A PARTIAL ECONOMIC WARRANT FOR CLIMBING LANES

K M Wollhuter  Pr.Eng
CSIR/Transportek, P O Box 395 Pretoria, 0001

A variety of warrants for climbing lanes exist. Typically, they relate to variations in truck speeds on grades either as absolute values or relative to the speeds of passenger cars. In essence, reference is either to safety or to convenience. In neither case, is any consideration given to the cost of creating a road environment or the cost of operating in it. This study suggests that the warrants employed should be expanded from the safety- and convenience-related warrants to include an economic warrant. A procedure for analysing the economics of climbing lane provision is offered.

The only parameter considered in the proposed economic warrant is the cost of delay to passenger cars. Hence the reference to a partial economic warrant. The argument offered is that, if warranted on the basis of delay alone, there is no need to consider the other economic factors such as vehicle operating costs.

Data were acquired spanning a variety of gradients, traffic flows, traffic stream compositions and directional splits. These were used to calibrate TRARR, a micro simulation model developed by the Australian Road Research Board. Multiple linear regression of the outcome of the micro simulation provided a relationship between vehicle speeds and the identified independent variables. The difference between journey times at the passenger car space mean speed under the various flow regimes and under the “zero” regime of no competing traffic constitutes delay.

Using ADT and the peak hour factor, β, developed by Jordaan, the flow in any given hour of the year is derived. The delay associated with this flow and the road gradient can then be calculated assuming the other traffic factors remain constant. The calculation is repeated for each hour of the year and summed for succeeding years over the design life of the climbing lane. Finally, a monetary value is attached to the total delay and discounted to the anticipated date of construction.

The benefit accruing from the provision of the climbing lane is the elimination of this total delay. The benefit is compared to the sum of the cost of construction and the discounted cost of maintenance of the climbing lane.

Software to carry out these calculations has been developed.

1 INTRODUCTION

The volume of freight moved on the South African road network has increased dramatically over the last decade. It follows that, with an increasing percentage of trucks in the traffic stream, levels of service have declined implying increases in user costs in terms of delay. Resolving this problem is difficult given that spending on infrastructure has been diminishing in real terms. After three decades of steady decline, the 2001 Budget shows a modest increase in the allocation to infrastructure. It may therefore be possible, once we have caught up with the maintenance backlog, to do something about actual improvements to the service provided by the network.
The two questions that simultaneously spring to mind are: “How can we utilise this network better?” and “How can we make optimum use of our scarce financial resources?”

One possible option with regard to the first question is to use climbing lanes that will have the effect of improving the Level of Service on gradients to match those prevailing on the relatively flat portions of the two-lane cross-section where, in general, the Level of Service is adequate. In exercising this option, the answer to the second question is to provide these climbing lanes only where they are economically justified.

This paper offers a method based on reduction of delay for considering the economic merits of providing a climbing lane on a two-lane road.

2 CURRENT WARRANTS FOR CLIMBING LANES

Current warrants tend to favour construction of climbing lanes on steeper grades at relatively low traffic flows in preference to construction on flatter grades at higher traffic volumes. In the former case, the steeper gradients are often associated with rugged topography and generate correspondingly high construction costs. The relatively low traffic flows suggest that little total delay is eliminated by the provision of the climbing lane. The second case, on the other hand implies low construction costs and a significant reduction in total delay. It is thus inferred that current design practices are directed more towards convenience than economic considerations.

In the literature the underlying reasons for the selection of a given warrant are often not stated or are obscure.

2.1 Volume-based warrants

The Highway Capacity Manual\(^1\) considers climbing lanes to be warranted if upgrade volumes exceed 200 veh/h and truck volumes exceed 20 veh/h in the design hour. The design hour is typically selected as the 30\(^{th}\) highest flow occurring in the design year, which is commonly accepted as being the last year of the design life of the road often 20 years after completion of construction.

Reference to the Comprehensive Traffic Observations\(^2\) indicates that many South African roads have traffic counts higher than the warranting flows so that the traffic volumes quoted would not, by themselves, eliminate many routes in consideration of the provision of climbing lanes.

