MATRIX ESTIMATION FROM TRAFFIC COUNTS: AN ALTERNATIVE APPROACH BASED ON PROPORTIONAL PATH AVERAGES

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ABSTRACT

In attempting to match assigned volumes with traffic counts, several different methods are commonly used in transportation and traffic modelling to adjust demand matrices within iterative traffic assignment procedures. This paper describes an alternative approach based on proportional path averages, implemented using a simple algorithm translated into an Excel VBA macro. The algorithm can be applied independently of the assignment technique, as it requires as input only three text files: a demand prior matrix, a set of link and/or turn traffic counts and the assigned volumes along all Origin-Destination paths. An iterative adjustment is applied to the proportional path volumes where firstly, all fractional volumes passing through a count station are adjusted proportionally to match the specific count, and secondly, each fractional OD path volume is adjusted to match the average of the fractional path counts at all stations along the OD path. The two steps are repeated inside an inner loop until convergence, requiring at most 3 traffic assignments within the outer loop. Practical application of the principle is illustrated via two project case studies.

1. INTRODUCTION

There is no doubt that the estimation of Origin Destination (OD) demand matrices using available traffic counts is a complex topic. Over the last five decades, transportation and traffic modelling specialists have developed several different approaches. Many of these techniques have found their way into commercial macro-, meso- and microscopic strategic transportation and traffic simulation software packages such as Emme, TransCAD, Saturn, Aimsun, Vissim or similar. These matrix adjustment algorithms are implemented either as built-in modules or as stand-alone external macro procedures that access one or more intrinsic modules.

Despite the wide variety of algorithms being applied, the common thread running through most of the simulation models comprises the following components and steps:

1. Availability of a prior demand trip matrix, derived from surveys or via a gravity, entropy, or similar distribution model.
2. An abstract representation of the road network, consisting of links and intersections, along which road traffic will flow between origins and destinations.
3. A traffic count dataset containing both link and intersection turn volumes.
4. An assignment algorithm that assigns the trip matrix to the road network, in an iterative manner to attain equilibrium between traffic volumes and travel times.
5. Comparison of modelled traffic volumes with the input traffic counts.
6. Matrix adjustment, to improve the alignment between the modelled volume and traffic counts, using a demand adjustment algorithm.
7. Iterative application of steps 4 to 6, until no further improvement can be obtained, as measured in step 5.

The demand adjustment algorithms invariably require the minimisation of an objective function subject to constraints. An additional complication arises in that an infinite number of OD demand matrices satisfying the conditions are possible. Thus, to ensure that the solution is feasible, the adjusted demand must also remain as close as possible to the original OD demand matrix.

Table 1 lists a subset of the models and the objective function parameters, as summarised from the literature reviews cited in Abrahamsson (1998) and Lindström/Persson (2018).

Table 1: Objective functions used in matrix adjustment algorithms

Approach	Method(s)	Objective function parameters and constraints	References
Traffic modelling	Minimum information / Entropy maximisation	• Observed and modelled link flows • Proportionate matrix • Synthetic or outdated prior matrix	• Willumsen (1978) • Erlander et al (1979) • Van Zuylen and Willumsen (1980) • Erlander et al (1984) • Fisk (1988) & (1989) • Tamin and Willumsen (1989) • Kawakami et al (1992) • Sherali et al (1994)
Statistical inference	Maximum likelihood	• Congested case with user-equilibrium as constraint • Prior matrix obtained from surveys	• Spiess (1987)
	Generalized least squares	• Prior matrix obtained from estimated OD demand with probabilistic error term • Traffic counts treated as stochastic	• Cascetta (1984) • Bell (1991) • Yang et al (1992) &(1994) • Bierlaire and Toint (1995) • Fujita et al (2017)
	Bayesian inference	• Counts and matrix with MVN probability distribution	• Maher (1983)
Gradient based	Steepest descent	• Congested case: user-equilibrium • Prior matrix	• Spiess (1990) • Drissi-Kätouni and Lundgren (1992) • Florian and Chen (1993) • Chen (1994) • Denault (1994) • Cipriani (2011)

Solution of such minimisation problems, regardless of their formulation, requires advanced mathematical techniques which, for the non-mathematician, are quite daunting and in some cases almost incomprehensible.

