Forecasting Realized Volatility of International REITs: The Role of Realized Skewness and Realized Kurtosis

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Abstract

We use an international dataset on 5-minutes interval intraday data covering nine leading markets and regions to construct measures of realized volatility, realized jumps, realized skewness, and realized kurtosis of returns of international Real Estate Investment Trusts (REITs) over the daily period of September, 2008 to August, 2020. We study out-of-sample the predictive value of realized skewness and realized kurtosis for realized volatility over and above realized jumps, where we also differentiate between measures of "good" realized volatility and "bad" realized volatility. We find that realized skewness and realized kurtosis significantly improve forecasting performance at a daily, weekly, and monthly forecast horizon, and that their contribution to forecasting performance outweighs in terms of significance the contribution of realized jumps. Our results have important implications for investors and policymakers.

JEL Classifications: C22, C53, G15.

Keywords: REITs; International data; Realized volatility; Forecasting

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1 Introduction

Real Estate Investment Trusts (REITs), as an investment vehicle (operating through asset allocation, risk reduction, and diversification channels), have grown substantially during the last decade, with a total market capitalization of US $1.7 trillion (Nazlioglu et al., 2020). Although the United States (US) continues to be the leader among the REITs markets (with a market capitalization of US $ 1.15 trillion), the number of countries now offering REITs stands at 40 countries.[1] The success in attracting such a massive scale of investment capital is mainly because REITs are accessible to all investors irrespective of portfolio size. Naturally, accurate forecasting of REITs volatility is an important issue for investors, given that volatility, as a measure of risk, plays a critical role in portfolio diversification, derivatives pricing, hedging and financial risk management. Further, REITs returns do not suffer from measurement error and high transaction costs compared to other real estate investments, and provide a perfect high-frequency proxy for the overall real estate market, since REITs earn most of their income from investments in real estate being exchange-traded funds, and also because trading occurs as common stocks (Marfatia et al., 2017). Given these properties, and the fact that the Global Financial Crisis had its roots in the collapse and the resulting uncertainty of the US real estate sector, forecastability of REITs volatility, which is possible at a high-frequency unlike the housing market, is an important issue for policymakers as well in designing appropriate policies to circumvent the potential negative impact of uncertainty in the REITs sector on the real economy.

In this regard, given the current emphasis that intraday data leads to more precise estimates and forecasts of the volatility of the REITs returns (Zhou 2017, 2020a, 2020b),[2] we contribute to this burgeoning line of research by predicting the realized volatility (RV) of the US and other developed and developing REITs markets, where we estimate RV by using 5-minute-interval intraday data for the period from September 2008 to August 2020, based on a modified version of the popular Heterogeneous Autoregressive (HAR) model introduced by Corsi (2009). More specifically, we extend the basic HAR-RV model to incorporate information on daily realized skewness and realized kurtosis for forecasting RV of international REITs markets.

The motivation to look at the role of realized skewness and realized kurtosis in forecasting REITs RV emanates from the large theoretical literature, starting with Kraus and Litzenberger (1976) and continuing with the macroeconomic disaster research by Rietz (1988), Longstaff and Piazzesi (2004), and Barro (2006), which hypothesises that heavy-tailed shocks in general, and left-tail events in particular have an important role in explaining asset-price behaviour. In the process, realized

[1]See: https://www.reit.com/investing/global-real-estate-investment and Global REITs Market, EY Global Real Estate Report of 2018, for further details.

[2]Earlier studies on modeling and forecasting of REITs volatility were primarily based on Generalized Autoregressive Conditional Heteroscedasticity (GARCH)-type models (see, for example, Devaney (2001), Stevenson (2002), Cotter and Stevenson (2006), Bredin et al. (2007), Lee and Pai (2010), Zhou and Kang (2011), and Pavlova et al. (2014)).

skewness and realized kurtosis aims to captures asymmetry and extreme movements in REITs) returns, and act as an empirical proxy for the theoretical concept of rare disaster risks.[3] Note that, such risks can be easily associated with the ongoing COVID-19 pandemic, and the resulting tremendous variability of the global financial markets (Bouri et al. 2020), including REITs, which witnessed a loss of nearly 30% worldwide and 32% in the US (Akinsomi, 2020). Empirically, Mei et al. (2017) was the first study to highlight the role of realized skewness and realized kurtosis for forecasting stock-market RV for China and the US. And then Gkillas et al. (2019) built on this paper to depict evidence of forecastability for six (Australian dollar, Canadian dollar, Swiss franc, euro, British pound, and Japanese yen) major currencies relative to the United States (US) dollar-based on realized skewness and realized kurtosis.[4]

Against this backdrop, our paper aims to extend this line of research associated with forecasting RV with realized skewness and realized kurtosis to international REITs markets for the first time, given the importance of this issue to market participants and policy authorities. The remainder of the paper is organized as follows: Section 2 outlines our international dataset, which covers nine leading markets and regions, and the methodology, Section 3 presents the results, and Section 4 concludes.