2.2 Speed-based warrants

Speed reductions adopted around the world vary typically in the range of 15 km/h to 25 km/h\(^3\) and are usually intended to be applied to a single grade. The Australian approach\(^4\) bases the need for climbing lanes on examination of a considerable length of road. The justification for climbing lanes is based on traffic volume, percentage of trucks and the availability of passing opportunities along the road. Speed reduction is to a final speed of 40 km/h and not through 40 km/h.

South African practice, as described in TRH175\(^5\), uses a combination of the speed and traffic volume as a warrant, requiring both to be met before the climbing lane is considered to be warranted. The speed reduction applied is 20 km/h from an initial 80 km/h. The volume warrant is given in Table 1 below.
### 2.3 Level of Service Warrants

Level of Service is a descriptor of operational characteristics in a traffic stream. An important feature is that it is purely a representation of the driver’s perception of the traffic environment and is not concerned with the economics of modifying that environment. The warrants suggested by the Highway Capacity Manual are:

- A reduction of two or more Levels of Service in moving from the approach segment to the grade; or
- Level of Service E exists on the grade.

Polus et al\(^7\) are of the opinion that the Highway Capacity Manual warrant is too severe. They suggest further that this warrant does not, in any event, match the speed reduction warrant so that the provision of a climbing lane at a specific site is dependant on the type of warrant selected. They accordingly suggests that a climbing lane is warranted by a reduction of one or more Levels of Service.

### 2.4 The consequences of adoption of current warrants

Truck speed reductions without reference to traffic volumes have merits in terms of safety. Their principal benefit lies in reduction in the speed differential in the through lane, thereby reducing the probability of the occurrence of an accident. It is, however, theoretically possible that a climbing lane would be considered warranted merely because it would lead to the required truck speed reduction even if total traffic volumes were very low with virtually no trucks in the traffic stream.

The addition of a volume warrant increases the likelihood of a reasonable economic return on the provision of a climbing lane. Analysis using ANDOG (Analysis of Delay on Grades)(more of which later) and as shown in Table 2 demonstrates the value of time savings per kilometre achieved over the design life of the climbing lane when warranting volumes, converted to AADT, and truck speed reductions are applied.

### Table 2: Time savings on climbing lanes justified by current warrants

<table>
<thead>
<tr>
<th>Gradient (%)</th>
<th>ADT (veh/d)</th>
<th>Time saving (R/km)</th>
<th>Cost of convenience (R/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1 590</td>
<td>73 150</td>
<td>36 850</td>
</tr>
<tr>
<td>6</td>
<td>1 180</td>
<td>53 210</td>
<td>56 790</td>
</tr>
<tr>
<td>8</td>
<td>960</td>
<td>51 320</td>
<td>58 680</td>
</tr>
<tr>
<td>10</td>
<td>810</td>
<td>70 580</td>
<td>39 420</td>
</tr>
</tbody>
</table>

The difference between the value of total road user savings and the construction plus maintenance cost represents an economic disbenefit that, effectively, is the value attached to convenience. The table also implies that the cost of construction has deliberately been kept very low.
The Level of Service warrant was also tested using ANDOG. Table 3 demonstrates that the total delay suffered before a climbing lane is warranted on the flatter gradients is far greater than on the steeper slopes. Unfortunately, the reduction in benefit (through the elimination of delay by provision of the climbing lane) is somewhat anomalous. Steep gradients usually suggest rugged terrain with correspondingly increased construction costs.

Table 3 : Total delay for LOS D on various gradients

<table>
<thead>
<tr>
<th>Gradient (%)</th>
<th>Service flow rate (veh/h)</th>
<th>Individual delay (s/veh/km)</th>
<th>Total delay (h/h/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1 705</td>
<td>31.5</td>
<td>6.56</td>
</tr>
<tr>
<td>4</td>
<td>1 431</td>
<td>30.2</td>
<td>5.28</td>
</tr>
<tr>
<td>5</td>
<td>1 104</td>
<td>29.0</td>
<td>3.91</td>
</tr>
<tr>
<td>6</td>
<td>701</td>
<td>27.5</td>
<td>2.36</td>
</tr>
<tr>
<td>7</td>
<td>493</td>
<td>25.9</td>
<td>1.56</td>
</tr>
</tbody>
</table>

The intention has been to demonstrate that warrants currently in use do not necessarily lead to the provision of economically justifiable climbing lanes.

