Stochastic geometry approach towards interference management and control in cognitive radio network: A survey

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Abstract

Interference management and control in the cognitive radio network (CRN) is a necessity if the activities of primary users must be protected from excessive interference resulting from the activities of neighbouring users. Hence, interference experienced in wireless communication networks has earlier been characterized using the traditional grid model. Such models, however, lead to non-tractable analyses, which often require unrealistic assumptions, leading to inaccurate results. These limitations of the traditional grid models mean that the adoption of stochastic geometry (SG) continues to receive a lot of attention owing to its ability to capture the distribution of users properly, while producing scalable and tractable analyses for various performance metrics of interest. Despite the importance of CRN to next-generation networks, no survey of the existing literature has been done when it comes to SG-based interference management and control in the domain of CRN. Such a survey is, however, necessary to provide the current state of the art as well as future directions. This paper hence presents a comprehensive survey related to the use of SG to effect interference management and control in CRN. We show that most of the existing approaches in CRN failed to capture the relationship between the spatial location of users and temporal traffic dynamics and are only restricted to interference modeling among non-mobile users with full buffers. This survey hence encourages further research in this area. Finally, this paper provides open problems and future directions to aid in finding more solutions to achieve efficient and effective usage of the scarce spectral resources for wireless communications.

Keywords:
Cognitive radio, handover, NOMA, queueing theory, spatiotemporal analysis, stochastic geometry.

1. Introduction

Cognitive radio network (CRN) is expected to play an essential role in the implementation and expansion of next-generation networks. Because of continuous demand for wireless technologies, many devices or users will require access to transmit on the scarce spectrum resources. Preliminary research, however, shows that the existing fixed spectrum allocation policy is ineffective and inefficient owing to its inability to accommodate the present evolution in wireless communication systems [1] –[3]. Hence, CRN has continued to receive increasing attention over the past two decades. In CRN, unlicensed devices or users can transmit on the channels belonging to the licensed devices or users with the aim of satisfying their own spectrum demands, while ensuring that the interference resulting from such spectrum usage does not disrupt the activities of the licensed users. These unlicensed users are known as secondary users (SUs) and licensed users are called primary users (PUs).

Three CRN models are widely discussed in the literature. These are the underlay [4] –[10], overlay [11] –[13] and hybrid [14] models. The most frequently considered of these models is the underlay approach because of its low implementation complexity. In the underlay model, SUs access the channels concurrently with PUs, as long as their activities do not generate disruptive or excessive interference in the network. In the overlay model, SUs are granted access to make use of channels in the absence of PUs, while the hybrid model involves the combination of both underlay and overlay models [15]. Similarly, analysis of CRN models is normally obtained under either downlink or uplink scenarios. Under the downlink scenario [4] –[6], the desired signal and interference powers are both affected by users' location, hence equal transmit signal power is often assumed to avoid complicated analysis, while power control at the secondary network is usually ignored. In the uplink scenario [1], [16], [17] however, the interference signal power received at any tagged receiver is not affected by its location, hence the interfering nodes may be closer to the tagged receiver than its pair tagged transmitter, making power control a necessity in the uplink scenario. While the adoption of CRN can significantly improve spectrum reuse and enable efficient resource allocation [18] –[24], interference control remains a major issue [15] and the activities of SUs must be well coordinated to reduce interference in the network. It is hence not surprising that interference management and control have been the focus of much research in the past few decades.

For effective protection of PUs' activities via interference management and control, proper and accurate interference modeling is required. A traditional grid approach was previously adopted to characterize interference in the network. Such an approach, however, often results in intractable and

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inaccurate analysis. In fact, such models require complex and extensive Monte Carlo simulations and the analyses obtained from them often require unrealistic assumptions, leading to inaccurate results. It is worth noting that the practical deployment of users is not expected to follow a grid-based approach owing to diverse demands for access across various service areas. Although the hexagonal grid model was able to mitigate interference in the simplified wireless network, the model lacks scalability when multiple users are involved and is ineffective in enhancing spectral usage efficiency because of its inability to model the locations and distributions of users in more practical terms. Hence, the stochastic geometry (SG) approach is now being adopted in CRN [7], [16], [26], [27].

The SG is a useful approach that aims at providing efficient mathematical and statistical mechanisms that are useful in the study and analysis of random spatial patterns. Such random spatial patterns (otherwise called point processes) - defined as the set of locations distributed within a certain designated region - are normally generated using stochastic mechanisms and are known to be important elements of SG. Using random point patterns, the distribution of users in CRN can be realistically captured, while the analyses resulting from such a process can be accurate and tractable, albeit subject to various simplifications and assumptions. Notwithstanding, the accuracy of the SG methods has been well demonstrated in many research works and is known to provide more realistic users' distribution unlike the traditional grid models. SG based models are usually verified using less complex Monte Carlo simulations and have been successful in modeling many practical applications and systems, such as cellular networks, internet of things (IoT) networks, cognitive networks, etc. Furthermore, the limitations of the grid models are well addressed in the SG models with the ability to offer scalability when multiple users are involved. This paper presents a comprehensive survey of the applications of SG to effect interference modeling in CRN.

Owing to its importance in wireless communications and networks, various publications exist on the methods and applications of CRN and it is unsurprising that many surveys have been carried out to present a diverse state of arts in CRN. To be specific, surveys on variations of underlying taxonomy of CRN as well as its architecture [28] –[30], sensing methods [31], [32], resource allocation [33], [34], communication mechanisms [35], channel-sharing methods [36], network-coding approaches to CRN [37], PU activity modeling [38] and queueing applications to CRN [39], etc. have been presented. Although a large body of literature exist on the applications of SG in interference management and control in CRN, the various analyses presented in the literature adopt diverse approximations and simplifications that are necessary to obtain a tractable and accurate analysis. Despite the fact that SG is known to be capable of representing important aspects of the wireless networks owing to its ability to capture the distribution of users in more practical terms, as well as its ability to provide a better understanding of the network, the difficulty of obtaining tractable yet accurate analysis forms the major challenge when characterizing interference in CRN using the tool of SG. It is therefore imperative to provide a comprehensive overview of the existing SG-based interference management and control models to understand the effects of these approximations and simplifications on the accuracy and tractability of the interference models.

1.1. Motivation

Building on the existing SG-based interference models proposed for traditional wireless networks, various research works on CRN often seek to obtain models that reflect the key characteristics of CRN by capturing two essential properties – priority and dependence properties – which are distinct characteristics of CRN that are often not captured in most traditional networks. These two characteristics are briefly highlighted as follows:

- Priority: The notion of priority is an important aspect of CRN owing to the requirement of the SUs to vacate the band before the arrival of the PUs when transmitting in the overlay mode. Similarly, SUs are required to adopt a proper power control technique when transmitting in the underlay mode to ensure that the interference generated at any PU receiver is below the maximum allowable interference threshold in the network. Hence, priority cannot be ignored when characterizing interference in the domain of CRN. The notion of priority ensures that PUs do not experience degraded services when sharing their assigned spectrum with the SUs. It then means that PUs must always be given higher priority when modeling the network.

- Dependence property: In CRN, SUs are not allowed to transmit except their locations suggest that they are far from the locations of active PU receivers or their transmissions will not cause excessive interference at the PU receivers. Hence correlations exist in CRN. This implies that the independence property commonly assumed in traditional wireless networks may not be suitable in CRN, where the distributions of SU transmitters are expected to reflect the dependence property to reduce interference to the activities of PUs. Furthermore, PU transmitters (e.g. TV transmitters, radars, and cellular base stations) are normally deployed with sufficient repulsion to reduce intra-network interference in the primary network, hence the distributions of active PU transmitters as seen in real-life deployment are not independent.

This paper hence presents the current state of the art on the applications of SG-based solutions to effect interference modeling in the domain of CRN. Despite diverse approaches to interference characterization in CRN, to the best of our knowledge, a comprehensive survey and classification of SG-based interference models in CRN have not been presented before. SG-based models for cellular wireless networks were surveyed in [40], while a tutorial on the application of SG in the modeling of the cellular network was given in [41]. The models formulated for cellular networks, however, cannot be adopted fully in CRN owing to the difference in the architectures of these two networks. Meanwhile, if the uniqueness of CRN is carefully identified, various models developed for the cellular networks can be carefully modified and adapted to capture the characteristics of CRN. Similar to [40],[41],

Table 1: The topics of the related survey and tutorial papers

Reference	Focus	Modeled interference?
[28]–[30]	Architecture of CRN	No
[31], [32]	Spectrum sensing mechanisms	No
[33], [34]	Resource allocation in CRN	No
[35]	CRN communication mechanisms	No
[36]	CRN channel sharing methods	No
[37]	Network coding techniques in CRN	No
[38]	Users' activities modeling in CRN	No
[39]	Queueing applications in CRN	No
[40]	Stochastic geometry application in cognitive cellular wireless networks	Yes
[41]	Stochastic geometry modeling and analysis of cellular networks	Yes

this survey review some of the SG based interference models with a focus on CRN. We believe that this survey will aid future research, as efforts continue to achieve accurate and tractable analysis to enable interference control in CRN. The main contributions of the existing related survey and tutorial papers are compared with this survey in Fig. 1 and Table 1.

1.2. Contributions

This survey is structured to reflect the progress in the development of SG based interference models with a focus on how various characteristics of CRN were captured in the system modeling. The characteristics of CRN considered in this survey include dependence property, priority, mobility, cooperation and non-orthogonal multiple access (NOMA) for CRN. The key contributions of this survey paper are summarized as follows:

- An SG-based approach to spectrum occupancy modeling in CRN: We carried out an overview of the SG-based approach to occupancy modeling in the domain of CRN. This occupancy modeling was shown to depend on effective channel sensing and is important to interference management and control in CRN.

- A comprehensive discussion of the impact of users' distribution on interference models: Commonly adopted users' distribution models in CRN were reviewed, while basic analyses of common performance metrics were presented. We further discussed the limitations of each of these distribution models that led to the adoption of other distribution models.

- Application of queueing theory in SG-based interference modeling: The importance of queueing theory to accurate interference modeling in CRN was discussed. We noted that, despite its usefulness in obtaining more realistic interference management and control models, the application of queueing models in this regard has been limited in CRN.

- Impact of cooperation, mobility, and handover on network performance: The effects of cooperation, mobility and handover on users' coverage as well as network analysis in CRN were discussed. We further discussed the application of a power control technique in the secondary network, which is capable of enhancing spectral usage efficiency.

- Open problems and future direction: Based on the current state of the art, we discussed some open problems and possible area of research which can be useful to readers.

To avoid unnecessary repetition, the basics of SG are not discussed in this paper. These have been extensively discussed in [42] –[45]. We focus on related literature on SG models for interference characterization in CRN.

1.3. Related survey articles in CRN

Our present survey on interference management and control in the domain of CRN differs from existing surveys and tutorials as we comprehensively survey SG models for interference characterization in CRN. Although, various surveys in CRN have been carried out on different interesting areas of CRN such as spectrum sensing techniques, security challenges and methods, resource allocation mechanisms and challenges, queueing methods and applications, spectrum assignments among cognitive users, etc., to the best of our knowledge, no prior detailed survey that comprehensively covers SG-based models for interference management and control in CRN has ever been presented.

A survey on the variations of underlying taxonomy of CRN as well as network communication functionalities were presented in [28] where various issues relating to CRN such as spectrum sensing, sharing, and management were discussed. In [29], issues including CRN models, spectrum opportunity detection and tracking with major technical and regulatory concerns were reviewed, while advances in dynamic spectrum access were presented in [30]. Also, various sensing methods and challenges were reviewed in [31], [32], while resource allocation methods and challenges in CRN was extensively discussed in [33], [34]. The review in [33] focused on various methods of allocating the existing limited spectrum to SUs through the integer programming-based optimization solutions, while the survey provided in [34] considered resource allocation process and its components for underlay CRNs. Similarly, a survey on full-duplex communication mechanisms applications in CRN was presented in [35], while a comprehensive review of channel sharing methods was also presented in [36].

Other important surveys in CRN include network coding approaches and applications [37], where an in-depth review of network coding schemes in CRNs was provided from the perspective of white spaces and PU's activity modeling in [38] with the aim of providing a combined source in the form of a survey paper that contains existing research on modeling methods of PU's activities in CRN. Applications of queueing theory to CRN was also surveyed in [39] where various queueing models and queueing theoretic tools were classified, while the importance of priority-based queueing models to CRNs was discussed. Although interference modeling was mentioned by some of these survey papers as an important issue in CRN, it has not been the focus of any of the existing surveys. Interference modeling was considered in [40],

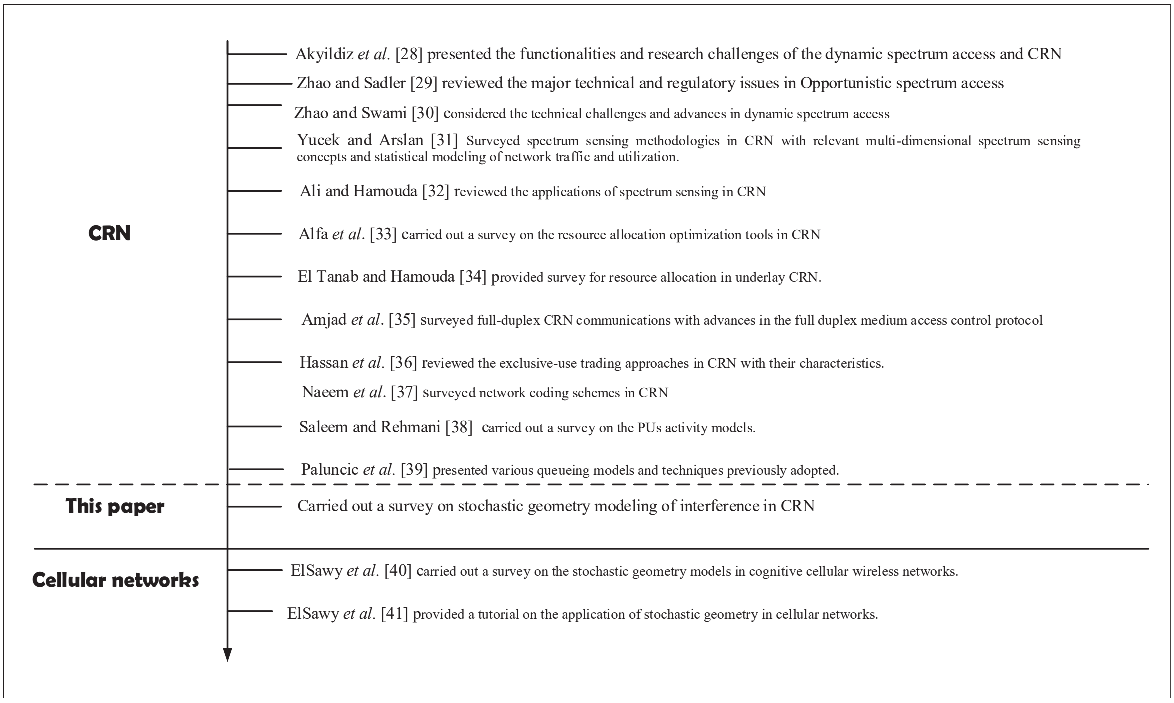

Figure 1: The main contributions of the existing related survey and tutorial papers.

