and Stephens’s (2014) words, “It is naïve to imagine that a program developed in one culture can be employed in a different culture without adaptation” (p.3). Different countries have made adaptations on the original version of Japanese LS to suit their own cultural needs. This section briefly outlines how four different countries have adapted the Japanese LS model to implement in their own cultural contexts. Furthermore, LS is an overarching context within which this study is located. For that reason, I go further to provide details of the LS model adopted and used in South Africa and how it fits within the broader framework of this study. LS has evolved over time and the countries chosen in this study are merely a demonstration of how each one adapted this teacher-led professional development model. I have considerdered the versions of LS in different cultural contexts, namely, Denmark (Europe), Indonesia (Asia), Zambia (Africa), and South Africa (homeland) as representative of non-Japanese cultures. The choice of these countries was motivated by their unique LS models which they adopted.

According to Winsløw et al. (2018) LS in Denmark is conceptualised as a system of paradidactic situations whereby teachers interact within a didactic situation. The paradidactic system consists of the teachers taking part in the LS, along with the numerous didactic stages. Thus, in the context of countries in Denmark, as Winsløw et al. (2018) outline, the LS stages conventionally labelled plan, observe and reflect have been re-named as predidactic situations (PrS), observational situations (ObS) and postdidactic situations (PoS) respectively. LS serves as a platform for generating paradidactic knowledge, what to predict, observe and analyse in a multi-cycled LS. The LS cycle in the Danish context unfolds in these steps.

The LS cycle in Indonesia unfolds in a three step model broadly defined as *plan*, *do* and *see* (Ilma, 2011). The first stage in a cycle is *Planning.* Key things are given consideration during planning stage in the Indonesian context which are Realistic Mathematics Education (RME), Constructivist perspectives and contextual teaching and learning. The second stage is *Implementation and observation* stage during which the identified teacher implements the planned lesson while others observe and take notes during classroom interactions. The third stage in the cycle is *Reflection*. Teachers and other experts who observed the lesson share data they collected on learners' learning. Ilma (2011) used the model to examine the performance of mathematics primary school teachers in developing and implementing teaching material. Findings revealed that LS was useful in helping teachers develop the skill of designing and implementing teaching materials.

Jung et al. (2016) reported that the LS cycle in Zambia is carried out in eight stages which include: (1) Defining a challenge; (2) Planning of a demonstration lesson; (3) Implementation of a demonstration lesson; (4) Discussion and reflection; (5) Revise the lesson; (6) Conduct the revised lesson; (7) Discuss lesson and reflect on its effect ; and finally (8) Compare and share reflections. After the fourth stage the process is repeated within one cycle where lesson improvement is done based on the comments made during the reflection meeting. According to Jung et al. (2016) the eight-step model of LS is perceived unique to the Zambian context to localize the programme, and to give teachers a time intensive teacher improvement model. The goal is to change a culture of teachers and teaching through a prolonged LS cycle.

The LS model which is currently used in South Africa is characterised by five stages namely *diagnostic assessment/analysis*, *lesson planning*, *lesson presentation* and *observation*, *post-lesson reflection* and *lesson improvement* stage (Sekao & Engelbrecht, 2021) (see Figure1). Diagnostic analysis is a distinct feature in the LS model presented and that is what makes the South African LS model unique.

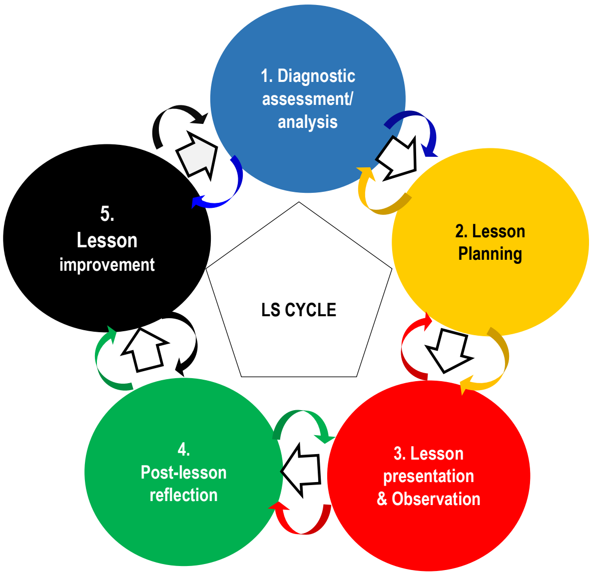


Figure 3.1: LS model in SA (source Sekao & Engelbrecht, 2021)

The LS model which is implemented by most schools in South Africa affords teachers the opportunity to experience the unique feature of each stage of an LS cycle (Sekao & Engelbrecht, 2021). Adaptations of the Japanese LS in the countries is driven by the focus in each country. LS in South Africa is gradually gaining momentum and as expected, researchers continue to make efforts to explore how useful LS can be in enhancing the teaching of mathematics in continuing professional development programmes and in pre-service teacher education. Sekao and Engelbrecht (2021) explored the views of primary school teachers, who engaged in an LS project in South Africa.

Ono and Ferreira (2010) examined how instrumental LS can be in establishing a school-based in-service system to accelerate the quality of teaching in mathematics and science. Findings revealed that teachers who participated in the study improved their mathematics lessons. Mhakure (2019) and Posthuma (2012) explored how LS can be used as a tool for enhancing school-based Continuous Professional Development for teachers who are located in rural disadvantaged areas. Their studies posit that implementation of LS as a Continuing Professional Development can assist teachers in rural areas to develop teachers' Mathematical Knowledge for teaching (Mhakure, 2019) and can enhance reflective practices of teachers in rural settings (Posthuma, 2012). Research on the effectiveness of LS has also been extended to pre-service teachers, particularly on the lesson reflection experiences of pre-service teachers (Botes et al., 2022). Findings revealed that the LS approach enabled the team of prospective teachers to engage in collaborative practices which afforded them the opportunity to share creative lesson ideas in a small-group format. Biccard (2020) explored how an LS oriented professional development programme can be instrumental in developing mathematics teacher-noticing in primary schools. Teachers who participated in the study expressed their appreciation for the time they were given to collectively observe and take note of how students were learning mathematics.

Adler and Pournara (2019) explored how useful LS can be in providing a supportive environment for professional development practice where the notion of examples in mathematics (exemplification) is heightened. Conversational reflections revealed that the LS approach provided an enabling environment on the choice of mathematics examples, reasons for choosing those examples and how to utilize them. While LS in South Africa is gaining traction, more research is still needed to focus on areas that concern the quality of mathematics education. Moreover, little or no attention has been paid to how LS teams consider questions and questioning. This study illuminates this missing point.

3.8. TRANSPOSING LS TO DIFFERENT CULTURES

It is important to acknowledge that cultural backgrounds can either promote or hinder successful implementation of LS in non-Japanese contexts. Cheng (2018) uses the metaphor of a plant and its soil arguing that there is a need to prepare the soil that LS requires to enable the seeds of LS to successfully germinate and grow. These comments have implications for the cultural orientations on which Japanese LS is transposed. Researchers have reported cultural barriers that impeded successful implementation and issues of sustainability of LS. have been reported (Ebaeguin & Stephens, 2014). Teachers are perceiving LS as an extra responsibility which consumes their time (Sekao & Engelbrecht, 2021). Huang and Shimizu (2016) categorised cultural obstacles broadly into two categories i.e. macro-level conditions, which encompasses education systems (constraints including cultural values, the culture of teaching and learning, teacher professional development systems, leadership from district officials). At Macro level challenges include attentiveness to developing quality lessons, teacher pedagogical content knowledge and mindset of developing plans for lessons. These issues in general act as impediments for the successful implementation of LS.

LS teams in non-Japanese cultures not only need courageous undertaking with unshakable commitment to implement LS but also need to understand what it is and why it has been a useful social fabric and a timeless tool of professional culture to the Japanese teachers. Foreign implementers will need the resilience demonstrated by Japanese teachers when the winds of change were blowing. Non-Japanese cultures need to understand that LS is not a quick fix, because the results take time to manifest (Ebaeguin & Stephens, 2014).

3.9. CHAPTER SUMMARY

This chapter explored historical origins of LS in Japan. The chapter shed light on valuable lessons that can be drawn from the resilience of Japanese teachers as LS evolved. Without engaging in revolutionary energies, Japanese teachers demonstrated unwavering commitment to the professional development model in which their identities as professionals were taking shape. The chapter further explored mechanisms by which LS improves instructional practice in mathematics. LS models across different cultures were discussed in this chapter to give a glimpse on how adaptation takes place across cultures. Finally, challenges involving ‘cultural fit’ of LS in non-Japanese cultures were briefly discussed.

CHAPTER FOUR: THEORETICAL ORIENTATIONS

4.1. INTRODUCTION

This chapter presents a theoretical orientation of the study. The chapter chronicles the origin and background of the Variation Theory - the theory that guides the study. The theoretical orientation provided is used to demonstrate how compatible Variation Theory is to this study and how relevant it is in responding to the research questions. Discussions throughout this chapter broadly explore the intersections of Variation Theory and a phenomenon of questioning. The chapter culminates in a detailed conceptual framework which brings together all three dimensions that constitute the conceptual framework, i.e., Variation Theory (theoretical lens), LS (the context of the study) and Emanuelsson's categories of classroom interactions.

4.2. ORIGIN AND BACKGROUND OF VARIATION THEORY

Variation Theory evolved from phenomenographic research tradition (Pang, 2003). Pang describes how the concept phenomenography was derived asserting that it comes from Greek words 'phainemon' and 'graphen' which imply appearance and description. In the 1970s a group of educational researchers at the University of Goteborg in Sweden developed phenomenography with an attempt to respond to questions, "(1) 'What does it mean that some people are better at learning than others?' and (2) "Why are some people better at learning than others?" (Pang, 2003, p. 146). Marton (2015) maintains that individuals have unique ways of experiencing the same phenomena differently. Central to phenomenographic research is to recognize and identify the differences in experiences that a particular group of people has of a particular occurrence (Bussey et al., 2013). Phenomenography was later criticized by researchers in the 1990s, arguing that it is rather too descriptive and theoretical (Bussey et al., 2013). The argument they put forth was that phenomenography and its methodological stance could be used to identify and describe series of experiences a particular group of people had with a given occurrence, however it could not be used to give details of why the differences in experience existed. This led to the development of what was referred to as new phenomenography (Pang, 2003), which marked the shift from "methodological to theoretical questions, and characterized a way of experiencing something in terms of the critical aspects of the phenomenon as discerned by the learners" (p.145). Variation

Theory was then conceptualized as a more theoretical dimension of phenomenography in that it made effort to elucidate how people (learners in particular) can understand the same phenomenon differently and how that knowledge can be used to advance instruction (Bussey et al., 2013).

4.3. THE BASIC ASSUMPTIONS OF VARIATION THEORY

The line of reasoning pursued by Variation Theory is that to learn, we have to make nuanced judgement (discern) about the properties of a phenomenon being studied (Bussey et al, 2013; Lo, 2012; Marton, 2015). To discern the intended ideas learners must be presented with carefully structured variation, to notice what is to be learned. The presence of variation makes a possibly observable difference within or between the critical features of a phenomenon (Bussey et al., 2015). Variation Theory helps us to understand why some teaching enactments help or do not help learners learn effectively (Lo, 2012). This is attributed to the kinds of variations that are being enacted in the classroom. Marton (2015) and Lo (2012) explain this by saying that there are certain conditions that are necessary for learning.

Experiencing variation against a background of invariance is central to the Variation Theory (Lo, 2012 & Marton, 2015). Marton (2015) expresses it in this way, "In order to separate something from something else, you must experience variation (difference) in the former, against a background of invariance in the latter" (p. 56). Put differently, meaning is properly conveyed only when there is sufficient differentiation between two concepts. The kind of learning dealt with in Variation Theory is that of being able to tell things apart (making distinctions) (Marton, 2015). Making a distinction is thus a very important aspect in variation theory because without attending to it the learning aimed at cannot happen. For instance, to understand what linear equations are, we can compare them with quadratic equations.

4.4. DEFINING KEY CONSTRUCTS OF VARIATION THEORY

Variation theory has key pedagogical constructs which must be clearly understood by a reader. Practical examples are used to explain these key pedagogical terms.

4.4.1. Critical features and critical aspects

Lo (2012) differentiates between critical aspects and critical features as follows. “Critical aspects refer to a dimension of variation, whereas critical features are a value of that dimension of variation” (p. 65). For example, the number 250 000 has many features i.e. it is: composite, even, a perfect square and it can be expressed in a decimal form as 250 000.0. To understand an object from a particular point of view only features that correspond to that view are critical (Bussey et al., 2013; Lo, 2012, Marton, 2015). For individuals seeking to write the number in a scientific notation, a way of seeing 250 000 as 250 000.0 becomes a critical feature in the context of scientific notation because of the requirement to figure out the number of place values in between the decimals and the first non-zero digit which in this case is 2. If mathematics teachers follow this example, they can help learners make judgements of what is considered critical and key in the description of a number that meets the criteria to be written in scientific notation. This is so because unless teachers are fully aware of the concept of magnitude or size of numbers (critical aspects) they cannot talk about big number or small number (critical features).

4.4.2. Discernment

To discern is to see or recognize clearly. Discernment is defined by Bussey et al. (2013) as “...the ability to hold an aspect of a phenomenon in focal awareness and contrast it with its environment in order to construct meaning for that aspect and, subsequently, for the phenomenon” (p. 11). In other words, humans can recognise or see clearly if their consciousness is fixated on an aspect of an occurrence and if they were able to make distinction between the occurrence and its environment to construct meaning. From the perspective of Variation Theory, meaning does not arise from sameness but rather from difference, i.e., difference rather than sameness is key (Lo, 2012 and Marton, 2015). For example, in a pedagogical situation designed to help learners recognize the effect of gradient on a linear graph, the critical feature (gradient) can be varied (contrasted) while the y intercept is kept constant (invariant) so that a dimension of variation (effect of changing gradient) can easily be noticed by learners. Marton (2015) asserts that awareness of a single feature cannot exist without the awareness of differences (variation) between features: there can be no discernment without experienced difference, and there can be no experienced difference without a simultaneous experience of at least two things that differ.

4.4.3. Simultaneity

Teaching and learning by embracing the insights of Variation Theory is described as changes in the capability of seeing a phenomenon in certain ways, in which a way of seeing can be defined in terms of the aspects that are discerned and attended to concurrently at a certain point in time (Xu, 2019). Awareness of a single feature by an individual learner cannot enable them to fully understand a phenomenon (Lo, 2012; Marton, 2015; Xu, 2019). Learners must be simultaneously aware of multiple features of the phenomenon and able to discern the phenomenon from its environment. When a person is capable of being simultaneously and focally aware of more aspects of a phenomenon, then from the Variation Theory perspective that person is said to have learned (Lo, 2012). To fully understand an object of learning, one must discern all of the critical features and their relationships simultaneously. Simultaneity then refers to concurrent discernment of the critical features and their relationship to develop awareness of the parts and the whole.

4.4.4. Object of learning

Teaching is not only about attending to people's cognitive needs and experiences, but it has always been about 'something'. This view is confirmed by Bussey et al. (2013) when they indicate that the pursuit of learning implies that there is something to be learned. From the perspective of Variation Theory this 'something' is referred to as the *object of learning* (Bussey et al., 2013; Huang and Li, 2017; Lo, 2012; Marton, 2015). The object of learning is the centrepiece of Variation Theory, and it is what makes this theory different from all the other theories of learning (Bussey et al., 2013). The object of learning attends to the question of, "What is to be learned?" in three ways: it defines (1) the content, (2) the educational objective, and (3) what needs to be learned (critical aspects) (Kullberg et al., 2017; Lo, 2012; Marton, 2015). Variation theory focuses on the object of learning and is interested in how learners' experience and understand it (object of learning).

Variation Theory emphasises that for a genuine mathematical experience to occur, conditions of learning must be clearly spelled out (Lo, 2012; Marton, 2015). From the perspective of this theory, learning failures are described in specific ways: when learners do not learn what was intended, and they have not discerned the necessary aspects of the object of learning (Marton, 2015). Researchers (Bussey et al., 2013; Lo, 2012;

Marton, 2015) differentiate between the object of learning and the learning objectives. Bussey et al. (2013) caution us not to mistake an object of learning for a physical object, but rather to understand that it is a target concept, phenomenon or experience that revolves around a learning event. Lo, (2012) defines the object of learning as, "...what the students need to learn to achieve the desired learning objectives" (p. 43). This definition of the object of learning is contrasted with the learning objective which Lo (2012) describes as statements of what learners are expected to learn at the end of the learning process and are pre-determined. The object of learning is not the same as the notes, texts, or teaching materials that teachers use while teaching (Lo, 2012).

These descriptions of the object of learning and the learning objective help us to understand what the object of learning is and what it is not. Although the learning objectives stipulated in curriculum policy statements serve as important references for the teacher, they cast a shadow of doubt since they only point to the result, and as such tend to undermine the dynamic nature of the object of learning (Marton, 2015). The striking difference between the object of learning and the learning objective sharpens our awareness of what we should do as teachers to develop the capabilities we desire in our learners. The critical role of the object of learning is captured in the following words:

> Learning must be directed towards an object (i.e., an object of learning), and so even if the learning environment is luxurious and high tech, the teachers are kind and caring and the students highly motivated, if the object of learning is very complex and difficult, learning is still unlikely to take place without the teachers' help to tease out the critical aspects and make them available for students (Lo, 2012, p. 4).

Mathematics lessons embedded in games, flashy digital devices, attractive digital presentations or even a sense of humour from the teacher are not substitutes for a well-thought-out lesson where the object of learning is made explicit. All these will draw learners' attention but may not satisfy the conditions of learning (Marton, 2015). I am not suggesting that digital devices, humour, attractive digital presentations, and mathematical games are inappropriate or irrelevant in a lessons, rather I am implying that if what is intended for learning is not well communicated, learning will still not take place. The flashiness, humour and games will not attend to learners' cognitive needs to make sense of the world.

4.5. DIFFERENT KINDS OF THE OBJECTS OF LEARNING

An examination of the object of learning in Variation Theory is done from three different perspectives which are discussed here under:

- *Intended object of learning:* Mathematics teachers enter classrooms with intentions about what they wish their learners to learn (Lo, 2012; Marton, 2015). In the context of LS as is the case in this research, these intentions are collectively pondered, shared and agreed upon by an LS team (Fujii, 2019). These collective intentions and aspirations represent the teacher perspective about the object of learning (Marton, 2015). The assumption made by a researcher in this study is that the mathematical content intended for learning cannot be discussed independent of the questions that will be used to facilitate learning.
- *The Enacted object of Learning* is defined by Marton, (2015) as, "The learning that is made possible" (p. 116). Variation Theory reminds us that what is intended for learning is not always what unfolds because there are many other factors that influence learning such as learners' level of preparedness to engage with content, their previous knowledge and interest in learning (Lo, 2012). In other words, even though teachers have the responsibility to take actions that are necessary to make learning happen, there is no guarantee that learning will take place since they can only make learning possible. These actions, amongst others, include development of instructional material for learning, creating a conducive environment for learning and asking focused questions. It is questions and questioning that is of interest to this study. Variation Theory allows the researcher to examine how the questions that were planned for a lesson filter into the lesson. It is also possible that questions may be planned, but follow-up and probing questions will depend on what learners are saying or how they are responding to the teachers' questions. However, the main assumption of this study was that productive questions will be planned and written on a lesson plan and even on the worksheets if any.
- *The Lived object of Learning* refers to what learners have actually learned. In other words, the object of learning refers to the knowledge learners have acquired and understanding they have developed at the end of the lesson and beyond (Bussey et.al., 2013; Lo, 2012; Marton, 2015). However, this dimension

is not of interest to this study because the focus is on the attributes of teachers' questions.

4.6. WAYS OF SEEING

The phrase 'ways of seeing' is frequently used in Variation Theory to explain that people see and describe the same phenomenon differently (Bussey et. al., 2013; Lo, 2012; Marton, 2015). The different experiences that individuals bring to the learning environment largely account for the different ways of seeing (Marton, 2015). The assumption made by Variation Theory is that it is natural for teachers to see the content to be taught differently from the way learners might see it (Lo, 2012; Marton, 2015). Lo (2012) encourages teachers to consider practical measures (e.g., using several methods such as allowing learners to express themselves verbally, conducting pre-lesson interview) to find out how learners see and understand the critical features of the phenomenon in question (Lo, 2012). The notion of seeing is made explicit in the following quotation:

> Teachers must override their natural attitude and recognise that it is quite natural for students to see the object of learning in a different way to the way that they themselves see it. The only way that teachers can help students learn to see the object of learning in the same way that they do is to first analyse and identify the critical features that they themselves focus on to arrive at the meaning that they have acquired of the object of learning. (Lo, 2012, p. 68).

It is evident from these comments that the 'different ways of seeing' described in Variation Theory have far-reaching implications in that they challenge teachers to transcend their 'natural attitude' (Lo, 2012). Teachers then must accept that the pre-conceived ideas learners have about a particular concept, will always differ from their own experiences. From the perspective of Variation Theory, learning is said to have occurred if the learner changes his/her perceptions or even develops a totally new way of seeing what was intended for learning (Lo, 2012; Marton, 2015). The implication is if teachers wish to see learners developing the same understanding of the object of learning, they must help them recognise the critical features and grasp the relationship among them holistically. I argue that one way of achieving this is through attentiveness to questions and the art of questioning. Marton (2015) has indicated that for learners to develop powerful ways of seeing, they must, “decompose the object of learning and bring it together again” (p.145).

4.7. VARIATION THEORY ON QUESTIONS AND QUESTIONING

In the literature, discussions about the characterisation of questions gravitate towards the desired learning outcomes. Several researchers (Aizikovish-Udi & Star, 2011; Dahal et al., 2019; Dong et al., 2015; Shahril, 2013; Tofade et al., 2013; Webb & Ing, 2019) are of the view that different questions can be asked during the lesson to address all cognitive levels if questioners are mindful of the desired educational outcomes (learning outcomes). Variation Theory opposes this view arguing that learning goals cannot be solely described by the learning outcomes (Lo, 2012). For a lesson unit to achieve the intended learning, all its other aspects such as the choice of material, teaching strategies, the actual teaching enactments should be considered and carefully inspected (Lo, 2012).

The upcoming discussions go further to justify the relevance of Variation Theory in this study by exploring the intersections of Variation Theory and a phenomenon of questioning. The line of reasoning which this study is accentuating is that if teachers have a clear vision of the mathematical message (object of learning) they want to convey to learners, then they should be able to prepare questions (especially productive questions) and tasks that will make it possible for learners to discern the critical features and aspects of the object of learning. Literature has demonstrated that teachers are unable to ask productive questions that engage learners because they are difficult to produce spontaneously during a lesson (Chen et al., 2017). The main assumption made by Variation Theory is that learning implies seeing or experiencing critical aspects of an object of learning (Bussey et al., 2013; Kullberg et al., 2017; Lo, 2012; Marton, 2015). How, then, do teachers develop questions that make it possible for learners to become aware of critical features of a given phenomenon? Can the attributes of such questions and tasks be detached from critical features and critical aspects of the object of learning? I invoke the insights of Holmqvist and Selin (2019), Lo (2012), and Marton (2015) to explore these questions as an attempt to demonstrate how compatible Variation Theory is to this study.

Teachers’ questions in mathematics lessons are supposed to pursue a particular agenda. From the perspective of Variation Theory, this agenda is to help learners see in powerful ways and to discern critical features in order to make the object of learning their own (Åkerlind, 2015; Bussey, et al., 2013; Lo, 2012; Marton, 2015). Discussions throughout this section attempt to demonstrate intersections of a phenomenon of questions and questioning and perspectives offered by Variation Theory. Variation Theory highlights the need for teachers to empirically determine the critical features of the object of learning (Åkerlind, 2015; Bussey., Orgill and Crippen, 2013; Lo, 2012; Marton, 2015). Lo (2012) expresses it like this, "If teachers expect students to learn the intended object of learning, then in choosing that object, they must ask themselves why their students need to learn it" (p. 47). This implies that the reason to choose what students must learn should be compelling and justifiable. This is because if the object of learning is too abstract, learners can easily disengage and lose motivation if they do not find the content worth knowing. Efforts to empirically determine the difficulties learners have about the critical features amounts to finding learners' ways of seeing the object of learning through questions and questioning.

To resolve this challenge, Lo (2012) posits that a carefully designed diagnostic pre-test and post-test be conducted and learners' answers analysed thoroughly to establish the pre-conceived ideas and difficulties they have about the critical features. Lo (2012) further stresses that the questions in the tests should be analytical and focus on the critical features of the object of learning. Variation Theory adopts a particular stance regarding the development of questions and tasks that can help learners to discern the critical features of novel situations. This is conveyed in the following quotation:

> In order to explore the different ways in which students see novel situations, and to find out if they have developed more powerful ways of seeing in everyday life as a result of their studies, the questions asked have to be rather simple and straightforward, and neutral, with regard to the difference between every day and scientific conceptualizations (it should be possible to answer them from either perspective). And above all, the questions should not point out the relevant aspects of the problem to be solved, as this is exactly what the students are supposed to find out (discern) (Marton, 2015, p. 92).

The mentioning of the words 'simple and straightforward' in the above quotation should not be misconstrued to imply that questions asked should not stimulate learners' intellectual curiosity. Rather it should be understood to imply that ambiguity of any kind should be avoided in mathematics questions. This is because there are already demands placed upon learners to open up a dimension of variation (e.g. applying rules, making conjectures) and to discern the critical features of the object of learning (Lo, 2012 & Marton, 2015). The fact that a learner should be able to answer these questions from either perspective is an important condition which suggests that questions should be a rich problem-solving type, foster high level of reasoning and must present opportunities for varied approaches as this is precisely what Marton (2015) referred to as development of “powerful ways of seeing” (p. 15).

But what makes critical features become critical features? Lo (2012) proposed an answer to this question as follows, “Critical features are critical because the learners participating in the study have problems with them” (p. 63). It follows that learners cannot be overburdened with ambiguous and poorly structured questions when they still must unravel the critical features and open up dimensions of variations. Lo, emphasised that critical feature is a source of learning complexity for most learners. Questions should rather be structured to attend to learners’ cognitive needs.

Variation Theory makes a strong demand on teachers to ask questions in order to establish how their ways of seeing the critical features of an object compares with that of the learners (Åkerlind, 2015; Bussey et al., 2013; Lo, 2012; Marton, 2015). However, Marton (2015) argued that, “Some questions and some tasks are obviously more useful for finding out others’ ways of seeing the world than other questions and other tasks” (p. 95). Implied here is that not all questions and tasks developed to support learning can successfully do so. It follows that if teachers want to understand how learners see and understand the learning that is intended for them and to learn more about the distinctions they are making, they must skilfully develop questions and tasks that ‘fit’ the purpose of the intended mathematical message (object of learning). Marton (2015) makes explicit the attributes of those questions indicating that they must be novel and open-ended.

It is interesting to note that Variation Theory clearly articulates the attributes of questions that should be aligned to the objects of learning. In all the instances where the use of questions is advocated, attributes of such questions are detailed, i.e., questions should be simple, straight forward, and neutral; novel and open-ended and should not point out the relevant aspects of the problem to be solved (Marton, 2015). Questions should also be analytical and focus on the critical features of the object of learning (Lo, 2012). Marton (2015) seals the conversations on questions and questioning in this manner “But in order to formulate a question, a problem, you must have some idea of what kind of differences you (as the author of the question) are interested in" (p. 93). What this translates to is that thinking about the questions we (as teachers) intend to use to accelerate learning is important, but equally important is thinking about the information we wish our questions to convey. A critical exploration of the intersections of a phenomenon of questioning in mathematics discourse and Variation Theory is necessary in this study to dispel doubts that there may be missing links. It is possible to view a phenomenon of question and questioning through the lens of Variation Theory to deepen our understanding of what we want learners to learn.

I argue that, just as the quality of the lesson can be judged by its effect on student learning Lo (2012), the quality of questions prepared and used to facilitate learning can be judged by its effect on how learners open up a dimension of variation and how they develop new ways of seeing. Just as the choice of the object of learning affects the quality of learners' learning, the choice of questions to guide the intended learning affects the quality of classroom conversations when the object of learning is handled. Of course, researchers such as Lo (2012), Marton (2015) and Kulberg et al. (2017) have argued that there are several variables that determine how different learners experience the enacted object of learning. Learners may have different backgrounds, experiences, and understandings of the object of learning, which can obviously influence how they learn what was intended. Emanuelsson (2001) supports the views expressed by Variation Theory arguing that it is possible to learn how teachers' questioning behaviour makes it possible to learn about how learners learn and understand the mathematical content taught in classrooms. More details of Emanuelsson's (2001) insights are presented in section 4.8.2.

4.8. THE CONCEPTUAL FRAMEWORK

This study was guided and informed by the conceptual framework presented in Figure 4.1. The conceptual framework incorporates Variation Theory (theoretical lens), LS (the context of the study) and Emanuelsson's categories of classroom interactions. Review of literature, design of data collection instruments, analysis and interpretation of data were all done with the conceptual framework in mind. I discuss in detail the aspects that constitute the conceptual framework and explain how each component forms part of the conceptual framework.

