Predictability of Tail Risks of Canada and the U.S. Over a Century: The Role of Spillovers and Oil Tail Risks

Afees A. Salisu*, Rangan Gupta**, and Christian Pierdzioch***

Abstract

Motivated by the long standing strong economic ties between Canada and the United States (U.S.), we examine whether such relations can be extended to their stock-market tail risks using over a century of monthly data, while also accounting for the role of tail risks of other advanced economies such as France, Germany, Japan, Italy, Switzerland, and the United Kingdom (U.K.) as well as the role of oil-market tail risk. We employ the Conditional Autoregressive Value at Risk (CAViaR) model developed by Engle and Manganelli (2004) to measure tail risks, where we estimate four variants (Adaptive, Symmetric absolute value, Asymmetric slope and Indirect GARCH) of the CAViaR model to compute the 5% Value-at-Risk (VaR). We then use model diagnostics such as the Dynamic Quantile test (DQ) test, %Hits and Regression Quantile (RQ) statistic to determine the model that best fits the data. Relying on the "best" tail-risk model and a predictive model that additionally accounts for the salient features of the tail-risk data, we find a strong positive relation between the stock-market tail risks of Canada and the U.S., consistent with risk spillovers between the two economies. Our findings hold for various out-of-sample forecast horizons. We also find contrasting evidence for the oil-market tail risk, whose effect is positive for Canada (being a net oil exporter) and negative for the U.S. (being a net oil importer). Further results obtained after accounting for the role of tail risks of other advanced economies combined using a principal-component analysis reveal a positive relation with the U.S. and negative one for Canada, supporting the diversification potential of the latter in the presence of tail risks of advanced economies other than the U.S. Our findings have implications for investors and policymakers, and are robust to alternative VaR measures.

Keywords: Tail Risks, Equity and Oil Markets, Spillovers, Predictability

JEL Codes: C22, C32, C53, G15, Q02

* Corresponding author. Centre for Econometric and Allied Research, University of Ibadan, Ibadan, Nigeria. Email address: adebare1@yahoo.com.

** Department of Economics, University of Pretoria, Private Bag X20, Hatfield, 0028, South Africa. Email: rangan.gupta@up.ac.za.

*** Department of Economics, Helmut Schmidt University, Holstenhofweg 85, P.O.B. 700822, 22008 Hamburg, Germany. Email: macroeconomics@hsu-hh.de.

1. Introduction

In the wake of multiple recent episodes of financial distress, like the Lehman default, the "Great Recession" followed by the European debt crisis, and the Chinese stock market crash, and currently the ongoing COVID-19 pandemic, the issue of tail risks has emerged as an important research question (Baker et al., 2015; 2020; Adrian et al., 2019). This is mainly because tail risks have been shown to predict not only equity returns, but also real economic variables, such as employment, investment and output (Kelly and Jiang, 2014; Chevapatrakul et al., 2019; Hollstein et al., 2019; Salisu et al., 2021a). Naturally, determining which factors drive the future evolution of tail risks is an important question for both investors and policymakers. In this regard, note that, tail risk is the additional risk which, commonly observed, fat-tailed asset return distributions have relative to normal distributions (Li and Rose, 2009).

The objective of this research is to analyse (both in and out-of-sample) the link between Canadian and U.S. stock-market tail risks, where we control for the influence of stock-market tail risks of other advanced economies and in particular for the predictive role of oil-market tail. In our empirical analysis, we use data that cover the monthly period of 1916:M02 to 2020:M10. Our decision to relate oil-market tail risks to stock-market tail risks is motivated by the large literature that documents the relationship between these two markets (see, Degiannakis et al., (2018) and Smyth and Narayan (2018) for detailed reviews). Oil-price movements in general, and extreme price movements in particular can affect stock-market tail risk through multiple channels like stock-market valuations, monetary and fiscal policy responses, and output and uncertainty dynamics. In other words, oil-price movements (including tail risks) by itself tend to contain leading information for a gamut of macroeconomic and financial variables (Lombardi et al., 2012; Gupta et al., 2021) that can drive stock-market tail risks (Mensi et al., 2017). Besides, oil-market tail risks have been shown to predict first- and second-moments of oil returns in general (Ellwanger 2017; Salisu et al., 2021b).

Our decision to look at the U.S. and Canada is not only due to the availability of data spanning over a century of stock-market and oil-markets movements, which allows us to avoid any sample-election bias and control for a wide array of historical crises (such as the Spanish Flu, the two World Wars, the "Great Depression", the oil-price shocks, Black Monday, the Gulf War, the Asian Financial Crisis, the dot-com bubble, the Iraq invasion, besides the recent crises mentioned above), but also enables us to study the possible differences between the two major players in the oil market. Specifically, according to the Central Intelligence Agency (CIA) World Factbook,[1] Canada is the 6th largest net oil exporter, while the U.S. ranks the 2nd in terms of net oil imports.

While we shed light on the role of oil-market tail risks in predicting stock-market tail risks of Canada and the U.S., we also highlight possible spillovers of stock-market tail risks between these two countries, and we control for the role of a common factor that captures the tail risks of other major stock-markets, namely those of France, Germany, Italy, Japan, Switzerland, and the United Kingdom (U.K.), in line with the evidence provided by Das et al., (2019) and Ji et al., (2020). In this context, it is important to recall that there is a large literature on stock-market correlations, documenting the presence of a conditional pattern in return correlations with respect to market conditions. The so-called correlation asymmetry phenomena reported in a number of studies (Longin and Solnik; 1995, 2001; Ang and Bekaert 2002; Campbell et al., 2002; Goetzmann et al. 2005; Bekaert et al., 2009; among others) refers to the asymmetric pattern according to which correlation of stock-market returns tends to strenghten during bear-market regimes (i.e., lower tails) as well as during periods of extreme price fluctuations observed during episodes of crises.

As far as the econometric approaches are concerned, it must be mentioned that there are primarily two approaches for computing tail risks. One is associated with option-implied measures, while the other is based on the underlying returns data (Gkillas et al., (2020)).. Understandably, due to unavailability of such long-spans of historical data on options, we take the second route, whereby we estimate tail risk using the popularValue at Risk (VaR) metric by employing the conditional autoregressive VaR model as proposed by Engle and Manganelli (2004). In this regard, the models considered are: (i) the adaptive model; (ii) the symmetric slope model, (iii) the asymmetric slope model, and; (iv) the indirect generalized autoregressive conditional heteroscedasticity (GARCH) model with an autoregressive mean. In our empirical analysis, we then use the tail-risk model that statistically best-fits the eight stock markets and oil returns. Equipped with the optimal tail risks models, in terms of the predictive framework, we rely on the approaches of Westerlund and Narayan (2012, 2015), which allows us to account for persistence, endogeneity, and conditional heterocsedasticity effects, which are typical features of oil and financial markets (Narayan and Gupta, 2015; Salisu et al., 2021a, b, c).

[1] See: https://www.cia.gov/the-world-factbook/.

To the best of our knowledge, this is the first paper to predict the tail risks of the stock-markets of Canada and the U.S. based on monthly data that covers over a century, by accounting for spillovers and oil-market tail risk. The only somewhat related paper is that by De Nicolò and Lucchetta (2017), whereby the authors presents a set of multi-period forecasts of indicators of tail real (industrial production and employment growth) and financial (distance to insolvency measures of corporate and banking sectors) risks obtained using a large database of monthly U.S. data for the period 1972:M1–2014:M12. The key finding of De Nicolò and Lucchetta (2017) is that forecasts obtained with autoregressive (AR) and factor-augmented vector autoregressive (VAR) models significantly underestimate tail risks, while quantile projections deliver fairly accurate forecasts and reliable early-warning signals for tail real and financial risks up to a 1-year horizon.
The remainder of this paper is organized as follows: Section 2 outlines the methodologies and the data, while Section 3 presents the results, with various robustness tests, and Section 4 concludes the paper.

