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The effectiveness of smoothed bonus portfolios for mitigating investment risk in defined contribution pension funds

by

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DECLARATION

I, Corlia Petronella Laue, declare that the dissertation which I hereby submit for the degree MSc Actuarial Science at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other institution.

Signature: ______________________________

Date: ______________________________

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I would like to acknowledge and thank my two supervisors, Dr Beyers and Ms Venter. Their advice and comments steered me in the right direction, and helped me to think through the problems I wrestled with in this study. I thank my brother, Heinrich Laue, for his help with the more practical aspects related to this study. He willingly shared insights and learnings gained from his own research. I am grateful to my parents, Edgar and Ronelle Laue, for their prayers, encouragement and listening ears. Glory and honour to the one true and living God, for 'of Him and through Him and to Him are all things' (Romans 11.36, NKJV).

Abstract

The aim of this study is to investigate whether smoothed bonus portfolios (SBPs) are effective at managing the investment risk that members of a defined contribution pension fund are exposed to. Investment risk arises from the uncertainty of the performance of the assets invested in by the fund during the accumulation phase. This creates uncertainty for a member as to what the outcome at retirement will be. It is measured as the value at risk as well as conditional tail expectation, calculated on a member's simulated savings at retirement. The effectiveness of an SBP is investigated through applying three methodologies, namely 1) a return/risk analysis where the contribution of each of the features of an SBP to its return and return/risk ratio is analysed; 2) comparing the simulated outcome at retirement of an SBP with the outcome of two types of notional benchmark portfolios that apply simpler investment strategies, but are set up to have the same level of risk as the SBP; and 3) applying first-order stochastic dominance (FSD) rules.

On a risk adjusted basis, the guarantee and smoothing mechanism of an SBP make positive contributions to its performance. However, when comparing the outcome of the notional benchmark portfolios with that of the SBPs, the former consistently outperform the SBPs modelled. Applying FSD rules, the notional benchmark portfolios are found to be preferred to a greater extent than the SBPs.

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List of Abbreviations

Abbreviation	Description
APN	Actuarial practice note
ARIMA	Autoregressive integrated moving averages
ARCH	Autoregressive conditionality heteroscedastic
ASSA	Actuarial Society of South Africa
BSR	Bonus smoothing reserve
CDC	Collective defined contribution plan
CDF	Cumulative distribution function
CPPI	Constant proportion portfolio insurance
CRRA	Constant relative risk aversion
CTE	Conditional tail expectation
DB	Defined benefit
DC	Defined contribution
ESG	Economic scenario generator
FSB	Financial Services Board
FSD	First-order stochastic dominance
GARCH	Generalised autoregressive conditionally heteroscedastic
GMAB	Guaranteed minimum accumulation benefit
GMDB	Guaranteed minimum death benefit
GMIB	Guaranteed minimum income benefit
GMWB	Guaranteed minimum withdrawal benefit
iCPPI	Individualised constant proportion portfolio insurance
IRR	Internal rate of return
JSE	Johannesburg Stock Exchange
LPM	Lower Partial Moment
MGWP	Maturity Guarantees Working Party
MVA	Market value adjuster
PPFM	Principles and Practices of Financial Management

Abbreviation	Description
SAP	Standards of actuarial practice
SBP	Smoothed bonus portfolio
SETAR	Self-exciting threshold autoregressive
TAR	Threshold autoregressive
UK	United Kingdom
VaR	Value at risk

1 Introduction

1.1 Introduction

As a member of the ABC pension fund your benefit statement as at 31 October 2007 indicates that your retirement savings built up over the last 30 years amounts to R4m. You are looking forward to retirement and are feeling satisfied that you are now going to reap the rewards of the persistence and diligence shown throughout your working life in saving for retirement. One year later and you want to retire from the fund, only to hear with shock that the value of your savings has dropped by 31% to R2.8m. 'How is this possible?', you cry in devastation. This seems so unfair, but there is nothing to be done.

1.2 Background

South Africa experienced widespread conversions from Defined Benefit (DB) to Defined Contribution (DC) occupational pension fund structures in the 1980s and 1990s (Andrew, 2004). With this conversion came a significant change in the way in which risk is shared among the various stakeholders of a pension fund.

In a DB fund, members are guaranteed a level of pension based on their individual salary history and employment duration (Bodie et al., 1988). All contributions are managed collectively such that pre- and post-retirement members collectively share the investment and mortality risks of the fund. However, ultimately it is the employer who carries the risk associated with the guaranteed replacement rate of income at retirement offered to members, as well as the risk associated with the members' longevity.

In a DC fund, the retirement income of a member depends on the investment performance realised from the investment of contributions during the accumulation phase, i.e. pre-retirement phase, the wage path of the member (which influences the level of contributions made, since this is normally expressed as a percentage of salary), and the interest rate applicable at the member's retirement date (Bodie et al., 1988). One of the implications of such a structure is that it is the members who carry the annuitisation and investment risk in full.

In South Africa, a DC fund registered under the Pension Funds Act of 1956 operates as a separate legal entity and is managed by a board of trustees. Trustees have a fiduciary duty to ensure that the fund is financially healthy and is managed responsibly (Southern African Legal Information Institute, 2014). Regulation 28 requires a pension fund to have an investment policy statement in place. It further requires that the trustees ensure that the fund's assets are appropriate for its liabilities and that the trustees consider what factors may materially affect the sustainable long term performance of the assets (Southern African Legal Information Institute, 2014).

It is common practice for trustees to delegate the asset management function to a third party manager. These managers could either be appointed on a specialist mandate basis, or taking a balanced (or multi-asset) approach (Actuarial Education Company, 2015). Where a balanced mandate approach is taken, the fund can either opt to invest in a pooled portfolio with a predefined investment strategy, or in a segregated portfolio where the trustees of the pension fund itself decides the investment strategy according to which the portfolio must be managed.

Smoothed bonus portfolios (SBPs) are savings products that are designed for the investment of assets underlying DC pension funds. These portfolios apply risk mitigation strategies to reduce the investment risk members of the pension fund are exposed to. These strategies take the form of a guarantee attached to the level of savings paid on exit from the fund and a smoothing mechanism applied to portfolio returns.

Section 2.9 discusses the features of SBPs in greater detail. Appendix A provides a summary of the features of most of the SBPs currently open to new business in South Africa.

1.3 Purpose of the study

1.3.1 The objective of SBPs

SBPs are marketed as aiming to meet the following objectives:

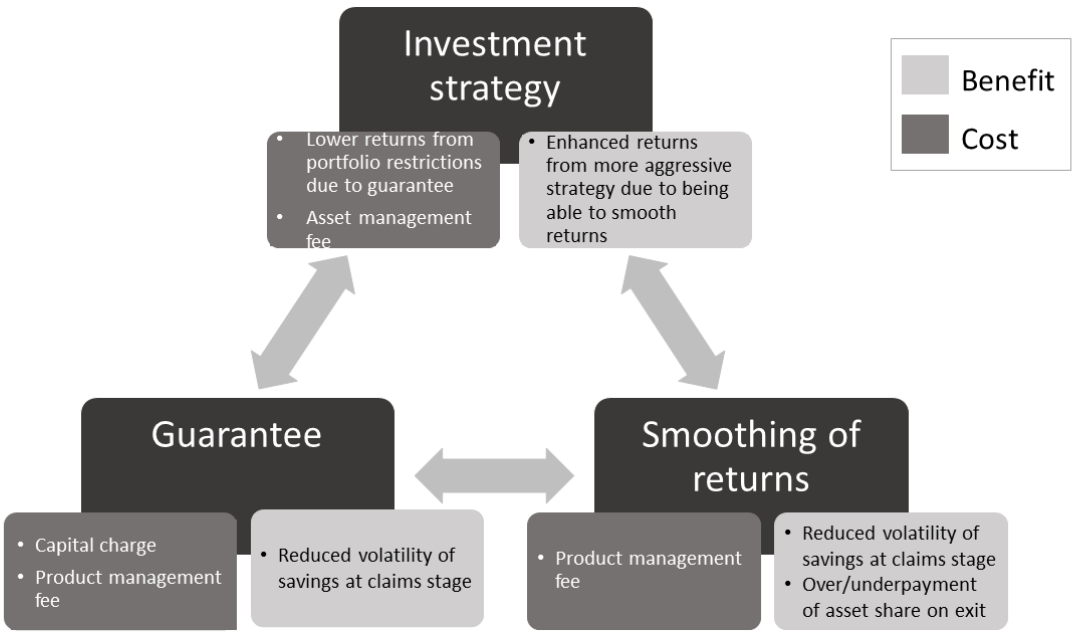
- 'grow retirement savings while protecting investors from the risk of volatile markets', and 'manage volatility without reducing their real return targets' (Old Mutual, 2016, p. 1)

- 'investors seeking to plan with confidence for retirement' and 'seeking downside protection for their benefits' (Momentum, 2016, p. 1)
- 'protects investors against short term volatility by smoothing out investment returns, whilst providing valuable guarantees on benefit payments' (Sanlam, 2016, p. 1)

The objectives of a rational pension fund member is discussed in section 2.3.1. Comparing those objectives with that of an SBP suggests that these portfolios are a possible option that DC pension funds could consider for the investment of the assets in the fund. Whether these portfolios are an effective option though, is what this study aims to investigate.

1.3.2 Features of SBPs

Figure 1 demonstrates the interrelationship between the three key features of an SBP. These features together aim to grow retirement savings for members, while at the same time aiming to reduce risk exposure. The figure illustrates the benefits as well as the costs associated with each feature in achieving its aims.



Source: Author's own illustration

Figure 1: Illustrative interaction of the features of an SBP

Investment returns are passed on to members in the form of regular bonuses that are declared. Therefore, even though the investment performance of the assets underlying an SBP is not directly passed on to pension fund members, the investment strategy followed does remain a significant determinant of the bonus rates that will be declared, and therefore also of the outcome at retirement. Passing returns on to members in a smoothed manner reduces their exposure to the volatility of the assets underlying the product (Alexander Forbes, 2010). In addition, maintaining a bonus smoothing reserve (BSR) as part of the smoothing mechanism employed, allows for a more aggressive investment strategy to be followed even as a member approaches retirement. This should improve the final outcome for members when, *ceteris paribus*, compared against a strategy where as a means of managing risk, members start moving to less risky asset classes as they approach retirement. By guaranteeing that bonus rates cannot fall below a certain set minimum, i.e. guaranteeing that a set portion of contributions plus declared bonuses will be paid out on exit from the fund, the level of investment risk members are exposed to is further reduced. They do not fully share in the downturns.

On the downside, if the guarantee restricts the permissible investment strategies insurers are willing to select, as they in turn try to manage their obligation in terms of the guarantee, an implicit cost of potentially lower returns will be borne by members (Jensen and Sørensen, 2001). This is in addition to an explicit cost in the form of a capital charge levied for the guarantee offered. Insurers charge a product management fee to cover the administrative and operational costs associated with the management of an SBP (Sanlam, 2016; Old Mutual, 2015). The asset manager charges a fee for managing the underlying assets of the product and this is deducted directly from investment returns before the returns are passed on to the fund.

The balance of the BSR at any point in time is influenced by the extent to which there is a misalignment between the returns earned on the investment of the underlying assets of the product and the bonuses allocated to members' savings. Holding a BSR implies that should a member exit the fund when the BSR is at a higher absolute level than when they joined the fund, they will be leaving behind some of their asset share. A member's asset share in this instance is taken to mean the sum of all contributions made, plus returns earned on the investment of those contributions, less fees. In the reverse, should a member withdraw when the BSR is negative (given that it was positive when they joined), they will be receiving more than their asset share on withdrawal. It will effectively be the remaining members who will foot the bill in this case.

The faster the pace at which the BSR is transferred to members, or the closer the link between actual investment returns earned on the underlying assets and the bonuses allocated to a member, the closer aligned a member's payout on retirement is likely to be to their asset share. However, a closer alignment translates to a lesser degree of smoothing and therefore less of a reduction in investment risk observed by the member.

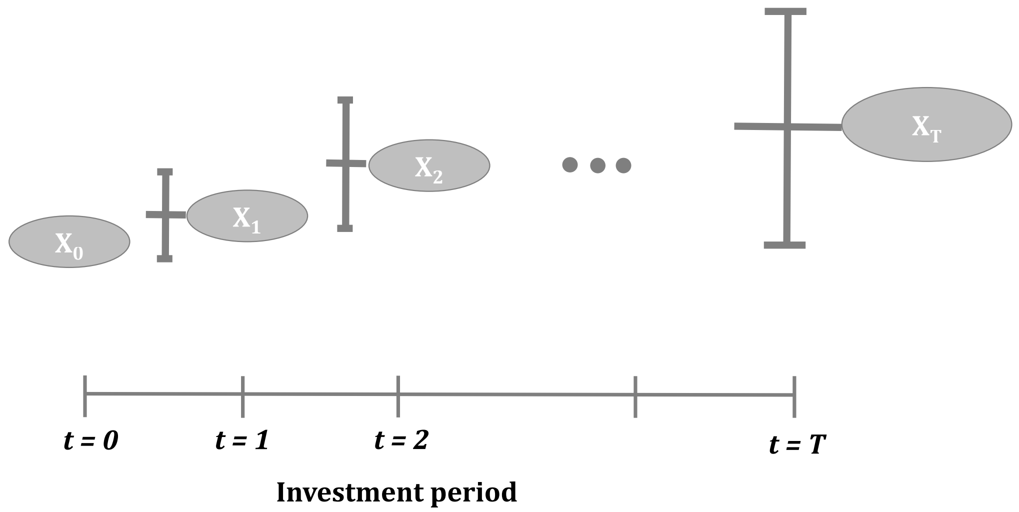
All of these features and their interactions need to be considered in an investigation into the effectiveness of SBPs.

1.3.3 Performance assessment of an investment strategy

There is an interconnectedness between the risk assumed by an investment strategy and the outcome from that strategy. Figure 2 illustrates this. This illustration assumes the investor has set an end date to the investment period. This is appropriate in our context where we are considering members saving for retirement. In our case, the investment period represents the accumulation phase over which a member contributes to a pension fund, while time T represents the retirement date.

During the investment period, the value of the investment observed at times t, X_i for $i \in [0, T]$, will be influenced by the risk assumed in the choice of assets invested in. This risk will influence both the level of X_i, as well as the possible range of values of X_i, i.e. the volatility of the outcomes. The risk assumed during the investment period will of course also influence the level and range of possible values of X_T at the end of the investment period.

Retrospectively, an investor is able to determine the value of his investment, X_t, at any time t prior to the current time. He can also measure the volatility of those outcomes. Prospectively,



Source: Author's own illustration

Figure 2: Illustration of the interconnectedness of the outcome and risk of an investment

the value of his investment, X_t, at any time t up to time T is uncertain. This uncertainty can be modelled in order to estimate what the expected final outcome at time T is. If a stochastic model is used to model the uncertainty, the investor is able to determine how wide the range of values is within which he can expect the final outcome to fall, i.e. the expected volatility of the outcome at time T can be determined. The greater the level of risk assumed in the choice of assets invested in during the investment period, the wider the range of likely outcomes at time T will be.

In any discussion on the performance of an investment strategy, it is essential that the value of the outcomes either at times t or time T, not be considered in isolation of the risk, in all of its facets, inherent in the investment strategy. If this is not done it might lead to investors drawing inaccurate conclusions about the effectiveness of an investment strategy and therefore making suboptimal investment decisions. Examples of these suboptimal decisions are provided next.

If only the level of the final outcome expected, i.e. the expected value of X_T, is considered, a strategy that is expected to produce a higher final outcome might be seen as a better performing strategy. This conclusion could be reached without taking into consideration whether an excessive amount of risk needs to be assumed during the investment period in order to to achieve this outcome. If this is the case, the greater amount of risk assumed during the investment period will reduce the certainty that this higher outcome expected at time T will indeed be realised. The investor could therefore be left with a much lower outcome than anticipated at time T.

If investors are only concerned with achieving a more certain outcome at time T, they might opt for a strategy that offers more stable returns during the investment period. However, lower risk investments are likely to lead to a lower expected outcome at time T. Investors might therefore, in choosing this more stable outcome, be sacrificing a significant amount of upside without realising it. If more risk is assumed, it is true that it would widened the range of likely outcomes at time T. However, it needs to be investigated whether the lowest bound of this wider range of outcomes is not so close to the expected outcome from the more stable strategy that the investor is likely to have been in a better or similar position if they had chosen the more risky strategy.

1.4 Research objectives

The objective of this study is to investigate whether SBPs are an effective means of managing the investment risk that DC fund members are exposed to. The aim is to investigate whether the outcome from an SBP is sufficient, taking into consideration the level of risk reduction it is able to achieve and the costs, both implicit and explicit, associated with that. This involves the following:

- Split and analyse the return on an SBP between its various features to determine whether each feature adds to or detracts from its overall performance.
- Calculate the riskiness, as well as the return/risk ratio for an SBP.
- Analyse the extent to which each feature of an SBP contributes to or detracts from overall performance on a risk-adjusted basis by analysing the return/risk ratios.
- Establish whether an SBP is able to generate a better outcome for members relative to two types of simple notional benchmark portfolios that have a similar degree of risk as an SBP.

- Determine whether an SBP exhibits first-order stochastic dominance over these notional benchmark portfolios.

The methodology applied is discussed in greater detail in chapter 3.

1.5 Importance and benefit of the proposed study

The following are some of the reasons why this study makes an important contribution in this field.

1.5.1 Shift from DB to DC

The shift from DB to DC funds implies that a significant change has taken place in terms of the level of investment risk members saving for retirement are now being forced to bear. This study will make a contribution to the discussion on how to deal with this shift in risk exposure by investigating one mitigation tool available in the South African market.

1.5.2 Alternative risk management strategies

Members of DC pension funds are faced with the conundrum of wanting to protect the value of their savings on the date of exit on retirement, while at the same time also wanting to ensure that they retire with a sufficient amount of money. The latter implies maximising the returns on their savings. The former goal however requires investment in less risky, and therefore poorer performing asset classes.

The 2014 Sanlam Research Insights survey found that 63% of the stand-alone fund respondents regarded cash portfolios as providing good guarantees. This reflects an increase from the 47% found in the 2011 survey (Sanlam Employee Benefits, 2014). The 2015 survey found that 54% of the funds of members due to retire in a year's time is invested in a cash portfolio (Sanlam Employee Benefits, 2015). Looking back since 1925, the real return offered by cash has been estimated to have been 0.8% p.a. (Lambrecht, 2015). This supports Lamprecht's view that 'cash is trash', particularly for funds with a long-term investment horizon.

The 2015 Sanlam Research Insights survey found that only 5% of retirement funds use SBPs as their 'end stage' portfolio, i.e. the investment strategy used a few years prior to retirement. The findings of this study should benefit trustees and fund members in determining whether greater attention should not be given to SBPs as an alternative option to achieve the members' objectives.

1.5.3 Cross-subsidisation between members

During the 2015 Actuarial Society of South Africa's (ASSA) Life Assurance Seminar, Paul Truyens presented results from an investigation into the extent to which cross subsidisation between different cohorts of policies is taking place within SBPs (Truyens, 2015b,a). He set out to address the question of whether the level of cross subsidisation in SBPs is fair towards fund members. This study, though not directly addressing this issue, will provide a fresh perspective on how to view SBPs by taking into account factors not directly considered by Truyens. This study is also based on the features of two types of SBPs offered in the market, whereas his investigation solely focused on Old Mutual's Absolute Growth Portfolios.

1.6 Delimitations and Limitations

This study is limited in the scope of the benefits of SBPs that it considers. In particular, the following two benefits of the product are not considered:

- Exit prior to retirement: The guarantee offered by these portfolios, depending on the product rules, also applies should a member exit the fund prior to retirement, due to events such as resignation or retrenchment. The guarantee therefore offers value to members on the occurrence of these events too. By only focusing on retirement, this additional benefit and its impact on members is not considered or allowed for in this study.

- Impact on members' behaviour: The impact of the reduced volatility from the smoothing of returns and the guarantee offered on members' willingness to save for retirement is not investigated. This impact should however not be dismissed and can be significant. Consiglio et al. (2015) go as far as saying that the success of a DC fund structure depends on the design of appropriate guarantees, since this will help make retirement income safer, which in turn will encourage participation and lead to increased levels of savings.

This would of course apply less in an environment where membership is compulsory, which is largely the case in South Africa.

The effectiveness of SBPs relative to the other alternatives used to manage investment risk in South African DC funds (discussed in section 2.8) is not investigated.

Human capital can be defined as the present value of an individual's future labour income (Bodie and Treussard, 2007). Human capital risk is the risk associated with the uncertainty of future income to be received from employment. This is just one of the additional types of risk that will affect the level of savings a member has at retirement, but that is not considered in this study.

Once a member reaches retirement, annuitisation risk is realised. Blake et al. (2014) show how the optimal investment strategy to follow during the decumulation, i.e. retirement, period links to the accumulation phase investment strategy. This study will not consider ways of managing annuitisation risk nor take into consideration the impact such mitigation strategies could have on the management of pre-retirement investment risk.

This study proposes three methods of assessing the effectiveness of SBP. Should alternative techniques of assessing effectiveness be applied it could potentially lead to different conclusions.

As with any modelling of a real world scenario, the outcomes are only as valuable as the accuracy of the model and assumptions made. Model risk would therefore affect the level of usefulness of the outcome of this study.

1.7 Conclusion

With the move from DB to DC pension fund structures, members of pension funds are now greatly exposed to investment risk as they save for their retirement. Managing exposure to investment risk is complex because by reducing investment risk assumed during the accumulation phase in an attempt to ensure a more stable outcome at retirement, the member increases their risk of retiring with less than expected. The question facing Boards of Trustees of DC pension funds is therefore how to ensure a more certain outcome for members while at the same time ensuring that the expected outcome is as large as possible given a certain contribution rate. SBPs is one option available to DC pension funds to manage members' exposure to investment risk. This study sets out to assess the effectiveness of these products.

The remainder of this thesis is structured as follows. Chapter 2 provides an overview of the international and South African approaches being taken to manage the investment risk that members of DC funds are exposed to. Chapter 3 sets out in detail the methodology applied in investigating the objectives of this study. The results from the modelling process is set out in chapter 4. The outcome from a sensitivity analysis done is also covered in this chapter. The summarised findings and conclusions of the research and areas for further research are presented in chapter 5.

2 Literature review

2.1 Introduction

This literature review starts by considering the widespread conversion from DB to DC pension funds, which led to a change in the risk exposure many members saving for retirement face. The objectives of a DC pension fund member is discussed in section 2.3 followed by a discussion on utility theory. Section 2.5 discusses how investment risk is defined in this study. It also sets out the desirable properties of a risk measure. A high-level overview of the risk mitigation strategies used in the management of investment risk in DC funds is provided in section 2.6. This is followed by a more detailed description of approaches taken both internationally and in South Africa, covered in sections 2.7 and 2.8 respectively. Section 2.9 provides a detailed outlay of the workings and features of SBPs being sold in South Africa.

2.2 The move from DB to DC structures

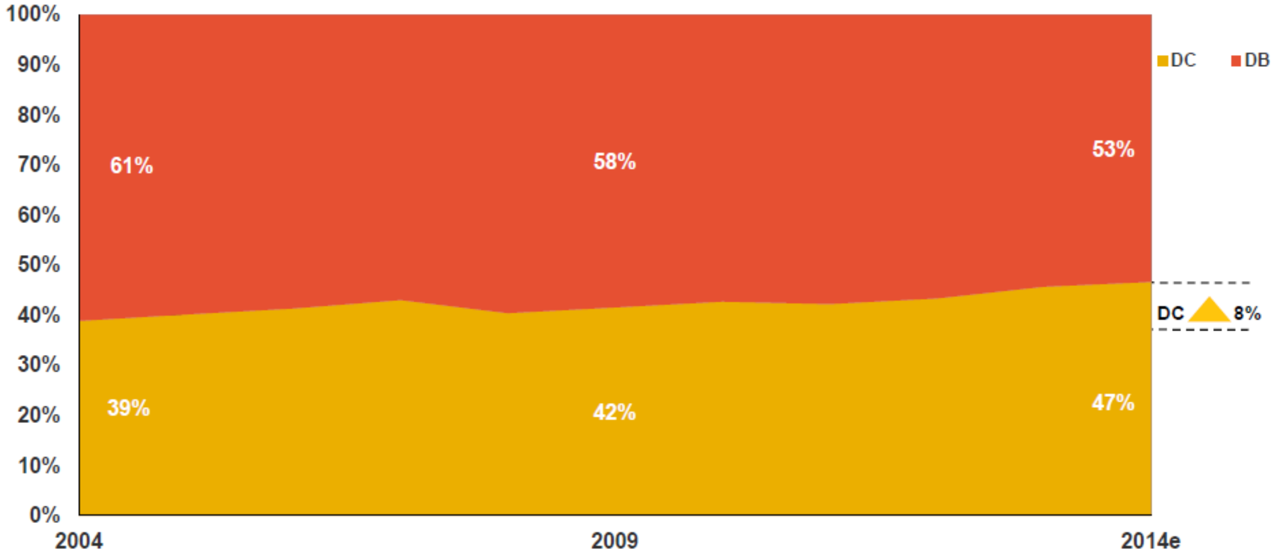
In July 2012, the then United Kingdom (UK) pensions minister, Steve Webb said (Department for Work and Pensions, 2012),

> I am convinced people have a huge appetite for certainty about their pension savings, and this demand will drive the shape of pension provision in the future. I want industry to innovate and think hard about this.

This was said at the dawn of automatic enrollment in the UK which makes it mandatory for employers to make contributions on behalf of certain employees and was thus expected to lead to increased flows into DC-type funds.

The global shift

Globally a shift from DB to DC arrangements has been and continues to be observed as reflected in figure 3.



Note: Data pertaining to seven countries namely, Australia, Canada, Japan, the Netherlands, Switzerland, the UK and the US are included in the above graph.
Source: Towers Watson (2015)

Figure 3: DB/DC asset split: Change over the ten year period up to 2014

The reasons for this shift include (Broadbent et al., 2006):

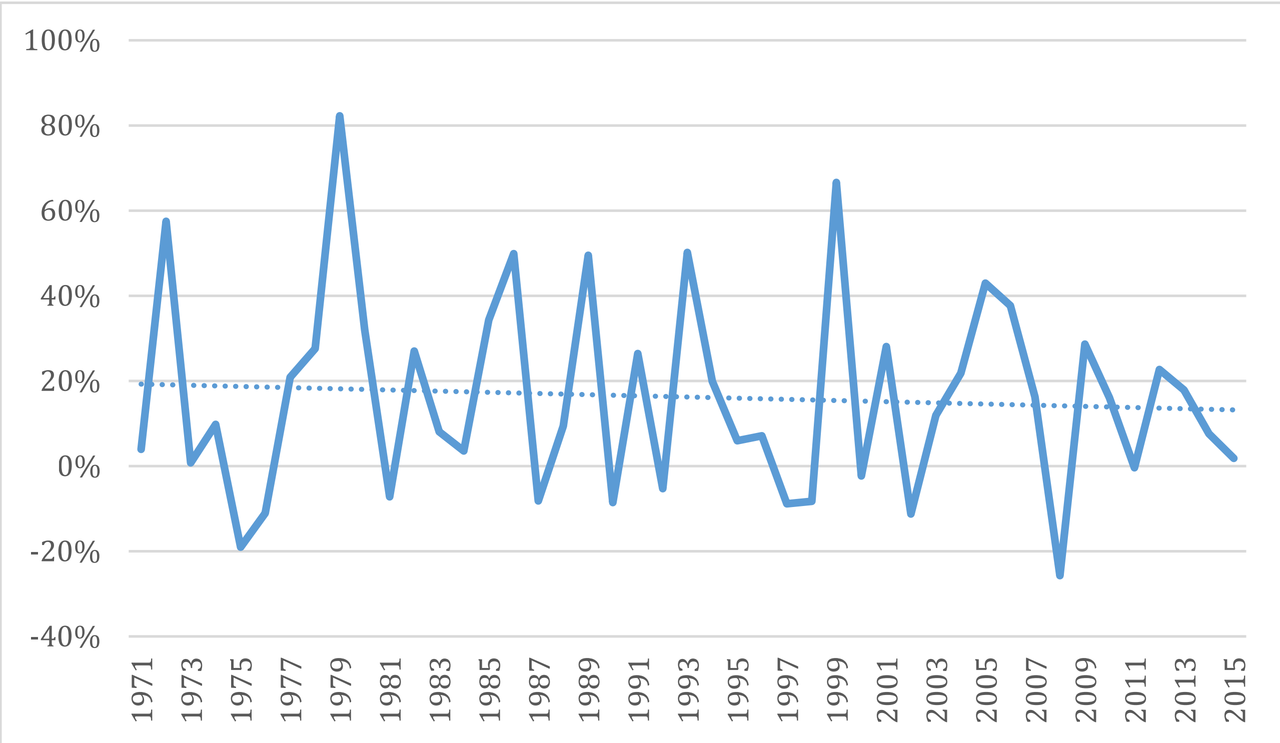
- Worker mobility: Leaving one place of employment will necessitate exiting the employer's pension fund. Members exiting a DB fund is subject to "accrual risk" since the calculation of benefits favours members who have had a longer tenure in the fund. This is not the case in a DC fund where the member's exit value is calculated merely as the sum of their contributions plus allocated returns.

- Regulatory burden: Changes in accounting standards resulted in employers having to incorporate their pension fund liability and changes thereof in their financial statements. Pension funds therefore affect employers' reported profits and require sufficient assets to be held to back the liability. There has been a move to a more market based approach in setting valuation assumptions, which affects the volatility of pension funds' solvency levels.

- Increased cost: A fall in long-term interest rates as well as improved longevity has led to increased costs of pension benefits provided by employers.

The South African shift

One of the reasons why about 80% of South African DB fund members opted to convert to a DC structure in the 1980s and 1990s, as discussed in section 1.2, was because of the high real returns earned in the South African economy as the Johannesburg Stock Exchange (JSE) entered a bull market (Andrew, 2004). Figure 4 illustrates the returns earned on the JSE All Share Index over about a 45 year period (Firer and McLeod, 1999).



Source: Firer and McLeod (1999); iNet BFA

Figure 4: Annual change in JSE All Share Index values

Andrew (1994) estimates that real returns of 4.25% p.a. were being passed on to DC fund members at the time of the conversions and the risk of poor investment performance seemed remote at the time. Employers in turn were motivated by the fact that investment and other risks would be transferred to the employees in an environment that was politically uncertain and in which AIDS was expected to lead to significant increases in insurance costs. It was the increased volatility of returns observed towards the end of the 1990s that started to awaken members to the level of investment risk they had actually accepted (Andrew, 2004).

2.3 The DC pension fund and its members

2.3.1 Objective of a DC pension fund member

The pure time value of money principle states that when individuals decide to put away funds today and sacrifice present consumption for future consumption, they expect to receive compensation in the form of interest from the borrowers of those funds (Reilly and Brown, 2003). In addition, should there be any uncertainty associated with the payout of the investment in future, the investor requires additional returns in the form of a risk premium.

Considering that a DC pension fund is one mechanism through which individuals can save, pension fund members would have this same requirement to earn a return on their savings that is in line with the risk assumed.

It is assumed that when making a choice between alternative investment options, members act rationally and will always prefer more to less. However, the outcome that can be obtained is constrained by the level of risk the members is willing to take on.

Markowitz's portfolio theory states that a portfolio can be considered as efficient if no other portfolio of assets offers a higher expected return for the same level of risk, and *vice versa* (Reilly and Brown, 2003). From an investor's point of view an optimal portfolio is one that is able to generate the highest possible level of returns subject to the constraint of not exceeding the level of risk the investor is willing to assume. The objective of the investor in this study, who in this case is a DC pension fund member, is taken as the desire to maximize savings at retirement subject to the risk appetite of the member.

2.3.2 Behavioural finance

Behavioural research challenges the notion that individuals are rational, autonomous and apply unbiased judgment when making decisions pertaining to their retirement savings (Mitchell and Utkus, 2003). The outcome of conventional expected utility models seem to fail to explain the behaviour exhibited by individuals who are saving for their own retirement. The following are some examples of these behavioural biases.

Pension fund members have been found to be follow 'the path of least resistance' (Choi et al., 2002, p. 68). This passive approach to decision-making implies that employers and trustees can have a significant influence (and need to be aware of this) on the savings members will retire with. For example if the path of least resistance is the path requiring no action to be taken at all, members following this path can be expected to select the default investment strategy decided on by the trustees.

As members get closer to retirement age, inertia and procrastination may lead to them not taking responsibility for actively managing their retirement savings and for reducing their exposure to more risky assets (Mitchell and Utkus, 2003). Overconfidence may also lead to members being overoptimistic about the future and therefore not be willing to reduce their risk exposure as they approach retirement. The outcome of such an approach may be much lower than expected if markets take a downturn not thought possible by the member.

Regret can be defined as 'the disutility of not having chosen the ex post optimal alternative' (Braun and Muermann, 2004, p. 738). A DC fund member's asset allocation decision will be influenced by the possible regret he could feel if the decision taken turns out to have been suboptimal. Through applying expected utility theory, Muermann et al. (2006) illustrate how differences in individual's averseness to regret will influence their portfolio selection decision and thus outcome at retirement.

Where members exhibit an aversion to risk, they are likely to allocate their retirement savings conservatively, leading to a potentially lower outcome at retirement (Bajtelsmit and VanDerhei, 1997). The 2009 UK Department of Work and Pensions survey towards attitudes into pensions in the UK found that 68% of respondents said that for the sake of the safety of their pensions savings, they would rather invest more conservatively even if a more risky strategy could earn them more (Department for Work and Pensions, 2012).

2.4 Utility theory

2.4.1 Background

Utility theory allows for the transformation of a series of possible outcomes into a series of utility values attached to those outcomes. Expected utility theory states that 'the decision maker chooses between risky or uncertain prospects by comparing their expected utility values' (Mongin, 1997, p. 1). Therefore, the choice made between alternative prospects will not depend on the expected outcome itself, but rather on the expected level of utility attached to that outcome.

A member's expected utility can be calculated as the sum of the utilities associated with the possible outcomes, multiplied by the outcome's probability of occurrence (Mongin, 1997). Alternatives can then be ranked by comparing the expected utility values associated with those alternatives.

A utility value in this context, reflects the level of utility a member attaches to a unit of savings at retirement.

2.4.2 Utility theory as a normative theory

Thomson (2003a) provides a detailed defense for the use of expected utility theory as a normative theory when making recommendations with regard to an appropriate investment strategy in a DC pension fund. A normative theory aims to describe the behaviour of a rational individual who subscribes to the axioms underlying that theory. If the individual does not subscribe to the axioms underlying it, then the theory will not apply to him. A descriptive theory on the other hand aims to describe observed behaviour.

Numerous difficulties are faced in developing descriptive theories. For example, when conducting field experiments, individual behaviour may change simply because these individuals are aware that they are not facing a real life choice, but an artificially created scenario. Normative theories on the other hand are easier to derive, because its usefulness and the level of validity depends on the extent to which individuals prescribe to the axioms underlying the theory.

Applying expected utility theory in this study allows us to incorporate attitudes towards risk in determining what an effective strategy for a pension fund member is. Applying it as a normative theory will however not allow the conclusions to be extrapolated to the whole population of pension fund members in SA. This is important to remember when interpreting the results.

Briefly, the axioms of expected utility theory as defined by von Neumann-Morgenstern and assumed in this study are as follows (Thomson, 2003a):

1. Completeness: An individual is always able to express at least a weak preference when faced with a choice between alternatives.

2. Transitivity: Preferences can be ranked such that if A is preferred to B, and B is preferred to C, then A is preferred to C.

3. Continuity: If B is preferred to A, and C is preferred even more, no value of probability $p \in (0, 1)$ exists such that B is preferred more than p of A and $(1 - p)$ of C.

4. Reduction of compounding: What is importance to an individual is the probability that an outcome will occur. The intermediary steps taken to get to that outcome is irrelevant to them.

5. Substitutability: If a decision maker is indifferent between A and B, and also between A and C, then the decision maker will also be indifferent between B and C.

6. Monotonicity: A is preferred to B if it has a greater probability of having a better outcome than B, or has a lesser probability of receiving a worse outcome.

2.4.3 Utility functions

A utility function in this context reflects the level of satisfaction that a member attaches to various outcomes at retirement. An assumption needs to be made regarding the shape of a member's utility function. Let U be a a utility function on some random variable, S. The following are some of the most commonly used utility functions (Luenberger, 1998):

1. Exponential: $U(S) = -e^{-aS}$ for some parameter value $a > 0$

2. Logarithmic: $U(S) = ln(S)$

3. Power: $U(S) = bS^b$ for some parameter value $b \leq 1$, $b \neq 0$

4. Quadratic: $U(S) = S - cS^2$ for some parameter value $c > 0$

We are able to scale a utility function, $U(S)$, by multiplying it with a constant, $d > 0$, and/or to translate it by adding a constant, e, such that $V(S) = dU(S) + e$ if and only if the ratios of the second derivatives divided by the first derivative are equal, i.e. $\frac{V"(S)}{V'(S)} = \frac{U"(S)}{U'(S)}$. Such a utility function is said to isoelastic (Norstad, 1999).

Since an isoelsatic utility function is scalable and transformable the utility an individual attaches to a particular outcome is relative and not dependent on his initial wealth. The log and power utility functions above are examples of isoelastic utility functions. Individuals with such utility functions are said to have relative risk aversion. Where the level of utility attached to a particular outcome depends on the initial wealth, individuals are said to have absolute risk aversion. The exponential function above is an example of such a utility function.

Pézier and Scheller (2011) assumes the risk attitude of members of a pension fund can be characterized by an exponential utility function with a coefficient of risk tolerance, $a = 0.025$. Making the assumption that members are concerned with the income they will be receiving in retirement and not just the lump sum available, Cairns et al. (2006) defined a terminal utility function, $U(S(T), Y(T))$. This utility function takes into account both the retirement savings at the retirement date T, defined as $S(T)$, and the final salary received at the point of retirement, defined as $Y(T)$. $U(S(T), Y(T))$ is defined by applying power and log utility functions such that:

$$U(S(T), Y(T)) = \begin{cases} \frac{1}{\gamma}\left(\frac{S(T)}{Y(T)}\right)^{\gamma} & , \gamma < 1, \gamma \neq 0 \\ log\left(\frac{S(T)}{Y(T)}\right) & , \gamma = 0 \end{cases} \tag{1}$$

where γ is an assumed risk appetite measure.

Jensen and Sørensen (2001), Deelstra et al. (2003), Gollier (2008) and Cui et al. (2011) assume a power utility function. When γ is a measure of risk aversion this is called a von Neumann-Morgenstern utility function. The utility at time T is calculated as follows:

$$U(S(T)) = \begin{cases} \frac{S(T)^{1-\gamma}}{1-\gamma} & , \gamma \neq 1 \\ log\left(S(T)\right) & , \gamma = 1 \end{cases} \tag{2}$$

The larger the value of γ, the more conservative or risk averse an individual is assumed to be. Cui et al. (2011) and Gollier (2008) assume $\gamma = 5$, while Deelstra et al. (2003) assigned γ a value of -1. Jensen and Sørensen (2001) show results for the risk parameter ranging between between 0.25 and 4. No motivation could be found for the choice of values assigned to γ in those studies. Assuming γ is constant and given that the power and log utility functions are reflective of the utilities of members with relative risk aversion, equation 2 can be classified as a constant relative risk aversion (CRRA) utility function.

2.5 Risk measurement

2.5.1 Definition of investment risk

Investment risk for any investor arises from the uncertainty of payments from, or stated otherwise, the returns earned on an investment (Reilly and Brown, 2003). During the accumulation phase in a pension fund, investment risk arises from the uncertainty of the performance of the investment vehicles where the assets underlying the pension fund are invested in. This uncertainty experienced during the accumulation phase leads to the following two risks a member faces at retirement, namely:

- The volatility of the savings at retirement

  A wide range of potential outcomes could be observed at retirement, creating uncertainty as to what the final outcome will be. The greater the uncertainty of returns observed during the accumulation phase from the choice of assets invested in, the greater the range of possible outcomes at retirement will be.

- The level of savings at retirement

Vigna and Haberman (2001, p. 233) defines investment risk in the context of defined contribution pension funds as,

> the risk that poor investment performance during the active membership leads to a lower than expected accumulated fund.

Therefore included in the uncertainty of the outcome is the risk that the value of a member's savings could turn out to be lower than anticipated given the performance of the chosen investment vehicle during the accumulation phase.

The definition of investment risk in this research encompasses all of these elements. The focus when measuring investment risk will however be on the outcome at retirement rather than on points during the accumulation phase. Bodie et al. (1988), Artzner et al. (1999) and Guillén et al. (2013) take a similar approach.

The following are some reasons for taking this approach:

- Guillén et al. (2013) argues that focusing only on the annual change in the value of the fund fails to take into account the long-term horizon or objective of a pension fund and its members.

- Focusing on the final outcome allows one to incorporate additional factors that affect the outcome at retirement such as changes in the investment strategy or level of contributions made into the fund (Guillén et al., 2013).

- Artzner et al. (1999) argues that because risk is related to the variability of the final outcome, it is better to focus on future outcomes only. This is consistent with the fact that although pension fund members are constantly exposed to investment risk while members of a fund, the impact on consumption will only be felt once they reach retirement.

- Due to the 'portfolio size effect', as Basu and Drew (2007) referred to it, the effect of experiencing poor returns in later years will be worse than if it was experienced in earlier years. This is because a member's pool of savings is expected to grow larger as they approach retirement due to the following,

  - investment returns are earned over time;

  - the number of contributions that have been made increases with time; and
  - salary growth is likely to be observed during the accumulation phase.

  Focusing on the volatility of annual returns rather than the final outcome on retirement will not capture this effect.

- If it is assumed that in most instances members retire when they reach the retirement age of their fund, the date on which they will exit the fund is fixed in advance. Focusing on the outcome on that retirement date will encapsulate the added risk arising from members being obliged to exit at times that may be deemed inappropriate.

Thomson (2003b) provides the following motivation for focusing on the exit value on retirement rather than the level of annuity a member would be able to purchase on retirement. He provides this motivation within the context of applying utility theory to determine an appropriate investment strategy for a member of a DC fund.

- One cannot assume that a member will annuitise his/her total accumulated savings on retirement. Where the option exists to take part of the benefit in cash, members may be more concerned with the size of the pool of savings than the income it can purchase. This might be the case where for example the bequest motive is strong, members still have significant financial obligations outstanding on retirement or where they want to invest their savings in unconventional assets such as cattle.

- If it can be argued that long-term interest rates are fairly stable and basically fixed, it should make no difference whether one considers the utility of the savings at retirement or the utility of an annuity.

The way in which investment risk, or what Vigna and Haberman (2001) referred to as 'a lower than expected accumulated fund' is measured, is discussed next.

2.5.2 Coherency of risk measures

In order to measure risk, some statistical distribution is applied to the outcomes from a random variable. A risk measure then aims to assess the riskiness of the random variable by considering a feature of the distribution of that random variable (Sweeting, 2012).

Suppose we have two loss random variables, X and Y, and that we define $\mathcal{H}$ as our risk measure. Calculating the risk measure for X and Y, we then get $\mathcal{H}(X)$ and $\mathcal{H}(Y)$. It is desirable that any risk measure used be coherent. A risk measure is said to be coherent if it satisfies the following four properties (Sweeting, 2012; Hardy, 2006),

1. Monotonicity: If X is always smaller than Y, then the risk measure of X should always be smaller than that of Y. This can be expressed as follows. If $X \leq Y$, then $\mathcal{H}(X) \leq \mathcal{H}(Y)$.

2. Subadditivity: The risk should not be reduced when the portfolio is split in two parts, i.e. $\mathcal{H}(X + Y) \leq \mathcal{H}(X) + \mathcal{H}(Y)$. This property encapsulates the requirement that diversification should reduce risk.

3. Positive homogeneity: The unit in which the loss random variable is expressed should not change the outcome of the risk measure. I.e. for any non-negative, non-random value λ, the risk measure, $\mathcal{H}(X)$, scales linearly such that $\mathcal{H}(\lambda X) = \lambda \mathcal{H}(X)$.

