A LOT SIZING PROBLEM IN REVERSE LOGISTICS FOR TWO ITEMS OF DIFFERENT QUALITY GRADES WITH AN IMPERFECT MANUFACTURING PROCESS, TIME VARYING DEMAND AND RETURN RATES

by

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EXECUTIVE SUMMARY

Natural resource scarcity has accelerated considerations of return logistics in manufacturing processes. Most supply chain designs now consider a closed-loop design where demand can be satisfied by both newly manufactured goods as well as remanufactured returns, allowing for maximum value creation over the entire life cycle of a product. This dissertation proposes two inventory systems, one a dynamic lot sizing model and the other a closed form solution. It is also assumed that some items fail during manufacturing and these items are treated as returns that can be remanufactured to satisfy one of two types of demand. Returns are remanufactured to one of two states such that items that may not be remanufactured to an as-good-as-new state of the first product can satisfy a secondary customer demand of a lower grade. Returns are constrained by expressing customer returns as a percentage of demand and items that fail during manufacturing as a percentage of the manufacturing batch. In the dynamic lot sizing model, the remanufacturing processes require some other components to be procured to bring the returned items back to either of the two states of reuse. A modified Wagner/Whitin model for the alternate application of remanufacturing and manufacturing for the satisfaction of the top range item demand and supplemented by a modified reverse Wagner/Whitin model for the remanufacturing of the lower variety items is derived to solve the dynamic lot sizing model proposed. The closed form solution considers that additional feedstock for the top range item can be procured in the case that the demand cannot be fully satisfied by the manufacturing and remanufacturing processes respectively. Both models aimed to minimize cost across a horizon and the total cost proves to be very sensitive to the manufacturing setup cost and the proportion of demand returned for remanufacturing in the case of the dynamic model and Type A serviceable holding cost and the Type A product yield rate from the manufacturing process in the case of the closed form solution.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. Introduction

1.1 Background

Natural resources, such as water, land, plants, animals, ecosystems, and minerals, drive the economy and various human activities. The effective measurement of resource use is vital in guaranteeing sustainable growth. Natural resource scarcity has been emphasized and studied extensively, since 1929, where the use of scarce resources originally gave organisations a competitive advantage, resulting in practical methods of responding to the negative aspects of natural resource scarcity not being developed for a long period (Brander, 2007; Friedrich, 1929; Krautkraemer, 1998; Kronenberg, 2008).

Resource-advantage (R-A) theory is used in the research of Bell et al., (2013) as a theoretical basis in examining closed-loop supply chains (CLSC) as a possible solution to natural resource scarcity. The aim of CLSC management is to maximise the value creation over the entire product life cycle (Guide and van Wassenhove, 2006a). In the late 1990s, European legislation required manufacturers to recover products to reduce or circumvent landfilling (Flapper et al., 2005). It was during this time that CLSC management gained international attention. CLSC management does not only deliver a cost reduction benefit due to reuse and remanufacturing, but there is a strategic aspect to it as well as it responds to the natural resource scarcity threat. Environmental conscious materials management and logistics is achievable with reuse. From an economic point of view, the environmental load is reduced, and exploitation of natural resources minimised through the return of used items in the manufacturing process (Imre, 2006).

Remanufacturing is the process in which a product is taken apart, cleaned, repaired, and reassembled for further use (Inc.com, 2019). Remanufacturing requires that the condition of the part be assessed to determine whether remanufacturing is worthwhile to restore the part to an as-good-as-new condition. Figure 1.1 illustrates an inventory system where demand can be satisfied from the manufacturing of new items or remanufacturing of returned items.

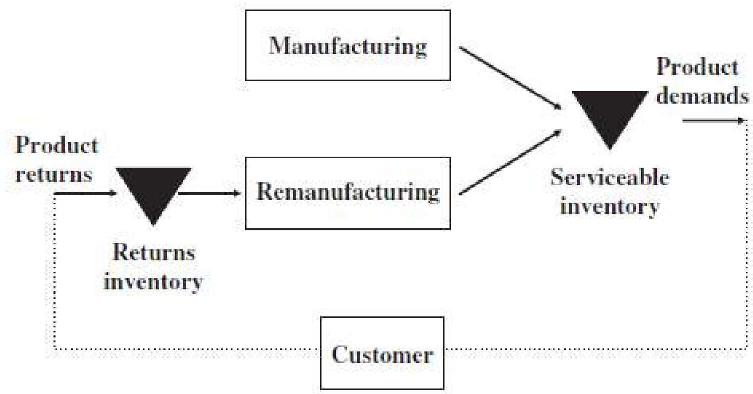

FIGURE 1.1 INVENTORY SYSTEM WITH REMANUFACTURING (TEUNTER ET AL., 2006)

Traditional production planning and inventory management models do not typically consider return and remanufacturing of an item to satisfy the same demand. Models that consider the use of remanufactured products to satisfy demand for a different product are even more uncommon. With sustainability now a key consideration in many industries due to the pressure on the environment and the natural resources it supports, recycling and remanufacturing is necessary. This is particularly so in an environment like the pulp and paper industry that draws heavily on many such natural resources: water, land, and fibre rich plants like wood. This has spurred the development of many processes that are able to reclaim used resources and process such back into the original form and grade of the products or into another form in which such may still be utilised even if it cannot be utilised for the topmost grade product.

At the start of the paper recycling process, returned paper is sorted into types and grades. The properties of paper depend on the qualities of the pulp from which it is made. Pulp may be made through different processes and the process choice goes a long way in affecting the quality of pulp produced. Chemical pulping tends to produce longer grains of fibre than mechanical pulping, usually leading to better strength for paper made from this pulp. Even when chemical pulping is used, all fibres will not be of the same length. This may be further aggravated by the de-lignification process that is used to improve the brightness of the pulp, and hence, paper. Consequently, there may be the need to separate the pulp produced into different grades through the process of fractionation so that the quality of paper may be engineered as desired, depending on the intended use.

Generally speaking, one may consider that the paper product may be intended as a top-quality writing and/or printing paper or a lower quality newsprint. Newsprints are usually used to produce cheaper and disposable products like newspapers, general tissue papers and others.

In addition, after paper products have been used, they are usually collected and re-pulped so that fibre is reclaimed from the collected papers, which would be reprocessed into papers again. This reduces the quantity of virgin wood stock that needs to be chopped and pulped. This reclamation process helps with the conservation of the environment, but in many instances, also helps to reduce the production cost of paper (Smith, 1997).

The process of pulp recovery includes softening and de-inking (or bleaching) the used paper to bring it back into a good state of recovery of the pulp for reuse in paper products. It is often the case that when this pulp is being recovered, a proportion of the long fibre grains of the pulp gets damaged and may no longer be useable in the production of the premium grade paper products in which it was initially used, but the damaged pulp may be fractionated and mixed with the newsprint stock to produce the lower quality products. Generally, however, recovered newsprint paper product may only be used to produce newsprints again because their pulp quality is inadmissible for the premium grade products (Smith, 1997).

1.2 Research objective

The paper industry and the need to conserve natural resources presents the background for the lot sizing problem considered in this dissertation. There is a need for a model to determine the optimal lot size for the alternate application of manufacturing and remanufacturing of a top-grade item, while also considering that returned items cannot always be remanufactured to an as-good-as-new state, returned items can be remanufactured to satisfy demand of a lower variety item as well, such as newsprint in the case of paper.

Two models are proposed in this dissertation to address the problem of remanufacturing to satisfy a secondary demand. The first model is a linear programming model solved with the use of a modified Wagner/Whitin heuristic. The second model is a closed form EOQ model. Both models consider two types of demand, the first demand type is for a top variety product that can be fulfilled by either manufacturing or remanufacturing, and the second type of demand is that of a lower variety item, where returned items that cannot be remanufactured to an as-good-as-new top variety item but is still good enough to satisfy the lower variety item demand. The models represent, without any loss of generality, the paper manufacturing context presented. It is, however, useful in any other production environment that has similar characteristics like in the processing and reprocessing of leather products amongst others. The objective of this paper is to present a lot sizing model that can be adopted in the management of such systems, such that the overall inventory management cost is minimised. While this is not the first work in the area of lot sizing of return items, its application in this context is rather interesting as it creates a practical scenario in which there are two items of different quality grades but with inter-dependent structure of replenishment, especially when they further share a production facility.

Although the two models address the same problem, there are a few subtle differences apart from the formulation and solution type. The linear programming model considers that manufacturing and remanufacturing for the top variety item is performed on the same resource and the remanufacturing of the lower variety item is performed on a separate resource. The remanufacturing processes for both the top and lower variety items require additional input materials. For the closed form solution, the manufacturing and remanufacturing of the top variety item as well as the remanufacturing of the lower variety item is performed on the same resource. A cycle consists of all three processes. For this model, no additional input materials are considered for the remanufacturing processes, top variety feedstock is however considered to fulfil demand in the case where there is a shortage in the output of the manufacturing and remanufacturing processes of the top variety item.

1.3 DISSERTATION STRUCTURE

The remainder of this dissertation is organised as follows: Chapter 2 provides a review of related work in this area after which the respective models are presented in Chapter 3 and Chapter 4. A detailed system definition with assumptions and notations are presented in each of the model chapters respectively, followed by a full numerical analysis and sensitivity analysis to end each of these chapters. The dissertation is then concluded in Chapter 5.

2. LITERATURE REVIEW

There seems to have been a general awareness in both the manufacturing and consuming circles about the need to reduce the human footprint on the environment. This has led to the blossoming of research in various areas related to the management of products at the end of their life and the popularisation of many terms like reverse logistics, closed loop supply chain, circular economy, and remanufacturing processes amongst others. Reverse logistics is seen as an environmentally friendly way to deal with products at the end of their life span and has attracted an increasing amount of attention in the last couple of years (Zhalechian et al., 2016; Govindan and Soleimani, 2017; Rajeev et al., 2017; Govindan and Bouzon, 2018). Integrating the remanufacturing process into the manufacturing process is seen as an opportunity to improve profits and confirm sustainability by many companies (Wei and Zhao, 2014). Although traditional production planning and inventory management models do not typically consider the return and remanufacturing of an item to satisfy the same demand, remanufacturing of end-of-life products to an as-good-as-new state has attracted considerable attention in recent years (Jiang et al., 2016; Lee et al., 2017; Paterson et al., 2017; Zlamparet et al., 2017; Jin et al., 2018; Lu et al., 2018). Models that consider the use of remanufactured products to satisfy demand for a different product are still relatively uncommon. Before discussing the pertinent literature on lot sizing in return logistics, a brief discussion of the recycling in the paper industry is presented.

2.1 RECYCLING IN THE PAPER INDUSTRY

The need for the conservation of the ecological system of the earth has increased the need to reduce the exploitation of the natural resources. While discussing the sources of carbon dioxide accumulation in the atmosphere, Woodwell et al. (1983) mentioned two important sources: combustion of fossil fuel and deforestation. They concluded that evidence from their research indicated that deforestation appears to have had the dominant biotic effect on the atmospheric carbon, and by extension, global warming, and its possible effects on the earth. The implications of the impacts of human activities on the climate has necessitated the need for the global world, and in particular many governments, to implement several control policies in many industries, a number of which implies the design of a closed loop supply chain to enforce the collection of used items. The paper industry is one such industry where collection of return is enforced like the mandatory collection programme in the US, of which collection of wastepaper (especially Old Newsprint – ONP) is a key component of the legislation (Smith, 1997).

It was stated that the circular process of recycling can be construed to consist of three main processes viz: the recovery process, the consumption of the recovered process and the sale of the product remanufactured from the recovered items. They opined that the main focus of the government of the US, for instance, has initially been on the input substitution implication of the recovery process, thereby putting emphasis on the collection process; and later, on the sale of the items made from the inputs, thereby considering the demand and supply ends of the circular system; with little considerations given to the middle process of usage (remanufacturing), which is where the transformation of the collected items is. Most activities of the paper processing industry are within the scope of the middle process, which is the conversion process (Smith, 1997).

The paper industry is said to be structured into three main areas: pulp production; paper and paperboard production; and finished products conversion. About one third of the manufacturing facilities in the US is believed to be fully integrated (Smith, 1997). The remaining plants are either stand-alone mills (involved mainly in pulp production) or semi-integrated manufacturers (with some extent of backward integration). In addition, most manufacturers, rather than making sole use of virgin or recycled input materials, extend their process capability (and/or capacity) to be able to utilise wastepaper in their production, (Smith, 1997). The foregoing makes it important to model the production system with a multi echelon approach, integrating recovery, remanufacture and probably input procurement. This is what this paper seeks to achieve.

Approximately fifty percent of the fibre used in the paper making process today come from purposely harvested wood, where the rest of the fibres come from sawmill wood fibres, recycled newspaper, cloth as well as vegetable matter (Madehow.com, 2021). There are two commonly known processes used in pulp production, the first of which is a mechanical process, where logs are tumbled in a drum to remove the bark. The logs are then sent to grinders, where the logs are pressed between large revolving slabs to break the wood down into pulp. All foreign objects are filtered from the pulp. The second process is a chemical process, where wood chips from the debarked logs are cooked in a chemical solution. The wood chips are cooked in digesters at high pressure in a solution of sodium hydroxide and sodium sulphide. The solution dissolves the wood chips into pulp. The pulp is filtered, after which bleach or colourings are added before the pulp is sent to the paper plant (Madehow.com, 2021).

The paper and paperboard production starts with a beating process, where the pulp is subjected to machine beaters in a large tub. Various filler materials, such as chalk, clay, or other chemicals, can be added at this point to influence the density and various other qualities of the final product. To finally turn the pulp into paper, the pulp is fed into large, automated machines on a moving belt of mesh screening, where the pulp is squeezed through a series of rollers and suction devices below the belt is used to drain off any excess water. The paper is then moved onto the press section of the machine, where it is pressed between rollers of wool felt. Any remaining water is removed from the paper by passing it over a series of steam-heated cylinders. The dried paper is then wound onto large reels, waiting to be further processed depending on its end use.

Almost all paper products can be recycled, although brown and craft envelopes, carbon paper, paper towels, tissues, candy wrappers, coffee cups and pizza boxes are not typically accepted in collection bins. Some of the most commonly recycled paper items include cardboard, newsprint and magazines, manuals and booklets and assorted office paper. The paper recycling process starts with the collection of used paper. It is very important that used paper is kept separated from other recycled goods as contaminated paper is not accepted for recycling. The paper is then sorted into different categories such as cardboard, papers, newspaper, magazine paper, office paper etc. The different categories are treated differently to create different types of recycled paper products (Isustainrecycling.com, 2021).

The paper remanufacturing process also starts with a pulp production process after which all non-fibrous contaminants such as staples, plastic and glass are removed. The fibres are gradually cleaned, and the pulp is filtered and screened several times through screens with holes of different sizes and shapes to remove contaminants such as glue and plastic. The whiteness and purity of the paper is increased by the de-inking process. This is achieved through a combination of mechanical actions such as shredding and the addition of chemicals. Ink is removed in a flotation process where air is blown into the solution. After the ink has been removed, the fibre is bleached. In the final stage of the paper recycling process, the cleaned paper pulp is ready to be used in the manufacturing of new paper. The pulp is combined with virgin wood fibres to make it smoother and stronger. This is not always the case as recycled paper fibres can be used on its own as well (Isustainrecycling.com, 2021).

It is believed that the growth of the paperless movement would not take away the use of papers. Global competition is stiff, affecting profit margins, and might have forced the evolution in the industry, with some manufacturers diversifying to specialty products while a number remain in the price driven environment (Lamberg et al., 2012). The industry manufactures both specialty products like those used in special sanitary products and printing money, quality general products like copier and office papers as well as other lower quality use products like newsprints and packaging products. According to Lamberg et al., (2012) while technological advancement appears to have also advanced in the paper industry, like in many other industries, more focus seems to have been placed on the improvement of productivity and operations due to the nature of competition, implying the need to improve on utilisation of resources and planning of operations. All this makes it important to develop models that can support daily operations in the pulp and paper industry.

The recovery rate of wastepaper for use is another interesting issue. It was noted in Smith (1997) that determining the recovery rate of wastepaper is difficult in the US. Such measures are not even existent in many countries, including South Africa. It was noted that the wastepaper recovery rate seemed highest around the World War II era at about 50 percent but has since declined steadily. It currently stands at less than 40 percent in the US, even with the recent gains in collection (Smith, 1997). This figure has significant impact on the quantity of virgin wood that would be required for pulping, including its attendant effect on the environmental carbon footprint. There is, thus, the need to develop cost efficient measures of managing the pulp and paper industry in an integrated manner, and hence, this work. This leads to the discussion of remanufacturing strategies in lot sizing models.

2.2 Lot sizing in return logistics

Reverse supply chain models can be broadly classified into two groups namely deterministic (Atasu and Çetinkaya, 2006; Feng et al., 2014; Konstantaras and Papachristos, 2008; Özceylan et al., 2014; Tang and Teunter, 2009; Teunter et al., 2009; Zanoni et al., 2012) and stochastic (Aras et al., 2004; Behret and Korugan, 2009; Fleischmann et al., 2002; Karaer and Lee, 2009; Timmer et al., 2013; van Donselaar and Broekmeulen, 2013; Vlachos and Dekker, 2003; Zolfagharinia and Haughton, 2012), where stochastic models can be further classified as continuous or periodically reviewed models. Akçali and Çetinkaya (2010) went even further in classifying periodic review models based on the number of stock points, dependency of return rates on demand, length of planning horizon and the consideration of lead time. For the remainder of this section, models are classified by the assumptions made in the models, for ease of identifying gaps in current literature.