It is conceded that performance-based warrants such as those described above are not intended nor are ever likely to be fully economic. In view of the economic restraints on new construction, it is suggested that a compromise between convenience and cost effectiveness is required. The compromise proposed is that, while delay – seen as a major criterion of Level of Service – is employed in determination of the need for provision of a specific climbing lane, the delay considered would not be that suffered by the individual vehicle but rather the delay inflicted on the entire traffic stream.

3 MODELLING OF CLIMBING LANES

Having already hinted that delay, rather than arbitrarily selected truck speed reductions with or without equally arbitrarily selected traffic flows should be the Measure of Effectiveness of a climbing lane, it follows that a major paradigm shift is being introduced into the consideration of the provision of climbing lanes.

The current warrants tend to focus on the capabilities of the trucks in the traffic stream with it being assumed that this will impact in some or other unspecified fashion on the passenger cars in the stream. It is suggested that it would be more realistic to focus on the performance of the passenger cars in the traffic stream and directly quantify the impact of the truck traffic on the passenger cars.

The problem with delay is that it cannot be measured. The reason for this is that it is not a single quantity but is the difference between an actual – and hence measurable – state and a desired state which does not present itself at the time when other measurements, e.g. traffic flow, are being made. It must, therefore be modelled in some or other fashion. Botha⁶ refers to various models that have been used in the consideration of the provision of climbing lanes and suggests that these can be broadly divided into three groups being empirical models, analytical models and simulation models.

Empirical models, such as contained in the Highway Capacity Manual are usually simple but have the weakness of requiring much data to be reliable and then are both space-bound, i.e. appropriate only to the area where they were developed, and time-bound, i.e. cannot accommodate changes in vehicles or driver characteristics. Analytical models rely on classical mathematics and theory of probability. The randomness of the variables is considered only in the broad aspect of employing
distributions of these variables and microscopic interactions are not considered. These models are inexpensive to develop but tend to be restrictive insofar uniform highway sections have to be assumed as well as steady traffic state conditions. The introduction of a climbing lane is a departure from a uniform highway section and traffic behaviour in the vicinity of a climbing lane cannot be described as being uniform so that analytical models are not appropriate to climbing lanes.

Microscopic simulation models on the other hand are used to replicate the actual highway system and the behaviour of individual vehicles on the highway. They require substantial computational ability but the data requirements of empirical modelling are replaced by sample data for calibration purposes, representing a distinct saving in the cost of data collection. Furthermore, such models can be updated fairly easily for use in different environments or with different vehicles and drivers. Simulation, therefore, is the logical answer to an attempt to model climbing lanes.

Various simulation models were considered and it was ultimately decided that TRARR (Traffic on Rural Roads) would be the most suitable. Joubert had exhaustively considered this model under South African conditions and had concluded that, with some minor modifications, it could be used with confidence.

### 4 DATA ACQUISITION

Seven sites were selected in order to cover the range of gradients. A prime criterion for their selection was that data acquisition had to take place where speeds had stabilised to match the gradient. The grades had, therefore, to be substantially longer than the critical length. The sites are listed in Table 4.

**Table 4 : Description of data sets**

<table>
<thead>
<tr>
<th>Site</th>
<th>Cross-section</th>
<th>Gradient ( % )</th>
<th>Distance along grade (m)</th>
<th>Sample size (veh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cornelia</td>
<td>2-lane</td>
<td>3.62</td>
<td>1 600</td>
<td>9 298</td>
</tr>
<tr>
<td>Colenso</td>
<td>2-lane</td>
<td>5.21</td>
<td>3 000</td>
<td>17 384</td>
</tr>
<tr>
<td>Long Tom</td>
<td>Freeway</td>
<td>8.38</td>
<td>2 000</td>
<td>16 200</td>
</tr>
<tr>
<td>Rigel North</td>
<td>Freeway</td>
<td>3.54</td>
<td>3 100</td>
<td>16 149</td>
</tr>
<tr>
<td>Rigel South</td>
<td>Freeway</td>
<td>4.45</td>
<td>4 000</td>
<td>15 985</td>
</tr>
<tr>
<td>Ben Schoeman</td>
<td>Freeway</td>
<td>4.97</td>
<td>1 400</td>
<td>21 017</td>
</tr>
<tr>
<td>Krugersdorp</td>
<td>Freeway</td>
<td>6.44</td>
<td>1 800</td>
<td>21 302</td>
</tr>
</tbody>
</table>

As can be seen, the total sample size was substantial, comprising nearly 120 000 vehicles.

