Supporting the teaching of mathematics by information technology in a co-operative learning environment

by

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Abstract

Supporting the teaching of mathematics by information technology in a co-operative learning environment

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This study presents the computer-supported co-operative mathematics learning environment (CSCML) as a catalyst for change of an inappropriate educational system. CSCML is seen as a way to enhance social skills and critical understanding of mathematics, thereby preparing learners for the demands of the technological society.

The need for an overall view of the CSCML environment was pointed out in the study. This is the need addressed by this study i.e. the main aim of this work was to develop a theoretical framework for CSCML which would give an overall view of the most important features and components of this complex learning environment. It is the belief that such an overall view could facilitate better design and understanding of the learning environment.

A social constructivist model for the learning of mathematics was developed with the focus on the self-organising nature as well as the social aspects of mathematics learning.

Two case studies were conducted guided by the social constructivist model for mathematics learning. The data collected from the case studies was then used to enhance the social constructivist model, mentioned above, to a model for CSCML.

This enhanced model proved to be inadequate. It failed to capture the dynamic nature of the learning environment, the complexities of social constructivism and the perceived openness and unpredictability of this learning environment.

Giddens' theory of structuration was then suggested as a suitable theory to use, to enable better understanding of this learning environment. Giddens' theory reconceptualises the reciprocal influence between the structural aspects of social systems and human action as a duality. With the aid of this theory, social constructivism was then reconceptualised as a duality.

The CSCML environment was interpreted from a structurational perspective as a *social system* in which interpretations of *organisational and mathematical structures* of signification, legitimation and domination are represented by the *CL methods/principles*, *IT, objective mathematical knowledge and the learning task.* These components act as *modalities of structure* upon which actors can draw in *co-operative actions* to reconstitute or change the structural properties of the components. This takes place by the *changing or reaffirmation of the agent's mental schemas* in a specific *organisational context*.

This framework was applied to one of the case studies to illustrate how it can be used as a tool to enable better understanding of the CSCML as a whole. The framework sheds more light on the dynamics of social constructivism and recognises the unpredictable nature of the learning environment. It also recognises one of the emergent properties of the CSCML environment as its evolutionary nature, i.e. the application of this learning environment could act as a catalyst to reshape and redirect traditional educational practices.

Opsomming

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Rekenaar gesteunde koöperatiewe wiskunde leer (RKWL) word in hierdie studie voorgestel as 'n moontlikheid om verandering binne bestaande uitgediende onderrigsisteme te bevorder. RKWL kan die dieper verstaan van wiskunde bewerkstellig. Dit kan ook sosiale vaardighede bou en leerders sodoende beter voorberei vir die eise van die tegnologiese samelewing. Die studie beklemtoon die tekort aan 'n samevattende beskrywende teoretiese raamwerk vir RKWL. Hierdie studie spreek hierdie tekort aan, en het ten doel om 'n teoretiese raamwerk vir RKWL te ontwikkel wat 'n holistiese beeld gee van die belangrike aspekte en komponente waaruit die betrokke leeromgewing bestaan.

'n Sosiale konstruktiwistiese model wat die leer van wiskunde beskryf, is ontwikkel. Die model fokus op die teenwoordigheid van beide individuele sowel as sosiale prosesse in die leer van wiskunde. Twee gevallestudies is daarna gedoen. Sekere aspekte van die

beplanning en uitvoering van die gevallestudies is gebaseer op die bogenoemde model. Die data wat versamel is vanuit die gevalle studies, is hierna gebruik om die model aan te pas tot 'n beskrywende model vir RKWL.

Dit het egter geblyk dat hierdie aangepaste model onvoldoende is. Die model het nie die dinamiese aard van die leeromgewing gereflekteer nie. Dit het ook nie die kompleksiteite van sosiale konstruktiwisme genoegsaam beskryf nie. Verder het dit die betrokke leeromgewing as 'n voorspelbare sisteem beskryf met seker invoere, prosesse en uitkomste.

Die struktureringsteorie van Giddens is gevolglik voorgestel as 'n gepaste teorie wat gebruik kan word om tot 'n beter verstaan te kom van die dinamiek van die RKWL omgewing. Hierdie teorie rekonseptualiseer die wedersydse wisselwerking tussen die strukturele aspekte van sosiale sisteme en menslike aksie, as 'n dualiteit. Met behulp van hierdie teorie kon sosiale konstruktiwisme ook as 'n dualiteit gekonseptualiseer word.

Vanuit 'n struktureringsteorie perspektief kan die RKWL omgewing dus soos volg beskryf word: Dit is 'n sosiale sisteem waarbinne interpretasies van organisatoriese en wiskundige strukture verteenwoordig word deur koöperatiewe leer beginsels, inligtingstegnologie, objektiewe wiskunde kennis and die leertaak. Hierdie komponente dien as modaliteite van struktuur waarop agente hulle koöperatiewe aksies baseer. Die aksies bevestig of verander die strukturele eienskappe van die komponente. Dit alles vind plaas deur die verandering of bevestiging van die agent se kognitiewe skemas binne 'n spesifieke organisatoriese konteks. Die woord 'agent' verwys hier na die mens se vermoeë om 'n verskil te maak. Die mens is dus 'n agent in die sin dat hy/sy een of ander vorm van mag kan uitoefen.

Dit het duidelik geword dat hierdie teoretiese raamwerk meer lig werp op die dinamiek binne die RKWL omgewing en dat dit ook die onvoorspelbaarheid van hierdie sisteem erken. Die model identifiseer ook een van die eienskappe van RKWL as sy evolusionêre aard. Hiermee word bedoel dat die toepassing van hierdie leeromgewing, tot veranderinge en nuwe rigtings in die tradisionele onderriggebruike kan lei. In hierdie studie word hierdie eienskap dan ook uiteindelik geïdentifiseer as een van die belangrikste bydraes wat die toepassing van RKWL kan lewer.

Contents

Chapter 1

An introduction

1.1 Introduction

Rapid change, the accelerated pace of life, temporariness, instability, variation, novelty - these are some key words used by Toffler (1971) to describe the life of the 21st century person. A totally new society is foreseen, activated by technology. This highly technological society will be characterised by computer-based information technology which will have (and is already having) an effect on the fundamental structuring principles of society (Skovsmose, 1994).

Some of the predictions that Toffler made more than a decade ago, have already come true: not only amoeba but sheep have been cloned, the Internet already provides ' ... *near-instantaneous communication across the globe* ... ' (Toffler, 1971:363), and in organisations we see the demise of bureaucracy. However, his prediction about '*education in the future tense*' is far from being realised, leaving young people unprepared for the immense challenges facing them in adult life.

Businesses and other organisations are changing from bureaucracies to organisations characterised by Bennis (in Toffler, op.cit.) as ' ... *temporary systems, where problems will be solved by task forces composed of 'relative strangers who represent a set of diverse professional skills* ... '' (Toffler, op.cit.:137). Also, ' ... *skills in human interaction will become more important, due to the growing needs for collaboration in complex tasks* ... ' (Toffler, op.cit.:137).

The majority of schools around the world still follow the model of industrial bureaucracy. The system was adequate in the industrial era, since students left school to enter a similar adult society. However, the future world does not need ' ... *millions of* *lightly lettered men, ready to work in unison at endlessly repetitious jobs, it requires not men who take orders in unblinking fashion ... but men who can make critical judgements, who can weave their way through novel environments ...* ' (Toffler, op.cit.:364). Machines will do the routine work, leaving humans free to concentrate on the intellectual and creative tasks.

What would a more appropriate educational system than the present one look like? A more adequate educational system would be characterised by a less clear separation between formal and informal education (Bannon, 1995), cross-disciplinary curricula (Newman, 1995), a move from public education to home instruction (Debenham & Smith, 1994), part-time schooling and innovative teaching techniques (seminars, role-playing, group projects, microworlds, computer-based learning, etc.). Toffler is of the opinion that an educational system must increase the individual's ' ... *cope-ability* ... ' (Toffler, op.cit.:364).

There is generally an acute awareness of the need for change in the educational system. For example, information technology has been used in education for more than three decades. Many teachers experiment with different teaching techniques. Still, the changes in schools seem to be made more slowly than the changes in society. Most schools are still dominated by teacher-centred lectures in factory-like environments. Changes and new ideas are adopted as isolated innovations into a closed system that stays the same (Jost & Schneberger, 1994). Papert blames it on the ' ... *innate intelligence of School, which acted like any living organism in defending itself against a foreign body.'* (Papert, 1993:40).

The computer-supported co-operative learning environment is seen by some researchers as a possible catalyst for change. Van Weert states that '(c)*ollaborative learning in multi-disciplinary teams, with integrated use of Information Technology, is expected to have growing importance in education, and in the end to change its organization.'* (Van Weert, 1995:9). De Villiers (1995) proposes computer-supported co-operative learning

(CSCL) as an organisational idea that has the potential to connect educational and working environments, thereby making school more relevant to the demands of adult life.

Mathematics and related sciences underlie the dominant technology, since many aspects of information technology consist of applications of mathematical models. Mathematics and its formal methods are thus becoming an integral part of today's, and the future's, society. Some understanding of the role of mathematics is necessary for a person not to become a victim of the social processes of which mathematics is a component (Niss in Skovsmose, 1994). Niss adds that ' ... *the purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also perform the application of mathematics in society, in particular to situations which are of significance to their private, social and professional life.'* (op.cit., 1994:57).

The computer-supported co-operative learning environment provides an ideal organisational structure in which mathematics can be not only understood but also applied, judged and reflected upon both collectively and individually. Learners experience mathematics as both *social power* and *social construction*. On the one hand mathematical principles are built into the software, constraining and mediating social and mathematical behaviour, while on the other hand new mathematical meanings are created through social negotiation.

This study will thus focus on the *support of the teaching of mathematics by information technology in a co-operative learning environment*. Not only is this learning environment seen as a way of preparing learners for adult life but also as a way of enhancing learners' critical understanding of mathematics.

1.2 Problem statement

Co-operative Learning is described as a way of structuring the learning environment such that groups of students pursue academic goals through collaborative efforts. Research shows that co-operative learning results in more higher-level reasoning, generation of new ideas and solutions, and better transfer of learning (Slavin, 1991a).

Research on Co-operative Mathematics Learning (CML) shows that benefits include the fostering of positive attitudes towards mathematics, the development of problem-solving skills, better social skills and the promotion of higher self-esteem and motivation (Leikin & Zaslavsky, 1997; Good, T.L., Mulryan, C. & McCaslin, M.; 1992; Dees, 1991). It was also found that CML fosters on-task verbal interaction which has a positive effect on mathematics learning (Leikin & Zaslavsky, 1997).

Learning in a co-operative mathematics learning environment can be enhanced by the integration of information technology (IT) to provide computer-supported co-operative mathematics learning (CSCML). Research on CSCML includes studies on the effect of heterogeneous/homogeneous grouping on the gaining of complex learning skills, studies contrasting mathematical interaction in computer and non-computer contexts, and studies contrasting mathematics learning supported by different software (Hooper & Hannafin, 1988; Emihovich & Miller, 1988; Hoyles et al., 1994). Other studies reflect on the development of appropriate software to assist the co-operative mathematics learning process (Denning & Smith, 1997; Croisy et al., 1994).

McConnel (1994) points out that CSCL is a new area of research and that there are no definite answers to questions of design. This is also the case for CSCML. O'Malley (1995) identifies a need for an agreed framework for comparing and contrasting research on CSCL which might provide guidelines or principles for design. An overall view could help to identify important variables that need investigation in future research. Such theoretical frameworks do exist for CSCL environments but some researchers ask for more local theories for CSCL taking into account the particular knowledge domain (Mandl & Renkl, 1992). Good et al. (1992) also identify the need for a structure or overall view of the mathematics co-operative learning environment that identifies important variables that need investigation. They believe that this could serve as a basis for planning other research and as an aid in examining and interpreting existing research. Livni and Laridon (1993) feel that the application of learning theories to instructional design is too narrow and that there is a lack of attention given to the nature of the discipline being treated and the power of the computer itself.

It is thus clear that although researchers in general realise the interdependence of the different features and variables of the CSCML environment, no attempt has been made to describe this interdependence. This study addresses this need by providing a theoretical framework for the CSCML environment.

1.3 Contribution of this study

This study contributes to mathematics education by providing a theoretical framework for the design of a computer-supported co-operative mathematics learning environment. The findings from the case studies (discussed in Chapter 4) and the findings implicated by the framework (discussed in Chapter 5),

i) enhance the understanding of the complex web of activities and processes involved in CSCML,

ii) enable instructors to design more effective CSCL mathematics lessons by knowing what the features are and how they interact,

iii) provide more insight into the cognitive processes involved in the learning of mathematics in groups, and

iv) provide more insight into how the use of technology could enhance mathematics group learning.

1.4 Research philosophy

The topic of inquiry, viz. *supporting the teaching of mathematics by information technology in a co-operative learning environment*, places this study in the field of Information Systems. Roode defines the field of Information Systems as ' ... *an inter-disciplinary field of scholarly inquiry, where information, information systems and the integration thereof with the organization are studied in order to benefit the total system (technology, people, organization and society).'* (Roode, 1993:2). This definition recognises the fundamental social nature of information systems and challenges the sometimes implicit assumption that information systems is a purpose unto itself.

One can thus say that in this study, Information Technology and the integration of it in the mathematics classroom, or more specifically, the co-operative mathematics learning environment, will be studied. It is hoped that this will benefit the total system which includes the elements of technology, the learners, the effectiveness of the co-operative learning environment, the greater mathematics community as well as society in general.

The nature of the topic points to research philosophies within Information Systems (IS) as a possible source of ways to approach this inquiry.

1.4.1 Research in Information Systems

Smith (1990) argues that epistemological assumptions and issues surrounding social science research, guide the choosing of appropriate methods of research. Such an epistemological view that dominated social sciences until recently, is positivism. Positivists are working and thinking in the manner of natural scientists (Belt and Newby in Smith, 1990). Behind it lies the belief that social sciences can be investigated in a similar manner to the natural sciences. The general assumption is that reality is objectively given and that it can be described by measurable properties. These properties are independent of the observer. The observer is thus seen as objective and the scientific method tries to annihilate the individual scientist's standpoint. Also, the aim of science is regarded as the discernment of causal relationships pertaining to the social context and formulation thereof as universal scientific laws (Lamprecht, 1997).

Smith (1990) argues for the use of appropriate methods for the research problem - the researcher has to consider the purpose of the research and the nature of the phenomenon under investigation. If the assumption is made that information systems is social rather than technical, several reasons can be given why the underlying philosophy of positivism is inappropriate for Information Systems research. Lamprecht (1997) summarises a few of these points:

- The premise that the social world operates according to fixed general causal laws is questionable since empirical generalisations are made based on past experience alone which states nothing about the future.
- Observation is not a theoretically neutral matter - the observer's assumptions influence his/her observations. Also, the values of the observer influence the process and hence produce value-laden science (in contrary to positivism's claim of value-free science).
- The meaningful character of social life is largely ignored by positivist views. Social action is described and redescribed in the light of new evidence. Meaning can be understood hermeneutically and thus ascribes importance to language .
- The acquisition of knowledge is a social process in which knowledge becomes intersubjectively determined within the rules and conventions held by a particular community, which constitute a scientific discipline.
- Social wholes are metaphysical entities (not directly observable or verifiable against direct experience), giving them a non-scientific status in the positivistic view.
- Social systems are fundamentally unstable because of their ability to adapt in several ways (Lamprecht, 1997:39,40).

The above points explain why several researchers choose more inductive and interpretive approaches, using qualitative methods including case studies, action research, etc. For example, Banville and Landry (1989) view Management Information Systems (MIS) as a pluralistic scientific field and emphasise that it can only be understood and analysed with pluralistic models - not only models grounded on positivism.

Other researchers argue for the application of the interpretive paradigm to the study of Information Systems (IS). Interpretive researchers view the world as created through meanings and believe that access to reality is through social constructions such as language, consciousness, and shared meanings (Myers, 1997). For example, Boland (1985) believes that the design and study of information systems is best understood as a hermeneutic process. The Information System's output is text that must be read and interpreted by other people. Also, in the studying of IS, researchers study the interaction during system design and use in order to interpret the significance and potential meanings they hold (Boland, 1985:196). Also, ' ... *whereas positivist science pretends meanings are not problematic, phenomenology accepts meaning as the central problem on which all the knowledge of the social world will depend.*'(Boland, 1985:196).

Hughes (in Lamprecht, op.cit.), sees the aim of social sciences as reconstruction of particular fragments of the social reality from the elements of the structural mechanism. It is not concerned with prediction but rather with structures which exist independently of our knowledge and experience. It places less emphasis on the discovery of invariant empiricist causal generalisation than on the construction of models which will account for the patterns found among phenomena.

1.4.2 Research approach

The field of inquiry has an inter-disciplinary nature, borrowing research methods and background from sociology, psychology, management, anthropology, mathematics education, mathematics, and philosophy. It is thus a multi-dimensional, complex whole that cannot be reduced to its composite observable and 'measurable' parts.

Although this inquiry is about mathematics, it is not mathematics: nothing can be proved, only suggestions can be made; there are no predefined dependent and independent variables; no definition has mathematical precision since no such formal and unambiguous language exists in which these phenomena can be described.

It is assumed that mathematical knowledge is a social construction, that social interaction and negotiations enhance mathematics learning, that information technology is a human construction, subject to social interpretation. If knowledge is a social construction, then it can only be accessed through social constructions such as language, consciousness and shared meanings (Myers, 1997).

The above discussion provides the motivation for the choice of the philosophical perspective of this study, namely **interpretivism** - i.e., it will be assumed that phenomena can only be understood through the meanings that people assign to them.

With this underlying philosophy in mind, a model is developed which accounts for the patterns found amongst phenomena in the computer-supported co-operative mathematics learning environment.

Two theories are used to obtain a better understanding of the social system in which learning takes place, namely, **Gidden's structuration theory** and **social constructivism**. The latter is a philosophy of knowledge acquisition and the former a meta-theory of the constitution of social society.

Both theories try to overcome a dualism. Social constructivism tries to marry a dualism between the psychological subject and mathematical object. The underlying principle of social constructivism is that the social mathematical domain has an impact on the individual's developing of mathematical concepts and that the individual appropriates his or her meanings in response to his or her experiences in social context (Ernest, 1994a).

Gidden's structuration theory reconceptualises the dualism between objectivism (focus on society) and subjectivism (focus on the human agent). According to Gidden's duality of structure, social activity is enabled and constrained by social structures produced and reproduced through human agency (Lyytinen & Ngwenyama, 1992).

Case studies are used to inductively refine the developed theory. According to Yin (in Myers, 1997), '*(a) case study is an empirical inquiry that: investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident.*'(Myers, 1997:6). Goode and Hatt (in Smith, 1990) put it like this: ''*The case study, then, is not a specific technique. It is a way of organising social data so as to preserve the unitary character of the social object being studied. Expressed somewhat differently, it is an approach which views any social unit as a whole.*''(Smith, 1990:127).

The greatest criticism against case study use in research is its unrepresentativeness. Representativeness implies the applicability of conclusions derived from samples to the population as a whole. According to Smith (op.cit.), one can overcome the problem by either viewing case studies as appropriate to exploratory work only, or by applying quantitative procedures. Alternatively, representativeness can be viewed as irrelevant: i.e., that validity does not depend on the representativeness of cases in a statistical sense, but on the plausibility of the logical reasoning used in describing the results from the cases, and on drawing conclusions from them (Walsham, 1993:15). In the interpretivist view '' *... every particular social relation is the product of generative forces of mechanisms operating at a more global level, and hence the interpretive analysis is an induction (guided and couched within a theoretical framework) from the concrete situation to the social reality beyond the individual case.*''(Orlikowski & Baroudi in Walsham, 1993:15).

Case studies are thus chosen, not on how typical they may be, but on their explanatory power (Smith, 1990).

1.5 Research questions

Burrell and Morgan (1979) emphasise that all social sciences are approached via explicit or implicit ontological and epistemological assumptions. These include assumptions on human nature, e.g. are human beings conditioned by their external experiences or are they the creators of their own environment? These assumptions have implications for methodology. The researcher could view the world like the natural world or stress the importance of the subjective experience in the creation of the social world. Others emphasise the importance of overthrowing the limitations of social arrangements. Burrell and Morgan (1979) identify four paradigms defined by meta-theoretical assumptions that underlie the frame of reference, mode of theorising, and strategies of researchers working within the paradigms (Burrell & Morgan, 1979:23). These are the functionalist, interpretivist, radical humanist, and radical structuralist paradigms. According to Burrell and Morgan (op.cit.) they are mutually exclusive and cannot be operated in simultaneously - in accepting the conditions of one, the assumptions of the others are defied.

1.5.1 Process-based framework

Du Plooy, Roode and Introna (1993) (see also Roode (1993a)) see a way in which the deliberate use of the different paradigms mentioned above can assist researchers in their task. By switching assumptions, different facets of the problem are highlighted which enhances its holistic appreciation. They propose a framework called process-based which refers to the deliberateness behind the use of different sets of assumptions in the viewing of the research problem.

The framework entails the posing of different questions to explore different aspects of the problem. The researcher does not necessarily accept the assumptions associated with one question, or defy the assumptions of the other questions. It is merely an inquiry about different facets of the research problem to obtain as much information as possible. The importance and order of the questions (utilising certain assumptions from the Burrell paradigms) will be determined by the problem. The generic research questions are given in Figure 1.1.

What is?:

These type of questions explore the fundamental nature and essence of the problem. The structure of the problem as well as the underlying concepts are determined. The questions draw from the interpretivist paradigm in that they attempt to uncover relations and meanings.

How does?:

These questions deal with the observation and description of the problem as it manifests itself in reality.

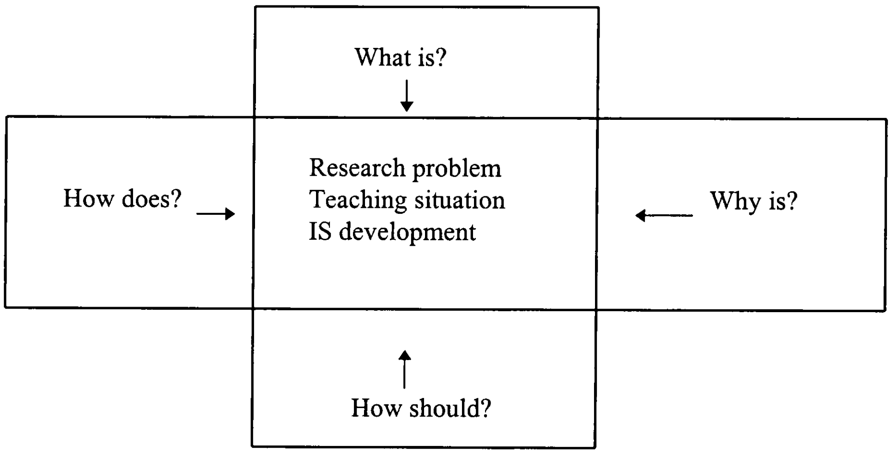


Figure 1.1

Generic research questions

(Source: Roode, 1993:11)

Why is?:

Real-life behaviour of phenomena are explained here. Relationships between variables are determined and described to provide possible explanations. The underlying assumption is that the relationships can then be used to generalise.

How should?:

The conclusions, implications and normative aspects of the research results are reviewed here. It might provide prescriptive conclusions or just enhance understanding of the problem domain.

By applying the different generic questions discussed above to the topic of inquiry of this study, the following research questions are defined.

1.5.2 The research questions

What is?

The key concepts are explored and defined.

- *What is mathematics?* This question not only involves a definition for mathematics but also assumptions about its nature, e.g. what is the nature of mathematical objects, how can the robustness and stability of mathematics be explained, is mathematics discovered or constructed?
- *What is mathematics learning?* Questions are asked here on assumptions about learning in general and more specifically mathematics learning, e.g. how important is the social environment for mathematics learning to take place, what is the role of the learner in his/her learning process, how are mathematical concepts stored in the mind of the learner?
- *What is a co-operative mathematics learning environment? What is a computer-supported co-operative mathematics learning environment?* These questions deal with the main 'ingredients' of the CML and CSCML environment and more

specifically the nature of mathematics learning in these specific learning environments.

How does?

Although not unrelated to the 'what is' questions, these questions deal with the modelling of real-life phenomena.

- *How can the learning of mathematics be modelled?* Questions are asked here on the process of mathematics learning based on the assumptions made in the 'what is' questions. All important aspects and features of mathematics learning need to be identified and related in this model.
- *How can the model be enhanced to reflect the CSCML environment?* The shortcomings of the model need to be identified and addressed by interpreting data obtained from case studies. Questions need to be asked about the relationship between different features and components of the learning environment and how it can be clarified using different social theories.
- *How does the introduction of mathematics in the CSCL environment influence that environment?* These questions deal with the uniqueness of mathematics and what this uniqueness brings to the CSCL environment.
- *How does the CSCL environment influence the mathematics curricula and learning?* Questions are asked here on the influence on the role of the learner, the role of the teacher, the rate and order of learning, the curricula, and the attitudes towards mathematics.

Why is?

Real-life behaviour is explained. Most of the 'how does' questions also deal with explanations of relationships between variables.

- *Why would one want to apply CSCL to the teaching and learning of mathematics?* This question looks for reasons why one would use CSCL in the mathematics classroom. Benefits on cognitive, affective and social levels are considered.

How should?

An evaluation of research results will provide a prescriptive conclusion on how the environment should be designed.

- *How should the CSCML environment be designed to enhance effectiveness and production.* This question addresses the practical implications of the developed theory for designers of the CSCML environment.

The chapters that follow are structured around the research questions. The *what is* and *why is* questions are addressed in Chapters 2 and 3. Preliminary work on the *how does* questions are done in Chapter 2 and finalised in Chapter 5. The *how should* question is addressed in par. 6.3. The research questions are revisited in par. 6.3 and this paragraph shows how the work done in this study answers the questions.

1.6 Layout

The main aim of this study is to develop a theoretical framework for CSCML which will give an overall view of the most important features and components of this complex learning environment. The development of this framework proceeds in the following way.

Chapter 2 describes the development of a social constructivist model for the learning of mathematics. It is assumed that mathematics is a human invention and that mathematics learning is both a process of self-organisation and enculturation. Different theories for the learning of mathematics are presented from which a final model is derived.

Chapter 3 gives an overview of important literature on CL, CML, CSCL and CSCML. The fragmented nature of research on CSCML is pointed out. This chapter also addresses problems and obstacles to the implementation of CSCML as well as design issues.

Chapter 4 describes two case studies that were conducted, guided by the theory developed in Chapter 2 as well as the theory described in Chapter 3. The data collected from the case studies is then used to refine the social constructivist model given in Chapter 2. Shortcomings of this model are pointed out and addressed in Chapter 5.

Chapter 5 summarises the main points of the structuration theory of Giddens and gives applications of the structuration theory to CSCW (Lyytinen & Ngwenyama, 1992) and CSCL (De Villiers, 1995). Finally, a theoretical framework for CSCML is developed using theory discussed in Chapter 5. This is followed by an illustration of ways in which the framework can enhance the better understanding of the dynamics of the CSCML environment by applying it to one of the case studies.

Chapter 6 evaluates the developed theory described in Chapters 2 to 5. It answers the research questions stated in this chapter, gives a critical evaluation of the contribution made to the body of knowledge and discusses further research. Figure 1.2 shows the interrelatedness of the different chapters of this study.

1.7 Summary

This chapter focuses initially on the social, technological and mathematical needs of the technological society, and the inadequacy of most of the educational systems to meet them. It offers CSCML as a possible catalyst for the change of outdated practices. It further highlights the need for an overall view of the CSCML environment to facilitate better design and understanding of the learning environment. The aim of this study is then to address this need. The philosophical approach is discussed. This approach, grounded in interpretivism, involves the development of an initial model for the learning of mathematics and the enhancement of it by case studies, learning theories and structuration theory. The research questions are then discussed based on the process-based framework of Du Plooy et al. (1993) (see also Roode, 1993a).

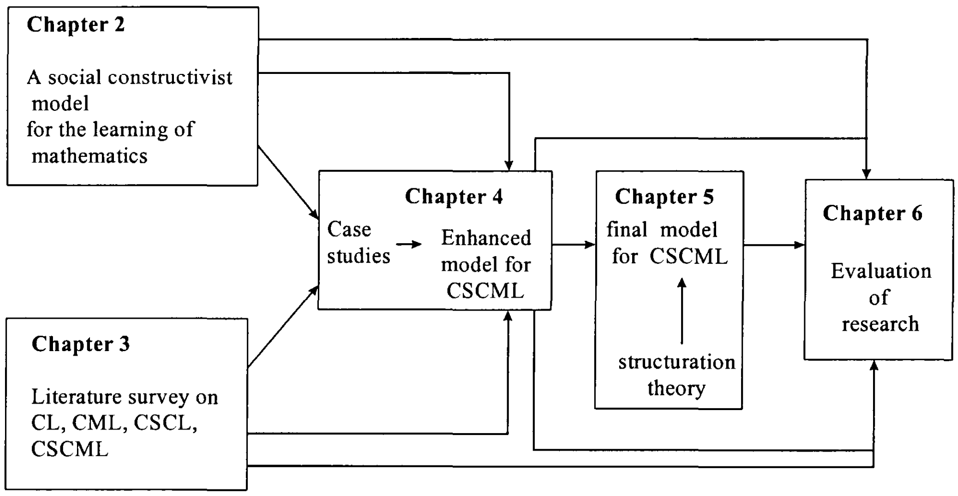


Figure 1.2

Inter-relatedness of the chapters

Chapter 2

Mathematics Learning

2.1 Introduction

In this chapter a social constructivist theoretical framework will be developed to describe the learning of mathematics. The framework is based on certain ontological and epistemological hypotheses. The ontological hypothesis is that mathematical objects are social-cultural-historical entities. The epistemological hypothesis is that of social constructivism, incorporating human and cultural aspects in epistemological questions.

The first part of the chapter presents an explanation and justification for the choice of ontological and epistemological hypotheses. The second part gives existing theories for the learning of mathematics and the last part explains the theoretical framework for the learning of mathematics. Existing theories are thus investigated through the lens of social constructivism and used in the development of the theoretical framework.

2.2 The philosophy of mathematics

2.2.1 Historical overview

Until as late as the nineteenth century, geometry was regarded by everybody as the most reliable branch of knowledge. Analysis derived its legitimacy from its link with geometry (Hersh, 1979). Therefore, the axioms on which the arguments were based, were accepted as basic truths which needed no justification. It was accepted that theorems that were derived from assumed axioms were true, since logical proof preserves truth and the axioms were self-evident truths. This view, called **Platonism**, considers mathematical activity as relating to the discovery of mathematical objects and relations that have an independent and objective existence.

It was found, however, that the denial of some of the 'true' postulates, leads to new bodies of knowledge (e.g. the denial of the parallel postulate of Euclid lead to the discovery of non-Euclidean geometries, showing that there was more than one thinkable geometry).

The development of analysis surpassed geometrical intuition, exposing its vulnerability. This loss of certainty in mathematical knowledge was perceived as intolerable, since it implied the loss of certainty in human knowledge (Hersh, 1979).

With great fervour, mathematicians set out to repair the foundations. The quest for certainty turned from geometry to arithmetic, and infinite sets were now introduced into the foundations of mathematics (Davis & Hersh,1981).

Four major views resulted from the quest for certainty. The fathers of **Logicism**, Russell, Frege and Whitehead, attempted to establish mathematics upon logic as foundation. However, in order to do this, they needed certain set theoretic axioms such as the axiom of infinity. Russell's paradox in 1902 shattered this illusion by exposing certain contradictions in set theory and the search for certainty continued (Devlin,1988).

Followers of **Intuitionism**, the school of the Dutch topologist Brouwer, argued that the natural numbers were reliable and needed no deeper foundation (Hersh, 1979). They are given to us by a fundamental intuition, and should therefore be seen as the starting point for all mathematics. This school views the only meaningful (and existent) mathematical objects as those that are given by a construction, in a finite number of steps, starting from the natural numbers. A more streamlined intuitionism was produced by Errett Bishop called **Constructivism** (Hersh, 1994). To most mathematicians, the criteria of intuitionism and constructivism seemed unreasonable: these programmes reject the Law of Excluded Middle and therefore any 'reduction ad absurdum' proofs. A strict adherence to this approach would prevent the creation of many fundamental results (especially in Analysis). The majority of mathematicians thus continued to work nonconstructively (Hersh, 1979).

Formalism, the creation of Hilbert, originated from his response to the dilemma by inventing proof theory. Hilbert's idea was to regard mathematical proofs as sequences of formal symbols, rearranged and transformed according to certain rules which correspond to the rules of mathematical reasoning. Finite, combinatorial arguments would then be found to show that the axioms of set theory would never lead to a contradiction (Hersh, 1979). Thus, mathematics is seen as only a matter of choosing the right axioms and examining the logical consequences of these axioms (Rucker, 1982).

This was Hilbert's way of providing a secure foundation to mathematics, although not a 'true' one in the sense that geometry had been believed to be true. The formalists tried to make mathematics safe by turning it into a meaningless game.

However, in 1930, Gödel showed that no finite describable theory can codify all mathematical truth. The theorem, known as Gödel's Incompleteness Theorem, implies that mathematics is open-ended (Hersh, 1979). There can thus never be a final, best system of mathematics. The message sent to the thinkers of the Industrial Revolution who regarded the universe as a vast pre-programmed machine, was that Man will never know the final secret of the universe.

The schools described above are known as foundationist schools because of their attempts to establish a foundation for mathematical indubitability (Davis & Hersh, 1981). This absolutist view of mathematical knowledge sees it as consisting of certain and unchallengeable truths.

A radically different alternative was offered by Lakatos. Based on the philosophy of science of Karl Popper, Lakatos holds that informal mathematics grows by a process of successive criticism and refinement of theories and the advancement of new and competing theories (thus not by the deductive pattern of formalised mathematics) (Davis & Hersh, 1981). His masterpiece *Proof and Refutations* (Lakatos, 1976) is and was an inspiration for many people working on a new tradition in the philosophy of mathematics. The new tradition sees a broadening of the scope of the philosophy of mathematics by admitting the human nature of mathematics and the consequent incorporation of social and cultural aspects in the understanding of the nature of mathematics.

2.2.2 The fallibilist view

This view rejects the absolutist view by saying that mathematical knowledge is not an absolute truth and not beyond correction and revision (Ernest, 1991). Hersh (1979) claims that the verification of the correctness of a proof is not only a mechanical procedure, but always includes intuitive reasoning (whether verbal or diagrammatic). This explains why mathematicians often disagree, make mistakes, or are uncertain whether a proof is correct or not.

A term also associated with this view is **quasi-empiricism**. Tymoczko (in Koetsier, 1991) loosely defines quasi-empiricism as a set of related philosophical positions, re-characterising mathematics experiences, taking into account the actual practice of mathematicians: '*If we look at mathematics without prejudice, many features will stand out as relevant that were ignored by the foundationalists: informal proofs, historical development, the possibility of mathematical error, mathematical explanations (in contrast to proof), communication among mathematicians, the use of computers in modern mathematics, and many more.*' (Tymoczko in Koetsier, 1991:2).

Another term associated with this view is **social constructivism**. The term social constructivism applied to the work of sociologists of science and knowledge from the late 1960's. The term was later also used to describe work done on psychology by Harré, Shotter and the Soviet Activity Theorists, as well as work done on ordinary language and speech act philosophy by Wittgenstein and the philosophy of science with contributors Kuhn, Hesse and Foucault (Ernest, 1994a).

The underlying principle of social constructivism is that the social domain has an impact on the developing individual, and that the individual appropriates his or her meanings in response to his or her experiences in social context (Ernest, 1994a). Applied to the philosophy of mathematics, social constructivism focuses the attention on conversation and interpersonal negotiation in the construction of mathematical knowledge. (A more detailed discussion of constructivism will be given in par. 2.4.2.)

Hersh (1994) lists a few questions that a philosophy of mathematics should be able to answer. A few of these questions will now be looked at from the perspective of the contributors to the fallibilist view of mathematics:

What is mathematics?

A survey on the beliefs of mathematicians and mathematics educators regarding a definition of mathematics, gives several views: the reduction of complexity to simplicity, problem solving, a tool for other sciences, logic, rigour, accuracy, deductive reasoning, a language, and so on (Mura, 1995). However, Rucker (1987) gives a compelling argument in describing mathematics as the study of pure pattern. He groups the patterns of mathematics into five groups: number, space, logic, infinity and information. He further describes how logic and infinity can be seen as tools to bridge the gap between number and space: logic combines the facts about space patterns into a few symbols whereas infinity connects number and space by breaking up space into infinitely many distinct points. However, he argues that the new world view (which is partly a result of the computer revolution), describes reality as a pattern of information. Rucker makes this world view relevant to mathematics by describing it as problem solving: '*Mathematics turns shapes into areas, conjectures into theorems, equations into solutions.*

Mathematics turns questions into answers; this is a process of generating information.' (Rucker, 1987:25).

What is the nature of mathematical objects?

The objects are non-empirical and inaccessible to sense-perceptions. Hersh (1994) provides an argument to characterise mathematical objects as social entities. He proposes that objects can be divided into not only mental or physical, but also social categories. Mental is thought, individual consciousness, wishes, hopes, etc. Matter is what takes up space, has weight and can be studied by scientific instruments. '*Is there anything that is neither mental nor physical? Yes! : sonatas; poems; churches; morality; the profit motive; armies; wars; academies of science.'* (Hersh, 1994:15). According to Hersh, these are social entities.

He expands mathematics as social entities to social-cultural-historical entities. Social, because mathematical objects are created by humans and because of the necessity of a 'mathematical social environment' for work to be criticised and refined; cultural, because of the growth of mathematical knowledge in response to pressures in society and the needs of science and daily life, and historical, because of the origins of mathematics in the self-creation of the human race.

Why is mathematical knowledge stable and reproducible?

Hersh explains it as follows: '*I believe that there are social or intersubjective concepts which have the rigidity, the reproducibility, of physical science.'* (Hersh, 1994:19). He calls mathematics the study of lawful, predictable parts of the social-conceptual world. On the question of why there is such a lawfulness, Hersh provides no answer and considers the question as fruitless as the question, 'Why is there a universe?'

In an admittedly superfluous attempt to answer this question, the constructivist Piaget's theory of operational constructivism will now be examined. (Constructivism will be discussed in more depth in par. 2.4.2.)

Piaget distinguishes between two types of experience at elementary stages: physical experience (which comes from actions on objects) and logico-mathematical experience (experience whose source is the general co-ordination of these actions). This is, according to Piaget, where the roots of mathematical knowledge lie. Later on, in the formal operations stage, as the child grows older, he/she becomes capable of deductive thinking and reflective abstraction, based on the above two kinds of experience (Reynolds et al., 1995).

Piaget makes it clear that genetic analysis has convinced him of the non-empirical character of the origin of mathematics:

- The subject initially discovers logico-mathematical truths through experience by manipulating objects; truth is not derived from the objects, but from the actions carried out on the objects.
- Logico-mathematical relationships are constructed by the subject starting from a schematisation of the general co-ordination of action, which is neither perceptible nor the object of direct experience.
- This schematisation is common to all subjects and not dependent only on the characteristics of individual action.
- The subject becomes aware of the structures by reconstruction. *'The "reflective abstraction" by means of which the subject discovers the laws of the co-ordination of actions, consists of projecting or reflecting on to a new plane what is abstracted from the structure to be discovered, so as to reconstitute it in order to use it.'* (Beth & Piaget, 1966:282). The reconstruction enriches the initial structure and implies new operations that free the initial structure from its concrete context to provide a more general and abstract model of it.

Still, why the accordance of mathematics with reality, why the robustness and stability of mathematical knowledge? Piaget emphasises that individuals' mathematical constructions have common origins. '*And this common origin is simply the co-ordination of the subject's actions. But as this general co-ordination of actions itself depends on the laws of neural co-ordinations, and the latter on the laws of organic co-ordination in general and as the organisms originate (in a way still unknown to us) out of interaction with the physico-chemical environment, this common origin of reason and experience assumes from the start a fundamental interaction between the subject (organism) and the objects (environment).*' (Beth & Piaget, 1966:284).

This viewpoint implies a subtle shift towards Platonism in that ' ... *any structure, however elementary, provided that it be of a logico-mathematical nature (that is, that it be either operational or pre-operational but, free or capable of being freed, from any connection with objects or actions of the individual subject, as opposed to general co-ordinations) involves a whole system of possible developments, and that the novelty of later structures consists merely of actualising some of them* ... ' (Beth & Piaget, 1966:301). However, no transition from the possible to the real entity can take place without actualisation by an effective construction.

Ernest (1991) presents a social constructivist philosophy of mathematics. One of the premises is that the mathematical knowledge embedded in language usage provides a basis for informal mathematical knowledge which eventually leads to formal mathematical knowledge. This happens through the construction of a series of language games of which the basis is embodied in the rules of natural language and where the upper reaches of the hierarchy are formalised and axiomatised theories. At this level the rules of the games become explicit.

Piaget disagrees with this linguistic interpretations of mathematics: ' ... *to introduce in this connection the general co-ordination of actions as the starting point of logico-*

mathematical structures forms a guarantee of autonomy, which is neither more nor less reliable than a reference to linguistic syntax and semantics, but it is a question of a deeper origin from which the linguistic co-ordination themselves are derived.' (Beth & Piaget, 1966:296). Piaget agrees that there is order in language, but ascribes it to human behaviour which enters at all stages, which depends in turn on biological organisations which are yet more primitive.

How does mathematical knowledge grow and how do we know it is true?

Mathematics has been defined as a subject that exhibits the pattern of assumption-deduction-conclusion (Davis & Hersh, 1981). (This is still the acceptable way of presenting new mathematical knowledge). The deduction part of this process usually takes the form of proof. Proof is a process of moving from a hypothesis to a conclusion on the basis of mathematical rules and logical manipulation. Several mathematicians, philosophers and psychologists emphasise the fact that the assumption-deduction-conclusion pattern reflects only the final ordering of thoughts and gives no clear way of the actual working and thought processes taking place in the mathematician's mind.

Lakatos (1976) distinguishes between Euclidean theories and quasi-empirical theories. Euclidean theories are considered to be theories in which the characteristic flow of truth of the whole system goes from the axioms down to conclusions. Quasi-empirical studies on the other hand, are theories in which the flow of truth is an upward transmission of falsity from basic statements to axioms (Koetsier, 1991). Ernest summarises Lakatos' logic of mathematical discovery as a cyclic process ' *... in which a conjecture and an informal proof are put forward (in the context of a problem and an assumed informal theory). In reply, an informal refutation of the conjecture and/or proof is given. Given work, this leads to an improved conjecture and/or proof, with a possible change of the assumed problem and informal theory.'* (Ernest, 1994a:42).

Lakatos took fallibilism seriously: not only may mathematical theories be overthrown, but he considers attempts to overthrow theories by falsification as a necessary condition for progress (Koetsier, 1991:11).

Hanna (1996) questions the adequacy of Lakatos' scheme to explain some important cases of mathematical discovery (e.g. the development of abstract group theory, the emergence of non-standard analysis). Indeed, Lakatos himself said (in Hanna (1996:8)): '*Not all formal mathematical theories are in equal danger of heuristic refutations. For instance, elementary group theory is scarcely in any danger; in this case the original informal theory has been so radically replaced by the axiomatic that heuristic refutations seem to be inconceivable.*'

It is important to realise, though, that this model is first and foremost an attack on formalism and does not claim to give answers to all philosophical questions: '*The core of this case-study will challenge mathematical formalism ... Its modest aim is to elaborate the point that informal quasi-empirical mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proof and refutations.*' (Lakatos, 1976:5).

One of the premises of Ernest's social constructivist philosophy of mathematics, is that the basis of mathematical knowledge is linguistic knowledge, conventions and rules (Ernest, 1991). Ernest also distinguishes between objective and subjective mathematical knowledge. Objectivity is social - knowledge becomes objective after public acceptance. Newly generated mathematical knowledge can either be subjective or objective and is linked in a cyclic process. In this cycle, subjective and objective knowledge of mathematics contribute to the renewal of each other. This cyclic process is illustrated in Figure 2.1.

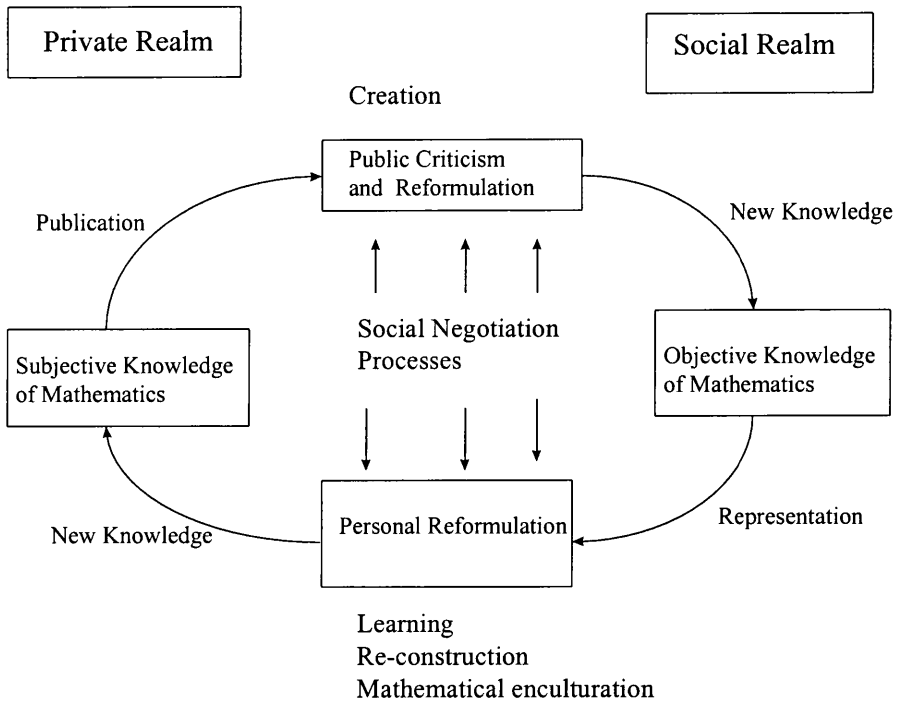


Figure 2.1

The relationship between objective and subjective knowledge

(Source: Ernest, 1991: 85)

Figure 2.1 can be explained as follows:

- An individual possesses subjective mathematical knowledge. The mathematical thought of an individual is subjective thought which are unique representations of mathematical knowledge.
- Publication is necessary (not sufficient) for subjective knowledge to become objective knowledge. To be accepted, subjective knowledge must be physically presented (in print, in writing or as the spoken word).

- Published knowledge becomes objective knowledge through Lakatos' heuristics model. These heuristics depends on objective criteria. They include shared ideas of valid inference and other basic methodological assumptions.
- The objective criteria (referred to above) are based on objective knowledge of language as well as mathematics. The criteria depend not only on shared mathematical knowledge, but ultimately on common knowledge of language (linguistic convention).
- Subjective knowledge of mathematics is largely internalised, reconstructed objective knowledge. Individual contributions can add to (new conjectures, proofs, concepts or definitions), restructure (generalisations, synthesis), or reproduce existing knowledge (textbooks).

The social constructivistic view of mathematics regards subjective and objective knowledge as mutually supportive and dependent. In a creative cycle, subjective knowledge leads to the creation of mathematical knowledge. Therefore, objective knowledge rests on the subjective knowledge of individuals. On the other hand, representation of objective knowledge allows the genesis and re-creation of subjective knowledge. The constraints at work throughout this cyclic process include the physical and social worlds, and the linguistic and other rules embodied in social forms of life.

Koetsier (1991) recognises the 'proof' aspect of the methodology of mathematics but adds tactical and strategic considerations. Similar to Ernest, he distinguishes between the micro- and macro-levels of mathematical activity. The micro-level is that of the individual, busy with tactical aspects, posing him-/herself problems and publishing articles. He/she is limited by his/her own talent and knowledge as well as subjective personal preferences. On the other hand, the macro-level is the level of the mathematical community as a whole. The long-range development of mathematics is important here as well as strategic issues which play a role (recognising main problems, setting up new trends, etc.). Survey papers, e.g. reflect developments on this level. There is a wider overview, and more knowledge and talent involved. Subjective factors play a lesser role, and aspects and factors on which there exists consensus play an important role. Journal editors and referees for example act as filters. There is also feedback from the macro- to the micro-level: the individual researcher does not work in isolation and takes notice of new results and trends in the greater mathematical community (op.cit.:13,14).

A final remark on the issue of objective and subjective knowledge: traditionally 'objective meaning' refers to meaning that exists independently of any human agency. If we accept that no meaning exists independently of its attribution by the human brain, we have to redefine objective meaning as concepts that are **fit** or **sound.** Certain patterns of meaning are just indispensable. '*Human capacities for attributing meaning show regularities that keep them from being radically private or arbitrary not because they are grounded in objective meaning but because the human brain develops under what we might call necessary biology and necessary experience.'* (Turner, 1994). (This definition of 'objective truth' also explains why what is now considered as truth can be considered false in the future, e.g., for thousand of years people believed the earth to be flat.)

2.2.3 Conclusion

The following essentials are extracted from the preceding discussion to serve as a premise for the rest of the argument in this chapter.

Ontological hypothesis

The ontological hypothesis is that mathematical objects are social-cultural-historical entities. Social, because of the necessity of a 'mathematical social environment' for work to be criticised and refined; cultural, because of the growth of mathematical knowledge in response to pressures in society; and historical, because of the origins of mathematics in the self-creation of the human race (Hersh, 1994). The origin of mathematics lies in the co-ordination of human actions. The commonality of the co-ordination of human actions lies in a universal which is that of biological organisation itself. This common origin explains the robustness of mathematics as well as its accordance with reality. Also, a quasi-Platonism is accepted: logico-mathematical structure involves a system of possibilities, and new constructions are the actualising of these possibilities.

Epistemological hypothesis

The epistemological hypothesis is that of social constructivism, namely that

- Mathematics is a human invention;
- Objectivity is social in the sense that there is agreement on what is true. This agreement is based on a shared basis of linguistic knowledge, conventions and rules of logic (with biological origin). However, language is a necessary ' ... *condition for the achievement of the structures of a certain level (hypothetico-deductive and propositional) but it is not a sufficient condition for any operational construction.'* (Piaget in Beth & Piaget, 1966:289);
- Mathematics grows not only through the top to bottom deductive process (proof), but also through the upwards transmission of falsity from basic statements to axioms (Lakatos' heuristics), and other tactical and strategic considerations;
- Mathematical knowledge is not infallible. It can advance by making mistakes, correcting them and recorrecting them (Hersh, 1994; Ernest, 1991; Lakatos, 1976; Beth & Piaget, 1966). Davis (in Hersh, 1979), suggests that the length and interdependence of mathematical proof mean that truth in mathematics is probabilistic.

2.3 Mathematics education

Mathematics educators realise that the information age presents new challenges of which some are yet unknown. Bell (in Skovsmose, 1994), describes the coming century as one in which technological development will provide new social structures and where the highly technological society is characterised by the dominance of computer-based information technology. According to Skovsmose (1994), mathematics and related subjects provide a condition for the development of information technology (every application of a computer can be seen as an application of a mathematical model). Skovsmose argues further that mathematical competence constitutes a major part of democratic competence in a highly technological society. Morgan Niss (in Skovsmose, 1994:57) states that ' ... *the purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also perform the application of mathematics in society, in particular to situations which are of significance to their private, social and professional life.'*

Schoenfeld (1990), emphasising the technological sophistication needed for the non-dead-end jobs of the information society, sees the goal of mathematics instruction as the development of the abilities to

- understand mathematical concepts and methods,
- discern mathematical relations,
- reason logically, and
- apply concepts, relations and methods to solve nonroutine problems.

2.4 The learning of mathematics

Mathematics education researchers study how people learn and are taught mathematics, including the phenomena that influence teaching and learning (Selden & Selden, 1993). Research focuses on several aspects including mathematical aspects, cognition, teaching methods, psychological factors and culture, borrowing research methods and background from sociology, psychology, philosophy and artificial intelligence.

Skemp (1971) considers problems of learning and teaching as psychological problems and feels that we need to know how mathematics is learnt before we can make much improvement in the teaching of mathematics. Since the psychology of the learning of mathematics borrows concepts from learning theory, some of these learning theories and their influence on the learning of mathematics will now be discussed. Also, an in-depth discussion on the philosophy of constructivism will be given to provide a suitable background for the development of the social constructivist model for mathematics learning.

2.4.1 Learning

The interest of educationists in the study of learning lies in the implications it can have for educational practices. However, in a certain way, learning is inaccessible: most learning theorists agree that learning cannot be studied directly. Rather, changes in behaviour can shed light on the nature of learning. Thus, with a few exceptions, learning theorists view learning as a process that mediates behaviour. Hergenhahn (1988) elaborates further: ' ... *learning is a relatively permanent change in behavior or in behavior potentiality that results from experience and cannot be attributed to temporary body states such as those induced by illness, fatigue, or drugs.'* (Hergenhahn, 1988:7).

Other descriptions of learning include, ' ... *a process of adaptation. It encompasses creativity, problem solving, decision making and attitude change.'* (Kolb in Shedletsky, 1993:7,8).

Yet others emphasise the social nature of learning (Schwen, Goodrum & Dorsey, 1993). They feel that pedagogy should emphasise both active participation by teachers and students, and a more democratic relationship between them.

There exist numerous viewpoints concerning the learning process, possibly because of the relatively covert nature of learning. Three broad theoretical perspectives will be highlighted here: **behavioural, information processing and contructivist.** It is important to understand that this is a rather crude categorisation and that most theories share elements of each other.

2.4.1.1 The behavioural perspective

The father of behaviourism, J.B. Watson (1878-1958), felt that consciousness can only be studied through introspection which is an unreliable research tool. Instead, behaviour can be seen and be dealt with directly. His main concern was thus with behaviour and how it varies with experience. Some behaviourists concentrated on behaviour related to survival and others focus on behaviour in terms of laws of association.

Thorndike (in Hergenhahn, 1988), being a behaviourist of the first kind, tried to discover how human actions as well as thought processes contribute to adaptation and survival. He called the association between sense impressions (stimuli) and impulses to action (responses), a connection. The stimulus (S) and response (R) are connected by a neural bond. He viewed the most basic form of learning as trial-and-error which is not insightful but direct (not mediated by thinking).