4. Translation invariance: If a constant amount is added to a loss random variable, the risk measure should increase by that same constant amount. If any non-random number c, is added to the random variable, X, the risk measure should increase exactly by c such that $\mathcal{H}(X + c) = \mathcal{H}(X) + c$.

The coherency of the risk measures used in this study will not be tested for.

2.5.3 Types of risk measures

Two of the measures used by Eason et al. (2013) to assess the performance of a strategy calculated on the savings at retirement, was the probability of a capital loss and the probability of beating a risk-free rate of return. Guillén et al. (2013) used value at risk (VaR), conditional tail expectation (CTE) and the price of issuing a put option on the remaining savings at the final age as their risk measures.

Sweeting (2012) lists the following possible risk measures. For each measure a brief description is provided as well as an indication as to whether or not the measure is used in this study. In discussing these risk measures, it is assumed that the random variable on which a risk measure is to be calculated is a loss random variable, i.e. that a higher outcome of the variable represents a worst outcome.

- Standard deviation

  This measures the volatility of the outcomes of a random variable. Standard deviation as a risk measure is only useful if the underlying distribution of the loss random variable is symmetrical (Sweeting, 2012). Because of the guarantee offered on an SBP, the distribution of the outcome at retirement is not expected to be symmetrical. This is indeed confirmed in figure 21. Also, standard deviation does not incorporate the second part of the definition of investment risk discussed in section 2.5.1, namely the risk of observing a lower than anticipated outcome. It only measures volatility. This measure is therefore not used in this study.

- VaR

  The VaR is the maximum amount that the loss is not expected to exceed within a specific confidence level, $(1-\alpha)$, where $0 \leq \alpha \leq 1$. The outcome of the random variable is therefore expected to be less than the VaR with an α probability of certainty. Artzner et al. (1999) shows that VaR is not a coherent risk measure because it doesn't satisfy the subadditivity property. Danielsson et al. (2005) however showed that the failure of asset distributions to satisfy this property is the exception rather than the rule. They show that this property, specifically in the tails of the distribution, only fails if the tails are exceptionally fat. VaR is also still a widely popular measure of risk. VaR will therefore be used as a risk measure in this study.

- Probability of ruin

  The probability of ruin is simply the inverse of VaR, i.e. instead of starting with the desired probability level at which the maximum loss is expected to be observed, it starts with the amount at which financial ruin is observed, and then determines the probability of that outcome being achieved. Applying this risk measure in this context would be not be sensible because it would require the lowest level of retirement savings at which financial ruin will be experienced, to be determined.

- Tail VaR

  The tail VaR measures the expected loss in the worst $(1-\alpha)$ part of the distribution of the random variable (Hardy, 2006). Tail VaR, also referred to as CTE, will be used as a risk measure in this study. It is a widely popular risk measure and satisfies the definition of coherency. Due to the CTE reflecting the mean loss, it is a more conservative measure of risk than VaR (Hardy, 2006).

- Expected shortfall

  This risk measure multiplies the losses in the tail of the distribution, i.e. the worst $(1-\alpha)$ part of the distribution of the random variable, with the probability of observing that loss (Sweeting, 2012). So, where the CTE provides the expected shortfall in the *tail*, this risk measure calculates the expected shortfall across the *whole distribution* of the random variable. Since this measure is so similar to the tail VaR, it will not be used in this study.

In addition to the risk measures discussed by Sweeting (2012), there is another class of risk measures called Lower Partial Moments (LPMs). LPMs only consider the negative deviations from a certain target outcome. Any outcome observed above the target outcome is considered to be desirable by the investor and therefore does not contribute to the investor's risk exposure (Unser, 2000). For a continuous distribution of outcomes, x, where $x\epsilon[-\infty;t]$, each LPM can be calculated as follows,

$$LPM_n^t(x) = \int_{-\infty}^{t} (t-x)^n dF(x) \tag{3}$$

where t represents the target outcome, x is the outcomes from its probability distribution with density function $f(x)$, and n reflects the weight an investor places on deviations from t (Unser, 2000). The difficulty in using LPMs is that assumptions are required as to what appropriate values for the average member of a DC pension fund are for the target outcome, t, as well as the weighting coefficients (Unser, 2000). Assigning values to t and n will introduce an additional element of subjectivity and LPMs are therefore not considered further.

2.6 Risk mitigation strategies

Investment risk in a DC pension fund can broadly be managed in the following ways (The Pensions Authority, 2015):

1. Asset allocation decision

   This relates to the setting of the investment strategy. Different asset classes have varying exposures to risk factors and are imperfectly correlated. By allocating funds across various asset classes, the benefits of diversification can be realised and the overall riskiness of the portfolio reduced (Reilly and Brown, 2003).

2. Asset selection decision within asset classes

   By investing in a diversified group of securities within an asset class, the risk that is unique to an individual security in the portfolio, i.e. the unsystematic risk, can be diversified away.

3. Derivatives and other hedging strategies

   Regulation 28 of the Pension Funds Act of 1956 makes provision for pension funds to invest in derivative instruments with a restriction on investing in such a way that more can be lost than initially invested (National Treasury of the Republic of South Africa, 2011).

Particular examples of the above strategies and alternative approaches to managing investment risk in DC pension funds, abroad and in South Africa, are covered in the following two sections, i.e. sections 2.7 and 2.8.

2.7 International approaches to the management of investment risk in DC funds

Intergenerational smoothing and the factors influencing its effectiveness, as well as two current products applying this principle, is discussed in section 2.7.1. Section 2.7.2 considers products with explicit guarantees. Some of the lessons learned from the past failures related to guaranteed products are also discussed. Section 2.7.3 discusses different types of lifestyle strategies.

2.7.1 Intergenerational smoothing

Intergenerational risk-sharing within a DC fund takes place when a fund uses its reserves to smooth returns earned on the underlying assets before passing it on to its members. By spreading risks across all members, including retirees, and current and future employees, time diversification can be achieved. This creates risk capacity and an opportunity to exploit equity risk premiums through increasing the allocation to more risky asset classes (Gollier, 2008; Cui et al., 2011).

Van der Lecq and Van der Wurff (2011) found that the level of contributions into a DC scheme in the Netherlands would need to be between 1.65 and 2.5 times more than the contributions into a DB scheme to achieve a similar outcome at retirement within a 97.5% confidence interval. They attributed this huge difference to the fact that DB schemes have institutionalised risk sharing whereas in a DC fund each member has to bear the volatility of investment returns themselves.

The Pension Scheme Act 2015 in the UK, which received royal assent in March 2015 aims to enable greater risk sharing between employers, members and third parties (The UK government, 2014). One way in which this is done is by making allowance for defined ambition schemes in the act (discussed later in this section).

Numerous studies have found intergenerational risk-sharing in funded pension schemes to be welfare enhancing (Cui et al., 2011; Gollier, 2008; Westerhout, 2011). Gollier (2008) applied a von Neumann-Morgenstern utility function of the form,

$$u(z) = \frac{z^{(1-\gamma)}}{1-\gamma} \tag{4}$$

where γ is a measure of the level of risk aversion of a pension fund member.

The certainty equivalent of a random variable, S, measuring savings at retirement can be defined as the level of certain savings, CE, that has a utility level equal to the expected utility of S (Luenberger, 1998). The certainty equivalent retirement savings, CE, can be calculated by solving,

$$U(CE) = E[U(S)] \tag{5}$$

Gollier (2008) found an improvement in the certainty equivalent yield rate on savings from

3.23% for a scheme where individuals save in autarky, i.e. independently, to 4.05% when intergenerational risk-sharing was introduced.

The following factors impact and could hamper the benefits of intergenerational smoothing experienced:

- The extent to which participation is mandatory

  Beetsma et al. (2012) found that the benefits of intergenerational risk-sharing are only likely to be realised if participation in the scheme is mandatory. Where participation is voluntary, younger members are not likely to be willing to join a scheme which finds itself in a poor financial position. Westerhout (2011) calls this a time inconsistency problem. Gollier (2008, p.1464) likewise states that 'current generations are more reluctant to build the fund reserves than to consume the surpluses of the system'. Westerhout (2011) notes that a change in the risk averseness of future generations of members may also impact the value they attach to intergenerational risk-sharing and therefore their willingness to participate in a scheme.

- Homogeneity of a fund

  This speaks to the extent to which the distribution of returns between pension fund members is fair, equitable and in line with reasonable expectations. Consider a case where market conditions have changed significantly. The question needs to be answered as to whether it would be fair to expect of younger members in a non-voluntary scheme to contribute to the benefits paid to pensioners if market conditions have changed to such an extent that younger members cannot reasonably expect to receive the same level of support when they are older.

- The setting of surplus distribution rules

  One of the criticisms in the Lord Penrose (2004) report on the failure of Equitable life (which is discussed in more detail in section 2.7.2) is the lack of a surplus distribution policy according to which distribution decisions were made. The report however also acknowledges that determining what an equitable level of reserves to keep is, in order to reduce the risk of members being over or underpaid on maturity, is no easy task.

- The impact of solvency restrictions

  Restrictions on the extent to which the funding level of a scheme may be negative, i.e. the market value of the fund's assets being less than it's liabilities, restricts the extent to which smoothing can take place. During periods of under-performance, the fund would be limited in its ability to transfer risk to future cohorts of members. Gollier (2008) found a drop in the certainty equivalent yield rate from the 4.05% to 3.76% after allowing for a solvency requirement that the market value of a fund's assets must be larger than the value of its liabilities.

Collective Defined Contribution Plans (CDCs)

CDCs (also known as defined ambition or target benefit schemes) are currently available in the Netherlands and Canada [Slaughter and May]. In the UK, the Pension Scheme Act 2015 has in addition to the DB and DC definitions of a pension scheme made specific allowance for defined ambition schemes (The UK government, 2014). CDCs broadly have the following characteristics (Aon Hewitt, 2013a):

- Employers contribute a fixed percentage of salary, similar to the case for DC funds.
- Rather than keep a separate account for each member, the assets are pooled and invested on an aggregate basis. No individual member investment strategy decisions are thus needed.
- Benefits are expressed in terms of the level of pension a member can expect to receive during retirement rather than in terms of a capital value of savings.
- At retirement a member will receive an annuity that is paid by the fund. The pension received during retirement will depend on the funding level and is not guaranteed.
- Longevity and investment risks are borne by the employees and retirees as a group.

In the event of funding losses, either employee contribution rates (but not employer rates) can be adjusted, or bonuses (and ultimately even benefits) will be reduced.

Because pensions are paid from the plan during retirement and no annuity needs to be purchased on retirement, and because the income during retirement is not dependent on the individual's accumulated savings at retirement, a larger portion of assets can be retained in growth assets as a member approaches retirement. This is expected to lead to a superior outcome for members compared to a DC fund where members are concerned with managing their risk exposure as they approach retirement (Aon Hewitt, 2013b).

TimePension

TimePension is a product that was launched by a Danish insurer in 2002 (Guillén et al., 2013). This product is a savings vehicle for pension fund assets. The product maintains a 60% allocation to equity during the pre- and post-retirement phases. Policyholders are allocated returns via a return smoothing mechanism that has been explicitly defined in mathematical terms (Petcher, 2013). Instead of being allocated the same returns as that earned on the investment of the assets underlying the product, the returns earned by policyholders consist of two parts (Guillén et al., 2006):

1. A well defined reference policy interest rate taken as a long-term government bond yield.

2. After allowing for the rate in (1), a certain proportion of the difference between the market value of assets underlying the product, which includes the returns earned on the investment of the underlying assets, and the amount due to the policyholder, is allocated to the policyholder's account. The proportion of the surplus allocated can be seen as the smoothing parameter. The greater the proportion allocated, the less the degree of smoothing taking place, and *vice versa*.

The product design allows for the return-smoothing principle of with-profit products to be maintained, while addressing some of the drawbacks of with-profit products, namely (Guillén et al., 2006),

- The lack of transparency: The application of an explicitly defined bonus formula implies an improvement in the level of transparency.

- The onerousness of guarantees: The smoothing parameter discussed above allows for negative returns to be allocated to policyholders. This will be the case when the market value of the underlying assets is less than the policyholder liability. This removes the onerousness of the guarantees embedded in traditional with-profit policies and reduces the capital burden imposed by regulatory requirements on the insurer.

2.7.2 Products with explicit guarantees

Eason et al. (2013) investigated whether there is a place in the UK DC market for guaranteed savings products. Their findings provide details on the methods used for managing investment risk in pension funds in a number of different countries. In summary, their investigation found that a return-of-contribution guarantee is mandatory in a number of countries including Japan and Switzerland. Where a return guarantee is offered, they found that savings tend to be invested more cautiously and that members often only receive the guaranteed rate.

Pézier and Scheller (2011) argue that a conflict of interest exists between the sponsor or product provider, and fund members in terms of the investment strategy the sponsor/product provider is likely to implement where a guarantee is offered, impacting the returns members will receive. Jensen and Sørensen (2001) argue that where offering the guarantee places a restriction on the permissible investment strategies for the fund, fund members pay an extra implicit cost, in addition the the explicit charge for the guarantee, equal to returns forgone from following a lower risk investment strategy. Members may not be aware that they are paying this implicit cost.

Muermann et al. (2006) investigated the impact that regret bias has on the value that members place on guarantees in DC funds. Guarantees, through the protection that they offer, can reduce the regret risk for a member, but it comes at a cost. Muermann et al. (2006) found that the value placed on a guarantee by a regret-averse member is higher than for a risk-averse member when the allocation to more risky assets is higher. Conversely when the allocation to more risky assets is lower, a lower value is placed on the value of the guarantee by the regret-averse investor. It is thus not only the (implicit and explicit) costs of the guarantee or the fund's exposure to risky assets that influence how valuable a guarantee is, but also the extent to which a member is subject to regret avoidance bias. Guarantees can thus also be seen as a regret risk mitigation tool.

With-profit policies

With-profit policies operate on a pooled basis and offer the benefit of both the smoothing of returns and an explicit guarantee. Returns earned on the pooled contributions are allocated to policyholders in a smoothed manner over time (Guillén et al., 2006). A guarantee on the level of benefits payable at maturity is provided which grows as returns are allocated. The insurance company issuing the policy retains the freedom to make decisions related to certain product features such as the investment strategy and the declaration of bonuses (Eason et al., 2013).

With-profit policies were popular in the UK in the 1980s when equity markets were buoyant (Jenkins and Beresford, 2016). However, during the 1990s the guarantees became more valuable to policyholders as interest rates fell and markets deteriorated, and the risks insurers were exposed to became more and more apparent. Many with-profits funds closed to new business. A notable event was the failure of one of the biggest mutual life assurers, Equitable Life Assurance Society (Equitable Life) in 2000 (Pollock, 2009). Following this failure, public confidence in with-profit policies was lost. The 36 with-profit funds open to new business in 1997 had consolidated into 19 funds by 2016, with only six of these funds remaining open to new business (Jenkins and Beresford, 2016).

The following are some lessons from the investigation performed by Lord Penrose into Equitable Life's failure which are worth noting in this context (Lord Penrose, 2004).

- The absence of bonus declaration and smoothing policies

  Without a functional bonus distribution policy, it is difficult to determine an appropriate target range for bonus declarations to ensure policyholders are not receiving more/less than is reasonably due to them. There needs to be a mechanism in place to ensure alignment between the asset shares (reflecting the duration of a policy and returns earned by the fund), and the value of benefits due to policyholders, taking into consideration their reasonable expectations.

- Opportunity for unfair or unequitable intergenerational cross-subsidy

  Significant changes in market conditions undermine the homogeneity of members being pooled together. This makes it difficult to ensure an equitable and fair distribution of surplus between different cohorts of policyholders without being perceived as biased in such a distribution. One way of dealing with this is to start a new bonus series in such an event.

Starting a new bonus series might be correct from a fairness point of view, but does have its drawbacks:

- If clients withdraw when the smoothing reserve is negative, i.e. they withdraw more than their asset share, the remaining policyholders will bear the cost of this since there are no new inflows to 'neutralise' the cost.

- Where a fund aims to maintain a certain minimum level of smoothing reserves, as the number of clients dwindle to a minimum, the question arises as to how the remaining reserves should be distributed. Would it be fair to allocate all of it to the remaining handful of policyholders?

Variable annuities

Variable annuities are unit-linked policies sold by insurance companies often used as savings vehicles for retirement or for providing a regular income stream during retirement (Ledlie et al., 2008). An important feature of variable annuities is the range of guarantees that are available as rider benefits. The four main types of guarantees are (Ledlie et al., 2008):

1. Guaranteed Minimum Death Benefits (GMDB): A variety of rules such as a return of principle invested, a return of principle plus a minimum rate of return, etc. are applied to determine the benefit payable on death.

2. Guaranteed Minimum Accumulation Benefits (GMAB): This is similar to GMDB, but with the guarantee applying on certain events such as a policy anniversary date. The level of the guarantee could again be for example a minimum rate of return.

3. Guaranteed Minimum Income Benefits (GMIB): Regular minimum income payments equal to for example an amount fixed in absolute terms or a percentage of premiums invested, are paid during retirement. No fund balance is payable on death.

4. Guaranteed Minimum Withdrawal Benefits (GMWB): An income is provided during retirement with a guarantee of for example returning the principle through paying a certain level of pension for a certain number of years regardless of investment conditions. On death the remaining fund balance, if any, is paid out.

Variable annuities have been successful particularly in the USA and Japan (Ledlie et al., 2008). Increased volumes have also been sold in the UK in response to the gap left by the decline in the popularity of with-profit policies (Eason et al., 2013). The Financial Industry Regulatory Authority, a self-regulatory organisation in the USA, has warned investors to beware of their long-term nature and the penalties associated with an early surrender of such a policy, as well as their product charges, which is estimated to reach up to 2% p.a. of the annuity value (Financial Industry Regulatory Authority, 2012).

One of the investment strategies used to manage the guarantees provided by variable annuities is a target volatility strategy (Chew, 2011). This strategy switches between risky and risky-free assets by considering the short-term realised volatility and comparing that to a volatility target (Eason et al., 2013). The allocation of assets will constantly be rebalanced to get the weighted volatility of the portfolio back to the target volatility level. The thinking behind this strategy is that by switching out of risky assets when its short-term volatility is higher than the target level, the portfolio is protected from a potential crash.

Sharing of risk with the product provider

Germany's Pensionskassen are retirement schemes typically offered by insurance companies that provide a guarantee on the rate of return earned. Every year contributions are accumulated at the higher of either a set guaranteed minimum rate of return, or a portion of the fund's annual return (Pézier and Scheller, 2011). In the latter instance, the insurer will retain the unallocated portion of the returns earned on the assets.

Pézier and Scheller (2011) demonstrate how the conflict of interest that arises in the selection of an investment strategy by these product providers could be reduced and the welfare of fund members improved if,

- Product providers' participation share of investment returns earned (in years where the guarantee doesn't bite) is increased from 10% to at least 25%;
- The guaranteed minimum return is set below the risk free rate; and
- The guarantee, rather than being linked to the annual performance and expressed as a floor on the annual returns that may be allocated to members, is linked to the cumulative returns that have been earned since inception.

Constant proportion portfolio insurance (CPPI)

CPPI is a risk management strategy that is used in savings products offering unit-linked guarantees. It involves allocating funds between risky assets, aimed at generating returns, and risk-free assets, aimed at providing capital protection (Macdonald, 2017). It is similar to deterministic lifestyling, discussed in the next section, but a dynamic allocation strategy is followed that allows for a switch between risky and risk-free assets based on market conditions rather than time to retirement.

The investor sets a guaranteed value which represents the minimum payoff at maturity, e.g. 95% of the initial fund level. The cushion value will in this case then equal 5%. A constant proportion of the cushion value, called the multiplier, is then invested in the risky asset. The multiplier is often the inverse of the maximum expected loss of the risky asset over a set period of time, e.g. a day (Eason et al., 2013). So if for example the maximum expected loss in one day is 10%, then the multiplier will equal 10. Fifty percent, i.e. the cushion value of 5% times the multiplier of 10, of the total funds would then be invested in the risky asset and the remaining 50% in the risk-free asset. If the daily volatility remains the constant, the fund is guaranteed to return at least 95% of the initial fund value.

To prevent constant rebalancing, switching only takes place when the actual exposure to the risky asset differs from the theoretical exposure by a certain number of percentage points

(Hirsa, 2009). It is expected that a greater proportion of assets are allocated to growth assets when markets perform well, whereas more will be allocated to protection assets when the markets perform poorly (Macdonald, 2017).

The downside of this strategy is that if the market suddenly drops and exhibits greater volatility, a switch out of the risky asset and into the risk-free asset will be made. This not only crystallises the loss incurred on the investment in risky assets, but removes the fund's ability to benefit from a rebound (Eason et al., 2013).

Individualised CPPI (iCPPI)

A strategy closely linked to CPPI and that is gaining momentum, is called iCPPI. This strategy operates on the same basis as CPPI except that rather than investors' funds being pooled into one fund, it is offered on an individual basis. This allows the insurer to offer the policyholder a product with an explicit guarantee that is tailored to their needs (Macdonald, 2017).

2.7.3 Lifestage strategies

Deterministic lifestage

Deterministic lifestage investment strategies, also called target-date, lifecycle, or lifestyle strategies, aim to protect a member from significant falls in the equity market close to retirement through progressively moving into more conservative assets as the member approaches retirement (Estrada, 2014). The way in which the portfolio is de-risked is referred to as the 'glidepath'. During the early years savings are invested in growth assets, targeting robust portfolio growth. Should markets exhibit volatility during those years the thinking is that the member has time to recoup losses. As a member approaches the target retirement date the allocation to less risky assets such as bonds and cash is increased over the remaining years to retirement, thereby aiming for preservation of accumulated savings close to retirement (Cairns et al., 2006).

Blows (2016) states that target-date mutual funds are the most popular pension investment style in the USA. Significant growth has been observed, as can be seen in figure 5.

Source: The Economist (2014)

Figure 5: Growth in US target-date mutual funds

It is likely that the simplicity of this strategy both in terms of explaining it to members and implementing it could be a reason for its growing popularity in the US and elsewhere. Also, since switches automatically take place as a member reaches certain age milestones, this strategy is thought to be an appropriate antidote to dealing with the investor inertia (Basu et al., 2011).

This strategy is however not without its drawbacks and lacks strong empirical evidence of being an optimal investment strategy (Blake et al., 2014). Target date funds have two objectives, namely maximising members' real savings and minimising the uncertainty associated with the outcome at retirement (Arnott et al., 2013). Estrada (2014) however questions whether risk can really only be defined as the volatility associated with the outcome at retirement, and asks whether the possibility of observing underperformance in the long run could not seen as posing a bigger risk to the member.

The outcome at retirement depends not only on the investment strategy, but also the frequency and level of contributions made into the fund. A lifestyle strategy proposes that when a member's level of contribution and built up pool of savings is still fairly low they should have a high exposure to risky assets, but when they are at a stage that their level of contributions have possibly grown and pool of assets accumulated is significant, they should move to a more conservative investment strategy. This seems counter-intuitive since by moving out of risky assets when their pool of assets is at its largest, the member could forego a significant amount of returns (Basu et al., 2011). Estrada (2014) found that even though lifestyle funds do offer greater certainty of the outcome than a contrarian fund that becomes more aggressive as retirement is approached, contrarian strategies offer members a greater outcome than lifestyle funds even in periods of poor market performance.

One way in which lifestyle funds can partially deal with this drawback, is that rather than reach the conservative asset allocation on the retirement date, members can opt to have the fund move to this allocation after the target date, which Estrada (2014) refers to as 'glide through' rather than 'glide to' [Financial Industry Regulatory Authority].

Since a lifestyle strategy is linked to a member's planned retirement date, should a member decide to change that date when already in the de-risking phase close to retirement, the effectiveness of the strategy could be compromised (Eason et al., 2013).

Bodie and Treussard (2007) found that the optimality of a target-date type strategy varied greatly when members' level of risk aversion and exposure to human capital risk was considered.

Dynamic lifestage strategies

Although the glidepaths implemented vary between service providers, a commonality they share is the fact that it is the number of years to retirement exclusively that is considered when making a switch between asset classes (Estrada, 2014).

Basu et al. (2011) proposed a dynamic switching strategy where the switch between growth and conservative assets, rather than only being based on a member reaching a certain age, also depends on the cumulative investment performance of the fund relative to a target that depends on the individual's accumulation objective, e.g. a cumulative return of 10% p.a.. A switch to less risky assets as the member approaches retirement is still made, but before each switch is made, a check is done to determine whether the accumulated savings match or exceed the target accumulation. If it does, the exposure to risky assets is reduced. If it does not, the allocation is weighted more, rather than less, heavily towards risky assets.

In their research the target accumulation values are calculated assuming an annual effective rate of return of 10%. The dynamic switching strategy achieves the target wealth level with about 50% certainty and thus has a 50% chance of exceeding this wealth level. The deterministic switching strategy reflected only a 25% probability of achieving an outcome exceeding the target wealth level.

Dahlquist et al. (2016), in their investigation into an optimal default strategy for a DC fund, found that the optimal allocation to equity varies substantially with the equity market's past performance. They thus found that positive gains could be made by accounting for past performance in the switching rule.

2.8 South African approaches to managing investment risk

Antolín et al. (2011) argues that the rationale for minimum investment return guarantees during the pre-retirement phase depends critically on the extent to which an individual will have access to other, more valuable forms of protection that offers a minimum level of income during retirement. The larger the part of the overall retirement income provided from DC funds, the more valuable the guarantee becomes. Examples of minimum level benefits includes old-age pensions, DB pensions, and other social safety nets.

The South African retirement funding system is based on a three pillars model (National Treasury of the Republic of South Africa, 2004):

- Pillar 1 comprises benefits provided by the government as part of their social security program. South Africa has no compulsory or national pension fund. However, an old age grant, capped at R 1 600 p.m. (during the 2016/2017 financial year) for an individual between ages 60 and 75, and equal to R1 620 p.m. for an individual aged 75 and above, is paid subject to a means test [South African Government].
- Pillar 2 includes all privately managed pension and provident funds. Where employers

offer this benefit employees are normally obliged to join the fund.

- Pillar 3 comprises the voluntary contributions made by individuals who for example want to save more than what is provided for via their employer's pension fund, or are self-employed and not able to belong to an occupational savings vehicle.

Given that the old age grants translate to only about US$4 per day, and that it is paid on a means-tested basis, most individuals would need to provide for retirement using the mechanisms of pillar 2 and 3, which offer less certainty. Applying the argument of Antolín et al. (2011) would then suggest that investment guarantees would be valuable to most South Africans.

The 2014 Sanlam Research Insights report found that 84% of the stand alone funds that were surveyed believed that providing stable investment returns to members is important (Sanlam Employee Benefits, 2014). The following strategies are used by South African pension funds to manage investment risk.

2.8.1 Lifestage strategies

The 2015 Sanlam Research Insights report found that where a default investment portfolio is offered, 61% of the trustees surveyed chose a lifestage strategy as the fund's default option (Sanlam Employee Benefits, 2015). Eighty-three percent of members were found to be invested in the default portfolio.

2.8.2 Absolute Returns Funds

Absolute return products have mandates with the dual objective of targeting a return linked to inflation, i.e. a real return target, and in addition seeking to preserve capital over shorter periods of time, usually 12 months. (Alexander Forbes, 2015). Capital preservation however is not guaranteed. The Alexander Forbes Manager Watch™ surveys includes funds with return targets of CPI plus 3% through to 6% p.a.

Absolute return funds use various strategies to achieve their objectives including asset allocation modeling, dynamic risk modeling, explicit hedging strategies, and fixed income strategies

(Alexander Forbes, 2015). Coronation Fund Managers, for example, use the following techniques in aiming to achieve this dual outcome (Burton, 2015):

1. Asset allocation models and stock selection processes are focused on meeting the objectives.

2. Derivatives can be used to reduce downside risk, but is only used when deemed necessary.

3. Diversification through the selection of uncorrelated assets classes and through investing offshore.

2.8.3 Smoothing of returns within a pension fund

The South African Pension Funds Act of 1956 makes allowance for trustees to smooth the returns allocated to members (Southern African Legal Information Institute, 2014). In such an event the Standards of Actuarial Practice (SAP) 201 requires the method of smoothing to be outlined in the valuation reports produced by the valuator (Actuarial Society of South Africa, 2013).

However, the ability to smooth is constrained by the following two requirements:

- Section 15G of the Act requires an equitable portion of any reserves held by the fund to be paid out to the member upon exiting the fund (Southern African Legal Information Institute, 2014). This implies that while returns earned during the membership period may be smoothed, no smoothing of the outcome at retirement is possible through this means.

- The fund is required to be fully funded at all times is implied by the fact that where a deficit is identified, section 18(1A) of the Act requires the fund to within a three month period submit a plan of remediation (Southern African Legal Information Institute, 2014).

This differs from retirement savings portfolios that are issued by long term insurers (e.g. SBPs). SAP 104 makes allowance for a negative smoothing reserve to be held on products that smooth bonuses (Actuarial Society of South Africa, 2012b). This can be allowed because long-term insurers hold regulatory capital to back these portfolios, whereas pension funds do not have this requirement.

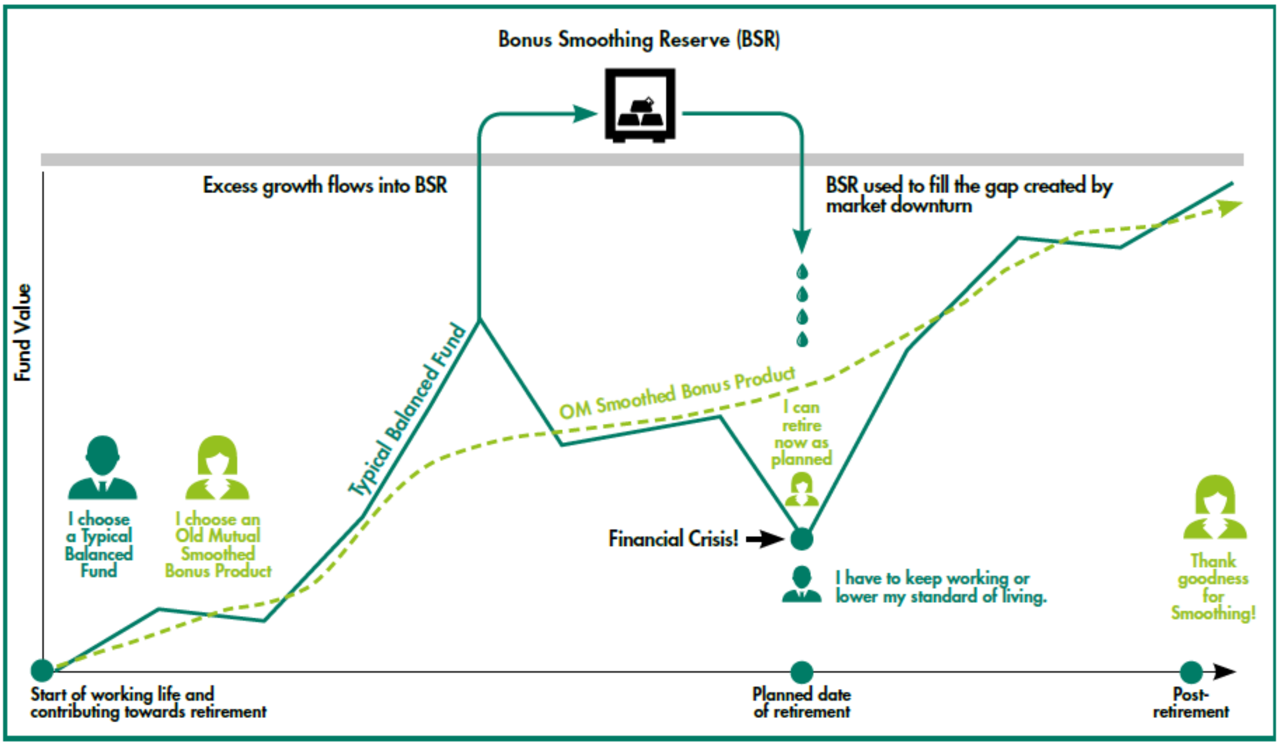
2.9 Smoothed bonus portfolios

The first smoothed bonus product was launched in South Africa in 1967 by Old Mutual (Old Mutual, 2017). The workings and features of SBPs are discussed in the following two sections. Where reference is made to the policyholder in these sections, it refers to a pension fund. It is the pension fund that has the contractual relationship with the insurer.

2.9.1 Smoothing of returns

SBPs aim to reduce return fluctuations by applying a smoothing methodology to the returns allocated to policyholders (Old Mutual, 2016). Investment returns earned on the underlying assets are pooled in a separate reserve called the BSR. Policyholders receive returns through bonus rates that are declared. Once calculated, the total amount of the bonus is transferred out of the BSR into the policyholder account.

The smoothing principle is illustrated in figure 6.



Source: Old Mutual (2016)

Figure 6: Illustration of the working of a bonus smoothing reserve

During periods when the underlying assets perform well, lower bonuses will be declared. When markets perform poorly however, SBPs will potentially still be able to declare a bonus (Alexander Forbes, 2010). This is because retained returns during periods of strong market performance will be available from which bonuses can be paid when markets perform poorly.

The smoothing of returns implies that a level of cross-subsidisation takes place between all the members whose savings are invested in SBPs via their pension funds. When a member joins an SBP and the BSR level is positive, that member will benefit from the distribution of the built-up reserve when bonuses are declared. Likewise, if they join when it is negative, they will contribute to building up the reserve through receiving lower bonuses than earned on the underlying assets.

Bonuses are normally declared at monthly or annual intervals depending on the product rules. Where annual bonuses are declared, an interim bonus will apply until the final bonus declaration is done.

Varying levels of discretion are applied in determining the bonus rate. In some instances the insurer retains full discretion, whereas in other instances bonus rates depend on a predetermined disclosed formula. In the latter case, the bonus formula is often linked to inflation (Old Mutual, 2016; Sanlam, 2016).

Appendix A contains details for the products open to new business in South Africa.

2.9.2 Guarantees

SBPs offer an explicit, though not necessarily full, guarantee on withdrawals made by pension funds for the purpose of having to settle benefit payments to its members. A benefit event on which the guarantee would apply is normally defined as retirement, resignation, death, disability or retrenchment (Old Mutual, 2016; Sanlam, 2016).

Some insurers maintain two separate accounts for each policyholder. This is a vested (or basic) account and a non-vested (or capital) account (Alexander Forbes, 2010). The bonuses declared are split between a vested or basic, and a non-vested or capital portion. The vested portion can be seen as the realised gains and increases the guaranteed benefit value of the policyholder. The non-vested portion can be seen as the unrealised returns that can still be lost or removed (Alexander Forbes, 2010).

The vested account balance equals all contributions less charges plus vested bonuses declared. The non-vested account balance equals all non-vested bonuses declared. The vested account balance is guaranteed, while the non-vested balance is not guaranteed and could be removed when deemed necessary. For example, when the BSR is severely negative and not likely to recover soon, the non-vested balance could be removed and used to improve the level of the BSR. Part of the non-vested account balance is expected to vest over the policy term (Alexander Forbes, 2010). The percentage vested is discretionary, but is expected to be set such that the guarantee level remains at levels consistent with policyholder expectation.

Alternatively some insurers maintain only one investment account for each policyholder. The guarantee level is then expressed as a fixed percentage of this account balance. When the BSR is negative, the bonus rate could be negative, but can never be less than one minus the guarantee percentage, e.g. if the guarantee level is set at 80%, the lowest bonus rate that can be declared is -20%.

On partial or full terminations, market value adjusters (MVAs) could be applied (Liberty; Old Mutual (2015)). MVAs are likely to be applied when market values are low and surrenders likely to be high. This is done to prevent anti-selection and to protect the interest of the remaining policyholders.

The guarantees offered by SBPs are underwritten by an insurance company registered under the Long-term Insurance Act, 1998. Shareholder capital is needed to back the guarantee offered and to meet statutory capital requirements. The reserves to be held by insurers on SBPs for statutory reporting purposes are covered in the Standards of Actuarial Practice (SAP) 104 and the Actuarial Practice Note (APN) 110 (Actuarial Society of South Africa, 2012a,b).

The assets held by the life insurer in respect of a SBP cover the following:

- Liability value

  This equals the market value of the assets underlying the SBP, i.e. equal to the sum of all vested and non-vested account balances plus the BSR.

- Capital adequacy requirement

  Insurers are required to hold capital that will act as a buffer to protect them against adverse experience. The level of the reserve to be held is calculated according to SAP 104 (Actuarial Society of South Africa, 2012b). The assets held in respect of this requirement will be invested separately from the assets of a SBP and based on a potentially very different investment strategy.

- Investment guarantee reserve

  APN 110 sets out the recommended methodology to quantify the reserves to be held to cover expected shortfalls related to embedded investment derivatives, which includes SBPs, in the event of adverse market experience (Actuarial Society of South Africa, 2012a).

In the event that the BSR is negative, SAP 104, section 3.4.6 (p. 8) states,

> it is acceptable to reduce the liabilities to reflect the amount that can reasonably be expected to be recovered through under-distribution of bonuses during the ensuing three years, provided that the Statutory Actuary is satisfied that if market values of assets do not recover, future bonuses will be reduced to the extent necessary.

If it is not deemed possible to restore the level of the BSR to zero, even after the removal of non-vested bonuses, shareholder support will be provided in the form of a loan. Once the BSR level has been restored, the capital is returned to the shareholders with or without interest [Liberty, Old Mutual, 2015; Sanlam Employee Benefits, 2014].

SBPs are classified according to the International Financial Reporting Standards as insurance contracts with discretionary participation features. Directive 148.A.i (LT) issued by the Financial Services Board (FSB) requires insurers who are selling discretionary participation products to disclose their Principles and Practices of Financial Management (PPFM) that are applied in the management of these portfolios (Financial Services Board, 2006). This requirement increases the transparency of these portfolios particularly relating to the investment strategy, the smoothing mechanism and the bonus policies applied.

From the table in Appendix A, the allocation to domestic growth assets, i.e. listed equity and property, ranges from approximately 35% to 80%, while the allocation to international assets ranges from zero to roughly 25% of assets. It is thus apparent that the level of risk assumed by these portfolios, and therefore their bonus objectives, differ between them. Policyholders are therefore offered alternatives to chose from, taking into consideration their own risk/return objectives.

As compensation for the required capital that shareholders have to provide to back an SBP, a capital charge is levied. This charge (for portfolios that disclose it) ranges from 1.0% to 2.7% p.a. for a full guarantee. The capital charge for funds that keep a separate vested and non-vested account is approximately 1% p.a.

SBPs that maintain a separate vesting and non-vesting account and that declare bonuses on a discretionary basis are very similar to the design of the with-profit policies that have now largely been closed to new business in the UK. The newer generation SBPs with set guarantee levels and explicit bonus formulae are similar in design to the TimePension product that is discussed in section 2.7.1, with the exception of the guarantee attached to the SBPs sold in SA.

2.9.3 Performance assessment of an SBP

Figure 7 is based on figure 1, but only shows those elements of an SBP that this study focuses on and that is incorporated in the analysis. The arrows labeled, 'risk', indicates that the feature to which it is attached mainly affects the volatility of the outcome for a member. The arrows labeled, 'return', indicate that the feature to which it is attached mainly affects the level of the outcome the member will receive on retirement.

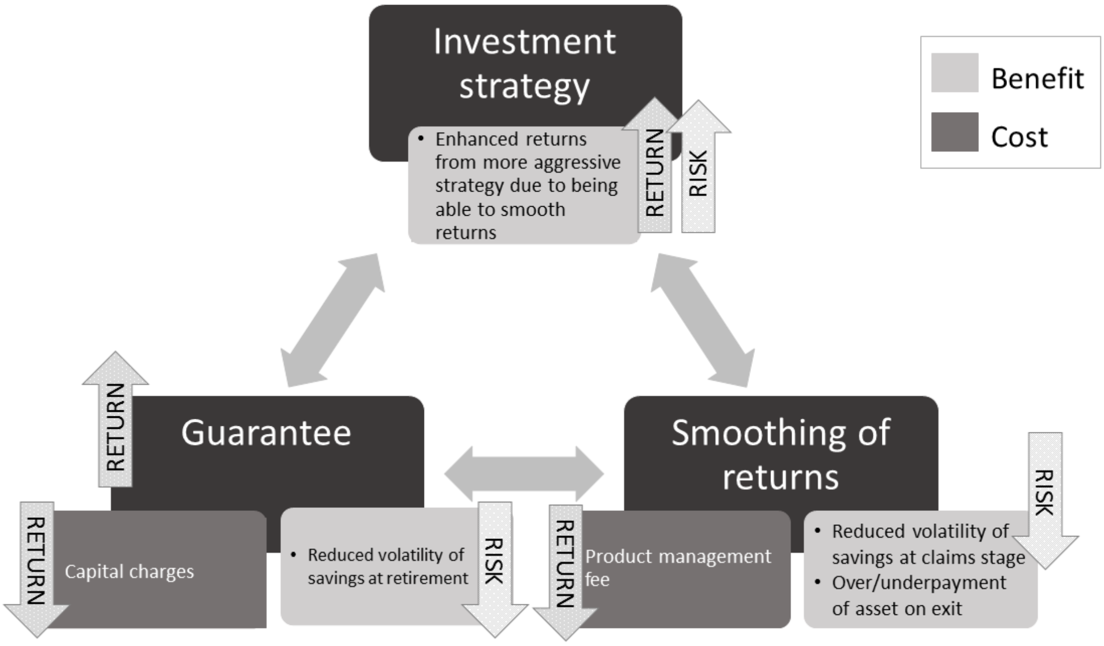


Figure 7: Impact of the features of an SBP on its riskiness and returns

The more aggressive the investment strategy chosen by the insurer is, the higher the expected returns, but also the greater the volatility of the outcome the member expects to receive, and *vice versa.* Figure 1 reflects an implicit cost associated with the choice of investment strategy arising from the insurer wanting to manage their risk exposure. This cost and the benefit reflected in figure 7 associated with the choice of investment strategy will not be split in this study, since this would simply be very subjective.

By not letting members share in negative performance, the guarantee is expected to enhance savings at retirement. At the same time, the guarantee reduces volatility. It is assumed that the only explicit charge associated with the guarantee is the capital charge.

Although the product management fee would in practice cover the management and administrative costs related to every aspect of the product, for the purposes of this study it is taken only as the cost of managing the smoothing mechanism.

The aim of smoothing is to pass the returns generated by the underlying assets on to members in a smoothed manner. Over the long-term one would therefore not expect the smoothing mechanism to impact the level of savings at retirement. Rather, it's greatest impact would be on the volatility of a member's savings during the accumulation phase.

2.10 Conclusion

The management of investment risk in DC pension funds is becoming more and more relevant as a greater portion of retirement savings are moved to DC pension fund structures. This chapter covered some of the methods being used to try and manage this risk, but it is clear that no one superior strategy currently exits to deal with this risk.

Section 2.9 set out in detail the workings of an SBP. These portfolios provide guarantees and apply a smoothing mechanism to reduce members' risk exposure while aiming to generate an appropriate level of returns. The next two chapters set out the methods applied to assess the effectiveness of SBPs, as well as the results from the investigation.

3 Methodology

3.1 Introduction

A basic principle of investment science states that a higher outcome is expected where a higher degree of risk is taken. This is because investors will require compensation in the form of increased returns for assuming increased risk (Reilly and Brown, 2003). In the same way, reducing risk in order to, for example protect pension fund members from adverse market movements, is expected to lead to a lower outcome. When assessing the effectiveness of a pension fund's investment strategy, it is important that attention not only be given to returns, or not only be given to risk exposure, but to consider both of these elements and the trade-off between them.

The next section provides an overview of the process followed and the methodology applied in this study.

3.2 Overview of the methodology

Figure 8 summarises the approach taken to assess the effectiveness of SBPs to manage investment risk in a DC pension fund.

