QUEUEING MODELS FOR ANALYSING AND MANAGING HARVESTED ENERGY IN WIRELESS SENSOR NETWORKS

by

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SUMMARY

QUEUEING MODELS FOR ANALYSING AND MANAGING HARVESTED ENERGY IN WIRELESS SENSOR NETWORKS

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The advancement of wireless technology has led to an increase in the employment of wireless sensor networks (WSNs). Traditionally, WSNs are powered by batteries. However, the high power consumption and the need to change the batteries regularly has made these networks costly to maintain. The nodes in the WSNs are increasingly strained as power consumption increases and the batteries are depleted faster. This has consequently decreased the overall lifetime of the WSNs.

Although many energy-conserving techniques exist, for example energy-efficient medium access control and energy-efficient routing protocols, energy consumption remains one of the significant constraints in the development of WSNs. A natural solution to this constraint is harvesting energy from the environment. However, unlike conventional energy, energy harvested from the environment is random in nature, making it challenging to realise energy-harvesting transmission schemes. Although energy harvesting might be considered a solution to many problems, it brings about new challenges with regard to the usage and management of the energy harvested. Some of these challenges include uneven consumption of power in the network, resulting in dead nodes in some portion of the network

and the batteries used in the network are being affected negatively by the energy usage; they may consequently sustain the nodes for long or short periods. To analyse the usage and consumption of energy, a number of techniques have been proposed, namely; information theory, game theory and queueing theory.

By this time, the performance of the sensor nodes in WSNs has been analysed making use of a queueing-theoretic model for each sensor. The aforementioned model inadequately expresses the physical constraints, namely, the energy drawing process and the finite battery capacity.

This research focuses on developing a model that captures the harvesting, accumulation and dissipation of energy, utilising queueing theory. A rechargeable battery with a finite storage capacity will be used. To ensure that the battery does not lose its capability to store charge after being recharged repeatedly, the *leaky* bucket model is proposed to check the network data flow as the harvested energy in the WSN is analysed.

To capture real-world WSNs with energy harvesting in which there is energy leakage, the energy-harvesting sensor node performance is analysed with two assumptions: data transmission and energy leakage occurring and the token buffer being subjected to a threshold. The system had finite buffers for the data and energy. To make it possible to have some influence over the system performance measures a threshold is imposed on the token buffer.

Four models are developed: a basic model, a basic model with leakage incorporated, a basic model with leakage and priority incorporated and a basic model with leakage, priority and threshold incorporated. The developed models are simulated and results for the performance measures are obtained.

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LIST OF ABBREVIATIONS

CTMC	Continuous-Time Markov Chain
DTMC	Discrete-Time Markov Chain
EH	Energy Harvesting
ENO	Energy Neutral Operation
FCFS	First Come First Serve
GD	General Queue Discipline
GT	Game Theory
HP	High Priority
IEEE	Institute of Electrical and Electronics Engineers
ILP	Integer Linear Programming
IoT	Internet of Things
LP	Low Priority
LTE	Long-Term Evolution
PH	Phase Type Distribution
QBD	Quasi-Birth-Death
QoS	Quality of Service
WSN	Wireless Sensor Network

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CHAPTER 1 INTRODUCTION

1.1 PROBLEM STATEMENT

1.1.1 Background and context of the problem

The growth of Wireless Sensor Networks (WSNs) in the last decade has been due to their use in numerous applications in many disciplines, namely agriculture, environmental studies, military systems, transportation systems, healthcare and security systems [1]. These applications may range from small scale to large scale.

A WSN consists of many sensor nodes that are deployed either within an event or in close proximity to it. A sensor node consists of four units, namely transceiver, processing, power and sensing units. The nodes in WSNs have sensing, computational and communication capabilities. These nodes are usually powered by batteries and leaving them to run autonomously for a long time leads to battery depletion. It is usually cost-prohibitive and tedious to replace these batteries. Therefore, a major constraint in WSNs is the amount of energy available to each sensor node. Uneven power consumption in the network will lead to a portion of the network having dead sensors and this will lead to network degradation [2].

The recent growth of WSN has prompted further research in the following areas with regard to WSN [3]:

1. Energy efficiency in WSNs
2. Prolonging network lifetime and reducing power consumption

3. Security defence on WSN transmission.

In this research the focus is on energy efficiency in WSNs, prolonging network lifetime and reducing power consumption. One of the viable solutions proposed to solve the problem of network lifetime is energy harvesting. Incorporating energy harvesting into nodes in WSNs allows them to be independent and in continuous operation [2]. Various natural sources of energy can be used in energy harvesting. These include thermal, wind, solar and electromagnetic sources. Furthermore, man-made sources can be utilised in the harvesting of energy.

In the modelling and analysis of WSNs, models have been developed. The models are largely classified into two distinct groups, namely stochastic and deterministic models [4]. Deterministic models are fit for applications with foreseeable or moderately changing sources of energy [4]. Transmitters in the system have information on the arrival of energy and the amount of energy that is in the system. The non-causal energy state of the transmitters is also required. This model relies on the precise forecasting of the profile of the energy over a long period. Nonetheless, modelling discrepancy occurs as forecasting intervals rise.

In a stochastic model, the process of renewing energy is regarded as random and a non-causal state of energy is non-essential, therefore rendering the aforementioned model fit for applications with unforeseeable state of energy. Many stochastic models are utilised in the harvesting of energy applications, including exponential process, Poisson process and Bernoulli models [4].

In addition to the primary groups of models used in the analysis of WSNs, there are other models. In one of these models RF signals are produced artificially by external devices. RF signals are classified as random or deterministic. The quantity of harvested energy relies on two parameters, namely the channels from the transmitters to the harvesting receivers and power transmitted by the transmitters [4]. The stochastic model is employed in this research. Stochastic models may employ queueing theory or game theory (GT).

GT is a theory of making decisions under conditions of interdependence and uncertainty. In GT, decision makers model situations that result in precise actions that might have mutual and possibly conflicting consequences. In WSNs, GT is used as a tool in the formation of co-operative schemes among nodes, networks or terminals [5], [6]. Many applications in WSNs can employ GT. Some of the roles of GT in the design of WSNs are power control, target tracking, routing protocol design, energy saving and data collection. However, when applying GT models to WSNs, problems are encountered in obtaining precise models and solutions [7]. This is attributed to the inability of existing game-theoretic models to cope with engineering-specific problems, for example modelling time-varying channels, developing performance functions dependent on restrictive communication metrics and conforming to specific standards such as IEEE 802.16 and LTE [7]. In addition, applications with many sensor networks result in large, complex WSNs. This results in difficulty in describing all possible strategies and the corresponding outcomes to which they will lead [8]. In addition, there is difficulty in assigning pay-off to the given outcomes.

Queueing addresses the issue of waiting, which is one of life's common and unpleasant experiences [9]. One of these unpleasant experiences includes waiting in line at a bank. Queueing models are mostly used in the detection of congestion. The employment of queues is mostly in the form of queue-assisted protocols. The aforesaid protocol's foremost focus is the queue length of the data at the sensor nodes, and these nodes use a technique known as rate adjustment to keep the queue length at a minimum.

1.1.2 Research gap

In stochastic models there are critical aspects with regard to parameters such as state transitions and the amount of energy available in a given state. Commonly, this is linked to the energy-harvesting data that are measured by the energy harvester of each node. The capability of harvesting energy is usually node-specific. Nonetheless, not much emphasises has been placed on the construction of data-driven energy-harvesting models and the energy that is harvested is assumed to be discrete.

Despite the fact that research has been conducted on queueing models in regard to energy harvesting in WSNs, a few of the models do not precisely encapsulate the process of drawing energy [10], [11], [12], [13], [14] and [15]. In [10], the authors study an energy-harvesting WSN with finite battery storage. The main focus is resource control with the emphasis on channel state information, energy and data queue state information. The results obtained provide details on the quantity of available harvested energy in the energy buffer and the flow of data. In [11], the main focus of the authors is the effect of powering each node by a rechargeable battery that is charged using renewable energy. Two queues are proposed, one for the packets and the other for energy. The authors compare the effect of having unlimited energy to the effect of having limited but harvested energy. The authors show that the harvested energy, which is stochastic in nature, enforces an energy availability restriction on each sensor node. In [12], the authors propose a model in which the harvested energy in the WSN is controlled. The control objective is to ensure that the battery energy level approaches a specified level. A controller that is non-linear, robust in nature and allows energy predictions is considered. The authors show that queue-based control techniques that are used in congestion control can be used in the management of energy in WSNs. In [13], the main focus of the authors is the design of WSNs capable of energy harvesting, in regard to the energy and the sizes of the data buffers. The authors develop a Markov model, which includes the process of energy harvesting, the arrival process of the event, the amount of energy in storage and number of events in the queue. The simulations are continuous in time and validate the analytical results. In [14], the authors propose a WSN with an energy harvesting-capability and a finite battery. To ensure that packets are not lost when the battery is empty, a threshold is imposed. When the battery level is equal to or greater than a specified level, packets are transmitted. The arrival process is assumed to be Poisson and the service process exponential. In [15], the proposed model is a discounted Markov decision process (MDP) with the energy-harvesting process being driven by the previous data records at the sensor node. The authors reveal that the ideal policy has a threshold structure dependent on the states of the battery. By analysing the bit rate, the performance of the energy-harvesting node can be analysed.

The models presented in the literature are used to independently analyse the accumulation, harvesting and dissipation of energy. A model that incorporates all three has not been presented; a gap therefore exists in this area. It is the aim of this research to propose and develop a model that captures the harvesting, dissipation and accumulation of energy. The proposed models capture a practical system with leakage imposed at different transitions. In addition, a threshold is imposed on the system to ensure that the analyst has some control over the system (that is, to ensure that the high-priority (HP) data packets are always transmitted). The models are developed in such a way, that the tokens are used by the packets and whatever is left in the system leaks and the HP packets are constantly transmitted.

1.2 RESEARCH OBJECTIVE AND QUESTIONS

In a system with priority, the harvested energy may be used up by the low-priority (LP) packets and the HP packets arriving will not be transmitted. The subsequent questions are posed:

- Is the selected approach capable of analysing the energy usage in the system? In the analysis, is it possible to capture the usage and leakage of energy?
- For cases where there is a limit on the amount of energy needed to transmit data, what minimum amount of energy will be considered to ensure that the system operates optimally?
- In the absence of HP data packets, what is the threshold below which the LP packets are not transmitted?

The objective of this research is to model a WSN capable of energy harvesting using queueing theory. The model will be designed to ensure the following:

- The total energy consumption will include energy consumed during sensing, transmission and processing by the sensor.
- It adequately captures the process of drawing energy, dissipation and accumulation of energy that is in the network.
- There is an overall increase in the efficiency of the WSN and extended network lifetime with the same battery capacity.
- Constant operation of the WSN continues without interruption caused by the death of a node.

1.3 APPROACH

To address the questions posed in this research, the hypothesis is split into three categories, namely

- Markov process tools are favoured in controlling systems of harvesting energy because of the capability to integrate heterogeneous components and support a wide range of applications [16], [17].

- The majority of queueing models can be set up as a Markov chain whose states are obtained from the harvested energy and the transition at a particular state.
- The token or leaky bucket procedure can be integrated into the queueing model to decrease the consumption of energy and set an equilibrium in data traffic[18].

The strategy that will be employed to deal with the questions posed in the research will comprise the following: definition, formulation, analysis and simulation of queueing models. To generate applicable results, the model will build on prior work. A discrete model will be developed assuming the use of a battery with a finite capacity. Many phases will be executed and are defined as follows:

- A sensor network with energy-harvesting capabilities from nature (such as wind and solar radiation) is considered; in addition, it will use a rechargeable battery with finite storage capacity. The energy harvested is modelled to arrive at the transmitter, denoted as TX, during transmissions in random time frames and is then stored in the energy queue [19]. The energy queue, also known as a battery, is regarded to have a finite capacity and can store a maximum of b_{max} units of energy. A battery with finite capabilities is taken into consideration so as to circumvent energy overflows, thereby economising on energy. Figure 1.1 shows the transmitter for energy harvesting [19].

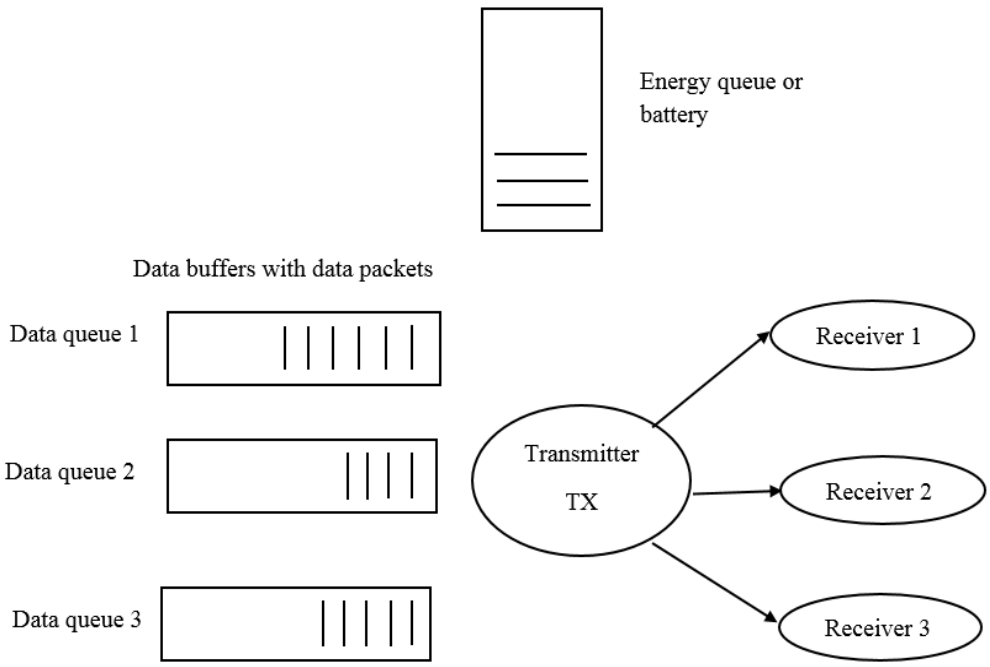

Figure 1.1. Transmitter for energy harvesting

- In Figure 1.2, multiple sensor nodes (S_n, n =2...N) are connected in parallel [19]. The sensor nodes measure $x_{n,t}$ of the occurrence H_t and then relay the occurrence to the fusion centre. The fusion center then makes a decision $\hat{H}_t$. During the aforementioned process, $w_{n,t}$ energy packets are used by the node. Subsequently the nodes harvest $e_{n,t}$ energy packets [19]. A queueing

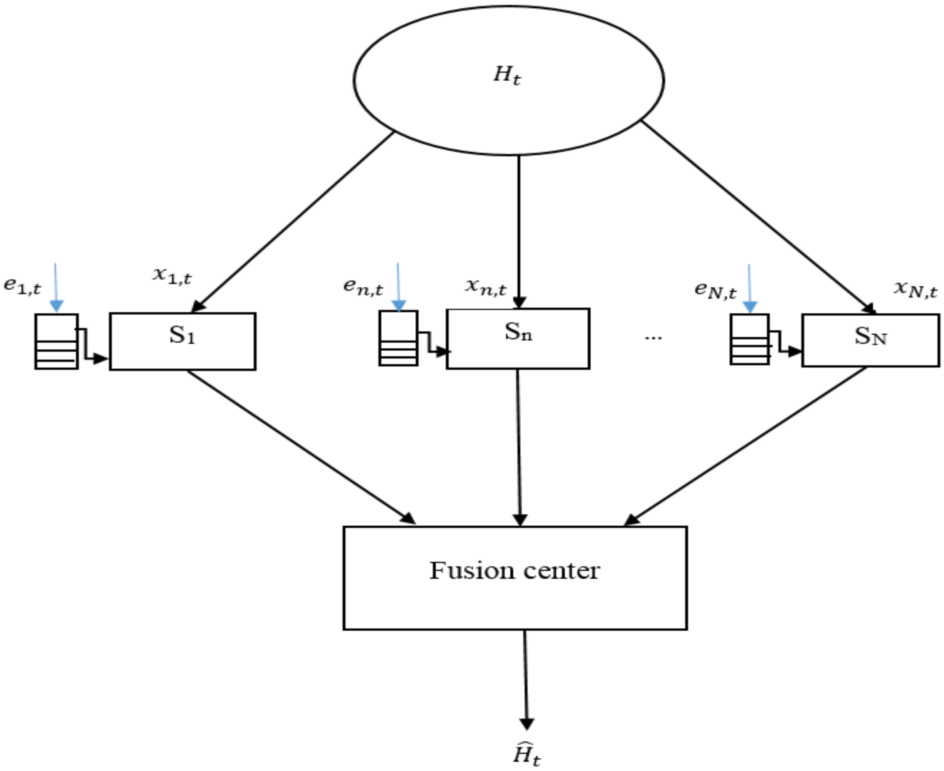

Figure 1.2. Energy-harvesting network

model that shows the dissipation, usage of energy and accumulation that relies on the harvest-use architecture is proposed.