2.2.1 Deterministic demand and returns with identical lot sizes

Schrady (1976) was the first to investigate a reverse logistics model in an EOQ framework. He studied the U.S. Navy Aviation Supply Office, where the repair of high-cost items as opposed to procurement could lead to possible cost saving. In the case where repair of an item is infeasible, demand is satisfied by newly procured products. Items that are not yet repaired are held in non-ready-for-issue (NRFI) inventory awaiting repair or overhaul. Repaired items along with newly procured items are sent to ready-for-issue (RFI) inventory awaiting demand. This inventory system is best described by Schrady's substitution inventory holding policy, where there is one procurement batch with multiple repair batches. In Schrady's substitution inventory policy, procurement and repair lead times are ignored. This model starts with a repair cycle, where the initial NRFI level is reduced by a repair batch size. This continues until the NRFI inventory level reaches zero after supply in the RFI inventory. Schrady's model was later enhanced by Imre (2006) and Helmrich (2013) by considering that the return lot sizes are integer values. Shortages are not allowed in Imre's model and procurement, and repair quantities are assumed to be equal. This is not ideal as the purpose of considering the manufacturing and remanufacturing of the same product is to supplement the manufacturing process with remanufacturing and in turn reduce waste and total cost. Assuming the quantities to the be equal might lead to excess of finished goods, resulting in an increased holding cost.

If returned parts are used in the place of newly manufactured or procured parts, leaving the production lot size as is and ignoring the additional parts supplied from returns, will result in additional returns inventory and other associated costs for the returned parts (Dekker et al., 2004).

Dekker et al. (2004) considered the impact of setup costs on the decision to manufacture or remanufacture. A netting approach can be used in the case where either the manufacturing or remanufacturing setup costs are negligible. If the remanufacturing setup cost is negligible, all returns can be remanufactured to satisfy demand and the net demand can be satisfied by determining the EOQ for manufacturing new parts. If the inverse is true, remanufacturing lot sizes can be determined first after which the remaining demand can be satisfied by manufacturing just in time. However, if both processes have a significant setup cost, there are several trade-offs to consider simultaneously, the setup cost of each process should be evaluated while also considering the holding cost of returned products and finished goods (Dekker et al., 2004).

Dekker et al. (2004) assume a deterministic and constant demand and return rate, with no backorders allowed. The return rate does not surpass the demand rate to ensure the stable behaviour of the system. This assumption is realistic as not all products will be returned by customers and not all returns will be in a condition to be remanufactured to an as-good-as-new state. There are two types of inventories to consider, namely serviceable parts that are ready for use and can satisfy customer demand as is and recoverable parts returned by users and not ready to satisfy customer demand yet. The serviceable parts inventory is replenished either form newly manufactured or procured parts or from remanufactured parts. Lead times are assumed to be negligible in both processes. This model does not allow for disposal and all returned items are reused, this results in no variable manufacturing or remanufacturing costs. The manufacturing and remanufacturing setup can be associated with respective setup costs. A holding cost per unit per unit of time is incurred for storing recoverable items. Serviceable items, whether remanufactured or newly manufactured are assumed to incur the same holding cost per unit per unit of time. The holding cost of serviceable parts is assumed to be greater than or equal to the cost of holding recoverable items. These assumptions stem directly from the traditional EOQ models. Two cases are considered, the first refers to the model developed by Schardy (1967) where one manufacturing batch is followed by a number of remanufacturing batches. All returned items are thus remanufactured, and manufacturing batches serve only to satisfy the difference between the demand and return rate. In order to determine the optimal lot sizes, total cost should be minimised, where total cost is expressed as the sum of production setup cost, remanufacturing setup cost, serviceable holding cost, and the holding cost of recoverable items. The optimal manufacturing and remanufacturing lot sizes are derived from the first differential of the total cost function. The demand rate is reduced by the return rate for the production lot size and the holding cost is weighted accordingly. The remanufacturing lot size on the other hand takes the gross demand rate into account and the holding cost is adjusted to equal the sum of the serviceable and recoverable holding cost rate.

The second model considered, is a model by Teunter (2001), where one remanufacturing batch is followed by a number of identical manufacturing batches. Total cost per unit of time is expressed as the sum of production setup cost, remanufacturing setup cost, serviceable holding cost and the holding cost of recoverable items. From the first differential of the total cost function the optimal manufacturing and remanufacturing lot sizes are derived. The manufacturing lot size remains unchanged from the traditional EOQ model. The remanufacturing lot size takes the return rate into account and the holding cost rate is weighted accordingly. The manufacturing batch number should once again be an integer; however, this constraint is not adhered to in Teunter's model. The integer setup numbers have later been addressed by Richter and Dobos (1999) and Minner (2002).

When comparing the cost functions in the models of Schardy (1967) and Teunter (2001), the single manufacturing batch followed by multiple remanufacturing batches is better than the single remanufacturing batch followed by multiple manufacturing batches when the return rate and demand rate is similar, as the remanufacturing batches will continue to consume the returned products, thus not leading to the prolonged storage of returned products while waiting for the next remanufacturing batch. When the demand rate is much higher than the return rate the single remanufacturing batch model is preferred (Dekker et al.,2004). This is because the number of remanufacturing batches possible is constrained by the available returns. If the demand rate is much higher than the return rate, the total demand that can be satisfied by remanufacturing is constrained and the manufacturing batch at the beginning of the period will have to be sufficiently large to fulfil the demand that cannot be met by the remanufacturing batches, resulting in holding cost incurred on serviceable inventory across the horizon.

2.2.2 Deterministic demand and returns with non-identical lot sizes

The property of identical batch sizes does not always hold true for the remanufacturing model (Dekker et al., 2004). This assumption is relaxed by allowing a number of consecutive manufacturing batches followed by a number of remanufacturing batches in a cycle of time. With the start of each inventory cycle or the placement of any order, the serviceable inventory is zero. To further reduce costs, the start of the next remanufacturing batch is delayed as far as possible as the cost of holding serviceable items is greater than or equal to the cost of holding recoverable items. Costs incurred per inventory cycle consists of setup cost for manufacturing and remanufacturing and the holding costs of both serviceable and recoverable items. The first order conditions determined for the optimal timing of the production batches results in equal lengths between production batches and therefore equal production batch sizes. The first order condition for all but the last recovery batches also imply identical batch sizes. The last remanufacturing batch is smaller. This finding supports the fact that identical remanufacturing batch sizes are never optimal, reason being that the items returned after the final equal sized remanufacturing batch will have to be stored until the next remanufacturing batch is started. Results up until this point assume unconstrained returns, the remanufacturing batch size should however be constrained by available recoverable items. This work will consider constrained returns in remanufacturing.

2.2.3 Delayed manufacturing and remanufacturing processes

Nahmias and Rivera (1979) extended Schrady's (1976) model by suggesting that the repair process requires time to complete, with a fixed repair rate that is constant over time. This model considers waste disposal in the reuse process. The original model by Nahmias and Rivera (1979) allowed only one procurement batch and has since been extended by Imre (2006) to allow more than one procurement batch, where the procurement and repair batch sizes are dependent on the rate of return. Inventory is held as recoverable and serviceable items, where demand is satisfied from the serviceable item inventory, also referred to as the supply depot by Imre. The assumption of constant demand still holds during the repair and procurement cycle, shortages are not allowed, and the procurement and repair lot sizes are assumed equal. The repair rate is assumed to be greater than the demand rate. Repaired products are sent to the supply depot and used as new products to fulfil demand. Purchasing and repair lead times are constant and thus not considered in the model. The inventory policy proposed for this model is the substitution policy developed by Schrady (1967). Although the model considers waste disposal, it is not a decision variable. The consideration of depot capacity as well as waste disposal costs are possible extensions to this model.

Another model to consider is a model by Koh et al. (2002). This model is similar to that of Nahmias and Rivera (1979) but uses the continuous supplement inventory policy by Schardy (1967) instead of the substitution policy. Batch sizes are not expressed explicitly in the model by Koh et al. and the remanufacturing rate has a capacity limit less than or equal to the rate of production and reuse. Inventory is held as serviceable and recoverable items, where serviceable inventory is replenished from both new purchases and repairs. Shortages are again not allowed. Procurement and repair batches are assumed to be equal. The return rate of used products is known, and the repair or overhaul capacity is assumed to be finite and greater than the rate of demand, which in turn is greater than the return rate. Lead times of procurement and repair is again disregarded. In this model the inventory holding cost of remanufactured and usable products is more than the inventory holding cost of reusable products, replenishment is thus economical with an inventory level of zero. The optimal production lot size remains the same for systems with an infinite and finite remanufacturing rate. From the Economic Production Quantity (EPQ) model (Silver et al., 1998) the optimal remanufacturing lot size is greater on the assumption of an instantaneous remanufacturing process.

The models studied thus far have the following common assumptions:

- Stock holding policies are known
- Demand for new and remanufactured products is constant and deterministic over time.
- The return rate is known and constant over time
- Ordering and setup costs are known.
- Stock holding costs of new and remanufactured products are known as well as the holding costs of items waiting to be repaired.
- Shortages in remanufactured and new products to fulfil demand is not considered.
- Remanufactured product quality is as-good-as new

2.2.4 Dynamic lot sizing models for varying demand and returns over time

Dynamic lot sizing in planning manufacturing and production is a widely researched topic in production and inventory management (Silver et al., 1998). Dynamic lot sizing in return logistics where the remanufacturing of returns is an alternative to manufacturing is however not as common yet. Static models no longer apply in a dynamic environment with varying demand and returns over time. Dekker et al. (2004), formulated a mixed integer linear programming model assuming a fixed planning horizon of discrete time periods, where time varying customer demand should be satisfied in each time period. Backordering is not allowed, and customers return used items in each period. This results in the two types of inventories, namely serviceable and recoverable inventory. Serviceable inventory is used to satisfy demand and replenished either by a manufacturing batch or a remanufacturing batch with the assumption that remanufactured goods are as-good-as-new. Excess returns are disposed of. Zero lead times are assumed for all processes. A setup cost is associated with each manufacturing, remanufacturing and disposal batch respectively. Variable manufacturing, remanufacturing and disposal cost per unit is incurred. The mixed integer linear optimization problem formulated by Dekker et al. minimises the total cost over the planning horizon, where total cost consists of the setup costs, variable costs and holding costs for both serviceable and recoverable inventory.

2.2.5 THE DEPENDENCY OF RETURN RATE ON DEMAND

Most of the models published thus far assume that demand and return rates are independent (Fleischmann and Kuik, 2003; Heisig and Fleischmann, 2001; Karaer and Lee, 2009; Kiesmüller and Scherer, 2003; Shi et al., 2011; Teunter and Vlachos, 2002). Although this assumption simplifies the model significantly, the dependency of the return rate on demand is a realistic assumption (Kim et al., 2013; Vercraene and Gayon, 2013). Kiesmüller and van der Laan (2001) studied a single stock model where the return rate is a function of product demand. The proposed problem was solved using a Markov-chain approach. Vlachos and Dekker (2003) proposed the first newsvendor model considering the probability of return and its dependency on demand. Sun et al. (2013) investigated a manufacturing and remanufacturing system where the return rate is dependent on demand and solved the proposed problem using a three-stage stochastic dynamic programming model. Schulz and Ferretti (2011) considered the meticulousness of the remanufacturing process itself, by including the disassembly process as an explicit recovery step. The remanufacturing process thus consists of two sub processes, the disassembly process in which returns are disassembled and a rework process, where items are reworked to an as-good-as-new state. Schulz and Ferretti considered random yield in components recovered from the disassembly process in conjunction with the demand dependency in the return rate. Zolfagharinia et al. (2014), was the first to explicitly consider the dependency of return rate on demand in a two-stock system where used or damaged products are returned to the original manufacturer and backordering is allowed. The two-stock system results in two types of inventories, namely serviceable and recoverable inventory. Serviceable inventory is used to satisfy demand and replenished either by a manufacturing batch or a remanufacturing batch with the assumption that remanufactured goods are as-good-as-new. Recoverable inventory is the returned goods inventory used for remanufacturing. By incorporating two stock points to separate serviceable and recoverable inventory, remanufacturing can be postponed as far as possible to take full advantage of the lower recoverable inventory holding cost. Zolfagharinia et al. used a simulation-based hybrid variable neighbourhood search to solve the proposed problem. Piñeyro and Viera (2015) considered a new theoretical approach that can be considered a generalisation of the well-known zero-inventory property.

2.2.6 THE POSSIBILITY OF REMANUFACTURED PRODUCTS SATISFYING A SECONDARY DEMAND STREAM

A mutual assumption across existing models is that all returned or defective items should be remanufactured to an as-good-as-new state, sold at a reduced rate or disposed of. The typical remanufacturing cost is between 40 to 60% of the manufacturing cost with 20% of the effort required (Dowlatshahi, 2000). In some cases, remanufactured products are perceived as a lower quality by the market compared to new products (Abbey et al., 2014). Hasanov et al. (2012), Helmrich (2013) and Jaber and El Saadany (2009) assumed the quality of remanufactured products to be incomparable with newly manufactured products, resulting in two demand streams. Piñeyro and Viera (2010) introduced a novel lot sizing model for a hybrid manufacturing and remanufacturing model. They proposed a one-way substitution option where demand for remanufactured items can be satisfied by newly manufactured items should there be a shortage of remanufactured products. Zouadi et al. (2019) investigated a joint pricing and lot sizing problem in a hybrid manufacturing and remanufacturing system with one-way substitution where the manufacturing and remanufacturing processes use the same resource. The remanufacturing process produces products of a lower quality, thus resulting in two demand streams for newly manufactured products and remanufactured products respectively. In the case of a shortage of remanufactured products, the demand for the lower quality product can be satisfied by newly manufactured products. Zouadi et al. proposed a mixed integer programming model to find the optimal production and pricing strategy over the planning horizon and a novel adaption of a cost benefit evaluation heuristic and memetic algorithm is proposed to find the near optimal solution.

2.2.7 LOT SIZING MODELS WITH DEFECTIVE YIELD AND REMANUFACTURING

The possibility of producing defective items during the manufacturing process was introduced by Porteus (1986). A manufacturing process can go ‘out of control’ with a given probability every time a unit in a lot is produced, resulting in the production of defective items. These defective units are either scrapped, sold at a lower price, reworked or other remedial actions are taken. The defective yield models reviewed thus far either assume that once a process reaches an ‘out of control’ state, all items produced thereafter will be defective up until the point where the process is returned to an 'in control' state or alternatively, a process does not enter a complete 'out of control' state but rather produces a defective item occasionally. Defective items in this case can be identified by making use of a threshold policy where items are classified as good, reworkable or non-reworkable with known probabilities. Good or non-reworkable items leave the system completely after inspection, whereas reworkable items are returned to be remanufactured (So and Tang 1995a, 1995b; Liu and Yang, 1996; Chern and Yang, 1999). The latter is assumed for this work.

2.2.8 Dynamic programming models

The first dynamic lot sizing model was introduced by Richter and Sombrutzki (2000) as an extension to the well-known Wagner/Within model (Wagner and Whitin, 1958). Richter and Sombrutzki solved for the optimal manufacturing and remanufacturing lot size heuristically by first deriving a modified reverse Wagner/Whitin model. The model operates in a simplified environment where used products at the beginning of the decision period is sufficiently large to allow for a full remanufacturing batch. They then considered a modified Wagner/Whitin model to heuristically solve for the alternate application of manufacturing and remanufacturing. The reason for the modified heuristic is due to the weakness in the standard Wagner/Whitin model to consider two supply types. Richter and Sombrutzki assumed that a remanufacturing batch can be executed at any time because the returned item inventory at the start of any given period is greater than or equal to the demand in that same period. In each period, the cost of remanufacturing is weighed against the cost of manufacturing, where the less of the two drives the decision on whether to remanufacture or manufacture. The manufacturing cost considers the holding cost incurred on the returned items that will not be remanufactured in a period of manufacturing. The model was extended by Richter and Weber (2001) to consider variable costs in the manufacturing and remanufacturing processes. Dekker et al. (2004), formulated a mixed integer linear programming model where customers return used items in each period. Teunter et al. (2006) proposed two models, the first considers one setup cost for both manufacturing and remanufacturing and the second considers different setup costs for the two processes. Zouadi et al. (2014) proposed two metaheuristic-based approaches in a hybrid manufacturing and remanufacturing system. This work was later extended by Zouadi et al. (2017) to consider the returns collection and supplier selection phase.

A comparison of the proposed inventory system and previously published relevant inventory models for return items in literature is provided in Table 2.1

TABLE 2.1 GAP ANALYSIS OF RELATED LITERATURE WORKS AND THE CONTRIBUTION OF THIS PAPER

References	**Characteristics of the inventory system**					
	Remanufactured product satisfies original product demand	Remanufactured product satisfies secondary demand	Imperfect yield	Return rate is a function of product demand	Shared resources	Remanufacturing process require dependent demand input
Chern and Yang, (1999)	√		√			
Dekker et al. (2004)	√					
Hasanov et al. (2012)	√	√				
Helmrich (2013)	√	√				
Jaber and El Saadany (2009)	√	√				
Kiesmüller and van der Laan (2001)	√			√		
Liu and Yang (1996)	√		√			
Porteus (1986)	√		√			
Richter and Sombrutzki (2000)	√					
Richter and Weber (2001)	√					
So and Tang (1995a)	√		√			
So and Tang (1995b)	√		√			
Sun et al. (2013)	√			√		
Teunter et al. (2006)	√				√	

References	**Characteristics of the inventory system**					
	Remanufactured product satisfies original product demand	Remanufactured product satisfies secondary demand	Imperfect yield	Return rate is a function of product demand	Shared resources	Remanufacturing process require dependent demand input
Vlachos and Dekker (2003)	✓			✓		
Zolfagharinia et al. (2014)	✓			✓		
Zouadi et al. (2014)	✓				✓	
Zouadi et al. (2017)	✓				✓	
Zouadi et al. (2019)	✓	✓			✓	
This dissertation	✓	✓	✓	✓	✓	✓

A review of current literature suggests that no work has been published on inventory modelling which considers remanufacturing to satisfy a primary and secondary demand, while also taking into consideration that a manufacturing process can yield defective items which are returned to be remanufactured. Additional dependent input items required in remanufacturing is also not considered in the extant literature. An attempt is made here to develop an inventory system that considers remanufacturing as an alternative to manufacturing for a top variety item. Where items cannot be remanufactured to a top variety item, they are remanufactured to a lower variety item to satisfy a secondary demand. Recoverable inventory consists of customer returns as well as a proportion of defective items produced during the manufacturing process, which is returned for remanufacturing to either one of the two item variety types depending on the extent of defect, which is a given proportion of new items manufactured. The model also considered that there may be dependent demand items for the remanufacturing processes, and such items need to be procured from outside the manufacturing system, and this process also needs optimisation. This integrated dependent demand structure is also not considered in extant literature.

