Contagion between Real Estate and Financial Markets: A Bayesian Quantile-on-Quantile Approach

Massimiliano Caporin[a], Rangan Gupta[b] and Francesco Ravazzolo[c]

February 2019

We study contagion between REITs and the equity market in the U.S. over four sub-samples covering January, 2003 to December, 2017, by using Bayesian nonparametric quantile-on-quantile regressions with heteroskedasticity. We find that the spillovers from the REITs on to the equity market has varied over time across the four sub-samples, though similarity is observed between the first and the last sub-samples. Further, barring the extreme ends of the two markets, contagion from REITs upon the stock market went down during the global financial crisis relative to the pre-crisis period, with the spillover picking-up during the European sovereign debt crisis.

[a] University of Padova Via Cesare Battisti 241, 35121 Padova, Italy; Email: massimiliano.caporin@unipd.it.

[b] Department of Economics, University of Pretoria, Pretoria, 0002, South Africa; Email: rangan.gupta@up.ac.za.

[c] Corresponding author. Free University of Bozen-Bolzano and BI Norwegian Business School, Universitätsplatz 1, 39100 Bozen-Bolzano, Italy; Email: francesco.ravazzolo@unibz.it.

1 Introduction

The benefits of including real estate in mixed-asset portfolios are now well-recognized (Hoesli et al., 2004; MacKinnon and Al Zaman, 2009; Bouri et al., 2018). However, investing in real estate can be problematic due to the high unit value and illiquidity of properties. Thus, it is not surprising that the importance of the securitized real estate market, i.e., Real Estate Investment Trusts (REITs) has grown substantially during the past decades. As indicated by the Nareit (the worldwide representative voice for REITs),[1] REITs of all types collectively own more than \$3 trillion in gross assets across the U.S., with stock-exchange listed REITs owning approximately \$2 trillion in assets. Moreover, U.S. listed REITs have an equity market capitalization of more than \$1 trillion, and more than 80 million Americans invest in REIT stocks (through their 401(k) and other investment funds). Indeed, the characteristics of REITs have overcome many of the drawbacks associated with direct real estate. Hence, an understanding of the nature of real estate stocks is crucial for investors.

In this regard, an important stream of research has examined the relationships of REITs with stocks, bonds and its underlying asset i.e., real estate (see for example, Li et al., (2015), Tsai (2015), Chiang et al., (2017), Damianov and Elsayed (2018)). More recently, the extreme events that unfolded in financial markets during the global financial and the European sovereign debt crises have strengthened the desire of researchers to better understand contagion, whereby, loosely speaking, contagion can be defined as a rapid shock spillover that increases cross-market linkages.[2] While there exists a vast literature on contagion involving bonds, stocks, currencies, and more recently hedge funds (see for example, Pericoli and Sbracia (2003), Dungey et al. (2005), Pesaran and Pick (2007), and Forbes (2012)), the literature disentangling contagion issues concerning real estate markets is limited. In this regard, few studies that test for financial contagion in

---

[1]See: https://www.reit.com/nareit.

[2]The existing literature has recognized at least three possible 'theories‚Äô of contagion, i.e., through financial linkages (which in turn has three channels, i.e., information correlation, liquidity correlation, and portfolio rebalancing), trade links, and herding behaviour (Hoesli and Reka, 2015).

REITs and are worth mentioning, involves the works of Kallberg et al., (2002), Gerlach et al., (2006), Fry et al., (2010), Hoesli and Reka (2013, 2015). In general, these studies confirm the existence of contagion involving real estate markets during the Asian crisis of 1997 and the global financial crisis of 2007-2008.

We aim to extend this limited literature associated with real estate markets, by studying contagion between REITs and the equity market (S&P500) in the U.S. based on an extended sample period of daily data covering 2003 till 2017, which in turn allows us to study the impact of not only the global financial crisis, but also the European sovereign debt turmoil. But more importantly, we aim to contribute to this literature by applying quantile-on-quantile (QQ) based nonparametric regressions to study the impact of the REITs market on U.S. equities. The QQ approach allows us to trace the effect of the entire unconditional distribution of REITs on the conditional distribution of the U.S. equity market. In the process, we are able to analyze how changes in the REITs returns from its initial state of bear (lower quantiles), normal (median), or bull (upper quantiles) regimes affect the entire conditional distribution S&P500 returns, i.e., capturing various corresponding states of the equity market.

Understandably compared to copula models used to analyze extreme tail dependence, and standard quantile regressions to study the conditional distribution of the equity markets as in as in Hoesli and Reka (2013), our QQ approach is more informative, as it studies contagion over all possible states associated with REITs and equity markets. In other words, our paper presents a more complete picture on stability of parameters associating real estate and equity markets of the U.S., and hence investigates the presence of (possible) shift-contagion during the crises periods. In this regard, we evaluate the parameter stability by also controlling for the existence of structural breaks. Shift-contagion is actually a special case of structural break, since in its presence, the coefficients linking variables tend to increase (or decrease) after the break date. If structural instability is ignored, we could mix data from different regimes, and thus quantiles are not those of a specific density but are recovered from a mixture of different densities (Qu, 2008; Caporin

2

et al., 2018). Given this, we conduct our analysis based on sub-samples of January, 2003 to July, 2007; August, 2007 to December, 2009; January, 2010 to December, 2012, and; January, 2013 to December, 2017. These break-ups also allow us to study the periods of pre, during, and post the financial and sovereign debt crises.

Note that, contagion is defined as the presence of a significant increase of cross-market linkage after a shock, i.e, departure from fundamentals (Forbes and Rigobon, 2002). In light of this, to analyze contagion, one should ideally assess the connections between markets after having controlled for economic fundamentals. But with contagion associated with high-frequency data, such type of data is not available for macroeconomic variables. To control for issues such as omitted variables (latent factors) and endogeneity, we supplement our QQ approach with a Bayesian heteroskedastic version, where the conditional variance of the residuals follows a Generalized Autoregressive Conditional Heteroskedasticity (GARCH(1,1)) specification, since biases due to omitted variables and endogeneity are strictly related to heterskedasticity effects (Chen et al., 2009; Caporin, et al., 2018).

To the best of our knowledge, this is the first paper to study contagion across REITs and equity markets of the U.S. surrounding the extreme events of the global financial and European sovereign debt crises, based on a QQ approach controlling for various types of biases due to omitted variables, endogeneity and structural breaks. Note that, our model can be considered as an extension of the quantile-GARCH approach of Caporin et al., (2018) to a corresponding QQ-version of the same. The remainder of the paper is organized as follows: Section2 discusses the econometric model and estimation methodologies, with Section 3 presenting the data and empirical results. Robustness analyses is performed in Section 4, and Section 5 concludes the paper.