Three learning laws formulated by Thorndike are:

- The law of readiness. Interfering with goal-directed behaviour causes frustration, and causing someone to do something they do not want to do is also frustrating.
- The law of exercise. Exercising the connection between a stimulating situation and a response strengthens the connections.
- The law of effect. If the response is followed by a satisfying/annoying state of affairs, the S-R connection is strengthened/weakened.

Thorndike had a low opinion of the lecture technique of teaching and urged the formation of S-R connections that life itself demands. In 1922 he published a book 'The Psychology of Arithmetic' which has had a great influence on American mathematics instruction for many years. His proposal that S-R connections that go together, should be taught together implies drill and practice techniques that are still used as an instructional approach (Schoenfeld, 1987).

In general, the behaviourist's view of the learning process can be summarised as follows: the environment presents an **antecedent** that prompts a **behaviour** that is followed by some **consequence** that then determines whether the behaviour will occur again (Newby, Stepich, Lehman & Russell, 1996:29).

Behaviourism now includes a variety of forms such as emergent behaviourism, inter-behaviourism, methodological behaviourism, paradigmatic and radical behaviourism. All these forms try to deal in their own way with the inability of the S-R formula to explain the divergent complexity of observed behaviour (Vargas, 1993).

The implications for teaching are:
Instruction in this case, focuses on environmental conditions that are arranged and presented to students. Teachers follow certain guidelines: objectives must be stated as learners' behaviours; goals must be broken down into observable simpler behaviours and should be arranged in a sequence of frames that guide the progress of students towards the goal; teachers should provide cues to guide students to desired behaviour; consequences (reinforcers) should be used to reinforce desired behaviour (Newby et al., 1996).

2.4.1.2 The information processing perspective

Metaphorically speaking, this approach views the mind as a channel through which information flows and which shapes and directs that flow (Viau, 1994). Learning is seen as a set of processes having the function of information processing (Gagné & Glaser, 1987). The approach assumes that information from the environment is acted upon by the cognitive structures before it is translated into behaviour. Information is processed from the short-term memory (STM) to the long-term memory (LTM).

Gagné & Glaser describe the process as follows:
STM receives input from the senses;

- Because of the limited capacity of the STM both in space and time, information progresses to the working memory (WM). The WM compares incoming information with knowledge already stored in the LTM;
- The task is to match and recognise items, and to compare and to integrate new material to be learned with an organised set of knowledge (schema) retrieved from the LTM. Other functions of the WM are rehearsal (the implicit repetition of material) and elaborate rehearsal (investing stored material with additional semantic meaning).

Figure 2.2, illustrates the process described above.

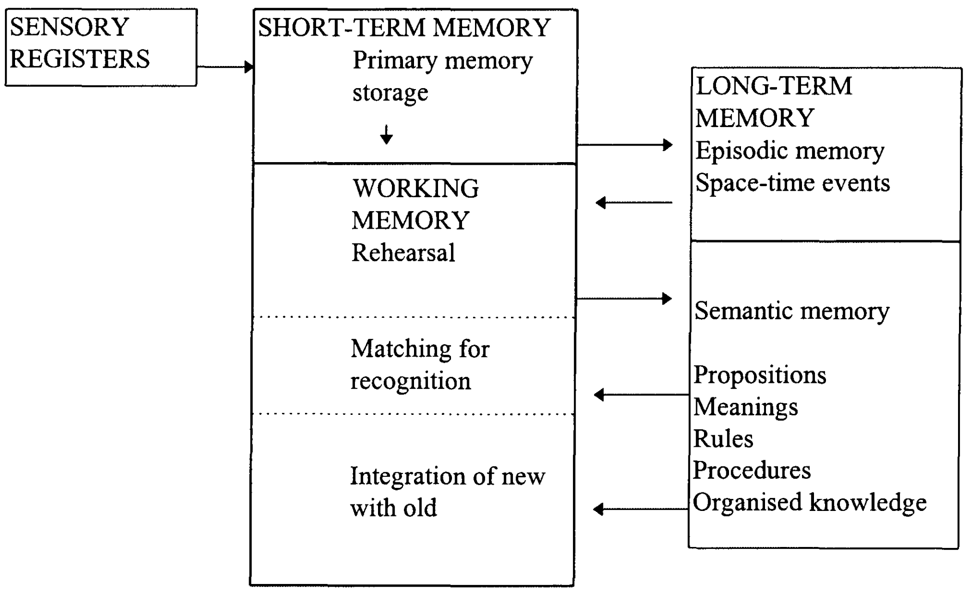


Figure 2.2

Flow of information in the short-term and long-term memory systems

(Source: Gagné & Glaser, 1987:55)

The implications for teaching are: The focus is on cognitive processes and the critical role memory plays in translating new information to meaningful forms that can be remembered. Teachers should organise new information; link new information to existing information and use techniques to aid students in accessing, encoding and retrieving information. These techniques include the use of focused questions, highlighting of important information, and the use of analogies, metaphors and mnemonics (Newby et al., 1996).

2.4.1.3 The constructivist perspective

Although constructivism is not a learning theory but a philosophy of knowledge acquisition, quite a few learning theories have their origin in the constructivist philosophy. What these theories have in common is the belief that we build our own interpretative framework for making sense of the world, and the world is then seen in the light of that framework (Schoenfeld, 1987). The constructivist perspective differs from other perspectives in that knowledge is defined as an individual interpretation of experience. Learning is also determined by the complex interplay between the students' knowledge, the social context and the problem to be solved. Piaget (1896-1980), a well-known constructivist, bases his analysis of thinking and learning on observable child-behaviour rather than adult introspection. His learning theory and how it applies to mathematical learning was discussed in par. 2.2.2 and will again be discussed in greater detail in par. 2.4.2.

The implications for teaching are:
The teacher has to pose good problems. A good problem is defined as one that requires students to make and test a prediction, that is realistically complex, that benefits from group efforts, and is seen as relevant and interesting by students. In this setting students will naturally explore their knowledge which would lead to continual refinement of the knowledge. One of the assumptions of this perspective is that students learn while interacting with others - by meeting challenging ideas that result in cognitive conflict and by interiorising more sophisticated thinking processes observed in peers and the teacher. The teachers should thus provide opportunities for collaboration as well as modelling and guiding the construction process (Newby et al., 1996).

2.4.1.4 More views

Members of the school of **Gestalt theory** believed that we experience the world in meaningful wholes. We do not see isolated stimuli. Rather, we see stimuli organised into meaningful Gestalten (configurations). Hence the saying *the whole is more than the sum of its parts* . Insightful learning to them, has certain characteristics among which are a) a solution to a problem gained by insight is retained longer, and b) a principle gained by insight is easily applied to other problems. Wertheimer (in Hergenhahn, 1988) criticised on the one hand teaching based on the teaching of correct S-R connections through drill and memorisation, and on the other hand, teaching that emphasises the importance of logic. According to Wertheimer, understanding also involves emotions, attitudes and perceptions. In his book 'Productive thinking', he argues his case with a number of examples that illustrate the ability of students to carry out arithmetic procedures without understanding the meaning of those procedures. His most famous examples deal with the parallelogram problem (finding the area of a given parallelogram with base B and height H). Unfortunately, the Gestaltists had little or no theory of instruction (Schoenfeld, 1987).

Bloom (1956) describes the different steps that must be taken for information to become meaningfully assimilated into cognitive structures in his Taxonomy of Educational Objectives (Bloom, 1956). Six levels in order of complexity are named: knowledge (recall of facts), comprehension (low level of understanding), application (use of abstractions in particular and concrete situations), analysis, synthesis and evaluations (quantitative and qualitative judgements).

De Villiers (1995) describes a model of the learning process incorporating ideas from research on information systems. It can be summarised as follows:

- Data are seen as basic facts.
- Data become information through a process of appropriation. This process should be guided by the teacher. Understanding now takes place.

- Through understanding, learners can apply, analyse synthesise and evaluate. At this stage, information has become knowledge.
- By applying, experiencing and judging the knowledge, the learner moves to a stage of competence.
- Teachers' knowledge must be converted into data to help the learner to go through the appropriation process, transforming the data into knowledge. This is where teachers can use models or other techniques to convey data.
- The learners follow an iterative process in the learning process. Information gained by the learner will at some stage revert back to data, asking for a new appropriation process.

The learning process as described by De Villiers is illustrated in Figure 2.3.

Currently, theories that stress the cognitive nature of learning dominate research on mathematics learning theory. Even so, behaviouristic learning principles are still being applied in classrooms all over the world. In practice, most educationists use learning principles from different perspectives in their classrooms.

However, the most influential and widely accepted philosophical perspective in mathematics education today is constructivism (Selden & Selden, 1993). According to Schoenfeld (1987), constructivism is not part of cognitive science per se, but rather a perspective that plays an important role in cognitive inquiries. In the next section, a detailed discussion will be given of the different forms of constructivism and its implications for mathematics teaching and learning.

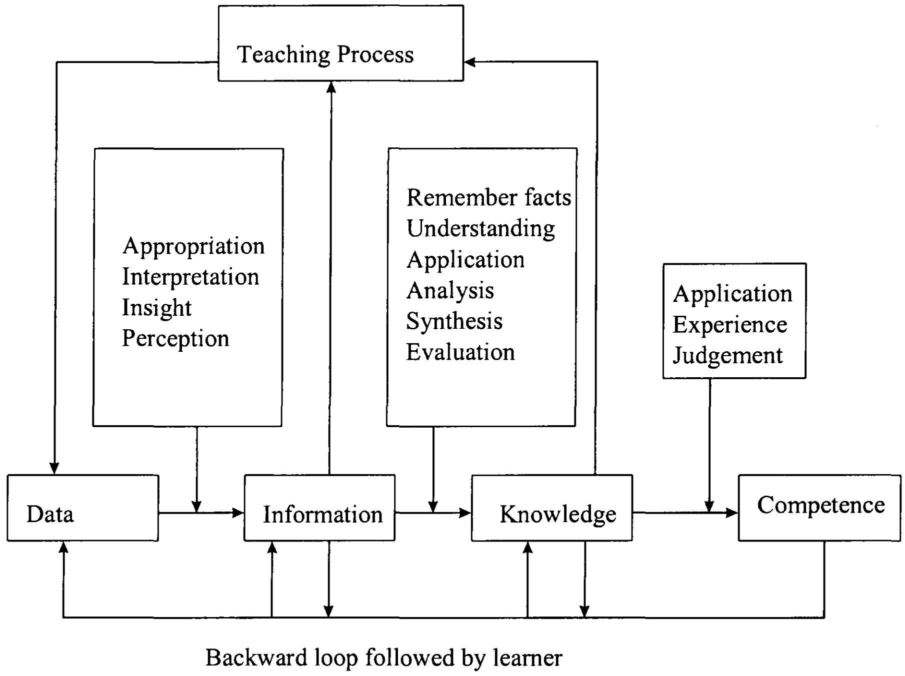


Figure 2.3

Model of the learning process

(Source: De Villiers, 1995:83)

2.4.2 Constructivism

According to the constructivist perspective, we build our own interpretative framework for making sense of the world, and the world is then seen in the light of that framework (Schoenfeld, 1987).

The biologists Maturana and Varela (1992), investigating the biological roots of human understanding, illustrate this by referring to optical illusions as well as the phenomenon of coloured shadows (first described by Otto van Guericke in 1672). In this illustration, certain shadows are observed as greenish-blue whereas if measured, no predominance of green or blue wavelengths is found. The authors propose that our naming of colours correlates with neuronal activity and not with wavelengths, and that our experience is dependent on our human structure in a binding way.

Piaget (in Jaworski, 1994) illustrates this perspective further using the 'conservation of volume' example: when equal amounts of liquid are poured from identical glasses into glasses of different shapes, children will say that there is more liquid in the glass with a higher level of liquid. This version of reality differs from the adult perception. He saw the construction of knowledge as taking place through cognitive adaptation in terms of the learner's assimilation and accommodation of experience into action schemas (Jaworski, 1994).

Piaget's theories are seen as forerunners of a more revolutionary form of constructivism called radical constructivism.

2.4.2.1 Radical constructivism

Von Glaserfeld (in Jaworski, 1994) defines radical constructivism as a theory of knowledge which asserts the following two main principles:

1. Knowledge is not passively received but actively built up by the cognising subject;
2. a) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
2. b) Cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality (Jaworski, 1994:16).

The 'radicality' of constructivism lies in the second principle: although the existence of an objective world is not denied, it is only possible to know 'that world' through experience.

Maturana and Varela put it this way: ' ... *the phenomenon of knowing cannot be taken as though there were "facts" or objects out there that we grasp and store in our head. The experience of anything out there is validated in a special way by the human structure, which makes possible "the thing" that arises in the description.'* (Maturana & Varela, 1992:26).

Knowledge and meaning

In contrast to the absolutist view of knowledge as true and matching ontological reality, constructivism sees knowledge as viable in the sense that it fits within the constraints of the 'real' world (Von Glaserfeld in Jaworski, 1994).

Thus, ideas, theories, rules and laws are exposed to the world from which they were derived, and if they do not hold up, they have to be modified to take the constraints into account. Since the human structure is more or less homogeneous, our experiences of the world are more or less homogeneous, which leads to more or less similar perspectives (within cultures), but the organising of the individual's experience is done in a unique and subjective way. Despite this individuality of knowledge, the sharing of meaning can take place through communication. Further 'fitting' and 'viability testing' of the individual's meaning structures take place through communication. As Piaget has noticed, the most frequent reasons for accommodation (change in a way of operating and acting) arise in social interaction where the individual's ways and means turn out to be insufficient in comparison to the ways and means of others (Von Glaserfeld, 1991).

Teacher and learner

Radical constructivism is not a pedagogy, but many constructivists have claimed consequences from a constructivist philosophy for the teaching of mathematics (Jaworski, 1994). Von Glaserfeld, for example, suggests five consequences:

1. There will be a radical separation between educational procedures that aim at generating understanding ('teaching') and those that merely aim at the repetition of behaviours ('training').
2. The researcher's, and to some extent also the educator's, interest will be focused on what can be inferred to be going on inside the student's head, rather than on overt 'responses'.
3. The teacher will realise that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in the process of guiding the student's construction.
4. The teacher will try to maintain the view that students are attempting to make sense in their experiential world. Hence he or she will be interested in students' 'errors' and indeed, in every instance where students deviate from the teacher's expected path, because it is these deviations that throw light on how students, at that point in their development, are organising their experiential world.
5. The previous point is crucial also for educational research and has led to the development of the teaching experiment, an extension of Piaget's clinical method, that aims not only to infer the student's conceptual structures and operations but also to find ways and means of modifying them (Von Glaserfeld in Jaworski, 1994:26,27).

The five points described above, emphasise the importance of the teacher/learner relationship. In the teacher's construction of a student's mathematical understanding, not only the behaviour but especially the student's construction of knowledge should be considered. By talking to the learner, setting tasks, analysing the outcomes of the task in a cyclical fashion, a picture could be built of the learner's construction (Jaworski, 1994).

Weaknesses of radical constructivism

The focus of radical constructivism is on the individual organising experiences in a way which best fits his/her needs and purposes. Critics view this individualistic emphasis as leading to an overly learner-centred, romantic progressivism and solipsism, where anything the child does is sanctioned as expressions of individual creativity (Ernest, 1993). The society's shared sets of concerns and values are largely ignored.

Both Cobb (1994a) and Ernest warn against the 'political correctness' surrounding constructivism in mathematics and science education, and point out that rote learning, drill and practice, and listening to lectures, can also give rise to learning (Ernest, 1993). Zevenbergen (1996) criticises the failure of constructivism to acknowledge that the schooling system only recognises particular constructions of meaning. Some students are thus at a distinct disadvantage when they enter the schooling system. Constructivism does not address this issue fully (Zevenbergen, 1996:111). However, radical constructivism offers a rich theory which is giving rise to innovative research into the learning of mathematics.

2.4.2.2 Sociocultural versus constructivist perspectives

The sociocultural perspectives have a social view of knowledge construction that is based on a Vygotskian view of learning: '*Human learning presupposes a special social nature and a process by which children grow into the intellectual life of those around them.*' (Vygotsky in Jaworski, 1994:25). His emphasis is on social and linguistic influences on learning and he defines the zone of proximal development as the distance between the actual development level and the potential development level of the learner in collaboration with others. He sees a mental function as being social before it is internalised.

The primacy of social versus individual processes in learning has caused much debate in recent years. Constructivists argue that socioculturalists do not adequately account for the process of learning, whereas socioculturalists blame constructivists for not being able to account for the production and reproduction of the practices of schooling and the society. Cobb (1994) gives some comparisons and contrasts between these two perspectives:

- Whereas both highlight the crucial role that activity plays in mathematical learning, the sociocultural perspective links activity to participation in culturally organised practices. The constructivists, on the other hand, focus on the individual's sensory-motor and conceptual activities.
- For the constructivists, the focus in analysing thought is on conceptual processes located in the individual, whereas the sociocultural theorists take the individual-in-social action as their unit of analysis (Minick in Cobb, 1994).
- The socioculturalists focus on social settings enabling students to participate in the activities of experts, rather than on cognitive processes.
- Bauersfeld (as a constructivist) characterises negotiation as a process of mutual adaptation, in the course of which teacher and learners establish expectations for the activities of others and obligations for their own activity. This implies that Bauersfeld's point of reference is the local classroom rather than the mathematical practices of the wider society (Bauersfeld in Cobb, 1994). On the other hand, Newman et al. (in Cobb, 1994) characterise negotiation as mutual appropriation where teacher and learner co-opt and use each other's contributions in the setting of the wider mathematical society.
- Sociocultural theorists use sociohistorical metaphors such as appropriation, whereas constructivists employ interactionist metaphors such as accommodation and adaption.

Cobb (1994) argues against a choice between the two perspectives, but sees the perspectives as complementary by referring to an analyses of arithmetical activity by Saxe and Steffe et al. (in Cobb, 1994). In analysing the body-parts counting systems developed by the Oksapmin people of Papua New Guinea, Saxe takes a developmental perspective that focuses on the individual's understanding while emphasising at the same time the influence of cultural practices and the use of sign forms and cultural artefacts.

From this study, it became apparent to Cobb that, on the one hand, both the process of individual construction and its products are social, while on the other hand, the various strategies (viewed as cultural forms) are cognitive, being results of the individual

Oksapmin's constructive activities. Cobb forms the view that learning is a ' ... *process of both self-organization and a process of enculturation that occurs while participating in cultural practices, frequently while interacting with others.* '(Cobb, 1994:18).

2.4.2.3 Social constructivism

The view of Cobb described above can be seen as a social constructivist theory in that it acknowledges that both social processes and individual sense making have essential and central parts to play in the learning of mathematics.

Ernest (1994) distinguishes between two types of social constructivism: social constructivism with a Piagetian theory of mind, and social constructivism with a Vygotskian theory of mind.

The 'Vygotskian' social constructivism views the individual and the realm of the social as indissolubly interconnected (Ernest, 1994). This version has no underlying metaphor for the isolated individual mind. According to Ernest (1994), the social constructivism of the first kind is either a radical constructivism acknowledging the importance of social interaction or a complementarist version adopting two complementary theoretical frameworks, namely the intra-individual and interpersonal frameworks.

Language, knowledge and meaning

All versions of social constructivism acknowledge the importance of language in the construction of meaning. Discourse can occur, because people have established a consensual domain. Maturana (in Richards, 1991) describes a consensual domain as a biological relationship established when two or more organisms interact recursively and where the conduct of each organism is interlocked with the other.

The importance of language is this: the medium of language is a necessary condition for the creation of most human consensual domains, albeit not sufficient. Not only conversation, but communication should take place. Maturana and Varela (1992:193) define communication as 'the co-ordinated behaviours mutually triggered among the members of a social unity'. Participants within consensual domains learn what they can take for granted (what is obvious) and only that which is not obvious is said (Richards, 1991). Misunderstandings occur within consensual domains and negotiation of meaning takes place via sharing of doubts, answers, questions, challenges and compromises. Language thus plays a crucial role in the meaning-making process, but within a certain context. According to Good et al. (1992), language acquires two functions: communication with others and self-direction (which is related to self-consciousness).

Meta-cognition / Reflective thinking

This capacity for self-awareness that leads to self-direction or self-regulation is, according to Maturana and Varela, mankind's most intimate experience. Self-directive inner speech is a mechanism for merging the affective with the intellectual in the pursuit of motivated thinking and learning (Good et al., 1992). Schoenfeld (1987) translates meta-cognition (self-regulation) into 'thinking about your own thinking' and relates meta-cognition to three distinct categories of mathematical behaviour:

- Knowledge about your own thought processes;
- Self-regulation, or control. Keeping track of what you are doing;
- Beliefs and intuitions. Knowing what ideas and beliefs about mathematics you bring to your work in mathematics, and how they influence your work.

Schoenfeld describes efficient self-regulation as ' *... to be good at arguing with yourself.'* (Schoenfeld, 1987:210) In his research on problem-solving, he found that a person with this ability takes on different roles (idea generator, critic, progress monitor) in solving problems, thus putting forth multiple perspectives on the problem. Schoenfeld argues that these skills which are part of higher order cognitive skills, are developed by the internalisation of the individual's interaction with others. Referring to Vygotsky's zone of proximal development (ZPD), Schoenfeld sees the individual as functioning at a somewhat higher level in collaboration with more able peers, or under adult guidance. In a group the learner has to defend his/her own views, listen and evaluate others' views which is precisely the kind of inner speech needed for self-regulation.

Skovsmose (1994) sees reflective knowing (thinking) as important in preparing pupils for critical citizenship. He views technology (which contains frozen mathematics) as a formatting power with constructive and destructive potentials. Reflective knowledge has to do with the evaluation and discussion of technological aims, including the social and ethical consequences of pursuing that aim. Since mathematics and specifically mathematical modelling underlies technology, Skovsmose extends reflective knowledge to mathematics education, assigning it the role of developing a critical conception of the use of mathematics.

Tasks involving problem solving activities, seem to provide a rich context for self-regulation and reflective thinking.

Problem solving

Using a constructivist definition of knowledge as viable and fitting the context, students should be provided with opportunities to test the viability of their knowledge. Problem solving tasks could provide such a setting. Cobb et al. (1992) agree that an implication of constructivism is that mathematics should be taught through problem solving. By problem solving they do not mean typical textbook word problems. Instead, they view teaching through problem solving as acknowledging that problems arise for students in their attempts to achieve their goals in the classroom. This is reiterated by Schoenfeld (1990) who views word problems as only a small part of the problem solving world. In conducting a teaching experiment, Cobb et al. found that genuine mathematical problems can arise from classroom interaction as well as from the individual's attempts to complete the instructional activities. Schoenfeld (1990) defines a mathematical problem for an individual as a task (a) in which the individual is engaged and interested, and (b) for which the individual has no ready access to a means of getting there. Thus, a given task will be experienced by an individual as a problem depending on what he/she knows.

Teacher and learner

The social interaction in the classroom as well as the teacher's overt use of the socio-cultural context to promote mathematics learning become important from the social constructivist perspective (Jaworski, 1994). The view of mathematics as a dynamically organised structure (located in a social and cultural context) identifies it as a problem solving activity. The teacher should facilitate these activities and deliberately set contexts where active, meaningful learning can take place.

The dialogic nature of the teaching/learning process implies a non-authoritarian disposition in the classroom. However, the teacher may act in an authoritative way, being an experienced member of the mathematical community. The students should thus be encultured into the conventions and discursive practices of the mathematical community (Magadla, 1996).

2.4.3 Theories of mathematics learning

By adapting the definition of learning theory of Newby et al. (1996) to mathematics learning, a mathematics learning theory can be defined as a set of related principles explaining changes in mathematical human performance or mathematical performance potential in terms of the causes of those changes (Newby et al., 1996:28). Mathematics learning theories not only explain mathematics learning but also predict it (i.e. if x, then y). Most of the learning theories that will be discussed in the next section, not only explain mathematics learning, but some explicitly predict the order of activities that should take place for successful learning to occur. It is clear that mathematics learning theories are strongly informed and influenced by ontological and epistemological assumptions. Most of these theories discussed here are strongly influenced by the constructivist philosophy discussed in the preceding section.

2.4.3.1 Piaget's operational constructivism

Piaget did extensive work on the development of children's mathematical understanding. His emphasis is on the role of overt activities in the construction of concepts. The process of conceptual abstraction is thus seen as a highly developed form of activity. This involves a complex learning process which is only possible at a relatively late age (Beth & Piaget, 1966).

The activities lead to the construction of 'operations' which are ' ... *action or system of bodily movements, which has become internalised in the form of thought activities.'* (Beth & Piaget, 1966:xvi). In his stage theory he describes the developing child's mind at each of four main stages:

The four main stages are the

- sensori-motor stage (pre-language sensory-motor activities of the young child can display some of the features of intelligence),
- pre-operational thought (in which language, symbolic play and invention occur),
- concrete operations (involving classifying, ordering and enumerating activities), and
- propositional or formal operations (verbal and formal logico-mathematical reasoning) (Mays in Beth & Piaget, 1966:xvi).

In different words, Piaget's view of the learning process can be summarised as:

- children are born with sensori-motor schemata;
- events are assimilated into these schemata;
- through experience, these schemata are modified;
- the cognitive structure of the child changes, allowing more experiences;

- later on, children are freed from needing to deal directly with the environment but can now deal with symbolic manipulations;
- the cognitive structures can now be said to construct the physical environment.

2.4.3.2 Other theories based on Piaget's operational constructivism

In line with Piaget's genetic epistemology, Steffe (in Von Glaserfeld, 1991) considers mathematical knowledge to be based on co-ordination of action such as throwing, pushing and lifting, into organised action patterns to achieve some goal. She argues that the mathematical knowledge of children can thus be understood as goal-directed action patterns where action is seen as mental as well as physical actions.

Dubinsky (1994) extends Piaget's view of learning, as described above, to college level and describes mathematical concept formation with the action-process-object idea. He describes concept formation as process, and object construction through a process of reflective abstraction.

- An **action** is seen as any repeatable physical or mental manipulations that transform objects to obtain objects.
- An action has become interiorised to become a **process** once the total action can take place in the mind of the individual without running through all the specific steps.
- When a process can be transformed by some action, then it has been encapsulated to become an **object**.

All mathematical objects are seen as encapsulated processes and any object can be de-encapsulated to become a process again. Existing objects and processes can be used to deal with new situations through generalisation (Dubinsky, 1994). Figure 2.4 illustrates this theory of learning.

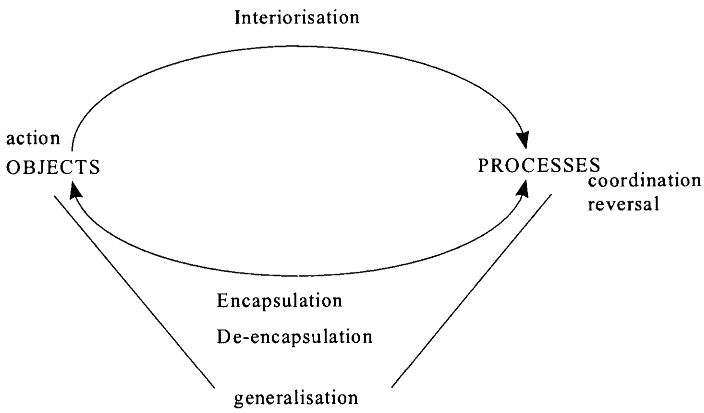


Figure 2.4

Reflective abstractions

(Source: Dubinsky,1994:229)

This learning theory can further be illustrated by an example:

- object: number,
- action: adding three to a number,
- process: when adding three to different numbers a tendency is detected and interiorised to become the process x + 3.

A single process can be encapsulated to become an object, e.g. x + 3 where the standard algebraic manipulations are now seen as actions on the objects.

Sfard (1991) tries to understand the psychology of the forming of mathematical concepts by asking epistemological questions on the nature of mathematical knowledge. She believes that there is more to mathematics than just the rules of logic or its abstractness, but that mathematical abstractness differs from other kinds of abstractions in its nature. Her main premise is that mathematical entities (concepts) have a dual nature which is illustrated by the history of mathematical concepts, manifested in several symbolic representations of the same concept. This has implications for the teaching and learning of mathematics.

Sfard constructs her argument as follows: The building blocks of mathematics are called 'concepts', whereas the internal representations and associations evoked by the concepts are called 'conceptions'. Like the physicist and biologist, the mathematician talks about a universe, populated by concepts with certain features which are subjected to certain processes governed by well defined laws (an objective reality). However, unlike material objects, these mathematical constructs can only be seen in the mind's eye. Being able to see these invisible abstract objects appears to be an essential component of mathematical ability.

This kind of conception is defined as the structural conception. However, this is not the only kind of conception. For example, functions are not only a set of ordered pairs but also a computational process. The computational process part of the definition of functions involves actions, processes and algorithms, which reflect an operational conception of a notion.

Interpreting a concept as a structure implies a static, integrative real thing. On the other hand, operational conceptions are potentialities (rather than actual entities), coming into existence in a sequence of actions. It is dynamic, sequential and detailed.

Sfard believes that there is an ontological gap between operational and structural conceptions, because it implies different basic beliefs about the nature of mathematical entities. Even so, the two conceptions are complementary (in the same sense as the dual nature of entities at sub-atomic level) and inseparable though different facets of the same thing.

Most mathematical concepts have this dual nature, e.g. a natural number can be seen as both a property of a set or as a result of counting; a rational number can be seen as both a pair of integers or as a result of division of integers. The dual nature of a mathematical construct is also illustrated by the various kinds of symbolic representations, e.g. a graphic representation tends to give an integrated whole whereas the algebraic representation can be seen as a concise description of a process.

Sfard argues that a prerequisite to a deep understanding of mathematics is the ability to see a mathematical concept as both an object and a process, and that real insight necessary for mathematical creation can hardly be achieved without the ability ' ... *to see abstract objects, and that, on the other hand the structural conception is very difficult to obtain ...* ' (Sfard, 1991:9).

Sfard explains the role that this duality plays in the formation of mathematical concepts as follows: for a person to get acquainted with a new mathematical concept, it seems that the operational conception must first be developed. Three hierarchical steps in the process of concept formation are identified:

- Interiorisation: the learner gets acquainted with the processes which will eventually lead to a new concept. The process has been interiorised if it can be carried out through mental representations (it no longer needs to be actually performed).
- Condensation: compacting lengthy sequences of operations into more manageable units. The learner becomes more capable of thinking about a given process as a whole. This is the point at which a new concept is 'officially' born.
- Reification: seeing the notion as a fully-fledged object. Something familiar is seen in a totally new light, where processes become objects.

Processes can now be performed on these new objects (interiorisation of higher level concepts). ' *... here is a vicious circle: on one hand, without an attempt at the higher-level interiorization, the reification will not occur; on the other hand, existence of objects on which the higher-level processes are performed seems indispensable for the interiorization - without such objects the processes must appear quite meaningless. In* *other words: the lower-level reification and the higher-level interiorization are prerequisite for each other!* ' (Sfard, 1991:31).

Figure 2.5 gives a model for concept formation.

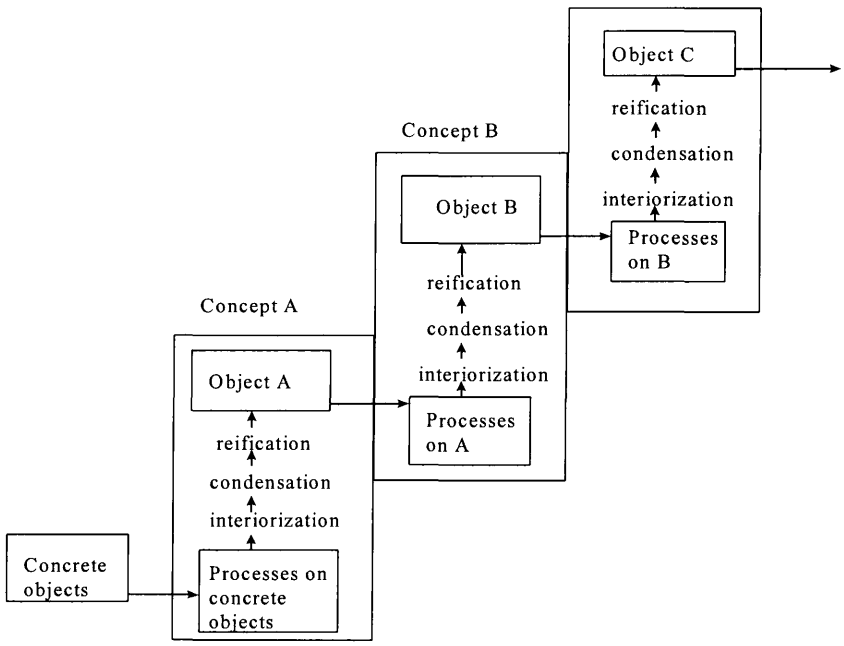


Figure 2.5

General model of concept formation

(Source: Sfard, 1991:22)

2.4.3.3 Problem Solving

There is a prevailing thought among mathematicians and mathematics educators that mathematical activity involves essentially the solving of problems. Balacheff (1990) assumes that problems are the source of the meaning of mathematical knowledge. Also, that mathematical productions turn into knowledge only if they prove to be ' ... *efficient* *and reliable in solving problems that have been identified as being important practically (they need to be solved frequently and thus economically) or theoretically (their solution allows a new understanding of the related conceptual domain).'* (Balacheff, 1990:259). Through reflective thinking, Polyá examined his own thoughts to identify patterns of problem solving behaviour (Schoenfeld, 1987). In his book 'How to Solve It', he proposes a four-phase model: understanding the problem, making a plan, carrying out the plan and looking back. Polyá does not refer explicitly to the managerial processes in problem solving (i.e. self-monitoring, self-regulating and self-assessment), but addresses it in the form of heuristic suggestions (Fernandez et al., 1994). Wilson (in Fernandez et al., 1994) presents a framework representing a dynamic and cyclic interpretation of Polyá's stages, including the managerial decisions implicit in the movement from one stage to the other.

Schoenfeld (1987) argues that these managerial processes, as well as problem solving activities, should be addressed since most students are unaware of their own thinking processes.

Other thoughts on mathematics learning that describe and predict the order of thinking that takes place in the construction of mathematical concepts are those of Romberg, Freudenthal and van Hiele.

Romberg (1994) recognises four related activities common to all of mathematics namely abstraction, invention , proof and application. These activities will be discussed in greater detail in par. 2.5.

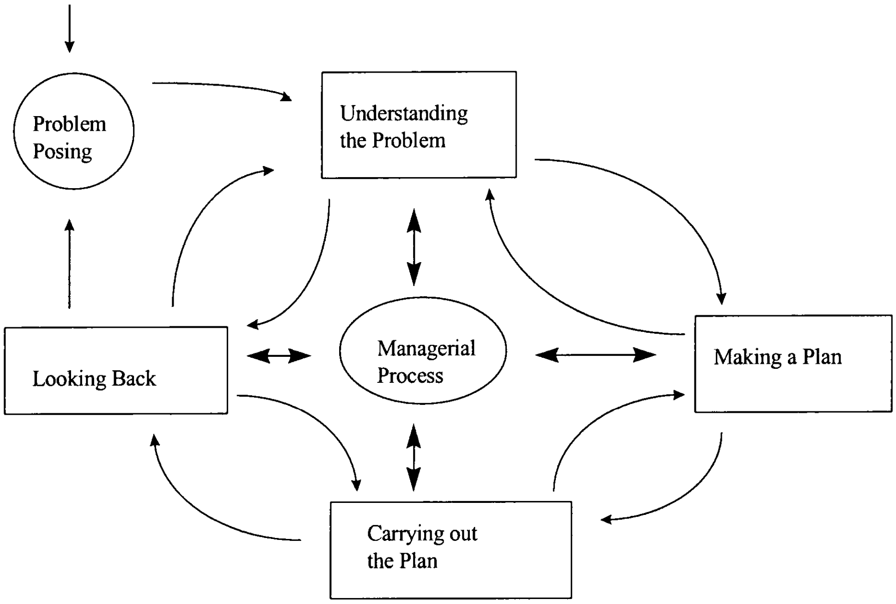


Figure 2.6

The dynamic and cyclic nature of problem solving activity

(Source: Fernandez et al., 1994:196)

Freudenthal (in Schoenfeld, 1994) believes that the central focus of a mathematics curriculum should be the learning of strategies of mathematizing. He describes mathematizing as follows: '*Mathematizing involves representing relationships with a complex situation in such a way as to make it possible to put them into a quantitative relationship with one another.'* (Schoenfeld, 1994:295).

The **Van Hieles** (in De Villiers, 1997) distinguish between five levels of argumentation in the development of pupils' understanding of geometry. The first four levels (considered to be most important in secondary school geometry) can be characterised as follows (op.cit.:1997):

Level 1 (Recognition): The objects of thought are individual figures and pupils recognise the figures by their global appearance and shape.

Level 2 (Analysis): Pupils think in terms of classes of figures. They analyse and name the properties of figures but do not interrelate figures or properties of figures yet. For example, a rectangle will not be considered as a parallelogram since a rectangle has ninety degree angles but a parallelogram not.

Level 3 (Ordering): Pupils think in terms of definitions of classes of figures. They logically order the properties of figures and understand the interrelationship between figures.

Level 4 (Deduction): Students are able to develop longer chains of deductions and they show an understanding of the importance of deduction, proof, theorems and axioms.

Freudenthal (Goffree, 1993) extended their theory to other mathematical areas. In investigating the difficulties experienced in the teaching of mathematical induction, he decided that the problems originate in the levels at which teachers begin their explanations being too high. Students are not given opportunity to go through previous, lower levels of the learning process. He saw mathematics as an organising activity and that through reflection on these activities higher levels of learning are reached.

2.4.3.4 Schematic learning

Using ideas of Piaget and the information processing approach discussed above, Skemp (1971) identifies schematic learning as learning which uses existing schemas as tools for the acquisition of new knowledge. He distinguishes between rote learning and intelligent learning. He defines the latter as the formation of conceptual structures communicated and manipulated by means of symbols. Networks of conceptual structures form schemas. Assimilation refers to the fitting of new material to an existing schema, with no great change in the schema itself. For example, the concept of number initially means counting numbers. The number 2000 requires assimilation since, although it is large, it is still countable. Accommodation, on the other hand, refers to a major reorganisation of the basic structure of the schema. For example fractions form a new number system, not an enlargement of the existing schema of number. A major accommodation of the schema is needed before it is understood.

Concept formation

A concept requires for its formation a number of experiences which have something in common. Language is closely linked to concept formation. Not only does it play a part in actual concept formation, but it also helps in classifying concepts into classes. Skemp distinguishes between primary concepts (experiences from the outside world) and secondary concepts (more abstract and removed from experiences from the outside world). Higher order concepts cannot be communicated by definition, but only through experiencing a suitable collection of examples. Since most mathematical concepts are of a higher order, the communication thereof is difficult. Man's superiority lies in the ability to detach concepts from experiences which gave rise to them and to attach them instead to language. Without language, primary concepts cannot be brought together to form higher order concepts. The construction of a conceptual system is done by the individual but is facilitated by social interaction and experiences.

Skemp (1971:32) summarises the consequences of the above discussion for mathematics educators as:

- Concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples.

- Since in mathematics these examples are almost invariably other concepts, it must first be ensured that these are already formed in the mind of the learner.

Schemas

Concepts are embedded in structures of other concepts. These networks of concepts (mental structures) are called schemas. However, the schemas could be unsuitable in certain contexts. The inappropriateness of a schema can be detected by testing the adaptability of the learner to new, though mathematically related, situations. An appropriate schema is one which takes into account the long-term learning task.

The responsibility of the educator is thus to facilitate the laying of a well-structured foundation of basic mathematical ideas (to help students to find basic patterns) and to teach them to be ready to accommodate their schemas.

Intuitive and reflective intelligence

Skemp (1971) describes intuitive intelligence as intelligence dealing with objects in the outside world, accessible to the sensory registers. Reflective intelligence, on the other hand, deals with mental objects (e.g. mathematics objects). Skemp (1971) suggests that primary concepts can be formed intuitively, whereas reflective intelligence is needed for the formation of higher order concepts. Also, once we become able to reflect on our own schemas, we can communicate them, set up new schemas and replace inappropriate schemas with new ones. A sophisticated and powerful result of reorganising schemas is mathematical generalisation, which involves the creation of enlarged concepts ahead of demands for assimilation of new situations. Communication has an important influence on the development of reflective intelligence, because of the linking of ideas with symbols and the clarification and justification of ideas that take place.

2.5 A social constructivist model for the learning of mathematics

A theoretical framework for the learning of mathematics will now be developed based on the ontological and epistemological hypotheses discussed in par. 2.2.3. Ideas will be used from Ernest's social constructivist philosophy of mathematics; from Skemp's schematic learning model and from the various theories for learning and mathematics learning that were discussed. The underlying social constructivist philosophy will be radical constructivism acknowledging the importance of social interaction.

In the discussion that follows, elements that comprise mathematics learning are identified. Different components that play a role in mathematics learning are highlighted and related to each other. The learning environment is seen as a micro-community set in a macro-community. The norms and the negotiation of the norms of these communities are discussed. Learning is then described as the transformation of data into information and finally into knowledge. The transformation process exhibits a cyclical nature in that 'publicised' private knowledge becomes data to other learners. The creation of subjective knowledge (the private knowledge of the learner) is described by considering the role of convergent and divergent thinking. Concepts related to divergent and convergent thinking are discussed in detail. These concepts include abstraction, generalisation, formalisation, logic, proof, analysis and inductive and deductive reasoning. The role of symbols are discussed and related to ways in which mathematical concepts are represented in the mind of the learner.

Macro-/micro - mathematical community

According to Ernest (1991), mathematical activity takes place in the private and social realm. This is also the case in the mathematics classroom.

Cobb et al. (1992) draw attention to the similarities between the mathematics classroom 'community' and the scientific research community. Both create traditions which influence and restrict perceptions of what count as problems, solutions, explanations and justifications. As is pointed out by Ponte and Matos (1992), in mathematical investigations, students play the role of mathematicians: in complex situations they try to understand, to find patterns, to generalise and to solve. Investigations could involve complex tasks or relatively simple ones dealing with small variations on well-established facts or procedures.

The mathematical research community is a community with well-established research traditions. Richards (1991) describes the discourse domain in which this community functions as 'research maths'. He regards this as the spoken mathematics of the professional mathematicians or scientists. The language used is technical and the discourse is structured according to the logic of proof and refutations. The mathematical research community differs from other research communities in their reliance on notions regarding the nature of proof.

It is also in this (fairly small) community where new mathematics is created and transmitted (Davis & Hersh, 1981). New mathematical knowledge in its final form (after it has been criticised, corrected and accepted) is presented through publication. This takes place, according to Richards, in another domain of discourse which he names 'journal maths' which is the language of mathematical publications and papers. It is a formal communication, based on reconstructed logic.

Mathematical activities are not restricted to the research community. Richards (1991) describes another domain of mathematical discourse as 'inquiry maths' which is the mathematics as it is used by mathematically literate adults. Although being similar to research mathematics, it includes more informal activities: participating in mathematical discussions, reading and challenging popular articles containing mathematical content, listening to mathematical arguments, solving new mathematical problems, etc. He further describes 'school maths' as the discourse of the standard mathematics classroom.

It usually takes on the form of the tripartite exchange (initiation - reply - evaluation) (Wood, 1996). According to Richards, this discourse, in many ways, does not produce mathematical discussions and exchanges. However, whatever the underlying traditions and conventions in the mathematics classroom are, the classroom community is set in the realm of the macro mathematical community. Any learning that takes place in the mathematics classroom, takes place against the background of accepted objective mathematical knowledge (in the sense of Ernest's philosophy). Thus, the role of the teacher includes that of mediating ' ... *between students' personal meaning and culturally established mathematical meanings of wider society.'* (Cobb, 1994:15).

On the other hand, the mathematics classroom can also be described as a micro-mathematical community. This community is characterised by certain patterns of interactions and practices which become taken-for-granted ways of acting, and on which teacher/learner draws to produce regularities in their actions and communications (Wood, 1994). Cobb et al. (1992) view these classroom traditions as having a great influence on students' experiences of meaningfulness. In a study on characteristics of classroom mathematics traditions, they analyse interactions in two different classrooms where the lessons that were given covered the same theme and where the same manipulative materials were used. They found two different established traditions. In the one class they noticed

- an implicit belief that mathematical interpretations do not need to be justified,
- a shared assumption that to engage in mathematical activity is to act in accordance with the teacher's expectations,
- that making mistakes was interpreted as ineffectiveness (inability to achieve the aim of acting in accordance with the teacher's expectations), and that making mistakes resulted in the student experiencing him/herself as an outsider to the classroom community,
- that the main goal of the children was to be effective by following procedural instructions,

- that the teacher regarded the mathematical procedures as fixed and self-evident, and that the children acted as though they believe the same, and
- that the teacher acted as the sole validator of the pupils' interpretations and solutions.

However, in analysing the interactions in the other class, they noticed

- a shared belief that one could challenge each other's interpretations,
- the constitution of mathematical truths by the teacher and children in the course of the social interactions,
- that the acts of explaining and justifying were central to the process of social negotiations,
- that the teacher and children acted together as validators of interpretations, and
- the teacher using the students' autonomous constructions to guide the constitution of their taken-as-shared mathematical reality towards the macro mathematical community's ways of knowing.

Cobb et al. (1992) use Richards' (1991) domains of discourse discussed above to describe the two cultures as 'school maths' and 'inquiry maths'. In both traditions, communication was a process in which the teacher and students mutually orient their own and each other's activity within a consensual domain of taken-as-shared mathematical meanings and practices. Also, in both traditions, teachers were authorities that initiated the students into particular interpretive stances (Cobb et al., 1992:597). However, it was the students' experiences of meaningfulness as they engaged in mathematical activity that differed. In one class, they experienced mathematical 'understanding' when they can follow procedural activities successfully, whereas in the other class, understanding was experienced when they could create and manipulate mathematical objects in ways that they could explain and justify. This last view of understanding is in line with the social constructivist view of mathematical knowledge as human constructions which become socially accepted through the negotiation of meaning.

One of the most important role players in the constitution of a micro-mathematical community where meaningful learning can take place, is the teacher. The ways in which teachers can guide the development and maintenance of such a consensual domain, will be discussed later.

Social Negotiation

Social constructivism recognises the importance of language and discourse in the creation of mathematical knowledge (Lakatos, 1976; Ernest, 1991). Bauersfeld (in Cobb, 1994) characterises social negotiation as a process of adaptation in the course of which teacher and learner establish expectations for each other's activity, and obligations for their own activity. This happens in the setting of the macro mathematical community. Much and Shweder (in Cobb et al., 1992) distinguish between five types of classroom norms: regulations, conventions, morals, truths and instructions.

- Regulations are established by an authority (e.g. only one person in a group may fetch the study materials).
- Conventions are similar to regulations but their source is not specifiable (e.g. it is customary for students to attempt to respond to the teacher's questions).
- Morals are traditional norms (e.g. the norm that students should not copy each other's answers).
- Truths are norms of which the transgression is seen as an error.
- Instructions are norms of which the transgression is seen as ineffectiveness.

Social negotiation acts are triggered by the moments in which a community member accuses another of having transgressed a norm. By using ideas of Much and Shweder, Cobb et al. (1992) call these moments, situations for justification (negotiations of mathematical meaning) and situations for explanation (the individual's clarification of aspects of his or her mathematical thinking). These situations are interactively constituted by the students and teacher, and can occur as students solve given tasks, explain their interpretations, validate explanations or discuss the legitimacy of particular mathematical constructions.

However, a situation for explanation or justification is interactively constituted only if there is a shared understanding that an interpretation or solution should be explained or justified (Cobb et al., 1992). Also, in order to negotiate a taken-as-shared interpretation of a given solution, community members must have the same understanding of what counts as an explanation and justification.

Again, social negotiation will only take place if it is part of the convention: i.e. if the students come to believe that mathematical constructs should and can be explained, and be justified to arrive at mathematical truths. The teacher's acts could establish these beliefs. This, again, has implications for the teacher's role. Steffe (in Von Glaserfeld, 1991) gives some guidelines to teachers who opt for social constructivism. They should try to

- communicate mathematically with their students,
- foster reflection and abstraction in the context of goal-directed mathematical activity,
- engage students in goal-directed mathematical activity,
- encourage students to communicate mathematically among themselves,
- learn the mathematics of the students they teach.

Also by creating opportunities for students to make connections between their explanations and the formal notational methods used by the wider mathematical society, students will begin to realise that symbols can convey mathematical meaning (Cobb et al., 1992).

Wood (1996), emphasises the importance of the negotiated norms as follows: '*It is the nature of these norms and the manner in which they are agreed upon and committed to by the participants that create opportunities for children to engage in the process of mathematical thinking and reasoning for themselves. These settings create opportunities* *for children to engage in what is alternatively referred to as reflective thinking or reflective abstraction.'* (Wood, 1996:102,103).

Data

Data can be seen, as described by de Villiers (1995), as basic facts. Van Loggerenberg (in de Villiers, 1995) describes data as unevaluated attributes. For the purpose of this study, data will be viewed as stimuli that are registered through the senses. In the proposed model for the learning of mathematics, distinctions will be drawn between two kinds of data: data that can be socially negotiated and fixed data. The fixed data can include physical objects, the physical environment, and objective mathematical knowledge. The data that are negotiable are facts generated within the community including the explanations, justifications, solutions, regulations, and conventions of the classroom community members.

Although some of the data are accidentally registered, most of the data are presented to the student by the teacher. The way of presenting mathematical data will depend on the level of the learners (as described by Piaget and van Hiele in par. 2.4.3). At the elementary school level, educators stress the use of concrete and iconic representations of concepts, whereas later, the focus shifts to symbolic and abstract representations of mathematics (Gadanidis, 1994). Also, by presenting examples in order to convey a new concept, the learner must already have the concepts on which these examples are based. However, in the presentation of data, the social constructivist view of learning as the active organisation of the experiential world, should be kept in mind.

This is where problem solving activities can provide a meaningful learning experience by enabling the learner to discover the relevance of data making it more meaningful.

Information

Through a process of personal reformulation and interpretation, data is put into perspective and context (van Loggerenberg, in De Villiers, 1995). This process will depend on the existing schemas of the learner. Through the processes of assimilation the new material is fitted into an existing schema. Understanding now takes place. These are covert processes and it is possible that the learner could have constructed an inappropriate schema. Being a member of the macro mathematical community, the teacher should identify an inappropriate schema and guide the learner to the construction of more appropriate schemas. This is only possible if the covert mental processes become public. By setting suitable tasks, asking suitable questions and encouraging students to justify and explain their own interpretations, the diagnosis can be made. By these activities, information becomes better settled in the network of schemas - it is becoming knowledge.

Creation of subjective knowledge

This refers to the new subjective mathematical knowledge created by the learner in the learning process. 'New' here is seen as new from the perspective of the learner and subjective mathematical knowledge refers to the individualised mathematical knowledge. This knowledge includes newly formed concepts and schemas which lead to reinterpretations of existing knowledge, new conjectures, proofs and definitions, and solutions to given problems. Dubinsky (1992:46) views an individual's subjective knowledge as ' ... *her or his tendency to respond to certain kinds of perceived problem situations by constructing, reconstructing, and organizing mental processes and objects to use in dealing with the situations.'*

By referring to Skemp's metaphor for learning (1971) - *schematic learning* - understanding is seen as the assimilation of data into an appropriate schema. However, for information to become knowledge, major reorganisation of basic structures of schemas should take place (accommodation). Earlier schemas are used (with their particular examples) but giving them a wider meaning (Skemp, 1971).

The question now is, through what processes does this major reorganisation of schemas take place?
A closer look into the different models for the creation of mathematical knowledge and the learning of mathematics reveals three common features: the **hierarchical nature** of mathematics (levels of sophistication), **convergent** and **divergent thinking**.

The **hierarchical nature** of mathematics follows from the fact that higher order concepts are built on, and with, already formed ones. Ernest (1991:77) describes it as follows: ' *... the generation of a hierarchy of increasingly abstract concepts reflects a particular tendency in the genesis of human mathematical knowledge. Namely, to generalise and abstract the shared structural features of previously existing knowledge in the formation of new concepts and knowledge.'* This has important implications for mathematical understanding: e.g. it is not possible to understand algebra without ever really having understood arithmetic since algebra can be seen as generalised arithmetic. Also, the level of the sophistication of concepts and its hierarchy depends on the developmental level of the learner (this was discussed in more detail in par. 2.4.3.2).

Fischer and Ellis (in Tromp, 1993) view **divergent thinking** as unconstrained thinking, not attempting to be efficient and rational. Related to the concept of divergent thinking is De Bono's lateral thinking which is described as provocative and probabilistic, with no fixed categories, classifications or labels, exploring all avenues (Tromp, 1993). For the purpose of this study certain concepts will be associated with divergent thinking, namely generalisation, inductive thinking, intuition and visualisation.

Convergent thinking, on the other hand, concentrates on validity and efficiency. Through a finite number of logical and analytical thought processes, a valid justification or solution is reached (Tromp, 1993). Related concepts are abstraction, deductive reasoning, formalisation, logic, proof , formulation (symbols), and analysis.

For example, in Lakatos' model for the heuristics of mathematical discovery (par. 2.2.2), the guesses by speculation and criticism involve divergent thinking where criticism is based on convergent thinking in the form of the logic of proof and refutation.

Also, the stage theories of Piaget and van Hiele (par. 2.4.3) show a clear hierarchical progress where the ability of convergent thinking develops with age. (Classifying, ordering, deductive thinking and formal logico-mathematical reasoning only occur at later stages.)

The problem solving models (par. 2.4.3.3) imply abstraction and analysis (being able to discard irrelevant information) which are converging thought processes. To make a plan (or build a model) requires inventive thinking. The validity and self-consistency of the plan are based on how well implementation of the plan worked as well as the logic of proof.

The models of Dubinsky and Sfard (par. 2.4.3.2) give actual descriptions of the elements of invention, called *processes* and *objects*. In the invention process (called reflective abstraction by Dubinsky), both divergent and convergent thinking is involved. Loosely described, the interiorisation activity can be seen as a divergent kind of thinking since a mental leap is made to a *process*, whereas the encapsulation/reification activity can be seen as a convergent kind of thinking which involves abstraction. These activities are reversible. Also, by this cyclic process, higher levels of conceptual sophistication are reached.