2 Methodology and Higher-Moments

For the forecasting analysis, we use variants of the widely-studied HAR-RV framework of Corsi (2009) to model and forecast daily realized REITs variance. While the HAR-RV model apparently has a simple structure, it has become increasingly popular in the literature because it is able to capture long memory and multi-scaling behavior of REITs market variance (Zhou, 2011, 2020a; Pavlova et al., (2014); Assaf, 2015). In our application, the benchmark HAR-RV model is given by:

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \epsilon_{t+h} \tag{1}$$

where the index h denotes the forecast horizon, and (for $h > 1$) RV_{t+h} denotes the average realized volatility over the h-days forecast horizon, with $h =$ 1,5 and 22 in our context. In addition, $RV_{w,t}$ is the average RV from day $t-5$ to day $t-1$, while $RV_{m,t}$ denotes the average RV from day $t-22$ to day $t-1$. In this regard, it must be pointed out that we use the classical estimator of RV, i.e., the

[3]Studies like Harvey and Siddique (2000), Ang et al. (2006), Kelly and Jiang (2014), Amaya et al. (2015), Shen et al. (2018), and Neuberger and Payne (2020) show that realized skewness and realized kurtosis could predict aggregate and cross-sectional stock market returns.

[4]One can refer to Demirer et al. (2018), Gupta et al. (2019a, 2019b), Gkillas et al. (2020), Bouri et al. (forthcoming) who have highlighted the role of rare disaster risks proxied by the International Crisis Behavior (ICB) database and El Niño-Southern Oscillation (ENSO) index in predicting returns and volatility of the various asset and commodity markets.

square root of the sum of squared intraday returns (Andersen and Bollerslev, 1998), expressed as

$$RV_t = \sqrt{\sum_{i=1}^{M} r_{t,i}^2} \tag{2}$$

where $r_{t,i}$ is the intraday return which is defined as the log-difference of two consecutive prices and $i = 1, ..., M$ is the number of intraday observations.

In addition, we also investigate an extended version of the HAR-RV model in Eq. (1) by incorporating jumps (RJ), the role of which has been highlighted by Odusami (2021) for REITs RV, as follows:

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \beta_1 RJ_t + \epsilon_{t+h} \tag{3}$$

Next, following Mei et al. (2017) and Gkillas et al. (2019), we extend equation (3) by first including RSK, and then adding RKU to the model as follows:

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \beta_1 RJ_t + \beta_2 RSK_t + \epsilon_{t+h} \tag{4}$$

and,

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \beta_1 RJ_t + \beta_2 RSK_t + \beta_3 RKU_t + \epsilon_{t+h} \tag{5}$$

We compute RSK and RKU as measures of the higher-moments of the daily REITs returns distribution. Like Amaya et al. (2015), we consider RSK as a measure of the asymmetry of the daily REITs returns distribution, and RKU as a measure that accounts for extremes. Given the intraday returns and realized variance, RSK on day t is

$$RSK_t = \frac{\sqrt{M} \sum_{i=1}^{M} r_{(i,t)^3}}{RV_t^{3/2}}, \tag{6}$$

while, RKU on day t is given by

$$RKU_t = \frac{M \sum_{i=1}^{M} r_{(i,t)^4}}{RV_t^2}. \tag{7}$$

The scaling of RSK and RKU by $(N)^{1/2}$ and M, respectively, makes sure that their magnitudes correspond to daily skewness and kurtosis.