In Figure 1.2 the following assumptions are made:

1. The sensor nodes (S_n) have harvesting capabilities of at most one token of energy during a time period.
2. $e_{n,t}$ is drawn from a set 1,0 with probabilities p_e.
3. 1-p_e is independent for each n and t.

If all the sensors have the same storage device with maximum b_{max}, then the energy available at transmission time *t + 1* cannot surpass b_{max} and is obtained as follows:

$$b_{n,t+1} = min(b_{n,t} - w_{n,t} + e_{n,t} b_{max}). \tag{1.1}$$

In Figure 1.2, communication between the sensors and the fusion centre is via a one-way parallel channel. The cost of sending a positive message is one energy packet, while a negative one incurs no cost through non-transmission.

- The *leaky bucket* model is utilised in the analysis of the harvested energy in WSNs. The aforesaid model can be used as a scheduling algorithm and to check data transmissions. The concept of the model is kindred to the functioning of a *leaky bucket* holding water. The leaky bucket takes data and collects it until it reaches maximum capacity and, thereafter releases it at a set rate; leakage is halted once the bucket is out of data [20]. In Figure 1.3 the *leaky bucket* procedure is illustrated.

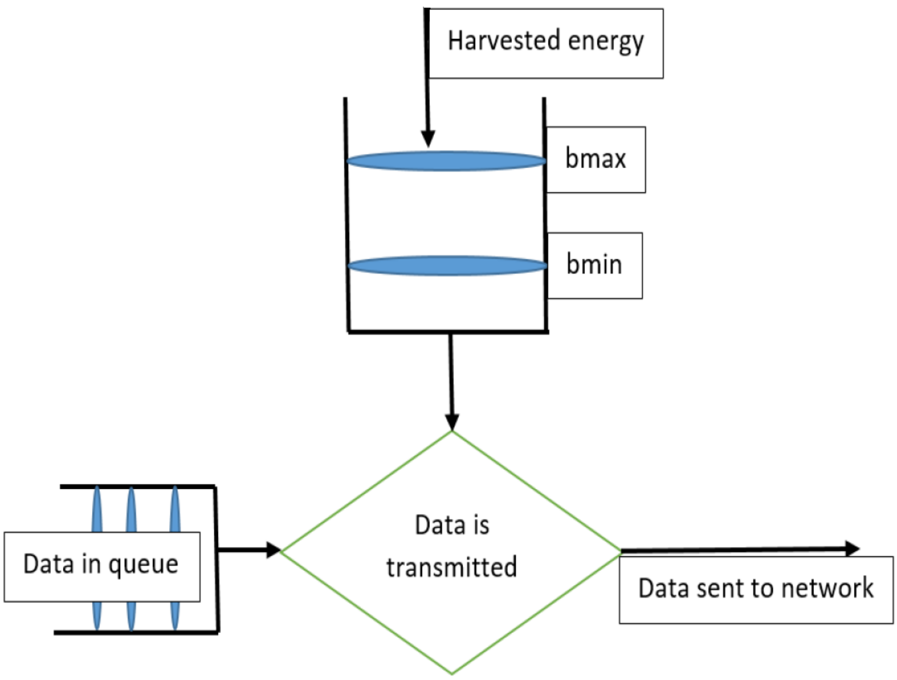

Figure 1.3. Leaky bucket procedure

The *leaky bucket* procedure is as follows:

1. An energy token is harvested and stored in the bucket.
2. The bucket containing the tokens can hold a maximum of b_{max} and a minimum of b_{min}.

3. On arrival, the data remove a certain amount of energy (token) from the bucket. Thereafter, the data is sent to the network.
4. If the bucket has an amount of energy, b, that is less than b_{min}, then no energy (token) is removed from the bucket and no data is transmitted. This data is then lost. b_{min} has to be established and should not be reached to prevent loss of information.
5. For data to be transmitted, the condition, $b_{min} \leqslant b \leqslant b_{max}$ should hold.

The proposed model is bivariate with variables Y_t (the amount of data to be transmitted) and X_t (the level of energy).

$$(X_t + 1, Y_t + 1) = [(X_t - 1) + energyharvested], [(Y_t - 1) + newdata] \tag{1.2}$$

The above equation is only applicable to the case in which $X_t \geqslant 0$ and $Y_t \geqslant 0$.

1.4 RESEARCH GOALS

The goals of this research are as follows:

- To create a model that accurately captures the energy-drawing process.
- To employ queueing theory in specific, preemptive priority that allows two data packets to be transmitted depending on the system requirements.
- To control the system by imposing a threshold on the token buffer and analyse the results.
- To develop and compare four models that incorporate leakage of energy, priority of data packets and a threshold imposed on the token buffer.

1.5 RESEARCH CONTRIBUTION

The major technical contributions of this research are as follows:

1. Bivariate model: A bivariate model consisting of Y_m packets and X_m tokens was developed. The following assumptions are considered:

- Each token of energy allows for transmission of one bit of data. If a token is waiting in the energy buffer, an arriving bit of data removes it from the energy buffer and enters the network.
- If there are no tokens waiting in the energy buffer, the incoming data packet waits in the data buffer of a given size; when the buffer is full the data packet is lost.
- A leakage is imposed on the token buffer. Leakage may be due to denial of sleep to the node (the node is constantly transmitting data and does not rest) [21], or age of the battery (battery degrades with age).

2. Priority: A model was developed with two sources sending packets, one source sending HP packets and the other sending LP ones. The assumption made is that at an instant, there may be no packets arriving or one packet arriving that is either an LP or an HP packet. In addition no more than one packet may arrive at a given time. The priority queues studied have both geometric arrivals and services. The state space models were developed and the transition matrices were generated.

The work presented closely relates to the models above. However, in contradiction of existing literature, leakage of energy is accounted for and a threshold is imposed on the energy buffer. This makes the model more complex as leakage and threshold are imposed on the energy harvesting node.

1.6 RESEARCH OUTPUTS

Based on the work presented in this research, the following paper has been published in a peer-reviewed international conference proceeding:

1. O.P. Angwech, A. S. Alfa, and B. T. Maharaj, "Analysing usage of harvested energy in wireless sensor networks: a Geo/Geo/1/k approach," in *9th International Conference on Sensor Networks*, Sensornets, pp. 71-77, 2020.

Based on the research work presented here, the following paper has been submitted to Energy Reports, a peer-reviewed journal:

1. O.P. Angwech, A. S. Alfa, and B. T. Maharaj, "Managing the harvested energy in wireless sensor networks: a priority Geo/Geo/1/K approach with threshold", *Energy Reports*, December 2020, in review.

1.7 DISSERTATION OVERVIEW

This research deals with aspects of WSNs. In this chapter (introduction), the background and context of the research were introduced. The research gap was identified, the research questions and objectives were documented and the approach employed to meet the research goals was briefly explained.

In Chapter 2, a literature study is performed. An extensive literature study is done on WSNs and energy harvesting. The problems, origins, future advancements and developments of WSNs are discussed. The current challenges faced in WSNs and the proposed solutions are also discussed. Advances in energy harvesting are discussed, highlighting past, present and future challenges. Queueing theory is the second major topic of this research. In Chapter 2, an introduction to queueing theory is provided. The relationship between queueing theory and WSNs is discussed. The proposed models that will be used in this study are also discussed. The classical Quasi-Birth-Death (QBD) matrix and Geo/Geo/1 queue are also studied in order to compare the existing models with the proposed model and observe the performance parameters.

In Chapters 3 and 4, the main research method is discussed. This consists of the methods used to develop the proposed model for this research. In Chapter 3, the mathematical background of the basic model is given, assumptions are made and explained. Transition matrices are developed and the behaviour of the queue is analysed. In Chapter 4, three models that are a build-up of the model in Chapter 3 are developed. The models are a basic model with leakage incorporated, a basic queueing model with leakage and priority incorporated and a basic queueing model with leakage, priority and threshold incorporated. The models with priority incorporated and are bivariate in nature.

In Chapter 5 the results are provided and discussed. In addition, the main observations are provided. In this section, the results are interpreted properly to explain the behaviour of the results obtained and analyse them based on theoretical knowledge.

In Chapter 6 concluding remarks and suggestions for further work are discussed. The overall contribution of the research is presented.

CHAPTER 2 LITERATURE STUDY

2.1 CHAPTER OBJECTIVES

In this chapter a literature study is provided. The study is of WSNs, energy harvesting, queueing theory and applications that employ WSNs.

2.2 WIRELESS SENSOR NETWORKS

In an Internet of Things (IoT) structure, the connected devices are typically equipped with sensors, wireless transceivers, processors and an energy source to monitor the surroundings and send or receive data [22].

The advancement in IoT technologies over the years has stupendously facilitated the growth and development of remote WSNs. WSNs are event-driven communications that play a very important role in various applications. WSNs are made up of many sensor nodes that may be positioned close to or far away from the phenomenon [23]. One of the unique features of a WSN is the ability to deploy the nodes in the network randomly. This feature means that the protocols and algorithms of the nodes have self-organising capabilities.

The nodes measure or sense physical data in the environment being monitored. The analog signal sensed by the sensors is then digitised by an analogue-to-digital converter and thereafter sent to the controllers for further processing [24]. Furthermore, the sensor nodes can cooperate with one other and have an on-board system that allows them to perform simple data processing, consequently transmitting the necessary data. This may occur for example, in environmental monitoring applications where numerous sensor nodes are deployed and must remain functional to facilitate data collection and data transfer from the environment to the base station [25].

2.2.1 Applications

The features of WSNs imply that numerous applications utilise sensor networks in areas such as security, health, environmental management and the military. The actualisation of the aforementioned applications need wireless and traditional ad hoc networking techniques. Nonetheless, WSN techniques are favoured over the traditional ad hoc networking techniques depending on [23]:

1. Dense deployment of the sensor nodes;
2. Frequent change of the topology of the WSN;
3. Difference in the number of the sensor nodes;
4. Method of communication. In WSNs the mode of communication is broadcast while in an ad hoc system a point-to-point method is employed [23].

WSNs are particularly useful in remote or hazardous environments or when a large number of sensor nodes are required to be deployed. A typical WSN has the following characteristics [24]:

1. Node mobility;
2. Nodes being powered by batteries or having an energy-harvesting capability;
3. Adaptability to the magnitude of deployment, either small or large scale;
4. Ability to handle node failures;
5. Capability of working in harsh or hazardous environmental conditions.

Figure 2.1 is an example of a WSN application [26].

2.2.2 Structure

The sensor nodes in WSNs are densely deployed. Data collection in the node is done by a sink node in either a single-hop or multiple-hop manner. The data is then sent to the internet or private Internet

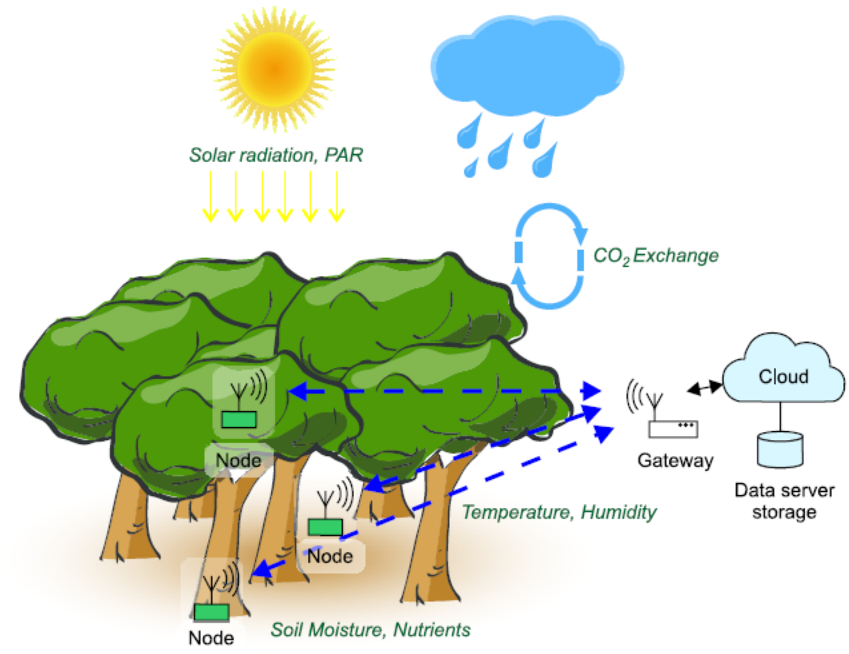

Figure 2.1. Monitoring of the environment. Taken from [26], with permission.

Provider (IP) networks via a gateway to the users [27]. Figure 2.2 shows a simple wireless sensor network [27].

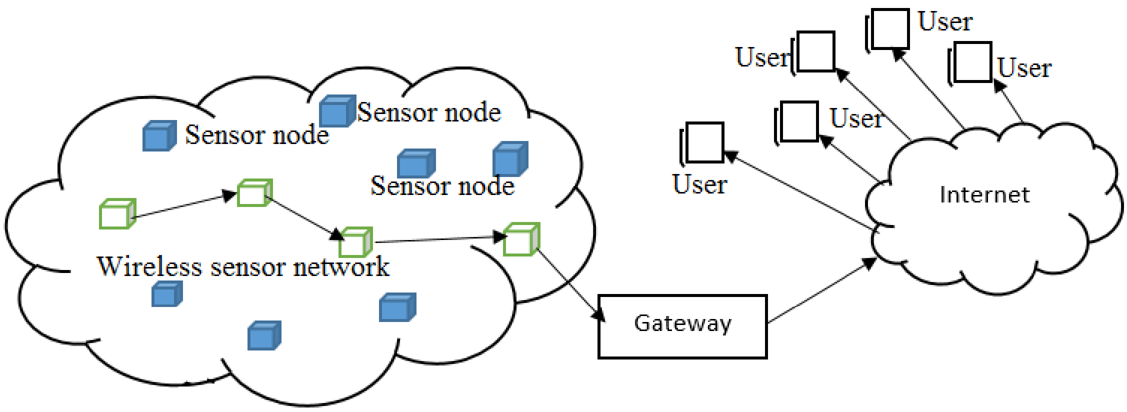

Figure 2.2. Typical structure of a WSN.

Each sensor node consists of three basic components, as shown in Figure 2.3:

1. Sensing unit;
2. Transmission unit;

3. Processing unit.

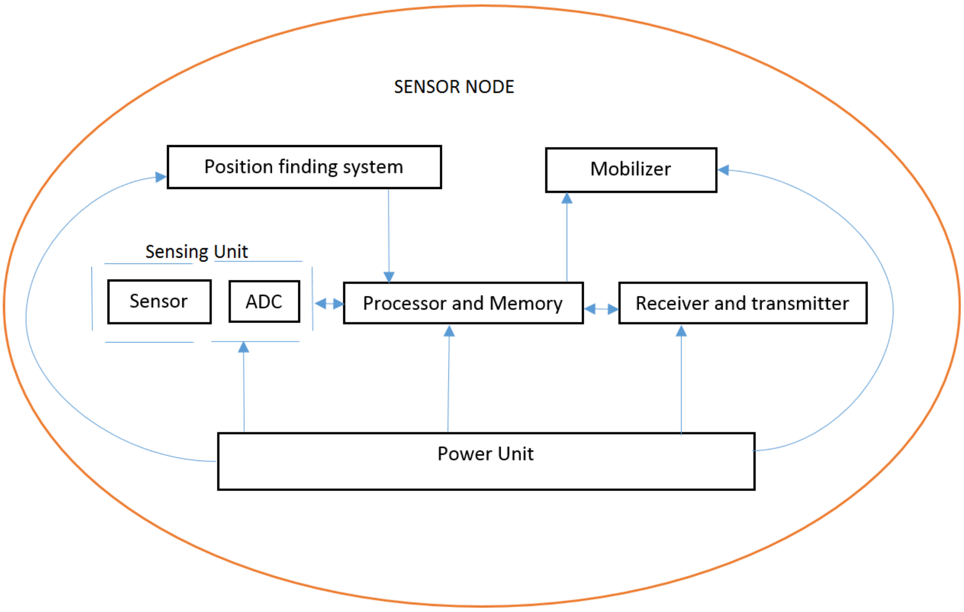

Figure 2.3. Components of a sensor node.

2.2.3 Challenges in WSNs

WSNs are often deployed in remote areas, usually without access to electricity from the mains [26]. In order to provide a long operational life for WSNs, the total average supply current to the system must be less than 30 μA [28]. The size limitation of the sensor node makes power a limited asset, since the source of power is typically small lithium cells batteries (2.5 cm in diameter and 1 cm thick), which have finite capacity [23], [28]. The batteries are typically negatively affected by the usage pattern of the energy and the node's level of activeness. The batteries can support the network for varying periods of time that may be short or long.

In addition to low power consumption, constant energy consumption is another challenge in WSNs. Varying energy consumption will lead to some of the nodes being depleted faster than the other nodes in the system. The depleted nodes are rendered useless and will inevitably kill the network [29].

In the interest of increasing the network's lifetime, harvesting energy has been employed in WSNs [2]. Assigning harvesting of energy abilities to nodes in WSNs allows the network to be independent and function continuously. Energy-harvesting models are critical in the evaluation and design of the systems of harvesting energy.