The data were acquired using the Traffic Engineering Logger developed by the then National Institute for Transport and Road Research (NITRR), now CSIR/Transportek. Data acquired included, for each vehicle, the time of arrival (to nearest 0.1 second), speed (to nearest 1 km/h) and class of vehicle.

### 5 CALIBRATION OF MODEL

Although only the speed of the passenger cars is of interest, it was necessary to derive relationships between the speeds of all the vehicle classes and gradient in order to calibrate the simulation model. In order to do this, only those speeds associated with headways of ten seconds or longer were selected from the database for each class of vehicle and aggregated into 5 km/h intervals.
Regression derived the following relationships between gradient and desired speed

\[ \begin{align*}
V_C & = 123.32 - 6.99G \quad (R^2 = 0.986) \quad \text{Eq.1} \\
V_T & = 76.89 - 4.79G \quad (R^2 = 0.994) \quad \text{Eq.2} \\
V_S & = 69.13 - 5.33G \quad (R^2 = 0.946) \quad \text{Eq.3}
\end{align*} \]

Where \( V_C \) = Passenger car speed (km/h)
\( V_T \) = Truck speed (km/h)
\( V_S \) = Semi trailer speed (km/h)
\( G \) = Gradient (\%)

These were compared to the performance of 17 of the 18 various classes modelled by TRARR, it being decided that the Class 1 vehicle (Extraordinary vehicle) by definition could not be considered to be a design vehicle. The condition of the runs were that the observation points in the simulation corresponded to the points at which the data were actually gathered on the various gradients and that flows were selected to represent headways of 10 seconds or longer.

A point to note is that TRARR models space mean speed whereas the Traffic Engineering Logger measures spot or time mean speed. The difference between the two is the coefficient of variance. Applying this modification to the data and testing for statistical significance led to the conclusion that the best correspondence between the simulation and the observed situation could be achieved by using:

- Cars: TRARR Class 11
- Trucks: TRARR Class 7
- Semi trailers: TRARR Class 6

6 DEVELOPMENT OF SPEED RELATIONSHIPS

The variables of interest in the determination of speed are flow, gradient, directional split and traffic composition. The last-mentioned variable is split into two sub variables, being the overall percentage of trucks and the percentage of these that are semi-trailers.

A series of simulation runs was thus set up as shown in Table 5

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value of variable</th>
<th>Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow (veh/h)</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 500</td>
<td>1 800</td>
</tr>
<tr>
<td>Gradient (%)</td>
<td>0,0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3,0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4,5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6,0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7,5</td>
<td></td>
</tr>
<tr>
<td>Directional split</td>
<td>30/70</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>40/60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50/50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60/40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>70/30</td>
<td></td>
</tr>
<tr>
<td>Fraction trucks</td>
<td>0,00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0,05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0,10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0,15</td>
<td></td>
</tr>
<tr>
<td>Fraction semi-trailers</td>
<td>0,00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0,20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0,40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0,60</td>
<td></td>
</tr>
</tbody>
</table>
There are 125 combinations of flow, gradient and directional split. Semi-trailers are expressed as a proportion of truck traffic so that, for each of the three ranges of truck traffic greater than zero, there are four possible ranges of semi-trailer traffic, i.e. twelve variations for each of the 125 combinations of flow, gradient and directional split. Zero truck traffic obviously allows only for zero semi-trailer traffic providing one further run for each of the 125 combinations. In consequence, a total of 1 625 runs were necessary to cover all the possible combinations of variables.

With a total of 1 625 lines of data, the data set could legitimately be described as large and the presence of five independent variables caused it to be complex. Because of its highly structured nature, the various variables could be categorised and tested by means of cluster analysis. The SAS program, X-AID was employed. This provided an indication of the variables that, in descending order of significance, provided the best descriptors of the dependent variables.