[41], where SG-based analyses were provided for single-tier, multi-tier, and cognitive cellular networks [40]. Although these works provide very useful insights into the application of SG techniques to cellular networks, important characteristics of CRN such as dependence property, priority queueing requirements, cooperation, etc. which are central to CRN modeling were not covered in these works.

Owing to its importance to CRN and with diverse existing literature in the domain, this survey presents a review of the existing approaches toward SG-based interference modeling in CRN. Several important issues, methods of modeling and challenges of the interference models are presented, while the analyses of common performance metrics of interest are discussed. The contributions of the existing related surveys and tutorials are presented in Fig. 1 and Table 1.

1.4. Organization

The rest of this paper is thus organized as follows: an SG approach to channel occupancy modeling in CRN is presented in Section 2, while Section 3 presents various network distributions in CRN and the analyses of common performance metrics of interest. Section 4 presents a spatiotemporal approach to network modeling in CRN, while Section 5 presents a stochastic analysis for CRN with cooperation and handover mechanisms. In Section 6, the application of NOMA mechanisms to SG-based interference models is presented, while Section 7 provides open problems and future directions. Section 8 concludes the paper. The taxonomy of the important aspects of interference modeling in CRN is given in Fig. 2.

2. Channel occupancy modeling

One important step towards interference control and management in the CRN is accurate channel occupancy modeling (COM), in which SUs obtain the state of the channel with the purpose of accessing such channels if not occupied by PUs or other active SUs. While avoiding interference with the activities of the PUs is non-negotiable in any CRN, a good CRN interference control model is also expected to reduce interference in the secondary networks so as to satisfy SUs' quality of service. Reliable and effective COM is thus important when allocating spectral resources in CRN. These spectral resources can be allocated based on the time, frequency or space domain. In the time domain, SUs are permitted to access spectral resources in the absence of PUs, while in the frequency domain, SUs transmit on PUs' idle frequency bands only. The spatial domain, however, allows SUs to access a channel at the same time and in the same frequency domain as the PUs by exploiting the spatial spectrum holes[1] [12], and is known to be the most efficient method for effective spectral reuse. These PUs' spectrum holes, otherwise known as spectrum opportunities, are conventionally referred to as the unused PUs' frequency bands at any particular time in a specific geographic area [31], [47] –[49]. We do not delve into various methods of spectrum sensing for channel occupancy detection, as these have already been discussed extensively in [31]. We focus on the detection of PUs' and SUs' signals in the presence of noise and interference, while capturing the spatial distribution of users in the network. It is worth noting that most research on COM do not capture the spatial distribution of users and are hence unsuitable for interference modeling analysis in CRN.

Spectrum opportunity detection for cognitive users can be obtained based on either primary receivers' (PRs) locations [2], [12], [14], [50] or primary transmitters' (PTs) locations

[1]A spectrum hole is a multi-dimensional region of the network within the time, frequency and space in which STs' transmissions result in limited interference with the activities of PRs [46].

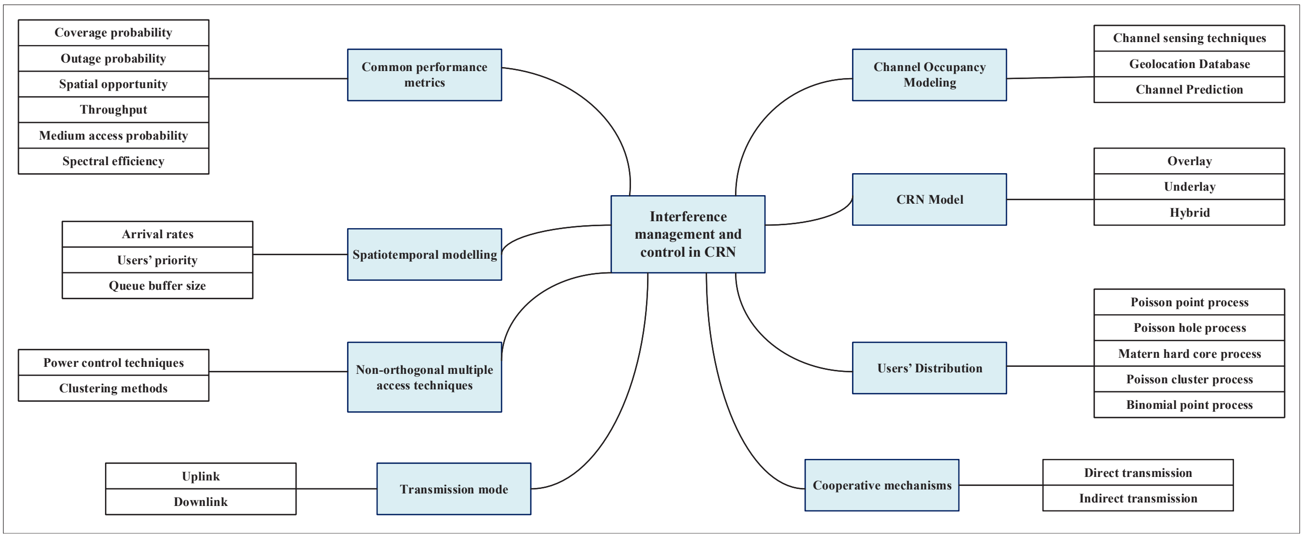

Figure 2: Taxonomy of important aspects of interference modeling in CRN

[12], [51]. The authors in [2], [12], [14], [50] conditioned that active PRs broadcast their respective unique beacons at a suitable power level. In order to reduce the effects of channel fading and shadowing, each beacon is assumed to be transmitted via a dedicated channel in [12]. Using the received beacons' power strength, secondary transmitters (STs) can detect a spatial spectrum hole if the received beacon signal is less than the predefined threshold. When spectrum opportunity for an SU is obtained following the distribution of PTs, STs detect spatial spectrum holes through the unique pilot signals sent from each active PT to its corresponding pair PR. This approach can be extended to the secondary network in which all active secondary receivers (SR) and STs broadcast their unique beacons when spectrum opportunity detection is based on SR location and ST location respectively. A newly arrived ST can then decide whether the channel is currently being used by a PT or another ST with an earlier arrival time. Similar to [52], each time slot can be divided into two phases: the spectrum opportunity detection phase and transmission phase. Upon discovery of a spectrum hole during the spectrum opportunity detection phase, an ST can proceed to the transmission phase.

2.1. *Detection of spectrum opportunity*

Detection of spectrum opportunity can be achieved through either channel sensing (see [31] for details) or by querying a dedicated geolocation database. Channel occupancy detection methods are summarized in Fig. 3 and are briefly discussed below;

2.1.1. *Channel sensing*

The need to mitigate network interference while effectively improving spectrum opportunity makes the use of a centralized control model for dynamic channel access infeasible, since the centralized control model suffers from unnecessary complexity and delay [40]. Hence, dynamic channel sensing is a popular method of spectrum hole detection in CRN [2], [52] –[56]. In [54], a cognitive carrier sense multiple access protocol is proposed in which each slot is divided into three phases: PU sensing, SU sensing (both forming the spectrum

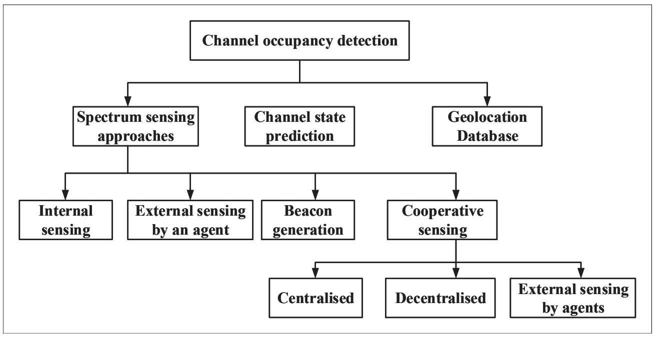

Figure 3: Channel occupancy detection methods

hole detection phase) and the transmission phases. Through the proposed carrier sense technique, SUs can detect spectrum holes similar to the medium access control protocol [53]. Spectrum sensing can also be internal – which can be either single-radio or dual-radio sensing architecture and external when sensing is done by external agents. Because of its importance to effective channel usage, cooperative sensing has also been considered to increase sensing accuracy further [2].

In cooperative channel sensing, multiple SUs simultaneously carry out channel sensing with the aim of obtaining more accurate sensing outcomes. Such a sensing method is said to be centralized when all sensing outcomes from each sensing node are forwarded to a centralized SU - a user that is mandated to make the final decision about the state of the channel at the time. Cooperative sensing can also be decentralized when multiple SUs obtain the state of the channel through sensing and exchange the outcome of their sensing with the aim of helping each SU to make more accurate decisions. Although cooperative channel sensing can achieve more accurate sensing outcomes, the approach may not be suitable for low-energy devices owing to the expected higher level of energy required for sensing, transmission and receiving of sensing outcomes.

Another common approach is known as the channel state prediction method, in which the state of the channel is normally predicted for the next time slot (say $n + 1$), given the state of the channel (obtained perhaps through channel sensing) in the present time slot (say n) [57] –[61]. In such an approach, the activities of PUs are predicted with the purpose of transmitting on PUs' channels when they are inactive, as in the overlay CRN model. The limitation of such an approach, however, is its dependence on accurate channel predictions and a higher false alarm rate, as well as a higher probability of misdetection, which heavily degrades the channel usage efficiency and increases interference in the network. While spectrum sensing can be very effective in the detection of PUs' spectrum holes, spectrum sensing may mean high consumption of energy for SUs that are expected to be energy-efficient devices or users. Other literature suggested the use of the geolocation database to avoid this limitation.

2.1.2. Geolocation database

The geolocation database approach in the detection of spectrum opportunity was considered in [62] –[64]. Although such an approach can be unrealistic in dynamic networks where the usage patterns of PUs are non-uniform, its ability to provide accurate channel states in static networks can make its usage considerable and useful in some systems where users' dynamism is not considered. Unlike the sensing model that depends on pre-defined thresholds to detect spectrum holes, channel states and information about channel usage patterns of PUs are made available to all users through a database maintained by the appropriate regulators. When the geolocation database is used properly, it is capable of preventing interference in CRN and has been recommended by the Federal Communications Commission because of its ability to enhance effective spectrum sharing [64].

In order to limit the inter-network interference experienced in any primary network, reliable up-to-date information on PUs' activities is provided to all SUs through the geolocation database and can be assessed by SUs through various query methods. The geolocation database, however, suffers from privacy issues, as the privacy of PUs can be compromised, hence making PUs' locations available every time may not be desirable in sensitive systems [64]. Similarly, when a geolocation database is adopted to limit intra-network interference in the secondary network, SUs' locations and activities can also be tracked by malicious users through access to the database with the purpose of compromising the secondary network.

Despite its limitations, which are now receiving much attention [62], [64], the use of a geolocation database ensures adequate energy saving in the network, since it does not require SUs to carry out channel sensing before accessing the channel for transmission. Hence SUs can reserve their energy for transmissions. When used in a dynamic network, its effectiveness and ability to ensure interference reduction in the network depends on the capability of the geolocation database to provide up-to-date information for users in any time slot, which may still require channels to be sensed, however, by dedicated users. Hence channel sensing is important and central to interference modeling in CRN.

2.2. Spectrum opportunity detection analysis

Considering a typical SU that requires access to transmission opportunity, if the received signal at such an SU is greater than the pre-set thresholds for primary and secondary transmissions, the presence of respectively PUs and SUs can be determined. The channel occupancy model can be obtained as [3]

$$C_o = \begin{cases} 1 & \text{if channel is busy} \\ 0 & \text{if channel is idle.} \end{cases} \tag{1}$$

The received signal at such a typical SU can be expressed in the form

$$y(n) = S(n) + W(n) + I(n), \tag{2}$$

where $S(n)$ is the actual signal power to be detected, $W(n)$ is the noise power signal and $I(n)$ is the aggregate intra-network and inter-network interference power received at the typical ST. $S(n)$ from any PT given as $S_p(n)$ can be obtained from (2) as

$$S_p(n) = P_p h_{X_{ps}} \|X_{ps}\|^{-\alpha}, \tag{3}$$

where P_p is the PT transmit power, $\|X_{ps}\|$ is known as the Euclidean distance between the active PT and the typical SU, $h_{X_{ps}}$ is defined as random channel gain (between the active PT and the typical SU) that captures the level of uncertainty that results from fading and shadowing, while the path loss exponent is represented as α. Similarly, the $S(n)$ from any active ST, given as $S_s(n)$, can be obtained as

$$S_s(n) = P_s h_{x_{ss}} \|x_{ss}\|^{-\alpha}, \tag{4}$$

where P_s is the ST transmit power, $\|x_{ss}\|$ is the Euclidean distance between the active ST and the typical SU and $h_{x_{ss}}$ is the random channel gain that captures the outcome of fading and shadowing between the active ST and the typical SU. Let the detection threshold of the PU signal be φ_p and the detection threshold of the SU signal be φ_s in the presence of interference and noise, then a typical channel C_i is busy serving a PT if $y(n) \geq \varphi_p$, while the channel can be said to be serving another ST if $y(n) \geq \varphi_s$. The thresholds φ_p and φ_s must be carefully selected to account for the interference and noise in the network. The parameter $I(n)$, however, depends on the distribution of users in the network and can be obtained through the tool of SG, as will be shown in the subsequent sections of this paper.