2. Methodology and Data

2.1 Methodology

We start by formulating an empirical model that allows us to examine the connection and predictive prowess of stock-market tail risks between Canada and the U.S. These two advanced economies are strong trading partners with the U.S. serving as Canada's largest export market and, therefore, negative market shocks to the U.S. often have negative effects on the Canadian economy (Nicar, 2015). Volatility spillovers have also been observed to be bi-directional for exchange-traded funds between the two countries (Krause and Tse, 2013). Consequently, we hypothesize that the tail risk in one stock market may contain significant predictive information for the tail risk of the other stock market. To test the hypothesis, we formulate a predictive model separately for the tail risks associated with the two stock markets while also accounting for the role of global market risk using oil-market tail risk[2] as well as the stock-market tail risks of other (six) advanced economies. To estimate the tail risk for each of the return series, we follow the approach developed by Engle and Manganelli (2004) as it utilizes the asymptotic form of the tail rather than modeling

[2] There is a huge body of literature linking stock returns to movements in oil price (see for a recent survey, Smyth and Narayan, 2018, Salisu et al., 2019a, 2019b).

the whole distribution.[3] This approach involves a conditional autoregressive quantile specification of Value-at-Risk (VaR)[4], which is also termed as conditional autoregressive value at risk (CAViaR)[5] and provides an alternative measure of market (systematic) risk used by financial institutions. Rather than modelling the whole distribution, Engle and Manganelli (2004)[6] provide a different approach to the quantile estimation of VaR. A generic CAViaR specification is given as

$$f_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\beta) + \sum_{j=1}^{r} \beta_j l(x_{t-j}) \tag{1}$$

where $f_t(\beta) \equiv f_t(x_{t-1}, \beta_\theta)$ denote the time t θ-quantile of the distribution of portfolio returns formed at $t-1$. Note that θ subscript is supressed from β_θ as in Eq. (1) for notational convenience. Also, $p = q + r + 1$ is the dimension of β and l is a function of a finite number of lagged values of observables. The autoregressive terms $\beta_i f_{t-i}(\beta)$, $i = 1, \ldots, q$, ensure that the quantile changes "smoothly" over time. The role of $l(x_{t-j})$ is to link $f_t(\beta)$ to observable variables that belong to the information set. We estimate four variants of the tail risks namely Adaptive, Symmetric absolute value, Asymmetric slope and Indirect GARCH and are respectively specified as follows:

Adaptive:

---

3 Recent studies using the same approach to estimate tail risk include Salisu et al., (2021a, b, c), among others.

4 Several attractions to the use of Value at risk (VaR) as a standard measure of market risk are well documented in Engle and Manganelli (2004). Chief among these attractions is its conceptual simplicity as it reduces the market risk associated with any portfolio to a single (monetary) amount.

5 The new approach is designed to overcome the statistical problem inherent in the standard VaR method. Since VaR is simply a particular quantile of future portfolio values, conditional on current information, and because the distribution of portfolio returns typically changes over time, the challenge is to find a suitable model for time-varying conditional quantiles, an issue that is ignored in the standard VaR but incorporated in the CAViaR.

6 There are other approaches of modelling tail risks (see Boudoukh, Richardson and Whitelaw, 1998; Danielsson and de Vries, 2000), however we favour the one proposed by Engle and Manganelli (2004) given the inherent shortcomings in the previous approaches and the ability of the latter to overcome them. For instance, the approach proposed by Danielsson and de Vries (2000) is not "extreme enough" to capture the tail of the distribution and more importantly, the quantile models are nested in a framework of iid variables, which is not consistent with the characteristics of most financial series, and, consequently, the risk of a portfolio may not vary with the conditioning information set (Engle and Manganelli, 2004).

$$f_t(\beta_1) = f_{t-1}(\beta_1) + \beta_1 \left\{ \left[1 + \exp\left(G\left[y_{t-1} - f_{t-1}(\beta_1) \right] \right) \right]^{-1} - \theta \right\} \tag{2}$$

Symmetric absolute value:

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 |y_{t-1}| \tag{3}$$

Asymmetric slope:

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 (y_{t-1})^+ + \beta_4 (y_{t-1})^- \tag{4}$$

Indirect GARCH (1,1):

$$f_t(\beta) = \left(\beta_1 + \beta_2 f_{t-1}^2(\beta) + \beta_3 y_{t-1}^2 \right)^{1/2} \tag{5}$$

where G is some positive finite number which makes the model a smoothed version of a step function and the last term in Eq. (2) converges almost surely to $\beta_1 \left[I\left(y_{t-1} \le f_{t-1}(\beta_1) \right) - \theta \right]$ if $G \to \infty$ with $I(\cdot)$ representing the indicator function. Note that Eqs. (3) and (5) are symmetric in nature while Eq. (4) is asymmetric as the response to positive and negative returns is identical for the former category but differs for the latter. While the adaptive model has a unit coefficient on the lagged VaR, the other three are mean reverting implying that the coefficient on the lagged VaR is not constrained to be 1.

We subject all returns series to the CAViaR test, where we use all four alternative specifications to produce results for the 5% VaR across the four variants of the CAViaR. Thereafter, we use model diagnostics such as the Dynamic Quantile test (DQ) test, the %Hits, and the Regression Quantile[7] to determine the model that best fits the data. The results obtained in this way are then used to study tail-risk predictability using the Westerlund and Narayan (2012, 2015) methods, which allow us to account for additional salient features inherent in the tail-risk data such as persistence, endogeneity, and conditional heterocsedasticity effects typical of most financial series. The methods rely on the following predictive model partitioned into three variants as follows:[8]

[7] These are standard test statistics for evaluating the relative performance of the alternative specifications of CAViaR test.

[8] See Westerlund and Narayan (2015) for computational details while several applications are evident in the literature as regards the use of this methodology for stock return predictability (see for example, Bannigidadmath and Narayan,

Case I: This is a single-predictor case where the stock-market tail risk of one country serves as a predictor in the predictive model for the stock-market tail risk of the other country. The effect of U.S. stock-market tail risk on Canadian stock-market tail risk is depicted in Eq. (6), while the converse is expressed in Eq. (7).

Stock Tail risk of Canada: $$tr_t^{can} = \omega^{can} + \phi^{us} tr_{t-1}^{us} + \alpha^{us}\left(tr_t^{us} - \rho^{us} tr_{t-1}^{us}\right) + \varepsilon_t^{can} \quad (6)$$

Stock Tail risk of US: $$tr_t^{us} = \omega^{us} + \phi^{can} tr_{t-1}^{can} + \alpha^{can}\left(tr_t^{can} - \rho^{can} tr_{t-1}^{can}\right) + \varepsilon_t^{us} \quad (7)$$

where tr is the best fit tail risk at period t obtained from the CAViaR framework; ω is the intercept; ϕ is the predictability slope coefficient; and ε_t is the zero mean idiosyncratic error term. Note that the superscript on the tail risk defines the return series used in calculating it, thus, superscripts "can", "us", and "$others$" (in Eqs. (8)-(11) discussed below) respectively denote the tail risks for the stock return series of Canada, United States, and other six advanced countries (namely France, Germany, Italy, Japan, Switzerland and UK) combined using the principal component analysis while "oil" is the tail risk of oil price returns. Note that in addition to the lagged predictor series - ϕtr_{t-1}, we include an additional term - $\alpha\left(tr_t - \rho tr_{t-1}\right)$ in all the predictive models in order to resolve any inherent endogeneity bias resulting from the correlation between the predictor series and the error term as well as any potential persistence effect (see Westerlund and Narayan, 2012, 2015, for technical details)[9]. Also, using high (monthly) data frequency over a century requires the need to account for conditional heteroscedasticity effect. We implement this by pre-weighting all the data with the inverse of standard deviation obtained from a GARCH-type estimation of equations (6) to (11). The resulting equations are estimated with the Ordinary Least Squares method to obtain the feasible quasi generalized least squares estimates.

Case II: Here, we extend equations (6) and (7) to include distinctly the stock-market tail risks estiamted for the remaining six advanced economies using a principal-component analysis to form an index. This additional regressor is motivated by the strong trade connections among the advanced economies (see Canzoneri et al., (2003); Arin and Koray(2009), and Nicar(2015),

---

(2015), Narayan and Bannigidadmath (2015), Narayan and Gupta (2015), Phan et al., (2015), Devpura et al., (2018), Salisu et al., (2019a,b) among others).