It was found that the principal factor influencing car speed was total flow, with gradient being the next best descriptor. The various subsets of flow data, being directional split, and fractions of truck and semi-trailer traffic followed in that order. This is illustrated in Table 6.

Table 6 : Impact of independent variables on passenger car speed

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Range of independent variable</th>
<th>Extent of influence (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow (veh/h)</td>
<td>100 to 1 000</td>
<td>- 47,8 to –77,9</td>
</tr>
<tr>
<td>Gradient (%)</td>
<td>0,0 to 7,5</td>
<td>0,0 to -15,3</td>
</tr>
<tr>
<td>Directional split</td>
<td>0,3 to 0,7</td>
<td>-5,4 to -12,7</td>
</tr>
<tr>
<td>Trucks</td>
<td>0,00 to 0,15</td>
<td>0,0 to -5,1-</td>
</tr>
<tr>
<td>Semi-trailers</td>
<td>0,00 to 0,09</td>
<td>0,0 to -4,9</td>
</tr>
</tbody>
</table>

This table suggests that truck performance as a warrant for climbing lanes is not appropriate.

It is obvious that the influence of trucks is not so great that they can be used as the sole descriptors of the need for climbing lanes, as has traditionally been the case. It has been stated previously that truck traffic is a relatively small percentage of the total flow. The likelihood that a car will actually be delayed by a truck is thus correspondingly low. This likelihood is determined by various factors such as the length of the upgrades as a percentage of the route length, and the catch-up rate between trucks and cars as described by density in vehicles/km and the speed differential between the two vehicles.

Obviously, when a car is impeded by a truck, the delay it suffers is substantial. Current warrants are essentially oriented towards the delay suffered by an individual vehicle. For this reason, it is proposed that consideration of delay suffered by the entire stream is a more legitimate argument in favour of the provision of a climbing lane.

The relationships between the means, μ, and standard deviations, σ, of speeds for the three classes of vehicles and the independent variables are given below.
Passenger car speed:

\[
\mu_C = 143,96 - 10,39 \ln Q - 0,04 (G^2 - 5,20) - 18,08 D - 33,89 P_T - 54,15 P_S \\
(\text{Eq 4(a)} \quad R^2 = 0,96)
\]

\[
\sigma_C = 24,40 - 0,007 Q + 0,018(G-11,15) G^2 - 9,64 D - 5,88 P_T \\
(\text{Eq 4(b)} \quad R^2 = 0,84)
\]

Truck speed:

\[
\mu_T = 89,31 - 3,55 \ln Q + (3,28 - 1,27 G + 0,05 G^2) G - 9,40 D - 20,8 P_S \\
(\text{Eq 5(a)} \quad R^2 = 0,86)
\]

\[
\sigma_T = 11,83 - 0,001 Q + (3,51 - 1,59 G + 0,13 G^2) G - 2,00 D \\
(\text{Eq 5(b)} \quad R^2 = 0,60)
\]

Semi-trailer speed

\[
\mu_S = 89,68 - 3,35 \ln Q + (4,38 - 2,19 G + 0,14 G^2)G - 7,52 D - 13,68 P_S \\
(\text{Eq 6} \quad R^2 = 0,86)
\]

\[
\sigma_S = \text{Poor model*} \\
* \text{This model’s residuals demonstrate a fan-shaped plot with increasing value of predicted standard deviation. Other transformations will have to be sought if the model is to be improved.}
\]

Where

\begin{align*}
Q & = \text{total flow (veh/h)} \\
G & = \text{gradient (\%)} \\
D & = \text{directional split as a decimal fraction} \\
P_T & = \text{fraction of trucks in stream} \\
P_S & = \text{fraction of semi-trailers in stream}
\end{align*}

7 **HOURLY DELAY**

Delay was defined earlier as being that period of time added to a trip duration by a reduction of speed to a value less than desired.

