In solving the above algebraic equations with the aid of Maple, we get

$$\begin{aligned}
&a_0 = \frac{-3}{2}k^2\gamma^2 + \frac{1}{2}(6k\gamma \pm \sqrt{6})k\gamma + \frac{1}{4}, k = k,\\
&a_1 = (6k\gamma \pm \sqrt{6})\rho k, a_2 = 6k^2\rho^2, c = \pm\frac{5}{6}\sqrt{6},\\
&\rho = \rho, \gamma = \gamma, \sigma = \frac{1}{24}\frac{6k^2\gamma^2 - 1}{k^2\rho}.
\end{aligned} \tag{5.18}$$

Hence, putting what we obtained in Eq.5.18 into Eq.5.16 we get

$$\begin{aligned}
\mathcal{U}(z) =& \frac{-3}{2}k^2\gamma^2 + \frac{1}{2}(6k\gamma \pm \sqrt{6})k\gamma + \frac{1}{4} +\\
&(6k\gamma \pm \sqrt{6})\rho k U + 6k^2\rho^2 U^2.
\end{aligned} \tag{5.19}$$

By substituting Eq.3.12 into Eq.5.19, we obtain the final solution of the Fisher equation to be

$$\begin{aligned}
u(x,t) =& \frac{1}{4} \pm \frac{k\sqrt{6}}{2}\sqrt{\gamma^2 - 4\rho\sigma}\tanh\left(-\frac{\sqrt{\gamma^2 - 4\rho\sigma}}{2}\left(kx \pm \frac{5k}{\sqrt{6}}t + A\right)\right) +\\
&\frac{3k^2\gamma^2}{2}\tanh^2\left(-\frac{\sqrt{\gamma^2 - 4\rho\sigma}}{2}\left(kx \pm \frac{5k}{\sqrt{6}}t + A\right)\right) -\\
&6k^2\sigma\rho\tanh^2\left(-\frac{\sqrt{\gamma^2 - 4\rho\sigma}}{2}\left(kx \pm \frac{5k}{\sqrt{6}}t + A\right)\right).
\end{aligned} \tag{5.20}$$

It is the travelling wave moving to the left or right if Eq.2.2 is satisfied. Hence, that can be achieved by considering different values of ρ, σ and γ. Here, we consider few cases:

- case 1: If $\rho = -1$, $\gamma = 0$, $\sigma = 1$, then

$$\begin{aligned} i)\ u(x,t) =& \frac{1}{4} - \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right), \\ ii)\ u(x,t) =& \frac{1}{4} + \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right). \end{aligned} \tag{5.21}$$

- case 2: On condition that $\rho = 1$, $\gamma = 0$, $\sigma = -1$ we get

$$\begin{aligned} i)\ u(x,t) =& \frac{1}{4} - \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right), \\ ii)\ u(x,t) =& \frac{1}{4} + \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right). \end{aligned} \tag{5.22}$$

- case 3: Given that $\rho = 4$, $\gamma = 3$, $\sigma = -1$, we get

$$\begin{aligned} i)\ u(x,t) =& \frac{1}{4} - \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right), \\ ii)\ u(x,t) =& \frac{1}{4} + \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right). \end{aligned} \tag{5.23}$$

- case 4: For $\rho = -1$, $\gamma = 1$, $\sigma = 0$, we obtain

$$\begin{aligned} i)\ u(x,t) =& \frac{1}{4} - \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right), \\ ii)\ u(x,t) =& \frac{1}{4} + \frac{1}{2}\tanh\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right) + \\ & \frac{1}{4}\tanh^2\left(\pm\frac{1}{2\sqrt{6}}\left(x \pm \frac{5}{\sqrt{6}}t\right) + A\right). \end{aligned} \tag{5.24}$$

In each case we got two travelling waves solutions, propagating in different directions. Therefore, Eq.5.20 becomes a travelling wave solution if

$$\gamma^2 - 4\rho\sigma > 0.$$

The four cases demonstrate that we get same result for $\gamma^2 - 4\rho\sigma > 0$.