2.2.4 Mechanisms employed to conserve energy in WSNs

In order to conserve energy in WSNs, a number of techniques have been employed [30].

2.2.4.1 Optimisation of wireless communication

Wireless communication is one of the major components leading to battery depletion in WSNs. To optimise the communication, some of the areas that have been studied include modulation optimisation, transmission power control and directional antennas [1] and [31].

2.2.4.2 Reduction of data and sleep/wake techniques

By reducing the amount of data from the sensors to the sink in Figure 2.2, energy can be saved in WSNs. This can be done in two ways: limiting the sensing tasks (transmission and acquisition of data) and limiting unnecessary samples.

In addition to data reduction, sleep/wake techniques may be employed. In this method, when the node is inactive it is put to sleep. This may be achieved using the following methods: duty cycling, control of topology and passive wake-up schemes [1].

2.2.4.3 Energy routing and charging of the storage device

Dense deployment of sensor nodes may also be employed to overcome some of the challenges in WSNs. It will ensure that the nodes are very close to one another, therefore enabling multi-hop communication that consumes less power in comparison to single-hop communication. In addition to less power consumption, multi-hop communication may also lead to a reduction in some of the effects of signal propagation usually observed in long-distance wireless communication [23].

However, multi-hopping may induce stress on the nodes closer to the sink as more packets are routed through them. This will also result in the batteries being depleted faster. To this end the following is proposed:

1. Organising the WSN into clusters, with each cluster being controlled by a cluster head. The cluster head is a node that ensures that there is co-ordination and communication between the clusters.
2. Employing multi-path routing [32]. The forwarding nodes are alternated, leading to energy balance in the WSN.
3. Using a base station that is mobile and changes position to collect information from the nodes in the WSN.

Another option that can be employed in saving energy is charging the storage device without the intervention of human beings. For this, two methods are proposed, wireless charging and harvesting energy [32]. For the sensors to operate perpetually and to prevent the WSN from breaking down, the energy consumed has to be less than the harvested energy. This is referred to as energy-neutral operation (ENO) [33].

2.3 ENERGY HARVESTING

To enable WSNs to be self-sustaining, energy harvesting has been proposed. Energy harvesting refers to the process of converting any source of energy, for example wind or solar radiation, into energy that can be utilised [33].

Figure 2.4 shows the architecture of a self-powered WSN node. The core part of the energy system is the collection and conversion unit. It collects energy from the energy harvester. A storage system is included to store energy in the system.

2.3.1 Energy sources

Energy sources are classified as follows [34]:

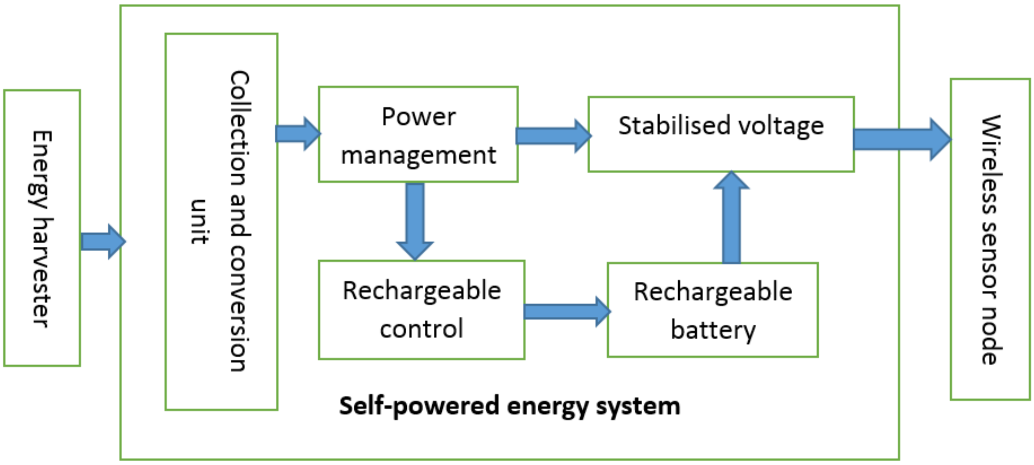

Figure 2.4. Architecture of a self-powered WSN node

1. Uncontrolled and predictable energy sources. These energy sources cannot be regulated to produce energy at a specific time but their behaviour can be modelled with some margin of error to predict energy at a specified time. An example of such an energy source is solar radiation.
2. Uncontrolled and unpredictable energy sources. These sources can neither be controlled nor have their behaviour modelled. The behaviour may be modelled but may be too complex to be implemented. An example of such an energy source is indoor vibrations.
3. Fully controlled energy sources. For these energy sources the desired energy is produced when required.
4. Partially controlled energy sources. For these energy sources the desired energy may be determined by the system designers however, the behaviour may be random. An example of such an energy source is radio frequency (RF) energy source.

In addition to the above classification, energy sources can also be classified as ambient or external sources. Ambient sources are readily available in the environment, for example solar radiation. External sources, on the other hand, are emitted to the environment, for example human sources.

Some of the types of energy that can be harvested include [35]:

- Mechanical energy. Mechanical energy from pressure, vibrations and stress, is converted to electrical energy [36]. Mechanical energy is classified into electrostatic, piezoelectric and electromagnetic energy.

- Photovoltaic energy. Photons from artificial light or solar radiation are converted into electrical energy. Photovoltaic cells are used to harness photovoltaic energy. This is the most commonly used type of energy [37].
- Wind energy. Air flow is converted into electrical energy. Wind turbines are used.
- Thermal energy. The source of energy is heat.
- Biochemical energy. This type of energy is generated by living organisms and is generally stored in plants.
- Hybrid energy. Energy is harvested from different sources of energy.

Table 2.1 shows the characteristics of wind and solar power harvesters [38].

Table 2.1. Types of harvesters and their characteristics

Energy harvester	Characteristics
Wind power harvesters	Changes in wind power make the system power availability random with increasing uncertainties. The power generated is alternating current (AC) power, which requires rectification for systems that operate using direct current (DC) power.
Solar power harvesters	Power is readily available. Energy can be harvested using devices that do not have movable parts, resulting in a more reliable system with lower maintenance cost.

2.3.2 Energy-harvesting models

The energy-harvesting models are generally split into two categories; that is, deterministic and stochastic [15]. Deterministic models are fit for applications with foreseeable or moderately changing sources of energy and rely on the correct estimation of the profile of the energy over a prolonged time span. Transmitters are used to determine the energy arrival instant and amounts in advance. These models are useful in characterising optimal energy-scheduling strategies in order to design approaches that require the casual energy state information (ESI). However, modelling discrepancies occur as forecasting intervals rise.

Stochastic models are either time-correlated or time-uncorrelated. Time-correlated models include first-order Markov chains, two-state Markov chains and generalised Markov chains. Time-uncorrelated models encompass the uniform process [34], Poisson process [39] and exponential process [40]. For most energy sources, time-correlated models are preferred, as they are adequate in capturing the temporal properties of the energy, making them fit for applications with uncertain states energy.

Other models include stochastic-geometry models, RF free space propagation and hybrid energy storage. In RF free space propagation models, for example, RF signals that are artificially produced by external devices are considered [41]. The RF signals are either random or deterministic. The quantity of harvested energy is reliant on two factors, namely power transmitted by the transmitters and the connection between the transmitters and the harvesting receivers. The aforementioned factors present a trade-off between energy and transfer of information in WSNs.

2.3.3 Energy storage

Energy storage is a term used to refer to the technology that facilitates the conversion of electrical energy to a form that can be stored, such as electrochemical energy [26]. The stored energy is then converted back to electrical energy when needed. Depending on the application, the energy storage units may be required to meet certain criteria related to adequate capacity, size and impact on environment. The following are the most commonly used storage devices:

1. Rechargeable or non-rechargeable batteries;
2. Super-capacitors;
3. Hybrid storage devices; these are usually a blend of rechargeable batteries and super-capacitors.

To meet the conditions for ENO, the harvesting system can be implemented as follows [34]:

1. With no energy storage: In this system the harvested energy is immediately used by the sensor node. Unused energy is not stored for later use. This system can also be referred to as harvest-use.
2. With an ideal energy buffer: In some applications, the consumption and generation profiles differ. The device employed has an ideal mechanism, with an efficient charging system and no leakage.
3. With a non-ideal energy buffer: For practical systems, a non-ideal energy buffer is employed. The energy buffer has limited capacity and there is leakage of energy: This system can also be referred to as harvest-use-store. Harvest-store-use is another system in which two storage buffers are used [42]. Harvest-store-use is also referred to as harvest-store-consume [43].

Table 2.2 shows the WSNs applications and the storage devices used [25].

Table 2.2. WSNs applications and storage devices used.

WSN application	Storage
Monitoring of water quality	12 V battery
Monitoring of climate change	Rechargeable battery and a 2 200 *mAh* lithium-ion battery
Monitoring of wildlife	1 *Ah* lithium battery
Monitoring of bridge structures	lithium-ion or lithium-polymer batteries with a 10 000 *mAh* capacity
Monitoring of aquatic environment	NiMH-cell battery
Buildings incorporated with WSNs	Hybrid storage
In-line river monitoring	Hybrid storage

Batteries are usually used to supply the system with energy and to store energy harvested from the environment. There are a number of specifications that are important when selecting a battery, namely storage technology, overcharging tolerance, internal resistance, discharge depth and self-discharge.

Batteries can be grouped into two categories, namely non-rechargeable and rechargeable. Non-rechargeable batteries cannot be charged and have to be replaced once they are depleted. They have many advantages, some of which include, being stable under varying temperatures and high capacity.

However, the drawback is that they require periodic maintenance. Non-rechargeable batteries are usually classified as alkaline and acidic batteries. Alkaline batteries yield better performance while acidic ones are less expensive and more dependable. Rechargeable batteries can be charged; however, the number of charge and recharge times is limited by cycling capacity.

Super-capacitors are constructed either as pseudo-capacitors or ultra-capacitors. Pseudo-capacitors use a redox reaction that transpires on the electrode. Charges are generated and thereafter transferred across a layer. Ultra-capacitors, on the other hand, employ the electrochemical principle. Pseudo-capacitors have a lower power density than their counterparts, the ultra-capacitors, but provide higher energy density and specific capacity [26]. Table 2.3 shows the advantages and disadvantages of some of the storage devices used [25], [26].

Table 2.3. Comparison of storage devices.

Energy storage device	Advantage	Drawback
Rechargeable battery	Economical Rate of self-discharge is low Energy per unit weight is high	Low life time Recharge life cycle is low
Super-capacitor	Fast charging process Long lifetime High recharge cycle life Wide operating temperature range (-40 to $65°C$) Low internal resistance Range of voltage and current is broad	Rate of self-discharge is high Low cell voltage Expensive Energy per unit weight is low Dielectric absorption is high Environmentally friendly, as it does not contain toxic materials such as lead

In this research, the storage device considered is the rechargeable battery. This is because super-capacitors have a high self-discharge rate ranging from 50 - 60 % per month [44]. This self-discharge can significantly reduce the operational time of the sensor nodes in the WSNs.

2.3.4 Batteries as the main storage device in EH WSNs

As mentioned earlier, the main energy storage device for EH in this research is a battery. However, there are a few drawbacks of using batteries, some of which include [45]:

- When not in use, a battery experiences leakage. This effectively reduces the amount of energy available for use by the packets [46].
- In extreme weather conditions, batteries may break down, resulting in pollution of the environment.
- The limited energy density of a battery may considerably reduce the operation of the sensor node over time.

2.3.5 Techniques employed to analyse WSNs with energy-harvesting capability

To analyse WSNs with energy-harvesting capabilities, a number of techniques have been proposed:

1. Information theory: Applied to the study of the systems that deal with mathematical modelling and analysis of a communication system.

   Algorithms such as mutual information, Chernoff information and entropy are used [47]. The theorems used are usually defined in terms of codes or channels [16], [48]. Information between a transmitter and receiver is analysed through a channel that may not be reliable.

   An algorithm that is used to obtain the ideal power allocation in the system is provided by the authors in [16].
2. Game theory: A theory of making decisions under two conditions namely, uncertainty and interdependence. The resulting actions have mutual and possibly conflicting consequences. Game theory has been employed in WSNs because of its autonomous nature in data transmission [49], [50] and [51].

Game theory is employed in WSNs for various reasons. Firstly, besides being fixed and resource-constrained, the nodes in WSNs have limited battery life. In order to ensure that the lifetime of the WSNs is maximised, the nodes in WSNs have a conflicting interest between quality of service (QoS) and energy conservation. Secondly, the designed solutions for WSNs are partially or fully distributed [5], [7]. However, for complex networks, describing all possible scenarios and their outcomes is usually not possible [8]. A number of challenges are encountered when employing game theory in WSNs, some of which include that assumption of rationality when dealing with nodes is not guaranteed and nodes may choose to act selfishly for their own benefit [6].

3. Energy management policies: In [52], the policy used is throughput-optimal and delay is minimised in the data queue.

2.4 QUEUEING THEORY

Queueing theory is one of the current approaches used to analyse energy harvesting in WSNs. It is used in the analysis of many problems found in real-life applications such as wireless networks, communication networks, integrated services digital networks (ISDN) and cognitive radio networks. In the analysis of queues, the QoS is obtained in terms of waiting time and queue length. The QoS helps researchers to study and optimise the queues.

2.4.1 Markov process

To simplify the analysis of the queueing system, the Markov process is used. A Markov process is classified as a stochastic process in which the future state of the system is independent of the past states of the system; it is only dependent on the present. A stochastic process is defined as a group of points that are indexed over a parameter usually representing time [53]. A system with a finite state space is referred to as a Markov chain and such systems are either discrete time or continuous time Markov chains.

Before the early 1990s, most queues in the literature were developed in continuous time. This was because researchers did not see any reason to study queues in discreet time unless it was to make the analysis of difficult models easier, in particular queues with time-varying parameters. However, with the development of digital communication systems, it is now possible to work in time slots. Discrete modelling has therefore become more suitable.

The difference between continuous and discreet time is that in continuous time the system cannot be observed at any time t. This means that in continuous time, it takes time to perceive an event and register its effect. One can therefore only observe time in intervals, for example between times t and Δt.

In discrete time, on the other hand, time can be divided into finite intervals. One assumes that the system is frozen at time points; for example, a system can be observed at $t_1, t_2, t_3, ..t_n$.

2.4.2 Queue types

A queueing system can be classified into different groups based on the available resources and applications. The following are some of the categories of queues:

- Single-node queues: A system in which an arriving packet is processed in one location and thereafter leaves the system and is not processed further. The packet may still re-enter the system at the same location and be served as a new packet. A single-node queue can have one or multiple parallel servers at one location. A single-node queue with multiple serves is called a single-node, multiple-server queue. An example of single-node queue with multiple parallel servers is a queue in a bank where customers are attended to by tellers. Another variation of a single-node is referred to as polling or contention. Here there are multiple parallel queues and one server. The queues are attended to by the server based on a rule that is pre-scheduled. A feedback queue is also a variation of a single-node queue. Here the packet is served and may return for service immediately at the same location.
- Tandem queues: A system in which several single-node queues are combined in series. The packet is processed and proceeds to the next queue for additional service. The packet goes through the single node queues in a sequential pattern without skipping any queue. An example of a tandem queue is a virtual representation of the circuit of a communication system [53].
- Network system of queues: A cluster of many single-node queues. The packet can arrive at any queue and after being processed may proceed to another queue or exit the system. Unlike

tandem queues, the packet does not go through the queues sequentially and can enter or leave the queue at any time. One can see that the basic structure of queues is a single-node, which is why systems are usually decomposed to single-node queues and used as an approximation for the purpose of analysis. To this end, in this research the focus is on single-node queues.

2.4.3 Characteristics of queues

The basic elements that are used to characterise queues are the inter-arrival process, the service process, the number of service channels, the service discipline, the system capacity, the population source size and the queue discipline.

The most frequently used inter-arrival and service distribution is the geometric distribution, and this is attributed to its lack of memory. The phase type (PH) distribution is also used in discreet time, as several distributions found in queueing are like it. In addition, PH distribution simplifies difficult problems.

When modelling a queueing system, it is important to find optimal conditions to ensure that the system output meets the desired expectations. In order to do this, the system is subjected to different conditions and observed. Some of the important performance measures that can be evaluated include [53]:

- Queue length is the number of packets that are waiting in some location to be served. The queue length is usually related to the performance of the queueing system. A long queue length correlates to poor performance from a user's view.
- System time is the total time spent by the packet from arrival to departure from the system.
- Departure time. Time taken by a packet to complete a service in a single-node system. This parameter is important in tandem queues where the packet must go into another queue for service.
- Waiting time. Time duration between the arrival of the packet and the time when it receives service.
- Age process. The period that is of interest in the age process is how long the packet that is currently receiving service has spent in the system.

- Busy time. Time duration from when an empty server begins serving packets to when it is empty again.