2   Model and Estimation Methodology

As we mentioned in the introduction, our purpose is to evaluate the impact of REITs returns on the equity market to uncover possible contagion occurrences. Within the econo-

metrics literature focusing on contagion tests, we decided to follow the recent view put forward by Caporin et al. (2018), that analyze the shift-contagion by adopting quantile regression.

The baseline model might might take the form of a single index model

$$R_{SP,t} = \alpha + \beta R_{REITs,t} + \varepsilon_t \tag{1}$$

where $R_{SP,t}$ is the return of the S&P 500 index at time t and $R_{REITs,t}$ is the return of the S&P REITs index at time t. The occurrence of contagion might be addressed by evaluating the statistical significance of the β parameter across the quantiles of the variable of interest, the S&P 500 return. However, this approach neglects the possible role of the location of the REITs index returns across their density support. In fact, the possible impact on the equity market of REITs movements might depend on both the equity market states, but also on the real estate market phases. Consequently, we generalize the approach of Caporin et al. (2018) and move toward a more flexible quantile regression approach, namely a non-parametric quantile regression (Koenker, 2005), also called quantile-on-quantile (Sim and Zhou, 2015). Within a non-parametric quantile regression, the estimation of the parameters of model (1) corresponds to the optimization of the following criterion function over a sample of size T:

$$\min_{\alpha,\beta} \sum_{t=1}^{T} \rho_\tau \left(\varepsilon_t\right) K \left(\frac{R_{REITs,t} - \theta}{h}\right) \tag{2}$$

where $\varepsilon_t = R_{SP,t} - \alpha - \beta R_{REITs,t}$, $\rho_\tau\left(u\right) = u\left(\tau - I\left(u < 0\right)\right)$ is the usual check function adopted in quantile regression, τ is the quantile of interest for the dependent variable, $K\left(.\right)$ is a kernel function, θ is a the unconditional quantile of the REITs return, and h is a bandwidth. The difference between quantile-on-quantile and non-parametric quantile regression is that in the latter the value of θ is given by a collection of pre-defined knots on the support of the conditioning variable, while in the former the values of θ are estimated and correspond to unconditional quantiles. In our case, as we follow a quantile-on-quantile

4

approach, we will set a collection of θ values associated with unconditional quantiles of the REITs returns.

Parameter estimation from the previous equation lead to the evaluation of the non-linear relation between the variables when focusing on the neighbourhood of the τ quantile for the S&P 500 and the θ quantile for the REITs. Therefore, by evaluating the variation of β over θ and τ we are able to monitor the existence (by significance) and strength (by size) of the relationship between the variables of interest.

Following from Caporin et al. (2018), the adoption of quantile regression provides a flexible approach for analysing how the explanatory variable influence the location, scale, and shape of the entire response distribution. In our analyses we make a further step as we allow that the influence of the conditioning variable changes across the distribution of the conditioning variable. However, we stress that, when the distribution of the variables of interest show evidences of different volatility properties over time, the estimation of the links across variables (over the joint support) might be biased or at least inefficient, leading to incorrect evaluations. This is particularly relevant at extreme quantiles, where the dynamic changes might be highly influenced by volatility dynamics. Hence, we take into account the possible presence of heteroskedasticity in the variables of interest, we follow Hiemstra and Jones (1994), Koenker and Zhao (1996), and Chen et al. (2009), and we allow for heteroskedasticity directly in the quantile regression.

Specifically, we follow Chen et al. (2009), and introduce in the criterion function adopted for quantile-on-quantile the heteroskedasticity characterizing the dependent variable and we resort to Bayesian estimation approaches. However, to simplify the computational burden, we slightly modify the criterion function and let the Kernel interact directly with the observed quantities:

$$min_{\alpha,\beta} \sum_{t=1}^{T} \left(\frac{\rho_\tau \left(R_{SP,t} K(R_{REITs,t}, \theta, h) - \bar{\alpha} - \beta R_{REITs,t} K(R_{REITs,t}, \theta, h) \right)}{\sigma_t(\tau)} + log(\sigma_t(\tau)) \right),$$
(3)

where $K(R_{REITs,t}, \theta, h)$ is the same Kernel function adopted above,[3] and $\sigma_t(\tau)$ is the square root of residual variance computed using quantile τ estimates of the parameters α and β together with the parameters $\delta = \{\theta_0, \theta_1, \theta_2\}$ appearing in the variance equation

$$\sigma_t^2(\tau) = \theta_0 + \theta_{1,}e_{t-1}^2 + \theta_2\sigma_{t-1}^2. \tag{4}$$

The extra logarithmic term in the criterion function ensures that the parameters do not converge to infinity. See Xiao and Koenker (2009) for an alternative criterion function. We stress that the volatility parameters and the causal effect parameters are estimated simultaneously, resulting in a vector of parameters that, similarly to the baseline case, depend on both the quantile of the dependent variable, τ, and the quantile of the explanatory variable, θ. We choose a Bayesian approach to estimate the parameters because we believe this method has several advantages including: (i) accounting for parameter uncertainty through the simultaneous inference of all model parameters; (ii) exact inferences for finite samples; (iii) efficient and flexible handling of complex model situations and non-standard parameters; and (iv) efficient and valid inference under parameter constraints.

Bayesian inference requires the specification of prior distributions. We chose weak uninformative priors to allow the data to dominate inference. As it is the standard approach, we assume a normal prior for $\Theta_\tau \sim N(\underline{\Theta}_{0,\tau}, \underline{\Sigma})$. $\underline{\Theta}_{0,\tau}$ is set equal to the frequentist estimates of model (1); and $\underline{\Sigma}$ is chosen to be a matrix with sufficiently "large" but finite numbers on the diagonal. The volatility parameters α_τ follow a jointly uniform prior, $p(\alpha_\tau) \propto I(S)$, constrained by the set S that is chosen to ensure covariance stationarity and variance positivity, as in the frequentist case. These are sufficient conditions to ensure that the conditional variance is strictly positive. See Nelson and Cao (1992) for a discussion of sufficient and necessary conditions on GARCH coefficients. Such restrictions reduce the role of the extra logarithmic term in equation (3).

---

[3]We repeat the basic idea: The check function is $r(u_t, \tau)$ with $u_t = y_{it} - \beta_{i0} - \beta_{i1}X_{it}$. The nonparametric QR minimizes $\sum_t r(u_t, \tau)K(X_t, \gamma)$, therefore $r(u_t, \tau)K(X_t, \gamma) = u_t \times I(u_t < 0)K(X_t, \gamma)$. This is equal to $r(u_t'(\gamma), \tau)$ with $u_t'(\gamma) = u_t K(X_t, \gamma)$. Then, we have $u_t'(\gamma) = y_{it}K(X_t, \gamma) - b0(\tau) - b1(\tau)X_t K(X_t, \gamma)$.

