

## APPENDIX A

### PROOF OF LEMMA 1

$$\begin{aligned}
I_{pp} &= \sum_{x_i^p \in \Phi_p^1 \setminus x_k^p} P_p h_X \|y_i^p\|^{-\alpha} \\
\mathcal{L}_{I_{pp}}(s) &= E \left\{ \exp \left( -s \sum_{x_i^p \in \Phi_p^1 \setminus x_k^p} P_p h_X \|y_i^p\|^{-\alpha} \right) \right\} \\
&= E_{\Phi} \left\{ \prod_{x_i^p \in \Phi_p^1} E_{h_X} \left( \exp(-s P_p h_X \|y_i^p\|^{-\alpha}) \right) \right\} \\
&\stackrel{(a)}{=} E_{\Phi} \left\{ \prod_{x_i^p \in \Phi_p^1} \frac{1}{1 + s P_p \|y_i^p\|^{-\alpha}} \right\} \\
&\stackrel{(b)}{=} \exp \left\{ -2\pi \lambda_p^1 \int_0^{\infty} \frac{r}{1 + \frac{r^\alpha}{s P_p}} dr \right\} \\
&\stackrel{(c)}{=} \exp \left\{ -\pi \frac{\gamma \lambda_p^1 (s P_p)^{\frac{\gamma}{\pi}}}{\sin(\gamma)} \right\}
\end{aligned}$$

(a) follows after Rayleigh fading assumption and (b) is obtained after applying PGFL of PPP in polar coordinate form. Solving the integral and making  $\gamma = \frac{2\pi}{\alpha}$ , (c) is obtained. This completes the proof.

## APPENDIX B

### PROOF OF LEMMA 5

$$\begin{aligned}
I_{sp} &= \sum_{x_i^s \in \Phi_s \cap \Xi_R^c \cap \Xi_d^c} P_s h_X \|y_i^s\|^{-\alpha} \\
\mathcal{L}_{I_{sp}}(s) &= E \left\{ \exp \left( -s \sum_{x_i^s \in \Phi_s \cap \Xi_R^c \cap \Xi_d^c} P_s h_X \|y_i^s\|^{-\alpha} \right) \right\} \\
&\stackrel{(a)}{=} \exp \left\{ -\lambda_s \int_{R^2 \setminus \Xi_R} \frac{1}{1 + \frac{\|y\|^\alpha}{s P_s}} dy \right\} \exp \left\{ \lambda_s \int_{\Xi_d} \frac{1}{1 + \frac{\|y\|^\alpha}{s P_s}} dy \right\} \stackrel{(c)}{\leq} E_{\Phi} \left\{ \prod_{x_i^s \in \Phi_s^{11}} \exp \left( 2 \int_{w-d}^{w+d} \left( \frac{-d^2 + w^2 + r^2}{2wr} \right) \frac{r}{1 + \frac{r^\alpha}{s P_s}} dr \right) \right\}
\end{aligned}$$

Note that our interest is in obtaining interference from SUs in the space except SUs located inside any disk  $b(y_i^s, d)$ , where  $y_i^s (i = 1, \dots, n)$  is a typical active secondary receiver assumed to be located at the origin of a disk of radius  $d$ . Since interfering SUs cannot be located inside active PUs' exclusion region, the idea is to capture interference generated outside all disks  $b(y_i^p, R)$  and  $b(y_i^s, d)$ , bearing in mind that there may be overlap of protection regions. To do this, we refer to Fig. 1. The first part of (a) gives

$$\begin{aligned}
&\stackrel{(b)}{=} \exp \left\{ -\pi \frac{\gamma \lambda_s (s P_s)^{\frac{\gamma}{\pi}}}{\sin(\gamma)} \right\} \times \\
&\exp \left\{ \lambda_s \prod_{x_i^p \in \Phi_p^1} \int \int_{r, \theta \in b(y, R)} \frac{r}{1 + \frac{r^\alpha}{s P_s}} dr d\theta \right\}
\end{aligned}$$

(b) involves application of PGFL of PPP in polar coordinate form. From Fig. 1, each PU has a circle centered at its primary receiver with protection region  $R$  denoted as  $b(y_i^p, R)$ . Hence,  $r$  should be bounded in the range  $v - R \leq r \leq v + R$  and for every  $r$  within that range,  $\theta$  should be bounded in

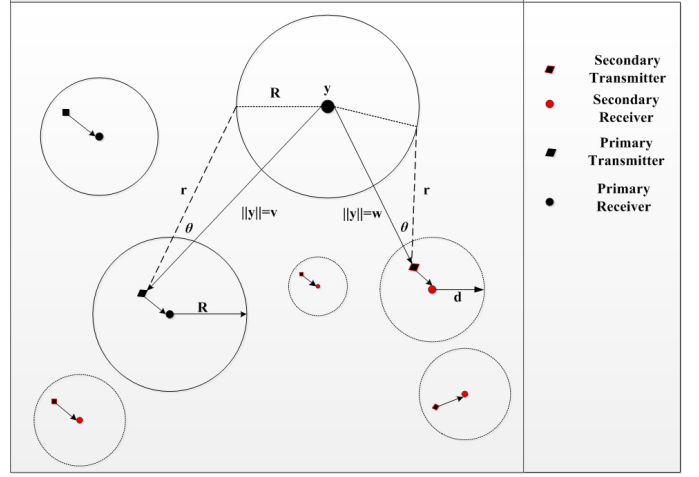


Fig. 1. Area of integration under PU with SU interference control

the range  $-\cos^{-1} \left( \frac{-D^2 + v^2 + r^2}{2vr} \right) \leq \theta \leq \cos^{-1} \left( \frac{-D^2 + v^2 + r^2}{2vr} \right)$  following the cosine rule. Hence,

$$\begin{aligned}
&\exp \left\{ -\lambda_s \int_{R^2 \setminus \Xi_R} \frac{1}{1 + \frac{\|y\|^\alpha}{s P_s}} dy \right\} \leq \exp \left\{ -\pi \frac{\gamma \lambda_s (s P_s)^{\frac{\gamma}{\pi}}}{\sin(\gamma)} \right\} \\
&\exp \left\{ -2\pi \lambda_p^1 \int_R^{\infty} \left( 1 - \exp(f(v)) \right) v dv \right\},
\end{aligned}$$

where  $f(v) = \int_{v-R}^{v+R} \cos^{-1} \left( \frac{-R^2 + v^2 + r^2}{2vr} \right) \frac{2\lambda_s r}{1 + \frac{r^\alpha}{s P_s}} dr$ . Now, we solve the second integral.

$$\int_{\Xi_d} \frac{1}{1 + \frac{\|y\|^\alpha}{s P_s}} dy = E_{\Phi^{11}} \left\{ \prod_{x_i^s \in \Phi_s^{11}} \int_{b(x_i^s, d)} \frac{1}{1 + \frac{\|y\|^\alpha}{s P_s}} \right\}$$

Hence,

$$\begin{aligned}
&\exp \left\{ \lambda_s \int_{\Xi_d} \frac{1}{1 + \frac{\|y\|^\alpha}{s P_s}} dy \right\} \stackrel{(d)}{\leq} \\
&\exp \left\{ -2\pi \lambda_s^{11} \int_d^{\infty} \left( 1 - \exp(f(w)) \right) w dw \right\}.
\end{aligned}$$

(c) and (d) follow applying PGFL of PPP in polar coordinate form. Substituting the solutions of the first and second integrals back into (a) gives Lemma 5. This completes the proof.