Before we delve into the details of network distributions that are central to interference management and control in CRN, we mention two channel cases that are common in the literature. The first is the single-channel assumption [8], [25], [53], [65], where a single channel is considered in order to avoid complicated analysis. While this assumption fails to capture the real practical systems, such an assumption is useful in investigating the performance of any developed model, which can then be extended to consider the case of a multichannel. Because of the need to capture more characteristics of practical systems, the single-channel assumption is normally relaxed to consider the multichannel scenario in order to demonstrate or investigate the reliability of any proposed interference model under the multichannel scenario. For instance, the interference models derived for a single-channel scenario in [52] were extended to capture a multichannel case in [66]. When a multichannel scenario is considered, inter-channel relationships can be properly captured in the interference analysis [7], [67]. In the next section, we present various network distributions that are central to interference management and control in CRN.

2.3. Summary

Occupancy modeling in CRN is an important process to achieve interference management and control among the network users. Owing to the channel access requirement to avoid interference in the primary networks, accurate and effective occupancy modeling is a necessity to ensure that interference at the primary networks is below the pre-set threshold. Similarly, an effective COM is not only expected to limit interference with the activities of the PUs, but is also required to ensure that the interference at secondary networks is properly controlled so as to guarantee users' quality of service. In this section, we have presented the relevant channel occupancy detection approach to interference control in CRN. One important parameter is the user's detection threshold, which must be carefully selected to avoid interference in the network, while ensuring that the spectrum holes are efficiently detected by the SUs.

When a single PU in a single channel scenario is considered, all SUs contend for the spectral resources among themselves. Access is granted to any SU if its transmission will not generate an excessive disruption of the PU. In such a scenario, intra-network interference in the primary network is avoided, leading to a simplified network analysis. Multiple PUs in a single-channel scenario means that PUs now contend to access the spectral resources – an approach that may not necessarily capture the PUs' independent access in CRN. Regardless of whether there is a single PU or multiple PUs, intra-network interference in primary and secondary networks can be neglected under a single-channel scenario if only one PU and SU respectively can access the channel at any time. More characteristics of the practical system are captured in the multichannel scenario in which multiple PUs and SUs access the multiple channels based on the pre-defined channel access rule. In such cases both intra-network (in primary and secondary networks) and inter-network interference are captured in the system analysis. More discussion on intra-network and inter-network interference is provided in the next section.

3. Network distribution and performance analysis

The oversimplification of users' distribution in d-dimensional space is the main limitation of the traditional grid model when used in modeling interference in any wireless network. As a result of this, attention is being drawn to the use of SG owing to its ability to capture statistical properties of random points within d-dimensional space through a point process – an important element of SG [68]. A point process with constant points' intensity within the Euclidean space R^d is said to be homogeneous, while it is non-homogeneous if the intensity is not constant. Also, a homogeneous point process with points that are invariant by translation is said to be stationary, while a homogeneous point process is said to be simple if there is a maximum of one point in the same location, i.e. $N(\{x\}) \leq 1, \forall x \in R^d$.

Accurate modeling of PUs' and SUs' distribution with the appropriate point process is important for proper interference control and management in CRN. The underlying challenge, however, is the possibility of some point processes complicating the analyses of various performance metrics of interest. Hence, various simplifications are usually resorted to in order to obtain a trade-off between tractability and accuracy. We present some commonly used point processes for users' distribution modeling and obtain a general analysis for various performance metrics of interest. The point processes considered in this paper include the Poisson point process (PPP), Matern hardcore point process (MHCP), Poisson hole process (PHP) and Poisson cluster process (PCP), while the performance metrics considered are limited to spatial opportunity, coverage probability, outage probability, spatial throughput, network throughput, spectral efficiency, and medium access probability, since most of the performance metrics in the literature are closely related. We provide the taxonomy of the existing literature based on the widely assumed point process and performance metrics adopted in Table 2. The summary of the surveys presented in this section is presented in Fig. 4.

3.1. Independent Poisson point process

The PPP is the most commonly assumed distribution in the literature owing to its ability to produce tractable analysis for various metrics of interests. Hence, its adoption is widely discussed in wireless networks and has been well adopted in the domain of CRN [1], [4] –[13], [16], [25], [53], [55], [56], [65] –[67], [69], [70]. Because of its importance in generating other point processes, PPP is the most important point process [40]. Although modeling users' location following PPP is analytically tractable, such distribution may not capture the actual effect of interference in the network, since the location of users in real practical systems does not necessarily follow HPPP, where users are arbitrarily closer to one another. Notwithstanding this, such a distribution leads to tractable analysis for various performance metrics of interest, hence its wide adoption.

Owing to its ability to produce tractable analysis in CRN, the distribution of PUs was assumed to follow HPPP in [4], [5], [7] –[9], [11] –[13], [16], [25], [53], [56], [65] –[67], [69], [70], while the distribution of SUs was considered to follow an independent PPP in [1], [4], [6] –[13], [16], [25], [53],[55], [65] –[67], [69], [70]. Such representation can be obtained through simulations as shown in Fig. 5. However, the distributions of active SUs are not totally independent of the distributions of active PUs, as SUs are expected to avoid interference in the primary network and are not permitted to access the channel in the presence of PUs as in the traditional overlay model and within a typical cell V of radius a in the presence of PUs as in an underlay model; hence authors often resort to the circular void probability techniques [81] – a technique that captures the repulsion among nodes using probability, in order to account for the required distance between PUs and SUs that was not well captured by the independent PPPs assumption. In CRN, we can define void probability as the probability that there are no SUs within a typical cell $V(0, a)$, which can be expressed from [81] as

$$V(0, a) = P(0, a) = \exp(-\lambda_p \pi a^2), \quad (5)$$

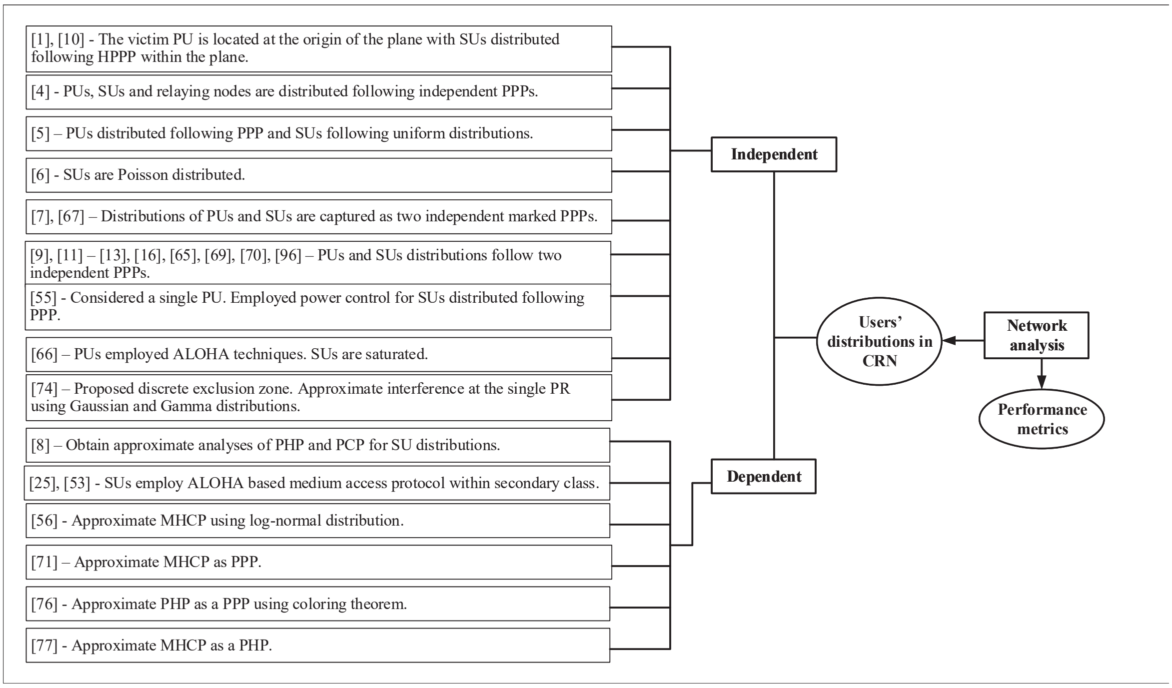

Figure 4: Summary of the reviews on network distributions.

Table 2: The taxonomy of existing literature based on the commonly considered point process and performance metrics

Point process	Poisson point process	[1], [2], [4] –[13], [16], [25], [52], [53], [55], [56], [65] –[67], [69], [70] –[75], [78], [79].
	Poisson hole process	[8], [52], [53], [73] –[78].
	Matern hard-core process	[25], [52], [56], [71], [77], [78].
	Poisson cluster process	[8], [76].
	Binomial point process	[80]
Performance metrics	Outage probability / coverage probability	[4], [5] –[16], [25], [52], [53], [55], [56], [65] –[67], [69], [71] –[77].
	Spatial opportunity	[12], [13].
	Spatial throughput	[12], [65].
	Network throughput	[7], [11], [17], [25], [52], [53], [56], [67], [69], [77].
	Medium access probability	[25], [52], [53], [66], [69].
	Spectral efficiency	[16]

where λ_p is the intensity of the active PUs. It is also possible to ensure minimum repulsion among PUs using the void probability technique. Next, we discuss how these distributions affect the expressions for common performance metrics.

One important parameter necessary for interference characterization in the network is the signal-to-noise plus interference ratio (SINR) or signal-to-interference ratio (SIR) when noise is neglected. Consider a typical CRN in which PTs are distributed following HPPP Φ_p of intensity λ_p and STs are distributed following independent PPP Φ_s of intensity λ_s. When the distribution of PTs and STs is considered to follow two independent PPPs and a bipolar network model [8] is assumed, such that each PR or SR is considered to be located at a fixed distance from its corresponding PT or ST, it is easy to observe from such a network that the distribution of PRs and SRs also follows independent PPPs of intensities λ_p and λ_s respectively. The SINR received at any typical primary receiver (PR) y_k^p located at the centre of a disk of radius D is expressed as

$$SINR_{y_k^p} = \frac{P_p h_{X_{pp}} \|X_{pp}\|^{-\alpha}}{\sigma^2 + I_{pp} + I_{sp}}, \tag{6}$$

where σ^2 is the noise signal power, $\|X_{pp}\|$ is the Euclidean distance between any active PT and the tagged PR, $h_{X_{pp}}$ is the random channel gain that captures the outcome of fading and shadowing between any active PT and the tagged PR, I_{pp} is the interference power from other active PTs $x_i^p \in \Phi_p$ and I_{sp} is the interference power from active STs $x_i^s \in \Phi_s$. A common approach when modeling interference in any wireless network is the equal transmit power assumption for all users belonging to the same network. For instance, in [75], PRs were considered to be located within a fixed distance from their respective pair PTs assumed to be transmitting with equal transmit power. From (6), the performance of the network can be investigated through various metrics.

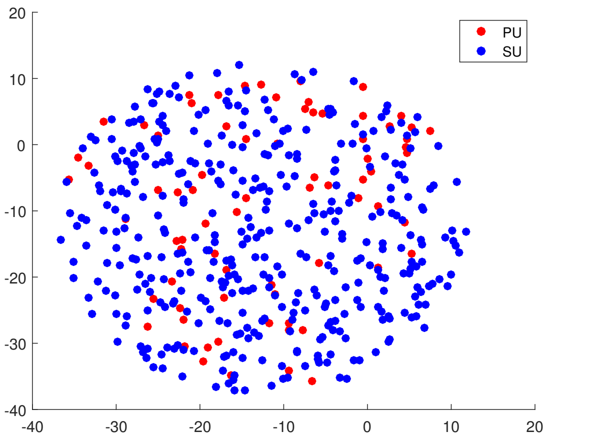

Figure 5: PUs and SUs distribution of intensities 0.03 and 0.2 respectively following independent PPPs.

3.1.1. Coverage probability

Coverage probability is the probability that the SINR received at the tagged receiver is greater than the pre-defined threshold. In the primary network, the probability that the SINR received at the y_k^p is greater than the pre-defined threshold θ_p is given as

$$P_{co}^p = P(SINR_{y_k^p} > \theta_p), \quad (7)$$
$$= P\Big(\frac{P_p h_{X_{pp}} \|X_{pp}\|^{-\alpha}}{\sigma^2 + I_{pp} + I_{sp}} > \theta_p\Big).$$

Under Rayleigh fading assumption of unit mean $h \sim \exp(1)$,

$$P_{co}^p = E\Big\{\exp\Big(\frac{-\theta_p \|X_{pp}\|^\alpha}{P_p}\sigma^2 - \frac{\theta_p \|X_{pp}\|^\alpha}{P_p} I_{pp} - \frac{\theta_p \|X_{pp}\|^\alpha}{P_p} I_{sp}\Big)\Big\}. \quad (8)$$

From the definition of Laplace transform (LT), we know that $L_z(s) = E[\exp(-sz)]$. By making $s = \frac{\theta_p \|X_{pp}\|^\alpha}{P_p}$,

$$P_{co}^p = \exp(-s\sigma^2)\mathcal{L}_{I_{pp}}(s)\mathcal{L}_{I_{sp}}(s). \quad (9)$$

The expression of outage probability is similar to the coverage probability and is defined as the probability that the SINR received at the tagged receiver is not more than the pre-defined threshold. Such a metric at any y_k^p is given as

$$P_{out}^p = P(SINR_{y_k^p} \leq \theta_p) = 1 - \exp(-s\sigma^2)\mathcal{L}_{I_{pp}}(s)\mathcal{L}_{I_{sp}}(s). \quad (10)$$

The definitions for $\mathcal{L}_{I_{pp}}$ and $\mathcal{L}_{I_{sp}}$ is provided in the following lemmas (for more clarity and understanding) under the assumption that any typical transmitter is connected with the nearest receiver [4] –[13], [71]. The nearest association under various assumption for users in CRN was provided in [10], [82].

Lemma 1. *The LT of $I_{pp} = \sum_{x_i^p \in \Phi_p \setminus x_k^p} P_p h_{X_{pp}} \|X_{pp}\|^{-\alpha}$ given that PTs are distributed following HPPP Φ_p of intensity λ_p is given as [8], [71]*

$$\mathcal{L}_{I_{pp}}(s) \triangleq \mathcal{L}(\lambda_p, P_p, s) = \exp\Big(-\pi\lambda_p \frac{\theta_p^\delta r_p^2}{sinc(\delta)}\Big), \quad (11)$$

where $\delta = \dfrac{2}{\alpha}$ and r_p is the distance between any tagged primary transmitter-receiver pair.

