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COGNITION, OPTIMISM, AND THE FORMATION OF AGE-DEPENDENT SURVIVAL BELIEFS*

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This article investigates the roles of psychological biases for deviations between subjective survival beliefs (SSBs) and objective survival probabilities. We model these deviations through age-dependent inverse S-shaped probability weighting functions. Our estimates suggest that implied measures for cognitive weakness increase and relative optimism decrease with age. Direct measures of cognitive weakness and optimism share these trends. Our regression analyses confirm that these factors play strong quantitative roles in the formation of SSBs. Our main finding is that cognitive weakness instead of optimism becomes with age an increasingly important contributor to the well-documented overestimation of survival chances in old age.

1. INTRODUCTION

Important economic problems, such as the decision about when to retire, how much to save for retirement, and whether to purchase life-insurance, depend on the formation of survival beliefs over an individual's life cycle. A rational individual would be modeled as a statistician whose survival beliefs are given as data-based estimates. For this benchmark, any differences between subjective survival beliefs (SSBs) and their objective counterparts can only result from an insufficient amount of data, and biases will decrease when the individual collects more data with age. Empirical studies, however, do not support this notion of convergence of perceived survival chances to objective survival probabilities (OSPs). Instead, the literature robustly documents a *flatness bias*, that is, respondents of age 50–70 express underestimation, whereas older respondents (older than age 75) express overestimation of survival chances on average by nonnegligible amounts.[1]

In this article, we provide a *structural interpretation* of these biases through transformations of objective probabilities known from experimental prospect theory (PT).[2] Accordingly, when plotting SSBs against OSPs, SSBs do not lie along the 45-degree line but rather exhibit an

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[1] Inspired by Hamermesh (1985), a growing body of economic literature documents such a *flatness bias*, cf., for example, Elder (2013), Ludwig and Zimper (2013), Peracchi and Perotti (2014), Heimer et al. (2019), Groneck et al. (2016), and Bissonnette et al. (2017).

[2] See Kahneman and Tversky (1979), Tversky and Kahneman (1992), and Wakker (2010).

inverse S-shape. We document that *psychological factors* such as optimism and cognitive weakness are important quantitative drivers of this transformation. Our findings suggest that age-increasing overestimation of OSPs is not due to increasing optimism as one may expect. It is rather a consequence of age-increasing insensitivity to objective likelihood leading to inverse S-shaped transformations of OSPs. When OSPs decrease with age, individuals therefore overestimate their OSPs more strongly.

As our first step, we compare SSBs to OSPs using data from the Health and Retirement Study (HRS). In the HRS, interviewees are asked about their beliefs to survive from the interview age to some target age. In order to construct objective counterparts, we estimate for each interviewee the corresponding individual-level OSP by using the information on actual HRS mortality and several conditioning variables. Plotting SSBs against OSPs over age, we document the *flatness bias* in the form of an average underestimation of survival chances by respondents of age 70 and younger, and an overestimation of survival chances by respondents of age 75 and older. *Within* a given age group, we find that respondents with low OSPs express overestimation, whereas respondents with high OSPs express underestimation.

For our structural interpretation of these biases we model SSBs as age-dependent inverse S-shaped Prelec (1998) probability weighting functions (PWFs). In line with the usual interpretation of the parameters of the Prelec function, cf. Wakker (2010), we assume that the motivational factor *relative optimism* is expressed through the *elevation* of the Prelec function and that the cognitive factor of *likelihood insensitivity* corresponds to its *flatness*. Likelihood insensitivity refers to a cognitive weakness according to which people cannot distinguish well among respective likelihoods of events. An extreme case of flattening-out are 50–50 probability judgments, which are well-documented in the psychological literature (Bruine de Bruin et al., 2000).[3] Estimating age-specific Prelec PWFs on our data of SSBs, we find that the *elevation* of the Prelec function weakly decreases with age, whereas its *flatness* increases. Thus, the implicit measure of relative optimism weakly decreases and the implicit measure of likelihood insensitivity increases with age.

We next analyze directly observable counterparts of these implicit cognitive and motivational factors. We use HRS data on *dispositional optimism* derived from the same statements as in the well-known Life Orientation Test-Revised (LOT-R). As a proxy for likelihood insensitivity, we consider HRS measures on the *cognitive weakness* of the respondent, which is motivated by a cognitive interpretation of likelihood insensitivity (Wakker, 2010). We show that these direct measures exhibit the same age trends as our indirect measures.

In order to quantify the impact of these direct measures on SSBs, we finally specify both parameters of the Prelec function—*relative pessimism* and *likelihood insensitivity*—as linearly dependent on dispositional optimism and cognitive weakness. Our according estimates give rise to a decomposition analysis with the following main findings. We identify a strong *base bias* in the form of a baseline inverse S-shaped transformation of OSPs, which captures the survival belief of the most pessimistic person with the lowest cognitive weakness. Thus, individuals apparently only partially use the information on their individual-level OSPs when forming SSBs to the effect that they express likelihood insensitivity with respect to the OSPs. This may reflect an initial degree of cognitive weakness, incomplete statistical learning, (rational) inattention with respect to OSPs, rounding,[4] or a statistical artifact from truncation of the data.[5] Since the baseline inverse S-shaped transformation of OSPs is constant over

[3] Gonzalez and Wu (1999) refer to these concepts as *attractiveness* and *diminishing sensitivity*, respectively.

[4] There exists a growing literature on rounding of subjective probability questions, including questions on perceived survival chances, and how to correct for potential rounding or focal point answers, cf., for example, Hurd (2009), Manski and Molinari (2010), Hudomiet and Willis (2014), Kleinjans and van Soest (2014), Ruud et al. (2014), Bissonnette et al. (2017), and Drerup et al. (2017). Kleinjans and van Soest (2014) conclude that reporting behavior—that is, rounding, focal point answers and item nonresponse—does not have a large effect on estimated subjective probability distributions.

[5] Underestimators cannot report SSBs less than zero and overestimators cannot report SSBs above one. Such truncation may induce overestimation, on average, for OSPs close to zero and underestimation for OSPs close to one,

age, changes in differences between SSBs and OSPs attributable to the base bias are caused by movements of the OSPs. Because OSPs are relatively high at age 65, the base effect induces an underestimation of long-horizon survival chances of approximately 6%p. At age 85, however, OSPs are relatively low and the base bias induces an overestimation of approximately 15%p. *Cognitive weakness* leads to a further age-increasing flatness of the PWF, which induces an additional underestimation of SSBs at age 65 by about −5%p, and an additional overestimation at age 85 also by 5%p. In contrast to these dynamic effects of cognition, the impact of the motivational factor *relative optimism* is constant in age leading to an upward bias by about 10%p. Thus, the age-increasing overestimation of survival probabilities can (partially) be explained by cognitive weakness and not by increasing optimism as one may expect.

The remainder of this article is organized as follows. Section 2 reviews related literature, Section 3 presents the main stylized facts on survival belief biases, Section 4 provides our structural interpretation of these biases, Section 5 looks at direct psychological measures elicited in the HRS, and presents our results on their quantitative roles for SSBs. Finally, Section 6 concludes with a discussion on the economic implications of our findings. Separate appendices contain further information on the data and additional results.

2. RELATED LITERATURE

We contribute to the literature on subjective expectations (Manski, 2004), particularly on SSBs, inspired by Hamermesh (1985). On the one hand, this literature documents that SSBs are broadly consistent with OSPs and covary with health behavior, for example, smoking, or health status, in the same way as OSPs (Hurd and McGarry, 1995; Gan et al., 2005), that SSBs serve as predictors of actual mortality (Hurd and McGarry, 2002; Smith et al., 2001a), and that individuals revise their SSBs in response to new adverse (health) shocks (Smith et al., 2001b). On the other hand, several authors document important biases in SSBs when comparing sample average beliefs to OSPs (Elder, 2013; Ludwig and Zimper, 2013; Peracchi and Perotti, 2014; Groneck et al., 2016; Bissonnette et al., 2017). We emphasize that motivational and cognitive factors are important contributors to these biases.

In this respect, our work relates to medical studies examining the link between psychosocial dispositions and health shocks (Kim et al., 2011) or subjective body weight (Sutin, 2013). Mirowsky and Ross (2000) and Griffin et al. (2013) study how incorporating motivational factors influences subjective life expectancy. We extend their work by controlling for OSPs.

Manifestations of biases driven by motivational factors have also been discussed in the behavioral learning literature in the form of *confirmatory* biases (Rabin and Schrag, 1999), *myside* biases (Zimper and Ludwig, 2009), *partisan* biases (Jern et al., 2014; Weeks, 2015), and *irrational belief persistence* (Baron, 2008) and in the literature on motivated beliefs (Bénabou and Tirole, 2016). People biased by motivational factors "only see/learn what they want to see/learn" so that any new information tends to confirm already existing beliefs. One would expect that motivational biases play an important role in the formation of survival beliefs, since "most of us prefer to minimize even our cognitive encounters with death" (Kastenbaum, 2000). Elderly people might express more *optimistic* attitudes toward their likelihood of surviving and an age-increasing motivational (confirmatory) bias in the form of optimism could accordingly explain the observed age increasing overestimation of survival chances. Although our analysis suggests that a confirmatory bias (optimism) is important for the formation of survival beliefs at all ages, we find that it leads to a roughly constant bias across age. Our findings instead suggest that cognitive weakness is an increasingly important quantitative contributor to the overestimation of survival chances over an individual's life cycle.[6] For this reason

which leads to a natural flatness of the PWF relative to the 45-degree line; see Erev et al. (1994) for a formal analysis of this phenomenon in a general context.

[6] Our finding of increasing likelihood insensitivity with age is also consistent with Booij et al. (2010).

we caution against using survival expectations to proxy optimism as in, for example, Puri and Robinson (2007) and Angelini et al. (2019), and rather recommend to use direct psychological measures.

In order to model age-dependent survival beliefs, we employ a Prelec PWF applied to OSPs, which is a prominent approach in PT. As a generalization of *rank-dependent utility theories*, pioneered by Quiggin (1981, 1982), modern PT has developed into a comprehensive decision theoretic framework that combines empirical insights—starting with Kahneman and Tversky (1979)—with theoretical results about integration with respect to nonadditive probability measures, cf. Schmeidler (1989) and Gilboa (1987). Our model of age-dependent biases in survival beliefs is related to the experimental PT literature, which shows that subjective probability judgments cannot be described as additive probabilities. According to experimental findings, inverse S-shaped beliefs are prevalent in decision situations under *risk*, but are even more pronounced in situations under *uncertainty*, cf. Wakker (2004). We contribute to this literature using survey instead of experimental data, where only few papers document evidence of inverse S-shaped probability judgments, for example, Polkovnichenko and Zhao (2013) and Andrikogiannopoulou and Papakonstantinou (2016). Since it is plausible to assume that assessments of long-run survival chances involve ample uncertainty, the strong quantitative role of the base bias we uncover can be interpreted as a confirmation that inverse S-shaped probability weighting is indeed very pronounced under uncertainty. It speaks to the robustness of the experimental PT findings that we confirm the typical inverse S-shape for survival beliefs, and our regression analyses support the conventional psychological interpretations of this shape.

With this emphasis on the role of uncertainty our work relates to Bago d'Uva et al. (2020), who analyze the accuracy of longevity expectations. They find that with higher cognitive weakness of respondents in the HRS, the SSBs of these respondents are less accurate in predicting their actual survival. Likewise, through this perspective, we relate to Hill et al. (2004), who show that uncertainty with respect to survival beliefs increases in cognitive weakness. Our work complements these papers by asking how individuals with a given noisy signal on OSPs assess SSBs. Also, these findings are fully consistent with our notion of cognitive weakness as a proxy for likelihood insensitivity leading to a flatter PWF in light of uncertainty.

3. AGE PATTERNS OF BIASES IN SURVIVAL BELIEFS

3.1. *Data and Sample Selection.* We use data of the HRS. The HRS is a national representative panel study of the elderly U.S. population. Individuals are interviewed on a biennial basis. The interviewees are individuals older than age 50 and their spouses regardless of age. Interviews of the first wave were conducted in 1992.

Our sample selection is guided by our main variables of interest: SSBs, estimated individual-level OSPs, and the psychological variables optimism and cognitive weakness. As a result, we use several subsamples. We here provide a brief overview on our sample selection with further details on the construction of the main variables used and with descriptive statistics relegated to the appendix.

3.1.1. *SSBs and main samples.* In the HRS, an interviewee is asked the following question in the Expectations section of the HRS Core questionnaire:

> [On a scale from 0 to 100, where "0" means that you think there is absolutely no chance, and "100" means that you think the event is absolutely sure to happen,] What is the percent chance that you will live to be [X] or more?,

where $X \in \{80; 85; 90; 95; 100\}$ is the target age, equal to the age of the respondent at the time of the survey plus an horizon ranging between 11 and 15 years for respondents aged 65–89. In contrast, respondents under age 65 are asked about living to age 85 implying a horizon ranging between 21 and 25 years for the age group 60–64. Due to this inconsistency in target age

TABLE 1
INTERVIEW AGE h AND TARGET AGE $m(h)$

Interview Age h	Target Age $m(h)$
65–69	80
70–74	85
75–79	90
80–84	95
85–89	100

SOURCE: Health and Retirement Study (HRS).

we focus on individuals of age 65 and older. Respondents aged 90 or older are not asked the question, and thus our sample selection based on age is individuals in age bracket 65–89. The assignment of target age $m(h)$ to interview age h for our sample is summarized in Table 1. For all respondents i of age at interview h and according target age $m(h) > h$ in our sample we denote the according individual SSB by $SSB_{i,h,m(h)}$.