However, these constructions have to be 'managed' by the learner. These managerial thinking skills are called reflective thinking or metacognition (par. 2.4.2.3). Most researchers whose theories were discussed above, stress the necessity of reflective thinking: Dubinsky and Piaget refer to reflective abstraction, whereas Schoenfeld emphasises the importance of reflective thinking in problem solving (par. 2.4.2.3).

From the discussion above a general descriptive model for the construction of subjective mathematical knowledge is now proposed and given in Figure 2.7.

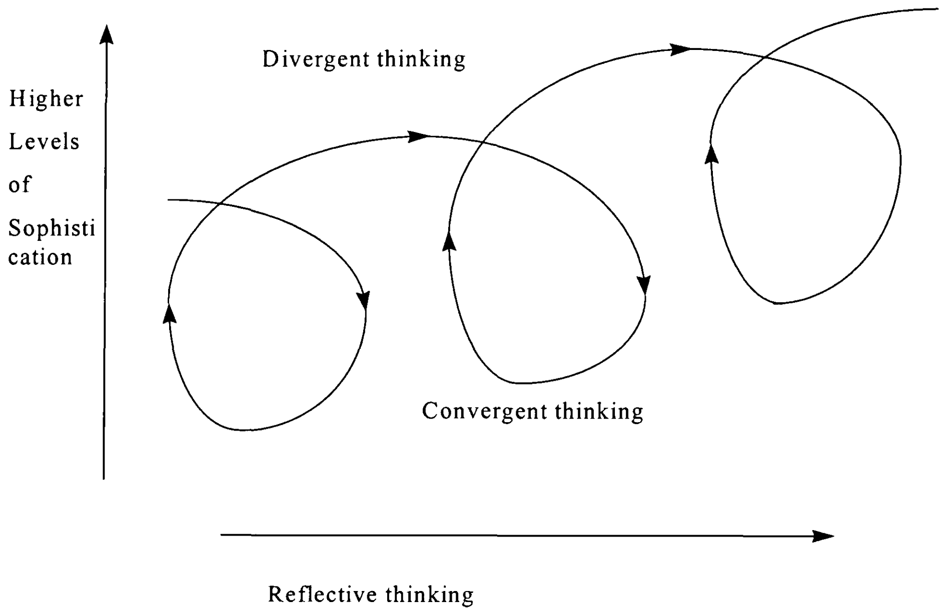


Figure 2.7

The construction of subjective mathematical knowledge

The construction of mathematical knowledge thus takes place through

- a constant fluctuation between divergent and convergent thought processes, which is
- managed by the learner through meta-cognition (reflective thinking),
- moving to higher levels of sophistication.

.

This fluctuation between divergent and convergent thinking is described by Beth (in Beth & Piaget, 1966:93) as ' ... *a curious methodological dualism which is distinguished, amongst other things by the traditional opposition between an* ars inveniendi *and an* ars disserendi. ' *Ars inveniendi* means the heuristic precepts which enable possible invention of solutions but which have no demonstrative power. It is also not guaranteed that a solution will be found, or that if it is found, that it will be correct. On the other hand, the *ars disserendi* provide principles to judge the validity of the solutions, but no way to discover the solution.

This fluctuation can be further described using Einstein's words: ' ... [the] *emotional basis*' [of this] '*rather vague play'* [with concepts (guesses), is] '*the desire to finally arrive at logically connected concepts*.' [Also, the] '*play with the mentioned elements is aimed to be analogous to certain logical connections one is searching for.'* (Einstein in Hadamard, 1945:143).

The concepts associated with convergent and divergent thinking will now be discussed in more detail.

Abstraction

According to Skemp (1971), abstraction is an activity by which we become aware of similarities among our experiences. This enables us to recognise new experiences as having similarities to an existing class of concepts and the consequent classification of them. The process of abstraction leads to abstractions called concepts that are networked to form schemas. The important role that language plays in concept formation has already been discussed in par. 2.4.2.1.

Davis and Hersh (1981) distinguish between abstraction as extraction, and abstraction as idealisation. An example of the first kind is where the number four is extracted as a common feature from the examples: four birds, four apples, etc. The second kind refers to what Aristotle has described as the stripping away of everything that is sensible (weight, hardness, heat) and leaving only quantity and spatial properties (Davis & Hersh, 1981). A simple example is that of the straight line: there is the straight line drawn by a carpenter across a board using a pencil, and there is the mental idea of the mathematical abstraction of an ideal straight line from which the imperfections have been eliminated. Through idealisation, mathematical models are built that again have implications for the real world situation, e.g. a system of differential equations can be seen as an idealisation of a complex set of physical conditions (Davis & Hersh, 1981).

Generalisation

The words abstraction and generalisation are sometimes used interchangeably but Dienes (in Plumpton, 1972) defines generalisation as ' '*the discovery that a general rule extends beyond the first few known cases' '*, whereas abstraction is ' '*the awareness that the rule applies in a number of other situations' '* (Plumpton, 1972:98). Skemp (1971) describes generalisation as a powerful activity, because it makes possible new schemas ahead of the demands for them, seeking or creating new examples to fit the enlarged concept. Plumpton views generalisation as a more efficient coding, while Davis and Hersh speak of it as a consolidation of information (or a synthesis).

Plumpton notes that the generalisation of a particular result should include the result itself as a special case (e.g. Pythagoras' rule as a special case of the cosine rule). However, Davis and Hersh point out that the general cannot include all aspects of the particular. As an example, they refer to the general theory of continuous functions which contains only a small amount of interesting information about a particular continuous function, e.g. the exponential function.

It is also through generalisation that the concept of infinity is formed. Although being a basic object in mathematics, intuitively understood by children, infinity is shrouded in mystery and paradoxes. Nevertheless, by naming the unnameable, and expressing facts about the infinite in a finite number of symbols, a powerhouse is created. Davis and Hersh (1981) issue a warning: where there is power, there is danger. The nature of infinity is open-ended, and the necessity for carefully scrutinising arguments involving infinity will always be there.

Inductive/Deductive reasoning

Two kinds of reasoning are usually associated with mathematical thinking, namely inductive and deductive reasoning. Although being mentioned as if separate from abstraction and generalisation, they are actually abstracting and generalising activities. Simon (1996:197) defines inductive reasoning as ' ... *the drawing of a generalised conclusion from particular instances* ... ' and deductive reasoning as conclusions drawn from a ' ... *logical chain of reasoning in which each step follows necessarily from the previous.'*

Deductive reasoning is usually associated with logic and that which gives mathematics a unique character, namely 'proof '. Proof is a formalised argument based on assumed truths (axioms), followed by propositions each of which is implied by the former through rules of logic. Proof is also a kind of validation and certification. By presenting a proof, it is subjected to a process of criticism, reformulation and acceptance, and then becomes new mathematics.

Inductive reasoning on the other hand usually precedes new mathematical ideas. It is through deductive reasoning that these ideas are validated and formalised, opening them up to public scrutiny and eventually to the creation of new mathematical knowledge. It seems though, that a lot more than only inductive and deductive thinking is involved in mathematical creativity.

Symbols and formalisation

Mathematics can be seen as a discourse dealing with a specific subject, having a specific nature. Communication takes place via the use of signs.

In the study of signs (semiotics), signs are classified as indexes (relating the sign to what it signifies by describing its intensity), icons (acting upon the senses in the same way as what it signifies do) and symbols (a looser connection to what it signifies). Due to the abstract nature of mathematics, the most frequently used signs are symbols.

However, there is agreement among the members of the mathematical community on the meaning of these symbols. Furthermore, mathematical discourse characteristically strives for effective communication (i.e. a high degree of correspondence between the intention of the sender and the receiver's interpretation) (Liebenau & Backhouse, 1990). This implies that in mathematics, symbols are there to designate with precision and clarity (Davis & Hersh, 1981) and that the meaning of each symbol should be unambiguous. Formalised texts can be seen as unambiguous and precise. The best-known example of a formalised text is a computer program where every logical step is included, leaving nothing to the imagination (Davis & Hersh, 1981). Mathematical text in general is only partly formalised. Sometimes the formalised parts are left out because they are seen as obvious, leaving behind the sentences explaining the not so obvious (Davis & Hersh, 1981).

Symbols also play an important role in learning, as Roman Jakobson (in Hadamard, 1945) has described: '*Signs are a necessary support of thought. For socialized thought (stage of communication) and for the thought which is being socialized (stage of formulation), the most usual system of signs is language properly called; but internal thought, especially when creative, willingly uses other systems of signs which are more flexible, less standardized, than language and leave more liberty, more dynamism to creative thought ... Amongst all these signs or symbols, one must distinguish between* *conventional signs, borrowed from social convention and, on the other hand, personal signs which, in their turn, can be subdivided into constant signs, belonging to general habits, to the individual pattern of the person considered and into episodical signs, which are established ad hoc and only participate in a single creative act.'* (op.cit.: 1945:96,97). Thus, symbols enable the learner to communicate his/her ideas, not only to others but also to him/herself. New concepts can also be communicated through symbols, where classes of concepts already known can be related to form new concepts.

Furthermore, symbols are used in the recording of knowledge which is a permanent record of mathematical ideas. Through these written and printed symbols, new generations can learn in a few years ideas that took generations to evolve. Another important function of symbols is to be able to record one's thoughts on paper as one progresses. The amount of information that we can keep in consciousness at a time is limited. A single mathematical symbol carries considerable information and this helps to reduce the cognitive strain. Symbols can thus also be described as both a label and a handle for identifying and manipulating concepts (Skemp, 1971).

Invention and discovery

Inspired by a celebrated lecture of Henri Poincaré given at the Societé de Psychologie in Paris, Hadamard wrote a book in 1945 entitled 'The psychology of invention in the mathematical field'. Several great mathematics inventors were asked for their opinion and experiences. Quite a few similarities were found: the suddenness of a new idea after long often fruitless, thinking described as ' ... *a gleam of light ...'* or a ' ... *sudden flash of lightning* ... ' (Hadamard, 1945:16,17), the emergence of a new idea at a moment when not consciously thinking about the problem, new ideas being preceded by constantly thinking about the problem or as put by Poincaré: ''*a consented, a voluntary faithfulness to an idea''* (Hadamard, 1945:44).

A frequent emotion accompanying new ideas, is the feeling of absolute certainty. Oliver (1972) observed similar reactions from children involved in a study on intuitive thinking. They were not aware of any mathematical thought before they made their guess, but were still certain about the correctness of it. Oliver also found that not all children had the same aptitude for intuitive thinking, but that intuitive thinking could be fostered by asking the right questions and creating an atmosphere where guesses and consequent mistakes are allowed and encouraged. Poincaré (in Hadamard,1945) illustrates the cognitive processes during and before invention with a comparison. He compares already formed concepts that are used in the invention process to hooked atoms which are at rest in the inactive mind. Studying a problem results in the mobilising of these ideas or 'atoms'. '*The mobilized atoms undergo impacts which make them enter into combinations among themselves or with other atoms at rest, which they struck against in their course.*' (Hadamard, 1945:47). Not all these combinations are helpful, nor relevant to the specific problem but new combinations were made that could lead to sudden insights later. These combinations can be likened to 'schemas' referred to earlier.

Although it is generally accepted that mathematical invention is not something that can be commanded at will, George Polyá shared his experiences on the principles of discovery and invention in a series of books. The best known of these, translated into 18 languages, is the famous 'How to solve it' published in 1945. His ideas have already been mentioned in the section on problem solving in par. 2.4.3.3.

Form of schemas

A question that could have been asked before, but that will now be discussed is: in what form are these schemas stored? Do we think in words, pictures or neither? Hadamard found some opposing views related to this question:

- *No thought is possible without words,* Max Muller, famous philologist;
- *We think in nouns,* Hegel;

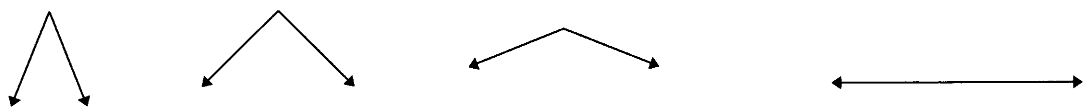
- *We cannot think without images,* Aristotle;
- *Thoughts die the moment they are embodied by words,* Schopenhauer.

The list is by no means complete and it reflects Galton's opinion that people differ greatly in cognitive style (Skemp, 1971). However, of the mathematicians that Hadamard contacted in his survey, practically all of them used vague images. Einstein mentioned that ' ... *the words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought.'* [He considered] ' *elements in thought'* [to be] '*signs and more or less clear images which can be voluntarily reproduced and combined.'* These elements are some ' ... *visual and some of muscular type.'* (Hadamard, 1945:142, 143).

The famous physicist Richard Feynman jokingly describes the same kind of visual imagery he used in his arguments with his mathematician colleagues: ' ... *the mathematicians would come in with a terrific theorem, and they're all excited. As they're telling me the conditions of the theorem, I construct something which fits all the conditions. You know, you have a set (one ball) - disjoint (two balls). Then the ball turns colors , grows hairs, or whatever, in my head as they put more conditions on. Finally they state the theorem, which is some dumb thing about the ball which isn't true for my hairy green ball thing, so I say, "False!".'* (Feynman, 1985:85).

Most mathematics educators share the experience that visual symbols are often more understandable than a verbal-algebraic representation of the same idea. Skemp (1971) views visual symbols as more primitive and basic but more difficult to communicate. This corresponds with Koopman's idea that the images appear in full consciousness, ' ... *while the corresponding arguments provisionally remains in the antechamber ... '* (Hadamard, 1945:86).

Although some of these visual images have to be translated into words to be communicated, some compelling arguments can be presented in a visual way. For example, Skemp (1971) argues that a straight line may also be considered as a particular kind of angle and present his argument visually in the following way:



Skemp summarises the difference between the two kinds of symbols in Table 2.1.

Table 2.1

Differences between visual and algebraic symbols

(Source: Skemp, 1971:111)

Visual	Verbal-algebraic
Abstracts spatial properties, e.g. shape, position	Abstracts properties independent of spatial configurations, e.g. number
More individual, harder to communicate	May represent socialised thinking, easier to communicate
Showing structure	Showing detail
Simultaneous, intuitive	Sequential, logical

In Hadamard's survey, a third kind of mental schema was mentioned, namely that of kinetic images. As an example, the above visual argument of Skemp has also a kinaesthetic and action-oriented mode (the enlarging of the angle is 'seen').

Simon (1996) develops a related concept in what he calls transformational reasoning. This is a reasoning that is involved when students want to get a sense of how a mathematical system in question works, by running the system. It thus involves the transformation of a mathematical situation and the interpretation of the results of the transformation. Simon illustrates this kind of reasoning with several examples, such as the case of the student whose understanding of an isosceles triangle consists of the idea of two people walking from the ends of a line at equal angles towards each other, having covered the same distance when they meet. He notices that the consequence of this kind of reasoning is often a sense of understanding of 'how it works'. According to Simon, mathematics educators have failed to recognise the importance of transformational reasoning in the mathematics classroom, to the detriment of insight. Encouraging this reasoning could lead to theorem generation, validation of ideas and the connection of different mathematical ideas (Simon, 1996). Schemas are thus stored in visual, kinetic or verbal-algebraic forms.

Overt and covert subjective mathematical knowledge

Maturana and Varela (1992:174) write about knowledge: '*We admit knowledge whenever we observe an effective (or adequate) behavior in a given context, i.e., in a realm or domain which we define by a question (explicit or implicit) ...* '.

This is a view shared by both behaviourists and cognitive theorists, but the latter are more interested in the cognitive processes behind the effective behaviour. The cognitive processes include the reorganisation of schemas to form new knowledge (images, conjectures, proofs, definitions, solutions, etc.)

Not all this knowledge will be made public, but it is by making it public that it becomes data to other community members. It is also only now that the teacher and others can admit knowledge if effective (or adequate) behavior is observed in the given context (using the definition of knowledge of Maturana and Varela).

'Making public' here is not restricted to a written communication or printed formalised proofs, but can include verbal informal statements. The other learners will start interpreting this incoming data and, depending on misfits with their own existing schemas and objective mathematical knowledge, will ask for justifications or validations. Again, the teacher's role here is to guide acts of creation, justification and criticism close to the practices of the wider mathematical community. The learner will reformulate and rethink his/her own creation via this social negotiation process which leads to new creations.

The new subjective mathematical knowledge that stays in the private realm, can also undergo the processes of criticism and consequent reformulation. By listening to other's explanations and justifications, and observing their effective behaviour, the learner can criticise and rethink his/her own ideas through inner speech (reflective thinking). However, the 'private' knowledge tends to be vague and sometimes lying in deeper layers of the unconscious (Hadamard, 1945). It is in the private realm that the vague combinatory play with ideas takes place before being made overt by the ' ... *connection with logical constructs in words or other kinds of signs which can be communicated to others.'* (Einstein in Hadamard, 1945:142). It is also through this formulation/reformulation and formalisation that the vague ideas are clarified and brought to consciousness.

Figure 2.8 shows the cyclical process of transformation of data into knowledge. It also shows how this process is accompanied by social negotiation as well as individual creation processes. The role of covert and overt subjective knowledge is shown and the learning environment is set against the background of the macro mathematical community.

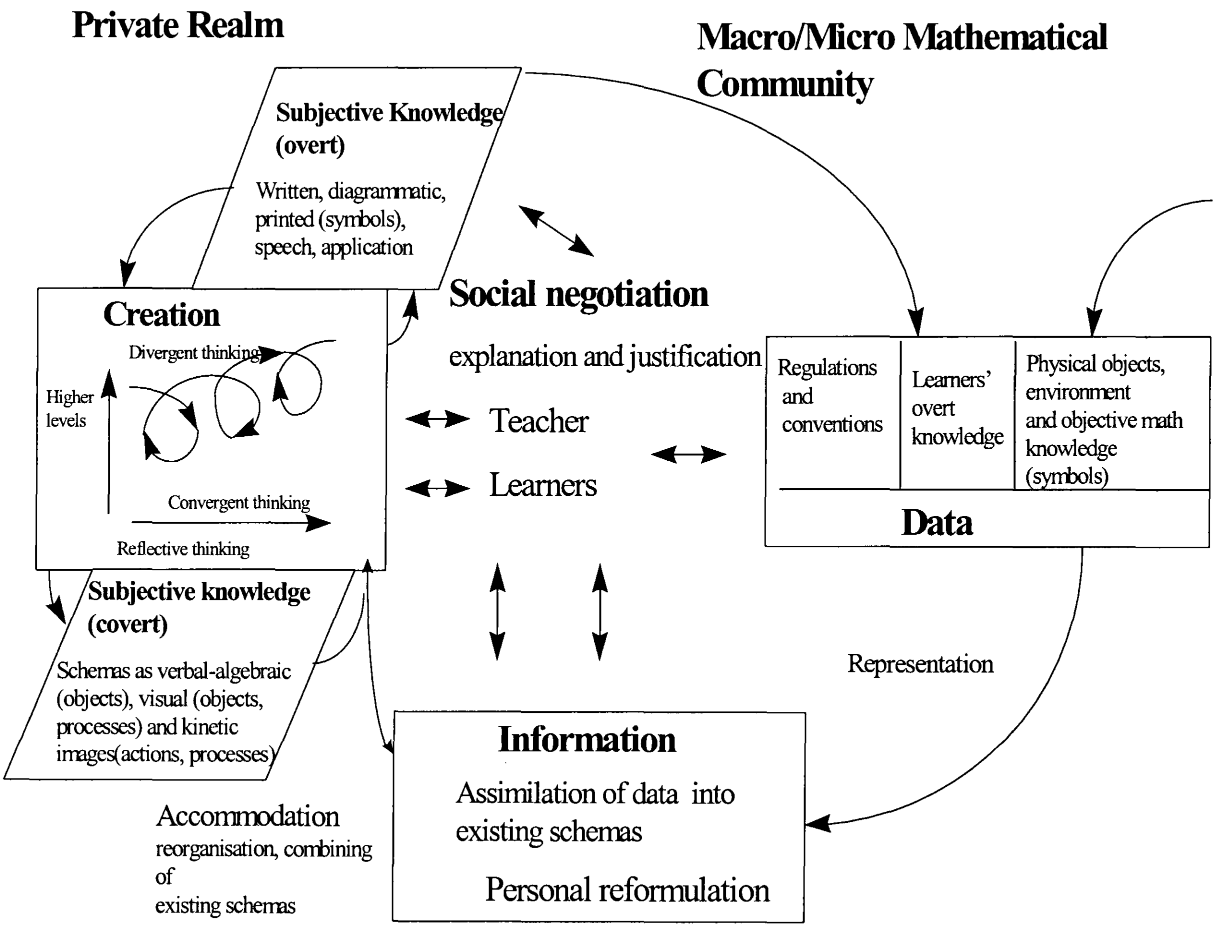


Figure 2.8

A social constructivist model for the learning of mathematics

2.6 Summary

In this chapter, a brief overview is given of selected theories regarding the philosophy and learning of mathematics. Mathematics is defined and the nature of mathematics discussed. Learning theories in general are described and constructivism extensively discussed, in particular the mathematics learning theory of Piaget and related theories. From these theories, a framework for the learning of mathematics is then developed. The framework is based on the following ontological and epistemological hypotheses.

The ontological hypothesis is that mathematical objects are social-cultural-historical entities. The epistemological hypothesis is that of social constructivism, namely that mathematics is a human creation and that objectivity is social in the sense that there is agreement on what is true.

The model for the learning of mathematics which was discussed in detail in par. 2.5., does not provide for co-operative learning nor computer-supported learning. These issues will be addressed in Chapter 4 where the existing model will be enhanced to provide for CSCML. Before that can be done a closer look into existing research on CL, CML, CSCL and CSCML is necessary. The next chapter will thus give a review of some of the existing literature available on these topics.

Chapter 3

Computer Supported Co-operative Mathematics Learning (CSCML)

3.1 Introduction

This chapter gives an overview of research done on CML and CSCML. The goal of this chapter is not only to define and clarify the most important concepts comprising the topic of this study, but also to highlight the need in existing literature for a conceptual framework for CSCML.

The chapter is divided into three parts. The first part (par. 3.2) covers the definition of CL, it methods , its rationale and principles. It then discusses research findings on CML and covers some design issues. The second part (par. 3.3) discusses the integration of IT in education in general and then more specifically, the integration of IT into mathematics education. This includes a survey of some of the research done on computer support to mathematics teaching and learning. The third part (par. 3.4) defines CSCL and discusses research done on CSCL. This includes the different technologies used in CL environments as well as the relevant research on CSCW and CSCL. CSCML is then defined as an organisational structure based on CL principles in which a group of students pursue mathematical academic goals through collaborative efforts supported by IT. Relevant research on CSCML is then summarised and design issues are highlighted.

3.2 Co-operative learning

3.2.1 Learning environment

The learning environment will be defined in terms of its constitutive 'ingredients', i.e. all relevant components, features and social aspects which create the environment in which learning takes place. For example, Collin, Brown and Newman (in Chung, 1991), list three dimensions that constitute a learning environment: content, teaching methods and sequence (phases of integration and generalisation of knowledge). De Villiers (1995), on the other hand, focuses on the social aspects created by the different interactions between the learners.

The structuring of the learning environment could have different effects on student-student interactions: they try to outperform their peers (competitive learning), students work towards a goal without needing to worry about the efforts of the other students (individualised learning), or they work together, ensuring that all group members master the given study material (co-operative learning).

Johnson and Johnson (1994) argue that each pattern of interaction has its place. An effective teacher will use all three appropriately and ensure that students learn how to work co-operatively, how to make competition fun and how to work on their own.

Co-operative learning is the most thoroughly researched of the three. The great interest in co-operative learning stems from the benefits that research findings show in social-affective outcomes (better self-esteem, better intergroup relations and the acceptance of academically handicapped students) (Slavin, 1991a). In addition, co-operative learning offers possible answers to the demands put to the student by a rapidly changing industrial world. It hopes to prepare the learner for an increasingly collaborative work force (Slavin, 1991a; Adams & Hamm, 1990).

Smith and MacGregor (in Bitzer, 1994) describe the broader concept of collaborative learning as an umbrella term for a collection of educational approaches that deal with joint intellectual efforts by students, or students and teachers together. Collaborative learning is one of several alternative names used by educationists for co-operative learning. Other names include small group learning, study circles and collective learning.

3.2.2 Definition of co-operative learning

There are numerous ways of defining co-operative learning. Johnson et al. (in Bitzer, 1994:40) define it as: ' ... *the instructional use of small groups so that students work together to maximize their and each other's learning.'*

Hilke (in De Villiers, 1995:90) describes it as: *'An organizational structure in which a group of students pursue academic goals through collaborative efforts. Students work together in small groups, draw on each other's strengths and assist each other in completing the task. This method encourages supportive relationships, good communication skills and higher-level thinking abilities.'*

This last definition draws the attention to the components that characterise co-operative learning: teachers and students participate actively; co-operation fosters a sense of community; knowledge is created, not just transferred, and that knowledge becomes community property (Whipple in Chung, 1991).

3.2.3 Principles of co-operative learning

A number of people working together is not necessarily a co-operative group. This is an opinion shared by several researchers (Johnson et al., 1994; Dockterman, 1991). There are certain basics that must be adhered to in designing a co-operative learning environment. Johnson et al. (1994) describe the basic principles or components of effective co-operative structuring as positive interdependence, individual accountability, face-to-face promotive interaction, interpersonal and small group skills, and group processing.

Positive interdependence

In his article 'A theory of co-operation and competition' M. Deutsch wrote: *'In a co-operative social situation the goals for the individuals or sub-units in the situation under consideration have the following characteristics: the goal-regions for each of the individuals or sub-units in the situation are defined so that a goal-region can be entered (to some degree) by any given individual or sub-unit only if all the individuals or sub-units under consideration can also enter their respective goal-regions (to some degree).'* (Deutsch, 1949:131,132).

Johnson et al. (1994) summarise the above paragraph with ' ... *group members have to know that they "sink or swim together".'* (Johnson & Johnson, 1994:81).

Johnson et al. (1994) name three steps to take in structuring positive interdependence. Firstly, learners must have a clear idea of what they are supposed to do. Secondly, a goal interdependence must be established - group members must know that they cannot succeed unless all the members succeed. Finally, goal interdependence could be combined with other types of interdependence: resource interdependence (dividing resources), role interdependence or identity interdependence. Salomon (1992) describes the characteristics of interdependence as the need to share information, meanings and conclusions provided by the group; a division of labour, and the pooling together of minds.

The effective structuring of positive interdependence could prevent the occurrence of *free-riders* (individuals perceiving their efforts not to be necessary) or *social loafing* (loss of motivation due to large groups) (Hooper,1992). Salomon (1992) names *status sensitivity* and *ganging up on task* as two more social effects that could lower positive interdependence. Status sensitivity describes the taking charge of the process by learners with perceived high status (high ability, popularity, etc.). Ganging up on task refers to group members working together to finish the task as fast as possible.

Individual accountability

Each student is individually accountable for his or her share of the work. This can be accomplished by rewarding a group in a way that is based on the assessment of the performance of each member of the group.

Face-to-face promotive interaction

Promotive interaction exists when group members encourage each other and guide each other's efforts to reach group goals. Johnson et al. (1994) see promotive interaction as characterised by help and assistance, exchanging information and materials, providing feedback, challenging conclusions and reasoning to promote higher quality decision making and greater insight, motivating each other to strive for mutual benefit, and acting in trusting ways.

Interpersonal and small group skills

Kagan (in Bennett, 1994) says that for pupils to benefit from group work, they need a degree of tolerance and mutual understanding, an ability to state a point of view, to discuss, to probe, and to question. Kagan is of the opinion that these skills are not innate, but need to be taught. The more socially skillful the student is, the higher the achievement in the groups. Some social skills that need to be taught are trust, communication, acceptance and support of each other, and conflict resolution (Johnson et al., 1994).

Group processing

Groups should reflect on how well they are functioning. Members could reflect on each other's actions and make decisions on what actions to continue or change. This should be done to improve the effectiveness of the member's contribution to the team's efforts to achieve the mutual goal.

The five basic elements need to be designed into the co-operative learning environment for it to be effective. These elements should also be kept in mind in using informal co-operative learning (group discussion combined with direct teaching) and other kinds or group work.

There is general agreement that of the five basic elements the most important is positive goal interdependence. Helen Block Lewis wrote in an article: '*A minimum requirement for cooperative behavior is not physical togetherness nor joint action, nor even synchronous, complementing behavior, but a diminution of ego demands so that the requirements of the objective situation and of the other person may function freely. In truly cooperative work, personal needs can function only if they are relevant to the objective situation, the common objective, in other words, is more important than any personal objective.*' (Block Lewis in Deutsch, 1949:135). Deutsch sees it as the group members occupying ' ... *the same relative positions with respect to their goals*.' (Deutsch, 1949:135). Salomon states that ' ... *for genuine collaboration to take place, you need genuine interdependence.*' (Salomon, 1992:64).

3.2.4 Rationale for co-operative learning

The value of interpersonal processes in both learning and relationships is widely recognised by educators. Reviews of studies of co-operative learning confirm this by showing positive results in affective and cognitive outcomes (Slavin, 1990). Research on the rationale for co-operative learning can be divided into two perspectives: the motivational and cognitive perspectives.

Motivational perspective

This emphasises the reward or goal structure under which group members operate. This approach emphasises the effect of co-operative rewards on students' achievements in co-operative learning.

The co-operative rewards could include individual or group points and create an interpersonal reward structure (group members give or withhold praise and encouragement among each other) (Bennett, 1994).

Cognitive perspective

This perspective relates to the effects of working together (with or without a mutual goal). The following developmental and cognitive elaboration theories fall under the cognitive approach (Slavin, 1990).

Developmental theory

In the last decade developmental psychology changed its concept of a learner from a 'lone scientist' to that of a 'social being' (Bennett, 1994:51). The theories of Piaget and Vygotsky (which dominate the developmental psychological research on co-operative learning) look at the learner from different perspectives.

The Piagetian approach underlines the necessity of the presence of cognitive conflict for learning to take place. Co-operative learning provides the ideal setting for socio-cognitive conflict through voicing of different opinions and strategies (Mandl & Renkl, 1992).

Related to the Piagetian approach is the controversy theory. This theory states that conceptual conflict creates reconceptualisation and a search for more information. This leads to more thoughtful conclusions (Johnson et al., 1994).

The Vygotskian approach, on the other hand, emphasises the internalisation of processes on the social level. According to Vygotsky, learning is seen as a relationship between language and cognition - this learning takes place in an interactive context in which cognitive development of children can be seen as the transition from other-regulation, to self-regulation, of behaviour. This transition is a function of mediated activity with origins in the social interaction between child and adult. These mediated strategies of the adult become the scaffold on which children cross the zone of proximal development (ZPD), i.e. children perform tasks which they do not exactly understand but come to internalise these strategies through adult/peer guidance (Emihovich & Miller, 1988).

Cognitive elaboration theory

Cognitive psychology research has found that for information to be accommodated into already existing schemas in the long-term memory, some kind of cognitive restructuring or elaboration of it must take place (Wittrock in Slavin, 1990). Research findings show that a very effective way of elaboration is to explain the study material to someone else (Devin et al.; Dansereau in Slavin, 1990).

De Villiers (1995) discusses two models that describe the group process that is relevant to co-operative learning. These are Allen's model of group behaviour and the model of Hackman and Morris.

Allan's model of group behaviour

Slavin et al. (in De Villiers, 1995), describe it as an input-output model. The input variables to the group are process, individual, group level and environmental characteristics. The output variables are the performance and social reactions of the learner. The interaction process takes place between the input and output in a task in a social environment that facilitates the task. Figure 3.1 illustrates this model.

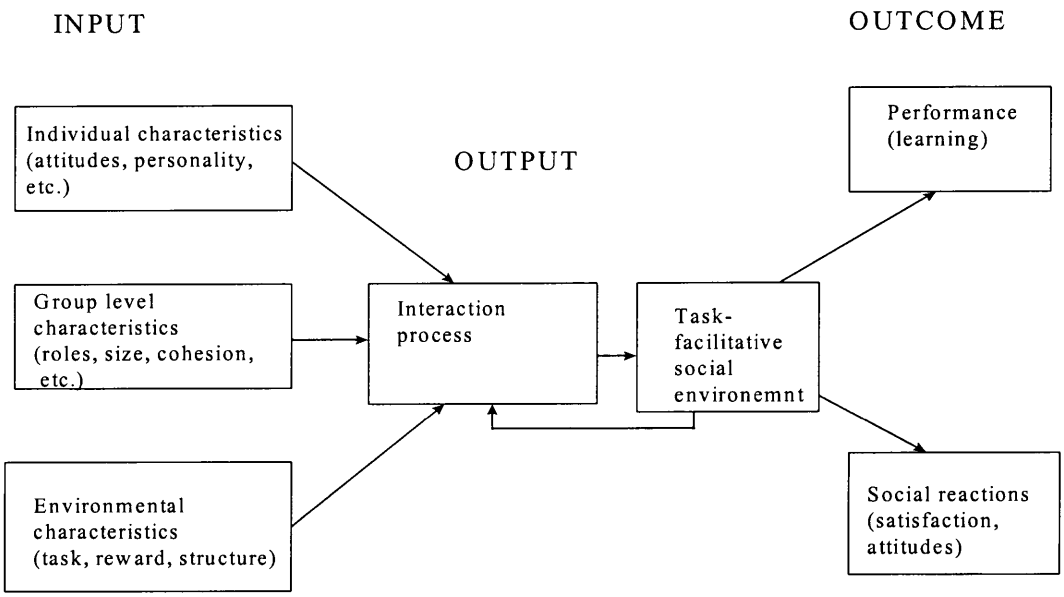


Figure 3.1

Allen's model for group behaviour

(Source: Slavin et al. in de Villiers, 1995: 100)

Hackman and Morris model

This model is a combination of an input-output model and a contingency model. Here, attention is put on the most important input variables from the individual (prior knowledge, task performance strategies and member effort) (Slavin in de Villiers, 1995). Although the relationship between group variables and individual variables is stated explicitly, the model neglects to show how the input factors operate in the interaction process (De Villiers, 1995). Figure 3.2 shows the relationship between the different variables.

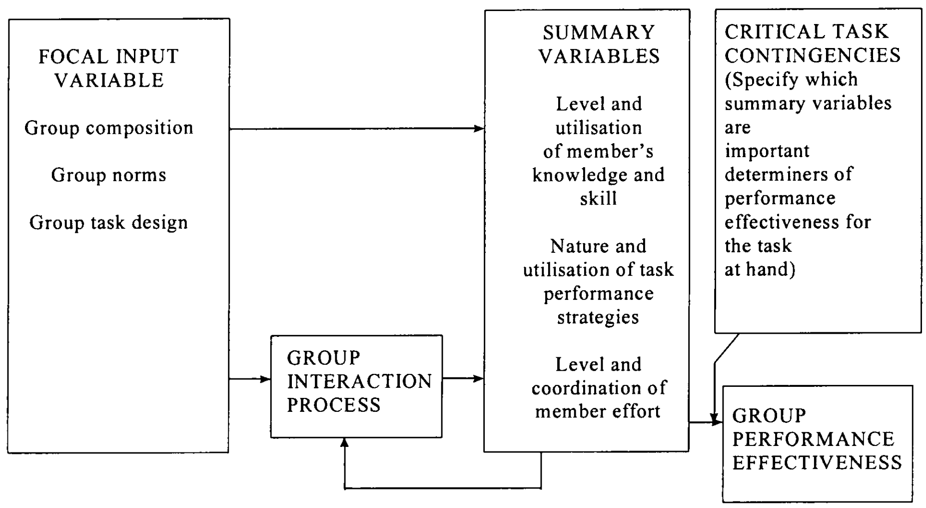


Figure 3.2

Hackman and Morris Model

(Source: Slavin et al. in De Villiers, 1995:101)

3.2.5 Methods of co-operative learning

There are many different forms of co-operative learning, but all of them require students to work in small groups or teams to help each other learn academic material. Co-operative learning could be incorporated in courses as informal groups, base groups or formal groups. Informal groups are short-term groups and are used with direct teaching (lecturing). This is where the teacher could invite discussion on a topic between students sitting next to each other. Base groups are long-term learning groups with the emphasis on social support and long-term accountability. Formal learning groups are groups where the emphasis is on a mutual goal and individual responsibility to achieve a goal (Johnson et al., 1994).

Some formal co-operative learning models that have been evaluated are described below:

Jigsaw

Students are assigned to six-member teams to work on academic material that has been fragmented into sections. Each team member studies his or her section after which the members of the different groups who have studied the same section meet in their expert groups to discuss that section. The experts then return to their teams and teach the others what they know.

De Villiers (1995) draws attention to important other aspects of this method: specially designed curriculum materials; group building and communication training; importance of a group leader; and heterogeneity of groups with respect to sex, race and ability levels.

Student Team Learning (STL)

Student team learning methods emphasise the use of group goals and group success. The student's task is not to do something but to learn something. Five STL methods that have been extensively researched will now be discussed.

Students Teams-Achievement Division (STAD)

The teacher presents a lesson after which the students work within their heterogeneous teams to make sure that all team members have mastered the lesson. Finally, students take individual quizzes on the material. The scores are compared to the individual's own past averages, and points are awarded based on the degree to which students can meet or exceed their own earlier performances. These points added together form team scores. High performing teams earn certificates or other forms of rewards.

This process could take three to five class periods. The method has been used with learners of all ages and in a variety of subjects, but it seems better suited to the mastery of factual material (De Villiers, 1995).

Team-Games-Tournament (TGT)

It has the same structure as STAD, but replaces the quizzes with weekly tournaments. Students from different groups compete against others with comparable ability. This method is appropriate for the same kind of objectives as STAD.

Team Assisted Individualisation (TAI)

TAI combines co-operative learning with individualised instruction. It was originally designed to teach mathematics to primary school children.

Students enter an individualised sequence according to a placement test and then proceed at their own pace. Group members thus work on different units but check each other's work and help each other. Final unit tests are taken. Rewards are given on a weekly basis to groups based on the total number of units completed by all group members. Extra points are given for perfect papers and completed homework. While students proceed at their own pace, the teacher teaches small groups drawn from the groups.

The use of TAI in the classroom enhances self-concept in mathematics, as well as achievement (Slavin, Madden & Stevens in De Villiers, 1995).

Co-operative Integrated Reading and Composition (CIRC)

CIRC was designed to teach reading and writing to young learners. In most CIRC activities, students follow a sequence of teacher instruction, group practice, group preassessments and quizzes. It also includes reading groups, pairs from different reading groups, partner checking, testing, direct instruction in reading comprehension, and independent reading (Slavin, in De Villiers, 1995).

Jigsaw II

This is a modification of the Jigsaw method by incorporating it into the STAD method. It combines individual and co-operative incentives: groups compete for group rewards and learner's grades are based on points obtained in quizzes (Sharan, in De Villiers, 1995).

Learning together

Students work together on assignment sheets in heterogeneous groups. A group hands in a single sheet and the reward is based on the group product. Slavin (in De Villiers, 1995) emphasises the training of learners to be effective group members and the evaluation of the functioning of the group by the members.

Group investigation

Learners in small groups take responsibility for deciding what to learn, how to organise the task and how to communicate their knowledge to the class. Sharan (in De Villiers, 1995) describes steps to be followed by the learners: topic selection, co-operative planning (procedures and goals), implementation, analysis and synthesis, presentation and evaluation.

Despite being the most complex of the methods, research findings show that implementation of this method leads to higher levels of achievement, and enhancement of co-operation and social interaction (Sharan & Sharan in De Villiers, 1995).

Reynolds et al. (1995) add to these categorisations three further methods which are variations on group investigation used in mathematics teaching: Coop-Coop (a combination of jigsaw and group investigation), small group laboratory approach, and the small group discovery method (with focus on the discovery of new ideas).

3.2.6 Research on co-operative learning in mathematics education

The effectiveness of co-operative learning in mathematics teaching and learning has been confirmed by research findings (Davidson & Lambdin Kroll, 1991). In their overview of research on co-operative learning related to mathematics, Davidson and Lambdin Kroll classify existing research studies into three groups. The first group consists of studies which investigate conditions under which co-operative learning raises students' achievement. The second group includes studies identifying the benefits (cognitive and affective) of co-operative learning, and the last group focuses on the processes and interactions taking place in a co-operative learning environment. A few research studies and results will be discussed under these headings.

Promotive conditions

Research findings indicate that the effects of co-operative learning vary considerably according to the particular methods used (Bennett, 1994). Slavin (1991a) is of the opinion that the two factors necessary for achievement gain are the use of group goals and individual accountability. Group goals are sometimes used synonymously with group rewards. In a study done by Yackel, Cobb and Wood (1991), no extrinsic incentives were provided for groups in the project classroom. It was found that the classroom norms (mutually constructed and reconstructed), supplied the incentives, or, in other words, the norms were inextricably related to mathematical activity. The norms included that students should co-operate to solve problems, that meaningful activity is valued over correct answers, that persistence on personally challenging problems is more important than completing a large number of activities, and that partners should reach consensus as they work on the activities (Yackel et al., 1991:397).

Benefits

Researchers have focused on diverse outcomes including achievement, higher level reasoning, motivation, student retention, transfer of learning, prejudice, social support moral reasoning, and many others. Johnson et al. (1994) divide these outcomes into three broad categories: effort to achieve, interpersonal relationships and psychological health.

Research shows that co-operative learning resulted in more higher-level reasoning, generation of new ideas and solutions, and better transfer of learning (Slavin, 1991a).

Studies done by Wimbish (in Reynolds et al., 1995), Leikin and Zaslavsky (1997) and Mulryan (in Good et al., 1992) have found a fostering of positive attitudes toward mathematics, the instructional experience, and further study in the subject area. Mulryan (in Good et al., 1992), reports a narrower and less differentiated view of co-operative learning among low achievers than among high achievers. Also, low achievers tend to view mathematics more as work with numbers and operations.

One of the earliest findings in co-operative learning, was that students who co-operate learn to like each other (Slavin, 1991a). This is also the case where group members have different ethnic backgrounds (Johnson et al., 1994).

Johnson et al. (1994) discuss research findings which compare the impact of individual, competitive, and co-operative learning on the student's self-esteem. These studies show consistently that co-operative efforts promote higher self-esteem than competitive and individual efforts. They ascribe this to the perception of students that they are liked by their peers, that they have contributed to the group's success, and that they were appreciated for their own abilities.

One of the major strengths of co-operative learning is to help students develop problem solving skills (Good et al., 1992). Good et al. define the problem solver as someone who maintains the intention to learn while trying different strategies in the face of uncertainty. This conception of the problem solver involves spheres of emotion, motivation, cognitive strategy, and metacognition. A study done by Dees (1991) involving college remedial mathematics students, shows an increase in students' problem solving skills through the use of co-operative learning, especially in word problems and proof writing in geometry. The findings also confirm other researchers' views that co-operative learning may not be significantly better than traditional methods for acquiring skills and facts not involving complex thinking.

Yackel et al. (1991) noticed that children engage in two types of problem solving activities as they work in small groups: solving the mathematical problem and solving problems of working productively together. The resulting interactions give rise to opportunities of learning in the following way: the use of the solution activities of others as prompts in developing one's own solution; the reconceptualisation of a problem for the purpose of analysing a wrong solution method, and the extending of one's own conceptual framework in an attempt to make sense of the solution activities of others for the purpose of reaching consensus (Yackel et al., 1991:406).

Quite a few researchers have employed a black box strategy in researching co-operative learning, i.e. more focus is put on the outcomes than the process. However, Good et al. (1992) notice a notable difference between studies that show achievement effects (black box designs) and studies involving observation of classroom processes. The latter report more problematic findings, including some that not always support the themes of co-operative learning (Slavin in Good et al., 1992). Webb (1991) is of the opinion that such studies that focus on the processes could contribute more by identifying the kinds of interaction among students that should be encouraged to maximise learning.

Process and interactions

Mulryan (in Good et al., 1992) conducted a study focusing on the attention and interactive behaviour of secondary school mathematics pupils in co-operative small groups. Mulryan found that opportunities provided by small-group instruction vary for different students. Also, low achievers had the most difficulty in adapting to the group context and that they asked more questions whereas the high achievers gave more explanations.

Webb (1991) identifies possible strategies for promoting effective small-group interaction. These include certain group compositions, changing the reward structure, providing training in desirable verbal behaviour, and structuring the group to enhance social negotiation (explanation and justification). For example, it seems as if the most productive group composition consists of high and medium ability students, or medium and low ability students. Also, groups with an equal number of boys and girls promoted more explanation than groups with unequal number.

Leikin and Zaslavsky (1997) divided mathematics classroom activities into active and passive on-task activities, and active and passive off-task activities. By observation and interviews they collected data from research experiments and found that co-operative learning induces a shift towards students' on-task verbal interactions. Webb (1991) found that task-related verbal interactions are closely related to learning outcomes.

Good et al. (1992) recognise a need for a structure or overall view of the mathematics co-operative learning environment that identifies important variables that require investigation. This could serve as a basis for planning other research and as an aid in examining and interpreting existing research. They also point out that several variables, such as task variables, teacher and instructional variables, individual and group variables, need further research.

3.2.7 Obstacles to co-operative learning and co-operative mathematics learning

Slavin warns that *'Every innovation in education carries within it the seed of its own downfall, and cooperative learning is no different in this regard.'*(Slavin, 1991:86).

He mentions a few problems:

- Teachers with inadequate knowledge may use ineffective forms of this approach which could result in failure.
- Undertrained teachers apply co-operative learning methods to subject areas for which it is not suitable (e.g. STAD and TGT are less suitable for subjects that lend themselves to discussion and controversy).

Dockterman (1991) adds:

- Teachers are used to structuring their lessons in such a way as to ensure maximum control. Co-operative learning activities could threaten classroom control.
- Direct teaching is still the best way to cover a large amount of material in a short time. Teachers with too much work to handle will not deviate from traditional lectures to avoid an additional burdening on their work load .
- Group members could waste time discussing irrelevant issues and should be carefully monitored.

Complaints from learners are:

- They spend more time on courses that are taught using co-operative learning strategies than on traditionally taught courses (Killen, in De Villiers, 1995).
- They find co-operative learning activities more demanding (Hiltz, in De Villiers, 1995).

Good, Reys, Grouws and Mulryan (in Good et al., 1992) conducted research that involved 63 observations of 15 elementary school mathematics teachers implementing co-operative learning. They identified several weaknesses of the method:

- Inadequate curriculum - the lack of curriculum material forced teachers to use unsuitable text books or develop their own material;
- Curriculum discontinuity - no continuity in content difficulty across grades;
- Designing appropriate tasks - too many topics and activities are forced into the model;
- Implementing new tasks - too little time was allowed for groups to work on assigned tasks;
- Assigning student roles - designation of students as leaders, recorders, etc. seemed artificial; and
- Lesson structure and accountability - lessons ended abruptly without time to summarise.

It is thus clear that grouping in itself does not improve lessons. Adams and Hamm (1992) see the individual teacher as a critical factor in the effective implementation of co-operative learning. Not only should the teacher be trained in the use of co-operative learning methods, but he/she should have experience of the various methods of co-operative learning to understand the power of the technique.

3.2.8 Designing the co-operative mathematics learning environment

Adams and Hamm (1990) assign the teacher a critical role in the designing of an effective co-operative learning environment. This is also the case for the co-operative mathematics learning environment. Co-operative learning groups differ from the traditional classroom in several ways: the role of the teacher changes to that of a facilitator and the role of the student to that of responsible creators of their own knowledge and meaning (Bitzer,1994). Learners become tutors, investigators and presenters.

The teacher's role

Johnson, Johnson and Holubec (in Johnson et al., 1994) describe five aspects of the teacher's role:

1. Specifying the objectives for the lesson.
2. Making preinstructional decisions, such as
   - deciding on the size of groups; the basic rule is the smaller the group, the better;
   - assigning students to groups; this includes deciding on heterogeneous or homogeneous groups;
   - arranging the room; students should be put in small groups with resources readily available;
   - planning the study material; Reynolds et al. (1995) emphasise the active nature of the learning of mathematics. In designing the learning task, the teacher should keep in mind that the purpose of the learning activity is to create a basis for group discussion of a concept, rather than to find a numerically correct, unique solution; and
   - assigning roles to ensure interdependence; as was said earlier, some researchers find the roles artificial (Good et al., 1992) and others are of the opinion that roles originate naturally because of negotiated classroom norms (Yackel et al., 1991).
3. Structuring the task and positive interdependence. Teachers should
   - explain the academic task,
   - explain criteria for success,
   - structure positive interdependence by creating goal interdependence,
   - frequently assess the level of performance of each group member, and
   - specify behaviours that are appropriate and desirable within the group.
4. Monitoring the co-operative lesson. Teachers should
   - keep the groups on task by stimulating discussion, giving information, sharing opinions and co-ordinating activities;
   - observe the thought processes of learners and intervene when necessary to help students understand what they are studying;
   - offer encouragement, foster communication and energise learners; and

- intervene to teach necessary social skills.

5. Evaluating learning and processing interaction.

   Students should be able to summarise what they have learned. Tests should be given, and papers and presentations should be evaluated. Teachers should use a variety of evaluation procedures . At the end of an assignment, the functioning of the group should be discussed. Students have to reflect on their experiences in the group process.

Teachers need to model attitudes, problem solving attitudes and inquiry. Learners should understand that they are active learners and that no one knows all the answers (Adams & Hamm, 1990). Zigurs and Kozar (1994), emphasise that the group process should not be underestimated by the initiator (teacher) of the co-operative activities. They should concentrate on group building exercises. Adams and Hamm (1990) argue that teachers should re-examine the concepts of the organisational process and grouping structure in order to structure co-operative groups and activities.

3.2.9 Summary

This part of the chapter defines CL as an organisational structure in which a group of learners work together to pursue academic goals (par. 3.2.2). It is made clear that CL is more than simply grouping learners together. To ensure successful implementation, the principles of CL should be adhered to (par. 3.2.3). The application of CL techniques to mathematics learning has certain benefits. Research studies mention positive attitudes towards mathematics, better self-esteem, improved motivation and the fostering of problem solving skills. Other research focuses on conditions under which CL raises students' achievement (par. 3.2.6). Several obstacles to CL and CML are pointed out and the teacher is identified as a crucial role player in the design and execution of CML (par. 3.2.7 and 3.2.8).

Most of the research studies discussed above concentrate on isolated features of the co-operative mathematics learning environment. Good et al. describe a need for an overall view of the CML environment that identifies and relates the important components of the learning environment. It will be shown that this need also exists for the CSCML environment (par. 3.4). The next section first deals with the integration of IT in mathematics education after which the integration of IT in CML environments is discussed in par. 3.4.

3.3 Information technology and education

'Technological tools can foster students' abilities, revolutionize the way they work and think, and give them new access to the world.' This is the view of Peck and Doricott (1994:11), shared by several educators.

3.3.1 Historical overview

Van Weert (1995) describes three consecutive stages of information technology development in education as the first stage of automation, the second stage of information and the third stage of communication:

- The automation stage aims at automating the teaching process and administration. Much emphasis was put on computer-assisted instruction (CAI) which Bork (in De Villiers, 1995) criticises as the mere transposing of books and lectures. The computer was at the centre of the teaching process with restricted interactivity, forcing the student to follow a sequence of frames (Mühlhäuser, 1995).
- The information stage views information technology as a tool which can be used to empower the individual. Much emphasis is put on simulations, computer literacy and development of suitable software.
- The communication stage focuses on the innovative educational uses of computer communication.

3.3.2 Reasons for the integration of information technology in learning environments

Van Weert (op. cit.) sees the last two stages described above, as supporting the new organisational structures in society brought about, and enabled, by the use of information technology.

According to van Weert, this organisational change is from a hierarchical, industrial organisation to a network organisation. Powerful personal computers are being integrated into local-area, wide-area and global networks, empowering individuals in business processes and supporting co-ordination in team-based network organisations. New competencies are thus needed in the workplace: inductive thinking, generalist and information technology competencies, decision making, handling of dynamic situations, communication and co-operation skills. Some of these competencies can be enhanced by using information technology in the learning process.

Bannon (1995) emphasises the fact that much learning takes place outside the formal classroom and that everyday social practices of people at work and play, offer rich opportunities for learning. He questions the separation of *formal* and *informal* education and feels that the use of information technology in the so-called *informal* education could have a significant impact (Bannon, 1995). By permitting and facilitating exchanges between 'teachers' of both environments, computer communications networks indeed blur the separation between learning in the formal and informal contexts (Kaye, 1992).

Peck and Doricott (1994) give several reasons why information technology should be used in learning:

- Technology can individualise instruction by offering individual learning paths through integrated learning systems via computer networks.

- Competencies such as assessing, evaluating and communicating information are fostered by information technology.
- Technology fosters problem solving by enabling students to independently organise, analyse, interpret, develop and evaluate their own work all of which assist in focused problem solving.
- Modern technology-based art forms encourage artistic expression by providing an alternative way of expression to students constrained by traditional options of verbal and written communication.
- Technological tools allow students to reach around the world and use resources that exist outside the school.
- Technology provides a global audience for students' work, enabling them to do work that has value outside school.
- Electronic media, laserdisc and CD ROM offer stimulating, interesting courses.
- Students should feel comfortable with using information technology, being such an integral part of their living environment.
- Productivity and efficiency in schools can be increased by using information technology for doing routine tasks, elevating the role of the teacher.

3.3.3 Information technology and mathematics education

3.3.3.1 Historical overview

Computers have been used in mathematics education for more than thirty years now. The stages of IT development in mathematics education follow the same pattern as is laid out by van Weert (1995), par. 3.2.5.1. In the stage of automation, educational computing in mathematics was dominated by Computer Assisted Instruction (CAI) and BASIC programming courses. Mathematics was seen as particularly suitable for the use of computerised teaching and learning (where the computer takes over or minimises the teacher's role) (Morgan, 1994). Nowadays (in the stage of information and communication), new learningware is continuously being developed to exploit the vast memory, logical structures and graphic capabilities of the computer. Powerful tools are available to empower the individual by enabling exploration, discovery learning and the building of intellectual structures.

3.3.3.2 Information technologies used in mathematics education

Rather than going into detail on all the software and technologies available, an attempt will be made to give a general overview of what is commonly used in mathematics education. The popular categorisation scheme developed by Robert Taylor (Newby et al., 1996) divides computers in education into three broad categories, namely tutor, tool and learner. Morgan (1994) and Kaput (1992) view this categorisation as not very successful since a particular piece of software can be used in various types of activities. Morgan sees the way in which software is used as dependent on the existing classroom culture (Morgan, 1994). Kaput (1992) realises that a categorisation of existing technologies for mathematics will quickly be outdated and uses the fundamentals of the underlying structures of users' interaction with mathematical notation in any medium (not considering the specific technologies) to categorise IT uses in mathematics education. This results in categories including dynamic/static media, interactivity of media, and the presence of procedure capturing and executing facilities.