5.3 Burgers-Fisher equation

The Burgers-Fisher equation is normally used to model fluid dynamics, number theory, heat conduction, elasticity and many more [27], [28]. The tanh method for generalized forms of Burgers-Fisher equations was presented by [28]. As an example, we provide solution of the generalized Burgers-Fisher equation by using Example 2. The generalized Burgers-Fisher equation is given by

$$\frac{\partial u}{\partial t} + u^2\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = u(1-u^2). \tag{5.25}$$

Transforming to the z variable yields

$$ck\frac{d\mathcal{U}}{dz} - k\mathcal{U}^2\frac{d\mathcal{U}}{dz} + k^2\frac{d^2\mathcal{U}}{dz^2} + \mathcal{U} - \mathcal{U}^3 = 0. \tag{5.26}$$

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After comparing the values of m using Eq.4.39 and Eq.4.40, we obtained m to be a fraction. Hence, we move to **Example 2** and Table 2 to find that $m = 1$ and $\alpha = 1/2$. Therefore, we can seek the solution of Eq.5.26 in the form

$$\mathcal{U}(U) = \sum_{i=0}^{1} a_i (U^{1/2})^i, \tag{5.27}$$

where $U = U(z)$ satisfies

$$\frac{dU}{dz} = U^2 - U.$$

Putting Eq.5.27 into Eq.5.26 together with Eq.3.3, collecting all terms with like powers U^j and $U^{1/j}$ and setting them to zero, we get the system of six equations,

$$\begin{aligned}
U^0 \, &: a_0 - a_0^3 = 0, \\
U^1 \, &: k a_0 a_1^2 - 3 a_0 a_1^2 = 0, \\
U^2 \, &: - k a_0 a_1^2 = 0, \\
U^{1/2} \, &: - 2cka_1 + k^2 a_1 + 2k a_1 a_0^2 - 12 a_0^2 a_1 + 4a_1 = 0, \\
U^{3/2} \, &: 2cka_1 - 4k^2 a_1 - 2k a_1 a_0^2 + 2k a_1^3 - 4a_1^3 = 0, \\
U^{5/2} \, &: 3k^2 a_1 - 2k a_1^3 = 0.
\end{aligned} \tag{5.28}$$

Solving the above equations, we get the following set of solutions:

$$\begin{aligned}
&\left\{ a_0 = 0, a_1 = 1, c = \frac{10}{3}, k = \frac{2}{3} \right\}, \\
&\left\{ a_0 = 0, a_1 = -1, c = \frac{10}{3}, k = \frac{2}{3} \right\}.
\end{aligned} \tag{5.29}$$

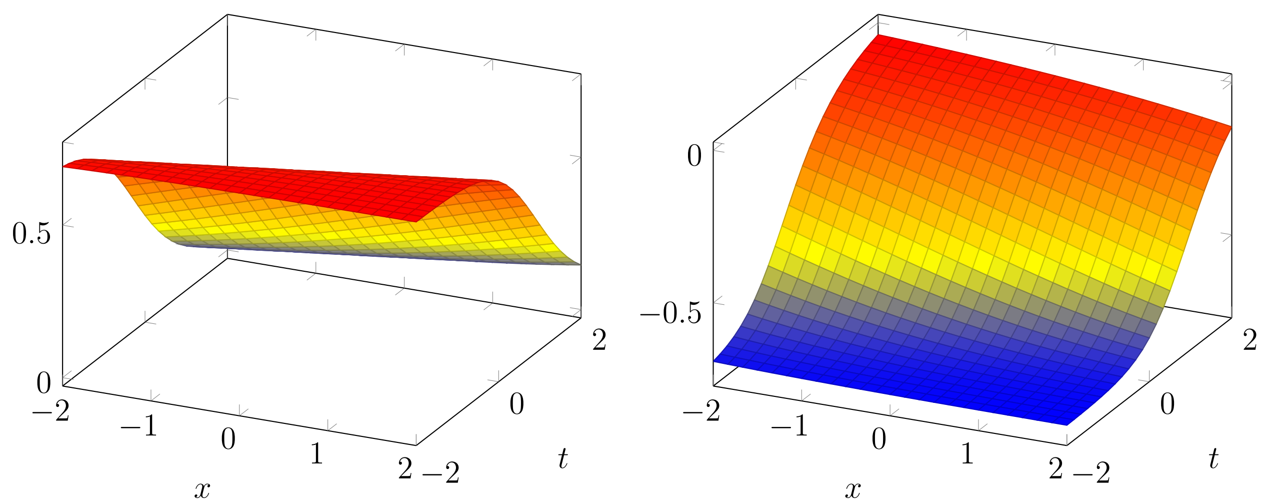
Substituting the above set into Eq.5.27, we get

$$\mathcal{U}(U) = \pm U^{1/2}. \tag{5.30}$$

Finally, after substituting Eq.3.12 into Eq.5.30, we get two travelling wave solutions:

$$\begin{aligned} u(x,t) &= -\frac{1}{2}\left(1+\tanh\left(\frac{x}{3}-\frac{10}{9}t\right)+\frac{A}{2}\right)^{1/2}, \\ u(x,t) &= \frac{1}{2}\left(1+\tanh\left(\frac{x}{3}-\frac{10}{9}t\right)+\frac{A}{2}\right)^{1/2}. \end{aligned} \tag{5.31}$$