As our main sample we use pooled HRS data of years 2006–14 (Waves 8–12). The restriction on these HRS waves is due to availability of psychological variables to measure optimism, which were first collected in the HRS in the year 2006 (Wave 8). The number of observations with nonmissing SSB values in our sample consists of 38,846 observations and is used in Subsection 3.3 and Section 4. The corresponding OSPs are out of sample predictions from the estimation described in Subsection 3.2. This large sample size allows us to analyze descriptive statistics for each age group defined by the target ages of SSBs, cf. Table 1. The sample sizes for each age group are 10,985 (age 65–69), 11,268 (age 70–74), 8,583 (age 75–79), 5,279 (age 80–84), and 2,731 for the oldest age group 85–89.

In our regression analyses in Section 5, the sample size is reduced by including psychological and further control variables. Psychological variables are only collected in each biennial wave from an alternating (at random) 50% subset of all core panel participants who were visited for an enhanced face-to-face interview (EFTF).[7] Additionally deleting observations with missing values of control variables, our sample used in Section 5 consists of 11,954 observations.

3.1.2. *Extended sample for estimating OSPs.* In our duration model estimation of individual-level OSPs we only use one observation per individual. To still have sufficient observations, we extend the year range by taking HRS data from years 1998 to 2014. This is required for construction of the duration variable, that is, when the individual died or left the sample. We thereby exclude earlier waves of the HRS due to inconsistently measured control variables across waves, which concerns, in particular, some health questions used to construct the mobility index and the muscle index.

In addition, we extend the age range when estimating individual-level OSPs. Notice from Table 1 that interviewees of age 85–89 are asked for their subjective beliefs to live until age 100. In order to increase the precision of our estimates of the OSPs for this age group, we therefore include respondents of up to age 98 (and we exclude even older ages due to very small sample sizes). Further deleting observations with missing values of the control variables used in the duration model, the sample employed for the estimation of individual-level OSPs consists of 15,373 observations.

3.2. *Estimating OSPs.* Comparing individual-level SSBs to survival probabilities extracted from aggregate cohort life tables as, for example, in Perozek (2008), Ludwig and Zimper (2013), Peracchi and Perotti (2014), and Groneck et al. (2016), is ill-suited because individual-level OSPs generally deviate from sample averages. In order to estimate individual-level

[7] Note, that we use lagged variables for psychological optimism and cognitive weakness, which implies that essentially, only Waves 9–12 are used, where in Wave 9 the psychological and cognitive variables from Wave 8 are used.

OSPs, we instead follow Khwaja et al. (2007), Khwaja et al. (2009), Winter and Wuppermann (2014), Kutlu-Koc and Kalwij (2017), Perozek (2008), Bissonnette et al. (2017), and Siegel et al. (2003) by adapting a mixed-proportional hazard (MPH) model, see van den Berg (2001). This allows us to estimate hazard rates conditional on a broad set of individual-level characteristics and to simulate survival probabilities for the full sample by making out-of-sample predictions.

Let T be a nonnegative random variable denoting the time to failure event, that is, the number of years to death. Further, let $f(t_i)$ be the density of $F(t_i) = P(T \leq t_i)$, where t_i is a realization of T. The survivor function defined as the probability of surviving beyond time t_i is given by $S(t_i) = 1 - F(t_i) = P(T > t_i)$ and the hazard function $\mathrm{h}(t_i) = \frac{f(t_i)}{S(t_i)}$ is the conditional, or age-specific, failure rate (force of mortality). The hazard rate is duration dependent if it changes with t_i. Specifically, we assume that mortality of individual i conditional on covariates x_i and unobserved heterogeneity η_i is given by the hazard function

$$\mathrm{h}(t_i|\mathbf{x_i}, \eta_i; \alpha, \beta) = \lambda_0(t_i; \alpha) \cdot \exp(\mathbf{x}_i'\beta) \cdot \eta_i,$$

where $\lambda_0(\cdot)$ is the baseline hazard, and $\exp(\mathbf{x}_i'\beta) \cdot \eta_i$ is a "mixed proportional hazard" with coefficient vector β and a multiplicative unobserved heterogeneity η_i. The individual-specific proportional hazard thus scales the common baseline hazard function with the underlying assumption that conditional on the baseline characteristics, the duration to death is given by the same baseline hazard function for all individuals (Cleves et al., 2008). For the baseline hazard function $\lambda_0(t)$, we assume a Weibull distribution[8]

$$\lambda_0(t_i) = \alpha t_i^{\alpha-1},$$

which allows for $\alpha > 1$ capturing positive duration dependence.

The unobserved heterogeneity η_i accounts for random differences of individuals not captured by observed variables and dynamic selection effects, that is, a potentially selected sample with rising age, see Kalwij et al. (2013) and van den Berg et al. (2006). The individual effect η_i essentially scales the no-frailty component of the survivor function. Individuals with above-average values of η_i have high hazard rates and the opposite occurs for individuals with below-average values of η_i.

In order to estimate the unobserved effect we assume a specific distribution of η_i so that its functional form is summarized in terms of parameters that can be estimated. As common in the literature since Lancaster (1979), η_i is assumed to obey a Gamma distribution with scale parameter κ and shape parameter σ, $\eta_i \sim \Gamma(\kappa, \sigma)$, where normalization is such that for reasons of identification, the mean is one ($\kappa = \frac{1}{\sigma}$) and the variance is thus σ^2. As a consequence the likelihood function refers to the shape parameter σ of the Gamma distribution instead of to each η_i.

The response variable in our data is the duration until death. However, some respondents do not die until the end of the observation time. These individuals are *right-censored*, that is, we only know the probability that they did not die before a certain period. To take right-censoring into account, denote by d_i an indicator variable, which is one if individual i is uncensored. Note that $S(t_i)$ is also the probability that t_i is right-censored. For a sample of size n, we can write the log-likelihood as

$$\begin{aligned} \ln L\left\{\beta, \alpha, \sigma|(t_1, d_1, \mathbf{x_1}), \ldots, (t_n, d_n, \mathbf{x_n})\right\} = \sum_{i=1}^{n} \big\{ & d_i \cdot \log\left[S(t_i|\mathbf{x_i}; \beta, \alpha, \sigma)\mathrm{h}(t_i|\mathbf{x_i}; \beta, \alpha, \sigma)\right] \\ & + (1 - d_i)\log\left[S(t_i|\mathbf{x_i}; \beta, \alpha, \sigma)\right]\big\}, \end{aligned} \tag{1}$$

[8] According to Perozek (2008), the Weibull and the Gompertz model are most widely used when estimating human mortality. Significant differences mainly occur at advanced ages past 85.

which we minimize with respect to β, α, and σ. Using the estimated parameters, we predict OSPs at all horizons t for each individual i of interview age h as

$$OSP_{i,h,h+t} = \exp\left[-\exp(\mathbf{x}_i'\beta)t^{\alpha}\right]. \tag{2}$$

From this, we can also construct the OSP until target age with horizon $t = m(h) - h$, $OSP_{i,h,m(h)}$, which we assign to the respective $SSB_{i,h,m(h)}$ of individual i.

As described in Subsection 3.1, when estimating the duration model, we extend the age range of our sample to individuals aged 65–98, and choose all observations when respondents enter the HRS (which can be at different waves) within our observed time period. The average age is 70.7 and 42% of individuals die within the observed time interval of at most 17 years. As covariates we use demographic variables (age, gender, marital status), a set of objective and subjective health variables, as well as a measure for the average age-specific survival probability that we estimate using the Lee and Carter (1992) procedure employing the life tables from the Human mortality database in order to account for the time trend in life expectancy, cf. Table A.2 in Appendix A.2. In addition, we allow a nonlinear relation between age and time to death by employing a third-order polynomial. Control variables are held constant at their respective values when individuals enter the sample.

The parameter estimate of the Weibull hazard function is $\alpha = 1.64$ significantly above one thus indicating positive duration dependence, that is, the probability of death increases the longer the individual was observed in the sample. The estimated variance of the Gamma distribution is significantly different from zero at $\sigma = 0.06$, thus our sample features unobserved heterogeneity of small size. This implies that our coefficient estimates are very similar to a model without unobserved heterogeneity. Both estimates are in line with the literature (e.g., Kalwij, 2014; Bissonnette et al., 2017; Kalwij et al., 2013).

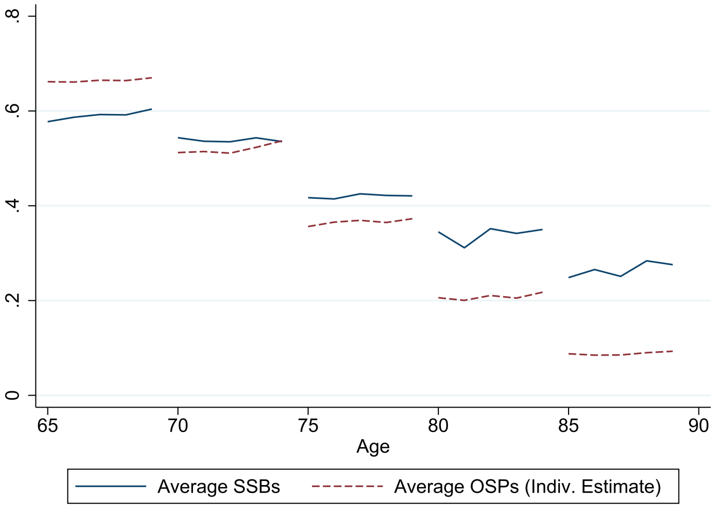
The signs of the coefficient estimates on the control variables reported in Table A.3 of Appendix A.2 are broadly in line with prior expectations and what has been found in the literature, cf. Khwaja et al. (2007), Khwaja et al. (2009), and Kutlu-Koc and Kalwij (2017). For example, most measures of health limitations are positively associated with the hazard of dying.[9] We also include cognitive weakness (see Section 5 for the variable's construction) as a control in our estimation of OSPs, which we find to be significantly different from zero. However, we cannot include the motivational variable optimism in the hazard model because this would reduce the sample size to only 2,108 observations.[10] If optimists were more likely to survive, then any deviation from SSBs caused by this motivational attitude would (at least partially) reflect additional information of respondents on their objective mortality risk instead of psychological biases. We address this concern in a smaller subsample by reestimating the hazard model with the inclusion of the motivational variable, using HRS data from Waves 8–12, with results shown in Table A.4 in Appendix A.2. We do not find that optimism has a significant effect on mortality. The coefficient estimates of other control variables are not much affected either. This supports our interpretation of the effects of optimism on SSBs as reflecting psychological biases that do not directly affect mortality.

Descriptive statistics of the distributions of OSPs and SSBs are given in Appendix A.3. There we also show that our model of OSPs fits actual sample mortality well.

3.3. *Biases.* As a first step of comparing individual-specific SSBs and OSPs, we replicate results of previous literature(e.g., Hamermesh, 1985; Elder, 2013; Ludwig and Zimper, 2013; Peracchi and Perotti, 2014) on the age patterns of survival beliefs in Figure 1. As a

[9] Exceptions are the muscle index and the variable ever drinking, which is likely due to collinearity with activities of daily living (ADLs).

[10] This stark reduction is due to the fact that we only observe motivational variables from Wave 8 onward, additionally containing many missing values and that the sample for OSP contains only one (the first) observation per individual.



NOTES: Average subjective survival beliefs (SSBs, solid line) and corresponding average objective survival probabilities (OSPs, dashed line), cf. Equation (2). SSBs are elicited in the HRS for a combination of the age at interview of the individual (which is shown on the abscissa) and a corresponding target age, cf. Table 1. The step function follows from changes in the interview age/target age assignment. Sample size: 38,846 observations. Sample size by age group: 10,985 (age 65–69), 11,268 (70–74), 8,583 (75–79), 5,279 (80–84), 2,731 (85–89).