The performance measures are affected by several factors determined by either the service provider or the packets. The factors determined by the service provider include [53]:

- The number of servers providing service will determine the performance. The system will provide faster service with more servers in parallel but at an increased cost to the service provider.
- The way the packets are selected from the queue for service is indicated by the queue discipline [54]. The queue discipline may be first in first out (FIFO), also referred to as first come first served (FCFS), last in first out (LIFO), also referred to as last come first served (LCFS), service in random order (SIRO) and priority (PR).
- A major factor that has to be considered in queues is the number of packets waiting at a time in a system when the server is busy. For all practical purposes, a waiting room that is large is assumed to be infinite [55].

In addition to the above factors, the arrival and service rates are major factors that affect the performance measures. A high arrival rate results in an increase in queue lengths, busy time and waiting time. These factors are therefore critical in understanding queueing.

The basic elements that are used to characterise queues are the arrival process (A), the service process (B), the number of service channels (C), the service discipline (D), the buffer size (E), the population source size (F) and the queue discipline.

Kendall's notation is used to represent a single-node queue as A/B/C/D/E/F. For example, for a system whose arrival process is geometric, service is phase type with one server, FIFO service discipline, K buffers and a population size that is infinite is represented as Geo/PH/1/FIFO/K/ infinity.

2.4.4 Queues in WSNs

Several models have been proposed for the analysis of WSNs with energy-harvesting capability. The most commonly used queueing system is the Markovian queueing system. The authors in [56] propose a Markovian queueing system. Two queues are used, one for the data and the other for energy. The system is described as a QBD process. The authors show that the energy harvesting has an influence on the performance measures obtained.

The authors in [57] employ the Markov model to analyse energy flow. The queues in the model are for data and energy. The probability of data overflow and battery degradation is derived. The main focus of the research is to provide insights to ensure an optimal design of WSNs capable of energy harvesting.

In [58] the authors propose a continuous time Markov model with the aim of computing the probability of battery outage in the energy-harvesting device. The authors in [59] also concentrate on the battery in energy-harvesting WSNs. However, their focus is the degradation status of the battery. In the proposed model the energy harvested is geometrically distributed and is only available for a specified time period.

In some works the authors propose an M/M/1 queueing model with priority [60]. In order to improve QoS, the packets are differentiated into different priorities. The authors show that an increase in the arrival rate of the HP packets results in an increase in the discard rate of LP packets. In other works, an M/G/1 queueing model is proposed [61]. The model was developed to investigate the performance of the system. The authors conclude that queueing theory can be used to solve problems faced in the real world.

Although a number of authors have carried out research in WSNs with energy harvesting, the queueing models that have been studied in this area are not adequate to capture the energy drawing process [62], [63], [64] .

2.4.5 Priority

In [65] a priority-based queue is employed and is based on the Geo/Geo/1 system in which two classes of data packets arrive in the system . The two classes are HP packets and the LP packets. Priority queues are classified into two groups based on the service discipline:

1. Pre-emptive priority: The HP packets have priority and when they arrive for service, and service to the LP packets is halted. The LP packets only receive service if there are no HP packets in the system and both queues are of infinite length [66]. The following other types of pre-emptive priority exist [65]:

    - Pre-emptive resume: The interrupted LP packet can resume service after the HP packet has been transmitted.
    - Pre-emptive repeat. The interrupted LP packet has to be transmitted again.

2. Non-pre-emptive priority: In this system, the LP packet is transmitted despite the arrival of HP packets. This means that if an LP packet is receiving service, service is not interrupted when an HP packet enters the system [65].

The authors in [67] propose a Geo/Geo/1 pre-emptive priority discrete time queue. The buffer for the HP packets is infinite, while the buffer for the LP packets is finite. To simplify the analysis of the system, a recursive procedure, numerical in nature. was used. The system parameter measures were analysed.

2.5 CONTROLLING THE MODEL

In order to ensure that there is always sufficient energy in the system, an optimal value of the energy buffer is selected. This is done by imposing a threshold on the energy buffer.

A Geo/G/1 queue modelled in discrete time is proposed in [68], using two policies. To ensure that consumption of energy by the sensor is minimised, an optimal threshold is found. In [14], the authors model a system with customers and energy units with random arrivals. The authors assume that for service to be rendered to a customer, a number of units of energy are required. Therefore, if the energy buffer is empty initially, service to the customer is terminated and the customer is lost. This situation introduces a control problem. The strategy for this problem is assumed to be a threshold type [14]. A threshold is imposed on the queue with customers to ensure that the following conditions are met:

1. The threshold is not too small to ensure that service to the customers is not halted owing to the absence of energy.
2. The threshold is not so large that customers are lost because of the limitation of the waiting time.

In the model the authors also assume that if the units of energy are below a specified threshold, service to the customers does not commence.

A number of authors propose imposing a threshold on the system. However, this is usually imposed on the data buffer [69].

To analyse the developed models, three methods are proposed, namely

- Standard algebraic approach: This approach involves solving linear equations and is employed in systems with non-exponential servers [70].
- Z-transform: This approach depends on inverting Laplace transforms and/or generating functions to obtain usable results [71].
- Matrix-geometric approach: It is most suited to Markov chains with a QBD structure. It is popular because it aids the construction and analysis of models in a way that is both unified and algorithmically tractable [72] and [73].

The effort involved when using the traditional methods (standard algebraic approach and z-transform) is high compared to the Matrix-geometric approach. For further descriptions of these methods and how they are used please refer to [53]. The method used in this research is the matrix-geometric approach.

2.6 CONCLUSION

From the literature review, it is observed that a gap exists in terms of a stochastic model for an energy-harvesting WSN that adequately captures the accumulation, usage and leakage of energy. To model and analyse the behaviour of the system, a Geo/Geo/1 priority queue with leakage is implemented and four models are presented. To control the system, a threshold is introduced and is imposed on the energy buffer in the fourth model.

CHAPTER 3 BASIC MODEL

3.1 CHAPTER OVERVIEW

Having done a literature study in Chapter 2 and established the technique that will be employed to analyse the WSN node, the first model will be presented. The first step in achieving the research objective is to develop a model that captures the energy-harvesting and drawing process. This model is referred to as the basic model, in which there are tokens and packets in the system, and is presented in this section with assumptions outlined. Firstly, we look at the system model that is used to describe the basic model.

3.2 SYSTEM MODEL

A single-server queueing system is considered in the modelling of the basic model. The following assumptions are made:

1. The energy buffer size is finite. This means that the number of tokens in the buffer is limited.
2. The data buffer size is finite. This means that the number of packets in the buffer is limited.
3. Each token of energy gives permission for transmission of one bit of data. If a token is waiting in the energy buffer, an arriving bit of data removes it from the buffer and enters the network.
4. If there are no tokens waiting in the energy buffer, the incoming data waits in the data buffer of a given size. When the buffer is full the data is lost. Similarly, if the energy buffer is full, the energy tokens are lost.

The following system parameters are considered:

1. Number of packets in the data buffer;
2. Number of tokens in the energy buffer;
3. Tokens used by the packets;
4. Packets transmitted.

In the system, the arrival process and service time are assumed to be discrete. The discrete-time queueing network has geometric inter-arrival time and geometric service.

In discrete time the system is observed at times t_0, t_1, t_2, ... For simplicity $t_0 = 0$, which is the system at time 0. The proposed model is shown in Figure 3.1.

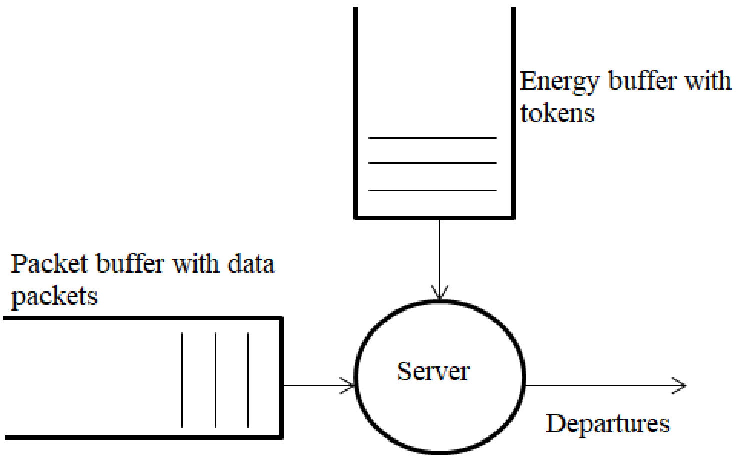

Figure 3.1. System model.

The increment or decrement in the content of the energy buffer is in line with a fluid-flow rate regulated by an environment that is a stochastic process with a finite state space. Whenever the buffer is empty it is topped up to a preset level immediately, and at the same time the environment state jumps to another state immediately with a given probability (or it may stay unchanged). A jump in the fluid level indicates an energy arrival or departure.

3.2.1 Single-server queue with one variable

A single-server queue with A arrivals, B departures and X_n in the buffer at an instant n is shown in the Figure 3.2. At $n+1$ the following applies:

1. $X_{n+1} = (X_n + A - B)^+$, where $(X)^+ = max(X, 0)$, and implies that the final value inside the bracket should always be greater than 0. The above equation implies that at $n+1$, there is a value X_n, A arrivals and B departures.
2. A second option is that there is a value X_n, A arrivals and no B departures.
3. Another option is that there is a value X_n, no A arrivals and B departures.

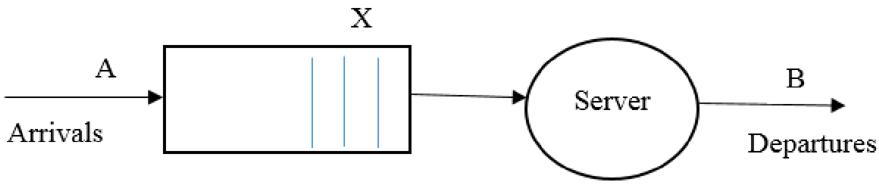

Figure 3.2. System model with only arrival of data packets.

The assumption is that the packets, i, can increase or decrease by one. The inter-arrival and service times of the data and tokens are discrete and follow a geometric distribution with probabilities a, arrival of a packet and b, arrival of a token. The probability of no packet arrival is $1-a = \bar{a}$ and that of no token arrival is $1-b = \bar{b}$. For $i \geq 2$ the probability, P_i is given as follows:

$$P_i = P_{i-1}a(1-b) + P_i(ab + (1-a)(1-b)) + P_ib(1-a). \tag{3.1}$$

To obtain the matrix, the current state and next states are considered and taken into consideration.

$$P_m = \begin{bmatrix} 1-a & a & & & \\ (1-a)b & ab+(1-a)(1-b) & a(1-b) & & \\ & (1-a)b & ab+(1-a)(1-b) & a(1-b) & \\ & & \ddots & \ddots & \ddots \end{bmatrix} \tag{3.2}$$

3.2.2 Single-server queue with two variables

If there are two variables i (data packets) and j (energy tokens), they can transition to the states indicated i(j-1), i(j+1), (i-1)j, (i+1)j, ij. The following assumptions are made:

1. The system can increase or decrease by one *step*.
2. A data packet and a token cannot be in the queue at the same time, as the data packet is automatically transmitted.

3. When there are no tokens in the system a data packet is not transmitted.
4. Inter-arrival and service times of the data packets and energy tokens follow geometric distributions.

The probabilities are only satisfied when there are more than zero tokens and packets in the system.

3.3 MODEL WITH AN ENERGY-HARVESTING CAPABILITY

The system can be described by a Markov chain that is two-dimensional at time n, $(L_n, J_n), n \geq 0$, L_n is the number of packets in the buffer, $0 \leq L_n \leq N$, J_n is the number of tokens in the buffer, $0 \leq J_n \leq K$,K is a finite number that represents the token buffer size. Each token gives permission for the transmission of one packet. The packet buffer size is N. When there are no energy tokens in the system, a data packet is not transmitted. The state space is defined as follows:

$$\Delta = (0, j) \cup (i, 0), 0 \leq j_n \leq K; 1 \leq i_n \leq N, n \geq 0. \tag{3.3}$$

The steady state equations of this model can be written as follows:

$$x_{0,0} = x_{0,0}(ab + \bar{a}\bar{b}) + x_{0,1}\bar{a}b + x_{1,0}a\bar{b} \tag{3.4}$$

$$x_{0,1} = x_{0,0}a\bar{b} + x_{0,1}(ab + \bar{a}\bar{b}) + x_{0,2}\bar{a}b \tag{3.5}$$

$$x_{0,j} = x_{0,(j-1)}a\bar{b} + x_{0,(j)}(ab + \bar{a}\bar{b}) + x_{0,(j+1)}\bar{a}b \tag{3.6}$$

$$x_{1,0} = x_{0,0}\bar{a}b + x_{1,0}(ab + \bar{a}\bar{b}) + x_{2,0}a\bar{b} \tag{3.7}$$

$$x_{2,0} = x_{1,0}\bar{a}b + x_{2,0}(ab + \bar{a}\bar{b}) + x_{3,0}a\bar{b} \tag{3.8}$$

$$x_{i,0} = x_{(i-1),0}\bar{a}b + x_{i,0}(ab + \bar{a}\bar{b}) + x_{(i+1),0}a\bar{b}. \tag{3.9}$$

Considering a state space of $\Delta = (i, j), i \geq 1, j \geq 0$, there are a number of ways to analyse the steady-state equations namely, the z-transform approach, the algebraic approach and the matrix-geometric approach. The matrix-geometric approach is preferred over the other mentioned approaches due to their complexity in the analysis of the system of equations. The transition matrix, P, is a classical QBD matrix. An entry in the matrix represents the transition from one state given in the row to the next state given in the corresponding column. An absence of an entry in the matrix implies that the two states are not accessible to each other. Figure 3.3 shows the resulting transition.

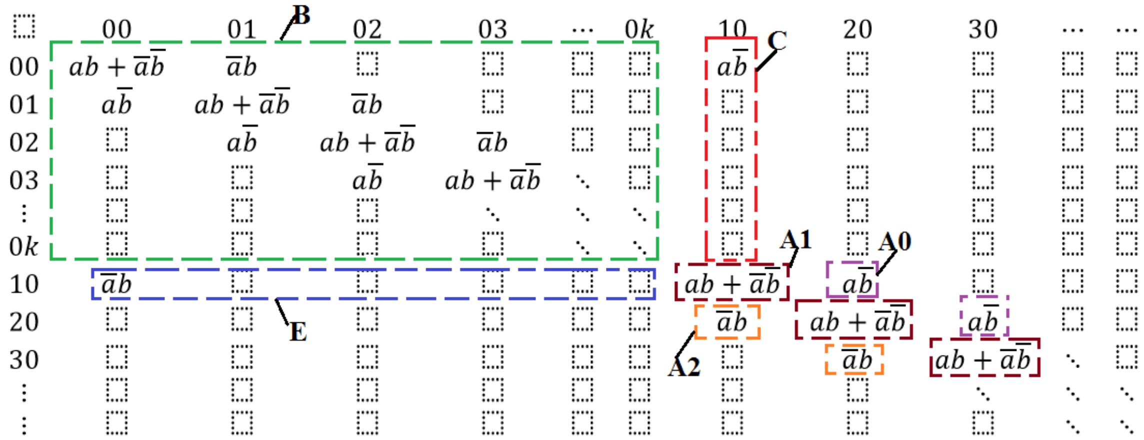

Figure 3.3. Transition matrix for the model.