This paper outlines a simple approach that is practical, computationally efficient, and scalable for application in large urban networks. Mathematical formulations are limited to fractions and averages, resulting in an understandable approach that can be applied quickly and with relative ease within an Excel spreadsheet, using the built-in VBA scripting.

2. MATRIX ADJUSTMENT ALGORITHM

The algorithm is formulated simply as follows:

- Step 1: For each turn where a count is available, adjust all modelled OD path volumes passing through the turn, pro rata to the individual path volumes, so that the total volume passing through the turn matches the count volume, as per example in Figure 1.

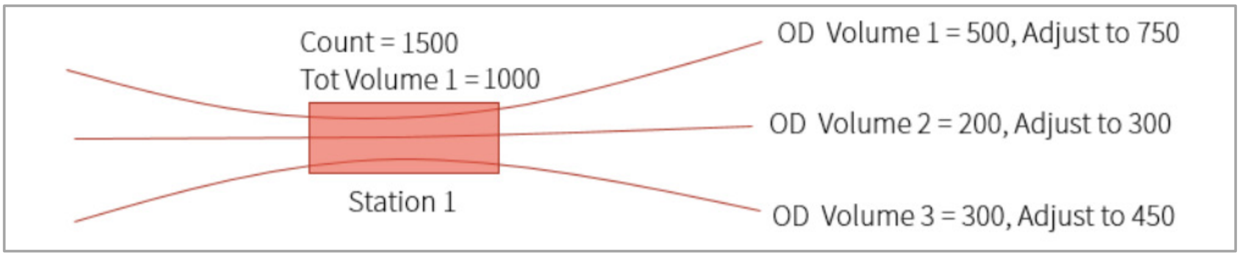


Figure 1: Adjusting Fractional OD Movements through Traffic Count Stations

- Step 2: For each OD movement path, adjust the OD path volume to match the individual partial path count of turn volumes along that path. Since the partial path counts should remain constant along the entire OD-path, any differences in the partial path count volumes are smoothed out by setting the path OD volume equal to the average partial count volume along that path, as per example in Figure 2. The adjustment is applied for all OD movements that pass through two or more count stations.

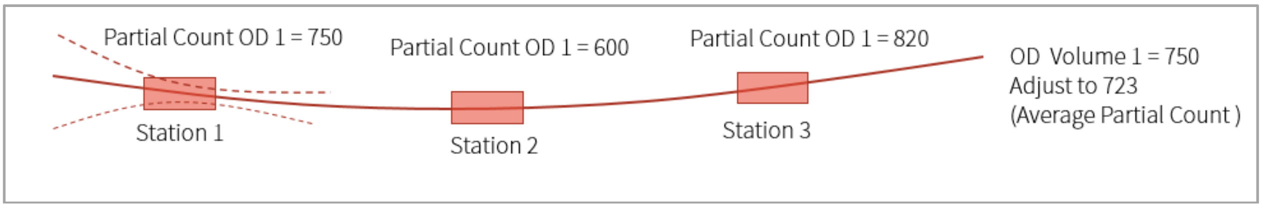


Figure 2: Adjusting OD Trips using Averaged Partial Counts along OD Paths

- Each step will change the OD volumes, so repeat steps 1 and 2 until no further changes occur and the calibration converges to a consistent solution.

3. ILLUSTRATIVE EXAMPLE

3.1 Network Layout

Figure 3 illustrates a hypothetical road network consisting of:

- Four zones numbered from 1 to 4.
- Eight nodes numbered from 11 to 18. Two nodes, 11 and 13, have multiple legs and are treated as intersections, but without incurring delays due to signal settings, turning movements etc.

- Four two-way centroid connectors, each with a nominal length of 100m and constant speed of 60km/hr.
- Eight two-way links, all 200m long. All links share the same attributes and typical congestion behaviour, with Speed = Free-flow Speed * (1+0.5 * (Volume/Capacity)^4).