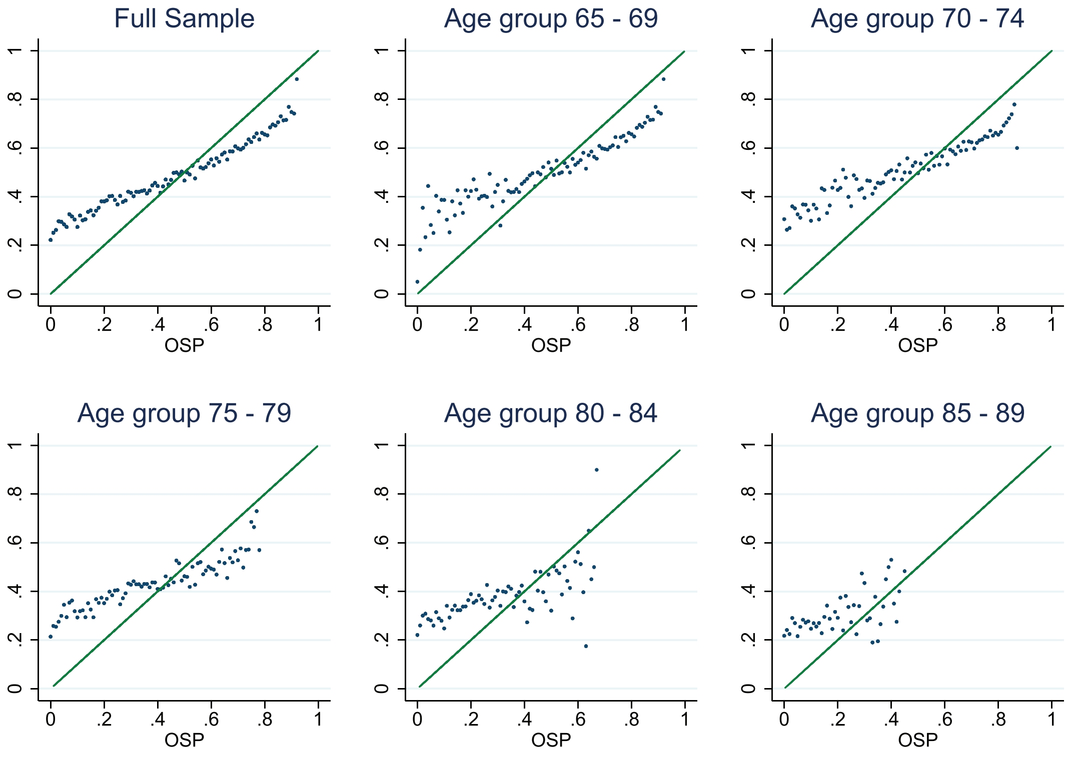
FIGURE 1

FLATNESS BIAS
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

crucial difference from this literature, we calculate average OSPs from our individual measures instead of using average (cohort) life tables. The step function in the figure is due to the change in assignment of interview and target ages, cf. Table 1. Our findings confirm the well-established *flatness bias* with individual-level data: At ages prior to 70, individuals, on average, underestimate their probabilities to survive, whereas for ages above 75, they overestimate it.

Next, we take a new perspective for which individual-level data are needed. Instead of computing averages over age, we average over OSPs, that is, for each OSP, we compute the average SSB. In the upper left panel of Figure 2, we show the corresponding results by plotting average SSBs against average OSPs. If SSBs are aligned along the 45-degree line, then there is no bias on average. However, we observe a very systematic pattern of misconception: Individuals with low OSPs, on average, overestimate their survival chance, whereas those with high OSPs underestimate it.

The two perspectives on the data taken in Figure 1 and the upper left panel of Figure 2 suggest a very simple explanation for the observed biases across age. Suppose that, irrespective of any cognitive notion on likelihood sensitivity, individuals were to always resolve any uncertainty about their survival chances in a 50–50 manner, that is, their response were a weighted average of a 50% chance of survival and the actual OSP, then such a 50–50 heuristic could obviously explain the pattern in the upper left panel of Figure 2. Furthermore, young respondents in our data have OSPs above 50%. If they were to apply such a simple heuristic, then they would, on average, underestimate their chances to survive. Old respondents, on the other hand, have long-run OSPs less than 50%, on average, and would accordingly overestimate their OSPs, on average. Hence, such a 50–50 bias could simultaneously explain the patterns in these graphs.



NOTES: SSB over OSP by age group. The upper left panel is for all ages. The remaining age group panels focus on different target ages according to Table 1. Sample size: 38,846 observations. Sample size by age group: 10,985 (age 65-69), 11,268 (70-74), 8,583 (75-79), 5,279 (80-84), 2,731 (85-89).

FIGURE 2

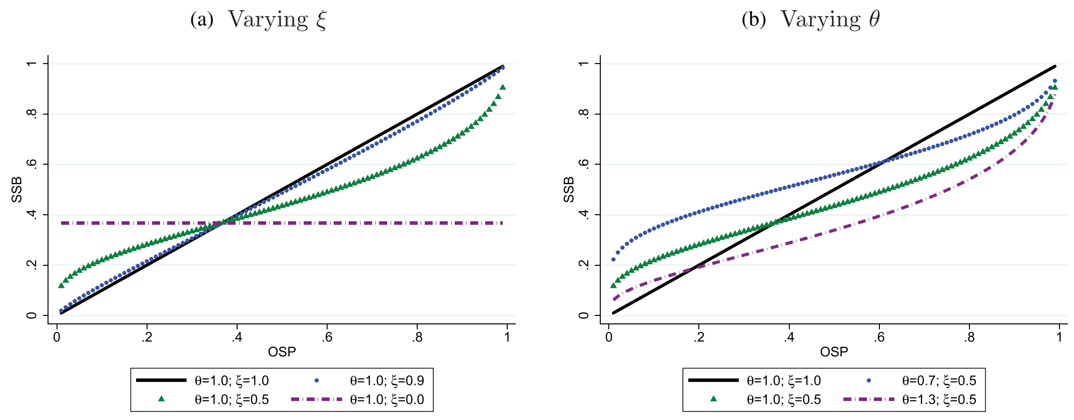
OBJECTIVE SURVIVAL PROBABILITIES AND SUBJECTIVE SURVIVAL BELIEFS BY AGE GROUP
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

However, there is more information content in the data, giving rise to alternative interpretations.[11] This can be illustrated by repeating the previous analysis for different target age groups in the remaining panels of Figure 2, which suggests that the flatness of SSBs against OSPs grows stronger with increasing age—compare, for example, age group 65–69 with age group 80–84. In addition, the intersection with the 45-degree line moves downward, from approximately 50% for age groups 65–69 and 70–74 to approximately 40% for age group 80–84. Therefore, the average tendency for underestimating a given OSP increases across age groups. Figure A.4 in Appendix A.5 further supports this view by showing that average SSBs by bins of OSPs weakly decrease over age.

4. MODELING SSBS

We interpret these biases in survival beliefs through the lens of PT by adopting age dependent inverse S-shaped PWFs to map OSPs into SSBs. This enables us to model the observed (age increasing) flatness of SSBs relative to OSPs. We use a parsimonious parameterization of PWFs, which, employing the terminology of Wakker (2010), gives rise to two

[11] The general notion of more information content beyond a mere 50–50 bias is also confirmed in the earlier work by Hurd and McGarry (1995), Hurd et al. (1999), Smith et al. (2001a), Smith et al. (2001b), Hurd and McGarry (2002), and Gan et al. (2005). We add to this literature by emphasizing the roles of cognitive and motivational factors.



NOTES: Stylized Prelec (1998) probability weighting functions. The left panel shows the impact of likelihood insensitivity, ξ, for $\theta = 1$ and $\xi \in [0, 0.5, 0.9, 1]$. The right panel shows the impact of pessimism for $\xi = 0.5$ and $\theta \in [0.7, 1, 1.3]$.

FIGURE 3

PESSIMISM AND LIKELIHOOD SENSITIVITY IN STYLIZED PRELEC PWF
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

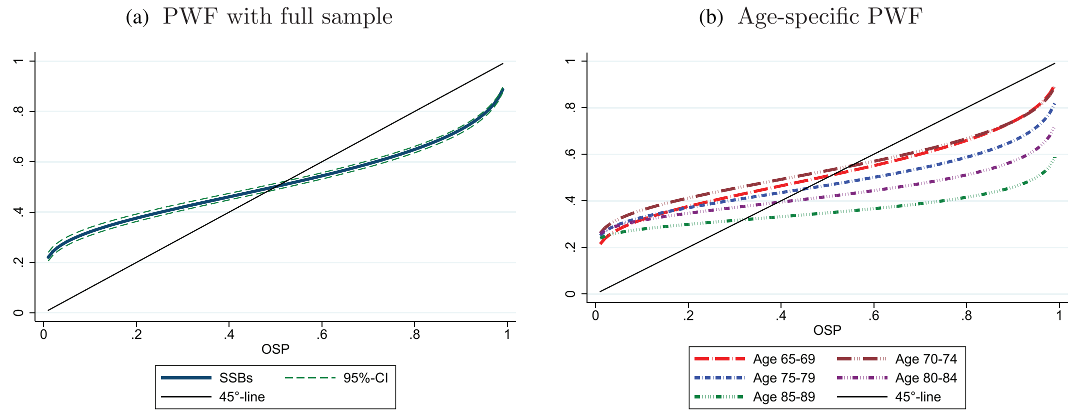
psychological interpretations. First, the increasing flatness of SSBs relative to the 45-degree line reflects, along a *cognitive* dimension, an increasing insensitivity to the objective likelihood of the decision maker (likelihood insensitivity). Second, the decreasing intersection of SSBs with the 45-degree line reflects decreasing optimism, and hence a *motivational* interpretation of the data.

4.1. *The Prelec PWF.* Specifically, we employ the PWF suggested by Prelec (1998) and accordingly map OSPs into SSBs by

$$SSB = \left(\exp\left(-(-\ln(OSP))^{\xi}\right)\right)^{\theta} \tag{3}$$

for parameters $\xi \geq 0, \theta \geq 0$. These two parameters control the elevation and the curvature of the function, which can be interpreted as measures of optimism and likelihood insensitivity, respectively. To see this, observe that for $\xi = \theta = 1$, function (3) coincides with the 45-degree line. Holding θ constant at one, an increase of ξ above one leads to an S-shaped pattern and a decrease below one leads to an inverse S-shape, where the intersection with the 45-degree line is at the objective probability of $OSP = \exp(-1) \approx 0.37$. This dependency on ξ is illustrated in Panel (a) of Figure 3, where we decrease ξ from one toward zero giving rise to an inverse S as in the data of Figure 2. In the limit where $\xi = 0$, the curve is flat. Hence, ξ can be interpreted as a measure of likelihood insensitivity. In Panel (b) we show that decreasing θ leads to an upward shift of the PWF, whereas increasing θ leads to a downward shift. Accordingly, θ can be interpreted as a measure of optimism/pessimism.

It is instructive to emphasize three different effects by use of Figure 3. Suppose that with age, relative pessimism θ increases. For all OSPs, this induces stronger underestimation (Panel (b)). At the same time, however, OSPs decrease with age to the effect that the mass of the population lives to the left of the intersection of the PWF with the 45-degree line. This movement of OSPs induces overestimation in old age. If with age also likelihood sensitivity, ξ, decreases, the corresponding flattening of the PWF leads to additional overestimation of OSPs at low OSP levels (Panel (a)).



NOTES: Estimated Prelec PWFs for the full sample in Panel (a)—including 95% confidence intervals—and for different age groups in Panel (b). Sample size: 38,846 observations. Sample size by age group: 10,985 (age 65–69), 11,268 (70–74), 8,583 (75–79), 5,279 (80–84), 2,731 (85–89).

FIGURE 4

ESTIMATED NONLINEAR PROBABILITY WEIGHTING FUNCTIONS
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

4.2. *Age-Dependent PWFs.* Due to the age pattern in the data, cf. Figure 2, we proceed by specifying an age-dependent PWF and accordingly model the subjective belief of individual i to survive from age h to some future age $h+t$ as

$$SSB_{i,h,h+t} = \left(\exp\left(-(-\ln(OSP_{i,h,h+t}))^{\xi_h}\right)\right)^{\theta_h}, \tag{4}$$

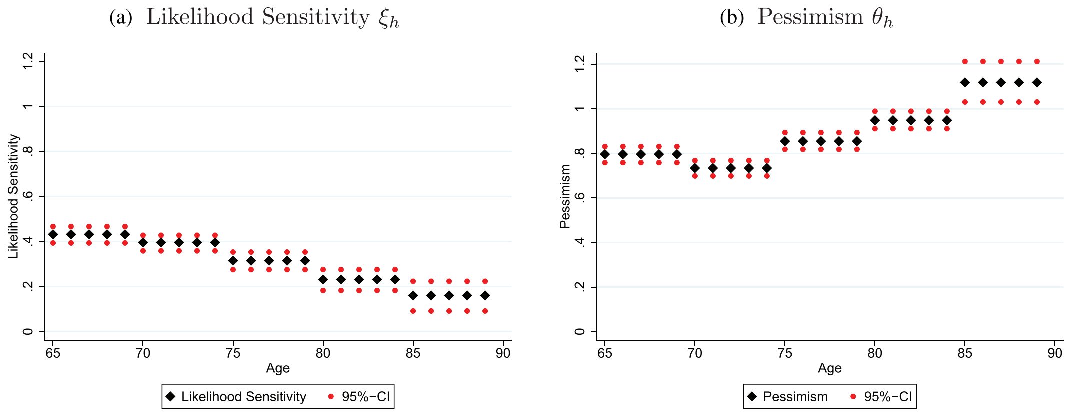
for a given $OSP_{i,h,h+t}$ with age-specific parameters θ_h, ξ_h. At the estimation we restrict parameters ξ_h, θ_h to be the same for each interview age h assigned with a specific target age $m(h)$, that is, we let $\xi_h = \bar{\xi}_{m(h)}$ and $\theta_h = \bar{\theta}_{m(h)}$. We identify these parameters by estimating (4) for $h + t = m(h)$, adding an additive error term $\epsilon_{i,h,m(h)}$ to the equation and minimizing the Euclidean distance between the predicted and reported SSBs for each individual in group $m(h)$. Figure 4 shows the predicted average PWF with corresponding bootstrapped 95% confidence intervals in Panel (a) and predicted target age group–specific PWFs in Panel (b).[12] For the full sample we observe a quite symmetric PWF intersecting the 45-degree line close to 0.5. The age-specific PWFs in turn become flatter with increasing age, and, with the exception of interview age group 70–74, their intersection with the 45-degree line is at lower values for older ages—it is at approximately 55% for age group 65–69 and at approximately 40% for age group 80–84.