By applying the lexicographical order, the probability transition matrix P of the discrete-time Markov chain (DTMC) is obtained as follows:

$$P = \begin{bmatrix} B & C & & & \\ E & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & \ddots & \ddots & \ddots \\ & & & A_2 & A_1 + A_0 \end{bmatrix}, \tag{3.10}$$

where

$$B = \begin{bmatrix} ab+\bar{a}\bar{b} & \bar{a}b & & \\ a\bar{b} & ab+\bar{a}\bar{b} & \bar{a}b & \\ & \ddots & \ddots & \ddots \\ & & a\bar{b} & ab+\bar{a} \end{bmatrix},$$

$$C = \begin{bmatrix} a\bar{b} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, E = \begin{bmatrix} \bar{a}b & 0 & \cdots & 0 \end{bmatrix},$$

$$A_1 = ab+\bar{a}\bar{b}, A_0 = a\bar{b}, A_2 = \bar{a}b.$$

3.4 NUMERICAL COMPUTATION OF THE INVARIANT PROBABILITY VECTOR

In this section, the matrix is analysed to obtain the time-invariant vector using the matrix-geometric method. The computation is done for bivariate DTMC and can be extended to a multivariate DTMC. The equations in the matrix are transformed to the standard form given below,

$$AX = b, \tag{3.11}$$

keeping in mind that the transition matrix can be written as

$$P = \begin{bmatrix} B & C & & & & \\ E & ab+\bar{a}\bar{b} & a\bar{b} & & & \\ & \bar{a}b & ab+\bar{a}\bar{b} & a\bar{b} & & \\ & & \ddots & \ddots & \ddots & \\ & & & \bar{a}b & ab+\bar{b} \end{bmatrix} \tag{3.12}$$

where $P_{i,j}, i = 1,2,....$ and $j = 0,1,2,...$ is a $M \times M$ block matrix. If x_i^n is defined as the probability that there are i packets in the system at a time n, then assuming, $x = x_0^n, x_1^n, ...$

$$x^{n+1} = x^n P. \tag{3.13}$$

For a stable system a relationship between the matrix P and the vector x in the form

$$x = xP, \qquad x\mathbf{1} = 1. \tag{3.14}$$

If P is irreducible and positive recurrent then an invariant vector exists which is equivalent to the limiting distribution, $x = x^n|_{n\to\infty}$ exists and the equation above then holds. To obtain the form $AX = b$, assume x_n has a state space given as $0,1,2,...N < \infty$. Let $\mathbf{1}_M$ be n $M \times 1$ vector of ones and $\mathbf{0}_M$ be n $M \times 1$ vector of zeros. The stationary equation can be written as

$$\begin{aligned} 0_M^T &= x_0(B_{00} - I) + x_1 B_{01} + x_2 B_{02} + \cdots + x_N B_{0N} \\ 0_M^T &= x_0 B_{10} \quad + x_1(B_{11} - I) + x_2 B_{12} + \cdots + x_N B_{1N} \\ 0_M^T &= x_0(B_{20}) \quad + x_1 B_{21} \quad + x_2(B_{22} - I) + \cdots + x_N B_{2N} \\ \vdots \quad &= \vdots \quad + \quad \vdots \quad + \quad \vdots \quad + \quad \dots \quad + \quad \vdots \end{aligned} \tag{3.15}$$

One then obtains the matrix given below

$$\begin{bmatrix} B_{00}-I & B_{01} & B_{02} & \dots & B_{0N} \\ B_{10} & B_{11}-I & B_{12} & \dots & B_{1N} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ B_{(N-1)0} & B_{(N-1)1} & B_{(N-1)2} & \ddots & B_{(N-1)N} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \dots & \mathbf{1} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix} = \begin{bmatrix} 0_M \\ 0_M \\ \vdots \\ 0_M \\ \mathbf{1}_M \end{bmatrix} \tag{3.16}$$

3.5 SYSTEM PERFORMANCE MEASURES

To determine the performance measures of a queueing system, the key characteristics of the system are specified. These are the arrival and service times [53]. The distribution of the service and inter-arrival times is important, as this is used to evaluate the performance of the proposed system in comparison to the existing Geo/Geo/1 queue. The system is assumed to be a FCFS system, as mentioned previously. The performance measures that will be focused on in this research are the mean number of packets and tokens in the queue and system. These performance measures are obtained as described below.

3.5.1 Mean number in system

The mean number of packets and tokens in the system for the model is given as:

$$E[X] = \sum_{i=1}^{N} iX_{i,0}, \qquad E[Y] = \sum_{j=0}^{K} jY_{j,0}. \tag{3.17}$$

3.5.2 Mean number in queue

The mean number of packets and tokens in the queue for each of the three models is given as:

$$E[X_q] = \sum_{i=2}^{N} (i-1)X_{i,0}, \qquad E[Y_q] = \sum_{j=1}^{K} (j-1)Y_{j,0}. \tag{3.18}$$

3.6 CONCLUSION

The model developed in this section is the basic model. To investigate the WSN node with energy-harvesting capability, the proposed model is modelled as a Geo/Geo/1/k system. Two queues are considered, one for data packets and the other for energy tokens. The proposed model can be described by a QBD and is considered to be FCFS unless stated otherwise. The performance measures are obtained using Equations 3.17 and 3.18. This model is for an "ideal" system. However, it does not adequately represent a practical system. Therefore, non-ideal models with attributes of a practical system such as leakage and emergency data are required and will be developed in Chapter 4.

CHAPTER 4 MODELS WITH FOCUS ON LEAKAGE, PRIORITY AND THRESHOLD

4.1 CHAPTER OVERVIEW

Having developed the ideal basic model with only tokens and packets in the system, it is also possible to develop models that reflect a practical system, in which tokens leak, for example.

Three models that are a build-up on the basic model are presented in this section. A basic model with leakage incorporated is first developed; thereafter, priority and threshold will be included respectively. The models presented are a basic model with leakage incorporated, a basic model with leakage and priority incorporated and a basic model with leakage, priority and threshold incorporated.

4.2 MODEL WITH LEAKAGE INCORPORATED

In a practical system, unused tokens often leak. To ensure that the basic model mimics a practical system, a leakage is imposed on it.

To cater for energy leakage in the system, a parameter θ is introduced. Energy leakage is expected when there is more than one token in the system. The following assumptions are made: firstly, when a token arrives, it stays until the next transition (at least one unit) before it leaks. Secondly, disaster arrivals are avoided. A disaster arrival occurs when a token arrives in the system and removes all present tokens (in the queue and in service) in the system. Disasters are also known as queue flushing, catastrophes, stochastic clearing systems and mass exodus [74]. Finally, energy cannot leak unless there is a token in the system. The probability of leakage is θ and that of no leakage is $1 - \theta$. The probability of having n tokens in the system and k tokens leaking is expressed as $l_{n,k}$ and is a binomial distribution, in the form.

$$l_{n,k} = \binom{n}{k}\theta^k(1-\theta)^{n-k}, \tag{4.1}$$

where k is the number of tokens that leak at that instant and is defined as $0 \leq k \leq n$. For example at state 01, there are no packets and one token in the system or queue. The number of tokens n, is 1 and either one token leaks or none leaks. The value of k is 0 when no token leaks and 1 when a token leaks.

$$l_{1,k} = \binom{1}{k}\theta^k(1-\theta)^{1-k} \tag{4.2}$$

k takes on two values 0 and 1. $l_{1,0} = 1-\theta$ and $l_{1,1} = \theta$. For $n = 2$, the leakage is given as follows.

$$l_{2,k} = \binom{2}{k}\theta^k(1-\theta)^{2-k}, \tag{4.3}$$

k takes on three values 0, 1 and 2. $l_{2,0} = (1-\theta)^2$, $l_{2,1} = 2\theta(1-\theta)$ and $l_{2,2} = \theta^2$. The steady state equations of this model can be written as follows:

$$x_{0,0} = x_{0,0}(ab+\bar{a}\bar{b}) + x_{0,1}\bar{a}b + x_{1,0}a\bar{b} \tag{4.4}$$

$$x_{0,1} = x_{0,0}((ab+\bar{a}\bar{b})l_{1,1} + a\bar{b}l_{1,0}) + x_{0,1}(\bar{a}bl_{1,1} + ab + \bar{a}\bar{b}l_{1,0}) + x_{0,2}a\bar{b}l_{1,0} + x_{1,0}a\bar{b}l_{1,1} \tag{4.5}$$

$$\begin{aligned} x_{0,j} = {} & x_{0,0}((ab+\bar{a}\bar{b})l_{j,j} + a\bar{b}l_{j,(j-1)}) + x_{0,1}(\bar{a}bl_{j,j} + (ab+\bar{a}\bar{b})l_{j,(j-1)} + a\bar{b}l_{j,(j-2)}) \\ & + x_{0,(j-1)}(\bar{a}bl_{j,2} + (ab+\bar{a}\bar{b})l_{j,1} + a\bar{b}l_{j,0}) + x_{0,j}(\bar{a}bl_{j,1} + (ab+\bar{a}\bar{b})l_{j,0} + a\bar{b}l_{j,0}) + x_{1,j}a\bar{b}l_{j,j} \end{aligned} \tag{4.6}$$

$$x_{1,0} = x_{0,0}\bar{a}b + x_{1,0}(ab+\bar{a}\bar{b}) + x_{2,0}a\bar{b} \tag{4.7}$$

$$x_{2,0} = x_{1,0}\bar{a}b + x_{2,0}(ab+\bar{a}\bar{b}) + x_{3,0}a\bar{b} \tag{4.8}$$

$$x_{i,0} = x_{(i-1),0}\bar{a}b + x_{i,0}(ab+\bar{a}\bar{b}) + x_{(i+1),0}a\bar{b}. \tag{4.9}$$

4.2.1 Transition matrix

Considering a state space of $\Delta = (i,j), i \geq 1, j \geq 0$, the matrix-geometric approach is employed in the analysis of this system of equations. Figure 4.1 shows the resulting transition matrix.

The probability transition matrix P of the DTMC is in the form:

$$P = \begin{bmatrix} B & C & & & \\ E & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & \ddots & \ddots & \ddots \\ & & & A_2 & A_1+A_0 \end{bmatrix}, \tag{4.10}$$

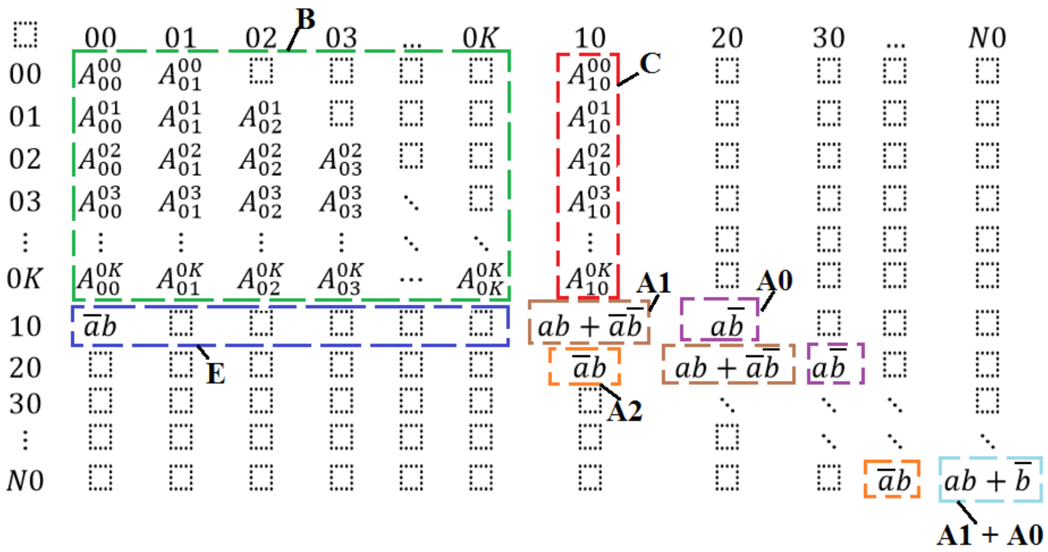

Figure 4.1. Transition matrix for the model with leakage incorporated.

where

$$B = \begin{bmatrix} B_{00}^{00} & B_{01}^{00} & & & \\ B_{00}^{01} & B_{01}^{01} & B_{02}^{01} & & \\ \vdots & \vdots & \vdots & \ddots & \\ B_{00}^{0K} & B_{01}^{0K} & B_{02}^{0K} & \cdots & B_{0K}^{0K} \end{bmatrix},$$

$$C = \begin{bmatrix} B_{10}^{00} \\ B_{10}^{01} \\ \vdots \\ B_{10}^{0K} \end{bmatrix}, E = \begin{bmatrix} \bar{a}b & 0 & \cdots & 0 \end{bmatrix}$$

with

$$B_{00}^{0K} = (ab + \bar{a}\bar{b})l_{K,K} + a\bar{b}l_{K,K-1} \tag{4.11}$$

$$B_{01}^{0K} = \bar{a}bl_{K,K} + (ab + \bar{a}\bar{b})l_{K,K-1} + a\bar{b}l_{K,K-2} \tag{4.12}$$

$$B_{02}^{0K} = \bar{a}bl_{K,K-1} + (ab + \bar{a}\bar{b})l_{K,K-2} + a\bar{b}l_{K,K-3} \tag{4.13}$$

$$B_{0K}^{0K} = a\bar{b}l_{K,1} + (ab + \bar{a}\bar{b})l_{K,K-K} + a\bar{b}l_{K,K-K} \tag{4.14}$$

$$B_{10}^{0K} = a\bar{b}l_{K,K} \tag{4.15}$$

$$A_1 = ab + \bar{a}\bar{b}, A_0 = a\bar{b}, A_2 = \bar{a}b. \tag{4.16}$$

When a token arrives at the buffer and finds that the buffer is at its maximum capacity, the arriving token is dropped and the present state is retained.

The probability of a token at $n = K$, is given as

$$B^{0K} = B^{0K}_{00} + B^{0K}_{01} + B^{0K}_{02} + ... + B^{0K}_{0K} + B^{0K}_{10}. \tag{4.17}$$

4.2.2 Performance measures

In order to analyse the system, the mean number of packets and the mean number of tokens in the system and queue are obtained as follows.

4.2.2.1 Mean number in system

The mean number of packets and tokens in the system for the model is given as follows:

$$E[X] = \sum_{i=1}^{N} iX_{i,0}, \qquad E[Y] = \sum_{j=0}^{K} jY_{j,0}. \tag{4.18}$$

4.3 MODEL WITH LEAKAGE AND PRIORITY INCORPORATED

In addition to leakage, a practical system will have emergency data (referred to as HP packets in this research), which have to be transmitted immediately. The model presented in this section is a build-up of the model with leakage incorporated. Leakage of energy and priority are captured in the model presented in this section.

4.3.1 Pre-emptive queue model

A system with two classes of packets that arrive according to the Bernoulli process is considered with parameters $a_{H,L}$. a_H refers to the arrival rate of HP packets and $1 - a_H$ refers to the no arrival rate of HP packets and a_L refers to the arrival rate of LP packets and $1 - a_L$ refers to the no arrival rate of LP packets.

For the two classes of data packets presented, it is assumed that there are two buffers. The proposed model is shown in Figure 4.2.

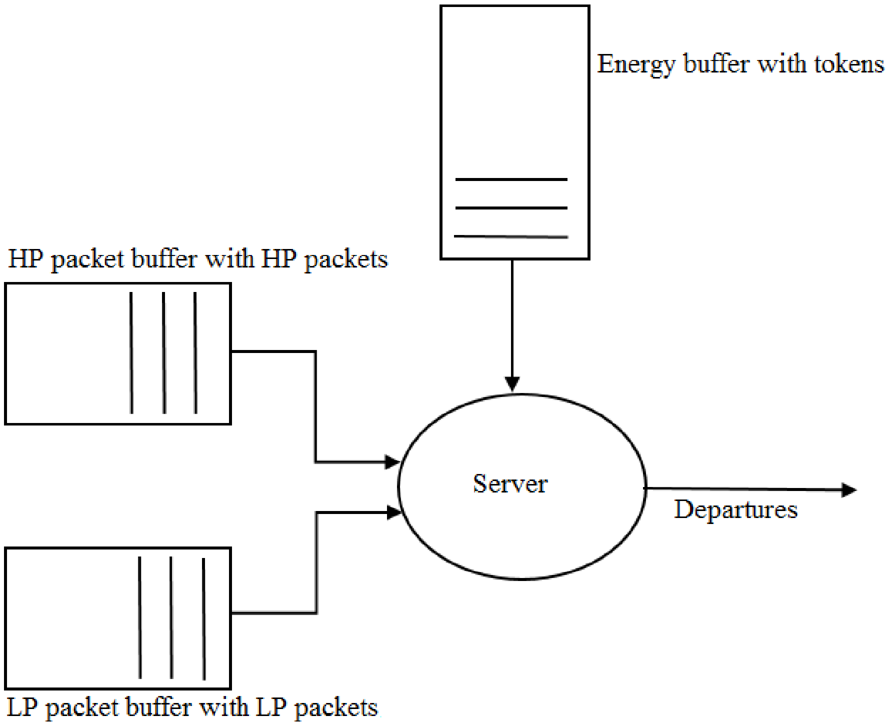

Figure 4.2. System model with priority incorporated.

The pre-emptive model has two separate packet buffers, one for the HP packets and the other for LP packets. In this research, our focus is on the pre-emptive resume discipline in respect of the following:

- No LP packets can start receiving service unless there is no HP packet in the system.
- If an LP packet is receiving service (in the absence of an HP packet in the system), the service of this LP packet will be interrupted on the arrival of an HP packet occurring before the completion of the LP service.

Each packet requires geometric service with parameter b. There is a possibility of having up to two packets of each type arriving, since one is dealing with discrete time. Hence, the probability of arrivals of H type HP and L type LP is defined as $a_{H,L}$, $H = 0,1; L = 0,1$. Therefore,

$$\begin{aligned} a_{0,0} &= (1-a_H)(1-a_L), \\ a_{0,1} &= (1-a_H)a_L, \\ a_{1,0} &= a_H(1-a_L), \\ a_{1,1} &= a_H a_L. \end{aligned} \tag{4.19}$$

In this model, the following assumptions are made:

1. Service time to both the HP and LP packets is assumed to be independent and geometrically distributed.
2. One packet is transmitted at a time. Therefore if there are two HP packets in the system in a given state, then the next state cannot have zero HP packets.
3. If a given state has both HP packets and LP packets, then the next state cannot have zero packets. Since one is dealing with a pre-emptive queue model, it is expected that the next state will have LP packets, as the HP packet will be transmitted.

To cater for leakage, the following is taken into consideration: The probability of leakage is θ and that of no leakage is $1-\theta$. The probability of having n tokens in the system and k tokens leaking is expressed as $l_{n,k}$ and is a binomial distribution given in Equation 4.1.