3. A DYNAMIC LOT SIZING MODEL FOR TWO ITEMS WITH IMPERFECT MANUFACTURING PROCESS, TIME VARYING DEMAND AND RETURN RATES, DEPENDENT DEMAND AND DIFFERENT QUALITY GRADES

3.1 INTRODUCTION

This chapter presents a linear programming model supplemented by a modified Wagner/Whitin heuristic as a solution to the time complexity of the linear programming model. This section is organised as follows: Section 3.2 provides a detailed description of the proposed system. The notation adopted and mathematical representation of the inventory system considered is given in Section 3.3. Numerical results are presented in Section 3.4 to illustrate the proposed solution procedure and to provide managerial insights through a sensitivity analysis.

3.2 SYSTEM DEFINITION

Consider a manufacturing system that produces two items, A and B. The demand rate for item A is D_A while that for item B is $D_{\boldsymbol{B}}$. A is a top-grade item, while B is also a good item, but not as good as item A. A is produced from both manufacturing from new input materials as well as remanufacturing of recovered materials, but B is manufactured only from recovered materials. The recovery process includes return of used items A and B as well as salvaging items that are damaged when being newly manufactured.

The material flow of the proposed model is depicted in Figure 3.1, where the flow of the top range items is illustrated by a solid line and the flow of the lower range items is illustrated by the dotted line. Two inventory types are considered, namely serviceable and recoverable inventory. Serviceable inventory is inventory available to service the demand of the end user and can be further split into two types, namely Type A and Type B, where Type A is inventory of the top range product and Type B is inventory of the lower range product. Recoverable inventory consists of repairable products which includes items returned by the end user as well as defective products produced by an occasional 'out of control' manufacturing process. Recoverable inventory is also split into Type A and Type B items respectively.

3.2.1. Model assumptions

Demands for Type A and Type B are satisfied under the following conditions:

- The manufacturing and remanufacturing processes for Type A items are performed on the same resources and the manufacturing and remanufacturing batches are alternated in such a way as to minimize the total cost over the planning horizon;
- Items are remanufactured into Type B products on a separate resource;
- The manufacturing process produces Type A items only;
- Some items may fail during manufacturing and are sent to recoverable inventory for possible remanufacturing;
- Both Type A and Type B used items can be returned by the end user to recoverable inventory for possible remanufacturing;
- A reparable Type A item can either be remanufactured to an as-good-as-new state to satisfy Type A demand once again, or alternatively, a reparable Type A item can be remanufactured to a Type B item;
- Type B items can only be remanufactured to an as-good-as-new Type B item;
- Demand for both items is deterministic, but may vary over time;
- The rates of return are deterministic, but may vary over time;
- Reparable items need some other input items that need to be procured to bring the returned items back to one of two states of reuse;
- Lower variety input items are used to produce Type B items during the remanufacturing process;
- Top variety input items are used during the remanufacturing processes of Type A items;
- Ordering and setup costs are known and constant;
- Shortages in remanufactured and new products to fulfil demand is not allowed;
- Stock holding costs of Type A and Type B serviceable inventory as well as the holding costs of items waiting to be repaired are known;
- Lead times for both manufacturing and remanufacturing processes are negligible.

3.3 MODEL FORMULATION

The model of this inventory system is developed in this section, but the notations adopted for the model development is presented first.

3.3.1. LIST OF VARIABLES

3.3.1.1. LIST OF DECISION VARIABLES

Q_{P_t} is the manufacturing batch size in time t;

$Q_{P_{A_t}}$ is the product yield quantity for Type A items from every manufactured batch of size Q_{P_t} in time t;

$Q_{r_{A_t}}$ is the remanufacturing batch size of Type A items in time t;

$Q_{r_{B_t}}$ is the remanufacturing batch size of Type B items in time t;

$I_{S_{A_t}}$ is the serviceable inventory level of Type A items at the start of time t;

$I_{S_{B_t}}$ is the serviceable inventory level of Type B items at the start of time t;

$I_{r_{A_t}}$ is the recoverable inventory level of Type A items at the start of time t;

$I_{r_{B_t}}$ is the recoverable inventory level of Type B items at the start of time t;

q_{A_t} is the Type A input item procurement batch size for use in the remanufacturing of Type A items in time t;

q_{B_t} is the Type B input item procurement batch size for use in the remanufacturing of Type B items in time t;

i_{A_t} is the Type A remanufacturing input item inventory level at the start of time t;

i_{B_t} is the Type B remanufacturing input item inventory level at the start of time t;

γ_{P_t} is the binary variable indicating the release of a manufacturing batch in time bucket t;

$\gamma_{r_{A_t}}$ is the binary variable indicating the release of a Type A remanufacturing batch in time t;

$\gamma_{r_{B_t}}$ is the binary variable indicating the release of a Type B remanufacturing batch in time t;

ω_{A_t} is the binary variable indicating the release of a Type A remanufacturing input item procurement batch in time t;

ω_{B_t} is the binary variable indicating the release of a Type B remanufacturing input item procurement batch in time t.

3.3.1.2. *LIST OF PARAMETERS*

D_A is the demand rate for Type A items;

D_B is the demand rate for Type B items;

r_{A_A} is the recovery rate of used Type A items that will be remanufactured to an as-good-as-new Type A item state, expressed as a percentage (or proportion) of the demand rate D_A;

r_{B_A} is the recovery rate of used Type A items that will be remanufactured to a Type B item, expressed as a percentage (or proportion) of the demand rate D_A;

r_{B_B} is the recovery rate of Type B items that will be remanufactured to an as-good-as-new Type B item state, expressed as a percentage (or proportion) of the demand rate D_B;

α is the failure rate of the manufacturing process for items that can be remanufactured to an as-good-as-new Type A item, expressed as a percentage (or proportion) of the manufacturing batch size Q_P;

β is the failure rate of the manufacturing process for items that can be remanufactured to a Type B item only, expressed as a percentage (or proportion) of the manufacturing batch size Q_P;

z_A is the number of Type A dependent demand items required in the remanufacturing of each Type A item;

z_B is the number of Type B dependent demand items required in the remanufacturing of each Type B item;

t is the length of a manufacturing/remanufacturing time (time bucket);

T is the total planning horizon (made up of t time buckets);

h_{s_A} is the holding cost rate of Type A serviceable inventory (per item per time);

h_{s_B} is the holding cost rate of Type B serviceable inventory (per item per time);

h_{r_A} is the holding cost rate of Type A recoverable inventory (per item per time);

h_{r_B} is the holding cost rate of Type B recoverable inventory (per item per time);

v_A is the holding cost rate of Type A dependent demand item inventory (per item per time);

v_B is the holding cost rate of Type B dependent demand item inventory (per item per time);

K_p is the manufacturing batch setup cost;

K_{r_A} is the Type A remanufacturing batch setup cost;

K_{r_B} is the Type B remanufacturing batch setup cost;

P_A is the Type A remanufacturing input item ordering cost;

P_B is the Type B remanufacturing input item ordering cost;

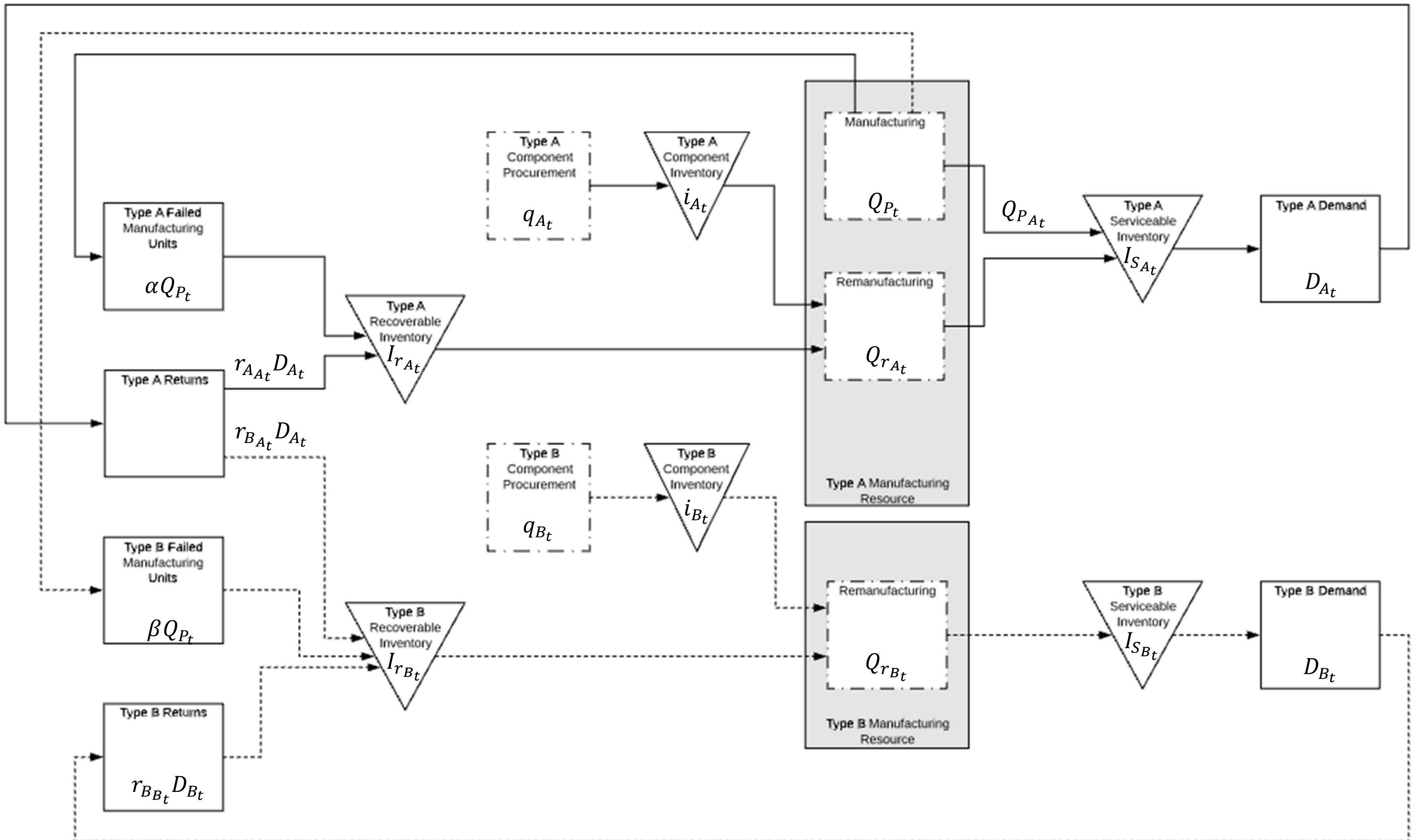

FIGURE 3.1 MATERIAL FLOW DIAGRAM CONSIDERING REVERSE LOGISTICS IN SATISFYING THE DEMAND FOR A TOP RANGE ITEM AS WELL AS A LOWER RANGE ITEM

3.3.2. THE TOTAL COST FUNCTION

The cost of holding recoverable inventory h_r is assumed to be lower than or equal to the cost of holding serviceable inventory h_s. Serviceable and recoverable inventory for Type A and Type B items attract different costs, due to the different values of the items. The value of Type B serviceable inventory is also assumed higher than the value of Type A recoverable inventory, the holding cost relationship is expressed in (3.1).

$$h_{s_A} \geq h_{s_B} \geq h_{r_A} \geq h_{r_B} \tag{3.1}$$

Similarly, it is assumed that the cost of holding inventory of dependent demand items for remanufacturing of Type B items, v_B, is lower than or equal to the cost of holding inventory of dependent demand items used during the remanufacturing of Type A items, v_A since lower variety input items are used in the remanufacturing of Type B items. The holding cost relationship for the dependent demand items is expressed in (3.2).

$$v_A \geq v_B \tag{3.2}$$

A dynamic lot sizing model is proposed. The proposed model is expressed as an integer linear programming model and will be solved with a modified Wagner/Whitin dynamic programming algorithm. The mixed integer linear programming formulation with the objective of minimising the total cost over the planning horizon is given in (3.3) constrained by (3.4) to (3.16).

$$\min C = \sum_{\tau=1}^{T} \begin{pmatrix} K_p \gamma_{P_t} + K_{r_A} \gamma_{r_{A_t}} + K_{r_B} \gamma_{r_{B_t}} + h_{s_A} I_{s_{A_t}} + h_{s_B} I_{s_{B_t}} + h_{r_A} I_{r_{A_t}} + h_{r_B} I_{r_{B_t}} \\ + P_A \omega_{A_t} + P_B \omega_{B_t} + v_A i_{A_t} + v_B i_{B_t} \end{pmatrix} \tag{3.3}$$

subject to:

$$I_{S_{A_t}} = I_{S_{A_{t-1}}} + Q_{P_{A_t}} + Q_{r_{A_t}} - D_{A_t}\,;\; t = 1,2, \dots T \tag{3.4}$$

$$I_{S_{B_t}} = I_{S_{B_{t-1}}} + Q_{r_{B_t}} - D_{B_t}\,; t = 1,2, \dots T \tag{3.5}$$

$$I_{r_{A_t}} = I_{r_{A_{t-1}}} - Q_{r_{A_t}} + d_{A_t}\,; t = 1,2, \dots T$$

(3.6)

$$d_{A_t} = r_{A_A} D_{A_t} + \alpha Q_{P_t}\,; t = 1,2, \dots T$$

(3.7)

$$I_{r_{B_t}} = I_{r_{B_{t-1}}} - Q_{r_{B_t}} + d_{B_t}\,; t = 1,2, \dots T$$

(3.8)

$$d_{B_t} = r_{B_A} D_{A_t} + r_{B_B} D_{B_t} + \beta Q_{P_t}\,; t = 1,2, \dots T$$

(3.9)

$$Q_{P_{A_t}} = [1 - (\alpha + \beta\,)]\; Q_{P_t}\,; t = 1,2, \dots T$$

(3.10)

$$\gamma_{P_t} + \gamma_{r_{A_t}} \leq 1$$

(3.11)

$$i_{A_t} = i_{A_{t-1}} + q_{A_t} - z_A Q_{r_{A_t}}\,;\; t = 1,2, \dots T$$

(3.12)

$$i_{r_{B_t}} = i_{r_{B_{t-1}}} + q_{r_{B_t}} - z_B Q_{r_{B_t}}\,; t = 1,2, \dots T$$

(3.13)

$$Q_{P_t}, Q_{r_{A_t}}, Q_{r_{B_t}}, I_{S_{A_t}}, I_{S_{B_t}}, I_{r_{A_t}}, I_{r_{B_t}}, q_{A_t}, q_{B_t}, i_{A_t}, i_{B_t} \geq 0\,;\; t = 1,2, \dots T$$

(3.14)

$$\gamma_{P_t}, \gamma_{r_{A_t}}, \gamma_{r_{B_t}}, \omega_{A_t}, \omega_{B_t} \in \{0,1\}\,;\; t = 1,2, \dots T$$

(3.15)

$$q_A \leq M\omega_A, q_B \leq M\omega_B, Q_P \leq M\gamma_P, Q_{r_A} \leq M\gamma_{r_A}, Q_{r_B} \leq M\gamma_{r_B}; t \in \{1, T\}$$
$$where\ M\ is\ a\ sufficiently\ large\ number$$

(3.16)

A finite planning horizon of T discrete time periods, $t = 1,2, \dots T$, is assumed. Customer demands D_{A_t} and D_{B_t} need to be satisfied in each time period t. Backordering is not allowed. Customers return $r_{A_A} D_{A_t}$, $r_{B_A} D_{A_t}$ and $r_{B_B} D_{B_t}$ used items in each period t. Although customer returns are expressed as a proportion of demand within a period, this does not mean that the exact item that has satisfied customer demand within period t is returned for remanufacturing in period t, this is simply a method of realistically constraining customer returns. Returns are thus, considered, to be available at the beginning of a period to be used in the remanufacturing process and not at the end. The feasibility of this assumption is guaranteed by the inclusion of sufficient opening balance at the beginning of the first period of the planning horizon. This can be construed to include the returns from the previous periods that have been carried over as the opening balance. This is a typical assumption of dynamic programming and aggregate planning models. A proportion of newly manufactured products α is returned to Type A recoverable inventory and a proportion β is returned to Type B recoverable inventory for remanufacturing due to imperfect yield in each period t. The inventory balances of the four inventory types, namely Type A serviceable, Type B serviceable, Type A recoverable and Type B recoverable items at the end of each period, t, are expressed in (3.6) and (3.8) respectively.

The serviceable manufacturing quantity per manufactured batch size is given in (3.10). Initial inventory levels for $I_{S_{A_0}}$, $I_{r_{A_0}}$, $I_{S_{B_0}}$ and $I_{r_{B_0}}$ are given. Type A serviceable, Type A recoverable, Type B serviceable and Type B recoverable inventory are subject to holding costs of h_{S_A}, h_{r_A}, h_{S_B} and h_{r_B}, per unit item per unit of time respectively. Setup costs of K_p, K_{r_A} and K_{r_B} are associated with each manufacturing batch and remanufacturing batch of Type A and Type B items respectively. The release of a batch is indicated by the respective binary variables γ_{P_t}, $\gamma_{r_{A_t}}$ and $\gamma_{r_{B_t}}$. The variable is equal to one if a batch of the respective type is released at time t, otherwise zero. The manufacturing and remanufacturing processes for Type A items are performed on the same resources and only one of the two batches are, thus, allowed per period. The release of a Type A manufacturing or remanufacturing batch is constrained by (3.11) in every period t.

The Type A dependent demand component inventory level, i_{A_t} is, therefore, given by (3.12) and the Type B dependent demand component inventory level, i_{B_t} is given by (3.13).