Figure 5.5 shows the shape of two exact travelling solutions, in which both waves are travelling at the same speed but in opposite directions. The same results have been obtained by many researchers. In [28] work, they found m



(a) Burger Fisher solution, $u(x,t) = \frac{1}{2}\left(1+\tanh\left(\frac{x}{3}-\frac{10}{9}t\right)\right)^{1/2}$, $A=0$

.

(b) Burger Fisher solution, $u(x,t) = -\frac{1}{2}\left(1+\tanh\left(\frac{x}{3}-\frac{10}{9}t\right)\right)^{1/2}$, $A=0$

.

Figure 5.5: Burger Fisher solution profiles

to be a fraction. Hence, they utilize substitution to get rid of that fraction. Here, we demonstrate a simple, general and straightforward method that can be used to solve problems that involve m as a fraction. **Example 2** together with Table 1 and Table 2 proved to be an effective way of solving such problems without any tedious calculations. Our results are totally different to what [25] obtained. The modified tanh $-$ coth method utilized in [25] does not produce travelling wave solutions.

Chapter 6

Second approach of finding exact travelling wave solutions

In Chapter 2, we stated that travelling wave solutions occur between two equilibrium. We also assume that $P(U)$ is a quadratic polynomial. Here, we want to show that $P(U)$ can be $U \ln U - U$. Consider a first order autonomous equation

$$\frac{dU}{dz} = U \ln U - U \quad , \quad U > 0. \tag{6.1}$$

We then find the exact expression for the solution to Eq.6.1. An ODE is separable. By arranging and integrating both sides, we get

$$\int \frac{1}{U(\ln U - 1)} dU = \int dz + A_1. \tag{6.2}$$

Let $h = \ln U - 1$, so $dh = \frac{1}{U} dU$, implies

$$\int \frac{1}{h} dU = \int dz + A_1, \tag{6.3}$$

where A_1 is the integration constant. By using partial fractions, we get

$$\begin{aligned} \ln|\ln U - 1| &= z + A_1, \\ |\ln U - 1| &= e^{z+A_1}, \end{aligned} \tag{6.4}$$

$$|\ln U - 1| = \ln U - 1, \quad \text{if} \quad U > e. \tag{6.5}$$

Finally, we have

$$U(z) = e^{e^{z+A_1}+1}. \tag{6.6}$$

In this case, Eq.6.6 does not need to satisfy Eq.2.2 but Eq.2.2 needs to be satisfied when Eq.6.6 is substituted into the following assumption. Let us assume that the solution of an ODE is of the form:

$$\mathcal{U}(z) = a_0 + \sum_{i=1}^{m}\sum_{j=1}^{n} a_i b_j U^{-i+1}(\ln U)^{-j}, \tag{6.7}$$

where $U = U(z)$ satisfies the first order nonlinear ODE:

$$\frac{dU}{dz} = U\ln U - U \quad , \quad U > 0. \tag{6.8}$$

We then demonstrate the application of the method by solving two well known equations, the Fisher equation and Korteweg-de Vries equation.

6.1 Korteweg-de Vries equation

The Korteweg-de Vries equation is a third order nonlinear partial differential equation. It was derived from fluid mechanics to describe shallow water waves in a rectangular channel. The equation is written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + b\frac{\partial^3 u}{\partial x^3} = 0, \tag{6.9}$$

where constant $b > 0$. After transformation and integrating once, we obtain

$$-ck\mathcal{U} + k\frac{1}{2}\mathcal{U}^2 + bk^2\frac{d^2\mathcal{U}}{dt^2} - B_1 = 0, \tag{6.10}$$

where B_1 is the integration constant. After balancing exponents, we again have $m = 1$ and $n = 2$. Then, Eq.6.7 leads us to

$$\mathcal{U}(z) = a_0 + a_1 b_1 (\ln U)^{-1} + a_1 b_2 (\ln U)^{-2} \tag{6.11}$$