[9] Some preliminary tests are rendered in this regard to establish the presence of these effects and the results can be provided by the authors upon request.

among others). The idea is to examine the influence of the risks associated with other advanced economies on the two economies being examined and whether such risks possess any predictive value for the predictability of the stock-market tail risks of Canada and the U.S. The extended predictive regressions are specified for Canada and the U.S. in (8) and (9):

Stock Tail risk of Canada:
$$tr_t^{can} = \omega^{can} + \phi^{us} tr_{t-1}^{us} + \alpha^{us}\left(tr_t^{us} - \rho^{us} tr_{t-1}^{us}\right) + \phi^{others} tr_{t-1}^{others} + \alpha^{others}\left(tr_t^{others} - \rho^{others} tr_{t-1}^{others}\right) + \varepsilon_t^{can} \quad (8)$$

Stock Tail risk of US:
$$tr_t^{us} = \omega^{us} + \phi^{can} tr_{t-1}^{can} + \alpha^{can}\left(tr_t^{can} - \rho^{can} tr_{t-1}^{can}\right) + \phi^{others} tr_{t-1}^{others} + \alpha^{others}\left(tr_t^{others} - \rho^{others} tr_{t-1}^{others}\right) + \varepsilon_t^{us} \quad (9)$$

Case III: Finally, we extend equations (8) and (9) to include the oil-market tail risk given the strong connections between oil and stock markets (Narayan and Gupta, 2015; Salisu and Isah, 2017; Smyth and Narayan, 2018; Salisu et al., 2019a, b). In doing so, we are able to test whether the Canada and the U.S: respond differently to oil-market tail risk given that one is a net oil exporter (Canada) and the other is a net oil importer (the U.S.). We present the extended predictive regressions in equations (10) and (11) for Canada and the U.S.:

Stock Tail risk of Canada:
$$tr_t^{can} = \omega^{can} + \phi^{us} tr_{t-1}^{us} + \alpha^{us}\left(tr_t^{us} - \rho^{us} tr_{t-1}^{us}\right) + \phi^{others} tr_{t-1}^{others} + \alpha^{others}\left(tr_t^{others} - \rho^{others} tr_{t-1}^{others}\right) + \phi^{oil} tr_{t-1}^{oil} + \alpha^{oil}\left(tr_t^{oil} - \rho^{oil} tr_{t-1}^{oil}\right) + \varepsilon_t^{can} \quad (10)$$

Stock Tail risk of US:
$$tr_t^{us} = \omega^{us} + \phi^{can} tr_{t-1}^{can} + \alpha^{can}\left(tr_t^{can} - \rho^{can} tr_{t-1}^{can}\right) + \phi^{others} tr_{t-1}^{others} + \alpha^{others}\left(tr_t^{others} - \rho^{others} tr_{t-1}^{others}\right) + \phi^{oil} tr_{t-1}^{oil} + \alpha^{oil}\left(tr_t^{oil} - \rho^{oil} tr_{t-1}^{oil}\right) + \varepsilon_t^{us} \quad (11)$$

In the final step of our empirical analysis, we evaluate the forecast performance of equations (6) to (11) relative to a benchmark (driftless random walk) model which ignores the tail risk-based predictor series in the predictability of the stock-market tail risks of Canada and the U.S.. We employ both the single (Root Mean Square Forecast Error) and pairwise forecast measure using the Clark and West (2007) for the forecast evaluation analysis, where we use a75:25 data split to obtain out-of-sample forecasts (that is, 75% of the full sample form the in-sample period and the remaining 25% are the out-of-sample period).[10]

2.2 Data sources

The data used in our empirical research are monthly stock price indices for Canada (S&P TSX 300 Composite Index) and the U.S. (S&P500 Index) as well as stock price data for other six advanced countries namely, France (CAC All-Tradable Index), Germany (CDAX Composite Index), Italy (Banca Commerciale Italiana Index), Japan (Nikkei 225 Index), Switzerland (All Share Stock Index), and the UK (FTSE All Share Index) while we use the West Texas Intermediate crude oil price as a proxy for the oil price. The indices and oil price are derived from the Global Financial Data[11] database and the datasets cover the period from 1916M02 to 2020M10, the choice of which is governed by the availability of data for Switzerland. We convert all variables into log returns in percentage, i.e., the first-difference of the natural logarithm of the indices multiplied by 100.

3. Empirical results

3.1 The results of tail risks

We begin our empirical analysis by generating the tail-risk data for the return series of all the stock-price indices and the oil price. We estimate the four CAViaR specifications described in Section 2 to obtain the 5% Value-at-Risk[12] and, thereafter, we compute relevant diagnostics such as the %Hits, the Dynamic Quantile (DQ) test, and the Regression Quantile (RQ) statistic to determine the "best" CAViaR specification (tail risk) for each return series. We expect the %Hits to be about 5% for 5% VaR[13]; the DQ test statistic is not expected to be significant, and the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. The DQ test, however, takes prominence over the %Hits and RQ statistics. In cases where

[10] Note that there is no theoretical guidance in the literature for data splitting in forecast analysis, however, studies have adopted 25:75, 50:50 and 75:25 respectively between the in-sample and out-of-sample forecasts (see Narayan and Gupta, 2015) and the outcome is observed to be insensitive to the choice of data split (Narayan and Gupta, 2015; Salisu et al., 2019a, b).

[11] https://globalfinancialdata.com/.

[12] We also consider 1% Value-at-Risk for robustness and the results are presented in the appendix.

[13] Similarly, we expect the %Hits to be about 1% for 1% VaR.

more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistics to determine the model with the best fit.

Tables 1, 2, 3, and 4 summarize the results for the Symmetric Absolute Value (SAV), Asymmetric slope (ASY), Indirect GARCH (GARCH), and Adaptive (ADAPT) models across the various return series under consideration. A cursory look at the results shows that the Asymmetric CAViaR model largely offers the "best" results as the model is favoured in six out of the nine return series implying an asymmetric response of investors to up and down swings in stock prices, while the Symmetric Absolute Value is the best fit model for oil returns. The Indirect GARCH model, in turn, is chosen for the stock returns of France and UK.[14] Regardless of the best fitting model, all the return series exhibit volatility clustering as measured by the statistically significant coefficient (Beta2) on the autoregressive term in the SAV, ASY, and GARCH specifications, which is a requirement for tail risks (see Engle and Manganelli, 2004).

3.2 Predictability and forecast evaluation of stock-return tail risks of Canada and the U.S.

After obtaining the best fit tail risks, we use the same in the predictability analysis following the sequence of the specified predictive models in equations (6) to (11) as structured into three cases in Section 2. Table 5 depicts the results for the three cases, where, just to recollect, Case I involves a single tail-risk factor (which allows us to evaluate the connection between the tail risks of Canada and the U.S.), Case II extends Case I to include the tail risk of the other advanced economies (owing to the increasing interdependencies among them, see Holmes and Pentecost (1992), Pesaran et al., (2004), Felmingham and Cooray (2008)), and Case III extends Case II to include oil-market tail risk given the strong connection between oil and stock markets (Narayan and Gupta, 2015; Salisu and Isah, 2017; Smyth and Narayan, 2018; Salisu et al., 2019a, b, among others).

The results for Case I reveal possible risk-spillover effects between Canada and the U.S. as the two slope coefficients are positive and statistically significant, implying a rise in the market risk associated with one economy (say U.S) has a spillover effect onto the other economy (Canada). This outcome further reinforces the strong economic ties between the two countries. For instance,

[14] The tail risk results for the 1% VaR are similar to those of 5% VaR particularly in terms of the superiority of the Asymmetric slope model over other models, with complete details of these results available upon request from the authors.

the U.S. is Canada's main market by a wide margin across nearly every export-oriented sector and, therefore, the effects of a minor U.S. slowdown are amplified by the Canadian reliance on the U.S. market for its own growth (Kenton, 2020).

In order to further motivate these findings, particularly from an investment perspective, we evaluate the in-sample and out-of-sample forecast performance of accounting for the tail risk of the U.S. (Canada) for the predictability of tail risk of Canada (theU.S.). This offers insights into the predictive value of stock-market tail risks crucial for diversification strategies. The out-of-sample forecast is evaluated across multiple horizons (h = 6, 12 and 24 months), using the RMSE and Clark-West test. The forecast evaluation results (for Case I) reported in Table 6 show a superior forecast performance of the tail-risk-based model over a random walk model as evidenced by the positive and statistically significant Clark-West test both for the in-sample and out-of-sample forecasts. A similar result is obtained for an alternative (1%) Value-at-Risk (see Table A1, Appendix). Thus, information about the risk exposure of two intertwined economies should be useful for an effective policy- and investment-decision making.