In the following overview, a short discussion will be given of well-known information technologies in mathematics education, with reference to the above categories.

- Microworlds: This is where mathematical phenomena are an integral part of the learner's environment. Noss, Healy & Hoyles (1997) describe it as ' ... *almost any exploratory learning environment which incorporates a computer ...* ' (Noss et al., 1997:203). As put by Papert (in Wiebe, 1993:146): '*The idea of "talking mathematics" to a computer can be generalized to a view of learning mathematics in "Mathland"; that is to say, in a context which is to learning mathematics which living in France is to learn French.*' Seymour Papert is also one of the developers of the programming language LOGO (and Turtle graphics) with which such microworlds can be created. In this microworld, certain concepts central to writing computer programs as well as mathematical concepts are mastered. Ernest (in Wiebe, 1993) identifies a few of these concepts as estimating distance, angles, shapes, symmetries, similarities, transformation, variables and problem solving strategies. Other computer languages used are Basic, Pascal and multimedia/hypermedia authoring tools such as HyperCard and Boxer (Newby et al., 1996). This is a good example of where the computer becomes the learner. The learner has to teach the computer to perform a task. In order to do that, the learner has to know how to perform a task and communicate it to the computer in an understandable way (i.e. to learn to program).

- Computer Algebra Systems (CAS): This is described by Smith (in Karian, 1992:2) as *' ... an integrated symbolic, numeric and graphical system with interactive and procedural interfaces.'* Unfortunately, it is not always immediately clear how to use CAS effectively. The most transparent of the CAS is Derive, but it is seen more purely as a tool and lacks real procedural interface or capability for creating students' learning environments. The most used CAS are Maple, Derive and Mathematica. The latter is described by Kaput as a potent set of mathematics tools coupled with a sophisticated command language. The user needs to be competent in mathematics and physics to be able to use it as a tool or learner.

- Mathematical programming languages: The well-known LOGO has already been mentioned above. Another language worth mentioning is ISETL. With this language mathematical concepts can be constructed on the computer. The syntax of ISETL language and its basic constructs are so close to standard mathematical notation, that learning the language is inseparable from learning mathematics (Dubinsky, 1997).

- Multi-representational software: This enables the student to see what the effects of changing a feature of one representation has on the others. Different aspects of complex ideas are exposed and the meaning of actions in one notation is linked to its consequences in other notations. For example, rate of change can be expressed in slopes of graphs or in formal algebraic derivatives; solving of equations can be done using tables, graphs or symbols (Kaput, 1992). Noss et al. (1997) describe a microworld called Mathsticks where they helped students to construct mathematical meanings by providing links between the actions, and symbolic and visual representations they developed.

- Graphing Facilities: Graphing calculators are very popular because of their portability and easy integration into classroom activities (Karian, 1992).

- Content free software: Spreadsheets are used not only in the teaching of statistics, but also in the teaching of division, geometry, problem solving skills and other activities involving generalisations (Wiebe, 1993).

- The Internet: The Web is already being used as a conveyer of mathematical material, mathematical software and as a former of educational communities. A consortium of American Universities, called NetMath, is investigating the establishment of mathematical learning environments (not correspondence courses) via the Internet (Klotz, 1997).

- Geometry Learning Environments: A widely used software package is the Geometer Supposer series. It provides easy construction of geometric shapes and easy measurement as well as the capture of students' constructions as general procedures that can be repeated. CABRI and The Geometer's Sketchpad provide more by enabling animation wherein a construction can be adjusted by dragging actions (Kaput, 1992).

In addition to the above classifications, Schoenfeld (in Karian, 1992) adds more categories which include drill-and-practice environments, tools that do the drudgery, simulations and intelligent tutoring systems.

3.3.3.3 Research on information technology in mathematics education

Research in this field focuses on different aspects of the learning environment. Because of the observed ability of IT to sustain social interaction, the social aspects of IT support with regard to mathematics education is increasingly becoming a point of interest to researchers. These aspects will be discussed in greater length in par. 3.4.

Some research studies deal with instructional design and the integration of the ideas in the development of new software. Laborde et al. (1991) describes the linking of the existing tools, viz. Cabri and Hypercard, to create an intelligent tutoring system. The design is based on observation of the tasks of a tutor guiding a construction task in geometry.

Other researchers develop theories for the learning of mathematics and use the theoretical framework to develop new software. Dubinsky (1992) and fellow researchers have been busy with specific undergraduate research for the past 12 years. This research is based on a paradigm that entails an analysis of learning; the consequent development of a general theory of learning based on Piaget's ideas (discussed in par. 2.4.3.2); designing the instruction using pedagogical strategies relating to the general theory; implementing instruction; and observing students while they learn. The observation is influenced by the theory, but the observation also influences the theory by identifying the necessity for revising it (Dubinsky, 1997). Roughly speaking, their learning theory looks like this: processes are built up out of actions on objects and ultimately converted into new objects which are used for new processes (Dubinsky, 1992). Dubinsky views the ' ... *primary goal of teaching to help students construct appropriate processes and objects and get them to reflect on and use these constructions in dealing with problem situations that arise in maths.'* (Dubinsky, 1992:48). He analyses topics to decide which construction one would like the students to make, and designs problem solving strategies accordingly. The students need to solve the problems using ISETL. The intent is (and studies show that it does happen), that students will make effective mathematical constructions in their minds as a result of making the computer constructions (Dubinsky, 1997).

The work of Papert has already been looked at briefly. Illuminating work resulted from Papert's epistemology and learning research group at MIT. Papert and his colleagues' ideas are based on a theoretical framework called constructionism. He describes it as standing on two legs, constructivism, and the construction of public entities. These ideas were implemented in research programs. One of these describes a learning environment where elementary school pupils are given the active role of teacher/explainer in designing software (Harel & Papert, 1991). The programme was called 'Instructional Software Design Project' (ISPD). The evaluation of ISPD was designed to examine learning of fractions and Logo through ISPD, versus learning of fractions and Logo through other pedagogical methods. Pupils had to design screens and representations to explain the difficult concepts of fractions to lower-grade fellow pupils. Quantitative results showed that students improved their ability to perform on standardised tests on fractions, more so than students not on the ISPD programme. Qualitative findings include the effectiveness of learning programming and fractions simultaneously rather than separately, a greater involvement with deep structure, rather than surface structure (algorithms) of rational number concepts, personal expression and social communication of ideas (partly due to the use of Logo). The researchers found that Logo facilitated communication about acts of cognition and learning. They realised, however, that the improvement of results could not be attributed to only one factor but to various factors including the affective side of cognition, personal appropriation of knowledge, the use of Logo, the learning-by-teaching principle, and many others.

Di Sessa (1991) uses different images of learning as a theoretical framework to study the role of pupils in the designing of representational forms for capturing motion with the aid of Boxer. Boxer is a general purpose computational system with all-time accessible resources of text and hypertext editing, for dynamic graphics and for programming (Di Sessa, 1995). By letting the different images of learning guide their observations, it was concluded that the image of the child as a designer is the most conducive to good learning. This image of the child as a designer will prove profitable because of the provision of certain imperative elements in the learning environment, namely continuity, externalisation (providing a physical artefact), investment (personal or group pride in ownership), goal clarity, involvement (preventing the danger of the 'teacher knows what is right') and co-operation and sharing (Di Sessa, 1991).

Kaput (1992) develops a theoretical framework for the interaction between mental processes and physical actions in structuring physical media. The theoretical framework is based on the following premises: The power of our mental ability lies in the interaction between two sources of organisation of experience, namely the structures inherent in our long-term knowledge and our ability to exploit physical means of organising experience. This can broadly be described as the interaction between thought and language. Kaput distinguishes thus between a world of mental operations (almost always hypothetical) and a world of physical operation, frequently observable. Interaction between these two worlds include on the one hand, reading and the evoking of mental phenomena by physical material, and on the other hand, the production of new structures and the projection of mental structures into existing material. Kaput further describes notation systems as systems of rules for identifying or creating characters, for operating on them and for determining relations among them. According to Kaput, most true mathematical activity involves the coordination of, and translation between, different notation systems, e.g. the function $y = x + 1$ may be translated to a coordinate graph system.

Kaput describes a case study which investigates the influence on the translation between two notation systems by computer-based notation systems. Wood Dines blocks and computer-based blocks were used to teach addition. It was found that the computer-based system helps to overcome cognitive overload problems by handling some of the translation activities. Also, computer-based systems keep records to help students remember previous acts.

According to de Villiers (1997), the availability of new dynamic geometry software such as Cabri and the Geometer's Sketchpad, is one of the most exciting developments in geometry since Euclid. It has the potential to encourage pupil-oriented research involving associated skills such as problem posing, explanation, conjecturing, refuting, reformulating and proof. It also has implications for teaching - Cabri/Sketchpad type investigations seem so convincing that students generally do not see the necessity of proof. Other conceptualisations of proof rather than verification are needed. De Villiers found that students agree that inductive verification lacks illuminating power, and that they are willing to give deductive arguments as a means of explaining how their findings are the consequences of other results (De Villiers, 1997).

Laborde (1995) believes that the way geometry is taught often presents geometry as theoretical knowledge, ignoring the relations between drawings and theory. She distinguishes between drawing (a material entity) and 'figure' (the theoretical referent from the drawing). Mathematicians ignore the imperfections of drawings and work on idealised drawings. However, this is not the case for pupils - they often misinterpret drawings as constructions. Laborde believes that the unique character of Cabri makes it a helpful tool for interpreting visual phenomena. The unique character stems from the dragging mode of free elements (primitives). Figures in this learning environment become sets of geometrical properties and relations attached to a drawing that are invariant through the dragging mode (Laborde, 1995:41). Other consequences of using Cabri include the generation of new kinds of problems due to the concretisation of abstract concepts (e.g. transformations become means of construction). Also, the visual feedback appears to play an important role in showing students the inadequacy of their strategies as well as giving evidence of some visual phenomena.

3.3.4 The integration of information technology in mathematics education

Tarragó (1994) describes the integration of Information Technology in education as '... *any situation in which Information Technology becomes a full and habitual part of the objectives of education, of teaching systems and models, of the learning activities of pupils and the teaching activities of teachers, and of any information management systems at the service of the educational community.*'(Tarragó, 1994:16).

Educational systems are professional bureaucracies; that is, organisations whose aim is to ensure the education of pupils by professionals with particular formal qualifications (Tarragó, 1994). The role of management in a professional bureaucracy is small. This is a characteristic unique to learning environments not generally seen in other organisations using information technology (Tarragó, 1994). Changes in educational organisations cannot be forced from the top downwards. The individual professional educators are thus fundamental role players in the innovation process (Jost & Schneberger, 1994; Barron & Orwig, 1995). This should be kept in mind by educational authorities in the integration of information technology in educational environments. Guidance, more flexible structures, and support should be provided to teachers.

Caftori (1994), and Jost and Schneberger mention the disparity between the way educational software (technology) is utilised and the way designers intend it to be used. These and other problems are mentioned by several researchers in their realisation that information technology has had a low impact on education with few implementation successes (Jost & Schneberger, 1994).

Vockell (1990) ascribes the problems experienced in the integration of technology in learning environments to several factors, among which is the application of computers to instruction without a sound theoretical framework.

Ridgway and Passey (in Cannings,1995), point out that information technology integration in the culture of schools takes a minimum of three years. They describe seven developmental stages in technology implementation in schools. The stages are:

- innovation (awareness of possible uses by person(s));
- firelighting (persuading influential people);
- promotion (supported by school administration);
- growth (other teachers use);
- co-ordination (maintaining student outcomes);
- integration (most teachers use technology); and
- extension (new uses are explored).

Tarragó (1994) emphasises the following points on technological innovations in educational organisations:

- The integration of information technology in education should be seen as an ongoing process occurring in a complex system.
- Technological change is a question of culture, an intrinsic part of the evolution of society.
- The integration of information technology and the goals of education (defined in terms of development of pupils' abilities) are compatible.
- Successful integration of information technology into education involves adapting organisational structures.
- The innovation process includes staff training, responsibility and job satisfaction. Alavi (1994) feels that the training of instructors and learners in the use of computers is not enough, but that a departure from the traditional mode of instruction is needed.
- Without teamwork, the innovation brought about by the integration of technology will not be assimilated by educational organisations.

From the discussion in par. 3.3.3.3, it seems as if the integration of IT in mathematics education could support a mathematics learning environment characterised by learner and teacher's interactivity. IT offers motivating, novel and dynamic possibilities for stimulating meaningful interaction with, and acquisition of mathematical concepts.

However, Hillel (1991) highlights some problems regarding the use of IT. Since feedback is quick, a solution strategy of trial-and-error becomes a dominant heuristic regardless of its effectiveness. The availability of the computer thus changes the character of problems. Attiyah (in Hillel) describes mathematics as ' ... *the art of avoiding brute-force calculation by developing concepts and techniques which enable one to travel more lightly.'* (Hillel, 1991:208). Students' actions show their disbelief of this statement. This also focuses the attention on the belief of students that problem solving entails the production of solutions and not the production of knowledge (De Villiers, 1997; Hillel, 1991).

Koblitz (1996) launches a fierce attack on the use of IT in Mathematics Education using arguments like immediate gratification leading to anti-intellectualism, etc. In their reaction to Neal Koblitz's attack, Dubinsky and Noss (1996) see IT as a medium of expression (comparing it to a piano or pen) of which the creative use depends on the user. The role of the teacher is thus of paramount importance in choosing appropriate teaching strategies to change the learning environment, allowing more appropriate uses of technology.

Simonsen and Dick (1997) regard one of the barriers to successful integration of IT in mathematical classrooms as the lack of access to IT. They see the more accessible, less expensive graphing calculators as a possible solution to this problem.

North recommends the teaching of information technology as a cross-curricular theme in preparing students for an information-based society. However, the integration of information technology may challenge the content and structure of a curriculum consisting of bounded subjects (North, 1991). Cross-curricular information technology also challenges formal, authoritarian didactic teaching styles and is more concerned with processes than outcomes. There are thus some areas where conflict can arise between the ideals of information technology and current practice. Bottino and Furinghetti (1996) describe the reluctance of mathematics teachers to accept informatics in the mathematics curriculum. They ascribe this to a view of mathematics as a rigid body of knowledge, not to be contaminated by outside elements.

Research on the adoption of information technology emphasises the user's acceptance and use of the system. Users (teachers and learners) should be involved in the designing process and their needs should be identified and addressed (De Villiers, 1995). In a study done by Simonsen and Dick (1997) on teachers' perceptions of integration of graphing calculators in the mathematics classroom, teachers identified a need for organisational support (relevant curriculum materials and technical assistance), as well as a supportive environment for their professional growth and development. Bottino and Furenghetti (1996) report effective integration of informatics in mathematics teaching, only when it is perceived as providing answers to questions already present in teachers' minds (even subconsciously).

The importance of successful integration of information technology in educational environments is stressed by Tarragó. He feels that an integration failure would amount to the missing of a '*... social opportunity whose effects are felt in the near future by society itself in the form of problems of job competitiveness and assimilating people into the workforce.*' (Tarragó, 1994:18).

3.3.5 Summary

This section gave a brief history of computers in education and more specifically in mathematics education (par. 3.3.1). It presented reasons for the use of IT in education in general and motivated the use of IT in mathematics education by describing some technologies used in mathematics education (par. 3.3.2 and 3.3.3.2). The research on IT support in mathematics education includes the use of ideas on instructional design and mathematical learning theories in the development of mathematics software. The work of Papert, Dubinsky and Kaput are highlighted in this regard. The research on new dynamic software is included and it reports the influence of the software on notions of mathematical proof and the creation of new mathematical problems (par. 3.3.3.3). The consequences of integration of IT into mathematics education and obstacles to it are then discussed. The section concludes by emphasising the need for the successful integration of IT in education to ensure a well-prepared and -equipped future workforce.

3.4 Computer supported co-operative learning (CSCL)

There is general agreement among researchers (including educational researchers) that the growing social and technological complexity of the workplace need the capacity for group work and communication skills (Jost & Schneberger, 1994; Van Weert, 1995, Tarragó, 1994). Not only are teamwork skills necessary for the successful integration of information technology in learning environments, but it seems as if technology integration in schools induces a shift from a competitive to a co-operative structure (Newman, 1992). Some views on the futuristic role of teamwork in education are:

- Schwen, Goodrum and Dorsey (1993) build a conceptual futuristic learning environment where people make sense individually and as a team by sharing, comparing and contrasting views. Learners arrange working documents in a personal and collective workspace, and construct their own meaning and knowledge structures.
- Debenham and Smith (1994) propose a radical vision of public education where individualised instruction through personal computers and networks will be done at home, while schools will focus on group-orientated instruction.
- Van Weert foresees the focus of education to be on the furthering of social understanding and community participation. He states that '*Collaborative learning in multi-disciplinary teams, with integrated use of Information Technology, is expected to have growing importance in education, and in the end to change its organization.*' (Van Weert, 1995:9)

Jost and Schneberger (1994) describe the educational system as a social system where the challenge is to learn from different perspectives and working together.

3.4.1 Definition

In his description of the nature of CSCL Bannon (1995) sees it as a multifaceted concept that involves learning, co-operative learning, support for co-operative learning and specifically computer support for co-operation between human learners. McConnel (1994) uses the term to encompass any form of co-operative learning that occurs over a network of computers.

The definition used in this study will be an extension of the definition for co-operative learning given by Hilke (par. 3.2.2): *An organisational structure based on co-operative learning principles in which a group of students pursue academic goals though collaborative efforts supported by the instructional use of IT.*

In this study, the term co-operative learning will be narrowed down to formal co-operative learning. Co-operative learning methods thus refer to the formal co-operative learning methods discussed in par. 3.2.5. However, in the review of research done on CSCL, studies on informal co-operative learning will be included.

3.4.2 Reasons for the use of information technology in co-operative learning environments.

Educators have been interested in the potentialities of co-operative learning and in computers for enriching learning for over a decade now. Both provide opportunities for the enhancement of cognitive and metacognitive skills, self-esteem and social development (Light & Mevarech, 1992).

Light and Mevarech (1992) describe the interest in CSCL as an *interesting intersection* of, on the one hand, the interest in co-operative learning and, on the other hand, the interest in the use of information technology in instruction.

Initially, software design focused on individualised instruction. The computer seemed a suitable medium for individualised instruction because of its potential to offer different presentation modes and alter instructional decisions on the basis of individual performance (Hooper, 1992). Most computer courseware still emphasises individualised instruction.

In the search for more effective ways of integrating information technology in education, educators find that co-operative learning represents a cost-efficient alternative to individual instruction, and a solution to the social isolation and possible sterile environment associated with computer-based education (Hooper, 1992).

Dockterman (1991) sees the computer as an aid in managing co-operative learning activities:

- As assistant classroom manager, the computer helps the teacher keep multiple teams of students directed and on task.
- As distributor of information, the computer enforces a level of co-operation among group members.
- As record-keeping device, the computer helps increase inter-group interactions during the activity - knowing the situation of each group at any given moment, competitive or co-operative messages can be sent to other teams.
- Telecommunications and computer networks offer additional ways in which technology can promote co-operative learning. Newman (1995) discusses the use of local area networks within the school. He points out that this technology can help in making cross-disciplinary connections, giving students flexible access to these connections and enabling teachers to collaborate. However, he warns that networks

can serve to isolate rather than bring students together. (Networks are traditionally used in schools to deliver individualised instruction.)

Salomon (1992) sees the most successful role of the computer in learning, as opening up new opportunities for co-operative learning and supporting it. He points out that the computer in the classroom shifts learning from recitation to exploration, from individualised learning to co-operative learning, and from separate disciplines to cross-disciplinary curricula. However, he stresses that a computer tool, *in and of itself,* cannot support the socially based process of meaning appropriation. This process is less dependent on technology and far more on other factors (Salomon, 1992:63).

Another reason for the interest in the support that information technology provides to co-operative learning, stems from the concepts, practices and technologies being developed in a related area of research, namely computer supported co-operative work (CSCW) (Davies, 1988). Some researchers are of the opinion that the application of CSCW technology in learning environments will yield significant advantages (Rüdebusch, 1995; Davies, 1988; Midoro & Briano, 1994).

3.4.2.1 Computer Supported Co-operative Work (CSCW)

This term is used to refer to the interests of a number of researchers involved in seeking new ways to assist groups in performing tasks co-operatively (Rüdebusch, 1995). Research in this area focuses on the design, delivery and evaluation of computer supported co-operative work applications.

The computer support ranges from message handling facilities or shared data spaces to describing rules for co-operatively solving complex tasks. The software systems that support user groups are referred to as *groupware.* Rüdebusch gives a more detailed definition of groupware and describes it as a ' ... *software system which support two or* *more, possibly simultaneous, users working on a common task and which provide an interface to a shared environment.'* (Rüdebusch, 1995).

O'Malley (1995) points out that in introducing technology to the social learning environment, it is sometimes done with the underlying assumption that the technology is neutral to the process. This deterministic view fails to realise the new structures and meaning brought about by technology, e.g. it could change the nature of the task (Pozzi, Hoyles & Healy, 1992). This deterministic view also ignores the rich social context of the CSCW environment.

Lyytinen & Ngwenyama (1992) remark that the traditional view of work is based on a mechanistic *theory of work*. This view underpins much of the research on information systems and focuses on an individual's task productivity while underestimating the importance of the social context.

They propose a theoretical framework for CSCW based on Giddens' structuration theory. They define co-operative work and CSCW within the structuration theory framework, describe CSCW applications as social structures and discuss distinguishing characteristics of CSCW applications. This framework will be discussed in par. 5.3.2.

3.4.3 Research on computer supported co-operative learning

Research in this area has stemmed from two paradigms, one focusing on co-operative learning methods and the other on computer assisted learning (Light & Maverich, 1992). Research questions asked about computer supported co-operative learning, thus involve both aspects. Light & Maverich (1992) ask two obvious questions being asked:

- Does peer interaction facilitate computer based learning?

- Do computers have anything special to contribute to fostering effective peer interaction?

As researchers in this area have discovered, these two questions are by far not the only important questions to ask. O'Malley (1995) points out that none of the factors in the computer-supported co-operative learning environment can be considered in isolation, which makes research in this area a complex task. Batson (1992) discusses the problematic nature of the evaluation of the effectiveness of CSCL practices.

This is due to the relative incomparability of CSCL with traditional classroom practices. Attempts to compare CSCL with traditional classroom practices, involve pairing variables of which any could bring about significant changes in teaching and learning. Other variables do not have counterparts in the traditional classroom.

Bannon (1995) is of the opinion that a theoretical framework that includes features influencing the CSCL process can assist comparing and contrasting different studies on CSCL. This will bring about better understanding of the process which will influence the success of future implementation. (Such a framework will be discussed in par. 5.3.3.)

As was discussed before (par. 3.2.4), the psychological research on co-operative learning is dominated by theories based on Vygotsky's and Piaget's ideas. Pozzi, Hoyles and Healy (1992) argue that the introduction of the computer to the co-operative learning environment, creates a new context in which learning is mediated through interaction with the computer as well as with peers.

Despite new variables being added to the process by the introduction of technology, research on CSCL is still based on Piaget's and Vygotsky's theories with preference being given to the sociocultural theory based on Vygotsky's ideas. Mandl and Renkl (1992) find research results more differentiated than these theories would predict. They ask for more local theories of co-operative learning that take the following aspects into account: the knowledge domain, the kind of learning objective, the psychologically and educationally relevant dimensions of the computer software, and the mechanisms of compensation and substitution in the social learning process.

3.4.4 Information technologies used in co-operative learning environments

A number of research studies focus on specific technologies and how they can mediate co-operation. Some of the technologies are described below:

Computer-assisted instruction (CAI)

CAI has been traditionally designed for individualised instruction. However, because of logistical and financial constraints, students often work in small groups at the computer. Seemingly students have much to gain from co-operating at the computer using CAI: Shlechter (1990) reports that learners who complete CAI lessons in a co-operative group perform as well as students who work alone. Other studies show that the performance is often better (Hooper, 1992). Shlechter also finds that co-operative practices can occur in minimally structured groups without group rewards or specially designed courseware.

Hooper (1992) points out that the nature of the instructional strategies for CAI may be unfit for group learning. Designers should reconsider issues including learner control, feedback timing, nature of feedback and instructional scripting.

Sloan and Koohang (1991) encourage the use of CAI through local area networks but emphasise that the CAI programs should be interactive, encouraging inquiry and discovery, and including game and computer simulations, peer teaching and problem-solving.

Roschelle & Teasley (1995) use the EM (the Envisioning Machine), a graphical simulation of concepts of velocity and acceleration that can be manipulated, to teach physics concepts to groups of students. They find that the computer assists in disambiguating language, resolving impasses, inviting and constraining students' interpretations. They describe the CSCL environment as a rich environment for studying learning.

Computer-mediated communication (CMC)

This is a term used to describe computer-based interactive message passing systems. Davies (1988) sees the main features of CSC systems as time independence (messages are stored centrally until accessed), distance independence, and centrally structured communication. The CMC systems can be locally based (Local Area Networks) or nationally/internationally based (Wide Area Networks) using computers networked together.

Hiltz et al. (1994) describe different modes in which CMC can be utilised to support education:

- As an adjunct to a regular face-to-face course in order to improve communication.
- As a mechanism for providing communication in a distance education.
- As a total means of delivery, without other communication modes.

Two examples of CMC systems are:

Electronic mail

The immediacy of the medium, the ability to go beyond classroom walls and the computerised trace of activities make it a suitable medium to support co-operative practices. However, social activities must be created to ensure co-operative practices. (Bannon, 1995). Riel (1992) describes the learning circle design where a learning circle refers to a small number of classrooms that interact electronically. Teachers involved work closely with students to plan their activities and become thus learners and problem solvers. It seems then that electronic networks provide the possibility for educational programme that bring students and teachers into working relationships with each other. Teachers working on educational networks rank their own learning (not that of their students) as the most important benefit.

Computer conferencing

Computer conferencing supports many-to-many communication. It also includes features designed to help in the organisation, structuring and retrieval of messages. Applications vary from virtual seminars where a small group exchanges ideas over a period of months, to multi-media distance education programmes (e.g. programmes run by Open University and EuroPACE) (Kaye, 1992). The great potential of computer conferencing for co-operative learning lies in the cumulative record of message contribution and the tools available for retrieving and organising messages, since thoughtful analysis and review of earlier contributions can take place (Kaye, 1992).

Group Decision Support Systems (GDSS)

The term refers to the integrated combination of specialised hardware, software and procedures to support group interaction and activities (Zigurs & Kozar, 1994).
GDSS supports groups through different mechanisms, such as

- process support (electronic messaging capabilities),
- process structure (techniques of rules directing the pattern, timing or content of group interactions),
- task structure (analytical techniques and models), and
- task support (information and computational infrastructure) (Alavi, 1994).

Alavi (1994) found in a study involving MBA students that those who used GDSS to support co-operative learning activities, perceived higher levels of skill development, learning and interest in learning, compared to students who did not use GDSS.

It also appears that the information structuring and information sharing feature of GDSS contribute to the co-operative learning process by enabling members to link and collect opinions and perspectives.

Hypermedia and multimedia

The non-linear structure of hypermedia systems makes hypertext-based collections of material powerful resources for collaborative discovery and learning. Midoro & Briano (1994) report that contrary to studies claiming that only gifted students gain in using hypermedia systems, most students found the use of hypermedia systems motivating and fun.

Hamm & Adams (1992) consider that multimedia systems in co-operative learning environments can be used:

- to illustrate and support lecture material,
- in courseware as simulations and problem solving techniques,
- to combine computer graphics and video through authoring systems,
- to provide material to students who miss a class to review complex concepts in the library, and
- to enable students to develop video projects.

Other technological developments used in co-operative learning environments include shared screen facilities and interactive video. Dillenbourg & Self (1995) describe the design of a system where collaboration takes place between human learner and artificial learner.

3.4.5 Research on computer-supported co-operative mathematics learning (CSCML)

Few studies done on CSCML involve formal co-operative learning methods. Studies vary from the effect of peer collaboration to the design of collaborative technologies. Hoyles, Healy and Pozzi (1994) identify important questions addressed by research on CSCML including a) the importance of structuring the task environment, b) the importance of socio-cognitive conflict on negotiation, c) the role of computers in stimulating formal mathematical expressions, d) the influence of prior individual experiences and interpersonal variables (e.g. gender, status, etc.). For the purpose of this study, the research on CSCML will be divided into studies investigating the effects of different variables and studies investigating IT support to co-operative learning.

Effects of variables/group processes

Hooper and Hannafin (1988) studied the effect that homogeneous and heterogeneous grouping (with respect to ability) has on the learning of complex concepts. This took place through computer-based tutorials on arithmetic operations. They found that the grouping strategies had little influence on high ability students (definitely not detrimental), but low ability students in heterogeneous groups performed better than those in homogeneous groups. This was on factual and application levels. There was however no significant gain in complex learning skills. They thought that it was unlikely to occur because of limited exposure to group work and that more long term research was needed.

Emihovich & Miller (1988) believe that successful learning of Logo concepts involves a careful structuring of the teaching context. Basing their ideas on Vygotsky's perspective, they hypothesised that the use of mediating teaching strategies, instead of discovery learning, will allow the groups of pupils to achieve more. They found that with time, teachers' directives decreased as peer collaboration increased and that children's talk became more task oriented.

Hoyles, Healy and Pozzi (1994) discuss in several articles the outcomes and findings of a research project that was part of a greater project called 'Group work with Computers'. They used a multi-site case study design in six schools which involved eight heterogeneous groups (with respect to ability and gender) of six pupils each (aged 9 - 12), undertaking three mathematical tasks, two using Logo and one a database. This encompassing study addresses several research questions. Some of them are:

- How effectively can a group function without a teacher?
- To what extent can groups of pupils take responsibility for task organisation and the articulation of mathematical ideas? (Hoyles, Healy, Pozzi, 1992).
- How do background and process factors of the group influence group work?
- What is the influence of the task and software?
- Is it possible to identify criteria for group management to achieve successful group work?
- How can tasks be designed that facilitate successful group work? (Hoyles, Healy & Pozzi, 1994).

In the analysis of the collected data, the researchers identified several variables to focus on. The background variables included previous experiences of software, co-operative work, prior knowledge, established inter-personal relationships and friendly relationships. Process variables included organisational style and patterns of interaction. Outcome variables included effectiveness (individual progress) and productivity (was the group goal reached?).

Their findings reveal yet again the complexity of the CSCML environment. These includes the influence of the form of individual involvement on the nature of the discussion taking place, on role-taking (pupils adopting a specific role for the remainder of the group work), and eventually on learning. This involvement includes not only learner-learner but also learner-computer interaction (typing, encoding, etc.) (Pozzi, Hoyles & Healy, 1992). They also found consistent patterns of working across tasks and software (these findings differ from other research findings (Hoyles, Healy & Sutherland, 1991)). The same pupil took on the role of pupil-teacher across the three tasks. Effective groups were characterised by the *emergence of a synergy between structured pupil interdependence and pupil autonomy* - a sharing of responsibility for successful task completion but a sharing in ways attainable by every pupil in the group (Hoyles, Healy & Pozzi, 1992:254). The task structure allowed students to structure a system of interdependence, and the software allowed pupils to construct and develop their own ideas. Furthermore, a good group outcome seems to be associated with the emergence of a pupil-teacher who is accepted by the group members. The pupil-teacher needs to manage group activities and to have access to relevant mathematics knowledge and software skills. A minimum amount of mutual respect and willingness to co-operate is important for co-operative learning to take place. In contrast to other research, this study did not indicate significant influences of gender differences, previous co-operative experiences and software experiences. The only significant influence was that of age difference. Older children (over 10 years old) managed the complexity of the task, the computer and human resources better than the younger children.

Hoyles, Healy & Pozzi (1994) summarise unsuccessful and successful group settings in Figure 3.3 and Figure 3.4 respectively.

In another research study titled 'The role of peer group discussion in Mathematics Environments', processes whereby pairs of pupils came to make mathematical generalisations were analysed (Hoyles, Healy & Sutherland, 1991). Interaction in computer and non-computer contexts were contrasted. Pairs of pupils worked in three environments, Logo, spreadsheet, and paper and pencil. All the tasks involved the construction and formalisation of mathematics generalisations. Hoyles et al. (1991) found that the Logo environment and Logo language itself assisted in the generalising process in contrast to the other two environments. Also, formalisation took on an important role in computer environments in contrast to the paper-pencil environments.

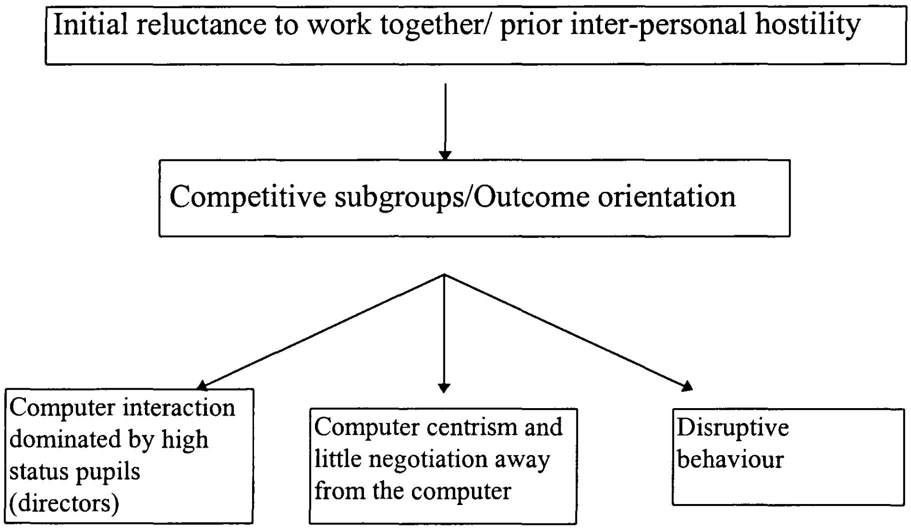


Figure 3.3

Unsuccessful group settings

(Source: Hoyles, Healy & Pozzi, 1994:213)

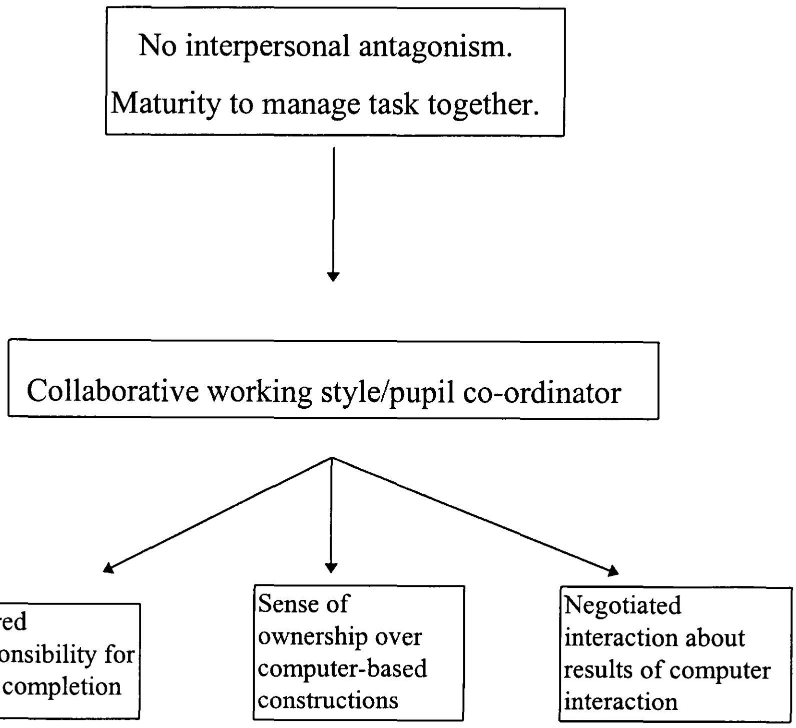


Figure 3.4

Successful group settings

(Source: Hoyles, Healy & Pozzi, 1994:213)

This could be ascribed to the necessity of providing formal representations in the computer environment. They also found that in both computer environments, the same person who adopted the exclusive role of encoder, also dominated the keyboard. In the paper-pencil environment, more so than in the computer environment, pupils seem to be more likely to share in the generalisation process.

IT support to co-operative learning

Most of the software and technologies available for use in mathematics education discussed in par. 3.3.3.2, are used in groups. However, the research studies reporting on these uses, focus on the software and its effect on learning rather than group processes. Kafai and Harel (1991) conceptualise collaboration somewhat differently from the usual way within their framework of ISPD (discussed in par. 3.3.3.3). Instead of groups working towards a group goal, all students have a common goal (to construct screens to explain difficult concepts of fractions to a lower grade peer). They work individually but are allowed to form partnerships on all or selected aspects of the task.

One way in which CAS is used in group work, is in the calculus laboratories. This concept resulted from calculus reform attempts where questions were asked about the active involvement of students in conceptual learning. This is a laboratory where observation, identification, experimental investigation, analysis and explanation take place. CAS assists in the investigation process. Students work with partners and have to hand in a report. There are quite a few manuals available on the use of Maple, Derive and Mathematica in the laboratory (Leinbach, 1992).

Clements and Nastassi (1991) conducted three separate studies in which students were observed working in pairs in Logo and Computer Assisted Instruction (CAI) environments. Behaviours reflecting conflict and its resolution were identified and observed. They found that peer interaction that is focused on learning and problem solving, is more likely to take place in Logo environments than in more traditional environments (e.g. CAI drills). Logo facilitated engagement in conflict resolution strategies and interpersonal co-ordination of divergent ideas. CAI on the other hand, structures the problem, solution process and interaction. Thus, the responses of students are not necessarily built on those of others.

Roschelle and Teasly (1995) emphasise the inherent fragility of the collaborative learning process - for learning to take place, individuals must make a conscious effort to produce shared knowledge. They believe that the most important resource for co-operative learning is talk. Collaborators use the overall turn-taking structure of talk, narration, questions, socially-distributed productions and repairs to enhance mutual understanding (Roschelle & Teasly, 1995:94). Roschelle et al. see computer support to co-operative learning as a resource that mediates collaboration. By conducting studies on teaching physics through the use of the Envisioning Machine, they identified several important support functions provided by the computer to co-operation:

- The computer is a means for disambiguating language by providing precise technical vocabulary,
- The computer mediates the resolving of impasses - students resolve their differences by trying out ideas on the computer to see what works,
- The computer invites and constrains students' interpretations - e.g. the display of the Envisioning Machine was designed to suggest appropriate interpretation but also constrains interpretation because it behaves according to Newtonian physics.

Other collaborative technologies are specifically designed to assist co-operative learning. For example, The Color Matcher, is a game designed to foster the ability to manipulate numerical variables in co-operation with other students so as to achieve a definite goal (Denning & Smith, 1997). The software and hardware are designed to encourage individual accountability and positive interdependence necessary to ensure academic gains.

Croisy, Clement and Barme (1994) built a multimedia environment for mathematics education. One of the four prototypes called RTMConf provides students with an integrated learning environment. Students have to solve a mathematical problem using different tools of the RTMConf which include public window, telepointer, opinion collector, time manager, audio tool and welcomer tool. They communicate through PC stations connected to a server which includes an audio channel. There are four sites (with four students at each site) and one teacher site.

Jehng, Shih, Liang and Chan (1994) developed a computerised learning environment called Turtlegraph to support co-operative geometry problem solving. Since knowledge sharing plays an important role in problem solving, the designers decided that the co-operative learning environment must be designed to enable group members to communicate and share their ideas effectively. Students sit individually at PCs and can communicate with peers or the computer via the software learning environment. It consists of different instructional areas: control panel, dialogue recorder, program editor, listener and Turtle window. The control panel has two buttons: the communication button and system button. When in need of help from peers, the communication button is pressed which activates all the communication buttons and collaboration can then take place.

It is clear that there are various features that influence the CSCML process and that these factors can be combined in numerous ways in research studies. Bannon (1995) lists a few features that play a role:

- the nature of the task,
- the nature of collaborators (peers, teacher-student, student-computer),
- the number of collaborators,
- the previous shared experiences between collaborators,
- the motivation for co-operation (intrinsic, money, etc.),
- the physical environment,

- the conditions of collaboration (computer-mediated, etc.), and
- the time-period of co-operation.

3.4.6 Obstacles to CSCL

Dockterman (1991) lists a few obstacles to co-operative learning and how it can be overcome by introducing the computer to the process (par. 3.2.7). Most of the obstacles to CSCL have to do with the incompatibility of the teaching styles called for by CSCL and the styles used in current teaching (Newman, 1995):

- Time frame: The impact of the CSCL process on students' achievement is cumulative and realised over longer periods of time (Alavi, 1994; Hooper & Hannafin, 1988).
- Curriculum: Effective CSCL sometimes implies decompartmentalisation of the curriculum of the school (Newman, 1995).
- Location: CSCL implies the distribution of workstations among classrooms rather than centralising them in the traditional computer laboratory.
- Evaluation: Usual evaluation procedures fail to capture the complexity of the co-operative learning process.

Bannon (1995) expresses his concern over the gap between available technologies and what can be expected to work in an ordinary educational setting. He found that in the course of his research study, much of the time was spent in overcoming technical problems regarding response time and interface issues.

3.4.7 The design of the CSCL and CSCML environment

Salomon (1992) emphasises the complexity of the co-operative learning process and points out that in his experience, success of co-operative teams in terms of learning outcomes and true collaboration are rare. A reason for this is the lack of genuine interdependence. He argues that the ' ... *whole learning environment, not just the* *computer program or tool, be designed as a well orchestrated whole.'* (Salomon, 1992:64).

The same principles as for the design of the co-operative learning environment, are applicable to the design of the CSCL and CSCML environments. The essential components discussed in par. 3.2.3 should be incorporated in the design.

McConnel (1994) raises several questions that should be asked in the process of designing the CSCL environment:

- How can the learners be encouraged to be active, to take control of their own learning, to contribute to the learning of the group, and to participate in the design, assessment and evaluation?
- What kind of learning is being designed for?
- What kind of knowledge are we trying to encourage and explore?
- What kind of educational philosophy are we involved in?

From the discussions on research in co-operative mathematics learning and computer-supported co-operative mathematics learning, it is clear that co-operative learning enhances the learning of certain mathematical knowledge. For example, problem solving activities seem particularly suitable for group exploration. Also, co-operative learning may not be significantly better than traditional methods for acquiring skills and facts not involving complex skills. Hoyles, Healy and Pozzi (1992) identify the difference between co-operative learning in mathematics and other subjects as the importance in the task solution process of clarifying and articulating what the problem space is and developing a language to describe it. The designer of the CSCML environment should keep these aspects in mind in the design process.

McConnell (1994) mentions six important aspects of CSCL design:

- Openness in the educational process - learners should feel free to make decisions about their learning and to exercise their choices.

- Self-determined learning - learners should become aware of how they learn through interaction with other learners.
- A real purpose in the co-operative activity - this is often best achieved through a problem-centred approach.
- A supportive learning environment - through interaction, learners need to encourage and facilitate each others' efforts.
- Collaborative assessment of learning - learners should have an important part to play in assessing each others, and their own, work.
- Evaluation of the ongoing learning process - this must be done by both the learners and the tutor and must be characterised by a real willingness to change.

CSCL is a new and untested area of research and there are no definite answers to questions regarding design (McConnel, 1994). O'Malley (1995) affirms this and identifies the need for an agreed framework for comparing and contrasting research on CSCL which might provide guidelines or principles for design.

De Villiers (1995) developed a theoretical framework for CSCL in an attempt to contribute to the understanding of the CSCL environment as a whole. She developed a generic model for CSCL and refined this using results from case studies and the structuration and adaptive structuration theories of Giddens' and DeSanctis and Poole respectively. This theoretical framework will be discussed in greater length in par. 5.3.3.

3.4.8 Summary

This section introduces CSCML as an organisational structure based on CL principles in which groups of students pursue mathematical academic goals through collaborative efforts supported by the instructional use of IT. Since some of the research on CSCML and CSCL overlaps, this section starts by defining CSCL and discussing some reasons for the use of IT in CL environments (par. 3.4.2). Not only does CL present a cost-efficient alternative to individual computer-based instruction, but the computer also seems to sustain social interaction. The application of findings from research on CSCW is believed to have important advantages for CSCL (par. 3.4.2.1). Technologies used in CSCW are also used in CSCL (e.g. Group Decision Support Systems, Computer Mediated Communication). Other technologies include CAI, hypermedia and multimedia (par. 3.4.4).

Some relevant research studies on CSCML are discussed. They range from studies on the effect of heterogeneous/homogeneous grouping on the gaining of complex learning skills, to studies contrasting mathematical interaction in computer and non-computer contexts, and studies contrasting mathematics learning supported by different software. Other studies reflect on the development of appropriate software to assist the co-operative mathematics learning process (par. 3.4.5).

This section also discusses implementation and integration problems with respect to CSCML (par. 3.4.6 and 3.4.7). Quite a few obstacles and problems are noted, most of which stem from the difference between the teaching styles called for by CSCL and the styles used in traditional teaching. Other problems are caused by ineffective design (mostly a lack of genuine interdependence) and uninformed teachers. The teacher is seen as playing a crucial role in the design of the learning environment. Salomon (1992:64) warns teachers that this learning environment should be designed as a ' *... well orchestrated whole.* '.

3.5 Conclusion

A number of researchers identified variables that play a role in the CSCML learning environment (Bannon, 1995; Good et al., 1992). In an encompassing study done by Hoyles et al. (1994), variables were grouped into background-, process- and outcome-variables (par. 3.4.5). Their work provides valuable remarks on the characteristics of successful groups, the role of pupil-teachers and task structure. Although researchers in general realise the interdependence of the different features or variables, no attempt has been made yet to describe this interdependence. Variables are at most paired or grouped and then investigated.

O'Malley (1995) identifies a need for an agreed framework for comparing and contrasting research on CSCL which might provide guidelines or principles for design. Good et al. (1992) identify a similar need for the mathematics co-operative learning environment. No such framework exists for the CML or CSCML environments. This study addresses this need by developing a theoretical framework for CSCML which will be presented in Chapter 5.

The next chapter reports two case studies, of which the design and execution were guided by the model for the learning of mathematics developed in chapter 2 and by the research findings presented in this chapter. The findings of the case studies are then used to enhance the model in Chapter 2 to a theoretical framework for CSCML. The shortcomings of this model are pointed out and addressed in Chapter 5 by the development of a final model for CSCML.

Chapter 4

Case studies

4.1 Introduction

This chapter describes two case studies that were designed and conducted, guided by the model for mathematics learning developed in Chapter 2 and the research findings presented in Chapter 3. The first case study was directed at mathematics teachers and formed part of a greater project which is described in par. 4.2.1. The other case study involved undergraduate Linear Algebra students.

Data were collected through questionnaires, observation lists, tests and tape and video recordings. The video tapes were subsequently transcribed and are available as two documents (Transcripts A and B, available from the author). The theoretical framework for mathematics learning discussed in par. 2.5 is generic in the sense that it does not provide for computer support to mathematics learning or the implementation of co-operative learning techniques. The findings from these two case studies and the results from other similar experiments in the literature (discussed in Chapter 3) were used to refine the model in par. 2.5 to provide for the computer-supported co-operative mathematics learning environment (par. 4.3.9). This enhanced model is criticised and improved in Chapter 5 using Giddens' structuration theory.

4.2 Case study: The development of computer supported co-operative mathematics learning at community learning centres

4.2.1 Background

The South African government has decided to implement a new education policy generally referred to as Curriculum 2005. This is a learner centred approach based on the principles of outcomes based education (OBE). It requires lecturers and students to focus their attention and efforts on the desired end results of education. Eight learning areas are defined and broken down into specific outcomes for classroom practice. Seven critical outcomes underpin changes in the new curriculum and feed into each of the eight learning areas. The seven critical outcomes include the ability to work effectively with others in groups as well as the ability to use science and technology effectively.

The Departments of Didactics, Informatics, Electrical and Electronic Engineering of the University of Pretoria have initiated a project that hopes to address the problems in Science and Mathematics Education in South Africa. Based on the principles of OBE, it plans to *facilitate computer-supported co-operative learning at community learning centres, through the development of a model to support teacher education, using interactive television and educational technology.*

The focus of this case study is on a part of the overall project, namely the development and deployment of Computer Supported Co-operative Mathematics Learning (CSCML) at community learning centres, with the emphasis on teachers. The pilot learning community centre will be established at SEIDET's community learning centre (Siyabuswa Education Improvement and Development Trust). SEIDET is a non-profit, non-governmental based education improvement institution and a registered development trust that was established at Siyabuswa (100 km north-east from Pretoria) in 1992.

The main objective of this project is the establishment of a computer-supported co-operative learning (CSCL) environment at the SEIDET community centre at Siyabuswa to complement the current teaching of subject areas such as Science, Mathematics, English and Biology (other subject areas are not excluded).

Ultimately, the goal is to establish several such community learning centres in rural South Africa where CSCL could form part of the available infrastructure. In order to achieve this, the (pilot) project at Siyabuswa will be utilised as a feasibility study to develop implementation plans for further rural learning centres.

The overall project is expected to last until December 2000. A concern of the project is the sustainability of the community learning centres that were created. The focus is on existing community centres and the proper training of persons involved in the operation of the centres.

With the first phases of the project completed the focus is now on the establishment of an extended and fully equipped computer laboratory at SEIDET centre, the continuation of in-service education of teachers, and the establishment of computer-supported co-operative distance learning environments through interactive television.

The completed part of the project involved laying the foundation for CSCML environments by introducing and practising the fundamental concepts related to co-operative learning (CL) and computer support to mathematics learning through contact tuition. The work was conducted as a case study and is described below.

4.2.2 The learners

The research group consisted of six male mathematics teachers, all teaching and living in the rural areas of South Africa near Siyabuswa in Mpumalanga. The case study was conducted in co-operation with SEIDET at their centre in Siyabuswa. The teachers participated voluntary in the case study on Saturday afternoons from 14:00 to 17:00. They were divided into three permanent groups of two each. However, this was changed in sessions 5, 6 and 7 when one of the teachers was absent.

4.2.3 Outline of the procedure

The work was distributed over nine sessions in the following way:

Session 1: An introductory and group building session where expectations of the course were discussed.

Session 2: A lecture session on the definition, elements and methods of CL as well as a hands-on experience of the application of CL in the mathematics classroom.

Sessions 3 and 4: Computer literacy teaching through CL methods using MSWord.

Session 5: Individual testing of teachers on MSWord and introduction to mathematical software.

Sessions 6 and 7: Teachers doing CSCML lessons.

Sessions 8 and 9: Teachers developed and presented their own CSCML lessons.

4.2.4 The content and materials used

The mathematical content of the lessons in sessions 2, 6 and 7 was built around quadrilaterals. The quadrilaterals included the six well-known ones (trapezium, parallelogram, rectangle, square, rhombus and the kite) as well as newly defined cyclic quadrilaterals. The lessons (worksheets) were developed using the ideas of De Villiers (1995) and curriculum material developed by Key Curriculum Press, accompanying the software. The worksheets are given in Appendix A. The mathematical software used in sessions 5 to 9 was a geometry package called *The Geometer's Sketchpad.*

4.2.5 The objectives

Two main objectives were defined:

At the end of the programme, teachers should be able to

1. show an understanding of the principles and techniques of co-operative learning and of the use of the particular software by being able to apply it in the development and conducting of a CSCML lesson; and
2. show an understanding of the definitions (and properties) of the trapezium, rectangle, square, parallelogram, kite, rhombus, cyclic quadrilateral, cyclic kite and cyclic trapezium, by being able to relate them in the form of a family tree.

4.2.6 The procedure

Session 1: Group building exercise

The researchers used two lecturers from the Department of Didactics to do a group building exercise with the teachers and to introduce them to the importance of the concept of co-operative learning in the new education policy that will be introduced into the South African educational system from 1998. This opportunity was also used to discuss the expectations of the course. It was clear that most teachers expected an emphasis and focus on the role of the computer in mathematics education. Computer literacy and how to design computer supported mathematics lessons were also mentioned as expectations.

Session 2: Co-operative learning

The second session was spent giving a lecture to the teachers on CL covering the definition of CL, elements of CL and different co-operative learning methods. The teachers were then asked to develop a vague CL lesson plan to revise the properties of the six familiar quadrilaterals. This was discussed and improved on in their groups. The teachers then had a chance to practice the concepts of CL by doing a CL mathematics lesson. The Jigsaw method was used and two expert groups were formed who investigated the properties of different quadrilaterals through tessellation with triangles. The experts also had to discuss the correctness or otherwise of an example of a family tree relating only a few of the quadrilaterals. After that the experts joined their groups and had to complete a table of properties of the six quadrilaterals: trapezium, parallelogram, rectangle, square, rhombus and the kite. The teachers then had to identify the different elements of CL in the lesson and give their input on how to improve it.

Session 3: Computer literacy

On the third Saturday a short introduction to the computer was given to the teachers, telling them about the components of the computer. They were then divided into groups of two each and had to work through two computer-based training (CBT) lessons developed by the University of South Africa. The lessons dealt with the use of the mouse and the keyboard, as well as general word processing.

Session 4: Word processing

An adaptation of the Jigsaw method was chosen as the co-operative learning method to teach the teachers something about word processing using MSWord.

Session 5: Testing of word processing skills and introduction to mathematics software

An assignment was given to each teacher to complete individually on MSWord. The purpose of this exercise was to determine whether they had really learnt something during the co-operative learning sessions. All six teachers received full marks for this assignment and had showed that they accomplished the objectives that the researchers had set out at the beginning of the case study.

The teachers were then introduced to *The Geometer's Sketchpad* by working through tutorials. The software is best described as an electronic compass, pencil and straight-edge with additional commands that allow translation, rotation and dilation. Measuring and graphing are possible which makes it suitable to do analytical geometry. Also, scripts (recordings) can be made of complicated sketches which extend the capabilities of the software. But most of all, by clicking and dragging, constructed relationships remain valid and geometry becomes dynamic (The Geometer's Sketchpad, Guide and Reference Manual, Key Curriculum Press, 1995).

Sessions 6 and 7: Teachers doing a CSCML lesson

Again, the Jigsaw method was used as the CL method. Each member of the group is given unique information on a subject which is then discussed with their counterparts in the other groups (Johnson & Johnson, 1991). The following describes the lesson according to the steps necessary for the successful implementation of the Jigsaw method.