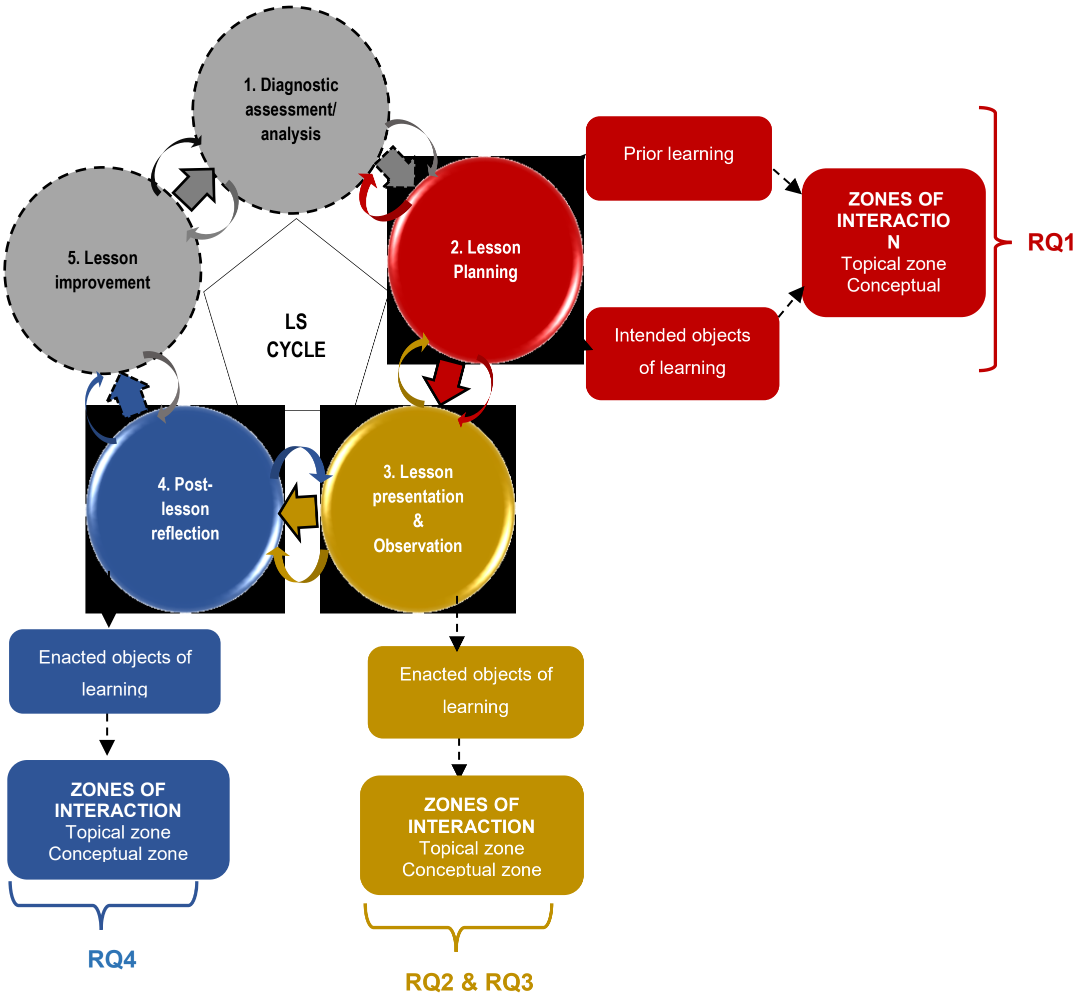


Figure 4.1: Conceptual framework

4.8.1. The Lesson Study cycle

LS provided the context for this study. In chapter 2, I discussed broadly what LS is and how it works as a teacher professional development model. In this chapter, I provide an overview of how it forms part of the conceptual framework and what the focus of observation was during each of the three stages as highlighted in Figure 4.2.

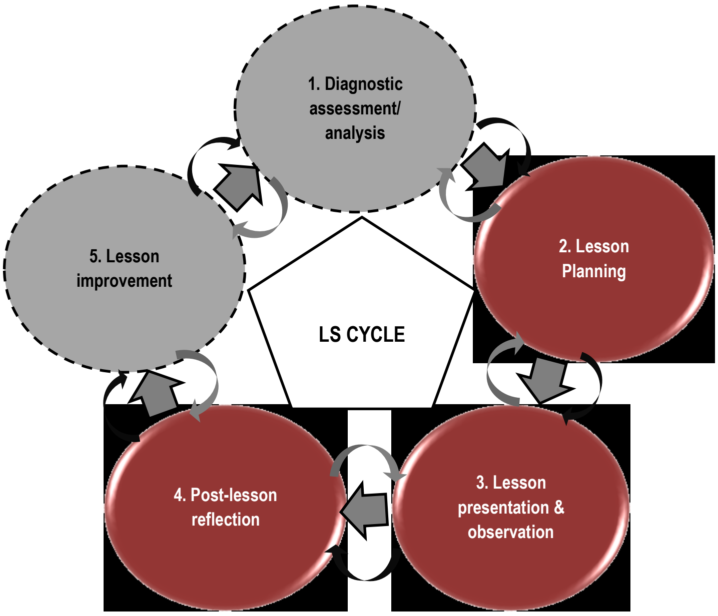


Figure 4.2: LS stages relevant to the current study

The focus of the study was on three of the five stages of LS. However, for the purpose of providing the reader with a holistic view of the entire LS cycle as adapted and implemented in South African context, all five stages are introduced but only three are flagged as seen in Figure 4.2. Presenting the entire cycle of LS is important to enlighten the reader on what a complete LS cycle looks like.

4.8.1.1. The focus of observation during the lesson planning stage

The focus of observation during the lesson planning stage was an exploration of the attributes that characterise questions planned during the collaborative lesson planning stage. The goal of observation was to establish whether questions are planned for. Secondly to establish if they are purposefully connected to what is intended for learning (intended object of learning). The questions prepared for a lesson during the lesson planning stage were described in terms of Emanuelsson's categories of classroom interactions known as topical and conceptual zone. As can be seen in Figure 4.1, the secondary research question being addressed in the lesson planning stage is RQ1 which is stated as: *What attributes characterise questions planned during collaborative lesson planning?* The assumption I made here was that questions collectively planned by an LS team to explore previous learning experiences about the intended object of learning will be considered during planning. One of the rituals of LS is the collaborative development of a research question (which translates into a research theme) to guide an LS team in their planning of research lesson in their entire cycle (Fuji, 2019 & Wei, 2019). This research question and aim will have a greater influence on collective intentions and aspirations of an LS team (Fujii, 2019). This aspect was observed at the first meeting to establish whether they incorporate the research theme to guide their planning.

4.8.1.2. Observation during the lesson presentation and observation stage

The focus of observation during the *lesson presentation and observation stage* was to establish how the planned questions were enacted during the lesson presentation stage, to establish the purposefulness of each question posed and to explore the attributes of such questions. The questions posed during the lesson presentation and observation stage were described in terms of Emanuelsson's categories of classroom interactions known as topical and conceptual zone. A dimension of how Grade 9 learners experienced teachers' questions was also explored during the lesson presentation and observation hence the procedural zone was included in the third stage of the LS. The focus on procedural zone was more on the kinds of ideas learners construct because of teachers' questions. As it can be seen on Figure 4.1, there are two secondary research questions being addressed in the lesson presentation and observation stage. It was RQ2 which was stated as: *How do questions planned during the collaboration planning stage permeate the lesson presentation?* RQ3 which was stated as: *How do learners experience teachers' questions during teaching?*

4.8.1.3. Observation during post-lesson reflection

Observation during post-lesson reflection focussed on the nature of comments and contributions made by the team members in relation to questions and questioning. I explored comments teachers made about how their plans unfolded but focussed more on what they say about questions. It was necessary to establish whether the LS team consider the need to raise issues about questions planned and posed during the lesson.

As seen in Figure 4.1, teachers' comments, and contributions to the discussions during post-lesson reflection were categorised in terms of Emanuelsson's (2001) zones of classroom interactions. The secondary research question being addressed during the post-lesson reflection was RQ4 which was stated as: *What critical features on questioning emerge during post-lesson reflection*?

4.8.2. Emanuelsson's categories of classroom interactions

Emanuelsson (2001) conceptualised the three categories of classroom questioning which he referred to as zones, i.e., topical, conceptual, and procedural zone.
Emanuelsson (2001) uses the phrase topical zone to refer to the kinds of interactions characterised by reproduction of mathematical facts, application of known procedures where learners' knowledge is categorised in terms of right or wrong. The second being the conceptual zone. Teacher questioning in the conceptual zone seeks to elicit learners' thoughts, understanding as well as their ability to reason. Interactions in this zone are more about digging deeper into learners' cognitive processes rather than reproduction of mathematical facts. The third category of Emanuelsson is referred to as procedural zone. Interactions in this zone are more about the form of learners' work rather than the content. In this study the observation of classroom interactions was extended to the observation of ideas learners construct because of the questions they were asked. More precisely the kind of knowledge that becomes transparent when learners respond to the teachers' questions. The procedural zone was thus included for the purpose of exploring ideas and thoughts learners construct as they respond to questions. The notion of the conceptual zone offers a complementary perspective on the views expressed by Variation Theory on questioning because questions in this zone seek for ways of understanding and making distinctions. This is because the attributes of questions advocated in the Variation Theory are aligned to Emanuelsson's category of classroom interactions known as the conceptual zone, a category of classroom interaction which promotes openness.

4.8.3. Variation Theory

The Variation Theory has been appropriated by many researchers in the field of mathematics, e.g., Adler and Pournara (2019); Kullberg et al. (2017); Wasserman (2015) and in the field of science education, e.g., Bussey et al. (2013) and Xu (2019), to explain qualitatively different ways of comprehending aspects of the world. It is also significant to mention that the Variation Theory like all other theoretical frameworks, has its own limitations, i.e., it may not be used to answer some of the empirical questions. Bussey et al. (2013) acknowledged this limitation when trying to respond to the question "How is it that two students who are sitting in the same class on the same day with access to the same materials can come to understand a chemical concept (or any concept for that matter) differently?" (p. 9). Apart from having some limitations, Bussey et al. (2013) considered the Variation Theory to be an appropriate framework because it tended to respond to the question from multiple perspectives, i.e., through an examination of the intended object of learning (the teacher perspective), enacted object of learning (the researcher's perspective of the potential for learners' learning created during classroom interactions) and lived object of learning (the learner perspective).

In this study, the key tenets of Variation Theory (intended and enacted objects of learning) are structured around the LS model which is implemented in South Africa (Sekao & Engelbrecht, 2021). The South African adapted version of LS presented by Sekao and Engelbrecht (2021) impacts mathematics teachers' practices in five ways. However, for the purpose of this study, only three of the five stages (i.e., collaborative lesson planning, lesson presentation & observation, and post-lesson reflection stages) were considered sufficient to help answer the research questions. being explored (Figure 4.1).

The goal of every learning event is to develop learner's understanding of certain abilities or phenomena. From the perspective of Variation Theory, these phenomena which in the context of LS are collectively pondered upon, are referred to as the objects of learning. Here I direct my attention to how questions are incorporated within the objects of learning (intended and enacted objects) during the three stages of each LS cycle which are Lesson Planning, Lesson presentation and post-lesson reflection stages of LS.

Variation Theory acknowledges the critical role of questions in the enactment of the object of learning. In a pedagogic situation questions are drivers of teachers' intentions for a learning event. While the role of questions and the purpose of purposeful questioning is heightened in the literature, I was mindful of the possibility that teachers may not recognise the role played by these important pedagogic tools in what they intend for their learners (intended object of learning) during the lesson planning session. In other words, the LS team may not recognise that oral questions collaboratively planned for a lesson can affect their intentions. For this reason, the context in which this study occurs (LS) was given consideration. In Figure 4.1 the second stage of the LS cycle is linked to learners' prior knowledge and to the intended objects of learning. The arrows from prior knowledge and intended object of learning converge on questioning zone (topical and conceptual zone).

Studies guided by Variation Theory have three common goals: (1) to describe distinctions in what teachers intend for their learners about a particular object of learning, (2) to identify the learning encountered by learners (experienced variation) created in the space of learning about a particular object of learning and (3) to explain the distinction in learners' understanding of a given object of learning after the learning event has taken place (Bussey et al., 2013; Lo, 2012; Marton, 2015). Questions are presumably entangled with all three kinds of objects of learning (intended, enacted, and lived). Variation Theory was applicable to this study for its potential to enable me to carefully inspect the attributes of mathematics teachers' questions from different perspectives to gain a holistic understanding of how questions and questioning evolve within the LS context. However, for the purpose of this study, the lived object of learning was not explored since the focus of the study was not primarily on questions used by teachers to assess distinctions in learners' understandings after a learning event has occurred.

The influence of teachers' questions on the enacted object of learning is direct and explicit. This is because questions make learners' knowledge transparent during classroom interactions (Marton, 2015). The interactions during the lesson-presentation stage enabled me to examine how learners experience questions ideas that learners construct as a result of teachers' questions. I take one of the key assumptions of Variation Theory in which researchers are reminded that:

> People live in a world which they-and not only the researchers - experience. They are affected by what affects them, and not by what affects the researchers. What this boils down to [...] is taking the experiences of people seriously and exploring the physical, the social, and the cultural world they experience.'' (Bussey et al., 2013, p. 15).

The views expressed here are consistent with the logic that in a typical pedagogic situation, a teacher will pose questions. These questions affect learners by way of influencing their thinking. Different learners are likely to present different experiences of those questions because their prior knowledge of the object of learning may be different from that of the teacher (Bussey et al., 2013; Lo, 2012; Marton, 2015). Hence, in this study the potential influence of learners' prior knowledge on the enacted object of learning is recognised. Both the teacher and learners are important actors in the enactment of the object of learning (Mahmud, 2020).

What then is the role of the researcher in this situation? The role of the researcher as Bussey et al. (2013) put it, is to explore the world experienced by people, and in this case the characteristics or attributes of teachers' questions during instruction. The intended object of learning was collaboratively conceptualised during the lesson planning stage of LS and the enacted object of learning was actualised during teaching and classroom interactions (Marton & Pang, 2013). The two objects of learning were deemed adequate to provide stronger evidence of the attributes of teachers' questions. The post-lesson reflection stage of the LS further provided the researcher with insights on what issues were raised by an LS team on the nature (attributes) of questions.

4.9. THE CONNECTIONS BETWEEN LS, VARIATION THEORY AND EMANUELSSON'S CATEGORIES

In the LS, mathematics teachers engage in prolonged, robust, and fruitful discussions to conduct a thorough investigation of the object of learning (Lewis & Perry, 2014; Maeda & Ono, 2019; Takahashi, 2014). The goal is to: gain a thoughtful understanding of what they intend for their learners; clearly articulate their own assumptions about effective instruction and predict learners' responses to the task/questions (Lewis, 2016). All these become enacted in the actualisation of the research lesson during which observers collect data on learners' responses and reactions to instruction to prepare for the post-lesson reflection stage.

These efforts of professional engagement resonate well with the principles of Variation Theory as expressed by Lo (2012), "An important part of teachers' self-learning and professional development is to be able to analyse their own lessons and know why they work and why they do not work" (p. 193). While Variation Theory provides mathematics educators with a very constructive guiding principle to do this, (Lo, 2012; Marton, 2015), LS provides them with a conducive environment to achieve this goal. The combination of Variation Theory and LS suited the purpose of this research study because they provided me with a strong explanatory power to explore the phenomenon being studied. A careful examination of the object of learning from different perspectives will be useful for teachers who wish to learn how their questions and questioning practices influence their teaching practice.

The first two categories (topical and conceptual zone) were used to categorise questions planned and posed, while the third category was used to categorise learners' responses, so that observations can be made on how learners experience the teacher's questions. Questions have a primary function to facilitate learners' construction and acquisition of mathematical ideas and as such the dimension of how they experience these questions cannot be left unattended.

Emanuelsson's categories of classroom interactions were relevant categorical tools because they are grounded within the Variation Theory. These categories were taken as a classification system to account for how classroom interactions can be framed while paying explicit attention to how questions evolve in a learning environment. Similarly, questions posed by the teachers or learners during the lesson lead to interactions. Questions therefore produce interactions and interactions produce questions. The different kinds of questions and the responses to these questions produce the different kinds of zones, i.e., procedural, conceptual, and topical zone (see Figure 4.1). Sometimes questions within a zone can point to another zone. A shift between zones is known as a vertical shift and such a question deepens a concept being discussed.

This newly developed conceptual model presented for this study in Figure 4.1, allowed me to examine a phenomenon of questioning by raising issues about the attributes of verbal questions posed by mathematics teachers participating in the LS. More precisely questions were raised about: 1) the attributes of questions intended for a learning event 2); how those questions filter into a live research lesson (enactment); (3) how learners experience those questions); and lastly 4) the comments made by LS team members about questions.

Remarkable advances have been made by several researchers (Aizikovitch 2013; Aziza 2015; Dong et al., 2015; Purdum-Cassidy et al., 2015; McCarthy et al., 2016) to understand a phenomenon of questioning from various perspectives. A critical synthesis of literature has shown that research on questions and questioning paid little attention to how teacher communities can be engaged in discussions that pertain to their questioning behaviour. Moreover, theoretical tools employed in these studies focussed solely on how questions can be used to achieve the learning objectives (Aizikovish-Udi & Star, 2011; Dahal et al., 2019; Dong et al., 2015; Shahril, 2013; Tofade et al., 2013; Webb & Ing, 2019) rather than on what to do to achieve the learning outcomes (Lo, 2012 and Marton, 2015). These untold stories of teacher questioning created a gap in conversations on classroom questioning. This study therefore extends the scope of existing studies by elevating conversations on classroom questioning in mathematics to a context which fosters collective responsibility and scientific activity (Pjanic, 2014). The theoretical lens employed in this study causes us to look at a phenomenon of questioning in a different light.

4.10. CHAPTER SUMMARY

The focus of this chapter was to discuss the origin and background of Variation Theory, define the key constructs of the theory and most importantly to explore the perspectives of Variation Theory on questions and questioning. The chapter explored the intersections of Variation Theory and Emanuelsson's categories of questions. The chapter demonstrated in a detailed way that a phenomenon of questions being explored in this study, occupy an important space in Variation Theory. Interestingly the theory makes explicit that the attributes of such questions should be met to fulfil what is intended. It is also clear that, the attributes of questions cannot be detached from what is to be learned (objects of learning) and what needs to be learned (critical features).

CHAPTER FIVE: RESEARCH METHODS AND METHODOLOGY

5.1. INTRODUCTION

Educational research is a highly negotiated process which is conducted in terms of purpose of the research, the philosophical position of the researcher, the research methodology and the methods used. This chapter outlines a set of methodological issues that were adhered to so that the chosen research study can be feasible and properly undertaken. The chapter begins with a detailed description of the research paradigm and paradigmatic assumptions that underpins this study. These include discussions of philosophical discourses underlying qualitative research such as ontology and methodological assumptions. The chapter goes further to detail issues related to the research approach and design, research site and sampling. Data collection instruments and the processes of data collection are clearly outlined. Furthermore, the chapter provides details of how data were analysed, how ethical issues were addressed and finally, the quality assurance criteria.

5.2. RESEARCH PARADIGM AND ASSUMPTION

5.1.1. Research paradigm

The paradigm of any research represents a lens through which the researcher views and interprets the world. A paradigm as defined by Cohen et al. (2018) is, “A way of looking at or researching phenomena, a world view, a view of what counts as accepted or correct scientific knowledge or a way of working, an accepted pattern or model” (p. 8). Johnson and Christensen (2017) describe a paradigm as, “A worldview or perspective about research held by a community of researchers that is based on a set of shared assumptions, concepts, values, and practices” (p.31). The different descriptions of the notion of paradigm have a common thread and that is; for researchers to fully understand the world they are researching, they have to adopt particular lenses through which they can conceptualise and execute research.

A suitable paradigm constitutes a key endeavour in any research project as it controls and guides the research process. The paradigm chosen for this study is interpretivist. The interpretive paradigm focuses on action (Cohen et al., 2018) and it puts construction and re-construction of meanings people make at the centre of inquiry (Leavy, 2017). The key assumption of an interpretive paradigm is for the researcher to understand the intentions of actors in a society. To understand those intentions, the researcher therefore needs to understand the meanings the actors construct and negotiate through their actions and interactions (Cohen et al., 2018; Creswell & Poth, 2018; Johnson & Christensen, 2017; Leavy, 2017). An interpretive paradigm is considered an appropriate choice for this study because the aim of this study is to comprehend, describe and interpret the attributes of mathematics teachers' questions in the context of LS. The interpretive paradigm fits the task of uncovering and interpreting meanings participants attach to a complex phenomenon of designing and posing questions which take place in the context of a professional development model, known as LS.

5.1.2. Paradigmatic assumptions

All research is structured within a set of paradigmatic assumptions which reflect implicit assumptions about the nature of reality and how knowledge is constructed. I discuss in detail the paradigm assumptions that shaped my understanding and view of the world.

Three philosophical assumptions, i.e. ontology, epistemology and methodology influence how researchers understand and view the world (Cohen et al., 2018). Ontology is a branch of philosophy that examines the nature of reality and asks whether there is a real world that exists independently of our understanding (Coe, 2021; Cohen et al., 2018; Creswell & Poth, 2018; Johnson & Christensen, 2017; Leavy, 2017). From the ontological position, reality is multifaceted as perceived from participants' various perspectives.

Cohen et al. (2018) have stated that: "Qualitative research regards people as anticipatory, meaning-making beings who actively construct their own meanings of situations and make sense of their world and act in it through such interpretations" (p.288). The ontological position adopted in this study is that of multiple realities being constructed. These multiple realities were those of interacting individuals of an LS community, the researcher and the readers. LS presents a social setting in which participants' subjective understandings and multiple meanings were socially constructed. I understood the phenomenon of question and questioning, what counts as a good question, and what characterises a question through spoken and written words, actions, gestures and meanings of interacting individuals during observations and interviews.

An epistemology as defined by Leavy (2017) is: “a philosophical believe system about how research proceeds and what counts as knowledge” (p.12). Epistemology compels the researcher to explore how can, what is assumed to exist out there, be known (Coe et al., 2021; Creswell & Porth, 2018). The epistemological assumption aligned to the interpretivist paradigm is social constructivism (Coe, et al., 2021; Cohen et al., 2018). The basic assumption of social constructivism is that knowledge is socially constructed through interactions, behaviour and thereby data are socially situated, context-related and context-dependant (Cohen et al., 2018). The meanings participants attach to situations, the ideas they construct and re-construct and how they make sense of the world in their own terms, all occur in a socio-cultural context (Coe et al., 2021; Creswell & Poth, 2018; Johnson & Christen, 2017). The researcher with an interpretivist point of view, lessens the distance between her/himself and what is being studied (Creswell and Poth, 2018), and acknowledges that knowledge is acquired through the process of observation, interpretation and reflection (Coe et al., 2021).The complexities involved in exploring the attributes of mathematics teachers’ questions cannot be studied from a distance (Johnson & Christensen, 2017). In order to understand the context of this study specifically and holistically, I was close to the participants throughout the entire period of data collection, observing, describing and explaining numerous and diverse interpretations.

The focus of this research was on subjective accounts, understandings and interpretations of the phenomenon, by both the participants and me. Prolonged engagements with participants enabled me to gather quality, rich and authentic data. The ideas constructed by an LS community in relation to the oral questions they intend to use for teaching, how these questions filter into the collaboratively planned lesson and how they are reflected upon during the post-lesson reflections, formed the basis of exploration to understand the subjective experiences of individual members of an LS team.This study is concerned with the uniqueness of how an LS community of mathematics teachers grapple with the processes of planning questions to be used to facilitate learning and how the plan ultimately unfolds in class during instruction. LS is a natural setting and as such a direct source of information. I agree with Fraenkel and Wallen (2019) who are of the view that “activities can best be understood in the actual setting in which they occur” (p.423).

The ontological and epistemological assumptions discussed have direct implications for methodological concerns in this study. Qualitative studies seek to understand human and social behaviour from an insider's perspective (Ary et al., 2006; Cohen et al., 2018; Johnson & Christensen, 2017). In this study, I sought to understand and portray the meanings constructed by the participants involved in a professional development programme known as LS. The social reality, experiences and phenomenon being explored here, arex the questioning culture that evolves within this community. To achieve this, Cohen et al. (2018) recommend that the researcher's approach to inquiry is to collect observational data acquired from the "natural, uncontrived and undisturbed real-world setting with participants speaking in their own terms and behaving naturally" (p.289). In this study, data were collected through observations, interviewing participants and document analysis. These methodologies yielded what Creswell and Poth (2018) refer to as providing 'rich and thick' descriptions.

5.2. RESEARCH APPROACH AND DESIGN

A well thought out study should have a well-constructed plan for organising the research and making it workable. I provide details of the research approach and design that were considered to make this study practicable.

5.2.1. Research approach

In order to explore the practices and culture of questioning in mathematics classrooms occurring in LS contexts, the researcher employs qualitative research as an approach for the study. Creswell (2018) describes qualitative research as "a means for exploring and understanding the meaning individuals or groups ascribe to a social or human problem" (p.4). Those who engage in qualitative research understand and appreciate a way of looking at research that honours an inductive style, a focus on meanings individuals attach to words and actions.

Qualitative methods are preferred because they enable the researcher to collect "rich and thick" descriptive data (Cohen et al., 2018). In this study, the researcher is of the view that if mathematics teachers are to concern themselves with designing learning experiences that make it possible for learners to learn what is intended, then they should equally be concerned about the quality of questions they will use for informal assessment to enact the object of learning. The rich and thick descriptions referred to in this study involved paying attention to how a group of teachers grapple with the process of planning questions to be used to facilitate classroom interactions, their pre-conceived ideas about questions, actions, feelings and meanings. An examination of how the plan ultimately unfolds in a classroom and to reflect and improve their practices, formed an important part of the analysis. These descriptions provided sufficient contexts so that a reader or an individual outside the LS culture can derive meaning and make their own judgments of the actions, words and activities that unfolded.

Incidents pertaining to questions unfolding in the cyclic processes of LS were explored and systematically discussed. The qualitative approach is preferred for its special attribute of enabling the researcher to explore a holistic picture and to gain a deeper understanding, which quantitative methods may not afford in this context (Coe et al., Cohen et al., Creswell & Poth, 2018).

5.2.2. Research design

Creswell (2018) describes research design as, "A plan and procedure for research that span the decisions from broad assumptions to detailed methods of data collection and analysis" (p. 4). The plan involved a number of crucial decisions which were informed by the paradigmatic assumptions I brought to the study. In this study, a case study design was chosen because it enabled me to respond to the broader research question as truthfully and as deeply as possible (Cohen et al., 2018). Cohen defines a case study as "a detailed examination of a small sample, and an in-depth investigation of a specific, real-life project, policy, institution, program or system from multiple perspectives in order to catch its complexity and uniqueness" (p.375).

In this study, the case was a single group of mathematics teachers in the Senior Phase, teaching mathematics in Grade 8 and 9 and who, at the time of data collection, had been participating in a LS for some time. The initial plan was to involve a group of mathematics teachers from eight secondary schools within the district. All of them had agreed to participate in the study and subsequent to that completed the consent form in 2020. The outbreak of the COVID 19 pandemic in 2020 necessitated a change of plans. Data collection could not take place as planned and as such was only collected in 2021. The construct being explored was the attributes of mathematics questions and questioning practices within LS contexts. A case study is defined by specific boundaries, referred to as a "bounded system" (Ary et al., 2006). This study was bounded by a single case (Senior Phase teachers) and a teacher development model (LS). This bounding of the study was consistent with instrumental case study. Cohen et al. (2018) explain that instrumental case studies are used to examine a specific case in order to gain insight into an issue or a theory. In this study, an LS team was a case study which was instrumental in helping me to understand how their questioning practices were shaped by their interactions.

A case study has its own weaknesses and strengths. One of its strengths is that it is known for capturing reality as it unfolds. One of the weaknesses of case study is that findings may not be generalised to the broader community. Therefore, generalisation becomes a drawback in that the findings are only applicable to that specific context. In this study the aim was not to generalise but to explore the character of questions planned by a group of teachers and how those questions permeated the lesson presentation and how they were reflected upon during the post-lesson session. An important dimension of the ideas and conceptions learners constructed as a result of teachers' questions formed part of the analysis.

5.3. RESEARCH SITE AND SAMPLING

Qualitative researchers use non-probability sampling techniques. Cohen et al. (2018) alluded to the fact that, "The selectivity which is built into a non-probability sample derives from the researcher targeting a particular group, in the full knowledge it does not represent the wider population; it simply represents itself" (p. 217). Five mathematics teachers from four secondary schools within the Sub District constituted an LS team that participated in the study. These teachers were purposively sampled because they had an idea of what LS is since they had been implementing it as a teacher professional development model (Cohen et al., 2018). This is consistent with the views expressed by Cohen et al. (2018) that, "Researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality or possession of the particular characteristic (s) being sort" (p. 218). For the purpose of this study, a school where all activities related to the research study were conducted was purposively selected (Cohen et al., 2018). The school was chosen because the school management team (SMT) and learners were familiar with LS since they had previously hosted LS activities for the Sub District. The school was also convenient for the other teachers from three other schools since it was central and accessible to them. I need to make a distinction here between 'school' and 'schools'. By school I refer to a research site where LS activities (lesson planning meetings, lesson presentations, observations and post lesson reflections) were held. By ‘schools’ I am referring to different schools whose teachers constituted an LS team.

A single Grade 9 class was sampled to participate in the study. This grade is an exit grade in the Senior Phase. The inclusion criteria are stated in Table 5.1. Ary et al. (2006) and Cohen et al. (2018) caution qualitative researchers that purposive sampling is prone to bias in that the researcher may have pre-judged the participants and mistakenly assumed that they are all knowledgeable about the phenomenon being explored. In this study, it was necessary to ensure that all participants were familiar with the context (LS) in which the study was conducted. I acknowledged that the newly appointed teacher, Ms Davidson who has just joined the school was not familiar with LS processes. Taking this limitation into account, I then requested participants to explain what LS is and how it works prior to the commencement of the study.

Table 5.0-1: Inclusion criteria

Participants	Reason for selection in the study
LS cluster	Commitment and quest to develop professionally through LS. The cluster has been implementing LS since 2018.
School	- It is a convenient meeting place for teachers within a cluster - The teachers and School Management Team are familiar with LS processes. - School Management Team [SMT] provide support to the teachers as required for the successful implementation of the LS approach.
Grade 9 learners	Exit grade in the Senior Phase.

5.4. DATA COLLECTION INSTRUMENTS AND PROCESS

Three strategies were used for data collection, namely: observations, informal conversational interviews and document analysis. Data were collected through an observation of teachers’ collaborative lesson planning processes, lesson presentation and observations and post-lesson reflections of all four research lessons. The three strategies and processes involved are outlined in the next paragraphs and an explanation of how instrumental each one was in helping with answering of research questions is provided.

5.4.1. Observation protocol

One of the powerful attractions to observation is its ability to enable the researcher to obtain fresh and uncontaminated data and to see things which participants may not disclose in an interview (Cohen et al., 2018). Observation goes beyond 'just looking' as it requires the observer to take a closer look and note situation, behaviours, settings and routines in a more structured manner. In this study, observations were conducted during three of five stages of LS, namely: collaborative lesson planning, lesson presentation and observation and the post-lesson reflection stage. Annexure A1 and Annexure A2 were used to collect observational data from lesson planning and lesson presentation and observation stage respectively, while Annexure A3 was used to guide collection of data from the post-lesson reflection stage. Video-recordings were used to support the observation processes (Cohen et al. 2018). The focus of observing an LS team during the collaborative lesson planning stage, was on exploring the attributes that characterise questions planned for teaching and learning. I used an observation sheet to guide my observations. One must have some directives before entering a class and just observe. The observation protocol for the collaborative lesson planning and lesson presentation and observation stage were used to:

- Establish whether teachers plan questions for the lesson and to explore the attributes/character of such questions
- Establish how the planned questions are enacted during the lesson presentation stage.
- Explore how learners experience teacher's questions.
- Establish the purposefulness of each question posed during the lesson.