Figure 5 depicts the corresponding parameter estimates $\xi_h = \bar{\xi}_{m(h)}, \theta_h = \bar{\theta}_{m(h)}$ with bootstrapped 95% confidence intervals. The coefficient estimates $\xi_h = \bar{\xi}_{m(h)}$, shown in Panel (a), are decreasing in h, reflecting increasing likelihood insensitivity. Similarly, estimates $\theta_h = \bar{\theta}_{m(h)}$, shown in Panel (b), are increasing between interview age groups 70–74 and 85–89, although there is a nonmonotonicity before. We can thus conclude that pessimism (or, the opposite of optimism) increases for ages above 70.

[12] Since our data are clustered, we perform a cluster bootstrap that samples the clusters with replacement. Thus, in each bootstrap, we solve

$$\min_{\bar{\xi}_{m(h)}, \bar{\theta}_{m(h)}} \left\{ \sum_{i=1}^{N^{m(h)}} \left[\epsilon_{i,h,m(h)}\right]^2 \right\}.$$

Standard errors are computed using the percentile method.



NOTES: This figure shows estimates of $\xi_h = \bar{\xi}_{m(h)}$ in Panel (a), estimates of $\theta_h = \bar{\theta}_{m(h)}$ in Panel (b), and the bootstrapped (1,000 replications) 95% confidence intervals, which are based on the percentile method. Sample size: 38,846 observations. Sample size by age group: 10,985 (age 65–69), 11,268 (70–74), 8,583 (75–79), 5,279 (80–84), 2,731 (85–89).

FIGURE 5

ESTIMATED PRELEC PARAMETERS: LIKELIHOOD SENSITIVITY AND PESSIMISM
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

These findings suggest that the overestimation of SSBs at ages 75 and older documented in Figure 1 cannot be explained by increasing optimism, as one may suspect. It is rather due to the flatness of the PWF and the reduction of OSPs, with age-increasing likelihood insensitivity further increasing the flatness of the PWF and thus additionally adding to the overestimation at low OSP levels. Thus, likelihood insensitivity is an (increasingly) important contributor to the overestimation of survival chances in old age.

However, our estimates may be biased by two features of the data. First, for the oldest two age groups our data are censored, because the long-run objective survival chances do not exceed 70%, respectively 50%, cf. Figure 2. This implies that our estimates of the PWFs extrapolate outside the sample for these age groups. Second, survival chances are naturally bounded from below by zero and from above by one so that respondents with very high (low) OSPs cannot overestimate (underestimate) their survival chances by much, whereas the respective other side is less limited. This may induce a flatness of the PWFs. We address these concerns in our subsequent regression analyses on the relationship between direct psychological measures and SSBs. In particular, we will show that flatness of the PWF is associated with cognitive weakness. In our view, this lends support to our extrapolation of PWFs for the oldest age group in Figure 2 and confirms that the observed flatness of the PWFs is not caused by the truncated support of survival beliefs (alone).

5. PSYCHOLOGICAL MEASURES AND SURVIVAL BELIEFS

Since our preceding structural interpretation of biased survival beliefs suggests that cognitive and motivational factors are important determinants for the formation of SSBs, we proceed by quantitatively evaluating the relationship between direct cognitive and motivational measures and SSBs. To this purpose we specify both parameters of the Prelec function—*relative optimism* and *likelihood insensitivity*—as linearly dependent on dispositional optimism and cognitive weakness and use proxies from the HRS for both variables. On the basis of our estimates we then decompose the quantitative impact of these variables on SSBs.

TABLE 2
REGRESSING COGNITIVE AND MOTIVATIONAL VARIABLES ON AGE

	Cognitive Weakness	Dispositional Optimism
Age	0.0428***	−0.0045**
	(26.88)	(−2.45)
Constant	−3.35***	0.37***
	(−27.82)	(2.72)
Observations	11,954	11,954

t statistics in parentheses; $* \; p < 0.10$, $** \; p < 0.05$, $*** \; p < 0.01$
NOTES: This table reports coefficients for simple regressions using *standardized* cognitive weakness and *standardized* lagged optimism as dependent variable and age as the independent variable. Sample size: 11,954 observations.
SOURCE: Own calculations, Health and Retirement Study (HRS).

5.1. *The Measures.* From Wave 8 onward, the HRS contains measures on *dispositional optimism* derived from the same statements[13] as in the LOT-R.[14] This psychosocial information is obtained in each biennial wave from a rotating (random) 50% of the core panel of participants who complete the EFTF. Respondents are given various statements regarding a specific latent variable, and answers to questions of the form *"please say how much you agree or disagree with the following statements* " are rated on a scale from 1 (*strongly disagree*) to 6 (*strongly agree*). We take average scores for each question normalized to the [0,1] interval to construct an index for relative optimism, so that higher values mean more optimistic attitudes.[15]

Based on our cognitive interpretation of *likelihood insensitivity* (Wakker, 2010), our proxy variable for it measures the cognitive weakness of the respondent. It is a version of a composite score taken from RAND[16] and combines the results of several cognitive tests, such as the ability to recall a list of random words and to count backwards. In total, 35 questions are summarized in an ability score. We use it to create our index of cognitive weakness by subtracting the cognitive ability score from the maximal achievable value of 35—so that a higher score indicates higher cognitive weakness—and normalize it to the [0,1] interval.

Details on both measures of cognitive and motivational variables are provided in Appendix A.1. We subsequently use lagged variables as controls, denoting by $c_{i,h-2}$ the lag of cognitive weakness, and by $o_{i,h-2}$ the lag of optimism, respectively. Using lags allows us to treat these measures as weakly exogenous to avoid spurious correlation,[17] and to interpret our findings on the relationship between cognitive and motivational measures and SSBs tentatively as causal.[18] We use the pooled HRS data from Waves 8–12, that is, years 2006–14, which consist of 11,954 observations. Table 2 shows the coefficients of simple regressions of standardized cognitive weakness and dispositional optimism on age. Interestingly, the age trends coincide with those of the implicit measures we backed out from the estimated PWFs in Section 4, and they are much stronger for cognitive weakness than for optimism.

5.2. *Parameterizing the Nonlinear PWF.* As our parameterized variant of the Prelec (1998) function we postulate that for each individual in the sample i and each age h, the implicit measures of cognition, $\xi_{i,h}$, and optimism, $\theta_{i,h}$, from Equation (4) are linearly dependent on the

[13] Such statements are, for example, "In uncertain times I usually expect the best."

[14] The LOT-R questionnaire was developed to measure dispositional optimism, that is, a generalized expectation of good outcomes in one's life (Scheier and Carver, 1987; Scheier et al., 1994).

[15] We construct the uniform optimism score on the basis of questions regarding optimistic as well as pessimistic personality traits.

[16] More precisely, the RAND HRS Longitudinal Files provided by the RAND Center for the Study of Aging.

[17] For example, health shocks may affect cognition and motivational attitudes directly and lead to adjustments of SSBs.

[18] Although the approach of using lags for causal identification is widespread in social sciences, it is not without criticism, cf. Bellemare et al. (2017). We therefore speak of a "tentative" causal interpretation.

TABLE 3

THE EFFECTS OF COGNITION AND MOTIVATIONAL MEASURES ON SUBJECTIVE SURVIVAL BELIEFS

Cognitive Weakness Intercept (ξ_0)	0.540
	[0.477; 0.603]
Cognitive Weakness Slope (ξ_1)	−0.399
	[−0.566;−0.250]
Optimism Intercept (θ_0)	1.140
	[1.075; 1.214]
Optimism Slope (θ_1)	−0.433
	[−0.515; −0.358]
OSP_0	0.368
	[0.368; 0.369]
SSB_0	0.320
	[0.297; 0.341]
AIC	4,125
Observations	11,954

NOTES: Column 2 shows the point estimates. Bootstrapped 95% confidence intervals in brackets (1,000 replications, computed with percentile method). AIC: Akaike (1973) information criterion. Sample size: 11,954 observations.
SOURCE: Own calculations, Health and Retirement Study (HRS).

respective psychosocial variable:

$$\xi_{i,h} = \xi_0 + \xi_1 c_{i,h-2}, \tag{5a}$$

$$\theta_{i,h} = \theta_0 + \theta_1 o_{i,h-2}. \tag{5b}$$

Replacing in (4) the age-specific parameters ξ_h and θ_h with the individual and age-specific parameters $\xi_{i,h}$, $\theta_{i,h}$ and using (5), our specification of survival beliefs is

$$SSB_{i,h,m(h)} = \left(\exp\left(-(-\ln\left(OSP_{i,h,m(h)}\right))^{\xi_0+\xi_1 c_{i,h-2}}\right)\right)^{\theta_0+\theta_1 o_{i,h-2}}. \tag{6}$$

We add error term $\epsilon_{i,j,m(h)}$ and estimate Equation (6) by nonlinear least squares.

Turning to the parameters of interest in specification (6), we refer back to our analysis of Section 4, in particular to the illustration in Figure 3. In light of our discussion there, parameters ξ_0 and θ_0 capture a *base effect* in subjective beliefs. With regard to the base effect in cognition, ξ_0, we conjecture that this base effect exists in the form of an inverse S, and we therefore expect $\xi_0 \in (0, 1)$. This may reflect an initial degree of cognitive weakness (with measured cognitive weakness index at zero), incomplete statistical learning, (rational) inattention with respect to OSPs, rounding, or a statistical artifact from truncation of the data. With regard to optimism recall that $\theta_0 < 1$ reflects rather optimistic beliefs, whereas $\theta_0 > 1$ reflects rather pessimistic beliefs. Since our measure of optimism is normalized to 0 for the least optimistic persons in the sample, we expect that $\theta_0 > 1$. Also, recall that a lower likelihood sensitivity leads to a flatter PWF. Therefore, if changes in cognitive weakness are relevant for the formation of subjective beliefs, we would find its coefficient to be negative, $\xi_1 < 0$. Finally, since increasing relative optimism reduces θ_h leading to a higher elevation of the PWF, we expect that $\theta_1 < 0$.

5.3. *Quantitative Roles of Motivational and Cognitive Measures.* Our baseline estimates summarized in Table 3 show that there is indeed a significant baseline inverse S-shaped transformation of OSPs, $\xi_0 = 0.54 < 1$, and the estimated PWF is downward shifted, $\theta_0 > 1$. Our estimates also show that increasing lack of cognition leads to increasing likelihood insensitivity, $\xi_1 = -0.39$, flattening the nonlinear PWF, and that increasing relative optimism leads to a significant upwards shift, $\theta_1 = -0.43$, of the nonlinear PWF. Thus, cognitive and motivational factors have significant effects on the formation of SSBs of the expected sign.

In order to separately quantify the impact of the respective variables of interest, we further decompose the PWF as

$$\text{(7a)} \qquad \text{base bias:} \quad SSB^{b}_{i,h,m(h)} = \Big(\exp\Big(-\big(-\ln\big(OSP_{i,h,m(h)}\big)\big)^{\xi_0}\Big)\Big)^{\theta_0},$$

$$\text{(7b)} \qquad \text{base + cogn. weakn.:} \quad SSB^{bc}_{i,h,m(h)} = \Big(\exp\Big(-\big(-\ln\big(OSP_{i,h,m(h)}\big)\big)^{\xi_0+\xi_1 c_{i,h-2}}\Big)\Big)^{\theta_0},$$

and thus we define as the *base bias*, the $SSB^{b}_{i,h,m(h)}$ for individuals with optimism and cognitive weakness indices at zero. The SSB for the *base bias plus cognitive weakness* $SSB^{bc}_{i,h,m(h)}$ additionally takes into account the effects of increasing cognitive weakness. We accordingly define the contribution of the respective factors on the SSB by the differences

$$\text{(8a)} \qquad \text{cogn. weakn.:} \quad \Delta SSB^{c} = SSB^{bc}_{i,h,m(h)} - SSB^{b}_{i,h,m(h)},$$

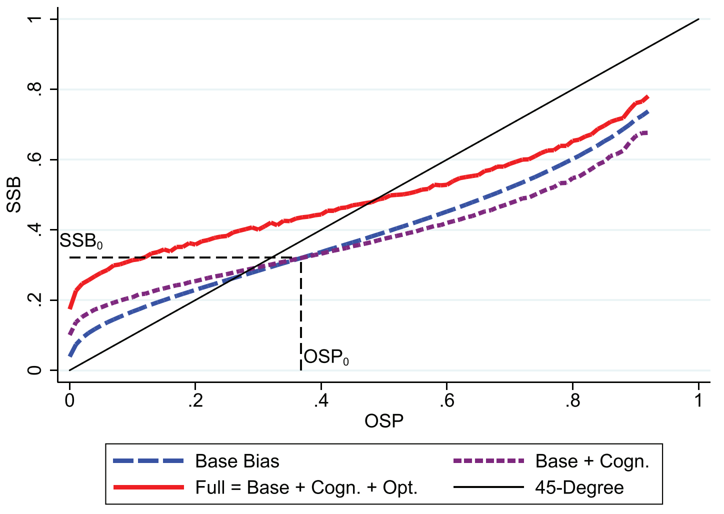
$$\text{(8b)} \qquad \text{optimism:} \quad \Delta SSB^{o} = SSB_{i,h,m(h)} - SSB^{bc}_{i,h,m(h)}.$$

In our decomposition analyses we first predict models (7) and the respective contributions (8) for each individual in the sample and then compute sample averages. We further define by OSP_0 the level of the OSP at which the base bias $SSB^{b}_{i,h,m(h)}$ intersects with the base bias plus cognition $SSB^{bc}_{i,h,m(h)}$ and the associated SSB is denoted by SSB_0—that is, to quantify the effects of cognition we pivot the PWF around point (OSP_0, SSB_0), with respective estimates as sample averages of the individual-specific intersections reported in Table 3.