1. *Divide into groups and do group building exercises.* As described in session 1, one afternoon was spent on this and the researchers used the same groups for the word processing part of the case study.
2. *Explain to the student the idea of group work.* Session 2 covered the lecture on co-operative learning that was given to the teachers before the process of computer literacy was started. At this point an explanation was given to the teachers about the Jigsaw method.
3. *Explain the goal and task.* The teachers received an envelope with learning materials and worksheets. The first page explained the overall goal, viz. Draw a family tree to show the relation between the following quadrilaterals: trapezium, rectangle, square, parallelogram, kite, rhombus, cyclic quadrilateral, cyclic kite and cyclic trapezium. It was explained to the teachers that in order to solve this problem, a deep knowledge of the properties and definitions of the different quadrilaterals was crucial. To acquire this knowledge, several activities were included in the lesson (see Appendix A, I):
    - Each member of the group had to join an expert group to revise the properties of

the first six quadrilaterals mentioned in the given problem. One expert group investigated the properties of the rectangle, kite and parallelogram, and the other expert group the properties of the rectangle, rhombus and square. These tasks were designed around the mathematics software (see Appendix A, II).

- After returning to their original groups, they had to read and discuss an extract of a document, 'A Classroom Episode' (De Villiers, 1996), which captures the discussion in a classroom where students are working on a similar task.
- Then they had to work on a worksheet (again involving the mathematics software) on cyclic quadrilaterals to investigate the properties of the different cyclic quadrilaterals, after which they were ready to draw the family tree (see Appendix A, III).

4. *Design special curriculum materials so that each member of the group has a unique source that can be used independently of other sources.* The two experts in their original groups had material on different quadrilaterals and could share their knowledge on the properties of those specific quadrilaterals in solving the problem.
5. *Use instructional material to promote interdependence among students.* Each group had only one computer to work on and only one copy of the worksheets. In the expert groups each member received a copy of the worksheet to take back to his home group.
6. *Assess the students' work.* The completed worksheets on cyclic quadrilaterals and the family tree were used to evaluate the teachers.
7. *Final assessment:* This was done by means of an individual assignment (see Appendix A, IV).
8. *Assess group functioning.* This was done through ongoing observation while the groups were busy with the course, as well as through the completion of a questionnaire by each individual at the end of the course.

The five basic elements of co-operative learning were implemented in the following ways:

a. Positive goal interdependence, which occurs when learners undertake a group task with a feeling of mutuality. This was achieved by having the group produce a single solution to the given problem. They also had to complete a single worksheet on cyclic quadrilaterals.

b. Face-to-face promotive interaction, which occurs when a verbal interchange takes place where learners explain how they obtained an answer or how a problem may be solved. The experts in the group had to explain to each other what they know. Also, by doing the given task as a group, group members gave their input and suggestions.

c. Individual accountability, which means taking responsibility for learning material. An individual test was given at the end of the course. This test focused on the one hand, on the ability to formally prove findings that were discovered and, on the other, the use of the software to investigate certain concepts.

d. Social skills, which involve knowing how to communicate effectively and how to develop respect and trust within a group. By this time the group members knew each other and certain group habits were formed. One group in particular had the problem of a dominating member which hampered effective communication.

e. Group processing to reflect on how well the group is working and to analyse their effectiveness and how it may be improved. Although the behaviour of the groups was monitored, the researchers only discovered a malfunctioning group when it started to work on difficult concepts. Up to that point, the other group members accepted the leadership of the 'natural' leader. When it came to the difficult concepts, it was clear that some of the other members would have been better leaders in deciding on strategies. By this time, however, group habits were set and the intervention of the researchers had little effect. This led to frustration and a less satisfactory solution to the given mathematics problem, which was reflected in the questionnaire.

Sessions 8 and 9: Teachers developed and presented their own CSCML lessons

The teachers were given copies of examples of lessons using the software from 'Exploring Geometry with The Geometer's Sketchpad' - D. Bennet. They now had to design a CSCML lesson in their groups using these examples to be presented (by the group) to the other groups at the next session.

In designing the lesson they had to keep in mind content, materials needed, learning objectives, procedure, evaluation, how to use the software, which CL method to use, and how to incorporate the basic elements of CL. After presenting the lessons the members of each group had to evaluate themselves and were also evaluated by their peers. This was done by means of evaluation lists (see Appendix A, V).

4.2.7 Results of the questionnaires

Two questionnaires were given to the teachers. The first questionnaire focused on co-operative learning, the use of computer-based training and MSWord. The second questionnaire focused on CSCML.

4.2.7.1 Questionnaire on CL, CBT and computer literacy

A total of 6 questionnaires completed by the teachers were evaluated. The following scale was used in the questionnaire:

4	Always / Definitely
3	Frequently / Nearly almost
2	Occasionally / Seldom
1	Never

The results were as follows (because of the small sample, the statistical analysis includes only the averages):

PART A	QUESTIONS ON THE STUDENT'S BEHAVIOUR IN A GROUP	Average
A1	I offer facts and relevant information in order to promote group discussion.	3.5
A2	I give my opinions and ideas and provide suggestions in order to promote group discussion.	3.16
A3	I express my willingness to co-operate with other group members.	4
A4	I expect other group members to be co-operative.	3.3
A5	I give support to group members who are struggling to express themselves intellectually.	3.8
A6	I keep my thoughts, feelings and reactions to myself during group discussions.	1.7
A7	I evaluate the contributions of other group members in terms of whether their contributions are useful to me and whether they are right or wrong.	2.8
A8	I take risks in expressing new ideas and my current feelings during group discussion.	2.5
A9	I communicate to other group members that I am aware of, and appreciate, their abilities, talents, skills and resources.	3.7
A10	I share any sources of information or other sources I have with the group members in order to promote the success of the individual members as well as the group as a whole.	4
A11	I offer help to anyone in the group in order to bring up the performance of everyone.	3.8

PART B	QUESTIONS ON THE LEVEL OF ACCEPTANCE OF THE STUDENT AS A GROUP MEMBER	Average
B1	My fellow group members are completely honest with me.	3.5
B2	My fellow group members understand what I am trying to communicate.	3.5
B3	My fellow group members accept me just the way I am.	3.7
B4	My fellow group members make it easy for me to be myself.	3.7
B5	My fellow group members include me in what they are doing.	4
B6	My fellow group members value me as a person, apart from my skills or status.	4

PART C	QUESTIONS ON GROUP COHESION	Average
C1	I try to make sure that everyone enjoys being a member of a group.	3.8
C2	I discuss my ideas, feelings and reactions to what is currently taking place within the group.	3.8
C3	I express acceptance and support when other members disclose their ideas, feelings and reactions to what is currently taking place in the group.	3.8
C4	I try to make all members feel valued and appreciated.	3.5
C5	I try to include other members in group activities.	3.8
C6	I take risks in expressing new ideas and my current feelings.	2.8
C7	I express liking, affection, concern for other members.	3.3
C8	I encourage group norms that support individuality and personal expression.	3.7

PART D	QUESTIONS ON GROUP WORK IN GENERAL	Average
D1	I have learnt more in the group than I would have learnt on my own.	3.8
D2	I enjoyed working in a group.	4
D3	The group motivated me to do my share of the work.	3.5
D4	The group work helped me understand the study material better.	3.7
D5	I learned to co-operate with other students.	3.7
D6	The group work caused me to be dependable and do my assignment.	3.0
D7	It was fun working in a group.	3.5
D8	In the group I got the benefit of everyone's ideas.	3.3
D9	I got help from group members with problems.	3.5
D10	The work got done faster and more work was done.	3.5
D11	The group work gave me an opportunity to talk and discuss the study material.	3.7
D12	The group work made the study material more interesting.	3.8

PART E: QUESTIONS ON SELF-DEVELOPMENT

E.1 I have worked in groups before.

YES =4	NO = 2

E.2 I would have liked to choose my own group members.

YES = 2	NO = 4

E.3 Why do you think group work was introduced in this course?

Some of the comments were:

It is a trend in South African education to introduce co-operative learning.
To share ideas and information with other people.
To find out how other people think and learn from them.
To show that it is easier to accomplish a task if you work with other people.

E.4 What do you like about working in groups?

Work becomes less burdensome.
It is fun.
Other group members help you to solve your problem.
Communication and social skills are developed.
Expression of support and acceptance within the group.

E.5 What don't you like about working in groups?

Some group members are not committed enough.
It takes time to get a chance to work on the computer.
When one group member wants to dominate the group.
It can be time consuming if the group does not agree on an issue.
Lack of respect for other people's ideas.

E.6 Do you think this experiment with group work was successful? Give reasons for your answer.

Yes, because the participants were happy and felt they have learnt something.
Yes, because help was available from other group members.
Yes, because we managed a lot of skills in a short period of time.

E.7 Would you like group work to be introduced in your school to teach children? Give reasons for your answer and if yes, in which subjects?

Children are able to learn from one another.
Learning becomes fun and enjoyable.
Children can share their experiences.
More skills are taught at one time.
Group work will develop pupils as a whole, academic as well as social skills.
Will develop problem solving skills.

Subjects mentioned are Mathematics and Science.

E.8 Explain what you liked about the computer-assisted instruction (keyboard and word processing) and what you didn't like.

The teachers liked the following:

Challenging and inspiring.
Accuracy, time saving and neatness of the lessons.
Liked it as a first exposure to the computer.

They didn't like:

The time allocated for this part of the case study was too short.

E.9 Do you think computers can be used successfully in groups as opposed to individual instruction? Motivate your answer.

Groups can cover a lot of work in a small space or time.
Members of a group are encouraged by the other members to continue.
Yes, provided there is a facilitator available to help with problems.
Not necessarily - the two methods must supplement each other.

E.10 What do you think will be the advantages and/or disadvantages of using computer-supported group work in a school environment?

Disadvantages:	Financial implications. Time constraints in the classroom. Unco-operative pupils may hinder the group progress. Lack of skills on the part of the teachers can be a problem.
Advantages:	Numerous advantages were listed as above (E.4, E.8, E.9).

4.2.7.2 Questionnaire on CSCML

The five completed questionnaires were evaluated. The same scale was used as in the questionnaire discussed above.

PARTS AA AND BB: INHERENT BELIEFS ON THE NATURE OF MATHEMATICS

Part AA

Describe your view of mathematics by using a comparison.

Teachers compared mathematics to games, a riddle, building a house and a toolbox.

Part BB

Prioritise the following descriptions of mathematics:

The creation and study of abstract structures and objects.
Logic, rigor and accuracy.
A kind of language, a set of notations and symbols.
A way of understanding and predicting real life phenomena.
Reduction of complexity to simplicity.
Problem solving.
The study of patterns.
Exploration, observation and generalisation.
An art, a creative activity.
A tool for other sciences.

Four out of the five students chose mathematics to be a kind of language, a set of notations and symbols characterised by logic, rigor and accuracy. Only one saw it as primarily a problem solving activity.

PARTS CC AND DD: CO-OPERATIVE MATHEMATICS LEARNING

Part CC

CC1. Name the aspects you liked most working in groups doing mathematics.

It creates opportunity for constructive arguments and positive conflict.
A variety of ideas are shared.
It encourages participation and promotes discussion.

CC2. Name the aspects you did not like working in groups doing mathematics.

It is time consuming (because of discussions and arguments).
The group is sometimes dominated by one person.
Unco-operative group members hamper group work.

Part DD		Average
DD1	I give my opinions and ideas when I disagree with the group members.	3.8
DD2	I offer explanations of my opinions.	4
DD3	The other group members ask me to explain and justify my ideas when I disagree with them.	4
DD4	I sometimes know what the answer is, but I find it difficult to say it (to verbalise it).	1.4
DD5	I think it is not necessary to verbalise my ideas.	1.8
DD6	Once I have verbalised my ideas, it is as if I understand it better.	3
DD7	The group members help me to verbalise my ideas.	2
DD8	The other group members correct me if I am wrong.	3.4
DD9	I could have done the same problem much quicker on my own.	2.2
DD10	I could have done the same problem much easier on my own.	2.2

PARTS EE, FF AND GG: COMPUTER SUPPORT TO CO-OPERATIVE MATHEMATICS LEARNING

Part EE

EE1. Name the aspects you liked most doing mathematics using computers.

The computer is fast and accurate in drawing, measuring and labelling.
Sketches can be saved for later use.
It is fun.
Instant feedback is given.
Constructions are made fast (e.g. parallel lines, perpendicular lines, etc.).

EE2. Name the aspects you did not like doing mathematics using computers.

If you do not know the software well, it is difficult and time consuming to make drawings.
On choosing the wrong tool, no feedback is given to correct that.
It does all the thinking, you do not have to think.

Part FF		Average
FF1	My view of mathematics has changed.	2.5
FF2	The software helped me to understand the study material better.	3.8
FF3	Working with the software helped me to formulate my ideas.	3
FF4	Working with the software helped me to prove my ideas formally.	3.2
FF5	Working with the software helped me to visualise possible solutions to the problem.	3.6
FF6	My intuitive ideas were confirmed by using the software.	3.8
FF7	The same lesson could be done as effectively without the software.	2.2
FF8	It would have been easier to do the problems individually using the computer.	1.4
FF9	I realised that my solution was correct if it was approved by the teacher.	2.2

Part GG

GG1. Explain your answer to question FF2.

The following ideas were mentioned:
The software helped me to understand the study material better because:
of its speed and accuracy,
I could investigate and discover properties from constructions,
it confirmed my intuition, e.g. it gave values as I predicted.

GG2. What skills do you need to solve problems?

Basic knowledge (know your tools, know the basics).
The ability to think analytically.
The ability to see patterns and relationships.

GG3. In what way did the use of the software aid you in solving the problems?

Relationships could be observed and generalisations could be made.
One could draw diagrams.
It confirmed suspicions and intuitions.
Deductive thinking skills were improved.

4.2.7.3 Analysis of questionnaires

An analysis of the completed questionnaires reveals the following points:

1. A sense of enjoyment (D2, D7, E4, E7, EE1).
2. A sense of belonging, trust, support and mutual respect (A5, A9, A11, B1, B2, B3, B4, B5, B6, C1, C3, C4, C7, E4, E6).

3. A willingness to co-operate and to ask for co-operation (A3, A4, C5, C8, C2, D3, D5, D9).
4. The offering of explanations, justifications and opinions, and the willingness to listen to each other's arguments and to adapt own views (A1, A2, A10, D8, D11, E4, DD1, DD2, DD3, DD8).
5. A perception of CSCML as advantageous:
   - It facilitates understanding (D1, D4, FF2, GG1).
   - More work and more difficult work is done (D10, E4, E6, DD9, DD10).
   - More interesting work is done (D12).
   - The computer helps with the formulation of ideas, proving ideas, visualising, confirming intuitions, aiding investigation, discovery and generalisation (GG1, FF2, FF3, FF4, FF5, FF6).

4.2.8 Other data collected from the case study

4.2.8.1 Independent evaluation

The Centre for Communication Research of the Human Sciences Research Council (HSRC) conducted an independent evaluation of the CSCML training at SEIDET. The evaluation showed that all the teachers involved consider *The Geometer's Sketchpad* to be useful and indicated a willingness to use it in their classrooms provided that they have computer facilities. However, some teachers expressed their concern about the effectiveness of CL methods in classes that are too big.

4.2.8.2 Worksheets (sessions 6 and 7)

De Villiers (1997) distinguishes between uneconomical and economical mathematical definitions. An uneconomical definition is one that gives unnecessary information, e.g. a rectangle is a quadrilateral with opposite sides parallel and equal, all angles 90 degrees, equal diagonals, half-turn symmetry, two axes of symmetry through opposite sides, and so on. It also tends to be partitional, e.g. a square will not be seen as an example of a rhombus. This understanding is at a lower level than the level of understanding economical definitions. An example of an economical definition is the definition of a rhombus as a quadrilateral with all sides equal. This is also a hierarchical definition - it includes squares. The understanding of hierarchical definitions lies at the heart of drawing the family tree. Students are forced to think in this way to be able to draw the family tree.

The worksheets of the expert groups were designed to lead the learner to the essential characteristics of the quadrilaterals (by using the properties to construct the quadrilaterals, they had to identify the minimum needed to do that). Also, in the cyclic quadrilaterals worksheet, pertinent questions are included to aid students to formulate hierarchical definitions, e.g. is a parallelogram also a cyclic quadrilateral? One group said no, whereas the other group answered it using the hierarchical definition - 'not always, only if the parallelogram is a rectangle'. Some groups thus showed an understanding of the hierarchical definition, although an inconsistent one (this is in line with the research done by De Villiers (1997)). Teachers still used partitional definitions for quadrilaterals in the development of the family tree. Both groups had great difficulty in drawing the family tree, one group especially so because of its ineffective functioning. In both cases the final product contained illogicalities. However, it is the belief that with time, the teachers would have become more fluent in this way of thinking - near the end of the session, one of the teachers said, 'I am not convinced yet, but I start to see the picture'.

The individual assignments revealed yet again the partitional definitions of quadrilaterals but showed good abilities to prove findings formally.

4.2.8.3 Teachers' lessons (sessions 9 and 10)

The teachers had to develop a CSCML lesson of 40 minutes to present to the other groups. After each lesson they assessed themselves and were assessed by their peers (see

Appendix A, V).

Group 1

- The content was the theorem: *The line joining the centre of the circle to the midpoint of the chord is perpendicular to the chord.* After students had discovered this by construction and measurement, they had to apply it to find an unknown angle in a sketch.
- In the lesson plan they gave the CL method as Jigsaw but did not use any of the principles of the Jigsaw method.
- Self-assessment: Both teachers saw the lesson as successful because 'learners were able to execute the individual task successfully'.
- Peer-assessment: They perceived the lesson as successful because 'we were able to prove the theorem', and because 'all benefited in the group'. They all indicated that they obtained new insights from the lesson. However, on answering the question 'How was positive goal interdependence promoted in this lesson?', the teachers showed only a vague understanding of the 'swim, or sink together' concept: remarks like 'we all helped each other, students worked together', were given. This was the case in all three lessons. All the teachers saw the software as necessary to do the lesson, because of the ease of making accurate constructions, and measuring angles and lengths.

Group 2

- The content dealt with the area of parallelograms. Teachers had to discover the relationship between the areas of rectangles and parallelograms by using a process called shearing (using the drag mode of the software).
- Practically no attention was given to the CL part of the lesson.
- The intention of the lesson plan was that individuals from each group would explain their findings at the end of the period, but because of a lack of time, this did not take

place.

- Self-assessment: Both teachers thought the lesson too long and the constructions too difficult. One teacher was of the opinion that clearer instructions would have made it easier.
- Peer-assessment: On questions about the necessity of the software to the lesson, some teachers commented on the time-saving aspect, while another mentioned the new insight he got from using the software: 'I was not aware that as long as the polygon is having the same height and same base, that the area does not change'.

Group 3

- The content dealt with trigonometric ratios. Students had to discover relationships between the sides and angles of right triangles by dragging one of the vertices.
- No real attention was given to CL techniques and principles.
- Individuals of each group were asked to explain their group's findings at the end of the lesson.
- Self-assessment: Both teachers perceived the lesson as successful. One of the teachers described the way in which the software assisted learning as 'the length of the side and the angles were changed without drawing new sketches'.
- Peer-assessment: All learners perceived the lesson as successful and interesting, and one gave the reason as 'findings were the same as in the table'.

4.2.8.4 Video-based analysis

An analysis of the transcript of the video-tapes revealed the following:

Role-taking

Hoyles et al. (1991) mention a tendency of specific pupils to take on a role for the duration of the lesson. The same was noticed here. Teacher A took on the role of operator in both the expert and home groups. Some teachers perceived the role of the operator as not part of the problem solving team: when A tried to see the worksheet, J reacted: 'Don't you look here, you look there' (pointing to the screen) 'and we will look here' (Transcript A, Tape 1,3:11). Teacher A was initially very quiet but eventually showed the most understanding in his home group.

Another teacher, J1 took on the role of leader in his group. He did this by verbalising his ideas all the time and giving instructions to the others. This proved to be detrimental to the group later on, because the other members had better strategies but were each time led back to inefficient strategies. It was only when the facilitator (M) intervened more forcefully that the pattern was broken and that J1 actually listened to the others' ideas:

M: 'Now what about a square, is a square also a cyclic trapezium, is it maybe also a cyclic kite?'
They discuss it. A leads the discussion.
J2: 'A square is also a cyclic trapezium.'
A draws a line.
M: 'What about a cyclic kite. What were the properties of a cyclic kite?'
J2: 'Give properties: one pair of opposite angles are equal and 90 degrees. Yes, for a square the opposite angles are also ninety degrees.'
M: ' ... and adjacent sides are equal.'
J2: 'Thus it is also a cyclic kite.'
M: (Points to J1), ' ... it seems that you are not convinced.'
J1: 'No, I'm not yet convinced but I get the picture.' (Transcript A, Tape 3, 34:00)

Problem solving

Schoenfeld (1994) mentions how new problems are created by students in the classroom while working on other problems. The facilitator should grasp such moments to create realistic problems. The Geometer's Sketchpad provides ample opportunity for this to happen:

M: ' ... and in this case, what are they, even more, these two they are both 90.
Now you must ask yourself the question, are there any other way to construct a cyclic kite?'
J: 'By the way what have we done here? … Another way is to draw circumscribed triangles.'
K: 'Another way is also, we have decided that a square is also a cyclic kite.' (Transcript A, Tape 2:6:42)

K: 'What do they mean by circumscribed circle.'
M: 'Let me show you the sketch here.' (Fetches a book.) 'You see this is the circumscribed circle.'
J: 'Does it mean the circle is not inside.'
M: 'Yes, that circle on which the four vertices lie. Exactly like this picture. And now you must say why the center is where you say it is. You can even, it is nice to draw a picture, but that is now a long story, so maybe we should not do it now. But if you draw a rectangle, you remember the way you constructed it, then you take the two diagonals, take the intersection. If you can construct the circle from there, it will go right through the four vertices. If you try this with the other quads it doesn't work, but for the rectangle it does. Why? ' (Transcript A, Tape 2, 00:48).

Beliefs of the teacher

The remark 'but that is now a long story, so maybe we should not do it now' from the facilitator in the above passage, indicates a belief that the product is still more important than the process. The facilitator, by trying to keep to the time limits and the original goal, discouraged learners from exploring meaningful new problems. The teachers should have been encouraged to try out all sorts of investigations. It also highlights the time problem with this approach that some teachers identified.

In the lessons offered by the teachers, J1 again took on the role of authoritarian figure and let the whole class make the constructions at the same pace. Groups were not functioning as individual units:

J1 (talking to the whole class): 'Thereafter we go to the midpoint of that chord, and how do you get that? You must select that line, that chord, you select it first.'
J1: 'Write down your findings, OK, after you have written down … ' (Transcript A, Tape 4, 4:00).

This kind of approach could lead to the belief that the main goal was to follow procedural instructions correctly. Indeed, one teacher wrote in his evaluation of the lesson 'we achieved our objectives by following the instructions'.

Inappropriate use of the software

Teachers mentioned in the questionnaire (EE2) that 'if you do not know the software well, it is difficult to make the drawings' and that 'on choosing the wrong tools, no feedback is given to correct that'. Most of the teachers had only a few hours exposure to computers when they started to work on this lesson. Teachers sometimes made inaccurate sketches by not using (out of ignorance) the facilities that guide one to points of intersection, etc.

J(1) 'So let us add it.'
M: 'Your sketch is not accurate enough. That is why it is different from what you expect.'
(Transcript A, Tape 1, 22:58).

Also, by not understanding the 'parent', 'children' concept of drawings, they often did not use the drag mode correctly. The success of group 2's lesson rested on the accurateness of the drawings (with respect to 'parent'' and 'children' elements). By making incorrect sketches at first, no clear findings could be made. All constructions had to be done again (with the help of the facilitator) before the investigation could lead to findings that was satisfactory.

The versatility of this software package lies in the dragging capabilities. When it was used appropriately, it lead to convincing illustrations of tendencies as in the case of the lesson of group 3:

K: 'This one is approaching zero, you see as the angle approaches zero, it approaches one.' (Transcript A, Tape 5, 5:29)

New Tools - new problems

Laborde (1995) mentions that CABRI (similar to The Geometer's Sketchpad) gives rise to new kinds of problems due to the concretisation of abstract concepts. For example, a property of the kite is that one of the diagonals is a symmetry axis. This is an abstract concept which, in this learning environment, becomes concrete by becoming a way of construction (using the reflect command). Teachers showed amazement when they used this command to construct figures.

Proof

From the observations it became clear that the teachers do not see the necessity to prove their findings formally. The measurements and findings from their constructions seem to be convincing enough:

J1: 'Is that your finding?'
J: 'So in a way you have proved the theorem.'
J1: 'Yeah, they have proved the theorem.' (Transcript A, Tape 4, 5:00)

Also, drawings become a way of intuitively deciding on the truth or otherwise of a statement, probably resulting from the use of the software (see underlined text below):

K:' ... the diagonals, then parallelogram'
Makes sketch with pen. Tries to fit parallelogram into circle.
J: 'Parm is not cyclic.'
K writes something down. Draws circle with quadrilateral (hand sketch). Shows opposite angles supplementary.
M: 'We have checked that last week.'
J: 'We say that opposite angles of the cyclic trapezium is supplementary. Do we have a cyclic parm?'
K: 'No.'
J: 'We do not have it.'.
W: 'it is not easy to draw.' (Transcript A, Tape 3, 8:00).

4.2.9 Remarks

It was mentioned above that teachers had difficulties using the software and that inaccurate sketches hampered the drawing of clear conclusions. It should be mentioned here that the difficulties experienced by the teachers could also be ascribed to the possibility that some of the teachers' geometry knowledge is still at Van Hiele level 1 (par. 2.4.3.3). The step-by-step instructions that they had to follow in order to construct the different quadrilaterals, would mean more to learners at a higher Van Hiele level. Once students have already explored the properties of a particular quadrilateral, they can 'see' how they are used in a construction. A more appropriate exercise for Van Hiele level 1 learners, would have been the dragging and exploring of a given construction.

One important result that emerged from the questionnaire completed by teachers was the fact that when they had problems assimilating the course material, they received help from the other group members. Another result was the idea that co-operative work will be advantageous in the school environment. The teachers continuously, throughout the questionnaire, expressed their enjoyment of the group work, the value of sharing ideas and feelings, and the motivation and support they experienced in their groups. The results of this case study also indicated that teachers had a better understanding of the study material and that they learned quicker. This shortens the educational life-cycle, which means that more people can use the system over a shorter period of time.

This case study clearly showed the teachers' enthusiasm about the concept of group work and of computer support thereof. This implies that one could expect positive results from employing CSCL to remove educational backlogs and to contribute towards development. However, any attempt to increase the efficiency of investment in education through technological support requires an indispensable component - the full support of teachers. Training in the technology used to support them only addresses part of the problem - if teachers do not accept the concept of technological support, no amount of training will make the particular attempt successful.

4.3 Case study: MATLAB, Linear Algebra and co-operative learning

4.3.1 Background

This study involved undergraduate mathematics students from Vista University. Vista University is a South African historically black university (HBU) and has a multi-campus structure. The case study was conducted on the Mamelodi campus in Mamelodi (now a suburb of Pretoria, but historically a black township). The majority of students at Vista University come from disadvantaged backgrounds.

4.3.2 The learners

Twelve students (4 female, 8 male) in the second year Linear Algebra class volunteered to participate. They were divided into four heterogeneous groups (with respect to ability) of three each. None of the participants can be described as high ability students and not one of them passed the first component of their second year mathematics course.

4.3.3 The content and material needed

The content deals with basic concepts of matrix operations, the solving of linear equations, vector spaces, linear independency, and basis. A preknowledge of the basic concepts of calculus was assumed. The learning tasks entailed revision of concepts already covered in the lectures during the year. The second learning task provided a different first exposure to the relations between the null space, row and column spaces of matrices. This was not very effective, since the students already knew most of the concepts from class and skipped investigations that were supposed to lead them to the findings. This was probably also because of time constraints and the belief that a final product had to be delivered.

The software used was the student version of MATLAB. The material used for the learning tasks and the tutorials came from Donelly (1995: 1-8, 19-35) and Leon, Herman and Faulkenberry (1996: 78-80, 82-84). The first version of MATLAB, written in the 1970s, was intended for use in courses on matrix theory, linear algebra and numerical analysis. Today, MATLAB is an interactive system and programming language for general scientific and technical computation. Its basic data element is a matrix. It has many routines for generating random matrices and various types of structured matrices which help to generate interesting examples in the classroom. The graphic capabilities help to visually illustrate the major theoretical concepts in linear algebra.

4.3.4 The objectives

Lesson 1: At the end of the lesson the learner must

- show a basic understanding of MATLAB usage by being able to carry out the basic matrix operations and drawing of graphs using MATLAB;
- show some problem solving skills by being able to analyse and interpret the given problem and generate ideas to solve it;
- show an understanding of differential calculus by being able to interpret the 30 degree angle in terms of the derivative of the polynomial; and
- show an understanding of matrix operations and the solving of systems of linear equations by being able to know which instructions should be carried out using the software.

Lesson 2: At the end of the lesson the learner must

- show a thorough understanding of dimension, basis and vector spaces by being able to, after completing the learning task, relate the dimensions of the row space, column space and null space of matrices, and to explain the relations.

4.3.5 The procedure

Session 1: Group building and getting to know MATLAB

Students were divided into their groups and given a group building exercise. This entailed the following: students had to individually prioritise aspects of problem solving from a given list, after which the group had to reach consensus on the four most important aspects of problem solving (see Appendix B, I). Each group then had to discuss their decision with the other groups. Groups were asked to point out negative and positive aspects of their fleeting experience of group work. The five principles of co-operative learning (par. 3.2.3) were then explained with reference to the feedback from the groups. After this, groups worked through a tutorial on MATLAB.

Sessions 2 and 3: Designing a ski jump

The Jigsaw method was used as the CL-method. The lesson is described below, according to the steps necessary for the successful implementation of the method:

1. *Divide into groups and do group building exercises*. This was done in session 1.
2. *Explain to the students the idea of group work.* This was also done in session 1.
3. *Explain the goal and task.* The groups received one set of materials per group. The first page explained the main goal: *Design a ski-jump that has the following specifications: the ski-jump starts at a height of 30,5 m and finishes at a height of 3 m. From start to finish the ski jump covers a horizontal distance of 36,6 m. A skier using the jump will start off horizontally and will fly off the end at a 30 degree angle from the horizontal. Find a polynomial whose graph is a side view of the ski-jump (check your answer visually by plotting the graph).* Since most students were unfamiliar with ski-jumps, pictures were shown and eventually they compared it with the more familiar slide. The second page contained another task to aid them in doing the given problem (see Appendix B, II).

   This task entailed:

   a) The acquiring of problem solving skills and getting familiar with the capabilities of MATLAB to draw graphs and solve linear equations: each member of the groups had to join an expert group. One expert group focused on problem solving skills, the other on how to solve linear equations with MATLAB and another on how to draw graphs using MATLAB (see Appendix B, III). It was made very clear to the students that they would be the only experts on these skills in their home groups and that the success of the group would depend on them.
   b) Revising the identification of the equation of a polynomial where three points on the curve are given. This eventually involved the solving of systems of linear

equations.

After returning to their home groups, they had to go through the activities described in b) above, after which they could proceed with the main problem.

4. *Specially designed curriculum material so that each member of the group has a unique source that can be used independently of other sources.* Each expert had his/her worksheet on the skills necessary to solve problems/solve linear equations/ draw graphs and shared their skills with the others when required by the group.
5. *Use instructional material to promote interdependence among students.* Each group had only one computer to work on and only one copy of the worksheets. In the expert groups each member received a copy of the worksheet to take back to his home group.
6. *Assess the students' work.* The group solution to the problem was handed in as well as a diary file of the work on the computers. This was used to assess the students.
7. *Final evaluation:* This was done by means of an individual assignment (see Appendix B, IV).
8. *Assessing group functioning.* This was done through ongoing observation while the students were working as well as by the completion of a questionnaire. Also, students completed an evaluation list at the end of the lesson to monitor their feelings as well as what they perceived they had learned (see Appendix B, V).

Session 4: Investigating the relation between dimensions of vector spaces

The CL-method 'learning together' was used in this lesson. This is a method where learners work in small groups to complete a single worksheet for which the group receives recognition. The lesson is described below, according to the steps necessary for the successful implementation of the method.

1. *Heterogeneous groups.* The students still worked in the same groups as the previous sessions. These groups were heterogeneous with respect to ability (according to marks obtained in tests during the linear algebra course). Students already knew what the desired behaviour within the group was and they were reminded of it during the lesson.

2. *Assign roles to the different group members.* Groups had to decide among themselves who to assign the role of scribe, operator and problem solving expert. However, the roles proved to be artificial and students did not adhere to their roles unless reminded to. This is in line with research done by Good et al. (1992) (see Appendix B, VI).
3. *Arrange groups in circles to facilitate communication.* Groups were arranged in half circles around the computer. This proved sufficient for communication to take place.
4. *Explain the goal and task.* The students were led through the investigation by a well structured worksheet that gave sequential steps that had to be followed. The same sequence of steps had to be carried out on different matrices to be able to recognise a pattern.
5. *Use instructional material to promote interdependence among students.* Each group had only one computer to work on and only one copy of the worksheet.
6. *Assessment.* The students were evaluated by the completed worksheet, as well as by the individual tests given at the end of the session (see Appendix B, VII).
7. *Evaluating group functioning.* An observation list was used to assess group functioning (see Appendix B, VIII). A check list was also included in the worksheet of the students through which they could evaluate their own group functioning (see Appendix B, V).

The five basic elements of CL were implemented in sessions 2, 3 and 4 in the following way:

a. Positive goal interdependence, which occurs when learners undertake a group task with a feeling of mutuality. This was achieved by having the group produce a single solution to the given problem.

b. Face-to-face promotive interaction, which occurs when a verbal interchange takes place where learners explain how they obtained an answer or how a problem may be solved. The experts in the group (in sessions 2 and 3) had to provide the group with their skills. Also, by doing the given task as a group, group members gave their input and suggestions.

c. Individual accountability, which means taking responsibility for learning material.

Individual tests were given at the end of sessions 3 and 4. These tests focused on aspects of MATLAB usage, on questions relating to the identifying of equations of polynomials, on relations between the dimensions of the fundamental spaces of a matrix, and on more elementary concepts of linear algebra.

d. Social skills, which involve knowing how to communicate effectively and how to develop respect and trust within a group. The students already showed some social skills in the group building exercises.

e. Group processing to reflect on how well the group is working and to analyse their effectiveness and how it may be improved. Groups were observed while they worked but sometimes intervention was necessary. However, there was one group with a dominating member and another group with two free-riders. In session 4, the worksheet contained one page with a check list of group functioning that the group had to complete. This forced the group to reflect on their own functioning.

4.3.6 Results of the questionnaires

The same two questionnaires that were given to the teachers in case study one, were given to the students (with a few changes).

4.3.6.1 Questionnaire on CSCL

A total of 12 questionnaires completed by the students, were evaluated. The following scale was used in the questionnaire:

4	Always / Definitely
3	Frequently / Nearly almost
2	Occasionally / Seldom
1	Never

The results were as follows (sd = standard deviation):

PART A	QUESTIONS ON THE STUDENT'S BEHAVIOUR IN A GROUP	Average	sd
A1	I offer facts and relevant information in order to promote group discussion.	3.5	0.7
A2	I give my opinions and ideas and provide suggestions in order to promote group discussion.	3.75	0.5
A3	I express my willingness to co-operate with other group members.	3.7	0.5
A4	I expect other group members to be co-operative.	3.7	0.5
A5	I give support to group members who are struggling to express themselves intellectually.	3.6	0.7
A6	I keep my thoughts, feelings and reactions to myself during group discussions.	1.4	0.8
A7	I evaluate the contributions of other group members in terms of whether their contributions are useful to me and whether they are right or wrong.	2.7	0.9
A8	I take risks in expressing new ideas and my current feelings during group discussion.	3.1	0.8
A9	I communicate to other group members that I, am aware of, and appreciate their abilities, talents, skills and resources.	3.2	1.1
A10	I share any sources of information or other sources I have with the group members in order to promote the success of the individual members as well as the group as a whole.	3.8	0.4
A11	I offer help to anyone in the group in order to bring up the performance of everyone.	3.7	0.7

PART B	QUESTIONS ON THE LEVEL OF ACCEPTANCE OF THE STUDENT AS A GROUP MEMBER	Average	sd
B1	My fellow group members are completely honest with me.	3.8	0.4
B2	My fellow group members understand what I am trying to communicate.	3.3	0.8
B3	My fellow group members accept me just the way I am.	3.7	0.7
B4	My fellow group members make it easy for me to be myself.	3.5	0.8
B5	My fellow group members include me in what they are doing.	3.6	0.8
B6	My fellow group members value me as a person, apart from my skills or status.	3.8	0.5

PART C	QUESTIONS ON GROUP COHESION	Average	sd
C1	I try to make sure that everyone enjoys being a member of a group.	3.8	0.5
C2	I discuss my ideas, feelings and reactions to what is currently taking place within the group.	3.7	0.5
C3	I express acceptance and support when other members disclose their ideas, feelings and reactions to what is currently taking place in the group.	3.5	0.5
C4	I try to make all members feel valued and appreciated.	3.6	0.7
C5	I try to include other members in group activities.	3.5	0.7
C6	I take risks in expressing new ideas and my current feelings.	3.1	1.1
C7	I express liking, affection, concern for other members.	3.3	0.5
C8	I encourage group norms that support individuality and personal expression.	3	1.1

PART D	QUESTIONS ON GROUP WORK IN GENERAL	Average	sd
D1	I have learnt more in the group than I would have learnt on my own.	3.9	0.3
D2	I enjoyed working in a group.	3.8	0.4
D3	The group motivated me to do my share of the work.	4	0
D4	The group work helped me understand the study material better.	3.8	0.4
D5	I learned to co-operate with other students.	3.9	0.3
D6	The group work caused me to be dependable and do my assignment.	3.3	1.0
D7	It was fun working in a group.	3.5	0.8
D8	In the group I got the benefit of everyone's ideas.	3.7	0.5
D9	I got help from group members with problems.	3.8	0.6
D10	The work got done faster and more work was done.	3.6	0.7
D11	The group work gave me an opportunity to talk and discuss the study material.	3.9	0.3
D12	The group work made the study material more interesting.	3.9	0.3

PART E: QUESTIONS ON SELF-DEVELOPMENT

E.1 I have worked in groups before.

YES =7	NO = 5

E.2 I would have liked to choose my own group members.

YES = 2	NO =10

E.3 Have you worked on a computer before doing this course?

YES =7	NO = 5

E.4 Why do you think group work was introduced in this course?

Some of the comments were:

Computer literate students can help illiterate students to work on the computer;
in groups we can use our time to really try to understand the work;
such that we can create experts in the groups;
it is easier to understand the work if we can work together;
to share different ideas;
to improve interpersonal skills;
verbalising makes your ideas clearer to yourself;
it takes less time to do the work in a group.

E.5 What do you like about working in groups?

Many points given in E.4 were repeated. Other points mentioned were:
If other group members understand better what the lecturer said, they can explain it to you;
a decision is made by compromise;
you learn how others think, and you learn to express yourself in a precise way;
if you verbalise something, you rarely forget it;
you are encouraged by others, you do not give up easily.

E.6 What don't you like about working in groups?

Unco-operative group members;
dominating group members;
being ignored by others;
unprepared group members;
if you find it difficult to say what you mean, but you know your idea is correct;
the blaming of each other and unwillingness to listen to each other.

E.7 Do you think this experiment with group work was successful? Give reasons for your answer.

Yes, because reasoning abilities increased;
I developed good communication skills;
if I get stuck, I do not have to wait for the lecturer to help me;
now I know my fellow students, I was shy, now I can talk in a full hall;
all members attended every Saturday;
we completed the problems successfully;
our group will still function as a group even after this course;

E.8 Do you think computers can be used successfully in groups as opposed to individual instruction? Motivate your answer.

Yes, it saves time and we can learn from each other, it never tells a lie, it is precise;
no, computers are like calculators, it will spoil us and we will not be able to solve problems by hand. It should not be allowed in the exam room.

4.3.6.2 Questionnaire on CSCML

The 12 completed questionnaires were evaluated. The same scale was used as in the questionnaire discussed above.

PARTS AA AND BB: INHERENT BELIEFS ON THE NATURE OF MATHEMATICS

Part AA

Describe your view of mathematics by using a comparison.
Students wrote:
Mathematics is like:
a motor car that needs to be checked to make sure that it is still perfect and correct;
a game, it has rules that have to be applied, a game in which we play with numbers that enable abstract thinking;
life, obstacles are encountered that must be overcome;
a rainbow - different colours can be grouped together and be one - different problems can be grouped together to obtain one solution;
an engine, without it life is difficult and you cannot solve even small problems.

Part BB

BB1. Prioritise the following descriptions of mathematics:

The creation and study of abstract structures and objects.
Logic, rigor and accuracy.
A kind of language, a set of notations and symbols.
A way of understanding and predicting real life phenomena.
Reduction of complexity to simplicity.
Problem solving.
The study of patterns.
Exploration, observation and generalisation.
An art, a creative activity.
A tool for other sciences.

Most students perceive mathematics as a problem solving activity involving rigor, logic and accuracy.

BB2. What is it that you like about mathematics?

It gives me insight in how to solve problems (even outside mathematics);
it keeps me busy, I forget my problems;
it makes me think logically;
it is challenging.

BB3. What is it that you do not like about mathematics?

I do not like the theorems, and the theory part;
it is too abstract, I cannot relate it to something in the real world;
it is nerve wrecking;
sometimes you are forced to cram the work without understanding.

PARTS CC AND DD: CO-OPERATIVE MATHEMATICS LEARNING

Part CC

CC1. Name the aspects you liked most working in groups doing mathematics.

A variety of ideas are shared;
we support each other;
cover a lot of work;
you are forced to prepare;
the group members challenge your ideas;
there is always only one solution, so even when we have different answers, we must agree on only one.

CC2. Name the aspects you did not like working in groups doing mathematics

It is time consuming and sometimes work is left undone;
arguments and disagreement amongst the group members;
unprepared group members hamper group work.

Part DD		Average	sd
DD1	I give my opinions and ideas when I disagree with the group members.	3.7	0.7
DD2	I offer explanations of my opinions.	3.9	0.3
DD3	The other group members ask me to explain and justify my ideas when I disagree with them.	3.8	0.4
DD4	I sometimes know what the answer is, but I find it difficult to say it (to verbalise it).	2.4	0.7
DD5	I think it is not necessary to verbalise my ideas.	1.3	0.8
DD6	Once I have verbalised my ideas, it is as if I understand it better.	3.7	0.9
DD7	The group members help me to verbalise my ideas.	3.3	0.8
DD8	The other group members correct me if I am wrong.	4	0
DD9	I could have done the same problem much quicker on my own.	2.4	0.8
DD10	I could have done the same problem much easier on my own.	1.9	1.0

PARTS EE, FF AND GG COMPUTER SUPPORT TO CO-OPERATIVE MATHEMATICS LEARNING

Part EE

EE1. Name the aspects you liked most doing mathematics using computers.

The computer is fast and accurate in graphing and solving of linear equations;
instant feedback is given;
we do not use our brains much, therefore, we do not have much stress.

EE2. Name the aspects you did not like doing mathematics using computers.

The technology does not always work;
if you do not know the software well, you make mistakes typing the commands;
you need to know how to arrive at an answer, but the computer does not show the steps, just solves the problem.

Part FF		Average	sd
FF1	My view of mathematics has changed.	3.8	1.2
FF2	The software helped me to understand the study material better.	3.8	0.4
FF3	Working with the software helped me to formulate my ideas.	3.3	0.8
FF4	Working with the software helped me to prove my ideas formally	3.2	0.8
FF5	Working with the software helped me to visualise possible solutions to the problem.	3.4	0.5
FF6	My intuitive ideas were confirmed by using the software.	3.3	0.8
FF7	The same lesson could be done as effectively without the software.	2	1.0
FF8	It would have been easier to do the problems individually using the computer.	1.9	0.8
FF9	I realised that my solution was correct if it was approved by the teacher.	2.9	1.3

Part GG

GG1. Explain your answer to question FF2.

The computer is reliable, I compare my answers to the computer's answers and then I continue, it always gives correct answers if you entered correctly and use correct commands;
it shows that you can do a lot with mathematics, it is more than just knowing theorems;
it is easy to draw the graphs;
I could try several ideas on a problem;
it decreases the work load;
you can see relations by drawing graphs.

GG2. What skills do you need to solve problems?

Persistence, patience;
divide the problem into subproblems;
read the problem carefully;
consider many alternatives;
creativity;
logical thinking.

GG3. In what way did the use of the software aid you in solving the problems?

It is quick in doing calculations (e.g. Gaussian elimination);
more problems can be tackled;
immediate feedback;
graphing capabilities.

4.3.6.3 Analysis of questionnaires

An analysis of the completed questionnaires reveals results similar to the first case study.

However, other aspects were also mentioned by the students:

- The motivational aspect of group work (CC1, D3).
- The importance of verbalising one's ideas and the encouraging thereof by group work (E4, E5, DD4, DD5, DD7).
- The changing of one's beliefs about mathematics because of CSCML (FF1, GG1).
- Additional advantages of computer support to mathematics co-operative learning, were the decrease of the work load (GG1), the many similar problems that can be generated using the software (GG1), the quick feedback given (EE1) and the facilitating of the drawing of relations and comparisons because of the graphing facilities (GG1).
- A perceived disadvantage of computer support to mathematics learning is the fear that the computer does all the thinking and that it gives the solution without giving the steps to get to the solution (EE2, EE1, E8).

4.3.7 Other data collected from the case study

4.3.7.1 Worksheets and individual assignments

Sessions 2 and 3:

All four groups' written solutions to the ski jump problem were correct. However, these correct solutions could only be found after hints (often very explicit) by the facilitator. Not a single group interpreted the 30 degree angle correctly and most tried to side-step that part of the information in their solutions. It was clear though, that the students could carry out basic commands of MATLAB.

The individual assignment given after the completion of the problem asked two questions about a MATLAB command and one question on further questions to ask about the finding of the equation of polynomials (see Appendix B, IV). The students complained (questionnaire, EE2, EE1, E8) about the non-transparency of the software. These first two questions tried to encourage students to ask themselves questions on what is going on behind the scene. Also, students were encouraged to use the command 'rrefmovi' to see a 'movie' of row-reduction on a matrix. The individual assignments were marked and the average obtained was 33%.

Session 4:

In analysing the group worksheets of session 4, it was clear that all groups could eventually identify the relation between the fundamental spaces of a matrix. However, they ignored the way in which the worksheet wanted to lead them there: they did not answer questions that asked for explanations and they found the column space using an algorithm they did in class during the year, instead of going through the instructions that used concepts from the null space to find the column space (which tested and reinforced more insight).

The individual assignment was an adaptation of part of a course test that they wrote earlier in the year (see Appendix B, VII). The average for that part of the course test written earlier in the course was 33% and the average for this individual assignment was 52%.

4.3.7.2 Evaluation lists completed by students

An evaluation list was given to the students to complete after each session. The following issues were addressed:

Sessions 2 and 3:

In you own words state today's goal or goals.

The students' perceived goals corresponded with the objectives stated in par. 4.3.4.

What was the topic/s of the day?

Most students just wrote 'designing a ski-jump' and others mentioned 'finding a polynomial' and 'graphing a polynomial'.

These are the strategies and concepts I learned today:

- How to analyse a problem, by using the little information given.
- How to plot graphs and solve linear equations using MATLAB.
- How to work in a group.

What was your AHA (now I understand) today?

- You can use the solved values to plot a graph.
- What 'format rat' does.
- Using the angle given in the problem, finding the last equation using the slope.
- Reducing a matrix using 'rref'.
- Plotting a graph using a computer, when the graph appeared on the screen.

I'm still confused about:

- Finding the points when given a question in words.
- Using the given angle.
- Element by element row operations.
- Why it is simpler doing the problem using MATLAB, than doing the problem on paper.
- How we reached the solution of the problem.
- Using some commands of MATLAB.

Today in class I felt:

- Helpful because I helped my group to solve linear equations using MATLAB.
- Like I am really working towards a goal, because I was deeply involved in different topics.
- Very happy because I solved some difficult problems.
- Very proud, because working in a group helps me to understand things that I never knew and increases my knowledge on what I know.
- Worried, because much of the work was done by the graphing expert and I did not get a chance to prove myself.

- Helpful, because I helped my group to plot the graph since I was the graphing expert.
- Lost and confused, because I missed the first session and did not feel prepared.

Session 4:

In you own words state today's goal or goals.

Most students stated the goal as the finding of dimensions of vector spaces and only three stated the goal as finding the relation between the dimensions of the fundamental spaces of a matrix.

What was the topic/s of the day?

Most students described the topic of the day as the same as the goal and a few wrote simply 'basis and dimension'.

These are the strategies and concepts I learned today:

- I learned how the row space, null space and column space are related.
- I learned what a null space, row space and column space is.
- How to find basis and dimension of vector spaces.

What was you AHA (now I understand) today?

- When I could 'see' the dimension of the null space in the reduced form of a matrix.
- To find the basis without thinking a lot.
- How the null space really works.

I'm still confused about:

- The difference between null space and nullity.
- Determining the basis of the null space.
- Nothing because we arrived at the conclusions without arguments.
- How to use the MATLAB commands.

Today in class I felt:

- Very pleased and grateful, because I can find the dimensions of the vector spaces; I could not do that before.
- Like I was really beginning to understand the concepts, because there were times where one had to make an input based on those concepts.

- Happy because we were co-operating.
- Great and hope that with group discussion I can get a distinction in mathematics, because my friends explained everything clearly and patiently to me.

4.3.7.3 Video-based analysis

An analysis of the transcript of the video-tapes revealed the following:

Group functioning/processing

There were two groups that did not function optimally. Group 1 had a dominating group member and group 2 had two group members that did not really participate.

In group 1, member 1b took on the role of operator and also dominated the discussions. She showed impatience with the other group members:

Group 1, 1b reads a sentence from the worksheet and points to it, 1a points to place on worksheet and 1b shoves her hand away. 1a laughs embarrassedly (Transcript B, Tape 2, 9:51).

This impatience with each other apparently became the norm:

Group 1, 1a sits in the middle, they are busy with solving of linear equations and 1a was a member of the expert group on that topic. They all have a look at what they have done last week.
1c rises. 1a hands 1b the worksheet and draws the keyboard nearer. She reaches over 1b for another document shoving her hand away. (Transcript B, Tape 3, 1:05).

Group 1, they have the system of linear equations on the screen. 1c is now looking at his 'graphing expert group' worksheet to see how to plot the graph.
1a takes it out of his hand. 1c tries to reach for it but 1a pulls away. 1b gives 1c another worksheet. (Transcript B, Tape 3, 8:45).

The facilitator tried to intervene in session 4:

M: 'Can I ask who the operator is?'
1b: 'I am.'
M: 'And the scribe?'
1a: 'I am.'
M: 'Don't you want to be the operator?' (Talking to 1a)
1a: 'OK.'

M: 'And you are the problem solver, you must ask them things like "have you read the problem carefully, those things… ' (Talking to 1c) (Transcript B, Tape 4, 5:56).

After this 1b left the room for about 10 minutes. It is interesting to note that most complaints about dominating group members and disagreements came from 1c. Also, he found session 4 more enjoyable because 'we were co-operating'.

Group 3 had two members who looked uninterested and confused. Also, the third member only joined them in sessions 3 and 4 and then tried to get the group going. However, she was not well informed and prepared, and this hampered group functioning. She mentioned this problem in the questionnaires and the self-evaluation lists (CC2).

3c (new member): 'I have this equation., 1 0 0 6.. OK we can solve it by using…Yeah.'
M: 'They have already solved this one, or which one is this.'
3c: 'It is this one.'
M: 'They have done this one, do you remember it?' (Directs questions to 3a and 3b).
3b: 'We haven't finished it.'
M: 'Yes, but it is OK, it was only to help you to solve this problem.'
M: 'Ask them they know how to solve this by using the computer, this is the idea we need to use the computer.'
3c: 'So we do not need to do this one?'
M: 'No, this is the example. This is the one I actually want you to solve.' (Points to the given problem).
(Transcript B, Tape 3, 24:22).

Problem solving strategies

A tendency was identified of students reading a problem, quickly choosing an approach and then taking off in that direction, instead of taking more time to make sense of the problem. This is in line with research done by Schoenfeld (1990). It could also be explained by the students' belief that in problem solving something must be done as soon as possible - this was one aspect of problem solving which students had to mark as important or not in their group building exercise in session one. Only two students considered it as not important at all. All the others found it rather or very important.

The following transcript shows how students combine numerical information from the problem in a haphazard way only minutes after they have read the problem:

Group 2, All look at THE PROBLEM. 2c fetches the calculator. 2b draws a sketch of a triangle showing an angle. 2a points to 2b's sketch.
2a: 'So it is this over this.'
2c: 'No, this thing squared, plus that thing squared is equal to the answer squared. So what we are actually doing...'
2c: 'So this is our 3m, this is our 30.5m and this one must be 36.6m.'
'So using Pythagoras' theorem, this 30.5m squared plus three squared is equal to this thing and this is equal to 30.'
2b:' ... the angle is 30 degrees.' (Transcript B, Tape 2, 6:28).

Mathematical actions and mathematical objects

None of the groups could interpret the 30 degree angle without help and tried several other strategies:

4c: 'We used the turning point.'
M: 'So you found the turning point? How did you do that? Remember the derivative is 0 at the turning point. So how did you get that? You see, let me just show you. They say at the end it flows off at an angle of 30 degrees. So we do not know anything here, OK we know the derivative is 0. Remember, the thing doesn't stop there, it has a little tip and it flies off here with an angle of 30 degrees.'
4c: 'Here?'
M: 'No, it is here, it is given, and you know this is the end, not this. So this is the ski jump ...' (Transcript B, Tape 2, 30:22).

