

## Annex B: The Mixed Logit Model (ML)

In accordance with random utility theory (McFadden, 1973, McFadden, 1974), we assume that the utility of individual  $n$  of choosing alternative  $j$  in choice situation  $t$  can be represented as:

$$U_{njt} = \boldsymbol{\beta}'_n \cdot X_{njt} + \varepsilon_{njt} \quad (1)$$

where  $X_{njt}$  is a vector of  $K$  observed attributes related to the alternative  $j$  of the choice situation  $t$ ;  $\boldsymbol{\beta}'_n$  is a vector of preference parameters which explain choices;  $\varepsilon_{njt}$  is the unobserved error term.

The preference parameters  $\beta_k$  are distributed in the population according to continuous random distributions  $f(\beta)$  to be chosen by the analyst (Train, 2009). For the full vector of  $K$  random coefficients in the model, we may write the full set of random parameters as:

$$\boldsymbol{\beta}_n = \boldsymbol{\beta} + \boldsymbol{\Gamma} \cdot \boldsymbol{v}_n \quad (2)$$

where  $\boldsymbol{\Gamma}$  is a lower-triangular matrix that takes care of the possible correlations among coefficients<sup>1</sup>, and  $\boldsymbol{v}_n$  correspond to the individual specific heterogeneity. If at least one of the elements below the diagonal of  $\boldsymbol{\Gamma}$  shows statistical significance, this is supportive of dependence across tastes (Scarpa and Del Giudice, 2004). As individuals were confronted to several choice situations (panel data), we have to consider a sequence of  $S$  observed choices ( $s_1, s_2, \dots, s_S$ ) by the same individual. The probability of observing a sequence  $s$  conditional on  $\boldsymbol{\beta}_n$  is:

$$L_{ns}(\boldsymbol{\beta}_n) = \prod_{s=s_1}^{s_S} \left[ \frac{\exp(\boldsymbol{\beta}_n' \cdot X_{njs})}{\sum_{q=1}^J \exp(\boldsymbol{\beta}_n' \cdot X_{nqs})} \right] \quad (3)$$

However, the researcher does not know  $\boldsymbol{\beta}_n$  and therefore cannot condition the probability of choosing one alternative on  $\boldsymbol{\beta}$ . The unconditional choice probability is therefore the integral of  $L_{ns}(\boldsymbol{\beta}_n)$  over all possible values of  $\boldsymbol{\beta}_n$  is  $P_{ns} = \int L_{ns}(\boldsymbol{\beta}) \cdot f(\boldsymbol{\beta}) \cdot d\boldsymbol{\beta}$ . Because the integral in  $P_{ns}$  equation does not have a closed form solution, the parameters of the model are estimated by simulated maximum likelihood estimation techniques (Train, 2009).

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<sup>1</sup> The matrix  $\boldsymbol{\Gamma}$  corresponds to the Cholesky decomposition of the covariance matrix:  $\boldsymbol{\Gamma} \cdot \boldsymbol{\Gamma} = \text{COV}$