The study requires the construction of a model projecting the fund values of a DC pension fund both at an aggregate and individual member level, discussed in section 3.3. The projections assume regular contributions are made into the fund. No allowance is made for new business or withdrawals. It is assumed that the contribution from new members is offset by any withdrawals made by existing members so that the number of members in the fund remains constant over the projection period. The projection of savings in the pension fund is based on the results from an economic scenario generator (ESG), discussed in section 3.4. It is assumed that the pension fund invests in either one of two types of SBPs. The modelling of the SBP is discussed in sections 3.5 and 3.6.

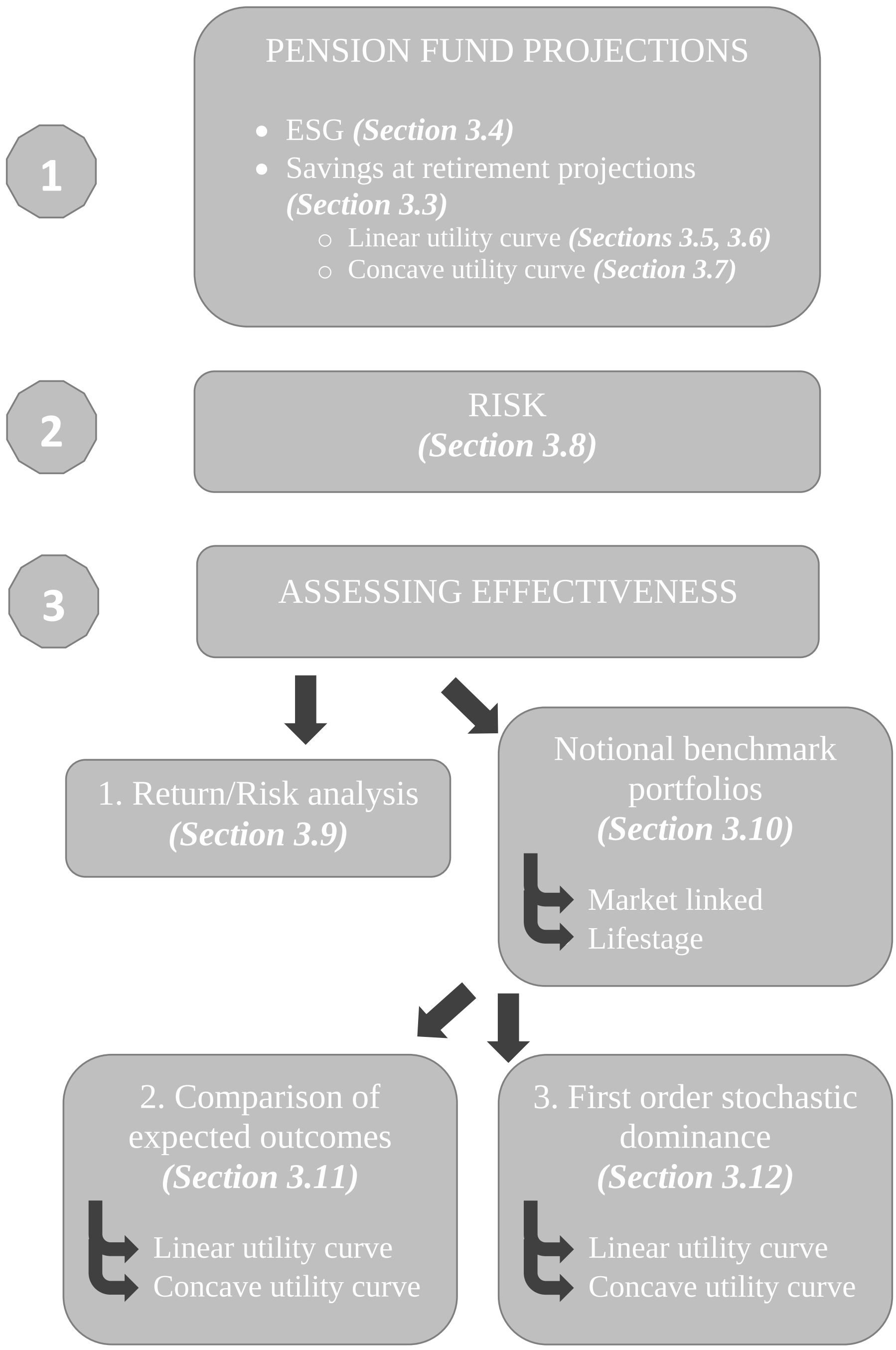


Figure 8: Overview of methodology

The methodology applied is based on the principle that, in order to assess the effectiveness of an investment strategy, it is imperative that the expected outcome be considered in conjunction with the investment risk exposure of a strategy. If this is not done, it could lead to incorrect conclusions being drawn and therefore suboptimal investment decisions being taken. This is discussed in greater detail in section 1.3.3.

Two measures of investment risk are defined and applied. This is discussed in section 3.8. Both elements of investment risk discussed in section 2.5.1, namely the level and volatility of the savings at retirement outcomes are incorporated in these measures.

Three methods of assessment are applied, namely a return/risk analysis, discussed in section 3.9, a comparison of expected outcomes with the notional benchmark portfolios, discussed in section 3.10, and first-order stochastic dominance, discussed in section 3.12.

In the return/risk ratios, the level of the outcome at retirement is incorporated in the numerator, while the volatility associated with that outcome is incorporated in the denominator. This links the return and risk elements of an SBPs that is needed in the assessment of its performance.

The manner in which the second and third methods of assessment is applied, is to evaluate the expected outcome of the SBPs relative to that of notional benchmark portfolios that are constructed to have similar degrees of risk as the SBPs being assessed. These notional benchmark portfolios however apply much simpler investment and risk management strategies than the SBP does. Two types of notional benchmark portfolios are constructed, namely one applying a constant weight asset allocation strategy, called a market linked portfolio, and the other a lifestage portfolio that tapers down the allocation to equity in the years prior to retirement. The construction of these notional benchmark portfolios is discussed in section 3.10.

The analysis in these last two methods are done assuming members have either linear or concave utility curves. The application of utility theory is discussed in section 3.7.

3.3 Simulation of the savings-at-retirement random variable for a DC pension fund

This study requires the stochastic modelling of a pension fund. The outcome of such an exercise is a range of values, and the outcome will therefore itself be a random variable whose distribution and characteristics can be analysed (Sriboonchita et al., 2010).

The random variable Guillén et al. (2013) based their investigation on was defined as the amount of savings remaining at age 90. This approach required assumptions to be made as to the type of annuity a pensioner would choose in retirement. Eason et al. (2013) based their analysis on the projected savings at retirement after 10, 20 and 40 years of regular contributions. The random variable, S, focused on in this study is the savings in the pension fund at retirement, after having made contributions into the fund for 35 years.

The projections of the pension fund is done at an aggregate, i.e. fund, level, with a separate record of the balance of an individual pension fund member's savings in the pension fund being kept. It is assumed that regular contributions growing with salary inflation are paid into the fund over the projection period of 35 years, at which point the member is assumed to retire and exit the fund. No new members or withdrawals are allowed for over the projection period.

It is assumed that the assets of the pension fund equal its liabilities at all points in time. In addition, no expenses associated with the management and administration of the pension fund itself are allowed for. Therefore, the assumption is made that the returns generated by the assets underlying the pension fund, which in the case of an investment in an SBP would be the bonuses declared by the product, is exactly equal to the returns allocated to members.

A pension fund can be fully described by referring only to the type of asset portfolio the pension fund is investment in. This is because of the above assumption that the pension fund's assets equal its liabilities. The only difference therefore between the pension funds being modelled is the portfolios in which their assets are assumed to be invested in.

Because of these reasons, whenever a pension fund is referred to in this study, it is referred to by stating the type of asset portfolio the pension fund is invested in. For example, if pension fund x's assets are invested in portfolio y, we will simply refer to portfolio y when wanting to speak about pension fund x.

A monthly time interval is used when performing pension fund projections. For each portfolio simulated, both the mean and median of the savings at retirement is calculated. Although the median outcome and the subsequent results based on the median is presented in chapter 4 along with those based on the mean outcome, the commentary on the results focuses more on the mean outcome.

3.4 ESG

ESGs allow future scenarios for financial and economic variables such as inflation or asset class returns to be generated. This process requires the variables to be calibrated and a relationship between them to be established prior to generating future scenarios (Baldvindsdottir, 2011).

3.4.1 Types of ESGs

ESGs can either be classified as market consistent or real world, each having different areas of application (Baldvindsdottir, 2011).

1. Market consistent ESGs

   Market consistent models generate future values for economic variables based on the current market prices of tradable assets (Actuarial Society of South Africa, 2012a). These models are used for example in actuarial valuations of liabilities and market consistent embedded values. APN 110 recommends the use of market-consistent stochastic models to calculate the reserves required to back the potential loss arising from offering guarantees (Actuarial Society of South Africa, 2012a). Market consistent ESGs however aim to calculate market prices on a particular day and are therefore not well suited for predicting future asset prices (Baldvindsdottir, 2011).

2. Real world ESGs

   Real world ESG scenarios aim to provide a realistic view of the future value of economic variables based on the probability distributions observed in the real world (which usually involves analysing historical data). The following represent examples of real world ESG models:

   (a) Linear models

- Lognormal model

  This model is based on the assumption that continuous time equity returns follow a geometric Brownian motion. The geometric increase in the equity price over a discrete time interval is therefore assumed to be lognormally distributed (Hardy, 2003). Although this model is simple and tractable, it is less suitable for longer-term projections due to the following reasons (Hardy, 2003):

  - The model assumes that returns in consecutive periods are independent of each other.
  - Empirical studies would seem to suggest that a lognormal model does not adequately capture extreme market movements.
  - Modelling financial and economic series independently of each other ignores expected correlations between the series.

- Autoregressive models

  These models assume that the process being modelled is mean-reverting. The parameters are normally calibrated using a time-series analysis of historical data and allows for serial correlation in the data (Hardy, 2003). Although this approach removes the independent increment assumption of the lognormal model, it still does not allow for correlations between series to be taken into consideration.

The Maturity Guarantees Working Party (MGWP) (1979) proposed an autoregressive integrated moving averages (ARIMA) model to predict dividends, yields and equity prices. Claassen (1993) applied an ARIMA model as the basis for the stochastic investment model used for asset/liability modelling of pension funds in South Africa. The Wilkie model (Wilkie, 1984, 1995; Wilkie et al., 2011) is a multivariate model very similar to what the MGWP proposed. The model itself is a collection of autoregressive models. It follows a cascading structure where inflation is the driving force influencing all other variables. Wilkie's intention was for the model to be appropriate for long-term projections.

Based on the Wilkie model, Thomson (1996) developed a stochastic investment model of inflation rates, short-term and long-term interest rates, dividend yields, dividend growth rates, rental growth rates and rental yields for South Africa. Maitland, in two unpublished papers, critiques Thomson's model (Maitland, 1996, 1997). He argues that the model suffers from a number of shortcomings that make the application of Box-Jenkins methods to South African economic time series data inappropriate. Based on his findings he does not believe the model to be appropriate for making long-term projections, as is desired of a stochastic investment model. It must be noted however that Thomson (1996) himself did not deem the model appropriate for projections exceeding more than ten years. This was, however, due to the relatively few year's data available on which the model was parameterised.

(b) Nonlinear models

The linear modeling of economic and financial variables as proposed by Claassen (1993), Wilkie (1995), and Thomson (1996) are based on the assumption that key variables are stationary. Where a time series exhibits non-linear effects, evidence of which has been found in numerous studies (Chan et al., 2004), non-linear models might provide more accurate predictions of future economic time series values. Non-linear models have the advantage of 'being able to capture asymmetries, jumps and time reversibility' (Chan et al., 2004, p. 38). Examples of such models that have been developed are:

- Autoregressive conditionally heteroscedastic (ARCH) and Generalised autoregressive conditionally heteroscedastic (GARCH) models

  The ARCH model, developed by Engle (1982) allows for the conditional variance as a linear function of past variances. Bollerslev (1986) extended this to the GARCH model, which allows for the lagged conditional variances as well.

- Threshold autoregressive models

  Tong (1990) introduced a threshold principle whereby a complex stochastic system is broken down into simpler sub-systems, were each sub-system or regime is identified by some threshold variable. Based on this principle, Whitten and Thomas (1999) apply non-linear stochastic modeling in the development of a threshold autoregressive (TAR) model for actuarial use. The basic or first class of these models is known as 'self-exciting' threshold autoregressive (SETAR) models. Chan et al. (2004) notes that there certainly are trade offs between non-linear SETAR models and linear autoregressive moving average models (such as the Wilkie model), and as such it is encouraged that suitable tests be used to assess which model best encapsulates the time series under consideration.

- Regime switching models

  Regime switching models (also known as Markov switching models) assume a discrete process switches between a number of regimes randomly, with each regime having its own parameter set. The probability of switching between regimes is assumed to follow a Markov process, i.e. it is dependent only on the current regime as opposed to the history of past states (Hardy, 2003).

The ESG used is a regime switching model. More specifically, Maitland's multiple Markov switching model, discussed in greater detail in the next section, is used. Asset management fees are ignored. If allowance was to be made for these fees, it would have been assumed that all portfolios are charged the same level of fees.

3.4.2 Maitland's multiple Markov switching model

Maitland developed a Markov switching framework that allows for the joint modeling of variables (Maitland, 2010). Four financial variables namely, inflation ($INFL_t$), zero-year nominal yields ($SINT_t$), 20-year nominal par yields ($LINT_t$) and total return on equities in excess of the risk-free rate ($XSEQ_t$) are modelled, where t reflects the number of quarters from $t = 0$. The model allows for each series to be in one of two states, each defined by a mean and volatility parameter. While allowing for the joint modelling of the variables, the model does not require all variables to switch between states simultaneously. Rather, a multiple-switch transition matrix was developed. This 16×16 matrix states what the probability is of each variable switching to the other state given its present state.

The table in Appendix B reflects the models that Maitland fit to the four time series modelled, as well as the parameter values for those series. Further detail on the model can be found in Maitland's article entitled, 'A multiple Markov switching model for actuarial use in South Africa' (Maitland, 2010).

One of the benefits of Maitland's model is that it can be recalibrated using South African economic and financial data (although Maitland does not stipulate exactly how one can go about doing this). Although this model has been used in South African research, a thorough critique of the model has not been done to identify its limitations and drawbacks.

Maitland modelled excess equity returns as the sum of a constant intercept term, the value of which is dependent on the state of the variable, plus an error term (see table 40 in Appendix B). The error term is assumed to follow a normal distribution with a zero mean and standard deviations of 62.1% or 30.6% depending on the state of the variable. Shapiro (2012), in his investigation on appropriate models to use to model long-term extremes of South African financial and economic variables, found that a random walk with normally distributed increments (and empirical estimation method) best fit extreme equity values. A random walk assumes that successive increments in a series are independently and identically distributed. Maitland's error term for excess equity returns assumes a similar distribution. However, due to the size of the standard deviations of the error term, the projected excess equity return values are very volatile. This is however not inconsistent with observed historical data on the South African equities market.

Table 1 presents the expected forces per variable from Maitland's results (Maitland, 2010). Inflation is based on the simulation of the variable $INFL_t$, long-term interest rates on the variable $LINT_t$, short-term interest rates on $SINT_t$, and the excess equity rates on the variable $XSEQ_t$. Maitland (rightly) pointed out that his transition matrix was developed based on past data and would thus not be appropriate for forecasting future values of the series. It is also widely accepted that future equity risk premiums in South Africa are expected to be lower than past observed returns (Maitland, 2010). Maitland's model therefore needs to be recalibrated to make it suitable for projection purposes.

Anderson and Empedocles (2016) provides expected real returns that they used to forecast the values of financial series. These were based on market conditions prevailing at the time of their paper. They tested the reasonability of these return expectations with one of the largest asset consulting houses in South Africa. The inflation expectation of Anderson and Empedocles (2016) is set at 6% p.a., i.e. a continuously compounded rate of 5.8% p.a. This is the upper limit of the South Africa Reserve Bank's inflation band [The South African Reserve Bank]. These rates, converted to continuously compounding rates to make it comparable to Maitland's values, are reflected in table 1.

Table 1: Annual forces of return from Maitland versus proposed by Anderson et al.

	Inflation	Long-term interest rates	Short-term interest rates	Excess equity rates
Anderson et al.	5.8%	8.2%	6.8%	4.4%
Maitland	8.2%	12.2%	10.2%	7.1%

Maitland's model is recalibrated before it is run to try and align the expected returns from the model with the values proposed by Anderson and Empedocles (2016). This is done as follows. For each of the four variables modelled, the input parameters, c_1 and c_2, reflected in table 40 in Appendix B, is scaled by dividing it by the annual forces from Maitland's results as per table 1, and then multiplying it by the annual forces proposed by Anderson and Empedocles (2016) as per the same table.

Maitland's model calculates annual forces at quarterly intervals. In this study, monthly effective rates are used. A monthly interval corresponds to the frequency at which bonus rates are declared in practice by the SBPs on which the SBPs in this study are based. To convert an annual force, δ to a monthly effective rate, i, it is assumed that the rate over any quarter remains constant such that for each month j, over the quarter, for $j = 1, 2, 3$, the monthly effective rate, i_j, is calculated as,

$$i_j = e^{\delta/12} - 1 \tag{6}$$

The outcome at retirement is expected to be less volatile due to this assumption that returns do not fluctuate over the quarter. This is a limitation, but avoids the need for an assumption to be made as to what an appropriate level of inter-quarter volatility is. The δ to be used in equation 6 when calculating the effective monthly rates for the bond and equity returns are determined as follows:

- Bond portfolio

  The constituents in the JSE All Bond Index were weighted as follows as at 2 February 2017 (The Johannesburg Stock Exchange, 2017). Bonds with terms of between one to three years were given a weight of 9% when calculating the index values. Bonds with terms longer than three years made up 91% of the index. Therefore, in this study the bond portfolio whose forces of return are calculated, is assumed to consist of 10% of the zero-year nominal yields plus 90% of the 20-year nominal par yields. It is assumed that the portfolio is rebalanced at the end of every month.

- Equity portfolio

  Rather than model the total return on equity, Maitland modelled the excess equity force of return. When fitting the time series, he deducted the zero-year nominal yields from the return on a total return index for equity to get the excess equity forces. The zero-year nominal yields served as a proxy for the risk-free rate. Therefore, to get the projected forces of return on an equity portfolio, the zero-year nominal yields from the simulations are added to the excess equity yields before converting it to effective rates as per the above equation.

3.5 SBPs

Two types of SBPs, referred to as SBP 1 and SBP 2, are modelled. The characteristics of SBP 1 and SBP 2 are based as far as possible on the features of two SBPs currently sold in the South African market. Both of these portfolios are assumed to have a 100% guarantee level, i.e. contributions plus bonuses (which cannot be negative) are guaranteed to be paid on exit.

Doing projections at a fund level allows the opening BSR level to be non-zero (it will be taken to be 5%), which is closer to a real-world scenario. We will assume that the pension fund being simulated is the only client invested in the SBP, although in reality it is a pooled product. The implication of this is that only the cashflows of the pension fund being modelled (in addition of course to investment returns and fees) will influence the level of the BSR in percentage terms. In practice the withdrawals and contributions made by other clients will also impact the BSR level and could thus impact bonus rates depending on the relative size of the cashflows.

The following three accounts of an SBP are modelled,

- Market value account: This reflects the value of the assets underlying the SBP, and equals the sum of the balance of the next two accounts.
- Book value account: This reflects the product liability or aggregate of members' savings invested in the product via their pension fund and depends on cashflows into or out of the product, as well as the level of bonuses declared.
- Bonus smoothing reserve: This is the reserve held for the product from which future bonuses will be paid. This reserve could be negative in a market downturn when what has been promised to policyholders, i.e. the book value, is greater than the value of the underlying assets, i.e. the market value.

These three accounts are discussed in greater detail in the following three sections.

3.5.1 Market value account

The market value of the SBPs at the end of every month is calculated as,

$$\begin{aligned} Closing\,market\,value \;=\; Opening\,market\,value \;+\; Contributions + \\ Investment\,returns \;-\; Capital\,charges \;-\; Product\,management\,fees \end{aligned} \tag{7}$$

Contributions are assumed to be paid in arrears at the end of each month up to retirement. The rate of return on the underlying assets is calculated as the strategic asset allocation multiplied by the rate of return per asset class earned over that month as per the ESG. The investment returns earned are then calculated as this rate of return multiplied by the value of assets at the start of the month. The implication of calculating returns in this manner is that it assumes the portfolio is rebalanced back to its strategic allocation at the start of every month. The capital charge is calculated as a percentage of the market value, and the product management fee as a percentage of the book value at the start of every month.

3.5.2 Book value account

The book value at the end of every month is calculated as,

$$\begin{aligned} Closing\,book\,value &= Opening\,book\,value\; + \; Contributions + \\ & Bonuses\,allocated\; - \; Product\,management\,fees \end{aligned} \tag{8}$$

The level of contributions and product management fees in the above equation equal the values of these variables in the calculation of the market value at the end of that same month. The bonuses allocated are calculated as the bonus rate declared multiplied by the opening book value, i.e. it is assumed that all cashflows take place at the end of each month.

3.5.3 Bonus smoothing reserve

At the end of every month, the following relationship holds,

$$\begin{aligned} Closing\,BSR &= Closing\,market\,value\,account\,balance - \\ & Closing\,book\,value\,account\,balance \end{aligned} \tag{9}$$

This can also be expressed as follows,

$$\begin{aligned} Closing\,BSR &= Opening\,BSR\; + \; Investment\,returns\; - \; Capital\,charges - \\ & Bonuses\,allocated \end{aligned} \tag{10}$$

The investment returns and capital charges equal the values used in the calculation of the market value at the end of this same month. The bonus rate applicable at time t is calculated at time $t-1$. The bonus formulae used to determine the level of the bonus are discussed next. These formulae are based to a great extent on the actual bonus formulae of the two SBPs in the market on which SBP 1 and SBP 2 are based. Both formulae declare bonuses that are linked to inflation plus a real return target. In addition, a BSR adjustment is added to the bonus rate to allow members to share in any returns generated above what is required to cover the inflation linked bonuses.

Bonus formula: SBP 1

The monthly bonus rate is linked to inflation plus a real return target. In addition, members also share in any outperformance, the extent of which depends on the level of the BSR. The level of the BSR at time t is calculated as $\frac{Market\,value_t}{Book\,value_t}$.

Bonus formula with guarantee

The bonus rate is calculated as follows:

$$\begin{aligned} Monthly\,bonus\,rate \;=\; & Average\,monthly\,inflation\,rate\,observed\,over\,past\,36\,months \\ & + Real\,return\,target \;+\; \tfrac{BSR\,level-103\%}{24-m} \end{aligned} \tag{11}$$

with,

- $m = 12$ when $BSR\,level > 110\%$
- $m = 0$ when $95\% < BSR\,level \leq 110\%$
- $m = 3$ when $92.5\% < BSR\,level \leq 95\%$

$Monthly\ bonus\ rate = 0$ when $BSR\,level < 92.5\%$

This bonus formula offers members a real return while the BSR is greater than 92.5%. This real return is then adjusted up or downwards based on the level of the BSR. The larger the BSR, the greater the size of the adjustment to the bonus rate, and *vice versa*. A zero percent return is allocated when the level of the BSR is less than 92.5%.

Bonus formula without guarantee

A scenario is modelled that assumes the SBP's guarantee is removed. This is discussed in section 3.9.1. For this scenario, the real return target is increased with the capital charge. Because the portfolios on which this study is based offers a guarantee, assumptions have to be made as to what the bonus formula would look like in the case where no guarantees are offered. When the $BSR\,level > 95\%$ the above formula is assumed to hold. However, when the $BSR\,level \leq 95\%$, the value of m is determined as follows:

- $m = 3 \times \left|\frac{BSR\,level - 95\%}{0.025}\right|$ when $80\% < BSR\,level \leq 95\%$
- $m = 18$ when $70\% < BSR\,level \leq 80\%$

When the $BSR\,level \leq 70\%$, the bonus rate is calculated as $(BSR\,level - 1)$.

In this instance, rather than set the bonus rate at zero percent when the BSR drops below 92.5%, the normal bonus formula, i.e. equation 11, is retained. The real return target is however increased by the capital charge which is now no longer payable. However, the negative BSR adjustment made to the real return becomes larger and larger the more negative the BSR becomes. When the BSR level drops below 70%, the normal bonus formula is abandoned and the bonus rate is set such that the level of the BSR is raised to 100% following the allocation of that bonus.

Bonus formula: SBP 2

Bonus formula with guarantee

For the second SBP modelled, the bonus rate is determined as follows. If the BSR is non-zero, the monthly bonus rate is calculated as follows, with the BSR adjustment as per table 2:

$$\begin{aligned} Monthly\,bonus\,rate \;=\; & Average\,monthly\,inflation\,rate\,observed\,over\,past\,36\,months \\ & + Real\,return\,target \;+\; BSR\,adjustment \end{aligned} \tag{12}$$

SBP 2 also offers members a real return, although at a different level than SBP 1, as reflected in table 5. The BSR adjustment is calculated a little different compared to SBP 1.

Table 2: BSR adjustment used in the bonus formula of SBP 2

	Average annual inflation rate observed over the past 36 months	
	$\leq 6\%$	$> 6\%$
$100\% \leq$BSR level$< 105\%$	0.00%	-0.50%
$105\% \leq$BSR level	$0.25\% \times \left(1 + \left\lfloor \frac{BSR\,level - 1}{0.05} \right\rfloor\right)$	$0.25\% \times \left(\left\lfloor \frac{BSR\,level - 1}{0.05} \right\rfloor - 1\right)$

The BSR adjustment increases in multiples of 0.25% as the BSR level increases. Therefore, the higher the BSR level, the greater the positive adjustment to the real return.

When inflation is high, i.e. greater than 6%, a smaller BSR adjustment is added to the real return because the bonus rate will already be larger by virtue of a higher inflation rate being used in the bonus formula. Also, periods of higher inflation might not be accompanied by higher returns in the market and insurers are concerned about maintaining solvency. Adding a smaller BSR adjustment to the bonus rate when inflation is high slows down the rate at which the BSR is distributed and depleted.

Bonus formula without guarantee

Here again an assumption is needed as to what the bonus formula would look like when the guarantee is removed. When the BSR is non-zero, the above formula would be used. However, the annual real return target is increased by the capital charge.

In the market, the SBP on which SBP 2 is based gives policyholders the option to select a level of guarantee set at 80%. The bonus rates allocated on that product when the level of the BSR is between 85% and 100% is used in this study. This is reflected in table 3. When the BSR level is less than 85%, a negative bonus rate is declared at a level that will restore the level of the BSR back to almost 100%.

Table 3: Bonus rates in the event of a negative BSR for SBP 2

	Bonus rate
97.5% $\leq$BSR level$<$ 100%	0.49%
95.0% $\leq$BSR level$<$ 97.5%	0.29%
92.5% $\leq$BSR level$<$ 95.0%	0.14%
85% $\leq$BSR level$<$ 92.5%	0.05%
BSR level $<$ 85%	$-0.05\% \times \left(\left\|\left\lfloor \frac{BSR level-1}{0.05} \right\rfloor\right\| - 1\right)$

Figure 9 illustrates the bonus rates at various BSR levels calculated using the above formulae for SBP 1 and SBP 2. An average annual inflation rate of 5% p.a. is assumed in the graphs. The graph on the left shows the bonus rates when the BSR is negative, while the graph on the right shows the bonus rates for positive BSRs.

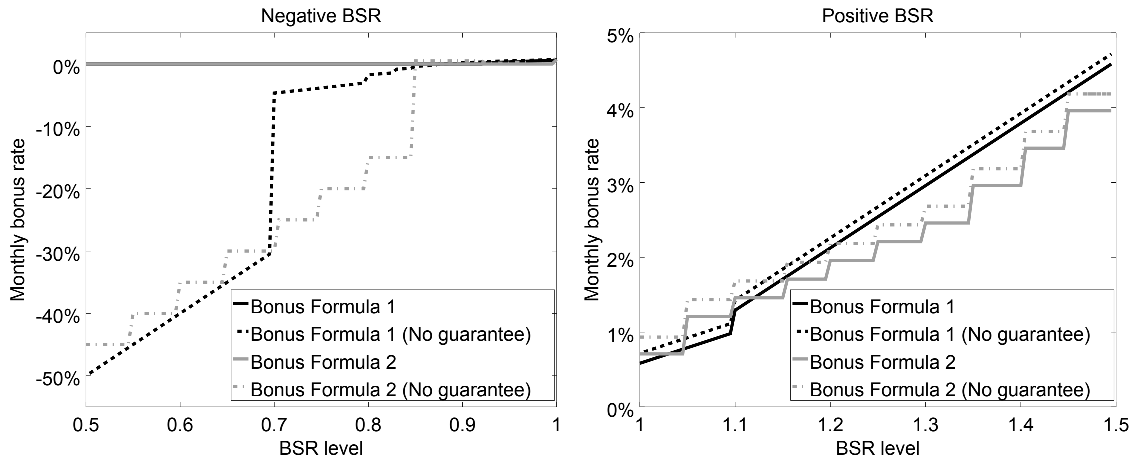


Figure 9: Illustrative bonuses at various BSR levels

When the guarantee holds, the bonus rates cannot be less than 0%. When the guarantee is removed and the level of the BSR is less than one, i.e. the BSR is negative, negative bonus rates can be declared and help to restore the BSR to positive levels. This is because part of the book value account will be transferred to the market value account. Increasing the bonus rates as the level of the BSR increases prevents a larger and larger amount of surplus from building up. This is fair since policyholders who exit while the BSR is positive, will not receive a share of any remaining surplus. They would however, most likely have contributed to it while invested in the product.

Capital injections

SAP 104 provides for a three year period during which a negative BSR must be restored to a non-negative position (Actuarial Society of South Africa, 2012b). If the BSR is negative for two consecutive years, a capital injection to raise the level of the BSR to 100% is made by shareholders.

Any capital injection into the product is treated as a loan to be repaid with interest. The capital charge is not levied to cover the cost of these capital injection, as seems to be a common misconception. The capital charge is levied to cover the cost associated with the solvency capital requirement of insurers as discussed in section 2.9.2. The capital requirement will be more onerous as a result of the guarantee offered on the product.

In this study, the bond returns from the ESG is used as the interest rate that is charged by the shareholders. This rate will be charged on any capital injected into the SBP for as long as the loan is needed, i.e. while the BSR is negative.

3.6 Internal rates of return

Similar to the approach taken by Guillén et al. (2013), for each portfolio simulated, the monthly internal rate of return (IRR), earned on a member's savings over the projection period is calculated. The IRR is taken as a measure of the performance of a particular portfolio. The IRR earned over T months, denoted as i, is calculated by solving for i in the following equation,

$$\sum_{t=0}^{t=T} c(t)(1+i)^{T-t} - E[S(T)] = 0 \tag{13}$$

where $c(t)$ reflects the member's monthly contributions and $E[S(T)]$ reflects the member's mean (and as a separate scenario, the median) savings at retirement.

It should be noted that the conversion of the mean level of savings at retirement to an IRR is merely converting the result from an absolute value to a result in percentage terms. This conversion is done to make the interpretation of results easier. The stream of contributions a member makes over the projection period up to time T is assumed to be the same across all simulations.

3.7 Utility theory

3.7.1 Shape of pension fund members' utility curves

As discussed in section 2.3.1, it is assumed that members act rationally. When considering the utility members attach to an outcome, acting rational implies that members attach a higher level of utility to a higher level of savings at retirement. They are therefore assumed to have non-decreasing utility functions. However, the shape of this non-decreasing utility function could differ between members depending on their perceptions of risk. Two utility curves with different curvatures that incorporate different member perceptions of risk is allowed for. These are defined as follows (Sriboonchita et al., 2010):

1. Linear utility curve: A member is indifferent between two uncertain options that have the same expected value. Their utility function is assumed to be linear, implying that the utility values attached to the savings at retirement increases linearly as the savings increases. Members are considered to be risk-neutral in the sense that as long as the level of the outcome is appropriate to the level of risk assumed in achieving that outcome, members are indifferent between a low and a high risk scenario.

2. Concave utility curve: If two outcomes have the same expected value, a member will choose the less risky, more certain of the two options. This implies that for each additional unit of the outcome received, a marginally lower utility is attached to that unit. Members in this case are considered to be risk-averse in the sense that, even if a higher outcome can be achieved that is commensurate to a higher level of risk assumed, members still prefer a less risky, and therefore lower outcome, option. This is because the additional utility they attach to a higher outcome is less than proportional to the additional risk assumed.

3.7.2 Risk parameter of the power utility function

Because of it's popular use and analytical convenience, the power utility function expressed in equation 2 in section 2.4.3 is assumed. The projected savings at retirement are converted to utility values using this equation. Figure 10 illustrates power utility curves assuming three different risk aversion parameters. The range of savings at retirement over which the utility values are calculated is based on the actual outcomes from the simulation of SBP 1 and SBP 2. Since the power utility function is iso-elastic, it is scalable and transformable. The utility values are scaled and transformed for comparison purposes in the figure.

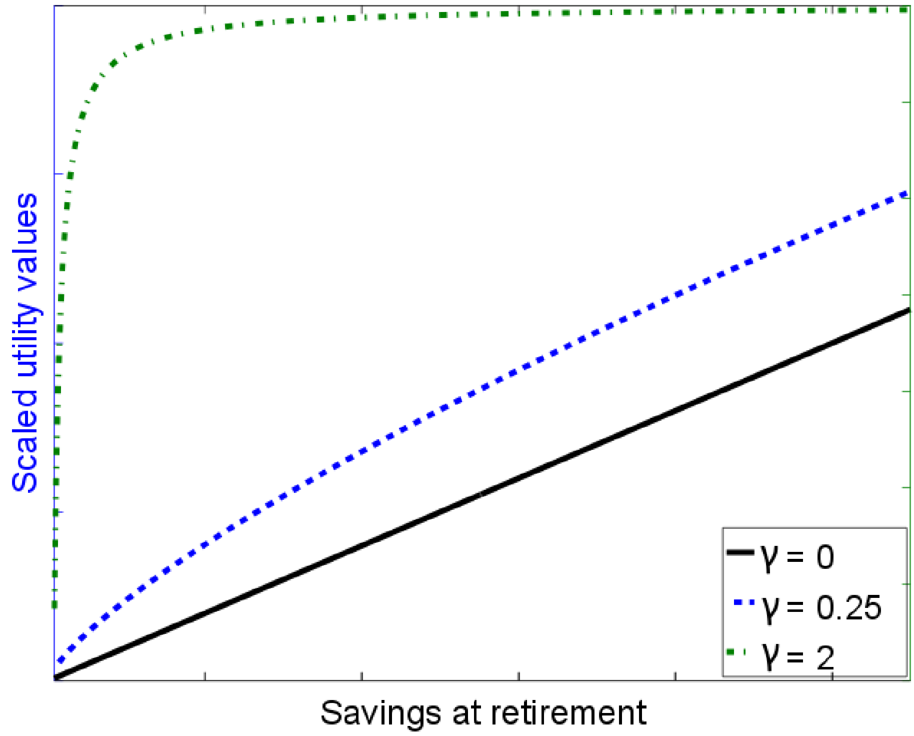


Figure 10: Utility values of savings at retirement assuming three different risk appetite parameter values

A member who is risk-neutral, i.e. has a has a risk appetite parameter of $\gamma = 0$, has a linear utility curve. In this instance, $U(S(T)) = S(T)$, for savings at retirement at time T, $S(T)$. Therefore, since the utility value attached to a level of savings at retirement is merely that savings at retirement value, evaluations are done on the simulated savings at retirement when members are assumed to have a linear utility curve.

The larger the value of γ, the more risk averse a member is assumed to be and the sooner the member is assumed to become indifferent to a larger amount of retirement savings. For the scenario where members are assumed to be risk-averse, i.e. have a concave utility function, a risk appetite parameter of $\gamma = 2$ is chosen in this study. From figure 10, when $\gamma = 2$ it can be seen that members become indifferent to additional units of savings at retirement fairly quickly. It was therefore decided that a parameter value larger than 2 would not render useful results. With a parameter value of 2, the utility function in equation 2 becomes, $U(T) = -S(T)^{-1}$ for savings at retirement at time T, $S(T)$.

Therefore, for the scenario where members are assumed to have a concave utility curve, the savings at retirement outcomes form the simulations are transformed into utility values attached to those particular levels of savings at retirement. The utility values calculated are scaled by multiplying each utility value by a factor of (1×10^{13}).

A sensitivity test is done by setting $\gamma = 0.25$. This is discussed further in section 3.15.

3.8 Riskiness of portfolios

3.8.1 The definitions of the risk measures

The random variable on which a risk measure is calculated in this study is the savings at retirement. This is a profit rather than a loss random variable, i.e. a higher outcome of the variable represents a better rather than a worse outcome. This therefore implies that a larger value of the risk measure will indicate a lower level of risk associated with the random variable.

Taking this definition of the random variable into consideration, the two risk measures used can be defined as follows, where S represents the savings-at-retirement random variable (Hardy, 2006):

- VaR

  For a given confidence level α, where $0 \leq \alpha \leq 1$ and α is expected to be closer to one, there is an α probability that the savings at retirement will not be lower than the VaR. The VaR can be defined as (Hardy, 2006, p. 5),

$$\mathcal{H}_1(S) = VaR_\alpha(S) = sup\,\{x\,:\,Pr[S < x] < (1-\alpha)\} \tag{14}$$

- CTE

  The CTE reflects the average savings at retirement in the lowest $(1-\alpha)$ part of the distribution of the savings-at-retirement random variable. Suppose the VaR is calculated at an α confidence level, then the CTE can be defined as (Hardy, 2006, p. 9),

$$\mathcal{H}_2(S) = CTE_\alpha(S) = E[S\,|\,S < VaR_\alpha(S)] \tag{15}$$

3.8.2 Advantage of using VaR and CTE

An advantage of using the VaR and CTE is that because it measures risk in absolute monetary terms, it encapsulates both elements of investment risk discussed in section 2.5.1. Figure 11 illustrates this with two portfolios where a higher outcome is preferred to a lower outcome, and a lower risk measure therefore indicates a less risky option, as is the case in this study. In both graphs the confidence level is set such that $\alpha = 95\%$.

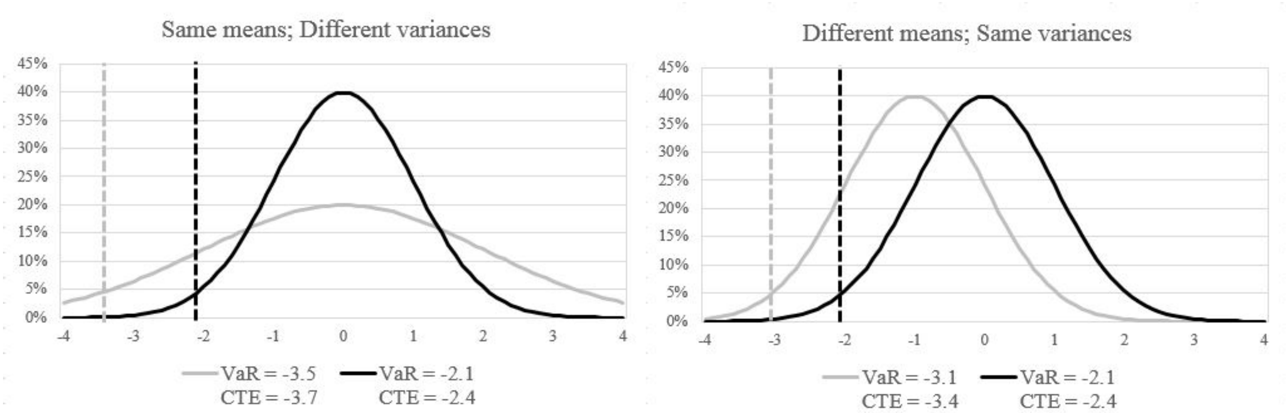


Figure 11: Example 1: VaRs and CTEs of two sample portfolios

Figure 11 shows the distribution for a black and a grey lined portfolio. First suppose these two portfolios have the same mean outcome (refer to the left hand graph in figure 11). The portfolio that is more volatile would be considered the more risky of the two, which in this instance is the grey lined portfolio. The higher level of risk inherent in the grey lined portfolio is reflected by it having a lower VaR and CTE than the black lined portfolio.

Now refer to the right hand graph in figure 11 which assumes that these two portfolios have the same volatility. The portfolio with the lowest expected mean has the greater risk of achieving a lower outcome on retirement, which in this instance is the grey lined portfolio. This is again reflected by the grey lined portfolio having a lower VaR and CTE than the black lined portfolio.

The fact that the VaR and CTE incorporates both the elements of investment risk implies that we are not, by solely considering the level of the VaR or CTE, able to establish which element makes the greater contribution to the portfolio's investment risk. Figure 12 illustrates this.

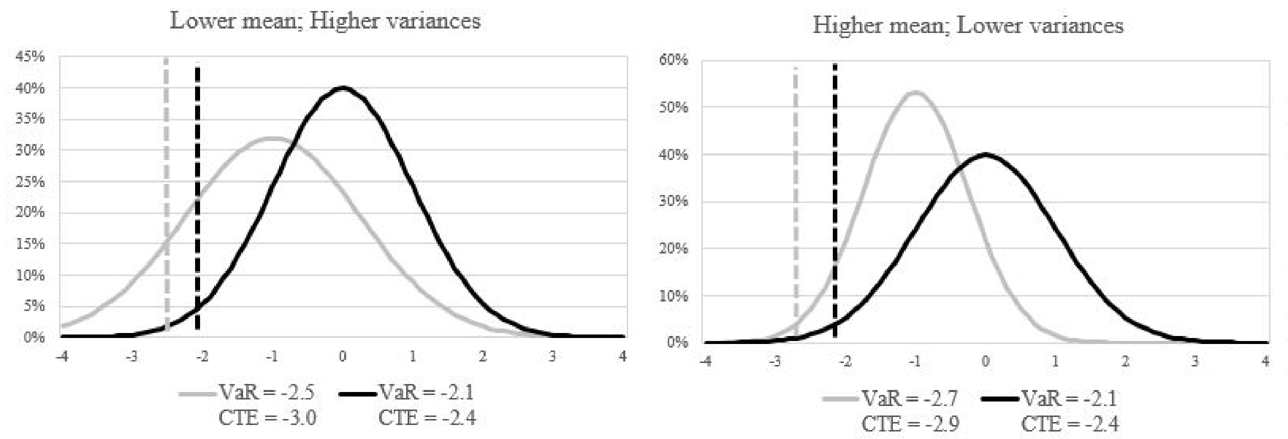


Figure 12: Example 2: VaRs and CTEs of two sample portfolios

Figure 12 again reflects the distribution for a black and a grey lined portfolio. In both graphs in the figure, the grey lined portfolio is found to be more risky than the black lined portfolio as can be seen from its lower VaR and CTE. In the left hand graph, the grey lined portfolio is more volatile than the black lined portfolio. However, the grey lined portfolio also has a higher mean and is therefore less at risk of observing a lower outcome than anticipated compared to the black linked portfolio is. The VaR and CTE however conceals this.

In the right hand graph, the grey lined portfolio has a lower mean than the black lined portfolio and is therefore more at risk of observing a lower expected outcome at retirement than the black lined portfolios is. That is inline with the lower VaR and CTE observed. However, the volatility of the grey lined portfolio is less than that of the black lined portfolio. The VaR and CTE conceals this.

3.8.3 Adjusted risk measures

In the previous section it is shown that VaR and CTE captures both the risk of observing a more volatile outcome, as well as the risk of observing a lower than anticipated outcome. Suppose we only want to measure the volatilty of the outcomes so that comparisson between portfolios with vastly differnt means in absolute terms is possible. This measurement of risk is needed in order to compare the return/risk ratios of portfolios, as discussed in section 3.9.

Standard deviation is a risk measure that calculates the volatility of outcomes. However, standard deviation as a risk measure assumes that the distribution of the outcomes from the variable being modelled is symmetrical, which is not likely to be the case in this study, as discussed in section 2.5.3. Since this assumption is not made by VaR and CTE as risk measures, the following adjustments are made to the VaR and CTE to ensure that it only captures volatility.