Lemma 2. *The LT of $I_{sp} = \sum_{x_i^s \in \Phi_s} P_s h_{x_{sp}} \|x_{sp}\|^{-\alpha}$ given that STs are distributed following independent PPP Φ_s of intensity λ_s is straightforward from (11) and can be approximated as[71]*

$$\mathcal{L}_{I_{sp}}(s) \triangleq \mathcal{L}(\lambda_s^I, P_s, s, D) = \exp\Big(-2\pi\lambda_s^I \int_D^\infty \frac{r}{1 + \frac{r^\alpha}{sP_s}}\Big), \quad (12)$$

where $\|x_{sp}\|$ is the Euclidean distance between any active ST and the tagged PR and $h_{x_{sp}}$ is the random channel gain that captures the outcome of fading and shadowing between any active ST and the tagged PR.

The solution to (12) is obtained based on the following information: In order to reduce interference received at the tagged PR y_k^p, a protection region of radius D is assumed around each PR such that the intensity of the ST within D is considered to be zero. In such a case, an ST is granted permission to transmit only if its location is not within the protection regions of PTs. Hence, active STs are said to be distributed following HPPP Φ_p^I of intensity $\lambda_s^I = \lambda_s p_s$, where $p_s = \exp(-\lambda_p \pi D^2)p_t$ represents the probability that a typical ST is not within the location of active PRs and p_t is the transmission probability of each ST. The definition and description of D in the bipolar model is given in [8].

The upper bound for the $\mathcal{L}_{I_{sp}}$ can be expressed as $\mathcal{L}_{I_{sp}}^I(s) \triangleq \mathcal{L}(\lambda_s, P_s, s, D)$ [8]. In [74], the authors considered a CRN with single PU and multiple SUs where all users were assumed to follow an m-dimensional HPPP. An alternative PU protection scheme called the discrete exclusion zone was proposed, since the existing geographical exclusion zone failed to capture the effect of fading, though the interference from SUs at the single PR was approximated using Gaussian and Gamma distributions.

When the protection region is used at the primary network, the distribution of SUs is no longer independent of the distribution of PUs and characterizing such distribution using HPPP becomes insufficient. It will be shown later that such distribution is better represented as PHP as in [8] or MHCP as in [71]. Meanwhile, when the Aloha protocol [66], [67] is employed in the primary network, the distribution of active PTs/PRs can be said to follow HPPP Φ_p^a of intensity $\lambda_p^a = \lambda_p p_p$ formed from independent thinning of Φ_p, where p_p represents the probability that any typical PT accesses its assigned channel at a particular time slot.

Similarly, when a secondary network is considered, the probability that the SINR received at any tagged SR y_k^s is greater than the pre-defined threshold θ_s is given as

$$P_{co}^s = P(SINR_{y_k^s} > \theta_s), \quad (13)$$
$$= P\Big(\frac{P_s h_{x_{ss}} \|x_{ss}\|^{-\alpha}}{\sigma^2 + I_{ss} + I_{ps}} > \theta_s\Big),$$

where I_{ss} is the interference from other active STs and I_{ps} is the interference from active PTs at the tagged SR. At $s = \dfrac{\theta_s \|x_{ss}\|^\alpha}{P_s}$,

$$P_{co}^s = \exp(-s\sigma^2)\mathcal{L}_{I_{ss}}(s)\mathcal{L}_{I_{ps}}(s). \quad (14)$$

The analysis for outage probability at the SR can similarly be obtained as shown in the case of PUs using the Laplace transform functions. The LT of $I_{ss} = \sum_{x_i^s \in \Phi_s \backslash x_k^s} P_s h_{x_{ss}} \|x_{ss}\|^{-\alpha}$, given that active STs are distributed following HPPP Φ_s of intensity λ_s is upper-bounded [8], [71] at $\mathcal{L}^I_{I_{ss}} \triangleq \mathcal{L}(\lambda_s, P_s, s)$. Similarly, the approximate expression for the $\mathcal{L}_{I_{ss}}$ is usually obtained by considering active STs as STs outside PUs' exclusion regions. Hence, we have $\mathcal{L}_{I_{ss}}(s) = \mathcal{L}(\lambda^I_s, P_s, s)$, while the LT of $I_{ps} = \sum_{x_i^p \in \Phi_p} P_p h_{X_{ps}} \|X_{ps}\|^{-\alpha}$ given that PTs and STs were distributed following two independent PPPs Φ_p and Φ_s can be approximated as $\mathcal{L}_{I_{ps}} \triangleq \mathcal{L}(\lambda_p, P_p, s, \bar{D})$.

Note that the disk of radius D is centred on each PR, hence the distance between a typical PR and the tagged ST is at least D. Also, let any primary transmitter-receiver pair be separated by a distance r_p and any secondary transmitter-receiver pair be separated by a distance r_s, then $D - r_p$ is the minimum distance between a PT and an ST. It is then clear that the minimum distance between any PT and the tagged SR is $\bar{D} = D - r_p - r_s$. It is worth noting that when the disk is centred on each PT as in a transmitter-centric scenario, the distance $\bar{D}$ is slightly modified. The minimum distance between any PT and the tagged SR in such a case is given as $\bar{D} = D - r_s$. Similarly, the Aloha principle was employed in [25] at the secondary network in order to obtain a more simplified analysis.

3.1.2. Spatial opportunity

This metric was first defined in [13] as the probability that a typical location within a PUs' geographical region is detected as a spectrum hole. Such a metric is useful when investigating the quantities of the spectrum that are available for secondary usage. With the respective users' distributions assumed to characterize interference in the network, it becomes important to investigate the effect of such measures on SUs' spectrum opportunities.

Depending on whether the beacons are generated at the PTs or PRs, a simple expression for the spatial opportunity can be obtained. Let Φ_p^a represent the set of active PUs[2]; the received beacon power B_j at any arbitrary location $x_k^s \in \Phi_s$ from active PU $x_k^p \in \Phi_p^a$ can be obtained as $B_j(x_k^s) = S_p(n)$. The maximum beacon power received from active PUs can, therefore, be represented as [13]

$$B_M(x_k^s) = \max_{x_i^p \in \Phi_p^a} B_j. \tag{15}$$

The expression for spatial opportunity is thus obtained in [12], [13] following

$$S_o = P\{B_M(x_k^s) \leq \varphi_p\}, \tag{16}$$

From the expressions of coverage probability provided in (9), (14) and spatial opportunity solution following (16), the expression for spatial throughput – which is the expected spatial density of successful primary/secondary transmissions [12], [65] – is obtained as

$$S_T^p = \lambda_p^a P_{co}^p, \; S_T^s = \lambda_s S_o P_{co}^s. \tag{17}$$

[2]PUs here mean PTs if beacons are generated at the active PTs and PRs if beacons are generated from the active PRs.

3.1.3. Network throughput

Network throughput is defined as the probability that a typical user successfully receives its intended packet in a particular time slot [7], [67]. It can also be seen as the mean number of transmissions that are correctly received in a network in any time slot [25], [56]. In the secondary network, throughput can be defined as the probability that the packet sent by the tagged ST is successfully received by the intended paired SR.

Consider a CRN in which any tagged ST $x_k^s \in \Phi_s$ can only transmit within a multi-dimensional region represented as cell c if such ST selects c with probability c_k, accesses c with probability p_k and finds c to be a spectrum hole. This scenario can be represented as

$$e_{x_k^s, c} = 1(c = c_k, p_{x_k^s} = p_k, I_{\Phi_p^a} < \varphi_p), \tag{18}$$

where $I_{\Phi_p^a}$ is the aggregate received signal at x_k^s from all PTs $x_i^p \in \Phi_p^a$ and can be expressed as

$$I_{\Phi_p^a}(x_k^s) = \sum_{x_i^p \in \Phi_p^a} P_p h_X \|X\|^{-\alpha}. \tag{19}$$

The spectrum hole is hence detected by x_k^s within c if $I_{\Phi_p^a}(x_k^s) < \varphi_p$. The active probability of x_k^s, i.e. the probability that x_k^s secures an opportunity to transmit within c, is straightforward from (18) and can be expressed as

$$P\{e_{x_k^s, c} = 1\} = c_k p_k P(I_{\Phi_p^a}(x_k^s) < \varphi_p). \tag{20}$$

The expression for SU active probability can be very complicated, though it can be approximated as S_o when only the PT that generates the highest signal at the tagged ST is considered. The throughput experienced by the tagged ST is obtained as [7], [67]

$$T_s = \sum_{c=1}^{N} c_k p_k P(I_{\Phi_p^a}(x_k^s) < \varphi_p) P_{co}^s(y_k^s), \tag{21}$$

where $P_{co}^s(y_k^s)$ is the coverage probability at the tagged SR y_k^s and N is the total number of cells available for secondary transmissions. Similarly, throughput in the primary network can be defined as the probability that the packet sent by the tagged PT is successfully received by its intending paired PR. Since PUs are licensed users in CRN, a tagged PT is considered to transmit on its assigned channel c with probability $p_k = 1$ in any time slot. Hence, the throughput experienced by such a PT is the same as the coverage probability received at its pair PR.

3.1.4. Medium access probability

As in the case of traditional CRN, PUs do not normally contend to use their respective assigned channels and are permitted to transmit on their channels at any time. Hence, the analysis for medium access probability (MAP) is only obtained for SUs.

In a transmitter-centric CRN [25], [53], [66] where each transmitter is considered to be located at the centre of the disk (or cell) of radius, say r, with its pair receiver uniformly distributed within the disk; a typical ST x_k^s is located within the protection region of any PU if

$$U_{x_k^s} = 1_{P_p h_{X_{ps}} \|X_{ps}\|^{-\alpha} > \varphi_p}. \quad (22)$$

Hence, the MAP of a typical ST x_k^s under Rayleigh fading assumption with mean $h \sim \exp(1)$ in a simple CRN[3] is obtained as follows:

$$P(U_{x_k^s} = 1) = P\{P_p h_{X_{ps}} \|X_{ps}\|^{-\alpha} < \varphi_p\}, \quad (23)$$

$$= 1 - \exp(-P_p \varphi_p \|X_{ps}\|^{\alpha}). \quad (24)$$

From (24), the access probability for various network models as shown in [25], [53], [66] can be obtained.

3.1.5. Spectral efficiency

Ensuring efficient usage of spectral resources is important in order to meet the need of the current evolution in wireless systems and communications by providing a higher data rate for users, hence the recent adoption of CRN. An important performance metric in CRN is spectral efficiency - a metric that captures the amount of information transmitted between any transmitter-receiver pair [83] –[86]. Spectral efficiency can be seen as the SINR received at any tagged PR/SR from its pair PT/ST and can be expressed through the principle of Shannon's theory [16].

In a typical primary transmitter-receiver pair where the signal $SINR_{y_k^p}$ is received at the PR, the spectral efficiency is known to be the probability that the Shannon capacity obtained between such a transmitter-receiver pair is greater than T, where T is the pre-defined threshold. The analysis for spectral efficiency can be expressed as

$$S_E^p = \int_0^\infty P[ln(1 + SINR_{y_k^p}) > T] dT,$$

$$= \int_0^\infty P\Big[\frac{P_p h_{X_{pp}} \|X_{pp}\|^{-\alpha}}{\sigma^2 + I_{pp} + I_{sp}} > E(T)\Big] dT, \quad (25)$$

where $E(T) = e^{T-1}$.

Another point process similar to PPP is the binomial point process (BPP), though its usage is not well proven in CRN. The realization of BPP is similar to the realization of PPP. The only distinction between a BPP and a PPP is the fact that different realizations of the PPP will consist of different numbers of points [87], [88].

3.2. Poisson point process with Poisson hole process

In PPP, node distributions within the realization are usually assumed to be conditionally independent of one another. However, such an assumption is known to be unsuitable for most practical networks where repulsions among network users are required. A more efficient way of modeling the distribution of users in CRN is achieved by characterizing users' distributions following PHP [8], [76], [89], [90]. PHP allows enough distance between users in the network in a manner more similar to practical system implementations. In real practical systems, PTs are implemented with considerable distance between one another. Each active PT is said to transmit to its intended PR located within the coverage region of such PT. These coverage regions (otherwise known as PUs' protection regions) are called the exclusion regions and no SU is allowed to transmit in such a region. With this constraint, SUs are forbidden inside active PUs' coverage regions and the distribution of SUs is not totally independent of the distribution of active PUs. In order to capture this dependence, the distribution of active STs was considered to follow PHP in [8], [76] though the distribution of active PTs was still considered to follow HPPP to ensure tractability.

To provide more understanding of users' distributions under this assumption, we present Fig. 6 obtained through simulations, which shows the dependence between users' locations in the network. It is clear from Fig. 6 that there is a minimum level of repulsion between active PUs similar to actual PUs' deployment, while the location of active SUs is not really independent of the location of active PUs. With such distributions, interference received at the primary network is reduced. In order to capture the required repulsions between active PUs, independent thinning of baseline PPP Φ_p of intensity λ_p is normally carried out to obtain a new PPP[4] Φ_p^a of intensity λ_p^a, while the baseline PPP for SUs was dependently thinned to ensure each PU protection zone of radius D is avoided, thereby forming a PHP Φ_s^{php} of intensity λ_s^{php}.

When modeling the repulsion between PUs and SUs, a common approach to enhance tractability is to rank the powers of various PTs (see (3)) received at the location of any typical user (ST) following $\{S_p^1, S_p^2, S_p^3, ..., S_p^n\}, \forall S_p^1 \geq S_p^2$. Since the impact of the path-loss factor is known to be more stable and dominant compared to the instantaneous multi-path channel fading effects, the order statistics of the distance is normally considered to outweigh the fading effects, which vary on a much shorter time scale. Therefore, the ranking of PUs using their distances as described above from any tagged PU or SU is normally considered as a reasonable approximation of their respective ranked received signal powers.

The way in which this distribution affects the performance of the network is presented next.

3.2.1. Coverage probability

Consider a typical CRN where active PTs are distributed following HPPP Φ_p^a of intensity $\lambda_p^a = \lambda_p \exp(-\lambda_p \pi D^2)$, while active STs are distributed following Φ_s^{php} of intensity λ_s^{php}. Let each PT be located within its protection zone of radius D centred on its pair PR, while each SR is uniformly distributed from its pair ST within a maximum distance r_s. We provide the definitions for $\mathcal{L}_{I_{pp}}$ and $\mathcal{L}_{I_{sp}}$ in the following lemmas.