Next, we turn our attention to the computations of RJ_t. Brandorff-Nielsen and Shephard (2004) show that realized variance converges into permanent and discontinuous (jump) components as:

$$\lim_{M \to \infty} RV_t = \int_{t-1}^{t} \sigma^2(s) ds + \sum_{j=1}^{N_t} k_{t,j}^2, \tag{8}$$

where N_t is the number of jumps within day t and $k_{t,j}$ is the jump size. This specification suggests that RV_t is a consistent estimator of the integrated variance $\int_{t-1}^{t} \sigma^2(s)ds$ plus the jump contribution. The asymptotic results of Brandorff-Nielsen and Shephard (2004, 2006) further show that:

$$\lim_{M\to\infty} BV_t = \int_{t-1}^{t} \sigma^2(s)ds, \tag{9}$$

where BV_t is the realized bipower variation defined as:

$$BV_t = \mu_1^{-2}\left(\frac{M}{M-1}\right)\sum_{i=2}^{M} |r_{t,i-1}||r_{i,t}| = \frac{\pi}{2}\sum_{i=2}^{M} |r_{t,i-1}||r_{i,t}|, \tag{10}$$

and

$$\mu_a = E(|Z|^a), Z \sim N(0,1), a > 0. \tag{11}$$

Having defined the continuous component of realized variance, a consistent estimator of the pure jump contribution can then be expressed as

$$J_t = RV_t - BV_t. \tag{12}$$

In order to test the significance of the jumps, we adopt the following formal test estimator proposed by Brandorff-Nielsen and Shephard (2006):

$$JT_t = \frac{RV_t - BV_t}{(v_{bb} - v_{qq})\frac{1}{N}QP_t}, \tag{13}$$

where QP_t is the Tri-Power Quarticity defined as:

$$TP_t = M\frac{M}{M-1}\left(\frac{\Gamma(0.5)}{2^{2/3}\Gamma(7/6)}\right)\sum_{i=3}^{M} |r_{t,i}|^{4/3}|r_{t,i-1}|^{4/3}|r_{t,i-2}|^{4/3}, \tag{14}$$

which converges to

$$TP_t \to \int_{t-1}^{t} \sigma^4(s)ds, \tag{15}$$

even in the presence of jumps. $v_{bb} = \left(\frac{\pi}{2}\right) + \pi - 3$ and $v_{qq} = 2$. Note that for each t, $JT_t \sim N(0,1)$ as $M \to \infty$.

As can be seen in Eq. (12), the jump contribution to RV_t is either positive or null. Therefore, in order to avoid having negative empirical contributions, we follow Zhou and Zhu (2012) and re-define the jump measure as

$$RJ_t = max(RV_t - BV_t; 0). \tag{16}$$

Finally, upward ("good", RVG) and downward ("bad", RVB) realized volatility can serve as measures of downside and upside risk, and capture the sign asymmetry in the price process. Thus, we also forecast RVG and RVB based on the information content of the EMVID, by replacing RV ($RVG+RVB$) in our forecasting equations by RVG and RVB in turn. In line with Barndorff-Nielsen et al. (2010), we compute bad and good realized volatility as:

$$RVG_t = \sqrt{\sum_{i=1}^{M} r_{t,i}^2 \ \mathbf{1}_{[(r_{t,i})>0]}}, \tag{17}$$

$$RVB_t = \sqrt{\sum_{i=1}^{M} r_{t,i}^2 \ \mathbf{1}_{[(r_{t,i})<0]}}. \tag{18}$$

3 Data and empirical results

3.1 Data

We use 5-minute-interval intraday data on the REITs indexes over a 24 hour trading day to construct daily measures of realized variance (RV), the corresponding good (RVG) and bad (RVB) variants, and the other covariates, i.e., realized jumps (RJ), realized skewness (RSK), realized kurtosis (RKU). Besides the FTSE Nareit All REITs (FNAR) Index for the US, which is the most prominent REITs market, we also investigate the role of RSK and RKU on the REITs markets covering other developed and developing countries and regions (for which intraday data is available) namely, the FTSE Nareit Developed Asia (EGAS) Index, FTSE Nareit Australia (ELAU) Index, FTSE Nareit Hong Kong (ELHK) Index, FTSE Nareit Japan (ELJP) Index, FTSE Nareit UK (ELUK) Index, FTSE Nareit Developed Markets (ENGL) Index, FTSE Nareit Eurozone (EPEU) Index, FTSE Nareit Emerging Markets (FENEI) Index. The price data for all these indexes, in a continuous format, are obtained from Bloomberg, with the final daily data coverage (derived based on the intraday data) being 2nd September, 2008 to 26th August, 2020.