Results on the predictions for the full model and its decomposition are displayed in Figure 6. The predicted base bias $\widehat{SSB}^{b}$ displays a pronounced inverse S reflecting the underlying misperception of survival chances mentioned above. Predictions for the base bias plus changes in cognitive weakness $\widehat{SSB}^{bc}$ lead to a clockwise rotation of the PWF around $(\widehat{OSP}_0, \widehat{SSB}_0)$. The additional effect of optimism shifts the PWF upward by more than 10%p. As a consequence of both mechanisms, the PWF in the full model is both flatter and shifted upward relative to the base PWF.

Figure 7 provides the corresponding decomposition over age. Panel (a) shows the data on SSBs and OSPs—that is, the data points of Figure 1—as well as the predicted values for the full model—displaying a very close match to the average SSBs by age—and for the base bias. Consistent with our findings in Figure 6, the base bias implies an age increasing underestimation relative to the full model. Panel (b) displays the sample average (conditional on age) contributions to the formation of SSBs of changes in cognitive weakness, optimism, and of both according to our respective definitions in Equation (8). Due to the age increasing cognitive weakness, individuals, on average, overestimate their survival chances increasingly more as they grow older: Relative to the base bias, cognitive weakness initially leads to a downward bias of almost −5%p because relatively young individuals have relatively higher objective survival chances on average and thus the clockwise tilting of the PWF leads to underestimation. Since with age objective survival rates decrease this initial underestimation turns into an overestimation by about +5%p for the oldest age group. Furthermore, over the life cycle, optimism leads individuals to overestimate their survival chances by roughly 10%p.

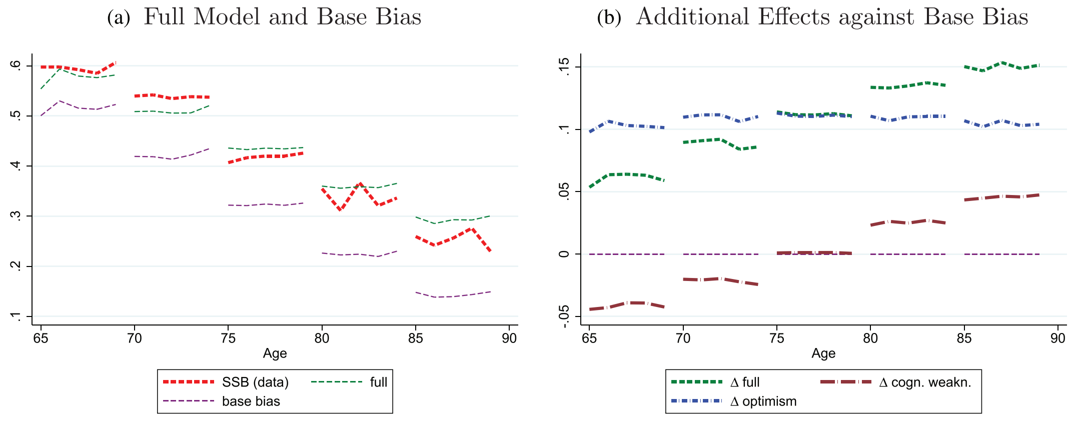
Overall, the effects of cognitive and motivational variables on SSBs are therefore quite strong. Importantly, the effects of cognitive weakness are changing with age, whereas the



NOTES: Sample averages of predicted nonlinear probability weighting functions according to Equations (6) and (7); "base bias": $\widehat{SSB}^b$; "base + cogn. weakn.": $\widehat{SSB}^{bc}$; "full": $\widehat{SSB}$. Sample size: 11,954 observations.

FIGURE 6

DECOMPOSITION OF NONLINEAR PWFS
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]



NOTES: Sample averages of predicted subjective survival beliefs according to Equations (6) and (7) by age; Panel (a): "full": $\widehat{SSB}$; "base bias": $\widehat{SSB}^b$; Panel (b): "Δ full": $\widehat{SSB} - \widehat{SSB}^b$; "Δ cogn. weakn.": $\widehat{SSB}^{bc} - \widehat{SSB}^b$; "Δ optimism": $\widehat{SSB} - \widehat{SSB}^{bc}$. Sample size: 11,954 observations.

FIGURE 7

NONLINEAR PWF: DECOMPOSITION OVER AGE
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

effect of optimism is constant. Therefore, lack of cognition instead of optimism plays an increasingly important role for the observed overestimation of SSBs.

5.4. *From Structural to Reduced Form Approaches.* Our identification of these mechanisms partially rests on our structural interpretation of the data combined with the nonlinear functional form assumption on the PWF. As sensitivity analyses we relax these assumptions in a stepwise manner, by considering a linear functional form and by subsequently moving toward reduced form approaches. As a linear model we consider a neoadditive PWF (Chateauneuf et al., 2007), which is linear for interior survival probabilities $OSP_{i,h,h+t} \in (0,1)$, thereby approximating the nonlinear model as

$$SSB_{i,h,h+t} = \left(1-\theta_h^l\right)\left(1-\xi_h^l\right) + \xi_h^l OSP_{i,h,h+t}, \tag{9}$$

where $\xi_h^l \in [0,1]$, $\theta_h^l \in [0,1]$ are parameters that are the analogues to parameters ξ_h and θ_h of the nonlinear specification in (4). To see this, observe that ξ_h^l controls the slope of the function, whereby for $\xi_h^l = 1$ the line in (9) corresponds with the 45-degree line; it can thus be interpreted as a measure of likelihood insensitivity. Likewise, $1-\theta_h^l \in [0,1]$ determines the intersection of (9) with the 45-degree line, whereby the intersection moves down when θ_h^l increases; it can thus be interpreted as a measure of relative pessimism.

Next, as for the nonlinear model let $m(h) = h+t$ and use (5) in (9) to get

$$SSB_{i,h,m(h)} = ((1-\theta_0) - \theta_1 \cdot o_{h-2})((1-\xi_0) - \xi_1 \cdot c_{h-2}) + (\xi_0 + \xi_1 \cdot c_{h-2}) \cdot OSP_{i,h,m(h)}. \tag{10}$$

Now, let $OSP_0 = 1-\theta_0$ and observe that the decomposition analogous to (7) is

$$SSB^b_{i,h,m(h)} = OSP_0 \cdot (1-\xi_0) + \xi_0 \cdot OSP_{i,h,m(h)}, \tag{11a}$$

$$SSB^{bc}_{i,h,m(h)} = SSB^b_{i,h,m(h)} + \xi_1 \cdot c_{h-2} \cdot \left(OSP_{i,h,m(h)} - OSP_0\right), \tag{11b}$$

$$SSB_{i,h,m(h)} = SSB^{bc}_{i,h,m(h)} - \theta_1 \cdot (1-\xi_0) \cdot o_{h-2} + \theta_1 \cdot \xi_1 \cdot (c_{h-2} \cdot o_{h-2}), \tag{11c}$$

and notice from (11a) and (11b) that—unlike in the nonlinear model—the intersection of SSB^b with SSB^{bc} is exactly on the 45-degree line at $OSP_0 = 1-\theta_0 = SSB_0$. Also observe from (11b) that the "pure" (i.e., ignoring interactions with the motivational variable optimism) marginal effect of an increase of cognitive weakness at a given OSP is $\xi_1(OSP - OSP_0)$. For $\xi_1 < 0$, we find that increasing cognitive weakness gives rise to stronger underestimation for $OSP > OSP_0$, and to stronger overestimation for $OSP < OSP_0$, just as in the nonlinear model. Likewise, from (11c) the marginal effect of an increase of optimism is given by $\theta_1(1-\xi_0)$, and we hence expect that $\theta_1(1-\xi_0) > 0$.

The reduced form specification follows from rewriting (10) as

$$\begin{aligned} SSB_{i,h,m(h)} = {} & \beta_0 + \beta_1 \cdot OSP_{i,h,m(h)} + \beta_2 \cdot \left(c_{h-2} \cdot OSP_{i,h,m(h)}\right) + \\ & \beta_3 c_{h-2} + \gamma_1 o_{h-2} + \gamma_2 \cdot (o_{h-2} \cdot c_{h-2}), \end{aligned} \tag{12}$$

where the structural model parameters map into the regression coefficients by

$$\beta_0 = OSP_0(1-\xi_0),\ \beta_1 = \xi_0,\ \beta_2 = \xi_1,\ \beta_3 = -OSP_0\xi_1,\ \gamma_1 = -\theta_1(1-\xi_0),\ \gamma_2 = \theta_1\xi_1. \tag{13}$$

Since the reduced form does not exactly identify all parameters of the structural model—there are six parameters in the reduced form and four parameters in the structural model—we impose at the estimation the additional restrictions implied by (13) of

$$\beta_3 = -\frac{\beta_0 \cdot \beta_2}{1-\beta_1} \quad \text{and} \quad \gamma_2 = -\frac{\beta_2}{1-\beta_1}\gamma_1. \tag{14}$$

TABLE 4

LINEAR MODELS: THE EFFECTS OF COGNITION AND MOTIVATIONAL MEASURES ON SUBJECTIVE SURVIVAL BELIEFS

	(1) Restricted Model	(2) Simple OLS Model	(3)
Constant (β_0)	0.090	−0.007	−20.194
	[0.068; 0.115]	[−0.082; 0.068]	[−37.429; −3.370]
OSP (β_1)	0.624	0.607	0.537
	[0.552; 0.691]	[0.535; 0.677]	[0.426; 0.637]
OSP × Cog. Weak. (β_2)	−0.384	−0.302	−0.359
	[−0.574; −0.202]	[−0.509; −0.114]	[−0.552; −0.174]
Cognitive Weakness (β_3)	0.092	0.319	0.205
	[0.047; 0.141]	[0.139; 0.509]	[0.032; 0.379]
Optimism (γ_1)	0.131	0.234	0.162
	[0.099; 0.169]	[0.139; 0.325]	[0.071; 0.241]
Optimism × Cog. Weak. (γ_2)	0.134	−0.118	0.041
	[0.070; 0.203]	[−0.349; 0.156]	[−0.184; 0.299]
OSP_0	0.242	0.923	0.589
	[0.195; 0.291]	[0.405;1.250]	[0.156; 1.065]
Additional Controls	No	No	Yes
AIC	4,102	4,087	3,522
Observations	11,954	11,954	11,898

NOTES: Column 2 shows estimates of the linear model, column 3 shows estimates of the linear model without restriction (14), column 4 adds control variables. Bootstrapped 95% confidence intervals in brackets (1,000 replications, computed with percentile method). AIC: Akaike (1973) information criterion. OSP_0 defined as the intersection between SSB^b and SSB^bc. Sample size: 11,954 observations.
SOURCE: Own calculations, Health and Retirement Study (HRS).

The results from estimating (12) subject to the restrictions (14) are summarized as Model 1 in Table 4. All coefficient estimates are of the expected sign and significantly different from zero. The decomposition shows very similar patterns to Figure 7. Relative to those results, the effects of cognitive weakness are slightly downward shifted—initially there is still a downward bias of about −5%p, but in the oldest age group there is an upward bias of only 3%p—and the effect of optimism is upward shifted—now leading to a constant overestimation by roughly 13%p, cf. Figure A.5 in Appendix A.6.

We next interpret (12) as a reduced form specification and correspondingly estimate it without imposing the additional restrictions in (14). Apart from the constant this mainly affects our estimate of the effects of cognitive weakness and of OSP_0, which we again identify as the intersection point of SSB^c with SSB^{bc}. As we show in the decomposition in Figure A.5 in Appendix A.6 not imposing the restrictions in (14) mainly implies that we lose an anchor of the base bias so that the additional effects of cognitive weakness are now significantly upward shifted, ranging from $+4\%p$ to $+13\%p$. Although thus the level of the cognitive weakness effect is shifted, the overall differential effect over the life cycle of about 9%p is unchanged relative to the baseline specification so that increasing cognitive weakness leads to increasing overestimation. We also again find that relative optimism induces to a relatively constant upward shift, now of about 14%p.