If the mean of the simulated outcomes of a portfolio is deducted from each of the outcomes individually, the distribution of the adjusted outcomes will be centered around zero. The VaR and CTE calculated on this adjusted distribution will then only reflect the volatility of the outcomes. This adjustment to the distribution makes the VaR and CTE relative rather than absolute risk measures. If the distributions of a number of portfolios are adjusted in this manner and the VaRs and CTEs of these adjusted outcomes are calculated, a comparison between the risk measures will then indicate which portfolio has a more volatile outcome.

The positive homogeneity and translation invariance properties of VaR and CTE allow for it to be multiplied by a constant, or for a constant to be added or subtracted from it, without changing the level of risk of the portfolio. Therefore, instead of first adjusting the outcomes from a simulation, and then recalculating the VaR and CTE, the adjusted VaR and adjusted CTE are calculated by simply deducting the mean savings at retirement from the VaR and CTE respectively. This is done for all portfolios simulated in this study.

3.8.4 Confidence level

Blake et al. (2001) tests the ability of DC funds to replicate DB funds using VaR as a risk measure with confidence levels of 50%, 80% and 95%. Haberman and Vigna (2002) in their investigation into optimal investment strategies for DC schemes used confidence levels of 90%, 95% and 99%. In this study a confidence level of 95% is used, as was used by Guillén et al. (2013). The volume of results from the study is quite large due to the application of three methodologies, four risk measures, two risk appetite parameters and two SBPs modelled. Therefore the impact a different confidence level would have on the results is left for further research.

3.8.5 The calculation of the risk measures

For each portfolio simulated, the VaR and CTE is calculated on the savings at retirement, or in the case of a concave utility curve, calculated on the utility values attached to the savings at retirement. From previous section, the confidence level, α, chosen in this study equals 95%. Since 10,000 simulations are performed there are 10,000 outcomes of the savings-at-retirement random variable on which the riskiness of the portfolio can then be assessed as follows. Sorting the outcomes at retirement from smallest to largest, the VaR will be the 500$^{\text{th}}$, i.e. $(1-\alpha) \times No.of\ simulations = 5\% \times 10,000$, lowest value in the range.

The CTE can be found by summing all the outcome at retirement that are smaller than the VaR, and dividing that by 499.

3.9 Return/risk analysis

The word 'return' in this context refers to the outcome from the savings at retirement of a pension fund simulated. The word 'risk' refers to the volatility of the outcome at retirement. Two types of analyses are performed in this section. The first involves considering the impact each of the three features of a SBP has on its IRR. The second step involves analysing the contribution each of the features of a SBP make to its return/risk ratio.

3.9.1 Return analysis

Five portfolios based on different permutations of SBP 1 and SBP 2 are modelled. These permutations are based on the elements of an SBP focused on, as discussed in sections 1.3.2 and 2.9.3. The performance of an SBP by product feature can then be analysed. The five portfolios modelled are described as follows:

1. SBP in its basic form

   This portfolio includes the interaction between all the features of an SBP that are reflected in figure 1.

2. SBP with no capital charge

This portfolio is similar to portfolio 1, but the cost associated with the guarantee is removed. This allows us to determine what the impact from the capital charge for the guarantee on the outcome at retirement is.

3. SBP with no guarantee and also no capital charge

   This portfolio assumes members share in downside performance by removing the guarantee. It can therefore be used to analyse the impact the guarantee has on the performance of an SBP.

4. SBP with no guarantee or any fees

   This portfolio is similar to portfolio 3, but without any product management fees. This portfolio allows us to analyse the impact the product management fee has on the performance of an SBP.

5. Market linked portfolio with a constant weight asset allocation strategy based on the investment strategy of the SBP

   This portfolio has none of the features of an SBP other than a similar investment strategy. The outcome of this portfolio therefore reflects the contribution the choice of the investment strategy for an SBP's underlying assets makes to its performance. In addition, the difference in outcome between this portfolio and portfolio 4 reflects the impact the smoothing mechanism has on the performance of an SBP.

For each of these portfolios, the mean (and as a separate scenario, the median) savings at retirement are calculated. The IRR based on the mean and median savings at retirement, as is discussed in section 3.6, is then calculated for each portfolio.

These return figures can then be used to calculate the contribution or detraction each of the features of an SBP make to its overall performance. This methodology is illustrated in figure 13, where the numbers indicate the IRR of the portfolios described above.

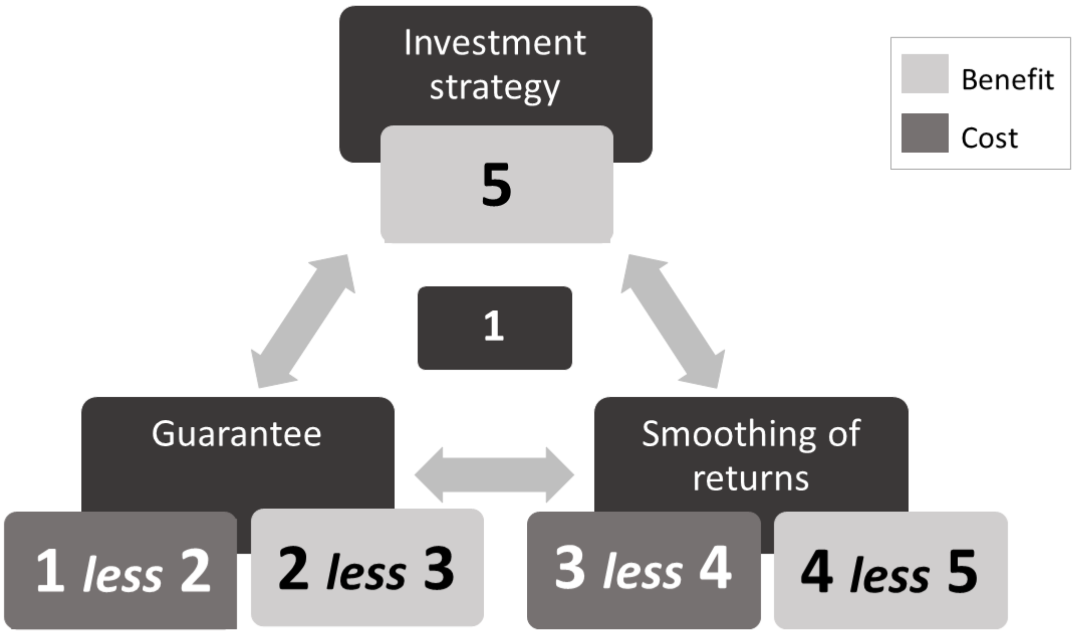


Figure 13: Methodology for calculating the impact of each feature of an SBP on it's performance

From the figure, the IRR of portfolio 1 can be split between the following impacts:

- Investment strategy, taken as the IRR of portfolio 5 described above.
- Cost of the guarantee, calculated by taking the difference in the IRRs of portfolios 1 and 2.
- Benefit of the guarantee, calculated by taking the difference in the IRRs of portfolios 2 and 3.
- Cost of the smoothing of returns, calculated by taking the difference in the IRRs of portfolios 3 and 4.
- Benefit of the smoothing of returns, calculated by taking the difference in the IRRs of portfolios 4 and 5.

Although this analysis renders interesting results, conclusions as to the effectiveness of SBPs cannot be drawn from this analysis alone because differences in the risk assumed during the accumulation phase by these portfolios, influencing the volatility of the outcomes at retirement, are not taken into account in this analysis.

The method will only be applied in the scenario assuming members have linear utility curves. Since the analysis is based on the differences in return, it wouldn't make sense to apply this methodology in the case where members are assumed to have a concave utility curve. In this instance, the order in which the impact of the various steps are calculated would significantly impact the results, making any conclusions drawn from a particular order chosen, nonsensical.

3.9.2 Return analysis incorporating risk

The next step entails calculating a return/risk ratio for each portfolio and then to conduct a similar exercise on these ratios as was done on the IRRs in the previous section. This analysis allows us to determine which feature of the SBP discussed in sections 1.3.2 and 2.9.3, contributes to or detracts from performance on a risk-adjusted basis.

Suppose we have two portfolios i and j with expected values, m_i and m_j respectively, and that these portfolios have levels of risk measured as r_i and r_j respectively. Further assume that the larger the value of r_i or r_j, the greater the riskiness of portfolios i and j respectively. Return/risk ratios for portfolios i and j can be calculated as $\frac{m_i}{r_i}$ and $\frac{m_j}{r_j}$ respectively. These ratios reflect the level of reward per unit of risk assumed that the portfolios are able to generate. An investor choosing between these two will opt for portfolio i if and only if $\frac{m_i}{r_i} \geq \frac{m_j}{r_j}$ (Tasche, 1999).

The return/risk ratio for a portfolio in this study is calculated by dividing the mean savings at retirement with the portfolio's riskiness, measured by the negative of the adjusted VaR (and as a separate scenario the adjusted CTE). (The method for determining the adjusted values of the VaR and CTE is discussed in section 3.8.3). The motivation for using the negative of the adjusted VaR and adjusted CTE is as follows.

Applying the formula above to calculate a return/risk ratio, a portfolio with a higher return/risk ratio is seen as either being able to generate a higher outcome for the same level of risk, i.e. has a higher numerator, or able to generate the same outcome for a lower degree of risk, i.e. has a lower denominator. The higher the savings at retirement for a given level of risk, the higher the return/risk ratio will be. For the risk measure, given the way in which the savings-at-retirement random variable is defined, a lower VaR or CTE indicates a more risky rather than less risky portfolio. This is because the savings-at-retirement random variable on which the risk measures are calculated is a profit and not a loss random variable. Therefore in order to ensure that a higher return/risk ratio is seen as a more positive outcome, the adjusted VaR and adjusted CTE used in this ratio is converted by simply taking the negative of the risk measure. Making this adjustment allows for the intuitive interpretation of the return/risk ratios as described above when comparing portfolios.

Figure 14 illustrates the methodology used to calculate the impact the features of an SBP has on its risk-adjusted performance. The numbers refer to the return/risk ratios of the portfolios described in section 3.9.1.

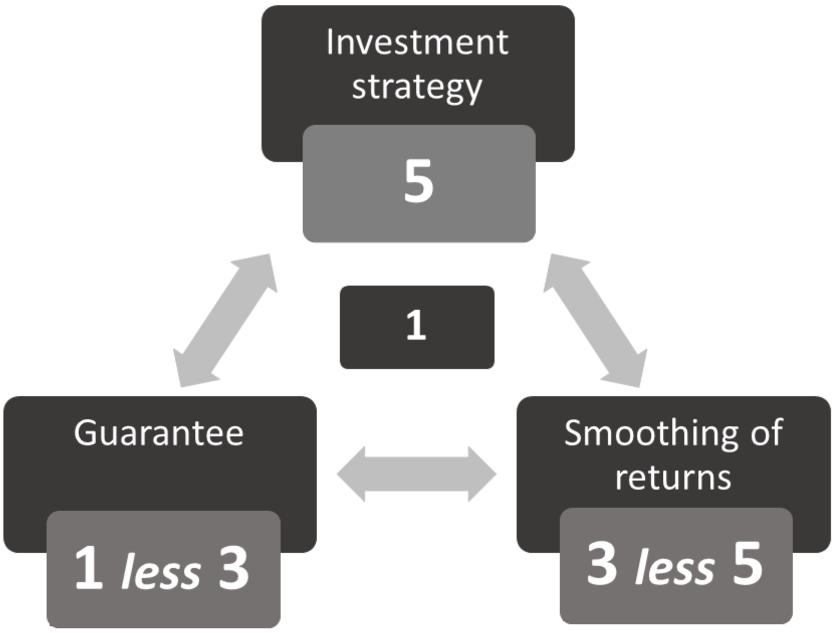


Figure 14: Methodology for calculating the impact of each feature of an SBP on it's return/risk ratio

The return/risk ratio of portfolio 1, i.e. of SBP 1 or SBP 2, can be explained as the sum of the following impacts:

- Investment strategy, taken as the return/risk ratio calculated on portfolio 5.
- Guarantee, taken as the difference in the return/risk ratios of portfolios 1 and 3.

- Smoothing of returns, taken as the difference in the return/risk ratios of portfolios 3 and 5.

Analysing the return/risk ratios of a SBP does have a limitation in that since it is not compared against the return/risk ratios of other pension fund investment strategies, the analysis does not allow a conclusion to be made as to the effectiveness of the SBP itself. We can only make conclusions about the effectiveness of each of the features of a SBP to its overall performance.

3.10 Notional benchmark portfolios

Guillén et al. (2013) takes an approach to assessing the performance of pension fund investment strategies by constructing notional benchmark portfolios against which the performance of the investment strategy is then compared. The notional benchmark portfolio is constructed in such a way that it has the same level of risk as the portfolio against which performance is being assessed. The investment strategy of these notional benchmark portfolios consists of an allocation of assets between a risk-free and risky asset class, the proportions of which is kept constant over the projection period.

The purpose of this is to determine whether a member invested in a simple market linked portfolio that has the same level of risk as the pension fund investment strategy, but achieves that level of risk exposure through no hedging strategy other than through the choice of the level of assets allocated to the risky asset, is able to generate a higher or lower return compared to the outcome of the pension strategy.

A similar approach is taken in this study.

3.10.1 Types of notional benchmark portfolios

The notional benchmark portfolios constructed by Guillén et al. (2013) have a constant weight asset allocation strategy. This strategy as well as a lifestage strategy is applied to construct notional benchmark portfolios against which the performance of an SBP is assessed. The investment strategies of these two notional benchmark portfolios look as follows:

1. Market linked portfolio: This portfolio follows a constant weight asset allocation strategy,

i.e. a fixed allocation of assets over the whole projection period is made between bonds and equities. The riskiness of this strategy is determined by the level of assets allocated to equity.

2. Lifestage portfolio: This portfolio is also assumed to follow a constant weight asset allocation strategy over the projection period, but as a member approaches retirement the portfolio is gradually de-risked by moving funds from the higher to the lower risk asset class. The Sanlam Employee Benefits (2015) survey found that the average phasing out period as members approach retirement is 5.6 years. A five year period is therefore assumed in this study. The riskiness of this strategy is determined by the initial level of assets allocated to equity, as well as the impact of reducing the exposure to this asset class during the five years preceding retirement.

3.10.2 Determining the asset allocation of the benchmark portfolios

After the riskiness of the investment strategy to be assessed is calculated, Guillén et al. (2013) worked backwards from the risk value that the benchmark portfolio must match to find the asset allocation weights of the benchmark portfolio. The fixed percentage allocation of assets between asset classes must be such that when simulating the outcome of the benchmark portfolio, the riskiness of that portfolio equals the riskiness of the investment strategy being assessed.

Given the two risk measures used in this study, i.e. VaR and CTE, the notional benchmark portfolios constructed must have a similar VaR (or CTE, depending on the risk measure in question) as the SBP against which performance is being assessed. Since there are two investment strategies for the notional benchmark portfolios and two risk measured that are used, four types of notional benchmark portfolios are constructed for SBP 1 and SBP 2 separately, namely,

1. Market linked VaR: A market linked notional benchmark portfolio with a similar VaR as the SBP;

2. Market linked CTE: A market linked notional benchmark portfolio with a similar CTE as the SBP;

3. Lifestage VaR: A lifestage notional benchmark portfolio with a similar VaR as the SBP; and

4. Lifestage CTE: A lifestage notional benchmark portfolio with a similar CTE as the SBP.

Notional benchmark portfolios for both SBP 1 and SBP 2 are constructed, for each of the two scenarios where members are assumed to have either linear or concave utility curves. Given the four types of notional benchmark portfolios described above, this gives a total of 64 notional benchmark portfolios are constructed. In addition, another 64 notional benchmark portfolios are constructed based on the adjusted VaRs and adjusted CTEs of the SBPs, described in section 3.8.3. It must be noted that in some instances, the notional benchmark portfolios do not exist due the constraints it has to meet.

The closing value of the notional benchmark portfolio at the end of each month is calculated as,

$$Closing\,value = Opening\,value + Contributions + Investment\,returns \tag{16}$$

where the level of contributions will be the same as the amounts used in the projection of the SBPs.

The strategic asset allocation strategies for each of the notional benchmark portfolios is determined as follows. Assume RS_{BM} is defined as the projected savings at retirement under the benchmark portfolio's investment strategy. Let RM_{SBP} and RM_{BM} reflect the risk measures for the SBP and notional benchmark portfolio respectively, calculated on the simulated savings at retirement as discussed in section 3.8.5. The constant asset allocation weights of the notional benchmark portfolio at inception, defined as w_i, for each asset class i, is determined by solving the following objective function:

$$\begin{gathered} \max_{w}(RS_{BM}),\ subject\,to \\ RB_{SBP} - RM_{BM} = 0 \\ \sum w_i = 1,\ and \\ w_i \geq 0,\ i = 1, 2. \end{gathered} \tag{17}$$

The same objective function holds for both the market linked and lifestage options, with the only difference being the way in which RS_{BM} is calculated.

After the asset allocation strategies of the benchmark portfolios are determined, simulations of the savings at retirement for each notional benchmark portfolio are done. The mean (and as a separate scenario, the median) savings at retirement is then used to calculated the IRRs of the portfolios, as is described in section 3.6. The IRRs are only calculated in the case where members are assumed to have a linear utility curve.

3.11 Comparison of expected outcomes

Since the level of risk of the SBP is the same as that of its notional benchmark portfolios, we can compare the outcomes of these portfolios to determine whether the SBP is able to generate a better or a worse outcome than a simple market linked or lifestage strategy can. This will then be used as an indication as to whether or not an SBP is able to effectively manage investment risk exposure.

A higher return generated by the SBP than its benchmark portfolios indicates that through the interaction of all of its features, the SBP is able to generate a higher return than a market linked and lifestage strategy for the same level of risk exposure. A lower return on the SBP would suggest that the level of risk reduction the SBP is able to achieve through applying its risk mitigation strategies, i.e. the guarantee and smoothing mechanism, comes at a greater cost than that of the simple risk mitigation strategies applied by the market linked and/or lifestage portfolios. In this case the market linked and/or lifestage portfolios are able to achieve the same level of risk exposure as the SBP and yet generate a higher outcome.

3.11.1 Linear utility curve

For each portfolio simulated, an IRR calculated on the savings at retirement is determined. The excess return of an SBP over its notional benchmark portfolio is calculated as the IRR of the SBP less the IRR of the notional benchmark portfolio. A positive value indicates that the SBP outperforms the notional benchmark portfolio and can therefore be considered to be a more effective option for a member. Such a result would suggest that the cost associated with the risk mitigation tools in an SBP, i.e. the implicit and explicit costs associated with the guarantee and smoothing mechanism, is less than the benefit observed in terms of reduced risk exposure.

If the difference in returns is negative, it indicates that the notional benchmark portfolio outperforms the SBP. Such a result would suggest that an SBP is not an effective means of managing investment risk since a simple market linked or lifestage strategy is able to achieve the same level of risk exposure, while generating a higher return, or stated alternatively, at a lower cost.

The analysis can be further enhanced by splitting the returns earned on the portfolios into the following components.

- SBP: The methodology discussed in section 3.9.1 allows for the return on an SBP to be split between its three features, namely its investment strategy, guarantee and the smoothing mechanism. (In this section the cost and benefit of the guarantee and smoothing mechanism respectively, are added together to get a net impact).

- Market linked notional benchmark portfolio: Since the only way in which the market linked notional benchmark portfolio manages risk exposure is through its choice of investment strategy, the returns generated on the market linked benchmark portfolios are not split further.

- Lifestage notional benchmark portfolio: For the lifestage notional benchmark portfolio, the return is split between the return earned on a portfolio that maintains the initial constant weight asset allocation strategy up to retirement, and the return lost from tapering down to a zero allocation to equity during the last five years prior to retirement.

The above exercise is repeated using the returns earned on the notional benchmark portfolios constructed based on the adjusted VaRs and adjusted CTEs of the SBP. In this instance a positive excess return indicates that the SBP is able to generate a higher return than a portfolio whose outcome is just as volatile as that of the SBP's. A negative excess return indicates that the risk mitigation strategies used by the SBP detracts from its performance because a market linked or lifestage portfolio is able to generate a higher return for an outcome that is just as volatile as that of the SBP.

3.11.2 Concave utility curve

The difference between the mean utility values of the savings at retirement of the SBP and its corresponding notional benchmark portfolios is calculated. A positive value suggests that the risk mitigation strategies applied by an SBP is effective at managing members' exposure to investment risk. A negative difference suggests that a simple market linked or lifestage strategy is able to achieve the same level of investment risk exposure while offering members a higher utility value on retirement than the SBP is.

3.12 First-order stochastic dominance

Stochastic dominance rules allow us to make decisions between, i.e. to rank, uncertain variables based on the distribution functions of those random variables (Sriboonchita et al., 2010). If we knew with certainty that the mean (or median) return would be realised, we could quite easily determine which portfolio will give the better outcome. We are, however, modelling an uncertain outcome, because the inputs to the model, particularly the future financial and economic variables, are uncertain. Therefore, rather than consider a single point of the probability distribution of the savings-at-retirement random variable as is done in section 3.11, the whole distribution of this variable is considered here to determine the effectiveness of SBPs relative to their notional benchmark portfolios.

Basu et al. (2011) uses stochastic dominance rules to rank various investment strategies. A limitation of their study however, is that differences in the level of risk assumed by those strategies being compared was not taken into account. This is addressed in this research by setting up the notional benchmark portfolios in such a way that it has the same level of risk as the SBPs.

Suppose X and Y are two random variables representing a DC pension fund member's accumulated savings on retirement. In addition, suppose X and Y have distribution functions F and G respectively such that $P(X \leq x) = F(x), \forall x$ and $P(Y \leq x) = G(x), \forall x$. Sriboonchita et al. (2010, p. 78) defines first-order stochastic dominance as follows,

> X is said to dominate Y in the FSD, denoted as X$\succeq_1$Y , if and only if $F(x) \leq G(x)$, $\forall x$

If the CDFs of X and Y are plotted, and if it is assumed that X dominates Y in the FSD, then the above implies that the CDF of X lies on or to the right of the CDF of Y at all points x.

Alternatively this definition can be stated as,

$$P[X \geq x] \geq P[Y \geq x], \forall x \tag{18}$$

Stochastic dominance rules do not require us to make assumptions about the nature of the distributions of the random variables (Basu et al., 2011). The strict definition of FSD could however fail to reveal a scenario where although *all* decision makers do not prefer one outcome over another, "most" do (Leshno and Levy, 2002). Leshno and Levy (2002) developed what they call, almost first-order stochastic dominance rules to deal with such scenarios.

Suppose X and Y are defined as above and that X dominates Y, but not at all values of x. X is said to exhibit almost FSD over Y if the total area where X fails to exhibit FSD over Y is relatively small in proportion to the total area in absolute terms between X and Y.

Mathematically this can be expressed as follows. Suppose S_1 represents the area where X fails to exhibit FSD over Y such that (Leshno and Levy, 2002, p. 1079-1080),

$$S_1(X, Y) = \{t \in [0, 1] : Y(t) < X(t)\} \tag{19}$$

Almost FSD can by defined as follows,

X dominates Y by ε-almost FSD iff $\int_{S_1} [X(t) - Y(t)]\, dt \leq \varepsilon \int_0^1 |X(t) - Y(t)| dt$

In this study, the area where FSD fails, is expressed as a percentage of the total absolute difference between the CDFs of the two random variables, i.e. the value of ε is calculated. The smaller the value of ε the greater the probability that X is preferred to Y. Where $\varepsilon = 0$, it indicates that FSD holds.

If an SBP exhibits FSD (or almost FSD) over its notional benchmark portfolios, it would suggest that the SBP is an effective means of managing the investment risk members are exposed to. If the notional benchmark portfolio exhibits FSD (or almost FSD) over the SBP, it would suggest that the SBP is not effective at managing investment risk since the outcome of the notional benchmark portfolios are preferred to a greater extent.

3.13 Assumptions

Table 4 lists the parameters and its assigned values used to project a DC pension fund.

Table 4: Parameters values used in all pension fund simulations

Parameter	Value
Opening pension fund value (book value)	R1 000 000
Opening BSR value	R 50 000
Starting monthly contributions (per member)	R 5 000
Monthly salary inflation rate	Inflation rate as per ESG
Number of fund members	20
Number of months to retirement	420 (i.e. 35 years)
Number of simulations	10,000

Table 5 lists the parameters and assumptions that define the features of the two SBPs simulated. Annual rates are taken to be nominal rates compounded monthly. The capital charges are in line with the capital charges levied for the actual portfolios on which SBP 1 and SBP 2 are based. The real return target for SBP 2 is similar to the return target of the actual product on which it is based, but due to a lack of disclosed information, an assumption is made for SBP 1.

Table 5: Product and investment features of SBP 1 and SBP 2

	SBP 1	SBP 2
Capital charge	1.6% p.a.	2.7% p.a.
Product management fee	0.35% p.a.	0.35% p.a.
Real return target	3.5% p.a.	3.5% p.a.
Strategic asset allocation strategy		
Equity	56%	80%
Bonds	44%	20%

SBP 1 and SBP 2 are assumed to have the same real return targets and product management fee levels, but the capital charge for SBP 2 is much higher than for SBP 1. This could be because the product follows a more aggressive investment strategy and is therefore expected to be more capital intensive from the insurer's perspective. The strategic asset allocation strategies for the two SBPs are based on the actual strategies for the portfolios on which SBP 1 and SBP 2 are based.

3.14 Limitations

Assumptions related to the bonus formulae to use in the event that the guarantee is removed and only the smoothing of returns is allowed for, have to be made. If in practice insurers had to develop an SBP excluding the guarantee, they might decide that a very different bonus structure would be appropriate, or change some of the product features.

Section 1.3.2 mentions that an implicit cost of an SBP could be the cost associated with the insurer opting for a less aggressive investment strategy in order to manage their solvency risk associated with the guarantee. This cost is not analysed separately as discussed in section 2.9.3.

The focus in this study is on the savings at retirement for reasons provided in section 2.5.1. However, in terms of the application of expected utility theory, Blake et al. (2014) argues that a long-term investor is not so much concerned about the final level of savings than about what consumption stream can be financed with that wealth. Changing the random variable on which the analysis is based could lead to a different set of results.

3.15 Sensitivity tests

A sensitivity analysis is done by making the following adjustments to the assumptions made, each in a separate scenario:

1. Allow for the investment of contributions in the SBP for only five years prior to retirement

   Projections of retirement savings are done over the full projection period, i.e. 420 months, but with the assumption that savings are invested in a market linked portfolio (with a similar investment strategy as that of the SBP) for the first 360 months, and then switched into the SBP for the last five years. This allows us to investigate the effectiveness of an SBP where members decide to use it only as an 'end stage' portfolio.

2. Allow for a risk parameter of $\gamma = 0.25$ in equation 2

   Reducing the value of gamma implies a scenario where members are assumed to be less risk averse.

If members are more concerned about the uncertainty associated with the outcome they will achieve at retirement, rather than with the investment risk faced during the accumulation phase, their focus will be more on managing their risk exposure at retirement rather than during the accumulation phase. This is why the scenario in the first sensitivity test is chosen and considered to be valuable. It needs to be noted that in practice there might be a restriction on a member buying into the SBP five years prior to retirement. This could happen if for example, markets are in turmoil and the BSR is severely negative. This possible restriction is not taken into consideration in this study.

The second scenario is chosen, because it was not clear from previous studies what an appropriate risk parameter value to use is. It is therefore appropriate to assess what the impact on the results are should a different parameter value be chosen.

Due to the computational intensity of running a scenario, only these two scenarios are tested. However, the following additional scenarios could also be tested,

- An opening BSR of 5% of the opening book value is assumed. By changing this variable, particularly assuming it has a negative value, could render interesting results.

- The ESG used is recalibrated based on the future returns expected by Anderson and Empedocles (2016). These returns are based on one view of the future state of financial markets and the economy. The impact of alternative views can be tested.

- This study made no allowance for withdrawals or new business during the projection phase. Stated differently, it is assumed the withdrawals offset the new business exactly at all points in time. However, both of these events on their own will effect the BSR level and therefore the expected future bonus rates for all policyholders of the product. A sensitivity test can therefore be done assuming new business levels exceed (or is less than) the level of withdrawals over the projection period.

3.16 Conclusion

The outline of the methodology applied in this study is reflected in figure 8. Two SBPs are modelled based on the features of two such products sold in the South African market. For the first methodology, permutations of the SBPs are modelled and an analysis performed of the returns and return/risk ratios of the SBPs. For the second and third methodologies, the riskiness of each SBP simulated is calculated and notional benchmark portfolios are set up. The IRRs of the SBP are then compared to the IRRs of its notional benchmark portfolios. FSD rules are also applied in the quest to determine whether SBPs are effective at managing investment risk exposure.

The results from this study is presented in the next chapter.

4 Results

4.1 Introduction

Figure 15 is a duplicate of figure 8 in section 3.2 and provides an illustrative overview of how the results of this study is presented in this chapter.

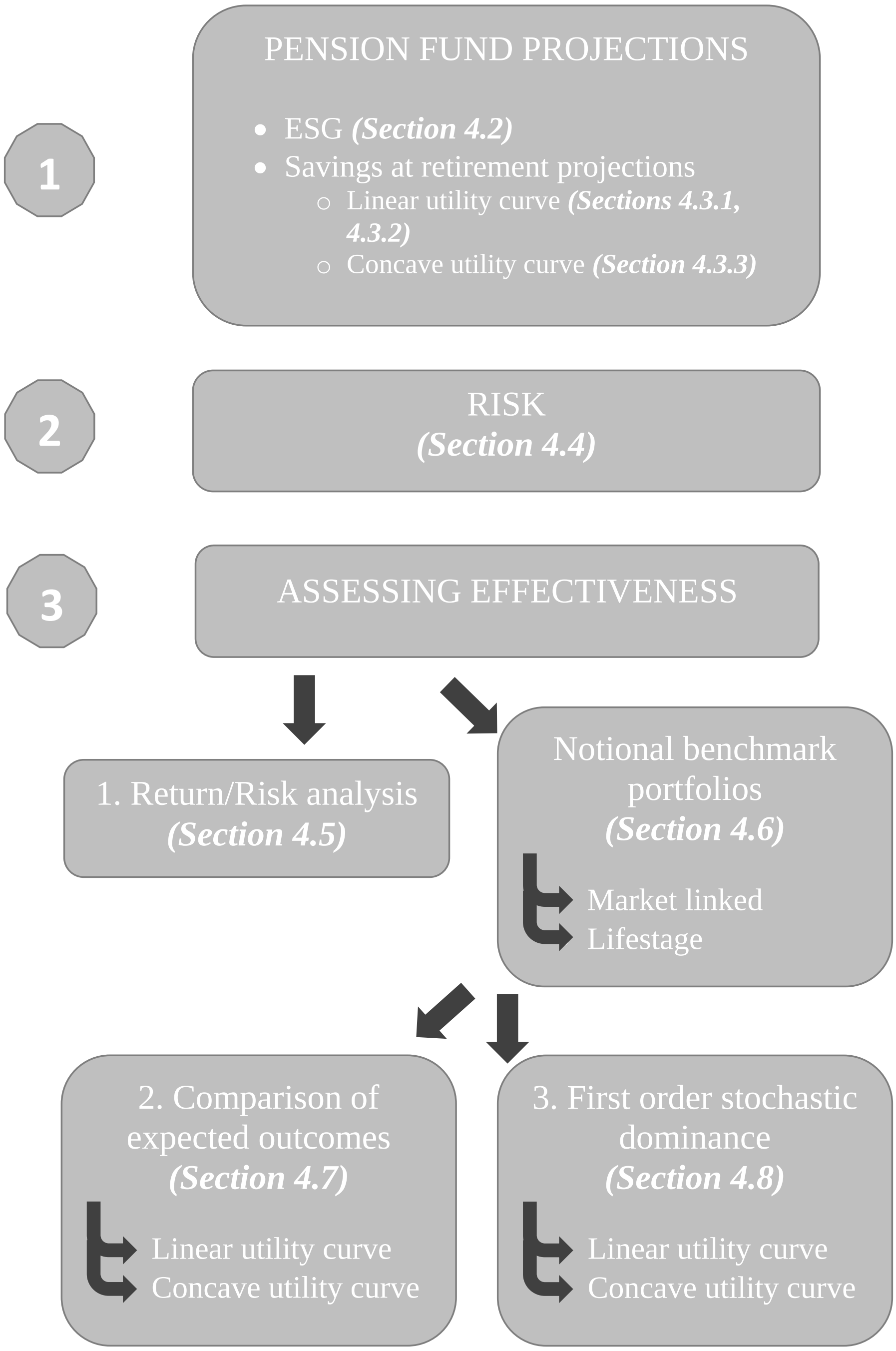


Figure 15: Overview of presentation of results

The figure indicates where the results of a particular step in the methodology is presented in this chapter.

4.2 Economic Scenario Generator

Figure 16 shows histograms of the distributions of the annual forces from the simulations generated by applying Maitland's multiple Markov switching model. The simulations were generated after the model was reparamterised to the long-term return expectations from Anderson and Empedocles (2016). The annual forces are calculated at quarterly intervals for the four variables discussed in section 3.4.2.

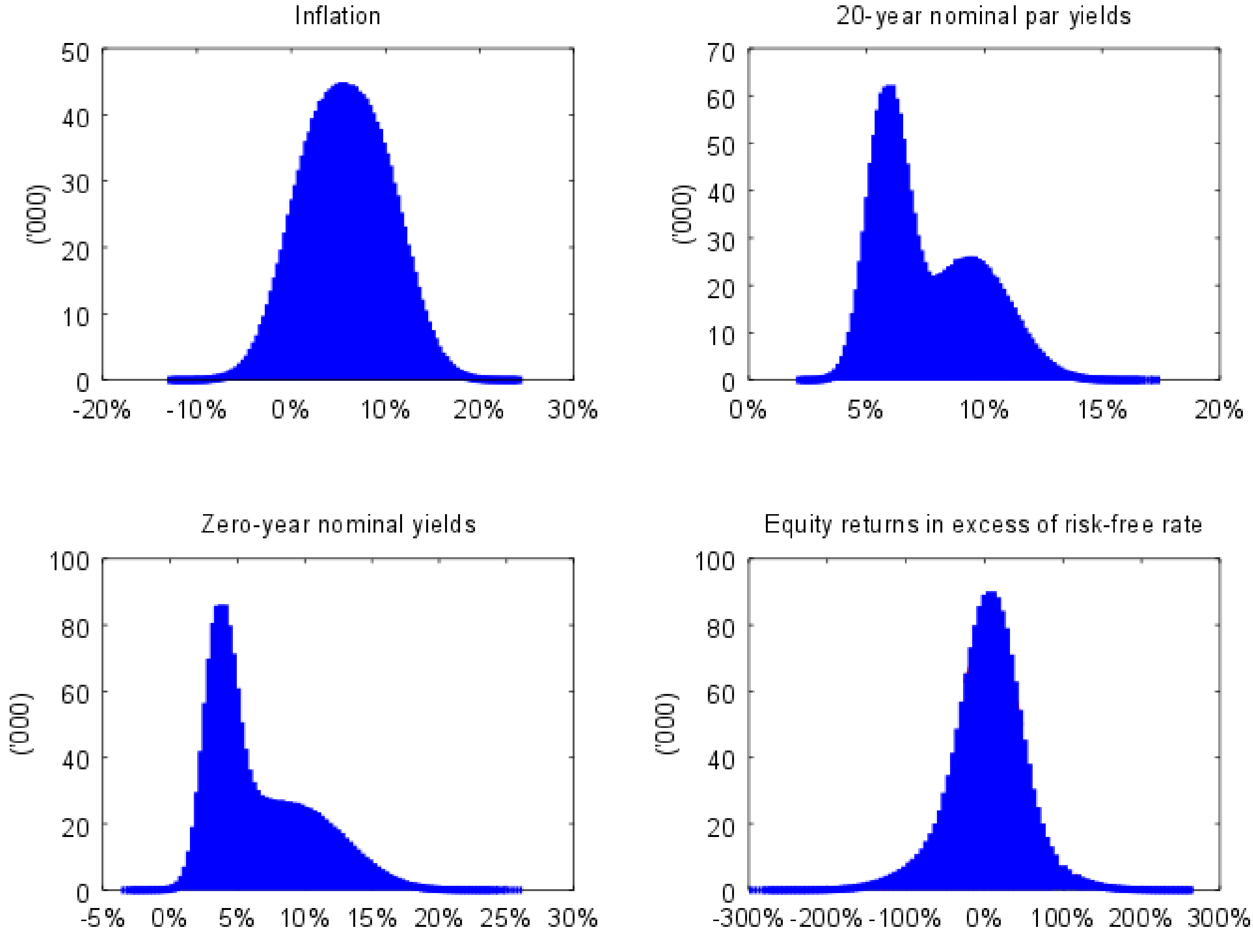


Figure 16: Histograms of the annual forces from Maitland's reparameterised model

The assumption in Maitland's model that short-term and long-term forces of interest are in either one of two states is clearly visible in the histograms for the zero-year nominal yields and the 20-year nominal par yields. The histograms for these two series have two visible peaks. The volatility of the excess equity forces can be seen from the length of the tails of its histogram.

Table 6 reflects the mean annual forces of the simulations on which figure 16 is based. The means are calculated over all simulations over all time periods. The expected annual forces from Anderson and Empedocles (2016) on which the reparametrisation of the model is based and that is presented in table 1 is again presented here for comparison purposes.

Table 6: Annual forces of return on each variable modelled

	Inflation	Long-term interest rates	Short-term interest rates	Excess equity returns
Anderson et al.	5.8%	8.2%	6.8%	4.4%
Maitland's reparameterised model (Mean)	5.7%	7.5%	6.6%	4.6%

The expected annual forces from the simulated results are fairly similar to the values proposed by Anderson and Empedocles (2016) for three of the four series modelled. The biggest difference observed is for long-term interest rates. This difference can possibly be attributed to the way in which Maitland's model is recalibrated, as is described in section 3.4.2. It is left for further research to investigate whether there is a more appropriate manner in which to adjust Maitland's model so that it is appropriate for projection purposes. We are comfortable that the simulated values produced by the model are reasonable and it is therefore used as is without making any further adjustments.

The annual forces for the four series modelled are used to calculate the monthly effective rates of inflation, bond returns and equity returns, as described in section 3.4.2.

Figure 17 reflects the mean effective monthly rates across all simulations at each month in the projection period for inflation, bond returns and equity returns. The volatility of the equity returns is clearly visible in the graphs. Table 7 reflects the mean monthly rates of return from figure 17 calculated across all simulations, as well as all time periods. A conversion of the expected forces from Anderson and Empedocles (2016) that are reflected in table 6, to monthly effective rates, is also presented for comparison purposes.

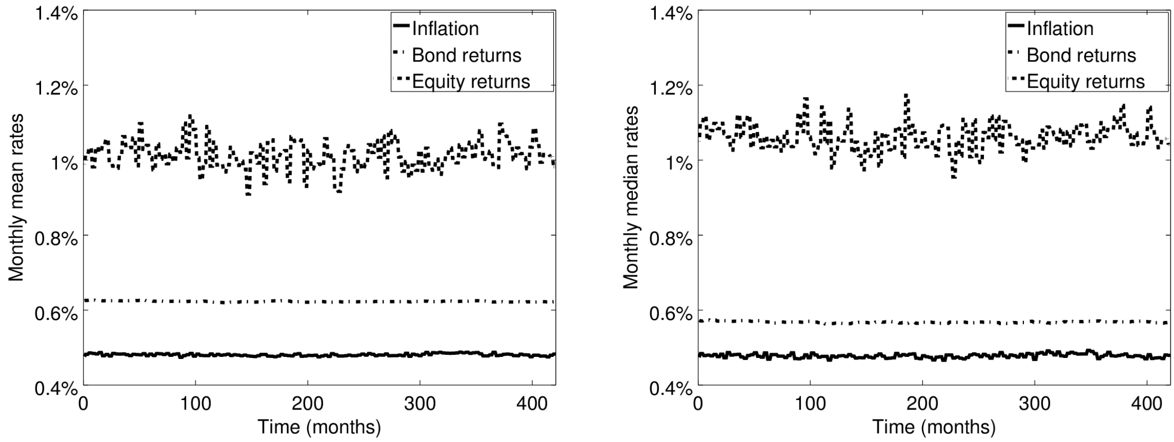


Figure 17: Mean and median effective monthly rates over projection period for each variable

Table 7: Monthly effective rates of inflation, bond returns and equity returns

	Inflation	Bond returns	Equity returns
Anderson et al.	0.49%	0.67%	0.93%
Maitland's reparameterised model (Mean)	0.48%	0.62%	1.01%

The mean equity return in table 7 from Maitland's model differs the most from the returns based on the rates proposed by Anderson and Empedocles (2016), i.e. 1.01% versus 0.93%. This could possibly be attributed to sampling error. Table 40 in Appendix B shows that the error term of the excess equity variable has the largest standard deviations, i.e. 62.1% and 30.6%, compared to the other three variables modelled. The error term of the inflation variable has a standard deviation of only 3.3%. The outcome from Maitland's model is therefore also found to be very close to the inflation value proposed by Anderson and Empedocles (2016), i.e. 0.48% versus 0.49%.

4.3 SBPs

4.3.1 SBP simulations

Figure 18 illustrates, over one simulation, the returns earned on the assets underlying the SBPs versus the monthly bonus rates declared by the SBPs. The returns earned are based on the outcome of the ESG. The bonus rates are calculated using the formulae in section 3.5.3.

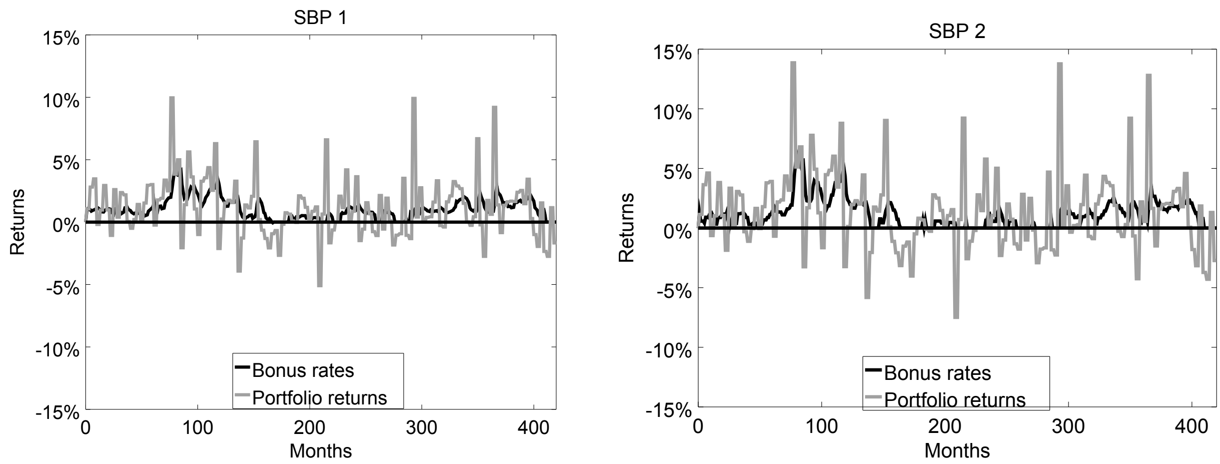


Figure 18: Returns earned on assets underlying SBP versus bonus rates declared by SBP

The reduced volatility of the bonus rates declared for members as a result of the guarantees and the smoothing mechanism compared to the actual returns earned, is clearly visible in the graphs.

Figure 19 illustrates, over one simulation, the BSR level at the end of each month over the projection period. The loan balance of capital injections received by the product is also reflected in the graphs.