While the process of bringing the object of learning to learners' focal awareness is done during the classroom interactions, the process of attaching meaning to participants' observations can only be done during reflection. The main focus of observation during the post-lesson reflection (Annexure A3) was on the following:

- To explore the nature of contributions and comments made in relation to questions.
- To establish whether the LS team consider the need to raise issues about questions posed during the lesson.

As Cohen et al. (2018) argue, observations are always theory-driven. What researchers observe depends on when, where and for how long they look.. The kind of observation that I was undertaking was pre-ordinate i.e., categories of what was to be observed was already known to the researcher. Emanuelsson's categories of classroom interactions outlined in Annexure A illuminated an agenda of issues to be observed. However, consistent with emergent design, categories of questions described could not be tightly prescribed i.e., other categories of questions emerged during the study (Johnson and Christensen, 2017).

5.4.2. Interview

An interview is a conversation between the interviewer and the interviewee (Creswell & Poth, 2018). Since it could not be possible to pre-empt what will emerge during the collaborative lesson planning stage and post-lesson reflections, informal conversational interviews were conducted where clarity was required in relation to questions teachers planned and reflected upon. Since interruptions are not allowed during the lesson presentation and observation, clarity seeking questions with regard to what transpired during each of the four research lessons were asked during post-lesson reflections. In an informal conversational interview, questions emerge from an observation are posed when the need arises (Cohen et al., 2018). In this study, I interjected without pausing a video while teachers were planning the lesson by asking questions to clarify or corroborate what I was observing. This type of interview increased the relevance of questions I asked during observation. Although there was no specific schedule to guide questioning during the interviews, I was guided by the conceptual framework in chapter 4, Figure 4.1 and the research questions. Conversational interviews were useful because they were time efficient and generated a wider range of responses from participants enabling me to gain more insights while simultaneously triangulating data (Cohen et al., 2018).

5.4.3. Document analysis

Cohen et al. (2018) make a distinction between two kinds of documentary sources referred to as 'primary' and 'secondary' analysis. Primary analysis involves interpretation of original first-hand materials whereas secondary analysis involves examination of what others have already interpreted. According to Cohen et al. (2018), a primary documentary source "encompasses every kind of evidence which people have left of their past activities produced during the period being studied" (p. 325). Secondary documentary sources are created sometime after the period of study. In this study, lesson plans of all the four research lessons which were collectively planned and documented by an LS team constituted a primary document analysis. These lesson plans were original and were created during the period of the research study. In the discussions that follow, I provide details of how the analysis was conducted.

The process of planning a research lesson in the LS cycle begins with broad discussions to brainstorm what is intended by the team. The LS team research, plan and design the learning experience and develop activities that will lead them towards the intended object of learning (Lo, 2012). The planning process during each cycle culminates in two crucial documents that represent collective intentions and aspirations of an LS team: lesson plan and learners' worksheets. The lesson plans for each of the four research lessons were perceived as primary documentary sources because they represented original records of evidence which were collectively and intentionally created by participants during their collaborative sessions (Cohen et al., 2018; Creswell & Poth, 2018). The focus of analysis was purely on questions written on these lesson plans which were intended to facilitate learning during teaching and learning.

In the context of LS, the lesson plan serves as a road map of what an LS team want to bring to learners' focal awareness. Questions written on lesson plans were first categorised according to Emanuelssons's categories of classroom interactions and analysed according to Variation Theory. Ary et al. (2006) point out that data extracted from documents can be very useful to researchers. In this study, lesson plans were analysed to establish whether discussions held about what questions would be used to facilitate learning eventually formed part of the lesson plan, and what key questions, if any, were put forth. It is the practice and culture of the LS team that was studied that, after discussions and thoughts are exchanged about what they intend for their learners, one member volunteers to type the first draft of a lesson plan during his spare time. The typed drafts were then emailed or sent through *WhatsApp* to each member including myself to establish whether the contents represent what was initially discussed. This was then followed by a meeting to discuss the final typed document and to engage in further discussions prior to the actual Lesson presentation and observation session.

One of the limitations of documents is that they are normally not created with data research agenda and may be lacking in terms of the phenomenon that the researcher intends to explore (Ary et al., 2006). In this study, I considered this method of data collection as a way of seeking convergence and corroboration in order to heighten data credibility (Cohen et al., 2018). The authenticity of documents can be challenged on many grounds such as: authorship, if there is inconsistency, if errors are detected and for whom a document was written (Cohen et al., 2018; Creswell & Poth, 2018). In this study, the documents were considered authentic because they were a first-hand document, final copies were consistent with initial drafts, and errors were minimal (Cohen et al., 2018).

5.4.4. Data collection process

Details of what happened, when and with whom are summarised in Table 5.2. During collaborative lesson planning meetings, teachers collaboratively prepared mathematics lessons based on what they considered challenging for learners generally. Such meetings were held at the research site during weekends i.e., Saturdays and sometimes on Sundays. This was necessary to protect the teaching time. There was however no consistency with regard to attendance of all the members of an LS team as some were unable to attend all the meetings due to personal commitments during weekends.

Lesson presentation and observations and subsequent post-lesson reflections were attended by all the five members consistently during school time. Conducting lesson observation and presentation and post-lesson reflections during school time was not peculiar to this study as this was normally how the team operated when their LS activities were not attached to the research study. As seen in Table 5.2 Lesson 1 and 2 and subsequent post-lesson reflections for such lessons were conducted on the same day i.e., 25th March 2021. Each lesson was immediately followed by the post-lesson reflection. The second lesson i.e., Lesson 2 was conducted in the afternoon followed by the post-lesson reflection. All lessons had breaks in between. The same pattern was followed for Lesson 3 and Lesson 4.

Table 5.0-2: Data collection process

Collaborative lesson planning meetings				**Lesson Presentation & observations**			**Post-lesson reflections**		
Date	**Duration**	**Members present (not their real names)**	**Purpose of the meeting**	**Date**	**Time**	**Activity**	**Date**	**Time**	**Activity**
28.02.2020	45min	Mr. Brown, Mr Adams Mr Mugu, Ms Elize Ms Davidson	Initial meeting to inform participants about the research study and to provide explanation of the procedures and processes						
18.01.2021	3hrs 5min 50sec	Mr. Brown,Mr Adams Mr Mugu	Planning and discussions of the 1^{st} research lesson	25.03. 2021	08:30-09:30	Lesson presentation & observation for Lesson 1 **(RATE speed, time & distance)**	25.03.2021	09:40 to 10:20	Post-lesson reflection for lesson 1
19.01.2021	2hrs 32min 6sec	Mr. Brown, Mr. Adams	Planning and discussions of the 2^{nd} research lesson	25.03.2021	11:00-12:13	Lesson presentation & observation for lesson 2 **(EXPONENTS-investigating laws of exponents)**	25.03.2021	12:20 to 12:58	Post-lesson reflection for lesson 2
04.02.2021	2hrs 24min 13sec	Mr. Brown, Mr. Adams Ms. Elize Ms. Davidson Mr. Mugu	To discuss the typed drafts for 1^{st} and 2^{nd} research lessons. To update members who were absent during initial planning meetings						
15.05.2021	5hrs 15min	Mr. Brown Mr. Adams Ms. Elize Mr. Mugu	Initial discussions based on 3^{rd} and 4^{th} research lessons.	07.06.2021	08:00 – 09:30	Lesson presentation & observation for lesson 3 **(ALGEBRAIC EXPRESSIONS-developing algebraic language**)	07.06.2021	09:40 to 10:10	Post-lesson reflection for lesson 3
06.06.2021	1hr 30min	Mr. Brown, Mr. Adams Mr. Mugu	Continue discussions on lesson plans for Lesson 3 and Lesson 4.						
07.06.2021	30min (wrapping)	Mr. Brown Mr. Adams Mr. Mugu Ms. Elize Ms. Davidson	Wrapping discussions on typed lessons. Lesson presentation & observation sessions.	07.06.2021	12:00-13:08	Lesson presentation & observation for Lesson 4 **(ALGEBRAIC EXPRESSION – add like & unlike terms).**	07.06.2021	15:26 to16:05 13:15-	Post-lesson reflection for lesson 4

The processes of data collection outlined in Table 5.2 were deemed fit for the criteria for establishing the characterisation of questions and questioning practices and how an LS community 'see' their role as questioners. This is consistent with Cohen et al.'s (2018) view that there is no specific prescription for planning research rather the research design is governed by fitness for purpose. The research study was conducted during the time when COVID 19 pandemic had caused serious constraints in schools. In such disruptive times school managers were forced to implement rotational time tables to comply with COVID 19 protocols. Participants then agreed that two separate research lessons be taught by two different teachers on the same day i.e. the first two research lessons be taught on 25 March 2022 and the last two be taught on 7 June 2022. This was because teachers could not frequently leave their schools as they had to protect teaching time.

5.5. DATA ANALYSIS STRATEGIES

According to Cohen et al. (2018) qualitative data analysis involves how we move from the raw data to making sense, explaining, and interpreting the phenomena being studied. Cohen et al. (2018) indicate that qualitative data analysis among other things includes, "organising, describing, and understanding, accounting for, and explaining data, making sense of data in-terms of participants' definition of the situation (of which the researcher is one), noting patterns, themes, categories, and regularities all of which are the tasks of the qualitative" (p. 643). In this study, the procedure of data analysis took place in two main steps, namely preparing and organising the data (section 5.6.1) and coding the data into categories (section 5.6.2).

5.5.1. Preparing and organising the data

The first task in data analysis was to ensure that data are in a form that can be analysed without difficulties. The first stage of preparing data for analysis involved transcribing the recorded data (observational and interview data) by converting video tape recordings into text data (Cohen et al., 2018; Drew et al., 2008). The video-recorded data were first transcribed verbatim using a Computer Assisted Qualitative Data Analysis Software (CAQDAS) known as Atlas.ti which is used to organise and structure qualitative data. Although video-recorded data were convenient because it enabled me to listen to the conversations repeatedly and to insert the non-verbal aspects into the text (Cohen et al., 2018) it was time consuming. Data for document analysis (four lesson plans) were already in word format and were thus loaded into the Atlas.ti software for analysis.

5.5.2. Coding data into categories

I read through the data and allowed the codes and themes to emerge. The approach was consistent with Cohen et al.'s (2018) advice that analysis of qualitative data involves preparation of the raw data, reading and re-reading, reflects on, and makes sense of raw data (transcripts) and what they imply. This is what constituted the inductive dimension of the analysis (Cohen et al., 2018). This was followed by category generation, revision and refinement involving proper coding and recoding. Organisation of data into categories was done with focus on research questions. In this study, the conceptual framework presented in chapter 4 Figure 4.1 was used to guide data analysis. The conceptual framework designed to guide the entire research study is a combination of LS, Emanuelsson's categories of classroom interactions and Variation Theory. Each one of these aspects serve a specific and complementary role to produce a synergistic effect on the study and analysis thereof.

Observational data i.e., questions planned during collaborative lesson planning and questions which were posed during lesson observation and presentations were categorised according to Emanuelsson's (2001) categories of classroom interaction. Questions were characterised as either a topical or conceptual zone. This categorisation of questions applied to document analysis i.e., questions written on all the lesson plans (documents) for all the four research lessons were also classified as either topical or conceptual. The study had interest in the ideas learners construct when responding to teachers' questions during classroom interactions. Learners' responses to the teachers' questions were categorised according to procedural zone which is Emanuelsson's (2001) third category of classroom interaction. Analysis and interpretation were responsive to the emerging data for example there were questions that characterised interactions in the topical zone. Those questions were listed under topical zone.

Data generated from conversational interviews were built into the lesson observation and presentation Analysis was incorporated into the analysis of observational data. By sifting, sorting, reflecting, and making inferences on data 'thick descriptions' emerged. The 'thick description' became helpful in assisting me to focus on what to describe and subsequently explain using literature. I then summarised what I found and moved from describing to explanation through Variation Theory, the phenomenon being studied. Analysis of qualitative data presents several challenges which include personal bias of the researcher (Cohen et al., 2018). In this study, I was interpreting the phenomenon of questioning as I saw it from participants' views (Cohen et al., 2018). Since I was part of the world I was researching, I ensured that reporting and analysis catches different definitions of the situation from different participants. This was necessary to avoid personal bias of which I was conscious as a researcher.

5.6. ETHICAL CONSIDERATIONS

Cohen et al. (2018) define ethics as, "A matter of principled sensitivity to the rights of others" (p.12). Implied in this statement is that researchers must consider the consequence of research on participants and to ensure that their dignity and integrity is not compromised. Fraenkel and Wallen (2019) state that researchers have to do all in their power to ensure that participants are protected from physical or psychological harm, discomfort or danger that may occur as a result of research procedures. Research on the other hand should promote ethical values such as trust, accountability; mutual respect and fairness, and consider social benefits of their endeavours.

Given the significance of ethics for conducting research, I accepted the responsibility to address ethical issues that were arising. In this study, I was mindful of all the ethical issues in the entire research process i.e., taking the responsibility to ensure that the research purposes, design, contents, methods, reporting and outcomes are aligned to ethical principles and practices (Cohen et al., 2018). I conducted the study being mindful of the fact that poorly designed research is tantamount to violation of ethical practice.

On the issue of gaining access and acceptance by those whose permission one needs, several researchers (Ary et al., 2006; Cohen et al., 2018; Drew et al., 2008) suggest that the researcher should seek permission to access the institution or organisation where the research is to be conducted. Since schools are hierarchical in nature, permission to conduct the study was first obtained from the North West Department of Education (Annexure D) and the school (research site) where LS activities were held (Annexure F). Permission was also obtained from four school principals whose teachers formed the LS team that participated in the study (Annexure E). Permission was obtained from both teachers (Annexure G and principals. Furthermore, ethics approval to collect data and ethical clearance to declare adherence to ethical practices after data collection were requested from the Ethics Committee of the University of Pretoria. This research study was conducted in accordance with the University of Pretoria's Ethics Guidelines. The ethical clearance and research approval were first obtained from the Faculty of Education Ethics Committee in accordance with University's regulations. Approval to conduct research in public school (chosen research site) as well as teachers was obtained from the North-West Department of Education.

In order to provide a fair explanation of the procedures and processes of a research, participants were invited to a meeting where information regarding the study, its purpose, the role of participants, obligations and responsibilities were clarified (Cohen et al., 2018; Creswell & Poth, 2018). Cohen et al. (2018) mention the concept of 'volunteerism' which entails ensuring that participants freely choose to take part or not in the study. The purpose of the initial meeting which was held with participants was to inform them about their right to freely decide whether or not they want to take part in the study and that they can withdraw their consent to participate at any point during the research process. Participants were not coerced to take part in the research. All participants were assured that their anonymity will be protected and confidentiality of data will be kept during data analysis and after dissemination of results.

This study also involved learners whose permission to participate was first obtained from their parents since they are minors (Annexure I). Cohen et al. (2018) emphasise that seeking informed consent with regard to minors is a two-stage process. First, the researcher should consult and seek permission from adults responsible for the minors. Secondly, the researcher should approach the young children themselves to obtain informed assent. Cohen et al. (2018) is of the view that children must be given a real and lawful opportunity to exercise choice in participating in research, i.e., they should be given the right to decide whether or not they want to take part. In this study parents of the targeted learners were given consent forms to willingly decide whether they allow their children to participate in the study (Annexure H) and learners were also given letters of assent to decide if they want to take part or not (Annexure I). Consent was received from all parents who were approached. All learners whose parents returned consent forms were given assent forms so that they could also decide if they wanted to participate in the study. All the learners whose parents gave consent assented.

Several researchers (Ary et al., 2006; Cohen et al., 2018; Fraenkel & Wallen, 2009) caution participants to exercise their rights to weigh the risks and benefits of being involved in research. In this study, there were no serious risks anticipated, because data were collected without adapting the implementation of the LS approach, i.e., the process of the LS including teaching and learning and classroom settings were kept natural as they normally unfold when the study is not being conducted. In addition, the fact that learners were exposed to different teachers during the research project is not peculiar to this study and it is therefore not regarded as a risk per se. It is a familiar practice in the schools within the Sub-district where data were collected that LS team members take turns to teach learners who do not belong to their schools and to study lessons which they planned collaboratively. This kind of arrangement or practice is preferred by LS teams and is perceived to be a method of strengthening collaboration. No alterations were therefore made on LS practice to suit the study. However, the following minor risks were managed:

- Potential risk 1: Video recording has the potential to expose facial identities of learners. To mitigate this, maximum caution was exercised to avoid capturing their faces in the video recording.
- Potential risk 2: Learners whose parents may opt not to return the consent forms and/or learners who may choose not to grant assent to participate in the study may miss out on the lessons presented. It turned out that all the parents of targeted learners returned the consent forms and learners also assented, therefore there was no need for mitigation.
- Potential risk 3: The uncertainties that were brought about by the COVID 19 pandemic were prevalent during the period of data collection. To mitigate challenges of potential health risks, all the observers, the teacher teaching the lesson, the researcher and learners wore face masks for the duration of the

lesson and during post-lesson reflections. Hand sanitiser was applied on learners and observers' hands. Social distancing was maintained throughout. Face masks were also worn during collaborative lesson planning.

5.7. QUALITY ASSURANCE CRITERIA

Cohen et al. (2018) mentioned a number of pathways which researchers can consider to ensure quality in qualitative studies. Trustworthiness in case studies is defined in terms of credibility, transferability, dependability and confirmability (Cohen et al., 2018). In the sections that follow, I provide details of how trustworthiness was ensured in this study.

5.7.1. Credibility

Credibility is defined as the confidence that can be placed in the truth of the research findings (Ary et al., 2006; Cohen et al., 2018; Johnson & Christensen, 2017). Credibility in qualitative research as Ary et al. (2006) maintain, concerns the truthfulness of the research findings. This concerns the extent to which the researchers' observations, interpretations and conclusions are believable or credible. Credibility in this study was ensured through prolonged engagements with participants in the field. Johnson and Christensen (2017) refer to this as extended field work. The process of data generation occurred for a duration of six months divided into seven sessions of data collection. Generation of data was carried out from January to June 2021. Data triangulation refers to the multiple use and different sources to provide corroborating evidence (Cohen et al., 2018; Fraenkel and Wallen (2019), Johnson & Christensen, 2017; Creswell & Poth, 2018).

In this study, triangulation was achieved through the use of multiple methods of data collection such as observations, conversational interviews, and document analysis. Another method that was employed to ensure credibility is a method described by Johnson and Christensen (2017) as participant feedback or member checking. This technique involves availing and discussing the researcher's interpretations and conclusions to the study's participants so that they can establish if they agree with what is said about them. In this study, this technique involved taking raw data (interview transcripts), analysis, interpretations and conclusions back to the participants so that they can make their own judgements of the truthfulness of the findings. I shared the findings with each member of the LS team to establish whether the data is the correct interpretation of their original views. Data transcripts were continuously shared with members to afford them opportunity to verify the non-verbal communication that was captured during video recording for possible misrepresentation by the researcher. The fact that findings were consistent with literature added to credibility.

The question of credibility as potential challenge on the use of documentary analysis needed to be settled. For the document to be reliable, the intended purpose has to be considered (Cohen et al., 2018). The reliability of the documents i.e., lesson plans for four research lessons that were analysed was seen through participant's eyes since they were circulated to them for each member to inspect for possible distortion, accuracy, representativeness and whether they (documents) catch the purpose as initially intended.

5.7.2. Transferability

Transferability in qualitative research is synonymous with replicability (Cohen et al., 2018). Unlike quantitative research, qualitative studies do not strive for replicability (Cohen et al., 2018). Transferability in qualitative research is of little importance because qualitative researchers do not strive for replicability or uniformity. Two researchers examining the same setting may arrive at different results but both sets of findings might still be reliable. Transferability in qualitative research is achieved by demonstrating to the readers that the research study's conclusions might be applied to different contexts, situations and times. In this study, I provided the data base that makes transferability judgements possible to potential appliers. The issue of transferability was addressed by providing thorough description of the context and working conditions of the participants. The 'rich and thick' descriptions provided will enable readers to make their own judgements about how well this fitted in with their situations.

5.7.3. Dependability

Ary et al. (2006) define dependability as the extent to which data and findings would be similar if the study were replicated under the same conditions with the same participants. Simply put, dependability refers to consistency of findings over a period of time. Since qualitative researchers are naturalistic researchers, they seek for dependability rather than reliability in establishing the value of the data (Creswell & Poth, 2018). Dependability in this study was achieved through keeping an audit trail of the whole research process, all transcripts, field notes, videos, initial and final drafts of typed lesson plans (Cohen et al., 2018; Johnson & Poth, 2017). All these documents were kept and made available to supervisors for comments and guidance and for possible use with another group in another context.

5.7.4. Confirmability

Confirmability is one of the criteria of trustworthiness that a qualitative researcher must demonstrate. This criterion has to do with the degree of confidence that the research study's conclusions are based on the participants' narratives and words rather than any researcher biases (Creswell & Poth, 2018; Johnson & Christensen, 2017). In this study I employed reflexivity, during data collection and analysis (Cohen et al., 2018). I kept a reflexive journal throughout the research proces where I reflected on what was happening in the research process. I was careful that my knowledge of the subject (mathematics) and LS do not influence the research process. This was necessary to ensure that findings are shaped by participants more than they were shaped by me (the researcher). However, qualitative researchers are aware of the constraints, challenges and potential pitfalls of replicating social phenomena (Creswell & Poth, 2018; Johnson & Christensen, 2017). This is because qualitative researchers admit that the exact conditions under which data were originally collected are difficult to replicate and so variability is expected (Ary, et al. 2006). In this study attention was paid to thick descriptions, and rich codes in interpreting the phenomenon.

5.8. CHAPTER SUMMARY

This chapter was detailed regarding research paradigms and assumptions, research approach, sampling criteria, data collection and instruments. The chapter went further to outline data analysis strategies. Attention was also drawn to the ethical considerations and quality assurance criteria. The approach that was considered is qualitative approach and research design was a case study. Case study was found to be appropriate because it enables me to generate in-depth information regarding the subject of questions and questioning in context that foster collaboration.

CHAPTER SIX: PRESENTATION OF THE FINDINGS

6.1. INTRODUCTION

This chapter presents details of the findings emanating from data collected through observations, interviews, and document analysis. The presentation of the findings provide clarity on the participants' experiences and their views about questions and questioning. The three stages of LS were useful to gather information that assisted in responding to the research questions. The findings presented in this chapter address the main question which states: What are the attributes of mathematics teachers' questions in the LS context?

I have divided the findings according to the secondary research questions. The findings for RQ1 which states: What attributes characterise questions planned during collaborative lesson planning were gathered during Stage 2 of LS cycle. I present questions (written and/or oral) planned during the collaborative lesson planning stage of the LS cycle. Particular attention is paid to the kinds of questions that have the potential to characterise interactions in the topical zone and questions that characterise interactions in the conceptual zone. Data regarding how prior knowledge were incorporated in the planning is also presented. This is because the conceptual framework presented in Figure 6.1 highlights prior learning as one of the considerations for lesson planning. Since coding of data thereof was both inductive and deductive, a number of aspects that emerged during the exploration of data are also presented to help answer research questions.

The findings for RQ2 and RQ3 are presented together. RQ2 states: How do questions planned during the collaboration planning stage permeate the lesson presentation? RQ3 states: How do learners experience teachers' questions during teaching? RQ3 is about how learners respond or react to the teachers' questions during teaching. Data for these two research questions were gathered during Stage 3 of the LS cycle. The findings based on RQ2 and RQ3 attend to a) how planned questions permeate the lesson presentation stage and, b) how learners experienced those questions**.** The purpose of RQ2 was to:

- establish how the planned questions were enacted during the lesson presentation and observation stage,
- establish the purposefulness of questions posed during the lesson, and

- explore the underlying attributes of questions posed by teachers during the lesson.

The purpose of RQ3 was to explore the ideas and thoughts that learners construct when they respond to teachers' questions. Findings presented to respond to RQ3 are aligned to interactions in the procedural zone.

RQ4 states: What critical features on questioning emerge during post-lesson reflection? Findings for this research question were gathered during Stage 4 of the LS cycle. The purpose of RQ4 was to:

- explore the nature of contributions made in relation to questions, and
- establish whether, as they reflect on the lesson taught, the LS team considers the need to raise issues about questions posed during the lesson presentation.

The names used throughout this chapter are fictitious. Since I was interested in the nature of questions across all four lessons, I did not disaggregate the findings according to the lesson number. It is important to point out that there were many questions that were posed by teachers during classroom interactions of all four research lessons. The questions and or tasks presented were those that were helpful in answering research questions.

6.2. THE ATTRIBUTES THAT CHARACTERISE QUESTIONS PLANNED DURING THE COLLABORATIVE LESSON PLANNING

This section addresses the secondary RQ1 which states: *What attributes characterise questions planned during collaborative lesson planning?* Data for RQ1 were gathered during Stage 2 of the LS cycle. The purpose of this question was to: establish whether questions are planned for mathematics lessons and explore the underlying attributes of such questions. Data collection was done through an observation protocol (Annexure A), document analysis (lesson plans) and unstructured interviews. Interviews were conversational and not scripted as a result there was no interview protocol. Clarity seeking questions were asked as and when the need arose during observation. I did not stop the video recording when interacting with teachers. This was important so that everything that transpired including the questions I asked and their replies could be captured in the recording. To gain a full understanding of how questions are conceptualised for teaching and learning during lesson planning sessions, I captured teachers' verbal records during moments where questions were intentionally or unintentionally discussed. The Lesson Planning stage is the stage during which the LS team collectively think about what they intend (plan) for their learners. This is the second stage in the LS cycle during which teachers work together to create a research lesson that addresses the goals which were set during the diagnostic assessment/analysis stage. The process of planning a 'research lesson' culminates in a detailed document which describes the content goals, concepts and skills.

In order to explore the attributes that characterise questions teachers intended to use to facilitate learning, I explored their conversations and focused on questions as they emerged during discussions that pertained to the intended object of learning. Questions that emerged from the discussions and questions that were documented in all the lesson plans became useful for data analysis during the collaborative lesson planning stage. Coding was then done on both oral and written questions. Unstructured interviews were conducted afterwards to corroborate and/or clarify data. The interview took the form of an informal conversational interview because the intention was to ask questions when the need arose or as the situation emerged from observation.

6.2.1. Written intended questions (topical and conceptual zone)

Information on Table 6.1 was extracted from all four lesson plans which were collaboratively planned by an LS team. Lesson plans serve as documents used to corroborate data. Examples of questions from each lesson plan have been indicated as seen in Table 6.1 I provide a brief description of what each lesson was about.

Lesson 1 focussed on solving problems in context involving ratio and rate. The task was typically a complex task, a characterisation which stems from the curriculum policy. Focus was on problems involving speed, time and distance.

Lesson 2 focussed on laws of exponents. Specific focus was on helping learners discover the three general laws of exponents which are :1) $a^m \div a^n = a^{m-n}$; 2) $a^0 = 1$; and 3) $a^{-m} = \frac{1}{a^m}$. The lesson was intended to be investigative in nature to enable learners to make such conjectures.

Lesson 3 focussed on developing algebraic language. The goal of the lesson was to assist learners to recognise and identify conventions for writing algebraic expressions. Teachers considered multiple representations as representing algebraic expressions in equivalent forms such as flow diagrams and words, to achieve this goal.

Lesson *4 focussed* on expanding and simplifying algebraic expressions. The lesson was intentionally designed to address learners' misconceptions in relation to like and unlike terms. The team planned a lesson hoping that errors will emerge from learners' initial responses to a small task administered at the start of the lesson. The plan was then to use those errors to inform the instruction.

Table 6.1: Information from four collaboratively planned lesson plans

Categorisation	Examples of questions
Written Questions on Topical Zone	**Questions for Lesson 1** (a) What is the meaning of the word 'Rate'? (b) What is the formula we use to calculate the speed? (c) What is the formula we use to calculate the time? (d) What is the formula we use to calculate the distance? **Questions for Lesson 2** Simplify the following: (e) $\frac{3^5}{3}$ (g) $x^2 \div x^3$ (h) $\frac{2^5x^3}{2^5x^2}$ (i) $\frac{x^2y^3}{xy^2}$ (j) $8x^6y^3 \div 2x^2y$ **Questions for Lesson 3** a) What are terms or words we can use to represent the following operations? $(\times;\div;\ +;\ -)$ b) Explain what you understand about a variable, expression and equation. **Questions for Lesson 4** a) What do you understand by like and unlike terms? b) Write an example of like and unlike terms c) Simplify the following expression: $6x + 2x$
Written Questions on Conceptual Zone	a) If Susan can run 2km in 8 minutes, how long will it take her to run 5km if she maintains her speed? b) A car travelling at a constant speed travels 60km in 18 minutes. How far, travelling at the same constant speed, will the car travel in 1hour 12 minutes? a) Relate the answer in exponential form in Question 3 with the example you provided in question 1. Is there anything you notice between the two, what do you think happened?

There was evidence of questions that had potential to stimulate interactions in the topical zone in all the lesson plans for four research lessons. Examples of such questions can be seen on lesson plans (see Annexures J and K). The description of what counts as a mathematical question from Smith and Julie's (2014) perspective was considered. This description has been detailed in chapter 2. Interestingly, only three questions that had potential to stimulate interactions in the conceptual zone were planned. Two of the three questions were planned for Lesson 1, while the third question was planned for Lesson 2.

6.2.2. Oral questions that emerged during the lesson planning stage

As I observed teachers' interactions during each lesson planning session, I was taking note of how they direct their attention to the mathematical message they intend to convey to learners and more precisely, how they infuse questions into what they intend for their learners. Questions documented in each lesson plan were coded and used to corroborate observed data. Because of the inductive and deductive approaches to data coding, other types of questions emerged from the data apart from those in the framework. Table 6.2 summarises data as they emerged from teachers' interactions.

Table 6.2: Summary of key issues that emerged during interactions during lesson planning

Zones of Interactions	Categorisation	Comments
Oral questions in the Topical zone	Typical recall, information seeking questions discussed by teachers to facilitate learning	Questions that were intentionally prepared that had the potential to stimulate interactions in the topical zone e.g., typical recall questions, demonstration of calculation procedures, etc.
Oral questions in the Conceptual zone	Thought-provoking questions discussed to facilitate learning	Questions that were intentionally prepared to stimulate interactions in the conceptual zone.
Other things that emerged during interactions	Questions of assumptions	Some questions revealed assumptions about learning. New revelations - different types of questions
	Questions about pedagogy	Questions that showed that teachers are thinking deeply about their teaching.
	Questions about questions learners may ask	Teachers thought about questions learners may pose during the lesson.