Lemma 3. *The LT of I_{pp} given that active PTs are distributed following HPPP Φ_p^a of intensity λ_p^a can be given as [8]*

$$\mathcal{L}_{I_{pp}}(s) \triangleq \mathcal{L}(\lambda_p^a, P_p, s). \quad (26)$$

Lemma 4. *The LT of I_{sp} given that active STs are distributed following a PHP Φ_s^{php} of intensity λ_s^{php} can be approximated at $s = \frac{\theta_p r_p^\alpha}{P_p}$ as [8]*

[3] Simple CRN simply refers to a CRN that has a single primary band with multiple SUs.

[4] Careful observation shows that the distribution of active PUs in this case is not actually a PPP, though this approximation has been proven to be accurate.

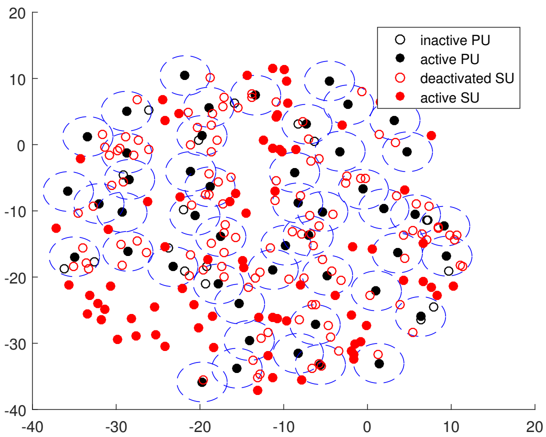

Figure 6: Users' distribution following dependence point process $\lambda_p = 0.03$, $\lambda_s = 0.03$.

$$\mathcal{L}_{I_{sp}}(s) \triangleq \mathcal{L}(\lambda_s^{php}, P_s, s), \tag{27}$$

where $\lambda_s^{php} = \lambda_s \exp(-\lambda_p \pi D^2)$.

The analysis following PHP is known to be mathematically non-tractable, while its probability generating functional (PGFL) is not known. As a result, PHP can be approximated by PPP through independent thinning of PPP as in [8], [76], as shown in (27), although such approximation has been reported to be inaccurate in [89], where tight bounds but less tractable analysis for the interference following PHP were presented. The analysis has since been adopted in various works. Following the analysis presented in [89], the LT of I_{sp} is given as Lemma 5.

Lemma 5. *Conditioned on $\|v\|$, the LT of I_{sp} is obtained following $I_{sp} = \sum_{x_i^s \in \Phi_s \cap b^c(x_k^p, D)} P_s h_{x_{sp}} \|x_{sp}\|^{-\alpha}$ as [89]*

$$\mathcal{L}_{I_{sp}}(s) \triangleq \mathcal{L}(\lambda_s, P_s, s, \|v\|, D)$$
$$= \exp\Big(-\pi\lambda_s \frac{(sP_s)^\delta}{sinc(\delta)}\Big) \exp\Big(\int_{\|v\|-D}^{\|v\|+D} \frac{2\pi\lambda(r)}{1+\frac{r^\alpha}{sP_s}} r dr\Big), \tag{28}$$

where $\lambda(r) = \frac{\lambda_s}{\pi} \cos^{-1}\big(\frac{r^2+\|v\|^2-D^2}{2\|v\|r}\big)$ and $b^c(x_k^p, D)$ represents a hole of radius D centred on x_k^p.

In the secondary network, the interference from the active STs at a tagged SR is difficult to obtain in its exact form owing to the location-dependent thinning of Φ_s to Φ_s^{php} and only its approximation has been obtained. The definitions for $\mathcal{L}_{I_{ss}}$ and $\mathcal{L}_{I_{ps}}$ are provided in the following equations.

Lemma 6. *The LT of $I_{ss} = \sum_{x_i^s \in \Phi_s \setminus x_k^s} P_s h_x \|x\|^{-\alpha}$ given that active STs are distributed following PHP Φ_s^{php} of intensity λ_s^{php} is often approximated at $s = \frac{\theta_s r_s^\alpha}{P_s}$ as [8]*

$$\mathcal{L}_{I_{ss}}(s) \triangleq \mathcal{L}(\lambda_s^{php}, P_s, s). \tag{29}$$

Similarly, we can say that the interference from active PTs at the tagged SR is the interference from a region $\bar{D}$, since the closest PT is at least a distance $\bar{D}$ from the tagged SR. The LT of I_{ps} under this scenario can be bounded at $s = \frac{\theta_s r_s^\alpha}{P_s}$ as [77]

$$\mathcal{L}_{I_{ps}}(s) \triangleq \mathcal{L}(\lambda_p, P_p, s, \|v\|, \bar{D}). \tag{30}$$

Obtaining exact expressions for these interference parameters remains a subject of future research. From (26) – (30), the expression for coverage probability is very straightforward for both primary and secondary networks. The analysis for outage probability is straightforward as well. The expression for spatial throughput is the same as given in (17) using the expression of coverage probability derived in this subsection, while the analysis of network throughput, MAP and spectral efficiency are straightforward following the definitions given in the last subsection.

3.3. Matern hard-core process

Despite ensuring minimum repulsion between PUs in Fig. 6, the distribution of active PUs was assumed to follow HPPP. This assumption is not valid, since the locations of active PUs are not independent of one another and can be better thought of as an MHCP. In MHCP, a minimum distance (known as hard-core distance) is implemented between nodes similar to the pattern obtainable in the practical implementation of primary base stations such as TV transmitters, where users are placed in a coordinated manner. This makes the MHCP a better point process when characterizing the distributions of active PUs in some literature. Similarly, the distributions of SUs can also be said to follow MHCP when capturing the dependence between PUs and SUs in the network [56], [71] similar to the case of PHP discussed in the previous subsection. In such a case, the distribution of SUs is well coordinated to ensure their locations are outside PUs' protection regions.

MHCP is formed through a dependent thinning of the initial PPP Φ of intensity λ such that there is a minimum distance of h between any two points. MHCP can be type I or type II [91]. In MHCP type I, all pairs of points with a pairwise distance less than h are removed, while type II MHCP eliminates points based on the randomly assigned mark and is known to provide a higher density of effective points [92]. For instance, a point x with a random variable $m_x, \forall m_x \sim \cup[0,1]$ in MHCP type II is retained if and only if there is no point with a smaller random variable than m_x located within a disk $b(x, h)$. The points in MHCP type II distributions are given as [56]:

$$\{x \in \Phi_{MHCP} | m_x < m_y, \forall y \in \Phi \cap b(x,h) \setminus x\}. \tag{31}$$

Hence, MHCP depends on the initial PPP spatial intensity [93]. The intensity of MHCP Φ_{MHCP} is given in [94] – [96] as

$$\lambda_{MHCP} = \frac{1-\exp(-\lambda\pi h^2)}{\pi h^2}. \tag{32}$$

In [56], [71], a modified MHCP was proposed for STs where dependent thinning was carried out twice to capture the required distance between active STs as well as the distance between active STs and active PTs. In such a case, the STs are said to be distributed following modified MHCP Φ_s^{mMH} of intensity λ_s^{mMH} given as

$$\lambda_s^{mMH} = \exp(-\lambda_p \pi D^2) \frac{1-\exp(-\lambda_s \pi d^2)}{\pi d^2}, \tag{33}$$

where d is the required minimum distance between two active STs (which can be referred to as the radius of each SU's protection region).

To ensure tractability, a common approach when modeling the repulsion between PUs and SUs under MHCP is the same as the one described for PHP where the powers of various PTs received at the location of any typical user (ST) are ranked accordingly. The received powers of various STs can also be ranked in a similar way if repulsion is required in the secondary network as in the case of modified MHCP. Based on the impact of the path-loss that is known to be more stable and dominant compared to the instantaneous multi-path channel fading effects, the effects of fading can be neglected. Therefore, the ranking of PTs and STs under MHCP and modified MHCP respectively using their distances is often considered as a reasonable approximation of their respective ranked received signal powers when characterizing interference in CRN.

Because of the non-availability of its PGFL, lower bound expressions for coverage probability following Jensen's inequality and MHCP are presented in [95], while the analysis for the nearest neighbor distance is presented in [82]. When MHCP is approximated as a PPP, the intensity of active PTs was given as $\lambda_p^a = \lambda_p \exp(-\lambda_p \pi D^2)$ in [97]. In such a case the LT of I_{pp} can be approximated as $\mathcal{L}^M_{I_{pp}}(s) \triangleq \mathcal{L}(\lambda_p^a, P_p, s)$. Similarly, the LT of I_{sp} can be approximated as $\mathcal{L}^M_{I_{sp}}(s) \triangleq \mathcal{L}(\lambda_s^I, P_s, s, D)$.

The LT of I_{ss} and I_{ps} can be obtained in a similar manner as provided in [8], [71]. Obtaining exact and closed-form expressions for various performance metrics of interest when adopting MHCP is very difficult. Hence, its adoption in the domain of CRN has been very limited. When adopted, MHCP has been mostly approximated using PPP in order to ensure tractability.

3.4. Poisson cluster process for secondary users

The realization of the point process in which users' locations follow the clustering pattern is known as PCP. Such a pattern is achievable in a typical CRN where the presence of SUs is restricted to a certain part of the network, for instance outside PUs' protection zones. In such a case where the distribution of active SUs exhibits clustering characteristics as obtained and presented in Fig. 7, adopting PPP becomes unsuitable and such distribution is better characterized using PCP [8]. PCP, however, is difficult to analyze and its adoption is also limited in CRN. Two common examples of PCP are the Thomas cluster process and the Matern cluster process. These two cluster processes were shown in [8] to be capable of closely approximating PHP for SUs' distribution in CRN owing to the non-availability of the PGFL for PHP.

PCP is formed by replacing each point $x \in \Phi$ of PPP Φ by a random finite point process Z_x known as cluster associated with x. The superposition of all clusters gives PCP $\Phi^{PCP} = \cup_{x \in \Phi}$ [8], [87], [98]. By approximating PHP using a PCP, the LT of I_{pp} is the same as in Lemma 1, while the LT of I_{ps} is the same as the approximate provided for the case of PPP. The LT of I_{sp} can also be accurately approximated by Lemma 2, since approximately the same interference is received at a tagged PR from active STs regardless of whether

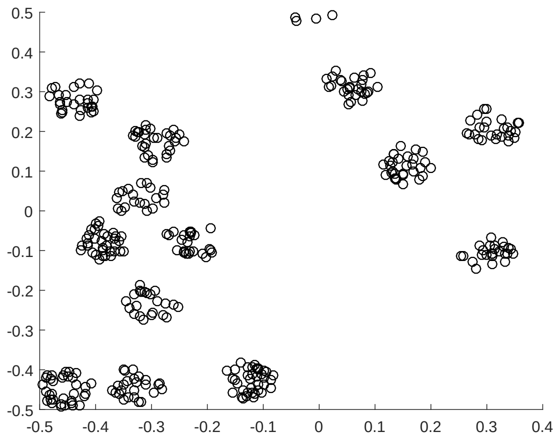

Figure 7: Realization of PCP: Parent intensity = 8, mean cluster size = 25, cluster radius = 0.05.

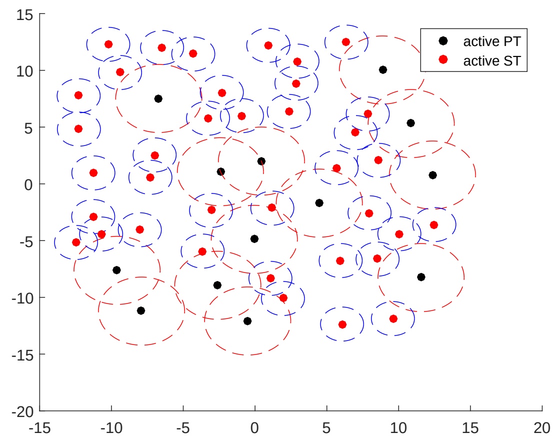

Figure 8: Intra-network and intra-network dependence $\lambda_p = 0.030$ $\lambda_s = 0.2$.

active STs' distribution is assumed to follow PPP or PHP [8]. In fact, the complementary cumulative density function of the interference among active SUs was shown to be the same for PHP and PCP in [8]. Interested readers are referred to [8] for more details.

3.5. Intra-network and inter-network dependence

Lastly, we consider the case where intra-network dependence and inter-network dependence among PUs and SUs are considered in the network modeling [71]. In such a case, active PTs' distribution, as well as active STs' distribution, can be modeled following either MHCP or PHP. Furthermore, the distribution of active STs can be modeled following a slightly modified version of PHP such that the location of any typical STs depends on the locations of active PTs and currently active STs. With that, both inter-network and intra-network interference can be properly captured. Such a distribution is presented in Fig. 8, as obtained through simulations.

3.6. Uplink transmissions in CRN

Besides the widely considered downlink transmissions where the power control is often ignored, another important but less considered transmission mode in wireless networks is the uplink transmissions. In such a transmission mode, the interference signal power received at any tagged receiver is not affected by its location. This implies that the interfering nodes may be closer to the tagged receiver than its paired tagged transmitter, hence interference control technique is generally adopted in the uplink transmission. Most interference models proposed for CRN often assume the presence of protection region around each receiver (especially PR), such that no other transmitting node is allowed to transmit in such an area. In such works, power control required for the uplink transmission of SUs is usually ignored. However, when the protection region is not used, power control is necessary for uplink transmissions of SUs. This power control is usually achieved through the adoption of the truncated channel inversion power control policy – a suboptimal transmission strategy.

When considering the uplink transmissions of SUs in CRN, each ST is required to ensure that the received signal power at its paired SR is equal to a certain cutoff threshold ρ_o by compensating for channel fading. As reported in [99], transmissions in the uplink mode depend on STs' maximum allowable transmission power $P_s^{\max}$, the transmission threshold ρ_o and the intensity of STs λ_s. The performance of the CRN is thus significantly impacted by the power ratio $\frac{P_s^{\max}}{\rho_o}$ [16]. If the required transmit power of any ST for the path-loss inversion is less than $P_s^{\max}$, such an ST is allowed to transmit, otherwise, such an ST does not transmit to reduce interference in the primary network, since PUs are generally subjected to interference by the uplink transmission of SUs whenever SUs are active [16].