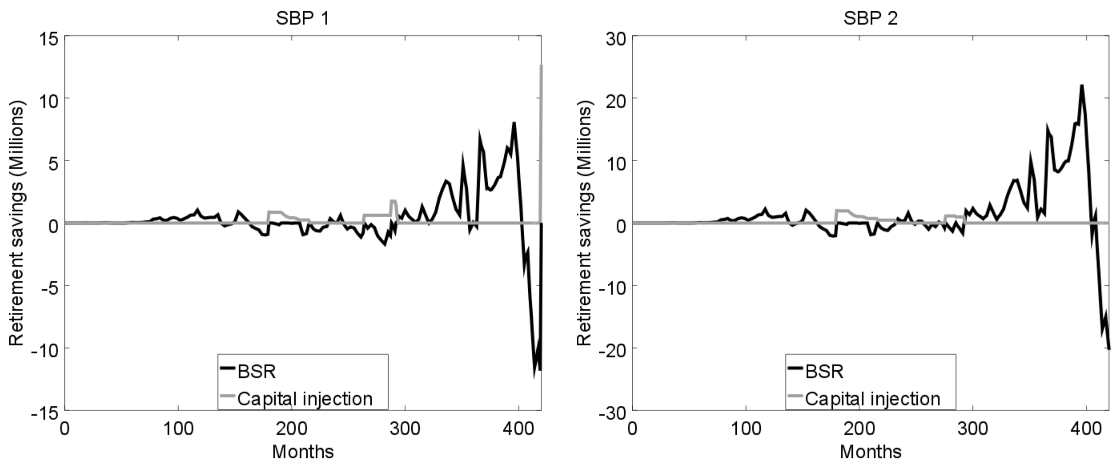


Figure 19: BSR account balances and capital injections to be repaid

In this simulation, the BSR is negative at retirement for SBP 2. For SBP 1, a capital injection is made at retirement, pushing the value of the BSR back to zero. For both SBP 1 and SBP 2 the member will benefit from receiving a book value that is greater than the market value of the assets underlying the product on retirement.

Figure 20 illustrates, over one simulation, the market values of the assets underlying the SBP, as well as the book values at the end of every month over the projection period. Again, the reduced volatility in the member's level of savings from the risk mitigation techniques applied by the SBP is clearly observable from the smoother book values curves.

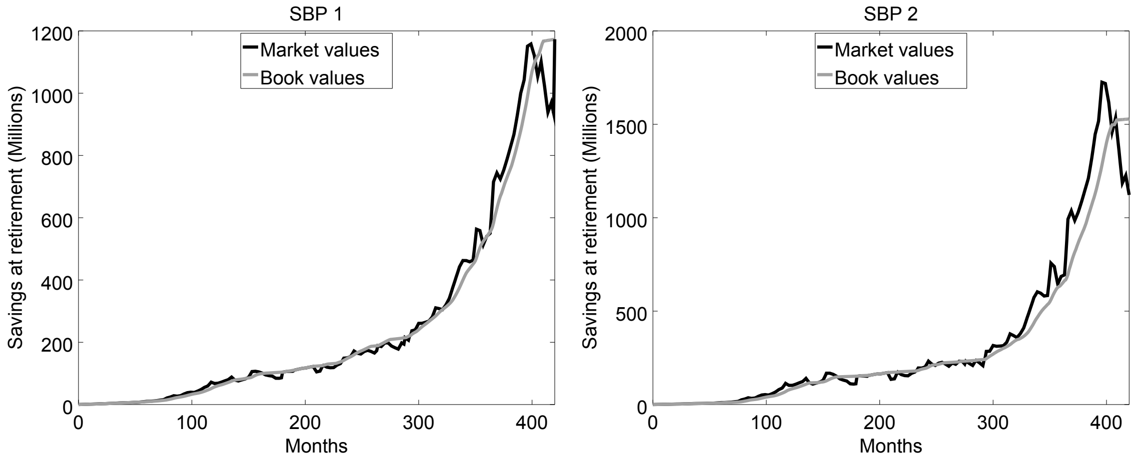


Figure 20: Market value versus book value account balances of SBP 1 and SBP 2

The months where the book value exceeds the market value are the times when a member benefits from not sharing in negative investment performance.

4.3.2 SBP scenarios modelled

Figure 21 reflects histograms for the distributions of the savings at retirement outcomes from the simulations of SBP 1 and SBP 2.

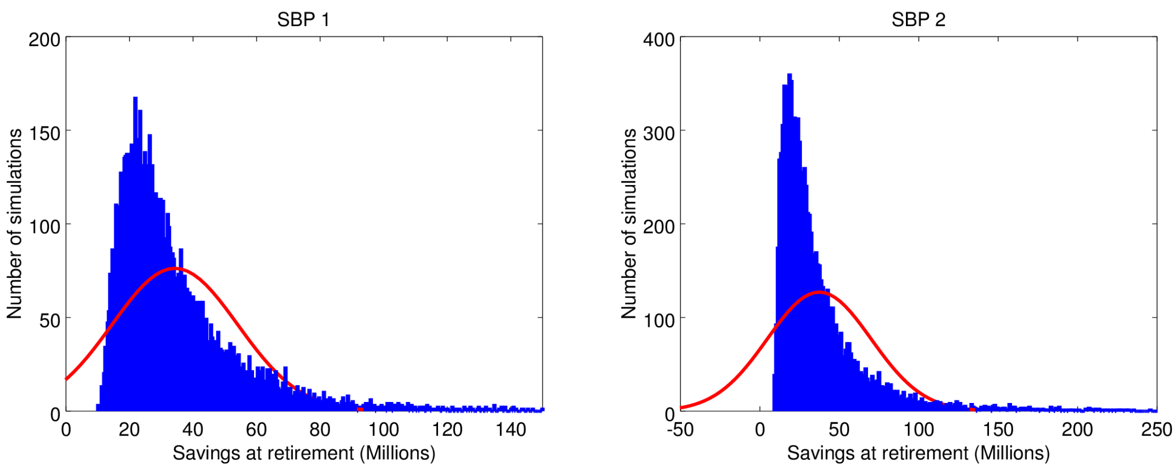


Figure 21: Histograms of the distributions of the savings at retirement for SBP 1 and SBP 2

The distribution of the savings at retirement of SBP 1 and SBP 2 are significantly positively skewed with coefficients of skewness of 2.9 and 6.7 respectively. This is because of the downside protection provided by the guarantee.

4.3.3 Application of utility theory

For each SBP simulated, the utility values of the savings at retirement are calculated by applying a CRRA power utility function with a risk parameter of $\gamma = 2$, as discussed in section 2.4.3. The utility values calculated are scaled for presentation purposes by multiplying it with a constant, as is discussed in section 3.7.2.

Figure 22 is a scatter plot of the savings at retirement for SBP 1 and SBP 2 against the utility value associated with those savings.

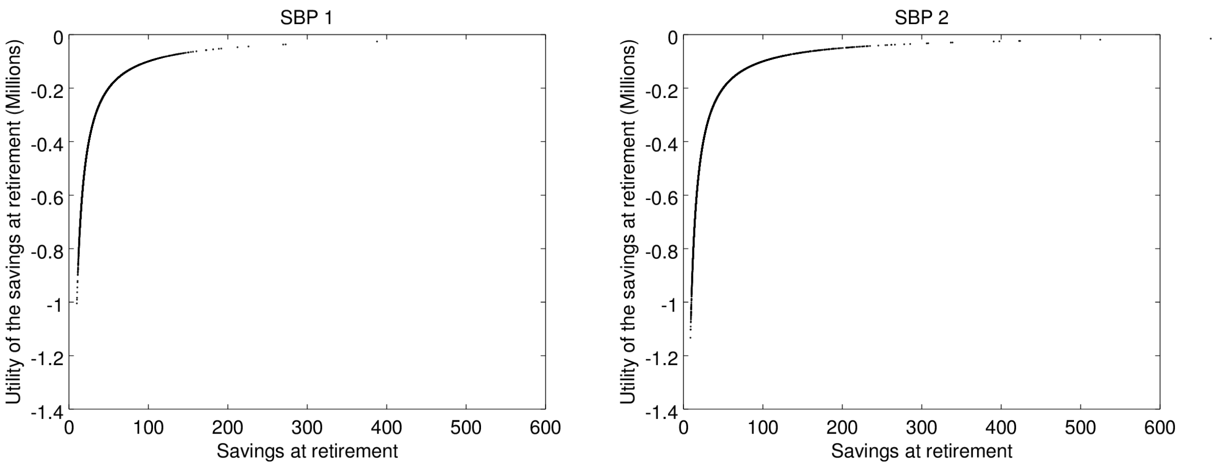


Figure 22: Scatter plot of savings at retirement against its utility values for SBP 1 and SBP 2

Assuming members have a concave utility curve implies assuming that they attach a higher utility value to a higher outcome, but that the marginal increase in utility as the savings at retirement increase, decreases. A lower marginal utility is therefore attached to each additional unit of retirement savings as the level of savings increase. At some point, i.e. where the curves in figure 22 flatten out horizontally, the additional utility attached to an additional unit of savings is so small, that it almost becomes negligible.

Figure 23 reflects histograms of the distributions of the savings at retirement for SBP 1 and SBP 2 that have been transformed into utility values.

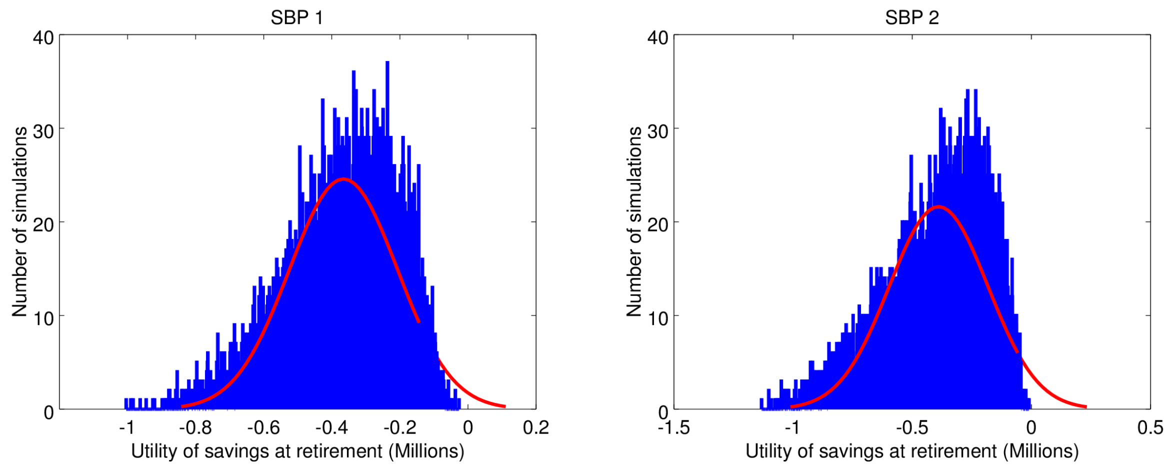


Figure 23: Histograms of the distributions of utilities values of the savings at retirement for SBP 1 and SBP 2

The histograms for both SBP 1 and SBP 2 are negatively skewed with coefficients of skewness equal to -0.6 and -0.7 respectively. This is because of the fact that at some point, i.e. where the curves in figure 22 flatten out, the marginal utility tends to zero.

4.4 Risk measures

4.4.1 Histograms of savings at retirement

Figures 24 and 25 are duplicates of figures 21 and 23, but with the riskiness of these portfolios now indicated on the graphs. The gray areas in the figures reflect the areas that are included in the calculation of the CTE, with the right edges of the gray areas reflecting the VaR.

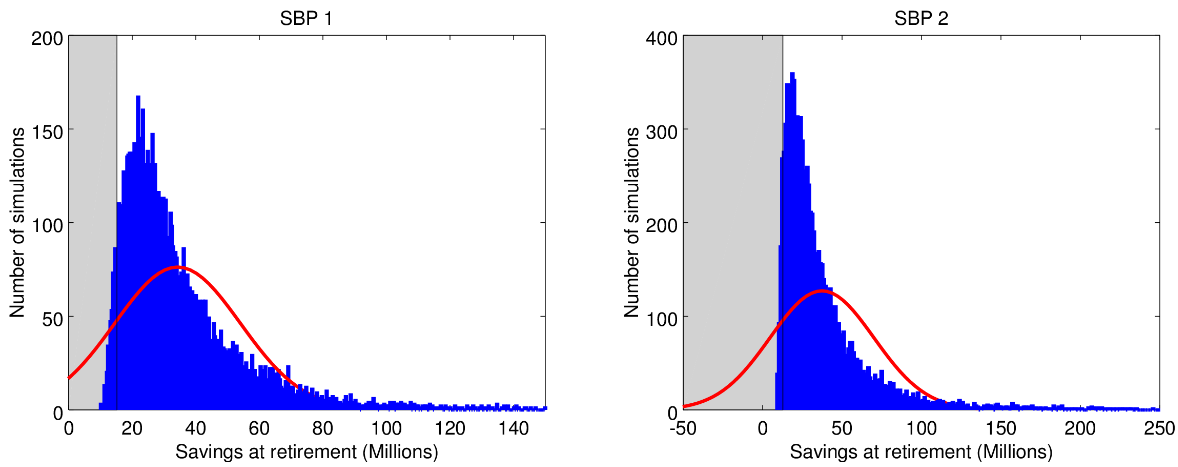


Figure 24: Histograms of the distributions of the savings at retirement for SBP 1 and SBP 2 and its risk measures

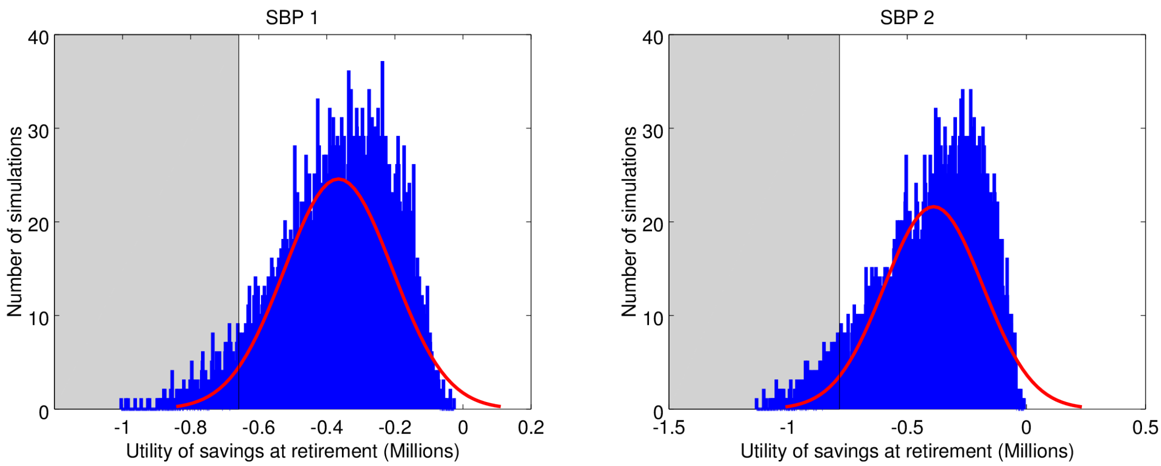


Figure 25: Histograms of the distributions of utility values of the savings at retirement for SBP 1 and SBP 2 and its risk measures

The actual values of the VaR and CTE in figures 24 and 25 are reflected in the column with the heading 'SBP' in figure 56 in Appendix C. These risk measures are used in the calculation of the return/risk ratios that are discussed in section 4.5, as well as the construction of the notional benchmark portfolios that are discussed in section 4.6.

4.5 Return/risk analysis

4.5.1 Return analysis

Table 8 reflects the IRRs calculated for each of the permutations of SBP 1 and SBP 2 discussed in section 3.9.1.

Table 8: IRRs of the permutations of the SBPs modelled

Portfolios	SBP 1		SBP 2	
	Mean	Median	Mean	Median
1. SBP in its basic form	0.77%	0.70%	0.81%	0.69%
2. SBP with no capital charge	0.90%	0.83%	1.03%	0.90%
3. SBP with no guarantee and no capital charge	0.88%	0.79%	1.00%	0.86%
4. SBP with no guarantee or any fees	0.91%	0.82%	1.03%	0.89%
5. Market linked portfolio	0.91%	0.83%	1.06%	0.91%

When capital charges are removed, i.e. in portfolios 2, the IRRs are higher than the IRRs of portfolio 1, as would be expected. When the guarantee is removed, i.e. in portfolio 3, the IRRs drop relative to the IRRs of portfolio 2, since the member in that instance shares in downturns. Removing the product management fee, i.e. in portfolio 4, increases the IRRs. The difference in the IRRs of portfolios 4 and 5 reflects the impact of the smoothing mechanism on the returns of the SBPs. The skewness observed in figure 21 is also reflected in table 8 in the differences in the mean and median IRRs observed.

Using these IRRs and calculating the differences between it, the cost and benefit impact of each of the three features of a SBP can be determined, as is discussed in section 3.9.1. Table 9 reflects the outcome from this analysis. The results from this table are illustrated in figures 26 and 27. These figures link back to figure 13 but in this case, is populated with results. The returns without brackets are based on the mean retirement savings, while the returns in brackets are based on median retirement savings.

Table 9: Analysis of the monthly IRRs of SBP 1 and SBP 2

	SBP 1		SBP 2	
	Mean	Median	Mean	Median
Investment strategy	0.91%	0.83%	1.06%	0.91%
Guarantee				
Cost	-0.13%	-0.13%	-0.22%	-0.21%
Benefit	0.03%	0.03%	0.03%	0.04%
Smoothing				
Cost	-0.03%	-0.03%	-0.03%	-0.03%
Benefit	-0.01%	-0.00%	-0.03%	-0.02%
Return on SBP	0.77%	0.70%	0.81%	0.69%

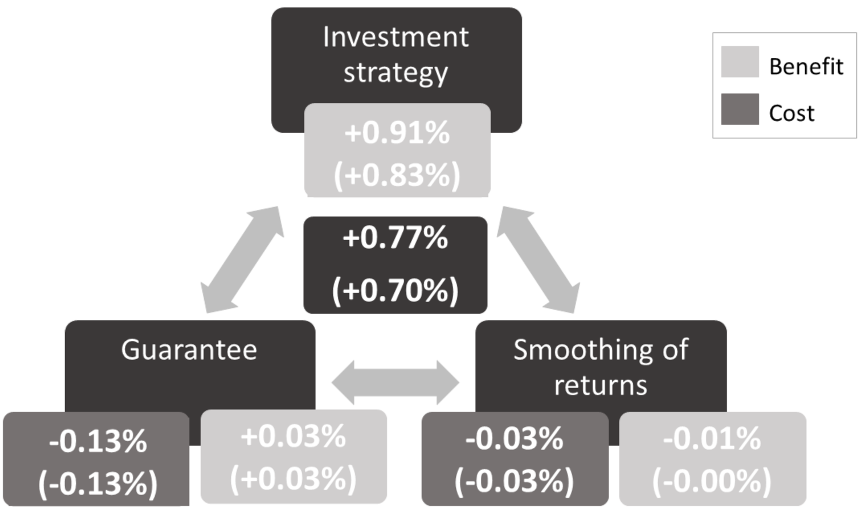


Figure 26: Analysis of monthly mean and median IRRs on SBP 1

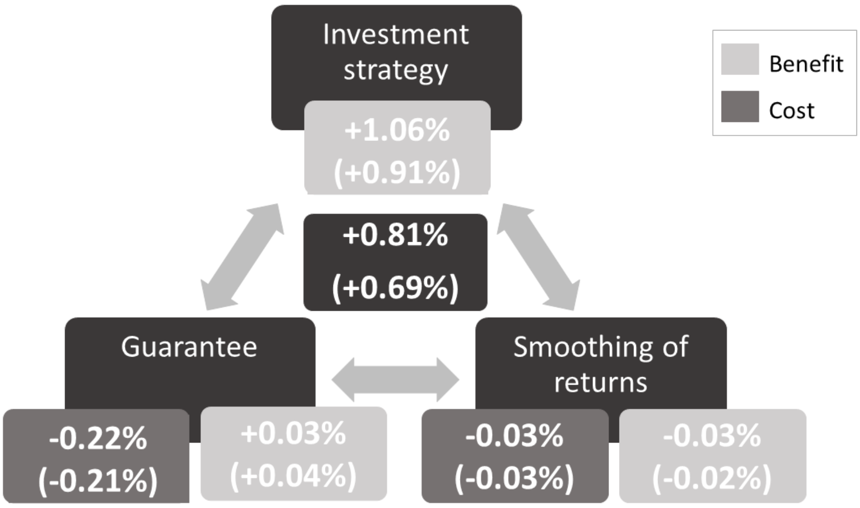


Figure 27: Analysis of monthly mean and median IRRs on SBP 2

The values in the middle of figures 26 and 27 are the IRRs of the SBPs based on the mean (and in brackets the median) savings at retirement. This is split between costs and benefits impacts for each features of a SBP, reflected in the five values around the middle value.

Being able to invest more aggressively does bring about the advantage of generating higher returns for members. The investment strategy contributes 0.91% p.m. (0.83% p.m. based on the median savings at retirement) for SBP 1, and 1.06% p.m. (0.91% p.m. on a median basis) for SBP 2.

From a return-only perspective, both the guarantee and smoothing mechanisms detract from performance. This is not unexpected since, as is discussed in section 3.1, reduced risk exposure is expected to be associated with a reduced return.

The charge for the guarantee exceeds the benefit of not sharing in negative performance by 0.1% p.m. on a mean and median basis for SBP 1, and by 0.19% p.m. and 0.17% p.m. for SBP 2 a mean and median basis respectively. The benefit of the guarantee is limited by the fact that the injection of shareholder capital in a downturn does not represent a permanent injection, but needs to be refunded once the BSR becomes positive again.

Over the long-run, if the bonus formula aims to allocate all of the returns earned on the underlying assets to a member, the smoothing mechanism should have no impact on the cumulative returns earned by the member other than the explicit cost, i.e. the product management fee, associated with it. The net impact of the smoothing mechanism in this case is found to be negative. This suggests that on average accross all simulations, a small part of a member's asset share is retained in the BSR when they retire.

As mentioned in section 3.9.1, the above analysis fails to relate the costs or benefits observed from a particular feature of an SBP with the change in risk exposure as a result of the inclusion of that particular feature. Limited conclusions can therefore be drawn from the above results.

4.5.2 Return analysis incorporating risk

In the previous section it is observed that applying the risk mitigation strategies of an SBP comes at a cost. However, the question is whether the level of upside sacrificed in achieving reduced risk exposure is appropriate.

A return/risk ratio for a portfolio is calculated by dividing the mean savings at retirement by its adjusted risk measure, as explained in section 3.9.2. The contribution each of the three features of an SBP makes to its performance on a risk adjusted basis can be calculated using the return/risk ratios calculated for each permutation of an SBP, as illustrated in figure 14. The outcome of this analysis is reflected in tables 10 and 11, for SBP 1 and SBP 2 respectively. The results from these tables are also illustrated in figures 28 to 31. These figures link to figure 14 but in this case are populated with results.

It needs to be taken into consideration that risk in this instance, because of the way in which the return/risk ratio is calculated, only refers to the volatility of the outcome at retirement.

Table 10: Analysis of the return/risk ratios of SBP 1

	Adjusted VaR		Adjusted CTE	
	Mean	Median	Mean	Median
Investment strategy	1.51	1.73	1.40	1.55
Guarantee	0.22	0.31	0.22	0.31
Smoothing	0.06	0.08	0.05	0.06
TOTAL	1.79	2.12	1.67	1.92

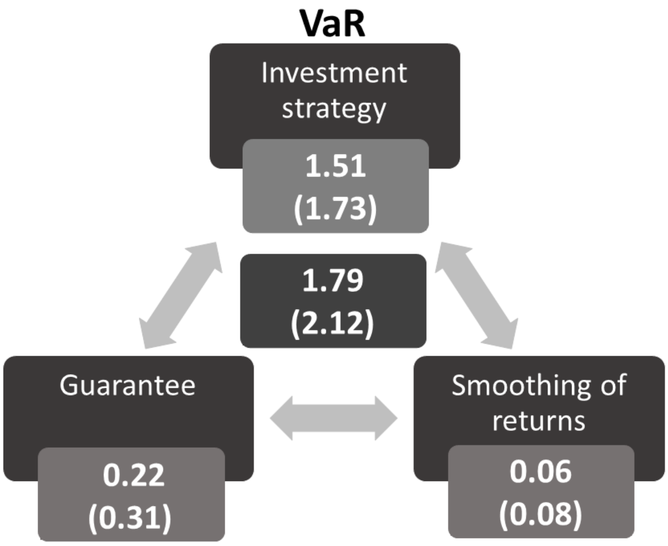


Figure 28: Analysis of the return/risk ratios of SBP 1 based on its VaR

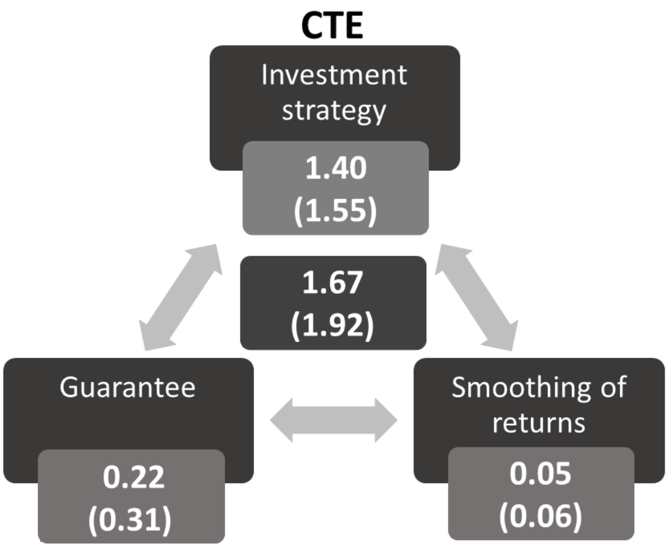


Figure 29: Analysis of the return/risk ratios of SBP 1 based on its CTE

The values in the middle of the figures are the return/risk ratios of the SBPs based on the mean (and in brackets the median) savings at retirement. This is split between the contribution made by the investment strategy, guarantee and smoothing mechanisms. The values linked to each of the three features give the contribution that each of the three features make to the return/risk ratio of the SBP. The middle value is therefore the sum of these three values around it.

On a risk-adjusted basis, using VaR as a risk measure, SBP 1 generates 1.79 units of return per unit of risk (2.12 using the median savings at retirement). Where CTE is used as a risk measure, the return drops to 1.67 units of return per unit of risk using the mean savings at retirement (1.92 using the median savings at retirement).

Table 8 shows that the risk mitigation tools of SBP 1 detract from its performance. However, taking the risk into consideration, the risk mitigation tools of an SBP are found to make a positive contribution to its return/risk ratio.

The guarantee adds 0.22 units of return per unit of risk (0.31 using the median savings at retirement) for both risk measures. The contribution of the smoothing mechanism, though still positive, is much smaller than the contribution the guarantee makes to the return/risk ratio of SBP 1. Therefore although both these features detract from returns in absolute terms, it makes a positive contribution to the risk-adjusted return of SBP 1.

Similar results are observed for SBP 2 compared to SBP 1, as reflected in table 11 and figures 30 and 31.

Table 11: Analysis of the return/risk ratios of SBP 2

	Adjusted VaR		Adjusted CTE	
	Mean	Median	Mean	Median
Investment strategy	1.26	1.45	1.20	1.32
Guarantee	0.23	0.35	0.22	0.34
Smoothing	0.03	0.03	0.02	0.03
TOTAL	1.52	1.83	1.44	1.69

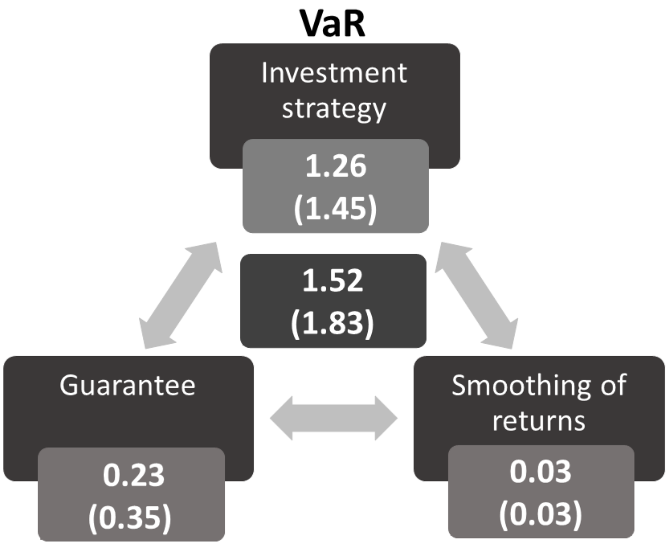


Figure 30: Analysis of the return/risk ratios of SBP 2 based on its VaR

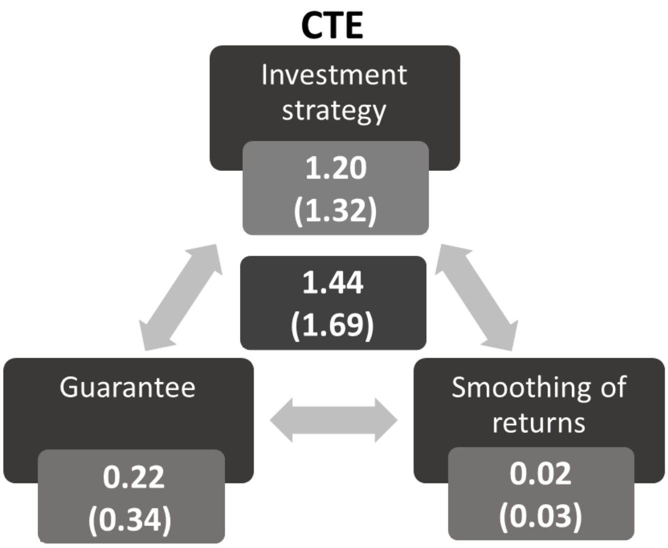


Figure 31: Analysis of the return/risk ratios of SBP 2 based on its CTE

The guarantee and smoothing mechanisms also both make positive contributions to the return/risk ratio of SBP 2, but the smoothing mechanism makes an even smaller contribution than is observed for SBP 1.

The investment strategy of SBP 2 allocates 80% of assets to equity compared to a 56% allocation for SBP 1 (as reflected in table 5). SBP 2's investment strategy however generates a lower return per unit of risk than SBP 1's does. This implies that an increased allocation to equity, and therefore an increased exposure to the risk of observing a more volatile outcome, is accompanied by a less than proportionate increase in returns. Figure 32 illustrates that this is indeed the case.

Constant weight market linked portfolios with an allocation of assets to equity ranging from zero percent to 100% are simulated. The riskiness and the return/risk ratios of these portfolios are then calculated and reflected in the figure. The x-axis in the figure reflects the portion of assets in a constant weight market linked portfolio allocated to equity. The y-axis reflects the return/risk ratios of those portfolios, where the returns are the mean savings at retirement and risk is measured by the adjusted VaR.

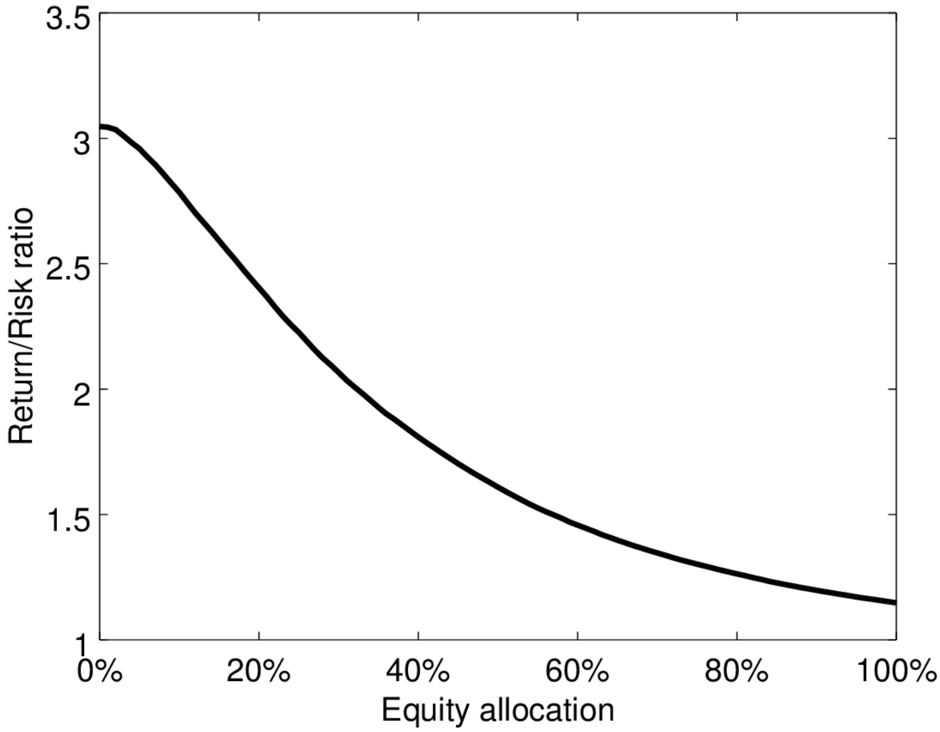


Figure 32: Return/risk ratios for constant weight market linked portfolios

A straight horizontal line would indicate that the return/risk ratio is constant regardless of equity allocation, i.e. that the return and risk both increase in the same proportion for each additional unit of funds allocated to equity. The downward sloping non-linear curve however, indicates that for each additional unit of funds allocated to equity, the return/risk ratio, i.e. the return per unit of risk, decreases. Therefore, as the allocation to equity increases, investors are compensated less and less on a risk adjusted basis.

Although this graph might suggest that one should not invest in equity at all, one needs to remember that the maximum return that a 100% bond portfolio is able to generate, might not be sufficient for most investors. Therefore to earn a satisfactory amount of returns, some risk will have to be taken.

Similar results are obtained where the adjusted CTE is used as a risk measure. This is observable from figure 60 in Appendix D. As mentioned in section 3.9.2, because the return/risk ratios of SBP 1 and SBP 2 are not compared against the return/risk ratios of other portfolios, we are not able to draw conclusions as to the SBP's effectiveness relative to other alternatives when applying this methodology.

4.6 Notional benchmark portfolios

The investment strategies for the two types of notional benchmark portfolios constructed can range from a zero percent allocation to equity and 100% allocation bonds, to a 100% allocation to equity and no allocation to bonds. The market linked portfolio's allocation to these asset classes is assumed to remain constant over the projection period up to retirement. The lifestage portfolio's allocation remains constant up to five years prior to retirement, after which the allocation to equity tapers down linearly to zero in the last five years prior to retirement.

4.6.1 Range of possible notional benchmark portfolios

Figure 33 plots the riskiness inherent in the range of possible investment strategies for the notional benchmark portfolios (on the y-axis) against its strategic allocation of assets to equity (on the x-axis). The balance of the assets is assumed to be invested in bonds. Risk in this instance is measured as the VaR and CTE of the portfolio. Figure 34 is a similar plot to figure 33. However, risk in this instance is measured by the adjusted VaR and adjusted CTEs, that are discussed in section 3.9.2.

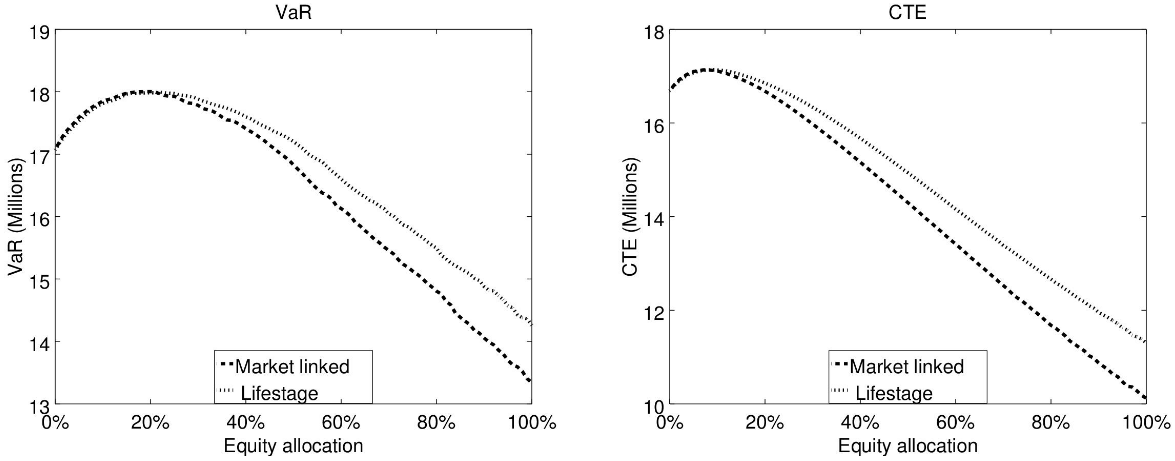


Figure 33: VaRs and CTEs of possible notional benchmark portfolios assuming $\gamma = 0$

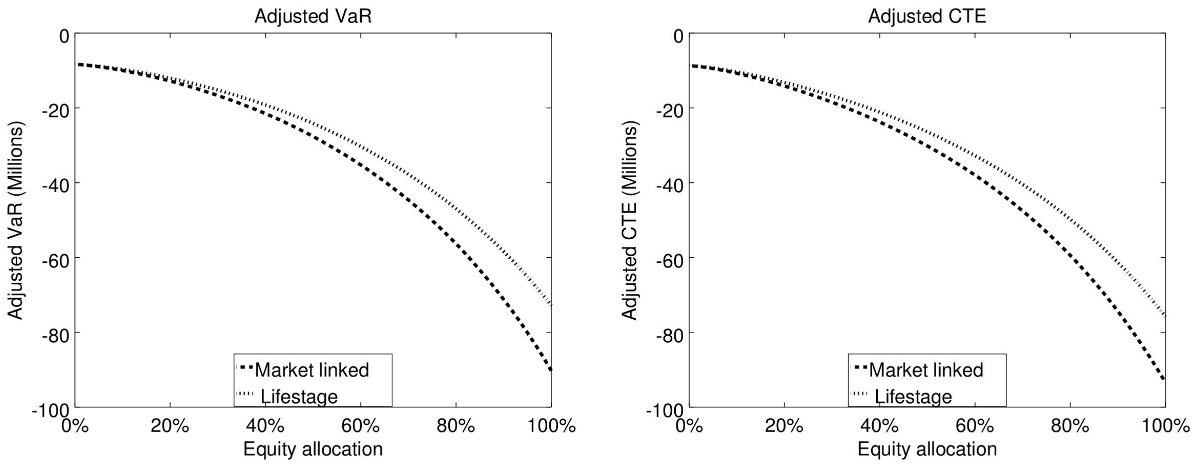


Figure 34: Adjusted VaRs and adjusted CTEs of possible notional benchmark portfolios assuming $\gamma = 0$

The downward slopes of these four curves of possible notional benchmark portfolios make intuitive sense. This is because as the allocation to equity is increased, the overall riskiness of the portfolio is expected to increase, which is reflected by lower VaRs, CTEs, adjusted VaRs and adjusted CTEs.

The curves for the lifestage notional benchmark portfolios consistently lie on or above the market linked benchmark portfolios, indicating that this strategy is the less risky of the two. This also makes intuitive sense because reducing the allocation of assets to the risky asset class over the five years prior to retirement, compared to a strategy that retains its allocation to equity, should reduce the overall riskiness of the strategy.

The above graphs assume members have a linear utility curve. The range of possible notional benchmark portfolios assuming members have a concave utility curve are reflected in the graphs in figures 61 and 62 in Appendix E. Those curves have similar shapes to the curves above.

4.6.2 Riskiness of SBP 1 and SBP 2

In trying to set up a notional benchmark portfolio that has the same level of risk as the SBP, the following opposing impacts play out on the choice of asset allocation strategy for the notional benchmark portfolio. The VaR and CTE of a distribution are affected by both the range of the values of the distribution, as well as the kurtosis of the distribution (refer back to the discussion on the two elements of investment risk captured by the VaR and CTE, discussed in section 2.5.1).

1. Matching the volatility of the outcome

   The risk mitigation strategies applied by the SBP reduces the risk of a more volatile outcome at retirement. Tables 10 and 11 show that the guarantee and smoothing mechanism make a positive contribution to the risk adjusted return. Therefore, for the benchmark portfolios to have a similar level of volatility as the SBP, or stated otherwise, to ensure its distribution of outcomes isn't wider than that of the SBP, a lower exposure to equity than that of the SBP is needed.

2. Matching the level of the outcome

   Table 9 shows that the risk mitigation strategies applied by SBPs come at a cost. The risk of observing a lower outcome at retirement is therefore greater because of these costs. If the distribution of the SBP is pushed to the left because of these costs, a lower VaR and CTE will be observed. For the notional benchmark portfolios to match these levels of the VaR and CTE, its distributions needs to be flattened so that it has values particularly at the lower end of the distribution. This implies having to increase its riskiness and therefore its exposure to equity.

We see these opposing impacts on the asset allocation strategies of the notional benchmark portfolios playing out in the next sections.

Linear utility curve

Figures 35 and 36 show the riskiness of market linked and lifestage portfolios with the allocation of assets to equity reflected on the x-axis. These graphs are similar to figures 33 and 34, except that included in these graphs are dots plotting the riskiness of SBP 1 and SBP 2 against their strategic allocation of assets to equity.

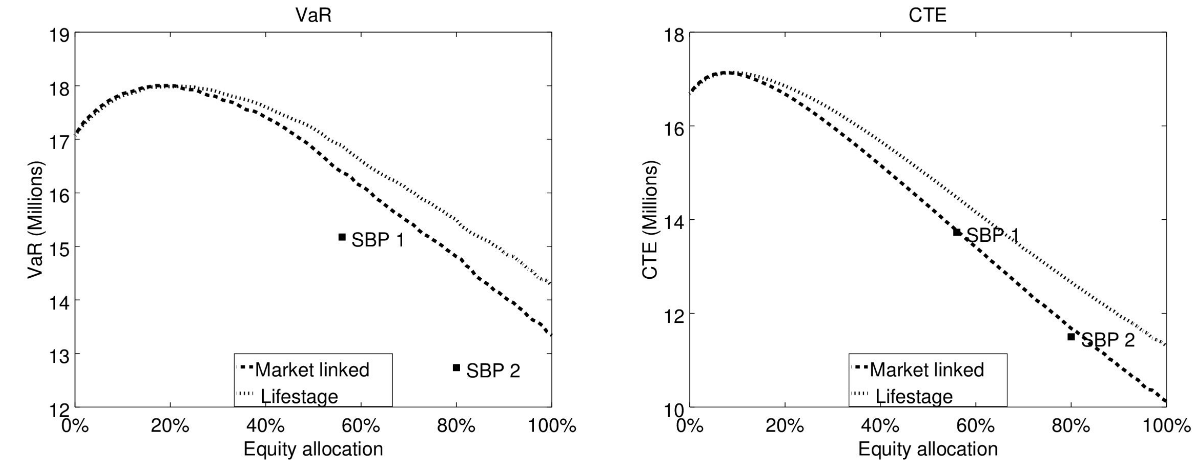


Figure 35: VaRs and CTEs of possible notional benchmark portfolios and the SBPs assuming $\gamma = 0$

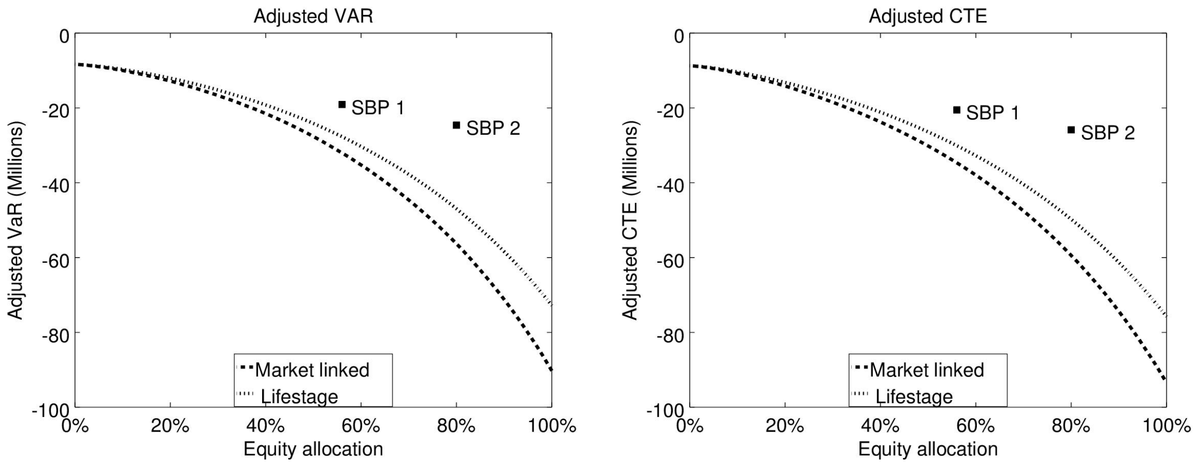


Figure 36: Adjusted VaRs and adjusted CTEs of possible notional benchmark portfolios and the SBPs assuming $\gamma = 0$

If horizontal lines are drawn through the dots for SBP 1 and SBP 2 respectively, the notional benchmark portfolios, which are required to have the same level of risk as the SBPs, would be the points where these straight lines cross the market linked and lifestage curves.