Substituting Eq.6.11 with Eq.6.6 into Eq.6.10 and collecting and equating coefficients of $U^{-i+1}(\ln U)^{-j}$ to zero, we obtain

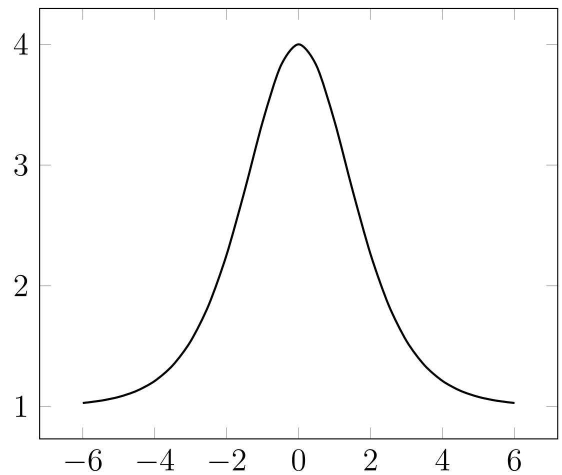
$$\begin{aligned}
U^0(\ln U)^0 &: -2cka_0 + ka_0^2 - 2B_1 = 0,\\
U^0(\ln U)^{-1} &: -cka_1b_1 + ka_0a_1b_1 + bk^2a_1b_1 = 0,\\
U^0(\ln U)^{-2} &: -2cka_1b_2 + 2ka_0a_1b_2 + ka_1^2b_1^2 - 6bk^2a_1b_1 + 8bk^2a_1b_2 = 0,\\
U^0(\ln U)^{-3} &: ka_1^2b_1b_2 + 2bk^2a_1b_1 - 10bk^2a_1b_2 = 0,\\
U^0(\ln U)^{-4} &: ka_1^2b_2^2 + 12bk^2a_1b_2 = 0.
\end{aligned} \tag{6.12}$$

Solving the above equations, we have

$$\begin{aligned}
&\Big\{B_1 = \frac{1}{2}b^2k^3 - \frac{1}{2}c^2k, a_0 = -bk + c, a_1 = -\frac{12bk}{b_2},\\
&b = b, b_1 = -b_2, b_2 = b_2, c = c, k = k\Big\}.
\end{aligned} \tag{6.13}$$

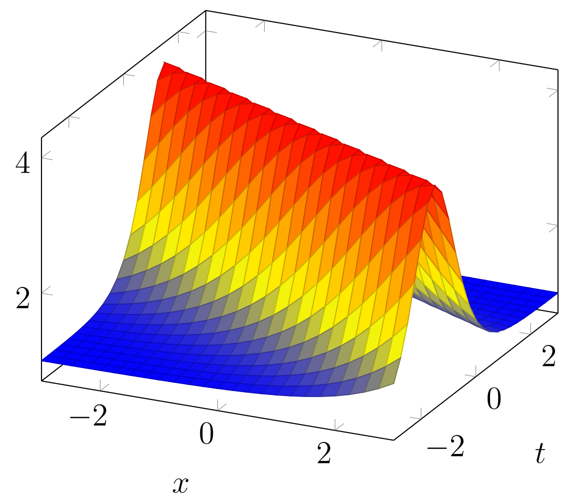
Hence, Eq.6.11 becomes

$$\begin{aligned}
\mathcal{U}(U(z)) &= (-bk + c) + 12bk(\ln U)^{-1} - 12bk(\ln U)^{-2},\\
\mathcal{U}(z) &= (-bk + c) + 12bk\left(\frac{e^{z+A_1}}{(e^{z+A_1} + 1)^2}\right)\\
&= (-bk + c) + 3bk\left(\frac{4}{e^{z+A} + 2 + e^{-z-A}}\right).
\end{aligned} \tag{6.14}$$



(a) $U(z) = 1 + \frac{12e^z}{(e^z+1)^2}$, $A_1 = 0$

.



(b) $u(x,t) = 1 + \frac{12e^{x-2t}}{(e^{x-2t}+1)^2}$, $A = 0$

.

Figure 6.1: Korteweg-de Vries equation profiles, where $A_1 = 0$, $c = 2$, $k = 1$, $b = 1$.

Comparing these results with the results we obtained in Chapter 2 (Eq.2.35), we have same results.