In [1], the authors derived the characteristic function and cumulants of the CRN interference at a PU and modeled the interference generated by SUs under power control policy within a finite region, taking into consideration the shape of the region and the position of the PU. Each SU's activities depend on the strength of the received uplink signal power transmitted by the PU. Hence, SUs are required to sense PUs' uplink channel in order to detect the presence of an active PU. The symmetric truncated-stable model for network interference was developed using cumulant expressions. Similarly, the authors in [16] obtained analyses for coverage probability and spectral efficiency in a single-tier uplink mode CRN where the truncated channel inversion power control for ST was employed with a cutoff threshold ρ_o at the SR owing to the limited transmission power constraints of STs. [100] also considered an uplink CRN where a SU base station is located at the center of a cell of radius R with multiple SUs considered to be uniformly distributed within the cell. Similarly, SUs transmit using a power control policy to reduce interference in the primary network.

In similar works, [99], [101] considered uplink cellular networks where user equipment employed a power control to ensure that the signal power received at the serving base station is equal to the cutoff threshold. The cell throughput of the SU in an uplink cognitive radio cellular network was analyzed in [17] subject to a minimum rate constraint of the primary network. According to [16], the SU coverage probability in the uplink mode CRN can be expressed as:

$$P_{cov} = P\{\frac{\rho_o h_o}{\sigma^2 + I_{ss} + I_{ps}} \geq \theta_s\}, \quad (34)$$

where $\rho_o h_o$ is the desired signal power that has been compensated by truncated inversion power control.

3.7. Summary

This section presents an overview of common users' distribution models for the characterization of interference in CRN. The presented network distributions and performance analyses techniques in CRN are given as Table 3. Although the adoption of PPP has received most attention to date, a cautious perception of users' distributions in practical systems reveals that PPP may be insufficient to characterize the distribution of primary and secondary users in CRN accurately. This section also provides a fundamental analysis of common performance metrics of interest, which rely heavily on the assumed distribution models.

Obtaining exact but tractable expressions for inter-network and intra-network interference among PUs and SUs in the network remains a subject of future research. Exact expressions have been obtained for some metrics when using PPP, but such expressions were obtained following some simplifications or assumptions, for instance, independent users' distribution assumption. Other point processes such as PHP and MHCP seem to model the distribution of users better; however, there is no known PGFL for such point processes, though the results presented in some of these works show that network parameters and performance can be obtained with less complexity if the approximation were carefully made. It is worth noting that there are other point processes, such as the Ginibre point process [102] and the Gibbs process, though their usability has not been well demonstrated in CRN. This section is fundamental to all analyses presented in the remaining part of this paper.

4. Spatiotemporal approach to cognitive radio network modeling

An implicit assumption in the preliminary work (as shown in Section III) is that the buffers of all transmitting nodes are always full. However, in practical systems, packets are known to arrive independently at each transmitting source in any typical time with a certain probability [103], hence the full buffer assumption fails to capture the relationship between the spatial location of users and temporal traffic dynamic [104]. In order to capture the spatiotemporal relationship between primary and secondary networks, there is a need to characterize the dynamics of both the spatial and temporal domains properly. It is therefore unsurprising that recent research has been exploring the adoption of both queueing theory and SG [72], though the spatiotemporal analysis of CRN has not received a lot of attention. In CRN, PUs are generally considered to have access to transmit on their assigned channels at any time and do not contend to use such assigned channels. When continuous and discrete-time queueing systems are adopted, PUs are allowed to access their assigned channels at any time if continuous time is assumed and at the start of any time slot if time is regarded as discrete, forcing SUs to vacate channels upon their arrival. As a result of this, PUs are always considered to exhibit pre-emptive priority over SUs.

Table 3: Network distributions and performance analyses techniques in CRN

Reference	PU distribution	SU distribution	Metrics	Transmission mode	CRN model
[1]	–	PPP	Bit error rate	Uplink	Overlay
[4]	PPP	PPP	Outage probability	Downlink	Underlay
[5]	PPP	Uniform	Outage probability	Downlink	Underlay
[6]	–	PPP	Outage probability, Amount of fading	Downlink	Underlay
[7]	PPP	PPP	COP, Throughput	Downlink	Underlay
[8]	PPP	PPP, PHP, PCP	Outage probability	Downlink	Underlay
[9]	PPP	PPP	Opportunistic prob., SE, Trans. prob.	Downlink	Underlay
[10]	–	PPP	Outage probability	Downlink	Underlay
[11]	PPP	PPP	Outage probability, Trans. prob.	Downlink	Underlay
[12], [13]	PPP	PPP	SOP, COP, Spatial throughput	Downlink	Overlay
[16]	PPP	PPP	COP, SE	Uplink	Underlay
[25], [53]	PPP	PPP	Medium acess prob., COP, Throughput	Downlink	Underlay
[55]	–	PPP	Outage probability, Spatial throughput	Uplink	Overlay
[56]	PPP	Modified MHCP	Throughput	Downlink	Underlay
[65]	PPP	PPP	SOP, COP	Downlink	Overlay
[66]	PPP	PPP	COP, Medium acess prob.	Downlink	Underlay
[67]	PPP	PPP	Active probability, COP, Throughput	Downlink	Underlay
[69]	PPP	PPP	Transmission capacity	Downlink	Overlay
[70]	PPP	PPP	Transmission capacity	Uplink	Overlay
[71]	PPP	PPP, MHCP	Transmission capacity, Outage probability	Downlink	Underlay
[76]	PPP	PHP, MCP	Outage probability	Downlink, Uplink	Underlay
[77]	MHCP	PHP	Outage probability, Throughput	Downlink	Underlay
[100]	PPP	PPP	Network capacity	Uplink	Underlay

COP = Coverage probability

SOP = Spatial opportunity

SE = Spectral efficiency

There are mainly two types of queues in such networks – a primary queue and a secondary queue [72]. All PTs are located in the primary queue, while all STs are located in the secondary queue. Since PUs possess pre-emptive priority, a typical active ST with non-empty buffer located on channel k will remain in the active state provided that the primary queue within such region is empty. Upon the arrival of any PT, the activity of the active ST is immediately interrupted to avoid interference and delay to the arriving PT. The interrupted ST can only resume its transmission (as in a pre-emptive resume system [105] –[108]) or repeat the entire transmission (in a pre-emptive repeat system [107], [108]) when the primary queue is empty. Hence a typical ST's service rate not only depends on the arrival rate at the secondary queue and the service rate of STs with earlier arrival time, but also on the PT's arrival rate and service completion rates. The main difficulty with queueing dynamics in such a network is, however, the interdependency of the interacting queue [109].

In the primary queue, each PT secures access to transmit following a first-come-first-served (FCFS) approach, while each ST in the secondary queue secures access to transmit when the primary queue is empty, following the FCFS approach. Accurate modeling of each queue is therefore necessary to understand the spatiotemporal relationships between users.

4.1. Analysis of primary queue

In any CRN, the spectral resources belonging to PUs are being used opportunistically by SUs. The queue status at each typical transmitter thus depends on other transmitters, resulting in an interacting queue; hence the active state of each transmitter is both spatial- and temporal-dependent. The SINR received at any typical receiver is governed by many stochastic processes, such as random spatial distribution of active users, random packet arrival and channel fading [109]. Hence, the SINR received at any typical receiver is a random variable that can only be modeled through distribution. The conditional coverage probability at any active PR y_k^p in time slot t can be expressed as [72]

$$\mu^{\Phi}_{y_k^p,t} = P(SINR_{y_k^p,t} \geq \theta_p | \Phi, \forall \Phi = [\Phi_p \cup \Phi_s]), \tag{35}$$

where $SINR_{y_k^p,t}$ is given as

$$SINR_{y_k^p,t} = \frac{P_p h_{X_o} \|X_o\|^{-\alpha}}{\sigma^2 + \sum_{x_i^p \in \Phi_p \setminus x_k^p} P_p a_{p,t} h_{X_i} \|X_i\|^{-\alpha} + \sum_{x_i^s \in \Phi_s} P_s a_{s,t} h_{x_i} \|x_i\|^{-\alpha}}.$$

The parameters $a_{p,t} \in \{0,1\}$ and $a_{s,t} \in \{0,1\}$ represent indicators showing whether a typical PT $x_i^p \in \Phi_p$ and a typical ST $x_i^s \in \Phi_s$ are transmitting at time t and are both spatial- and temporal-dependent. This SINR distribution depends on the PT packet arrival rate ξ_p and θ_p. It is obvious from the analysis in (35) that the coverage probability of a tagged PT x_k^p depends on the number of active transmitters at time t, hence it is important to derive the expression for the active probability of any typical PT conditioned on Φ. This is given as [109]

$$a^{\Phi}_{p,t} = \begin{cases} 1 & \text{if } \mu^{\Phi}_{y_k^p,t} \leq \xi_p \\ \frac{\xi_p}{\mu^{\Phi}_{y_k^p,t}} & \text{if } \mu^{\Phi}_{y_k^p,t} > \xi_p. \end{cases} \tag{36}$$

It appears from (36) that the active probability $a_{p,t}^{\Phi}$ depends on the PTs' service completion rate $\mu_{y_k^p,t}^{\Phi}$ – a parameter that also determines the average service rate at the primary network.

The PU service time μ_p comprises overall time spent waiting in the queue and the total time used for actual transmission. The traffic intensity at the primary queue can be obtained as $\tau_p = \frac{\xi_p}{\mu_p}$. Since access to any arbitrary channel k by the pre-emptive PTs is governed by FCFS, the PT with the longest waiting time w_t in each time slot is granted access to transmit, provided that no two PTs within the region considered have the same $w_t, \forall t = 1, 2, ...$, hence $w_1 > w_2 > w_3, ..., w_t$. Bearing this in mind, all PTs waiting to transmit can be said to be in a long queue arranged based on w_t. Considering a discrete-time system, the average service rate (expressed in terms of transmission success probability) at the primary network conditioned on Φ is given as [110]

$$\mu_p = pP(SINR_{y_k^p} \geq \theta_p|\Phi), \quad (37)$$

where p signifies the probability of a typical PT choosing to transmit at the time slot considered. The arrival rate can be modeled as a Poisson distribution [111] –[114] and Bernoulli process [72],[103], [109], [110], [115] –[117], depending on whether the time is considered continuous or discrete. Similarly, the distribution of the service time can be modeled as geometric, general, phase-type, Poisson, etc. distribution. More information on applications of queueing theory in CRN is provided in [118] –[120].

4.2. Analysis of secondary queue

The conditional coverage probability at any tagged SR y_k^s in any time slot, t, can be obtained from the SINR at the tagged SR given as [72]

$$SINR_{y_k^s,t} = \frac{P_s h_{x_o}\|x_o\|^{-\alpha}}{\sigma^2 + \sum_{x_i^s \in \Phi_s \setminus x_k^s} P_s a_{s,t} h_{x_i}\|x_i\|^{-\alpha} + \sum_{x_i^p \in \Phi_s} P_p a_{p,t} h_{X_i}\|X_i\|^{-\alpha}}.$$

Hence,

$$\mu_{y_k^s,t}^{\Phi} = P(SINR_{y_k^s,t} \geq \theta_s|\Phi, \forall\Phi = [\Phi_p \cup \Phi_s]). \quad (38)$$

We also know that the average service rate at the secondary network conditioned on Φ is obtained as $\mu_s = p_2 P(SINR_{y_k^s} \geq \theta_s|\Phi)$, where p_2 signifies the probability of a typical ST choosing to transmit in the time slot considered. The active probability of any typical ST conditioned on Φ depends on the traffic intensity at the primary queue, as well as the traffic intensity at the secondary queue $\tau_s = \frac{\xi_s}{\mu_s}$. Hence, the analysis of the secondary queue is known to be more difficult [72]. The active probability of ST is generally obtained through a solution to the steady-state distribution of the queue by solving (39),

$$x = xP, \; x1 = 1, \quad (39)$$

where x is the steady-state probability vector and P is the transition matrix for such a system. Similar to the case of the primary network, the arrival rate at the secondary queue can be modeled as a Poisson distribution if a continuous-time queueing system is considered and a Bernoulli process if a discrete-time queueing system is considered. The service time can be modeled as geometric, general, phase-type and Poisson distributions as well.

4.3. Non-saturation queueing behavior

In CRN, the saturation assumption is not sufficient, since the transmission rate is variable with idle periods. Hence the need to model the non-saturation queueing behavior. The preliminary way of capturing this behavior in SG-based interference modeling in CRN is by estimating the probability that a typical user is not transmitting - which may also be interpreted as the probability that a user has no data to send. With this, authors were able to put some restrictions to ensure that the non-saturation queueing behavior of traffic was indirectly captured so as to ensure that the network interference is not overestimated in the network. For instance, in [1], SUs join or exit the network with a certain probability, which is determined by the received uplink signal transmitted by the PUs, while PUs transmission probability was achieved in [66] through the Aloha principle, where each PU chooses a frequency band for transmission based on a certain probability and in [7] using Markov chain. Although all SUs were initially assumed to exhibit saturated behavior in [7], [66], [67], SUs transmission probability was later estimated based on the locations of SUs which signifies the non-saturated behavior required in CRN. Similarly, the non-saturated queueing behavior of SU traffic arrival was captured using the Aloha principle in [9], [25].

Similarly, the concept of the protection region has been used to model the non-saturated queueing behavior of traffic arrival, as presented in [8], [71], where users transmission probability was captured based on their location. However, when the traffic arrival of users is captured as independent of the service completion rate, the temporal correlation of interference is neglected (or at least not properly captured) and the information-centric interactions among network nodes are not captured. Accurately capturing temporal correlation of interference is, however, important since the service completion rate of users depends on the level of interference and pathloss in its location [109], which implies that the temporal correlation of interference affects system design and performance. Hence these methods are unsuitable to capture the non-saturated queueing behavior of traffic arrival expected in CRN.

Subsequent to this, several efforts are now focusing on understanding the non-saturated queueing behavior. This interplay between the temporal traffic dynamic and the spatial location was captured in [72] for CRN, [109], [115], [117], for small cell networks, [110], for the cellular network and [114] for IoT network. Capturing the non-saturated behavior of the traffic and the temporal correlation can complicate the network analysis, since memory is involved in the queues, although various reasonable assumptions can be made to reduce such complications. Existing contributions that considered the spatiotemporal analysis of wireless networks are compared in Table 4.