The model is estimated using the Metropolis-within-Gibbs MCMC algorithms. Similarly to Chen et al. (2009), we combine Gibbs sampling steps with a random walk Metropolis-Hastings (MH) algorithm to draw the GARCH parameters (see Vrontos et al. (2000) and So et al. (2005)). To speed the convergence and allow an optimal mixing, we employ an adaptive MH-MCMC algorithm that combines a random walk Metropolis (RW-M) and an independent kernel (IK)MH algorithm; see Caporin et al. (2018) for estimation details.

3   Data and Results

3.1   Data

As indicated above, our estimations involves two variables measuring the behavior of the overall equity market and the REITs sector. In this regard, we use the S&P500 equity and S&P REITs indices data, which in turn are obtained from Datastream of Thomson Reuters, and converted to log-returns. Our analyses cover the entire period of 2nd of January, 2003 to 29th of December, 2017 (i.e., a total of 3776 observations, and plotted in Figure A1 in the Appendix of the paper), but as our purpose is to show that relationship (spillover) from the REITs sector to the equity market is time-varying, we partition the entire period into four sub-samples: the calm period arriving up to the onset of the financial crisis covering 2nd of January, 2003 to 31st of July, 2007 (1152 observations); the crisis period of 1st of August, 2008 to 31st of December, 2009 (611 observations); 4th of January, 2010 to 31st of December, 2012 (754 observations), which includes the post subprime crisis period up to the European sovereign crisis, and; finally, 2nd of January, 2013 to 29th of December, 2017 (1259 observations) to capture the period post the global financial and the European Sovereign debt crises. Table A1 in the Appendix of the paper provides the summary statistics of each of these sub-samples. What stands out is the non-normality in the distribution of the log-returns of both the variables due to negative skewness and excess kurtosis. In addition, within each of the sub-samples, the S&P REITs

returns is consistently more volatile, and produces higher positive returns on average in the first and third sub-samples. In the second sub-sample, the mean return in the REITs sector as well as the overall equity market is understandably negative, with the former being higher in absolute terms - an indication of the origination of the crisis from the real estate sector. In the final sub-sample, the stock market yielded higher mean positive return than the S&P REITs, suggesting relatively stronger growth in the equity market in recent times.

3.2   Empirical Results

We proceed with the estimation of our main models and of the robustness checks on the various sub-samples. We report here the results focusing only on the parameter β of equation (1). Given that the parameter depends on two quantiles, the estimation output takes the form of a surface plot where we include only statistically significant coefficients. Figure 1 reports the estimated coefficient surfaces in the four subsamples with our main model, while Table 1 includes a subset of the estimated coefficients for the four samples of our analysis, with Table 2 reporting the differential effects evaluated with respect to the first sub-sample, i.e., the pre-crises calm period.

Figure 1 shows that the overall pattern is similar across periods, in particular when comparing the first and the fourth sub-samples, which is understandable given that these two periods correspond to the non-crises episodes of financial markets. But differences are observed by comparing the second and fourth sub-samples. Considering the sign of coefficients, we note that the impact of REITs return on the S&P500 return is positive across all quantiles of the two variables, but with differences in the size of the impact. In particular, the relation between quantiles is higher when both variables are in their tails. This suggests that, when the stock market is experiencing a negative, and probably turbulent phase, the impact of negative REITs return is higher compared to the impact of positive REITs return, thus contributing to the equity market instability. An opposite, and positive, behaviour is observed on the upper tails. The size of the impact

8

decreases when the two variables are located in opposite tails, and this is particularly evident in the second and fourth sub-samples. When considering the differentials, we note that, during the subprime crisis, the REITs impact suffer a general decrease apart when looking at extreme quantiles of both densities where we note a relevant increase of the beta coefficients. This might signal that, during the crisis, characterized mostly by daily volatile movements in the financial markets, the market experienced stronger spillover from the REITs sector onto the equity under both exceptionally bear- and bull-states. With financial markets, particularly equities (overall or sector-based) being vulnerable, this is understandable as tail-risk spillovers are likely to be stronger, since investor could be carrying out faster re-allocation in the portfolios between riskier assets and those that are considered safe-havens, especially during extreme movements.