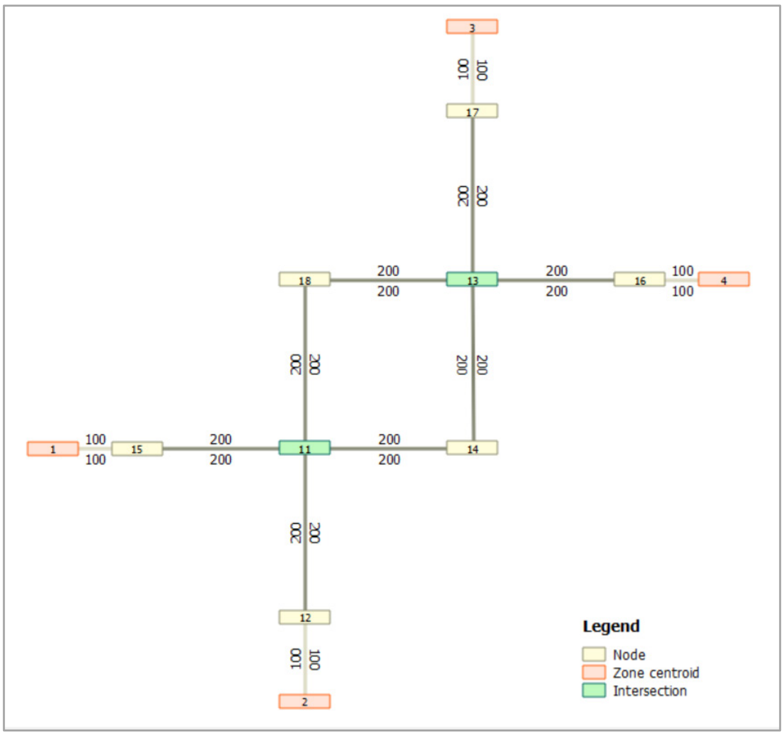


Figure 3: Illustrative Road Network Example

3.2 Prior Demand Matrix

Table 2 illustrates a typical prior demand matrix derived from first principles via a survey or a distribution model, with trip origins in rows and destinations in columns.

Table 2: Prior Demand Matrix (Vehicles/hr)

Zones	1	2	3	4	Total Orig
1		150	100	350	600
2	400		200	450	1,050
3	450	500		550	1,500
4	600	650	300		1,550
Total Dest	1,450	1,300	600	1,350	4,700

3.3 Equilibrium Traffic Assignment

Given the symmetrical layout of the hypothetical network, any equilibrium assignment of the prior matrix onto the road network should converge to the volume plot illustrated in Figure 4, where the trips between nodes 11 and 13 are split equally between the alternative paths via nodes 18 and 14.

Table 3 shows all available OD paths and the proportional trips, as inferred by inspection.