3.2 Empirical results

Table 1 summarizes the results (p-values) of the Clark and West (2007) test for an equal out-of-sample mean-squared prediction error (MSPE). We use a recursively expanding estimation window to compute out-of-sample forecasts, where we use the first 1000 initial observations to train the models. This training period comprises approximately one third of the total sample (and, thus, the number of out-of-sample forecasts is roughly 2000). We consider four competing models, corresponding to Equations (1), (3), (4), and (5). Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model. In addition, we report results for three different forecast horizons ($h = 1, 5, 22$), corresponding to daily, weekly, and monthly forecasts.

– Please include Table 1 about here. –

The test results show that realized skewness and realized kurtosis improve the forecast performance of the HAR-RV model in most cases, where the evidence of forecast improvements is stronger in the case of the two higher-order moments than in the case of jumps. For example, the HAR-RV-Jump model does not improve upon the baseline HAR-RV model for Australia, the UK, and the U.S. Besides, we note about the latter that all test results are insignificant. As can be seen from last column of the Table 1, adding realized kurtosis does not lead to further improvements of forecast accuracy relative to a model that already contains realized jumps and realized skewness for only emerging markets and the U.S at all forecast horizons. Figure 1 plots the actual values of realized volatility along with the forecasts that we obtain from Model 4. A closer examination of the Figure 1 shows that forecasts of the Model 4 capture turning points relatively well during the extreme volatility of the COVID-19 period, confirming the benefits of using realized skewness and realized kurtosis for capturing asymmetry and extreme movements in REITs.

– Please include Figure 1 about here. –

Next, we summarize in Table 2 results for the realized downward ("bad") volatility, and the realized upward ("good") volatility. On balance, the results resemble those given in Table 1. Evidence that realized jumps add predictive value is weak, though there are few more significant test results in the case of realized good than in the case of realized bad volatility. Evidence that realized skewness and realized kurtosis add to forecast accuracy, in turn, is stronger than in the case of realized jumps, where we observe some more significant test results for realized skewness than for realized kurtosis. As for realized kurtosis, we observe that the number of significant test results is somewhat larger in the case of realized good volatility than in the case of realized bad volatility, that is, in these cases, Model 4 has predictive value beyond the predictive value already added by Model 3.

– Please include Table 2 about here. –

As a robustness check, Table 3 reports the test results for realized volatility that we obtain when we vary the training period. Specifically, we delete 500 additional forecasts relative to the baseline scenario that we study in Table 1, implying that we use approximately half of the sample to initialize the recursive estimation and the other half for out-of-sample forecasting.[5] Again, we observe that realized jumps contribute to forecast performance only in the minority of markets/regions. Evidence of improvements in forecasting performance when we use realized skewness to extend the HAR-RV

[5]As an additional robustness check, we used a rolling-estimation window of length 1500 observations to compute out-of-sample forecasts. Results (not reported, but available from the authors upon request) corroborate that realized skewness and realized kurtosis have significant predictive value for realized volatility. Realized skewness has significant predictive value for realized bad and realized bad volatility in the majority of markets/regions. The test results for the model that features also realized kurtosis (that is Model 4), in turn, are mainly significant for good realized volatility.

model, in turn, is relatively strong. Also, we observe that the number of significant test results is larger when we study realized kurtosis than in the baseline scenario. For example, realized kurtosis (but not realized skewness) adds to the out-of-sample forecast performance of the model estimated on the U.S. data.

– Please include Table 3 about here. –

4 Conclusion

Motivated by the theoretical literature on rare disaster risks and asset market movements, we have assessed the importance of the realized skewness and the realized kurtosis of the daily returns distribution (capturing asymmetry and extremes) for international REITs realized-volatility forecasting, derived from 5-minutes-interval intraday data. We also differentiate between measures of good realized volatility and bad realized volatility estimated by upside and downside semi-variances, respectively. Based on the period of the analysis covering 2nd September, 2008 to 26th August, 2020, and using variants of the popular HAR-RV model, augmented to include realized jumps and then realized skewness and realized kurtosis, we report evidence that the two higher-order moments (that is, realized skewness and realized kurtosis) significantly improve forecasting performance at three different forecast horizons. Importantly, the contribution of the two higher-order moments to forecasting performance outweighs in terms of statistical significance the contribution of realized jumps. Finally, we have documented the predictive value of the two higher-moments for realized bad and realized good volatility.