On the contrary, when moving to the post-crisis period, the overall impact of REITs is observed to have increased compared to the first sample, with differentials being almost all positive. This is understandable in the sense that the real estate sector leading up to the crisis was doing so well that investors were reluctant to move funds out of the REITs investments into the overall equity market. But in the wake of the two back-to-back crises associated with the real estate sector and sovereign bond markets, market agents were probably more confident about the performance of the regular equity market, and hence, spillovers from the REITs sector is found to be stronger in the last sub-sample than the first one. This line of reasoning seems to hold water, as the average returns across the first and last sub-samples show higher returns for the REITs in the former and lower in the latter relative to the S&P500 return. Finally, the fourth period signals that, in most cases, the coefficients move back to pre-crisis levels, with the exceptions being the lower quantiles of the S&P500 return and the upper quantiles of REITs return. This again makes sense in general, as we are comparing here two calm periods, with one preceding the crisis and the other one following it. The stronger spillover during the last sub-sample from the REITs sector to the stock market only when the performer is performing better relative to the latter is again an indication of investors reallocating their portfolios in
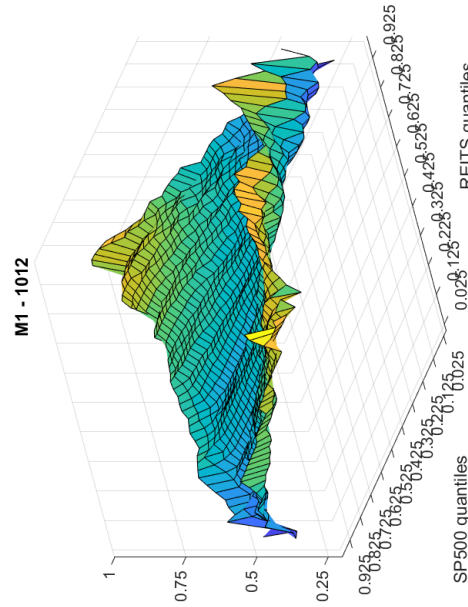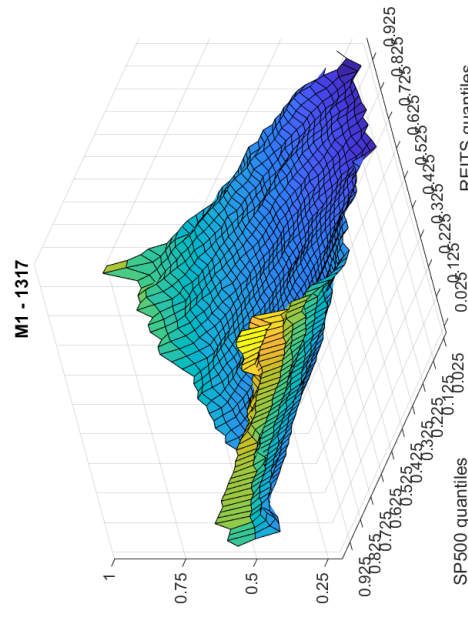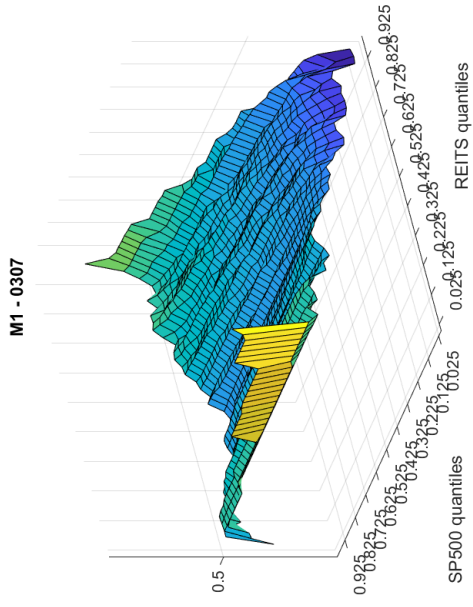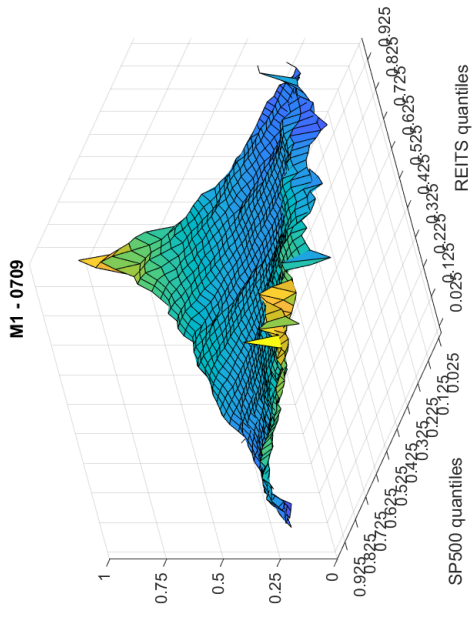
9

		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
		2003-2007					2007-2009				
Quantiles of the REITs return	0.05	0.521	0.469	0.426	0.391	0.343	0.765	0.578	0.392	0.260	*0.157*
	0.25	0.514	0.473	0.442	0.434	0.419	0.583	0.502	0.410	0.357	0.264
	0.50	0.514	0.473	0.449	0.455	0.471	0.492	0.433	0.393	0.368	0.346
	0.75	0.513	0.470	0.453	0.480	0.507	0.392	0.354	0.409	0.447	0.470
	0.95	0.500	0.461	0.462	0.516	0.541	*0.197*	0.251	0.366	0.347	0.750
		2010-2012					2013-2017				
	0.05	0.767	0.756	0.706	0.616	0.370	0.799	0.505	0.364	0.284	0.282
	0.25	0.713	0.661	0.572	0.475	0.505	0.773	0.468	0.404	0.367	0.405
	0.50	0.638	0.595	0.577	0.598	0.581	0.702	0.422	0.438	0.429	0.459
	0.75	0.552	0.539	0.580	0.668	0.721	0.598	0.418	0.487	0.508	0.532
	0.95	0.364	0.557	0.608	0.660	0.829	0.511	0.422	0.538	0.630	0.711

Table 1: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples. Coefficients are all statistically significant at the 1% level apart the two coefficients in italics in the sample 2007-2009.

favor of the equity markets possibly in fear of heating up of the REITs sector.

To verify the previous observations, we move to the analysis of coefficient surfaces, coefficient values and differentials when accounting for the heteroskedasticity that is present in financial returns (see Figure 2, and Tables 3 and 4), and in the process account for the possible omitted variables bias. Accounting for heretoskedasticity with Bayesian inference mainly increases tail effects, in particular when stock and REITs returns are in different phases see Table 3. Interestingly, the effect is quite mitigated across samples (in line with the findings of Forbes and Rigobon (2002)) and differences across the four samples in Table 4 are very small. The model captures the high volatility in the data in the second and in the third sub-samples as large shocks, but keeps similar transmission mechanisms between the two variables. Our toned down impact compared to the homoskedastic model is possibly an indication of the impact of the omitted variable in the benchmark model, which we are now accounting for in the heteroskedastic Bayesian QQ framework of ours. This result in particular, highlights the importance of accounting for GARCH-effects to obtain reliable results.[4]