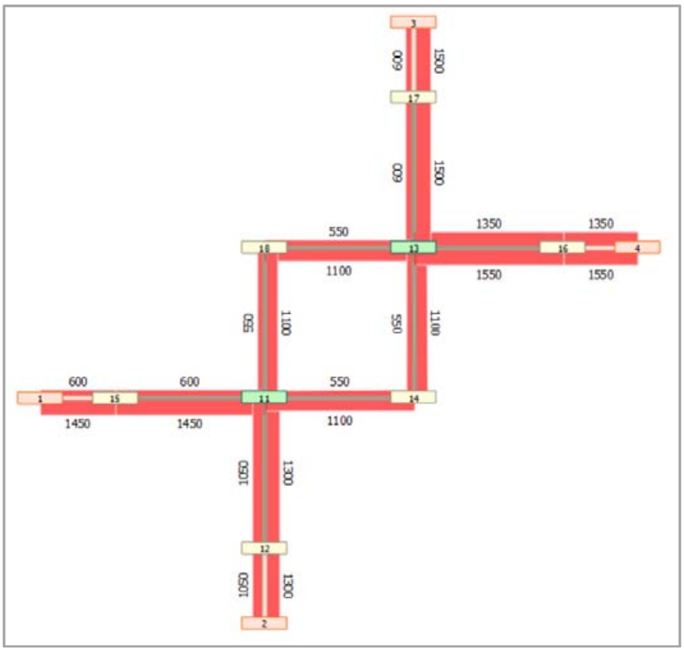


Figure 4: Traffic Volumes, Prior Matrix Equilibrium Assignment

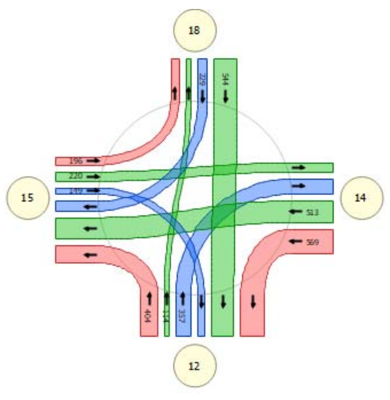
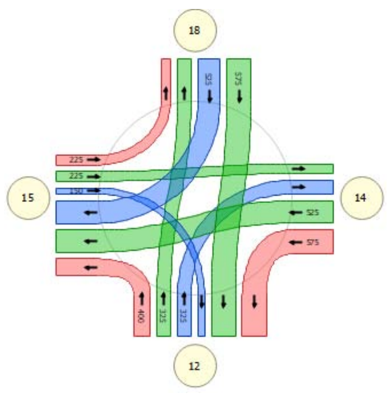
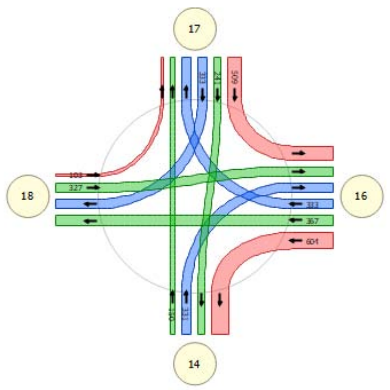
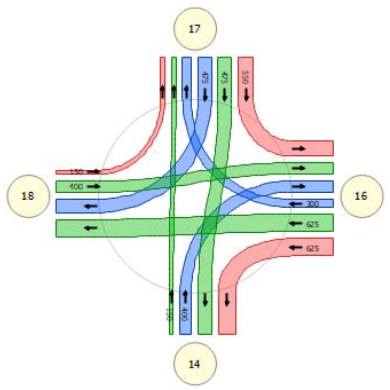
Table 3: OD Paths, Prior Matrix Equilibrium Assignment

Orig	Dest	Trips	Path via Nodes						
1	2	150	1	15	11	12	2		
1	3	50	1	15	11	18	13	17	3
1	3	50	1	15	11	14	13	17	3
1	4	175	1	15	11	18	13	16	4
1	4	175	1	15	11	14	13	16	4
2	1	400	2	12	11	15	1		
2	3	100	2	12	11	18	13	17	3
2	3	100	2	12	11	14	13	17	3
2	4	225	2	12	11	18	13	16	4
2	4	225	2	12	11	14	13	16	4
3	1	225	3	17	13	18	11	15	1
3	1	225	3	17	13	14	11	15	1
3	2	250	3	17	13	18	11	12	2
3	2	250	3	17	13	14	11	12	2
3	4	550	3	17	13	16	4		
4	1	300	4	16	13	18	11	15	1
4	1	300	4	16	13	14	11	15	1
4	2	325	4	16	13	18	11	12	2
4	2	325	4	16	13	14	11	12	2
4	3	300	4	16	13	17	3		

3.4 Traffic Counts

Table 4 and Figure 5 compare hypothetical traffic counts with the modelled volumes, after assignment of the prior demand matrix. Although the R-squared seems quite reasonable, the correlation is poor, as indicated by the scatter.

Table 4: Comparison of Traffic Counts and Modelled Volumes (Prior Demand Matrix)

From	Via	To	Count	Model	Count	Model
12	11	14	250	325		
15	11	12	149	150		
18	11	15	305	525		
12	11	18	204	325		
14	11	15	370	525		
15	11	14	220	225		
18	11	12	544	575		
12	11	15	404	400		
14	11	12	569	575		
15	11	18	196	225		
14	13	16	331	400		
14	13	17	150	150		
16	13	14	550	625		
16	13	17	333	300		
16	13	18	520	625		
17	13	14	370	475		
17	13	16	509	550		
17	13	18	333	475		
18	13	16	327	400		
18	13	17	103	150		
Total			**6,737**	**8,000**		