We also consider an alternative specification where the stock-market tail risks of the U.S. and Canada are replaced with the oil-market tail risk[15] in both equations (6) and (7) in order to evaluate their response to global risk. We find for this specification contrasting evidence for the two economies where it is positive for Canada (being a net oil exporting) and negative for U.S. (being a net oil importing country) (Appendix, Table A6 for the predictability of results). Being a net oil exporting country, a rise in oil-market market risk (particularly due to drops in global oil demand and the oil price) may have a spillover effect onto Canada as the country's oil and gas producers may face increasing challenges in competing with other, lower-cost producers (Erickson and Lazarus, 2020), while under the same condition (a drop in the oil price), a net oil importer like the U.S. realizes economic gains due to lower cost of production resulting in improved share value of stocks, *ceteris paribus* (Wang et al., 2013; Bouoiyour et al., 2017). The in-sample and out-of-sample predictive prowess of oil-market tail risk for stock-market tail risks of Canada and the U.S. is also evaluated and the results confirm this intuition (see Table A7, Appendix).

[15] In our main analysis, we utilize data covering 1916M02 to 2020M10. However, as a robustness check, we utilize data covering 1915M02 to 2020M10 for the analysis involving Canada and 1859M10-2020M10 for U.S. based on the start date for Canada (since oil price has a longer data span) in the former case and start date for oil price in the latter case (since U.S. stock data has a longer scope than that of oil price). The additional tail risk estimates required for the analysis of the oil tail risk-based model are presented in the appendix (see Tables A2, A3, A4 and A5 in the appendix).

3.3 Stock-return tail risks of other advanced economies and the predictability of stock - market return tail risks of Canada and U.S.

We next extend the single-predictor model (see Case I) to include the stock-market tail risks of other advanced economies (France, Germany, Italy, Japan, Switzerland and UK, Case II) as well as oil-market tail risk in a multiple setting (Case III) . To avoid proliferation of parameters, we use the principal component approach to develop an index that accommodates the best fit of tail risks for the stock returns of other advanced countries. Thereafter, the index is used as an additional predictor of tail risks for stock returns of Canada and the U.S. We employ the correlation matrix when computing the decomposition of the principal components because the alternative method involving the covariance matrix requires that the variances of the underlying variables of interest must be similar, which is not the case for our variables. The results, which are summarized in Table 7 for the 5% VaR and Table A8 (Appendix) for the 1% VaR, are shown in two panels. The first panel summarizes the information on eigenvalues while the second panel shows the estimated eigenvectors. The former are sorted in order of principality (importance), measured as the proportion of information explained by each principal component, while the latter summarizes the contributions of the tail risks to each of the principal components. As shown in Table 7, we find that the first principal component explains roughly 36% of the information contained in the correlation matrix, the second roughly 19%, the third about 20%, and so on. Furthermore, the cumulative proportion of information explained by the first four principal components is roughly 84% (36% + 19% + 17% + 12%). In other words, given that dimensionality reduction is desired, our analysis indicates that we can reduce the underlying dimensionality of the problem from 6 to 4, while retaining nearly 85% of the original information.[16] Thus, we use in our further analysis the average of the first four principal components (PC1, PC2, PC3 and PC4) because these components capture the majority of variations in the tail risks of the selected stock returns. Using this same approach, we find the first four principal components for the 1% tail risks retain almost 90% of the original information and, by extension, the average of these principal components (PC1, PC2, PC3 and PC4) is obtained and used in our subsequent analysis as an index for the tail risks of stock returns (see Table A8, Appendix).

[16] The dimensionality reduction problem involves finding first few directions that will capture the majority of variation, leaving the less principal directions to contribute information only marginally.

The predictability results of the tail risks of other advanced economies are presented in Table 5 for Cases II and III denoted as "Tail risk (Others)", and again we find contrasting evidence for Canada (negative) and the U.S. (positive). This outcome further reinforces the strong connection between the EU and the U.S. (Congressional Research Service, 2021) and, by extension, the exposure of the latter to the risk of the former (and vice versa) relative to Canada. This outcome is also instructive as it highlights the potential of Canada as a diversifier for EU tail risks, among other advanced economies.

4. Conclusion

Given the long-standing economic ties between the two advanced economies, we have studied the link between the stock-tail risks of Canada and the U.S. using a century of monthly data. We also have accounted for the role of oil-market tail risk as well as the stock-market tail risks of other advanced economies. We have employed to this end the Conditional Autoregressive Value at Risk (CAViaR) of Engle and Manganelli (2004) to measure tail risks as this model concentrates on the tail distribution rather the whole distribution. We have estimated the four variants (Adaptive, Symmetric absolute value, Asymmetric slope and Indirect GARCH) of the CAViaR model for every return series with the "best" tail-risk model obtained using diagnostics such as the Dynamic Quantile test (DQ) test, %Hits, and Regression Quantile (RQ) statistic. Consequently, we have utilized the best tail-risk model for analysing the predictive prowess of accounting for stock-market tail risk of one economy (say U.S.) for the predictability of the stock-market tail risk of the other economy (say Canada) for various out-of-sample forecast horizons.

We have found a strong positive relation between the stock-market tail risks of Canada and the U.S., showing that the risk spillovers between the two economies is sustained over multiple out-of-sample forecast horizons. Additional analyses involving the role of oil-market tail risk have revealed a positive association with the tail risk of Canada (being a net oil exporting economy) and a negative one for the U.S. (being a net oil importing nation). In order to assess the role of tail risks of other advanced such as France, Germany, Italy, Japan, Switzerland and the UK, we have employed a principal component analysis to combine their individual tail risks. We then have used the resulting index as an additional predictor for Canada and the U.S. The results obtained in this regard have revealed a positive connection with the U.S. and a negative one for Canada, thereby supporting the diversification potential of the latter in the presence of tail risks from advanced economies other than the U.S.

In sum, while evaluating tail risks for the design of optimal portfolios, investors in Canada and the U.S. should benefit by taking into account not only spillovers of tail risks, but also the differential impact of oil-market tail risks contingent on their position in the oil market. In addition, with heightened financial lower tail risks shown to predict economic recessions, policymakers would need to design policies in the U.S. and Canada that are state-dependent, with the states defined by the type of tail risks in the oil and equity markets of other advanced countries.

An extension of this study, dependent on data availability, is to investigate the predictability of historical tail risks of emerging stock markets.

References

Adrian, T., Boyarchenko, N., and Giannone, D. (2019). Vulnerable growth. American Economic Review, 109(4), 1263-1289.

Ang, A., and Bekaert, G. (2002). Regime switches in interest rates. Journal of Business and Economic Statistics, 20(2), 163-182.

Arin, K., and Koray, F. (2009). Beggar thy neighbor? The transmission of fiscal shocks from the US to Canada. Open Economics Review, 20(3), 425-434.

Baker, S., Bloom, N., Davis, S.J., and Sammon, M. (2015). What Triggers Stock Market Jumps? American Economic Association Session on "Shocks and Disasters", ASSA Meetings, Boston, 4 January 2015.

Baker, S., Bloom, N., Davis, S.J., Kost, K., Sammon, M., and Viratyosin, T. (2020). The unprecedented stock market reaction to COVID-19. The Review of Asset Pricing Studies, 10(4), 742-758.

Bannigidadmath, D., and Narayan, P. (2015). Stock return predictability and determinants of predictability and profits. Emerging Markets Review, 26, 153-73.

Bekaert, G., Hodrick, R.J., and Zhang, X. (2009). International stock return comovements. The Journal of Finance, 64(6), 2591-2626.

Boudoukh, J., Richardson, M., and Whitelaw, R.F. (1998). The Best of Both Worlds. Risk, 11, 64-67.

Bouoiyour, J., Selmi, R., Shahzad, S.J.H., and Shahbaz, M. (2017). Response of Stock Returns to Oil Price Shocks: Evidence from Oil Importing and Exporting Countries. Journal of Economic Integration, 32(4), 913-936.

Campbell, R., Koedijk, K., and Kofman, P. (2002). Increased correlation in bear markets. Financial Analysts Journal, 58(1), 87-94.

Canzoneri, M.B., Cumby, R.E., and Diba, B. (2003) New views on the transatlantic transmission of fiscal policy and macroeconomic policy coordination. In: Buti M (ed) Monetary and Fiscal Policies in EMU: Interactions and Coordination. Cambridge University Press, 283-311.