6.2.2.1 Oral questions in the topical zone

Questions that had the potential to characterise interactions in the topical zone were given more attention and documented in the lesson plan. Examples of questions that were intended for Lesson 1 which were about solving problems in context involving rate are listed.

a) A car travels at 80km/h, how far will the car travel in 10 minutes?
b) What is the meaning of the word rate?
c) What is the formula we use to calculate the speed?

d) What is the formula we use to calculate the time?
e) What is the formula we use to calculate the distance?

Some of the questions listed in 6.2.2.1 and 6.2.2.2 are the same as those indicated in Table 6.1. Some questions were discussed (oral questions) by a LS team members and were eventually documented (written questions) in the lesson plan. The point here is, some questions such as those written under questions for lesson 4 in Table 6.1 were documented on a lesson plan because they were decided upon by the teacher who volunteered to type the lesson plan. Such questions were not necessarily discussed and agreed upon.

As seen in the examples from (a) to (e), questions which were meant to stimulate discussions in the topical zone, required application of known procedures, explaining meanings of words, recall of formulas, definition of mathematical terms such as expressions, equations. However, during the discussions I was intrigued by the anticipated replies they were hoping to get from learners. Questions teachers ask, tell us more about how they understand the content they intend to teach their learners.

6.2.2.2 Oral questions in the conceptual zone

Questions that had the potential to stimulate interactions in the conceptual zone were made explicit during the planning of Lesson 2. For example, after discussions based on the general rule for division of powers with the same bases, Mr Brown posed the question, “So now is there anything you notice between the two? What do you think happened?” What was intriguing was that Mr Brown asked multiple questions, one after the other. This signalled the likelihood of asking questions without affording learners to cognitively process the answer.

Teachers discussed a task extensively that they intentionally prepared to develop problem solving skills. The task based on Lesson 1[taken originally as it was presented] is as follows:

A car is travelling at a constant speed travels 60km in 18 minutes [*sic*]. *How far, travelling at the same constant speed, will the car travel in 1hour 12 minutes?*

Teachers agreed unanimously that it is a complex task. This characterisation of a question as complex, stems from the curriculum policy document which teachers used as a guide during lesson planning. The complex nature of the task as perceived by teachers, involved specific keys that needed to be unpacked and understood by learners, for example expressing time in hours, understanding the question phrase “how far”, acknowledging the fact that there are two situations. Situation 1 requires learners to determine the speed when time is 18 minutes and distance is 60 kilometres and use that speed when time is varied i.e., 1 hour 12 minutes. Because of the complex nature of the task, the LS team agreed to simplify the task for learners. One would have expected the team to give attention to the variables involved in the task such as ‘do you think more distance will be covered when time is increased and speed is constant?’ A problem-solving task was turned into a procedural task as an attempt to simplify it for learners.

6.2.2.3 Other questions that emerged during interactions

The different zones of interaction (topical, conceptual, and procedural zone) which form part of the framework, were originally conceptualised to frame classroom interactions. It therefore makes sense that some of the questions that emerged during the lesson planning stage were more about other issues that concern teaching than questions that were meant to facilitate learning during classroom interactions. A careful inspection revealed different forms of questions which were about assumptions, pedagogy, and thinking about learners’ thinking. Although these questions were not the focus of the study, they were worth paying attention to because of the context (LS) in which this study is located. In the LS, teachers engage in robust discussions about how they intend to teach what they intend for their learners.

(a) Questions *of assumptions*

As they engaged in discussions, teachers posed questions in relation to the content they intended for their learners. Sometimes questions were posed to each other anticipating a reply and sometimes questions posed did not necessarily require a reply. Interactions during lesson planning stages became a source of insights about how questions get formulated, patterns of questioning that were observable and most importantly, the attributes underlying those questions.

During discussions to explore possibilities of making the learning of exponential laws possible for learners to discern, Mr Adams made the following remarks:

> Two to the exponent zero is what? And now they should notice that they are dividing a number with itself which the answer should be one. For example, if they take 5 to the exponent 2 divide by 5 to the exponent 2 is the same as 25 divided by 25 which is the same as one. So, in other words, automatically they should know that the answer to this 5^0 it should be 1.

Intrigued by Mr Adam's question: "two to the exponent zero is what?" and the subsequent illustration to respond to the question coupled with the assumption "automatically they should know" I wondered if these will stimulate further discussions. Later in the discussions one teacher said, "Yes, it is fine, let's hope they will see because with us we can see because we know". The team seemed reluctant to explore the comment.

Questions that were tied to assumptions such as learners will automatically know, automatically notice, automatically see, appeared frequently during the discussions. Ms Elize questioned "…So automatically they will know that if I have one divided by 2 to the exponent one it will be what?" She immediately answers her question "one divided by two with positive exponent one ($\frac{1}{2^1}$)." In an interview following the discussions on lesson planning I asked the team: "you said automatically they will know that two raised to exponent negative one (2^{-1}) is $\frac{1}{2}$. How will that connection be made? Mr Brown replied:

> That is the main idea in the lesson. I believe that at the end they should have the negative exponent. That's where now they should find the answer of the negative exponent as a positive exponent. Akere [isn't it] they should express a negative exponent as a positive exponent.

The question of how the connection will be made is not responded to, instead a new confusing phrase is thrown in "…they should find the answer of the negative exponent as a positive exponent". In the interview I probed further and asked the team how that crucial connection will be made. Ms Elize offers to explain further by illustrating the example on the board as follows:

> Akere already they worked out this [referring to $\frac{2^3}{2^4} = \frac{2\times2\times2}{2\times2\times2\times2} = \frac{1}{2}$]. Now they compare and connect these two i.e. [$\frac{2^3}{2^4} = \frac{2\times2\times2}{2\times2\times2\times2} = \frac{1}{2}$ and $2^{3-4} = 2^{-1}$]. So, they can connect $\frac{1}{2}$ to 2^{-1}. So automatically they will know that ok this means $2^{-1} = \frac{1}{2}$. And then you come to the other example that when you do expansion and then you compare with the answers that they get. They will realise that this is what is happening. They will see that if we have 2^{-1} that means we have $\frac{1}{2}$.

(b) Questions about pedagogy

Teachers directed important questions to each other during the discussions, for example Ms Tomson said, “Expected answer is two to the exponent negative two. How will they know that the answer is one over four?” This is a powerful pedagogic question because it has the potential to stimulate discussions in relation to varied techniques that can be collectively considered to make it possible for learners to see the relationship between 2^{-2} and $\frac{1}{4}$. While this question pertains to the object that will be handled during classroom interactions, the questioner seems to be conscious of potential constraints learners are likely to face in making such important connections. The question was responded to by Mr Adams as follows:

> That’s what we want them to find out to say a negative exponent it can be expressed in what? By one divided by that particular base raised to a positive exponent. Now as a teacher you can direct them to simplify straight forward without using the law.

Mr Adams’s reply shed light on some implications for teaching particularly on how in his opinion, the object of learning ought to be handled i.e., directing learners, simplifying straightforward and the wordy explanation he offers. Mr Brown added:

> We can go out of the box and explain and say maybe for example we might even want to change a negative exponent into a positive exponent. How are you going to do it? So sometimes you can even show that.

The contributions made by the two teachers do not necessarily demonstrate how learners will be assisted to untangle this seemingly abstract concept. Their contributions to the discussions do not offer a promising direction on how precisely learners will be assisted to make connections between different forms of representations 2^{-2}; $\frac{1}{2^2}$ and $\frac{1}{4}$ so that ultimately they can come to the culmination that a^{-n} is the same as $\frac{1}{a^n}$.

As they engage in discussions, the team also had a chance to reflect on how well the questions they intended for teaching are structured, although this aspect was given little attention during planning. After reading out the question, "What are words or terms that we can use to represent these operations?" He said, "I don't know how the question is, is it fine?" The team was satisfied and indicated that it is fine. During the lesson plan discussions, Mr. Adams indicated that a complex question can be transformed into a routine question. Mr. Brown asked a question "How so?" We may infer from this conversation that somehow questions were considered as pedagogic tools whose relevance needed to be negotiated and reflected upon.

While this study is concerned with questions teachers pose during teaching and learning, the reader is reminded of the context in which the study is undertaken i.e LS context. The question, "How are we going to sell the lesson to the learners?" became interesting because of the context. In the LS, innovative approaches to teaching are anticipated and in this instance the word 'sell' was promising. The question featured several times during the lesson planning sessions. This question had the potential to open up spaces for dialogues about how teachers can maximize instructional impact (an attitude that demonstrates willingness to promote cognitive engagement). Subsequent responses to this question, revealed that the LS team acknowledged their role as questioners. This is because teachers mentioned that questions can be used to attend to that aspect of selling the lesson. Afterwards, Mr. Brown asked his colleagues, "So now the introduction, how do we sell the lesson to the learners?" He went further and answered his own question, "We will start first by asking them to name the exponential laws".

In another session for lesson planning based on rates, Lesson 1, Mr. Adams commented, "Remember again we want to sell the lesson to the learners. Okay, you can advise me in this case. Okay, how are we going to sell the lesson to the learners? He went further and answered his own question:

> For an example the teacher who will be presenting this particular lesson he should sell the lesson to the learners by asking them engaging questions. Akere (isn't it) you are introducing the concept so you have to sell it to the learners. Make them

> own it right. So that is how you can make them own it. We ask those questions you see.

A question is proposed on the spot by Mr Adams, “What is the meaning of the word rate?” While the question, “How are we going to sell the lesson to the learners?” propels other members to tap into their intellectual resources on how they can stimulate learners’ interests into the lesson, it reveals a number of things that become valuable pointers in this study. These include what in their view an engaging question looks like. It indicates that teachers acknowledge their role as questioners. There are other methods like using videos to sell the lesson but in all instances when this question is posed, they choose to use questions. They acknowledge that questions are pedagogic tools that can help learners to take ownership of the lesson. Selling the lesson to learners seems to be a guiding principle for their research lessons because it is written at the introductory part of the standard lesson plan, they use for documenting research lessons.

(c) Instructions mistaken for questions

Distinctions between what qualifies as a mathematics question and all other prompts where reply is expected have been detailed in chapter 2. Another interesting observation was made during the discussions where instructions were referred to as questions as seen in Mr Brown’s comment when he said. “I don’t know whether the question is clear enough”. He went further and articulated what he referred to as the question, “Give any example dividing the same base written in exponential form”. Is it clear? Or does it need to be re-phrased?” In the lesson plan instructions were also labelled as questions e.g., Question 2: Simplify your problem/find the answer to $\frac{2^4}{2^2}$. This view resonates with what is documented in a lesson plan. Instructions were identified as questions. From the point of view of literature identifying instructions as questions is accepted. I will however interrogate this in the next chapter.

(d) Questions about questions learners may ask

As they discuss their learning intentions, teachers considered questions which may be posed by learners during classroom interactions. For example, Mr Brown made the following comment during the planning of Lesson 1 on rate, “This question can arise from those learners, to say Meneer [Sir] is fine but why do you convert by hours maybe to minutes or why do you convert hours to minutes? What is the difference?" This aspect of thinking about possible questions learners may ask during the lesson continued to feature in their discussions. During discussions related to lessons about laws of exponents Mr Adams mentioned that:

> Learners will get the exponent as zero if they apply the law correctly. And some may wonder what will be the answer to that. Most of them will stuck here. To say what is 5 to exponent zero? Some they might ask themselves such questions akere [isn't it so?]. Those who knows, the clever ones maybe they will find it.

The findings in this theme suggested that to some extent teachers were able to think about learners' thinking as they plan lessons.

6.2.3. Comments made by teachers in relation to questions during the collaborative lesson planning stage

The comments teachers made in relation to questions became important pointers/indicators of how they think about questions. Such comments were coded and analysed.

Table 6.3: Descriptors for comments made

Descriptor	Comments
Teachers' consciousness about question types	Comments that suggest that teachers are conscious of question types as they plan the lesson.
How teachers referred to mathematical objects in their questions	Mathematical objects not given their real identities e.g. "We convert this thing first".
Questions depicting Pedagogical Choices (PC) & Pedagogical Content Knowledge (PCK).	Questions that pertain to Pedagogical Choices which teachers plan to consider for enactment stage of the lesson.

6.2.3.1 Teachers' consciousness about question types

During the planning session for Lesson 2, teachers talked about different types of questions as described in the CAPS. They talked about questions that involve complex processes to solve which is referred to as complex procedure in the CAPS. Teachers shared their own understanding of what qualifies as a complex question. When I asked for clarity on what characterises a complex question, Mr Adams had the following to say regarding the question: Simplify $x^0 + 3^{-1} + 3^2$:

> Its more concept in one question. So, the complex part according to me or from my view I can say that's why I say its complex because it contains more concepts to be applied in that question that need to be learnt.

Discussions were held on how a complex question can be transformed to a procedural question. Their understanding of different types of questions according to levels of cognitive demand from the policy perspective evident in the comments made by Mr Mugu:

> Aha! So akere [isn't it] uhh, this type of problem neh [referring to a problem on rates], it can be either in routine or complex procedure. So, in order for her or him to convert it, it might be a problem that is why this question its under complex procedure. So, I believe we are going to make mistakes as educators.

6.2.3.2 How teachers referred to mathematical objects in their questions

My attention was caught by how teachers referred to mathematical objects. This was first noted during discussions for one of the research lessons when Mr Adams commented, "They will say maybe two multiplied by that thing". In yet another lesson preparation session Mr Mugu made this remark, "So, are we going to include this thing we have been talking about? The answer? The key words like terms and unlike terms. Term and terms. If we can define those things, I think we can". Naming mathematical objects (numbers, functions, and infinite examples of mathematical objects) as 'things' is not a proper way to denote them.

6.2.3.3 Questions depicting Pedagogical Choices (PC) and Pedagogical Content Knowledge (PCK)

Some questions that occurred during conversations pointed to the pedagogical choices teachers intended to consider for teaching. As they plan a lesson on rates, Ms Elise suggested, "Here we can make it simple by giving them a formula". "Then we can just say speed is distance over time". As the discussions continue Mr Brown commented:

> But ga re leka go e simplifier [If we try to simplify it] then we will err, maybe analyse it its fine". Since well we said err we simplified everything for them to recall the formulae. So, the triangle is there they can't struggle about the formula. It's fine.

The comments teachers made suggest that they see it pedagogically useful to simplify a problem-solving task in order to reduce the struggle. This, in their view, will make it easy for learners to tackle it. These comments raise important implications regarding the pedagogic decisions teachers collectively consider when planning 'research' lessons. For instance, they do not seem to see their goal as developing understanding, but rather as reducing the complexity of the task and simplifying it so that learners can do it. Teachers also pondered on how best they can approach the lessons they prepared. Mr Brown posed a question:

> But now how do we present it? That's the question". [Mr. Adams responded]: Exactly, that's the question. Wa bona ke ye gape [you see, here it is again]. Ha go na o o ka e tshwarang [no one will get it right]. Like they are very few those who can get it right. Wa e bona go re e boela gape mo go rona [you see it is coming back to us]. Like how do we present this lesson to these learners?"

Teachers were referring to the learner scripts of formal tasks previously administered in the district. Although the first stage of the LS (diagnostic analysis) and Collaborative lesson planning stage are two distinct stages in the complete cycle, it appeared that in practice the two stages could not be practically separated. This resembled the iterative process shown by the arrows in the LS cycle presented in chapter 4 Figure 4.1. Other questions revealed teachers' Pedagogical Content Knowledge (PCK) for example, Mr. Mugu asked, "Maar yaanong, ke sa itse gee. Is 1.2 the same as 1 hour 20 minutes. Or ha ke itse? Or it's 1.2 hours? [But now I'm not sure of what 1.2 means, does it mean 1 hour 20 minutes or 1,2 hours?]

6.3. HOW QUESTIONS PLANNED DURING THE COLLABORATION PLANNING STAGE PERMEATE LESSON PRESENTATION

The lesson presentation and observation stage is the stage of actualisation of the ideas that constitute the lesson plan. As the name of this LS stage suggests, there are two distinct activities that the LS team has to fulfil: one team member presents or teaches the lesson, while other members observe it (hence observers or observation panel). Lest there be confusion, I first explain the subtle differences between my role as an observer and the role of the LS team members as observers during the enactment stage. During this stage the other members of the LS team observed the lesson with the intention of collecting information about how learners learn and generally how the lesson impacts learners' learning. My role was that of an observer as a researcher, observing classroom interactions.

The Lesson presentation and observation stage of LS guided the findings for two research questions i.e., RQ2 and RQ3 (see Figure1). RQ2 states, *How do questions planned during the collaboration planning stage permeate lesson presentation?* The purpose of this research question is threefold, (1) to establish how the planned questions were enacted during the lesson presentation and observation stage, (2) to establish the purposefulness of questions posed during the lesson and (3) to explore the underlying attributes of questions posed by teachers during the lesson. This research question is tied to topical and conceptual zones.

RQ3 states: *How do learners experience teachers' questions during teaching?* The purpose of this research question was to explore the ideas and thoughts that learners construct when they respond to teachers' questions**.** This research question is connected to the procedural zone. In this study, interaction in the procedural zone is about the knowledge that become visible because of the questions asked by the teacher. From Emanuelsson's (2001) perspective, the content possible for the teacher to recognise is the form of learners' presentations such as clear presentations. In this study, the content possible for the teacher to find out is extended to the ideas learners formulate because of the teacher's questions. The findings presented here come from the data collected through observations, document analysis, and unstructured interviews.

Because of the nature of classroom interactions, findings on these two questions are presented simultaneously. In other words, when a teacher asks a question, a learner(s) immediately responds. Interactions lead to questions and questions lead to interactions. For that reason, findings on how learners experience or react to teacher's questions are dealt with or simultaneously presented with findings based on how planned questions permeate the lesson presentation and observation stage.

6.3.1 Shift between zones in classroom interactions

It is possible for interactions in the classroom to shift from one zone to the other. This can be done through the alterations of questions within the same question episode. A transition from a topical zone to a conceptual zone is termed vertical since it promotes deep understanding. This phenomenon was observed during classroom observations. Teachers tended to ask questions that characterised interactions within a topical zone and interactions within a conceptual zone simultaneously. Learners were however not afforded the opportunity to respond to such questions.

Excerpts from a transcript of Lesson 4 which was presented by Mr. Brown is used to demonstrate this shift between zones:

> Mr Adams: "Okay, what is algebra? What is it that you know about algebra? Anything that you know about algebra? Anything? I said that you must feel free to raise up your hand if you want to answer akere? (isn't it so?) Akere guys? (isn't it so?). Okay anything that you know or remember about algebra? Haa? About algebra? Yes (prompting a learner) Is there anything that you remember about algebra? What do we deal with in algebra? Okay, re ntse re le fela foo (while we are still there), what do we understand about the word expression? Haa? What do we understand about the word expression? Because the topic will be under algebraic expressions. Akere? Akere? Akere? (isn't it so?)".
> Learners: "Yes Sir"
> Mr Adams: "We have algebraic expressions and what else?"

In the subsequent excerpt, Mr Brown initiates interactions during Lesson 1 based on rate:

> Yaa but the first thing…. [he writes on a board the topic rate] followed by a question: What is rate? … We are talking about rate. I will hear from you …. What is rate? or what do you know about rate? or What is rate? I think it's not the first time you come across this word rate. I think so. So let me hear from you. What do you know about rate? Or What is rate? Bonolo say something. I think it's not for the first time you come across this word ne? Is it the first time? No, it's not for the first time.

The excepts are used to demonstrate how questions evolved during classroom interactions. More precisely, these interactions exemplified a shift from topical to conceptual zone (vertical shift). The question, "what is rate" require merely a description while the question "what do you know about rate?" invites a whole lot of posibilities, responses that may deepen understanding of what rate is. This aspect of vertical shift is more elaborated in Chapter 4. In these instances, learners were not afforded the opportunity to respond. It was therefore not possible for the teacher to learn about how the learners formulate their ideas.

6.3.2 How teachers asked questions

From observations of classroom interactions, it became clear that teachers have a particular way of asking questions. Teachers tended to repeat the same question more than five times within a short space of time. The more learners remain silent when teachers asked, the more the questions were repeated.

6.3.3 Interactions in the topical zone

Interactions in the topical zone are characterized by questions that call for straight recall of mathematical facts, recognition of correct mathematical formulae and procedures. Learners' responses in the topical zone are judged as either correct or incorrect. As I surveyed the data it became evident that questions that characterised interactions within the topical zone needed to be further categorized. Topical zone became the umbrella concept that encompasses diverse questions that typically did not open up to different ways of seeing. The table below summarizes what emerged in the data.

Table 6.4: Summary of questions in the topical zone that emerged during classroom interactions

	Question attribute	**Comments**
1	Fill in the missing words	Filling in the missing words in the teachers' talk during classroom interactions
2	Question prompts during teacher talk	This code depicts question phrases which when tagged to a statement prompt a 'yes' reply. Example: Akere which translates to "is it not so?
3	Linguistic demands on questions	Language became an obstacle to make questions mathematically sound
4	Questions of accuracy	Questions that required precision in the answer
5	Questions of assumptions	Questions posed with an assumption that the answer is too obvious and there is no cognitive effort required to process the answer.
6	Questions accompanied by actions	Teachers physical, written and verbal actions were tied to their questions.

6.3.3.1. Fill in the missing words

Some form of questioning emerged during classroom interactions. Learners were invited to fill in the missing words or sometimes the missing part of the word in the teachers' talk. Mr Brown begins a statement which he does not complete, "So when you are dealing with rate good people here, we are going to focus on kilo…" learners joined the teacher in a chorus: "meter". He then continues: "When you are talking about the speed you have also to know the units of the speed because you said speed, time, and distance. When you are talking about these things you are talking also about the …" Learners joined in chorus: "quantities".

In another questioning episode Mr Adams wanted learners to complete a word starting with q, "We have what we call…." he writes letter q on the board, faces the direction of learners and makes utterances, "Quo, quo, quo, quo". As he attempts to give learners a clue and invites them to complete the missing part of the word, the variation in the pitch of his voice could be ordered on a scale from low to high. One learner offered a response, "Cooperation". The teacher reacts to this response, "Monna wee cooperation?" meaning ["how can you say cooperation?"]. He then completes the word "quotient" and writes it on the board. "Do you remember a quotient?" "Haa?" "You don't remember it." Learners responded in a chorus, "No Sir"

What prompted learners to complete the word initiated by the teacher or to offer the missing word in the teacher's statement? From observation of classroom interactions, it is the tone [the degree of loudness with which the voice is pitched] that serves as an invitation to join in the talk. The teacher projects his voice in such a way that learners recognise that there is a missing word or part of the word which they need to complete.

6.3.3.2. Question prompts

As I observed patterns in the data, I was struck by the use of the phrase 'akere' which directly translates to 'isn't it?' or 'is it not so?' or 'are we together?' The phrase akere is a phrase commonly used in Setswana language – one of the eleven official languages in South Africa. The meaning of this phrase is shaped by the intention of the user, for instance, it can be tagged to the end of a declarative sentence and turn it into a question. It can also be used as an interjection or just at the start of the sentence where it is used loosely in a conversation without implying anything. The phrase 'akere' appeared many times during the conversations as I read and re-read transcripts for collaboratively planned lessons and for lesson presentations.

The phrase was sometimes tagged as a question, and sometimes used loosely not necessarily implying anything. During the enactment stage of the planned lesson the use of the phrase "akere" gained momentum. The excerpt below is used to demonstrate how it shaped classroom conversations when Mr Adams was teaching Lesson 3, "This is division. Okay now remember now in algebra in algebra remember we can either represent what an algebraic expression or equation in words akere? akere? akere guys?" Learners replied in chorus, "Yes Sir".

In some instances, the use of the phrase appeared to have a persuasive intent. The utterances in Mr Adam's speech (akere, akere, akere guys) was accompanied by a variation in the pitch of his voice from loud to loudest. He visibly directed his ear to the class with his eyes widened. Hidden in this phrase is a secret signal that invites an anticipated reply or an appeal for a confirmation. In several instances where the phrase was used during classroom interactions with a persuasive intent, it was repeatedly uttered until learners replied in a loud convincing chorus, "Yes sir".

6.3.3.3. Linguistic demands on questions

The mathematical message which the teachers wished to convey to learners was often constrained by the use of mathematical language. An exerpt from a lesson presented by Mr Adams is used to demonstrate and describe the complexities inherent in the mathematical language, "Ok so from what we have been doing, what is it that we can say about the law of dividing the same bases of exponents?"
In this example the phrase *law of dividing the same base of exponents* does not clearly resonate with what is actually intended as outlined in the curriculum policy which they referred to when planning this lesson. In the curriculum policy document, CAPS, the intended learning regarding concepts and skills are expressed in symbolic description in a more generalised form as follows:
Calculations using numbers in exponential form: revise the following general laws of exponents

- $a^m \div a^n = a^{m-n}\ if\ m > n$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

In the lesson plan which the teachers collaboratively developed, these curricular intentions were captured in words as follows:

- At the end of the lesson learners should be able to recognise the law of dividing same bases and be able to apply it in calculations.
- They should know that any number raised to exponent zero is one.
- They should be able to express a number with negative exponent as positive or vice versa.

The objective in words as captured in the lesson plan is not an accurate representation of the law $a^m \div a^n = a^{m-n}\ if\ m > n$ as indicated in the curriculum policy document. The wording of the question, "What is it that we can say about the law of dividing the same bases of exponents?" emanates from the incorrect wording of the lesson objectives as it appears in the lesson plan. The wording in the question suggests that it is the bases that are being divided not the powers which have the same bases as spelled out in the curriculum policy document. The missing word 'powers' makes the question mathematically incorrect. During the collaborative lesson planning stage teachers made comments such as *"law of division*"; "the *law of dividing the same base of exponents*"; *law of multiplication of exponent*". These mathematically incorrect utterances found their way into the enactment of the lesson through questioning. The incorrect use of mathematical language embedded in the teachers' question tell us more about their pedagogical content knowledge. The wording of the question had a direct implication on how learners reply and how they acquire knowledge of mathematical facts. This is conveyed in the following excerpt. Mr Adams, "Ok, according to your understanding can you just explain what is happening under this law of dividing the same base of exponent?" Boikhutso replied, "You can subtract the exponent". Mr Adams reiterated:

> He is saying that what he understand is that you can subtract the exponent. That is what Boikhutso is saying. What are you saying Mark? I'm saying what do you realise under the law of dividing the same bases?

Relebogile responded, "I understand that numerators of exponents and the denominators of exponents could change its answers to the exponents". Mr Adams probbed further, "How so if I may ask?" Relebogile replied: "I think that the first numerators, from the numbers that, the examples that we have been given them to you… and then subtract the number three and then subtract it from number nine" [refering to the expression $\frac{4^9}{4^3}$ which they were given to simplify at the start of the lesson]. Mr Adams, "From number nine and then it gives you that Ok". Another learner, Sammy offered another response, “I have noticed that under the law of dividing the same bases, bases don’t change and denominators are always small.” The excerpt below was said by Mr Adams at the start of the lesson when introducing the lesson:

> So for an example in the first one the first activity will take this one of the law of dividing same bases akere guys? Yaa in the first activity what I want us to do, I want us to give any example ra utlwana? (are we together?) whereby you are dividing exponents of the same base ra ultwana or powers of the same base ra utlwana? [are we together?]. Any example that you can give me dividing the same base in exponential form? Ok you can have any base ra utlwana? You can divide three by three having exponents ra utlwana [are we together?].

In another questioning episode that demonstrated constraints of mathematical language, Mr Brown’s attempts to make reference to mathematical objects raises interesting questions as seen in the excerpt. Let me check this one. He writes $x^2 + 3x + 2x$ on the board. Is this thing [referring to x^2] the same as this one [referring to $3x$?”]. In the question posed by the teacher, he refers to mathematical objects, terms x^2 and $3x$ as things as if they do not have identities. Mr Brown, “You must know all those things. Errr how far? How fast? How long? You must familiarise yourself with all these kinds of questions”.

6.3.3.4. Questions of accuracy

In the topical zone teachers ought to open up for possibilities to learn if their learners present anticipated responses and those responses teachers wish to hear. Specific types of questions which I labelled questions of accuracy emerged during the enactment stage. These kinds of questions appealed for precise and clear answers. Excerpts taken from Mr Adams’s lesson are used to elaborate further:

> Who can put it clearly? So, who can clearly state this law of dividing the same base of exponent? Remember you must use what, the same base because you have said that the base does not change akere?
>
> Learners: Yes Sir

Mr Adams went further: “And re rile ra reng? [What else did we say?] the exponents subtract each other akere? So now I want you to put it clearly. Try it Tshiamo”.
Tshiamo, “I think we always work on exponents

Mr Adams: When we are doing what?

Tshidiso: Law of dividing

Mr Adams: "(repeating what the learner said)" "Areng?" [What does she say?] the law of dividing the same bases we always work on the exponents. We leave the base as it is akere? And then what do we do on the exponents? We subtract the…

Learners: [join the teacher] "the exponents"

Mr Adams: Ehee [affirming what learners said] where do we get this one from? [referring to 1 on $\frac{1}{36}$ which resulted from simplifying 6^{-2}]. Now I want you to give me the exact answers to those particular powers ne? 5 to the exponent 2 is the same as what?

6.3.3.5. Questions of assumptions

Some questions were posed with an assumption that answers are too obvious and cognitive effort is not required to process the answer. Such questions were labelled questions of assumptions. Those questions were characterised by statements such as "automatically", "simply means", "obviously". Mr Adams asked learners: "It simply means speed is equal to distance divide by time ne?"