The fact that the dots for SBP 1 and SBP 2 lie above the market linked and lifestage curves in figure 36 imply that the horizontal line crosses these curves to the left of the dots for SBP 1 and SBP 2. The notional benchmark portfolios therefore have lower equity allocations than the SBPs. From the introduction to this section and given that this scenario takes risk only to include the volatility of the outcome at retirement, this lower allocation to equity is as expected.

Where the risk measure incorporates the risk of observing a lower return, as is the case in figure 35, the points for the SBPs lie on or below the market linked and lifestage curves. This implies that the notional benchmark portfolios with a similar degree of risk would lie on or to the right of SBP 1 and SBP 2, i.e. the horizontal lines through the dots for SBP 1 and SBP 2 would cross the market linked and lifestage curves to the right of those dots. The benchmark portfolios therefore have a higher allocation of assets to equity.

In this case, the cost associated with the risk mitigation tools of the SBPs lead to such a low VaR and CTE, that for the notional benchmark portfolios to have a similar VaR and CTE, its allocation to equity needs to increase in order for the kurtosis of its distribution to increase, i.e. to increase the number of outliers.

From figure 35 it can be seen that SBP 2 has such a low VaR, and is therefore so risky, that no notional benchmark portfolio can be constructed to match it's riskiness, i.e. a horizontal line through the dots for SBP 2 doesn't cross the market linked and lifestage curves. Even if 100% of the assets were allocated to equity for the market linked and lifestage portfolios, its distribution would not have a sufficient number of outliers that are as low as the values in the tails of SBP 2.

Concave utility curve

Figures 37 and 38 are similar to figures 61 and 62 in Appendix E, except that included in these graphs are dots indicating the riskiness of SBP 1 and SBP 2. This is plot against their strategic allocation of assets to equity.

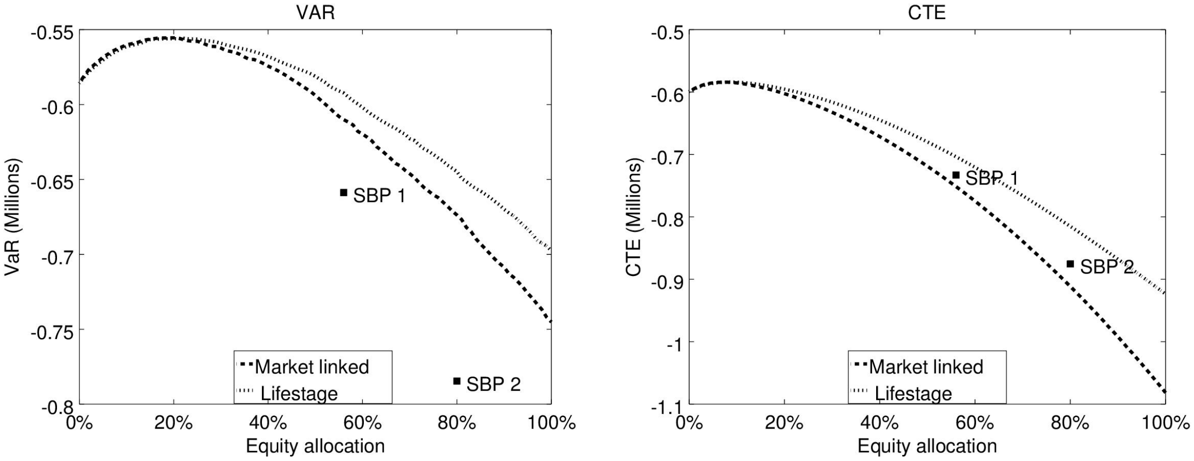


Figure 37: VaRs and CTEs of possible notional benchmark portfolios and the SBPs assuming $\gamma = 2$

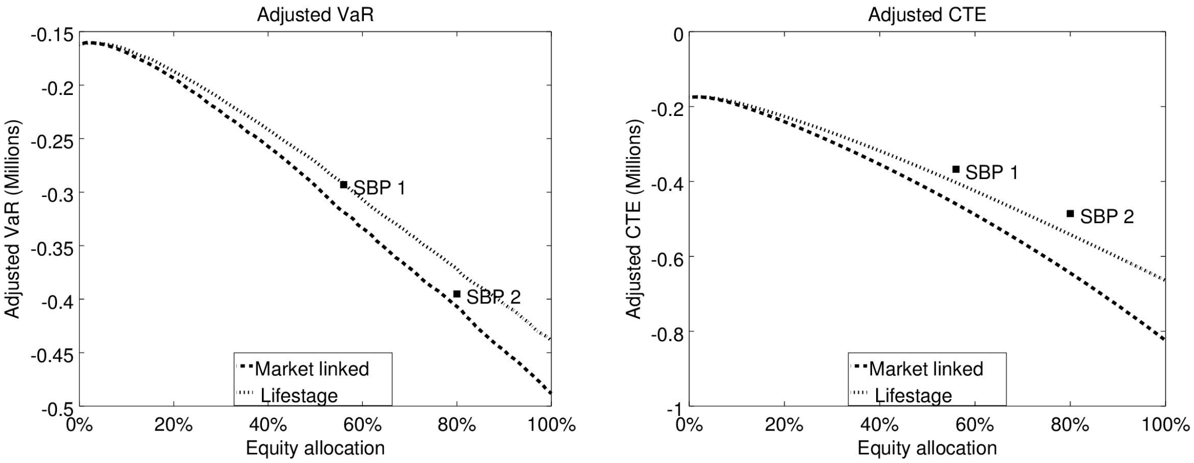


Figure 38: Adjusted VaRs and adjusted CTEs of possible notional benchmark portfolios and the SBPs assuming $\gamma = 2$

The location of the dots for SBP 1 and SBP 2 relative to the market linked and lifestage curve are fairly similar between the scenario assuming a linear utility curve in figure 35 and the scenario assuming a concave utility curve in figure 37. The main difference between the location of the dots is that in the latter scenario, the dots for both SBPs lie to the right of the market linked notional benchmark portfolio using CTE as a risk measure. This implies that for the notional benchmark portfolios to have the same level of risk as the SBP, using CTE as a risk measure, these portfolios will have a lower rather than a higher allocation of assets to equity. This suggests that the risk of observing a more volatile outcome at retirement makes a greater contribution to the CTE of the SBPs than the risk of observing a lower outcome.

Using the adjusted risk measures to construct notional benchmark portfolios, figures 36 and 38 are also fairly similar. The main difference is that the allocation of assets to equity of the lifestage portfolio, using the adjusted VaR as a risk measure, will be higher relative to the SBP when a concave utility curve is assumed than when a linear utility curve is assumed.

Appendix C contains the detailed results from the pension fund simulations. Tables 56 to 59 reflects the details of the notional benchmark portfolios that have similar degrees of risk as each of portfolios one to four described in section 3.9.1. The asset allocations, mean and median savings at retirement, and IRRs for the notional benchmark portfolios are provided. The relevant result are analysed in section 4.7 and section 4.8.

4.7 Comparison of expected outcomes

This section splits the analysis of results into two parts. Section 4.7.1 considers the case where members are assumed to have a linear utility curve, i.e. $\gamma = 0$, while section 4.7.2 assumes members have a concave utility curve, i.e. $\gamma = 2$.

4.7.1 Linear utility curve

Tables 12 to 15 reflect the excess return of SBP 1 and SBP 2 over its four notional benchmark portfolios, first based on the mean savings at retirement and then the median savings. In addition, the returns generated are split between the features of the portfolios. For the SBPs, the returns presented are the same as is reflected in table 9, only that the cost and benefit of the guarantee and smoothing mechanisms respectively are summed together.

The tables also reflect the strategic allocation of assets to equity of the notional benchmark portfolios, i.e. the values on the x-axis where the horizontal lines through the SBP dots in figures 35 and 36 cross the market linked and lifestage portfolio curves. The remaining balance of the assets is assumed to be invested in bonds.

SBP 1

The investment strategy for all four notional benchmark portfolios is either as or more aggressive than the SBP. This is seen from the higher allocation of assets to equity.

The excess return earned by SBP 1 is consistently negative over all four of its benchmark portfolios, based on both the mean and median savings at retirement. The market linked notional benchmark portfolios outperform SBP 1 because of the following reasons.

- The more aggressive investment strategies lead to higher or at least the same mean and median returns generated by the benchmark portfolios, i.e. 0.91% and 1.02% compared to 0.91%.
- In addition, the guarantee and smoothing mechanism detract from the performance of the SBP by 0.14% p.m. based on the mean savings at retirement and 0.12% p.m. based on the median savings at retirement.

Table 12: Analysis of IRRs of SBP 1 and its notional benchmark portfolios based on VaR and CTE

	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.77%	1.02%	0.91%	1.04%	0.93%
Investment strategy	0.91%	1.02%	0.91%	1.09%	0.97%
Guarantee	-0.10%				
Smoothing	-0.04%				
Lifestaging				-0.05%	-0.04%
Excess return of SBP		-0.24%	-0.14%	-0.26%	-0.16%
Median					
Portfolio return	0.70%	0.89%	0.83%	0.89%	0.83%
Investment strategy	0.83%	0.89%	0.83%	0.92%	0.86%
Guarantee	-0.09%				
Smoothing	-0.03%				
Lifestaging				-0.03%	-0.03%
Excess return of SBP		-0.19%	-0.13%	-0.19%	-0.13%
Equity allocation	56%	74%	56%	85%	66%

The lifestage portfolios generate a higher outcome than SBP 1, because of the following.

- A more aggressive investment strategy is followed, leading to higher returns, i.e. 0.97% and 1.09% compared to 0.91%.

- The cost of tapering down the allocation to equity close to retirement is very similar to the cost of smoothing in the SBP. The lifestage strategy however doesn't incur the additional cost associated with a guarantee.

Table 13 reflects the returns on the notional benchmark portfolios that have a degree of investment risk equal to the adjusted risk measures of SBP 1. The outcomes at retirement of all five portfolios included in the table are therefore equally volatile.

The more aggressive investment strategy followed by SBP 1, i.e. 56% allocation to equity versus an allocation of about 35% on the market linked notional benchmark portfolios, generates a higher return on both a mean and median basis compared to the market linked portfolios. However, the cost of managing the risk, i.e. the guarantee and smoothing mechanism, erodes the extra return generated to such an extent that the returns on the market linked portfolios are still higher than that of SBP 1.

Table 13: Analysis of IRRs of SBP 1 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.77%	0.80%	0.80%	0.80%	0.80%
Investment strategy	0.91%	0.80%	0.80%	0.83%	0.82%
Guarantee	-0.10%				
Smoothing	-0.04%				
Lifestaging				-0.02%	-0.02%
Excess return of SBP		-0.03%	-0.02%	-0.03%	-0.03%
Median					
Portfolio return	0.70%	0.75%	0.74%	0.75%	0.74%
Investment strategy	0.83%	0.75%	0.74%	0.77%	0.76%
Guarantee	-0.09%				
Smoothing	-0.03%				
Lifestaging				-0.02%	-0.02%
Excess return of SBP		-0.05%	-0.04%	-0.05%	-0.04%
Equity allocation	56%	35%	34%	40%	39%

In the case of the lifestage notional benchmark portfolios, the allocation to equity is approximately 40%, i.e. 16% lower than that of SBP 1. The more aggressive strategy followed by SBP 1 does generate a higher return than the asset allocation strategy of the lifestage portfolios. However, this outperformance is not sufficient to cover the cost associated with the guarantee and smoothing mechanism, even when the cost of tapering down the allocation of assets to equity in the lifestage portfolios is taken into consideration. The lifestage portfolios are therefore found to outperform SBP 1 in all instances.

In summary, considering the results from both tables 12 and 13, the analysis suggests that SBP 1 is not effective at reducing risk exposure. The notional benchmark portfolios are, with the simple risk management strategies it applies, able to achieve an equal or in most instances, higher level of return than what SBP 1 does.

SBP 2

Table 14: Analysis of IRRs of SBP 2 and its notional benchmark portfolios based on VaR and CTE

	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.81%	-	1.07%	-	1.11%
Investment strategy	1.06%	-	1.07%	-	1.17%
Guarantee	-0.19%				
Smoothing	-0.06%				
Lifestaging				-	-0.06%
Excess return of SBP		-	-0.26%	-	-0.30%
Median					
Portfolio return	0.69%	-	0.92%	-	0.93%
Investment strategy	0.91%	-	0.92%	-	0.96%
Guarantee	-0.17%				
Smoothing	-0.05%				
Lifestaging				-	-0.04%
Excess return of SBP		-	-0.23%	-	-0.24%
Equity allocation	80%	-	82%	-	97%

Only two notional benchmark portfolios, based on CTE as a risk measure can be constructed for SBP 2 (as is discussed in section 4.6.2). Both these portfolios have a higher allocation of assets to equity. This again seems to imply that the risk of observing a lower outcome at retirement, due to the costs associated with the risk mitigation strategies employed, makes a greater contribution to the overall riskiness of the product than the risk of observing a more volatile outcome (refer to the discussion in section 4.6.2).

As is the case with SBP 1, all the notional benchmark portfolios also outperform SBP 2. The more aggressive investment strategies generate higher outcomes, even when including the cost of tapering the allocation of assets to equity down in the case of the lifestage benchmark portfolio.

Table 15 reflects the returns on the notional benchmark portfolios that have a degree of investment risk equal to the adjusted risk measures of SBP 2, i.e. where only the volatility of the outcome at retirement is considered. Similar conclusions can be drawn when compared to table 13 for SBP 1.

Table 15: Analysis of IRRs of SBP 2 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.81%	0.85%	0.84%	0.86%	0.85%
Investment strategy	1.06%	0.85%	0.84%	0.89%	0.87%
Guarantee	-0.19%				
Smoothing	-0.06%				
Lifestaging				-0.03%	-0.03%
Excess return of SBP		-0.04%	-0.03%	-0.05%	-0.04%
Median					
Portfolio return	0.69%	0.78%	0.78%	0.79%	0.78%
Investment strategy	0.91%	0.78%	0.78%	0.81%	0.80%
Guarantee	-0.17%				
Smoothing	-0.05%				
Lifestaging				-0.02%	-0.02%
Excess return of SBP		-0.10%	-0.09%	-0.10%	-0.09%
Equity allocation	80%	45%	43%	51%	49%

Where risk is only taken to refer to the volatility of the outcome at retirement, four notional benchmark portfolios can be constructed for SBP 2. The allocation of assets to equity of the four notional benchmark portfolios is lower than that of the SBP, i.e. 80% versus between 43% and 51%. The benefit of the SBP being able to take on a more aggressive investment strategy up to retirement is evident. However, the additional return generated over its market linked notional benchmark portfolios is still less than the cost of the guarantee and smoothing mechanism. The market linked portfolios therefore outperform SBP 2.

In the case of the lifestage portfolios, the combined cost of tapering down the allocation of assets to equity prior to retirement, plus the returns lost from following a less aggressive investment strategy, is still lower than the cost of the guarantee and smoothing mechanism combined.

In summary, the analysis suggests that SBP 2 is also not effective at reducing investment risk exposure when compared to two simpler strategies, namely a market linked and a lifestage strategy. This is the case even when risk is only taken as the volatility of the outcome on retirement.

4.7.2 Concave utility curve

Tables 16 to 19 reflect the excess utility of SBP 1 and SBP 2 over its four notional benchmark portfolios, first based on the mean and then the median utilities of the savings at retirement.

The tables also reflect the strategic allocation of assets to equity of the notional benchmark portfolios. The remaining balance of the assets is assumed to be invested in bonds.

SBP 1

When assuming a concave utility curve, the notional benchmark portfolios all perform better than SBP 1, i.e. they have higher utility values.

Table 16: Analysis of utilities of savings at retirement of SBP 1 and its notional benchmark portfolios based on VaR and CTE

(Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio utility	-0.37	-0.27	-0.30	-0.27	-0.29
Excess utility of SBP		-0.09	-0.07	-0.10	-0.07
Median					
Portfolio utility	-0.35	-0.22	-0.26	-0.22	-0.26
Excess utility of SBP		-0.13	-0.08	-0.13	-0.09
Equity allocation	56%	74%	53%	85%	63%

For three of the four notional benchmark portfolios, the allocation of assets to equity exceeds that of SBP 1. Investing more aggressively leads to the mean and median utility values of the savings at retirement of these portfolios being higher than that of SBP 1.

Table 17: Analysis of utilities of savings at retirement of SBP 1 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures (Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio utility	-0.37	-0.30	-0.31	-0.30	-0.31
Excess utility of SBP		-0.07	-0.05	-0.06	-0.06
Median					
Portfolio utility	-0.35	-0.27	-0.29	-0.27	-0.29
Excess utility of SBP		-0.08	-0.06	-0.07	-0.06
Equity allocation	56%	50%	42%	56%	49%

Where risk is only measured as the volatility of the utility values of the savings at retirement, the allocation of assets to equity equals or is less than that of SBP 1. However, all of the notional benchmark portfolios still outperform the SBP. That implies that the utility attached to the additional returns earned from having a more aggressive investment strategy is more than eroded by the utility lost from offering a guarantee and applying a smoothing mechanism.

SBP 2

The results for SBP 2 are very similar to the results for SBP 1 when comparing tables 16 and 18, and tables 17 and 19.

Table 18: Analysis of utilities of savings at retirement of SBP 2 and its notional benchmark portfolios based on VaR and CTE

(Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio utility	-0.39	-	-0.27	-	-0.26
Excess utility of SBP			-0.12		-0.12
Median					
Portfolio utility	-0.36	-	-0.22	-	-0.21
Excess utility of SBP			-0.14		-0.15
Equity allocation	80%	-	75%	-	91%

Table 19: Analysis of utilities of savings at retirement of SBP 2 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures (Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio utility	-0.39	-0.27	-0.29	-0.27	-0.28
Excess utility of SBP		-0.12	-0.10	-0.12	-0.12
Median					
Portfolio utility	-0.36	-0.21	-0.25	-0.21	-0.24
Excess utility of SBP		-0.14	-0.11	-0.14	-0.11
Equity allocation	80%	77%	60%	87%	71%

The notional benchmark portfolios are in all instances again observed to have a higher mean and median utility than what SBP 2 has.

4.8 First-order stochastic dominance

Since the outcomes from simulating the SBPs and notional benchmark portfolios are random variables, i.e. there is uncertainty attached to the outcome, there would be value in analysing the distribution of that outcome and not just one point, i.e. the mean or median, on that distribution as is done in section 4.7. In this section we therefore apply first-order stochastic dominance rules to determine whether an SBP ranks higher than its notional benchmark portfolios that have similar degrees of risk as the SBPs, and is therefore preferred more than these benchmark portfolios.

Portfolio X will exhibit first-order stochastic dominance over portfolio Y, if at every point on the x-axis, the CDF of portfolio X lies on or to the right of the CDF of portfolio Y. In that case, portfolio X has a lower cumulative probability value at x than portfolio Y, and therefore has a higher probability of observing a value larger than x.

Tables are provided that reflect the percentage of the area where the SBP fails to exhibit FSD over its benchmark portfolios, ε. The tables also show the reverse scenario, i.e. the area where the notional benchmark portfolios fail to exhibit FSD over the SBP. In this instance, the percentage of the area is calculated as $(1 - \varepsilon)$. The smaller the value of ε, the greater the preference by *most* decision makers for the SBP over its notional benchmark portfolio. Alternatively stated, the smaller the value of $(1-\varepsilon)$, the greater the preference for the notional benchmark portfolio over the SBP.

A value of $\varepsilon = 0$ indicates that all decision makers prefer the SBP to the notional benchmark portfolio, and therefore that FSD is exhibited. What an acceptable value for ε is to conclude that almost FSD holds is subjective.

4.8.1 Linear utility curve

VaR and CTE

Figures 39 and 40 plot the CDFs of SBP 1 and SBP 2 respectively together with that of its four notional benchmark portfolios. Table 20 reflects the percentage of the area where SBP 1 and SBP 2 fail to exhibit FSD over its benchmark portfolios, ε, and *vice versa*.

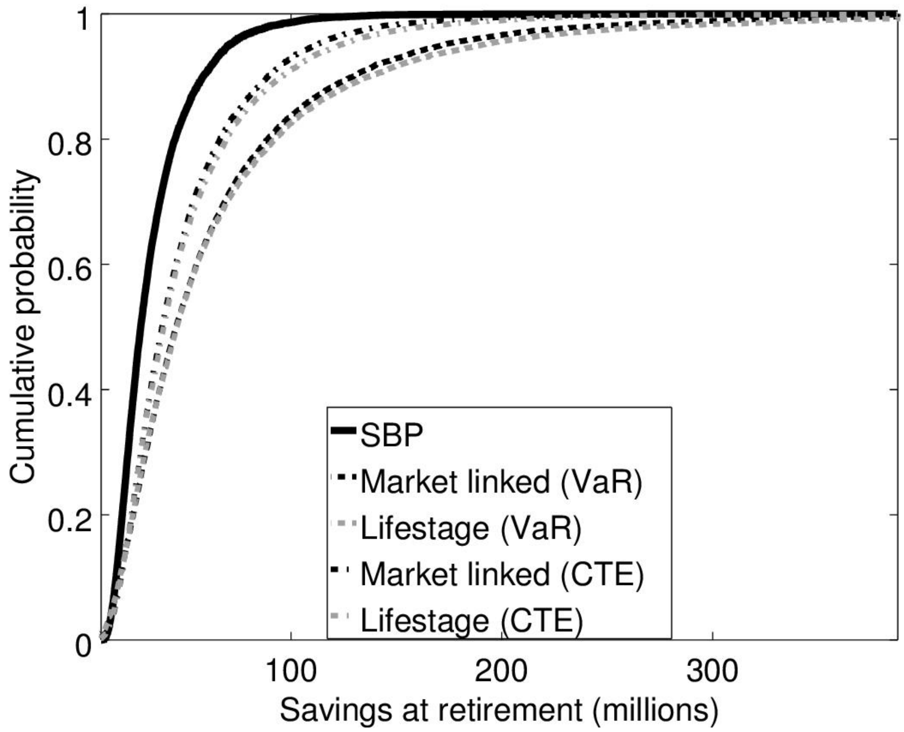


Figure 39: CDFs of SBP 1 and its benchmark portfolios based on VaR and CTE

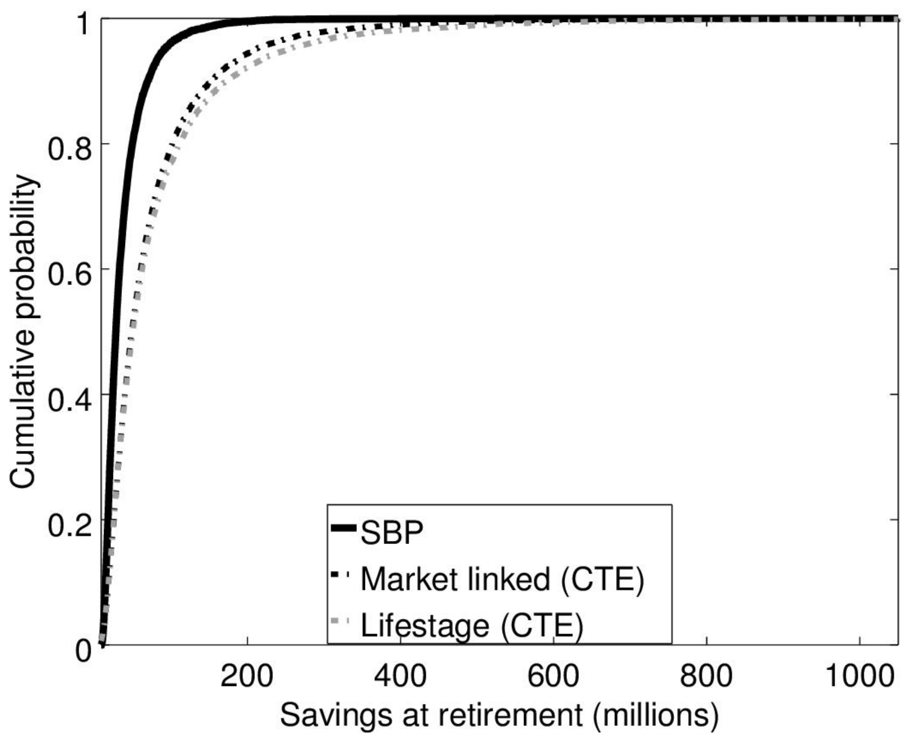


Figure 40: CDFs of SBP 2 and its benchmark portfolios based on VaR and CTE

As has been mentioned, SBP 2 has such a low VaR that no notional benchmark portfolio can be constructed based on this risk measure. Therefore, only the CDFs of the two notional benchmark portfolios based on the CTE of SBP 2 are reflected in figure 40.

For both SBP 1 and SBP 2 it appears that the SBPs rank lower than each of its notional benchmark portfolios. This conclusion is made because the CDFs of the SBPs lie above those of its notional benchmark portfolios at most points on the curves.

Table 20 confirms this result. The notional benchmark portfolios exhibit almost FSD over the SBPs in all cases. The area where the notional benchmark portfolios fail to exhibit FSD over the SBPs is less than 1% of the area between the curves.

Table 20: Percentage of area where FSD fails using VaR and CTE and assuming $\gamma = 0$

	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
SBP over Notional benchmark (ϵ)				
SBP 1	99.7%	99.8%	99.8%	99.9%
SBP 2	-	-	99.9%	99.9%
Notional benchmark over SBP (1 - ϵ)				
SBP 1	0.3%	0.2%	0.2%	0.1%
SBP 2	-	-	0.1%	0.1%

Adjusted VaR and adjusted CTE

Figures 41 and 42 reflect the CDFs of SBP 1 and SBP 2 respectively, with those of its notional benchmark portfolios, but where the portfolios are constructed based on the adjusted risk measures, i.e. incorporating only the volatility of the outcome on retirement.

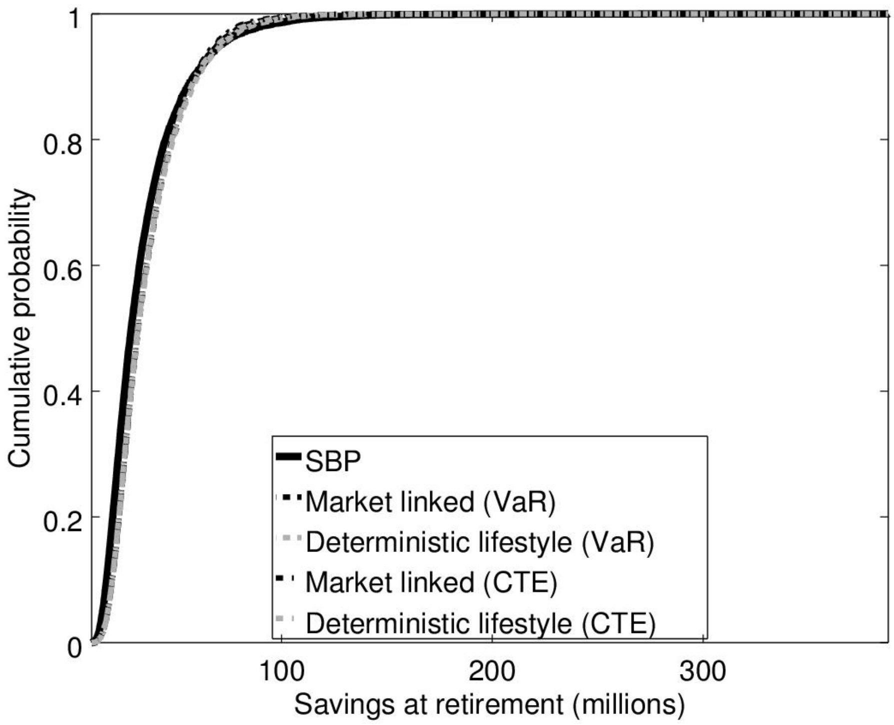


Figure 41: CDFs of SBP 1 and its benchmark portfolios based on adjusted VaR and adjusted CTE

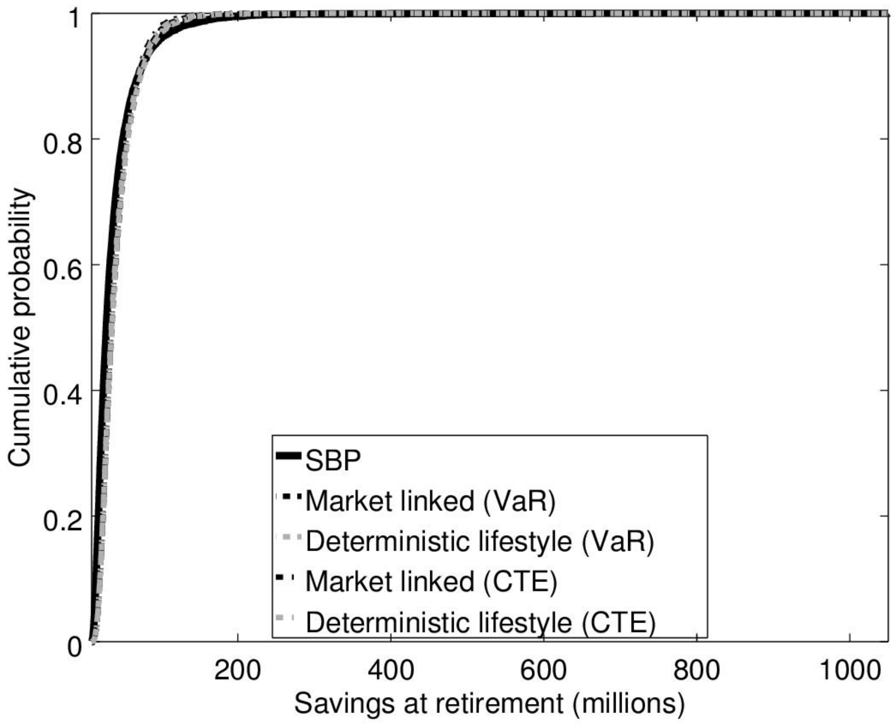


Figure 42: CDFs of SBP 2 and its benchmark portfolios based on adjusted VaR and adjusted CTE

It is very difficult to draw any conclusions from the above graphs since the CDFs lie on top of each other. Table 21 shows that the area where the notional benchmark portfolios fail to exhibit FSD over the SBPs ranges from between 10.3% to 27.2%, making conclusions difficult.

Table 21: Percentage of area where FSD fails using adjusted VaR and adjusted CTE and assuming $\gamma = 0$

Adjusted risk measures	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
SBP over Notional benchmark (ϵ)				
SBP 1	85.3%	89.7%	79.7%	85.6%
SBP 2	79.8%	83.1%	72.8%	76.7%
Notional benchmark over SBP (1 - ϵ)				
SBP 1	14.7%	10.3%	20.3%	14.4%
SBP 2	20.2%	16.9%	27.2%	23.3%

4.8.2 Concave utility curve

VaR and CTE

Figures 43 and 44 show the CDFs of the utility values of the savings at retirement for the SBP 1 and SBP 2 respectively, together with those of their notional benchmark portfolios.

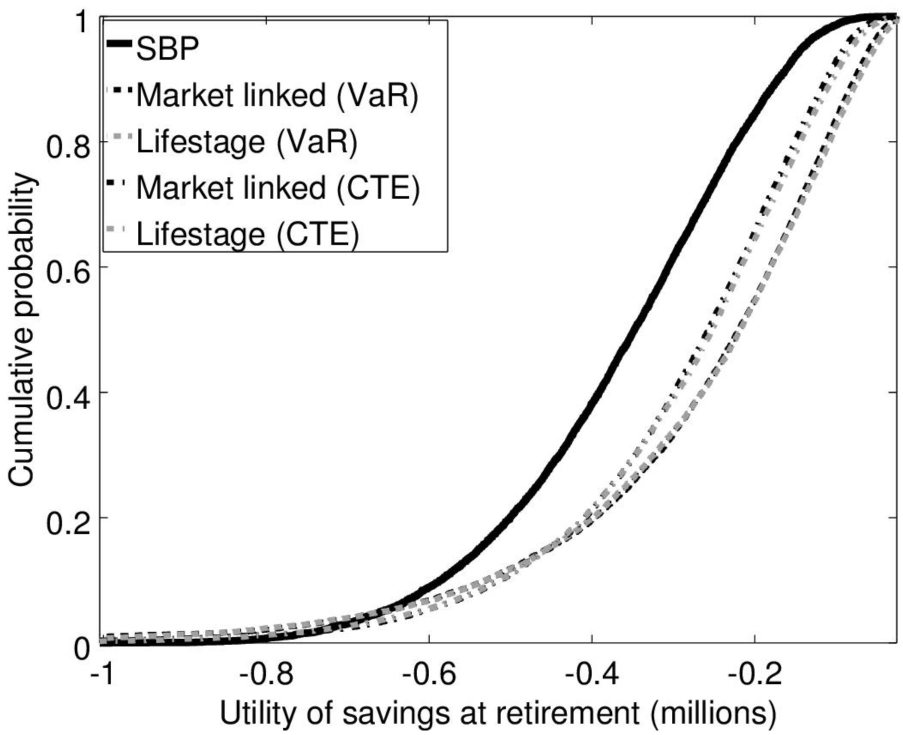


Figure 43: CDFs of utility values of the retirement savings of SBP 1 and its benchmark portfolios based on VaR and CTE

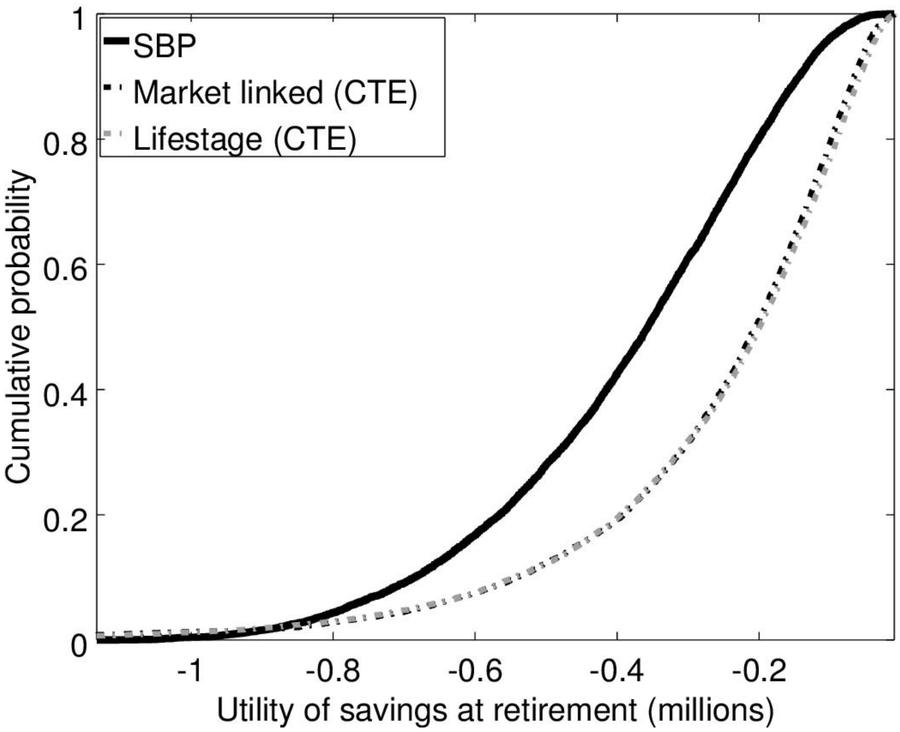


Figure 44: CDFs of utility values of the retirement savings of SBP 2 and its benchmark portfolios based on VaR and CTE

The CDFs of the utility functions have a flatter tail than the CDFs in figures 39 and 40 where a linear utility curve is assumed. This is consistent with the differences in the shapes of the histograms observed in figures 21 and 23. The notional benchmark portfolios rank above the SBPs in all instances accept in the lower tail.

Table 22 confirms that the notional benchmark portfolios exhibit almost FSD over the SBPs. The area where dominance fails ranges from between 1.8% and 6.2%.

Table 22: Percentage of area where FSD fails using VaR and CTE and assuming $\gamma = 2$

	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
SBP over Notional benchmark (ϵ)				
SBP 1	93.8%	95.0%	97.5%	98.2%
SBP 2	-	-	96.6%	97.7%
Notional benchmark over SBP (1 - ϵ)				
SBP 1	6.2%	5.0%	2.5%	1.8%
SBP 2	-	-	3.4%	2.3%

Adjusted VaR and adjusted CTE

Figures 45 and 46 reflect the CDFs of the utility values of the savings at retirement of SBP 1 and SBP 2 respectively, with those of its notional benchmark portfolios, but where the benchmark portfolios are constructed based on the adjusted risk measures, i.e. incorporating only the volatility of the outcome on retirement.

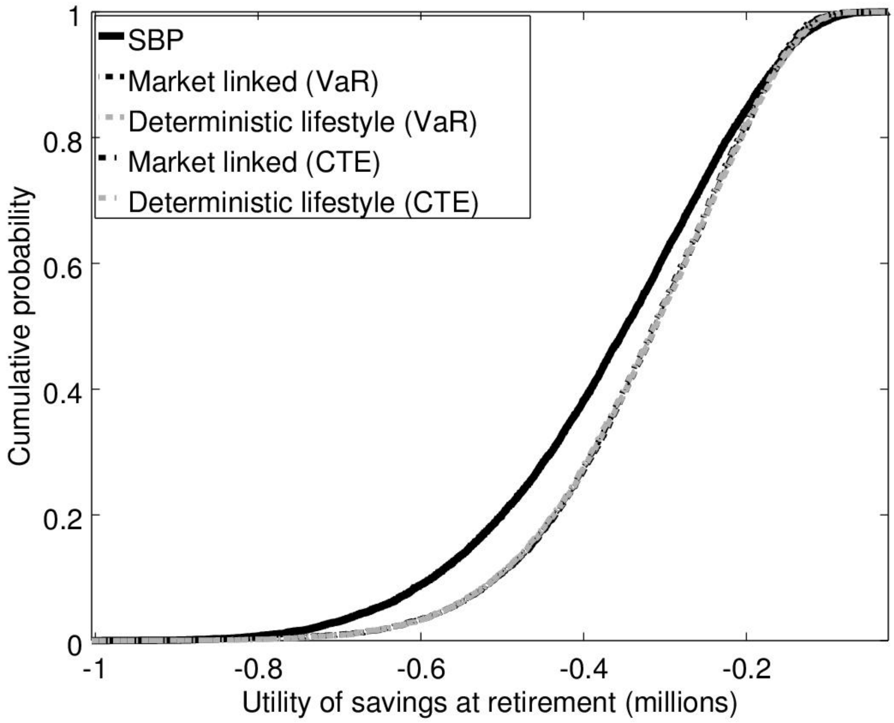


Figure 45: CDFs of utility values of the retirement savings of SBP 1 and its benchmark portfolios based on adjusted VaR and adjusted CTE

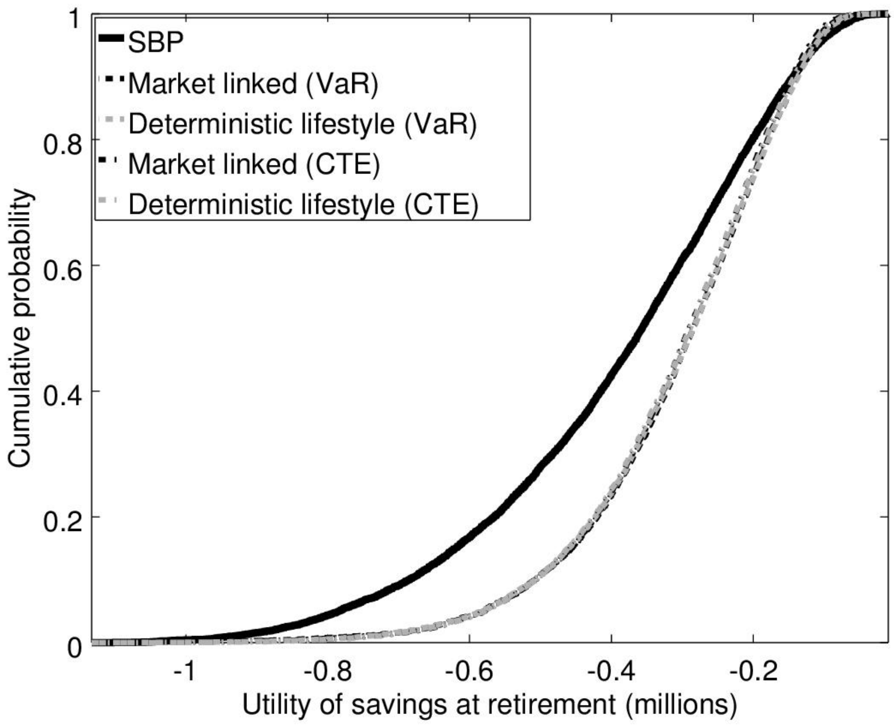


Figure 46: CDFs of utility values of the retirement savings of SBP 2 and its benchmark portfolios based on adjusted VaR and adjusted CTE

Table 23 shows that once again, the notional benchmark portfolios exhibit almost FSD over the SBPs. Comparing tables 22 and 23 the following observation is made. Where risk is measured only as the volatility of the utility values associated with the savings at retirement, the notional benchmark portfolios are preferred to a greater extent over the SBPs than when risk incorporates the risk of observing a too low outcome at retirement. This is the opposite of the scenario where members are assumed to have a linear utility curve. In that instance, the SBPs became slightly more attractive when risk is only considered as the volatility of the savings at retirement rather than when it includes the risk of observing a lower outcome.

Table 23: Percentage of area where FSD fails using adjusted VaR and adjusted CTE and assuming $\gamma = 2$

Adjusted risk measures	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
	SBP over Notional benchmark (ϵ)			
SBP 1	98.7%	99.3%	97.9%	98.9%
SBP 2	99.0%	99.4%	98.4%	98.9%
	Notional benchmark over SBP (1 - ϵ)			
SBP 1	1.3%	0.7%	2.1%	1.1%
SBP 2	1.0%	0.6%	1.6%	1.1%

4.8.3 Summary

Table 24 provides a summary of the results presented in section 4.8. It reflects the percentage of the area between the CDFs of the notional benchmark portfolios and the SBPs where the former fails to exhibit FSD over the latter.

Table 24: Summary of percentages of areas where notional benchmark portfolios fail to exhibit FSD over SBPs

(1 - ϵ)	Linear utility function		Concave utility function	
	SBP 1	SBP 2	SBP 1	SBP 2
Risk incorporating both level and volatility of retirement outcomes				
Market linked (VaR)	0.3%	-	6.2%	-
Lifestage (VaR)	0.2%	-	5.0%	-
Market linked (CTE)	0.2%	0.1%	2.5%	3.4%
Lifestage (CTE)	0.1%	0.1%	1.8%	2.3%
Risk incorporating only volatility of retirement outcomes				
Market linked (Adjusted VaR)	14.7%	20.2%	1.3%	1.0%
Lifestage (Adjusted VaR)	10.3%	16.9%	0.7%	0.6%
Market linked (Adjusted CTE)	20.3%	27.2%	2.1%	1.6%
Lifestage (Adjusted CTE)	14.4%	23.3%	1.1%	1.1%

In none of the scenarios do the SBP exhibit FSD over their notional benchmark portfolios. In all of the scenarios, the notional benchmark portfolios are preferred to a greater extent than the SBPs are, with the percentage of the area where FSD fails ranging from between 0.1% to 27.2%.