---

[4]Alternative to the Bayesian QQ model, we had also estimated the benchmark model with GARCH(1,1)-filtered stock and REITs returns. In general, and somewhat opposite to the Bayesian
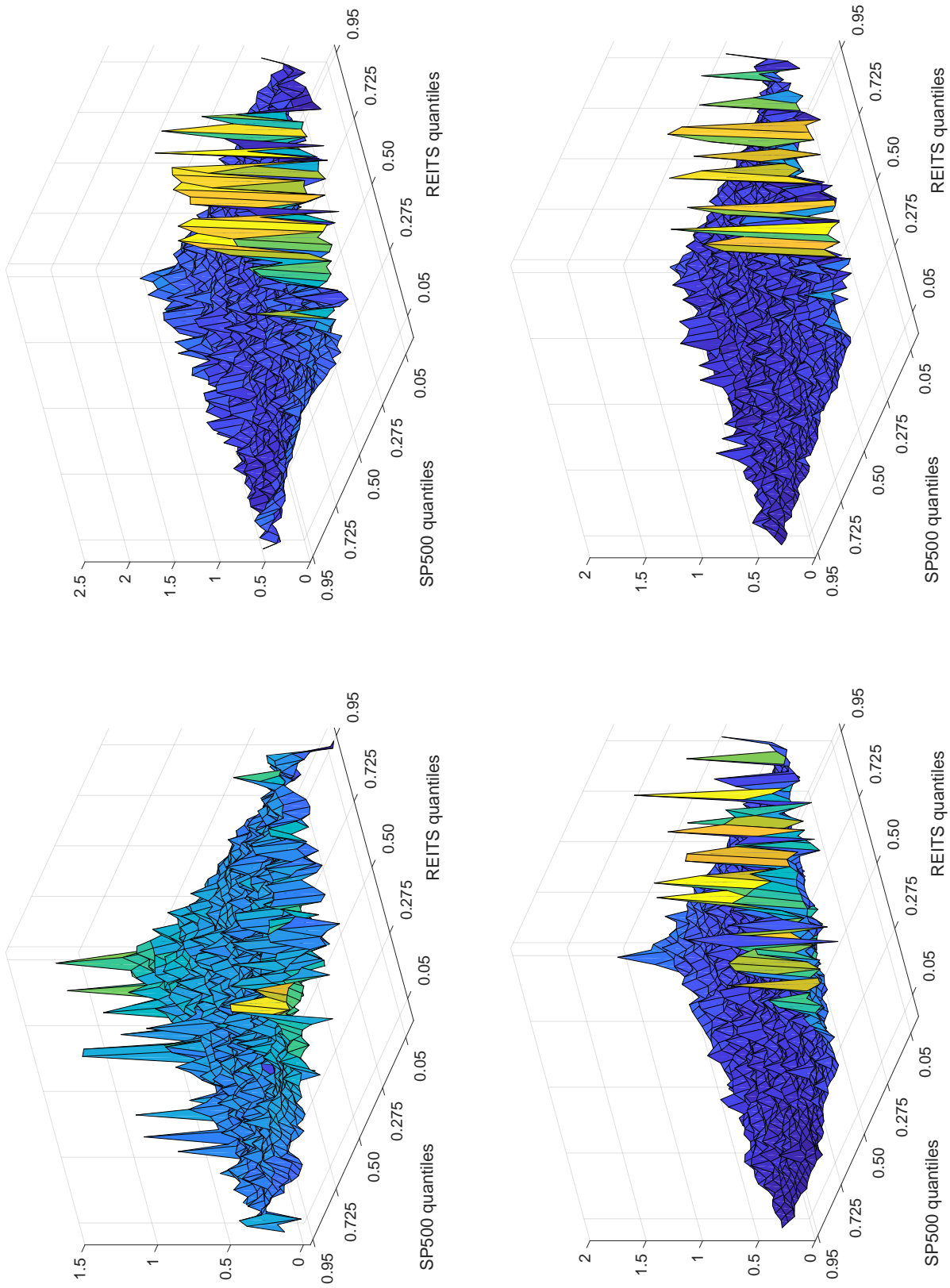
Figure 1: β parameter surface over quantiles of the REITs and S&P returns. The figure reports only statistically significant coefficients estimated from the four sub-samples.

	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05						0.244	0.109	-0.034	-0.131	-0.187
0.25						0.069	0.028	-0.032	-0.077	-0.155
0.50						-0.022	-0.040	-0.056	-0.087	-0.126
0.75						-0.122	-0.115	-0.044	-0.033	-0.038
0.95						-0.304	-0.211	-0.095	-0.169	0.209
	2010-2012					2013-2017				
0.05	0.246	0.287	0.279	0.225	0.027	0.278	0.036	-0.062	-0.107	-0.061
0.25	0.199	0.188	0.129	0.041	0.086	0.259	-0.005	-0.038	-0.067	-0.014
0.50	0.124	0.122	0.128	0.143	0.109	0.188	-0.051	-0.011	-0.025	-0.012
0.75	0.038	0.069	0.127	0.188	0.214	0.085	-0.052	0.034	0.028	0.024
0.95	-0.137	0.095	0.146	0.145	0.287	0.010	-0.039	0.076	0.115	0.170

(Quantiles of the REITs return over rows)

Table 2: Estimated differentials in the REITs return impact on S&P500 return for selected quantiles (S&P500 return quantiles over columns, REITs quantiles over rows) with respect to the impact observed in the first sample.

	Quantiles of the S&P500 return									
	0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
	2003-2007					2007-2009				
0.05	0.793	0.543	0.428	0.315	0.311	0.767	0.764	0.645	0.505	0.324
0.25	0.540	0.515	0.387	0.314	0.298	0.589	0.912	0.591	0.478	0.493
0.50	0.507	0.607	0.473	0.387	0.336	0.592	0.683	0.586	0.636	0.440
0.75	0.451	0.297	0.437	0.549	0.969	0.737	0.629	0.616	0.677	0.613
0.95	0.083	0.187	0.302	0.320	0.113	0.504	0.673	0.635	0.680	0.734
	2010-2012					2013-2017				
0.05	0.813	0.467	0.369	0.301	0.324	0.579	0.466	0.444	0.405	0.397
0.25	0.842	0.454	0.398	0.348	0.453	0.602	0.658	0.393	0.564	0.467
0.50	0.647	0.489	0.417	0.430	0.312	0.645	0.402	0.448	0.439	0.429
0.75	0.492	0.455	0.458	0.555	0.453	0.680	0.562	0.430	0.507	0.458
0.95	0.517	0.453	0.536	0.662	0.794	0.686	0.445	0.443	0.481	0.629

(Quantiles of the REITs return over rows)

Table 3: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity. The value 0 is not included in the 1% credible interval for all coefficients.

Figure 2: β parameter surface over quantiles of the REITs and S&P500 returns. The figure reports only statistically significant coefficients estimated from the four sub-samples based on the Bayesian heteroskedastic model.

Quantiles of the REITs return		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
		2003-2007					2007-2009				
	0.05						-0.025	0.221	0.217	0.191	0.014
	0.25						0.049	0.397	0.204	0.164	0.195
	0.50						0.084	0.077	0.113	0.249	0.104
	0.75						0.286	0.333	0.178	0.128	-0.355
	0.95						0.421	0.486	0.333	0.360	0.621
		2010-2012					2013-2017				
	0.05	0.020	-0.076	-0.059	-0.014	0.013	-0.214	-0.077	0.016	0.090	0.087
	0.25	0.302	-0.061	0.011	0.034	0.154	0.062	0.143	0.006	0.251	0.168
	0.50	0.140	-0.118	-0.056	0.042	-0.024	0.138	-0.205	-0.025	0.052	0.092
	0.75	0.041	0.158	0.021	0.006	-0.516	0.229	0.265	-0.007	-0.042	-0.511
	0.95	0.435	0.267	0.234	0.342	0.681	0.604	0.258	0.141	0.162	0.516

Table 4: Estimated differentials in the REITs return impact on S&P500 return for selected quantiles (S&P quantiles over columns, REITs quantiles over rows) with quantile regression with heteroskedasticity and with respect to the impact observed in the first sample.