6.3.3.6. Questions and actions

Findings revealed that some questions were accompanied by teachers' gestures such as intentionally directing his ears to the learners, widening eyes, straightening hands in parallel fashion etc. Here I refer to actions beyond gestures i.e., physical actions which accompanied questions teachers posed. A closer examination of classroom events revealed that teachers' questions are defined by their spoken, written, and physical actions. Mr Adams had just introduced Lesson 2 which was designed to guide learners to investigate the laws of exponents. A pedagogic decision that was collectively considered during the lesson planning stage involved asking learners to give examples of powers which they were subsequently required to simplify. Examples learners gave as per the teacher's instruction:

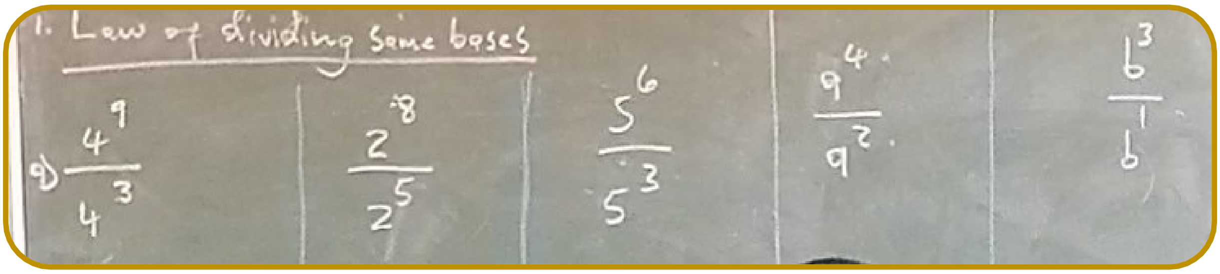


Figure 6.1: Examples of problems written on the boards

Learners displayed their responses on the board as seen in Figure 2

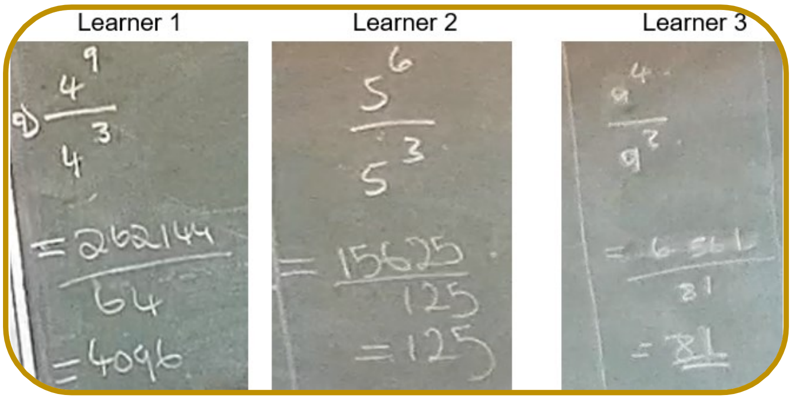


Figure 6.2: Learners' attempts to written questions on exponents

The teacher soon notices that what was intended to be letter *a* is perceived by Learner 3 in Figure 2 as number 9. This observation triggers a question from Mr Adams, "What is this?" The question is accompanied by the action of transforming what appeared as 9 to make it variable *a* as seen in Figure 3.

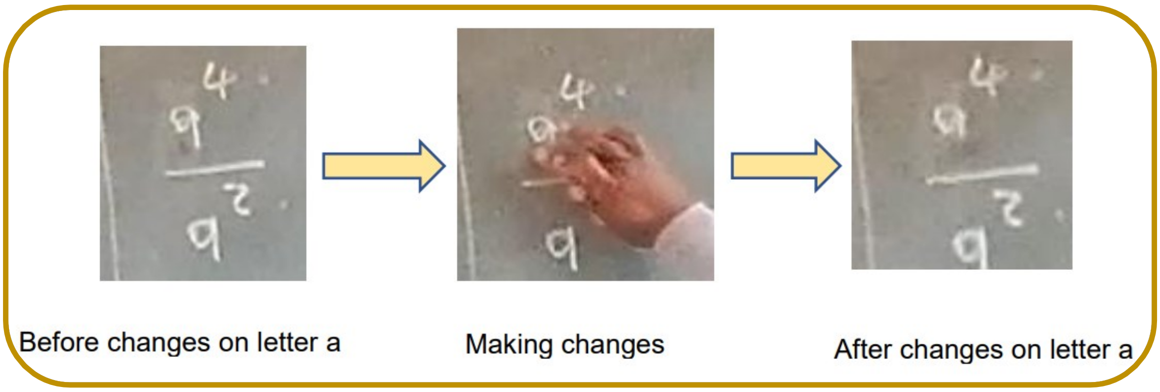


Figure 6.3: Changes made on number 9 to make it letter $\boldsymbol{a}$

Didi: It's letter *a*

Mr Adams: Ok come and correct it. Can you come and correct it ? [approaching L3]

Didi: Shaking head [a sign of dissaproval]

Mr Adams: Anyone who managed to do it?

The purpose of the question, "What is this?" and the teacher's action [adjusting 9 to make it a letter *a*] suggested that the learner must adjust his perception of the object $\frac{9^4}{9^2}$ and see it as $\frac{a^4}{a^2}$. The teacher thought by changing the objects learners will adjust their perception and see and engage with the task as it was intended which in this case was to simplify $\frac{a^4}{a^2}$.

Responses were presented by learners under the guidance of the teacher. Subsequent to that, learners were then requested to follow the examples and simplify the remaining expressions where variables were included. It became clear that what worked with powers of the same bases (where bases were numbers) could not work with powers of the same bases (where bases were variables). The question "Can you come and correct it?" meant the learner was to simplify the expression $\frac{a^4}{a^2}$. But Didi realised that he cannot do it after changes were made from 9 to letter *a*. Attempts made by learners to simplify are presented in Figure 6.4.

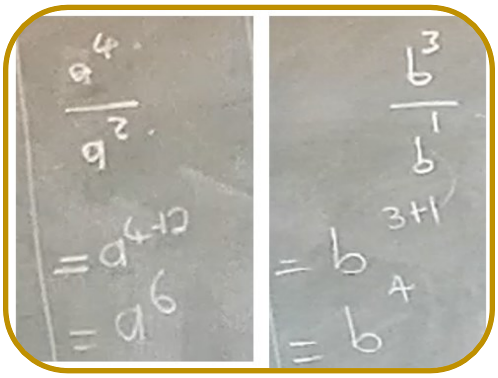


Figure 6.4: Learner responses

During the first meeting when this lesson was discussed a different pedagogic decision which involved expanding was considered as seen in Figure 5.

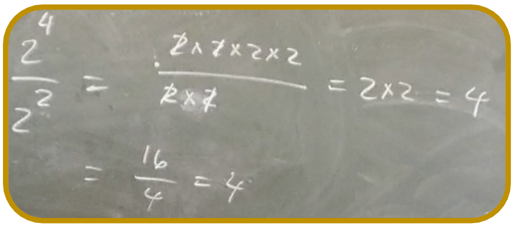


Figure 6.5: Demonstration of expansion method

In subsequent meetings where the lesson was further discussed, teachers reconsidered this decision and agreed on allowing learners to use calculators to determine products and express the results as a power. Big numbers result in ambiguity and delayed the process of discovering the general rules.

6.3.4 Interactions in the conceptual zone

Interactions in the conceptual zone are characterised by ways of comprehending rather than mere recall of mathematical facts or formulas. Questions and tasks in the conceptual zone typically promote cognitive rigor because they draw on learners thinking. Instead of focusing on memories, interactions focus on understanding methods.

In the quest to assist learners to successfully engage in tasks that demand more on their cognition, an LS team prepared a lesson based on rate (Lesson 1). This recognition could be confirmed in Mr Adam's comments during the final planning session for a lesson on rate, "Ok colleagues, remember LS when we prepare a lesson that means we have to target where learners have difficulties maybe in approaching some of the questions in their related grades". The task chosen for the lesson was as follows:

> *If Susan can run 2km in 8 minutes, how long will it take her to run 5km if she maintains her speed?*

From the point of view of CAPS which the team used as a guide, this is a typical complex task. Teachers' efforts to assist learners to overcome the difficulties of the task involved preparing learners to engage with the main task. The task in the lesson plan was broken down into smaller units which involved teaching learners how to interpret the necessary aspects of the tasks such as (1) using the formulae; (2)

expressing time in units; and (3) being able to recognize the missing information in the task. The complex task was thus routinised.

The engagements throughout the lesson, as I observed during classroom interactions, did not afford learners the opportunity to wrestle with the task. Instead, what was meant to be a problem-solving task, that afford learners the opportunity to unwrap the critical features of the object of learning was converted into a procedural task. More emphasis was on assisting learners to recall the formula for calculating speed, time and distance. Learners were provided with tools to remember the

formulae (see Figure 6.6).

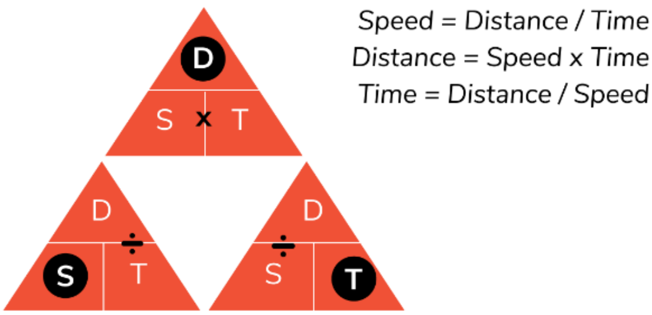


Figure 6.6: Formulae to calculate speed, time and distance

The teacher performed demonstrations to guide learners on how to use the triangle to get the correct formulae as seen in Figure 6.7.

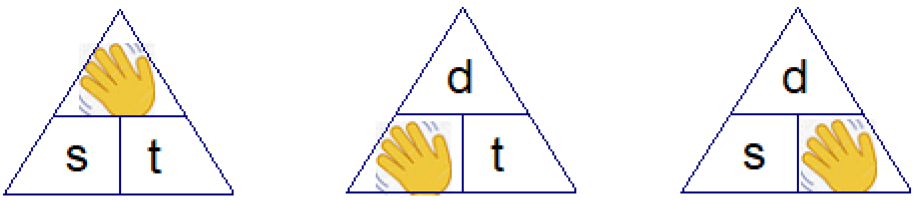


Figure 6.7: Demonstrations to guide learners how to use the triangle to get the correct formulae

Mr Brown made explicit the procedure for using the triangle to derive the correct formulae, “So the same thing to this one [pointing at an equation written earlier *d= distance*] when we talk about distance you hide it [uses his hand to hide d on the triangle] and when you see these [pointing at *s* and *t* on the triangle] you multiply ne?” Learners replied, “Yes Sir”.

The demonstrations and guidance provided, however, did not assist learners to successfully engage in a complex task. To engage successfully with problem solving tasks, learners needed factual knowledge which they lacked. Mr Brown eventually solved the task by himself and asked learners, “Can you copy correction from there? You have homework. Yaa lets be fast to copy correction guys”.

Instances where interactions in the conceptual zone were also observed in Lesson 2 (Laws of exponents). Excerpts from transcripts on classroom interactions are used to demonstrate how questions evolved. After several attempts by the teacher to guide learners to discover the rule a^{-n} is the same as $\frac{1}{a^n}$, learners remain clueless and could not respond to the open-ended question, “Anyone who can see what happened there?” The teacher decided to answer the question by correcting a learner’s response $\frac{3}{36}$ and make it $\frac{1}{36}$ as seen in Figure 6.8.

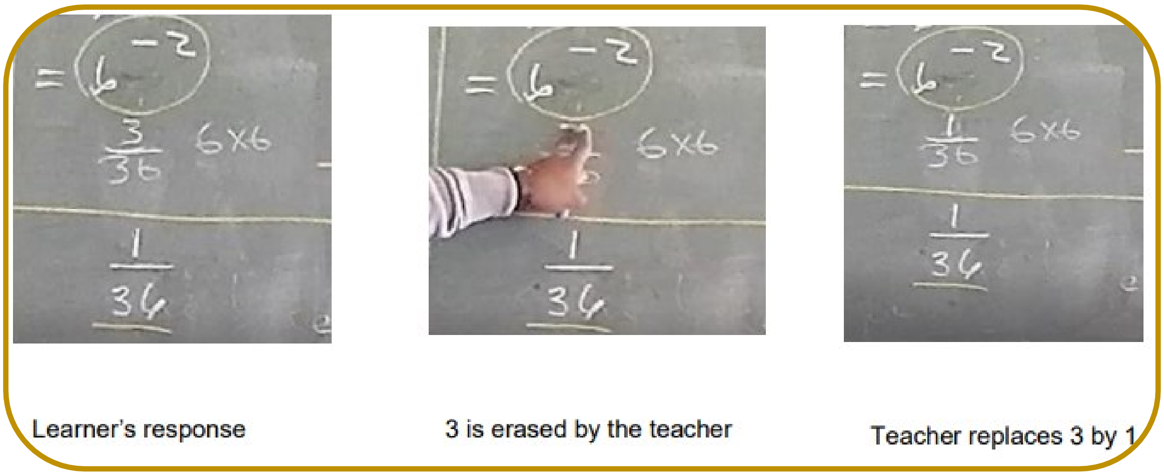


Figure 6.8: Correction made by Mr Adams on the learners’ response

What follows is an excerpt from a transcript of conversations between the learner and Mr Adams:

Mr Adams: Then where do we get that one? What will be at the top?

Tshiamo: It will be three

Mr Adams: To you its three? So that means you are disagreeing with the answer. Ok anyone who sees something else? Atleast he gave us a light ne?

Learners: Yes Sir

Mr Adams: Anyone? Ok guys what do we do? The answer here should be what? Should be the same as that one ne?

Learners: Yes Sir

Mr Aadms : It should also be …

Learners: One

Mr Adams : It should also be one, Ra utlwana akere? [Do you undertsand ?] guys

Learners : Yes Sir

Learners were unable to 'see' or to make connections or to respond to a question that had the potential to open up conversations in a conceptual zone.This marks an interesting twist of events. When a lesson on exponential laws was discussed during the lesson planning, Ms Elize expressed concern about how learners will be made to understand that two to negative two is the same as one over four. The response by Mr Adams was 'automatically they will know that two to the exponent negative two is the same as one divided by two with positive exponent two $(\frac{1}{2^2})$.' Further deliberations included explaining to learners, directing them to simplify straightforward.

Lesson 3 which was an investigative lesson, was based on like and unlike terms. This is a lesson in which teachers anticipated errors in learners' responses to simplification of algebraic expressions. The main purpose of the lesson was to engage learners in the process of making distinctions between like terms and unlike terms. The novel approach the LS team collectively considered involved presenting a task to learners (see Figure 6.9) and instructing them to simplify it with the hope that some will commit errors.

After leaners have completed a teacher may use the following table to show the equivalence of the given expression $(6x + 2x)$ based on learners responses.

x	1	2	3	4
Expression				
$6x + 2x$	8	16	24	32
$8x$	8	16	24	32
$8x^2$	8	32	72	128

Now after completing the table learners should be able to realise that $6x + 2x$ is equivalent to $8x$ as the two yields the same answer for all values of x and not equal to $8x^2$

Figure 6.9: An excerpt from a collaboratively planned lesson plan based on like and unlike terms

These errors were then to be used in an investigative approach to demonstrate to learners why for instance $6x + 2x = 8x$ is not $8x^2$. Although the lesson was investigative in nature, investigative questions were not planned or documented in the collaboratively designed lesson plan. The team anticipated that some learners may consider $8x^2$ to be a simplified form of $6x + 2x$. The plan was for the teacher to use those errors and correctly written responses to evaluate the expressions for different values of x. Interestingly, most learners successfully simplified the expression with the exception of one learner whose simplified expression $6x + 2x$ became $x^6 + x^2 = x^8$. Mr Brown made a comment after observing majority of learners' responses, "In fact most of us we wrote $8x$ ne?"

The table shown in Figure 6.10 was completed by learners in their own books, and later were asked to volunteer to put their responses on the board.

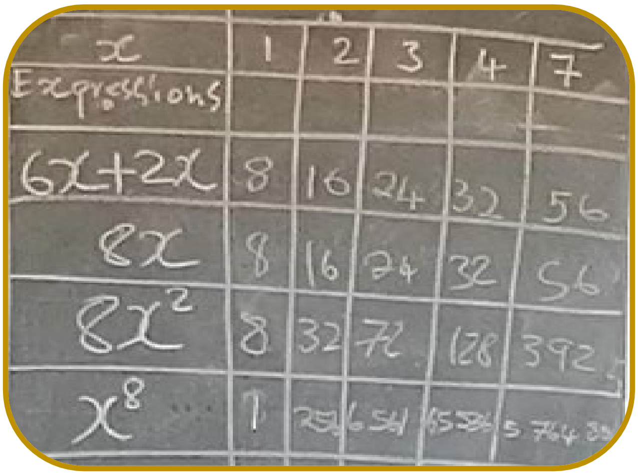


Figure 6.10: Worked solutions from learners

A completed table was then followed by a series of questions from Mr Brown:

> Then after this one [referring to a completed table] what have you noticed about the answers? Can you say something about this table? What can you notice? What can you say about that table? Did you check it thoroughly so? Tell us something about that. What can you say about that? What can you notice on this table? Just raise up your hand if you have noticed something. Check it compare them. Check everything then talk to us. What have you noticed there? [teacher's hand circulates around the entire table]. What have you noticed? I see a hand that come from the side

Kate responded, "On this table this side [pointing to the right side of the table] they are increasingThey increase". Mr Brown, "Okay numbers are increasing?" From there we have this number (pointing at 56) and this number (pointing at another 56)

….. is it increasing? The second learner, Kabelo offered a response, “I have noticed that every number multiplied by one is that number”. The third learner, Karabo responds, “I have realised that the variable x is replaced by the numbers on the row”. Mr Brown continues flooding learners with questions despite these responses:

> Okay. Besides? Anything that you have noticed there? The magic thing that you have noticed on the board there? There may be a thing that you have noticed on the board there. What about these rows? What can you say about the rows now? What can you say about them? What can you say when you compare them? When you compare this row and this one and this one and this one and you sum up everything what can you say now when you compare the whole rows? The fourth learner, Sue responds “I have noticed that a small exponent can make a big number.

The questions repeatedly asked, “What do you notice?” and “What can you say about this table?” “What can you say when you compare them?” which required learners to make their own connections to the object of learning. All the questions posed in this question episode had the potential to open up conversations in the conceptual zone.

The questions came with an expectation that, after the conditions of learning were made available to make it possible for learners to see that the objects $6x + 2x$ and $8x$ are equivalent expressions since evaluating them for different values of x yields the same results. This was not the case with other two terms i.e., $8x^2 \text{ and } x^8$. The process of substituting was intended to assist learners to notice that and to subsequently adjust their perception of the object.

Eventually Dan offers an anticipated response, “I have noticed that we get the same numbers. On row number one it’s just that we separate there with plus to get the sum”. Mr. Brown invites Dan to the board and asks him further, **“**Can you compare this one [referring to $6x + 2x$] and this one now [referring to $8x^2$?”] Dan replied, “You will have to write 4 and this one [pointing + sign] will change to multiply”. [What this learner implied is, to transform $6x + 2x$ to $8x^2$ will involve replacing 6 by 4 and replace a ‘+’ sign by $' \times '$ sign.

6.4. CRITICAL FEATURES ON QUESTIONING THAT EMERGED DURING POST-LESSON REFLECTION

The quality and level of reflection determines the quality of improvements on the lesson and in the practice of teaching as a whole. In this study, the aim of reflecting on teachers' reflections was to explore the nature of contributions made in relation to questions and comments based on how questions impacted learners' learning. Secondly the goal was to establish whether the LS team considered the need to raise issues in relation to questions posed during the lesson. Reflection of each lesson began with the teacher who presented the lesson, followed by reflection by each member of the team.

During the reflection session based on Lesson 1 dealing with rate, Mr Brown, the lesson presenter, reflects as follows on the lesson presentation:

> Oh yes thanks very much Sir. Eeehh I think the lesson went according to the plan. Eeehh as you know always the style of the lesson is always the challenge because maybe when you have to ask learners that question and how also maybe you ask them that question the problem sometimes you the teacher how you ask that question to those learners it becomes very difficult for them to answer you. Then if it's like that question comes back to you now and now you are forced to then apply what teacher centred method is you won't be able to up with learner centred-method. But the way you ask questions sometimes can change the way you planned your lesson but generally I think the lesson went very well. Ehhh thanks very much.

When probed further to elaborate more on what he meant by the statement, "The way you ask questions sometimes can change the way you planned your lesson." Mr Brown had this to say:

> Yaa, thanks very much for that one. Eeehh I was saying that the first thing maybe the way you ask questions to the learners they may need you maybe to clarify what you are asking. Like I said : what are rates? …So they know rates but they want just to define it as it is …without asking them to say whatever they know about rates. Because it seems like when you say what are rates. They don't know the definition, but they know rates. But if you can allow them to say what they know about rates, they can come up with something. You will see that they know something about the word rate.

Subsequent comments in relation to questions were made by other members during the post-lesson reflection for Lesson 1. As Mr Adams chairs the lesson reflection session, Ms Elize ask him to be next following Mr Brown. His comments were as follows:

I can start? How do you tell the chair to start? [they all laugh] Okay let me open eeh there are some things that I have noted down. For example, Meneer [Sir]did a good job at the introduction stage of the lesson trying to involve learners in the introduction of the rates. And learners were asked questions which is a good approach so that they can give responses to those particular questions. Even their interest they were too quiet that is what I noticed. Also, if we come to the lesson objective eeh what I remember, I didn't remember Meneer [Sir] introducing the topic for the lesson. I thought maybe during the lesson he will emphasise the objective of the lesson to say at the end of the lesson you should be able to do this and this. The lesson was learner-centred because most of the time, the teacher was transferring the questions to the learners. So, the learners were the once who in most cases were responding to the questions. And then also what is good about the lesson the teacher made learners to go to the board and write their responses and then he was also moving around the class checking learners while they were busy solving. Things that can be improved about the lesson eeh what I noticed is that most of the time the teacher was fast to correct the learners without taking them step by step throughout their errors. For example, if you look at this one (pointing at the board Figure 6.11)

Learners were supposed to calculate that. So, if you can check here for example the learner was calculating the speed. So, I thought maybe Meneer [Mr.] will go through the response of the learner and then maybe involve the learners to say what might be the error here. As we can see the learner is dividing by the minutes but

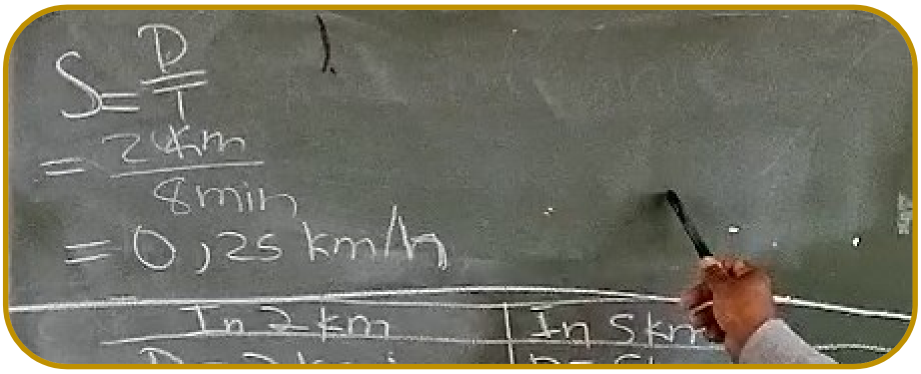


Figure 6.11: Learner's worked solution

look at the answer, Is it hours? Where does that particular hour come from? So, these are some of the errors tse e leng gore [which] a teacher should emphasise maybe try to deal with them before the lesson can continue. In most of the cases let us not rush to finish the lesson. He went too much on correcting the errors made by learners and providing the correct answer, but he didn't deal with those errors.

The following are Ms Elize's comments based on the same Lesson 1 about rate:

> Everything went well. What I liked the teacher at the beginning of the lesson he asked questions to check their prior knowledge if they know what is rate. Unfortunately, learners did not give the full definition of what is a rate. They gave examples of quantities like speed, time, and all that. At the end the teacher explained to learners what is the meaning of rate.

During the post-lesson reflection based on Lesson 2, laws of exponents, I asked Mr Adams [the lesson presenter] if there were instances where he asked questions he never intended to ask. This is how he responded:

> I really can't remember but as Mr. Brown was saying even that last part of negative exponent it was not easy but for an example if you can compare the two laws the law of dividing the same base and the law of zero exponent there were simple questions to pose to learners in order to relate what is happening. But now the last one it was like the rephrasing part of it, it was very difficult how can we rephrase it. So sometimes that's why Mr Brown was saying sometimes the way we pose questions to learners we may be misleading them or make them to not understand. So, I believe maybe you open up a question. Let it be open so that learners can come up with anything they know.

Comments pertaining to questions and questioning were more evident in post-lesson reflections for Lesson 1. This is because three of the five speakers made comments that were specific to questions and questioning. Question-specific comments were also made during post-lesson reflection for Lesson 4, algebraic expressions. When asked by Mr. Mugu, the chairperson of the post-lesson reflection for Lesson 4, what he would do differently if he is given the opportunity to re-teach the lesson Mr Brown said:

> The changes that I can make there is to come up with maybe with two more questions that are little bit tricky. I think it's simple to come up with something simple to the learners. After imposing it to the learners I realized that almost 90% of them they got it right. So, this one the only new thing that I can come up with is to come up with a more challenging task. So, you can see that when 100% of the learners know what you are coming with, you must come up with something different and if you didn't think about it before it becomes a challenge. So, this one the only thing I can come up with more challenging problems to the learners.

When probed further to explain what he meant by "questions that are little bit tricky", I realised that Mr Brown wanted to say a more cognitively demanding question for learners.

The comments made by Mr Adams during the post-lesson reflection for Lesson 4 also pointed to the need to consider questions during teaching. His comments on how he may have probed a learner during classroom interactions are as follows:

> And then also again there is one learner by the name of Bonolo. Bonolo was substituting x^8. And she wrote 1^8 and she said the answer is equal to 8 and then you stopped her. I think maybe you should have given her the opportunity to explain. You immediately opened it to the class to say let's hear what the class is saying about your thinking without first hearing her view about x to exponent 8 being equal to 8. By so doing maybe you could have identified the misconception of raising a number to an exponent because that number is multiplying itself. Maybe she was multiplying the exponent with the base that's why she got 8 you see. So now I believe what she wrote you immediately said check it with your calculator you see. And automatically when using their calculators obviously they will get the right answers but not knowing how we actually get the right answer. So yes, they can use their calculators but where it's necessary. But in that case the learner was supposed to explain or to respond to a question: How did you get this? She can leave the calculator out and explain: how did you get that? You see. Maybe she was going to say $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$ or maybe $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$ or maybe 1×8 is 8 you see. So that is one thing I think maybe you could have addressed in the substitution part.

Question-specific comments based on Lesson 3 which was about developing algebraic language were minimal. Ms Davidson commented as follows:

> I think the lesson was a good lesson. The only challenge that I saw was getting learners to engage in a lesson. The moment when you were asking a question and you are not getting a reply and obo botsa gape [*and ask again*] and you don't get a reply and you kept on asking without getting replies, you should have noticed that these learners are not comfortable. Then you should have thought to drop everything and find something that will engage them so that they can feel relaxed. I think again there was a learner one a araba (who was attempting to respond) on the board. I think the challenge was that the learner o ne a tshaba go araba because they thought if I get it wrong then what's going to happen? [The learner was uncomfortable to respond because she thought what if I get it wrong?]. Whereas if

ngwana a kreile [a child gets] something more interesting you can say they that's good. You can say to them good effort. But let's solve it. How do we solve it? So that the other one who got it wrong and willing to express his answer may not be discouraged to do so.

Table 6-5: Frequency of questions posed during lesson presentation and observation stage

Lessons	Questions in Topical zone		Questions in Conceptual zone	
	Frequency	Examples	Frequency	Example
Lesson 1	71	Soooo this simply means you need what to get to there?	3	• How did you arrive at the answer?
Lesson 2	83	Two to the exponent five is how much?	4	• How can we express this in words? • How can we simplify this?
Lesson 3	103	How many operations do we have in Mathematics?	5	• Algebraic equation and algebraic expression, How do we differentiate the two? • How can we rephrase that particular statement?
Lesson 4	79	Then 2 plus that 6 you get how much?	4	Can you come and explain for us why $8x$? Why do you say the answer is $8x$

In Table 6.5 frequency data are presented to demonstrate how frequently different question types (topical and conceptual questions) were asked during the presentation of each one of the four research lessons. Care was taken to ensure that the same question asked many times is counted once. The table shed light on how classroom interactions were dormiated by questions which stimulated interactions in the topical zone.

6.5. CHAPTER SUMMARY

In this chapter, findings in relation to the broader research question which guide the study were presented. I explored how teachers incorporate questions in their lesson planning, how those questions permeate the lesson presentation, how learners experience the teachers' questions. This chapter also explored the nature of contributions teachers made during the post-lesson reflection. The following chapter will discuss the research findings in relation to the literature review, the theory and framework that underpin this study.

CHAPTER 7: DISCUSSIONS, RECOMMENDATIONS AND CONCLUSIONS

7.1. INTRODUCTION

This chapter presents discussions of findings in relation to the broader research question that guided the study. The four secondary research questions were the organising principle that guided the organisation of this chapter. The discussions related to the secondary research questions assisted me to respond to the main research question. The discussions are presented such that they are compelling and responsive to the secondary research questions to which I responded. While the reader is referred to chapter 6 in certain sections of this chapter, there are instances where best examples of raw data are presented because they were considered more illuminative to highlight a point. Furthermore, the limitations, contributions to the research, recommendations for further research and conclusions are outlined. All names used throughout this chapter are fictious.

7.2. ATTRIBUTES THAT CHARACTERISE QUESTIONS PLANNED DURING COLLABORATIVE LESSON PLANNING

The question being addressed here is RQ1 which is stated as: *What attributes characterise questions planned during collaborative lesson planning?* The purpose of this question was to establish whether teachers include the planning of questions when they plan mathematics lessons, and what the underlying attributes of these questions are. In addition, the purpose was to establish whether questions planned are purposefully aligned to the intended learning. I first attend to a dimension of what teachers perceived as a question. What I discerned from the findings is that according to teachers' questions generally referred to tasks, problems, exercises, instructions, questions etc. These conformed to Smith and Julie's (2014) description of a question that, a question is a sentence worded or expressed so as to elicit information. From this perspective, even an instruction such as: *solve for y in the equation* $2y = 6$ is considered a question. The description of a question generally and a mathematical question specifically, as outlined in the literature chapter, suggests that a question mark may or may not be attached to a sentence to qualify it as a question (Smith & Julie, 2014). These perspectives resonate with how the LS team viewed what counts as a question.

From the observation of lesson planning sessions, questions to be used to facilitate learning featured during the discussions of what teachers intended for their learners. The initial draft of the lesson plan for each of the four lessons was typed by a team leader during his spare time and later shared with all members including myself on a *WhatsApp* platform. This was intentionally done to enable everyone to establish whether the contents represent what was initially discussed and to prepare for subsequent LS meetings. Some questions and tasks which were discussed during interactions found their way into the lesson plan. Some questions were decided upon by a teacher who typed the initial drafts of the lesson plan.