6.2 Fisher equation

Reconsider the Fisher equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(1-u), \tag{6.15}$$

After balancing the exponents, we get $m = 1$ and $n = 2$. Following all steps as in previous examples, we get the system of equations

$$
\begin{aligned}
U^0(\ln U)^0 :& a_0 - a_0^2 = 0,\\
U^0(\ln U)^{-1} :& a_1b_1 - 2a_0a_1b_1 - cka_1b_1 + k^2a_1b_1 = 0,\\
U^0(\ln U)^{-2} :& a_1b_2 - 2a_0a_1b_2 - a_1^2b_1^2 + cka_1b_1 - 2cka_1b_2 - 3k^2a_1b_1 + 4k^2a_1b_2 = 0,\\
U^0(\ln U)^{-3} :& -2a_1^2b_1b_2 + 2cka_1b_2 + 2k^2a_1b_1 - 10k^2a_1b_2 = 0,\\
U^0(\ln U)^{-4} :& -a_1^2b_2^2 + 6k^2a_1b_2 = 0.
\end{aligned}
\tag{6.16}
$$

From the above equations, we get the following results,

$$
\begin{aligned}
&\left\{a_0 = 0, a_1 = \frac{1}{b_2}, b_1 = 0, b_2 = b_2, c = \frac{5}{6}\sqrt{6}, k = \frac{1}{6}\sqrt{6}\right\},\\
&\left\{a_0 = 0, a_1 = \frac{1}{b_2}, b_1 = 0, b_2 = b_2, c = \frac{-5}{6}\sqrt{6}, k = -\frac{1}{6}\sqrt{6}\right\},\\
&\left\{a_0 = 1, a_1 = \frac{1}{b_2}, b_1 = -2b_2, b_2 = b_2, c = \frac{-5}{6}\sqrt{6}, k = \frac{1}{6}\sqrt{6}\right\},\\
&\left\{a_0 = 1, a_1 = \frac{1}{b_2}, b_1 = -2b_2, b_2 = b_2, c = \frac{5}{6}\sqrt{6}, k = -\frac{1}{6}\sqrt{6}\right\},
\end{aligned}
\tag{6.17}
$$

The the following exact travelling wave solutions are produced,

$$
\begin{aligned}
u(x,t) &= \frac{1}{\left(e^{\frac{\sqrt{6}}{6}\left(x - \frac{5}{6}\sqrt{6}t\right) + A_1} + 1\right)^2},\\
u(x,t) &= \frac{1}{\left(e^{-\frac{\sqrt{6}}{6}\left(x + \frac{5}{6}\sqrt{6}t\right) + A_1} + 1\right)^2},
\end{aligned}
\tag{6.18}
$$

and

$$
\begin{aligned}
u(x,t) &= 1 - \frac{2}{e^{\frac{\sqrt{6}}{6}\left(x - \frac{5}{6}\sqrt{6}t\right) + A_1} + 1} + \frac{1}{(e^{\frac{\sqrt{6}}{6}\left(x - \frac{5}{6}\sqrt{6}t\right) + A_1} + 1)^2},\\
u(x,t) &= 1 - \frac{2}{e^{\frac{-\sqrt{6}}{6}\left(x + \frac{5}{6}\sqrt{6}t\right) + A_1} + 1} + \frac{1}{(e^{-\frac{\sqrt{6}}{6}\left(x + \frac{5}{6}\sqrt{6}t\right) + A_1} + 1)^2}.
\end{aligned}
\tag{6.19}
$$

The presented solution of the Fisher equation solutions shows the same result as the previous method where $P(U)$ was treated as a second order polynomial. The intention of presenting the second approach was to emphasize that any function $P(U)$ (either a polynomial or not, but it must have more than one root) can be utilized to produce exact travelling wave solutions. That goal has been achieved; the Fisher equation and KdV equation produced travelling wave solutions in both cases. The application to established PDEs, demonstrated that the method technique is simple and capable of finding analytical travelling wave solutions for nonlinear evolution equations.

Chapter 7

Discussion

The general presented method of finding travelling wave solutions for nonlinear PDEs as linear combinations of functions satisfying certain assumptions provides a straightforward algorithm to work out nonlinear PDEs exact travelling wave solutions. It is shown that the methods like the G'/G, tanh-method and many more, are examples of th general method introduced in the dissertation. The main procedure of the method lies in the fact that $\mathcal{U}(z)$ must approach constant states, that is $\mathcal{U}(z \to -\infty) = u_1$ and $\mathcal{U}(z \to +\infty) = u_2$, which must be at equilibria of $P(U)$, hence $P(U)$ must have at least two roots.

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