M: 'Can I just ask, this third equation, where does it come from?'
M: 'OK, I understand this one, this one is ax^2 where x is 36.6 +bx. Can I just ask, this third equation, where does it come from?'
M: 'OK, I understand this one, this one is ax^2 where x is 36.6 + bx + c = y which is 3, and this last one where did you get the 21.1?'
2c: 'By using this thing.'
M: 'Just look at the logic behind what you are doing here. You say this polynomial goes through these three points. Can you see that if it is true it won't be a quadratic polynomial? Why not? Because what would it look like? At this point it would go straight up in the air, because this point is over there. So what your graph will look like is something like that instead of something like this.'
M: 'This information must be used in another way.' (Transcript B, Tape 3, 3:37).

It became clear that the students, understanding of a derivative at this stage only covered the operational aspects of it. The students could easily find the derivative of the general equation of the quadratic polynomial. However, they could not relate the 30 degree angle to the slope of the graph at that point. The word 'derivative' probably invokes in the mind's eye of the student a set of actions to be taken and not a holistic structure with certain properties. Sfard argues that a prerequisite to a deep understanding of mathematics is the ability to see a mathematical concept as both an object and a process, and that real insight necessary for mathematical creation can hardly be achieved without the ability *to see abstract objects* (Sfard, 1991:9).

Technology as a "black box"

The students complained that they did not know what was going on 'behind the scenes' when using MATLAB. The facilitator encouraged students to ask themselves questions about the computational processes of MATLAB:

M: 'Can I ask you something here, this A slash b, what do you think happens there? What does it mean, A slash b. What do you think are the operations used there? Because you know you cannot really divide by b, so what do you think goes on there?' (Transcript B, Tape 1, 5:10).

This problem would also be of less importance once the students are able to create their own macros.

Representational plasticity

Technology has the capability to support a variety of notational forms. Kaput (1992) draws attention to the ability of electronic media to enable us not only to create any manner of new notations for mathematical objects and actions, but also to create dynamic ones that can be linked. The representational plasticity observed here was not that sophisticated but nevertheless had great effect. It happened that certain numbers were given to students in scientific notation. They could change them to fraction notation by 'format rat'. Students no doubt learned again about scientific notation and one student even described this as her AHA experience:

2a reads to 2b what to type. They type in the matrix.
2c: 'Aikona!'
2b: 'What?'
They look surprised as the matrix is given in scientific notation.
2a points to values. They look confused.

M: 'OK, can I just explain to you what is happening here. You want 0 0 1 but it gives you 0 0 .001. But what this means is that they multiply each value by 1000. But if you find it difficult to work with, just say format rat. OK type that ' (Transcript B, Tape 3, 15:02).

4.4 Summary of the most important findings

The findings of both case studies confirm existing research results in that students reported a sense of enjoyment, belonging and trust. They were willing to co-operate and offer explanations and justifications, and showed an openness to each others' ideas. Students in case study 2 stressed the motivational aspects of group work, the importance of verbalising ideas and the changed attitudes towards mathematics resulting from their experiences in the groups.

Graphing facilities and its enhancement of visualisation were mentioned as an important aspect of computer support to mathematics learning. Students also mentioned the decreased work load, the many similar problems that can be generated using the software, and the quick feedback given. However, in both case studies, the perceived disadvantage of computer support to mathematics learning was that the computer does all the thinking. This perception can be explained by recalling the dual nature of mathematics concepts. In case study 2, the computer was used to do the computational or process part of the mathematics, leaving the students free to draw relations and concentrate on the structural side. Most students, however, consider the process part of the concepts the only part.

One of the important findings is thus that mathematical learning in the CSCML environment can be examined through the lens of the dual nature of mathematical concepts. This sheds light on most of the difficulties students experience with mathematical concepts. Another important finding was that the classroom culture or organisational context plays an important role in interactions and learning in the CSCML environment. The classroom culture consists of habitual patterns which are created through interactions and the underlying beliefs of the participants. Examples were given of the sometimes detrimental influence of the teachers' beliefs on meaningful problem solving.

The importance of true interdependence was confirmed. This was lacking in case study 1, although the Jigsaw method was used. The researcher exploited the idea of experts to the full in case study 2, which had the desired effect. It is thus clear that the success of CL implementation does not lie in the application of CL methods, but in the incorporation, and full understanding, of the CL principles.

The CSCML environments of the two case studies exhibited an open and unpredictable nature. Although goals were set, new ideas emerged during interaction and students negotiated alternative goals. This should be seen as an asset of this learning environment, which could lead to meaningful and sometimes surprising discussions and problems.

4.5 Enhancement of the generic social constructivist model for mathematics learning

4.5.1 A CSCML environment

The social constructivist model for mathematics learning described in par. 2.5 does not provide for groups of learners learning mathematics supported by technology. The interaction between teachers and learners (now groups of learners) is now influenced by information technology and the course material (this is indicated in Figure 4.1).

From the case studies it is clear that the CSCL process terminates at some point and that a certain outcome is desired. It will thus be more suitable to use the ideas of Hoyles, Healy and Pozzi (1994) and Slavin (in de Villiers, 1995) to describe the environment in terms of input-process-output functions. The input is considered as the data and other structures that are introduced to the learning and social process. The output is two-fold: on the one hand one hopes for an individual learner with subject knowledge (effectiveness) and on the other hand, one hopes for a productive, well functioning group that achieved the group goal (productivity) (Hoyles et al., 1994). This is indicated in Figure 4.1.

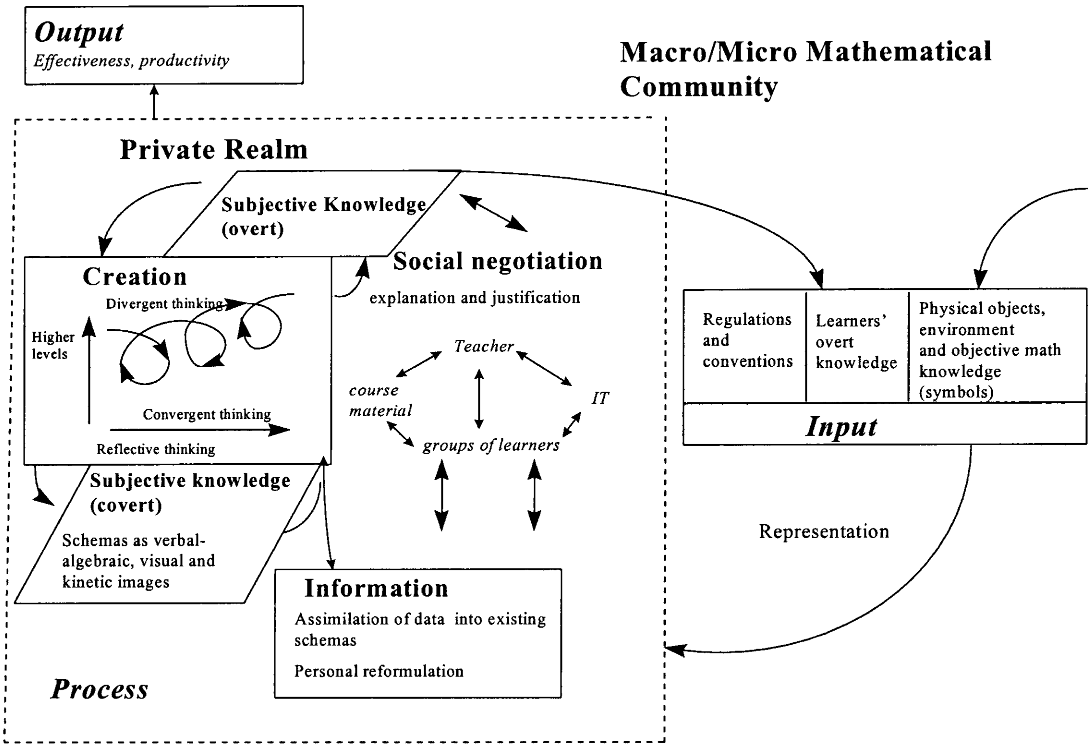


Figure 4.1
The input-process-output aspects of the CSCML environment

4.5.2 The dual nature of mathematical concepts

During and after the execution of the case studies, it became clear that the existing model for the learning of mathematics proves to be insufficient for understanding the learners' difficulties. For example, why was it so much more difficult for the teachers in case study 1 to use the definition of quadrilaterals in a comparative way than to use the definition to make constructions? Also, in case study 2 students had great difficulty in relating the 30 degree angle to the derivative of the function of the graph at that point, but no difficulty in finding the derivative of the general equation of the quadratic function (discussed in par. 4.3.7.3).

Sfard's (1991) theory for the forming of mathematical concepts highlights the dual nature of mathematical concepts. Her theory has been discussed in detail in par. 2.4.3.2. This theory provides a better explanation for the observed difficulties experienced by the learners in the case studies. For example, in case study 1, the construction of a parallelogram represents the operational aspects of it, whereas the economical definition represents the structural view of it (what is the least necessary for a quadrilateral to be a parallelogram and how does it relate to other quadrilaterals?). Sfard emphasises that the formation of a structural concept (object) is a lengthy and painfully difficult process. Also, previous operational understanding (processes) is a necessary condition for structural understanding. This could thus be a possible explanation for the difficulty learners experienced with the building of the family tree in case study 1. The existing model will now be enhanced by replacing the fluctuation between divergent and convergent thinking processes by a more specific fluctuation between mathematical processes and objects (see Figure 2.5).

Although it is shown as hierarchical, Sfard points out that any mathematical activity is an intricate interplay between the structural and operational aspects of the same mathematical concept. She also emphasises the important role that representations (symbols, graphs, names, etc.) can play in the condensation and reification processes (par. 2.4.3.2). Kaput (1992) sees true mathematical activity as involving co-ordination of and translation between, different notation systems (e.g. from algebraic-verbal to visual (par. 3.4.3). Sfard finds a (not necessarily one-to-one) correspondence between the operational side of mathematical concepts and verbal-algebraic inner representation of them on the one hand, and on the other, the structural conceptions and visual inner representation of them. This is in line with Skemps' classification of the two kind of symbols (par. 2.5). The kinetic images discussed in par. 2.5 can represent either processes or objects - it generally gives a feeling of the 'how it works' of mathematical concepts, thus involving the condensation process in Sfard's theory (par. 2.4.3.2).

There are several other theories which also recognise the dual nature of mathematical concepts (Dubinsky, 1997; Pirie & Kieren, 1994; Kaput, 1992). Pirie and Kieren (1994) make it clear that each knowledge level comprises a to-and-fro movement between acting and expressing. (They define more knowledge levels than Sfard's interiorisation, condensation and reification.) Acting refers to mental and physical activities, whereas expressing refers to making overt the activities to others and to the self. Acting usually precedes expressing and ' ... *encompasses all previous understanding, providing continuity with inner levels, and expressing gives distinct substance to that particular level.'* (Pirie & Kieren, 1994:175). The actions are directed to representations of mathematical concepts (e.g. seeing a graph, predicting certain trends, applying it). 'Representation' in the original framework will thus be replaced by 'actions on representations'.

Assimilation of new concepts into existing schemas will now be understood as the initial interiorisation of lower-level mathematical processes. Accommodation (major reorganisation of schemas) can now be seen as the interplay between the interiorisation, condensation and reification processes (as described in par. 2.4.3.2).

The subjective (overt) knowledge thus refers to expressing the mental and other activities and includes written, diagrammatic, verbal and printed reviewing, definition, recordings, justifications, prescriptions and proofs (Pirie & Kieren, 1994). These refinements to the existing model for the learning of mathematics are shown in Figure 4.2.

4.5.3 Classroom culture

Classroom culture is produced and reproduced by social interaction and negotiation, and includes regulations, conventions, morals, truths and instruction (par. 2.5). Truths are norms of which the transgression is seen as an error and instructions are norms of which the transgression is seen as ineffectiveness. The beliefs of the teacher and learners about the nature of mathematics play thus an important role. In case study 1, one teacher used an autocratic teaching style in the presentation of the group's lessons which undermined the discovering and exploring potential of both the technology and CL-environment (par. 4.2.8.4). This affirmed the already existing belief of some of the teachers that effectiveness in the mathematics classroom is to follow procedural instructions. (One teacher wrote in his evaluation of the lesson 'we have achieved our objectives by following the instructions'.)

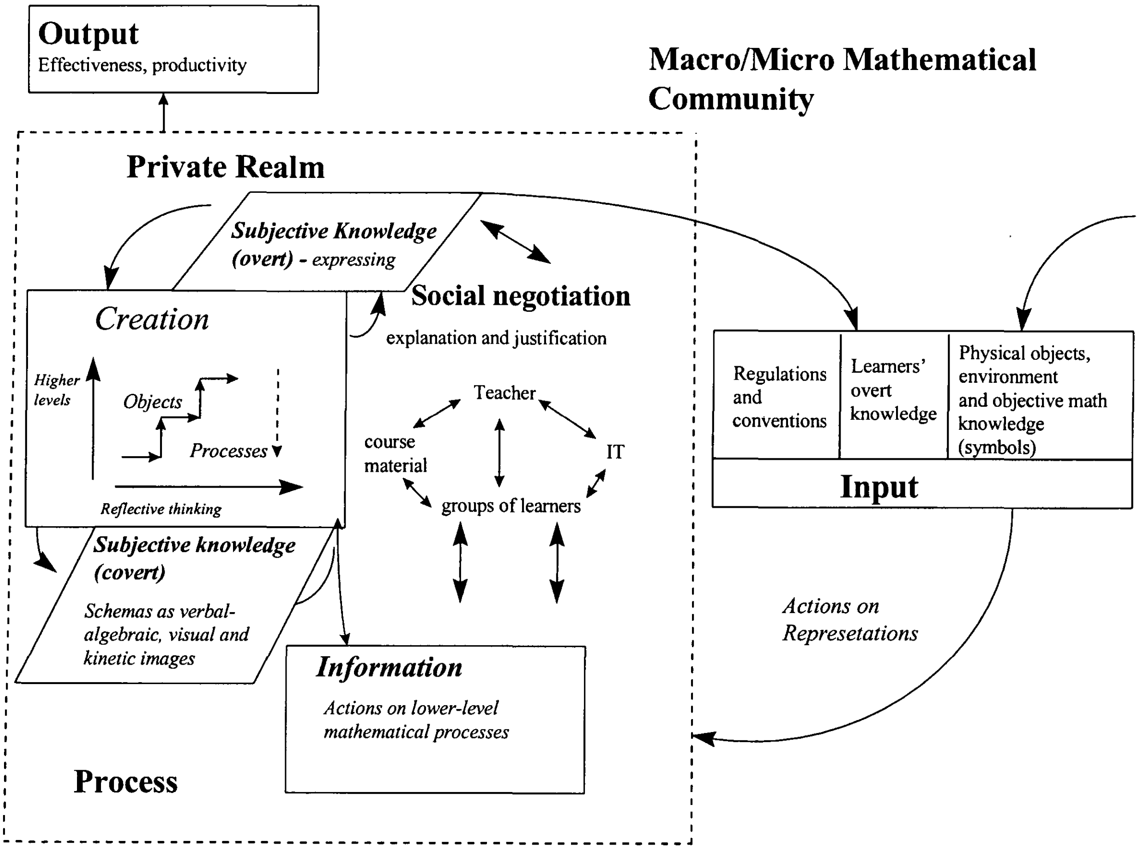


Figure 4.2

The dual nature of mathematical concepts in the CSCML environment

In case study 2, a product orientation was noticed, i.e. students were more interested in reaching a solution than in the process of getting there (par. 4.3.7.2). Also, students focused more on the operational side of mathematical concepts (par. 4.3.7.2). These beliefs had a detrimental effect on learning and attitudes towards the technology especially noticed in lesson 2 of case study 2.

On the other hand, the application of the CL-principles led to new social rules:

- The importance of verbalising one's ideas (par. 4.3.6.3).
- A change of attitude towards mathematics (par. 4.3.6.3 and 4.3.7.2).
- A willingness to co-operate and listen to others (par. 4.2.7.3).

It became clear that the CL environment provided ontological security (Lyytinen & Ngwenyama, 1994) which had an influence on motivation and attitude.

The classroom culture is thus part of the input to the learning and interaction process, influencing it and eventually being influenced itself by the interaction process (either reinforcing it or changing it). In the case studies, the CL principles and technology induced change in some of the initial beliefs and norms. This is indicated in Figure 4.3. It should be kept in mind that the learners involved in both case studies have the traditional mathematics classroom as a background, often involving poorly educated teachers and inadequate facilities. In these circumstances the 'traditional' way is often the easiest option.

4.5.4 Information technology

The CSCML environment involves the support of learning and teaching of mathematics by information technology. It can thus be seen as an input to the process of learning and interaction. This is indicated in Figure 4.3.

From the case studies it is clear that technology supports the learning of mathematics by aiding formulation, visualisation of intuitive ideas (graphs), easing the workload by doing the calculations, and providing immediate feedback (par. 4.2.7 and 4.3.6.2).

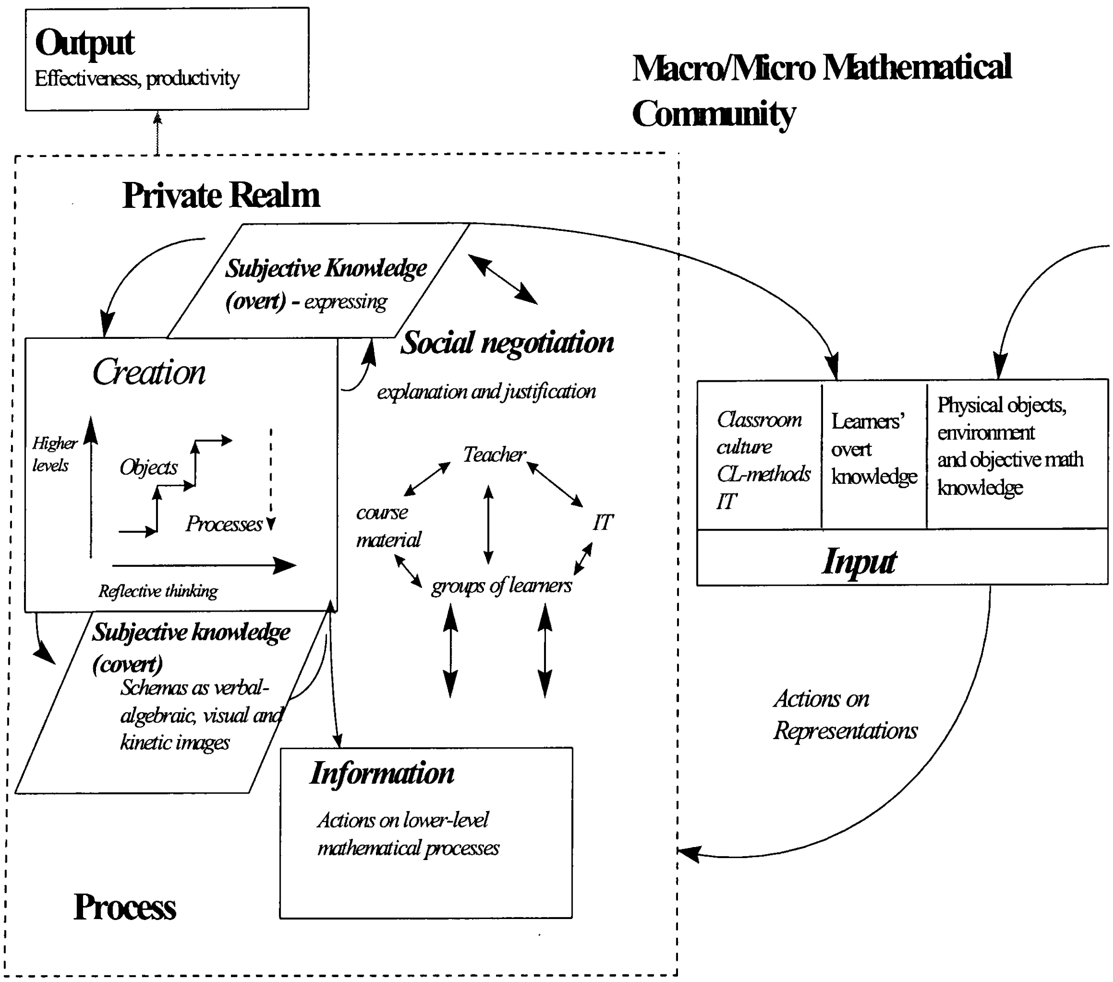


Figure 4.3

Information Technology and Classroom Culture as input to the CSCL-environment

Sfard (1991) describes mathematical activity as a fluctuation between the operational and structural sides of mathematical concepts. The Geometer's Sketchpad (used in case study 1) is a typical example of software which allows a simultaneous representation of operational and structural conceptions (e.g. after the construction of a kite, it remains the object 'kite' under the dragging mode - this construction becomes a generic construction). In this environment actions, representations and concepts become inseparable (Smith, 1994). '*Students create their own problematics in relation to posed problems where the* *problematic they construct is inseparable from their understanding of the actions which were possible using the available tools.'* (Smith, 1994:87). This type of environment also creates new kinds of mathematics problems (Laborde, 1995).

However, as was already mentioned (par. 4.2.8.4), the learners' inadequate knowledge of the possibilities of the software made them use it in an inappropriate way. In case study 2 students were dissatisfied with the feedback of the technology. The task was designed in such a way that the computer handled the operational aspects, leaving the students free to concentrate on the structural aspects of the mathematical concepts. Students considered the only part of mathematics to be the operational part. A reason for this could be that the structural view of the relevant mathematical concepts has not been fully developed yet. This could possibly explain their fears about the 'computer doing all the thinking and giving the solution without giving the steps to get to the solution'.

It is thus clear that the students' beliefs of mathematics as well as their knowledge of the rules and resources presented by the software, influence their interaction with the technology and its support to their learning.

A consequence of using technology in case study 1 was the assumption of teachers that formal proof is unnecessary. Their findings from the constructions seemed to be convincing enough. This was also noticed by de Villiers (1997). This tendency asks for alternative roles of proof (par. 3.4.3), which in the long run can have an influence on how proof is taught in the school environment as well as conceptualisations of proof in the professional environment.

The introduction of technology also influences the social interaction and production and reproduction of social norms. For example, in case study 1, it was observed that one student adopted the role of operator throughout the lessons (par. 4.2.8.4). The same happened in case study 2, but here it was more problematic since this person also dominated the discussions and activities of the group (par. 4.3.7.3).

4.5.5 Shortcomings of the social constructivist model for CSCML

The existing model for the CSCML environment in Figure 4.3, fails to provide for the delicate interplay between the components of the environment. For example, the influence of information technology on social interaction and the resulting altered social norms discussed above, are not provided for. Similarly, the change in learners' beliefs and attitudes towards mathematics as a result of the applications of CL methods is not described in a satisfactory way. The framework thus fails to provide for the production and reproduction of rules and resources during social mathematical interaction with technology.

This reciprocal influence between the structural aspects of social systems and human action has been conceptualised as a duality by Giddens (1984) in his structuration theory. In this theory Giddens views human action as being enabled and constrained by social structures which are at the same time produced and reproduced by human agency. Quite a few researchers adopted Giddens' structuration meta-theory to come to a better understanding of technology-induced change in organisations (Orlikowski, 1992; DeSanctis & Poole, 1994; Lyytinen & Ngwenyama, 1992). De Villiers (1995) uses structuration theory and adaptations thereof in her development of a theoretical framework for the CSCL environment. In the next chapter, the structuration theory as well as applications of it will be discussed. Lyytinen & Ngwenyama's framework for CSCW and De Villiers' framework for CSCL will then be explored after which a theoretical framework for CSCML will be developed.

4.6 Summary

This chapter presented two case studies of which the results and findings were used to enhance the model for mathematics learning presented in chapter 2. The findings of the case studies confirm existing research results (e.g. increased motivation, better self-esteem, enjoyment and a willingness to listen to each other's ideas) (par. 4.2.7.3 and

4.3.6.3).

The findings further highlight the importance of the influence of the classroom culture or organisational context on attitudes and learning in the CSCML environment. It also confirms the importance of real interdependence in the CL classroom for meaningful co-operation to take place. It is also concluded that some of the most important contributions of the computer to mathematics learning support, lie in its graphing facilities and representational plasticity.

The enhanced model, which is an input-process-output model, depicts a well-organised and predictable system. This is in contrast to the observed openness and unpredictability of this learning environment. The model also fails to provide for the influence of IT on social interactions and the resulting altered norms and rules. Finally, although it is called a social constructivist model, it does not show the process of self-organisation and enculturation clearly.

These shortcomings are addressed in the next chapter, by developing a theoretical framework for CSCML informed by Giddens' structuration theory and other applications of structuration theory.

Chapter 5

A Theoretical Framework for CSCML

5.1 Introduction

This chapter addresses the shortcomings of the enhanced model described in par. 4.5. This is achieved by focusing on the reciprocal influence between the structural aspects of the CSCML environment and human action/learning. Giddens has conceptualised this dualism as a duality in his structuration theory. By using structuration theory, a model is developed which accounts for the delicate interplay between the different components comprising this learning environment.

This chapter is divided into three parts. The first part consists of an in-depth discussion of Giddens' structuration theory. The second part presents two applications of structuration theory to conceptualisations of the use of Information Technology in organisations. These applications entail theoretical frameworks for CSCW and CSCL informed by structuration theory. A final theoretical framework for CSCML is developed and described in the third part. This is followed by an illustration of how the framework can assist in the better understanding of the dynamics of the CSCML learning environment by applying it to one of the case studies.

5.2 The theory of structuration

Giddens developed his social theory to ' ... *develop an ontological framework for the study of human social activities.'* (Bryant & Jary, 1991:201). It deals with conceptions of human being, human doing, social reproduction and social transformation (Giddens, 1984:xx).

5.2.1 Background

The development of Giddens' structuration theory can best be understood if it is seen as a reaction to development within the social sciences - structuration theory was formulated in part through ' ... - *the critical evaluation of a variety of competing schools of social thought.'* (op.cit.:xxxv). Until the 1960/1970s there was an apparent consensus on how social theory should be approached. Called by Giddens the 'orthodox consensus', human behaviour was seen as the result of forces that actors neither control nor comprehend (op.cit.:xvi). This view was rejected by many social scientists - they rather emphasised the active, reflexive character of human conduct and the importance of the role of language.

Structuration is offered as a social theory that is sensitive to the shortcomings of the orthodox consensus and to the importance of the new developments. As social theory it deals with ' ... *the nature of human action and the acting self; with how interaction should be conceptualised and its relation to institutions; and with grasping the practical connotations of social analysis.'* (Giddens, 1984:xvii). The focus is thus upon the understanding of human agency and social institutions. An important dualism in this regard is that between objectivism (focus on society) and subjectivism (focus on the human agent). Structuration theory attempts to reconceptualise this dualism as a duality, called the duality of structure. Giddens emphasises that although the theory recognises the importance of language and the 'linguistic turn' in social theory, it is not a version of interpretative sociology. In fact, Giddens acknowledges the call for the decentring of the subject, but does not imply with that the ' ... *evaporation of subjectivity into an empty universe of signs.' (op.cit.:xxii).*

5.2.2 The main premises of structuration theory

'(S)ocial practices, biting into space and time, are considered to be the root of the constitution of both subject and social object.' (op.cit.:xxii). The social structures of

societies (the social object) do not exist in a concrete sense, but are only instantiated in social activity over time. In this way the object/subject dualism is reconceptualised as a duality. It implies that ' ... *all social activity, ... can be viewed as enabled and constrained by social structures that are continually produced and reproduced via human agency.'* (Lyytinen & Ngwenyama, 1992:21).

5.2.3 Key concepts of structuration theory

The summary given above touches on some key concepts. These and several other important concepts comprising the theory will now be discussed. The concepts are closely related and will only be separated for analytical purposes in the discussion.

Human agents/agency

A main premise of the theory is the recursiveness (the self-reproducing nature) of human social activity, i.e. it is in, and through, the agent's activities that the conditions are reproduced that make these activities possible (op.cit.). Deeply involved in the recursiveness of action is the ' ... *reflexive form of the knowledgeability of human agents* ... '(op.cit.:3).

Knowledgeability refers to '*Everything which actors know (believe) about the circumstances of their action and that of others, drawn upon in the production and reproduction of that action, including tacit as well as discursively available knowledge.'* (op.cit.:375). Human actors are highly learned in respect of knowledge which they possess and apply in day-to-day conduct. This knowledge is more practical than theoretical. Giddens considers the knowledgeability of actors to be of fundamental significance and sees the actor's knowledge of daily social life, enabling them to 'go on' in diverse social contexts, as 'detailed and dazzling'. This knowledge is thus not incidental to the patterning of social life but an integral part to it (op.cit.:26).

The **reflexivity** refers to both a self-consciousness as well as the monitored character of the ongoing flow of social life. The ongoing flow of life not only presumes reflexivity but also enables it, by presenting a sameness across time and space. Furthermore, reflexivity presumes a purposive agent, who has reasons for her/his activities. However, action is not a combination of acts (where acts are seen as captured moments or, as described by Giddens (1984:3), a ' ... *discursive moment of attention* ... ') but a continuous flow of conduct. This implies that purposive action is not a series of separate intentions. From there the description of reflexivity as a continuous monitoring of action displayed (and expected to be displayed) by human beings. Giddens proposes a stratification model of action which is given in Figure 5.1.

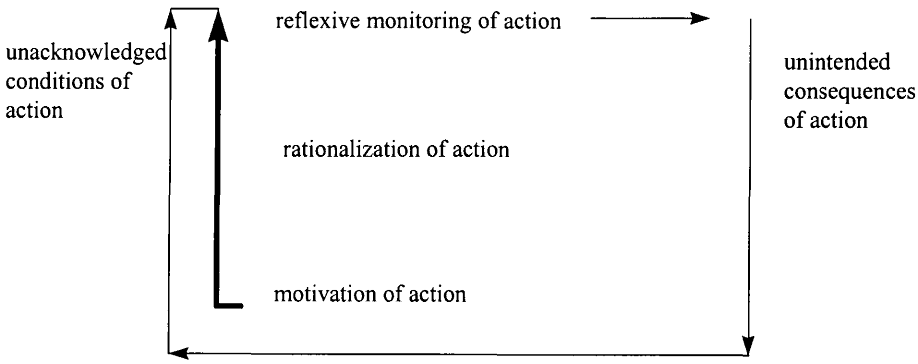


Figure 5.1

The stratification model of the agent

(Source: Giddens, 1984:5)

The model can be explained as follows:

Three layers of action are suggested: reflexive monitoring of action, rationalisation of action and motivation of action. Reflexive monitoring and rationalisation of action are directly involved with the continuity of action in the following way:

- actors **monitor** social and physical aspects of the context in which they move as well as the flow of their, and other's, activities,
- **rationalisation of action** refers to the continuing ' ... *capability competent actors have of "keeping in touch" with the grounds of what they do, as they do it, such that if asked by others, they can supply reasons for their activities.'* (op.cit.:376). These questions are usually asked only if activity seems puzzling or unconventional.

Since **motivation of action** refers more to the potential for action, rather than the modes in which everyday action is carried out, it is less involved in the continuity of action. However, unconscious motivation (the motives that cannot be reported discursively) plays a crucial role in human conduct. Giddens calls the non-discursive components of consciousness, practical consciousness (not to be confused with the unconscious). **Practical consciousness** consists of the things actors know tacitly about how to 'go on' in the context of social life without being able to give them direct discursive expression (op.cit.:xxiii). **Discursive consciousness** on the other hand, refers to what actors can say about the conditions of their own action. The **unconscious** refers here to the forms of cognition and impulsion which are repressed from consciousness or which appear only in distorted forms in consciousness. The unconscious can also motivate action but exclusive focus on these parts of motivational aspects of action, can lead to a reductive theory of consciousness which fails to explain the control agents characteristically have to sustain reflexively over their conduct. As was said before, actors' knowledge of social conventions, of oneself and of others, is an integral ingredient in the routinized character of daily social life. The knowledgeability is founded more upon practical than discursive consciousness. **Routinization** which is grounded in practical consciousness, is integral both to the continuity of personality of the agent and to the institutions of society. The **routinized** character of daily social life which is provided by the reflexive monitoring of social life provides an ontological security ' ... *based on an autonomy of bodily control* *within predictable routines and encounters.'* (Giddens, 1984:64). Giddens finds the motive behind this following of routines in the ' ... *prevalence of tact in social encounters, the repair of strains in the social fabric and the sustaining of "trust" ...* ', or differently put ' ... *a predominant concern with the protection of social continuity ...* ' (op.cit.:70).

Agency refers to the capability of the human actor to do things. '*Agency concerns events of which an individual is the perpetrator, in the sense that the individual could, at any phase in a given sequence of conduct, have acted differently. Whatever happened would not have happened if that individual had not intervened.'* (op.cit.:9). This definition implies that agency involves **power**. '*An agent ceases to be such if he or she loses the capability to "make a difference", that is, to exercise some sort of power.'* (op.cit.:14).

Agency refers to doing, not to the intentions behind the doing. Although daily social life occurs as a flow of intentional activity, acts have **unintended consequences** which could feed back to become **unacknowledged conditions** of further acts. Also, actors' knowledge about the condition of their actions and their reflexivity are bounded by the situated nature of action, the degree to which tacit knowledge can be communicated, unconscious motivation, and unintended consequences of action (Giddens, 1979). '*Human history is created by intentional activities but is not an intended project; it persistently eludes efforts to bring it under conscious direction.'* (Giddens, 1984:27).

Structure/ Structuration

Most social analysts view structure as some kind of 'social patterning' of social relations comparable to the skeleton of a building (op.cit.:16). Giddens, however, defines structure as a 'virtual order' of transformative relations which exists, as time-space presence, only in the instantiations in practices and as memory traces of human actors (op.cit.:17). **Structure** refers thus ' ... *to the structuring properties allowing the "binding" of time-space in social systems, the properties which make it possible for discernibly similar* *social practices to exist across varying spans of time and space and which lend them "systemic" form.'* (op.cit.:17). Structure is further described as rules and resources, that manifest in structural properties of the system. Two types of rules are defined, namely normative elements and codes of signification. Resources are divided into authoritative resources (co-ordination of the activity of human agents) and allocative resources (which stem from control of material products) (op.cit.:xxi).

Structural properties are structural institutionalised features of a social system giving solidity across space and time (op.cit.:23). These properties are institutionalised through the habitual use of rules and resources, in the ongoing human action. In this way structural properties are the medium of practices by providing the 'systemic' form of social systems which are drawn on by humans in their ongoing interaction. On the other hand, these structural properties are affirmed and reaffirmed through human action. **Structural principles** are those structural properties most deeply embedded, and implicated in the reproduction of societal totalities.

Giddens emphasises that whereas structure is *out of time and space*, **social systems** (through structuration) comprise ' ... *the situated activities of human agents, reproduced across time and space.'* (op.cit.:25). The analysing of the structuration of social systems implies the studying of the modes in which such systems, ' ... *grounded in the knowledgeable activities of situated actors who draw upon rules and resources in the diversity of action contexts, are produced and reproduced in interaction.'* (op.cit.:25). Thus, according to the notion of the **duality of structure**, ' ... *the structural properties of social systems are both medium and outcome of the practices they recursively organize.'* (op.cit.:25).

Structuration is thus seen as the *' ... structuring of social relations across time and space, in virtue of the duality of structure.'* (op.cit.:376).

Although structure was defined above as rules and resources, in a looser fashion structure can be spoken of as referring to principles of societal totalities (structural principles), rule-resource sets (structures), and structural properties. The identification of structural principles, '*... represents the most comprehensive level of institutional analysis. It refers to modes of differentiation and articulation of institutions across the "deepest" reaches of time-space.*' (op.cit.:185).

Modalities of structure

Giddens introduces the concept of 'modalities' of structuration to clarify the main dimensions of the duality of structure in interaction, by relating the knowledgeability of actors to structural features (op.cit.:28). '*Actors draw upon the modalities in the reproduction of systems of interaction, by the same token reconstituting their structural properties.*' (op.cit.:28).

Figure 5.2 shows the interdependence of the dimensions and modalities of the duality of structure.

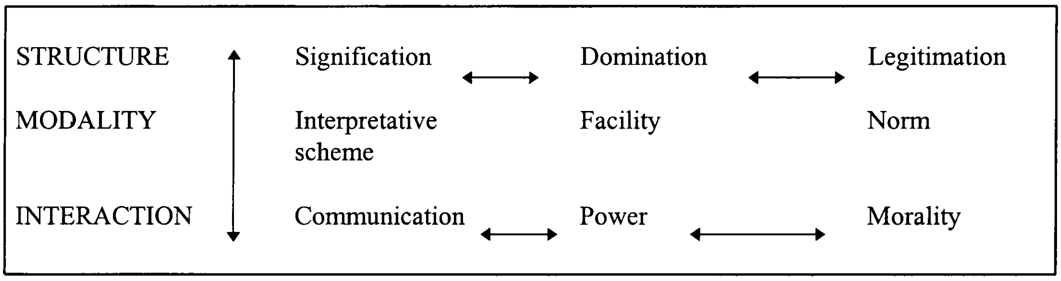


Figure 5.2

Elements of the Duality of structure

(Source: Giddens 1984:29)

Human interaction involves the communication of meaning. In this interaction, human actors draw consciously and tacitly upon the mutual stocks of knowledge called interpretive schemes (modes of typification). Also, interpretive schemes represent organisational structures of signification presenting rules that define and inform interaction. Giddens includes under signification semantic rules and '*all types of rules that are drawn upon as interpretive schemes to make sense of what actors say and do, and of the cultural objects they produce*' (Giddens in Bryant & Jary, 1991:10).

Action (and interaction) involves power in the sense of transformative capacity, i.e. the ability to intervene or not to intervene in the world. Power is being seen as the capacity to achieve outcomes, it is thus not necessarily an obstacle to '*... freedom or emancipation ...*', but its very medium (Giddens, 1984:257). Power in organisations (or social systems) is provided by resources on which actors can draw (and produce) in their interaction. Giddens distinguishes between allocative resources (forms of capabilities to generate commands over material phenomena) and authoritative resources (forms of capabilities to generate command over people) (op.cit.:83).

Also, these facilities constitute organisational structures of domination. The structures of domination depend on the symmetric properties between the two mentioned kinds of resources. A given structure of domination is reaffirmed if actors draw on a given asymmetry of resources. However, by explicitly changing the existing asymmetry of resources, structures of domination can be altered or undermined (Orlikowski, 1992:405).

Under the headings legitimation and morality, Giddens includes '*all types of rules that are drawn upon as norms in the evaluation of conduct*' (Giddens in Bryant and Jary, 1991:10). Interaction in organisations is ' *... guided by the application of normative sanctions, expressed through the cultural norms prevailing in an organisation.*' (Orlikowski, 1992:405).

On the other hand, norms constitute organisational structures of legitimation. This provides the moral code within an organisation which is sustained through morally accepted practices and traditions.

These elements are only analytically distinct and are actually interlinked in the production and reproduction of social relations. For example, the communication of meaning in interaction involves normative sanctions, since language use is sanctioned by the nature of its 'public character' (Giddens, 1984:28). Also, accountability involves interpretative schemes and norms since reasons must be explicated and normative grounds need to be supplied (op.cit.:30).

The dynamics of structuration are illustrated in Figure 5.3.

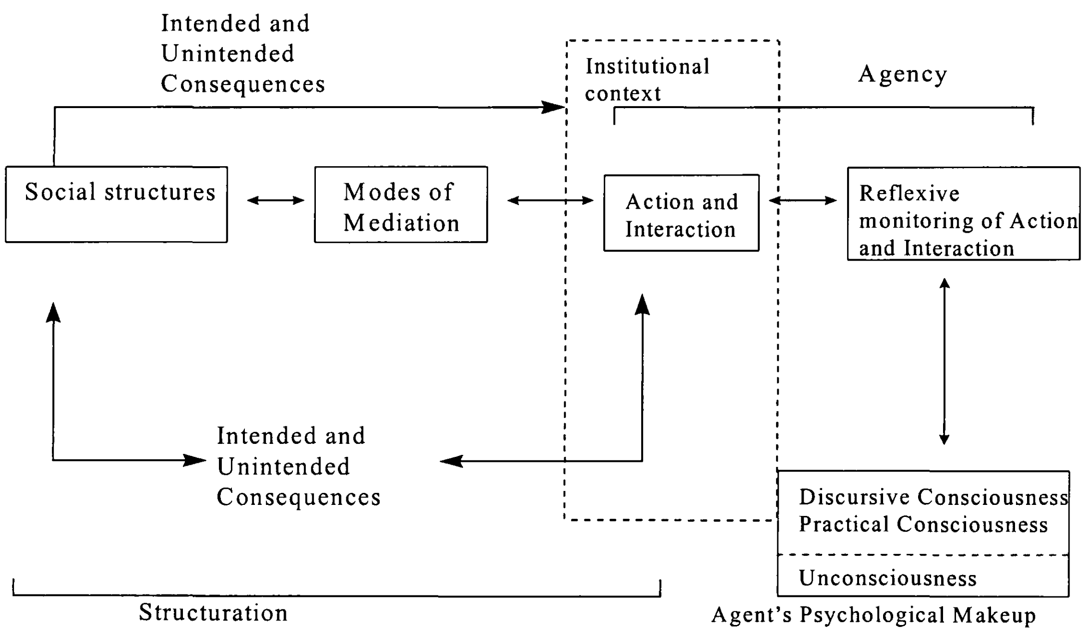


Figure 5.3.

The dynamics of structuration

(Source: Lyytinen & Ngwenyama, 1992:24)

Rules and Resources

As was said before, the rules and resources drawn upon in the production and reproduction of social action are also the means of system reproduction. However, when speaking of structure as rules and resources, there is a risk of misinterpretation because of certain dominant uses of 'rules' in other literature. Giddens makes it clear that

- the rules implicated in the reproduction of social systems are usually subject to far greater diversity of contestations than the rules of games (op.cit.:18),
- most of the rules are only tacitly grasped by actors, they know how to 'go on'. '*The discourse formulation of a rule is already an interpretation of it.*' (op.cit.:23),
- '*Rules cannot be conceptualized apart from resources, which refer to the modes whereby transformative relations are actually incorporated into the production and reproduction of social practices.*' (op.cit.:18),
- rules are procedures of action and therefore imply methodological procedures,
- rules have two sides to them, on the one hand the constitution of meaning, and on the other the sanctioning of modes of social conduct (op.cit.:18).

Giddens considers different examples of rules in trying to explicate the concept further. For example, the rule ' It is a rule that all workers must clock in at 8:00 am', is both constitutive and regulative - it regulates work practices and constitutes concepts about '... *industrial bureaucracy* ... ' (op.cit.:20). The rule $a_n = n^2 + n - 1$, according to Giddens, gives a very useful example of what the most analytically effective sense of 'rule' is in social theory (op.cit.:20). Here someone writes down a series of numbers and the other works out a formula supplying the numbers that follow. Understanding this formula involves the ability to apply it in the right context and way to continue the series. Giddens sees linguistic rules in a similar way. '*To understand a language means to be a master of a technique. This can read to mean that language use is primarily methodological and that rules of language are methodically applied procedures implicated in the practical activities of day-to-day life.*' (op.cit.:21).

As was said before, structures of signification are only analytically separable from domination and legitimation. Domination depends upon the mobilisation of two kinds of resources mentioned above, namely allocative and authoritative resources. Allocative resources might seem to have a real existence (a time-space presence), different from other structural properties as a whole (op.cit.:33). However, they still become resources (as transformative capacities) when incorporated within the process of structuration (op.cit.:3). Any co-ordination of social systems across space and time involves a combination of these two types of resources. Giddens classifies resrouces as follows:

Table 5.1

Classification of resources

(Source: Giddens, 1984:258)

Allocative Resources	**Authoritative Resources**
1. Material features of the environment (raw materials, material, power sources).	1. Organization of social time-space (temporal-spatial constitution of paths and regions).
2. Means of material production/ reproduction (instruments of production, technology)	2. Production/reproduction of the body (organization and relation of human beings in mutual association).
3. Produced goods (artefacts created by the interaction of 1 and 2).	3. Organization of life changes (constitution of changes of self-development and self-expression).

The storage of resources is closely involved with the stretching of social systems across time-space (time-space distanciation) and thus the generation of power (op.cit.:259). Indeed, the expansion of capitalism to form a new world economy would not have been possible without the development of techniques for the preservation and storage of perishable goods (op.cit.:259, 260). It sounds thus as if human history is a sequence of ' ... *enlargements of the "forces of 'production".'*(op.cit.:260). However, ' ... *allocative resources cannot be developed without the transmutation of authoritative resources, and the latter are undoubtedly at least as important in providing "levers" of social change as the former.'* (op.cit.:260).

The first category of authoritative resources refers to the forms of ' ... *regionalization within (and across) societies in terms of which the time-space paths of social life are constituted.'* (op.cit.:260). The second category, the production/reproduction of the body should not be seen as similar to material production/reproduction. The ' ... *co-ordination of numbers of people together in a society and their reproduction over time is an authoritative resource of a fundamental sort.'* (op.cit.:260). The third category, organisation of life-chances, is again not only dependent upon the material productivity of a society. '*The nature and scale of power generated by authoritative resources depends not only on the arrangement of bodies, regionalized on time-space paths, but also on the life chances open to agents.'* (op.cit.:261). 'Life chances' means not only chances for survival for human beings in different regions of society, but includes the whole range of aptitudes and capabilities, e.g. literacy.

Giddens sees the storage of allocative and authoritative resources as involving the retention and control of information or knowledge, that continue social relations across time-space. Storage presumes modes of information (e.g. books, files, films, etc.), modes of retrieval and modes of dissemination. The retrieval of the information depends on human memory and interpretative skills. Technology influences the dissemination of information . Also, ' ... *the character of the information medium, ... , directly influences* *the nature of the social relations which it helps to organize.'* (op.cit.:262). It is the containers which store the resources that generate the major types of structural principles in the constitution of societies.

Constraints

Structuration theory is based on the proposition that structure is both enabling and constraining. The various forms of constraints are thus also forms of enablement, opening possibilities of action at the same time that they restrict others. Giddens distinguishes between three types of constraint:

- Material constraint, which is constraint ' ... *deriving from the character of the material world and from the physical qualities of the body.*' (op.cit.:176). This includes the indivisibility of the body, the finitude of the life span, the limited capability of human beings to participate in more than one task at once, the fact that movement in space is also movement in time and the limited packing capacity of time-space (only one human being can occupy a space at one time) (op.cit.:111,112).
- (Negative) sanctions, which are constraints ' ... *deriving from punitive responses on the part of some agents towards others.'* (op.cit.:176). The constraining aspects of power are seen as sanctions of a different kind, ranging from application of force to the mild expression of disapproval (op.cit.:175).
- Structural constraint, which is constraints ' ... *deriving from the contextuality of action, i.e., from the "given" character of structural properties vis-á-vis situated actors.*' (op.cit.:176). Instead of seeing the structural properties of social systems as walls of a room from which individuals cannot escape, but inside which he or she is able to move around freely, structuration theory views structure as implicated in that very 'freedom of action' (op.cit.:174). Even in reacting to 'inevitable' social forces (social forces which actors are unable to resist), it only means that human actors are ' ... *unable to do anything other than conform to whatever the trends in question are, given the motives or goals which underlie their action.'* (op.cit.:178).

Society/Social systems

Social systems comprise ' ... *the situated activities of human agents, reproduced across time and space.'* (op.cit.:25). The situated activities are organised as regularised social practices, which with time, become reaffirmed as social properties - they start to appear as 'objectively given'. Reification, according to Giddens, refers not to the thing-like nature of the social properties, but to the consequences of thinking in this way. It is thus a form of discourse, ' ... *in which properties of social systems are regarded to have the same fixity as that presumed in laws of nature.'* (op.cit.:180). This apparent 'fixity' of social properties probably lies behind the popular view of societies as units with clearly demarcated boundaries. Giddens does not agree with this view and emphasises the variation in the degree of systemness of societies. According to him societies are social systems but also constituted by the intersection of multiple social systems. Such intersocietal systems are described as social systems which ' ... *cut across whatever dividing lines exists between societies or societal totalities, including agglomerations of societies.'* (op.cit.:375). Societies then, are ' ... *social systems which "stand out" in bas-relief from a background of a range of other systemic relationships in which they are embedded.'* (op.cit.:164). The societies stand out because structural principles serve to produce a specifiable overall clustering of institutions across time and space.

Social and System Integration

Structuration theory presumes that the conduct of individual actors reproduces the structural properties of larger collectivities. Giddens distinguishes between two ways in which elements of 'systemness' are created in interaction: **homeostatic causal loops** are causal factors that have a feedback effect which is mainly the outcome of unintended consequences; **reflexive self-regulation**, on the other hand, refers to causal factors with a feedback effect, which is mainly the outcome of the knowledge of actors and their use of it to control system reproduction. Most social theories acknowledge the effect of causal factors with largely unintended feedback on system reproduction, but structuration theory adds the importance of intended and reflexive self-regulation (Bryant & Jary, 1991:8).

Giddens adds two further important concepts relevant to system reproduction, namely social and system integration. Social integration refers to systemness on the level of face-to-face interaction, whereas system integration refers to the reciprocity between actors who are physically absent in time or space (Giddens, 1984:28). The mechanisms of system integration are based on the mechanisms of social integration but differ in some key respects.

Time/Space/Context

The study of contextualities of interaction is inherent in the investigation of social and system integration and thus of social reproduction (op.cit.:282). '*All social life occurs in, and is constituted by, the intersections of presence and absence in the "fading away" of time and the "shading off" of space.*' (op.cit.:132). **Contextuality** refers to the situated character of interaction in time-space, involving the setting of interaction; the co-presence of actors; and the communication between them (op.cit.:373). The terms *locale* and *regionalisation* are of importance here: **locale** refers to the use of space providing the settings of interaction, and **regionalisation** to the zoning of time-space in relation to routinized social practices (op.cit.:119). For example, locales can range from houses to areas occupied by nation-states. However, locales are internally regionalised (e.g. houses are regionalised into floors, halls and rooms) and the regions within them play important roles in the constitution of contexts of interaction.

The physical properties of the body and the surroundings in which it moves give social life a serial character and limit access to others not present. The positioning of the body in social encounters is thus fundamental to social life. The body is not only positioned in the immediate circumstances of co-presence in relation to others, but also ' ... *in the flow of day-to-day life; in the life-span which is the duration of his or her existence; and in the* *duration of "institutional time", the "supra-individual" structuration of social institutions.'* (Giddens, 1984:xxiv).

The human agent, and thus social interaction, is located in time-space. The 'locus' of the human agent is his/her body which has a finite or irreversible life-span. However, through structuration, social systems are recursively produced and reproduced giving it a 'reversible' life-span, i.e. through structuration (which is made possible by the recursiveness of the day-to-day practices), the temporality of human practices is transcended by the stretching of social relations across space and time (the reversible life-span of institutions). Giddens calls this phenomena **time-space distanciation**.

5.2.4 Remarks

The above gave a summary of the most important concepts of the structuration theory of Giddens. With this theory Giddens offers a way out of the conceptual divide between objectivism and subjectivism. Instead he presents the dualism as a duality, where social activity is regarded as being enabled and constrained by social structures that are produced and reproduced via human agency.

Central to this theory is the knowledgeable human agent and the recursiveness of human action. Giddens proposes a stratification model for the agent. The model suggests three layers of action, namely reflexive monitoring of action, rationalisation of action, and motivation of action. Reflexive monitoring of conduct presumes a purposive, self-conscious agent. Rationalisation of actions refers to the tacit continual theoretical understanding actors have of their own and other's actions. Motivation of action refers more to the potential for action, rather than the modes in which everyday action is carried out. Unconscious motivation plays a crucial role in human conduct. The three layers of consciousness linked to the three layers of conduct, are discursive consciousness (what actors can say about the conditions for their own actions), practical consciousness (what is known but cannot be communicated), and the unconscious.

Social structures have a 'virtual nature' in the sense that they exist only in memory traces in the individual's mind and become instantiated only in action. They are described as rules (normative elements and codes of signification) and resources (material and non-material).

By drawing upon the rules in everyday conduct, meaning is communicated, and conduct judged and evaluated. Drawing upon resources involves the exercise of power. Actors draw upon these rules and resources in their everyday conduct, thereby producing or reproducing the structures.

In contrast to social structures, social systems are situated in time and space, comprising of the situated activities of actors, reproduced across time and space. The systemness of social systems is created through homeostatic causal loops (feedback which is the effect of unintended consequences) and reflexive self-regulation (feedback which is the outcome of purposeful action).

It is the belief of the author that with the help of this theory a better understanding could be reached of the interplay between the different components of the CSCML environment. The production and reproduction of social rules and habits within the CSCML environment will be accounted for. Moreover, social constructivism shows similarities to structuration theory in that it tries to overcome the dualism between object and subject. With the help of structuration theory, the CSCML environment can be modelled, taking the dynamics of social constructivism fully into account. This will be dealt with in par. 5.4.

5.3 Information Technology

Information technology can be defined as the use of electronic equipment for storing, analysing and distributing information. Being a vast field of interest, there is little agreement on the definition and measurement of technology. However, there is no doubt that information technology and the integration of it in different environments is a complex issue involving numerous aspects.

5.3.1 Conceptualisations of technology

Orlikowski(1992) emphasises the lack of agreement on the definition and measurement of technology, despite years of research efforts. Prior conceptualisations of technology focused on specific aspects of technology, at the expense of others (op.cit.:398). Orlikowski (1992:398-403) gives an overview of different conceptualisations of technology which will be summarised below.

Two important aspects of technology are **scope** and **role**. **Scope** refers to that which comprises technology and **role** to the interaction between organisation and technology.

A set of studies focused on the **scope** of technology as 'hardware' i.e. equipment and machines used in productive activities. This led on the one hand to context-specific definitions of technology, making comparisons across studies and settings difficult. On the other hand, the use of broad definitions became too abstract, limiting informational value. The hardware view was extended to 'social technologies', including generic tasks, techniques and knowledge used when humans engage in productive activities. This conceptualisation of technology is useful in that it recognises technology to be an important variable in organisations. However, this approach creates boundary and measurement ambiguity. It also fails to look at the mediation of human action by machines, e.g. addressing questions like 'How do artefacts interact with human agents?', etc.

The research that focuses on the **role** played by technology in organisations can be divided into three streams which reflect the opposition between the objective and subjective realms discussed in Chapter 1. The earlier stream consisted of studies which view technology as an objective, external force with deterministic impacts on organisational properties such as structure. Later work focused more on the human aspects of technology, *seeing it as a product of shared interpretations or interventions.* According to Orlikowski (op.cit.), these studies sometimes rely too heavily on the capability of human agents and ignore the social and economic forces. The most recent work - the 'soft' determinism school - views technology as an external force having impacts, but where the impacts are influenced and moderated by human actors in organisations. For example, Barley (in Orlikowski, op.cit.), views technology as a potential changing force, intervening in the relationship between human agents and organisational structure. This view resulted from the observation that different organisations responding differently to the implementation of a similar technology. He proposes a role for technology as trigger, setting off social dynamics that lead to intended and unintended structuring consequences. Barley, however does not allow for the physical alteration of technology during use. Orlikowski stresses the point that some technologies can be modified during use, especially information technology.