4.4. Summary

The need to capture the interplay between the spatial location of network transmitters and their temporal traffic dynamic properly has paved the way for the adoption of queueing theory in SG-based interference modeling in wireless networks. Although capturing of the spatial and temporal param-

Table 4: Existing works on spatiotemporal modeling of wireless networks

Reference	Arrival process	Priority	Buffer	Network
[72]	Bernoulli	✓	Infinite	CRN
[103]	Bernoulli		Infinite	Poisson networks
[104]	Bernoulli	✓	Finite	IoT Networks
[109]	Bernoulli		Infinite	Small cell networks
[110]	Bernoulli		–	Cellular networks
[111]	Poisson		–	Cellular networks
[114]	Poisson		Infinite	IoT Networks
[115]	Bernoulli		Infinite	Poisson networks
[116], [117]	Bernoulli		Infinite	Small cell networks

eters leads to more complicated network analysis, such parameters are essential to the next generation systems, hence many preliminary works that assumed a full buffer for all transmitting nodes failed to capture the crucial effects of queueing delay on network users. In this section, we have presented the spatiotemporal approach to interference modeling in CRN, capturing the spatial and temporal relationship between primary and secondary queues. while stating how the queueing rules are incorporated into the system modeling. Service at the secondary queue (i.e. lower priority queue) is interrupted if a packet arrives at the primary queue (i.e. high priority queue) and will only resume transmission at the secondary queue when all PTs have been served and the primary queue is empty.

The two different techniques at the secondary queue are the pre-emptive resume and the pre-emptive repeat cases. In pre-emptive resume queueing systems, any interrupted ST resumes its transmission from the point of interruption, sending the next packet immediately the primary queue becomes empty, while a typical ST in pre-emptive repeat queueing systems repeats the whole transmission by resending all packets sent earlier each time it is interrupted by any PT. The common technique to determine which user (among many users with the same priority) is to access the channel is the FCFS technique. When both spatial and temporal dynamics are captured in system modeling, each user's quality of service can be determined through delay analysis.

5. Interference modeling in a cognitive radio network with cooperation and handover

Although the system performance and throughput can be significantly improved by capturing the spatial location of users and temporal traffic dynamic as described in Section IV, outage and handoff, two important parameters that can depend on the interference generated in large scale CRN, can significantly influence the performance of the network, especially under indirect transmissions (i.e. when a transmitter sends packets to its intended receiver via relays because its intended receiver is located outside its coverage region) that was not considered in Section IV. In order to improve the system performance in CRN further, several authors have taken advantage of user and channel diversities to propose cooperation of PUs and SUs to reduce aggregate interference, especially in the primary network.

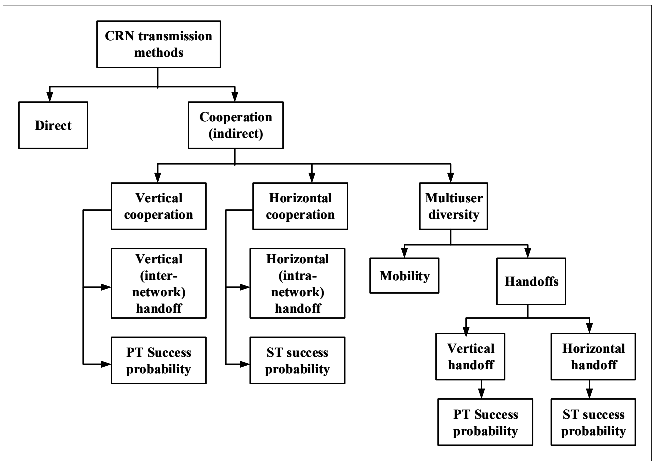

Figure 9: The taxonomy of cooperation mechanisms when modeling interference in CRN.

Cooperative transmission is achieved in CRN when any waiting ST or group of waiting STs serves as a relay node(s) between any primary transmitter-receiver pair - the case known as vertical cooperation - or between any secondary transmitter-receiver pair - the case known as horizontal cooperation - with the purpose of reducing the transmission period while improving coverage in the network [77]. Cooperation among CRN users can improve network throughput as well as PUs' power efficiency while providing spectrum opportunity for SUs [121]. By accepting to serve as a relay node, the ST can reduce its queueing delay. It is therefore unsurprising that cooperation among cognitive users has received a significant amount of attention in the literature [73], [77], [121] –[126]. The taxonomy of cooperation mechanisms when modeling interference in CRN is given in Fig. 9.

5.1. Analysis of cognitive networks with cooperation

Owing to the ability of transmitters in CRN to use cooperative transmission to ensure coverage at the intended receiver, it becomes imperative to model the interference experienced at the intended receiver in a cooperative environment. Modeling the interference experienced at any location in the network is necessary to determine the probability that the packet sent by a tagged PT is successfully received at its intended receiver (i.e. the tagged PR during direct transmissions or the selected relay nodes during indirect transmissions) and eventually at the tagged PR if sent through relay node(s). Similarly, for a secondary network, interference modeling is useful in understanding the probability that the packet sent by any tagged ST is successfully received at the intended receiver (i.e. the tagged SR during direct transmissions or selected relay node during indirect transmissions) and the probability that the packet sent by the selected relay node is successfully received at the tagged SR.

To avoid network congestion, the transmission of packets between a typical transmitter-receiver pair is normally depicted as a two-hop scheme [126] –[128] where a packet moves from the transmitter to any selected relay node(s) during the first hop and from the selected relay node(s) to the receiver during the second hop. Although a multi-hop relaying scheme for underlay CRN was considered in [129] while the mathematical analysis for transmission delay of multi-hop communications was in presented [130], these works did not consider network interference. The success probability in any two-hop scheme is defined as the probability of successful transmission between the transmitter-receiver pair over the two hops. Such a metric is obtained through the joint complementary cumulative distribution function of the received SINR at the selected relay and at the intended receiver [126], [131]. In the primary network, the success probability given as [77]

$$P^{p}_{suc} = P^{(PT-R)}_{suc} P^{(R-PR)}_{suc}, \tag{40}$$

where $P^{(PT-R)}_{suc} = P\{SINR_{(PT-R)} \geq \theta_R\}$ is the probability that the signal received at any selected relay is not less than the pre-defined threshold θ_R and $P^{(R-PR)}_{suc} = P\{SINR_{(R-PR)} \geq \theta_p\}$ gives the probability that the signal received at the tagged receiver is not less than the pre-defined threshold θ_p. In the secondary network, success probability can similarly be expressed as [77]

$$P^{s}_{suc} = P^{(ST-R)}_{suc} P^{(R-SR)}_{suc}, \tag{41}$$

where $P^{(ST-R)}_{suc} = P\{SINR_{(ST-R)} \geq \theta_R\}$ and $P^{(R-SR)}_{suc} = P\{SINR_{(R-SR)} \geq \theta_s\}$.

Interference in CRN with vertical and horizontal cooperation can include interference from primary networks to secondary and relay networks, interference from secondary networks to the primary network and relaying networks, as well as interference from relaying networks to primary and secondary networks [131]. Hence,

$$SINR_{(PT-R)} = \frac{P_p h_{X_0} \|X_0\|^{-\alpha}}{\sigma^2 + I_{pr} + I_{sr} + I_{rr}}, \tag{42}$$

$$SINR_{(R-PR)} = \frac{P_R h_{x_0} \|x_0\|^{-\alpha}}{\sigma^2 + I_{pp} + I_{sp} + I_{rp}}, \tag{43}$$

where I_{pr} is the interference from other active PTs at the tagged relay, I_{sr} is the interference from active STs at the selected relay, I_{rr} is the interference from currently transmitting relays at the tagged receiving relay and I_{rp} is the interference from all transmitting relays at the tagged PR. $P_R \approx P_s$ is the relay transmit signal power. Similarly,

$$SINR_{(ST-R)} = \frac{P_s h_{x_0} \|x_0\|^{-\alpha}}{\sigma^2 + I_{pr} + I_{sr} + I_{rr}}, \tag{44}$$

$$SINR_{(R-SR)} = \frac{P_R h_{x_0} \|x_0\|^{-\alpha}}{\sigma^2 + I_{ss} + I_{ps} + I_{rs}}, \tag{45}$$

where I_{rs} is the interference at the tagged SR from currently transmitting relays. The relaying technique can be based on various cooperative diversity protocols, such as fixed relaying - amplify and forward and decode and forward, selection relaying and incremental relaying [132]. The expressions for the success probability in both primary and secondary networks can be obtained by following any of the methods discussed in Section 3. In order to obtain a more simplified analysis, a single PU and multiple SUs were considered in [133] where primary transmitter-receiver pairs communicate through a line-of-sight link, while transmissions between secondary transmitter-receiver pairs are achieved via selected relays as a result of the path loss and shadowing, which makes direct transmission through the line-of-sight links impossible in secondary networks. In such a network, relays are not used in the primary network.

5.2. Multiuser diversity and handover

Multiuser diversity through packet relaying was considered in [134] where multiple nodes communicate among random source-destination pairs. In such a network, some of the nodes can be mobile and communication can be sustained between any transmitter-receiver pair using cooperative mechanisms when such pairs are separated by a large distance, for instance when a mobile receiver is located outside the coverage of its paired transmitter owing to mobility. The relaying node can be mobile as well (for instance, an unmanned aerial vehicle relay node [135] –[137]) and can ensure coverage between two fixed users. When mobility is considered as expected in cognitive networks, selecting a proper handoff mechanism is important in minimizing switching latency [138], though obtaining analysis for coverage probability when considering mobility is known to be very difficult and remains a subject for future research. When moving from one location/cell to another, the handoff process requires an effortless transfer of a connected pair with guaranteed quality of service [139]. As any mobile receiver moves away from the coverage region of its tagged transmitter, there is a need to perform either inter-network handoff in the case of vertical cooperation or intra-network handoff in the case of horizontal cooperation, provided that the currently paired transmitter no longer provides the highest signal at the mobile receiver. The effects of handover on network performance become important to study.

The probability that a typical transmitter does not remain the best transmitter for a mobile receiver is closely related to the handoff rate, especially for low velocities, given as [139]

$$H_{Rate} = E[\text{Number of handoffs per unit time}]$$

$$= \sum_{k=1}^{\infty} kP(\text{Number of handoffs per unit time} = k).$$

The handover rate is useful in quantifying handover delay per unit time $H_{D/T}$ [140] and can be obtained using $H_{Rate} = \frac{H_{D/T}}{H_{delay}}$, where H_{delay} represents handover delay. In a single-tier network where PPP is assumed, the handover rate is obtained following $H_{Rate} = \frac{4v}{\pi}\sqrt{\lambda}$ [141], where v represents the velocity and λ is the intensity of the transmitter available for the handover process.

5.3. Summary

With cooperation, coverage in the cognitive network can be significantly improved, while low-mobility users can sustain their communications with their respective pairs using vertical and horizontal cooperative measures. One important mechanism that can influence such a network is the handover mechanism - the process of switching the association to the transmitter that produces the best connection at the typical receiver. The effect of handoffs that result during this switching, however, can be significant in any network, while analysis that captures mobile nodes (when modeling interference in wireless networks) is known to be very difficult. The authors in [142] though provide an analysis of SIR in a moving network, which can be useful in the domain of CRN.

In this section, various interference modeling approaches in CRN with cooperation and handover were reviewed. The summary of the current related efforts when capturing various cooperation and handover issues in wireless networks is provided in Table 5. In order to sustain the transmission between any primary transmitter-receiver pair, vertical cooperation can be adopted. In vertical cooperation, any waiting secondary nodes within the coverage of an active PT can serve as a relaying node between such a primary transmitter-receiver pair. Similarly, the coverage between any secondary transmitter-receiver pair can be sustained through horizontal cooperation. Using these cooperative mechanisms, SUs' waiting time can be reduced, while channel usage efficiency can be significantly improved. To obtain an analysis of the overall success probability in such networks, we discussed that the success probabilities at each selected relaying node during the first hop and at the intending receiver during the second hop are required when the transmission between any tagged transmitter-receiver pair is an indirect transmission. When transmission between the tagged transmitter-receiver pair is direct, the success probability is only measured at the tagged receiver, as in the traditional setup. From the analysis of success probability (which is the same as coverage probability), the analyses for outage probability and other important metrics are straightforward.

6. Non-orthogonal multiple access-based cognitive radio network

A large volume of interference characterization models discussed in the Section III – V of this survey paper focused on conventional orthogonal multiple access where power control at the STs is often neglected; hence, most of the literature assumed equal transmit power for all STs. The introduction of the NOMA technique in CRN ensures more efficient channel usage by adopting the power control process at secondary networks. Hence, this section reviewed the application of the NOMA technique in the domain of CRN.

By continually making efforts to control SUs' interference in the primary network through various techniques discussed in Section 3 – 5, one may also observe that the channel access of SUs can be significantly affected. NOMA techniques seek to enhance fairness for the SU and compensate for the PU by supporting overloading transmissions while improving spectrum efficiency.

6.1. NOMA in cognitive networks

NOMA is a new technology proposed to ensure that multiple users are accommodated within the same spectrums, thus improving spectral usage efficiency. It has been recognized as an important tool for the next generation of wireless networks [143] –[152], considering the continuous increase in mobile data traffic volume. Meeting the system capacity required to accommodate the explosive growth being experienced in communication systems is one of the most important issues, owing to limited spectrum resources [153]. With such technology in CRN, STs can be allowed to transmit within the coverage of PTs by controlling their transmission powers through various power control mechanisms so as to avoid interference with the PRs, while SRs within the coverage of active PTs may adopt the use of successive interference cancelation (SIC) techniques [143], [154] to reduce interference from PTs.

NOMA technology is known to be capable of providing more connectivity opportunities for multiple users [144] in the downlink transmissions than the conventional orthogonal frequency division multiple access [145], [146] and its adoption has recently been considered in CRN. In NOMA-based CRN [5], an accurate characterization of interference is important for proper power control at each ST. Hence, STs do not transmit with constant or fixed transmit power as widely assumed, but must control the interference resulting from their transmission at the PRs. Also, there is an existence of underlay cognitive transmission without traversing the primary network and a higher priority is given to PU's QoS. This higher priority attributed to PU data transmission owing to the requirement in CRN means that the power allocation factor of the PU signal is greater than that of the SU signal [155].

In [4], [146], the analysis of outage probability derived using the SG technique was obtained in a NOMA-based underlay CRN, where STs (with lower priority) are allowed to transmit within the PUs' assigned channels as long as the interference resulting from such transmissions does not exceed the pre-defined interference threshold in the primary network, while NOMA application in downlink [147] and uplink [148] wireless networks has been considered. The application of the NOMA technique in mobile uplink and downlink networks was also considered in [149].