The duration of a trip over distance of L km at an actual speed of \( V_A \) is \( t_A \) and at a desired speed of \( V_D \) is \( t_D \), so that

\[
t_A = \frac{L}{V_A} \quad \text{and} \quad t_D = \frac{L}{V_D}
\]

From the definition, the delay, \( T_D = t_A - t_D \)

\[
= \frac{L}{V_A} - \frac{L}{V_D} = L \left( \frac{1}{V_A} - \frac{1}{V_D} \right) \\
= \left( \frac{1}{V_A} - \frac{1}{V_D} \right) \quad \text{Eq 7}
\]

The total delay occurring in one hour is

\[
T_{DH} = T_D n \quad \text{Eq 8}
\]

Where

\[
n = \text{Number of cars on the upgrade in 1 hour.} = Q D (1 - P_T - P_S)
\]

and other variables as previously defined

7.1 **The effect of variation in flow rate**

In the above equation it is tacitly assumed that all independent variables retain values that are constant for the duration of the hour. This is a legitimate assumption where the effect of the variable is small. The desired speed will obviously be constant.
In view of the major effect of flow on speed, it is necessary to consider the consequences of variations in flow rate within the hour on delay.

According to Gerlough and Huber\textsuperscript{10}, the arrival of vehicles at a point is a random occurrence so that the variation in flow tempo can be modelled by the Poisson distribution. They did not indicate whether this randomness applied also to gradients and it was deemed prudent to test this assumption.

The data collected at the sites listed in Table 1 were analysed by comparing the hourly flows to the equivalent rates for successive two-minute intervals. It was found that at flows of 100 veh/h or less the Poisson distribution provided a very good fit with the observations across the entire range of gradients. The highest flows recorded were in the range of 600 veh/h. At this flow, the Poisson distribution showed an $R^2$ value of 0.89, suggesting that, in the range of flows tested, the Poisson distribution applies also on the entire range of gradients likely to be encountered on rural roads. It is thus possible to calculate delays to individual vehicles on the basis of the flow rate corresponding to the arrival rate that they are experiencing as modelled by a Poisson process. And then to sum these delays to derive the hourly delay.

The intention was to establish if a relatively simple ratio of the form

$$T_{DP} = T_{DU} R_D$$

Eq.9

where

- $T_{DP}$ = Delay in terms of Poisson arrivals
- $T_{DU}$ = Delay in terms of uniform flows
- $R_D$ = Ratio between delays

could be used as this would considerably ease the computational burden. It was found that a ratio did exist and that this could be expressed as

$$R_D = e^{(0.046 + 50.51/Q)}$$

for $Q > 36$ veh/h \hspace{1cm} ($R^2 = 0.99$) \hspace{1cm} Eq 10

The total hourly delay to passenger cars is thus finally expressed as

$$T_{DH} = (1/V_A - 1/V_D) Q D (1 - P_T - P_S) e^{(0.046 + 50.51/Q)}$$

Eq 11

Where

- $T_{DH} =$ total hourly delay to all passenger cars (h/h)
- $V_A =$ actual speed (km/h)
- $V_D =$ desired speed (km/h)

and other variables as before.

The assumption being made is that all the delay suffered by not having the climbing lane is removed by its provision.

8 ANNUAL AND TOTAL DELAY

Having derived a process for calculating the extent of delay suffered by passenger cars with in a period of one hour, it is necessary to apply it to all the hours of one year and then to repeat the process for all the years of the design life of the climbing lane. As can be imagined, the computational effort is not trivial. However, once this total delay is known, it is then possible to compare it to the cost of construction and maintenance of the climbing lane and useful conclusions drawn regarding the desirability or otherwise of providing the climbing lane.
8.1 The relationship between flow and hour of year

Jordaan\textsuperscript{11} plotted the data from 65 permanent counting stations and established that, if the hourly counts are ranked from highest to lowest and plotted versus rank number on a log-log scale, the resulting plot is a straight line between the tenth highest and thousandth highest hour. Interestingly enough, the lines plotted all tend to pass through a common point in the region of the 1 030th hour so that the relationship given by Jordaan is

\begin{equation}
Q_N = 0.072 \text{ ADT } (N/1030)^\beta \tag{Eq 12}
\end{equation}

Where

- $Q_N$ = two-directional flow in N-th hour of year (veh/h)
- ADT = average daily traffic (veh/day)
- $N$ = hour of year
- $\beta$ = peaking factor.