Initial inventory levels, i_{A_0} and i_{B_0} are given. An ordering cost of P_A is applicable when purchasing a batch of input items used in the Type A remanufacturing processes. An ordering cost of P_B is applicable when purchasing a batch of input items used in the Type B remanufacturing process. The release of a purchasing batch is indicated by the respective binary variables ω_{A_t} and ω_{B_t}. The variable is equal to one if a batch of the respective type is released at time t, otherwise zero. Dependent demand items used in the Type A remanufacturing process is subject to holding costs of v_A per unit item per unit of time and dependent demand items used in the remanufacturing of Type B items are subject to holding costs of v_B per unit item per unit of time.

3.3.3. A DYNAMIC PROGRAMMING SOLUTION TO THE PROBLEM

A dynamic programming algorithm is formulated to aid in solving the optimization problem in this paper. In doing this, the optimization problem is broken down into simpler sub problems utilizing the fact that the optimal solution to the overall problem depends on the optimal solution to its sub problems. In the heuristic based on a modified Wagner/Whitin algorithm presented by Richter and Sombrutzki (2000) for the alternate application of manufacturing and remanufacturing batches, the assumption is made that $d_{A_t} \geq D_{A_t}$ and thus every period has a chance of manufacturing or remanufacturing occurring. In the model presented in this paper, returns consists of a combination of customer returns and items that fail during manufacturing. Customer returns are expressed as a proportion of demand and items that fail during manufacturing is expressed a proportion of the manufacturing batch size. The possibility of a remanufacturing batch is thus dependent on the accumulation of returns being enough to satisfy demand. The $d_{A_t} \geq D_{A_t}$ equation is thus modified for this paper and used to test whether a remanufacturing batch is possible in a period. Available recoverable inventory is calculated at the start of each period based on the manufacturing versus remanufacturing decision made in previous periods. Based on the available recoverable inventory at the start of the period, the possibility of a remanufacturing batch in that period can be determined as well as the possibility of remanufacturing to satisfy demand in future periods. The cost of manufacturing is compared to the cost of remanufacturing and the less of the two is selected as the preferred option.

A heuristic that builds on the modified Wagner/Whitin solution approach proposed by Richter and Sombrutzki is presented in (3.17) to (3.24).

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}, f_0 = 0$$

(3.17)

$$f_{A_t} = \min_{1 \le i < t} \{ f_{A_{it}} + c_{A_{it}} + f_{A_i} \}$$

(3.18)

$$f_{A_{it}} = min \begin{Bmatrix} K_{P_i} + \sum_{j=1}^{i} \beta Q_{P_j} h_{r_{B_{j,t}}} + (\alpha Q_{P_i} + r_{A_A} D_{A_{i,t}}) h_{r_{A_{i,t}}} \\ + I_{r_{A_{i-1}}} h_{r_A}(t-i), K_{r_{A_i}} + I_{r_{A_i}} h_{r_{A_{i,t}}} + min\{P_{A_i} + i_{A_i} v_{A_{i,t}}\} \end{Bmatrix}$$

(3.19)

where $I_{r_{A_t}} = I_{r_{A_{t-1}}} - Q_{r_{A_t}} + d_{A_t}$ and $I_{r_{A_t}} \geq 0$

(3.20)

$$c_{A_{it}} = D_{A_{i,t}} h_{S_{A_{i,t}}}$$

(3.21)

$$f_{B_t} = \min_{1 \le i < t} \{ f_{B_{it}} + f_{B_i} \}$$

(3.22)

$$f_{B_{it}} = K_{r_{B_i}} + D_{B_{i,t}} h_{S_{B_{i,t}}} + I_{r_{B_i}} h_{r_{B_{i,t}}} + min\{P_{B_i} + i_{B_i} v_{B_{i,t}}\}$$

(3.23)

where $I_{r_{B_t}} = I_{r_{B_{t-1}}} - Q_{r_{B_t}} + d_{B_t}$ and $I_{r_{B_t}} \geq 0$

(3.24)

The pseudo code for the main algorithm of the modified Wagner/Whitin algorithm is given in Figure 3.2. The pseudo code for each of the functions implementing the main algorithm can be seen in Appendix A.

Main Algorithm
Optimise the manufacture/remanufacture of type A
Explode plan for type A to determine quantity and timing of input components of type A
Optimise the procurement plan for input components of type A
Optimise the remanufacture of type B
        (If recoverable stock is insufficient, sub-optimize type A to get a feasible plan for B)
Explode plan for type B to determine quantity and timing of input components of type B
Optimise the procurement plan for input components of type B

FIGURE 3.2 PSEUDO CODE FOR MAIN ALGORITH OF MODIFIED WAGNER/WHITIN HEURISTIC

The total cost to manufacture for Type A demand also considers the Type B recoverable inventory cost incurred due to items that fail during manufacturing. Total Type B recoverable inventory is also considered in the cost minimization of remanufacturing for Type B demand. The calculation of f_t in (3.17) thus needs to correct for the duplication in Type B recoverable inventory cost due to items that fail during manufacturing. After the optimization of f_{A_t} in (3.18), the total recoverable inventory holding cost incurred on items that failed during manufacturing and can only be remanufactured to an as-good-as-new Type B item is subtracted for the calculation of f_t in (3.17) as the recoverable inventory holding cost of these failed items is considered in the total recoverable inventory cost in the optimization of f_{B_t}, in (3.22), along with the usage of recoverable inventory to remanufacture.

The possibility of remanufacturing for Type A demand is dependent on the availability of sufficient recoverable stock to make at least a batch of remanufactured Type A items, this is expressed in (3.20). If Type A recoverable inventory is not up to the quantity required for a remanufacturing batch of Type A items, a manufacturing batch will have to be released. A Type B remanufacturing batch is dependent on the availability of recoverable Type B stock that is at least enough to make a batch of remanufactured Type B items, as expressed in (3.24). In the case of a shortage of Type B recoverable inventory, a Type A manufacturing batch will have to be released to replenish Type B recoverable inventory with defective yield items from the Type A manufacturing process. This could potentially lead to f_{A_t} being reoptimized to ensure that there are no shortages in satisfying Type A and Type B demand.

3.3.4. Time complexity analysis of the solution algorithm

The worst-case time complexity of each of the algorithms are: $O(n^2)$ for optimising the manufacturing/remanufacturing of type A, $O(n)$ for requirement planning for components of type A, $O(n^2)$ for optimising the procurement plan of components of type A, $O(n^2)$ for optimising the remanufacture of type B (this includes the reoptimisation algorithm, which is $O(n)$), $O(n)$ for requirement planning for components of type B, and $O(n^2)$ for optimising the procurement plan of components of type B. Overall, the algorithm is $O(n^2)$, and consequently, should be much faster than the Linear Programming solution when the size of the input data, n, gets very large, and would be quite advantageous if the quality of the solution produced is not too bad, compared to the LP solution approach which in its basic form may be exponential in the worst case.

3.4 NUMERICAL ANALYSIS

Data is simulated for the purpose of the numerical work. The assumptions about the structure of the data are guided by Dekker et al. (2004), Dowlatshahi (2000), Richter and Sombrutzki (2000) and Zolfagharinia et al. (2014). The numerical example using the modified Wagner/Whitin heuristic was solved using Microsoft Excel. The sensitivity analysis was however done using the linear programming model coded in Python 3.8 to calculate the result of numerous parameter value changes. Each run was timed, and an average runtime of 0.7 seconds was observed for the linear programming model over the same planning horizon of 5 time buckets. Results of the two respective models are also compared in Section 3.4.3. For the numerical experiments performed in Section 3.4.4, more than one parameter is varied in each scenario. Each scenario is performed 5 times in which the demand for each scenario is varied, the same distribution is used for the demand generation.

3.4.1 NUMERICAL EXAMPLE

Let

$$K_P = 5\,000, K_{r_A} = 2\,000, K_{r_B} = 250, h_{S_A} = 1, h_{S_B} = 0.9, h_{r_A} = 0.8, h_{r_B} = 0.7, P_A = 2,$$
$$P_B = 1, v_A = 0.5, v_B = 0.2, \alpha = 0.1, \beta = 0.05, r_{A_A} = 0.5, r_{B_A} = 0.1, r_{B_B} = 0.25, z_A = 1,$$
$$z_B = 1, D_A = (1\,726, 1\,596, 1\,941, 1\,693, 1\,149), D_B = (199, 198, 193, 196, 141).$$

Then

$$r_{A_A} D_A = (863, 798, 971, 847, 575), r_{B_A} D_A = (173, 160, 194, 169, 115),$$
$$r_{B_B} D_B = (50, 50, 48, 49, 35).$$

A remanufacturing batch of Type A items is only possible in a period where Type A recoverable inventory is equal to or exceeds demand for Type A items in the same period. Based on the decision to manufacture or remanufacture in the previous period, Type A recoverable inventory is recalculated at the start of each period to determine if a remanufacturing batch is possible. Total recoverable inventory is shown in the first section of Table 3.1. The result of $f_{A_{it}} + c_{A_{it}}$ is provided in the second section of Table 3.1, where the cost of manufacturing and remanufacturing is compared in a period and the less of the two costs determine whether a manufacturing or remanufacturing batch will be executed in a period. The purchasing and holding cost of the dependent demand items required in the remanufacturing process is also minimised for each period in which a remanufacturing batch is executed and shown separately in the remanufacturing cost calculation in the second section of Table 3.1. The total cost of producing Type A items, f_{A_t}, is provided in the third section of Table 3.1.

TABLE 3.1 MODIFIED WAGNER/WHITIN HEURISTIC FOR THE CASE OF ALTERNATING MANUFACTURING AND REMANUFACTURING FOR TYPE A ITEM DEMAND

		$t =$				
	i	1	2	3	4	5
$I_{r_{A_t}}$	1					
	2		268			
	3			1 081		
	4				235	
	5					
$f_{A_{it}}$ $+ c_{A_{it}}$	1	5 924	**9 514**	16 739	26 228	35 548
	2		6 778 > 4 214 + 2 000	11 799 2 971	19 367 4 664	27 619 6 387
	3			7 817 > **2 865 +** 2 000	13 463 **2 847**	20 410
	4				7 045 > **2 188 +** 2 000	10 755 2 575
	5					**6 213**
f_{A_t}		5 924	9 514	14 379	17 414	23 627

Due to a constraint in Type A returns in period t = 1, the five-period cycle has to start with a manufacturing batch. The solution consists of a manufacturing batch of 3,322 units according to the Type A demand of the first two periods, including the provision for items that fail during manufacturing. The Type A demand in period t = 3 and t = 4 is satisfied by a remanufacturing batch of 1,941 and 1,693 units respectively, while the Type A demand in the last period is satisfied by a newly manufactured batch size of 1,149 units manufactured in period t = 5.

Type A dependent demand for the remanufacturing batches in period t = 3 and t = 4 is satisfied by a purchasing batch of 3,634 units ordered in period t = 3.

A remanufacturing batch of Type B items is only possible in a period where Type B recoverable inventory is equal to or exceeds demand for Type B items in the same period. Type B recoverable inventory is calculated at the start of each period, based on the decision to manufacture, or remanufacture Type A items to determine if a remanufacturing batch of Type B items is possible. Total recoverable inventory is shown in the first section of Table 3.2. The result of $f_{B_{it}}$ is provided in the second section of Table 3.2 where the cost of remanufacturing Type B items is minimised in each period. The purchasing and holding cost of the dependent demand items required in the Type B remanufacturing process is also minimised for each period in which a remanufacturing batch is executed and shown separately when calculating the cost of remanufacturing in the second section of Table 3.2. The total cost for producing Type B items, f_{B_t}, is provided in the third section Table 3.2.

TABLE 3.2 MODIFIED WAGNER/WHITIN HEURISTIC FOR THE CASE OF REMANUFACTURING FOR TYPE B ITEM DEMAND

		$t =$				
	i	1	2	3	4	5
$I_{r_{B_t}}$	1	219	21			
	2		230	37		
	3			279	83	
	4				302	161
	5					378
$f_{B_{it}}$	1	**403 +** 100	604 + **178**			
	2		411 + 100	**645 +** 256		
	3			445	813	
	4				461 + **100**	**754**
$f_{B_{it}}$	5					515
f_{B_t}		503	992	1 226	1 787	2 080

Type B demand is satisfied by a remanufacturing batch in period t = 1,2 and 4. Type B dependent demand for the Type B remanufacturing batches in period t = 1 and t = 2 is satisfied by a purchasing batch of 590 units ordered in period t = 1. The Type B dependent demand for the Type B remanufacturing batch in period t = 4 is satisfied by a purchasing batch of 337 units ordered in period t = 4.

The total cost of production can now be calculated with the use of (3.17), where the holding cost of the β items that failed during the Type A manufacturing batches in period t = 1 and t = 5 needs to the subtracted to avoid double counting of these costs. This results in a total cost of R24,976.

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}$$

$$f_5 = f_{A_5} + f_{B_5} - \beta Q_{P_1} h_{r_{B_{1,5}}} - \beta Q_{P_5} h_{r_{B_{5,5}}}$$

$$f_5 = 23\,627 + 2\,080 - 0.05\ \times 3\,322\ \times 0.7 \times 5 - 0.05 \times 1\,149\ \times 0.7 \times 1$$

$$f_5 = R\ 24\,976$$

3.4.2 Sensitivity analysis

Sensitivity analysis is conducted on the output of the linear programming model based on the small sample size used in the numerical analysis of the heuristic. The sensitivity analysis is conducted on selected input parameters considered relevant in order to investigate the effects that changes in those parameters have on the expected total cost and the number of Type A remanufacturing batches. The sensitivity analysis was conducted on 16 input parameters, serviceable and recoverable holding costs, setup costs, remanufacturing component ordering costs, recovery rates, remanufacturing component holding costs and manufacturing failure rates.

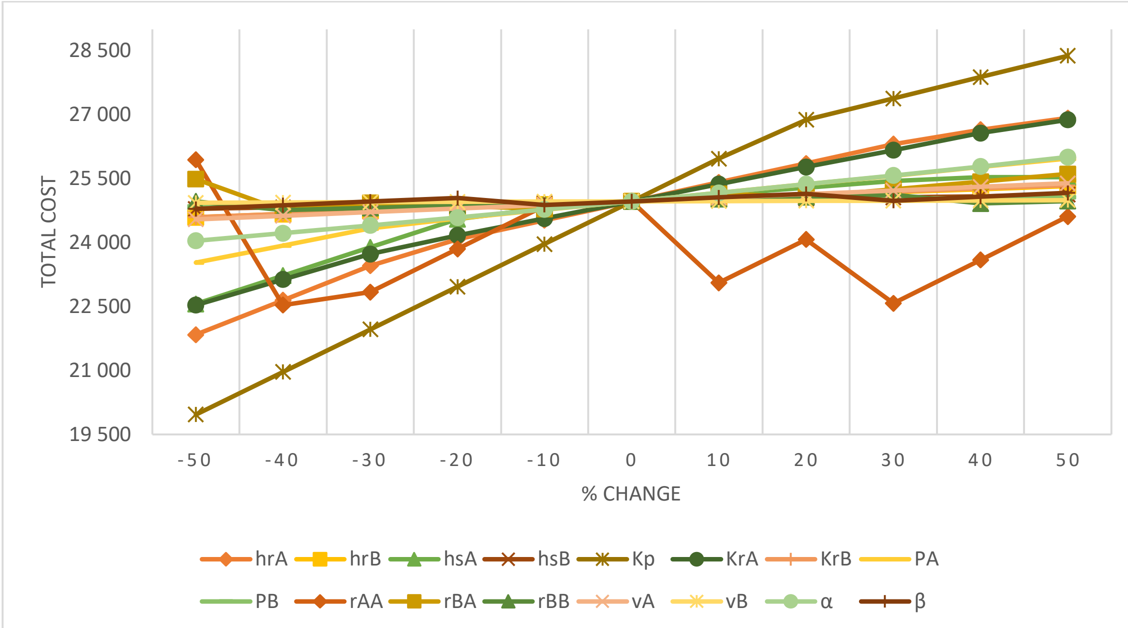

FIGURE 3.3 TOTAL COST IMPACT DUE TO PERCENTAGE CHANGE IN INPUT PARAMETERS

The following observations are made based on Figure 3.3, which shows the results of the sensitivity analysis of the total cost:

- The total cost is most sensitive to the manufacturing setup cost. As the manufacturing setup cost increases/decreases, total cost increases/decreases. This is because if all other parameters remain unchanged, the number of remanufacturing batches, as a possible cost reduction, cannot change due to constrained returns. The sensitivity to a change in the manufacturing setup cost will thus remain, whether the remanufacturing setup cost is higher or lower than the manufacturing setup cost.
- The total cost sensitivity to the remanufacturing setup cost and Type A recoverable inventory holding cost is also significant. The percentage change in these two parameters respectively results in a similar increase in total cost. A decrease of more than thirty percent in Type A recoverable inventory holding cost results in a larger decrease in total cost compared to the same percentage decrease in remanufacturing setup cost.
- A reduction in Type A serviceable inventory holding cost leads to a total cost reduction, as the Type A serviceable holding cost is applicable to both the manufactured and remanufactured Type A items.

- An increase in the Type A return rate leads to a reduction in total cost. This is not a linear relationship. This is because the increase in Type A return rate results in an increase in the number of periods in which demand is fulfilled by a remanufacturing batch. A reduction of thirty percent or more in the Type A return rate results in a reduced number of periods in which demand is fulfilled by remanufacturing batches. This holds true with the assumptions adopted from the literature on the relationship between manufacturing and remanufacturing setup costs in this paper. This also supports the sustainability of reverse logistics and how customers should be encouraged to return end-of-life products to enable the remanufacturing process.

The impact of the change in the Type A return rate on the total cost and the number of demand periods fulfilled by remanufacturing is shown in Figure 3.4.