4 Robustness analysis

In order to verify the impact of the model specification on the surface of the β parameters (its value depend on both τ and θ) we consider as robustness checks, several alternative specifications. We consider several possible cases applied both on the Bayesian QQ model accounting for heteroskedasticity, as we consider this particular model to be more general and robust. We describe here the various models starting with the baseline specification that, for comparison purposes, we report in the first line below:

model, the coefficients were found to increase under the GARCH(1,1)-filtered model, relative to the benchmark one. We however, believe that the results from the Bayesian QQ approach to account for heteroskedasticity directly in the error structure, is more robust relative to the GARCH(1,1)-filtered approach. This is because, the former accounts for not only linear and the heteroskedastic effects simultaneously, but also possible differences across quantiles in the heteroskedastic behavior. Complete details of the results based on GARCH(1,1)-filtered data is available upon request from the authors.

$$M0 \rightarrow R_{SP,t} = \alpha + \beta R_{REIT,t} + \varepsilon_t \tag{5}$$

$$M1 \rightarrow R_{SP,t} = \alpha + \beta R_{REIT,t} + \delta R_{REIT,t}^2 + \varepsilon_t \tag{6}$$

$$M2 \rightarrow R_{SP,t} = \alpha + \beta R_{REIT,t} + \gamma_1 R_{SP,t-1} + \gamma_2 R_{REIT,t-1} + \varepsilon_t \tag{7}$$

$$M3 \rightarrow R_{SP,t} = \alpha + \beta R_{REIT,t} + \gamma_1 R_{SP,t-1} + \gamma_2 R_{REIT,t-1} + \delta_1 R_{REIT,t}^2 \tag{8}$$

$$+ \delta_2 R_{REIT,t-1}^2 + \varepsilon_t \tag{9}$$

$$M4 \rightarrow R_{SP,t} = \alpha + \beta R_{REIT,t} + \delta R_{REIT,t}^2 + \phi R_{SP,t-1}^2 + \varepsilon_t \tag{10}$$

$$M5 \rightarrow R_{SP,t} = \alpha + \beta R_{REIT,t} + \gamma_1 R_{SP,t-1} + \gamma_2 R_{REIT,t-1} + \delta_1 R_{REIT,t}^2 \tag{11}$$

$$+ \delta_2 R_{REIT,t-1}^2 + \phi R_{SP,t-1}^2 + \varepsilon_t. \tag{12}$$

The first generalization ($M1$) controls for the non-linearity, at each single quantile, in the impact of the REITs return on the S&P500 return. Different from $M1$, $M2$ takes into account lagged effects of both the dependent and explanatory variables. Specification M3 combines the elements put forward in $M1$ and $M2$. In case $M4$, we extend the baseline model by including a component that proxy heteroskedasticity effects of the dependent variable at quantiles.[5] Finally, $M5$ combines all the possible effects, i.e., taking $M3$ and $M4$ together. Note that, in the latter cases we control for additional heteroskedastic effects not captured by standard GARCH models.

Tables A2 to A6 in the The Appendix of the paper contain the estimated β coefficients over selected quantiles of the S&P500 and the REITs returns for models $M1$ to $M5$ estimated using the Bayesian approach. Overall, by changing the model specifications, the impact on the size and surface (i.e., pattern of behavior) of the coefficients are limited. In particular, they appear to be concentrated in the upper quantiles of both variables and only for the last two sub-samples. We thus, find robust confirmation of the results obtained

---

[5]If we do have heteroskedasticity in the S&P 500 return, with the conditional variance depending on its past values, the quantile of the S&P500 return depend on lagged conditional variances. We proxy the latter by lagged squared returns.

in the previous section.

5 Concluding Remarks

In this paper, we study contagion between REITs and the equity market in the U.S. based on an (extended) sample period of daily data covering 2003 till 2017, which in turn allows us to study the impact of not only the global financial crisis, but also the European sovereign debt turmoil. We apply quantile-on-quantile (QQ) based nonparametric regressions to study the impact of the REITs market on U.S. equities. Realizing that, if structural breaks are ignored, we could mix data from different regimes, we conduct our analysis based on sub-samples of January, 2003 to July, 2007; August, 2007 to December, 2009; January, 2010 to December, 2012, and; January, 2013 to December, 2017. These break-ups also allow us to study the periods of pre-, during-, and post-financial and sovereign debt crises.

We find that the spillovers from the REITs on to the equity market has varied over time across the four sub-samples, though similarity is observed to certain extent under the calmer periods captured by the first and last sub-samples. This is not surprising given that these two sub-periods correspond to the pre-financial and post European sovereign debt crises. Interestingly, barring the extreme ends of the two markets, the contagion from REITs upon the stock market went down during the global financial crisis relative to the pre-crisis period. The spillover however, seemed to have picked-up during the European sovereign debt crisis. Importantly though, allowing for heteroskedasticity using a Bayesian approach, reduced the overall size of the impact of REITs return on the S&P500 return, except at the tails of the distributions. But, in general, our results were found to be robust to various model specifications.

In sum our results have two major implications: First, from an econometric point of view, we highlight that the role of heteroskedasticity, aiming to control for the omitted variable bias in high-frequency contagion analysis between securitized real estate and

equity markets, should not be ignored. If neglected, we are likely to get overestimates of the contagion effect leading to inaccurate inference. Second, from the perspective of an investor, our contagion results imply that, irrespective of the general macroecomic scenario, i.e., whether the U.S. economy is in recession or not, diversification benefits are not possible, especially under extreme bearish and bullish-situations of both real estate and equity markets. Given this, a policy-maker worried about contagion of especially the negative shocks, which in turn can deepen economic crises given the leading role of asset prices, should aim to revive the economy based on expansionary policies.

Reference

Bouri, E. Gupta, R., Wong, W-K., and Zhu, Z., 2018. Is Wine a Good Choice for Investment? Pacific-Basin Finance Journal, 51, 171-183.

Caporin, M., L. Pelizzon, F. Ravazzolo and R. Rigobon, (2018) Measuring Sovereign Contagion in Europe, Journal of Financial Stability, 34, 150-181.

Chen, C., R. Gerlack and D.C.M. Wei, (2009) Bayesian causal effects in quantiles: accounting for heteroskedasticity, Computational Statistics and Data Analysis 53(6), 1993-2007.

Chiang, M-C., Sing, T.F., and Tsai, I-C., 2017. Spillover Risks in REITs and other Asset Markets. Journal of Real Estate Finance and Economics, 54, 579‚Äì604.

Damianov, D.S., and Elsayeda, A.H., 2018. On the transmission of spillover risks between the housing market, the mortgage and equity REITs markets, and the stock market. Finance Research Letters, DOI: https://doi.org/10.1016/j.frl.2018.03.001.

Dungey, M., and Fry, R., Gonzalez-Hermosillo, B., Martin, V.L., 2005. Empirical modelling of contagion: a review of methodologies. Quantitative Finance, 5, 9‚Äì24.

Forbes, K.J., 2012. The „ÄúBig C‚Äù: Identifying and mitigating contagion. NBER Working Paper No. 18465.