Jordaan sums up his conclusion by saying that:

“There is, therefore, only one parameter, $\beta$, that determines the peaking characteristic of a given road. Or, in other words, only $\beta$ is needed over and above the ADT to determine any hourly volume from the highest to the 1 030th highest.”

The parameter, $\beta$, is thus a descriptor of the traffic on a given road and depends on factors such as the percentage and incidence of holiday traffic, the relative sizes of the daily peaks, etc. A value of –0.1 indicates a virtual lack of seasonal peaking and a value of –0.4 suggests very high seasonal peaks with –0.2 being a very typical value.

There are 8 760 hours in the year whereas Jordaan’s model addresses only the first 1030 of these. Admitting that this is not a very good model, he suggests that the balance of the hours be modelled by extending a straight line between the point on the graph representing flow in the 1030th hour and zero flow in the 8 760th hour. $Q_N$ in this region is expressed as

\begin{equation}
Q_N = 9.31 \cdot 10^{-6} \text{ ADT}(8 760 – N) \tag{Eq 13}
\end{equation}

8.2 The summation of delay for one year

The relationships developed by Jordaan do not preclude flows from assuming values greater than capacity. Three flow regimes must therefore be considered in the derivation of annual delay:

(a) Flow equal to capacity

It is necessary to determine the number of hours of the year in which capacity flow can be expected to occur and then to apply the capacity flow to these hours to determine the delay for the period in question. According to the Highway Capacity Manual, the capacity of a two-lane road under the “ideal” conditions of a 50/50 directional split achieves a peak value of 2 800 veh/h. Applying this value of flow to Jordaan’s relationship:

\begin{equation}
2 800 = 0.072 \text{ ADT } (N_C/1030)^\beta \tag{Eq.14}
\end{equation}

where

- $N_C$ = the last hour in which capacity flow occurs so that

\begin{equation}
N_C = \left(\frac{38.9 \cdot 10^2}{\text{ADT}}\right)^{1/\beta} 1030 \tag{Eq 15}
\end{equation}
From Eq 7, the cumulative delay for this flow regime is

\[
\sum_{N=1}^{N_C} T_{DH} = \left[ \frac{1}{A - 82.47} - \frac{1}{B} \right] \times 3.074 \times 10^6 \times C \times \left[ \frac{38.9 \times 10^3}{ADT} \right]^{1/7}
\]

Eq 16

where 
\[
A = 143.96 - 0.04 (G^2 - 5.20)G - 18.08 D - 33.89 P_T - 54.15 P_S
\]

\[
B = 97.69 - 0.04(G^2 - 5.20)G
\]

\[
C = D (1,0 - P_T - P_S)
\]

(b) Flow between the hours of \( N = N_C + 1 \) and \( N = 1030 \)

In this flow regime, total delay is the integral of the expression derived for hourly delay between the limits of \( N = N_C \) and \( N = 1030 \)

\[
\int_{N=N_C+1}^{1030} T_{DH} \, dN = C \times \int_{N=N_C+1}^{1030} \left( \frac{1}{V_A} - \frac{1}{B} \right) Q_N e^{-\left(\frac{0.046e^{50.51}}{Q_N^{0.86}}\right)} \, dN
\]

Eq 17

where \( V_A \) is as defined in Eq 1 and \( Q_N \), the flow in the \( N \)th hour, in Eq 8. This expression, as expanded, does not lend itself to an analytical solution and is resolved by means of arithmetic integration.

(c) Flow in hours beyond \( N = 1030 \)

At some or other point during the year, the flow will become less than 36 veh/h. Applying the relationship for delay beyond this point will result in a negative delay. This hour of the year, \( N_{36} \), is calculated as being

\[
N_{36} = 8760 - 3,8710^6 \times 1/ADT
\]

Eq 18

Hours beyond \( N_{36} \) are ignored in the further development of annual delay. Eq 11 is applied to this flow regime except that \( Q_N \) is now as defined in Eq 9 and the integral extends from \( N = 1031 \) to \( N = N_{36} \)

As in the preceding case, integration is arithmetic rather than analytical

Having derived expressions for the delay under the three flow regimes described above, the total delay for the year is thus

\[
T_{DY} = \sum_{N=1}^{N_C} T_{DH} + \int_{N=N_C+1}^{1030} T_{DH} \, dN + \int_{N=1031}^{N_{36}} T_{DH} \, dN
\]

Eq 19

8.3 Summation of delay over several years

The process of calculation of the delay that is to be eliminated by the provision of a climbing lane was initiated by calculation of the delay suffered by a single vehicle and then expanded to encompass the delay to all passenger cars with in the span of one hour. The following step was to expand the calculation still further to encompass the delay suffered by all passenger cars during the course of one year. The final step is expansion to cover the entire design life of the climbing lane.