Finally, we add the same control variables to the RHS of (12) as used in the estimation of our OSP, cf. Table A.2 of Appendix A.2. The relevance of control variables can be motivated by the notion that in a decision situation under uncertainty individuals may only be imperfectly informed about the respective OSP and instead condition their assessment of their SSB also on other variables. A related interpretation is based on formal statistical learning models according to which individuals learn their individual OSP by obtaining more information. This suggests that they base their survival beliefs on the OSP and additional variables as well as cognitive and motivational factors. Thus, adding control variables can be interpreted as a *snapshot* of a reduced form learning model, as in Viscusi (1985) and Smith et al. (2001b), and for biased beliefs in Ludwig and Zimper (2013) and Groneck et al. (2016). We include the same set of control variables that we use for the estimation of the objective survival beliefs. Results are reported as Model 3 in Table 4 and estimates for the control variables are contained in Table A.5 of Appendix A.7 (which are of the expected sign and are in line with findings in the literature). As the main effect, the estimate on the objective survival rate decreases, which reflects that now the additional controls soak up objective survival information. All other coefficient estimates are close to those from the structural Model 1, respectively, the confidence intervals overlap. With the additional control variables, we now fit average subjective beliefs by age in the full model quite well. Otherwise, the decomposition in Figure A.5 is similar to what we have seen for the structural Model 1 and our baseline results in Figure 7. The effects of cognitive weakness range from −2% to 7%—thus a similar range as before—and the effect of optimism is roughly constant at 12%p.

Additional robustness analyses (i) with ad hoc reduced form specifications, (ii) for quantile regressions, (iii) with respect to focal point answers, and (iv) for an extension of the nonlinear model are presented in our Online Appendix. These findings confirm our main results in a sense that the quantitative contribution of cognition is monotonically increasing over the life cycle with a differential effect of about 9%p and a roughly constant overestimation through optimism. If anything we find that the effect of optimism is decreasing with age (in robustness analysis (iv)).

6. CONCLUDING DISCUSSION ON ECONOMIC IMPLICATIONS

This article analyzes the effects of cognitive weakness and optimism on the formation of SSBs in the HRS through the lens of inverse S-shaped PWFs. Our main finding suggests that the age patterns of biases in survival beliefs documented in many studies are driven by increasing cognitive weakness inducing a monotonically increasing bias in survival misconception from an underestimation of survival beliefs by −5%p at age 65–69 to an overestimation by 5%p at age 85−89. On the contrary, the quantitative effect of optimism is roughly constant leading to an overestimation by about 10%p for all age groups. Thus, cognitive weakness instead of optimism becomes with age an increasingly important contributor to overestimation of survival chances.

What are the economic implications of our findings? If we were to use our parameter estimates in a life-cycle model of consumption and savings in order to calibrate SSBs we would conclude with similar findings as in Groneck et al. (2016) and accordingly report that life-cycle models with biased survival beliefs substantially improve the model fit to data on life-cycle asset holdings, relative to a rational expectations benchmark. The key mechanism is that the overestimation of survival beliefs in old age leads households to hold on to their assets and thus partially resolves the old-age dissaving puzzle. However, psychological attitudes may also bias other beliefs—for example, income expectations (Dominitz and Manski, 1997; Rozsypal and Schlafmann, 2017)—and may directly affect pure time discounting through cognitive processes (Binswanger and Salm, 2017; Gabaix and Laibson, 2017). If cognitive weakness leads to an increase of presence bias as in the theoretical work by Gabaix and Laibson (2017) then this constitutes an opposing force on effective time discounting to the one induced by increasing overestimation of survival beliefs. Related, optimism may induce households to overestimate their retirement incomes leading to lower savings, which is again a countervailing force to the effect of optimism on effective time discounting through the overestimation of survival beliefs. We therefore caution against the use of SSBs in life-cycle models of consumption and savings to study their implications for savings behavior in a ceteris paribus manner. Our results rather suggest that an important avenue of future research is to apply our methods to other expectations data, to study the empirical relationship between cognition and pure time discounting and to explore simultaneously the consequences of various psychological mechanisms in calibrated life-cycle models.

TABLE A.1
COGNITIVE AND MOTIVATIONAL VARIABLES

	Min	Max	Mean	*SD*	*N*
Cognitive Variable					
Cognitive Weakness, normalized	0	1	0.387	0.149	48,081
Lagged Cognitive Weakness, normalized	0	1	0.377	0.142	42,445
Motivational Variable					
Dispositional Optimism, normalized	0	1	0.695	0.188	21,182
Lagged Dispositional Optimism, normalized	0	1	0.700	0.188	16,532

NOTES: This table summarizes the sample moments our measure of cognitive weakness and dispositional optimism.
SOURCE: Own calculations, Health and Retirement Study (HRS).

TABLE A.2
CONTROL VARIABLES FOR THE HAZARD MODEL

	Mean	Std. Dev.	Obs.	Min	Max
Average Survival Probability					
Average 12-year survival probability (Lee–Carter)	58.89	23.93	21,435	0	83
Demographic Variables					
Age	70.68	7.07	21435	65	98
Age^2	5,046	1,066	21435	4,225	9,604
Age^3	364193	121798	21435	274625	941192
Male	0.43	0.50	21435	0	1
Married/Partnered	0.64	0.48	21421	0	1
Health Variables					
Self-rated health (excellent)	0.10	0.30	21424	0	1
Self-rated health (very good)	0.26	0.44	21424	0	1
Self-rated health (good)	0.32	0.47	21424	0	1
Self-rated health (fair)	0.22	0.41	21424	0	1
Smoke (ever)	0.58	0.49	21251	0	1
Smoke (now)	0.13	0.33	21375	0	1
Drink (ever)	0.46	0.50	21432	0	1
Limitations: ADL index	0.24	0.65	21435	0	3
Limitations: Mobility index	1.19	1.56	18644	0	5
Limitations: Muscle index	1.29	1.34	19434	0	4
Cognitive weakness (normalized)	0.36	0.15	19113	0	1
Ever had high blood pressure	0.53	0.50	21405	0	1
Ever had diabetes	0.19	0.39	21406	0	1
Ever had cancer	0.13	0.34	21390	0	1
Ever had lung disease	0.09	0.29	21411	0	1
Ever had heart disease	0.25	0.43	21408	0	1
Ever had stroke	0.09	0.29	21415	0	1

NOTES: Average survival probability in percent, estimated with the Lee–Carter procedure using data from the HMD and SSA life tables. Category health variables (ADL, Mobily, and Muscle indices) scaled such that higher values imply worse health conditions. *Ever had*-variables indicate diagnosed cases.

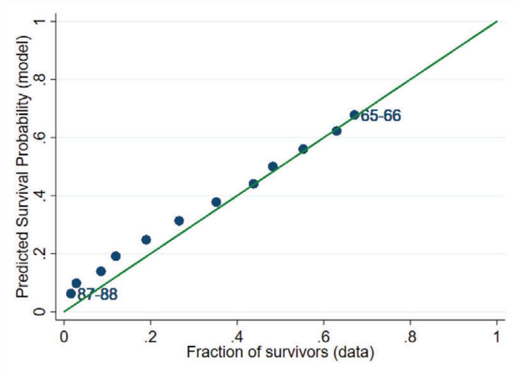
APPENDIX A

A.1 *Psychological and Cognitive Measures in the HRS.* The main data used in this article are the HRS. The HRS is a national representative panel study on a biennial basis, see Juster and Suzman (1995) for an overview. [19] Whenever possible, we make use of the so-called RAND HRS Longitudinal Files provided by the he RAND Center for the Study of Aging. The RAND files are user-friendly files derived from all waves of the HRS. It contains cleaned, processed, and imputed variables with consistent and intuitive naming conventions.

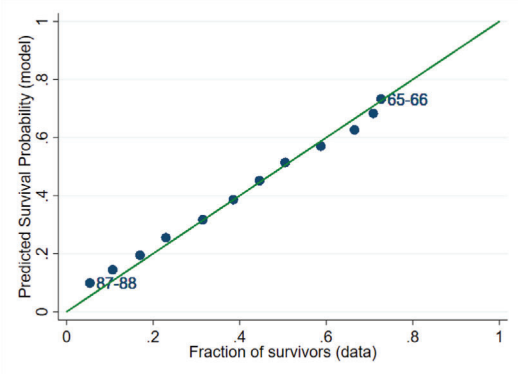
The main purpose of the HRS is to contribute a rich panel data set to the research of retirement, health insurance, saving, and economic well-being. Since 2006 (Wave 8), the HRS is

[19] The survey is administered by the Institute for Social Research (ISR) at the University of Michigan and mainly funded by the National Institute of Aging (NIA).

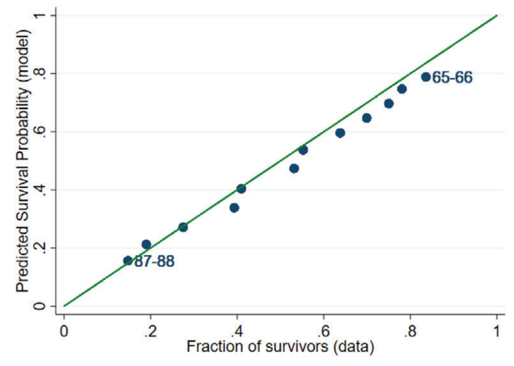
(a) 14-year survival



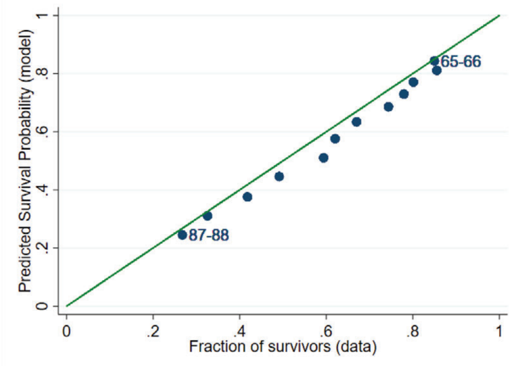
(b) 12-year survival



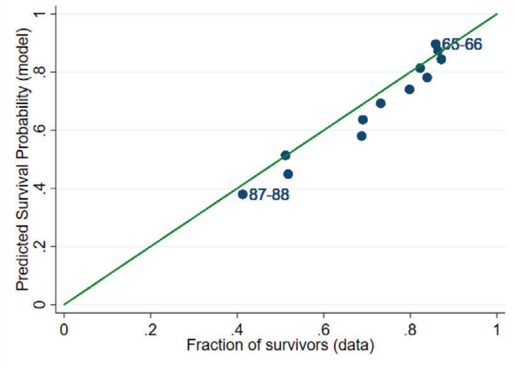
(c) 10-year survival



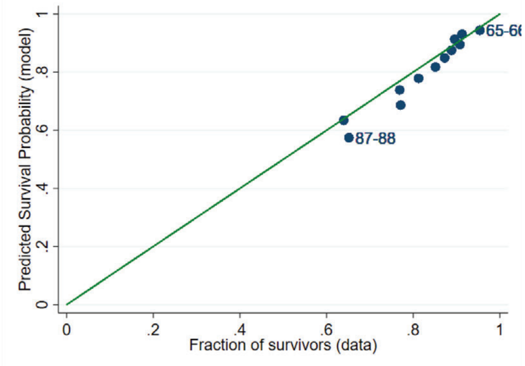
(d) 8-year survival



(e) 6-year survival



(f) 4-year survival



NOTES: Estimated average survival probabilities for different two-year agebins (blue dots) against the fraction of survivors from the data. The 45-degree line is depicted in green. Panels show different time intervals ranging from 14- to 4-year survival probabilities, where the final year is always 2014, for example, the 14-year survival probability corresponds to the fraction of people surviving between 1998 and 2014. Sample sizes by time interval: 6,156 (14 years), 6,369 (12 years), 6,637 (10 years), 7,077 (8 years), 7,921 (6 years), 8,025 (4 years).

FIGURE A.1

OBJECTIVE SURVIVAL PROBABILITIES: MODEL VS. DATA
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

TABLE A.3
MIXED PROPORTIONAL HAZARD MODEL

Average Survival Probability		
Average 12-year survival probability (Lee–Carter)	−0.0206***	(−2.75)
Demographic Variables		
Age	2.792***	(3.14)
Age^2	−0.036***	(−3.02)
Age^3	0.0002***	(2.99)
Male	0.125	(1.50)
Married/Partnered	−0.069**	(−2.12)
Health Variables		
Self-rated health (excellent)	−0.489***	(−6.57)
Self-rated health (very good)	−0.449***	(−7.29)
Self-rated health (good)	−0.307***	(−5.61)
Self-rated health (fair)	−0.186***	(−3.67)
Smoke (ever)	0.291***	(8.72)
Smoke (now)	0.584***	(13.50)
Drink (ever)	−0.160***	(−5.21)
Limitations: ADL index	0.027	(0.94)
Limitations: Mobility index	0.150***	(10.58)
Limitations: Muscle index	−0.061***	(−4.13)
Cognitive weakness	0.978***	(9.17)
Ever had high blood pressure	0.126***	(4.24)
Ever had diabetes	0.378***	(10.16)
Ever had cancer	0.322***	(8.23)
Ever had lung disease	0.530***	(11.33)
Ever had heart disease	0.311***	(9.56)
Ever had stroke	0.173***	(3.61)
Constant	−76.52***	(−3.60)
Observations	15,373	
Log Likelihood	−10,172	
Duration dependence parameter α	1.644**	
Variance of Unobserved Heterogeneity	0.064**	

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
NOTES: The duration dependence parameter α refers to the baseline hazard $\lambda_0(t) = \alpha t_i^{\alpha-1}$. The unobserved heterogeneity estimate refers to the variance of the Gamma distribution of unobserved heterogneity. Limitations variables are defined such that higher values imply higher limitations. Positive (negative) signs of the coefficients imply a positive (negative) impact on the hazard of dying.

complemented by a rich set of psychosocial information. These data are collected in each biennial wave from an alternating (at random) 50% of all core panel participants who were visited for an EFTF.[20] Thus, longitudinal data are available in four-year intervals and therefore the first panel with psychosocial variables is provided in 2010.