4.3.2 Transition matrix

The probability transition matrix P is obtained as described subsequently. The state space of the system is described by a Markov chain that is three-dimensional at time n, $(I_n, J_n, K_n), n \geq 0$, I_n is the number of HP packets in the buffer, $0 \leq I_n \leq M$, J_n is the number of LP packets in the buffer, $0 \leq J_n \leq N$, K_n is the number of tokens in the buffer, $0 \leq K_n \leq K$, K is a finite number that represents the token buffer size. Each token gives permission for the transmission of one packet. The HP packet buffer size is M and the LP packet buffer size is N. Equation 4.1 is used to cater for leakage of tokens in the system. The development of the transition matrix of this model is given in Addendum A.1.

The transition matrix is given as,

$$P_l = \begin{bmatrix} B & C & & & & \\ E & A_1 & A_0 & & & \\ & A_2 & A_1 & A_0 & & \\ & & A_2 & A_1 & A_0 & \\ & & & \ddots & \ddots & \ddots \\ & & & & A_2 & A_1 + A_0 \end{bmatrix}, \tag{4.20}$$

where

$$B = \begin{bmatrix} B_{00} & B_{01} & & & \\ B^2 & B^1 & B^0 & & \\ & B^2 & B^1 & B^0 & \\ & & \ddots & \ddots & \ddots \\ & & & B^2 & B^1 + B^0 \end{bmatrix},$$

with

$$B_{00} = \begin{bmatrix} B_{000}^{000} & B_{001}^{000} & & & & \\ B_{000}^{001} & B_{001}^{001} & B_{002}^{001} & & & \\ B_{000}^{002} & B_{001}^{002} & B_{002}^{002} & B_{003}^{002} & & \\ & \vdots & \vdots & \vdots & \ddots & \\ B_{000}^{00K} & B_{001}^{00K} & B_{002}^{00K} & B_{003}^{00K} & \cdots & B_{00K}^{00K} \end{bmatrix},$$

with

$$B_{000}^{00K} = (a_{1,0}b + a_{0,1}b + a_{0,0}\bar{b})l_{K,K} + a_{1,1}\bar{b}l_{K,K-1} + (a_{1,0}\bar{b} + a_{0,1}\bar{b})l_{K,K-2} + a_{1,1}\bar{b}l_{K,K-2} \tag{4.21}$$

$$\begin{aligned} B_{001}^{00K} &= a_{0,0}bl_{K,K} + (a_{1,0}b + a_{0,1}b + a_{0,0}\bar{b})l_{K,K-1} + a_{1,1}\bar{b}l_{K,K-2} \\ &+ (a_{1,0}\bar{b} + a_{0,1}\bar{b})l_{K,K-3} + a_{1,1}\bar{b}l_{K,K-3} \end{aligned} \tag{4.22}$$

$$B_{00(K-1)}^{00K} = a_{0,0}bl_{K,K-(K-2)} + (a_{1,0}b + a_{0,1}b + a_{0,0}\bar{b})l_{K,K-(K-1)} + a_{1,1}bl_{K,K-(K-K)} \tag{4.23}$$

$$B_{00K}^{00K} = a_{0,0}bl_{K,K-(K-1)} + (a_{1,0}b + a_{0,1}b + a_{0,0}\bar{b})l_{K,K-K} + (a_{1,0}\bar{b} + a_{0,1}\bar{b})l_{K,K-K} \tag{4.24}$$

$$B^2 = a_{0,0}b, \quad B^1 = a_{0,1}b + a_{0,0}\bar{b}, \quad B^0 = a_{0,1}\bar{b} \tag{4.25}$$

$$B_{01} = \begin{bmatrix} B_{010}^{000} \\ B_{010}^{001} \\ B_{010}^{002} \\ \vdots \\ B_{010}^{00K} \end{bmatrix},$$

with

$$B_{010}^{00K} = (a_{1,0}\bar{b} + a_{1,1}b)l_{K,K} \tag{4.26}$$

$$C = \begin{bmatrix} B_{100}^{000} & B_{110}^{000} & & & & \\ B_{100}^{001} & B_{110}^{001} & & & & \\ B_{100}^{002} & B_{110}^{002} & & & & \\ \vdots & \vdots & & & & \\ B_{100}^{00K} & B_{110}^{00K} & & & & \\ C^2 & C^1 & C^0 & & & \\ & C^2 & C^1 & C^0 & & \\ & & \ddots & \ddots & \ddots & \\ & & & C^2 & C^1+C^0 \end{bmatrix},$$

with

$$B_{100}^{00K} = (a_{1,0}\bar{b} + a_{1,1}\bar{b})l_{K,K}. \tag{4.27}$$

$$B_{110}^{00K} = (a_{1,0}\bar{b} + a_{0,1}\bar{b})l_{K,K-1} + a_{1,1}\bar{b}l_{K,K-1} \tag{4.28}$$

$$C^2 = a_{1,0}b, \qquad C^1 = a_{1,0}\bar{b} + a_{1,1}b, \qquad C^0 = a_{1,1}\bar{b}, \tag{4.29}$$

$$E = \begin{bmatrix} E^1 & \cdots & 0 & E^0 & & & \\ \vdots & & \vdots & E^1 & E^0 & & \\ \vdots & & \vdots & & E^1 & E^0 & \\ \vdots & & \vdots & & & \ddots & \ddots \\ 0 & \cdots & 0 & & & & E^1+E^0 \end{bmatrix},$$

with

$$E^1 = a_{0,0}b, \qquad E^0 = a_{0,1}b, \tag{4.30}$$

$$A_1 = \begin{bmatrix} A_1^1 & A_1^0 & & & \\ & A_1^1 & A_1^0 & & \\ & & A_1^1 & A_1^0 & \\ & & & \ddots & \ddots \\ & & & & A_1^1+A_1^0 \end{bmatrix},$$

with

$$A_1^1 = a_{1,0}b + a_{0,0}\bar{b}, \qquad A_1^0 = a_{1,1}b + a_{0,1}\bar{b}, \tag{4.31}$$

$$A_0 = \begin{bmatrix} A_0^1 & A_0^0 & & & \\ & A_0^1 & A_0^0 & & \\ & & A_0^1 & A_0^0 & \\ & & & \ddots & \ddots \\ & & & & A_0^1+A_0^0 \end{bmatrix},$$

with

$$A_0^1 = a_{1,0}\bar{b}, \qquad A_0^0 = a_{1,1}\bar{b}, \tag{4.32}$$

$$A_2 = \begin{bmatrix} A_2^1 & A_2^0 & & & \\ & A_2^1 & A_2^0 & & \\ & & A_2^1 & A_2^0 & \\ & & & \ddots & \ddots \\ & & & & A_2^1 + A_2^0 \end{bmatrix},$$

with

$$A_2^1 = a_{0,0}b, \qquad A_2^0 = a_{0,1}b. \tag{4.33}$$

The basic model with leakage and priority incorporated was developed using the matrix in Figure 4.3 and thereafter grouped as shown in order to compute the stationary distribution vector using MATLAB.

4.3.3 Performance measures

In order to analyse the system, the mean number of packets and the mean number tokens in the system and queue are obtained as follows.

$$E[X_i] = \sum_{i=1}^{N} iX_{i,0,0} \tag{4.34}$$

$$E[X_j] = \sum_{j=1}^{N} jX_{0,j,0}, \qquad E[Y_k] = \sum_{k=0}^{K} kY_{0,0,k} \tag{4.35}$$

where $E[X_i]$ is the mean number of HP packets in the system, $E[X_j]$ is the mean number of LP packets in the system and $E[Y_k]$ is the mean number of tokens in the system.

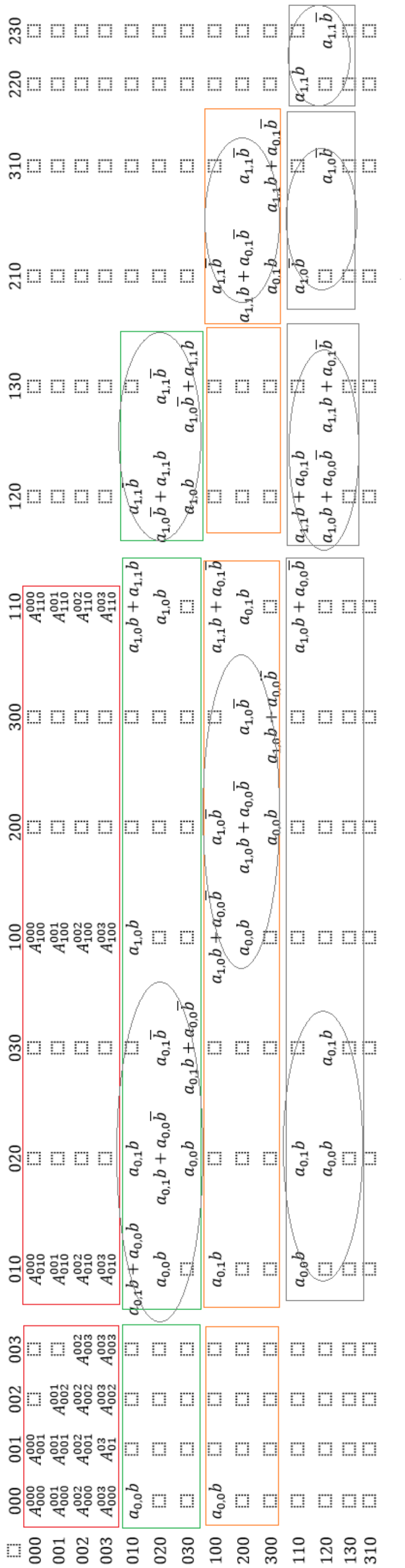

Figure 4.3. Matrix used to develop basic model with leakage and priority incorporated.

4.4 MODEL WITH LEAKAGE, PRIORITY AND THRESHOLD INCORPORATED

The model presented in this section is a build-up of the model with leakage and priority. The proposed model is shown in Figure 4.4. Below a specified threshold, no LP packets are transmitted, while HP packets are continuously transmitted. The LP packets are only transmitted when the value of the token buffer size is equal to or greater than the threshold. The assumption is that packets are transmitted before tokens leak.

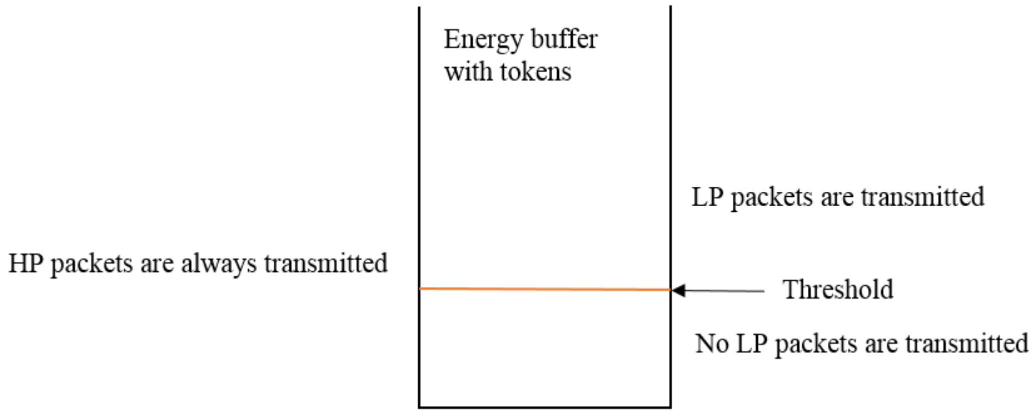

Figure 4.4. Model of system with threshold incorporated.

4.4.1 Transition matrix

The state space of the model is a three-dimensional Markov chain described in the priority model. A threshold is imposed on the token buffer, $0 \leq t \leq thresh$.

To cater for energy leakage in the system, a parameter θ is introduced in the system. Equation 4.1 is used. The development of the transition matrix of this model is given in Addendum A.2. The transition matrix of the model with leakage, priority and threshold incorporated is given as follows:

$$P_t = \begin{bmatrix} B & C & & & \\ E & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & \ddots & \ddots & \ddots \\ & & & A_2 & A_1 + A_0 \end{bmatrix}, \tag{4.36}$$

where

$$B = \begin{bmatrix} B_{00}^{00} & B_{01}^{00} & & & \\ B_{00} & B_{0,1} & B_{0,2} & & \\ & B_{0,0} & B_{0,1} & B_{0,2} & \\ & & \ddots & \ddots & \ddots \\ & & & B_{0,0} & B_{0,b} \end{bmatrix},$$

with

$$B_{00}^{00} = \begin{bmatrix} B_{000}^{000} & B_{001}^{000} & & & & \\ B_{000}^{001} & B_{001}^{001} & B_{002}^{001} & & & \\ B_{000}^{002} & B_{001}^{002} & B_{002}^{002} & B_{003}^{002} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B_{000}^{00K} & B_{001}^{00K} & B_{002}^{00K} & B_{003}^{00K} & \cdots & B_{00K}^{00K} \end{bmatrix},$$

with

$$B_{000}^{00n} = (a_{1,0}b + a_{0,0}\bar{b})l_{n,n} + a_{1,0}\bar{b}l_{n,n-1} \tag{4.37}$$

$$B_{001}^{00n} = a_{0,0}bl_{n,n} + (a_{1,0}n + a_{0,0}\bar{b})l_{n,n-1} + a_{1,0}\bar{b}l_{n,n-2} \tag{4.38}$$

$$B_{00t}^{00(t-1)} = a_{0,0}bl_{n,n-1} + (a_{1,0}n + a_{0,0}\bar{b} + a_{0,1}b)l_{n,n-2} + (a_{1,0}\bar{b} + a_{0,1}\bar{b})l_{n,n-3} \tag{4.39}$$

$$B_{00t}^{00(t)} = a_{0,0}bl_{n,n-2} + (a_{1,0}n + a_{0,0}\bar{b} + a_{0,1}b)l_{n,n-3} + (a_{1,0}\bar{b} + a_{0,1}\bar{b})l_{n,n-4} \tag{4.40}$$

$$B_{00K}^{00K} = a_{0,0}bl_{K,1} + (a_{1,0}n + a_{0,0}\bar{b} + a_{0,1}b)l_{K,0} + a_{0,0}bl_{K,0} \tag{4.41}$$

where

$$B_{01}^{00} = \begin{bmatrix} B_{010}^{000} & B_{011}^{000} & & & & \\ B_{010}^{001} & B_{011}^{001} & B_{012}^{001} & & & \\ B_{010}^{002} & B_{011}^{002} & B_{012}^{002} & B_{013}^{002} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B_{010}^{00K} & B_{011}^{00K} & B_{012}^{00K} & B_{013}^{00K} & \cdots & B_{01K}^{00K} \end{bmatrix},$$

with

$$B_{010}^{00n} = (a_{0,1}\bar{b} + a_{1,1}b)l_{n,n} + a_{1,1}\bar{b}l_{n,n-1} \tag{4.42}$$

$$B_{011}^{00n} = a_{0,1}bl_{n,n} + (a_{0,1}\bar{b} + a_{1,1}b)l_{n,n-1} + a_{1,1}\bar{b}l_{n,n-2} \tag{4.43}$$

$$B_{01K}^{00(K-1)} = a_{0,1}bl_{n,n-1} + (a_{0,1}\bar{b} + a_{1,1}b)l_{n,n-2} + a_{1,1}\bar{b}l_{n,n-3} \tag{4.44}$$

$$B_{01K}^{00K} = a_{1,1}bl_{K,t} + a_{1,1}bl_{K,(t-1)} + ... + a_{1,1}bl_{K,(t-t)} \tag{4.45}$$

where

$$B_{00} = \begin{bmatrix} B^{000}_{010} & B^{001}_{010} & & & & \\ B^{000}_{011} & B^{001}_{011} & B^{002}_{011} & & & \\ B^{000}_{012} & B^{001}_{012} & B^{002}_{012} & B^{003}_{012} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B^{000}_{01t} & B^{001}_{01t} & B^{002}_{01t} & B^{003}_{01t} & \cdots & B^{00K}_{01t} \end{bmatrix},$$

with

$$B^{00(t-1)}_{01(t-1)} = a_{0,0}bl_{t-1,0} \tag{4.46}$$

$$B^{00(t-1)}_{01t} = a_{0,0}bl_{t,1} + a_{0,0}\bar{b}l_{t,0} \tag{4.47}$$

$$B^{00t}_{01t} = a_{0,0}bl_{t,0} \tag{4.48}$$

where

$$B_{0,0} = \begin{bmatrix} B^{010}_{020} & B^{011}_{020} & & & & \\ B^{010}_{021} & B^{011}_{021} & B^{012}_{021} & & & \\ B^{010}_{022} & B^{011}_{022} & B^{012}_{022} & B^{013}_{022} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B^{010}_{02t} & B^{011}_{02t} & B^{012}_{02t} & B^{013}_{02t} & \cdots & B^{01t}_{02t} \end{bmatrix},$$