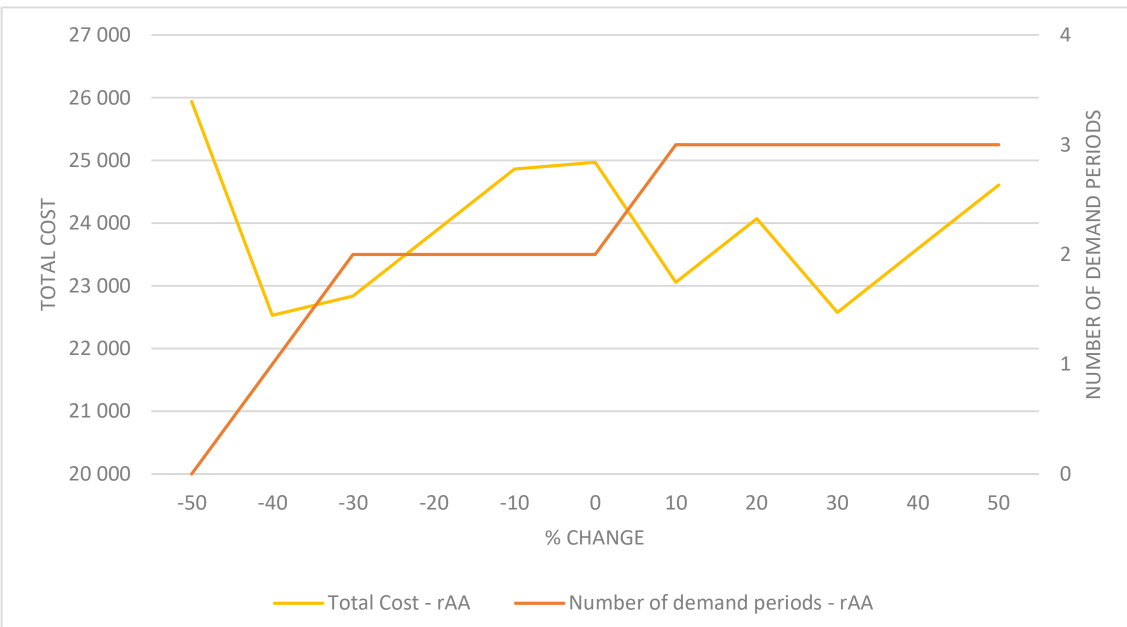

FIGURE 3.4 IMPACT OF CHANGE IN TYPE A RETURN RATE ON TOTAL COST AND NUMBER OF DEMAND PERIODS FULFILLED BY REMANUFACTURING

The following observations are made based on Figure 3.4:

- The relationship between the total cost and the change in type A return rate is not a linear relationship.
- The highest total cost is noted at a 50% reduction in Type A return rate and the lowest total cost is noted at a 40% reduction in Type A return rate, making the impact of Type A return rate on total cost, inconclusive.
- With an increase in the Type A return rate, there is an increase in the number of demand periods fulfilled by remanufactured items. This results in a lower total cost due to the reduction in total Type A and Type B recoverable inventory. As mentioned previously, this is not a linear relationship.
- With a decrease in the Type A return rate, there is a decrease in the number of demand periods fulfilled by remanufactured items. This results in an increase in the total cost due to the increase in both Type A and Type B recoverable inventory from items that fail during manufacturing.

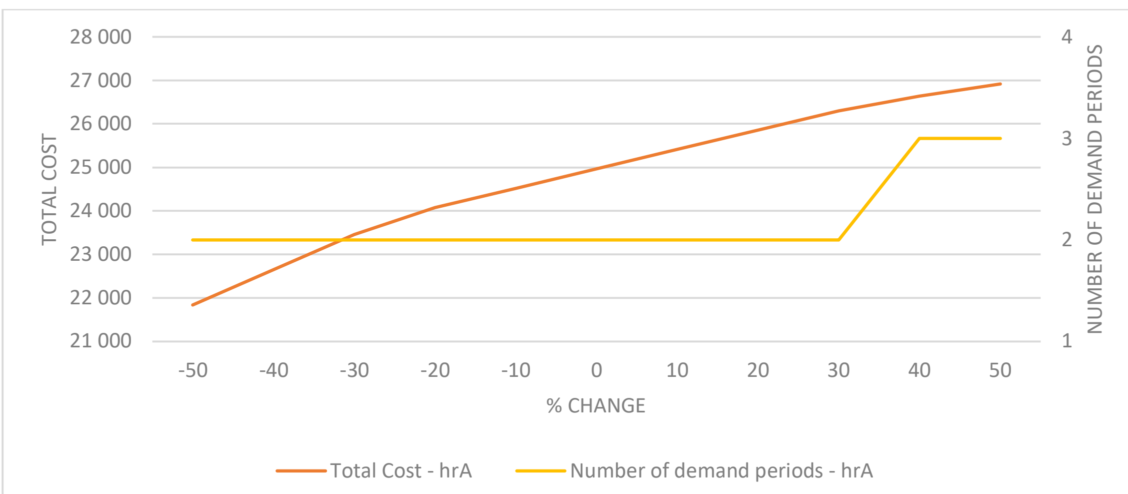

FIGURE 3.5 IMPACT OF CHANGE IN TYPE A RECOVERABLE INVENTORY HOLDING COST ON TOTAL COST AND NUMBER OF DEMAND PERIODS FULFILLED BY REMANUFACTURING

The following observations are made based on Figure 3.5:

- With an increase in the Type A recoverable inventory holding cost there is an increase in the number of demand periods fulfilled by remanufactured items. With the increase in recoverable inventory holding cost, the trade-off between consuming and holding recoverable inventory would lean more toward consuming.
- The increase in the number of demand periods fulfilled by remanufactured items, results in a slight reduction in the rate at which the total cost increases, due to the reduction of recoverable inventory in the system by introducing an additional remanufacturing batch.

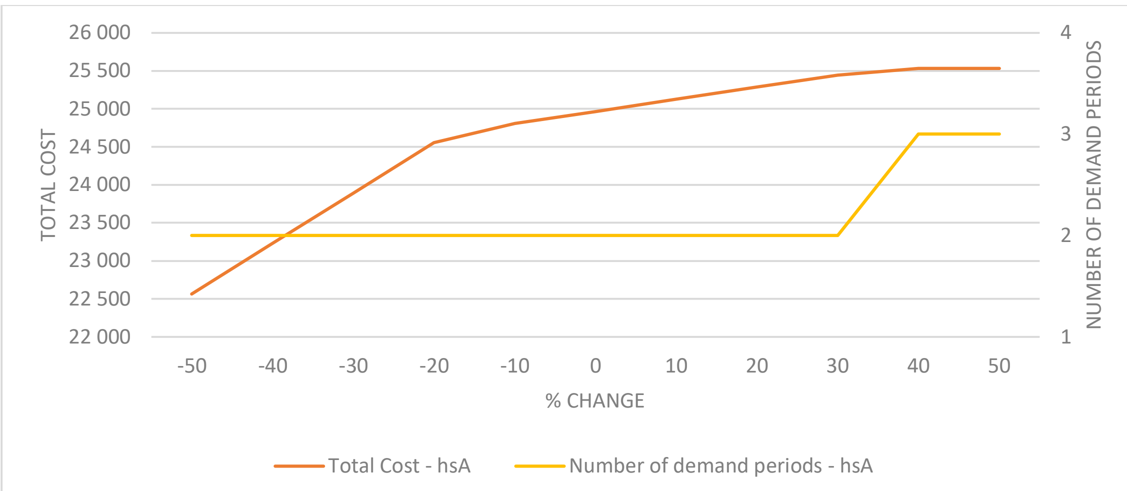

FIGURE 3.6 IMPACT OF CHANGE IN TYPE A SERVICEABLE INVENTORY HOLDING COST ON TOTAL COST AND NUMBER OF DEMAND PERIODS FULFILLED BY REMANUFACTURING

The following observations are made based on Figure 3.6:

- With an increase in the Type A serviceable inventory holding cost, there is an increase in the number of demand periods fulfilled by remanufactured items.
- With an increase in Type A serviceable inventory, the model adjusts to this increase by no longer manufacturing and remanufacturing for future periods. Thus, the trade-off at a certain point is simply between the cost of manufacturing and an increase in recoverable inventory or the cost of remanufacturing and a reduction in recoverable inventory. At a

40% increase in Type A serviceable inventory holding cost, total cost is minimized by adding an additional remanufacturing batch.

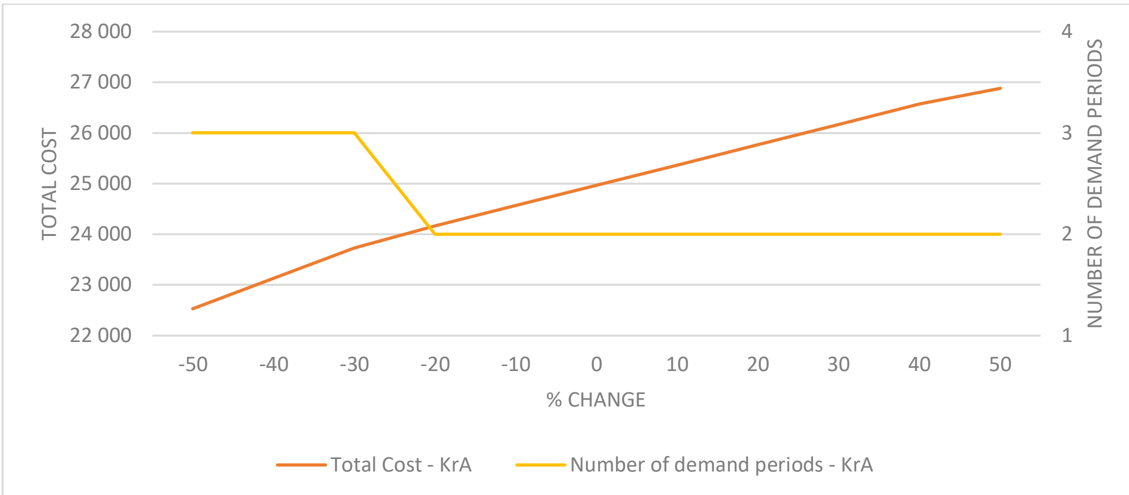

FIGURE 3.7 IMPACT OF CHANGE IN TYPE A REMANUFACTURING SETUP COST ON TOTAL COST AND NUMBER OF DEMAND PERIODS FULFILLED BY REMANUFACTURING

The following observations are made based on Figure 3.7:

- The point at which the number of demand periods fulfilled by remanufactured items increases, is noted at a reduction of 30% in remanufacturing setup cost.
- With an increase in the Type A remanufacturing setup cost, there is a decrease in the number of demand periods fulfilled by remanufactured items.
- The increase in the Type A remanufacturing setup cost results in a linear increase in the total cost with no clear correlation as the number of demand periods fulfilled by remanufacturing does not remain constant.

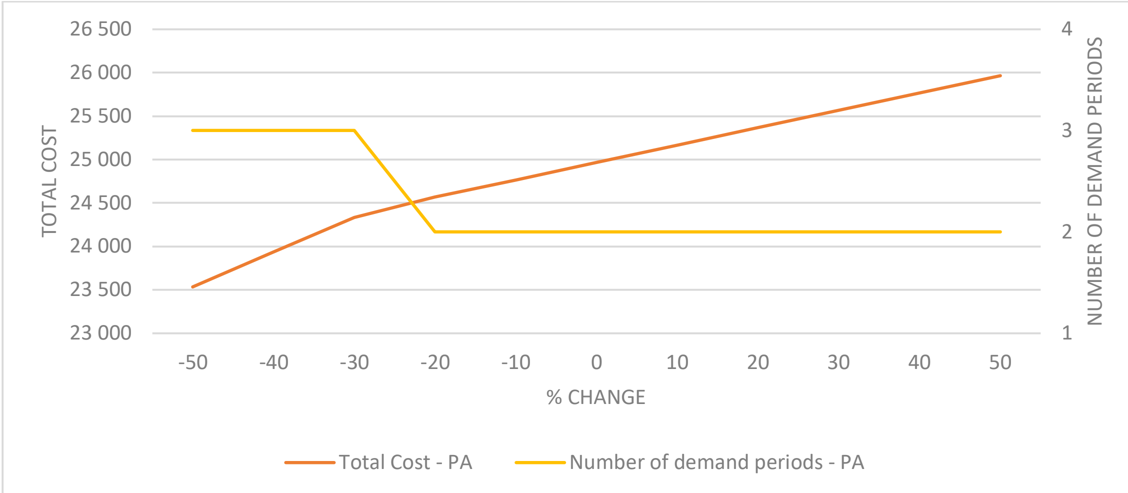

FIGURE 3.8 IMPACT OF CHANGE IN TYPE A REMANUFACTURING DEPENDANT DEMAND ORDERING COST AND NUMBER OF DEMAND PERIODS FULFILLED BY REMANUFACTURING

The following observations are made based on Figure 3.8:

- With a decrease in the Type A remanufacturing dependant demand ordering cost, there is an increase in the number of demand periods fulfilled by a remanufacturing batch. This is observed at a 20% decrease in the Type A remanufacturing dependant demand ordering cost.
- The increase in the Type A remanufacturing dependant demand ordering cost results in an increase in the total cost as the number of demand periods fulfilled by remanufacturing remains unchanged.

3.4.3 Model result comparison

Richter and Sombrutzki (2000) compared the result of their modified Wagner/Whitin heuristic to the result obtained when using the well-known Silver/Meal heuristic, the results from the Silver/Meal heuristic coincided with the results of the modified Wagner/Whitin heuristic. For this paper the result of the modified Wagner/Whitin heuristic derived in this paper is compared to the linear programming model result for the small sample used in the numerical analysis. The linear programming model resulted in a total cost of R 24 966, a R10 difference to the total cost result obtained by the heuristic of R 24 976. The difference is due to better decisions made in the remanufacturing of Type B items as the nature of a linear programming model allows for the testing of each manufacturing and remanufacturing scenario. The modified Wagner/Whitin heuristic will always favour the local optimum and move on to the next period. A Wagner/Whitin algorithm applies a forward-looking calculation to determine all possible alternatives, the optimal solution is then selected by looking backward, resulting in the local optimum (Wagner-Whitin algorithm - zxc.wiki, 2022).

A further result comparison is conducted on selected input parameters that the linear programming model proved to be most sensitive to in order to investigate the effects that changes in those parameters have on the expected total cost output from the heuristic. This will determine whether the modified Wagner/Whitin model is a suitable alternative to solve the total cost function. The comparison was conducted on 4 input parameters, Type A recoverable holding costs, Type A remanufacturing setup cost, manufacturing setup cost and Type A return rates.

TABLE 3.3 LINEAR PROGRAMMING MODEL RESULT COMPARISON TO HEURISTIC RESULT

Parameter	% Change	Parameter Value	Heuristic Total Cost	LP Total Cost	% Variance in Total Cost
r_{A_A}	-50%	0.25	25,939	25,939	0%
	0%	0.5	24,976	24,966	0%
	50%	0.75	24,607	24,607	0%
K_p	-50%	2500	19,976	19,966	0%
	0%	5000	24,976	24,966	0%
	50%	7500	29,966	28,378	5%
K_{r_A}	-50%	1000	25,128	22,533	10%
	0%	2000	24,976	24,966	0%
	50%	3000	26,966	26,878	0%
h_{r_A}	-50%	0.4	22,740	21,839	4%
	0%	0.8	24,976	24,966	0%
	50%	1.2	30,167	26,920	11%

The following observations are made based on Table 3.3:

- With no change in the parameter values of the numerical example, the linear programming model resulted in a total cost of R 24 966. With only a R10 difference due to more optimal decisions made in the remanufacturing of Type B items.
- A fifty percent decrease in the remanufacturing setup cost results in a ten percent variance in the total cost result of the two models. This is caused by a local optimum in period t = 2 of the modified Wagner/Whitin heuristic that recommends remanufacturing, this is however overridden by a lower cost decision to manufacture in period t = 1 for period t = 1,2 and 3. This increases the total cost for the full horizon, due to the increase in recoverable inventory.
- A fifty percent increase in the Type A recoverable inventory holding cost results in an eleven percent variance in the total cost result of the two models. This is again caused by a local optimum in period t = 2 of the modified Wagner/Whitin heuristic recommending a remanufacturing batch. This is once again overridden by a lower cost decision to manufacture in period t = 1 for period t = 1,2 and 3. This increases the total cost for the full horizon, due to the increase in recoverable inventory and the increase in the Type A recoverable inventory holding cost.
- On average the results of the two models differ by 3.33%, with the best-case scenario variance being as low as 0% and worst-case scenario variance being 11% on the small sample size evaluated.

The modified Wagner/Whitin heuristic seems to the be slightly lacking in considering the impact of Type A recoverable inventory holding costs incurred in future periods due to manufacturing versus remanufacturing decisions made in a period. The modified Wagner/Whitin heuristic finds the cost of manufacturing favourable in a period, however the compounding effect of multiple demand periods satisfied by manufacturing results in a increased level of Type A recoverable inventory. The heuristic output still produces the same trends in the total cost output. With an average variance of 3.33% in total cost, the heuristic proves to be a suitable alternative in calculating the total cost of manufacturing and remanufacturing for Type A and Type B items and determining the periods in which to manufacture and remanufacture for Type A and Type B items.

3.4.4 Numerical experiments with changing more than one parameter at a time

To compare the performance of the heuristic to the simplex solution, the experiment was designed at two levels. The first is to test the quality of solution obtained by the heuristic against that of LP, and the other is to check the resolution time of the heuristic against that of LP. These two experiments were separated because large number of replications were made to test the heuristic against the LP solution, and the resolution time for LP was getting quite long as the planning horizon exceeded 10 time buckets.

To guarantee a fair comparison between the linear programming model and the proposed modified Wagner/Whitin heuristic, a representative number of instances were examined on 3 levels of input data for the return rates for item type A and item type B, 2 levels of setup costs for manufacturing of A, remanufacturing of A and remanufacturing of B, 1 level for the holding cost for serviceable stock A, 2 levels each for serviceable stock B, recoverable stock A, and recoverable stock B. Altogether, we created 24 possible scenarios from the combinations. For each scenario, a planning horizon of 5 was used and each scenario was replicated 5 times (scenario sample size of 5). The demand was varied for each sample using the same distribution to generate demand. This leads to 120 replications for both the heuristic approach and the LP solution (240 in total). To analyse the results, each replication of 5 instances for each scenario was analysed for the mean cost value and the variance of cost for the replications within each scenario. The sample average and standard deviation for the heuristic and LP solutions were then compared to understand how the heuristic differs from the exact LP solutions.

