

Annex B: The Mixed Logit Model (ML)

In accordance with random utility theory (McFadden, 1973, McFadden, 1974), we assume that the utility of individual n of choosing alternative j in choice situation t can be represented as:

$$U_{njt} = \boldsymbol{\beta}'_n \cdot X_{njt} + \varepsilon_{njt} \quad (1)$$

where X_{njt} is a vector of K observed attributes related to the alternative j of the choice situation t ; $\boldsymbol{\beta}'_n$ is a vector of preference parameters which explain choices; ε_{njt} is the unobserved error term.

The preference parameters β_k are distributed in the population according to continuous random distributions $f(\beta)$ to be chosen by the analyst (Train, 2009). For the full vector of K random coefficients in the model, we may write the full set of random parameters as:

$$\boldsymbol{\beta}_n = \boldsymbol{\beta} + \boldsymbol{\Gamma} \cdot \boldsymbol{v}_n \quad (2)$$

where $\boldsymbol{\Gamma}$ is a lower-triangular matrix that takes care of the possible correlations among coefficients¹, and \boldsymbol{v}_n correspond to the individual specific heterogeneity. If at least one of the elements below the diagonal of $\boldsymbol{\Gamma}$ shows statistical significance, this is supportive of dependence across tastes (Scarpa and Del Giudice, 2004). As individuals were confronted to several choice situations (panel data), we have to consider a sequence of S observed choices (s_1, s_2, \dots, s_S) by the same individual. The probability of observing a sequence s conditional on $\boldsymbol{\beta}_n$ is:

$$L_{ns}(\boldsymbol{\beta}_n) = \prod_{s=s_1}^{s_S} \left[\frac{\exp(\boldsymbol{\beta}_n' \cdot X_{njs})}{\sum_{q=1}^J \exp(\boldsymbol{\beta}_n' \cdot X_{nqs})} \right] \quad (3)$$

However, the researcher does not know $\boldsymbol{\beta}_n$ and therefore cannot condition the probability of choosing one alternative on $\boldsymbol{\beta}$. The unconditional choice probability is therefore the integral of $L_{ns}(\boldsymbol{\beta}_n)$ over all possible values of $\boldsymbol{\beta}_n$ is $P_{ns} = \int L_{ns}(\boldsymbol{\beta}) \cdot f(\boldsymbol{\beta}) \cdot d\boldsymbol{\beta}$. Because the integral in P_{ns} equation does not have a closed form solution, the parameters of the model are estimated by simulated maximum likelihood estimation techniques (Train, 2009).

¹ The matrix $\boldsymbol{\Gamma}$ corresponds to the Cholesky decomposition of the covariance matrix: $\boldsymbol{\Gamma} \cdot \boldsymbol{\Gamma} = \text{COV}$