Chevapatrakul, T., Xu, Z., and Yao, K. (2019). The impact of tail risk on stock market returns: The role of market sentiment. International Review of Economics & Finance, 59(C), 289-301.
Clark, T.E., and West, K.D. (2007). Approximately normal tests for equal predictive accuracy in nested models. Journal of Econometrics, 138(1), 291-311.
Congressional Research Service (2020). U.S.-European Relations in the 117th Congress. Retrieved from: https://fas.org/sgp/crs/row/IF11094.pdf.
Das, S., Demirer, R., Gupta, R., and Mangisa, S. (2019). The effect of global crises on stock market correlations: Evidence from scalar regressions via functional data analysis. Structural Change and Economic Dynamics, 50(C), 132-147.
Danielsson, J., and de Vries, C.G. (2000). Value-at-Riska nd Extreme Returns. Annales d'Economiee t de Statistique, 60, 239-270.
Degiannakis, S.A., Filis, G., and Arora, V. (2018). Oil Prices and Stock Markets: A Review of the Theory and Empirical Evidence. The Energy Journal, 39(5), Article No. 4.
De Nicolò, G., and Lucchetta, M. (2016). Forecasting Tail Risks. Journal of Applied Econometrics, 32(1), 159-170.
Devpura, N., Narayan, P.K., and Sharma, S.S. (2018). Is stock return predictability time varying? Journal of International Financial Markets, Institutions and Money, 52, 152-72.
Ellwanger, R. (2017). On the tail risk premium in the oil market. Staff Working Papers 17-46, Bank of Canada.
Erickson, P., and Lazarus, M. (2020). Examining risks of new oil and gas production in Canada. Stockholm Environment Institute (SEI) Report, https://www.sei.org/publications/examining-risks-of-new-oil-and-gas-production-in-canada/.
Felmingham, B., and Cooray, A.V. (2008). Real interest rate interdependence among the G7 nations: does real interest parity hold? Journal of International Finance and Economics, 8(1), 14-22.
Gkillas, K., Konstantatos, C., Tsagkanos, A., and Siriopoulos, C. (2020). Do economic news releases affect tail risk? Evidence from an emerging market. Finance Research Letters. DOI: https://doi.org/10.1016/j.frl.2020.101727.
Goetzmann, W.N., Li, L., and Rouwenhorst, K.G. (2005). Long-term global market correlations. Journal of Business, 78(1), 1-38.
Gupta, R., Sheng, X., Pierdzioch, C., and Ji, Q. (2021). Disaggregated Oil Shocks and Stock-Market Tail Risks: Evidence from a Panel of 48 Countries. Department of Economics, University of Pretoria, Working Paper No. 202106.
Hollstein, F., Nguyen, D.B.B., Prokopczuk, M., and Wese Simen, C. (2019). International Tail Risk and World Fear. Journal of International Money and Finance, 93, 244-259.
Holmes, M.J., and Pentecost, E.J. (1992). Changes in the extent of financial interdependence between the G7 Countries in the 1970s and 1980s. International Economic Journal, 6, 495-105.
Ji, Q., Liu, B-Y., Cunado, J., and Gupta, R. (2020). Risk spillover between the US and the remaining G7 stock markets using time-varying copulas with Markov switching : evidence from over a century of data. The North American Journal of Economics and Finance, 51, 100846.
Kelly, B., and Jiang, H. (2014). Tail risk and asset prices. Review of Financial Studies, 27(10), 2841-2871.

Kenton, W. (2020). Spillover Effect. Retrieved from: https://www.investopedia.com/terms/s/spillover-effect.asp.

Krause, T., and Tse, Y. (2013). Volatility and return spillovers in Canadian and US industry ETFs. International Review of Economics & Finance, 25, 244-259.

Li, X-M., and Rose, L.C. (2009). The tail risk of emerging stock markets. Emerging Markets Review, 10, 242-256.

Lombardi, M., Osbat, C., and Schnatz, B. (2012). Global commodity cycles and linkages: a FAVAR approach. Empirical Economics, 43(2), 651-670.

Longin, F., and Solnik, B. (1995). Is the correlation in international equity returns constant: 1960–1990? Journal of International Money and Finance, 14(1), 3-26.

Longin, F., and Solnik, B. (2001). Extreme correlation of international equity markets, The Journal of Finance, 56(2), 649-676.

Mensi, W., Hammoudeh, S., Shahzad, S.J.H., and Shahbaz, M. (2017). Modeling systemic risk and dependence structure between oil and stock markets using a variational mode decomposition-based copula method. Journal of Banking & Finance, 75, 258-279.

Narayan, P.K., and Gupta, R. (2015). Has oil price predicted stock returns for over a century? Energy Economics, 48,18-23.

Narayan, P.K., and Bannigidadmath, D. (2015). Are Indian stock returns predictable? Journal of Banking and Finance, 58, 506-31.

Nicar, S.B. (2015). International spillovers from US fiscal policy shocks. Open Economies Review, 26(5), 1081-1097.

Pesaran, M., Schuermann, T., and Weiner, S. (2004). Modeling Regional Interdependencies Using a Global Error-Correcting Macroeconometric Model. Journal of Business & Economic Statistics, 22(2), 129-162.

Phan, D.H.B., Sharma, S.S., and Narayan, P.K. (2015). Stock return forecasting: Some new evidence. International Review of Financial Analysis, 40, 38-51.

Salisu, A.A., Gupta, R., and Ji, Q. (2021b). Forecasting Oil Price over 150 Years: The Role of Tail Risks. Department of Economics, University of Pretoria, Working Paper No. 202120.

Salisu, A.A., Gupta, R., and Ogbonna, A.E. (2021a). Tail Risks and Forecastability of Stock Returns of Advanced Economies: Evidence from Centuries of Data. Department of Economics, University of Pretoria, Working Paper No. 202117.

Salisu, A.A., and Isah, K. (2017). Revisiting the oil price and stock market nexus: A nonlinear Panel ARDL approach. Economic Modelling, 66(C), 258-71.

Salisu, A.A., Pierdzioch, C., and Gupta, R. (2021c). Geopolitical Risk and Forecastability of Tail Risk in the Oil Market: Evidence from Over a Century of Monthly Data. Department of Economics, University of Pretoria, Working Paper No. 202122.

Salisu, A.A., Raheem, I.D., and Ndako, U.B. (2019b). A sectoral analysis of asymmetric nexus between oil price and stock returns. International Review of Economics and Finance, 61(C), 241-59.

Salisu, A.A., Swaray, R., and Oloko, T.F. (2019a). Improving the predictability of the oil–US stock nexus: The role of macroeconomic variables. Economic Modelling, 76, 153-71.

Shaeri, K., Adaoglu, C., and Katircioglu, S.T. (2016). Oil Price Risk Exposure: A Comparison of Financial and Non-Financial Subsectors. Energy, 109, 712-723.

Smyth, R., and Narayan, P.K. (2018). What do we know about oil prices and stock returns? International Review of Financial Analysis, 57, 148-156.

Wang, Y., Wu, C., and Yang, L. (2013). Oil price shocks and stock market activities: Evidence from oil-importing and oil-exporting countries. Journal of Comparative Economics, 41(4), 1220-1239.

Westerlund, J., and Narayan, P.K. (2012). Does the choice of estimator matter when forecasting returns? Journal of Banking and Finance, 36, 2632-40.

Westerlund, J., and Narayan, P.K. (2015). Testing for predictability in conditionally heteroskedastic stock returns. Journal of Finance and Economics, 13 (2), 342-75.

Table 1: Estimates and Relevant statistics for the country-specific CAViaR specification [Symmetric Absolute Value]

5% VaR	Canada	France	Germany	Italy	Japan	Switzerland	UK	US	**Oil**
β_1	0.0619	1.0400	0.8670	0.4580	0.9610	0.4390	0.7300	0.4580	**0.0000**
Standard errors	0.1540	0.7610	0.2670	0.2440	0.4060	0.5080	0.3090	0.2440	**0.0001**
P values	0.3440	0.0854	0.0006	0.0303	0.0090	0.1940	0.0092	0.0303	**0.3510**
β_2	0.9230	0.7520	0.6010	0.8740	0.7650	0.8260	0.7770	0.8740	**0.8900**
Standard errors	0.0475	0.1250	0.1020	0.0508	0.0762	0.1270	0.0805	0.0508	**0.0002**
P values	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
β_3	0.1820	0.2590	0.6800	0.1690	0.2830	0.2860	0.2790	0.1690	**0.2630**
Standard errors	0.1090	0.0904	0.1910	0.0601	0.1130	0.1220	0.1050	0.0601	**0.0002**
P values	0.0478	0.0021	0.0002	0.0025	0.0064	0.0099	0.0039	0.0025	**0.0000**
RQ	573.0000	580.0000	828.0000	655.0000	651.0000	540.0000	530.0000	655.0000	**517.0000**
Hits in-sample (%)	4.9700	5.0600	4.9700	4.9700	4.9700	5.0600	5.0600	4.9700	**5.0600**
Hits out-of-sample (%)	3.6000	6.0000	8.0000	6.4000	7.6000	6.4000	6.0000	6.4000	**8.8000**
DQ in-sample (P values)	0.0625	0.7620	0.1470	0.0538	0.3370	0.0874	0.3090	0.0538	**0.0000**
DQ out-of-sample (P values)	0.6040	0.0043	0.0160	0.6460	0.0952	0.0501	0.3790	0.6460	**0.0000**