Given the tremendous growth of REITs as an asset class globally and, hence, the importance of accurate volatility forecasts as inputs for optimal asset-allocation decisions, our findings suggest that incorporating realized skewness and realized kurtosis, over and above volatility jumps, in forecasting models can help to improve the design of portfolios that include REITs across various investment horizons and countries. Further, with the future path of REITs volatility providing a high-frequency measure of uncertainty in the housing sector for which only low-frequency data is traditionally available, would allow policymakers to design timely responses to circumvent the negative influence on the real economy, given that the real estate sector is known to lead macroeconomic variables (Segnon et al., 2021). As part of future research, it would be is interesting to extend our study to other assets and commodities, besides the equity, currency, and REITs markets considered thus far.

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Table 1: Out-of-Sample Tests for Realized Volatility

Region / horizon	Model 1 / Model 2	Model 2 / Model 3	Model 3 / Model 4
EGAS / h=1	0.011	0.002	0.000
EGAS / h=5	0.053	0.015	0.000
EGAS / h=22	0.395	0.151	0.111
ELAU / h=1	0.103	0.028	0.011
ELAU / h=5	0.911	0.091	0.033
ELAU / h=22	0.830	0.110	0.427
ELHK / h=1	0.005	0.004	0.770
ELHK / h=5	0.203	0.000	0.004
ELHK / h=22	0.954	0.002	0.041
ELJP / h=1	0.017	0.000	0.000
ELJP / h=5	0.032	0.000	0.000
ELJP / h=22	0.120	0.113	0.027
ELUK / h=1	0.143	0.000	0.000
ELUK / h=5	0.144	0.000	0.000
ELUK / h=22	0.265	0.004	0.000
ENGL / h=1	0.705	0.061	0.000
ENGL / h=5	0.059	0.095	0.010
ENGL / h=22	0.028	0.426	0.002
EPEU / h=1	0.085	0.000	0.000
EPEU / h=5	0.028	0.001	0.002
EPEU / h=22	0.022	0.005	0.006
FENEI / h=1	0.088	0.024	0.102
FENEI / h=5	0.951	0.007	0.242
FENEI / h=22	0.149	0.058	0.562
FNAR / h=1	0.843	0.453	0.835
FNAR / h=5	0.803	0.852	0.813
FNAR / h=22	0.048	0.866	0.394

Note: This table reports results (p-values) of the Clark-West test for an equal mean-squared prediction error (MSPE) for alternative forecast horizons. Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model. The more parsimonious HAR-RV model is the benchmark model, and the extended HAR-RV model is the rival model. The alternative hypothesis is that the rival model has a smaller MSPE than the benchmark model. Realized volatility is defined as the square root of the realized standard variance. The p-values are based on robust standard errors. Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model.

Table 2: Out-of-Sample Tests for Bad and Good Realized Volatility

Panel A: Bad RV

Region / horizon	Model 1 / Model 2	Model 2 / Model 3	Model 3 / Model 4
EGAS / h=1	0.059	0.001	0.462
EGAS / h=5	0.161	0.002	0.109
EGAS / h=22	0.476	0.022	0.289
ELAU / h=1	0.110	0.286	0.017
ELAU / h=5	0.858	0.022	0.250
ELAU / h=22	0.818	0.055	0.237
ELHK / h=1	0.080	0.020	0.080
ELHK / h=5	0.527	0.007	0.298
ELHK / h=22	0.613	0.237	0.266
ELJP / h=1	0.050	0.002	0.000
ELJP / h=5	0.101	0.001	0.000
ELJP / h=22	0.819	0.000	0.050
ELUK / h=1	0.143	0.007	0.000
ELUK / h=5	0.309	0.018	0.000
ELUK / h=22	0.594	0.025	0.000
ENGL / h=1	0.571	0.000	0.618
ENGL / h=5	0.285	0.000	0.164
ENGL / h=22	0.104	0.006	0.035
EPEU / h=1	0.148	0.008	0.004
EPEU / h=5	0.280	0.009	0.011
EPEU / h=22	0.065	0.038	0.025
FENEI / h=1	0.073	0.532	0.166
FENEI / h=5	0.901	0.544	0.298
FENEI / h=22	0.046	0.886	0.605
FNAR / h=1	0.155	0.758	0.841
FNAR / h=5	0.840	0.019	0.839
FNAR / h=22	0.772	0.023	0.829