Most questions planned for all the four research lessons were lower order questions that could only stimulate interactions in the topical zone. Examples of questions that were extracted from lesson plans required learners to simply exercise their memory or to perform routine procedures (Table 6.1 in Chapter 6). This finding was consistent with literature that teachers tend to plan easy questions that do not promote cognitive rigor (Chen et al., 2017). From the perspective of Variation Theory, these questions were not worthwhile because they did not present opportunities for learners to develop powerful ways of seeing (Lo, 2012; Marton, 2015). Questions that are worthile from Variation Theory perspective demands highly on learners' cognition, and they present opportunities for varied approaches (Marton, 2015). Questions such as : "speed is equal to distance divided by time ne?" are typical questions that seek choral confirmation. Such questions dorminated classroom interactions. Questions that are worthwhile from the perspective of Variation Theory would have required learners to think deeply about the relationship between speed, time and distance e.g. how does increase in speed affect time taken to complete the trip?

Of the four research lessons that were planned, Lesson 1 about rate was the only lesson based on problem solving. The task had the potential to stimulate mathematical thinking (Livy et al., 2018; Warshauer, 2015; Webb et al., 2019; Sayster & Mhakure, 2020) and consequently interactions in the conceptual zone (Emanuelsson, 2001). This was not the case because the team agreed to turn it into a procedural task with the intention of making it easy for learners. What this tells us is that, although at some point teachers were able to recognise their role as questioners as they planned lessons, they were not aware of what Warshauer

(2015) referred to as ‘productive struggle’. When cognitively demanding tasks are presented to learners, a struggle is anticipated. The LS team then have the responsibility to figure out during planning meetings, how they will provide support to learners if they struggle to engage with the task. This can be done through questions as suggested by (Mason, 2020).

Important implications can be drawn from the teachers’ decisions i.e., changing a problem-solving task to a procedural task. It became apparent that the goal was not to present tasks that stimulate learners’ intellectual curiosity. In order for learners to successfully engage with a question or task that demands highly on their cognition, they need to possess basic mathematical facts and procedures associated with the task. In this study, teachers were aware of potential struggles learners were likely to encounter when solving the task, hence the decision to simplify the task by planning to teach procedures and facts before allowing learners to solve it. The challenges faced by mathematics teachers in implementing problem-solving tasks is well documented in the literature (McCormick, 2016; Ishak et al., 2021). The finding is consistent with the claim made by (McCormick, 2016) that teachers find it difficult to prepare lessons for problem solving when learners lack basic mathematical knowledge and facts. Important implications emerge from this finding that any group of teachers who attempt to implement LS need to be aware of what is at the heart of this professional development model. That is, the goal of LS is not necessarily to teach problems, rather to use these problems to cultivate independent and critical thinking (Warshauer, 2015).

The fact that teachers’ collaborative efforts resulted in planning lessons whose objects of learning were not worthwhile (e.g., Lessons 3 and 4), strongly suggests that they were not yet aware of what LS is used for, i.e., designing lessons that attend to higher order thinking skills (Fuji, 2015). During the conversations in their planning meetings, teachers kept making comments such as: they [learners] will ‘automatically see’, they will ‘automatically notice’ and they will ‘automatically realise’. Marton (2015) is aware of the assumptions teachers make indicating that, teachers always think that learners will see content the same way as they (the teachers) do. These phrases of assumptions filtered into the enactment stage of the lesson in the form of questions.

I now respond to the RQ1 which states, *What attributes characterise questions planned during collaborative lesson planning?* My observations were: (1) most questions planned for research lessons were questions that had the potential to stimulate interactions in a topical zone. (2) A question that could stimulate interactions in the conceptual zone was intentionally converted into a procedural task during the lesson planning stage. (3) Phrases that depicted assumptions about learners' learning featured prominently during lesson planning meetings.

7.3. PERMEATION OF PLANNED QUESTIONS INTO THE LESSON PRESENTATION

The question being addressed here is RQ2 which is stated as: *How do questions planned during the collaboration planning stage permeate the lesson presentation?* The purpose of this research question was three-fold: (1) To establish how the planned questions were enacted during the lesson presentation and observation stages; (2) To establish the purposefulness of questions posed during the lesson; and (3) To explore the underlying attributes of questions posed by teachers during the lesson presentation. As I carefully explore a dimension of how planned questions permeated the lesson presentation stage, I was mindful of the fact that lessons presented in this study were not scripted. The LS team were aware that the lesson presenters will incorporate their professional judgements when the situation emerges (Di Teodoro, 2011).

Several interesting occurrences were observed during the lesson presentations. The question that was planned for the introductory part of Lesson 1, "What is the meaning of the word rate?" was discussed and documented on a lesson plan. During the lesson presentation, things took an interesting twist because the questioner created another question, "or what do you know about rate?" The conjunction 'or' made it sound like the two questions were the same version of another or they appealed for the same description.

The two questions were repeatedly asked interchangeably. The question, "What is the meaning of the word rate?" which was conceptualised during the lesson planning stage was attached to a purpose which was explicitly communicated. This question was given as an example of a question which can be an engaging question for the presenter to sell the lesson to the learners. This purpose gave me an idea of what, in their view, is an engaging question. Their conception of an engaging question was in contrast with the views offered by Variation Theory. From the perspective of Variation Theory, an engaging question fosters high-level reasoning and must present opportunities for diverse approaches because this is precisely what Marton (2015) refers to as "powerful ways of seeing" (p. 15). The fact that an LS team unanimously agreed that a question, "What is rate?" is an appropriate question to engage learners, has implications on how they differentiate between productive and unproductive questions. What this implies is, these teachers may never be able to promote productive classroom discussions through quality and thought-provoking questions because they seem to have misconceptions about the attributes that characterise them. The question "what is rate" is a typical recall question which does not require deep thinking. Interestingly it was perceived by teachers as a question that can provoke deep thinking.

In a lesson taught by Mr Adams, a series of questions were posed at the introductory stage of the lesson of which none was planned or documented in the lesson plan or discussed during planning. I raised questions about those questions. For instance, what precisely was the object of attention in all those questions? Where were learners' attention directed to? Teacher tendency to vary the objects of the question was observed. The key issue here was, the questions repeatedly asked tended to evolve, taking up several forms along the process and appealing for different appearances and descriptions of the object (algebra, word, expression, anything), different actions on the object (know, understand, remember, do, come across) and the identity of the object (what is algebra?). The object of attention was out of focus which could be attributed to the turning and twisting of questions. As was observed throughout the classroom interactions, whenever multiple questions were posed without a wait time, the learners tended to lose focus. Changing questions resulted in changing the object of attention. The implication is, if key questions are not incorporated in the lesson during the lesson planning stage, the lesson is likely to get derailed and the lesson presenter is likely to waste time on unnecessary things.

I noticed particular features of the classroom interactions of lessons taught by Mr Brown and Mr Adams and closely examined them as they occurred during the enactment stage of each lesson to find potential underlying order of questioning across the lessons. Notably was the movement between the zones (topical and conceptual zone) within a questioning episode (Emanuelsson, 2001). In a lesson presented by Mr Brown the question, “What is rate?” (a question in a topical zone) was posed interchangeably with a question, “What do you know about rate?” which is a question in a conceptual zone. In Lesson 4 presented by Mr Adams a question, “What is algebra”? which is in a topical zone was posed interchangeably with a question, “What is it that you know about algebra?” (a question in a conceptual zone). Teachers also frequently asked learners if it is for the first time they hear about those concepts. This suggested that their intention was to tap into learners’ prior knowledge. Why do these teachers keep varying questions and alternating between the zones? The answer is located in Marton’s (2015) description of the dynamic nature of the object of learning. The questions, ‘What is algebra?’ and ‘What is rate?’ were about specific answers while the questions “What is it that you know about algebra?” and “What do you know about rate?” calls for learners to open a dimension of variation of range of ideas, possibilities for variations in descriptions of these concepts (Lo, 2012; Marton, 2015).

The manner in which learners were invited to the lessons through questions exhibited lack of questioning skills. A skilled questioner is conscious of the time he gives learners to cognitively process the answer (Aziza, 2018). From Emanuelsson’s (2001) perspective, movement between the zones is commendable because it deepens understanding of the content being taught. However, in the case of lessons taught by Mr. Brown (Lesson 1) and Mr. Adams (Lesson 4), too many questions were asked appealing for different descriptions and learners were not afforded the opportunity to respond. Movement between the zones was not purposefully done.

As I observed teachers engage in discussions during the lesson planning sessions, I could hear words in their conversations such as ‘akere’ [isn’t it so?], ‘Automatically they will see’, ‘automatically they will notice’, ‘they will realise’, and instances where they indirectly invited each other to complete the missing words or part of the word. These phrases or question prompts dominated their discussions as they discussed lessons during planning. Interestingly, these phrases found their way into the enactment stage of lessons, this time in the form of questions**.** We may deduce from these enactments that the way teachers think and communicate to their peers or even in general is the way they will do with others such as their learners. The question is, how then can LS be used to interrupt this cycle of poor teaching?

The purpose of question prompts with a persuasive intent 'akere' [isn't it so?] was to persuade learners to agree with what the teacher was saying. This was confirmed by the teacher's gestures, directing his ear to the class, widening his eyes. Shahrill and Clarke (2014) are aware of this form of questioning indicating that it features when teachers want to hear specific things from learners. On the other hand, Mason (2020) helps us to understand the question prompt which was accompanied by Mr Adams's gestures (directing his ear to the class, widening his eyes). It is because he was listening *for* an anticipated response which in this case was a loud convincing chorus ["Yes Sir"]. Indeed, learners can experience the teachers' questions through their actions. What this implies for the teaching and learning of mathematics is, perhaps we are still far from realising the dream of 'educating for good questioning' which (Watson, 2018) strongly advocates. For as long as learners are still habituated to questions that lead to thought-stopping answers, mathematics education will never achieve what it is meant to i.e., to encourage logical reasoning and promote cognitive rigor through quality questions.

At a surface level we may be convinced that the kinds of questions outlined in the findings chapter (Table 4) keep classroom conversations fluid. However, if we shift our focus to how these engagements provoke cognitive rigor, we begin to notice that learners have little or no distinctions to make (Marton, 2015). Distinctions were already made in the questions posed by the teachers and the role of learners was to agree. Variation Theory opposes these kinds of engagement where learners are not afforded the opportunity to make the object of learning their own (Marton, 2015). In several instances a dimension of variation was opened and closed by the teachers as they posed and answered their own questions (Lo, 2012; Marton, 2015). The kinds of questions outlined in the findings chapter (Table 4) did not in any way assist learners to see in powerful ways what was intended. The questions in topical zone which were documented in the lesson plans, appearing on the findings chapter (Table 6.1) and those that emerged during classroom interactions (Table 4) were the kinds of questions that did not promise to optimise learning opportunities and/or stimulate intellectual curiosity (Ulleberg & Solem, 2018; Watson, 2018). Classroom interactions observed throughout this study, exhibited traditional teaching practice which was characterised by dominance of questions that required learners to recall information. Such occurrences perfectly fit the description of what Marton (2015) referred to as “broken didactic contract” (p. 13).

The assumptions teachers made about learners’ learning can be explained in terms of the perspectives offered by Variation Theory. For instance, Marton (2015) has argued that teachers think that the new way of seeing the object of learning will occur effortlessly and learners will see it in the same way as they (teachers) do. Teachers still believe that their role is to transfer their knowledge to learners. This was demonstrated, for example, by Mr Adams’s question accompanied by his action of deleting the digit 9 to make it the letter *a* (Figure 3) in the findings chapter, hoping that learners will adjust their perception and successfully attend to it as required. In Lesson 4, the conditions of learning to make it possible for learners to distinguish *like terms* from *unlike terms* were created through variation (varying x values substitute in an expression and observe the evaluated term/expression). While this effort represents a novel approach, the choice of the object of learning perhaps was not so powerful because learners were already familiar with the intended mathematical message (Marton, 2015). The expected ‘noticing’ did not happen and this further propelled Mr Brown to add flavour to his questions, “The magic thing that you have noticed on the board there?”. The word ‘magic thing’ resonates with the phrases which prominently featured in lesson plan discussions such as ‘automatically they will see, automatically they will notice’, ‘they will realise’. The magic did not come. This was not surprising for several reasons. Firstly, the purpose of the investigation task was not communicated to learners. Questions were therefore not tied to any purpose. Secondly, it was observed from the onset that almost all the learners in class were able to simplify the expression $6x + 2x$ without difficulties. This is because the critical features of the object of learning were not empirically determined (Åkerlind, 2015; Bussey et al., 2013; Lo, 2012; Marton, 2015). Perhaps the teacher could have refined or adjusted his question to be: Which expressions amongst those appearing on the table are equivalent? How do you know?

The question, “What is it that we can say about the law of dividing the same bases of exponents?” posed by Mr Adams in Lesson 2’s presentation, emanated from the incorrect wording of the lesson objectives documented on a lesson plan. This question evolved from the conversations during the lesson planning sessions. The question was not planned *per se*, but resulted from an attempt to represent in words the law of exponents which was captured in symbolic form ($a^m \div a^n = a^{m-n}$) in the curriculum policy document. The incorrect wording could also be traced in the lesson objective documented on a lesson plan for Lesson 4. This incorrectly structured question tells us more about these teachers' pedagogical content knowledge [PCK]. McCarthy et al. (2016) cite teachers' pedagogical content knowledge as one of the factors that make it difficult for teachers to become good questioners.

The words that featured prominently in Mr. Adams's questions, such as 'clearly state'; put it clearly; 'give me the exact answers', became pointers of the accuracy and precision that was required in the replies that learners were supposed to offer. These questions were not structured to bring about variation so that the phenomenon in question can be explained in qualitatively different ways (Lo, 2012; Marton, 2015). The gestures that accompanied Mr. Adams's questions suggested an emphasis on the correctness and precision with which the desired answers were to be communicated. He went further and reminded them to use specific words in their answers to achieve the required exactness. In Lesson 2, Mr. Adams's question failed to open up for qualitatively different ways of understanding what was intended for a learning event. Despite the laid-out prescriptions for crystal clear answers, the qualitative differences in ways of seeing the object of learning could still be traced in learners' responses (Lo, 2012).

What implications do all these kinds of questions have for teaching and learning of mathematics in general and for LS communities in particular? Questions that stimulate interactions in the conceptual zone enable us to bring the insights of Variation Theory into meaningful dialogue. In this study a single task which pointed to interactions in a conceptual zone, was reduced to a procedural task with the intention to make it easy for learners. The implication for the teacher professional development occurring in LS communities emerges. Noting this pedagogical choice which a LS team collectively embraced, it seems reasonable to suggest that, LS teams should be made aware of what is at the heart of this professional development, i.e., it should be perceived as an effort to help learners develop powerful learning journeys (Fuji, 2018; Lewis, 2016; Takahashi & McDougal, 2016). If teachers can be made to understand in a deep sense their role as questioners, they will embrace the insights (as outlined in chapter 2) of Warshauer (2015) and Mason (2020). A productive question leads to a productive struggle and learners need support from the teacher.

Questions that stimulated interactions in the topical zone in this study, invoked some concerns. For instance, the fact that teachers mistook instructions for questions, exhibited the limits of their knowledge on what a mathematical question should look like. There appears to be little dialogue on the subject of what precisely counts as a mathematical question.

What these findings are communicating to us is that LS communities should be aware that, inter alia, quality and productive questions should be given sufficient attention during planning sessions (Chikiwa, & Schäfer, 2018; Di Teodoro et al., 2011; Etemadzadeh et al., 2013; Nappi, 2017; Shahrill, 2013). This is because quality questioning is at the heart of quality teaching. A fundamental question that is Variation Theory inspired is: Did the LS team have an idea of the kinds of differences they (as the author of the questions) collectively wanted to see? I ask this question because the findings in this study are indicating that the questioning practices I witnessed represent an extension of what these teachers do in their daily interactions with learners.

I now respond to the RQ2 which states, *How do questions planned during the collaboration lesson planning stage permeate lesson presentation?* (1) From observation of classroom interactions, I could confirm that indeed to some extent, planned questions found their way into the lesson presentation and observation stage. (2) Phrases that depicted assumptions about learners' knowledge and learning which were uttered by teachers during planning, evolved into questions of assumptions during the enactment of the lesson. (3) Questions prompts which constantly featured during conversations in lesson planning sessions as teachers discussed content, evolved into question prompts with a persuasive intent during the enactment stage. (4) Teachers' attempts to represent in words what was captured in symbolic form in the policy, resulted in a distorted message in the questions, making it mathematically irrelevant during the lesson enactment. (5) A productive task, which had the potential to stimulate interactions in a conceptual zone, during the lesson, was turned into a procedural task. These findings indeed have serious implications for policy makers in terms of the guidance mathematics teachers require to successfully orchestrate productive classroom conversations.

7.4. LEARNERS' EXPERIENCES OF TEACHERS' QUESTIONS DURING TEACHING

The question being addressed here is RQ3 which is stated as: *How do learners experience teachers' questions during teaching?* The purpose of this question was to explore the ideas and thoughts that learners construct when they respond to teachers' questions. When learners respond to teachers' questions during instruction, their knowledge become transparent and therefore become accessible to the teacher and in the context of this study,also to observers (Kullberg, 2017). The setting on its own created valuable opportunities for teachers to act upon. Interactions in the procedural zone are about learners' articulations (Emanuelsson, 2001). To be more precise, interactions in this zone provided me with the opportunity to observe the ideas learners construct in their attempts to respond to questions that were asked during interactions.

From observing classroom interactions, learners were able to respond to typical recall questions, for example, they were able to complete the missing words in the teachers' talk. However, there were instances where learners were unable to complete the word the teacher had started. As Mr. Adams attempted to give learners a clue and invite them to complete the missing part of the word that begins with quo…, a learner responded by saying *cooperation*. The expectation was that they will say *quotient*. What the learner saw, was in contrast with what the teacher expected them to see. This phenomenon is explained by Marton (2015) that teachers and learners will always see the content to be learnt in different ways. No effort was made by Mr. Adams to find out why the learner responded by saying cooperation not quotient.

During the classroom interactions for Lesson 2, Mr Adams posed a question, "What is it that we can say about the law of dividing the same bases of exponents?" This question was constrained using mathematical language. This question revealed how the object of learning appeared to three learners who responded. Learners had developed different perspectives of the object of learning (the general rule for division of powers with same bases). The conditions that were laid out (as outlined in the findings chapter) did not assist them to adjust their perception of the object of learning and develop new meanings that were intended.

The incorrect words in Mr Adams' question, "Under the law of dividing the same bases" could be traced in Sammy's response. The words contained in Relebogile's response clearly demonstrated that she missed out on the critical features of the object of learning. The critical features involved consideration of the operation to be perfomed on the exponents of two powers whose bases are the same. Her reply lacked mathematical structure and coherence. The difficulty that learners were supposed to overcome rested upon their abilty to unpack the critical features which in this case involved making a conjecture for dividing powers of the same bases. However, this learning was not made possible for them and it was further perplexed by the teacher's questions which I viewed as not mathematically sound.

What does this tell us about the knowledge displayed by these learners as a consequent of questions and questioning? The mathematical knowings which were captured in the three learners' verbal records are quite revealing and serve as evidence that each one of them has developed a different perspective of the object of learning (Kullberg et al., 2017; Lo, 2012; Marton, 2015). To Sammy, the bases do not change and denominators are always small. To Boikhutso, it is about subtracting the exponents. It is difficult to trace the key message in Relebogile's reply because her ideas seemed scattered. I saw Relebogile as coming from an inappropriate place of knowing how to articulate her thoughts and communicate mathematically when she mentioned that, "I understand that numerators of exponents and the denominators of exponents could change its answers to the exponents". Of course correctness is difficult to judge, but it is also difficult to understand what she implied. However, the scattered ideas found in her response could also be traced in the teacher's talk which were captured throughout the lesson.

It is possible that the learners' situations were aggravated by the teacher's inappropiately constructed questions which lacked explicit attention to the critical features (exponent values in the powers that were being divided) and how they connected to the big idea within the lesson. What are the implications for learning? The challenges that teachers encountered as they attempted to formulate questions, point to the significance of language in the teaching and learning of mathematics. Language is a vehicle for thought because we organise our thoughts through language. Chikiwa & Schäfer (2018) have long recognised the constraints of questioning in multilingual classrooms. In this instance, the wording of the question had direct implication on how learners replied and how they acquired knowledge of mathematical facts. The inappropriate choice of words in a question made it poorly worded. To become good questioners, teachers must be mindful of the choice of words.

Towards the end of Lesson 4 which was presented by Mr Brown, a series of questions were posed to the class, “What do you notice” and “What can you say about this table?” “What can you say when you compare them?” The questions came with an expectation that, after the conditions of learning were made available, learners will see that the objects $6x + 2x$ and $8x$ were equivalent expressions since evaluating them for different values of x yields same results.

Learners presented diverse responses. Kate said numbers were increasing while Kabelo indicated that he noticed that every number multiplied by one is that number. Karabo said the variable x is replaced by numbers. Sue’s reply was, “I have noticed that a small exponent can make a big number.” All four learners’ replies demonstrated that the object of learning appeared differently for each one of them. The investigation that was conducted (details in the findings chapter) did not help the learners to adjust their perception of the object of learning in the way Mr Brown expected.

Eventually Mr Brown changed his question, “Can you compare this one referring to $6x + 2x$ and this one? referring to $8x^2$?” Dan replied, “You will have to write 4 [meaning replace 6 by 4] and this one [pointing + sign] will change to multiply. What Dan implied was that, to transform $6x + 2x$ to $8x^2$ will involve replacing 6 by 4 and replace a ‘+’ sign by $'\times'$ sign. In this instance, Dan made the teacher aware of the transformations that needed to be performed on the algebraic expression $6x + 2x$ to create an equivalent term. On the spot he altered the expression $6x + 2x$ to create the factors $(4x \times 2x)$ that are equivalent to $8x^2$). This demonstrated deep conceptual understanding.

What lessons can be drawn from these interactions and learners’ conceptions in particular? A Variation theory-inspired response to this question would be: the conditions of learning do not guarantee learning, but they can only make learning possible (Lo, 2012; Marton, 2015). As an observer, responses offered by learners made it possible for me to see the missing bricks in learners' foundational knowledge, as in the case of Relebogile. If what is to be learned is not made explicit to learners, they are likely to get lost. The series of questions posed were not tied to any purpose. Learners' responses to questions were shaped and influenced by how the questions were worded. The case of Sammy given as an example of how the wording of the question influences how the answer is conceptualised. It is possible for learners to demonstrate novel approaches if teachers adjust the questions to re-direct learners' attention. This was evident in the response offered by Dan.

What implications do all these lessons have for teaching and learning in general and for LS communities in particular? Questions posed during classroom interactions should be well thought out and be connected to a purpose. Marton (2015) puts it elegantly like this, "But to formulate a question, a problem, you must have some idea of what kind of differences you (as the author of the question) are interested in" (p. 93). What this translates to is that thinking about the questions we (as teachers) intend to use to accelerate learning is important, but equally important is thinking about the information we wish to elicit through our questions. Teachers should also take note that learners' responses to questions are a window into their minds. In general, teachers should take into cognisance that their questions matter most during the key pedagogical teaching moments when the object of learning is handled.

I now respond to the RQ3 which states: *How do learners experience teachers' questions during teaching?* In several instances learners' responses to questions confirmed Marton's (2015) logic, i.e., what is taught is not synonymous with what is learnt. This is so because learning is different for every learner. How learners respond to teachers' questions is dependent upon many factors such as: (1) The experiences the learners bring to the learning environment (how learners appropriate the object of learning) depends on state of mind, level of readiness to engage with content, past experiences on the topic. (2) How teachers' questions are formulated have direct influence on how learners formulate their conceptions. (3) The connections between questions posed and the intended purpose. (4) How teachers adjust their questions to re-direct learners' attention

These pointers have been found to be important indicators on how learners experience teachers' questions during instruction. Productive and purposeful questioning can be heightened if teachers can embrace the insights of Variation Theory, (Marton, 2015).

7.5. CRITICAL FEATURES ON QUESTIONING THAT EMERGED DURING POST-LESSON REFLECTION

The question being addressed here is RQ4 which is stated as: *What critical features on questioning emerge during post-lesson reflection*? In this study, the aim of reflecting on teachers' reflections was to explore the nature of contributions made in relation to questions and comments based on how questions impacted learners' learning. Secondly the goal was to establish whether the LS team considered the need to raise issues in relation to questions posed during the lesson. Reflection of each lesson began with the teacher who presented the lesson, followed by reflection by each member of the team.

During the post-lesson reflection of Lesson 1, Mr Brown made detailed comments in relation to his way of questioning. He seemed to have come to terms with the fact that he was perplexed by his ways of asking questions. He voiced the dilemma he faced during the lesson and this could be captured in the statement he made when he was given the opportunity to speak first as the lesson presenter:

> When you have to ask learners that question and how also maybe you ask them that question the problem sometimes you the teacher how do you ask that question to those learners it becomes very difficult for them to answer you.

These comments demonstrate self-introspection and an honest admission of his own limitations on asking questions. He further alluded, "But the way you ask questions sometimes can change the way you planned your lesson". This remark further revealed an examination of his own consciousness on how he experienced his questioning skills during instruction and how it interfered with the mathematical message he intended to deliver. We can deduce from this remark that Mr Brown's questioning became the object of attention when he was given the opportunity to reflect.

I must say that these detailed comments made by Mr Brown during post-lesson reflection of Lesson 1 took me by surprise. This is because in all the post-lesson reflection sessions that I observed outside this study, teachers generally never incorporate questions in their reflections. Nevertheless, important implications can be drawn from Mr Brown's reflective comments. Personal reflection, according to Gutierrez (2015) is the most effective strategy for teachers to develop their capability. In Watson's (2018) words, Mr Brown's reflective comments demonstrate 'intellectual humility'. This was so because he entered conversations willing to admit the limit of his knowledge on how to ask questions. We may infer from his comments that, somehow his participation in the LS gradually impacts his practice.

Contrarily, other members of the team did not seem to have interest to explore Mr Brown's comments or to find ways in which the concerns he raised in relation to his questioning behaviour can be addressed in future lessons. Instead, their comments suggested that there were no challenges. For example, Mr Adams' comment during post-lesson reflection of Lesson 1 was, "…learners were asked questions which is a good approach so that they can give responses to those particular questions". He went on to say, "The lesson was learner-centred because most of the time, the teacher was transferring the questions to the learners. So, the learners were the ones who in most cases were responding to the questions". Judging from Mr Adams's comments based on Lesson 1, the concerns raised by Mr Brown regarding how puzzled he was by his ways of questioning, do not seem to matter because they remained uninterrogated. To Mr Adams, the fact that questions asked were responded to, painted a picture of learner-centredness. His reluctance to comment on the nature of questions suggests that he may have not paid attention to how questions shaped classroom conversations.

Mr Adams expressed similar concerns about how difficult it was to construct a question during the lesson for Lesson 2. He made specific reference to a moment where he wanted learners to see the connection between 6^{-2} and $\frac{1}{6^2}$. The question he posed during the lesson, "Anyone who can see what happened there?", could not assist learners to grasp the connection between these two representations. The challenges raised by the two teachers are consistent with the observation made by Cheng et al. (2017), that quality questions are difficult to create in the heat of the moment during instruction. This is precisely why productive questions have to be planned ahead of the lesson presentation. Purdum-Cassidy et al. (2015) are aware of the reasons for teachers' reluctance to plan questions ahead of the lesson.

It is because they think questioning comes naturally and they need not practice. From the perspective of Variation Theory, if teachers do not have a clear vision of the object of learning, then they will not prepare quality questions that will make it possible for learners to discern the critical features (Marton, 2015). One advantage of having several members of the LS team is that these extra pairs of eyes and ears, which Kotelawala (2012) talks about, serve an important purpose of closely examining each part of the process. The question now becomes, how useful were the multiple extra eyes and ears in the entire process?

The comments made by Mr Brown and Mr Adams in my view presented an opportunity for a LS team to engage in discussions on how the quality of questions and questioning can be improved in future lessons. They both developed consciousness about questioning and they saw the need to elevate their concerns to the relevant platform, LS community. Posthuma (2012) argued that for LS teams to achieve the deep reflection that is needed, other teachers are needed in the reflective process. It did not seem to be the case in this study. Ono et. al.'s (2013) question, "How reflective are LS discussion sessions?" becomes significant now. But what precisely made the two teachers (Mr. Brown and Mr. Adams) reflect on their questioning? And why is it that other observers were unable to detect the dilemmas they faced? The two teachers have travelled the journey which their peers did not experience. They were directly involved in the situation (lesson presentations). From the perspective of Variation Theory, we can understand why the concerns raised by the lesson presenters during the post-lesson reflection do not seem to matter to observers. In Marton's (2015) words, "…nobody can experience difference for someone else" (p. 220). What this translates to is: experience is a personal encounter and no person can experience someone else's experience. Mr. Brown and Mr. Adams had the first-hand experience of what it feels like to be faced by a dilemma of being unable to construct an appropriate quality question on the spot when the need arises. That was the possible reason why the issues they raised about questions and questioning did not seem to matter to other observers.

Nevertheless, there were other comments that were made by observers that pertained to questioning. For example, the question prompt '*akere*', which featured quite prominently during the enactment stage was somehow noticed by Mr Mugu during the post-lesson reflection for Lesson 2. He advised the team to refrain from using the word ‘akere’ because it gives an impression that what the teacher says cannot be interrogated by learners. During the post-lesson reflection for Lesson 3, Ms. Davidson indicated that Mr. Adams should have noticed that learners were not ready to respond to a series of questions which came one after the other. She further suggested that he could have paused and thought of alternative ways to invite learners into the lesson. The construct of noticing is echoed by researchers such as Biccard (2020), Shack et al. (2021) and Dreher and Kuntze (2015). These researchers are in consensus that noticing in the teaching profession is specialised. According to Dreher and Kuntze (2015), teachers’ ability to notice rests upon their professional knowledge and perspectives. This view is supported by Biccard (2020) who maintains that decisions made by teachers will be influenced by what they notice, which could change how they conduct lessons. What Ms Davidson implied was that, Mr Adams missed the opportunity to act appropriately because he continuously asked questions even in the midst of learners’ silence.

During post-lesson reflection for Lesson 4, Mr. Adams recognized the need for the teacher who presented the lesson (Mr. Brown) to open conversations and optimize learning through a quality question. He (Mr. Adams) indicated that the learner could have been given a chance to explain how he got the answer. The question specific remarks made by the three teachers, Mr. Mugu, Ms. Davidson and Mr. Adams during the post lesson reflections, open a window for some element of hope that LS can be used as a platform to interrogate some of the taken-for-granted aspects of the teaching practice. Consistent with Gutierez’s (2015) view, these remarks are deemed to be “constructive utterances of opinion and feedback” (p. 315). Indeed, LS has the potential to turn teachers into researchers of their own practice (Biccard, 2020; Fujii, 2015; Takahashi & McDougal, 2016).