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit

Table 2: Estimates and Relevant statistics for the country-specific CAViaR specification [Asymmetric Slope]

5% VaR	**Canada**	France	**Germany**	**Italy**	**Japan**	**Switzerland**	**UK**	**US**	Oil
β_1	**0.2310**	1.0400	**0.6530**	**0.3730**	**1.7900**	**0.6090**	0.6490	**0.3730**	0.0000
Standard errors	**0.3810**	0.7720	**0.8210**	**0.1900**	**0.8450**	**0.5470**	0.3230	**0.1900**	0.0000
P values	**0.2720**	0.0887	**0.2130**	**0.0247**	**0.0169**	**0.1330**	0.0224	**0.0247**	0.4840
β_2	**0.8740**	0.7190	**0.6290**	**0.8530**	**0.5030**	**0.7820**	0.8050	**0.8530**	0.7310
Standard errors	**0.0757**	0.1180	**0.2350**	**0.0411**	**0.1330**	**0.0977**	0.0987	**0.0411**	0.0000
P values	**0.0000**	0.0000	**0.0037**	**0.0000**	**0.0001**	**0.0000**	0.0000	**0.0000**	0.0000
β_3	**0.1080**	0.2490	**0.3440**	**0.1880**	**0.3860**	**0.1080**	0.0801	**0.1880**	0.2000
Standard errors	**0.1470**	0.1170	**0.2480**	**0.0585**	**0.0869**	**0.1010**	0.0866	**0.0585**	0.0000
P values	**0.2300**	0.0168	**0.0832**	**0.0007**	**0.0000**	**0.1420**	0.1770	**0.0007**	0.0000
β_4	**0.3080**	0.3190	**0.8530**	**0.2380**	**0.8230**	**0.4390**	0.3440	**0.2380**	0.9190
Standard errors	**0.1560**	0.1090	**0.5940**	**0.0798**	**0.1710**	**0.0817**	0.1850	**0.0798**	0.0000
P values	**0.0239**	0.0016	**0.0755**	**0.0014**	**0.0000**	**0.0000**	0.0310	**0.0014**	0.0000
RQ	**568.0000**	578.0000	**797.0000**	**652.0000**	**648.0000**	**526.0000**	520.0000	**652.0000**	478.0000
Hits in-sample (%)	**4.8700**	4.8700	**4.9700**	**4.9700**	**4.7700**	**4.9700**	5.0600	**4.9700**	5.9600
Hits out-of-sample (%)	**4.8000**	7.2000	**8.8000**	**6.4000**	**7.6000**	**5.2000**	7.6000	**6.4000**	10.0000
DQ in-sample (P values)	**0.3860**	0.8310	**0.9860**	**0.1260**	**0.9630**	**0.6010**	0.4240	**0.1260**	0.0000
DQ out-of-sample (P values)	**0.8860**	0.0183	**0.0497**	**0.7010**	**0.1210**	**0.8770**	0.5360	**0.7010**	0.0023

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit

Table 3: Estimates and Relevant statistics for the country-specific CAViaR specification [Indirect GARCH]

5% VaR	Canada	**France**	Germany	Italy	Japan	Switzerland	**UK**	US	Oil
β_1	-0.3840	**7.0000**	4.5900	3.3200	7.3400	1.5100	**5.0700**	3.3200	0.0000
Standard errors	0.3780	**6.7200**	3.4600	1.8200	4.0000	2.3800	**2.9500**	1.8200	0.0000
P values	0.1550	**0.1490**	0.0923	0.0341	0.0332	0.2630	**0.0428**	0.0341	0.4930
β_2	0.9300	**0.7040**	0.4560	0.8780	0.7710	0.8040	**0.7090**	0.8780	0.7320
Standard errors	0.0159	**0.1320**	0.1340	0.0277	0.0573	0.0763	**0.0740**	0.0277	0.0005
P values	0.0000	**0.0000**	0.0003	0.0000	0.0000	0.0000	**0.0000**	0.0000	0.0000
β_3	0.2250	**0.4350**	1.2000	0.1810	0.3080	0.5020	**0.4580**	0.1810	0.7210
Standard errors	0.1320	**0.2320**	0.7270	0.2900	0.1540	0.2800	**0.4720**	0.2900	0.0060
P values	0.0437	**0.0303**	0.0499	0.2670	0.0228	0.0364	**0.1660**	0.2670	0.0000
RQ	575.0000	**580.0000**	834.0000	662.0000	654.0000	538.0000	**533.0000**	662.0000	503.0000
Hits in-sample (%)	5.0600	**5.0600**	5.0600	5.0600	5.0600	5.0600	**5.0600**	5.0600	4.6700
Hits out-of-sample (%)	6.0000	**6.8000**	9.2000	6.8000	8.0000	6.0000	**7.6000**	6.8000	8.0000
DQ in-sample (P values)	0.0835	**0.8830**	0.8970	0.0134	0.4620	0.3770	**0.4430**	0.0134	0.0000
DQ out-of-sample (P values)	0.1370	**0.0116**	0.0006	0.5660	0.0240	0.0363	**0.5170**	0.5660	0.0000

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit

Table 4: Estimates and Relevant statistics for the country-specific CAViaR specification [Adaptive]

5% VaR	Canada	France	Germany	Italy	Japan	Switzerland	UK	US	Oil
β_1	0.6870	-0.1740	3.5500	1.0400	1.3600	-0.0020	1.0600	1.0400	5.8900
Standard errors	0.2580	0.0982	0.0002	0.1580	0.0428	0.1200	0.1570	0.1580	0.0000
P values	0.0039	0.0382	0.0000	0.0000	0.0000	0.4930	0.0000	0.0000	0.0000
RQ	622.0000	604.0000	905.0000	736.0000	711.0000	582.0000	560.0000	736.0000	494.0000
Hits in-sample (%)	4.3700	4.4700	4.6700	5.0600	5.1600	4.2700	5.0600	5.0600	3.7700
Hits out-of-sample (%)	4.0000	6.0000	4.8000	4.8000	4.4000	4.4000	4.4000	4.8000	6.0000
DQ in-sample (P values)	0.0000	0.0109	0.0032	0.0285	0.0319	0.0000	0.0012	0.0285	0.0126
DQ out-of-sample (P values)	0.0102	0.0023	0.7030	0.2260	0.3130	0.0036	0.2250	0.2260	0.0001

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit

Table 5: Predictability results for the stock and oil tail risks [1916M02-2020M10]

	Canada	US
Case I		
Tail risk (Canada)	-	0.9045[a]
	-	(0.0230)
Tail risk (US)	0.4006[a]	-
	(0.0343)	-
Case II		
Tail risk (Canada)	-	0.9512[a]
	-	(0.0201)
Tail risk (US)	0.4129[a]	-
	(0.0323)	-
Tail risk (Others)	-0.3456b	0.5614a
	(0.1721)	(0.1610)
Case III		
Tail risk (Canada)	-	0.4258[a]
	-	(0.0332)
Tail risk (US)	0.9927[a]	-
	(0.0201)	-
Tail risk (Others)	-0.6634[a]	0.8331[a]
	(0.1737)	(0.1695)
Tail risk (Oil)	0.0580[a]	-0.1208[a]
	(0.0174)	(0.0195)

Note: For Case I, we estimate equations (6) and (7); for Case II, we estimate equations (8) and (9); while for Case III, we estimate equations (10) and (11), respectively for Canada and United States. The predictability coefficients reported here are those of the first lags in all the estimated equations while the values in parentheses capture the standard errors. "a", "b" and "c" represent significance levels at 1%, 5% and 10%, respectively.