Panel B: Good RV

Region / horizon	Model 1 / Model 2	Model 2 / Model 3	Model 3 / Model 4
EGAS / h=1	0.075	0.000	0.000
EGAS / h=5	0.165	0.002	0.000
EGAS / h=22	0.045	0.031	0.043
ELAU / h=1	0.124	0.079	0.011
ELAU / h=5	0.127	0.001	0.000
ELAU / h=22	0.101	0.053	0.580
ELHK / h=1	0.020	0.000	0.813
ELHK / h=5	0.001	0.000	0.138
ELHK / h=22	0.002	0.000	0.078
ELJP / h=1	0.104	0.000	0.000
ELJP / h=5	0.278	0.000	0.018
ELJP / h=22	0.776	0.010	0.512
ELUK / h=1	0.339	0.000	0.000
ELUK / h=5	0.497	0.000	0.000
ELUK / h=22	0.946	0.001	0.000
ENGL / h=1	0.235	0.001	0.001
ENGL / h=5	0.762	0.002	0.009
ENGL / h=22	0.711	0.007	0.005
EPEU / h=1	0.072	0.000	0.000
EPEU / h=5	0.083	0.000	0.003
EPEU / h=22	0.147	0.002	0.001
FENEI / h=1	0.062	0.001	0.313
FENEI / h=5	0.335	0.000	0.366
FENEI / h=22	0.417	0.000	0.579
FNAR / h=1	0.839	0.151	0.838
FNAR / h=5	0.790	0.126	0.823
FNAR / h=22	0.047	0.025	0.007

Note: This table reports results (p-values) of the Clark-West test for an equal mean-squared prediction error (MSPE) for alternative forecast horizons. Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model. The more parsimonious HAR-RV model is the benchmark model, and the extended HAR-RV model is the rival model. The alternative hypothesis is that the rival model has a smaller MSPE than the benchmark model. Realized volatility is defined as the square root of the realized standard variance. The p-values are based on robust standard errors. Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model.

Table 3: Results for a Shorter Out-of-Sample Period

Region / horizon	Model 1 / Model 2	Model 2 / Model 3	Model 3 / Model 4
EGAS / h=1	0.052	0.008	0.019
EGAS / h=5	0.136	0.027	0.044
EGAS / h=22	0.548	0.149	0.362
ELAU / h=1	0.098	0.039	0.024
ELAU / h=5	0.900	0.110	0.065
ELAU / h=22	0.830	0.114	0.548
ELHK / h=1	0.020	0.017	0.838
ELHK / h=5	0.213	0.001	0.011
ELHK / h=22	0.996	0.007	0.044
ELJP / h=1	0.065	0.018	0.000
ELJP / h=5	0.103	0.009	0.007
ELJP / h=22	0.299	0.142	0.114
ELUK / h=1	0.119	0.000	0.000
ELUK / h=5	0.124	0.000	0.001
ELUK / h=22	0.272	0.013	0.000
ENGL / h=1	0.056	0.045	0.000
ENGL / h=5	0.140	0.066	0.036
ENGL / h=22	0.061	0.439	0.007
EPEU / h=1	0.070	0.000	0.001
EPEU / h=5	0.025	0.001	0.012
EPEU / h=22	0.028	0.011	0.009
FENEI / h=1	0.157	0.004	0.007
FENEI / h=5	0.918	0.000	0.067
FENEI / h=22	0.112	0.010	0.596
FNAR / h=1	0.260	0.872	0.000
FNAR / h=5	0.099	0.161	0.000
FNAR / h=22	0.096	0.770	0.002

Note: This table reports results (p-values) of the Clark-West test for an equal mean-squared prediction error (MSPE) for alternative forecast horizons when 500 additional forecasts are deleted relative to the baseline scenario. Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model. The more parsimonious HAR-RV model is the benchmark model, and the extended HAR-RV model is the rival model. The alternative hypothesis is that the rival model has a smaller MSPE than the benchmark model. Realized volatility is defined as the square root of the realized standard variance. The p-values are based on robust standard errors. Model 1: Baseline HAR-RV model. Model 2: HAR-RV-Jump model. Model 3: HAR-RV-Jump-RSK model. Model 4: HAR-RV-Jump-RSK-RKU model.

Figure 1: Realized Volatility and Forecasts