Type A manufacturing setup cost and Type A remanufacturing setup cost can take on values of 2000 and 5000, while Type B remanufacturing setup cost can take on values of 250 and 2000. While the rate of keeping a Type A serviceable item in stock is set to one, holding a Type B serviceable item for one period can cost 0.9 and 0.7, holding a Type A recoverable item for one period can cost 0.8 and 0.4 and holding a Type B recoverable item for one period can cost 0.7 and 0.3. The return rate of Type A items can take on values of 0.6, 0.5 and 0.3, the return rate of Type B items from Type A items can take on values of 0.15, 0.1 and 0.05, while the return rate of Type B items from Type B items can take on values of 0.5 and 0.25. These values allow for groupings of instances to be evaluated in the numerical experiments, these groupings consist of instances with high, medium and low return rates, instances with high and low holding cost variances, instances where the Type A manufacturing setup cost is greater than the Type A remanufacturing setup cost as well as instances where the Type A remanufacturing setup cost is greater than the Type A manufacturing setup cost and lastly instances of high and low variance in the Type A and Type B setup costs. The heuristic is evaluated by using the percentage gap to the optimal solution as a performance measure. The results of the numerical experiments are presented in Table 3.4.

TABLE 3.4 PERFORMANCE OF THE MODIFIED WAGNER/WHITIN HEURISTIC

	% Cost error to the optimal solution		
	Average	Standard deviation	Maximum
All instances	**3,8%**	**2,6%**	**7,6%**
Return rate			
High return rates	5,5%	2,4%	7,2%
Medium return rates	1,4%	0,4%	1,8%
Low return rates	4,2%	2,9%	7,6%
Holding cost			
High holding cost variance	3,5%	3,1%	7,2%
Low holding cost variance	4,1%	2,5%	7,6%
Setup cost			
Type A manufacturing cost higher than remanufacturing cost	4,2%	2,9%	7,6%
Type A remanufacturing cost higher than manufacturing cost	3,5%	2,8%	7,2%
High variance in Type A and Type B setup costs	3,3%	3,2%	7,6%
Low variance in Type A and Type B setup costs	4,3%	2,4%	7,2%
K_p			
5 000	4,2%	2,9%	7,6%
2 000	3,5%	2,8%	7,2%
K_{r_A}			
5 000	3,5%	2,8%	7,2%
2 000	4,2%	2,9%	7,6%
K_{r_B}			
2 000	4,3%	2,4%	7,2%
250	3,3%	3,2%	7,6%

The following observations are made based on Table 3.4:

- With an average percentage gap of 3.8% in all instances and a maximum percentage gap of 7.6%, the heuristic proves to be a suitable alternative in calculating the total cost of manufacturing and remanufacturing for Type A and Type B items and determining the periods in which to manufacture and remanufacture for Type A and Type B items. The percentage cost error to the optimal solution is to be expected when using a heuristic.
- The lowest percentage cost error is experienced with medium return rates
- The greatest percentage cost error is experienced with high return rates.

3.4.5 Resolution time of the models

To evaluate the resolution time of the two solution approaches, a sample size of 5 was run for time buckets varying from 5 to 15 for both the heuristic and the LP solutions from which it could be seen that the resolution time of the LP solution picked up rapidly after about 10 time buckets when compared to that of heuristic. This implies that as the planning horizon becomes longer, solving with LP may gradually become unrealistic.

The resolution time of the linear programming model is shown in Figure 3.9.

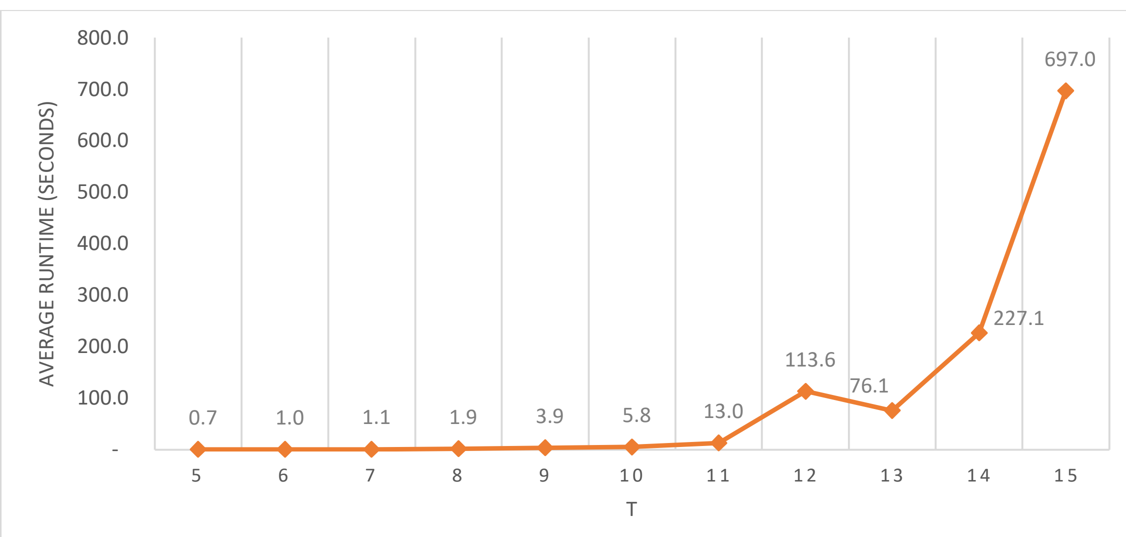

FIGURE 3.9 RESOLUTION TIME OF THE LINEAR PROGRAMMING MODEL

The following observations can be made based on Figure 3.9:

- The resolution time increases significantly from a total planning horizon of $T = 12$.
- The average runtime increases by 200% with each time bucket added from $T = 13$.

The increase in resolution time is to be expected from a linear programming model as the solution space increases. As the percentage cost error of the heuristic is relatively low and the resolution time of the linear programming model increases substantially from $T = 12$, the development and use of the modified Wagner/Whitin heuristic is supported. The runtime comparison of the two models is shown in Table 3.5.

TABLE 3.5 RESOLUTION TIME COMPARISON

T	Average LP runtime (seconds)	Average heuristic runtime (seconds)
5	0.7	0.0004
6	1.0	0.0012
7	1.1	0.0012
8	1.9	0.0020
9	3.9	0.0024
10	5.8	0.0024
11	13.0	0.0046
12	113.6	0.0040
13	76.1	0.0052
14	227.1	0.0058
15	697.0	0.0076

Comparing the quality of solutions obtained, it is apparent that the heuristic result should be quite close to optimal, while the solution continues to produce answers quite quickly as the horizon lengthens while LP needs a lot more time for resolution. This shows the quality of the solution approach to the problem, and makes it a good approach to consider when the planning horizon is long, which is not unrealistic in a number of planning environments, and even more so in instances where the organisation might need to analyse their plan in more granular time units, e.g., when moving from weekly to daily time buckets.

4. An economic order quantity model for two items with imperfect manufacturing process, time varying demand and return rates, dependent demand and different quality grades

4.1. Introduction

In this section a closed form solution is derived to solve a similar problem to the model presented in Section 3. The premium grade paper products are referred to as Type A item and the lower quality product as Type B item. With this notation, the general form of the system is presented next, but before the system is properly described, the model assumptions are presented in Section 4.2. After that, all the notations adopted are presented in Section 4.3 to foster a clear context of the modelling environment. The detailed system description is then presented after this in Section 4.4. Numerical results are presented in Section 4.5 to illustrate the proposed solution procedure and to provide managerial insights through a sensitivity analysis.

4.2. Model assumptions

The following assumptions are made for the two item production system presented of which the paper manufacturing context is a particular instance.

- The manufacturing and remanufacturing processes for Type A items are performed on the same resources and the manufacturing and remanufacturing batches are alternated in such a way as to minimize the total cost over the planning horizon;
- Items are remanufactured into Type B products on a separate resource;
- The manufacturing process produces Type A items only;
- Some items may fail during manufacturing and are sent to recoverable inventory for possible remanufacturing;
- Both Type A and Type B defective items can be returned by the end user to recoverable inventory for possible remanufacturing;

- A reparable Type A item can either be remanufactured to an as-good-as-new state to satisfy Type A demand once again, alternatively, a reparable Type A item can be remanufactured to a Type B item;
- Type B items can only be remanufactured to an as-good-as-new Type B item;
- Demand for the two items are independent of each other;
- Demand for both items are deterministic, but may vary over time;
- The rates of return are deterministic, but may vary over time;
- Reparable items need some other components that need to be procured to bring the returned items back to one of two states of reuse;
- Lower variety components are used to produce Type B items during the remanufacturing process;
- Top variety components are used during the remanufacturing processes of Type A items;
- Ordering and setup costs are known and constant;
- Shortages in remanufactured and new products to fulfil demand is not allowed;
- Stock holding costs of Type A and Type B serviceable inventory as well as the holding costs of items waiting to be repaired are known;
- Lead times for both manufacturing and remanufacturing processes are negligible.

4.3. Model notations

4.3.1. List of variables

4.3.1.1. List of decision variables

Q_{M_A} is the manufacturing batch size;

Q_{P_A} is the procurement batch size for material used in the manufacturing of Type A items;

Q_{R_A} is the remanufacturing batch size of type A items;

Q_{R_B} is the remanufacturing batch size of type B items;

Q_{C_A} is the return collection batch size of type A items;

Q_{C_B} is the return collection batch size of type B items;

I_{S_A} is the serviceable inventory level of type A items;

I_{S_B} is the serviceable inventory level of type B items;

I_{r_A} is the recoverable inventory level of type A items;

I_{r_B} is the recoverable inventory level of type B items;

v is the proportion of the recovered item A, r_A, that is remanufactured to an as good as new item A;

x is the quantity of raw material input procured, as a proportion the demand for item A, to fill gaps due to both insufficient materials recovered type A items and due to manufacturing yield loss while making a new item A. This ensures that the demand for A is met;

n_{P_A} is the number of manufacturing/remanufacturing cycle time in a cycle of procured item used in manufacturing item A;

n_{C_A} is the number of manufacturing/remanufacturing cycle time in a cycle of recovered batch used of item A to be used in the remanufacturing items A and B;

n_{C_B} is the number of manufacturing/remanufacturing cycle time in a cycle of recovered batch used of item B to be used in the remanufacturing item B;

T is the common cycle time for manufacturing and remanufacturing in a common facility;

t_{M_A} is the processing time for manufacturing a batch of new item A;

t_{R_A} is the processing time for remanufacturing a batch of recovered item A;

t_{R_B} is the processing time for remanufacturing a batch of recovered item B;

4.3.1.2. *List of parameters*

D_A is the demand rate for Type A items;

D_B is the demand rate for Type B items;

R_{M_A} is the manufacturing rate for type A items;

R_{R_A} is the remanufacturing rate for type A items;

R_{R_B} is the remanufacturing rate for type B items;

r_A is the recovery rate of Type A items;

r_B is the recovery rate of Type B items;

y is the product yield rate for Type A items from a manufactured batch of size type A item, recorded as a proportion of the manufactured batch size Q_{M_A};

u is the proportion of defective manufactured item A, $1-y$, that is remanufactured to an as good as new item A;

h_{s_A} is the holding cost of type A serviceable inventory (per item per time);

h_{s_B} is the holding cost of type B serviceable inventory (per item per time);

h_{R_A} is the holding cost of type A recoverable inventory (per item per time);

h_{R_B} is the holding cost of type B recoverable inventory (per item per time);

K_{M_A} is the manufacturing batch setup cost;

K_{R_A} is the type A item remanufacturing batch setup cost;

K_{R_B} is the type B item remanufacturing batch setup cost;

K_{C_A} is the collection cost for a batch of recoverable type A item;

K_{C_B} is the collection cost for a batch of recoverable type B item;

K_{P_A} is the ordering cost for a batch of new input items for manufacturing type A item;

S_{M_A} is the set up time for manufacturing a batch of new item A;

S_{R_A} is the set up time for remanufacturing a batch of recovered item A;

S_{R_B} is the set up time for remanufacturing a batch of recovered item B;

4.4. DETAILED SYSTEM DESCRIPTION

Consider a manufacturing system that produces two items, A and B. The demand rate for item A is D_A while that for item B is $D_{\boldsymbol{B}}$. A is a top-grade item, while B is also a good item, but not as good as item A. A is produced from both manufacturing from new input materials as well as remanufacturing of recovered materials, but B is manufactured only from recovered materials. The recovery process includes return of used items A and B as well as salvaging items that are damaged when being newly manufactured.

A proportion r_A of the total demand for item A, D_A, is recovered. These recovered items are sorted into two grades for use as feedstock for remanufacturing: top grade return, which is used as feedstock to remanufacture as-good-as-new item A and sold again; acceptable grade return, which can no longer be processed into A, but is processed into item B. A proportion,

v, of the returned item A, r_A, gets remanufactured back as item A, while the remainder, $1 - v$, of r_A gets remanufactured to item B. A proportion, r_B, of the total demand, D_B, for item B is also recovered, but such returns can only be processed to as-good-as-new item B again. If the proportion of item B recovered, r_B, is more than what is needed for remanufacturing to item B, the remainder is simply discarded.

Item A is manufactured in batches of Q_{M_A} and remanufactured in batches of Q_{R_A} while item B is remanufactured in batches of Q_{R_B}. New top-grade feedstock is procured for manufacturing of each item A batch of Q_{M_A}. The process of manufacturing new items is not perfect and produces defective items. The yield rate, y, is the proportion of good items recovered from every batch of Q_{M_A}. Based on the severity of the defect, the defective items from the manufacturing process are recovered and sorted to be reused as feedstock for remanufacturing of item A or item B.

The manufacturing and remanufacturing processes are all done using the same resource. A processing cycle time, T, consists of a single set up each for a batch, Q_{M_A}, for manufacturing item A from new feedstock, a batch, Q_{R_A}, of remanufactured item A, and a batch, Q_{R_B}, of remanufactured item B. The quantity of the number of manufactured items A from the new feedstock is dependent on the level of shortage observed from meeting the demand for item A as a result of the losses due to the less than perfect manufacturing system, $1 - y$, and the proportion of returned items A allocated to the remanufacturing of item B, $1 - v$. As a result, a proportion of the demand, xD_A, needs to be met by procuring new input stock for item A to augment the shortage in each manufacturing cycle.

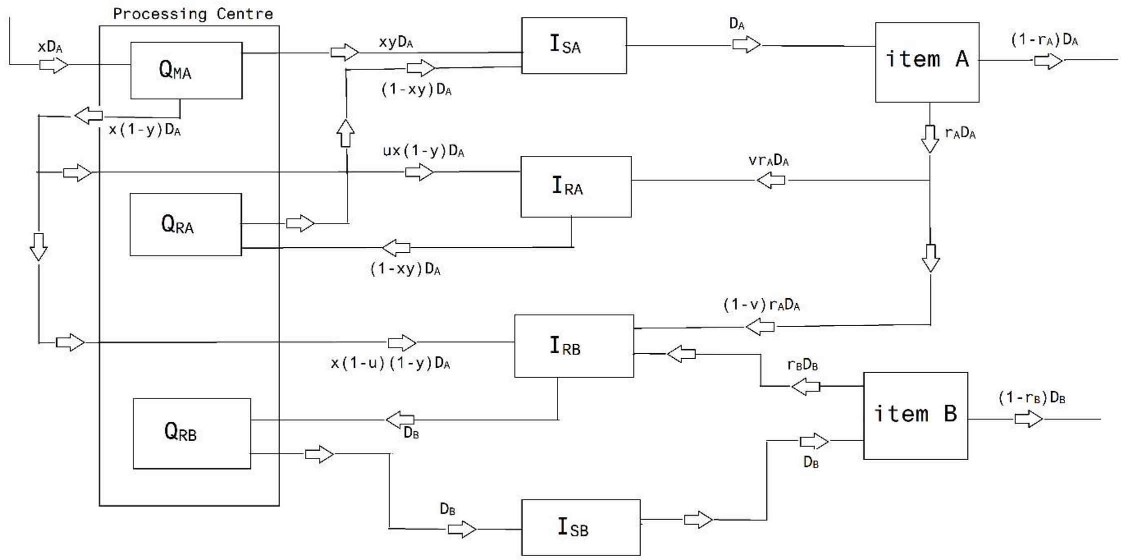

FIGURE 4.1 FLOW OF MATERIAL IN THE RECOVERABLE INVENTORY SYSTEM WITHIN A TIME INTERVAL

Figure 4.1 presents the flow of materials in the recoverable inventory system within a time interval. Quantity D_A are produced in aggregate to meet the demand of item A. Of this quantity, only the proportion r_A $(0 \leq r_A \leq 1)$ is recoverable, and the proportion $1 - r_A$ of the quantity D_A produced is lost or considered to be no longer useful for remanufacturing. A proportion, v $(0 \leq v \leq 1)$, of the recovered stock, r_A, is considered useful for remanufacturing item A, while the remaining proportion, $1 - v$, is only useful for manufacturing item B. Depending on the value of r_A and v, it is possible that the recovered stock is not enough to meet the demand for item A, D_A, hence, the need to procure some new feedstock as proportion of the demand for item A, xD_A, to make up for the shortages $(0 \leq x \leq 1)$. Manufacturing of item A from new feedstock produces some defective items. The yield rate of this process is y $(0 \leq y \leq 1)$, stated as a proportion of the quantity manufactured within the time interval, Q_{M_A}. The entire manufacturing reject is salvaged with the proportion u $(0 \leq u \leq 1)$ of the reject proportion $(1 - y)$ remanufactured back as item A, and the remaining proportion $(1 - u)$ remanufactured as item B.