with

$$B^{00(t-1)}_{02(t-1)} = a_{0,0}bl_{t-1,0} \tag{4.49}$$

$$B^{00(t-1)}_{02t} = a_{0,0}bl_{t,1} + a_{0,0}\bar{b}l_{t,0} \tag{4.50}$$

$$B^{00t}_{02t} = a_{0,0}bl_{t,0} \tag{4.51}$$

where

$$B_{0,1} = \begin{bmatrix} B^{010}_{010} & B^{010}_{011} & & & & \\ B^{011}_{010} & B^{011}_{011} & B^{011}_{012} & & & \\ B^{012}_{010} & B^{012}_{011} & B^{012}_{012} & B^{012}_{013} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B^{01t}_{010} & B^{01t}_{011} & B^{01t}_{012} & B^{01t}_{013} & \cdots & B^{01t}_{01t} \end{bmatrix},$$

with

$$B^{01t}_{010} = (a_{1,0}b + a_{0,0}\bar{b})l_{n,n} + a_{1,0}\bar{b}l_{n,n-1} \tag{4.52}$$

$$B^{01t}_{011} = a_{0,0}bl_{n,n} + (a_{1,0}b + a_{0,0}\bar{b})l_{n,n-1} + a_{1,0}\bar{b}l_{n,n-2} \tag{4.53}$$

$$B^{01(t-1)}_{011t} = a_{0,0}bl_{n,n-n} \tag{4.54}$$

$$B^{01t}_{01t} = a_{0,0}bl_{n,n-2} + (a_{1,0}b + a_{0,0}\bar{b})l_{n,n-n} + a_{0,0}bl_{n,n-n} \tag{4.55}$$

$$B^{01K}_{01(K-1)} = 0 \tag{4.56}$$

$$B^{01K}_{01K} = (a_{1,0}b + a_{0,1}b)l_{K,0} \tag{4.57}$$

where

$$B_{0,2} = \begin{bmatrix} B_{020}^{020} & B_{021}^{010} & & & & \\ B_{020}^{021} & B_{021}^{011} & B_{022}^{011} & & & \\ B_{020}^{022} & B_{021}^{012} & B_{022}^{012} & B_{023}^{012} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B_{020}^{01t} & B_{021}^{01t} & B_{022}^{01t} & B_{023}^{01t} & \cdots & B_{02t}^{01t} \end{bmatrix},$$

with

$$B_{020}^{01t} = (a_{1,0}\bar{b} + a_{1,1}b)l_{n,n} + a_{1,1}\bar{b}l_{n,n-1} \tag{4.58}$$

$$B_{021}^{01t} = a_{0,1}bl_{n,n} + (a_{1,0}\bar{b} + a_{1,1}b)l_{n,n-1} + a_{1,1}\bar{b}l_{n,n-2} \tag{4.59}$$

$$B_{02t}^{01(t-1)} = a_{0,1}bl_{n,n-n} \tag{4.60}$$

$$B_{02t}^{01t} = a_{0,1}bl_{n,n-2} + (a_{0,1}\bar{b} + a_{1,1}b)l_{n,n-n} + a_{0,1}bl_{n,n-n} \tag{4.61}$$

$$B_{02(t-1)}^{01t} = 0 \tag{4.62}$$

$$B_{02t}^{01t} = a_{1,1}bl_{t,0} \tag{4.63}$$

where

$$B_{0,b} = \begin{bmatrix} B_{0N0}^{0N0} & B_{0N1}^{0N0} & & & & \\ B_{0N0}^{0N1} & B_{0N1}^{0N1} & B_{0N2}^{0N1} & & & \\ B_{0N0}^{0N2} & B_{0N1}^{0N2} & B_{0N2}^{0N2} & B_{0J3}^{0N2} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ B_{0N0}^{0Nt} & B_{0N1}^{0Nt} & B_{0N2}^{0Nt} & B_{0N3}^{0Nt} & \cdots & B_{0Nt}^{0Nt} \end{bmatrix},$$

with

$$B_{0Nt}^{0N0} = (a_{1,0}b + a_{0,0}\bar{b} + a_{0,1}\bar{b} + a_{1,1}b)l_{n,n} + (a_{1,0}\bar{b} + a_{1,1}\bar{b})l_{n,n-1} \tag{4.64}$$

$$B_{0Nt}^{0N1} = (a_{0,0}b + a_{0,1}b)l_{n,n} + (a_{1,0}b + a_{0,0}\bar{b} + a_{0,1}\bar{b} + a_{1,1}b)l_{n,n-1} + (a_{1,0}\bar{b} + a_{1,1}\bar{b})l_{n,n-2} \tag{4.65}$$

$$B_{0Nt}^{0N(t-1)} = (a_{0,0}b + a_{0,1}b)l_{n,n-1} + (a_{1,0}b + a_{0,0}\bar{b} + a_{0,1}\bar{b} + a_{1,1}b)l_{n,n-2} + (a_{1,0}\bar{b} + a_{1,1}\bar{b})l_{n,n-3} \tag{4.66}$$

$$B_{0N(t-1)}^{0Nt} = 0 \tag{4.67}$$

$$B_{0Nt}^{0Nt} = (a_{1,0}b + a_{0,0}b + a_{0,1}b)l_{t,0} \tag{4.68}$$

where

$$C = \begin{bmatrix} B_{100}^{000} & B_{110}^{000} & & & & \\ B_{100}^{001} & B_{110}^{001} & & & & \\ B_{100}^{002} & B_{110}^{002} & & & & \\ \vdots & \vdots & & & & \\ B_{100}^{00K} & B_{110}^{00K} & & & & \\ & C_{1,1} & C_{1,2} & & & \\ & & C_{1,1} & C_{1,2} & & \\ & & & \ddots & \ddots & \\ & & & & & C_{0,b} \end{bmatrix},$$

with

$$B_{100}^{00n} = a_{1,0}\bar{b}l_{n,n} \tag{4.69}$$

$$B_{110}^{00n} = a_{1,1}\bar{b}l_{n,n} \tag{4.70}$$

where

$$C_{1,1} = \begin{bmatrix} B_{110}^{010} & B_{110}^{011} & B_{110}^{012} & \cdots & B_{110}^{01t} \end{bmatrix}^{\mathrm{T}},$$

with

$$B_{110}^{01t} = a_{1,0}\bar{b}l_{n,n} \tag{4.71}$$

where

$$C_{1,2} = \begin{bmatrix} B_{120}^{010} & B_{120}^{011} & B_{120}^{012} & \cdots & B_{120}^{01t} \end{bmatrix}^{\mathrm{T}},$$

with

$$B_{120}^{01t} = a_{1,1}\bar{b}l_{n,n} \tag{4.72}$$

where

$$C_{0,b} = \begin{bmatrix} B_{1N0}^{0N0} & B_{1N0}^{0N1} & B_{1N0}^{0N2} & \cdots & B_{1N0}^{0Nt} \end{bmatrix}^{\mathrm{T}},$$

with

$$B_{1N0}^{0Nt} = (a_{1,0}\bar{b} + a_{1,1}\bar{b})l_{n,n} \tag{4.73}$$

where

$$E = \begin{bmatrix} E^1 & E^0 & & & \\ & E^1 & E^0 & & \\ & & \ddots & \ddots & \\ & & & E^1 & E^0 \\ & & & & E^b \end{bmatrix},$$

with

$$E^1 = \begin{bmatrix} a_{0,0}b & \cdots & 0 \end{bmatrix},$$
$$E^0 = \begin{bmatrix} a_{0,1}b & \cdots & 0 \end{bmatrix},$$
$$E^b = \begin{bmatrix} a_{0,0}b + a_{0,1}b & \cdots & 0 \end{bmatrix}$$

where

$$A_1 = \begin{bmatrix} A_1^1 & A_1^0 & & & \\ & A_1^1 & A_1^0 & & \\ & & A_1^1 & A_1^0 & \\ & & & \ddots & \ddots \\ & & & & A_1^1 + A_1^0 \end{bmatrix},$$

with

$$A_1^1 = a_{1,0}b + a_{0,0}\bar{b}, \tag{4.74}$$

$$A_1^0 = a_{1,1}b + a_{0,1}\bar{b}, \tag{4.75}$$

where

$$A_0 = \begin{bmatrix} A_0^1 & A_0^0 & & & \\ & A_0^1 & A_0^0 & & \\ & & A_0^1 & A_0^0 & \\ & & & \ddots & \ddots \\ & & & & A_0^1 + A_0^0 \end{bmatrix},$$

with

$$A_0^1 = a_{1,0}\bar{b}, \tag{4.76}$$

$$A_0^0 = a_{1,1}\bar{b}, \tag{4.77}$$

where

$$A_2 = \begin{bmatrix} A_2^1 & A_2^0 & & & \\ & A_2^1 & A_2^0 & & \\ & & A_2^1 & A_2^0 & \\ & & & \ddots & \ddots \\ & & & & A_2^1 + A_2^0 \end{bmatrix},$$

with

$$A_2^1 = a_{0,0}b, \tag{4.78}$$

$$A_2^0 = a_{0,1}b. \tag{4.79}$$

4.4.2 Performance measures

For a stable system the output, x is obtained such that

$$x = xP, \qquad x\mathbf{1} = 1, \tag{4.80}$$

where

$$x = [x_{000}, x_{001}, x_{002}, ..x_{00K}, x_{010}, x_{011}, x_{012}, ...x_{01K}, x_{020}, x_{021}, x_{022}, ...x_{0Nk}, x_{100}, x_{110}, ...x_{1N0}, \\ x_{200}, x_{210}, ...x_{2N0}, ...x_{MN0}]. \tag{4.81}$$

The performance measures are obtained using Equation 4.34 and Equation 4.35.

4.5 CONCLUSION

The WSN node is further investigated to observe the effect of leakage, priority and a threshold imposed on it. Three models are proposed to observe these aspects. In the models, a token is used by the packet before leakage occurs. The two models with leakage and priority incorporated are Geo/Geo/1 pre-emptive systems. In the model with threshold incorporated, there are three buffers, two for data packets (HP and LP) and one for energy tokens. To ensure that there are tokens in the system for transmission of HP packets, a threshold is imposed on the token buffer. When the number of tokens are above or equal to the threshold both the LP packets and HP packets are transmitted. Below the specified threshold only the HP packets are transmitted.

The models proposed are developed to illustrate a practical system with unused tokens leaking and transmission of emergency data and are categorised as a QBD processes that will enable the researcher to determine the performance measures by applying matrix-geometric methods. The performance measures obtained are for the system. The simulated results of the models developed in Chapters 3 and 4 are presented in Chapter 5.

CHAPTER 5 RESULTS AND DISCUSSION

5.1 CHAPTER OVERVIEW

After developing the models in Chapters 3 and 4 and the transition matrices, the proposed models are analysed in depth. Simulations are provided to assess the performance of the proposed models. To observe the behaviour of the queue in each of the four models, variations are applied.

The results presented are obtained subject to the following: a, the rate of arrival of the packets, b, the rate of arrival of the tokens, F, buffer of the packet and K, buffer of the token. For the priority model the following parameters are used: a_H, the rate of arrival of HP packets, a_L, the rate of arrival of LP packets, M, buffer for HP packets, N, buffer for LP packets.

The token and packet buffers have an inverse relationship. When one buffer is full, the other is empty. Furthermore, as the rate of arrival of the data packets in the system rise, the mean number of packets in the system increases and the mean number of tokens decreases.

The queueing results obtained in simulation are discussed and explanations are given. The simulated results are discussed and the effect of imposing a threshold on the queue is explained in detail.

5.2 BASIC MODEL

The model whose transition matrix was obtained in Chapter 3 is presented. Figure 5.1 and Figure 5.2 show the mean number of packets and tokens in the system and queue respectively when the probability of packet arrival is varied. The results presented subsequently are for the mean number of packets and tokens in the system. The mean number in the queue is less than the mean number in the system by 1.

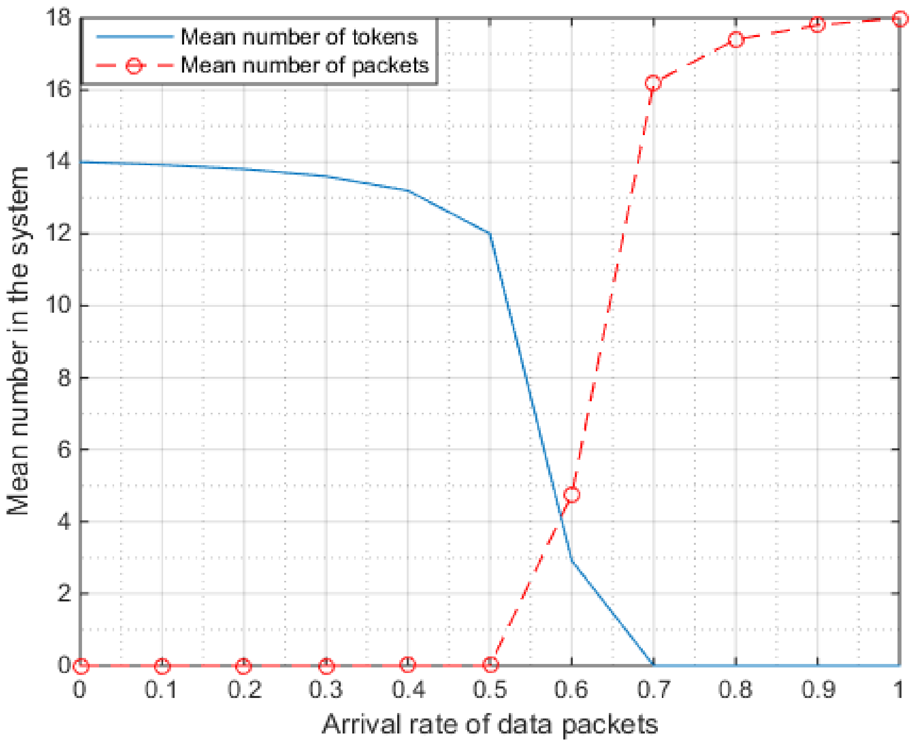

Figure 5.1. Effect of varying a on the system. Here $b = 0.6$, $F = 18$ and $K = 14$.

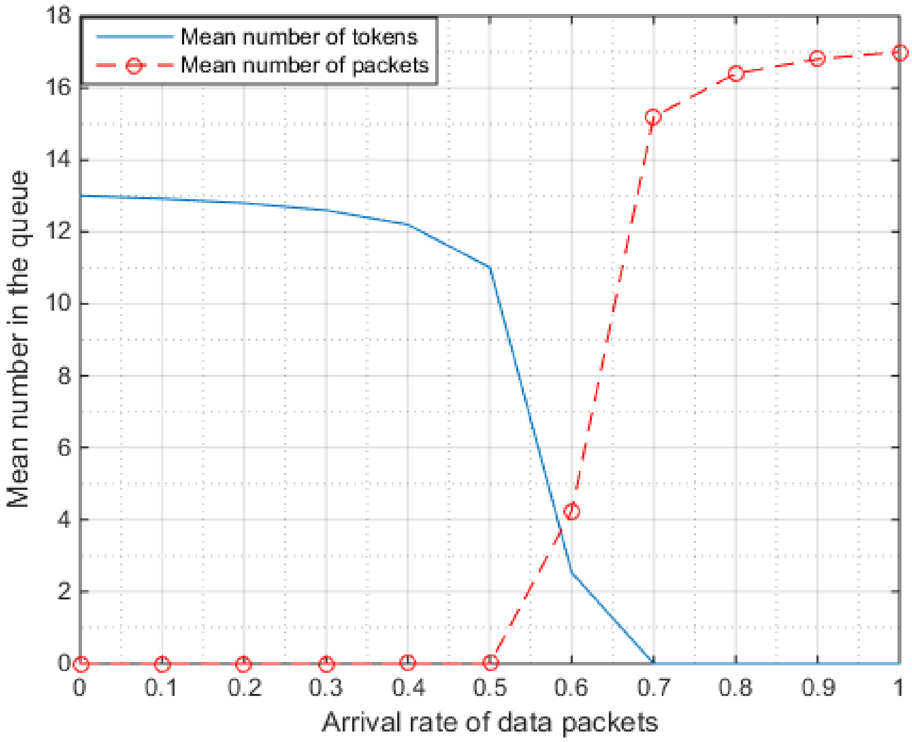

Figure 5.2. Effect of varying a on the queue. Here $b = 0.6$, $F = 18$ and $K = 14$.

Table 5.1 shows the results obtained for the arrival rate of tokens (b) of $0.4, 0.6$ as the arrival rate of the data packets (a) is varied. It is observed that a lower arrival rate of tokens results in a faster increase in the mean number of packets in the system. At $a = 0.5$ and $b = 0.4$, the mean number of packets in the system is 16 and at $a = 0.5$ and $b = 0.6$, the mean number of packets in the system is 0.002.

Table 5.1. Comparison of results when a is varied in the system and b is kept constant at two values.