While the reflective comments made by the three teachers signal and lead us into a promising direction of engaging in reflective practice, there are deeper issues concerning questions and questioning that deserved attention during post-lesson reflections. Different kinds of questions that emerged during the presentations of four research lessons, e.g., there were questions that required learners to fill in the missing words, while some questions posed linguistic demands on questioning. Some questions appealed for accuracy while some were based on assumptions.

One would have expected these different characterisations of questions to find expression during the post-lesson reflection stage of each lesson. The questions of assumptions which filtered into the lessons, should have probed them to examine what they took for granted. This did not happen.

I now respond to the RQ4 which states: *What critical features on questioning emerge during post-lesson reflection*? In responding to this research question, I direct my attention to the attributes that characterised questions outlined in the findings chapter (Table 4). These critical issues in relation to questioning did not receive the attention they deserved during the post-lesson reflection. These different questions that emerged in my view are critical issues on questioning because they acted as impediments on autonomous construction of mathematical knowledge by learners. The attributes of questions outlined in Table 4, in my view, defeat the whole purpose of LS which is to promote intellectual autonomy (Pjanić, 2014). Observing interactions during the post-lesson reflection stages have sharpened my awareness of what matters most to teachers. It is not how questions shape classroom interactions. Considering how unproductive questions dominated classroom interactions in all four lessons, the question that cannot be escaped remains: what was the role of the extra eyes and ears in the lesson observation and presentation?

7.6. REFLECTING ON THE UTILITY OF THE CONCEPTUAL FRAMEWORK

The conceptual framework developed to guide this study is an amalgam of LS, Emanuelsson's categories of classroom interactions and Variation Theory. Each one of these dimensions serves a specific and complementary role to produce a synergistic effect on the study. A conceptual framework was an explanatory devise that made it possible for me to explore the intersections of Variation Theory and a phenomenon of questioning. Review of literature was guided by the theoretical framework. For instance, the attributes of questions were broadly discussed while foregrounding the insights of Emanuelsson's categories of classroom interactions which featured prominently in the theoretical framework. Emanuelsson's categories of classroom interactions served as categorical tools i.e., findings of the study were structured in terms of Emanuelsson's categories of classroom interactions. In this case, questions planned for lessons and enacted during teaching and learning were categorized in terms of whether they stimulate interactions in the topical zone or in a conceptual zone. The questions teachers pose during classroom interactions have a function of assessing how learners learn the content and make their knowledge accessible, and most importantly, questions influence how learners formulate their conceptions.

In this study, interaction in the procedural zone was about how teachers' questions influence the way learners see, experience and understand what is to be learnt. The research questions that guided the study were grounded in the theoretical framework. The design of data collection instruments was done with the conceptual framework in mind. Most importantly, the conceptual framework was instrumental in assisting me to rigorously and appropriately answer the research questions. The conceptual framework guided the path of data analysis and in a very succinct way it enhanced the discussions of findings for this study. The choice of research design (qualitative approach) and data analysis plan were guided by the conceptual framework. A conceptual framework was fit for the purpose of structuring the research questions, discussing of literature review, as well as organizing and discussing the findings, conceptualising the research instruments.

There were however instances where the conceptual framework was not a perfect match about the analysis of data. For instance, from Emanuelsson's (2001) category of classroom interactions, procedural fluency is more about the form of learners' presentations. What this means is, how responses to questions are presented (posters, booklets, oral) is of interest to the teacher because they are able to say whether the presentation is clear or not so clear or the presenter is loud or not so loud. This description did not accommodate my approach because of more interest to me were the conceptions learners formulated as a result of the questions teachers asked.

7.7. CONTRIBUTIONS OF THE STUDY

What contribution did this study made to the on-going conversations regarding classroom questioning within LS contexts? Even though LS is a well-established teacher development model in the Japanese education system (Fernandez & Yoshida, 2012; Fuji, 2018; Pjanić, 2014) adoptive cultures are still grappling to understand how to effectively implement it. Research on questions and questioning in the LS context have never been conducted in a South African educational context. This means literature in this area is limited. This study is significant because it contributed to literature on how teachers participating in the LS use questions to facilitate learning. There are key important areas where the findings from this study contribute to building a knowledge base for future research on LS in the South African Education context.

7.7.1. Collaborative lesson planning

The fact that to some extent teachers thought of questions to include in their lessons was something that emerged during observation. Thinking about questions prior to the implementation of a research lesson helped teachers to think ahead of critical moments during the lesson. Although in some instances teachers were unable to focus their attention to desirable quality questions, it was found that participation in this study stimulated teachers' awareness on the subject of questions and questioning. LS is regarded as a venue for professional development of in-service teachers. In this study, the challenges that surround classroom questioning were elevated to an environment that fosters collective responsibility. This study contributes to knowledge and understanding of how secondary school teachers incorporate questions in their research lessons. When mathematics teachers prepare formal tasks (tests/examination) they refer to the curriculum policy for guidance on how to structure and categorise questions. This is so because there is a framework dedicated for providing guidance to teachers for the development of questions for formal assessment tasks. During the collaborative lesson planning, it occurred to me that teachers did not have an explicit guide to refer to when they plan questions for formative assessment. This study revealed this policy implementation gap. The Department of Education is not currently addressing this oversight and gap in policy implementation.

7.7.2. Lesson presentation and observation

There were significant pointers throughout the research processes which indicated that participation in the LS have contributed to teachers' development. This research study has carefully inspected how an LS team intentionally and unintentionally incorporates questions in what they intend for their learners and how such questions filter into the lessons. This was done by collecting a rich set of data obtained through observation, documents analysis (lesson plans) and conversational interviews. This study therefore contributes to a body of knowledge on how LS communities can begin to think deeply about how informal assessment can be considered for each LS cycle. The study shed light on how the character of questions posed by teachers during instruction can either make or break the lesson.

7.7.3. Post-lesson reflection

LS enhances mathematics teachers' reflection on their teaching practice generally and their questioning behaviour specifically. Although teachers did not pay attention to other attributes of questions that deserved interrogation during the post-lesson reflection, it was evident that participation in the LS have to some extent stimulated discussions around the subject of questions and questioning. Participation in LS has proven to enhance mathematics teachers' reflection in relation to questions during the post-lesson reflection stage.

7.7.4. The proposed conceptual model for questions and questions within LS

One of the key aspects that teachers consider when planning a research lesson is predicting learners thinking or responses. But questions are drivers of teachers intentions during the teaching and learning process i.e. it is through questions that teachers can access learners' thinking. The challenges observed throughout this study call for a strengthening of LS model that promote questions that are used to facilitate learning. The types of questions such as topical and conceptual questions, and the opportuned moments during which they could be asked, should intentionally be discussed and included in the lesson plan. Such questions should permeate the lesson presentation as well as the post-lesson reflection. Put differently, questions (especially key questions) should be explicitly stated during the collaborative planning sessions and intentionally posed during the classroom interactions. Questions should also be a subject of discussion during the post-lesson reflection stage. In fact, I propose that, contrary to the conventional view that post-lesson reflection should focus on learning, questions and questioning (teaching practice) should also be a subject of post-lesson reflection. Although focusing on learner thinking during post-lesson reflection is important, teachers are likely to learn more from their practice if they also reflect on their questions and questioning attributes.

7.8. LIMITATIONS OF THE STUDY

Learners were not interviewed to gain an in-depth understanding of how teachers' questions influenced their thinking. Instead, learners' experiences of teachers' questions were based on my observation of how they responded to the questions. Perhaps, the findings would have been different if the learners were interviewed. Data collection for the study was carried out during a time when the country was still experiencing the impact of COVID 19, as such restrictions were still imposed. Schools in general and at the district where the study was conducted were not functioning normally e.g., rotational time tables were used to allow for more space for social distancing. Time constraints became a serious issue for teachers participating in the study. Because of time constraints three teachers who formed part of the team were unable to attend all the planning sessions since they had to provide extra tutoring for Grade 12 learners during weekends and after school to make up for learning losses caused by COVID 19 pandemic.

This was an effort to make up for learning losses caused by COVID 19 pandemic. Perhaps their expertise on the subject matter knowledge and their knowledge on how to question would have added value to the quality of the conversations held during lesson planning sessions. As a way of responding to the COVID 19 safety protocols, face masks were used during all data collection sessions i.e., lesson planning, lesson presentation and observation and post-lesson reflections. This made it difficult for me to accurately capture some of the teachers' and learners' utterances. Although teachers demonstrated commitment to engage in the LS activities, they lacked understanding of some of the core principles of LS. For instance, LS is mainly used to rethink how to teach a difficult topic or concept. In this study, teachers used LS to teach mathematical concepts which may have been taught outside the context of LS. It is possible that their limited knowledge on the principles underpinning LS may have compromised the quality of data.

7.9. RECOMMENDATIONS FOR FUTURE STUDIES AND POLICY MATTERS

During this study, I found that teachers extracted a mathematics task (solving problems in context involving rate) from the curriculum policy document which they used during lesson planning. Although the LS team was able to recognise that it is a typical complex task, they missed the opportunity to apply their minds and judge the reasonableness of the task itself. From observation, the task did not make mathematical sense and the realistic conditions in the tasks were questionable. Teachers' reluctance to interrogate questions they obtained from the material they used could be an indication of lack of awareness of one of the fundamentals of LS, i.e., a closer examination of the teaching material. In Japanese this is referred to as (*kyozaikenkyu)*. Further research studies should be conducted to explore how LS communities interrogate questions and tasks from textbooks and all other resources they use as they plan what they intend for their learners.

A critical synthesis of literature by Ong et al. (2010) and Amirullah (2018) has shown that research on questions and questioning paid little attention to how teachers participating in the LS can be engaged in discussions that pertain to their questioning behaviour. Moreover, theoretical lenses employed in these studies focussed solely on how questions can be used to achieve the learning objectives, rather than on what can be done to achieve the learning objectives. More research on questioning should be conducted through the lens of Variation Theory to explore how LS impacts teachers' practice of questioning when intervention is introduced. In other words, a questioning model can be introduced during the study so that the aspect of questioning can be brought to the attention of LS communities.

Results from this research study also points out the need for LS workshops to be conducted for any group of teachers who show interest to consider it as a model for teacher development. For instance, certain key aspects that are fundamental to the successful implementation of LS were not considered by the team that I studied. These included:

- The initial meeting that was held did not incorporate formulation of a research theme/question to guide the team through the LS cycles. This guiding theme is critical in Japanese LS.
- The Framework for Mathematics Teaching and the curriculum policy document, CAPS, to a certain extent helped us to understand the significance of teachers' questions. Important as these documents are, they do not provide guidance on how productive classroom conversations can be planned and executed. The curriculum policy in particular has dedicated a section for the description and examples of questions to be used for formal assessment while minimal guidance is provided for formative assessment.

Teachers' inability to plan and pose proper questions may be attributed to the silence in the curriculum policy on the subject of questions and questioning. I therefore recommend that the review of curriculum policy should consider questions and questioning and provide explicit guidance on how informal assessment conducted orally and in written form during classroom interactions can be planned for and managed by teachers during teaching.

7.10. RESEARCHER'S PERSONAL REFLECTIONS

My research journey began in 2018. At that time, I was not yet registered with University of Pretoria. I attended World Association for LS (WALS) in Beijing Normal University, China. At that time, what I intended to explore on questioning had not yet crystalized. My contribution to the proceedings was just an attempt to gather ideas from the LS community regarding possible theories that may resonate with research on questioning. Indeed, participants who attended my presentation made valuable contributions for which today I am still grateful.

During this study, I engaged with five mathematics teachers who willingly participated in this study and gave me access to all their collaboration sessions as they planned their research lessons, presented and reflected upon them during the post-lesson reflection of all four cycles of LS. Though I may have misinterpreted data without being aware, I constantly reminded myself to be unbiassed. I made every effort to ensure the credibility of this study through methodological triangulation by using observation, conversational interviews and document analysis. To further improve the trustworthiness of my research study, I applied member checking, also referred to as participant validation. In this case the transcripts and interpretation of findings were returned to participants to check whether what is documented is an accurate representation of their experiences. To confirm my data interpretation, my supervisors and I held consensus meetings. We also confirmed my findings with those found in the literature.

I have personally experienced the value of questions and questioning in my entire research journey. My interactions with my supervisors were mainly through my written work and their questions (orally over telephonic conversations) and in written form on what I have documented. As I ponder through my entire research journey, I now realise that I have undergone some transformation by appreciating questions even more through the research process. Feedback from my supervisors came frequently in the form of questions. Not just questions, but purposeful and deep questions that provoked my thoughts, directed my attention, re-oriented my perspective and sometimes even revealed what would have not occurred to my mind if it was not of their questions. Frequently, I found myself wrestling with their questions with resilience, tapping into my inner resources. Of course, occasionally I fell in and out of love with their questions. But one thing became certain, their questions not only acted as a form of support, but they tapped on my intellectual engine and revealed to me what I was sometimes taking for granted. As I move towards the end of this study, I had to pause and reflect: How else can research supervisors provide support to their students if not through asking the right questions on their written work?

Now, regarding the classroom context: How else can teachers become masters of their practice if not through advancing their art of questioning? What if mathematics teachers adopt the same spirit of research supervisors of not rushing to explain and give answers but ask good questions? What if they are made to experience how research supervisors operate with their students? LS provides a promising platform to transform mathematics teachers' teaching practices in general and their questioning practices in particular. Questions are significant to a lawyer in a courtroom, a teacher in the classroom, a medical practitioner who want to gather details about the health of a patient, a police officer who collects evidence at a crime scene. Questions and I mean not just questions, do matter to all these and many other professionals. Through engagement in a research journey, I have also learned that questions are useful to research supervisors since they use them to help students reorganise their thinking. There is no universe without questions. As a researcher and an experienced mathematics teacher, I learnt much during this research study. My understanding of how teachers' questions shape classroom conversations extended to how supervisors' questions helped me to tap into my inner resources and reflect deeply throughout my PhD journey.

Further research in this area could consider using the findings of this study to propose and implement a conceptual model on questioning which LS communities may consider as a guide for navigating classroom conversations. This model should aim to assist LS teams to improve their questioning skills.

7.11. CHAPTER SUMMARY

This research study emanated from a quest to provide professional support as part of my responsibility to ensure that teachers implement the curriculum. The study was an effort to respond and explore the attributes that characterise mathematics teachers' questions in the LS context. I offered an account of why mathematics teachers' questions are worth paying attention to from multiple lenses i.e curriculum policy and The Mathematics Teaching and Learning Framework for South Africa and extensive review of literature.

Qualitative research was employed as a research approach. This approach was considered appropriate for its potential to yield 'rich descriptive data (Cohen et al, 2018). To explore the practices and culture of questioning in mathematics classrooms occurring in LS contexts, a case study design was chosen because it enabled me to respond to the broader research question in ways that quantitative methodologies may have not.

LS was the context in which the study occurred. While the adapted model of LS from a South African context impacts teachers' professional development in five stages only three of the five stages were considered for this study. The rich and thick descriptions referred to in this study involved paying attention to how an LS team grappled with the process of planning questions they intended to use to facilitate learning, their pre-conceived ideas about questions, actions, feelings and meanings.

Variation Theory (theoretical lens), LS (the context of the study) and Emanuelsson's categories of classroom interactions constituted the conceptual framework which guided the entire research process, including the design of data collection instruments, data collection and analysis. Findings in relation to the study were structured according to the secondary research questions. Although to some extent questions were given consideration during collaborative lesson planning, the kinds of questions that dominated classroom interactions were those that stimulated interactions in a topical zone.

It was also found that the LS context provides a fertile platform for teachers to deliberate on matters that concern their teaching practice generally and their questioning practices in particular. Findings also revealed that the notion of questions and questioning should be linked to a broader concern of teacher quality.

The questioning practices of teachers who were studied at three levels (planning, enactment and reflection stages), exhibited some form of traditional form of teaching characterised by transmission of knowledge as opposed to knowledge construction. Indeed, for questions to fulfil their full educational purpose, mathematics teachers in general and LS communities in particular, will first have to acknowledge that they do matter and so they are worth paying attention to. Until teachers realise that their questions matter, they will continue to promote ‘unconsciously’ traditional instruction in LS contexts.

REFERENCES

Abramovich, S., Grinshpan, A., & Milligan, D. L. (2019). Teaching Mathematics through Concept Motivation and Action Learning. *Education Research International 2019,* 3745406. https://doi.org/10.1155/2019/3745406

Adler, J. & Pournara, C. (2019). Exemplifying with Variation and Its Development in Mathematics Teacher Education. *International Handbook of Mathematics Teacher Education, 1*, 329-353.

Aizikovitsh-Udi, E. & Star, J. (2011). The skill of asking good questions in mathematics teaching. *Procedia-Social and Behavioral Sciences, 15*, 1354-1358.

Aizikovitsh-Udi, E., Clarke, D., & Jon, J. (2013). Good questions or good questioning: An essential issue for effective teaching. *CERME8: 8th Congress of the European Society for Research in Mathematics Education*. Antalya, Turkey.

Åkerlind, G. (2015). From Phenomenography to Variation Theory: A review of the development of the Variation Theory of Learning and implications for pedagogical design in higher education. *HERDSA Review of Higher Education, 2*,5-26.

Almeida, P.A. (2012). Can I ask a question? The importance of classroom questioning. *Procedia-Social and Behavioral Sciences, 31*, 634-638.

Arani, M.R.S., Keisuke, F., & Lassegard, J.P. (2010). “Lesson Study” as Proffesional Culture in Japanese Schools: An Historical Perspective on Elementary Classroom Practices. *Japan Review, 22*, 171-200.

Arias de Sanchez, G., Gabriel, M., Anderson, A., & et al. (2018). Code-switching explorations in teaching early number sense. *Education Sciences, 8*(1), 38.

Ary, D., Jacobs, L.C., Razavieh, A., & Sorensen, C. (2006). *Introduction to Research in Education* (7th ed.). Vicki Knight.

Amirullah, A.H. (2018). LS: An Approach to Increase the Competency of Out-of-Field Mathematics Teacher in Building the Students Conceptual Understanding in Learning Mathematics. *Journal of Educational Sciences, 2*(2), 1-13.

Aziza, M. (2017). "The use of open-ended question pictures in the mathematics classroom." *New Trends and Issues Proceedings on Humanities and Social Sciences, 4*(9), 1-9.

Aziza, M. (2018). An analysis of a teacher’s questioning related to students’ responses and mathematical creativity in an elementary school in the UK. International *Electronic Journal of Elementary Education, 10*(4), 475-487.

Biccard, P. (2020). The development of noticing in primary school mathematics teachers. *The Independednt Journal of Teaching and Learning, 15*(2), 92-106.

Bussey, T.J., Orgill, M., & Crippen, K. (2013). Variation theory: A theory of learning and useful theoretical framework for chemical education research. *Chemistry Education Research and Practice, 14*(1), 9-22.

Bjørkli, K. (2014). The impact on learning outcomes in mathematics of mobile-enhanced,combined formative and summative assessment. *International Journal of Technology Enhanced Learning, 6*(4), 343-360.

Botes, W., Moreeng, B., & Mosia, M. (2022). Pre-service teachers' experiences of a lesson study approach as a form of student support. *Issues in Educational Research, 32*(1), 57-70.

Campbell, J., & Mayer, R.E. (2009). Questioning as an instructional method: Does it affect learning from lectures? Applied Cognitive Psychology. *The Official Journal of the Society for Applied Research in Memory and Cognition, 23*(6), 747-759.

Coe, R., Waring, M., Hedges, L.V., & Arthur, J. (2017). *Research Methods & Methodologies in Education* (2nd ed.). SAGE Publications Ltd.

Coe, R., Waring, M., Hedges, L.V., & Ashley, L.D (2021). *Research Methods & Methodologies in Education* (3rd ed.). SAGE Publications Ltd.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education*, (8th ed.). Routledge.

Coenders, F., & Verhoef, N. (2019). Lesson Study: Professional Development (PD) for beginning and experienced teachers. *Professional Development in Education, 45*(2), 217-230.

Chen, Y., Hand, B., & Norton, L. (2017). Teacher roles of questioning in early elementary science classrooms: A framework promoting student cognitive complexities in argumentation. *Research in Science Education, 47*(2), 373-405.

Cheng, E.C.K., (2018). *Successful Transposition of Lesson Study: A knowledge Management Perspective*. Singapore, Springer.

Chikiwa, C., & Schäfer, M. (2018). Promoting critical thinking in multilingual mathematics classes through questioning. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(8), 1305-8223.

Chirinda, B., & Barmby, P. (2017). The development of a professional development intervention for mathematical problem-solving pedagogy in a localised context. *Pythagoras, 38*(1), 1-11.

Chong, J.S.Y., & Shahrill, M. (2014). The refinements needed to improve the teaching of conditional probability. *Age, 50*(59), 30-39.

Choy, B.H. (2013). Productive Mathematical Noticing: What It Is and Why It Matters. *Mathematics Education Research Group of Australasia*.

Church, I.M. (2016). The doxastic account of intellectual humility. *Logos & Episteme 7*(4): 413-433.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). RoutledgeFalmer: London.

Creswell, J.W. & Poth, C.N. 2018. *Qualitative Inquiry and Research Design: Choosing Among Five Approaches,* SAGE Publications: Singapore.

Dahal, N., Luitel, B.C., & Pant, B.P. (2019). Understanding the use of questioning by mathematics teachers: A revelation. *International Journal of Innovative, Creativity and Change, 5*(1), 118-146.

Daher, W., Amal Tabaja-Kidan, A., & Gierdien, F. (2017). Educating Grade 6 students for higher-order thinking and its influence on creativity. *Pythagoras, 38*(1), 1-12.

D'Ambrosio, U., & D'Ambrosio, S.D. (2013). The role of ethnomathematics in curricular leadership in mathematics education. *Journal of Mathematics Education at Teachers College, 4*(1), 19-25.

da Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2018). Fitting lesson study to the Portuguese context. *Mathematics lesson study around the world: Theoretical and methodological issues,* 87-103..

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for research in Mathematics Education, 28*(3), 355-376.

DBE (2011). National Curriculum Statement Senior Phase Grades 7-9: Curriculum and Assessment Policy Statement. Mathematics. Pretoria: Government Printers.

DBE. (2018). Mathematics teaching and learning framework for South Africa: Teaching mathematics for understanding, Pretoria: Government Printers.

Desli, D. & Galanopoulou. E. (2015). Questioning in primary school mathematics: an analysis of questions teachers ask in mathematics lessons. *Procedings from the 3rd International Symposium on New Issues on Teacher Education*.

Di Teodoro, S., Donders, S., Kemp-Davidson, J., Robertson, P., & Schuyler, L. (2011). Asking good questions: Promoting greater understanding of mathematics through purposeful teacher and student questioning. *The Canadian Journal of Action Research, 12*(2), 18-29.

Shack, E.O., Dindyal, J., Choy, B.H., Sherin, M.G. (2021). Exploring the terrains of mathematics teacher noticing. *ZDM–Mathematics Education,53*, 1-16.

Doig, B., & Groves, S. (2011). Japanese Lesson Study: Teacher professional development through communities of inquiry. *Mathematics Teacher Education Development, 13*(1), 77-93.

Dong, L., Seah, W.T., & Clarke, D. (2015). A Case Study of the Pedagogical Tensions in Teacher's Questioning Practices When Implementing Reform-Based Mathematics Curriculum in China. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics Education Research Group of Australasia*, 197-204. Merga.

Drew, C.J., Harman, M.L., & Hosp, J.L. (2008). *Designing and Conducting Research in Education*. Sage Publication: Singapore.

Dreher, A., & Kuntze, S. (2015). Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. *Educational Studies in Mathematics, 88*(1), 89-114.

Ebaeguin, M., & Stephens, M. (2014). Cultural Challenges in Adapting Lesson Study to a Phillippines Setting. *Mathematics Teacher Education and Development, 16*(1), 2-25.

Elder, L., & Paul, R. (1998). The role of Socratic questioning in thinking, teaching, and learning. *The Clearing House, 71*(5), 297-301

Emanuelsson, J. (2001). A question about questions: how teachers' questioning makes it possible to learn about the students' ways of understanding the content taught in mathematics and science. In *Swedish with summary in English*. Göteborg: UniversitatisGothoburgensis.

Etemadzadeh, A., Seifi, S., & Far, H.R. (2013). The role of questioning technique in developing thinking skills: The ongoing effect on writing skill. *Procedia-Social and Behavioral Sciences, 70*, 1024-1031.

Fatah, A., Suryadi, D., Sabandar, J., Turmudi.. (2016). Open-Ended Approach: An Effort in Cultivating Students' Mathematical Creative Thinking Ability and Self-Esteem in Mathematics. *Journal on Mathematics Education, 7*(1), 11-20.

Feldman, J. (2020). The role of professional learning communities to support teacher development: A social practice theory perspective. *South African Journal of Education, 40*(1), 1-8.

Fernandes, C., & Yoshida, M. (2012). Lesson Study: A Japanese approach to improving mathematics teaching and learning. *Asia Pacific Journal of Educational Development, 1*(1), 25-32.

Flick, U. (2018). An introduction to qualitative research (6th ed.) SAGE Publications Ltd.

Fraenkel, J.R. & Wallen, N.E. (2009). *How to design and evaluate research in education*. (7th ed.). McGraw-Hill. Boston.

Franke, M.L., Webb, N.M., Chan, A.G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Fujii, T. (2015). The critical role of task design in lesson study. Task design in mathematics education: *An ICMI Study, 22*, 273-286.

Fuji, T. (2018). Lesson study and teaching mathematics through problem solving: The two wheels of a cart, Mathematics lesson study around the world, In M. Quaresma et al. (Eds.). *Mathematics Lesson Study Around the World ICME-13 Monographs*, https://doi.org/10.1007/978-3-319-75696-7

Fujii, T. (2019). Designing and adapting tasks in lesson planning: A critical process of LS. *Theory and Practice of LS in Mathematics*, Springer, 681-704.

Gutierez, S.B. (2015). Collaborative professional learning through lesson study: Identifying the challenges of inquiry-based teaching. *Issues in Educational Research, 25*(2), 118-134. http://www.iier.org.au/iier25/gutierez.html

Haggard, M., Haggarda, M., Rowatt, W.C., Leman, J.C., Meagher, B., Moore, C., Fergus, T., Whitcomb, D., Battaly, H., Baehr, J., Howard-Snyder, D. (2018). Finding middle ground between intellectual arrogance and intellectual servility: Development and assessment of the limitations-owning intellectual humility scale. *Personality and Individual Differences, 124*, 184-193.

Hokanson, B. (2015). The Technology of the Question: Structure and Use of Questions in Educational Technology. *Educational Technology, 55*(6), 14-24.

Houen, S. (2016). I wonder what you know…'teachers designing requests for factual information. *Teaching and Teacher Education, 59*, 68-78.

Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teachers and teacher educators, and linking theory and practice through lesson study in mathematics, An International perspective. *ZDM, 48*(4), 393-409.

Huang, R., & Li, Y. (2017). *Teaching and learning mathematics through variation: Confucian heritage meets western theories*, Springer.

Hussin, H. (2006). Dimensions of Questioning: A Qualitative Study of Current Classroom Practice in Malaysia. *TESL-EJ, 10*(2), 1-18

Holmqvist, M., & Selin, P. (2019). What makes the difference? An empirical comparison of critical aspects identified in phenomenographic and variation theory analyses. *Palgrave Communications, 5*(1), 1-8.

Ilma, R. (2011). Proffesional development of mathematics primary school teachers in Indonesia using lesson study and reaslistic mathematics education approach. *Proceedings of ICSEI 2011*, Lymasol, Cyprus.

Ishii, T. (2017). Historical overview of lesson study. In *Curriculum, instruction and assessment in Japan.* Taylor & Francis.

Jamaludin, R. & Osman, Z.M. (2014). The use of a flipped classroom to enhance engagement and promote active learning. *Journal of Education and Practice, 5*(2), 124-131.

Jung, H., Kwauk, C., Nuran, A., Robinson, J.P., Schouten, M., Tanjeb, S. (2016). Lesson Study: Scaling up peer to peer learning for teachers in Zambia.

Johnson, R.B., & Christensen, L. (2017). *Educational research: quantitative, qualitative, and mixed approaches* (6th ed.). SAGE Publications Asia-Pacific

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

King, J. R., & Chetty, R. (2014). Codeswitching: Linguistic and literacy understanding of teaching dilemmas in multilingual classrooms. *Linguistics and Education 25*, 40-50.

Koizumi, Y. (2013). Similarities and differences in teachers' questioning in German and Japanese mathematics classrooms. *ZDM, 45*(1), 47-59.

Kotelawala, U. (2012). Lesson Study in a methods course: Connecting teacher education to the field. *The Teacher Educator, 47*(1), 67-89.

Kullberg, A., et al. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM, 49*(4), 559-569.

Kuno, H. (2017). Japanese progressivism and continuing cultural encounters. *Educational progressivism, cultural encounters and reform in Japan*, 169-174.

Larson, L. R., & Lovelace, M. D. (2013). Evaluating the efficacy of questioning strategies in lecture-based classroom environments: Are we asking the right questions? *Journal on Excellence in College Teaching, 24*(1).

Lewis, C. (2011). Response to Part IV: Seeing the Whole Iceberg-The Critical Role of Tasks, Inquiry Stance, and Teacher Learning in Lesson Study. *Lesson Study research and practice in mathematics education*, 235-240.

Lewis, C., Perry, R., & Hurd, J. (2004). A deeper look at LS. *Educational leadership, 61*(5), 1-6.

Lewis, C. C., Perry, R. R., & Hurd, J. (2009). Improving mathematics instruction through LS: A theoretical model and North American case. *Journal of mathematics teacher education, 12*(4), 285-304.

Lewis, C., & Perry, R. (2014). Lesson Study with Mathematical Resources: A sustainable model for Locally-Led Teacher Proffesional Learning. *Mathematics Teacher Education and Development, 16*(1), 1-20.

Lewis, C. (2016). How does lesson study improve mathematics instruction*? ZDM, 48*(4), 571-580.

Lo, M. L. (2012). Variation theory and the improvement of teaching and learning: Göteborg: ActaUniversitatis Gothoburgensis.

Livy, S., Muir, T., Sullivan, P. et al. (2018). Challenging tasks lead to productive struggle! *Australian Primary Mathematics Classroom, 23*(1), 19-24.

Maeda, M., & Ono, Y. (2019). Diffusion of LS as an educational innovation. *International Journal of Comparative Education and Development, 21*(1), 46-60.