Optimism. From Wave 8 onward, the HRS contains measures on optimism and pessimism, in section LB, the leave-behind questionnaires. Measures on *dispositional optimism* are derived from the same statements as in the well-known LOT-R.

The following six questions are asked to the respondent:

Please say how much you agree or disagree with the following statements

(1) If something can go wrong for me it will.

[20] In 2006 (Wave 8) respondents were sent an additional questionnaire in case they were part of this random 50% subsample—provided they were alive and either they or a proxy completed at least part of the interview in person. In 2008 (Wave 9), respondents who were not selected for the EFTF interview in 2006 were automatically selected in 2008. As in 2006 they were sent a questionnaire in case they were alive or a proxy completed at least part of the interview in person. In 2010 (Wave 10), respondents who had completed the EFTF interview in 2006 were again chosen to participate in this mode of data collection. As a result the first panel is available in 2010.

TABLE A.4
IMPACT OF PSYCHOLOGICAL VARIABLES IN THE HAZARD MODEL

Psycho and Cognitive Variables				
Optimism			−0.591	(−1.30)
Cognitive weakness	1.300*	(1.83)	1.107*	(1.69)
Average Survival Probability				
Average 12-year survival probability (Lee–Carter)	−0.181	(−1.12)	−0.174	(−1.11)
Demographic Variables				
Age	25.320	(1.33)	23.66	(1.29)
Age^2	−0.342	(−1.33)	−0.319	(−1.29)
Age^3	0.002	(1.35)	0.001	(1.30)
Male	−0.770	(−0.57)	−0.726	(−0.55)
Married/Partnered	−0.075	(−0.36)	−0.081	(−0.43)
Health Variables				
Self-rated health (excellent)	−1.355***	(−2.663)	−1.281***	(−2.59)
Self-rated health (very good)	−1.658***	(−4.57)	−1.614***	(−4.77)
Self-rated health (good)	−1.056***	(−3.75)	−1.022***	(−3.89)
Self-rated health (fair)	−0.568**	(−2.50)	−0.570**	(−2.55)
Smoke (ever)	0.571**	(2.46)	0.553**	(2.41)
Smoke (now)	0.486*	(1.92)	0.466**	(2.46)
Limitations: ADL index	0.400***	(2.69)	0.388***	(2.69)
Drink (ever)	−0.311*	(−1.70)	−0.298	(−1.64)
Limitations: Mobility index	0.147*	(1.68)	0.140*	(1.83)
Limitations: Muscle index	−0.160*	(−1.68)	−0.155*	(−1.83)
Ever had high blood pressure	−0.149	(−0.80)	−0.145	(−0.79)
Ever had diabetes	0.484***	(2.64)	0.465***	(2.58)
Ever had cancer	1.086***	(4.35)	1.080***	(6.13)
Ever had lung disease	0.828***	(3.78)	0.835***	(4.17)
Ever had heart disease	0.380**	(2.10)	0.380**	(2.14)
Ever had stroke	0.240	(0.95)	0.214	(0.87)
Constant	−604.4	(−1.34)	−564.7	(−1.30)
Observations	2,108		2,108	
Log Likelihood	−485.9		−485.1	
Duration dependence parameter (α)	1.459		1.457	
Variance of Unobserved Heterog.	0.029		0.000	

t statistics in parentheses
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$
NOTES: Comparison of model (1) without optimism and model (2) including optimism on a smaller subsample of individuals where optimism is available.

(2) I'm always optimistic about my future.
(3) In uncertain times, I usually expect the best.
(4) Overall, I expect more good things to happen to me than bad.
(5) I hardly ever expect things to go my way.
(6) I rarely count on good things happening to me.

The answer scale is given by: 1 = strongly disagree, 2 = somewhat disagree, 3 = slightly disagree, 4 = slightly agree, 5 = somewhat agree, and 6 = strongly agree.

We follow the documentation report, cf. Smith et al. (2017), to construct a measure of optimism from these questions. To this end, we recode Questions 1, 5, and 6 by reversing the answer pattern. Then, we build a six-item optimism score by averaging the scores across all questions. We set the score to missing if there is more than half of the answers missing.

In some studies, *optimism* and *pessimism* are measured separately, that is, respondents are asked questions with negative connotations (pessimism) or positive connotations (optimism). The reason for separate measures is that these two concepts tend to display bidimensionality (Herzberg et al., 2006). In our sample, the two distinct measures (optimism consisting of Questions 1, 5, and 6 and pessimism consisting of Questions 2, 3, and 4) have their peak at 1 (pessimism) and 5 and 6 (optimism) implying no strong bipolarity. Hence, for the sake of

TABLE A.5

LINEAR MODEL: THE EFFECTS OF COGNITION AND MOTIVATIONAL MEASURES ON SUBJECTIVE SURVIVAL BELIEFS: PARAMETER ESTIMATES ON CONTROL VARIABLES

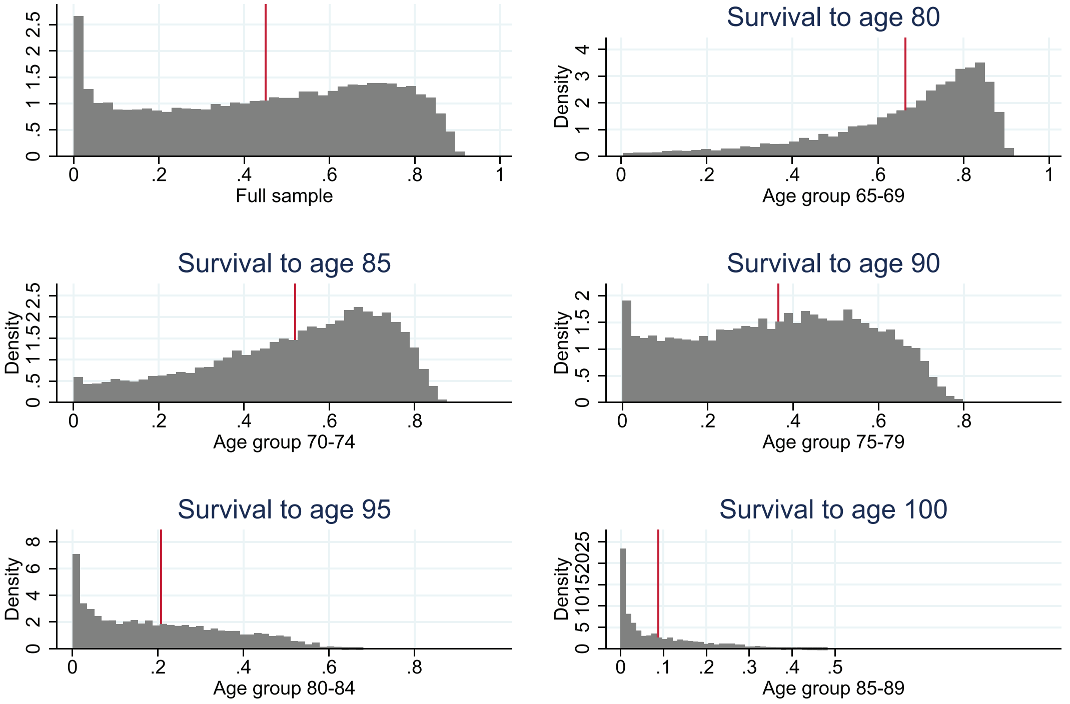
	Coefficient	CI−	CI+
Average Survival Probability			
Average 12-year survival probability (Lee–Carter)	−0.003	−0.008	0.002
Demographic Variables			
Age	0.817	0.128	1.514
Age^2	−0.011	−0.020	−0.002
Age^3	0.000	0.000	0.000
Male	−0.020	−0.070	0.029
Married/Partnered	−0.020	−0.033	−0.005
Health Variables			
Self-rated health (excellent)	0.216	0.179	0.252
Self-rated health (very good)	0.154	0.123	0.185
Self-rated health (good)	0.108	0.082	0.138
Self-rated health (fair)	0.052	0.026	0.080
Smoke (ever)	0.035	0.019	0.052
Smoke (now)	0.011	−0.016	0.036
Drink (ever)	0.004	−0.011	0.018
Limitations: ADL index	−0.004	−0.018	0.010
Limitations: Mobility index	0.007	0.000	0.014
Limitations: Muscle index	−0.005	−0.012	0.001
Cognitive weakness	0.105	0.038	0.164
Ever had high blood pressure	−0.004	−0.018	0.010
Ever had diabetes	0.028	0.009	0.046
Ever had cancer	0.002	−0.016	0.019
Ever had lung disease	0.035	0.011	0.057
Ever had heart disease	0.007	−0.008	0.023
Ever had stroke	0.033	0.009	0.054
Constant	−20.194	−37.429	−3.370
Observations	11,898		

NOTES: Column 2 shows the point estimates, columns 3 and 4 the respective bounds of 95% confidence intervals (CI− and CI+), which are calculated with the percentile method (1,000 replications).

simplicity, we follow the literature and treat optimism and pessimism as one dimensional, cf. Carver and Scheier (2014).

Cognitive weakness. We take the cognitive functioning total score ("RxACOGTOT") from RAND, which summarizes a set of cognitive functioning measures into one index. One set of questions include immediate and delayed word recall. The variables count the number of words the respondent was able to recall correctly from a list of 10 words directly and again after a delay of 5 minutes. Hence, the respondent can have a maximum score of 20 if all words are recalled. The second set of questions is a so-called mental status summary. This measure includes the serial 7 test, counting backwards, and naming tasks. The serial 7 test asks the individual to subtract 7 from the prior number, beginning with 100 for five trials. Counting backward asks the respondent to count backward for 10 continuous numbers from 20 and 86, respectively. The naming tasks comprise of correctly stating today's date, the name of the president and vice president, as well as naming certain objects (a cactus and scissors), and to give definitions of five given words (e.g., repair, fabric, domestic, remorse, plagiarize). The maximum score for the mental status summary is 15. The total cognition score from RAND adds up the scores from the total word recall and from mental status summary, resulting in a range of 0–35. We reverse the RAND score by subtracting the cognitive ability score from the maximal achievable value 35. As a result, a higher score indicates higher cognitive weakness.

We normalize both variables to the [0; 1] interval; a summary of the variables is given in Table A.1.



NOTES: The red vertical line indicates the average objective survival probability. Sample size: 38,846 observations. Sample size by age group: 10,985 (age 65–69), 11,268 (70–74), 8,583 (75–79), 5,279 (80–84), 2,731 (85–89).

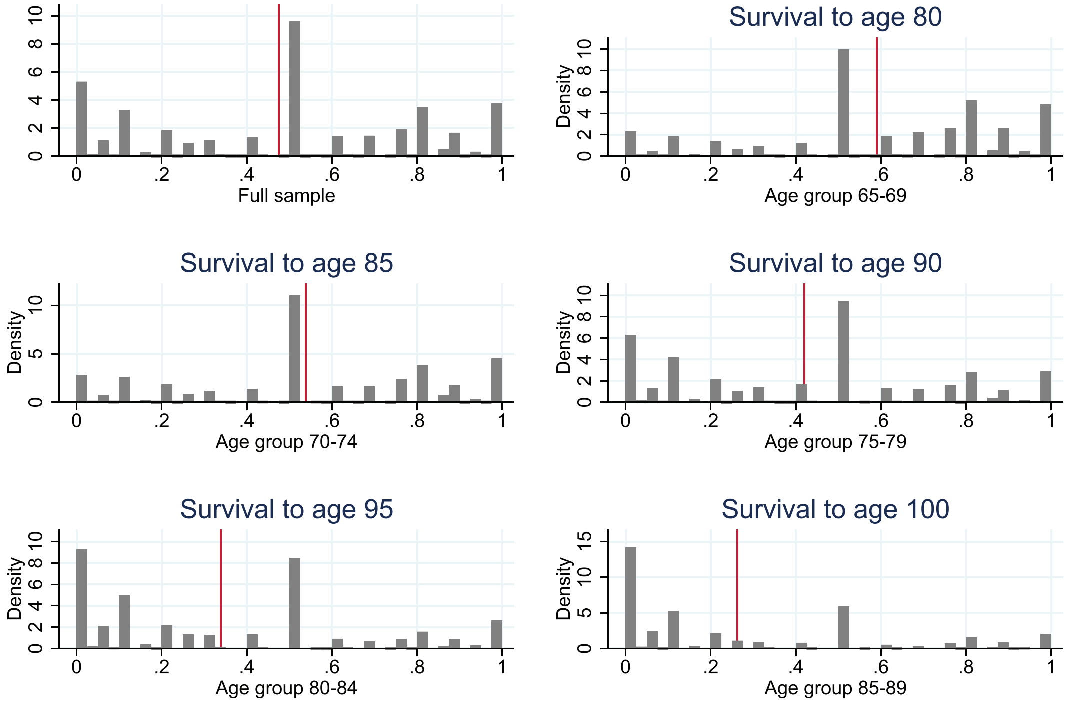
FIGURE A.2

HISTOGRAMS OF OSPS
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

A.2 *Estimation of OSPs.* We estimate the OSP as a counterpart for the subjective belief to survive (SSB) to a certain target age. We use the HRS to estimate conditional hazard rates for mortality. These hazards are estimated conditional on various characteristics of the individual and on a trend-adjusted average OSP. The hazard rates are used to compute individual specific OSPs.