Forbes, R., and Rigobon, R., 2002. No contagion, only interdependence: measuring stockmarket comovements. Journal of Finance, 57(5), 2223‚Äì2261.

Fry, R., Martin, V.L., and Tang. C., 2010. New Class of Tests of Contagion With Applications. Journal of Business and Economic Statistics, 28(3), 423-437.

Gerlach, R., Wilson, P., and Zurbruegg, R., 2006. Structural Breaks and Diversification: The Impact of the 1997 Asian Financial Crisis on the Integration of Asia-Pacific Real Estate Markets. Journal of International Money and Finance 25(6): 974-991.

Hiemstra, C. and J. D. Jones, (1994) Testing for Linear and Nonlinear Granger Causality in the Stock Price-Volume Relation, Journal of Finance 49, 1639-1665.

Hoesli, M., Lekander, J., and Witkiewicz, W., 2004. International evidence on real estate as a portfolio diversifier. Journal of Real Estate Research, 26(2), 161‚Äì206.

Hoesli, M. and Reka, K., 2013. Volatility Spillovers, Comovements and Contagion in Securitized Real Estate Markets. Journal of Real Estate Finance and Economics, 47, 1-35.

Hoesli, M. and Reka, K., 2015. Contagion Channels between Real Estate and Financial Markets. Real Estate Economics, 43(1), 101, 138.

Kallberg, J.G., Liu, C.H. and Pasquariello, P., 2002. Regime Shifts in Asian Equity and Real Estate Markets. Real Estate Economics, 30(2), 263-291.

Koenker, R., (2005) Quantile Regression, Econometric Society Monographs, n. 38, Cambridge University Press, New York.

Koenker, R. and Q. Zhao, (1996) Conditional quantile estimation and inference for ARCH models , Econometric Theory 12, 793-813.

Li, X-L., Chang, T., Miller, S.M., Balcilar, M., and Gupta, R. 2015. The Co-Movement and Causality between the U.S. Housing and Stock Markets in the Time and Frequency Domains. International Review of Economics and Finance, 38, 220-233.

MacKinnon, G. H.,and Al Zaman, A., 2009. Real estate for the long term: the effect of return predictability on long-horizon allocations. Real Estate Economics, 37(1), 117,Äì153.

Nelson, D.B. and C.Q. Cao, (1992) Inequality constraints in the univariate GARCH model, Journal of Business and Economic Statistics 10(2), 229-235.

Pericoli, M., and Sbracia, M., 2003. A primer on financial contagion. Journal of Economic Surveys, 17, 571,Äì608.

Pesaran, H., and Pick, A., 2007. Econometric issues in the analysis of contagion. Journal of Economic Dynamics and Control, 31, 1245,Äì1277.

Qu, Z., 2008. Testing for structural change in regression quantiles. Journal of Econometrics, 146, 170-184.

Sim, N. and H. Zhou (2015). Oil prices, us stock return, and the dependence between their quantiles Journal of Banking and Finance 55, 1-8.

So, M. K.P., C. W.S. Chen and M. Chen, (2005) A Bayesian Threshold Nonlinearity Test for Financial Time Series, Journal of Forecasting 24, 61-75.

Tsai, I-C., 2015. Dynamic information transfer in the United States housing and stock markets. North American Journal of Economics and Finance, 34, 215,Äì230.

Vrontos, I.D., P. Dellaportas and D.N. Politis (2000) Full Bayesian inference for GARCH and EGARCH models, Journal of Business and Economic Statistics 18, 187-198.

Xiao, Z. and R. Koenker, (2009), Conditional quantile estimation for GARCH models, Journal of American Statistical Association 104, 1696-1712.

Appendix

Figure A1: Data Plot of S&P REITs and S&P500 Returns
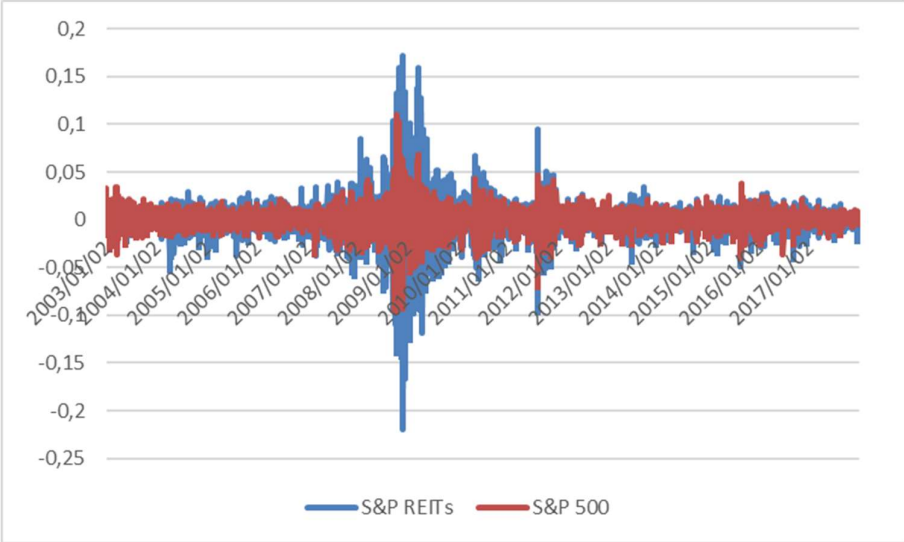
Table A1: Summary statistics

Statistic	02/01/2003 to 31/07/2007		01/08/2008 to 31/12/2009		04/01/2010 to 31/12/2012		02/01/2013 to 29/12/2017	
	S&P REITs	S&P 500	S&P REITs	S&P 500	S&P REITs	S&P 500	S&P REITs	S&P 500
Mean	0.0006	0.0004	-0.0007	-0.0004	0.0005	0.0003	0.0002	0.0005
Median	0.0011	0.0008	-0.0011	0.0008	0.0012	0.0006	0.0008	0.0005
Maximum	0.035	0.0348	0.1712	0.1096	0.095	0.0463	0.0337	0.0383
Std. Dev.	0.0101	0.0078	0.0419	0.0206	0.0158	0.0117	0.0092	0.0075
Skewness	-0.5794	-0.0703	-0.0495	-0.133	-0.0878	-0.4274	-0.5174	-0.4097
Kurtosis	4.8339	4.7339	6.2861	7.8105	7.7694	6.6809	4.9926	5.8921
Jarque-Bera	225.9005	145.2575	275.1658	590.9345	715.6069	448.6242	264.4549	474.000
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	1152		611		754		1259	