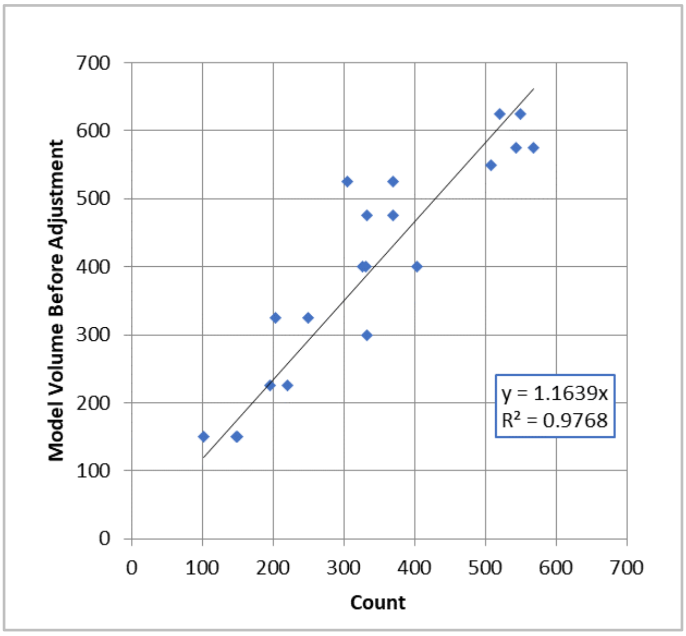


Figure 5: Count vs Model Scatter Diagram Before Adjustment

3.5 Applying the Demand Adjustment Algorithm

The matrix adjustment algorithm outlined in Section 2 was applied to the example, after expanding the steps within an Excel VBA macro as follows:

- Start-up: Manually enter the prior OD matrix, traffic counts, modelled volumes, OD paths and the fractional volumes along each OD path, through all the count stations. Create a chart with a scatter diagram illustrating the comparison between the counts and modelled volumes.

- Iterate: Apply the two step algorithm to adjust the model volumes within a loop, while monitoring the scatter diagram after each iteration.

- On convergence, calculate the cells of the adjusted matrix by summing the volumes along the adjusted OD paths.

Table 5 illustrates the adjusted OD matrix, after four iterations of the adjustment algorithm.

Table 5: Adjusted Demand Matrix (Vehicles/hr)

Zones	1	2	3	4	Total Orig
1	-	150	97	316	563
2	400	-	150	314	864
3	271	435	-	509	1,215
4	417	639	333	-	1,389
Total Dest	1,088	1,224	580	1,139	4,031

An equilibrium assignment of the adjusted demand matrix to the road network then created new modelled turn volumes flows. These differ from the adjusted turn volumes by at most 1 trip, with identical scatter diagrams. Normally, in larger networks, the entire process (comprising equilibrium assignment + matrix adjustment + re-assignment) would need to be repeated two to three times, to ensure alignment between the adjusted and final assigned traffic volumes.

Figure 6 illustrates the scatter diagrams, after adjustment and re-assignment.