Mahmud, M.S., (2020). Enhancing Mathematical Language through Oral Questioning in Primary Schools. *International Journal of Learning, Teaching and Educational Research, 19*(5), 395-410.

Makinae, N. (2010). The origin of lesson study in Japan. The 5th East Asia Regional Conference on Mathematics Education: In Search of Excellence in Mathematics Education, Tokyo. *Teaching Innovations, 27*(3), 83–93.

Mhakure, D. (2019). School-based mathematics teacher professional learning: A theoretical position on the lesson study approach. *South African Journal of Education, 39*(1), 1-8.

Marton, F. (2015). *Necessary conditions of learning*. New York, Routledge.

Marton, F., & Pang, M.F. (2013). Meanings Are Acquired from Experiencing Differences against a Background of Sameness, Rather than from Experiencing Sameness against a Background of Difference: Putting a Conjecture to the Test by Embedding It in a Pedagogical Tool. *Frontline Learning Research, 1*(1), 24-41.

Mason, J. (2020). Questioning in Mathematics Education. Encyclopedia of Mathematics Education. S. Lerman. (Ed.), *Springer International Publishing*, 705-711.

McCarthy, P., Sithole, A., McCarthy, P., Cho, J., & Gyan, E. (2016). Teacher Questioning Strategies in Mathematical Classroom Discourse: A Case Study of Two Grade Eight Teachers in Tennessee, USA. *Journal of Education and Practice, 7*(21), 80-89.

McCormick, M. (2016). Exploring the Cognitive Demand and Features of Problem Solving Tasks in Primary Mathematics Classrooms. Mathematics Education Research Group of Australasia. Nappi, J. S. (2017) (Eds.), The importance of questioning in developing critical thinking skills. *Delta Kappa Gamma Bulletin, 84*(1), 30-41.

Nurkaeti, N. (2018). Polya's strategy: an analysis of mathematical problem solving difficulty in 5th grade elementary school. *Edu Humanities Journal of Basic Education Cibiru Campus, 10*(2), 140-147.

Ishak, A.H.N., Osman, S., Wei, C.K., Kurniati, D., Ismail, N., A. & Nanna, W.I. (2021). Teaching Strategies for Mathematical Problem-Solving through the Lens of Secondary School Teachers. *TEM, 10*(2), 743-750.

Ong, E. G., Lim, C. S., & Ghazali, M. (2010). Examining the changes in novice and experienced mathematics teachers' questioning techniques through the LS process. *Journal of Science and Mathematics Education in Southeast Asia, 33*(1), 86-109.

Ono, Y., & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education, 30*(1), 59-74.

Ono, Y., Chikamori, K., & Rogan, J.M. (2013). How reflective are lesson study discussion sessions? Developing an instrument to analyze collective reflection." *International Journal of Education, 5*(3), 52.

Ottmar, E.R., Rimm-Kaufman, S., Larsen, R.A., Berry, R.Q., (2015). Mathematical knowledge for teaching, standards-based mathematics teaching practices, and student achievement in the context of the responsive classroom approach. *American Educational Research Journal, 52*(4), 787-821.

Pang, M.F. (2003). Two faces of variation: On continuity in the phenomenographic movement. *Scandinavian Journal of Educational Research, 47*(2), 145-156.

Parks, A.N. (2010). Explicit versus Implicit Questioning: Inviting All Children to Think Mathematically. *Teachers College Record, 112*(7), 1871-1896.

Penazzi, D., McCready-Fallon, J., & Rosser, S. (2022). Experiential and Outdoor Learning in the Transition to University courses-The Mathematics Case. *PRIMUS*, 1-14.

Pianta, R.C., Hamre, B.K., & Allen, J.P. (2012). Teacher-student relationships and engagement: Conceptualizing, measuring, and improving the capacity of classroom interactions. S.L. Christenson et al. (Eds.), *Handbook of research on student engagement*, Springer: 365-386.

Pjanić, K. (2014). The origins and products of Japanese LS. *Inovacije u nastavi-časopis za savremenu nastavu, 27*(3), 83-93.

Purdum-Cassidy, B., Nesmith, S., Meyer, R.D., (2015). What are they asking? An analysis of the questions planned by prospective teachers when integrating literature in mathematics. *Journal of mathematics teacher education, 18*(1), 79-99.

Posthuma, B. (2012). Mathematics teachers' reflective practices within the context of adapted lesson study, *Pythagoras, 33*(3), 1-9.

Pohl, K.A. (2016). "I'm Just Not Good at Math!" Rethinking What You Know About Mathematics. *Learning to Teach, 5*(1).

Radford, L. (2015). Of love, frustration, and mathematics: A cultural-historical approach to emotions in mathematics teaching and learning. *From beliefs to dynamic affect systems in mathematics education: Exploring a mosaic of relationships and interactions*, 25-49.

Rakoczy, K., Pinger, P., Hochweber, J., Klieme, E., Schütze, B., & Besser, M. (2019). Formative assessment in mathematics: Mediated by feedback's perceived usefulness and students' self-efficacy. *Learning and Instruction, 60*, 154-165.

Rundgren, C.J., & Tibell, L.A. (2010). Critical features of visualizations of transport through the cell membrane an empirical study of upper secondary and tertiary students'meaning-making of a still image and an animation. *International Journal of Science and Mathematics Education, 8*(2), 223-246.

Sayster, A., & Mhakure, D. (2020). Students' Productive Struggles in Mathematics Learning. *Pedagogy in Basic and Higher Education-Current Developments and Challenges*, IntechOpen.

Skovsmose, O. (2018). Critical constructivism: Inpterpreting mathematics education for social justice. *For the learning of mathematics, 38*(1), 38-43.

Shahrill, M. (2013). Review of effective teacher questioning in mathematics classrooms. *International Journal of Humanities and Social Science, 3*(17), 224-231

Shahrill, M., & Clarke, D.J. (2014). Brunei Teachers' Perspectives on Questioning: Investigating the Opportunities to Talk in Mathematics Lessons. *International Education Studies, 7*(7), 1-18.

Sharma, S. (2015). Promoting risk taking in mathematics classrooms: The importance of creating a safe learning environment. *The Mathematics Enthusiast, 12*(1), 290-306.

Sekao, R.D. (2019). The LS cycle. Accessed 20 May 2022 at https://www.up.ac.za/lesson-study.

Sekao, D., & Engelbrecht, J. (2021). South African Primary Mathematics Teachers' Experiences and Perspectives About LS. *International Journal of Science and Mathematics Education*, 1-23.

Smart, J. B. & Marshall, J.C. (2013). Interactions between classroom discourse, teacher questioning, and student cognitive engagement in middle school science. *Journal of Science Teacher Education, 24*(2), 249-267.

Smith, C., & Julie, C. (2014). Mathematical question types: Proceedings of the 20th annual National congress of the association for mathematics education of South Africa – Demystifying Mathematics.

South African Schools Act of 1996. Government Gazette, 29474.

Sorto, M.A., Mccabe, T., Washauer, M., & Warshauer, H. (2009). Understanding the value of a question: An analysis of a lesson. *Journal of Mathematical Sciences and Mathematics Education, 4*(1), 50-60.

Steyn, G., & Adendorff, S.A. (2020). Questioning techniques used by Foundation Phase Education students teaching mathematical problem-solving. *South African Journal of Childhood Education, 10*(1), 1-9. https://doi.org/10.4102/sajce.v10i1.564

Stigler, J.W. and J. Hiebert (2016). Lesson study, improvement, and the importing of cultural routines. *ZDM Mathematics Education, 48*(4), 581-587.

Takahashi, A., & Yoshida, M. (2004). Ideas for establishing lesson-study communities. *Teaching children mathematics, 10*(9), 436-444.

Takahashi, A. (2014). The Role of the Knowledgeable Other in LS: Examining the Final Comments of Experienced LS Practitioners. *Mathematics Teacher Education and Development, 16*(1).

Takahashi, A., Lewis, C., & Perry, R. (2013). A US lesson study to spread teaching through problem solving. *International journal for lesson and learning studies, 2*(3), 237-255.

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: Maximizing the impact of lesson study. *ZDM, 48*(4), 513-526.

Thompson, I. & Mackiewicz, J. (2014). Questioning in writing center conferences. *The Writing Center Journal, 33*(2), 37-70.

Tienken, C. H., Goldberg, S., & Dirocco, D. (2009). Questioning the questions. *Kappa Delta Pi Record, 46*(1), 39-43.

Tofade, T., Elsner, J., & Haines, S.T. (2013). Best practice strategies for effective use of questions as a teaching tool. *American journal of pharmaceutical education, 77*(7), 155.

Ulleberg, I., & Solemn, H. (2018). Which questions should be asked in classroom talk in mathematics? Presentation and discussion of a questioning model. *Acta Didactica Norge, 12*(1). 1-21.

Vangrieken, K., Meredith, C., & Kyndt, E. (2017). Teacher communities as a context for professional development: A systematic review. *Teaching and Teacher Education, 61*, 47-59.

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development, 41*, 121-130.

Viseu, F., & Oliveira, I.B. (2017). Open-ended tasks in the promotion of classroom communication in mathematics. *International Electronic Journal of Elementary Education, 4*(2), 287-300.

Warshauer, H.K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of mathematics teacher education, 18*(4), 375-400.

Wasserman, N.H. (2015). Unpacking teachers' moves in the classroom: navigating micro- and macro-levels of mathematical complexity. Educational Studies in Mathematics. *An International Journal, 90*(1): 75-93.

Watson, L. (2018). Educating for good questioning: A tool for intellectual virtues education. *Acta Analytica, 33*(3), 353-370.

Webb, N.M., & Ing, M. (2019). The role of teacher practice in promoting academically productive student dialogue. *International Journal of Educational Research, 97*, 154-156.

Wei, G. (2019). How Could Cultural-Historical Activity Theory Inspire LS? *Theory and practice of Lesson Study in mathematics*, Springer, 39-58.

Winsløw, C., Bahn, J., & Rasmussen, K. (2018). Theorizing lesson study: Two related frameworks and two Danish case studies. *Mathematics lesson study around the world: Theoretical and methodological issues*, 123-142.

Whitcomb, D., Battaly, H., Baehr, J., & Howard-Snyder, D. (2017). Intellectual Humility. *Philosophy and Phenomenological Research, 94*(3): 509-539.

Xu, L. (2019). Relating Teaching and Learning of Science Through the Lens of Variation Theory. *Scandinavian Journal of Educational Research, 63*(1), 145-162

Yamasaki, Y., & Kuno, H. (2017). *Educational progressivism, cultural encounters and reform in Japan*. Routledge.

ANNEXURES

Annexure A: Observation tool for lesson planning (stage 2 of LS cycle)

Subject: Mathematics Lesson topic: __________Date: ________Duration: _______

Lesson presenter: __________________ Number of teachers: _________

RESEARCH TOPIC: Teachers' questions matter: exploring the attributes of mathematics teachers' questions within the Lesson Study context.

This observation protocol was used to collect data to respond to the following question:

What attributes characterize questions planned during the collaborative lesson planning stage?

	What attributes characterise questions planned during collaborative lesson planning?	
	Topical zone	**Conceptual zone**
Comment	Oral questions planned for a lesson during collaborative lesson planning according to interactions within zones	

Annexure B: Observation tool for lesson presentation and observation (Stage 3 of LS)

Subject: Mathematics Lesson topic: __________Date: ________Duration: _______

Lesson presenter: __________________ Number of teachers: _________

RESEARCH TOPIC: Teachers questions matter: Exploring the attributes of mathematics teachers' questions in the context of Lesson Study.

This observation protocol was used to collect data to respond to the following research questions:

- How do questions planned during the collaboration planning stage permeate lesson presentation?
- How do learners experience teachers' questions during teaching?

Written questions planned for the lesson during collaborative lesson planning according to interactions within zones			
	Topical zone	**Conceptual zone**	**Procedural Zone**
Comment			
Oral questions planned for a lesson during collaborative lesson planning according to interactions within zones			
	Topical zone	**Conceptual zone**	**Procedural zone**
Comments			

Annexure C: Observation tool for post-lesson reflection (Stage 4 of LS)

Subject: Mathematics   Lesson topic: __________Date: ________Duration: _______

Lesson presenter: __________________ Number of teachers: _________

RESEARCH TOPIC: Teachers questions matter: Exploring the attributes of mathematics teachers' questions in the context of Lesson Study.

____

This observation protocol was used to collect data to respond to the following research question:

What critical features on questioning emerge during post-lesson reflection?

Contributions made by lesson observation panel in relation to questions		
Issues raised by observers in relation to questions that were posed	**Conclusion by the team**	**Researcher's comments/inputs**

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

Annexure D: Permision from NWED

REFEDU065/19

Enq: Ms L.B. Mokotedi

2339 BM Mokitme Drive

Unit 8 Mmabatho

2735

Superintendent General

North West Department of Education and Sport Development

Garona Building

Mmabatho

2735

Dear Superintendent General

Request for permission to conduct research in a school

I am a student at University of Pretoria and I am conducting a doctoral study titled ***Teachers' questions matter: Exploring the attributes of mathematics teachers' questions in the context of lesson study***. The purpose of the study is to explore the questioning practices of mathematics teachers who implement a professional teacher development model known as Lesson Study. This letter serves to request the office of the Superintendent General to grant me permission to use a Secondary school within Ngaka Modiri Molema District office in Ratlou Sub District Office as a research site for the duration of the research project.

If permission is granted all ethical principles such as anonymity, confidentiality and safety of all participants will be adhered to.

The study will take place as follows:

- A total of four (4) lessons will be observed and video recorded. The study will not affect the teaching time
- Collaborative planning of such research lessons is also expected to take place at the school during weekends
- Participants are mathematics teachers teaching grade 8 and 9 from the four local Secondary schools within a cluster in the Sub District.
- A single class of Grade 9 will also be invited to participate after their parents have given consent

For any additional information, you may contact me, Lesego Mokotedi, at (082 7150313) or my supervisor, Dr RD Sekao at 012 420 4640 or david.sekao@up.ac.za

Yours sincerely

Ms LB Mokotedi

Dr RD Sekao (Supervisor)

Faculty of Education

Annexure E: Permision from Principals of four schools

Ref:EDU065/19
Enq: Ms L.B. Mokotedi
2339 BM Mokitime drive
Unit 8 Mmabatho, 2735
Email:
brendamokotedi@msn.com

Dear Principal

Request for permission to release a mathematics teacher to participate in the research study

I am a student at University of Pretoria and I am conducting a doctoral study titled ***Teachers' questions matter: Exploring the attributes of mathematics teachers' questions in the context of lesson study***. The purpose of the study is to explore the questioning practices of mathematics teachers who implement a professional teacher development model known as Lesson Study. This letter serves to request you to grant a Grade 9 mathematics teacher permission to participate in the research study.

If you agree to release the teacher, all ethical principles such as anonymity, confidentiality and safety will be adhered to.
The study will take place as follows:

- A total of four (4) lessons will be observed and video recorded. The study will not affect the teaching time
- Collaborative planning of such research lessons is also expected to take place at the school during weekends to protect teaching time
- Participants are mathematics teachers teaching grade 8 and 9 from the four local high schools within a cluster in the Sub District.

For any additional information, you may contact me, Lesego Mokotedi, at (082 7150313) or my supervisor, Dr RD Sekao at 012 420 4640 or david.sekao@up.ac.za
Yours sincerely

________________ ____________________

Ms LB Mokotedi Dr RD Sekao (Supervisor

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Annexure F: Consent by the principal of host school

Ref:EDU065/19
Enq: Ms L.B. Mokotedi
2339 BM Mokitime drive
Unit 8 Mmabatho, 2735
Email:
brendamokotedi@msn.com

Dear Principal

Request for permission to conduct research study at school

I am a student at University of Pretoria and I am conducting a doctoral study titled ***Teachers' questions matter: Exploring the attributes of mathematics teachers' questions in the context of lesson study***. The purpose of the study is to explore the questioning practices of mathematics teachers who implement a professional teacher development model known as Lesson Study. This letter serves to request the school management team to allow me to use the school as a research site for the entire duration of the research study.

If you agree to avail the school as a research site, all ethical principles such as anonymity, confidentiality and safety of participants will be adhered to.

The study will take place as follows:

- A total of four (4) lessons will be observed and video recorded. The study will not affect the teaching time
- Collaborative planning of such research lessons is also expected to take place at the school
- Participants are mathematics teachers teaching grade 8 and 9 from the eight local high schools within a cluster in the Sub District.
- A single class of Grade 9 will also be invited to participate after their parents have given consent

For any additional information, you may contact me, Lesego Mokotedi, at (082 7150313) or my supervisor, Dr RD Sekao at 012 420 4640 or david.sekao@up.ac.za

Yours sincerely

________________ ________________

Ms LB Mokotedi Dr RD Sekao (Supervisor)

Annexure G: Consent by teachers

Ref: EDU065/19
Enq: Ms L.B. Mokotedi
2339 BM Mokitime drive
Unit 8 Mmabatho, 2735
brendamokotedi@msn.com

Dear Mathematics teacher

INVITATION TO PARTICIPATE IN THE RESEARCH STUDY

I am a student at University of Pretoria and I am conducting a doctoral study titled ***Teachers' questions matter: Exploring the attributes of mathematics teachers' questions in the context of lesson study***. The purpose of the study is to explore the questioning practices of mathematics teachers who implement a professional teacher development model known as Lesson Study. This letter serves to request you to participate in the aforementioned research study.

You are therefore invited to participate in this study by:

1. Participating and being observed during the three of the five stages of Lesson Study cycle i.e. *lesson preparation stage*, *lesson presentation & observation* and *post-lesson reflection* stage. A total of four (4) lessons will be observed and video recorded. The study will not affect the teaching time.
2. Being part of **interviews** that will be recorded.
3. Availing the collaboratively planned lesson plan for further analysis.

Note that your *participation is* completely *voluntary* and that you may withdraw at any stage of the study. If you agree to participate I will ensure that the following ethical principles are adhered to:

- *Informed consent:* your consent to participate is based on your understanding of the purpose and process of the study as I have explained them.
- *Safety in participation:* you will not be exposed to any risk or harm of any form.
- *Privacy*: Any information you provide will be kept anonymous. We also would like to request your permission to use your data, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on

future research studies. Your *na*mes and the data you provide will be kept confidential and anonymous.

- *Trust*: you will not be subjected to any act of deception or betrayal in the research process or its published findings.
- For any additional information, you may contact me, Lesego Mokotedi, at (082 7150313) or my supervisor, Dr RD Sekao at 012 420 4640 or david.sekao@up.ac.za

Yours sincerely

________________ ____________________

Ms LB Mokotedi Dr RD Sekao (Supervisor)

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

Annexure H: Consent by parents

Ref: EDU065/19

Enq: Ms L.B. Mokotedi

2339 BM Mokitime drive

Unit 8 Mmabatho, 2735

brendamokotedi@msn.com

Dear Parent

Request for permission for your child's participation in research study

I am a student at University of Pretoria and I am conducting a doctoral study titled ***Teachers' questions matter: Exploring the attributes of mathematics teachers' questions in the context of lesson study***. The purpose of the study is to explore the questioning practices of mathematics teachers who implement a professional teacher development model known as Lesson Study. This letter serves to request you to allow your child to participate in the aforementioned research study.
If you give consent for your child to participate in the study, he/she will be observed when his/her teacher teaches mathematics lesson. A total of four (4) lessons taught by four different teachers will be observed and video recorded. Your child's identity will not be revealed through video recording. The study will not affect the teaching time.

Note that your decision to give consent for your child to take part i*s* completely *voluntaries he/she will not be coerced* into participating in the study and he/she may withdraw at any time. If you agree I will ensure that the following ethical principles are adhered to:

- *Informed consent:* based on your understanding of the purpose and process of the study as I have explained them, you give consent for your child to participate. *Safety in participation:* your child will not be exposed to any risk or harm of any form.
- Privacy:
  - The name of your child and the data he/she will provide will be kept confidential and anonymous. We also would like to request your permission to use the data, confidentially and anonymously, for further research purposes, as the data sets are the intellectual property of the University of Pretoria. Further research may include secondary data

analysis and using the data for teaching purposes. The confidentiality and privacy applicable to this study will be binding on future research studies.

- Video recording has potential to expose facial identities of learners. To mitigate this, maximum caution will be exercised to avoid capturing your child's face in the video recording since the primary focus is on teaching/teachers; however if their faces may feature in the video, they (faces) will be concealed

- *Trust*: your child will not be subjected to any act of deception or betrayal in the research process or its published findings.

For any additional information, you may contact me, Lesego Mokotedi, at (082 7150313) or my supervisor, Dr RD Sekao at 012 420 4640 or david.sekao@up.ac.za

Yours sincerely

Ms LB Mokotedi

Dr RD Sekao (Supervisor)

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

Annexure I: Consent by learners

Ref: EDU065/19
Enq: Ms L.B. Mokotedi
2339 BM Mokitime drive
Unit 8 Mmabatho, 2735
brendamokotedi@msn.com

Dear Ms Mokotedi

INVITATION TO PARTICIPATE IN THE RESEARCH STUDY

I, ____ understand that I have been asked to participate in a study titled: ***Teachers' questions matter: Exploring the attributes of mathematics teachers' questions in the context of lesson study***. I understand that my parent/guardian have given consent for me to participate in the aforementioned study. I will be observed when a mathematics lesson is presented. I also understand that a total of four (4) one-hour lessons to be taught by four different teachers will be observed and video recorded. The study will not affect the teaching time.

I declare that I understand, as you explained to me, the purpose of the study and that you subscribe to ethical research principles, including the following:

- *Informed consent*: my parent/guardian has given consent for me to participate in the research study.
- *Safety*: I will not be exposed to any risk or harm of any form.
- *Privacy*: My parents, teachers, or anyone else will not know what I have said or done in the study. My name will not be revealed in any publication or any other method through which the findings of this study will be disseminated. My facial identity will not be video recorded. If it happens to appear in the video it will be concealed.
- *Trust:* I will not be subjected to any act of deception or betrayal in the research process or its published findings.
- *Assent:* Given all the above ethical considerations, I grant assent to be included and participate in the study. I understand that I may withdraw at any stage of the study.

For any additional information, I may contact, Lesego Mokotedi, at (082 7150313) or her supervisor, Dr RD Sekao at 012 420 4640 or david.sekao@up.ac.za. When

I sign my name, this means that I agree to participate in the study since my parents/guardian has granted me permission to do so.

__________________________ ____________________ _________________

(Name and surname) Signature Date

Annexure J: Lesson plan – Research lesson 1

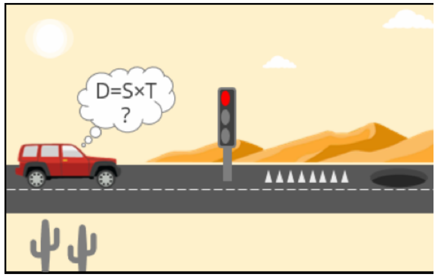
MATHEMATICS RESEARCH LESSON STUDY PLAN

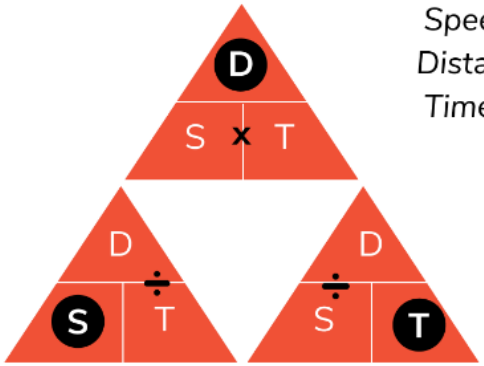
GRADE 9 TERM 1

FACILITATOR	DATE	DURATION	GRADE	HOSTING SCHOOLS
[redacted]		1hr	9	[redacted]

Content Area	Numbers, operations and relationships
Topic	Whole numbers
Concepts & Skills	Solving problems in context involving **Rates**
Lesson objectives	At the end of the lesson learners should be able to use relevant formulas to calculate **speed**, **time** and **distance**.
Resources	Textbook, DBE workbooks
Prior Knowledge	Proportion, fractions,

The team that Researched/prepared a lesson on the 18th January 2021
[redacted]

	Lesson development	Teaching and Learning activities	Learning points
introduction 10 mini	**Engagement Stage** Sell the lesson to the learners (A very interesting activity to capture learners interest)	**Rates** Ask learners to answer the following questions **Q1 :** What is the meaning of the word "**Rate**" **Exp Ans: Rate is a comparison of quantities which are measured in different units** **e.g. Speed, Time and Distance**  **Q2 :** What is the formula we use to calculate the speed? : What is the formula we use to calculate the time? : What is the formula we use to calculate the distance?	

Lesson presentation **40 min**	**Exploration & Explanation** Experience key concepts	• The triangular method can be used to derive formulas for different quantities  Speed = Distance / Time Distance = Speed x Time Time = Distance / Speed • Each quantity can be expressed in many different units - **Speed** is measured in kilometres per hour**(km/h)** - **Time** is measured in hours **(h)** - **Distance** is measured in kilometres **(km)** • Hence we can also use conversion of units to write the above units if we are given different ones e.g. Given the Time in minutes, we can convert the minutes to hours	

Activity 1

Learners will be given time to work out the following problems on their own and reflect on them with supervision of the teacher.

1. **In the following statements identify what is given and what is not given based on the three quantities you have learned (speed, time, and distance) together with their values and put question mark to the unknown quantity.**

   a. If a car drives 200km in 2 hours, how fast was the car in km/h?

   ____

   ____

   b. A car travels at 80km/h, how far will the car travel in 10 hours?

   ____

   ____

   c. If Susan can run 2km in 8 minutes, how long will it take her to run 5km if she maintains her speed?

In 2km race	In 5km race

Activity 2

- Learners will learn to analyse the questions.
- Convert units
- Use the formulas to calculate the unknown by substituting what is given.

		2. Now in the class-work book calculate the unknown quantities in the above activity. a to c	

	Evaluation	Extra work A car travelling at a constant speed travels 60km in 18 minutes. How far, travelling at the same constant speed, will the car travel in 1hour 12 minutes?	
		Summary and conclusion Key notes that may be confusing to learners: • How fast = speed • How long = time • How far = distance • Constant = not changing	

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Annexure K : Lesson plan (Research lesson number 2)

MATHEMATICS RESEARCH LESSON STUDY PLAN
GRADE 9 TERM 1

FACILITATOR	DATE	DURATION	GRADE	HOSTING SCHOOLS
[redacted]		1hr	9	[redacted]

Content Area	Numbers, operations and relationships
Topic	EXPONETS
Concepts & Skills	- Law of dividing the same base - Zero Exponent - Negative Exponent
Lesson objectives	- At the end of the lesson learners should be able to recognise law of dividing same base and be able to apply it in calculations. - They should know that any number raised to exponent zero is one. - They should be able to express a number with negative exponent as positive exponent or vice versa
Resources	Textbook
Prior Knowledge	- Expressing numbers in exponential form.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Duration	Lesson Development	Teaching and learning activities	Learning points
introduction 10 mini	Engagement Stage Sell the lesson to the learners (A very interesting activity to capture learners interest)	ACTIVITY 1: LAW OF DIVIDING SAME BASES Ask learners to do the following: Q1 : Give any example, dividing the same base written in exponential form. NB: IN THIS ACTIVITY TRY TO GUIDE LEARNERS TO PROVIDE EXAMPLE THAT HAVE A BIGER EXPONENT IN THE NUMERATOR AND SMALLER EXPONENT IN THE DENOMINATOR SO THAT THEY CAN SIMPLY DETERMINE THE LAW! Expected possible Ans: a) $\frac{2^4}{2^2}$ b) $\frac{3^4}{3^2}$ c) $\frac{2^3}{2^5}$ Q2 : Simplify your problem/find the answer Expected possible Ans: a) $\frac{2^4}{2^2} = \frac{16}{4} = 4$ b) $\frac{3^4}{3^2} = \frac{81}{9} = 9$ Q3 : Write your answer in question 2 in exponential form. Expected possible Ans: a) $4 = 2^2$ b) $9 = 3^2$ Q4 : Relate the answer in exponential form in question 3 with example you provided in question 1. Is there anything you notice between the two, what do you think happened? Expected possible Ans: Here learners should be able to determine the law of dividing the same base: That when you divide powers of the same bases you subtract the exponents! e.g. for Q1. a) $\frac{2^4}{2^2} = 2^2$ simply means $\frac{2^4}{2^2} = 2^{4-2} = 2^2$ [Simply write the base as it is and subtract the exponents]	

NOW LEARNERS SHOULD PROVIDE EXAMPLES WHICH THEY DIVIDES SAME BASES WITH THE EXPONENT OF THE NUMERATOR BEING SMALLER THAN EXPONENT OF THE DENOMINATOR.

Expected possible Ans:

a) $\frac{2^3}{2^4}$ b) $\frac{2^4}{2^6}$

Learners should also apply law of dividing same bases to simplify given examples:

Expected possible Ans:

a) $\frac{2^3}{2^4} = 2^{3-4} = 2^{-1} = ?$

b) $\frac{2^4}{2^6} = 2^{4-6} = 2^{-2} = ?$

Here learners will get the exponent of the base as negative exponent if they used the law correctly and some may wonder what will be the answer to that.

Now they can try simplifying it straight forward without using the law and see what they get.

Advise them that the denominator of the answer should be written in exponential form.

e.g. $\frac{2^3}{2^4} = \frac{8}{16} = \frac{1}{2}$

$\frac{2^4}{2^6} = \frac{16}{64} = \frac{1}{4} = \frac{1}{2^2}$

Therefore they will know that the answer to:

c) $\frac{2^3}{2^4} = 2^{3-4} = 2^{-1} = \frac{1}{2}$

d) $\frac{2^4}{2^6} = 2^{4-6} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Now they should know that any number raised to the exponent negative number can be written as one divide by a base raised to the same exponent but the exponent being positive, In which the denominator in exponential form can be simplified.

		<u>**CLASS WORK**</u> **SIMPLIFY THE FOLLOWING:** a) $\frac{3^5}{3}$ b) $x^2 \div x^3$ c) $\frac{2^5x^3}{2^5x^2}$ d) $\frac{x^2y^3}{xy^2}$ e) $8x^6y^3 \div 2x^2y$ f) $(-12x^5y^4)(-6x^3y^2)$	
		Summary and conclusion Emphasize to learners: • Law of dividing the same base states that: When you divide the same base you subtract the exponents! • Any number/variable raised to the exponent zero is always 1! • A number raised to the exponent that is negative can be written as one divided by that number raised to the same exponent but being positive, in which the denominator in exponential form can be simplified! e.g. $2^{-5} = \frac{1}{2^5} = \frac{1}{35}$	