It is clearly not correct to calculate the delay for a given year and then to assume that the traffic growth rate applies equally to the growth in delays. This is tantamount to suggesting that delay in
one year generates delay in the following year in the same way that interest earned in one year, generates interest in the following year. It is necessary to calculate the delays for each individual year.

These delays are then converted to their present day equivalent in terms of hours, as expressed in the function

\[ T_{PW} = \sum_{i=1}^{n} \frac{T_{DN}}{(1+i)^{t}} \]  

where

- \( T_{PW} \) = Present day equivalent delay
- \( T_{DN} \) = total delay during the Nth year
- \( n \) = duration of analysis period
- \( i \) = annual discount rate
- \( t \) = any year within analysis period

\[ \text{Eq.20} \]

\[ \text{9 THE PARTIAL ECONOMIC WARRANT} \]

The warrant is simply whether or not the cost of delay eliminated by the climbing lane provides a Benefit/Cost ratio greater than 1 when compared to the construction and maintenance costs involved in provision of a climbing lane.

Alternatively, the worth of a single hour of delay eliminated that will provide a Benefit/Cost ratio equal to 1 can be derived and compared to some or other value deemed appropriate by the road authority concerned. This approach may be more convenient insofar it offers a degree of flexibility to the authority in prioritising the construction of climbing lanes across several roads. For example, it may be considered desirable to have different values of time attached to various roads in the network.

The warrant is referred to as being partial because the only benefit considered is the timesaving provided by provision of the climbing lane. Benefits in terms of running costs and reduction in accident rates and/or the severity of accidents have been totally excluded. It is, however, believed that, if the climbing lane is warranted on grounds of timesavings only, any other benefits would only strengthen the argument in favour of provision of any specific climbing lane.

\[ \text{10 CONCLUSIONS} \]

Analysis of warrants currently in use indicates that they involve a “cost of convenience”, i.e. they do not lead to an economic end result. It has been suggested that, in the present era of limited financial resources, warrants based on individual convenience are not appropriate. The recommendation is that they should be replaced by warrants attuned to economic benefits to road users in general.

A methodology has been derived whereby the delay eliminated by a climbing lane can be estimated. As shown in the preceding sections, the relationships between delay and the independent variables of flow, gradient, directional split and percentages trucks and semi-trailers in the traffic stream are extremely laborious.
The ANDOG (Analysis of Delay on Gradients) program was written to carry out the analysis. The software calculates delay for a kilometre length of climbing lane beyond the critical length of grade. It will be necessary to modify the system to derive delays for the length of climbing lane upstream of the point at which critical length is achieved. The software does however provide an indication even for this situation in the sense that if a climbing lane is not warranted beyond the critical point it will definitely not be warranted upstream of this point.

11 BIBLIOGRAPHY

A PARTIAL ECONOMIC WARRANT FOR CLIMBING LANES

K M Wolhuter  Pr.Eng

CSIR/Transportek, P O Box 395 Pretoria, 0001

CV : Keith Wolhuter

Mr Wolhuter got his undergraduate education at Stellenbosch University, acquiring his BSc BEng in 1959. His career encompasses the period 1960 to 1968 at the then Cape Provincial Roads Department, 1969 to 1982 as an associate and then senior partner of the practice Kantey and Templer, and 1982 to the present at CSIR. He completed his MEng at Pretoria University in 1992. His main interest has always been the geometric design of roads and, to prove it, can point to TRH17 Geometric Design of Roads, Chapter 8 of the Department of Housing’s Red Book, and the SATCC Code of Practice for the Geometric Design of Trunk Roads. He is currently a member of the CSIR team writing the revised G2 Manual for SANRAL.