The serviceable inventory stock for item A, I_{S_A}, receives input from both the remanufacturing and the manufacturing processes and supplies the point of consumption of item A. The returnable inventory stock for item A, I_{R_A}, receives its input stocks from two sources also; a portion of the parts damaged during manufacturing, and a portion of the used and recovered item A, and it only supplies the remanufacturing process of item A. The recoverable inventory stock for item B, I_{R_B}, receives input from three sources; a portion of item A that was damaged during manufacturing, a portion of item A that was recovered after use and all of the item B that was recovered after use. It only supplies the remanufacturing process of item B. The serviceable inventory stock of Item B, I_{S_B}, receives input from only the remanufacturing process of item B. The quantity flow rate for each of these inputs and outputs from each type of stock are indicated on the lines in Figure 4.1. The collection cycle times for the procured items of the manufacturing input, the recovered used items of A and the recovered used items of B, are integer multiples n_{P_A}, n_{C_A} and n_{C_B} of the manufacturing cycle T by design, where the multiple can also be unity.

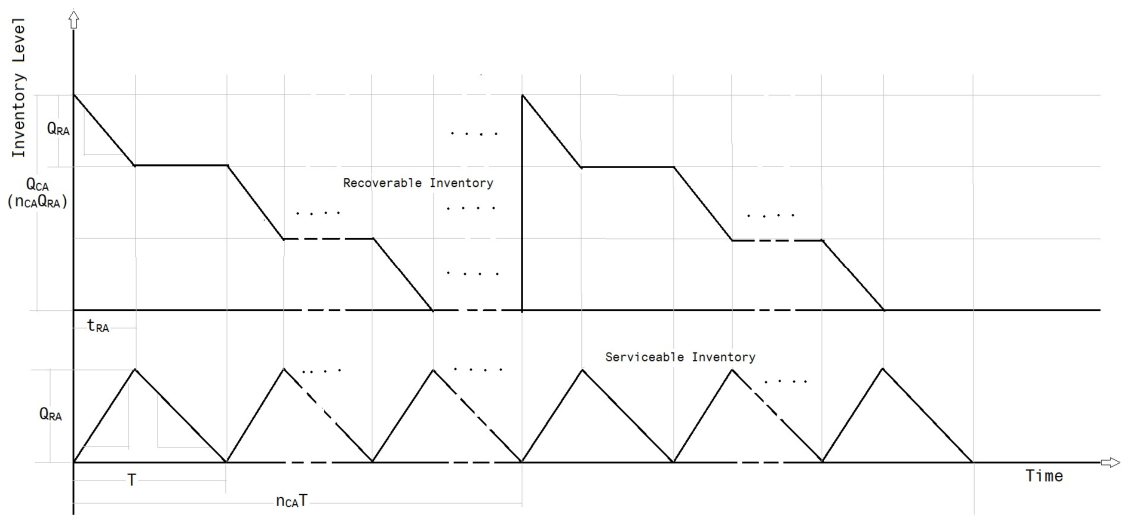

FIGURE 4.2 INVENTORY POSITION OF THE REMANUFACTURING PROCESS FOR ITEM A

The graph of the inventory position of the remanufacturing process for item A, showing the movements of the serviceable items of A produced from the recovered item A and the remanufactured item A is shown in Figure 4.2. Similar graphs can be drawn for the manufacturing process of item A and the remanufacturing process of item B. However, only graph of the remanufacturing process of item A would be discussed without any loss of generality.

A lot size of Q_{C_A} of recoverable item A were collected. This is equal to the quantity needed to produce serviceable stock of item A over the next n_{C_A} remanufacturing cycles. This quantity is gradually drawn down until it reaches zero over the time interval of $n_{C_A}T$ by which time a new batch of recovered item A would have been shipped in again. During each remanufacturing cycle, T, the remanufacturing process is completed from start after time t_{R_A} period, by which time the quantity of remanufactured item A that is needed to meet the portions of the demand for item A from the remanufactured process till the end of the cycle, T, would have been produced. The remainder of item A is produced by the manufacturing process.

Figure 4.3 shows the integrated graph of the inventory level positions for the three processes using the same resource for manufacturing item A and remanufacturing item A and item B. In this example, without any preference for any order, the common cycle time for the three processes is T, and the three manufacturing and remanufacturing processes must all be completed within this cycle for the schedule to be feasible. This example starts with the manufacturing of new item A serviceable materials (illustrated by the blue line) starting with the resource set up for this process, S_{M_A}. This is followed by the operations time, t_{R_A}, for the manufacturing process, after which there is a change over to the remanufacturing process for item A (illustrated by the red line), also consisting of set up and processing times as well. The resource is then set up for remanufacturing of item B (illustrated by the green line), all completed within this common cycle time, T, after which the cycle must be repeated. The rate of growth of inventory until when the maximum inventory level is attained per cycle for each of the processes is indicated in the graph.

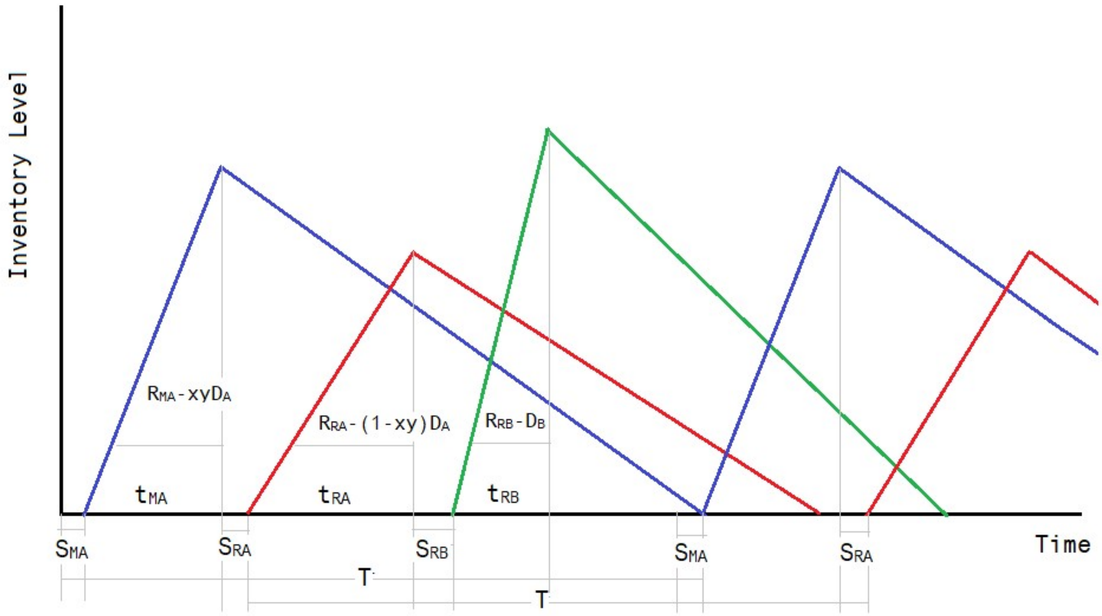

FIGURE 4.3 INTEGRATED GRAPH OF THE INVENTORY LEVEL POSITIONS FOR THE THREE PROCESSES

Before proceeding to derive the cost rate for the system, it would be pertinent to state some relationships that are apparent from the figures of the system presented.

The quantity if items are produced (manufactured and remanufactured) within the period, T, must meet the demand during the period. Only y proportion of the manufactured batch is good for use while all of the remanufactured items are considered good for use, hence

$$Q_{M_A}y + Q_{R_A} = D_A T$$

(4.1)

The quantity of good item A manufactured during the period, T, which is obtained after accounting for yield loss is (4.2), and by subtracting (4.2) from (4.1), the quantity of remanufactured item A used to meet the demand for item A in period T is (4.3). Also, since the entire demand for the item B in period T is met through the remanufacturing process of B only, it leads to (4.4).

$$Q_{M_A}y = xyD_A T$$

(4.2)

$$Q_{R_A} = (1 - xy)D_A T \tag{4.3}$$

$$Q_{R_B} = D_B T \tag{4.4}$$

Since a single return batch of procured item, Q_{P_A}, would be used for n_{P_A} batches of the manufactured item A, this leads to (4.5). Similar considerations for return batches of items A and B respectively leads to (4.6) and (4.7).

$$Q_{P_A} = n_{P_A} Q_{M_A} \tag{4.5}$$

$$Q_{C_A} = n_{C_A} Q_{R_A} \tag{4.6}$$

$$Q_{C_B} = n_{C_B} Q_{R_B} \tag{4.7}$$

The cost rate for the system can be calculated from the aggregation of those of the sub processes. For the manufacturing and remanufacturing processes for item A, there would be the production resource setup cost, the input feedstock ordering cost, the batch collection cost for recoverable item A to be remanufactured, the serviceable item holding cost, the manufacturing input feedstock holding cost and the recoverable item holding cost.

The batch setup cost per manufacturing and remanufacturing cycle are K_{M_A} and K_{R_A} respectively, and hence cost rate (which may be assumed as annual cost rate everywhere) are K_{M_A}/T and K_{R_A}/T respectively. For the cost rate for collecting recoverable used items of A for the remanufacturing process and the ordering cost of procuring converted feedstock for the manufacturing process, it should be noted that there are n_{M_A} and n_{R_A} possible cycles of the manufacturing and remanufacturing for each collection and procurement cycles. The annual ordering cost for the feedstock would, thus, be $K_{P_A}/(n_{C_A}T)$. For the recovered item A, however, it should also be observed that only the proportion, v, of the recovered items is used for the remanufacturing of A, the remainder being used for item B, but paid for in the collection cost of item A. The annual collection cost for recoverable stock would also be $K_{P_A}/(vn_{P_A}T)$. The total fixed cost for item A's subsystem becomes

$$\frac{1}{T}\left(K_{M_A} + K_{R_A} + \frac{K_{C_A}}{vn_{C_A}} + \frac{K_{P_A}}{n_{P_A}}\right) \tag{4.8}$$

For the serviceable inventory holding cost for item A, observe that this stock is built up from two processes: the manufacturing and the remanufacturing process. For the manufacturing and the remanufacturing processes, the maximum inventory levels attained respectively are

$$xyD_AT\left(\frac{R_{M_A} - xyD_A}{R_{M_A}}\right) \tag{4.9}$$

$$(1-xy)D_AT\left(\frac{R_{R_A} - (1-xy)D_A}{R_{R_A}}\right) \tag{4.10}$$

The average inventory for the serviceable item A stock is therefore,

$$\frac{D_AT}{2}\left[xy\left(1-\frac{xyD_A}{R_{M_A}}\right) + (1-xy)\left(1-\frac{(1-xy)D_A}{R_{R_A}}\right)\right] \tag{4.11}$$

(4.11) can be manipulated algebraically and multiplied with the unit holding cost rate for the serviceable inventory item A, h_{S_A}, to obtain the average holding cost rate per annum as

$$T\frac{h_{S_A}D_A}{2}\left[1 - D_A\left[\frac{(xy)^2}{R_{M_A}} + \frac{(1-xy)^2}{R_{R_A}}\right]\right] \tag{4.12}$$

The average stock of the recoverable inventory held for item A can be deduced from the graph in Figure 4.2. The batch of used stock brought in would last n_{C_A} remanufacturing cycles. There are n triangular cycles of height Q_{R_A} held over a period t_{R_A} and $n-1$ rectangular blocks of height Q_{R_A} held over periods ranging from T to $(n_{C_A}-1)T$. From this, the total inventory carried in period $n_{C_A}T$ can be calculated as

$$\frac{n_{C_A}}{2}Q_{R_A}t_{R_A} + \sum_{i=1}^{n_{C_A}-1} iTQ_{R_A}$$

(4.13)

It can also be seen from Figure 4.2 that t_{R_A} can be determined from (4.14) and T can be obtained by inverting (4.3). Substituting both into (4.13) and manipulating leads to (4.15).

$$t_{R_A} = \frac{Q_{R_A}}{R_{R_A}}$$

(4.14)

$$\frac{n_{C_A}Q^2{}_{R_A}}{2R_{R_A}} + \frac{n_{C_A}(n_{C_A}-1)Q^2{}_{R_A}}{2D_A(1-xy)}$$

(4.15)

Substituting (4.3) for Q_{R_A}, dividing (4.15) by $n_{C_A}T$ to get the average inventory for the recoverable inventory for item A in period n_{C_A}, rearranging and multiplying by the unit holding cost for item A's recoverable inventory, h_{R_A}, leads to

$$T\left[\frac{h_{R_A}\big(D_A(1-xy)\big)^2}{2}\left[\frac{1}{R_{R_A}} + \frac{n_{C_A}-1}{D_A(1-xy)}\right]\right]$$

(4.16)

In the same manner, the average inventory for the procured input feedstock for the manufacturing of the item A follows a similar pattern to that of the recoverable inventory items for A. Hence, by multiplying the average inventory held with the unit holding cost for the procured feedstock, h_{P_A}, (4.17) is obtained as

$$T\left[\frac{h_{P_A}(D_Axy)^2}{2}\left[\frac{1}{R_{M_A}} + \frac{n_{P_A}-1}{D_Axy}\right]\right]$$

(4.17)

The annual cost rate for the operation of the remanufacturing process for item B will consist of the production resource setup cost for remanufactured item B, the batch collection cost for recoverable item B to be remanufactured, the serviceable item holding cost and the recoverable item holding cost. In a manner similar to the derivation of (4.8), the total fixed cost for this subsystem will be

$$\frac{1}{T}\left(K_{R_B} + \frac{K_{C_B}}{n_{C_B}}\right)$$

(4.18)

The average inventory held for the serviceable inventory items for B can be seen from Figure 4.2 as

$$\frac{Q_{R_B}}{2}\left(\frac{R_B - D_B}{R_B}\right)$$

(4.19)

Substituting (4.22) for Q_{R_B} and multiplying the average inventory held by the annual holding cost rate for serviceable inventory items for A, h_{S_B}, the annual serviceable holding cost rate is

$$T\left[\frac{h_{S_B} D_B}{2}\left(1 - \frac{D_B}{R_{R_B}}\right)\right]$$

(4.20)

The average inventory for the recoverable items for B follows a similar pattern to that of the recoverable inventory items for A and can be derived to be

$$T\left[\frac{h_{R_B} D_B^2}{2}\left[\frac{1}{R_{R_B}} + \frac{n_{C_B} - 1}{D_B}\right]\right]$$

(4.21)

The total cost function for the system can be obtained by adding (4.8), (4.12), (4.16), (4.17), (4.18), (4.20) and (4.21) to obtain (4.22).

$$\frac{1}{T}\left[K_{M_A}+K_{R_A}+K_{R_B}+\frac{K_{C_A}}{vn_{C_A}}+\frac{K_{P_A}}{n_{P_A}}+\frac{K_{C_B}}{n_{C_B}}\right] + \frac{T}{2}\left[h_{S_A}D_A\left[1-D_A\left[\frac{(xy)^2}{R_{M_A}}+\frac{(1-xy)^2}{R_{R_A}}\right]\right]\right. + \left[h_{R_A}\left(D_A(1-xy)\right)^2\left[\frac{1}{R_{R_A}}+\frac{n_{C_A}-1}{D_A(1-xy)}\right]\right]\Bigg] + \left[h_{S_B}D_B\left(1-\frac{D_B}{R_{R_B}}\right)\right] + \left[h_{R_B}D_B^2\left[\frac{1}{R_{R_B}}+\frac{n_{C_B}-1}{D_B}\right]\right] \tag{4.22}$$

The optimum cycle time, T, to minimise the cost can be obtained by differentiating (4.22) and equating to zero as

$$T^* = \sqrt{\frac{2\left[K_{M_A}+K_{R_A}+K_{R_B}+\frac{K_{C_A}}{vn_{C_A}}+\frac{K_{P_A}}{n_{P_A}}+\frac{K_{C_B}}{n_{C_B}}\right]}{\left[h_{S_A}D_A\left[1-D_A\left[\frac{(xy)^2}{R_{M_A}}+\frac{(1-xy)^2}{R_{R_A}}\right]\right]+\left[h_{R_A}\left(D_A(1-xy)\right)^2\left[\frac{1}{R_{R_A}}+\frac{n_{C_A}-1}{D_A(1-xy)}\right]\right]\right]+\left[h_{S_B}D_B\left(1-\frac{D_B}{R_{R_B}}\right)\right]+\left[h_{R_B}D_B^2\left[\frac{1}{R_{R_B}}+\frac{n_{C_B}-1}{D_B}\right]\right]}} \tag{4.23}$$

The optimal basic cycle time is shown in (4.23). This can be used to calculate the optimal quantity to manufacture, remanufacture, collect, and purchase from (4.2) to (4.7).

4.4.1. Proof of optimality of the cycle time

To show that (4.23) is a minimum for (4.22), it suffices to show that the second derivative of (4.22) is positive definite, leading to (4.24).

$$\frac{2}{T^3}\left[K_{M_A}+K_{R_A}+K_{R_B}+\frac{K_{C_A}}{vn_{C_A}}+\frac{K_{P_A}}{n_{P_A}}+\frac{K_{C_B}}{n_{C_B}}\right] \tag{4.24}$$

It can be seen that (4.24) is always greater than zero since it is impossible not to have at least one of all the terms in bracket to be greater than zero, and none of them can be less than zero. Also, (4.24) would be defined since T must always be a positive non-zero number.