Table 6: Forecast evaluation results using the Clark and West (2007) test

In-sample forecast	US			Canada		
	Case I	**Case II**	**Case III**	**Case I**	**Case II**	**Case III**
CW test	106.0767[a]	103.6288[a]	102.8354[a]	74.8914[a]	88.5968[a]	77.0432[a]
	[30.3764]	[29.7451]	[29.2505]	[38.7580]	[33.4418]	[41.5853]
RMSE	3.3443	3.1401	3.1336	1.9530	1.8404	2.0478
Out-of-Sample forecast						
CW test	**Case I**	**Case II**	**Case III**	**Case I**	**Case II**	**Case III**
15	105.8266[a]	103.3827[a]	102.6053[a]	74.8559[a]	88.4838[a]	76.9884[a]
	[30.4831]	[29.8493]	[29.3587]	[38.9826]	[33.6050]	[41.8149]
30	105.5052[a]	103.0502[a]	102.2926[a]	74.7438[a]	88.3205[a]	76.8866[a]
	[30.5596]	[29.9172]	[29.4335]	[39.1576]	[33.7428]	[42.0111]
60	104.9566[a]	102.4912[a]	101.7713[a]	74.5659[a]	88.0326[a]	76.6997[a]
	[30.7465]	[30.0918]	[29.6192]	[39.5350]	[34.0341]	[42.4113]
RMSFE						
15	3.3544	3.1502	3.1460	1.9479	1.8356	2.0427
30	3.3593	3.1523	3.1506	1.9419	1.8305	2.0366
60	3.3648	3.1549	3.1575	1.9303	1.8208	2.0251

Note: For Case I, we estimate equations (6) and (7); for Case II, we estimate equations (8) and (9); while for Case III, we estimate equations (10) and (11), respectively for Canada and United States. For the Clark and West (2007) [CW] test, the null hypothesis of a zero coefficient is rejected if the t-statistic is greater than +1.282 (for a one sided 0.10 test), +1.645 (for a one sided 0.05 test), and +2.00 for 0.01 test (for a one sided 0.01 test) (see Clark and West, 2007), and are denoted by [c, b] and [a], respectively; and the values of the t-statistic are in square brackets. RMSFE denotes Root Mean Square Forecast Error.

Table 7: Principal Components Analysis for the tail risks of other advanced economies

Eigenvalues: (Sum = 6, Average = 1)

Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	2.1721	1.0214	0.3620	2.1721	0.3620
2	1.1507	0.1445	0.1918	3.3227	0.5538
3	1.0061	0.2790	0.1677	4.3289	0.7215
4	0.7272	0.2398	0.1212	5.0560	0.8427
5	0.4874	0.0308	0.0812	5.5434	0.9239
6	0.456557	---	0.0761	6	1

Eigenvectors (loadings):

Variable	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
FRANCE	0.5014	-0.2727	0.0661	0.4009	-0.6911	-0.1779
GERMANY	0.1893	0.7512	-0.0033	0.5763	0.1148	0.2335
ITALY	0.3494	-0.3462	0.6497	0.1920	0.5374	0.1017
JAPAN	0.3207	0.4382	0.4706	-0.6368	-0.2761	0.0447
SWITZERLAND	0.5104	0.1238	-0.3727	-0.1406	0.3725	-0.6533
UK	0.4769	-0.1850	-0.4617	-0.2122	0.0744	0.6889

Appendix

Table A1: Forecast evaluation results for 1% VaR

In-sample forecast	**US**			**Canada**		
	Case I	**Case II**	**Case III**	**Case I**	**Case II**	**Case III**
CW test	371.7388[a]	376.6124[a]	380.5184[a]	167.4329[a]	177.1638[a]	179.0815[a]
	[24.9403]	[25.4808]	[25.7950]	[27.8407]	[29.9488]	[29.7391]
RMSE	5.7146	5.8919	6.0449	9.6737	3.3869	3.6251
Out-of-Sample forecast						
CW test	**Case I**	**Case II**	**Case III**	**Case I**	**Case II**	**Case III**
15	370.5861[a]	375.4673[a]	379.3425[a]	167.4491[a]	177.2217[a]	179.0564[a]
	[25.0077]	[25.5511]	[25.8641]	[28.0195]	[30.1479]	[29.9229]
30	369.3440[a]	374.1599[a]	378.0266[a]	167.3718[a]	177.2462[a]	179.0219[a]
	[25.0657]	[25.6052]	[25.9188]	[28.1822]	[30.3415]	[30.1050]
60	367.1851[a]	371.8823[a]	375.6398[a]	167.4250[a]	177.5208[a]	178.9897[a]
	[25.2067]	[25.7396]	[26.0462]	[28.5400]	[30.7618]	[30.4709]
RMSFE						
15	5.7152	5.8943	6.0460	9.6713	3.3772	3.6139
30	5.7097	5.8862	6.0379	9.6642	3.3715	3.6054
60	5.6988	5.8703	6.0177	9.6557	3.3603	3.5852

Note: For Case I, we estimate equations (6) and (7); for Case II, we estimate equations (8) and (9); while for Case III, we estimate equations (10) and (11), respectively for Canada and United States. For the Clark and West (2007) [CW] test, the null hypothesis of a zero coefficient is rejected if the t-statistic is greater than +1.282 (for a one sided 0.10 test), +1.645 (for a one sided 0.05 test), and +2.00 for 0.01 test (for a one sided 0.01 test) (see Clark and West, 2007), and are denoted by [c, b] and [a], respectively; and the values of the t-statistic are in square brackets.

Table A2: CAViaR analysis for stock returns of Canada, 1915:02 – 2020:10

	SAV		ASY		GARCH		ADAPTIVE	
	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR
β_1	0.1620	0.0453	**0.0613**	**0.1807**	-0.2365	-0.3143	0.1832	1.4507
Standard errors	0.2637	0.1175	**0.3395**	**0.3017**	2.0906	0.5064	0.6032	0.0500
P values	0.2695	0.3498	**0.4113**	**0.2746**	0.4550	0.2674	0.0249	0.0000
β_2	0.9274	0.9114	**0.9282**	**0.8721**	0.9308	0.9266		
Standard errors	0.0467	0.0494	**0.0495**	**0.0560**	0.0287	0.0243		
P values	0.0000	0.0000	**0.0000**	**0.0000**	0.0000	0.0000		
β_3	0.2730	0.2385	**0.1550**	**0.1026**	0.6124	0.2393		
Standard errors	0.1146	0.1028	**0.0736**	**0.1335**	0.4759	0.1241		
P values	0.0086	0.0102	**0.0176**	**0.2211**	0.0991	0.0269		
β_4			**0.4200**	**0.3935**				
Standard errors			**0.3136**	**0.1482**				
P values			**0.0902**	**0.0040**				
RQ	118.8014	410.0103	**117.6338**	**403.8464**	119.1051	412.5605	150.7064	434.364
Hits in-sample (%)	0.9103	4.9415	**1.3004**	**5.0715**	0.9103	5.0715	0.5202	4.9415
Hits out-of-sample (%)	1.0000	3.6000	**1.2000**	**4.6000**	1.4000	4.8000	1.2000	4.6000
DQ in-sample (P values)	0.0502	0.0095	**0.1767**	**0.1984**	0.9953	0.0204	1.0000	0.0024
DQ out-of-sample (P values)	0.0024	0.6765	**0.0127**	**0.5878**	0.0180	0.4226	0.0001	0.0907

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit.

Table A3: CAViaR analysis for oil returns, 1915:02 – 2020:10

	SAV		ASY		GARCH		ADAPTIVE	
	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR
β_1	**0.2930**	**0.0000**	0.6060	0.0000	0.0005	0.0000	10.6000	2.9500
Standard errors	**0.0817**	**0.0000**	0.2850	0.0000	0.0011	0.0000	20.5000	0.5730
P values	**0.0002**	**0.2780**	0.0168	0.4620	0.3130	0.4900	0.3020	0.0000
β_2	**0.8270**	**0.8580**	0.8220	0.7910	0.8160	0.7330	0.0000	0.0000
Standard errors	**0.0252**	**0.0010**	0.0847	0.0000	0.2430	0.0004		
P values	**0.0000**	**0.0000**	0.0000	0.0000	0.0004	0.0000		
β_3	**0.7990**	**0.3660**	0.3290	0.0613	1.2800	0.7140		
Standard errors	**0.0645**	**0.0034**	0.6090	0.0000	206.0000	0.0021		
P values	**0.0000**	**0.0000**	0.2940	0.0000	0.4980	0.0000		
β_4			0.7880	0.9110				
Standard errors			0.0878	0.0000				
P values			0.0000	0.0000				
RQ	**109.0000**	**322.0000**	107.0000	283.0000	103.0000	314.0000	116.0000	321.0000
Hits in-sample (%)	**1.0400**	**5.0700**	0.9100	4.9400	1.1700	3.7700	0.3900	2.9900
Hits out-of-sample (%)	**0.8000**	**6.6000**	1.0000	8.2000	2.0000	8.0000	2.0000	6.0000
DQ in-sample (P values)	**0.0271**	**0.0000**	0.0000	0.0000	0.9830	0.0000	0.0000	0.0002
DQ out-of-sample (P values)	**0.9910**	**0.0000**	0.0019	0.0000	0.0000	0.0000	0.0000	0.0000

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit.