As part of the last perspective, several researchers have proposed models for the role of technology (and specifically IT) in organisational processes, based on the structuration theory of Giddens (discussed in par, 5.2). Structuration theory conceptualises the importance of both structure and human agency as a duality. Two research studies which use structuration theory as meta-theory to enhance the understanding of the role of technology in organisations will now be discussed. The first study gives a structurational analysis of CSCW (Lyytinen and Ngwenyama, 1992) and the second study looks at the complexities of the computer-supported co-operative learning environment (De Villiers, 1995).

5.3.2 A structurational analysis of Computer Supported Co-operative Work (CSCW)

Lyytinen and Ngwenyama (1992) contribute to the research body on CSCW, by presenting a theoretical foundation for further research on CSCW informed by Giddens' structuration theory.

From a structurational perspective ***co-operative work*** is defined as co-operative practices drawing upon rule, and resource, sets that are produced and reproduced through shared, recurrent interactions among individuals. It is also characterised by complex and intense interdependencies of activities, which are dependent on the shared understanding of the work process. The relationships between human agents are formed through planned structuring and deliberate discursive action. It differs thus from a spontaneous linking. Co-operative work relies on the reciprocity of practices and the development of a reservoir of taken-for-granted activities enabling frictionless encounters and effective interaction. The co-operative effect rests on established systems of signification (interpretive schemes). Giddens' concept of agency holds implications for this analysis: co-operative work provides for ontological security expressed on an unconscious level; on the level of practical consciousness, co-operative work enhances the development of rich stocks of knowledge that sustain and provide rationalisations of the co-operative behaviour of agents; and on the level of discursive consciousness, co-operative work involves the capabilities of agents to refine and evaluate their practices.

Lyytinen and Ngwenyama (1992) use an ontologically focused approach to define ***Computer Supported Cooperative Work*** that conceptually characterises the nature of computer support in co-operative practices. This is in an attempt to avoid the one-sided analyses of technologies at the expense of understanding the social nature of computer supported co-operative practices. They define CSCW as: '*Computer Supported Cooperative Work applications are open evolutionary structures embedding organizational and linguistic rules and serving as resources that mediate and transform cooperative interactions via recurrent use-processes (procedures and practices) within specific organizational contexts.*' (Lyytinen & Ngwenyama, 1992:26).

Social structures

CSCW applications act as resources by embodying means and materials of labour, around which co-operative work can be structured. Computers also embed rules in codified form which mediate and transform communication practices, organisational rules and norms, hierarchies and role expectations. The division between rules and resources as two modes of computer support is not clear-cut: any CSCW application entails the presence of both.

Human agents are assumed to be skilful and knowledgeable actors in drawing upon CSCW applications to support their co-operative activities. The level of support is influenced by the scope of the activity in 'co-operative work', the organisational context within which the social interactions are computer mediated, and the actors' capacities and involvement in the interactions (op.cit.:26).

Characteristics of CSCW applications

Lyytinen and Ngwenyama (op.cit.) believe that CSCW applications are distinguished from other computer applications by three characteristics: emergent properties, use-processes and organisational contextuality.

Emergent properties

This refers to features such as openness, evolutionary nature and goal ambiguity. These features instantiate combinations of rules and resources that support innovation and reinvention of co-operative work practices. An *openness* must be displayed in the application of CSCW practices to provide for rules (that are tacit and informal), the practical consciousness of agents and unintended consequences of implementation.

CSCW has an *evolutionary nature* due to the continuous reshaping and redirecting of organisational practices as a result of the adaptation of CSCW applications. '*As users interact over time via these applications they will produce and reproduce structures of meaning, create and recreate new facilities to support emerging work patterns, and new norms and standards of co-operative work.'* (op.cit.:29).

From a structuration theory perspective, organisations have no *goals:* agents can never describe their actions completely, and thus goals are always ambiguous and negotiable. The goals of every co-operative work situation are negotiated and renegotiated by actors to adapt them to their own needs. In this way then CSCW applications differ from more traditional information systems: they do not support clearly definable organisational goals. It is also difficult to identify quantifiable benefits from the use of CSCW applications.

Use-processes

Three use-processes are defined: collective, autonomous and interpretive.

Collective use implies that the system should provide an arena for significant computer mediated interactions among the operators. Actors should view CSCW applications both as mediums for, and products of, meaningful social interactions (e.g. bulletin boards). The collective use-processes also increase the level of system integration where actors are located at different time-space co-ordinates.

In contrast to traditional information systems, there is a high degree of voluntary participation in CSCW applications. This could be referred to as the *autonomous use*. Although an individual can decide to use applications or not, social pressure and high social visibility sometimes motivate participation. Thus, the use of CSCW applications is often legitimised by group pressure and symbolic value rather than by institutionalised rules.

The *interpretive mode* of use implies that the meaning of data is not fixed but is interpreted and reinterpreted by the users. Consequently, the type of rules are intensive, tacit, informal and weakly sanctioned.

Organisational contextuality

The CSCW applications are media and products of co-operative interactions that are embedded in a larger organisational context. The use of the system is enabled and constrained by the organisational structures (e.g. the organisational context could include contradictory, deep-seated practices that influence the co-operative practices.) Organisational practices could explain obstacles to implementation of CSCW applications.

The interactions between the characteristics and structures of CSCW within the context of structuration theory are given in Figure 5.5.

This framework could assist in understanding the nature of computer support in co-operative learning practices, that is, if co-operative learning is seen as a dialectical process between reflexive monitoring and rationalisation of action toward changing a situation. This duality shapes organisational reality, and thus the frameworks of meaning through which individuals make sense of their actions. (Sheather, Martin & Harris, 1993)

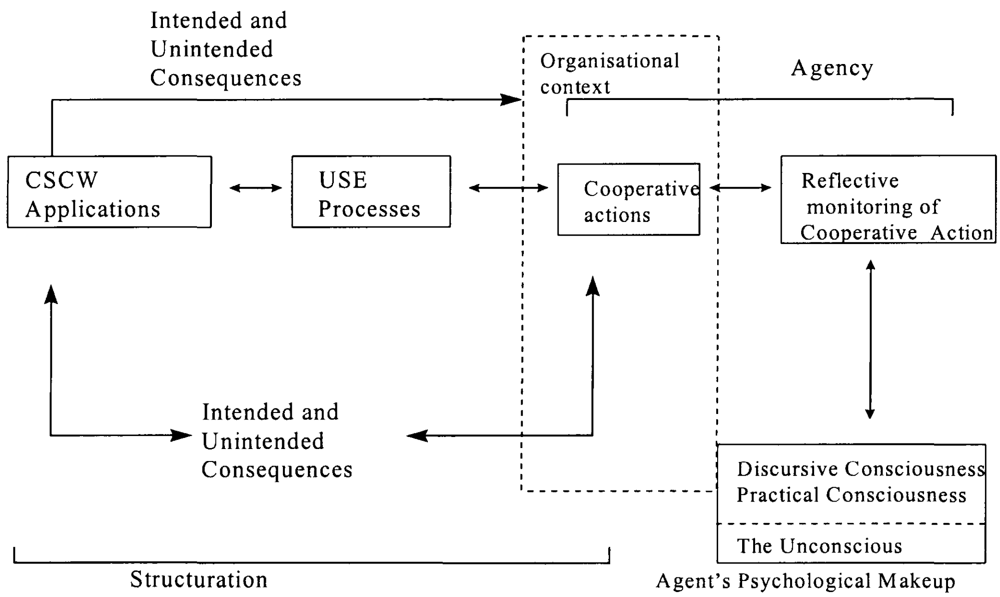


Figure 5.5

Structuration dynamics of CSCW

(Source: Lyytinen & Ngwenyama, 1992:27)

5.3.3 A theoretical framework for CSCL

De Villiers (1995) proposes CSCL as an organisational idea that has the potential to connect educational and working environments. She draws on research on systems theory and CSCW (specifically applications of structuration theory and adaptive structuration theory (DeSanctis & Poole, 1994)) to propose a theoretical framework for CSCL. This framework describes the CSCL process as a whole. The main concepts comprising the framework will now be discussed.

CSCL as social system

The social structures provided by CSCL not only support the learning environment (as traditional uses of IT), but also co-ordination and communication between learners and teacher. The CSCL environment is described as a system, recognising the activities of the components but at the same time considering the activity of the whole system that contains them.

The components

The following components are of importance:

- Groups of learners - groups must be formed, keeping in mind group dynamics and the basic elements of co-operative learning.
- Teacher - the teacher should mediate learning through dialogue and co-operation, and should execute the following tasks: present a learning task; control the group dynamics; keep the groups on task; unify the groups by offering encouragement; organise the activities, materials, equipment, etc.; monitor group work; be available to individuals; and observe the thinking processes.
- Co-operative learning methods - the methods should be chosen with the objectives, subject matter, evaluation methods and level of learners in mind.
- Information technology - technologies should be chosen in line with the topic and subject to be taught.
- Physical learning environment - the arrangement of desks and computers should encourage group discussion.
- Course materials - course material should be collected or developed by teachers.

Functions

This system has five basic interacting functions:

- *Input* (the elements that enter the process): learners with little or no subject knowledge, the level of the learners, study material, training of learners, the learning task and the planning of the groups.
- *Processing* (the transformation of input to output): the learning and teaching process (summarised in par. 2.4.1.4).
- *Output* (the goal of the process): learners with subject knowledge, output from CSCL (skills), output from the learning task (new information generated) and the output from the organisational environment.
- *Feedback*: data about the performance of the system; the teacher monitors the group processes, adjusts and coaches the groups accordingly.
- *Control:* evaluation of the system to determine whether the system is achieving its goal. This includes the evaluation of the learners and groups to ensure that the objectives have been achieved.

Organisational learning

Organisational learning can take place in the evaluation process. Organisational learning takes place when members of an organisation learn by responding to changes to the internal and external environments of the organisation by detecting and correcting errors in organisational theory-in-use. This results must become shared maps of the organisation (Argyris & Schön in de Villiers, 1995). It takes place as:

- Single-loop learning: e.g. a teacher realises during evaluation and feedback that course material should be adapted before continuing the process.
- Double-loop learning: e.g. changing and correcting norms and assumptions of the CSCL environment in an attempt to correct errors.
- Deutero-learning: e.g. teachers learn more about CSCL and develop their methods accordingly.

De Villiers uses the Adaptive Structuration theory (AST), to describe both the social as well as the technical character of the CSCL environment. The key concepts of AST are structuration and appropriation.

Structuration

Structural features are the specific types of rules and resources offered by the CSCL environment (e.g. co-operative learning methods and the formation of groups of learners). Other sources of structures are the learning task, the level of learners and the organisational environment in which learning takes place. These features bring meaning (signification) and control (domination) to group interaction. New sources of structure emerge as CSCL, the learning task and organisational structures are applied.

The *spirit* of the social structures is the general intent with respect to values and goals underlying that set of structural features. The spirit provides legitimation to CSCL by providing a basis for appropriate behaviour in the context of the CSCL environment. De Villiers finds that the goal and values that are promoted and supported by the CSCL environment are: better self-esteem, improved human relations and improved communication skills. Spirit is identified by analysing the CSCL method, the features incorporated in the environment, the study materials, and the training and assistance provided by the mediator to the learners.

Appropriation of Structure

The appropriations of the CSCL environment, the learning task and organisational environment are the immediate, visible actions that evidence deeper structuration processes. These actions make up the learning process. The learning process as described by De Villiers (1995) is discussed in par. 2.4.1.4. The teacher plays an important role in guiding the process of appropriation.

The appropriation of structures is influenced by:

- The degree of knowledge and experience of the group members with regard to the structures of the CSCL environment.

- The style of interaction among group members.
- The degree to which group members believe that other members know and accept the use of the structures.
- The degree of agreement on which structures to appropriate and the willingness to participate in the learning process.

Learning outcomes

Effective and quality learning, commitment amongst learners to the learning process and consensus within the groups are seen as desired learning outcomes. These outcomes also have an influence on the social interactions. Desired learning outcomes are more likely to occur if appropriations are faithful to the spirit of CSCL, if the number of technology appropriations is high, if there are more on-task instrumental uses of technology and if attitudes are positive towards appropriation.

Figure 5.6 summarises the main components and its interplay in the CSCL environment.

5.3.4 Some critique

An important premise of structuration theory is that the social structures of societies (the social object) do not exist in a concrete sense, but are only instantiated in social activity over time. In this way the object/subject dualism is reconceptualised as a duality. De Villiers and Lyytinen & Ngwenyama disregard this important concept by equating structure (rules and resources) to information systems. Information systems in their physical form only represent interpretations of rules. They become rules and resources when instantiated within social action through the agency of individual actors (Lamprecht, 1997:201).

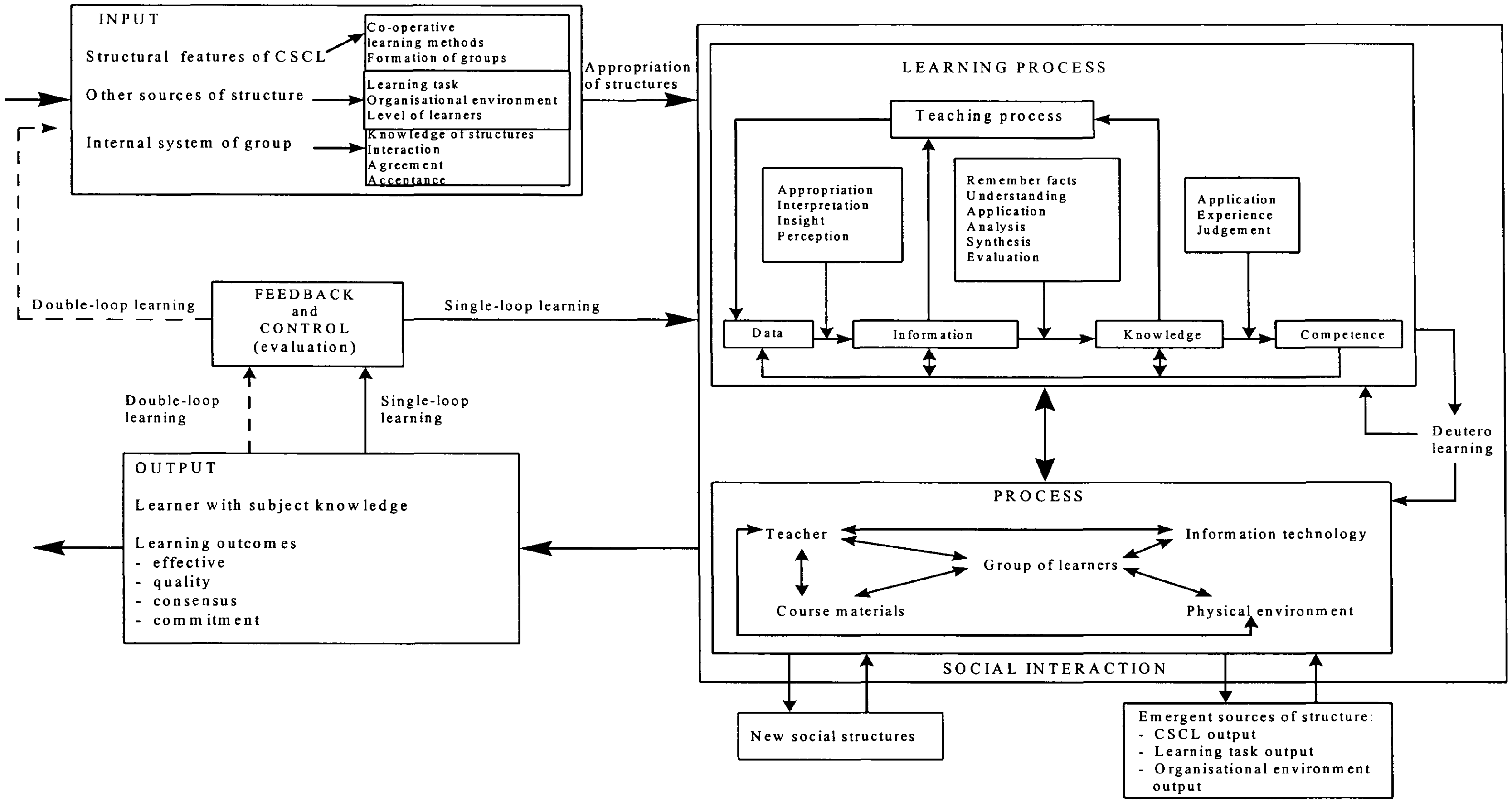


Figure 5.6, A Theoretical Framework for CSCL, (Source: De Villliers, 1995:225)

5.4 A theoretical framework for the CSCML environment

As was pointed out in par. 5.2.4, it is the belief of the author that with the help of structuration theory a better understanding could be reached of the interplay between the different components of the CSCML environment. Since mathematics is seen as a social construction, the theory of Giddens could provide useful insights into the social and private learning activities taking place in the CSCML environment. The production and reproduction of social rules and norms within the CSCML environment could also be accounted for. Since CSCL can be seen as a special application of CSCW, Lyytinen and Ngwenyama's (1992) application of structuration theory to CSCW, discussed in par. 5.3.2, will also be used in the development of the model for CSCML.

Giddens views the social structures of societies (the social object) as not existing in a concrete sense, but as 'memory traces' in the agent's mind. The structures are only instantiated in social activity over time. In this way the object/subject dualism is reconceptualised as a duality: the social object exists within the realm of the social subject. Structures are produced, and reproduced, by the eventual accommodation (Skemp, 1971) or reaffirmation of the 'memory traces' (or mental schemas) and the instantiation of them through action. It is thus clear that learning accompanies the structuration process.

A structurational perspective of the CSCML environment

The CSCML environment is interpreted from a structurational perspective as a *social system* in which interpretations of *organisational and mathematical structures* of signification, legitimation and domination are represented by the *CL methods/principles*, *IT, objective mathematical knowledge and the learning task.* These components act as *modalities of structure* upon which actors can draw in *co-operative actions* to reconstitute or change the structural properties of the components. This takes place by the *changing or reaffirmation of the agent's mental schemas* in a specific *organisational context*. Figure 5.7 illustrates the description.

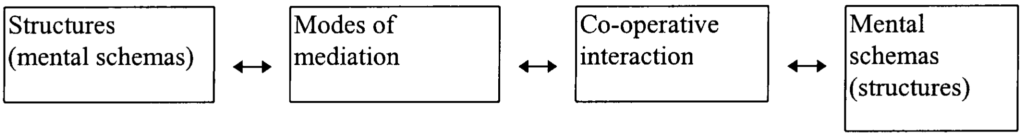


Figure 5.7

The structuration and learning process

Although the elements in the above discussion are inseparable, they will now be separated for the purpose of analytical discussion.

Social system

Giddens describes social systems as ' ... *the situated activities of human agents, reproduced across time and space.'* (Giddens, 1984:20). The CSCML environment as a social system consists of the co-operative interaction (and individual activities) of the teachers and learners in a mathematical learning environment. The social activities have a routinized and regularised character, providing it with time the appearance of 'objectively given'.

The system can in a certain sense be described as an intersocietal system since it shares rules and norms with the macro mathematical community (the professional mathematical community). The shared rules include the accepted mathematical knowledge and certain working practices. The teacher is seen as the representative of the macro-community and thus has to mediate between the individual's personal meanings and the culturally established mathematical meanings of the macro community.

Systemness is created through homeostatic loops and reflexive self-regulation. Reflexive self-regulation refers to the causal factors having a feedback effect which is the outcome of the actors' use of their knowledge to control system reproduction. By applying CL methods, the norm of 'group processing' (par. 3.2.3) is instantiated. This is a form of self-regulation where groups reflect on their functioning and introduce ways of improving it. The teacher, in particular, uses his/her expert knowledge of CL methods and mathematical knowledge to control system reproduction. The concept of organisational learning can be applied here (De Villiers, 1995).

Organisational learning takes place when members of an organisation learn by responding to changes to the internal and external environments of the organisation by detecting and correcting errors in organisational theory-in-use. These results must become shared maps of the organisation (Argyris & Schön in de Villiers, 1995). It takes place as:

- Single-loop learning: e.g. the teacher realises during evaluation and feedback that course materials should be adopted before continuing the process.
- Double-loop learning: e.g. changing and correcting norms and assumptions of the CSCL environment in an attempt to correct errors.
- Deutero-learning: e.g. teachers learn more about CSCL and develop their methods accordingly. (After conducting case study 1, the facilitator realised that problem solving skills need to be made explicit and consequently introduced the concept of a problem solving expert in case study 2).

Social and system integration is not only encouraged by the CL principles 'face-to-face promotive interaction' and 'positive interdependence' (par. 3.2.3), but also by the IT applications (e-mail, video-conferencing, etc.).

The co-operative mathematics learning process

The assumption of social constructivism is that learning is a ' ... *process of both self-organization and a process of enculturation that occurs while participating in cultural practices, frequently while interacting with others.'* (Cobb, 1994:18). Social constructivism is seen here as a philosophy that recognises both the importance of the individual's sense making in his/her experiential world and the effect of the social environment and culture on this sense making process. Too strong a focus on the individual leads to solipsism and ignores the society's shared sets of values and concerns. Too strong a focus on culture and society, on the other hand, leads to the ignoring of individual differences and the active role of the individual in the constitution of classroom culture. The social constructivist view adopted here (a radical constructivism with the recognition of the importance of the social context), has been criticised as the mixing of incompatible theories that lead to linguistic knowledge slides (Smith, 1994). This difficulty can be overcome by using the structuration theory of Giddens to obtain a better understanding of the social context of mathematics learning.

Learning is seen as an effect of interaction (Voigt, 1996): through social interaction (drawing on structures), subjective mathematical knowledge becomes compatible with the structures. By this process the structures are reinforced. 'Structures' indicate here the accepted rules and resources represented by objective mathematical knowledge as well as the rules of accepted mathematical and classroom practices.

The interaction process can also be seen as social negotiation which is a process of adaptation in the course of which teacher and learner establish expectations of each others' activity and obligations for their own activity (Bauersfeld in Cobb, 1994).

The co-operative mathematics learning process supported by IT proceeds in more detail as follows:

- The teacher presents the group with a mathematics learning task. The learning task involves the use of IT and objective mathematical knowledge.
- By individual and collective action on representations of mathematical knowledge, private mental mathematical concepts are 'activated'. The representations of the mathematical knowledge could be in the form of verbal-algebraic, visual or kinetic images that are presented electronically, verbally or in writing.

- However, mathematical concepts are ambiguous: the dual nature of mathematical concepts has already been discussed in par. 2.4.3.2. Also, individuals have unique ways of representing subjective mathematical knowledge.
- The CL methods provide norms of co-operation and goal-directed activity. By drawing on these norms, groups work towards the unambiguous understanding of mathematical concepts. Individuals experience mathematical concepts as unambiguous in the following way: interaction in the classroom is seen as more than a series of separate acts, but as a continuous flow of conduct, where the actor reflexively monitors his/her own actions in accordance with what he/she assumes the others' background understanding and expectations are. The others interpret the actions of the individual, adopting what they believe to be the actor's background understanding and expectations. In this way a background understanding is taken as 'objectively given' and mathematical concepts are experienced as unambiguous (Voigt, 1996).
- However, this does not mean that the individual group members have the same conception of the mathematical concepts, even if they collaborate without conflict. The more the background understanding of learners differ, the more likely it is that their understanding of mathematical concepts will differ.
- Because of the disparity between the background knowledge of the teacher and learners as well as the teacher's role of mentor, learners and teachers have to negotiate mathematical meanings.
- Through negotiation (between learner and learner, and teacher and learner), a shared stock of mathematical meaning is formed (called a 'theme' by Voigt (op.cit.)). The theme is not fixed but is interactively constituted through negotiation.
- With time, the theme gains stability because mathematical discussions are constrained and guided by mathematical structures and because of the need for routine to ' ... *protect social continuity ...* ' (Giddens, op.cit.:70). The knowledge now becomes institutionalised.

Even though a consensus was reached, misconceptions can still exist in the mind of the individual. A part of the individual's mathematical knowledge lies in the private realm, or practical consciousness. It is only by making this knowledge public, or shifting it to the discursive consciousness that the misconceptions can be identified by the teacher and other learners. This is usually a difficult process since the 'private' knowledge tends to be vague. It is in the practical consciousness that the vague combinatory play with ideas takes place before it is shifted to the discursive consciousness by the ' ... *connection with logical constructs in words or other kinds of signs which can be communicated to others.'* (Einstein in Hadamard, 1945:142).

In par. 2.4.3.2 , the formation of mathematical concepts was described as a fluctuation between processes and objects. Cobb et. al. (1997) believe that the negotiation process supports and stimulates the formation of these dual conceptions of mathematical concepts. They observe and define a kind of interaction which they call 'reflective discourse' which is characterised ' ... *by repeated shifts such that what the students and teacher do in action subsequently becomes an explicit object of discussion.'* They also define a related concept of 'collective reflection' - referring to the ' ... *joint or communal activity of making what was previously done in action an object of reflection.'* (Cobb et. al., 1997:258). This collective process supports similar individual mental activities (interiorization, condensation and reification (Sfard, 1991)) in the individual's mind.

Modalities of structure

In the above discussion the CSCML environment was described as a social system where the interaction of the learners and teacher produce and reproduce structures which are eventually reaffirmed to become structural properties of the system. This paragraph will take a more in depth look at the interpretation of structures represented by the different components of the CSCML environment.

CL methods

The essential principles of CL (par. 3.2.3) are built into the CL methods and bring the constitution of meaning and the sanctioning of modes of social conduct. The more formal CL methods (e.g. the STL method) impose more constraint on social behaviour, whereas the more informal methods (e.g. learning together) encourage exploration and discovery. Also, in drawing on the norms provided by CL principles in interaction, the norms are reaffirmed. So for example, in both case studies, learners realised the importance of verbalising their ideas and they showed a willingness to co-operate and listen to others. They also exhibited goal-directed activity and on-task verbal interaction.

The norms 'positive interdependence', 'face-to-face promotive interaction' and 'good social skills' help to develop the learners' psychological security which influences attitudes and motivation. This is probably one of the reasons why co-operative learning is such a good setting for creative problem solving. The problem solver needs to show a certain sense of endurance and self-confidence in trying different strategies in the face of uncertainty.

By drawing on the facilities provided by CL methods (e.g. one worksheet/computer per group, or the appointing of experts in groups) in interaction, structures of domination are produced and reproduced. In case study 2, in particular, the 'experts' awareness of themselves as agents, increased their motivation and their willingness to co-operate. The term 'agents' is used here in Giddens' sense: '*An agent ceases to be such if he or she loses the capability to 'make a difference', that is, to exercise some sort of power.'* (op.cit.:14).

The facilities represented by CL methods specifically to enhance positive interdependence (e.g. one computer per group) can be used in ways that alter the original norms. For example, in case study 2, the 'group leader' dominated the keyboard as well as the worksheet, which led to frustration and the establishment of a new norm of intolerance within the group (par. 4.3.7.3). The keyboard became the symbol of a power struggle within the group.

Information Technology

Technology acts as a resource and interpreter of rules: interaction supported by technology draws on the interpretive schemes designed into the IT, the facilities provided by IT, and the underlying norms represented by the system. Through interaction these structural properties are either reproduced or changed.

Mathematical software both enables and constrains the learners' mathematical understanding: mathematical structures are built into the software and thus restrict the learners' interpretation of what is accepted as true mathematical knowledge (Roschelle & Teasly, 1995). For example, certain software facilitates more rigid formalisations and thus initiates the learner into the formal mathematical language. It also facilitates group consensus in that it needs consensus on the formal language before it can proceed (Clements & Nastassi, 1991). Open-ended computer environments facilitate co-ordination of divergent perspectives and the resolving of impasses (Roschelle & Teasly, 1995).

The representational plasticity of IT referred to in par. 4.3.7.3, supports the formation of both kinds of conceptions of mathematical concepts (par. 2.4.3.2). It provides environments in which mathematical processes and mathematical objects become inseparable. In this way IT supports the most intimate aspects of mathematics learning.

Mathematical norms are being changed or questioned as a consequence of the use of mathematical software in learning. For example, alternative roles of proof need to be put forward (par. 3.4.3), which in the long run can have an influence on how proof is taught in the school environment as well as on conceptualisations of proof in the professional environment.

Another consequence of the use of IT in mathematics learning is the enriching of mathematical structures by the creation of new kinds of problems and new mathematical objects (par. 3.3.3.3 (Laborde, 1995)).

Giddens states that the character of the information medium (in this case IT) directly ' ... *influences the nature of the social relations which it helps to organize ...* ' (Giddens, 1984:262). The public nature of IT not only seems to sustain social interaction but also makes the groups' mathematical processes and products visible, aiding the difficult shift of knowledge from the practical to discursive consciousness.

Some technologies are specifically designed to support co-operative learning. The norms of individual accountability and positive interdependence are designed into the software and hardware to enable effective interaction (par. 3.4.5).

Objective mathematical knowledge

Objective mathematics represents mathematical rules. These include interpretive schemes and norms (procedures, definition, algorithms, proof, etc.). This knowledge is built into the CSCML environment through the mathematical software and the learning task. The teacher, group members and other written sources also provide representations of mathematical structures that can be drawn upon in interaction.

What is meant by mathematical structures (or mathematical rules and norms) in the sense of structuration theory? Although mathematics, the study of pattern, describes more than social patterns, it is a social construction (par. 2.2.2). It is thus constructed across time and space, through human interaction enabled and constrained by mathematical structures. By keeping in mind Giddens' definition of rules as procedures of action, one would interpret mathematical structures as only mathematical procedures and processes. However, by viewing mathematical objects as reified procedures, one could define mathematical structures as accepted mathematical objects and processes existing as mental schemas in the learner's mind.

The learning task

The learning task represents interpretations of both mathematical structures as well as organisational structures. It organises the groups' activity: with which subject matter they will deal, in which way (problem solving, application, discovery, etc.), the time they will need, etc. The CL norms must be designed into the task, e.g. the inclusion of evaluation forms for group functioning. Complex tasks allowing multiple inputs and viewpoints seem most suitable for the CL environment (Good et al., 1992).

Organisational context

The CSCML components represent structures that can be drawn upon in interaction that ' *... help to maintain social identity, achieve meaningful social interactions, and develop self-esteem and psychological security.'* (Lyytinen & Ngwenyama, 1992:25). The nature of the structures represented by the CSCML components encourages the formation of the following norms: ' *... that meaningful activity is valued over correct answers, that persistence on a personally challenging problem is more important than completing a large number of activities,'* [that individuals] '*will figure out solutions that are meaningful to them*', [that they] '*explain their solution methods to'* [their group members and that] '*they try to make sense of their partner's problem solving attempts.'* (Yackel et. al., 1991: 397, 398).

It was seen in the case studies (par. 5.2 and 5.3) and also observed by Voigt (1996) that the norms of the traditional teacher-centred classroom can be restructured within this new social organisation of the classroom. The organisational environment which provides the setting for the CSCML environment, has an influence on the patterns of interaction. Teachers and learners draw upon the underlying norms of the organisational environment in which they operate during their interaction, to establish a certain classroom culture (par. 4.4). This is described in a study done by Gregg (1995), where he investigated the acculturation of a new mathematics teacher into the mathematics tradition of a specific school. The school's approach to school mathematics was the traditional approach. Through interaction with students (who also accepted this as objectively given) and colleagues, the patterns of interaction soon established the traditional norms in the new teacher's classroom. Gregg emphasises that new teachers are not 'funnelled' into ' ... *traditional practices, but that teachers, students and administrators actively participate in the production and reproduction of these practices.' (*Gregg, 1995:461).

The motives and goals lying behind the conforming behaviour is the wish to act competently within the tradition. According to Gregg (op.cit.), this means to produce students who score well in university and college admission tests and who are adequately prepared for undergraduate mathematics. Traditional mathematics classrooms are thus characterised by a compartmentalised curriculum, assessment through test grades, time restrictions, and so on. By drawing on the rules and resources presented by the CSCML components, these traditional practices are challenged: teachers rethink assessment (e.g. screen output, verbal explanation (Morgan, 1994)) and existing curricula prove inadequate (concepts can be learned in different orders and at different rates, difficult concepts can be introduced earlier, etc. (Morgan, 1994)). It is thus clear that most of the obstacles to the effective implementation of CSCML has to do with the differences in the underlying norms of the traditional learning environment and the CSCML environment.

Agency

The knowledgeable agent stands central to Giddens' structuration theory. Giddens (1984) defines **knowledgeability** as ' ... *everything which actors know (believe) about the circumstances of their action and that of others, drawn upon in the production and reproduction of that action, including tacit as well as discursively available knowledge.'* (op.cit.:375). Since the learning of mathematics in the CSCML environment becomes a social activity, this knowledge does not only include the 'detailed and dazzling' knowledge of learners of how to 'go on' in daily social life (op.cit.), but also their mathematical knowledge. However, learners' knowledge about conditions for their actions is bounded by the situated nature of action, the degree to which tacit knowledge can be communicated, unconscious motivation, and unintended consequences of action.

The assumption is that learners are skilful and knowledgeable and that they know more than what can be communicated. However, the degree to which learners can draw upon CSCML components to support their co-operative interactions, is influenced by the internal system of the group (DeSanctis & Poole, 1992). The internal system of the group is determined by:

- The group members' degree of knowledge of, and experience with, the components of CSCML. It was seen in case study 1 that inadequate knowledge of the software led to their inappropriate use of it. Also, the degree of knowledge of the students (in case study 2) of calculus included only the operational aspects of it. This influenced the execution of the group tasks.
- The style of interaction between group members. Both case studies included disfunctioning groups that had a detrimental effect on learning and on attitudes towards the process.
- The degree to which group members believe that other members know and accept the use of the components of CSCML.
- The degree of agreement on which strategies to follow and the willingness to participate in the learning process.

Emergent properties

Lyytinen and Ngwenyama (1992) list one of the characteristics of CSCW applications as *emergent properties*. They describe emergent properties as *'... those features ... which support innovation and reinvention of co-operative work procedures and practices over time by instantiating different combinations of rules and resources ...'* (op.cit.:28).

One of the emergent properties noticed in the CSCML environment is that of **goal ambiguity**: although the facilitator sets a learning task with a specific objective, the reality is that the goal is renegotiated by the group members in interaction. Lesson 2 of case study 2 showed clearly that the groups' goal differed from the original intention of the facilitator and that the intervention by the facilitator did not change their direction (par. 4.3.7.3). Also, the application of IT sometimes has unintended consequences. The learners' changed view of the role of proof in case study 1 is an example. Another positive unintended consequence of applying CSCML is the more positive attitudes towards mathematics. Then there is the issue of mathematics learning. Can one say that effective mathematics learning takes place? The CSCML environment is no doubt designed to enhance effective learning, but to really assess whether it takes place is a difficult matter. The learner still has the choice to participate actively in the interaction and consequently be initiated into the mathematics culture. The individual also has to make personal sense of the interaction and discussions taking place. As was said already, this sense making process is influenced by several factors including the unintended consequences of actions. The CSCML environment is a rich learning environment that provides ample opportunity for incidental learning to take place. Although the average of the post-test in case study 2 was 20% better than the pre-test, many students still regarded their gain in social skill and self-esteem as the most important result of the programme (par. 4.3.6.3).

Another emergent property of CSCML is its **evolutionary nature**: the application of this learning environment could act as a catalyst to reshape and redirect traditional organisational practices. It was seen in the case studies as well as other research studies (Leikin & Zaslavsky, 1997) that participation in the CSCML environment fosters positive attitudes towards mathematics and mathematics learning. It also changes views of mathematics curricula, ways of assessment, and the role of the teacher.

5.5 Summary

This theoretical framework offers IT, CL methods/principles, the learning task and objective mathematical knowledge as modalities of structure upon which learners draw in their interaction to change or affirm the structural properties of these components. Learning is seen to be an effect of the structuration process which takes place in a specific organisational context.

The overt and covert knowledge of the individual is described as residing in the practical and discursive consciousness. The individual's creation process is described as a to-and-fro movement between mathematical objects and mathematical processes, moving towards higher levels of sophistication. This process is managed by the learner through the reflexive monitoring of his/her own mental and other actions.

Learners are recognised as knowledgeable agents who have the power to participate in or bypass the system. The teachers are seen as an important controller of system reproduction through assessment and critical evaluation of the process. The concepts of single-loop, double-loop and deutero-learning can be applied here. The effect of the internal system of the groups on its functioning (DeSanctis & Poole, 1994) is also recognised. Some actions have unintended consequences. These unintended consequences of action feed back into the system to become unacknowledged conditions for further acts, changing the course of original plans. This model thus recognises the unpredictability of the CSCML environment. Figure 5.8. gives an illustration of these dynamics.

The model developed in par. 5.4 is presented as a tool that can be used to enable better understanding of the CSCML environment as a whole. This is envisaged to have implications for design. The CSCML environment in case study 2 will now be revisited to illustrate the use of the theoretical framework.

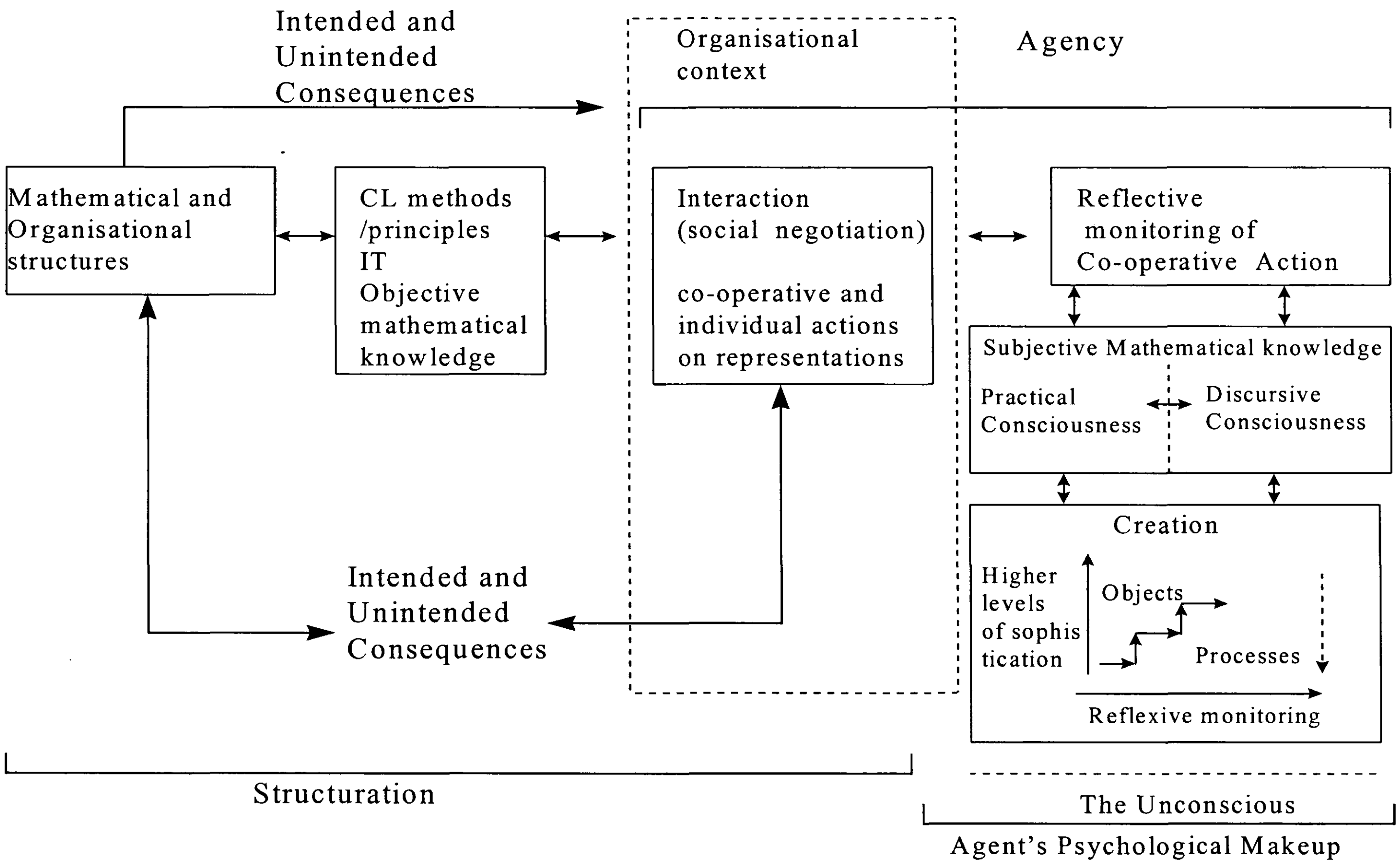


Figure 5.8, A Theoretical Framework for CSCML

5.6 The theoretical framework applied to case study 2

Social system

The CSCML environment consisted of the situated activities of the students and facilitator, which were produced and reproduced across time.

Initially, patterns of interaction were characterised by uncertainty, but with time regularised behaviour patterns were observed. These regularised interaction patterns differed from group to group. Examples of social integration within groups included taking turns, roles within groups, and specialised humour (Lyytinen & Ngwenyama, 1992). Group 4, in particular, was characterised by lively and humorous discussions. In group 1, it became the norm to 'fight' for a chance to participate. This was brought about by the dominance of one group member and the inability of the other two group members to negotiate better co-operation. Group 2 and 4 reinforced the CL norms of co-operation and willingness to listen to others. It also became part of the routinized behaviour of group 2 to accept 2b as operator throughout the course. Group 3 failed to establish conducive patterns of interaction: in all sessions, one member always came late because of transport problems and 3b only attended 3 sessions. At the end, 3b dominated discussions, while the other two listened without interest.

Group 2, and particularly group 4, practised reflexive self-regulation. For example, when the group reached an impasse, 4a reminded the group of the problem solving skills (albeit in a humorous way). For the facilitator, single loop learning took place when she intervened to change the established practices of group 1. The usual operator (and dominating member) was given a less conspicuous role in lesson 2. This had a positive influence on the group's co-operation. Also, deutero-learning took place when the facilitator realised from her experience in case study 1, that the CSCML environment is particularly suited to problem solving tasks, but that these skills need to be made explicit. She consequently used the topic of problem solving in the group building exercises of session 1, and introduced problem solving experts in lesson 1. In this way system reproduction was controlled.

The mathematics learning process

The following excerpt from the transcription of the videotapes shows the negotiation process between all three group members of group 4 and the facilitator M, to establish a mathematical theme. In this discussion, students are trying to understand the given problem graphically. The students have to design a ski-jump with a given vertical height and which covers a given horizontal distance. They need to identify the polynomial whose graph is a side view of the ski-jump.

Group 4, animated discussion is going on. 4b is showing with hands something perpendicular.
4b: 'This is perpendicular'.
4c:' Uhu, Uhu' (4c shakes her head saying no.)
4b: 'Oh, is it this way?' (4b shows with hands).
4a and 4c: 'Yes.' (4a and 4c nod heads).
4b: 'All right.'
All look carefully at THE PROBLEM. 4b underlines given values.
M looks at THE PROBLEM. Starts making a hand sketch.
M: 'This is the horizontal distance.'
4c: 'The distance is...'
4a: 'So this distance is the horizontal.'
4b:'From start to finish.'
4c: 'Yeah, see... '(4c starts drawing a hand sketch).
M: 'So it goes like this, she says that this distance is 36.6.'
4a and 4b: 'Oh' (Transcript B, Tape 2, 15:24).
.

The negotiation processes of groups 2 and 4 showed maturity and respect for others' opinions. However, the facilitator played an important role in this negotiation process in all four groups, because of their inability to interpret the information on the 30 degree angle. As was said before, this could be ascribed to the students' exclusive view of derivatives in the operational sense. This was discussed in full detail in par. 4.3.7.3.

Modalities of structure

The components of the CSCML environment are modalities of structure which students draw upon in interaction to produce and reproduce structures of signification, legitimation and dominance. This structuration process goes hand in hand with learning. The different components will now be discussed in more detail.

CL methods/principles

The five principles (or norms) of CL as well as the CL method (Jigsaw) were discussed in the group building session and built into the learning task (see par. 4.3.5).

Group 2 and 4 reaffirmed these norms through interaction. They realised the importance of verbalising their ideas and they showed a willingness to co-operate and listen to others. They also exhibited goal-directed activity and on-task verbal interaction. However in group 1, the facilities represented by CL methods to specifically enhance positive interdependence (e.g. one computer per group) were used in ways that altered the original norms. The 'group leader' dominated the keyboard as well as the worksheet, which led to frustration and the establishment of a new norm within the group of intolerance (par. 4.3.7.3). The keyboard became the symbol of a power struggle within the group.

Information Technology

The mathematical software enabled and constrained the students' mathematical understanding: the mathematical rules built into the software restricted and guided their activities (e.g. an error message was given if two matrices of incompatible sizes were multiplied).

The representational plasticity of the software had a great effect on the learners. They could see their ski-jump in a verbal-algebraic and graphical representation - some students indicated there AHA-experience as 'seeing the graph'. In this way both conceptualisations (operational and structural) were represented.

The public nature of the computer (screen output) made it easier for the facilitator to check the progress of the groups.

Some students, however, had a different expectation from IT support to their learning. It was remarked that 'the computer does all the thinking and gives the solution without showing the steps to get to the solution'. Also, 'we do not use our brains much, therefore, we do not have much stress'. Another student wrote: 'why is it so much simpler doing the problem using MATLAB, than doing the problem on paper?'. (In these lessons, the computer was used to do the usually tedious computations).

The learning tasks were designed to enhance the formation of structural conceptions of mathematics concepts. It was assumed, since students had already done the work in class earlier in the year, that the operational conceptions were already formed. It became clear, however, that students saw the operational side of mathematical concepts as the only side. When the computer did the procedural parts, leaving them free to interpret and 'objectify' procedures, they felt that there was nothing left for them to do. It is probably also the case that the students view the computer as a substitute for the teacher or textbook and not a tool (as it was used in the course). (Most of the students had only experienced 'drill' software before.) Hence the complaint about the computer not showing the steps.

These complaints about the IT, illustrate the 'reflexive monitoring' and 'rationalisation' of actions. Students tried to '*… keep in touch with the grounds of what they do …*' (Giddens, 1984:376). They were thus trying to understand the role of IT in their activities. This environment thus did not show a seamless integration of IT.

The facilitator encouraged questions about 'what is going on behind the scenes' of MATLAB. In this way the mathematical structures built into the software were exposed and could facilitate learning.

Objective mathematical knowledge

The objective mathematical knowledge included basic concepts of matrix operations, the solving of linear equations, vector spaces, linear independency, bases and calculus. The knowledge that represents interpretive schemes and norms, was presented in text books, and in the learning task, and could be discussed with the facilitator. Lesson 1 was a good example of how meaningful problems often exhibit a cross-curricular character.

Learning task

The CL principles were designed into both learning tasks. Task 1 encouraged positive interdependence by dividing the activities into problem solving, solving of linear equations, and graph plotting. Experts were then allocated to gain knowledge on these different topics which they eventually had to use in their home groups. Task 2 included a check list on group functioning to motivate students to assess their progress as a group.

Organisational contextuality

The CSCML environment was set in the organisational context of Vista University which is a historically black South African university. Vista University, being in a state of transition between different management structures, is still characterised by instability, uncertainty and disruptions. A further lack of computer and seminar facilities force lecturers to use precious time to deliver subject matter in the traditional lecture style. This is not questioned by the students, since most of them come from overcrowded schools with underqualified teachers, where the traditional teaching methods are often the easiest option. The outcome and algorithm centred approach of the students thus comes as no surprise. Still, new norms were established in interaction: the students' view of mathematics changed, they realised the importance of verbalising their ideas, they challenged and questioned others' solutions and offered explanations of their own ideas.

Agency

The actors (the learners) are assumed to be knowledgeable and skilful. The facilitator thus realised the students' need for ontological security and that they knew more than what can be said. Also, the facilitator viewed the students as agents who have the capability to learn.

The assumption was also that through group discussion, knowledge in the practical consciousness could be shifted to the discursive consciousness. In this way misconceptions could be identified and discussed.

However, the language used in the discussions between the facilitator and the students was English which is a second language to all parties involved. This hindered the degree to which tacit knowledge could be communicated (Giddens, op.cit.). Also, the internal system of the group influenced the degree to which students could draw upon the components in their activities:

- The degree of knowledge of the structures represented by the CSCML components. The degree of knowledge of the students (in case study 2) of calculus included only the operational aspects of it. This influenced the execution of the group tasks.
- The style of interaction between group members. Groups 1 and 3 were examples of disfunctioning groups that had a detrimental effect on learning and the attitude towards the process.
- The degree to which group members believe that other members know and accept the use of the components of CSCML. In group 3 one of the members did not have faith in the other members' mathematical abilities and consequently did all the work.
- The degree of agreement on which strategies to follow and the willingness to participate in the learning process. Group 3 had two members who did not really participate in the process. This could be the result of the patronising behaviour of the third member.

Emergent properties

The facilitator designed the learning task with certain objectives in mind (par. 4.3.4). However, a certain goal ambiguity was displayed:

- In lesson 2, students renegotiated the goal, adapting it to their own views of the mathematical concepts.
- Although learning did take place, it is difficult to assess to what extent it took place. In a post-test, students obtained marks of which the average was 20% higher than the pre-test. They also indicated in the self-evaluation lists, for example, that 'now I understand how the nullspace really works', etc. There are no doubt still students who have difficulties with the concepts. Unintended consequences (e.g. disfunctioning groups) had an effect on learning. Also, some students regarded their gain in social skills and self-esteem as the more important results of the course.
- The choices of individuals are often random and constrained by their knowledge of the possibilities (Lyytinen & Ngwenyama, 1992). Learners are also agents. They thus have a choice of bypassing the system or participating actively in the interaction. Since the social rules in this environment are often more informal, participation depends more on group pressure and the need for ontological security than on the rules from the organisational context.

It is thus clear that the learners' participation and learning can be guided and facilitated, but not determined.

The evolutionary nature of CSCML was seen in the change of attitude of students towards mathematics. Also, students commented that some of the groups will carry on functioning after the course. In this way the structural properties of the CSCML environment are carried into the organisational context.

5.7 Conclusion

This chapter dealt with the development of a theoretical framework for CSCML. It argues for the appropriateness of the use of Giddens' structuration theory for the development of the model, by recognising mathematics as social structures. Several ideas from Lyytinen and Ngwenyama's work on CSCW and De Villiers' theoretical framework for CSCL are used. However, the model differs from these applications of Giddens' structuration theory by viewing IT not as structures, but as either resources or representations of interpretations of rules.

This model (which is summarised in par. 5.5) provides for the dynamics of social constructivism. It also shows how rules and resources can be altered through interaction and unintended consequences, and it provides for the unpredictability and goal ambiguity of the process. It furthermore recognises the evolutionary nature of CSCML as one of its emergent properties, implying that this learning environment could act as a catalyst to reshape and redirect traditional organisational practices.

An illustration was given in par. 5.6 of how the model can be used to examine and analyse a CSCML environment. It is envisaged that this theoretical framework will enhance better understanding of the dynamics of CSCML, which will have implications for better design.

Chapter 6

Evaluation of research

6.1 Introduction

In this study, a theoretical framework was developed to describe the CSCML environment as a whole. This is expected to lead to a better understanding of the dynamics of this complex learning environment and to improved design practices. This chapter gives an evaluative look at the theory that was developed in the previous chapters. It takes the form of answering the research questions put in Chapter 1, a critical evaluation of the contribution made to the body of knowledge, and a discussion of possible further research. Before this is done, a brief summary will be given of the content of the first five chapters.

6.2 Summary

Chapter 1 motivates this study by focusing on the potential of the CSCML environment to act as a catalyst for change of an inappropriate educational system. This learning environment is seen as a way to enhance social skills and critical understanding of mathematics, thereby preparing learners for the demands of the technological society. Chapter 1 further highlights the need for an overall view of the CSCML environment to facilitate better design and understanding of the learning environment. This is precisely the need addressed in this study. The philosophical premise underlying the research approach is that of interpretivism (rather than positivism) which holds that phenomena can only be understood through the meanings that people assign to them. The development of the framework using learning theories, case studies and Giddens' structuration theory was subsequently undertaken, with this underlying assumption in mind.

Chapter 2 shows the development of a social constructivist model for the learning of mathematics. Mathematics is defined, the nature of mathematics is discussed, and the stability and reproducibility of mathematics are contemplated using Piaget's theories. The philosophies of mathematics are then briefly discussed to justify the epistemological hypothesis of social constructivism. Social constructivism holds that mathematics is a human invention and that objectivity is social in the sense that there is agreement on what is true. The important implication of social constructivism for mathematics learning is that learning is seen as both a process of self-organisation and a process of enculturation, that occurs while participating in cultural practices, frequently while interacting with others (Cobb, 1994:18). Different theories for mathematics learning are discussed with special interest in the theories of Piaget, Sfard and Skemp. The theoretical framework is then described, based on the different hypotheses as well as related mathematics learning theories.

The main features of the model include the following: learning is seen as the transformation of data (unevaluated attributes) into information (data put into perspective and context) and then into knowledge (newly formed concepts which lead to reinterpretations of existing knowledge). This happens through the fitting of new material into existing schemas in the brain (assimilation) and the eventual major reorganisation of the basic structures of schemas (accommodation) (Skemp, 1971).

A distinction is drawn between objective and subjective knowledge. Objective knowledge ('true knowledge') is publicly accepted knowledge whereas subjective knowledge refers to personal knowledge creations (not tested yet).

This learning takes place in a micro- mathematical community (e.g. the classroom) with a classroom tradition of freedom to create and manipulate mathematical objects in ways that can be explained and justified. However, the classroom community is set in the realm of the macro- mathematical community. Any learning that takes place in the mathematics classroom, takes place against the background of accepted objective knowledge.

It was mentioned that mathematical knowledge is accepted as true through public agreement. In the classroom this takes the form of social negotiation between teacher and learners, which is characterised by situations for explanation and justification.

To understand the acquisition of knowledge in the private realm better (subjective knowledge), different theories for the learning of mathematics are considered. A closer look into these theories reveals three common features: the hierarchical nature of mathematics (levels of sophistication), convergent and divergent thinking. The hierarchical nature of mathematics follows from the fact that higher order concepts are built on, and with, ones that have already been formed.