4.4.2. Feasibility conditions

For (4.23) to be feasible, it needs to satisfy several constraints. The first constraint is that all manufacturing and remanufacturing processes must be completed within the cycle time, hence, the sum of the set-up time and processing time for the three processes must be less than or equal to cycle time calculated in (4.23), that is

$$S_{M_A} + S_{R_A} + S_{R_B} + t_{M_A} + t_{R_A} + t_{R_B} \leq T \tag{4.25}$$

Equations (4.26) and (4.27) can be derived for t_{M_A} and t_{R_B} in a similar manner to (4.14) as

$$t_{M_A} = \frac{Q_{M_A}}{R_{M_A}} \tag{4.26}$$

$$t_{R_B} = \frac{Q_{R_B}}{R_{R_B}} \tag{4.27}$$

Substituting (4.14), (4.26) and (4.27) for processing times, (4.2) to (4.4) for batch sizes as appropriate and manipulating algebraically, (4.28) can be derived as

$$T \geq \frac{S_{M_A} + S_{R_A} + S_{R_B}}{\left[1 - D_A\left(\frac{xy}{R_{M_A}} + \frac{1 - xy}{R_{R_A}}\right) + \frac{D_B}{R_{R_B}}\right]} \tag{4.28}$$

The second and third constraints [(4.29) and (4.30)] are that the rate at which the recoverable items A and B accumulate must be greater than or equal to the rate at which they are to be withdrawn for the remanufacturing of items A and B respectively, leading to

$$xu(1-y)D_A + vr_A D_A \geq (1-xy)D_A \tag{4.29}$$

$$x(1-u)(1-y)D_A + (1-v)r_A D_A + r_B D_B \geq D_B \tag{4.30}$$

The fraction x is the proportion of demand for item A that is manufactured above the normal demand to accommodate possible shortages that could occur. This is because some proportions of items A and B $(1-r_A)$ and $(1-r_B)$ respectively are always lost in each cycle. Moreover, some portion of returned item A $(1-v)$ and some portion of damaged item A $(1-u)$ would be converted to item B. To maintain the constant flow of as-good-as-new items to meet the demand for A, there is a need to manufacture some extra units of item A to accommodate the losses and conversions. This implies that for the cycle time and lot sizes to be feasible, x may be made the subject in (4.29) and (4.30) as

$$x \geq \frac{1-vr_A}{u(1-y)+y} \tag{4.31}$$

$$x \geq \frac{\frac{D_B(1-r_B)}{D_A} - (1-v)r_A}{(1-u)(1-y)} \tag{4.32}$$

The two equations provide lower limits for the choice of x that would make the solution derived for T feasible. The values of x would be determined based on these two equations and the higher of the two becomes admissible. These two equations would be used later in the creation of solution procedure for the problem.

If we assume the value of v, the proportion of return of item A remanufactured back as A, is not fixed, but to be determined based on the level of return of item A and item B during each period, a good choice would be to select a value of x that satisfies both (4.31) and (4.32). This may be reasonable because there would usually be a sorting process when all returns have been received, and while some of the returned A items can be easily classified as either clearly suitable or unsuitable for remanufacturing serviceable item A, a number of such may not fall clearly into either category and a decision may need to be made about where to place such. This is where being able to choose v is useful. If (4.31) and (4.33) are solved simultaneously, the appropriate value for v can be obtained from (4.33). This is the value of v that would ensure that all returned items are fully utilised with nothing disposed of as excess return. This would also be utilised in the design of the solution algorithm.

$$v = [u(1-y)+y]\left[1-\frac{D_B(1-r_B)}{D_A r_A}\right]-\frac{(1-u)(1-y)}{r_A} \tag{4.33}$$

4.4.3. Solution algorithm for problem

To solve the problem, it is important to ensure that the cycle time determined using (4.23) is feasible, and if not, the closest feasible alternative must be found. To develop a solution procedure, therefore, two main questions need to be answered: whether the solution obtained from (4.23) is feasible or not and whether the value of v is fixed or can be chosen from a range of admissible values. The solution procedure is, therefore, developed based on the answer to these two questions and by exploiting the nature of the problem as highlighted in the inherent constraints indicated earlier in (4.24) to (4.33). The solution procedure is outlined in Figure 4.4, guiding the user's way through the two questions and equations (4.24) to (4.33).

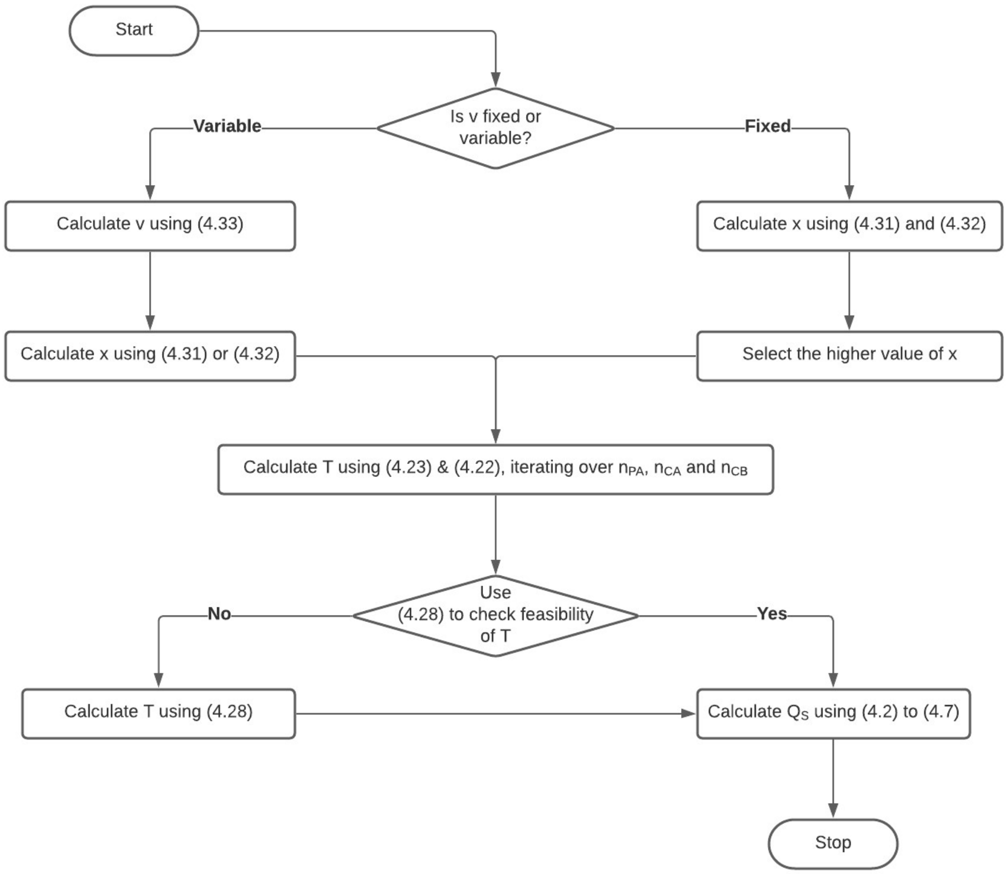

FIGURE 4.4 SOLUTION PROCEDURE

4.5. NUMERICAL ANALYSIS

4.5.1. NUMERICAL EXAMPLE

Data is simulated for the purpose of the numerical work. The numerical example using the proposed solution procedure shown in Figure 4.4 was solved using Microsoft SQL Server to easily iterate over n_{P_A}, n_{C_A} and n_{C_B}. The sensitivity analysis was also done using Microsoft SQL Server to calculate the result of numerous parameter value changes. The input parameter values are presented in Table 4.1.

TABLE 4.1 INPUT VARIABLES

Parameter	Value
D_A	1 621 / week
D_B	185 / week
r_A	0.5
r_B	0.35
y	0.9
u	0.1
h_{s_A}	R 1 / item / week
h_{s_B}	R 0.90 / item / week
h_{R_A}	R 0.8 / item / week
h_{R_B}	R 0.7 / item / week
K_{M_A}	R 5 000
K_{R_A}	R 2 000
K_{R_B}	R 250
K_{P_A}	R 1 000
K_{C_A}	R 500
K_{C_B}	R 50
R_{M_A}	15 000
R_{R_A}	15 000
R_{R_B}	15 000
S_{M_A}	5
S_{R_A}	3
S_{R_B}	1

Using the solution procedure outlined in Figure 4.4 and assuming v is variable, the calculated results are presented in Table 4.2, with a total cost of R11,242 per week and optimal cycle time of 13.78 weeks. In order to minimize the total cost function, (4.23) and (4.23) is used to iterate over n_{P_A}, n_{C_A} and n_{C_B}. All three parameters are first set to one, the total cost is calculated and then n_{P_A} is increased by one to determine whether this results in a reduction of the total cost. This is done until the total cost reaches a minimum, after which the iteration over n_{C_A} starts. This is followed by the iteration over n_{C_B} to determine the optimum. This iteration process resulted in a n_{P_A} value of 112 and a n_{C_A} and n_{C_B} value of 1 each. The impact of n_{P_A} on the total cost is almost negligible, thus the reason for the high n_{P_A} value. Iterating over n_{P_A} reduced the total cost by R106 per week from R11,348 to R11,242. The implication of this is that n_{P_A} can be fixed at any much smaller value without a significant change in the total cost. The optimal cycle time is used to calculate the respective optimal order quantities during the 13.78-week cycle. The optimal order quantities are provided in Table 4.2.

TABLE 4.2 CLOSED FORM SOLUTION OUTPUT USING THE PROPOSED SOLUTION PROCEDURE

Decision variable	Value
$\boldsymbol{v}$	0.59
$\boldsymbol{x}$	0.77
$\boldsymbol{T}$	13.78 weeks
$\boldsymbol{TC}$	R 11 242.33 / week
$\boldsymbol{Q_{MA}}$	15 517
$\boldsymbol{Q_{RA}}$	6 816
$\boldsymbol{Q_{RB}}$	2 549
$\boldsymbol{Q_{PA}}$	1 737 896
$\boldsymbol{Q_{CA}}$	6 816
$\boldsymbol{Q_{CB}}$	2 549
$\boldsymbol{n_{PA}}$	112
$\boldsymbol{n_{CA}}$	1
$\boldsymbol{n_{CB}}$	1

4.5.2. Sensitivity analysis

Sensitivity analysis is conducted on the output of the closed form solution based on the simulated data in the numerical analysis. The sensitivity analysis is conducted on selected input parameters considered relevant to investigate the effects that changes in those parameters have on the expected total cost. The sensitivity analysis was conducted on 17 input parameters, serviceable and recoverable holding costs, setup costs, collection costs, ordering cost, recovery rates, manufacturing and remanufacturing rates, the product yield rate for Type A items from a manufactured batch as well as the proportion of defective manufactured Type A items that are remanufactured to an as good as new Type A item.

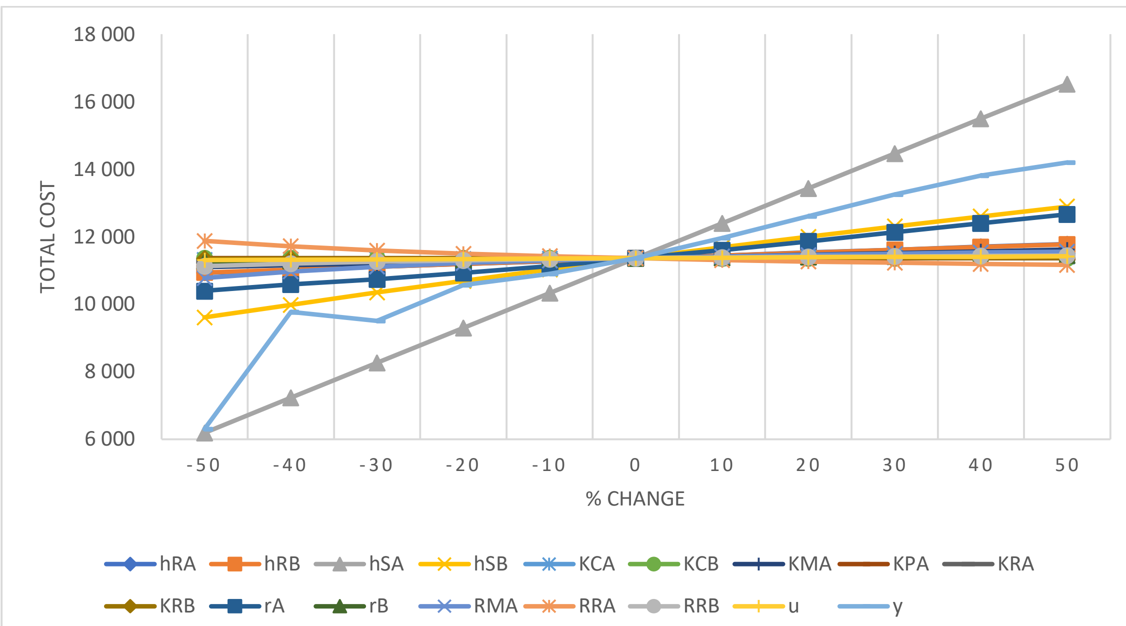

FIGURE 4.5 TOTAL COST IMPACT DUE TO PERCENTAGE CHANGE IN INPUT PARAMETERS

The following observations are made based on Figure 4.5, which shows the results of the sensitivity analysis of the total cost:

- The total cost is most sensitive to the serviceable holding cost of Type A items. As the Type A serviceable holding cost increases/decreases, total cost increases/decreases. This is to be expected as the total demand should be satisfied and the manufacturing and remanufacturing resource offers constraint capacity, since all three processes are performed on the same resource.

- The total cost sensitivity to the product yield rate for Type A items from a manufactured batch is also significant. With an increase/decrease in the yield rate, there is an increase/decrease in the total cost. With a higher product yield rate and the fact that Type A items are produced at the beginning of a cycle, the period of holding Type A serviceable inventory is increased and thus the total cost is increased.
- The total cost is inversely correlated to the remanufacturing rate for Type A items. As the remanufacturing rate for Type A items increases, the total cost decreases and vice versa. This is the ideal results and support the narrative to drive remanufacturing. This is however driven by the assumptions made in this paper and supported by Dowlatshahi (2000), that the cost to remanufacture is lower than the cost to manufacture.

5. CONCLUSION AND RECOMMENDATIONS

This work is an effort in modelling a reversed logistics system with two items with in which the recovered stock is used as an input in the remanufacturing two items of different quality grades. Two scenarios were considered: one in which the demand is discrete, where the solution makes use of a modified Wagner Whitten solution approach; and the other in which the demand in continuous and for which a closed form function is obtained, and a solution algorithm is presented. This model is useful in production environments like the manufacture of papers or similar products with recoverable input feed stock. Both models were illustrated with examples and sensitivity analysis was also done.

The major contribution made by the research presented here is the incorporation of constrained returns and taking into consideration that not all items can be remanufactured to an as-good-as-new state of the original item. Some portion of the recovered item top quality product can be used to satisfy a lower variety secondary demand. Each of the respective models consider some form of additional input items, where the dynamic lot sizing model considers that the remanufacturing process requires additional input items, and the closed form model considers that the manufacturing process can be supplemented by additional virgin feedstock for the top-quality item where the demand cannot fully be satisfied by the manufacturing and remanufacturing processes for the top-quality item.

The derived models take into consideration that items fail during manufacturing items are returned to be remanufactured to satisfy either of the two demand types. The proportion of demand that is returned for remanufacturing has a significant impact on the total cost function for the dynamic lot sizing model. This finding should motivate production and operations managers to encourage the customers to return and recycle used products. The manufacturing setup cost also has a significant impact on the total cost and can also be reduced by the increase in the proportion of demand that is returned. More returns result in more remanufacturing batches with a lower setup cost compared to that of a manufacturing batch. However, there will always be a need for manufacturing, as the items that fail during manufacturing are a vital input into the remanufacturing batches of both types of (top grade and lower quality) items being manufactured by the system.

The serviceable holding cost of the top-grade items have a significant impact on the total cost function of the closed form solution along with the product yield rate for top grade item from a manufactured batch. However, one important finding that should motivate the remanufacturing behaviour of production and operations managers in this case is that the total cost is inversely correlated to the remanufacturing rate for the top-grade items.

5.1. Future research considerations

This research can be extended in many other areas in the future, such as the consideration to include partial manufacturing and remanufacturing batches to counter constraint returns within a period. Shared remanufacturing resources for the top grade and the lower grade items in the case of the dynamic lot sizing model is also something to consider in future research. In the case of shared remanufacturing resources, a shortage cost should be considered in the case that demand for the respective items cannot be met within the same period.

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APPENDIX A

ALGORITHMS FOR OPTIMISATION OF ITEM TYPES AND COMPONENTS PLAN

<u>Manufacturing/Remanufacturing planning algorithm for type A</u>

For stage = 1 to planning horizon
Optimise stage or manufacturing using WW algorithm
Save all paths and the corresponding costs
Set the minimum cost and the corresponding path as manufacturing optimum
If there is sufficient recoverable stock for remanufacturing of A at stage
Optimise stage for remanufacturing using WW algorithm
Set the minimum cost and the corresponding path as remanufacturing optimum
Set the minimum of manufacturing optimum and remanufacturing optimum as the stage optimum
Else
Set the manufacturing optimum as the stage optimum
EndIf
EndFor
Return optimum cost and lot size plan for type A

<u>Explosion of type A for its input components quantities</u>

For i = 1 to planning horizon
If a remanufacturing batch for A is scheduled for i
Explode with BOM for component requirement for remanufacturing of type A
EndIf
EndFor
Return component requirement schedule for type A

<u>Procurement planning for input components of type A</u>

For stage = 1 to planning horizon
Optimise component purchase for A using WW algorithm
EndFor
Return optimal procurement cost and lot sizing plan for type A components

<u>Remanufacturing planning algorithm for type B</u>

For stage = 1 to planning horizon
If there is sufficient recoverable stock to remanufacture B
Use WW to optimise remanufacturing plan for B
Else
Reoptimise type A for feasible type B in stage
Optimise B using WW algorithm
EndIf
Return reoptimized manufacturing cost and lot sizes for A
EndFor
Return remanufacturing cost and lot size plan for B

Reoptimisation/Suboptimisation Algorithm

Sort manufacturing plans at stage in non-decreasing order of costs
Select the first manufacturing plan that makes remanufacturing of B feasible
Set current manufacturing plan as the optimum for A for current stage

<u>Explosion of type B for its input components quantities</u>

For i = 1 to planning horizon
If a remanufacturing batch for B is scheduled for i
Explode with BOM for component requirement for remanufacturing of type B
EndIf
EndFor
Return component requirement schedule for type B

<u>Procurement planning for input components of type B</u>

For stage = 1 to planning horizon
Optimise component purchase for B using WW algorithm
EndFor
Return optimal procurement cost and lot sizing plan for type B components