4.9 Sensitivity analysis

4.9.1 SBP as an 'end stage' portfolio

The methodology applied in this study as reflected in figure 8, is also applied for this sensitivity test. However, rather than assume the pension fund assets are invested in the SBP over the full projection period, it is assumed that for the first 30 years of the projection period, contributions are invested in a market linked portfolio. Only after that the assets are switched into an SBP and they then remain in the SBP for the last five years up to retirement. The aim of this test is to ascertain whether an SBP is an effective savings vehicle to use as an 'end stage' portfolio only, rather than investing in it over the full projection period.

In section 3.3 it is stated that a pension fund is referred to only by referring to the asset portfolio that the pension fund is invested in. In the rest of this section, SBP 1 (and similarly for SBP 2) is taken to refer to a pension fund whose assets are invested as follows:

- First 30 years: Market linked portfolio with an asset allocation strategy similar to that of SBP 1, as is reflected in table 5.
- Last five years: SBP 1, switching 100% of the assets exactly five years prior to retirement.

Return/risk analysis

Return analysis

The IRRs of SBP 1 and SBP 2 are reflected in table 25. The IRRs are split between the impact on the IRRs from the asset allocation strategy followed throughout the projection period, as well as the impact from having the guarantee and smoothing mechanism apply during the last five years of the projection period.

Table 25: Analysis of the monthly IRRs of SBP 1 and SBP 2

	SBP 1		SBP 2	
	Mean	Median	Mean	Median
Investment strategy	0.91%	0.83%	1.06%	0.91%
Guarantee				
Cost	-0.03%	-0.03%	-0.03%	-0.03%
Benefit	0.01%	0.01%	0.01%	0.02%
Smoothing				
Cost	-0.01%	-0.01%	-0.01%	-0.01%
Benefit	0.01%	0.02%	0.00%	0.01%
Return on SBP	0.90%	0.82%	1.04%	0.91%

The IRRs generated by SBP 1 and SBP 2 are quite a bit higher compared to the scenario where members are invested in the SBP for the full projection period, as is reflected in table 9. This is largely as a result of the cost of the guarantee and smoothing mechanism only being incurred for five as opposed to 35 years.

The benefit of the guarantee is about a third of the size of the benefit in table 9, i,e. 0.01% versus 0.03%. This is despite it only being in place a seventh of the length of the projection period, i.e. five years as opposed to 35 years. The assets in the pension fund would be largest closer to retirement. This therefore suggests that the benefit of the guarantee becomes greater as the size of a member's savings in a pension fund increases.

Return analysis incorporating risk

Table 26 is similar to table 10 in section 4.5.2, but assuming a shorter period of investment in SBP 1.

Table 26: Analysis of the return/risk ratios of SBP 1

	SBP 1			
	VaR		CTE	
	Mean	Median	Mean	Median
Investment strategy	1.51	1.73	1.40	1.55
Guarantee	0.04	0.05	0.04	0.05
Smoothing	0.04	0.05	0.03	0.04
Return on SBP	1.59	1.84	1.47	1.64

In table 25 it is observed that the IRRs where SBP 1 is only as an 'end stage' portfolio is higher than the IRRs of the portfolio invested in the SBP full term, as reflected in table 9, i.e. 0.9% and 0.82% versus 0.77% and 0.7% for SBP 1. However, the impact on the return/risk ratio from shortening the investment period in the SBP, as reflected in table 26, is negative, i.e. the ratios in this table are consistently lower than the ratios in table 10. The following observations can be made.

When investing in the SBP full term, as is assumed in table 10, the guarantee makes approximately a 13% contribution to the return/risk ratio of the SBP. When the SBP is used as an 'end stage' portfolio only, the guarantee only makes a 3% contribution. There is no real difference in the contribution made by the smoothing mechanism.

Table 27 reflects the results for SBP 2. It is very similar to SBP 1. The ratios are again consistently lower than the ratios in table 11 where the SBP is used over the fill projection period. In this instance the drop in the contribution the guarantee makes to the ratio goes from approximately 15% where the SBP is used over the full projection period, to 3% in the case where the SBP is used as an 'end stage' portfolio.

Table 27: Analysis of the return/risk ratios of SBP 2

	SBP 2			
	VaR		CTE	
	Mean	Median	Mean	Median
Investment strategy	1.26	1.45	1.20	1.32
Guarantee	0.04	0.06	0.03	0.05
Smoothing	0.02	0.03	0.02	0.02
Return on SBP	1.32	1.53	1.25	1.40

Comparison of expected outcomes

Linear utility curve

Table 28 reflects the excess return of SBP 1 over its four notional benchmark portfolios assuming the SBP is used only as an 'end stage' portfolio.

Table 28: Analysis of IRRs of SBP 1 and its notional benchmark portfolios based on VaR and CTE

	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.90%	0.84%	0.84%	0.65%	0.86%
Investment strategy	0.91%	0.84%	0.84%	0.65%	0.89%
Guarantee	-0.02%				
Smoothing	0.00%				
Lifestaging				-0.00%	-0.03%
Excess return of SBP		0.05%	0.05%	0.24%	0.04%
Median					
Portfolio return	0.82%	0.78%	0.78%	0.62%	0.79%
Investment strategy	0.83%	0.78%	0.78%	0.62%	0.81%
Guarantee	-0.02%				
Smoothing	0.01%				
Lifestaging				-0.00%	-0.02%
Excess return of SBP		0.04%	0.04%	0.19%	0.03%
Equity allocation	56%	43%	43%	2%	51%

In section 4.6.2 it is noted that the costs associated with the SBP when investing in this product for 35 years shifts the distribution of the savings at retirement so far to the left, that for the notional benchmark portfolios to have a similar degree of risk as the SBP, its allocation to equity needs to be higher than that of the SBP. In this instance where the SBP is only used as an 'end stage' portfolio, there is less of a risk of achieving a lower outcome because the cost associated with the risk mitigation tools of the SBP is less. The notional benchmark portfolios are therefore found to have a lower allocation of assets to equity than that of SBP 1.

In this instance we find that being able to retain a more aggressive investment strategy right up to retirement, and managing the additional risk exposure in the last five years through applying its two risk mitigation tools, enables SBP 1 to generate a higher return for members than a simple market linked or lifestage strategy. This is concluded from the positive excess returns reflected in the table. Using the SBP as an 'end stage' portfolio is therefore found to be an effective means of managing investment risk exposure.

When risk only reflects the volatility of the outcome at retirement, as is assumed in table 29, SBP 1 again outperforms its four notional benchmark portfolios. The outperformance in this instance however, is very small. It is observed that the allocation of assets to equity in the notional benchmark portfolios is very similar to the allocation of SBP 1.

Table 29: Analysis of IRRs of SBP 1 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.90%	0.89%	0.89%	0.89%	0.89%
Investment strategy	0.91%	0.89%	0.89%	0.92%	0.92%
Guarantee	-0.02%				
Smoothing	0.00%				
Lifestaging				-0.03%	-0.03%
Excess return of SBP		0.01%	0.01%	0.01%	0.01%
Median					
Portfolio return	0.82%	0.81%	0.81%	0.81%	0.81%
Investment strategy	0.83%	0.81%	0.81%	0.83%	0.83%
Guarantee	-0.02%				
Smoothing	0.01%				
Lifestaging				-0.03%	-0.03%
Excess return of SBP		0.00%	0.00%	0.01%	0.01%
Equity allocation	56%	52%	52%	58%	58%

The results for SBP 2 are reflected in tables 41 and 42 in Appendix F. The conclusions that can be drawn are very similar to those of SBP 1 discussed above.

Concave utility curve

Table 30 reflects the excess utility of SBP 1 over its notional benchmark portfolios.

Table 30: Analysis of utilities of savings at retirement of SBP 1 and its notional benchmark portfolios based on VaR and CTE

('000)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	-0.291	-0.312	-0.312	-0.418	-0.308
Excess utility of SBP		0.021	0.021	0.127	0.017
Median					
Portfolio return	-0.264	-0.290	-0.290	-0.414	-0.284
Excess utility of SBP		0.027	0.027	0.150	0.020
Equity allocation	56%	43%	43%	2%	51%

SBP 1 outperforms all four of its notional benchmark portfolios. A similar result is found where risk is referred to only as the volatility of the outcome at retirement, except that the proportionate level of outperformance is less. These results are reflected in table 43 in Appendix F.

Tables 44 and 45 in Appendix F reflects the excess utility of SBP 2 over its notional benchmark portfolios.

First-order stochastic dominance

Linear utility curve

Figures 47 and 48 reflect the CDFs of the savings at retirement of SBP 1 and SBP 2 respectively, as well as the CDFs of their notional benchmark portfolios. Table 31 reflects the percentage, ϵ, of the area where the SBP fails to exhibit FSD over its benchmark portfolios.

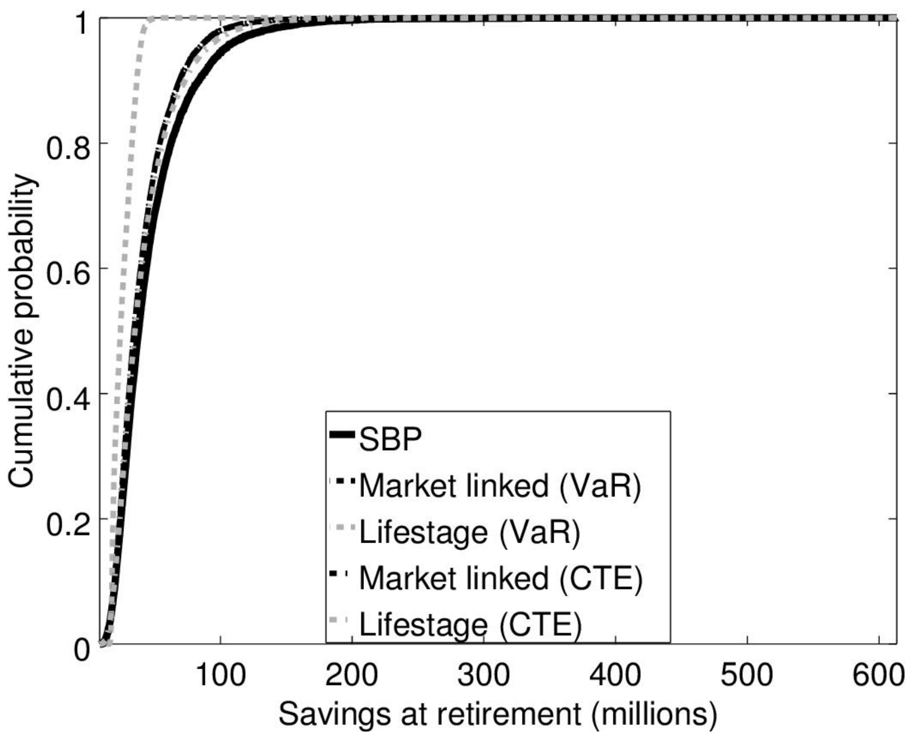


Figure 47: CDFs of SBP 1 and its benchmark portfolios based on VaR and CTE

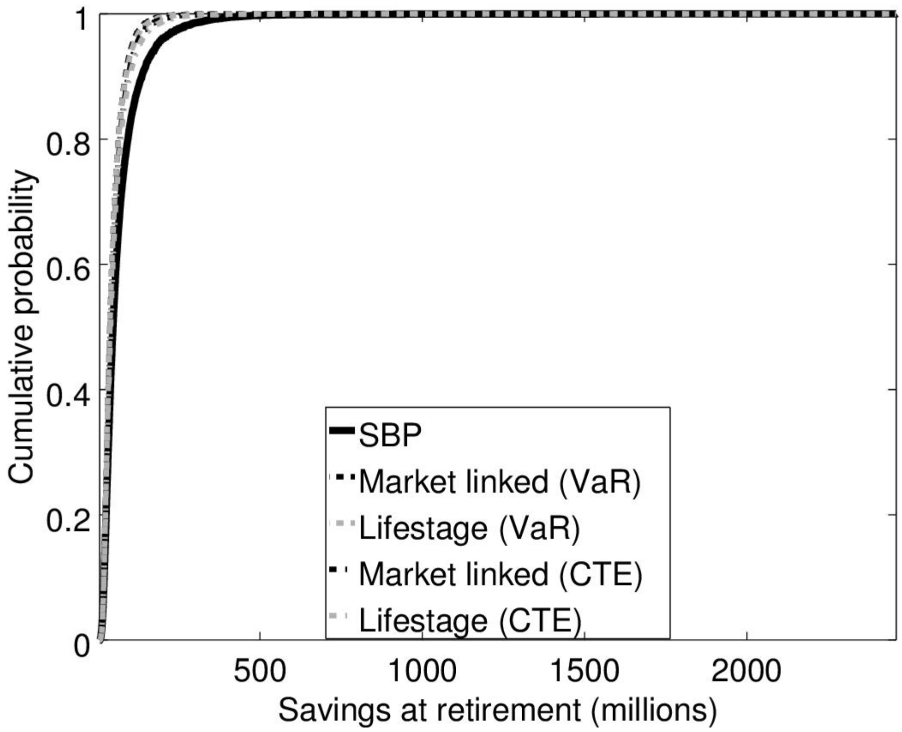


Figure 48: CDFs of SBP 2 and its benchmark portfolios based on VaR and CTE

Table 31: Percentage of area where FSD fails using VaR and CTE and assuming $\gamma = 0$

	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
SBP over Notional benchmark (ϵ)				
SBP 1	0.1%	0.5%	0.1%	0.0%
SBP 2	0.1%	0.1%	0.0%	0.0%
Notional benchmark over SBP (1 - ϵ)				
SBP 1	99.9%	99.5%	99.9%	100.0%
SBP 2	99.9%	99.9%	100.0%	100.0%

The CDFs of the SBPs in figures 47 and 48 lie on or to the right of the CDFs of the notional benchmark portfolios. Where risk is measured by the CTE, table 31 indicates that SBP 1 exhibits FSD over its lifestage notional benchmark portfolio, and SBP 2 exhibits FSD over both its market linked and lifestage notional benchmark portfolios. In all other instance, SBP 1 and SBP 2 exhibit almost FSD over its notional benchmark portfolios.

This result is the opposite to that found in the case where the SBP is the investment portfolio over the full projection period, as reflected in table 20. In the latter instance it was the notional benchmark portfolios that dominate the SBPs.

The results based on the adjusted VaR and adjusted CTEs are reflected in Appendix F in figures 63 and 64, and table 46. The results are very similar to the results above.

Concave utility curve

Figures 49 and 50 reflect the CDFs of the utility values of the savings at retirement of SBP 1 and SBP 2 respectively, as well as the CDFs of their notional benchmark portfolios. Table 32 reflects the percentage, ϵ, of the area where the SBP fails to exhibit FSD over its benchmark portfolios.

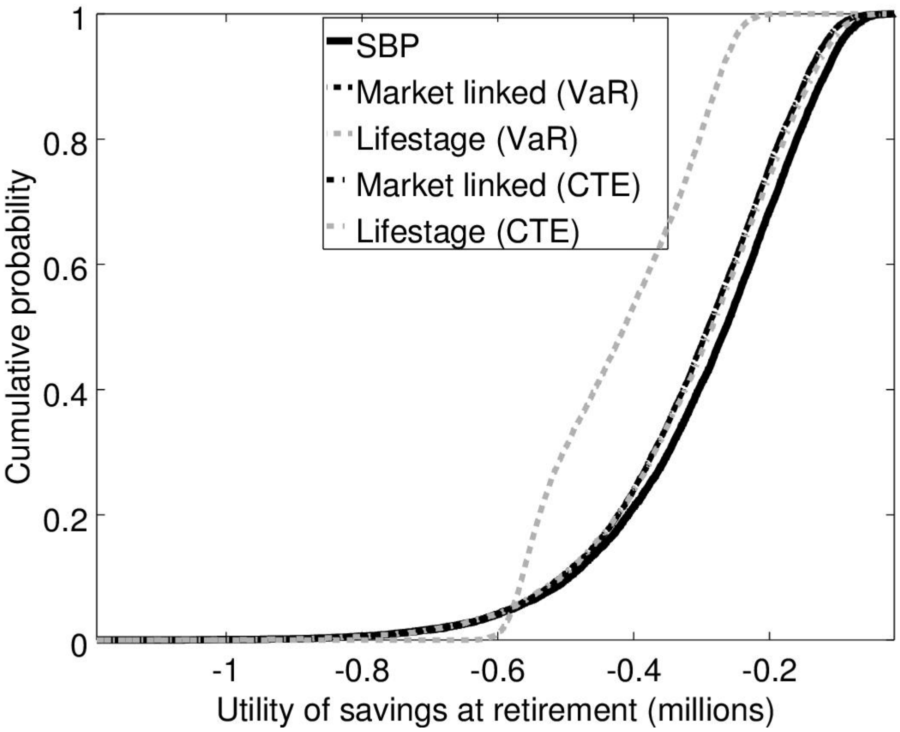


Figure 49: CDFs of utility values of the retirement savings of SBP 1 and its benchmark portfolios based on adjusted VaR and adjusted CTE

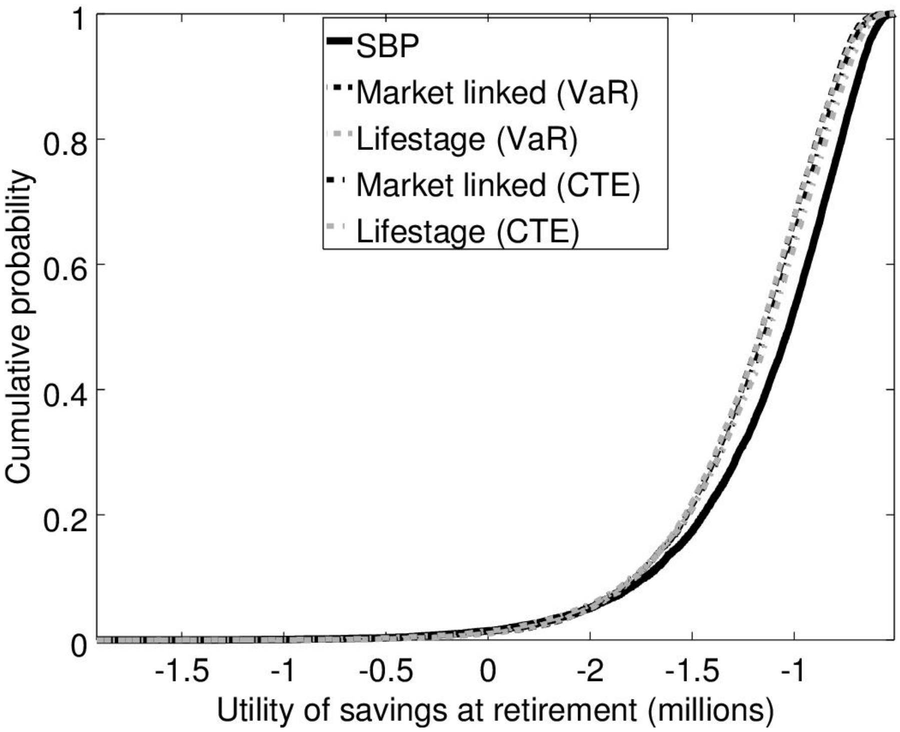


Figure 50: CDFs of utility values of the retirement savings of SBP 2 and its benchmark portfolios based on adjusted VaR and adjusted CTE

Table 32: Percentage of area where FSD fails using VaR and CTE and assuming $\gamma = 2$

	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
	SBP over Notional benchmark (ϵ)			
SBP 1	0.9%	3.4%	0.9%	0.7%
SBP 2	2.7%	4.1%	1.0%	0.8%
	Notional benchmark over SBP (1 - ϵ)			
SBP 1	99.1%	96.6%	99.1%	99.3%
SBP 2	97.3%	95.9%	99.0%	99.2%

In this case where the utility curve is assumed to be concave, the CDFs of the SBPs lie on top of or to the right of its notional benchmark portfolios at most utility values on the x-axis. Table 32 confirms that the SBPs exhibit almost FSD over all of its notional benchmark portfolios. The percentage of the area where dominance fails is between 0.7% and 4.1%.

This again is the opposite of what is found where the SBPs is used as the investment portfolio over the full projection period, as reflected in table 22.

The results based on the adjusted VaR and adjusted CTEs are presented in Appendix F in figures 65 and 66, and table 47. The results are very similar to the results above.

4.9.2 Risk appetite parameter of 0.25 (i.e. $\gamma = 0.25$)

This sensitivity tests the impact on results from assuming members have a risk appetite parameter of $\gamma = 0.25$. They are therefore assumed to have a concave utility curve which flattens out less quickly than when $\gamma = 2$, i.e. members are assumed to be less risk averse than the scenario where $\gamma = 2$. The shape of this utility curve is reflected in figure 10.

Histograms of the utility values of the savings at retirement

The simulated values of the savings at retirement on which the histograms in figure 21 are based, are transformed into utility values, but in this instance assuming $\gamma = 0.25$. The histograms of these utility values are reflected in figure 51.

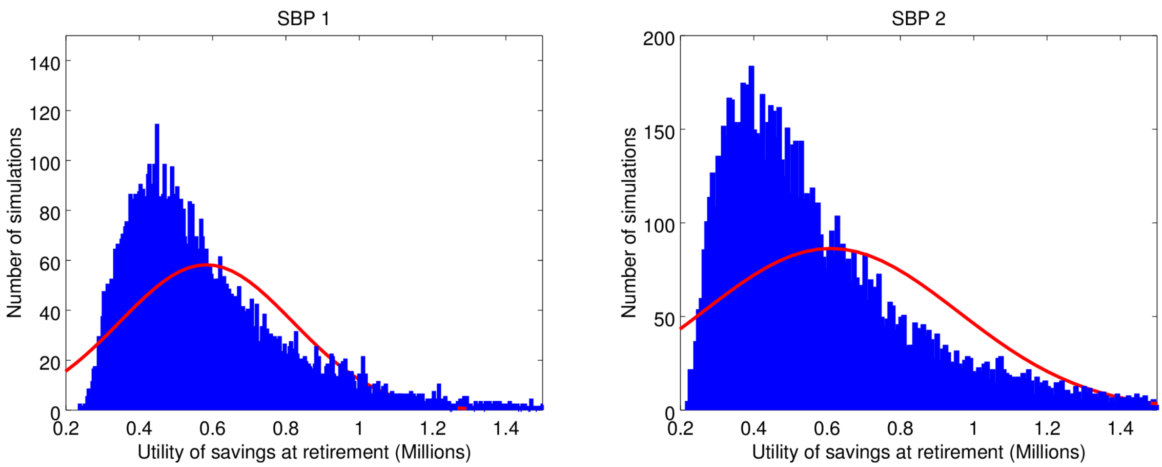


Figure 51: Histograms of the distribution of utility values of the savings at retirement outcomes for SBP 1 and SBP 2

These histograms look more similar to those in figure 21 where $\gamma = 0$, than those in figure 23 where $\gamma = 2$.

Comparison of expected outcomes

Table 33 reflects the excess utility of the SBP 1 over its notional benchmark portfolios where risk is measured by VaR and CTE.

Table 33: Analysis of utilities of savings at retirement of SBP 1 and it notional benchmark portfolios based on VaR and CTE

(Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.58	0.90	0.75	0.93	0.76
Excess utility of SBP		-0.32	-0.16	-0.35	-0.18
Median					
Portfolio return	0.52	0.74	0.66	0.74	0.66
Excess utility of SBP		-0.22	-0.13	-0.22	-0.14
Equity allocation	56%	74%	56%	85%	65%

The notional benchmark portfolios in all instance again outperform SBP 1, as is found when $\gamma = 2$. However, the following can be noted.

The strategic allocation of assets to equity in the notional benchmark portfolios is very similar to what is reflected in table 16 where members were assumed to be more risk averse, i.e. have a risk appetite parameter of $\gamma = 2$. However, the proportionate differential between the utilities generated by SBP 1 compared to its notional benchmark portfolios is much bigger in this instance compared to when members are assumed to be more risk averse in table 16. This suggests that as members become more risk averse, although the notional benchmark portfolios still offer a greater outcome, the outperformance of the notional benchmark portfolios relative to SBP 1 becomes smaller.

The results for SBP 2 are reflected in table 48 in Appendix G and are very similar to the results for SBP 1 discussed above, i.e. the more risk averse the member is, the smaller the outperformance of the notional benchmark portfolio relative to that of SBP 2.

Table 34 reflects the excess utility of the SBP 1 over its notional benchmark portfolios based on the adjusted VaR and adjusted CTE.

Table 34: Analysis of utilities of savings at retirement of SBP 1 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures (Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio utility	0.58	0.62	0.61	0.62	0.61
Excess utility of SBP		-0.04	-0.03	-0.12	-0.12
Median					
Portfolio utility	0.52	0.57	0.56	0.57	0.57
Excess utility of SBP		-0.05	-0.04	-0.05	-0.04
Equity allocation	56%	36%	34%	40%	39%

In this instance the notional benchmark portfolios also outperform SBP 1. However, the excess utility as a proportion of the mean utility of SBP 1 is much smaller than observed in table 33. This implies that where risk is measured only as the volatility of the utility values of the outcomes at retirement, the difference in utility, and therefore the attractiveness of the notional benchmark portfolios relative to SBP 1, is less than when the wider definition of risk is used.

The results for SBP 2 are reflected in table 49 in Appendix G and are very similar.

First-order stochastic dominance

Figures 52 and 53 reflect the CDFs of the utilities of the outcomes on retirement of SBP 1 and SBP 2. Table 35 reflects the percentage of the area where the SBPs fail to exhibit FSD over its notional benchmark portfolios.

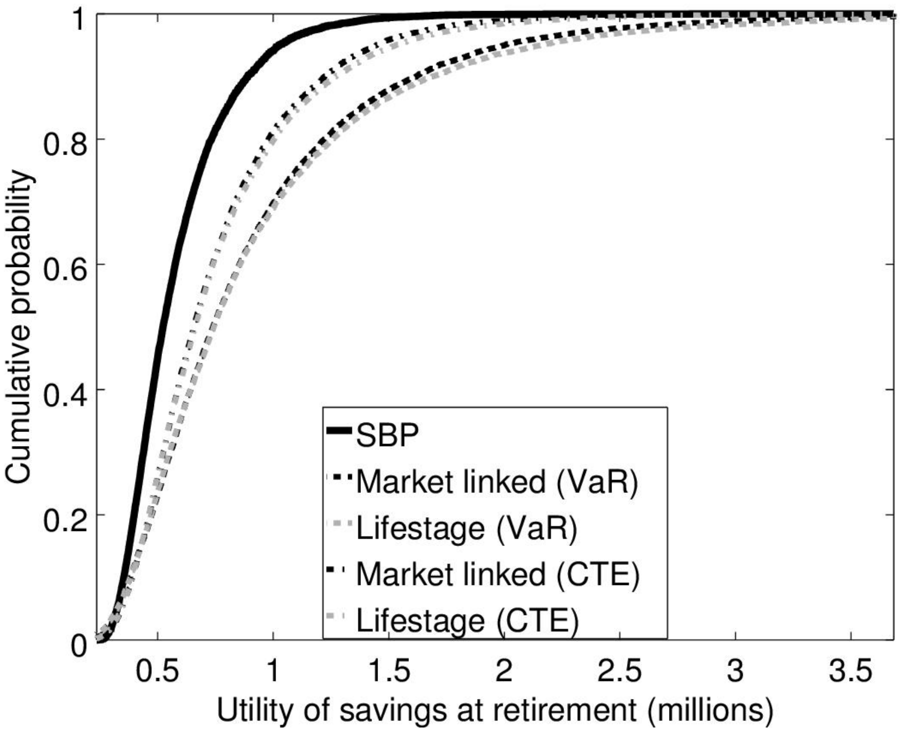


Figure 52: CDFs of utility values of the retirement savings of SBP 1 and its benchmark portfolios based on VaR and CTE

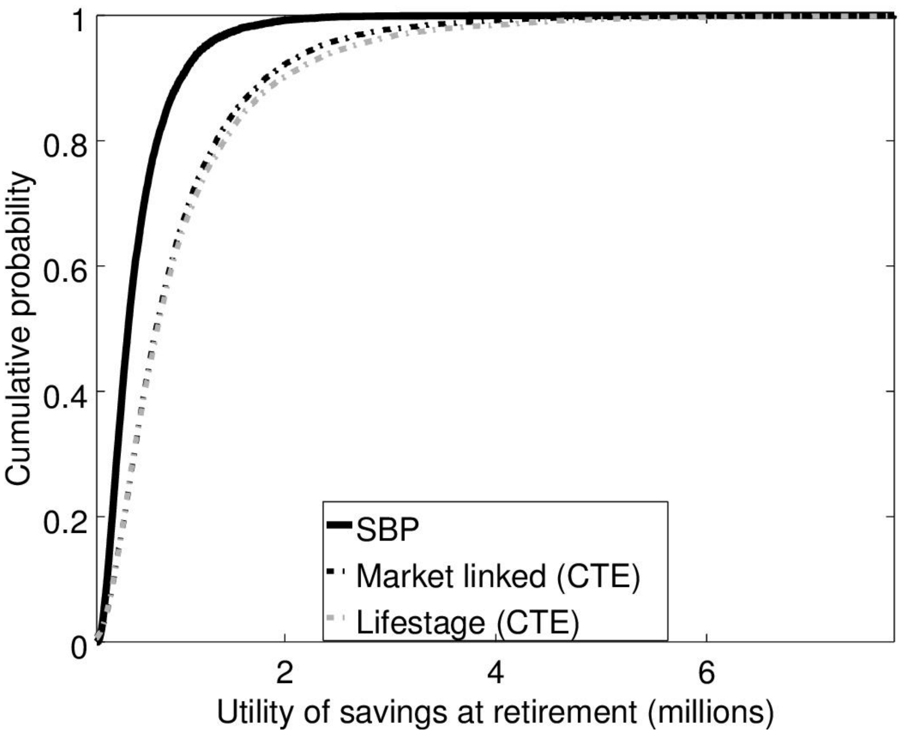


Figure 53: CDFs of utility values of the retirement savings of SBP 2 and its benchmark portfolios based on VaR and CTE

In both figures the notional benchmark portfolios seem to dominate the SBPs at all points on the x-axis. Table 35 confirms this observation. The notional benchmark portfolios exhibit

1%-almost FSD over the SBP in all the scenarios in the table.

Table 35: Percentage of area where FSD fails using VaR and CTE and assuming $\gamma = 0.25$

	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
SBP over Notional benchmark (ϵ)				
SBP 1	99.7%	99.7%	99.8%	99.9%
SBP 2	-	-	99.9%	99.9%
Notional benchmark over SBP (1 - ϵ)				
SBP 1	0.3%	0.3%	0.2%	0.1%
SBP 2	-	-	0.1%	0.1%

Figures 54 and 55 reflect the CDFs of the utility values of the outcomes on retirement of SBP 1 and SBP 2, where risk is measured by the adjusted VaR and adjusted CTE, i.e. only incorporating the volatility of the utility values of the savings at retirement. The areas between the CDFs of the SBPs and its notional benchmark portfolio curves are smaller than the areas between these curves in figures 52 and 53.

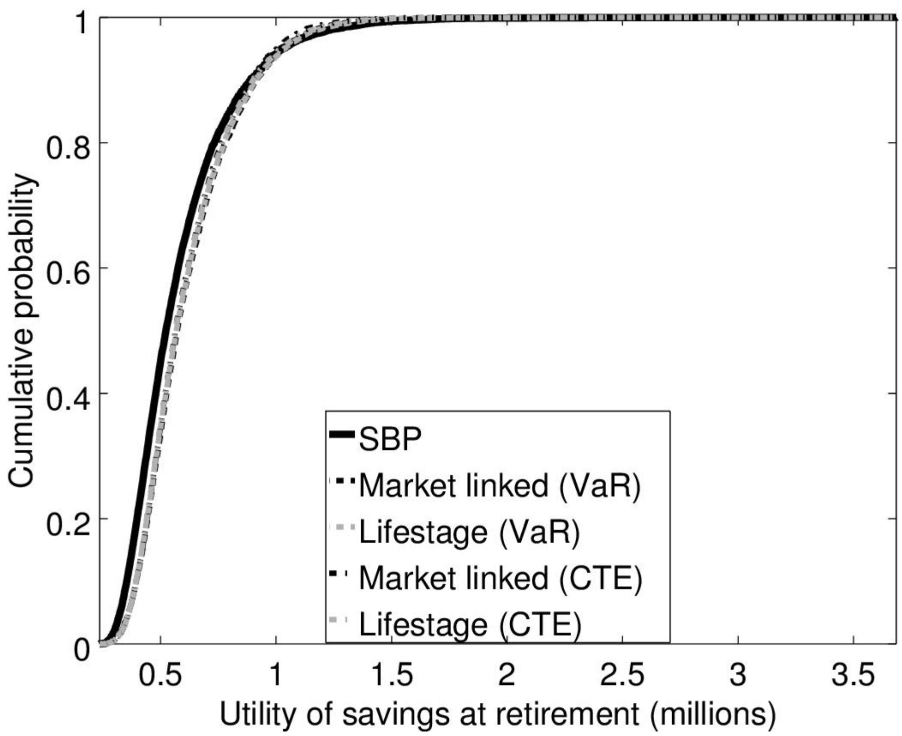


Figure 54: CDFs of utility values of the retirement savings of SBP 1 and its benchmark portfolios based on adjusted VaR and adjusted CTE

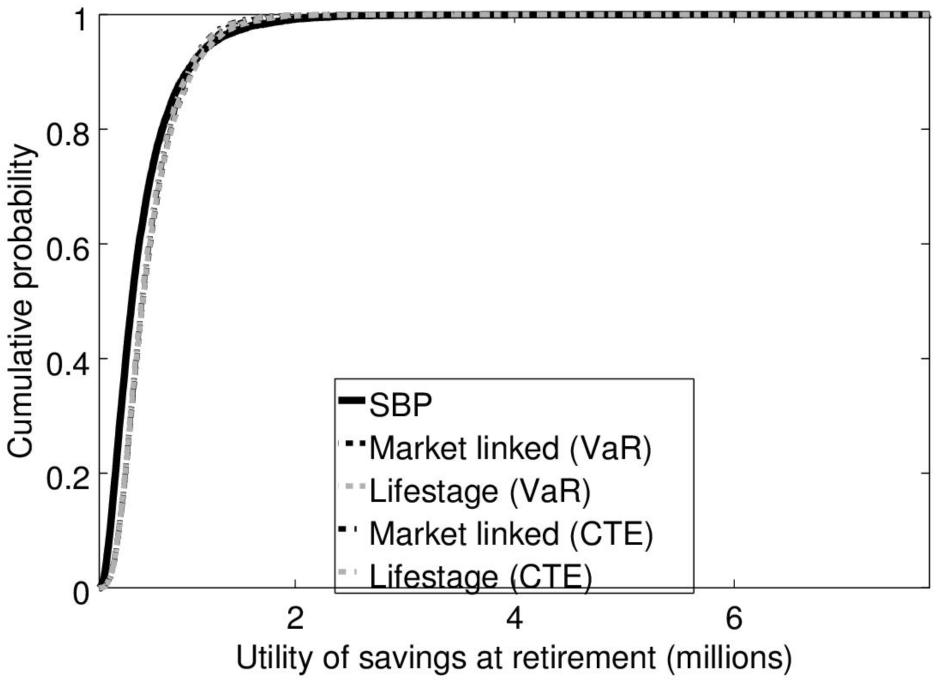


Figure 55: CDFs of utility values of the retirement savings of SBP 2 and its benchmark portfolios based on adjusted VaR and adjusted CTE

Table 36 reflects the area where the SBPs fail to exhibit FSD over its notional benchmark portfolios. The area where the notional benchmark portfolios fails to exhibit FSD over the SBP is greater than in table 35. Therefore, the notional benchmark portfolios are preferred to a greater extent relative to SBP 1 when the risk measure incorporates both definitions of investment risk.

Table 36: Percentage of area where FSD fails using adjusted VaR and adjusted CTE and assuming $\gamma = 0.25$

Adjusted risk measures	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
	SBP over Notional benchmark (ϵ)			
SBP 1	92.3%	92.5%	84.4%	89.2%
SBP 2	88.1%	90.4%	83.0%	83.4%
	Notional benchmark over SBP (1 - ϵ)			
SBP 1	7.7%	7.5%	15.6%	10.8%
SBP 2	11.9%	9.6%	17.0%	16.6%

4.10 Conclusion

This chapter presents and discusses the results of this study in detail. The guarantee and smoothing mechanisms of SBP 1 and SBP 2 are found to make a positive contribution to the products' return/risk ratios. Using the SBP as an 'end stage' portfolio, SBPs are found to be effective at managing investment risk exposure because the SBPs have higher IRRs and are preferred to a greater extent than its notional benchmark portfolios. The results from the application of the second and third methodologies suggests that SBPs are not effective at managing investment risk exposure.

An overview of the methodology applied in this study, as well as a summary of the results is presented in chapter 5. Areas for further research are also presented in the next chapter.

5 Conclusion

5.1 Introduction

Members of DC pension funds are exposed to investment risk by virtue of the way in which these funds operate. SBPs are available in South Africa as a savings vehicle for the assets underlying a DC fund. These products aim to manage investment risk exposure for members through offering a guarantee, which is attached to contributions made plus bonuses allocated, as well as applying a smoothing mechanism when declaring bonuses. The products also aim to grow members' retirement savings, which depends to a significant degree on the choice of investment strategy for the assets underlying the product.

The purpose of this study is to investigate whether SBPs are effective at reducing the investment risk that members of DC funds are exposed to, taking into consideration all of the implicit and explicit costs associated with the product. Two types of SBPs are constructed. Simulations of the savings of a member in a DC fund over a projection period of 35 years is done assuming the savings are invested in an SBP. Investment risk is defined as the uncertainty of the outcome at retirement encompassing both the volatility of that outcome, as well as the risk of observing a relatively lower outcome. Investment risk is measured by the VaR and CTE of the savings at retirement across all simulated outcomes.

This chapter summerises the results that are presented in chapter 4.

5.2 Overview of results

Three methodologies are applied in assessing effectiveness. A high level summary of the results is presented in table 37.

Method 1 is an analysis based on return/risk ratios, the detailed results of which are discussed in section 4.5.2. A tick indicates that the risk mitigation tools of the SBPs, i.e. the guarantee and smoothing mechanism, make a positive contribution to the return/risk ratio of the SBP. A cross indicates that it detracts from its performance. This method does not allow us to draw conclusions as to whether or not the SBP is effective relative to other possible investment strategies, since it only considers whether its risk mitigation tools make a positive contribution to its performance. Its overall performance is not compare to that of other investment strategies.

Method 2 is the comparison of the outcome of the SBP on retirement with that of its notional benchmark portfolios that have a similar degree of risk, using the two different risk measures. The results of this method are discussed in section 4.7. A tick indicates that the SBP outperforms its notional benchmark portfolios in all or most instances, while a cross indicates the reverse, i.e. that the notional benchmark portfolios outperform the SBPs in all or most instances.

Method 3 uses the first-order stochastic dominance rules, the results of which are discussed in section 4.8. A tick indicates that the SBPs are preferred over the notional benchmark portfolios in all or most of the scenarios modelled. A cross indicates that the notional benchmark portfolios are preferred over the SBPs in all or most of the scenarios modelled.

Table 37: Summarised results of the methodologies applied to assess the effectiveness of SBPs

Method	Linear utility curve		Concave utility curve	
	SBP 1	SBP 2	SBP 1	SBP 2
1. Return/risk analysis	✓	✓	✓	✓
2. Comparisson of expected returns				
VaR and CTE	×	×	×	×
Adjusted VaR and CTE	×	×	×	×
3. First-order stochastic dominance				
VaR and CTE	×	×	×	×
Adjusted VaR and CTE	×	×	×	×

The guarantee and smoothing mechanisms are found to make a positive contribution to the return/risk ratios of the SBP. However, when compared against its notional benchmark portfolios, the latter outperforms the former in all scenarios modelled.

Tables 38 and 39 summarises the results from the sensitivity tests. Sensitivity test 1 allows for the SBP to be invested in only during the last five years prior to retirement. Sensitivity test 2 assumes a risk appetite parameter of $\gamma = 0.25$. The interpretation of the tables are the same as for the above table.

Table 38: Summarised results of the methodologies applied to assess the effectiveness of SBPs assuming the SBPs are used as 'end stage' portfolios

Method	Linear utility curve		Concave utility curve	
	SBP 1	SBP 2	SBP 1	SBP 2
1. Return/risk analysis	✓	✓	✓	✓
2. Comparisson of expected returns				
VaR and CTE	✓	✓	✓	✓
Adjusted VaR and CTE	✓	✓	✓	✓
3. First-order stochastic dominance				
VaR and CTE	✓	✓	✓	✓
Adjusted VaR and CTE	✓	✓	✓	✓

Using the SBP only as the end stage portfolio is the only case in this study where positive results are rendered. However, as mentioned in section 3.15, implementing this strategy in practice might not be possible in all conditions.

Table 39: Summarised results of the methodologies applied to assess the effectiveness of SBPs assuming $\gamma = 0.25$

Method	Concave utility curve	
	SBP 1	SBP 2
2. Comparisson of expected returns		
VaR and CTE	×	×
Adjusted VaR and CTE	×	×
3. First-order stochastic dominance		
VaR and CTE	×	×
Adjusted VaR and CTE	×	×

As for the case where the risk appetite parameter is assumed to equal 2, the SBP does not appear to be an effective means of managing investment risk when either of the methodologies are applied.

A more detailed summary of the results is presented in sections 5.3 to 5.5.

5.3 Return/risk analysis

The effectiveness of an SBP is analysed by considering the impact each of its three main features, namely the guarantee, the smoothing mechanism and the choice of investment strategy has on its performance, both on an absolute and risk adjusted basis. The guarantee and smoothing mechanism both detract from the returns generated by the simulated SBPs. This is however not unexpected since reducing risk exposure is expected to come at a cost of reduced returns.

When incorporating risk, where risk is referred to only as the volatility of the outcome at retirement, it is found that both the guarantee and smoothing mechanism make a positive contribution to the return/risk ratios of the SBPs. Interestingly it is found that as the allocation of assets to equity increases, i.e. the underlying investment strategy becomes more aggressive, the investment strategy makes a smaller and smaller contribution to the return/risk ratio of the SBP. This is seen from the fact that the investment strategy of SBP 2, which is more aggressive than SBP 1, makes a lower contribution to the return/risk ratios of SBP 2 compared to that of SBP 1.

The size of the contribution that the guarantee makes to the return/risk ratios of SBP 1 and SBP 2 is fairly similar. However, considering this value as a percentage of the total return/risk ratios of SBP 1 and SBP 2, the guarantee makes a proportionately larger contribution for SBP 2 than for SBP 1.

The guarantee and smoothing mechanisms also make positive contributions to the return/risk ratios of the SBPs in sensitivity test 1.

5.4 Comparison of expected outcomes

The notional benchmark portfolios that are constructed to have the same VaR and CTE, and also the same adjusted VaR and adjusted CTE as SBP 1 and SBP 2, are found to consistently outperform the SBPs.

During the accumulation phase, intuitively a member will be more concerned about the amount of savings they will retire with than with the volatility of the outcome at retirement. Where investment risk is taken to incorporate both the level and volatility of savings at retirement, and where risk is measured by VaR and CTE, the allocation of assets to equity for the notional benchmark portfolios is found to be higher than what the SBPs have allocated in all instances. This implies that a member can generate a higher return for the same level of risk as the SBP by choosing an even more aggressive investment strategy than that of the SBP.