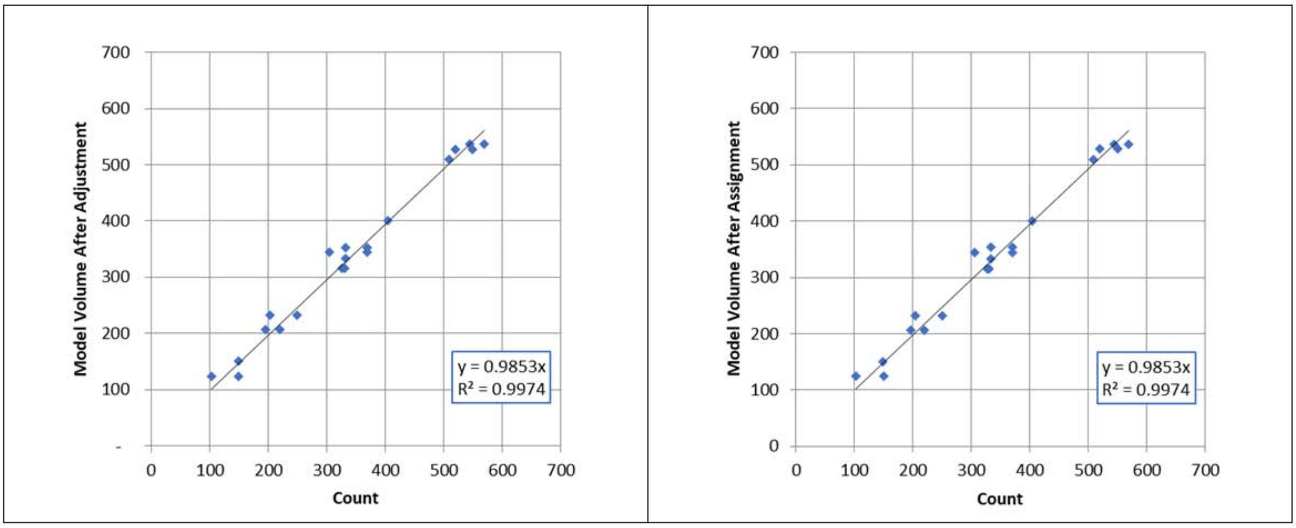


Figure 6: Counts vs Adjusted and Assigned Modelled Volumes

The averaging of fractional counts along the separate OD paths offers the additional benefit of providing a "best-fit" between conflicting counts located on the same OD path. Such averaging also minimises changes to the original matrix. In contrast, other techniques mentioned in the literature are invariably based on a "brute force" approach, where the adjustments to the demand matrix are done in combination with iterative equilibrium traffic assignments, resulting in excessive computer run-times.

4. CASE STUDIES

The two case studies outlined below illustrate successful application of the algorithm using variants of the Excel VBA macro spreadsheet described in Section 3.

4.1 Inyaninga Integrated Human Settlements Development

Tongaat Hulett Developments appointed Hatch Africa (Pty) Ltd to undertake a Strategic Transport Assessment and several Traffic Impact Assessments within the Inyaninga area located north-west of King Shaka International Airport between Tongaat and Verulam.

The 2017 Base Year model comprised:

- 99 Zone centroids.
- Classified intersection counts at 16 locations, totalling 120 individual turning movements.

Figure 7 illustrates the extent of the Aimsun micro-simulation model created to analyse road traffic forecasts associated with the phased roll-out of this large scale development.

Figure 7: Inyaninga Aimsun Model

The algorithm was applied successfully to adjust a sub-area demand matrix extracted from the eThekwini Transport Master Plan Emme/4 model. Figure 8 illustrates the resultant fit between modelled volume and turn counts, for the AM and PM peak hours.

In both cases, convergence was obtained after 15 iterations of the demand adjustment algorithm, within each of the 3 outer loop Aimsun micro-simulation assignments.