Divergent thinking is unconstrained thinking, not attempting to be efficient and rational. Concepts associated with divergent thinking are generalisation, inductive thinking, intuition, and visualisation. Convergent thinking, on the other hand, concentrate on validity and efficiency. Through a finite number of logical and analytical thought processes, a valid justification or solution is reached. Related concepts are abstraction, deductive reasoning, formalisation, logic, proof , formulation (symbols), and analysis.

However, these constructions have to be 'managed' by the learner. These managerial thinking skills are called reflective thinking or metacognition. Finally, private knowledge can either be made public or not. There is thus a distinction between overt and covert knowledge. 'Making public' does not mean restricted to a written communication or printed formalised proofs, but can include informal verbal statements. The covert knowledge tends to be vague and sometimes lies in deeper layers of the unconscious (Hadamard, 1949). It is in the private realm that the vague combinatory play with ideas takes place before being made overt by the ' *... connection with logical constructs in words or other kinds of signs which can be communicated to others.'* (Einstein in

Hadamard, 1949:142). It is also through this formulation/reformulation and formalisation that the vague ideas are clarified and brought to consciousness.

Chapter 3 presents an overview of some of the literature available on CL, CSCL and CSCML. The lack of a theoretical framework of the CSCML environment in current research is pointed out throughout chapter 3. CL is described as a way of structuring the learning environment such that groups of students pursue academic goals through collaborative efforts. Research shows that co-operative learning results in more higher-level reasoning, generation of new ideas and solutions, and better transfer of learning. It is also made clear that CL is more than just a number of people working together. The five basic components or principles of CL need to be adhered to in the design of the learning environment. There is general agreement that of the five basic principles the most important is positive interdependence.

Research on CML shows benefits that include the fostering of positive attitudes towards mathematics, the development of problem solving skills, better social skills, and the promotion of higher self-esteem and motivation. It was also found that CML fosters on-task verbal interaction which has a positive effect on mathematics learning. Some researchers of CML focus more on the outcomes than the process. Studies focusing on the process report more problematic findings but are considered more valuable for identifying types of interaction that foster learning.

Research shows that the introduction of computer support to co-operative mathematics learning enhances the potential of an already powerful learning environment. Apart from the fact that it prepares the learner for the technological and social demands of a technological society, it enhances mathematics learning in several ways. Powerful software tools empower the learner by enabling visualisation, formalisation, exploration, discovery learning, and the building of intellectual structures. Computers also sustain social interaction and mediate collaboration (Roschelle et al., 1995).

Research on CSCML includes studies on the effect of heterogeneous/homogeneous grouping on the gaining of complex learning skills; studies contrasting mathematical interaction in computer and non-computer contexts; and studies contrasting mathematics learning supported by different software. Other studies reflect on the development of appropriate software to assist the co-operative mathematics learning process.

McConnel (1994) points out that CSCL is a new area of research and that there are no definite answers to questions of design. This is also the case for CSCML. O'Malley (1995) identifies a need for an agreed framework for comparing and contrasting research on CSCL which might provide guidelines or principles for design. Good et al. (1992) identified a similar need for the mathematics co-operative learning environment. No such framework exists for the CML or CSCML environment. A number of researchers identified a list of variables that play a role in the CSCML learning environment (Bannon, 1992; Good et al., 1992). In an encompassing study done by Hoyles et al. (1994), variables were grouped into background, process and outcome variables. Their work provides valuable remarks on the characteristics of successful groups, the role of pupil-teachers, and task structure. Although researchers in general realise the interdependence of the different features or variables, no attempt has yet been made to describe this interdependence. Variables are at most paired or grouped and then investigated.

Chapter 3 also discusses implementation and integration problems with respect to CSCML. A number of obstacles and problems are noted, most of which stem from the difference between the teaching styles called for by CSCL and the styles used in traditional teaching. Other problems are caused by ineffective design (mostly a lack of genuine interdependence) and uninformed teachers. The teacher plays an imperative role and should design this learning environment as a *' ... well orchestrated whole.'* (Salomon, 1992:64).

Chapter 4 describes the two case studies that were conducted, based on the theory developed in Chapter 2 and the existing research described in Chapter 3. The first case study was directed at mathematics teachers and the second case study involved undergraduate mathematics students. Data were collected through questionnaires, observation lists, tests and video recordings. The interpretation of the data is described and subsequently used in the refinement of the social constructivist model for the learning of mathematics developed in Chapter 2.

The first refinement involves the changing of the model to that of an input-process-output system in order to provide for the intended outcome, namely effective mathematics learning and productive groups. The second refinement addresses the inadequacy of the existing model in aiding the understanding of the learners' difficulties with mathematics. The fluctuation between divergent and convergent thought processes was thought to be too vague. Sfard's (1991) ideas of the dual nature of mathematical concepts described in Chapter 2 are used to give a better picture of the more intimate processes of the creation process. The creation process is then described as the to-and-fro movement between mathematical objects and processes accompanied by a to-and-fro movement between acting and expressing.

Three components are then identified as the 'input' to the process, namely classroom culture, CL methods/principles and IT. Classroom culture is produced and reproduced by social interaction and negotiation. It includes regulations, conventions, morals and beliefs about the nature of mathematics. It became clear through the case studies that the classroom culture influenced, and was influenced by, the interaction process. For example, students believed that they had to reach a solution at all costs. This had an effect on their attitudes towards the technology as well as learning. The application of CL methods led to new social rules, e.g. the importance of verbalising one's own ideas, a change in attitude towards mathematics, and a willingness to listen to other's ideas. It also provided psychological security which had an influence on their attitudes and motivation. IT supported mathematics learning in several ways. It aided formulation and visualisation of intuitive ideas and provided immediate feedback. It also supported the dual conceptualisation through its representational plasticity, i.e. students could see concepts in both verbal-algebraic and graphical representations. However, students did not see the necessity of mathematical proof anymore in the first case study. The constructions they made were proof enough. Also, new mathematics problems were created which were not part of the initial objective but which were very meaningful. Power structures were formed around the keyboard and hostile feelings existed towards the IT in the second case study - students accused the computer of 'doing all the thinking'.

Chapter 4 closes with an argument for the insufficiency of the enhanced model. It is concluded that although it is called a social constructivist model, it still does not clearly show the process of self-organisation and enculturation. Also, it fails to provide for the dynamic nature of the learning environment. For example, it does not show the influence of IT on social interaction and the resulting altered norms and rules. Finally, it was clear from the case studies that there is an openness, and an unpredictability, about this environment. The enhanced model depicts a well organised and predictable system.

Chapter 5 offers Giddens' structuration theory as an appropriate meta-theory upon which a conceptual framework for the CSCML environment can be founded. An in-depth discussion of Giddens' structuration theory is given followed by conceptual frameworks for CSCW (Lyytinen & Ngwenyama, 1992) and CSCL (De Villiers, 1995), both based on structuration theory. A final theoretical framework for the CSCML environment is then developed and described. This is followed by an illustration of how the framework can assist in the better understanding of the dynamics of the CSCML learning environment by applying it to one of the case studies.

Giddens views social structures of societies (the social object) as not existing in a concrete sense, but as 'memory traces' in the agent's mind. The structures are only instantiated in social activity over time. In this way the object/subject dualism is reconceptualised as a duality. Structures are produced or altered by the reaffirmation or change of memory traces of the individual. In this way learning can be seen to accompany the structuration process. The CSCML environment is thus interpreted from a structurational perspective as a *social system* in which interpretations of the *organisational and mathematical structures* of signification, legitimation and domination are represented by the *CL methods/principles*, *IT, objective mathematical knowledge, and the learning task.* These components act as *modalities of structure* upon which actors can draw in *co-operative actions* to reconstitute or change the structural properties of the components. This takes place by the *changing, or reaffirmation, of the agent's mental schemas* in a specific *organisational context.*

From a structurational perspective, the co-operative mathematics learning process supported by IT proceeds as follows:

- The teacher presents the group with a mathematics learning task. The learning task involves the use of IT and objective mathematical knowledge.
- By individual and collective action on representations of mathematical knowledge, private mental mathematical concepts are 'activated'. The representations of the mathematical knowledge could be in the form of verbal-algebraic, visual or kinetic images presented electronically, verbally or in writing.
- However, mathematical concepts are ambiguous: the dual nature of mathematical concepts has already been discussed in par. 2.4.3.2. Also, individuals have unique ways of representing subjective mathematical knowledge.
- The CL methods provide norms of co-operation and goal-directed activity. By drawing on these norms, groups work towards the unambiguous understanding of mathematical concepts. Individuals experience mathematical concepts as unambiguous in the following way: interaction in the classroom is seen as more than a series of separate acts, it is seen as a continuous flow of conduct, where the actor reflexively monitors his/her own actions in accordance to what he/she assumes the others' background understanding and expectations are. The others interpret the actions of the individual, adopting what they believe to be the actor's background

understanding and expectations. In this way a background understanding is taken as 'objectively given' and mathematical concepts are experienced as unambiguous (Voigt, 1996).

- However, this does not mean that the individual group members have the same conception of the mathematical concepts, even if they collaborate without conflict. The more the background understanding of learners differ, the more likely it is that they will have different understandings of mathematical concepts.
- Because of the disparity between the background knowledge of the teacher and learners as well as the teacher's role as mentor, learners and teachers have to negotiate mathematical meanings.
- Through negotiation (between learner and learner, and teacher and learner), a shared stock of mathematical meaning is formed (called a 'theme' by Voigt (op. cit.)). The theme is not fixed but is interactively constituted through negotiation. This negotiation process supports the individual creation process which is described here as a to-and-fro movement between objects and processes.
- With time, the theme gains stability because mathematical discussions are constrained and guided by mathematical structures and because of the need for routine to '... *protect social continuity.'* (Giddens, op. cit.:70). The knowledge now becomes institutionalised (or reified).

Another feature of importance is that of *agency*. This implies that learners are actors in the sense that they have a choice of bypassing the system or participating actively. The assumption is also that learners are skilful and knowledgeable, and that they know more than what can be communicated. The knowledge of learners does not only include the 'detailed and dazzling' knowledge of how to 'go on' in daily social life (Giddens, 1984), but also their mathematical knowledge. However, the learners' knowledge about conditions for their actions is bounded by the situated nature of action, the degree to which tacit knowledge can be communicated, unconscious motivation, and unintended consequences of action. Also, the degree to which actors can draw upon CSCML components to support the co-operative interactions, is influenced by the internal system of the group (DeSanctis & Poole, 1992).

A part of the individual's mathematical knowledge lies in the private realm, or practical consciousness. It is only by making this knowledge public, or shifting it to the discursive consciousness that the misconceptions can be identified by the teacher and other learners. This is usually a difficult process since the 'private' knowledge tends to be vague.

This model gives a better depiction of the delicate interplay between the components of the environment: it provides for the dynamics of social constructivism, it shows how rules and resources can be altered through interaction and unintended consequences, and it provides for the unpredictability and goal ambiguity of the process. It also recognises the evolutionary nature of CSCML as one of its emergent properties, implying that this learning environment could act as a catalyst to reshape and redirect traditional organisational practices. It was seen in the case studies as well as other research studies (e.g. Leikin & Zaslavsky, 1997) that participation in the CSCML environment fosters positive attitudes towards mathematics and mathematics learning. It also changes views of mathematics curricula , ways of assessment, and the role of the teacher.

The most important findings of this study will be discussed in par. 6.4.

6.3 Research questions

This paragraph evaluates the research done, by examining the extent to which the questions stated in Chapter 1 have been answered.

What is?

What is mathematics? Mathematics is seen as the study of pure pattern. It is seen as social constructions by acknowledging that it is a human invention and that objectivity is social (in the sense that there is agreement on what is true). According to Piaget, the roots of mathematics lie in the action on objects and the eventual abstractions of the general co-ordination of these actions. The co-ordination is shared by human beings, because they depend on the laws of neural co-ordination. This explains the robustness of mathematics. A quasi-platonism is accepted, i.e. any structure of mathematical nature involves a whole system of possible developments and the novelty of later structures consists merely of actualising some of them. It is further assumed that mathematical concepts exhibit a dual nature, i.e. any concept can be seen as a process as well as an object.

What is mathematics learning? Mathematics learning is seen as both a process of self-organisation as well as enculturation which occurs while participating in mathematical practices, frequently while interacting with others. This self-organisation or creation process involves the to-and-fro movement between the process and object parts of concepts, moving to higher levels of sophistication. This process is managed by the learner through reflective thinking. Concepts are stored in the mind in visual, verbal-algebraic or kinetic form and most of them are tacit knowledge, which needs to be made explicit before learning can be acknowledged.

What is a Co-operative Mathematics Learning Environment? A CML environment can be seen as a learning environment with an organisational structure based on CL methods/principles in which groups of students pursue mathematical academic goals through collaborative efforts. In this environment, mathematics learning is seen as an effect of interaction: through interaction subjective mathematical knowledge becomes compatible with objective mathematical knowledge.

What is a computer-supported co-operative mathematics learning environment? A Computer-supported Co-operative Mathematics Learning Environment can be seen as a learning environment with an organisational structure based on co-operative learning methods/principles in which groups of students pursue mathematical academic goals through collaborative efforts supported by the instructional use of information technology. De Villiers (1995) emphasises the necessity of CL methods to enable real co-operation. This research found that the application of CL methods does not guarantee success (e.g. artificial roles in lesson 2, case study 2). Rather, it is the CL principles that need to be designed into the learning task. A number of studies show the successful use of 'informal' groups where individual accountability and interdependence are put in place by the values and norms of mathematics research practices (Good et al., 1992).

How does?

How can the learning of mathematics be modelled? Chapter 2 deals with the development of a *social constructivist model for the learning of mathematics.* The main features of this model are given in the summary of Chapter 2 in par. 6.2.

How can the model be enhanced to reflect the CSCML environment? The *enhanced final model for CSCML* (Figure 5.8), resulted from attempts to address issues raised by interpretation of data obtained from the case studies, as well as the use of Giddens' structuration theory. This theory recognises IT, CL methods/principles, the learning task, and objective mathematical knowledge as modalities of structure which learners draw upon in their interaction to change or affirm the structural properties of these components. Learning is seen to be the emergent result of the structuration process which takes place in a specific organisational context. This model gives a better depiction of social constructivism, i.e. how mathematics learning is both a process of self-organisation and at the same time a process of enculturation.

The overt and covert knowledge of the individual is described as residing in the practical and discursive consciousness. The fluctuation between convergent and divergent thought processes that characterise the individual's creation process, is now replaced by the to-and-fro movement between mathematical objects and mathematical processes, moving towards higher levels of sophistication. This process is managed by the learner through the reflexive monitoring of his/her own mental actions.

Other important features of this model include the concept of agency which recognises learners as skilful agents who have the power to participate in, or bypass, the system. The teacher is seen as an important controller of system reproduction through assessment and critical evaluation of the process. The concepts of single-loop, double-loop and deutero-learning can be applied here. The effect of the internal system of the groups on its functioning (DeSanctis & Poole, 1994) is also recognised. This model also recognises the unpredictability of the system by recognising the existence of unintended consequences of action. These unintended consequences of action feed back into the system to become unacknowledged conditions for further acts, changing the course of original plans.

How does the introduction of mathematics in the CSCL-environment influence that environment? The *introduction of mathematics into the CSCL environment* influences the environment in several ways: The dual nature of mathematical concepts seems to be unique to mathematical abstractions (Sfard, 1991). The representational plasticity of IT should be fully utilised to find different ways of representing mathematical knowledge (e.g. graphs, sequences, tables, verbal-algebraic sentences, etc.). Also, negotiation processes should include shifts in the discourse ' ... *such that what the students and teacher do in action subsequently becomes an explicit object of discussion.'* (Cobb et al., 1997: 258).

Hoyles, Healy and Pozzi (1992) identify the difference between co-operative learning in mathematics and other subjects as the importance in the tasks solution process to clarify and articulate what the problem space is and to develop a language to describe it. This language is a more formalised language which can be modelled by the software. For example, certain software facilitates more rigid formalisations and thus initiates the learner into the formal mathematical language. Some of the software also facilitates group consensus in that it needs consensus on the formal language before it can proceed.

Although mathematics is a social construction, it provides workable and stable methods to describe and control some aspects of the living environment. This characteristic also facilitates group consensus especially on mathematical problem solving. Although the next statement, made by a student in case study 2, is not quite true, it partly illustrates the way in which consensus was reached, 'There is always only one solution, so even when we have different answers, we must agree on only one'. This statement could be rephrased by, 'No matter how many opinions, we will all recognise and accept a workable solution'.

How does the CSCL environment influence the mathematics curricula and learning? The CSCL environment has a certain influence on mathematics teaching. The underlying norms and values of CSCML differ from that of the traditional mathematics classroom: learners are given more control over their own learning and they are seen as tutors, investigators and presenters, whereas the teacher's role changes to that of facilitator. It was also found that CSCL fosters positive attitudes towards mathematics.

Also, the computer makes the learners' activities and difficulties more visible, aiding the teacher to identify misconceptions. It enables learners to learn concepts at different rates and in different orders. This have implications for mathematics curricula. Because of computer support to mathematics learning, teachers tend to view assessment differently. More and more experimentation is being done into non-written, classroom-based assessment of computer supported mathematics learning (Morgan, 1994). A consequence of computer-supported mathematics learning, is that new problems are created and old practices questioned (Laborde, 1995; De Villiers, 1997). Teachers are thus challenged to come up with new reasons for old practices, which forces a thorough examination of the underlying foundations of mathematics.

Why is?

Why would one want to apply CSCL to the teaching and learning of mathematics? Throughout this study, the CSCML environment has been described as a potentially powerful learning environment. Research on CML shows benefits including the fostering of positive attitudes towards mathematics, the development of problem solving skills, better social skills, and the promotion of higher self-esteem and motivation. It was also found that CML fosters on-task verbal interaction which has a positive effect on mathematics learning.

Computers enhance mathematics learning in several ways. Powerful software tools empower the learner by enabling visualisation, formalisation, exploration, discovery learning, and the building of intellectual structures. Computers also sustain social interaction. It is envisaged that CSCML will prepare learners for the technological and social demands of the existing technological society. It is also seen as a catalyst for change of the current educational systems. However, the case studies and research make it clear that CSCML contains many pitfalls and that CL does not consist of the mere grouping of learners. The principles of CL need to be adhered to, especially that of goal interdependence and individual accountability.

Most of these positive attributes were confirmed by the case studies described in Chapter 4. However, because of the experience gained from the case studies, the following important aspect needs to be highlighted.

Increased motivation

This is one of the greatest assets of CML and CSCML. From the perspective of structuration theory, it is believed that CSCML increases learners' motivation and self-esteem because it addresses the need for ontological security. Learners feel that they belong to the group and that their opinions are respected. Also, CL recognises learners as agents. It gives the learners more power to control and determine their own learning. It was said before that problem solving seems to be ideal for the CSCML setting. Through the solving of meaningful problems, learners tend to change their view of mathematics as dull and uninteresting.

How should?

How should the CSCML environment be designed to enhance effectiveness and production? This final question addresses the practical implications of the developed theory for designers of the CSCML environment.

Interaction (social negotiation)

The core activity of the co-operative mathematics learning process lies in the negotiation of mathematical meanings and the consequent formation of a mathematical 'theme'. Meaningful discussion can be enhanced by designing a real purpose into the co-operative activity which is often best achieved through a **problem-centred approach** (McConnell, 1994). The teacher thus has to pose good problems. A good problem is defined as one that requires students to make and test a prediction, that is realistically complex, that benefits from group efforts, and is seen as relevant and interesting by students. In this setting students will naturally explore their knowledge which would lead to continual refinement of the knowledge.

The negotiation process is triggered by individual and co-operative actions on mathematics representations. Because of the different forms in which schemas are stored as well as the various meanings that the same concept can carry (Dubinsky and Sfard, par. 2.4.3.2), the teacher has to take a **multi-representational approach** in presenting knowledge.

Organisational context

Teachers and facilitators need to be aware of the difference between the norms represented by CL principles and technology, and the norms of the traditional classroom. Since the potential of the CSCML environment lies exactly in these new norms, the structures represented by the CSCML components should not be undermined but drawn upon and encouraged in interaction. The following norms should be encouraged:

- a shared belief that one could challenge each other's interpretations,
- that mathematical truths are constituted by the teacher and learners in the course of the social interactions,
- that the acts of explaining and justifying are central to the process of social negotiations,
- that the teacher and learners act together as validators of interpretations, and
- that the teacher should use the students' autonomous constructions to guide the constitution of their taken-as-shared mathematical reality towards the ways of knowing of the macro- mathematical community.

Social system

The system can be described as an intersocietal system since it shares rules and norms with the macro- mathematical community (the professional mathematical community). The shared rules include the accepted mathematical knowledge and certain working practices. The teacher, as an experienced member of the mathematical community, needs to mediate ' *... between students' personal meaning and culturally established mathematical meanings of wider society.'* (Cobb, 1994:15). The students should thus be encultured into the conventions and discursive practices of the mathematical community. The teacher needs to control system reproduction through reflexive self-regulation and assessment in the form of organisational learning (discussed in par. 5.4).

Modalities of structure

The learning task, technology, objective mathematical knowledge and CL principles are different representations of mathematical and organisational structures. However, in designing the CSCML environment, the different structures should be integrated to form a ' ... *well orchestrated whole.'* (Salomon, 1992:64). The content and structure of the learning task and the role that IT will play in the process need to be of such a nature that they enhance negotiation and interdependence (e.g. lesson 1 of case study 2). The CL principles need to be explained and made explicit but where possible they should be designed into the learning task and facilities.

Agency

Teachers need to acknowledge the fact that learners are skilful and knowledgeable, and that they know more than what can be communicated. Also, teachers need to realise that the learners' knowledge about conditions for their actions is bounded by the situated nature of action, the degree to which tacit knowledge can be communicated, unconscious motivation, and unintended consequences of action.

Learners need to experience themselves as agents, i.e. as individuals who have the capability to 'make a difference'. This awareness leads to increased motivation and willingness to co-operate. By encouraging students to participate in the design, assessment and evaluation of the learning environment, this fact is acknowledged. The CL principle of 'positive interdependence' acknowledges learners as agents. The application of this principle thus determines to a great extent the success of group functioning.

By realising that some of the learner's individual knowledge lies in the practical consciousness, the teacher's interest should also be focused on what can be inferred to be going on inside the student's head, rather than on overt 'responses'. The teacher should be interested in the students' errors and indeed, in every instance where students deviate from the teacher's expected path, because it is these deviations that throw light on how students, at that point in their development, are organising their experiential world (Von Glaserfeld in Jaworski, 1994:26,27). Learners should also get the opportunity to make their knowledge overt; either by speech or writing.

The degree to which learners can draw upon CSCML components to support their co-operative interactions, is influenced by the internal system of the group. Teachers should be aware of what determines the internal system and be prepared to intervene where necessary (see par. 5.4). Although learners are always reflexively monitoring their own actions, this 'action' needs to be seen as a skill that can be acquired. The reflexive monitoring of mathematical 'actions' (or reflective thinking) is considered to be very important in the learning of mathematics and should be made explicit. For example, problem solving activities should be taught explicitly since most students are unaware of their own thinking processes. Also, reflective thinking processes and methods should be modelled by the teacher.

The hierarchical and dual nature of mathematical concepts

In par. 2.4.3.2, the formation of mathematical concepts was described as fluctuating between processes and objects. The teacher should be aware that learners should understand the process side of mathematical concepts before these can be objectified. Students should be given opportunity to go through previous, lower process-levels of the learning process. Concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples.

By constantly keeping in mind the dual nature of mathematical concepts, teachers can have greater understanding of the learners' difficulties. Cobb et al. (1997) believe that the negotiation process supports and stimulates the formation of these dual conceptions of mathematical concepts. They observe and define a kind of interaction which they call 'reflective discourse' which is characterised ' ... *by repeated shifts such that what the students and teacher do in action subsequently becomes an explicit object of discussion.'* They also define a related concept of 'collective reflection' - referring to the ' ... *joint or communal activity of making what was previously done in action an object of reflection.'* (Cobb et al., 1997:258). This collective process supports similar individual mental activities (interiorization, condensation and reification (Sfard, 1991)) in the individual's mind. The teacher should steer the negotiation process towards 'reflective discourse' and 'collective reflection'.

Emergent Properties

Because of the unpredictability of the co-operative learning environment, the teacher should display an openness towards the educational process. Learners should feel free to make decisions about their learning and to exercise their choices. The teacher, on the other hand, should provide opportunity for the creation of new problems through interaction and the consequent slight deviations from the original goals.

6.4 Evaluation of contribution of this study

This study addressed the need for an overall view of, or conceptual framework for, the CSCML environment. Existing theories for the learning of mathematics were considered and subsequently used in the development of a social constructivist model for the learning of mathematics. This model guided the execution of two case studies of which the data were used to refine the social constructivist model for the learning of mathematics. The shortcomings of this model were pointed out and subsequently addressed by developing a final theoretical framework informed by Giddens' structuration theory.

The findings of this study consist of, on the one hand, findings from the case studies, and on the other, the theory developed based on these findings and other theories. The most important findings (divided into the above mentioned two groups) are given below.

Findings obtained from the interpretation of the case studies are:

- Existing research results were confirmed in that students reported a sense of enjoyment, belonging and trust. They were willing to co-operate and offer explanations and justification, and showed an openness to each other's ideas. The learners stressed the motivational aspects of group work, the importance of verbalising ideas, and the changed attitudes towards mathematics resulting from their experiences in the groups.

- Representational plasticity, graphing facilities and their enhancement of visualisation, were found to be important aspects of computer support to mathematics learning. Students also mentioned the decrease of work load, the many similar problems that can be generated using the software, and the quick feedback given.

- The learners' belief about the nature of mathematics has an influence on their perception of computer support to their learning. In both case studies, the perceived disadvantage of computer support to mathematics learning was that the computer does all the thinking. This perception can be explained by referring to the dual nature of mathematics concepts. In case study 2, the computer was used to do the computational or process part of the mathematics, leaving the students free to draw relations and concentrate on the structural side. Most students, however, consider the process part of the concepts the only part.

- One of the important findings is thus that mathematical learning in the CSCML environment can be examined through the lens of the dual nature of mathematical concepts. This sheds light on most of the difficulties students experience with concepts and computer support.

- Classroom culture or organisational context plays an important role in interactions and learning in the CSCML environment. The classroom culture consists of habitual patterns which are created through interactions and the underlying beliefs of the participants. Examples were given of the sometimes detrimental influence of the teachers' beliefs about meaningful problem solving.

- The importance of true interdependence in co-operative learning was confirmed. From the experience gained in the case studies, it became clear that the success of CL implementation does not lie in the application of CL methods, but in the incorporation and full understanding of the CL principles.

- The CSCML environments of the two case studies exhibited an open and unpredictable nature. Although goals were set, new ideas emerged during interaction and students negotiated alternative goals. This should be seen as an asset of this learning environment, which could lead to meaningful, and sometimes surprising, discussions and problems.

Findings from the development of the theory are:

- The existing theoretical frameworks for CL and CSCL are input-process-output models (see par.5.3.3 and par.3.2.4). They depict these learning environments as well organised and predictable systems. This is in contrast to the observed openness and unpredictability of these learning environments. The enhanced model that was developed in par 4.5 (based on previous similar models), proved to be inadequate because of this reason. It was subsequently enhanced by using Giddens structuration theory to provide for the openness of the system.

- Giddens' structuration theory proves to be a useful tool in trying to overcome the difficulties surrounding social constructivism. The duality presented by social constructivism is conceptualised as a dualism.

An evaluative look into the contribution which these findings make to current thinking, yields the following.

It has been pointed out by several researchers that there is a need for an overall view, or conceptual framework, to describe the CSCL and CML environment. Other researchers ask for more local theories for CSCL that take into account the particular knowledge domain (Mandl & Renkl, 1992). The developed framework addresses these concerns by identifying components and features of the CSCML environment and showing the relationships between them. This could serve as a basis for planning other research and as an aid in examining and interpreting existing research.

Although Giddens' structuration theory has been used before in research on Information Systems, this theory involves a first introduction of the structuration theory to the field of mathematics education research. The theories of Giddens, Sfard, De Villiers, DeSanctis & Poole, and Lyytinen & Ngwenyama are linked to contribute to mathematics learning theory through CSCML.

By understanding Giddens' structures as mental schemas that exist in the individual's mind, and the affirmation or changing of structures as the affirmation or changing of mental schemas, learning emerges from the structuration process. This constitutes a considerable departure from the traditional approach to learning through single and double loop correction of actions. Also, social constructivism is reconceptualised as a dualism using structuration theory: learning is seen as an effect of interaction (Voigt, 1996). Through social interaction (drawing on structures), subjective mathematical knowledge becomes compatible with the structures. The structures are reinforced through this process. 'Structures' indicate the accepted rules and resources represented by objective mathematical knowledge, as well as the rules of accepted mathematical and classroom practices.

Mathematical structures as rules and resources, constrain and enable mathematical social behaviour. One needs to be reminded that Giddens views rules as procedures of action. By relating this to Sfard's theory on the dual nature of mathematical concepts, one would interpret mathematical structures as only mathematical procedures and processes. However, by viewing mathematical objects as reified procedures, one could define mathematical structures as accepted mathematical objects and processes, existing as mental schemas in the learner's mind.

This theoretical framework differs from other theories that describe the CL and CSCL environment in that it does not predict the process or outcome, but provides for the openness and goal ambiguity experienced in the case studies and reported by several researchers (Good et al., 1992). Unintended consequences are drawn upon in interaction, to eventually change original goals or intentions. For example, the structural properties of mathematical software enable totally new ways of looking at mathematics problems and concepts (Laborde,1995). Learners could thus create and ask new questions not provided for in the original planning. Teachers should display an openness towards the shifting of goals and objectives to provide for these new meaningful problems. Teachers should also be ready to provide new motivation for old techniques, e.g. students using mathematical software sometimes do not see the necessity for proof.

The theory that has been developed also differs from some applications of Giddens' structuration theory to change induced by technology (Lyytinen & Ngwenyama, 1992; De Sanctis & Poole, 1992), by heeding the warning of Lamprecht (1997) that the view of technology as structures annuls the subject/object duality proposed by Giddens. Rather, IT is seen as either resources or representations of interpretations of rules.

The theoretical framework for CSCML produced a new perspective of the CSCML environment. This theory identifies IT, CL principles, the learning task, and objective mathematics knowledge as modalities of structure which actors can draw upon in their interaction. This implies that the choice and design of these components will have an important influence on the type of interaction taking place. On the other hand, interaction and action (mental or physical) are seen as a pre-condition for mathematics learning to take place. This learning process will either change or affirm the structural properties of the components. It also identifies the important role of the teacher as that of controller of system reproduction. This takes place through single-loop and double-loop learning as well as deutero-learning.

Lastly, this theory shows the importance of the organisational context in providing the setting for the CSCML environment. The structural properties of the organisational context play a constraining role on the interaction taking place. The underlying norms and rules of CSCML differ from those of the traditional mathematics classroom. To fit CSCML into a traditional educational context will imply the adaption of some of the underlying norms. However, CSCML shows an evolutionary nature, i.e. it carries the potential to act as a catalyst for change in the traditional mathematics classroom.

6.5 Further research

Although the theoretical framework was developed for mathematics learning, one could ask the valid question whether it can be applied and investigated in other contexts. For example, what will be the change to this framework if applied to learning in other subject areas (if any)? Also, now that the important components and features were defined and described, some of them can be isolated and investigated with respect to the influence on the system as a whole. For example, the effect of a specific mathematical software package on the learning task, the choice of CL methods, social interaction, and learning, can be studied.

More empirical research is necessary to refine the understanding of the individual creation process in the CSCML environment. The theoretical framework describes the individual creation process as a fluctuation between mathematical processes and objects. Research is needed into how the components should be chosen and designed to enhance this process. Also, more research is necessary on the work of Cobb et al. (1997) applied to the CSCML environment. They identified types of discourse in mathematics group learning (without computers) that support the objectifying of mathematical processes. Valid questions could be asked on the influence of the computer on these types of discourse.

This framework also tries to overcome the difficulties surrounding social constructivism, by conceptualising it as a dualism, using Giddens' structuration theory. This idea needs further investigation and development by comparing it to other theories in mathematics education trying to resolve the object/subject duality (e.g. Voigt's (1996) use of symbolic interactionism).

The changed views of established mathematical practices (e.g. the learners' views of mathematical proof), were indicated as one of the unintended consequences of CSCML. It is clear that the introduction of IT to mathematics learning has the potential to change the face of mathematics education: Bottino and Furinghetti (1996) are of the opinion that past changes in mathematics education were triggered by discussion on content, whereas the current changes (triggered by the presence of computers) lead to changes in methodology and new classes of problems. This is also true for mathematics research: the presence of computers leads to new classes of problems in numerical and discrete mathematics, formal languages, and chaos theory. Not only are new areas and problems created, but new questions are asked about the foundations of mathematics. For example, the computer-based proof for the four-colour theorem triggered great debate. This proof is not surveyable (because of its size) nor formalisable in the traditional sense. Computers and computer programs are assumed to be fallible, so that these assertions can never be more than provisionally true (unless the computer programs are formally verified - a route which, in most cases, would be humanly impossible). Tymoczko (1980) argues that computer-assisted proofs thus underline the need for a more realistic philosophy of mathematics allowing fallibility and empirical elements. Many mathematicians are willing to settle for this 'semi-rigour' whereas others are not willing to give up the idea of absolute proof. The fact is that many mathematicians use experimental computer methods in their research and more mathematicians have come to appreciate the power of computers in communicating mathematical concepts (Hanna, 1996:4). A valid question, and a question that needs further research, is: how do these developments influence the teaching of mathematics at both school and undergraduate level?

Finally, by being based on Giddens' structuration theory, the model described in par. 5.4, carries within it the same shortcomings as structuration theory. The role of the agent is probably overplayed and the influence of structural constraints underplayed. Still, this model gives a useful way of viewing learning as both an individual and a social process. Valid questions can be asked about new insights that can be obtained from developing similar theoretical frameworks for CSCML based on other theories. For example, the *general theory of communication* of Habermas could prove a suitable premise, because of the observed importance of interaction and negotiation in the CSCML environment. Habermas defines three conditions for ideal speech situations: communication need to be true to the objective world, right in the social world and sincere to the internal world. A well designed CSCML environment should thus enhance communication, understanding and rational discourse (Dahlbom & Mathiassen, 1993).

6.6 Final remarks

The theoretical framework presented in this study provides an overall view of the dynamics and components of the CSCML environment. It was illustrated in par. 5.6 how this model can be used to obtain a better understanding of the CSCML environment. Better understanding will lead to better design.

One of the properties of CSCML, as described by the model, is its evolutionary nature. In the case studies this was illustrated by the students' change of attitude towards mathematics and the fact that some of the groups carried on functioning after the course. CSCML thus has the potential to act as a catalyst for change in the traditional mathematics classroom. In the CSCML environment, learners experience mathematics both as social power and as social construction. The mathematical principles built into the mathematical software and course material enable, and constrain, social and mathematical behaviour while new mathematical meanings are created through social negotiation.

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Appendix A

Case Study 1

I

The Problem

Draw a family tree to show the relation between different quadrilaterals including

Trapezium
Rectangle
Square
Parallelogram
Kite
Rhombus
Cyclic quadrilaterals
Cyclic kites (kites with four vertices on circumference of circle)
Cyclic trapezia (trapezia with four vertices on circumference of circle)

The following activities will aid you in solving *The Problem*

Activities

- Each of you will join an expert group to revise the properties of the first six quads. (30 min)

After you have returned to you original group

- Read and discuss the contents of the document *A Classroom episode.*
  (15 min)
- Now do the exercises given in *Cyclic Quads* and hand it in at the end of the period.
  (45 min)
- Decide now as a group (through discussion), what your family tree will look like and draw it on a sheet of paper. Hand this in at the end of the period (with group members' names on it).
  (30 min)
- Each individual group member must complete an individual worksheet and hand it in at the end of the period.
  (15 min)

II

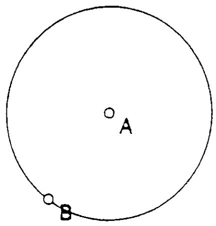
Expert Group

In this exercise you will revise some properties of the rhombus, square and rectangle by using Geometer's Sketchpad.

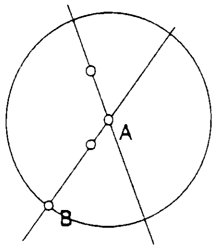
The rectangle

Construct a rectangle by doing the following:

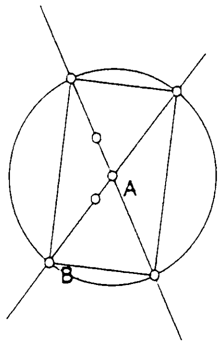
Step 1: Construct circle AB



Step2: Choose the line tool and draw two diameters of circle AB.



Step3: Join the points of intersection of the two diameters and the circle. EFGH now forms a rectangle.



Which properties of the rectangle were used to make this construction?

Use your knowledge of the properties of the rectangle to find another way of constructing a rectangle (and construct it).

The Rhombus

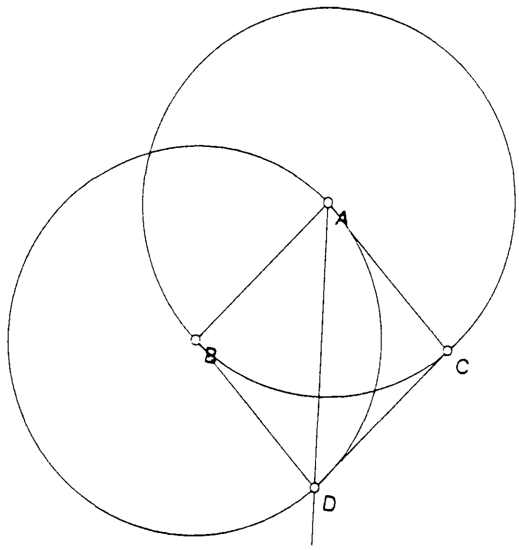
Construct a rhombus by doing the following:

Step 1: Construct a circle AB and C on the circle.

Step 2: Bisect the angle BAC in the following way: select angle BAC by selecting B, A and C in that order. From the *construct* menu, choose *Angle Bisector*.

Step 3: Construct circle BA and point D, the intersection of this circle and the bisector.

Step 4: Construct segments to form rhombus ABCD.



Which properties of the rhombus were used to make this construction?

Use your knowledge of the properties of the rhombus to find another way of constructing a rhombus (and construct it).

The square

Construct a square by doing the following:

Step 1: Draw circle AB and circle BA.

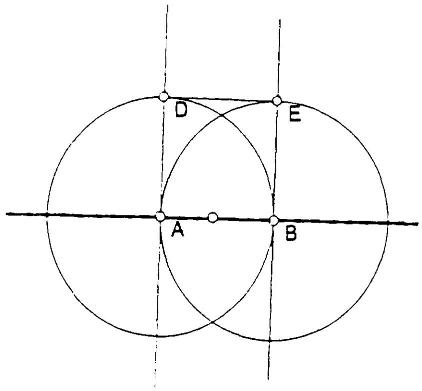
Step2: Choose the line tool and draw a line through both centers.

Step 3: Select the line and point A. From the *construct* menu, choose *Perpendicular line*. You now have a perpendicular line through A to the diameter.

Step 4: Construct a perpendicular line to the diameter through B by using the same method as in step 4.

Step 5: Construct point D at the intersection of the perpendicular line through A and the circle AB. Construct E at the intersection of the perpendicular line through B and the circle BA.

Step 6: Join D and E. ABED is now a square.



Which properties of the square were used to make this construction?

Use your knowledge of the properties of the square to find another way of constructing a square (and construct it).

Expert Group

In this exercise you will revise some properties of the kite, rectangle and parallelogram.

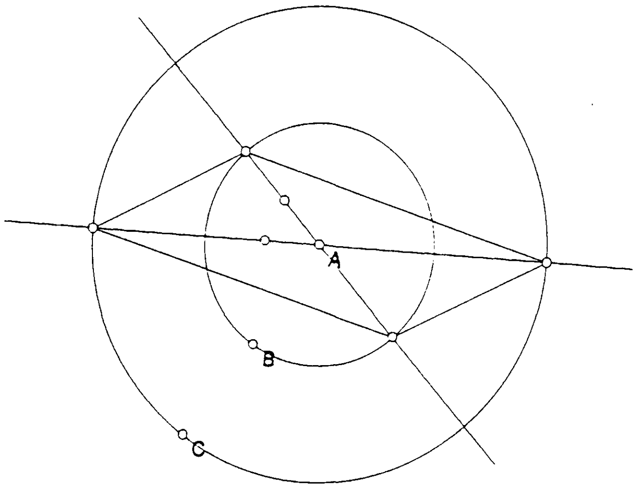
The Parallelogram

Construct a parallelogram by doing the following:

Step 1: Draw circle AB and circle AC with different radii.

Step 2: Choose the line tool and construct two different diameters for circle AB (and AC).

Step 3: Joint points of intersection of the diameters and circles as indicated on the sketch. The figure you now have is a parallelogram.



Which properties of the parallelogram were used to make this construction?

Use your knowledge of the properties of the parallelogram to find another way of constructing a parallelogram (and construct it).

The Kite

Construct a kite by doing the following:

Step 1: Draw any triangle ABC.

Step 2: Select any side of the triangle.

Step 3: Go to the *transform* menu and choose *Mark Mirror*.

Step 4: Now select the other two sides and choose *Reflect* from the *Transform* menu. You now have a kite.

Which properties of the kite were used to make this construction?

Use your knowledge of the properties of the kite to find another way of constructing a kite (and construct it).

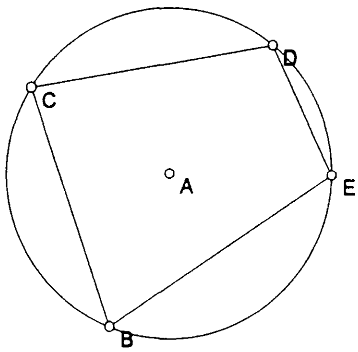
III

Cyclic Quads

Do you remember the properties of cyclic quadrilaterals?

<u>Investigate the properties again by doing the following:</u>

- Draw circle AB
- Construct a cyclic quad BCDE (name the vertices).
- Measure angles $B\hat{C}D$ and $D\hat{E}B$.
- Find $B\hat{C}D + D\hat{E}B$

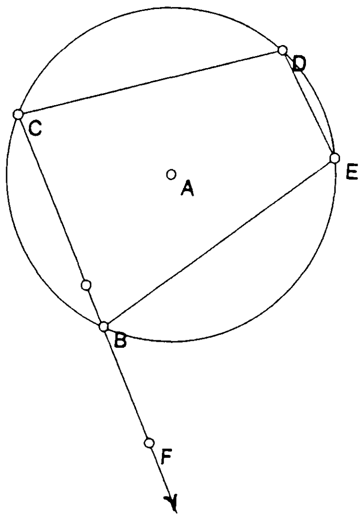


What do you find?

What can you conclude about $B + D$?

- Select $\overline{CB}$, delete it and replace it by a ray $\overrightarrow{CB}$.
- Construct a point F on the ray as indicated in the sketch.
- Measure $F\hat{B}E$ and compare it with $E\hat{D}C$.

What do you find?



- Why can one say that all rectangles and squares are also cyclic quadrilaterals?

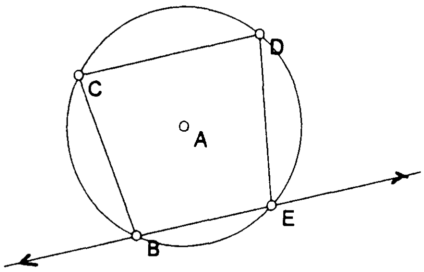
- Where would one place the compass to construct the circumscribed circle of a rectangle or a square? (Look at the sketch of the rectangle given in the worksheet of the expert groups).

- Is a parallelogram also a cyclic quadrilateral? Explain your answer.

Cyclic Trapezium

Construct a cyclic trapezium by doing the following:

- Construct circle AB.
- Draw a chord $\overline{CD}$ as indicated in the sketch.
- Select $\overline{CD}$ and B and from the construct menu choose *parallel lines.*
- Complete the quad CBED

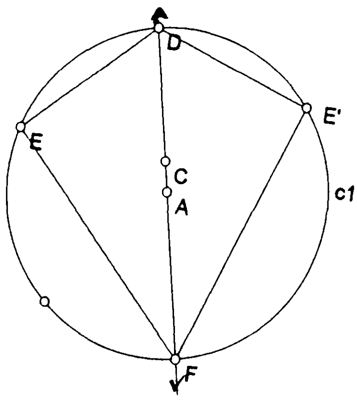


Discover the properties of a cyclic trapezium by measuring different sides and angles. Write down your findings.

Cyclic Kites

Construct a cyclic kite by doing the following:

- Construct circle AB.
- Choose the line tool and construct a diameter.
- Draw a triangle DEF as indicated on the sketch.
- Select the diameter and choose on the *Transform* menu, *Mark Mirror*.
- Select now $\overline{DE}$, point E and $\overline{EF}$.
- On the transform menu, now choose *Reflect*.



Discover the properties of the cyclic quad by measuring different angles. Write down your findings.

IV

Individual Worksheet

You now have to prove in a formal way that any cyclic trapezium will have the same properties as the one your group had investigated.

V **Self-evaluation, "Teachers"**

A. Content, didactical aspects and evaluation

A1. Did you have a clear idea of what your pupils must know at the end of the lesson when you designed the lesson?

Yes More or less No

A2. Do you think your list of instructions are clearly formulated?

Yes More or less No

A3. Are you satisfied that you evaluation methods will help you to get a clear indication whether your students have achieved the learning objectives?

a) as a group Yes Maybe No
b) as individuals Yes Maybe No

B. Co-operative learning

B1. How was positive goal interdependence promoted in this lesson?

....

B2. How was individual accountability promoted in this lesson?

....

B3. How did you make sure that the groups functioned effectively?

....

C. Software

C1. I think the software assisted learning in this lesson in the following way(s):

....

C2. Do you think that this lesson was successful?

....

C3. What will you change if you have to do it again?

....

VI Questionnaire "Pupils"

1. Content, didactical aspects and evaluation

	Yes	Maybe	No
Can you state clearly what you have learned today?			
Did the teacher give a small summary at the end on what you have learned today?			
Were there times in the lesson that you felt lost because of unclear instructions?			
Did you get new insights from this lesson?			
Was it expected from you to apply what you have learned in another setting? (e.g. prove something formally, do another similar problem, etc.)			
Were you evaluated individually?			

2. Co-operative learning

	Yes	Maybe	No
Have you felt dispensable at any stage of the lesson? (that is, have you felt that you group members didn't need you)			
Did you give feedback and assistance to you group members?			
Did you challenge each other's knowledge?			
Would you say your group functioned effectively?			

How was positive goal interdependence promoted in this lesson?

....

How was individual accountability promoted in this lesson?

....

3. Software

In what way did the use of Geometer's Sketchpad support your learning?	Yes	Maybe	No
a) Visualization			
b) Generalization (by seeing many examples of the same concept, a general rule can be formulated).			
c) Intuition (it confirms your "feeling")			
d) Formulation (it helps you to formulate in words your vague ideas).			
e) Proof (it helps you to prove something formally).			

Was it really necessary to use the software here?

……………………………………………………………………………………….

Would you say this lesson was successful?

………………………………………………………………………………………

Appendix B

Case study 2

I

Group building exercises

Prioritise the following descriptions of mathematics, that is, mark the description that you think is the most accurate with 1, the second most accurate with 2 etc.

The creation and study of abstracts structures and objects	
Logic, rigor, accuracy	
A kind of language, a set of notations and symbols	
A way of understanding and predicting real life phenomena	
Reduction of complexity to simplicity	
Problem solving	
The study of patterns	
Exploration, observation and generalization	
An art, a creative activity	
A tool for other sciences	

Prioritise the following aspects of **solving problems** by circling the appropriate numbers.

1. Very important
2. Rather important
3. Not important at all

To get to the right answer	1	2	3
To read the problem carefully	1	2	3
To analyse the problem statement	1	2	3
To divide the problem into sub-problems.	1	2	3
To have a look at similar problems	1	2	3
To try something as soon as possible	1	2	3
To verify afterwards that the solutions is correct	1	2	3
To try several ideas	1	2	3
Not giving up easily	1	2	3
To ask someone for help	1	2	3

What is it that you like about mathematics?

What is it that you do not like about mathematics?

II Lesson 1

The Problem

Design a ski jump that has the following specifications: The ski-jump starts at a height of 30.5 m and finishes at a height of 3 m. From start to finish the ski-jump covers a horizontal distance of 36.6 m.

A skier using the jump will start off horizontally and will fly off the end at a 30 degree angle from the horizontal.

Find a polynomial whose graph is a side view of the ski jump.

Check your answer visually by plotting the graph.

The following will help you in solving **the problem**:

1. Each member of your group will go to a different expert group. Decide amongst yourselves who to send to the expert group on
   A. Problem solving
   B. Graphing with MATLAB
   C. Solving of linear equations.

2. After having returned from the expert groups you have to work together on **the problem.**
   At the end of the period the group has to hand in a diary file of your work on MATLAB as well as a written solution to **the problem.**

Here is some more useful information:

A convenient way to draw curves of a desired shape is to select some points on the curve and find a polynomial whose graph goes through these points. Two points, for example, determine a unique line, which is the graph of a polynomial of degree 1 (if the line is not vertical). Three noncolinear points determine a unique parabola, which is the graph of a polynomial of degree 2 (if the points have distinct x co-ordinates). Four points determine a unique polynomial of degree 3, and so on.

Suppose we want a curve through the points (0,7) , (1,6), (2,9). We will find the unique quadratic polynomial

$$P(x) = ax^2 + bx + c$$

whose graph goes through these points. Substituting the values for x and y, we get a linear system:

$$\begin{aligned} 0a + 0b + 1c &= 7 \\ 1a + 1b + 1c &= 6 \\ 4a + 2b + 1c &= 9 \end{aligned}$$

1. Ask the expert in you group on solving linear equations, to help you solve this system.

2. Ask the expert in your group on graphing to help you graph the polynomial.

3. Try to solve **the problem** now.

III Expert group worksheet

Problem solving

1. Answer the following questions:
   a) What do you understand under *a problem*?

   b) Describe the general steps you would take in solving a given maths problem.

2. What is the definition of a problem ?

Schoenfeld defines a mathematical problem for a student as a task (a) in which the student is engaged and interested and (b) for which the student has no ready access to a means of getting there. Thus, a given task will be experienced by an individual as a problem depending on what he/she knows.

3. Are there general steps in solving mathematical problems?

Through reflective thinking, Polyá examined his own thoughts to identify patterns of problem solving behaviour In his book *How to Solve It*, he proposes a four-phase model:

understanding the problem, making a plan, carrying out the plan and looking back

Metacognition / Managerial processes

This has to do with 'thinking about your own thinking' and can be categorized as follows:

- Knowledge about your own thought processes;
- Self-regulation, or control. Keeping track of what you are doing;
- Beliefs and intuitions. Knowing what ideas and beliefs about mathematics you bring to your work in mathematics, and how it influence your work.

Efficient self-regulation is to be *good at arguing with yourself.*

If you want to become good at arguing with yourself, here are a few questions you can ask yourself while solving problems:

Individual	In a group
What am I doing?	What are we doing?
Can I describe precisely what I am doing?	Can we describe precisely what we are doing?
Why am I doing this?	Why are we doing this?
How does it fit into the solution?	How does it fit into the solution?
How will this help me?	How will this help us?
Can I divide this problem into smaller parts?	Etc.
Do I really understand what is asked?	
Can I verify this solution?	
Will this always work?	
Don't I know of a similar problem that can be used here?	
Which information is unnecessary?	

5. Try the following, while keeping in mind the discussions on metacognition:

Can you place the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 in the box, so that the sum of the digits along each row, each column, and each diagonal is the same? (This is called a magic square).

Expert group

Solving of systems of linear equations

Donelly (1995), pp. 19-35.

Expert group

Plotting graphs

Donelly (1995), pp. 19-35.

IV Individual worksheet

1. MATLAB has an operation to solve systems of linear equations Ax=b, called the matrix division operator "\". That is, to solve Ax=b, you type x=A\b. However, this operation can only be used under certain conditions. What is it?

2. What do you think are the actual computations done by the computer if you type x=A\b?

3. You solved a problem where three points were given and you had to plot a quadratic polynomial going through those points. This led to the solving of a system of linear equations. Can you think of a situation where three points are given but where you cannot find such a quadratic polynomial going through those points (i.e., where the system is inconsistent)?

V Evaluation list (lesson 1 and lesson 2)

In your own words state today's goal or goals.

What was the topic/s of the day?

These are the strategies and concepts I learner today:

What was your AHA (now I understand) today?

I'm still confused about[1]

Today in class I felt

because

(From Bagley & Gallenberger, 1992).

VI Lesson 2

List of instructions:

1. Decide who of you will take the role of
   a) Scribe ________________________
   b) Operator______________________________
   c) Problem solving expert______________________

2. Complete the given worksheet and hand it in at the end of the lesson.

3. Complete the individual worksheets and hand in at the end of the lesson. The individual worksheets may be completed while you work in the groups.

Group worksheet

Leon et al. (1996), pp. 78-80, 82-84.

VII Individual worksheet

1. Suppose that A is an m x n matrix of rank r. Complete the following:

FUNDAMENTAL SPACE	DIMENSION
Row space of A	
Column space of A	
Nullspace of A	
Nullspace of A^T	

2. Using the Gaussian elimination procedure determine h and k so that the solution set of the given system

$$x_1 + 2x_2 = k$$
$$4x_1 + hx_2 = 5$$

i) is empty,
ii) contains a unique solution,
iii) contains infinitely many solutions.

3. Mark each statement true or false. If true, prove it. If false, produce a counterexample.

i) If a system Ax = b has more than one solution, then so does the system Ax = 0.
ii) Every matrix has a unique row-echelon form.
iii) If A and B are both m x n matrices then $A^T B$ is a square matrix.
iv) $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ is invertible.

VIII

Evaluation of social skills and group processing	Yes	Not well enough	No
We checked each other's knowledge.			
We helped each other.			
We encouraged each other.			
We finished all the given tasks.			
We learned something new.			