		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
		2003-2007					2007-2009				
Quantiles of the REITs return	0.05	0.793	0.599	0.360	0.271	0.192	0.750	0.775	0.796	0.664	0.364
	0.25	0.580	0.625	0.413	0.300	0.287	0.624	0.663	0.579	0.446	0.339
	0.50	0.487	0.458	0.457	0.334	0.203	0.708	0.690	0.588	0.518	0.415
	0.75	0.257	0.413	0.436	0.659	0.705	0.510	0.659	0.660	0.733	0.632
	0.95	*-0.086*	0.050	0.341	0.368	*0.107*	0.755	0.658	0.701	0.652	0.749
		2010-2012					2013-2017				
	0.05	1.069	0.513	0.338	0.210	0.298	0.486	0.468	0.323	0.225	0.245
	0.25	0.904	0.488	0.365	0.336	0.130	0.431	0.653	0.378	0.354	0.364
	0.50	0.479	0.444	0.477	0.425	0.298	0.635	0.499	0.477	0.495	0.512
	0.75	0.492	0.443	0.594	0.601	0.785	0.403	0.394	0.460	0.585	1.055
	0.95	0.138	0.608	0.737	0.877	1.393	0.353	0.281	0.499	0.720	1.068

Table A2: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M1$. A number in *italics* indicates that the value 0 is included in the 1% credible interval.

		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
				2003-2007					2007-2009		
	0.05	0.735	0.608	0.462	0.371	0.435	0.750	0.715	0.634	0.596	0.352
	0.25	0.454	0.475	0.428	0.429	0.338	0.649	0.832	0.577	0.545	0.615
	0.50	0.422	0.421	0.390	0.346	0.252	0.695	0.714	0.559	0.502	0.463
	0.75	0.406	0.360	0.417	0.397	0.522	0.686	0.493	0.595	0.678	0.870
	0.95	0.199	0.301	0.306	0.268	-0.056	0.590	0.699	0.625	0.733	0.708
				2010-2012					2013-2017		
	0.05	0.768	0.508	0.348	0.300	0.331	0.518	0.480	0.443	0.365	0.442
	0.25	0.687	0.480	0.407	0.429	0.332	0.505	0.507	0.428	0.408	0.467
	0.50	0.672	0.532	0.437	0.488	0.431	0.401	0.516	0.412	0.363	0.408
	0.75	0.547	0.387	0.474	0.512	0.478	0.614	0.395	0.611	0.418	0.518
	0.95	0.394	0.465	0.551	0.610	0.792	0.537	0.437	0.440	0.501	0.834

(Quantiles of the REITs return labels the rows.)

Table A3: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M2$.

		Quantiles of the S&P500 return									
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
				2003-2007					2007-2009		
	0.05	0.603	0.480	0.394	0.294	0.386	0.679	0.793	0.742	0.636	0.637
	0.25	0.594	0.480	0.450	0.355	0.249	1.224	0.628	0.552	0.495	0.425
	0.50	0.556	0.400	0.387	0.375	0.222	0.767	0.636	0.701	0.711	0.447
	0.75	0.343	0.375	0.430	0.476	0.620	0.608	0.611	0.667	0.718	0.780
	0.95	0.471	0.354	0.369	0.241	0.068	0.650	0.816	0.698	0.740	0.604
				2010-2012					2013-2017		
	0.05	0.969	0.492	0.343	0.263	0.301	0.493	0.454	0.349	0.199	0.190
	0.25	0.803	0.516	0.366	0.368	0.172	0.560	0.513	0.414	0.349	0.448
	0.50	0.714	0.630	0.417	0.417	0.437	0.467	0.403	0.436	0.393	0.505
	0.75	0.437	0.352	0.520	0.622	0.688	0.537	0.429	0.462	0.541	0.720
	0.95	0.196	0.694	0.689	0.819	0.921	0.237	0.353	0.418	0.693	1.117

(Quantiles of the REITs return labels the rows.)

Table A4: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M3$.

			Quantiles of the S&P500 return							
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
				2003-2007					2007-2009		
Quantiles of the REITs return	0.05	0.635	0.633	0.388	0.308	0.139	0.707	0.746	0.755	0.643	0.310
	0.25	0.562	0.587	0.499	0.352	0.370	0.847	0.657	0.562	0.453	0.420
	0.50	0.461	0.435	0.390	0.452	0.343	0.656	0.678	0.799	0.631	0.414
	0.75	0.334	0.317	0.524	0.732	1.088	0.420	0.524	0.763	0.645	0.696
	0.95	*-0.110*	*0.013*	0.197	0.087	0.551	0.663	0.748	0.681	0.650	0.791
				2010-2012					2013-2017		
	0.05	0.945	0.463	0.324	0.219	0.258	0.557	0.449	0.320	0.204	0.313
	0.25	0.866	0.551	0.331	0.332	0.216	0.787	0.458	0.429	0.553	0.471
	0.50	0.507	0.434	0.437	0.419	0.431	0.621	0.471	0.488	0.410	0.546
	0.75	0.403	0.411	0.564	0.617	0.772	0.454	0.392	0.582	0.564	0.870
	0.95	0.212	0.609	0.713	0.797	1.211	0.344	0.286	0.492	0.681	1.064

Table A5: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M4$. A number in *italics* indicates that the value 0 is included in the 1% credible interval.

				Quantiles of the S&P500 return							
		0.05	0.25	0.50	0.75	0.95	0.05	0.25	0.50	0.75	0.95
				2003-2007					2007-2009		
Quantiles of the REITs return	0.05	0.640	0.487	0.471	0.284	0.385	0.638	0.710	0.735	0.606	0.226
	0.25	0.545	0.517	0.428	0.486	0.363	1.068	0.662	0.575	0.422	0.355
	0.50	0.501	0.421	0.406	0.384	0.225	0.451	0.705	0.560	0.622	0.494
	0.75	0.419	0.340	0.561	0.477	0.401	0.448	0.559	0.596	0.785	0.708
	0.95	0.510	0.433	0.349	0.133	0.161	0.782	0.811	0.732	0.714	0.591
				2010-2012					2013-2017		
	0.05	0.948	0.520	0.350	0.282	0.245	0.618	0.448	0.380	0.224	0.218
	0.25	0.857	0.461	0.349	0.301	0.187	0.655	0.454	0.428	0.341	0.536
	0.50	0.427	0.389	0.385	0.396	0.307	0.667	0.440	0.443	0.425	0.542
	0.75	0.470	0.414	0.499	0.568	0.769	0.534	0.441	0.426	0.559	0.708
	0.95	0.206	0.606	0.729	0.745	1.065	0.287	0.313	0.397	0.681	1.036

Table A6: Estimated coefficients for selected quantiles (S&P500 return quantiles over columns, REITs return quantiles over rows) and the four samples with quantile regression with heteroskedasticity using model $M5$.