Closer to retirement, a member's concern is likely to shift more towards wanting to reduce the risk of there being a drop in their savings, and therefore wanting to reduce the volatility of the outcome. If risk is taken to only refer to the volatility of the outcome at retirement, the member would still be better off investing in the notional benchmark portfolios than the SBPs.

In this case, where only the volatility is considered, the allocation of assets to equity in the notional benchmark portfolios is lower than that of the SBP. Because of this, the notional benchmark portfolios based only on the volatility of the outcome at retirement consistently underperforms the notional benchmark portfolios based on the more comprehensive definition of risk.

Therefore, if members do not have a clear understanding of what the investment risk is that they are trying to manage,e.g. only focus on the volatility of the outcome and not the risk of observing a too low outcome, it could lead to them taking on a less aggressive investment strategy and in that process be sacrificing substantial upside. They would thereby, in the attempt to reduce their exposure to investment risk,

When the risk appetite parameter value is reduced to 0.25, i.e. when it is assumed that members are less risk averse than when $\gamma = 2$, the notional benchmark portfolios continue to outperform the SBPs. The scenario where the SBP is only used as an 'end stage' portfolio is the first instance observed where the SBPs outperform the notional benchmark portfolios.

5.5 First-order stochastic dominance

In applying FSD rules, the whole probability distribution of a portfolio, rather than only a single point on the curve is considered. The notional benchmark portfolios are found to be preferred to a greater extent than the SBPs in all the scenarios modelled, both when assuming linear and concave utility functions.

Using VaR and CTE as risk measures, the notional benchmark portfolios that are constructed to have similar degrees of risk as the SBP are preferred to a greater extent relative to the SBPs when members are assumed to be risk neutral, i.e. have a linear utility function, than when they are assumed to be risk averse, i.e. have a concave utility function. However, the values of $(1-\varepsilon)$, i.e. the percentage of the area where the notional benchmark portfolios fail to exhibit FSD over the SBPs, are still relatively small in both cases.

In the scenario where a risk appetite parameter of 0.25 is assumed, the notional benchmark portfolios continue to exhibit almost FSD over the SBPs.

It is only in the scenario where the SBP is used as an 'end stage' portfolio that positive results are observed. In this case the SBPs exhibit almost FSD over the notional benchmark portfolios in all of the scenarios simulated.

5.6 Future research

The results are dependent on the choice and outcome of the ESG. It would be interesting to see how a different view of the long-term expected returns for each financial series would impact on the results of this study. A critique of Maitland's model would also be beneficial for future studies such as these. A very simple approach is followed to adjust the model in order to align the simulated results with the long-term expectations of the series modelled. An investigation can also be done to determine if there is a better way of reparameterising Maitland's model.

An investigation can be done to determine how the features of the SBPs modelled can be amended in order for the product to outperform its notional benchmark portfolios. Here thought can be given to features such as the real return targets, fees, and the investment strategy underlying the product.

Behavioural finance teaches that individuals do not always act rationally. For example, even if a market linked or lifestage portfolio with a more aggressive investment strategy than an SBP has the same level of investment risk inherent in it, members may not be willing to take on those strategies. This is because they may not be believe that an increased allocation to equity will not be leading to a greater exposure to risk if compared against a perceived lower risk strategy such as investing in an SBP. It would be useful to consider how the results from this study would be impacted if these subjective tendencies individuals have, were to be incorporated. Also, assessing the behavioural impact of the risk mitigation tools applied by SBP on people's willingness to save for retirement could be researched.

Members may attach different levels of importance to the two elements of investment risk considered. An investigation into members' risk appetites and what tools to best manage the different elements of risk could therefore be done. The investigation can also be repeated using LPMs as risk measures rather than the two risk measures used in this study.

Some of insurance companies sell SBPs with a less than 100% guarantee. Some of these options are a lot cheaper than the 100% guarantee option that is modelled in this study. It would be interesting to investigate how these products rank in terms of effectiveness relative to the SBPs with a 100% guarantee.

Since it was only the sensitivity test that allowed for the SBP to be used as an 'end stage' portfolio that gave positive results, it would be interesting to investigate whether, if all pension fund members were to use the SBP only as their 'end stage' portfolio and not at any other time over the projection period, the product would be viable from an insurer's perspective.

A more comprehensive investigation into the value of SBPs could include considering the benefit and impact of offering a guarantee that applies not only at retirement, but also on events such as resignation.

5.7 Conclusion

This study sets out to investigate whether SBPs are effective at managing investment risk in DC funds. The guarantee and smoothing mechanism are found to make a positive contribution to the performance of an SBP on a risk-adjusted basis, where risk only refers to the volatility of the outcome at retirement.

However, when comparing the performance of the SBP to much simpler strategies, namely a market linked and a lifestage strategy, that have similar degrees of investment risk as the SBP, the SBP is found to consistently underperform these simpler strategies. The results therefore suggest that members would be better off choosing these alternative strategies than investing their funds in SBPs. Although the notional benchmark portfolios do not exhibit FSD over the SBPs, the areas where dominance fails is less than 28% across all scenarios modelled.

The only instance where positive results for the SBP are returned, is when the SBP is only used as an ‘end stage’ portfolio. In this instance the SBPs are found to have higher IRRs than portfolios based on market linked or lifestage strategies with similar degrees of risk. The area where FSD of the SBPs over its notional benchmark portfolios fail is less than 5% across all scenarios modelled.

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Annexure A: Smoothed bonus portfolios in South Africa

The following tables summarise the features of (most of) the smoothed bonus portfolios currently sold in South Africa. Although some of the portfolios that are closed to new business still have significant levels of assets invested in them, they have not been included in this summary. The values are taken from fact sheets dated February 2017.

Company	Fund name	Size R'm	Objective	Strategic asset allocation	Capital charge	Guarantee level	Bonus frequency
Old Mutual	Absolute Smooth Growth Portfolio (2009 series)	108 800	CPI + 6% (over LT)	Domestic equities: 45.1% Property: 6.2% Interest bearing: 16.9% Alternative: 6.4% African assets: 1.0% Offshore equity: 21.0% Offshore interest bearing: 0.8% Offshore alternative: 2.6%	0.20%	50%	Monthly in advance
Old Mutual	Absolute Stable Growth Portfolio (2009 series)		CPI +5.5% (over LT)		0.70%	80%	Monthly in advance
Old Mutual	Absolute Secure Growth Portfolio (2009 series)		CPI +3.5% (over LT)		2.70%	100%	Monthly in advance
Old Mutual	CoreGrowth 100	8 400	CPI + 3% to 3.5% (over the LT)	Domestic equities: 25.7% Property: 6.2% Interest bearing: 36.5% Alternative: 6.4% African assets: 1.0% Offshore equity: 18.2% Offshore interest bearing: 3.4% Offshore alternative: 2.6%	1.80%	100%	Monthly in advance
Old Mutual	CoreGrowth 90				0.80%	90%	Monthly in advance

Company	Fund Name	Size R'm	Objective	Strategic Asset Allocation	Capital Charge	Level of Guarantee	Bonus Frequency
Investment Solutions (underwritten by various insurers)	FullVest	1 500	CPI (over 3 year period)	Local allocation: 75%-100% Offshore allocation: 0%-25%		100%	Monthly
Investment Solutions (underwritten by Metropolitan)	Full Vest (Local)	57	CPI + 2%			100%	Monthly
Investment Solutions (underwritten by Old Mutual)	Old Mutual Fully Vesting	55	CPI (over 3 year period)	Equities: 40% Property: 10% Interest bearing: 45% Alternative: 5%		100%	Monthly in advance
Investment Solutions (underwritten by Metropolitan)	Metropolitan Fully Vesting	93	CPI + 2%			100%	Monthly in advance
Liberty	Corporate Balanced Bonus Portfolio	914	Discretionary	Domestic equities: 55% Property: 2.5% Interest bearing: 17.5% Offshore equity: 17% Offshore interest bearing: 3% Alternatives: 5%	1.00%	Vesting/ Non-vesting	Annually with interim bonus

Company	Fund Name	Size R'm	Objective	Strategic Asset Allocation	Capital Charge	Level of Guarantee	Bonus Frequency
Sanlam	Sanlam Multi-Manager International Vesting Portfolio	263	Expected net long-term monthly return + (FL - 103%)/(24-m)	Domestic equities: 34.8% Property: 9.4% Interest bearing: 33.3% Hedged: 4.1% Offshore allocation: 18.5%	1.60%	100%	Monthly in advance
Sanlam	Stable Bonus Portfolio	7 300	Linked to performance of investment strategy	Domestic equities: 36.1% Property: 7.2% Interest bearing: 35.7% Alternative: 1.0% Offshore equity: 11.2% Offshore other: 8.8%	0.90%	Vesting/Non-vesting	Monthly in advance
Sanlam	Monthly Bonus Fund	7 421	Expected net long-term monthly return + (FL - 103%)/(24-m)	Domestic equities: 36% Property: 7.2% Interest bearing: 35.6% Alternative: 1.0% Offshore equity: 11.3% Offshore other: 8.9%	1.60%	100.00%	Monthly in advance
Momentum	Smart guarantee +3	-	CPI + 3% (over 7 year period)	Domestic equities: 55% Property: 10% Interest bearing: 10% Offshore equity: 20% Offshore interest bearing: 5%	0.50%	100%	Monthly in advance
Momentum	Multi -manager Smooth Growth Fund - Local	-	CPI + 4% (over 5 year period)	Domestic equities: 54.5% Property: 7.5% Interest bearing: 18% Multi-asset class: 20%	1%	Vesting / Non-vesting	Monthly in advance

Company	Fund Name	Size R'm	Objective	Strategic Asset Allocation	Capital Charge	Level of Guarantee	Bonus Frequency
Momentum	Multi-manager Secure Growth Fund - Global (2013 series)	-	CPI + 2% (over 5 year period)	Domestic equities: 21.5% Property: 12.5% Interest bearing: 28.3% Multi-asset class: 20% Africa equity: 2.5% Africa property: 0.2% Global equity: 15%	1.50%	100%	Monthly in advance
Momentum	Multi-manager Secure Growth Fund - Global	-	CPI + 2% (over 5 year period)	Domestic equities: 21.5% Property: 12.5% Interest bearing: 28.3% Multi-asset class: 20% Africa equity: 2.5% Africa property: 0.2% Global equity: 15%	1.50%	100%	Monthly in advance
Momentum	Multi-manager Smooth Growth Fund - Global	-	CPI + 4% (over 5 year period)	Domestic equities: 31.26% Property: 7.5% Interest bearing: 14.8% Multi-asset class: 25% Africa equity: 2.5% Africa property: 0.2% Global equity: 18.75%	1%	Vesting / Non-vesting	Monthly in advance
Momentum	Smooth Growth Fund - Global	-	CPI + 4% (over medium to LT)	Domestic equities: 49% Property: 8% Interest bearing: 23.5% SRI: 1.5% Offshore equity: 12.6% Offshore interest bearing: 5.4%	1%	Vesting / Non-vesting	Monthly in advance

Annexure B: Models fitted by Maitland

The following table reflects the models fit to the four variable that Maitland modelled (Maitland, 2010). The parameter values are also reflected. The values in brackets are the standard errors.

Table 40: Models fitted per variable by Maitland

1. Inflation rate	$INFL_t = c_{s_t} + \phi_1 INFL_{t-1} + \epsilon_t$	$c_1 = 0.0299\,(0.005)$	$\phi_1 = 0.24\,(0.097)$	$\epsilon_1 \sim N(0, 0.033^2)$
		$c_2 = 0.0944\,(0.013)$		
2. Zero-year nominal yield	$SINT_t = c_{s_t} + \phi_1 SINT_{t-1} + \epsilon_t$	$c_1 = 0.0072\,(0.002)$	$\phi_1 = 0.866\,(0.037)$	$\epsilon_1 \sim N(0, \sigma^2_{s_t})$ where
		$c_2 = 0.0234\,(0.006)$		$\sigma_1 = 0.0051$ and $\sigma_2 = 0.0155$
3. Twenty-year par yield	$LINT_t = c_{s_t} + \phi_1 LINT_{t-1} + \epsilon_t$	$c_1 = 0.0129\,(0.004)$	$\phi_1 = 0.852\,(0.041)$	$\epsilon_1 \sim N(0, \sigma^2_{s_t})$ where
		$c_2 = 0.0214\,(0.006)$		$\sigma_1 = 0.0041$ and $\sigma_2 = 0.0083$
4. Excess return on equities	$XSEQ_t = c_{s_t} + \epsilon_t$	$c_1 = -0.0333\,(0.093)$	-	$\epsilon_1 \sim N(0, \sigma^2_{s_t})$ where
		$c_2 = 0.1404\,(0.041)$		$\sigma_1 = 0.62057$ and $\sigma_2 = 0.30581$

Annexure C: Detailed results from the pension fund simulations

Tables 56 to 59 provide summaries of the results from the simulations performed in this study.

Each of the four horizontal blocks in a figure presents the results from the simulations based on a particular SBP and utility function. Block one is based on SBP 1 and assumes members have a linear utility function. Block two is based on SBP 2 and assumes members have a linear utility curve. Blocks three and four are based on SBP 1 and SBP 2 respectively but assume members have a concave utility curve.

The following results from the SBP simulations are presented in the first results column. The mean and median of the savings at retirement projections is reflected for each of these four above scenarios. The IRRs are calculated using these mean and median values together with the stream of contributions assumed to be made into the pension fund over the projection period. The equity weight is the strategic asset allocation to equity as per table 5. The remainder of the assets are assumed to be invested in bonds. The VaR and CTE are value of the risk measures calculated on the projected savings at retirement.

Columns two to four of the results reflect the outcome from the simulations of the notional benchmark portfolios. Columns two and three are based on the market linked and lifestage strategies respectively and are constructed to have the same VaR as the SBP. Columns four and five are based on the market linked and lifestage strategies respectively but in this instance are constructed to have the same CTE as the SBP. The values of the VaRs and CTEs reflected in the table are therefore very close to the VaR and CTE of the SBP.

The results of the SBP and notional benchmark portfolios in table 57 are based on the simulations that assume the capital charge of the SBP is removed and the real return target adjusted accordingly, i.e. the results from portfolio 2 described in section 3.9.1. The results in table 58 are based on the simulations assuming the guarantee of the SBP and its charge is also removed, i.e. portfolio 3 described in section 3.9.1, and the results in table 59 are based on the simulations assuming in addition that the product management fee is also removed, i.e. portfolio 4 from section 3.9.1.

		SBP	Market linked - VaR	Lifestage - VaR	Market linked - CTE	Lifestage - CTE
SBP 1 - linear utility	Mean	**34 265 694**	64 168 627	67 538 056	48 428 027	50 876 682
	Median	**28 726 878**	45 546 877	45 846 719	38 828 564	39 629 915
	Weight (Equity)	**56%**	74%	85%	56%	66%
	VaR	**15 175 701**	15 189 176	15 168 412	-	-
	CTE	**13 730 102**	-	-	13 762 674	13 696 600
	IRR (mean)	**0.77%**	1.02%	1.04%	0.91%	0.93%
	IRR (median)	**0.70%**	0.89%	0.89%	0.83%	0.83%
SBP 2 - linear utility	Mean	**37 343 365**	-	-	73 660 402	82 571 200
	Median	**28 048 030**	-	-	48 790 905	50 245 598
	Weight (Equity)	**80%**	0%	0%	82%	97%
	VaR	**12 736 917**	-	-	-	-
	CTE	**11 498 610**	-	-	11 533 982	11 468 681
	IRR (mean)	**0.81%**	0.00%	0.00%	1.07%	1.11%
	IRR (median)	**0.69%**	0.00%	0.00%	0.92%	0.93%
SBP 1 - concave utility	Mean	**-365 726**	-271 586	-269 031	-295 537	-291 381
	Median	**-348 106**	-219 554	-218 118	-264 152	-258 643
	Weight (Equity)	**56%**	74%	85%	53%	63%
	VaR	**-658 751**	-657 807	-657 746	-	-
	CTE	**-733 016**	-	-	-734 736	-733 746
SBP 2 - concave utility	Mean	**-389 398**	-	-	-270 739	-264 541
	Median	**-356 531**	-	-	-217 674	-207 927
	Weight (Equity)	**80%**	0%	0%	75%	91%
	VaR	**-784 581**	-	-	-	-
	CTE	**-875 245**	-	-	-874 333	-873 196

Figure 56: Results from simulations of SBP 1 and SBP 2

		SBP (No capital charges)	Market linked - VaR	Lifestage - VaR	Market linked - CTE	Lifestage - CTE
SBP 1 - linear utility	Mean	**47 394 364**	-	-	-	-
	Median	**39 079 970**	-	-	-	-
	Weight (Equity)	**56%**	0%	0%	0%	0%
	VaR	**19 264 879**	-	-	-	-
	CTE	**17 173 330**	-	-	-	-
	IRR (mean)	**0.90%**	0.00%	0.00%	0.00%	0.00%
	IRR (median)	**0.83%**	0.00%	0.00%	0.00%	0.00%
SBP 2 - linear utility	Mean	**65 415 851**	35 282 172	35 626 893	37 961 546	38 968 310
	Median	**46 493 623**	31 251 407	31 374 145	32 954 143	33 483 030
	Weight (Equity)	**80%**	32%	37%	38%	45%
	VaR	**17 710 340**	17 711 632	17 707 526	-	-
	CTE	**15 304 504**	-	-	15 330 771	15 309 020
	IRR (mean)	**1.03%**	0.79%	0.79%	0.82%	0.83%
	IRR (median)	**0.90%**	0.74%	0.74%	0.76%	0.76%
SBP 1 - concave utility	Mean	**-274 434**	-	-	-384 503	-411 768
	Median	**-255 886**	-	-	-381 481	-407 899
	Weight (Equity)	**56%**	0%	0%	12%	4%
	VaR	**-518 883**	-	-	-	-
	CTE	**-587 658**	-	-	-587 552	-587 378
SBP 2 - concave utility	Mean	**-249 418**	-335 329	-334 060	-320 833	-317 727
	Median	**-215 083**	-323 094	-321 188	-303 452	-298 659
	Weight (Equity)	**80%**	31%	36%	38%	45%
	VaR	**-563 691**	-563 897	-563 817	-	-
	CTE	**-662 640**	-	-	-662 906	-661 638

Figure 57: Results from simulations of SBP 1 and SBP 2 without capital charges

		SBP (No guarantee)	Market linked - VaR	Lifestage - VaR	Market linked - CTE	Lifestage - CTE
SBP 1 - linear utility	Mean	**44 105 978**	52 960 473	54 594 877	49 870 417	52 314 975
	Median	**35 873 565**	41 107 645	41 198 780	39 608 947	40 256 945
	Weight (Equity)	**56%**	62%	71%	58%	68%
	VaR	**16 014 911**	16 009 344	16 011 021	-	-
	CTE	**13 583 702**	-	-	13 586 478	13 549 797
	IRR (mean)	**0.88%**	0.95%	0.96%	0.92%	0.94%
	IRR (median)	**0.79%**	0.85%	0.85%	0.83%	0.84%
SBP 2 - linear utility	Mean	**60 694 092**	93 818 440	-	83 674 840	-
	Median	**42 206 345**	54 408 442	-	51 607 684	-
	Weight (Equity)	**80%**	95%	0%	89%	0%
	VaR	**13 722 692**	13 698 175	-	-	-
	CTE	**10 990 132**	-	-	10 976 585	-
	IRR (mean)	**1.00%**	1.15%	0.00%	1.11%	0.00%
	IRR (median)	**0.86%**	0.96%	0.00%	0.94%	0.00%
SBP 1 - concave utility	Mean	**-309 439**	-283 766	-282 107	-290 010	-285 422
	Median	**-278 757**	-243 264	-242 726	-255 094	-248 404
	Weight (Equity)	**56%**	62%	71%	57%	68%
	VaR	**-624 194**	-624 326	-623 980	-	-
	CTE	**-754 813**	-	-	-757 045	-756 567
SBP 2 - concave utility	Mean	**-295 823**	-258 685	-	-263 002	-
	Median	**-236 931**	-183 795	-	-198 449	-
	Weight (Equity)	**80%**	95%	0%	86%	0%
	VaR	**-727 146**	-726 076	-	-	-
	CTE	**-959 880**	-	-	-958 765	-

Figure 58: Results from simulations of SBP 1 and SBP without the guarantee

		SBP (No guarantee or fees)	Market linked - VaR	Lifestage - VaR	Market linked - CTE	Lifestage - CTE
SBP 1 - linear utility	Mean	**47 488 905**	42 680 825	42 898 885	43 860 767	45 122 969
	Median	**38 555 107**	35 843 318	35 696 601	36 530 270	36 873 277
	Weight (Equity)	**56%**	47%	53%	49%	57%
	VaR	**16 999 113**	17 023 248	17 009 358	-	-
	CTE	**14 385 308**	-	-	14 383 925	14 396 845
	IRR (mean)	**0.91%**	0.86%	0.87%	0.87%	0.89%
	IRR (median)	**0.82%**	0.79%	0.79%	0.80%	0.80%
SBP 2 - linear utility	Mean	**65 358 059**	74 984 755	81 144 985	72 368 775	79 753 318
	Median	**45 341 764**	49 155 760	49 888 260	48 314 699	49 558 600
	Weight (Equity)	**80%**	83%	96%	81%	95%
	VaR	**14 542 820**	14 585 496	14 537 499	-	-
	CTE	**11 600 503**	-	-	11 616 131	11 621 545
	IRR (mean)	**1.03%**	1.08%	1.10%	1.06%	1.10%
	IRR (median)	**0.89%**	0.92%	0.92%	0.91%	0.92%
SBP 1 - concave utility	Mean	**-289 344**	-304 761	-305 027	-301 559	-297 907
	Median	**-259 369**	-278 992	-280 139	-273 746	-268 948
	Weight (Equity)	**56%**	47%	53%	49%	58%
	VaR	**-588 099**	-587 247	-587 808	-	-
	CTE	**-713 190**	-	-	-713 766	-711 998
SBP 2 - concave utility	Mean	**-277 525**	-264 834	-261 264	-266 871	-260 067
	Median	**-220 547**	-203 435	-200 448	-208 802	-197 462
	Weight (Equity)	**80%**	83%	96%	80%	98%
	VaR	**-686 138**	-684 776	-686 654	-	-
	CTE	**-910 999**	-	-	-911 417	-912 251

Figure 59: Results from simulations of SBP 1 and SBP 2 without the guarantee and fees

Annexure D: Risk and return plots using CTE as a risk measure

This curve plots the return/risk ratios for a range of constant weight market linked portfolios using CTE as a risk measure.

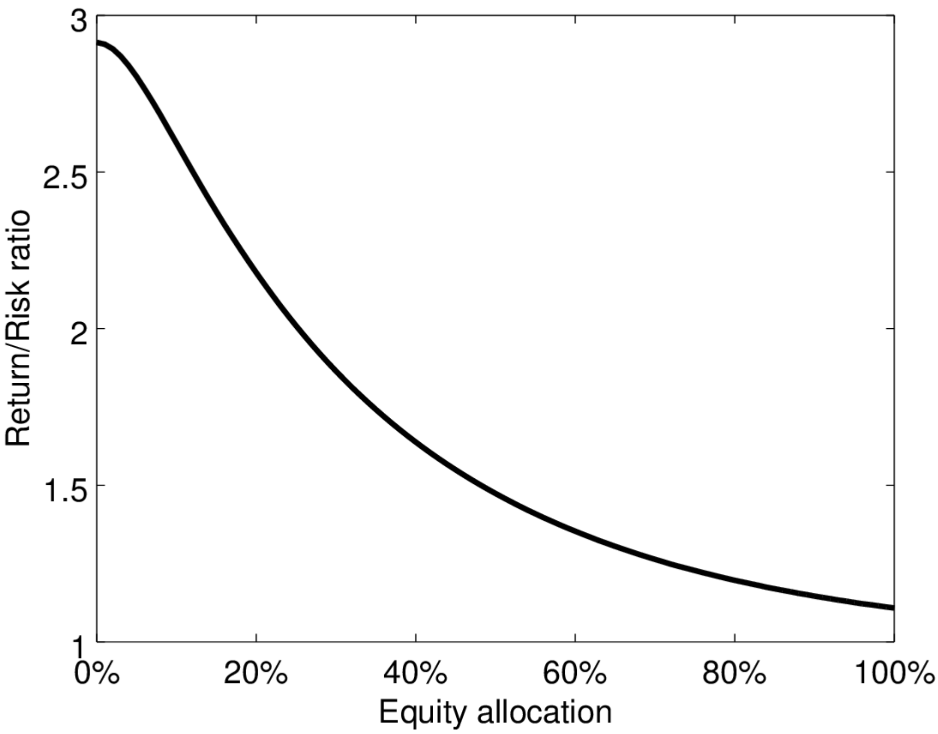

Figure 60: Return/risk ratios for constant weight market linked portfolios

The downward slope of the curve indicates that the return per unit of risk decreases as the allocation of assets to equity increases.

Annexure E: Range of possible notional benchmark portfolios

Figure 61 reflects the riskiness of the range of possible notional benchmark portfolios where it is assumed that members have a concave utility curve and that risk measured by the VaR and CTE. Figure 62 is a similar but based on the adjusted VaR and CTE.

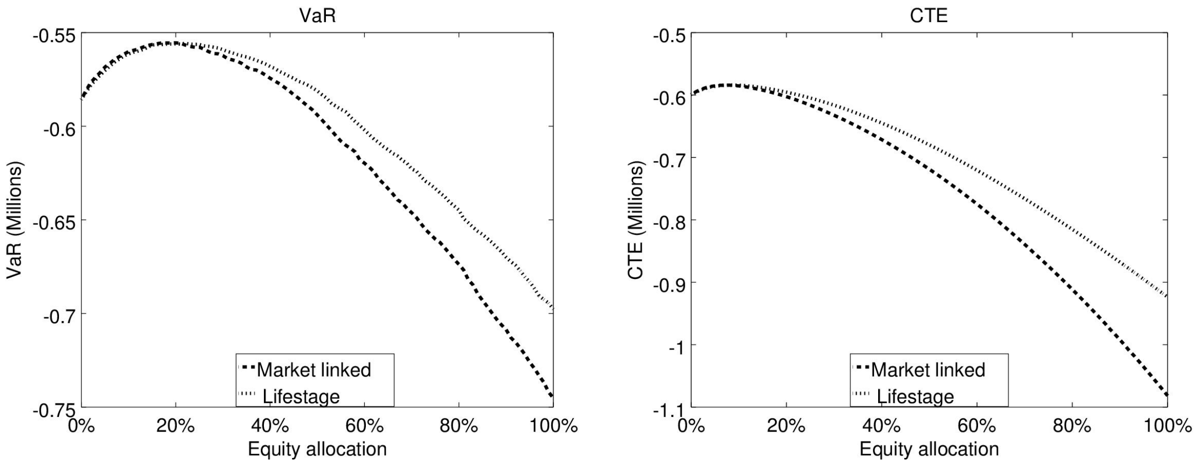

Figure 61: VaRs and CTEs of possible notional benchmark portfolios assuming $\gamma = 2$

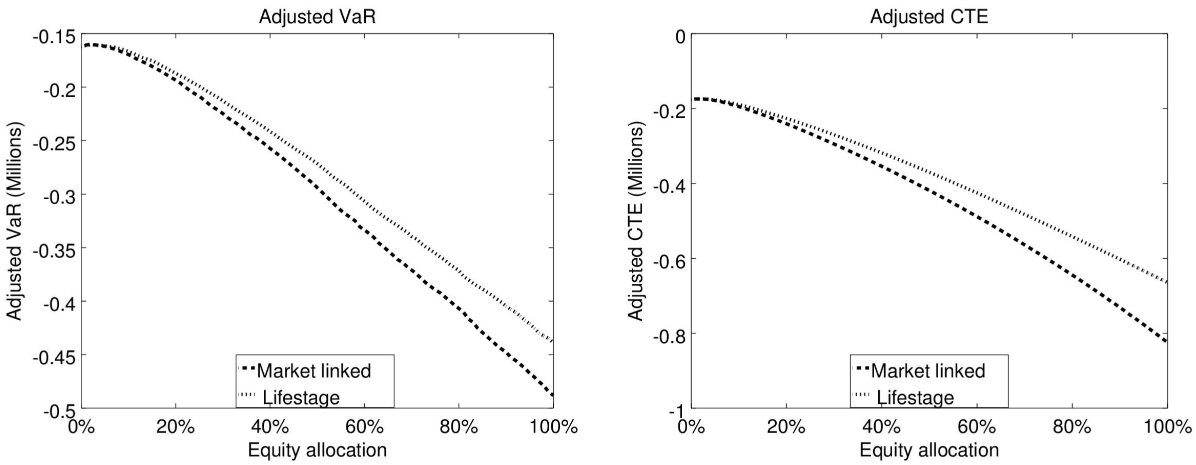

Figure 62: Adjusted VaRs and adjusted CTEs of possible notional benchmark portfolios assuming $\gamma = 2$

Annexure F: Additional results from sensitivity test 1

Comparison of expected outcome

Linear utility curve

Tables 41 and 42 reflect the excess return of SBP 2 over its four notional benchmark portfolios assuming SBP 2 is only used as an 'end stage' portfolio. In the first figure, risk is assumed to be measured by the VaR and CTE of the SBP. In table 42 risk only refers to the volatility of the outcome at retirement.

Table 41: Analysis of IRRs of SBP 2 and its notional benchmark portfolios based on VaR and CTE

	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	1.04%	0.90%	0.93%	0.91%	0.94%
Investment strategy	1.06%	0.90%	0.93%	0.94%	0.98%
Guarantee	-0.01%				
Smoothing	-0.00%				
Lifestaging				-0.04%	-0.04%
Excess return of SBP		0.14%	0.11%	0.13%	0.10%
Median					
Portfolio return	0.91%	0.82%	0.84%	0.82%	0.84%
Investment strategy	0.91%	0.82%	0.84%	0.84%	0.87%
Guarantee	-0.01%				
Smoothing	0.00%				
Lifestaging				-0.03%	-0.03%
Excess return of SBP		0.09%	0.07%	0.09%	0.07%
Equity allocation	80%	54%	59%	61%	68%

Table 42: Analysis of IRRs of SBP 2 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	1.04%	1.03%	1.03%	1.03%	1.03%
Investment strategy	1.06%	1.03%	1.03%	1.08%	1.08%
Guarantee	-0.01%				
Smoothing	-0.00%				
Lifestaging				-0.05%	-0.05%
Excess return of SBP		0.01%	0.01%	0.01%	0.01%
Median					
Portfolio return	0.91%	0.90%	0.90%	0.89%	0.89%
Investment strategy	0.91%	0.90%	0.90%	0.92%	0.92%
Guarantee	-0.01%				
Smoothing	0.00%				
Lifestaging				-0.03%	-0.03%
Excess return of SBP		0.01%	0.01%	0.02%	0.02%
Equity allocation	80%	76%	76%	84%	84%

Concave utility curve

Tables 43 and 45 reflect the excess return of SBP 1 and SBP 2 respectively over its four notional benchmark portfolios where risk only refers to the volatility of the outcome at retirement. Table 44 reflects the results based on SBP 2 and assuming risk is measured by the VaR and CTE.

Table 43: Analysis of utilities of savings at retirement of SBP 1 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures ('000)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	-0.291	-0.303	-0.306	-0.302	-0.304
Excess utility of SBP		0.012	0.015	0.011	0.013
Median					
Portfolio return	-0.264	-0.276	-0.282	-0.276	-0.278
Excess utility of SBP		0.013	0.018	0.012	0.014
Equity allocation	56%	48%	46%	55%	54%

Table 44: Analysis of utilities of savings at retirement SBP 2 and its notional benchmark portfolios based on VaR and CTE

('000)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	-0.255	-0.294	-0.289	-0.294	-0.284
Excess utility of SBP		0.039	0.034	0.039	0.030
Median					
Portfolio return	-0.210	-0.262	-0.252	-0.263	-0.247
Excess utility of SBP		0.052	0.042	0.053	0.036
Equity allocation	80%	54%	58%	61%	69%

Table 45: Analysis of utilities of savings at retirement of SBP 2 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures ('000)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	-0.255	-0.281	-0.284	-0.280	-0.279
Excess utility of SBP		0.027	0.029	0.025	0.024
Median					
Portfolio return	-0.210	-0.239	-0.243	-0.239	-0.237
Excess utility of SBP		0.029	0.033	0.029	0.027
Equity allocation	80%	64%	62%	73%	74%

First-order stochastic dominance

Linear utility curve

Figures 63 and 64, and table 46 are based on the adjusted risk measures for SBP 1 and SBP 2.

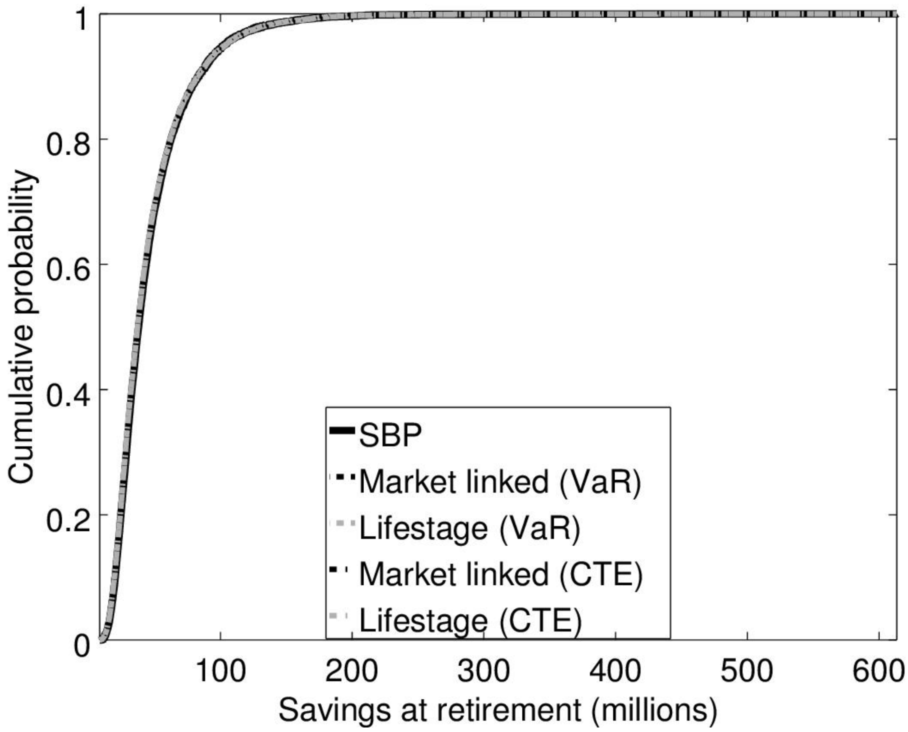

Figure 63: CDF of SBP 1 and its benchmark portfolios based on adjusted VaR and adjusted CTE

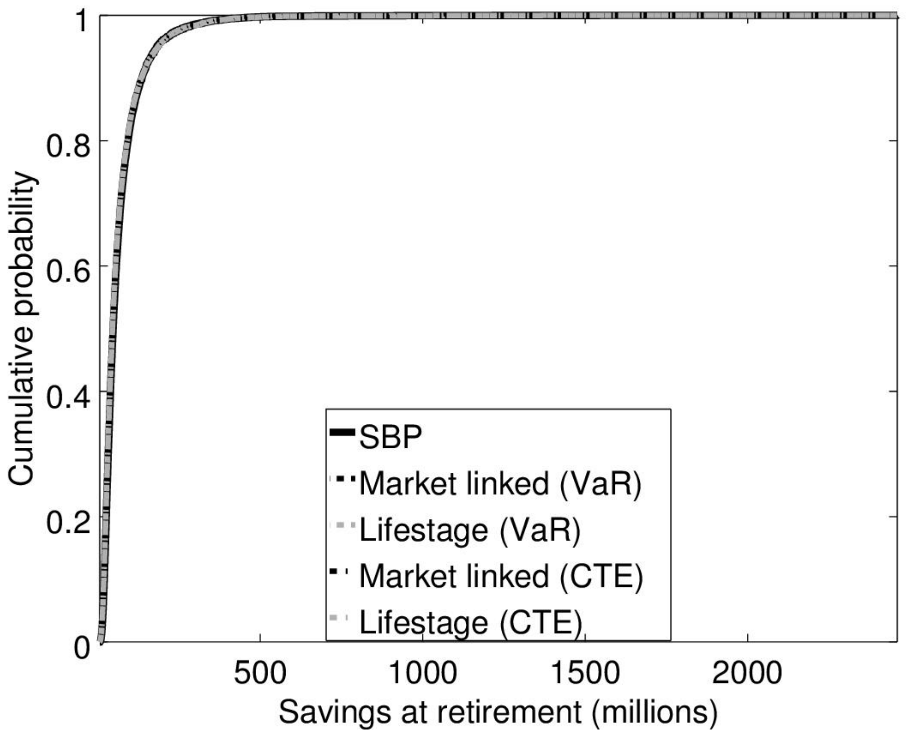

Figure 64: CDFs of SBP 2 and its benchmark portfolios based on adjusted VaR and adjusted CTE

Table 46: Percentage of area where FSD fails using adjusted VaR and adjusted CTE and assuming $\gamma = 0$

Adjusted risk measures	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
	SBP over Notional benchmark (ϵ)			
SBP 1	0.9%	4.4%	0.9%	4.4%
SBP 2	1.2%	11.7%	1.2%	11.7%
	Notional benchmark over SBP (1 - ϵ)			
SBP 1	99.1%	95.6%	99.1%	95.6%
SBP 2	98.8%	88.3%	98.8%	88.3%

Concave utility curve

Figures 65 and 66, and table 47 are based on the adjusted risk measures for SBP 1 and SBP 2.

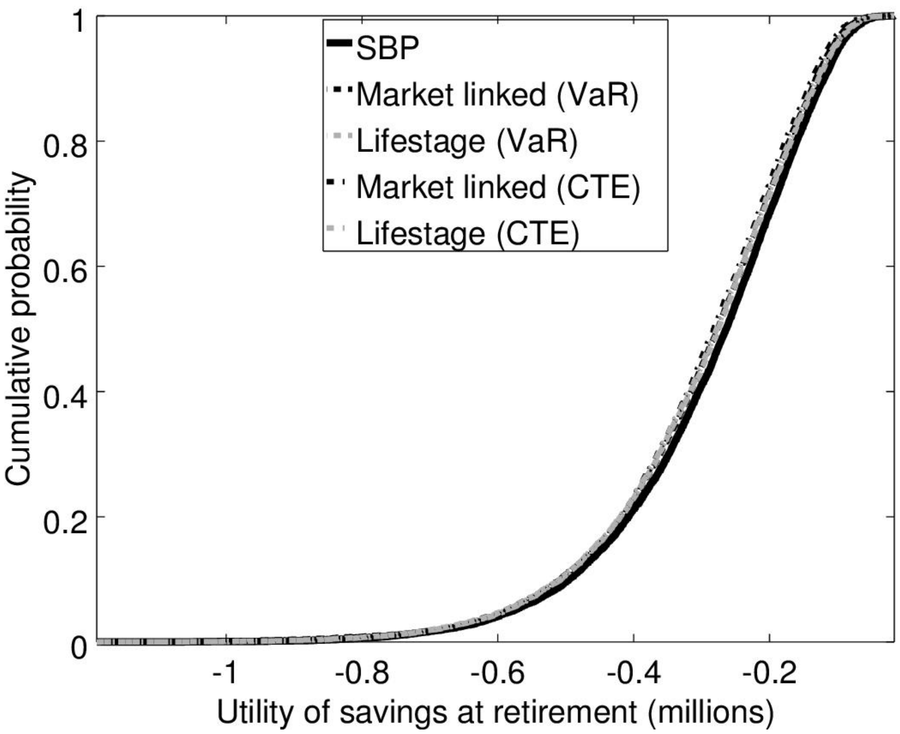

Figure 65: CDFs of utility values of the retirement savings of SBP 1 and its benchmark portfolios based on adjusted VaR and adjusted CTE

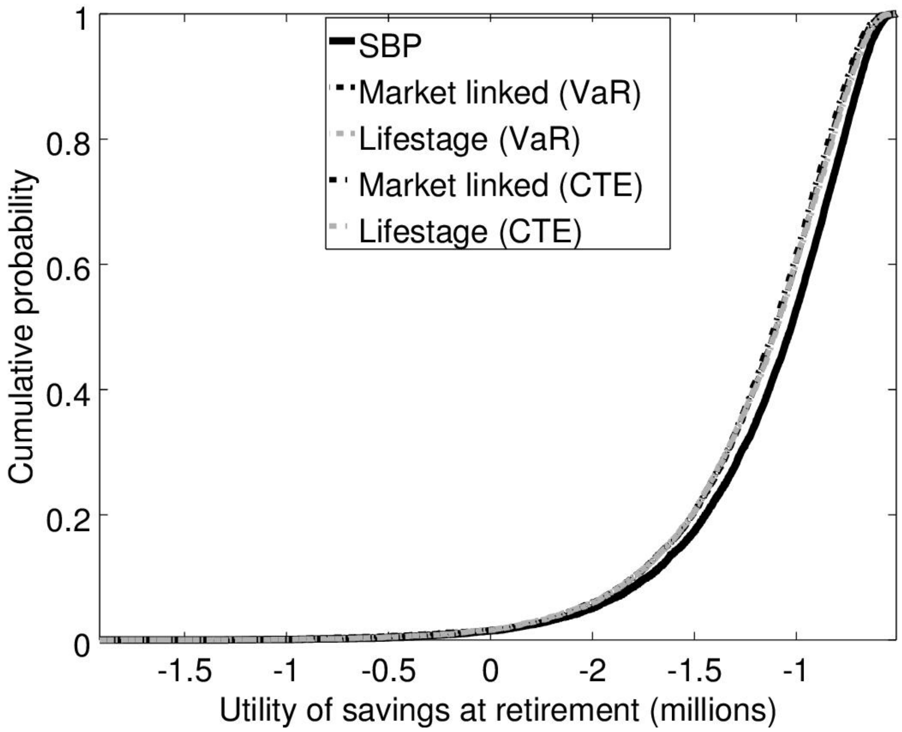

Figure 66: CDFs of utility values of the retirement savings of SBP 2 and its benchmark portfolios based on adjusted VaR and adjusted CTE

Table 47: Percentage of area where FSD fails using adjusted VaR and adjusted CTE and assuming $\gamma = 2$

Adjusted risk measures	VaR		CTE	
	Market linked	Lifestage	Market linked	Lifestage
	SBP over Notional benchmark (ϵ)			
SBP 1	0.0%	0.0%	0.1%	0.0%
SBP 2	0.0%	0.0%	0.0%	0.0%
	Notional benchmark over SBP (1 - ϵ)			
SBP 1	100.0%	100.0%	99.9%	100.0%
SBP 2	100.0%	100.0%	100.0%	100.0%

Annexure G: Additional results from sensitivity test 2

Comparison of expected outcome

Figures 48 and 49 reflect the excess utility of SBP 2 over its notional benchmark portfolios, first based on the VaR and CTE of SBP 2, and then based on the adjusted VaR and adjusted CTE of SBP 2.

Table 48: Analysis of utilities of savings at retirement of SBP 2 and its notional benchmark portfolios based on VaR and CTE

(Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio return	0.61	-	0.99	-	1.05
Excess utility of SBP		-	-0.38	-	-0.44
Median					
Portfolio return	0.51	-	0.78	-	0.79
Excess utility of SBP		-	-0.26	-	-0.28
Equity allocation	80%	-	82%	-	96%

Table 49: Analysis of utilities of savings at retirement of SBP 2 and its notional benchmark portfolios based on adjusted VaR and adjusted CTE

Adjusted risk measures (Millions)	SBP	Notional benchmark portfolios			
		Market linked		Lifestage	
		VaR	CTE	VaR	CTE
Mean					
Portfolio utility	0.61	0.68	0.67	0.68	0.66
Excess utility of SBP		-0.07	-0.06	-0.26	-0.26
Median					
Portfolio utility	0.51	0.61	0.60	0.61	0.60
Excess utility of SBP		-0.10	-0.09	-0.10	-0.09
Equity allocation	80%	46%	44%	52%	49%