Table A4: CAViaR analysis for stock returns of U.S., 1859:10– 2020:10

	SAV		**ASY**		**GARCH**		**ADAPTIVE**	
	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR
β_1	0.3470	0.4140	**0.2820**	**0.4980**	2.7700	3.0200	1.0200	1.2500
Standard errors	0.2770	0.1570	**0.2480**	**0.1520**	2.7700	1.9400	0.2590	0.0604
P values	0.1050	0.0042	**0.1280**	**0.0005**	0.1590	0.0593	0.0000	0.0000
β_2	0.9010	0.8560	**0.8790**	**0.8150**	0.8210	0.7540		
Standard errors	0.0762	0.0690	**0.0492**	**0.0503**	0.0374	0.0686		
P values	0.0000	0.0000	**0.0000**	**0.0000**	0.0000	0.0000		
β_3	0.2940	0.2030	**0.2160**	**0.0839**	1.2400	0.5200		
Standard errors	0.1960	0.1290	**0.1150**	**0.0506**	0.8680	0.2040		
P values	0.0668	0.0574	**0.0302**	**0.0485**	0.0762	0.0053		
β_4			**0.5320**	**0.3670**				
Standard errors			**0.2350**	**0.1150**				
P values			**0.0118**	**0.0007**				
RQ	218.0000	703.0000	**211.0000**	**673.0000**	210.0000	696.0000	250.0000	738.0000
Hits in-sample (%)	1.0500	5.0200	**1.0500**	**4.9500**	0.9760	5.0200	1.0500	5.0200
Hits out-of-sample (%)	1.6000	3.8100	**1.4000**	**4.8100**	1.8000	3.6100	1.0000	4.8100
DQ in-sample (P values)	0.0795	0.0002	**0.2750**	**0.7160**	0.9740	0.0076	0.0000	0.0009
DQ out-of-sample (P values)	0.7990	0.8440	**0.9550**	**0.9900**	0.5760	0.7510	0.0001	0.1730

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit.

Table A5: CAViaR analysis for oil returns, 1859:10– 2020:10

	SAV		ASY		GARCH		ADAPTIVE	
	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR	1% VaR	5% VaR
β_1	0.6428	0.0894	**1.3322**	**0.2309**	1.0007	0.0000	20.1375	4.0488
Standard errors	0.2149	0.0331	**0.3324**	**0.0944**	0.7132	0.0000	0.0000	0.0000
P values	0.0014	0.0035	**0.0000**	**0.0072**	0.0803	0.4858	0.0000	0.0000
β_2	0.8364	0.8668	**0.7977**	**0.7416**	0.7519	0.7529		
Standard errors	0.0258	0.0386	**0.0254**	**0.0377**	0.0102	0.0021		
P values	0.0000	0.0000	**0.0000**	**0.0000**	0.0000	0.0000		
β_3	0.5609	0.2934	**0.2415**	**0.2059**	1.7883	0.7438		
Standard errors	0.0939	0.0882	**0.0675**	**0.0687**	0.4637	0.0061		
P values	0.0000	0.0004	**0.0002**	**0.0014**	0.0001	0.0000		
β_4			**0.7180**	**0.7376**				
Standard errors			**0.1603**	**0.0892**				
P values			**0.0000**	**0.0000**				
RQ	392.2562	1311.7546	**357.0882**	**1235.0178**	379.3325	1288.5509	427.7278	1363.500
Hits in-sample (%)	1.0695	5.0505	**1.0101**	**5.0505**	1.2478	4.6940	0.6536	3.9810
Hits out-of-sample (%)	2.0000	9.2000	**1.6000**	**10.4000**	2.8000	8.0000	2.0000	6.0000
DQ in-sample (P values)	0.0002	0.0000	**0.1135**	**0.0001**	0.2793	0.0000	0.0034	0.0000
DQ out-of-sample (P values)	0.0018	0.0000	**0.9625**	**0.0011**	0.0475	0.0000	0.0024	0.0000

Note: SAV = Symmetric Absolute Value; ASY = Asymmetric slope; GARCH = Indirect GARCH; ADAPT = Adaptive. The tail risk that best "fits" the return series is put in bold. The criteria used are the DQ test and %Hits for the in-sample. For the "best" tail risk variant, we consider three criteria: (i) %Hits; (ii) DQ test; and (iii) RQ statistic. We expect the %Hits to be 1% for 1% VaR and 5% for 5% VaR; the DQ test statistic is not expected to be significant while the parameters are expected to minimize the RQ loss function, so the smaller the RQ statistic, the better. Nonetheless, the DQ test takes prominence over other statistics. In cases where more than one tail risk is statistically insignificant in terms of the DQ test, we consider both the %Hits and the RQ statistic to determine the model with the best fit.

Table A6: Predictability results for oil tail risk

	Canada [1915M02-2020M10]		US [1859M10-2020M10]	
	1%	5%	1%	5%
Oil tail risk	0.1556[a]	0.0897[a]	-0.0792[a]	-0.0143[c]
	(0.0334)	(0.0190)	(0.0135)	(0.0082)

Note: The results only capture oil tail risk as a predictor of the stock tail risks of Canada and United States.

Table A7: Forecast evaluation results for oil tail risk

	In-sample forecast		Out-of-sample forecast					
	1%	5%		1%			5%	
CW test			15	30	60	5% 15	30	60
US	524.0382[a]	153.3342[a]	524.4497[a]	524.5732[a]	525.6067[a]	153.3432[a]	153.3180[a]	153.5959[a]
	[62.9738]	[78.6477]	[63.2692]	[63.5427]	[64.1328]	[78.9732]	[79.2824]	[79.9702]
Canada	371.2357[a]	176.4929[a]	372.7624[a]	374.3758[a]	376.4152[a]	176.7880[a]	177.1248[a]	177.1372[a]
	[34.0324]	[42.7802]	[34.3284]	[34.6286]	[35.1947]	[43.0968]	[43.4230]	[43.9595]
RMSFE								
US	17.1955	8.1227	17.1939	17.1982	17.1929	8.1235	8.1258	8.1193
Canada	13.0164	8.0248	13.0396	13.0513	13.0752	8.0325	8.0361	8.0539

Note: RMSFE denotes the Root Mean Square Forecast Error. For the Clark and West (2007) [CW] test, the null hypothesis of a zero coefficient is rejected if the t-statistic is greater than +1.282 (for a one sided 0.10 test), +1.645 (for a one sided 0.05 test), and +2.00 for 0.01 test (for a one sided 0.01 test) (see Clark and West, 2007), and are denoted by [c, b] and [a], respectively; and the values of the t-statistic are in square brackets.

Table A8: PCA results for tail risks of other advanced economies with 1% VaR

Eigenvalues: (Sum = 6, Average = 1)

	Value	Difference	Proportion	Value	Proportion
1	2.139851	0.891306	0.3566	2.139851	0.3566
2	1.248545	0.090663	0.2081	3.388396	0.5647
3	1.157882	0.446731	0.1930	4.546278	0.7577
4	0.711152	0.315486	0.1185	5.257430	0.8762
5	0.395666	0.048761	0.0659	5.653095	0.9422
6	0.346905	---	0.0578	6.000000	1.0000

Eigenvectors (loadings):

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
FRANCE	0.524178	0.117562	-0.348256	0.368953	-0.034935	-0.672896
GERMANY	0.130238	0.323984	0.698371	0.601606	0.118054	0.120352
ITALY	0.297984	0.608848	-0.443181	0.019340	-0.069911	0.582099
JAPAN	0.330909	0.406333	0.395318	-0.705056	0.044081	-0.264706
SWITZERLAND	0.499263	-0.409488	0.195510	-0.025266	-0.698049	0.238580
UK	0.510605	-0.421582	-0.009344	-0.061864	0.700528	0.258646