We use nine waves of the HRS (years 1998–2014).[21] We restrict our sample to individuals older than 64 and younger than 99. We choose all observations when respondents enter the HRS (which can be at different waves) within our observed time period. The average age is 70.7 and 43% of them are males; 42% of individuals die within the observed time interval of at most 17 years. The covariates used for the estimation are summarized in Table A.2. We use demographic variables (age, gender, and marital status), and a wide set of health variables. We choose four sets of health variables. First, we use self-reported health, a measure where the individual can rate its general health on a scale of one (= "excellent") to five (= "poor"). We use indicator variables for each value where the reference group are individuals with a value of one (= "excellent"). Second, we construct three indices measuring functional limitations. The ADLs index collects information whether the individual is able to bath, dress and eat alone. The Mobility index counts limitations with certain kinds of mobility (walking across a room, walking several/one blocks, climbing several/one flights of stairs). The limitation in muscle index counts whether the individual is able to sit for two hours, get up from a chair, being able to stoop, and/or push/pull large objects. All categorial variables are scaled such they can be interpreted as a higher number representing more limitations. Finally, we take

[21] We exclude earlier waves due to consistency problems in how some variables were measured.



NOTES: The red vertical line indicates the average subjective survival belief. Sample size: 38,846 observations. Sample size by age group: 10,985 (age 65–69), 11,268 (70–74), 8,583 (75–79), 5,279 (80–84), 2,731 (85–89).

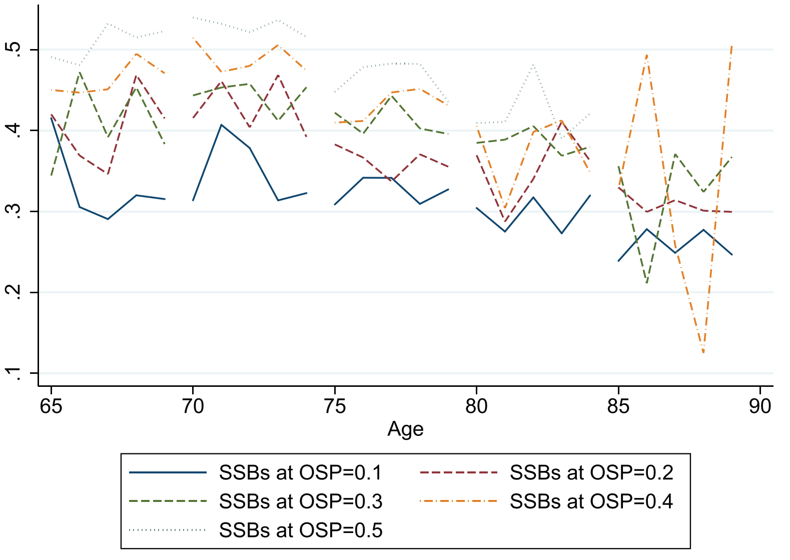
FIGURE A.3

HISTOGRAMS OF SSBS
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

cognitive limitations into account (see further below for a detailed definition of this variable). Third, we take lifestyle variables into account (ever smoke, smoke now, ever drink). Fourth, we take a set of variables indicating the incidence whether the respondent ever had a certain (chronic) disease. In addition to demographic and health variables, we include estimated average survival probability by gender and year of birth. This probability is estimated using the Lee and Carter (1992) procedure employing the life tables from the Human mortality database.[22] Through this we construct and predict cohort life tables, which in contrast to the cross-sectional life tables appropriately takes into account predicted trends in life-expectancy. For all control variables, we take the first observation when the individual enters the sample. Hence, we treat the covariates as constant over time. The final number of observations used in the estimation reduces to 15,373 due to many missing values for the health variables, see Table A.2.

A.3 *Descriptive Statistics on OSPs and SSBs.* Figure A.1 compares various average predicted survival probabilities from our model with the fraction of survivors in the data, ranging from 4- to 14-year survival probabilities. Note, that in our main analysis, we are mainly concerned with 10- to 15-year survival probabilities. The model fits the data well indicated by the points being close to the 45-degree line. It is important to note, though, that our model accounts for right-censoring and is therefore not meant to perfectly match the (right-censored) data.

[22] The Lee–Carter procedure decomposes mortality into a vector of age-specific constants and age-specific drift terms. These trends are then used to predict future survival probabilities until age 2090 to complete life tables on the basis of these estimates.



NOTES: This figure shows average SSBs over age by OSP bins of [0.05,0.15), [0.15 – 0.25), [0.25 – 0.35), [0.35 – 0.45), and [0.45 – 0.55). Sample size by age groups and OSP bins (in ascending order): 180,256,416,598 (age 65–69), 524,664,908,1267 (age 70–74), 992,1045,1212,1373 (age 75–79), 1138,1002,831,637 (age 80–84), 747,404,197,50 (age 85–89).

FIGURE A.4

SUBJECTIVE SURVIVAL BELIEFS BY AGE HOLDING CONSTANT OBJECTIVE SURVIVAL PROBABILITIES
SOURCE: OWN CALCULATIONS, HEALTH AND RETIREMENT STUDY (HRS).
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

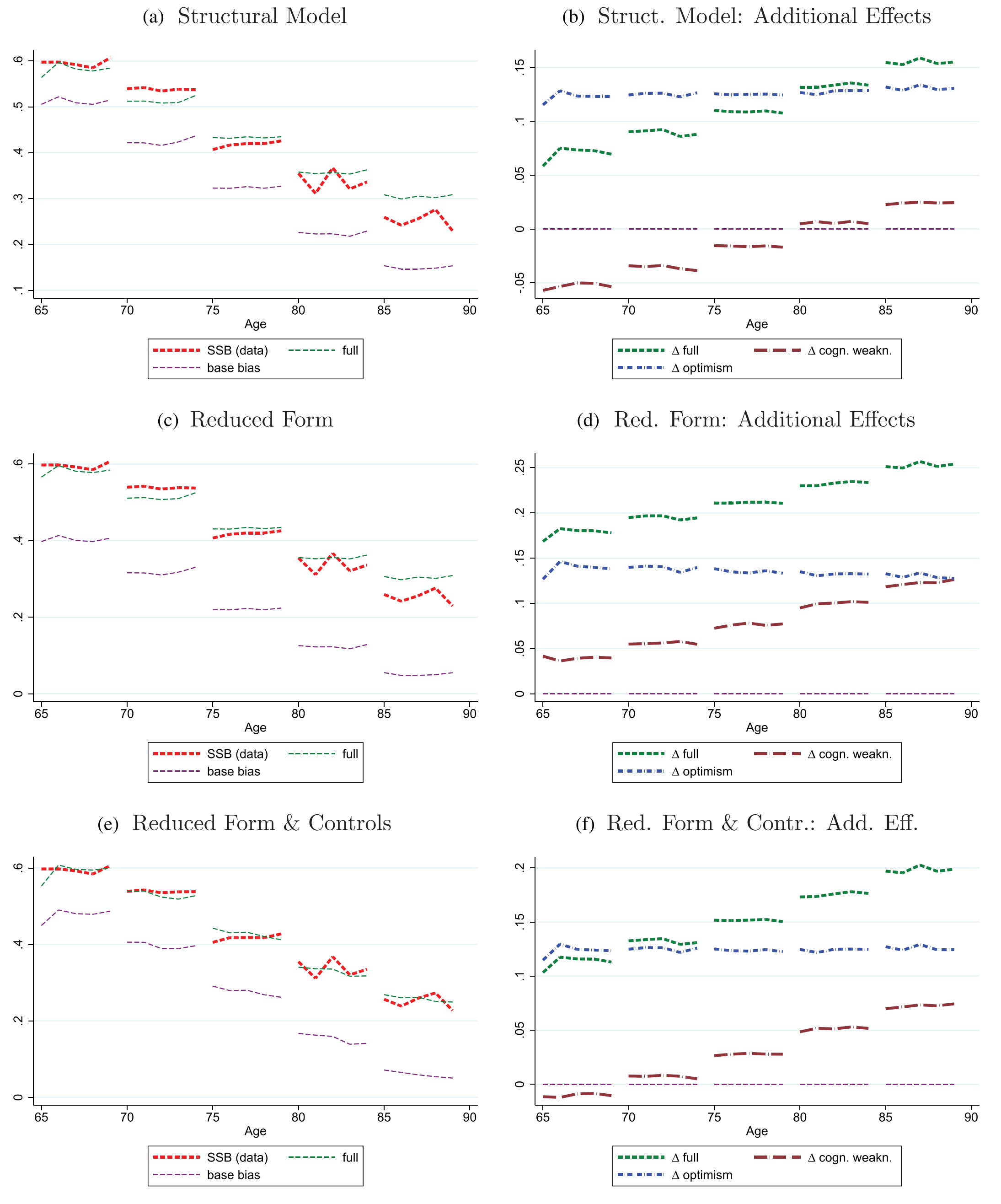
Figure A.2 shows the distribution of OSPs for the full sample and each interview age group. Each subfigure also contains a red vertical line indicating the average OSP for the respective age group. The histograms reveal that there is a significant dispersion of OSPs.

Figure A.3 shows the corresponding distributions of SSBs. Average SSBs decrease as we move up across target age groups, as with OSPs. However, the movement is not as pronounced as for the OSPs and the difference in the averages depicted by the red lines in both figures just reflects the facts shown in Figure 1. Second, there are focal point answers at SSBs of 0, 0.5, and 1. Observe that the fraction of individuals providing a focal point answer at 1 decreases, whereas the fraction giving answer 0 increases when the target age increases. This indicates that focal point answers do have information content that goes beyond simple heuristics that individuals may apply when being confronted with such complicated questions about survival prospects.

A.4 *Bootstrap.* Standard errors of the parameters of our regressions have to be corrected in order to account for the estimation variance of OSPs. We accommodate this by implementing a two-sample bootstrap procedure with 1,000 replications to estimate the standard errors of our coefficient estimates.[23] In this procedure we correct for the estimation variance in OSPs as follows.[24] In each bootstrap replication we (i) draw a sample with replacement from the HRS sample used to estimate OSPs, (ii) estimate the OSPs, (iii) draw a sample with replacement from the cross-sectional sample used for regression analysis, and (iv) perform regression analysis. Based on the resulting estimates we compute standard errors with the percentile method.

[23] We discard 8.5% of the bootstrap iterations that did not converge.

[24] Note, that our two samples are both based on the HRS dataset. The first sample is based on the sample used to estimate the OSPs and the second sample is used in the overall regression analyses.



NOTES: Sample averages of predicted subjective survival beliefs according to Equations (12) and (11) by age; "full": $\widehat{SSB}$; "base bias": $\widehat{SSB}^b$; "Δ full": $\widehat{SSB} - \widehat{SSB}^b$; "Δ cogn. weakn.": $\widehat{SSB}^{bc} - \widehat{SSB}^b$; "Δ optimism": $\widehat{SSB} - \widehat{SSB}^{bc}$.

FIGURE A.5

NEO-ADDITIVE (LINEAR) PWF: DECOMPOSITION OVER AGE
[COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

A.5 *Additional Results: Subjective Beliefs by OSP.* Figure A.4 shows SSBs by bins of OSPs over age. It shows that SSBs for given OSPs are decreasing in age from interview age group 70–74 on.

A.6 *Additional Results: Decomposition of Linear Model.* Figure A.5 shows the decomposition of the linear model (12). Panels (a) and (b) are for the structural interpretation of the linear model additionally imposing the restrictions (14). Panels (c) and (d) are the respective results where we give Equation (12) a full reduced form interpretation by accordingly not imposing these restrictions. Finally, Panels (e) and (f) show the results of that reduced form model with additional control variables.

A.7 *Additional Results: Control Variables in Linear Model.* Table A.5 shows the results of our estimation for the control variables.

Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Table A.10: OLS Estimates
Figure A.13: Quantile Regression: Coefficient Estimates
Figure A.14: Non-Linear PWFs: Excl. All Focal Point Answers
Figure A.15: Non-Linear PWFs: Excl. Focal Point Answers at 0.5
Figure A.16: Non-Linear PWF: Decomposition over Age, Excluding Focal Point Answers at 0.5
Table A.11: Focal Point Answers, Marginal Effects of Logit Regression
Table A.12: OLS Regressions
Figure A.17: Decomposition of Non-linear PWFs with Equation (17)
Figure A.18: Non-Linear PWF with Equation (17)

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