Consider a typical underlay CRN where the distribution of PTs follows HPPP Φ_p of intensity λ_p. Each PR is considered to be located within the coverage of its pair PT and the distribution of PRs can also be said to follow PPP Ψ_p of intensity η_p. Assume that each active ST is situated at the centre of a disk of radius d with its pair SR uniformly distributed within the disk. Owing to the NOMA technique, the ST transmission power P_{NO} is subject to power control in order to limit interference at the PRs. This, under Rayleigh fading assumption, can be given as [146]

$$P_{NO} = \min\Big(\frac{I_p}{\max_{y_i^p \in \Psi_p} |g_l|^2}, P_s\Big), \tag{46}$$

where I_p is the maximum allowable interference power at the PRs and $|g_l|^2$ is the overall channel gain from the ST to PRs. The message received at the SR y_i^p from the ST x_i^p can be obtained as [146]

$$m_{y_i^p} = h_{y_i^p} \sum_{n=1}^{M} \sqrt{a_n P_{NO}} x_n + \sigma_{y_i^p}, \tag{47}$$

where $h_{y_i^p}$ is the channel coefficient between y_i^p and x_i^p, M is the number of SRs within the x_i^p coverage of radius d (note that $M = 1$ in a single transmitter-receiver pair connection), a_n is defined as the nth SR power allocation coefficient with $\sum_{n=1}^{M} a_n = 1$, x_n is the actual message for $y_i^p \in \Psi_p$ and $\sigma_{y_i^p}$ is known as the additive white Gaussian noise at the y_i^p.

[5]NOMA-based CRN [156] is any CRN where the NOMA technique is applied to improve spectrum efficiency.

Table 5: The summary of the current related efforts when capturing various cooperation and handover

Reference	Cooperation	Handoffs	Multiuser diversity	Mobility	Network
[73]	✓	X	✓	X	CRN
[77]	✓	X	✓	✓	CRN
[121]	✓	X	X	X	CRN
[122]	✓	X	X	X	CRN
[124]	✓	X	X	X	CRN
[126]	✓	X	X	X	Cellular
[127]	✓	X	X	X	Wireless
[128]	✓	X	X	X	CRN
[132]	✓	X	✓	X	Wireless
[133]	✓	X	X	X	CRN
[134]	✓	X	✓	✓	Ad hoc
[135]	✓	X	✓	X	CRN
[138]	X	✓	X	✓	CRN
[140]	X	✓	✓	✓	Cellular
[141]	X	✓	✓	✓	Wireless
[142]	X	X	✓	✓	Cellular

The power allocation technique in a NOMA-based CRN with multiple PUs and SUs was presented in [156]. The performance of SUs was measured by the SINR in the network. The interference constraints [156] can be represented as

$$\sum_{x_i^s \in \Phi_s} P_{x_i^s} |g_l|^2 \leq I_p, \tag{48}$$

where $P_{x_i^s}$ is the transmit power at x_i^s. As presented in [156], (48) can be simplified as $P^s|g_l|^2 \leq I_p$, with $P^s = \sum_{x_i^s \in \Phi} P_{x_i^s}$ representing STs' overall transmit power. Another interesting area is cooperative NOMA where both the cooperation and NOMA techniques are combined to improve the system performance. A cooperative relay selection criterion was proposed in [155], [158] where the multicast user group acts as a secondary network underlaying a PU. SUs with the best channel gain forward messages belonging to PUs and SUs, though inter-network interference was not considered. The analyses of various performance metrics in NOMA-CRN are similar to the ones presented in the previous sections, though integration of SIC may further complicate these analyses.

6.2. Clustered CRN with NOMA

In clustered CRN with NOMA, SUs are modeled using the clustering process with the aim of improving the spectrum efficiency. A clustered CRN with NOMA is suitable in large-scale CRN, where massive connectivity and higher throughput are required and can be classified under two cases: the first case is usually observed in a large-scale CRN where multiple SUs require different access requirements in order to satisfy their QoS constraints. In such a network, some SUs with higher connectivity requirements (for instance, require fewer interruptions owing to PUs activities) are usually allowed to transmit within the part of the spectrum with less presence of PUs. SUs with lower connectivity requirements are allowed to transmit in areas with more presence of PUs in the underlay mode. The QoS constraints and priorities of PUs, as well as the connectivity demands of SUs force SUs to concentrate in some areas within the spectrum band. A second possible case where a clustered CRN with NOMA can be useful is in a network, where a secondary network is made up of single transmitter-multiple receivers' pair connections with each ST (or SU coordinator in some cases) locating at the center of each cluster as in [76].

The clustering is formed as follows: The parent points are firstly distributed following HPPP Φ of intensity λ and uniformly distributed within the considered area B such that the number of parent points $N_p = |\Phi|$ satisfies $P[N_p = n] = \frac{(\lambda B)^n}{n!} \exp(-\lambda B)$. The offspring points are then distributed independently and identically around each parent point following symmetric normal distributions with mean zero and form a cluster. In the first case, the center of each location within the channel where the presence of SUs is allowed is modeled as the parent point, while SUs are modeled as offspring points. In the second case, STs (or SU coordinators) are modeled as parent points and SRs connected to each ST are modeled as offspring points. The distance between each offspring point and its parent point can then be considered to follow two-dimensional Gaussian distribution. In such a network, inter-cluster interference exists though intra-cluster interference can be neglected.

Clustered CRN with NOMA still remains an open area of research although similar techniques have been used in mmWave networks [162] where interference and distance distributions were characterized among NOMA users. Similarly, clustered cooperative NOMA in the cellular network was considered in [163] where each cluster is connected to the core network. SUs can be identified and clustered based on specific tasks, priority, spectrum demands, etc. The introduction of clustering in CRN means the spatial distribution of SUs will better be captured as PCP, which was discussed in Section 3. More details on PCP are provided in [8].

6.3. Summary

By serving multiple users at different power levels using the power domain, spectral resources can be used in a more efficient manner. In NOMA-based CRN, multiple SUs can be granted access to the spectral resources belonging to the PUs within the same time, frequency and code block. At each ST, the power control scheme is adopted, while SIC is employed at each SR to reduce interference from the nearby PTs and STs. In this section, we have presented the application of NOMA in CRNs' interference management. The most significant deviation of the performance metrics analyses, in this case from conventional orthogonal medium access, is the power control mechanism required at each ST. With SIC at each SR, SRs with better channel conditions can filter out interference experienced at their respective locations.

The capability of NOMA techniques to achieve better performance in both downlink and uplink networks has been well reported in the literature, though its adoption is capable of further complicating the analyses of various performance metrics in SG-based interference modeling. Furthermore, its adoption has also been demonstrated in cooperative communication networks. The contributions of the related papers on the application of NOMA techniques when modeling interference in CRN is summarized in Table 6.

7. Open problems, future direction and discussion

In this section, we present the open problems in SG-based interference modeling in CRN and suggest possible future directions. We conclude this section by providing a brief discussion of the analyses and descriptions presented in this paper.

7.1. Open problems and future direction

Despite the importance of CRN to achieve successful implementation of next-generation networks, relatively limited research has been carried out on interference characterization among cognitive users, especially when important network parameters such as independence among users, mobility, queueing dynamics, etc. are captured in network modeling. Other interesting contributions to interference modeling in wireless networks were made in the cellular communication networks (a few recent ones are [99],[157], [164] — [168], etc.), the THz network [169] and the three-Tier wireless sensor networks [170]. The uniqueness of CRN, however, means that complete adoption of cellular network models in CRN is essentially impossible, owing to the distinct differences between the networks. In CRN for instance, the SUs are usually permitted to transmit on the bands/channels belonging to the PUs as long as the interference resulting from such transmissions does not disrupt the activities of the PUs. This notion of priority, a distinct feature of CRN, however, does not exist in cellular networks, since all users are authorized to utilize the spectrum [40].

In order to ensure efficient and effective usage of the scarce spectrum resources in any wireless networks, all users, especially SUs, are expected to have a sense of cognition, such that essential users or services (relating to PUs) that require urgent or immediate access to the spectral resources can secure immediate access, while non-essential services or users (relating to SUs) can desist from disrupting the transmissions of such essential users. It is therefore not out of place to understand interference modeling in CRN properly, knowing that this will enhance its applications in other networks such as the IoT and device-to-device communication networks where many devices are expected to contend for channel usage. It is worth noting that cognitive cellular networks - a network that combines the uniqueness of cellular networks and CRNs - have been receiving significant attention (see [17], [171] –[174]).

In cognitive IoT-based networks, many systems, such as connected health, connected cars and military applications, may require urgent attention or access to spectral resources and must hence take priority over less essential applications, though the channel access requirements of the less essential applications must also be satisfied. The adoption of CRN mechanisms in IoT-based networks is therefore important to control the level of disruption caused by non-essential applications to prioritized applications. With such adoption, the spectral access delay of essential services will be reduced, while the quality of service of non-essential services is also satisfied.

While the characterization of users' distributions has been achieved using the point processes discussed, most of the efforts have adopted PPP owing to its tractability. There is still a need to find a trade-off between tractability and accuracy. In addition, and to the best of our knowledge, none of the existing publications on interference modeling in CRN except [72] considered the effect of interacting queues, while most of the spatiotemporal approaches proposed for heterogeneous cellular networks failed to capture the concept of priority - an essential part of CRN. More research is therefore needed to find more suitable point processes, while accurately capturing the cognitive network priority and other important system parameters such as mobility and power control in system modeling.

Security is another major area that requires further investigation. Malicious users can generate higher interference in the network with the purpose of disrupting the stability in the network [175]. By transmitting at transmit power higher than the specified transmit power for all secondary transmission or by violating the protection region policy, a malicious SU can generate excessive interference in both primary and secondary networks, degrading the channel usage efficiency in the process.

Cognitive sensing is also an important aspect of interference modeling in any wireless network. Cognitive users are expected to sense channels properly so as to limit interference with the activities of active PUs. However, cognitive users, which are generally low-energy devices or users, may use most of their energy during the sensing phase, especially in densely populated networks, with little energy remaining for the transmission phase. Hence, more energy-efficient sensing/transmission models or techniques are required to enable cognitive users to use less energy during the sensing phase in order to meet their energy requirement for transmission.

7.2. Discussion

The capability and suitability of SG in characterizing users' spatial location in increasingly practical terms are some of the reasons for its continuous adoption in modeling interference in wireless networks. This suitability in various wireless networks, such as cellular communication networks and CRNs, has been widely presented in many related kinds of literature through analyses of various performance metrics of interest.

Table 6: The summary of the current related efforts when capturing various cooperation and handover

Paper	Focus	Transmission mode	CRN model
[76]	Obtained the analysis for the aggregate interference received at a secondary receiver because of the PU and SU transmissions.	Downlink, Uplink	Underlay
[146]	Derive outage and diversity analysis for a NOMA-based large-scale underlay CRNs with randomly deployed users.	Downlink	Underlay
[155]	Proposed a cooperative multicast NOMA scheme where the multicast user group acts as a secondary network underlaying a primary user.	Downlink	Underlay
[156]	Presented a power allocation algorithm capable of exploiting the important characteristics of NOMA-based CRN.	Downlink	Underlay
[158]	Proposed a cooperative transmission scheme that is capable of exploiting the inherent spatial diversity provided through the application of NOMA in CRN.	Downlink	Underlay
[159]	Proposed a cooperative scheme for a NOMA-based CRN to enhance the outage performance for user fairness and spectrum efficiency.	Downlink	Underlay
[160]	Proposed a NOMA CRN, where ST transmits simultaneously to both a PR and an SR using spatial modulation technique.	Downlink	Overlay
[161]	Proposed a NOMA assisted overlay spectrum sharing framework for multiuser CRN for an improved spectrum utilization.	Downlink	Overlay

Although PPP has been shown to be the most analytically tractable among all types of point processes, its limitations, as discussed in the literature [8], [40], [71], continue to encourage consideration of other point processes with the possibility of capturing the distributions of users in more practical terms. Hard-core point processes, such as MHCP and PHP, though capturing more system parameters, are not tractable and require some approximations to derive expressions for performance metrics.

Nevertheless, the tool of SG when appropriately adopted in CRN can produce significant analytically tractable solutions to enable effective spectral resource allocation and reuse [15] with limited interference or disruption of the activities of PUs and other users. This has been proven in different CRN scenarios, for instance in underlay and overlay CRN with single PU and multiple SUs, as well as CRN with multiple PUs and SUs [40]. We believe that more research will enhance system modeling further. This survey reveals that most of the existing approaches in CRN fail to capture the relationship between the spatial location of users and temporal traffic dynamics and are restricted to only interference modeling among non-mobile users with full buffers, hence the need for further research in this area. Finally, this paper provides open problems and future directions to aid in finding more solutions to achieve efficient and effective usage of the scarce spectral resources for wireless communications.

CRN improves spectrum sharing among different network users and interference characterization in such networks has been receiving a lot of attention to improve spectral usage efficiency further. However, obtaining the exact analysis of aggregate interference at any typical node in a large network is very difficult owing to the non-availability of the probability distribution function for such a parameter, hence the need to characterize aggregate interference following the LT of its probability distribution function or equivalently characteristic function and marginal generating function [40]. In order to reduce the number of assumptions and simplifications required, it may be necessary to investigate other possible means of obtaining the expression for aggregate interference outside the use of LT, characteristic function and marginal generating function.

8. Conclusion

In this paper, we present a concise survey of CRN focusing on interference characterization and various point processes for users' distribution. Existing literature in CRN with a focus on interference management and control using SG has been appropriately examined and categorized with the aim of presenting a brief but comprehensive survey. The categorization was based on various point processes assumed for users' distribution in both primary and secondary networks, as well as distinct assumptions made to obtain tractable, though accurate, analyses. The existing approaches to COM, analysis of common performance metrics of interest, spatiotemporal analysis of a network model, cooperation and handover mechanisms and NOMA mechanisms were presented.

For each aspect of the network modeling considered, suggestions for future directions were presented, while the emphasis on the importance of a proper understanding of the interference modeling in CRN to the next generation networks, as well as its importance to the continuous demand for spectral resources, was discussed. We believe that this survey paper presents an up-to-date state of the art in CRN interference modeling using the tool of SG and is useful in encouraging more research in this direction. More research in this domain will aid further solutions to ensure efficient and effective usage of the scarce spectral resources for wireless communication.

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