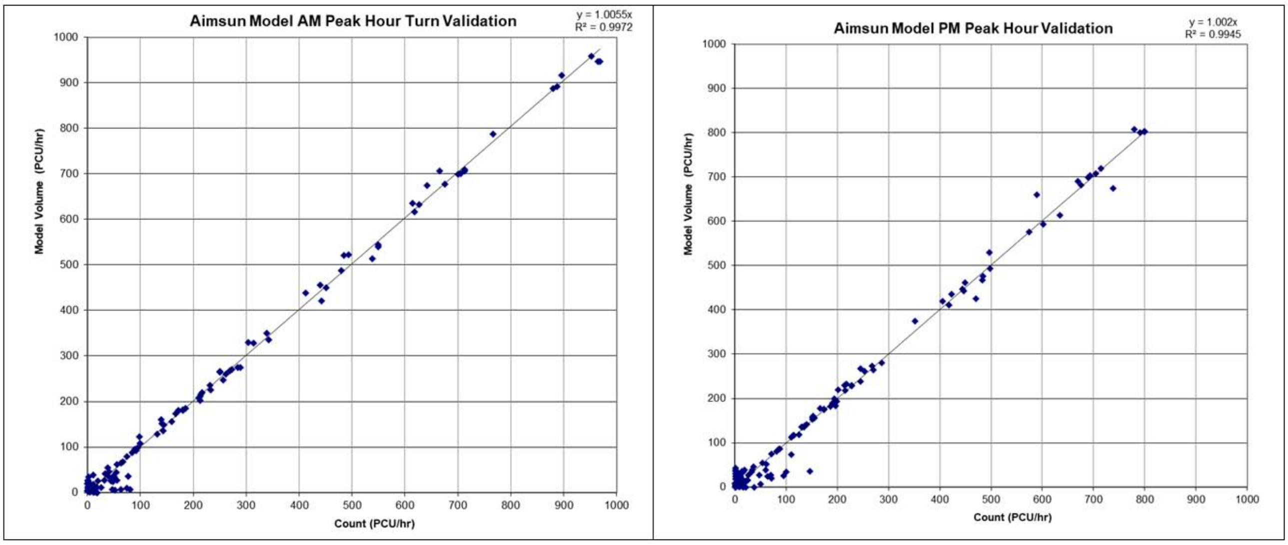


Figure 8: Inyaninga Aimsun Model Comparison after Demand Adjustment

4.2 Proposed Umhlathuze Waste Water Reuse Facility

The City of Umhlathuze appointed Escongweni BPH Engineers in 2019 to prepare and submit a Basic Assessment Report for environmental approval of the Proposed Umhlathuze Waste Water Reuse Facility located west of the N2, about 3km north of the Empangeni town centre. A Traffic Impact Assessment was required to support the application.

The 2019 Base Year covering Richards Bay and Empangeni and built on an Emme/4 platform comprised the following:

- 80 Traffic zones.
- Classified intersection turn counts at 14 locations.
- CTO link counts on the N2 and surrounding road network.

Figure 9 illustrates the road network extent.

The algorithm was applied successfully to adjust the base year demand matrix derived from first principles. Figure 10 illustrates the resultant fit between modelled volume and turn counts, for the AM peak hour, for the 145 individual turn or link volumes included in the adjustment process. Convergence was obtained after 5 iterations of the demand adjustment algorithm, within each of the 3 outer loop Emme/4 equilibrium assignments.

Figure 9: Umhlathuze Base Year Emme/4 Network

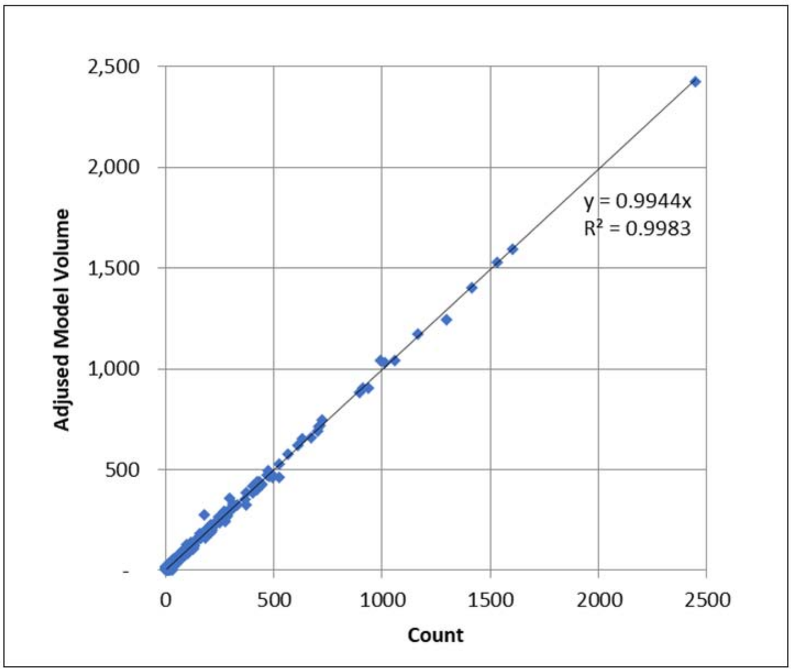


Figure 10: Umhlathuze Emme/4 Model Comparison after Demand Adjustment

5. CONCLUSIONS

The matrix estimation algorithm based on proportional path averages is a viable alternative to the traditional techniques applied in commercial transportation and traffic engineering macro-, meso- and microscopic simulation software packages.

The method is computationally efficient, easy to understand and produces results comparable with conventional techniques. This paper illustrated successful application of the method using an illustrative hypothetical example, as well as practical application within two recent traffic engineering projects.

Excel VBA was used to code the algorithm into a macro. Further research will be required to translate/migrate the algorithm to more efficient programming languages such as python or similar, for direct integration into existing software.

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