Arrival rate of data packets	Arrival rate of tokens $b = 0.4$		Arrival rate of tokens $b = 0.6$	
	Mean number of packets	Mean number of tokens	Mean number of packets	Mean number of tokens
0	0	14	0	14
0.1	9.14×10^{-14}	13.8	2.47×10^{-14}	13.92
0.2	3.44×10^{-8}	13.4	2.51×10^{-14}	13.8
0.3	9.82×10^{-4}	12.2	2.25×10^{-10}	13.6
0.4	4.75	2.92	8.24×10^{-7}	13.2
0.5	16.0	3.93×10^{-4}	0.002	12
0.6	17.2	3.22×10^{-8}	4.75	2.917
0.7	17.6	1.5×10^{-12}	16.2	1.66×10^{-4}
0.8	17.8	6.15×10^{-16}	17.4	6.8×10^{-10}
0.9	17.92	1.89×10^{-17}	17.8	1.31×10^{-15}
1	18	0	18	0

Figure 5.3 shows the expected mean number of tokens and packets in the system as the arrival rate of tokens is varied.

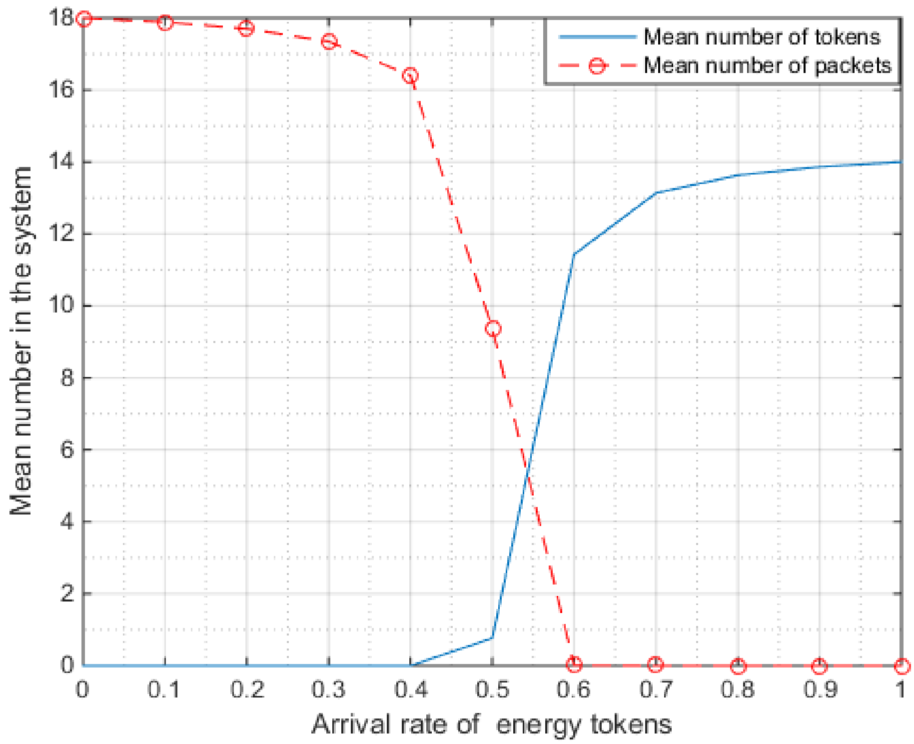

Figure 5.3. Effect of varying b. Here $a = 0.4$, $F = 18$ and $K = 14$.

Table 5.2 shows the results obtained for the arrival rate of packets a of $0.4, 0.52$ as the arrival rate of the tokens (b) is varied. It is observed that a lower arrival rate of packets results in a faster increase in the mean number of tokens in the system. At $b = 0.5$ and $a = 0.4$, the mean number of tokens in the system is 12 and at $b = 0.5$ and $a = 0.52$, the mean number of packets in the system is 0.764.

Table 5.2. Comparison of results obtained when b is varied and a is kept constant at two values.

Arrival rate of tokens	Arrival rate of packets $a = 0.4$		Arrival rate of tokens $a = 0.52$	
	Mean number of packets	Mean number of tokens	Mean number of packets	Mean number of tokens
0	18	0	18	0
0.1	17.8	5.84×10^{-16}	17.9	2.14×10^{-15}
0.2	17.4	6.81×10^{-10}	17.7	1.36×10^{-14}
0.3	16.2	1.66×10^{-4}	17.35	2.83×10^{-9}
0.4	4.75	2.92	16.4	5.90×10^{-5}
0.5	0.002	12	9.37	0.76
0.6	8.24×10^{-7}	13.2	0.01 5	11.43
0.7	2.25×10^{-10}	13.6	1.876×10^{-6}	13.13
0.8	2.23×10^{-14}	1.8	8.43×10^{-11}	13.63
0.9	1.88×10^{-14}	13.92	2.73×10^{-14}	13.86
1	0	14	0	14

5.3 MODEL WITH LEAKAGE INCORPORATED

In this section, the results for the model that was developed in Section 4.2 are presented. Figure 5.4 shows the mean number of tokens and packets in the system as the arrival rate of the packets is varied.

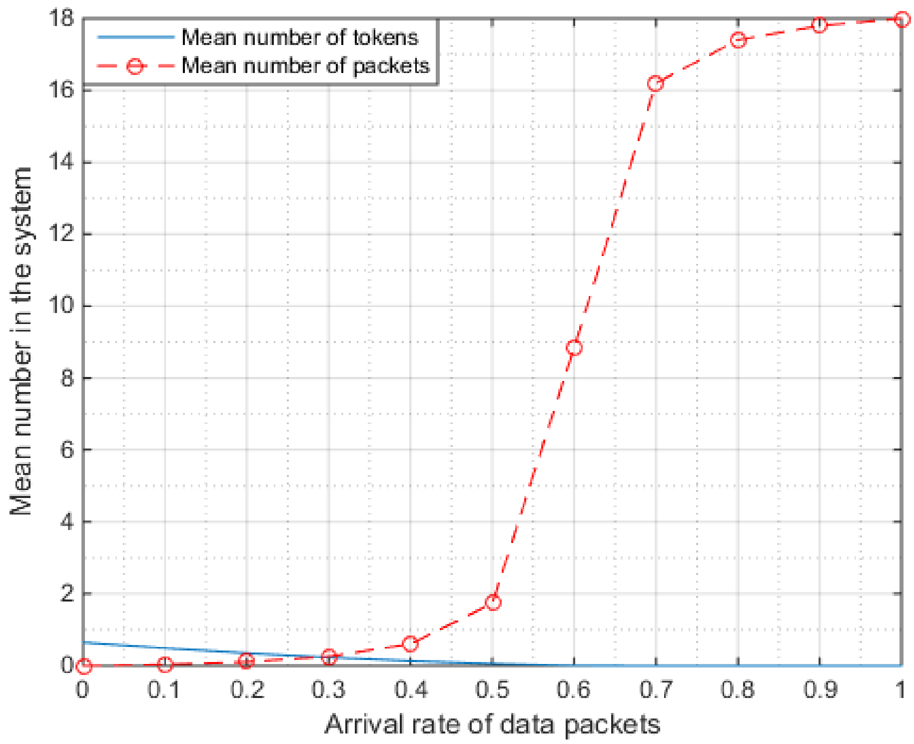

Figure 5.4. Effect of varying a. Here $b = 0.6$, $\theta = 0.4$, $F = 18$ and $K = 14$.

Table 5.3 shows the results obtained for the rate of leakage (θ) of 0.2 and 0.4 as the arrival rate of the packets (a) is varied.

Table 5.3. Comparison of results obtained when a is varied, b is 0.6 and θ is kept constant at two values.

Arrival rate of data packets	Rate of leakage $\theta = 0.2$		Rate of leakage $\theta = 0.4$	
	Mean number of packets	Mean number of tokens	Mean number of packets	Mean number of tokens
0	5.703×10^{-14}	2.024	1.00×10^{-14}	0.643
0.1	0.009	1.57	0.035	0.487
0.2	0.042	1.15	0.107	0.35
0.3	0.135	0.774	0.256	0.231
0.4	0.408	0.451	0.6	0.133
0.5	1.453	0.191	1.735	0.056
0.6	8.675	0.019	8.851	0.006
0.7	16.2	1.99×10^{-5}	16.2	6.75×10^{-6}
0.8	17.4	7.43×10^{-10}	17.4	2.86×10^{-10}
0.9	17.8	1.51×10^{-15}	17.8	1.37×10^{-15}
1	18	0	18	0

Figure 5.5 shows the mean number of tokens and packets in the system as the arrival rate of the tokens is varied.

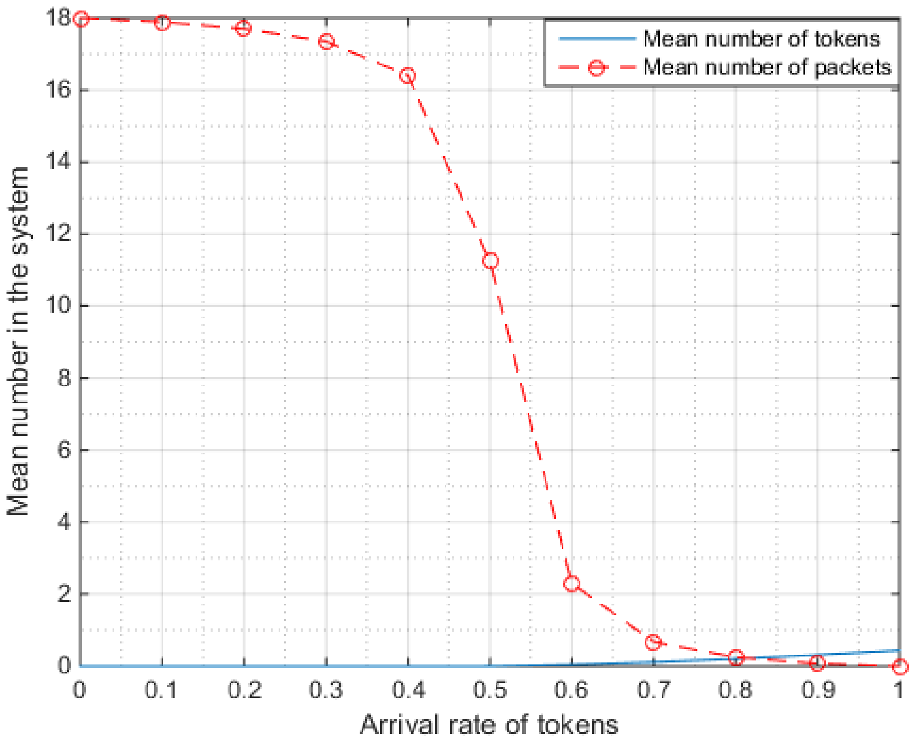

Figure 5.5. Effect of varying b. Here $a = 0.52$, $\theta = 0.4$, $F = 18$ and $K = 14$.

Table 5.4 shows the results obtained for the rate of leakage (θ) of 0.2 and 0.4 as the arrival rate of the tokens (b) is varied. From the results it is observed that a lower rate of leakage results in a higher mean number of tokens in the system and a lower mean number of packets. At $b = 0.5$ and $\theta = 0.4$, the mean number of tokens in the system is 0.0024 and the mean number of packets in the system is 11.232. At $b = 0.5$ and $\theta = 0.2$, the mean number of packets in the system is 0.212 and the mean number of tokens in the system is 0.224 .

Table 5.4. Comparison of results obtained when b is varied, a is 0.52 and θ is kept constant at two values.

Arrival rate of data packets	Rate of leakage $\theta = 0.2$		Rate of leakage $\theta = 0.4$	
	Mean number of packets	Mean number of tokens	Mean number of packets	Mean number of tokens
0	18	0	18	0
0.1	17.2	2.93×10^{-9}	17.9	6.97×10^{-14}
0.2	8.89	0.003	17.7	3.16×10^{-14}
0.3	1.21	0.047	17.35	9.94×10^{-10}
0.4	0.448	0.123	16.4	3.61×10^{-6}
0.5	0.212	0.224	11.23	0.002
0.6	0.107	0.349	2.29 5	0.04
0.7	0.054	0.497	0.67	0.113
0.8	0.024	0.663	0.244	0.202
0.9	0.008	0.847	0.075	0.309
1	2.043×10^{-14}	1.047	2.73×10^{-14}	0.434

Figure 5.6 shows the mean number of tokens and packets in the system as the rate of leakage is varied.

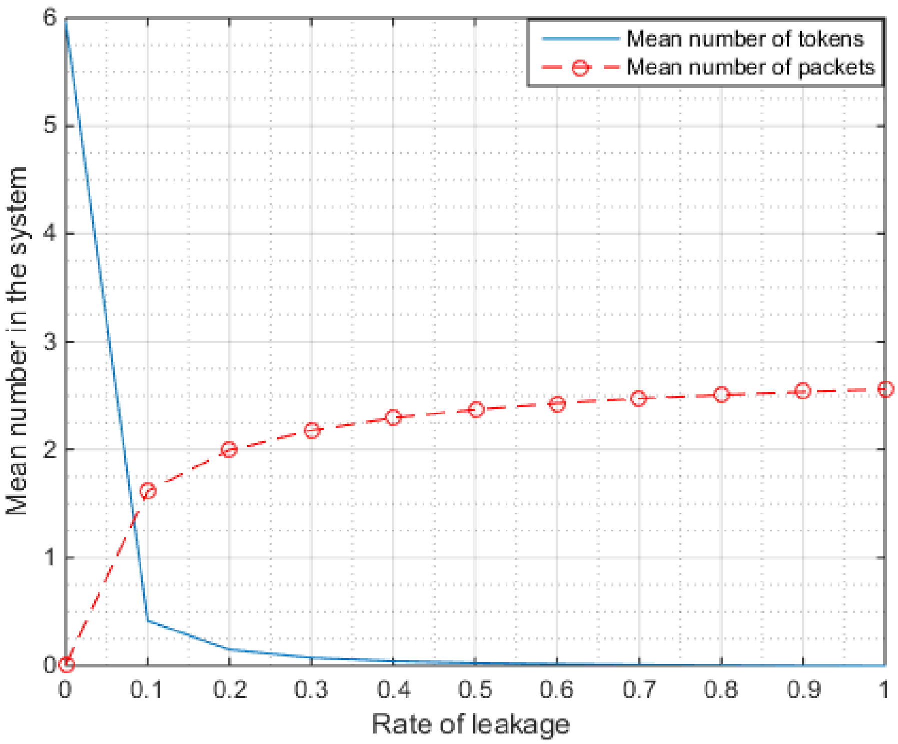

Figure 5.6. Effect of varying θ. Here $a = 0.52$, $b = 0.6$, $F = 18$ and $K = 14$.

Figure 5.7 shows the probability of the packet buffer being full as the rate of leakage is varied. It is observed that the probability of the packet buffer being full increases with the rate of leakage.

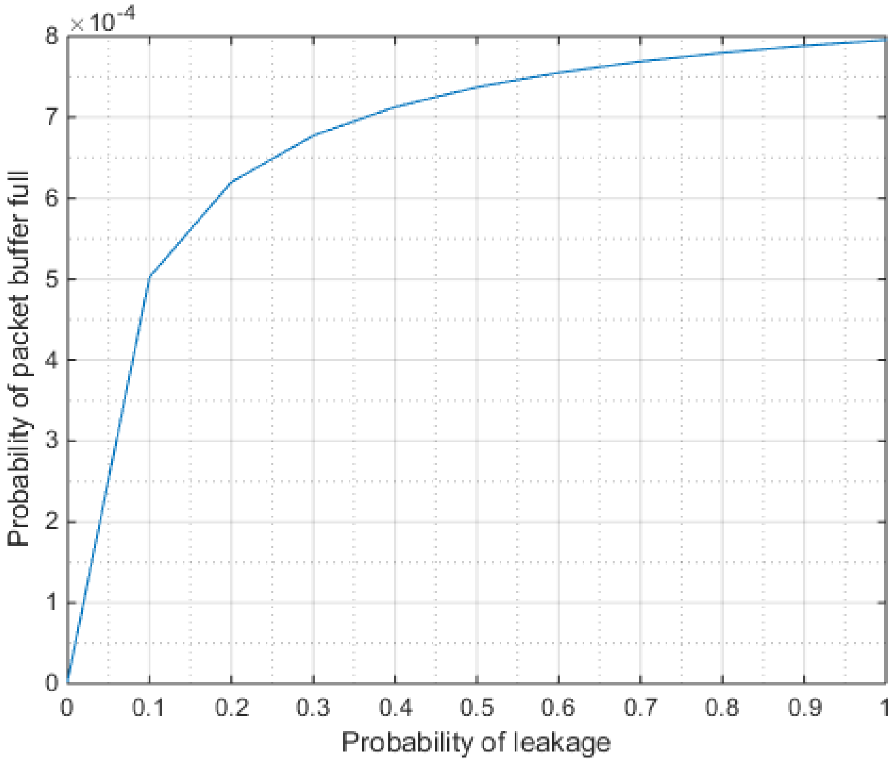

Figure 5.7. Probability of the data packet being full. Here θ is varied, $a = 0.52$, $b = 0.6$, $F = 18$ and $K = 14$.

5.4 MODEL WITH LEAKAGE AND PRIORITY INCORPORATED

In this section the results for the model that was developed in Section 4.3 are presented. Figure 5.8 shows the effect of varying the arrival rate of HP packets in the system when all the other parameters are kept constant.

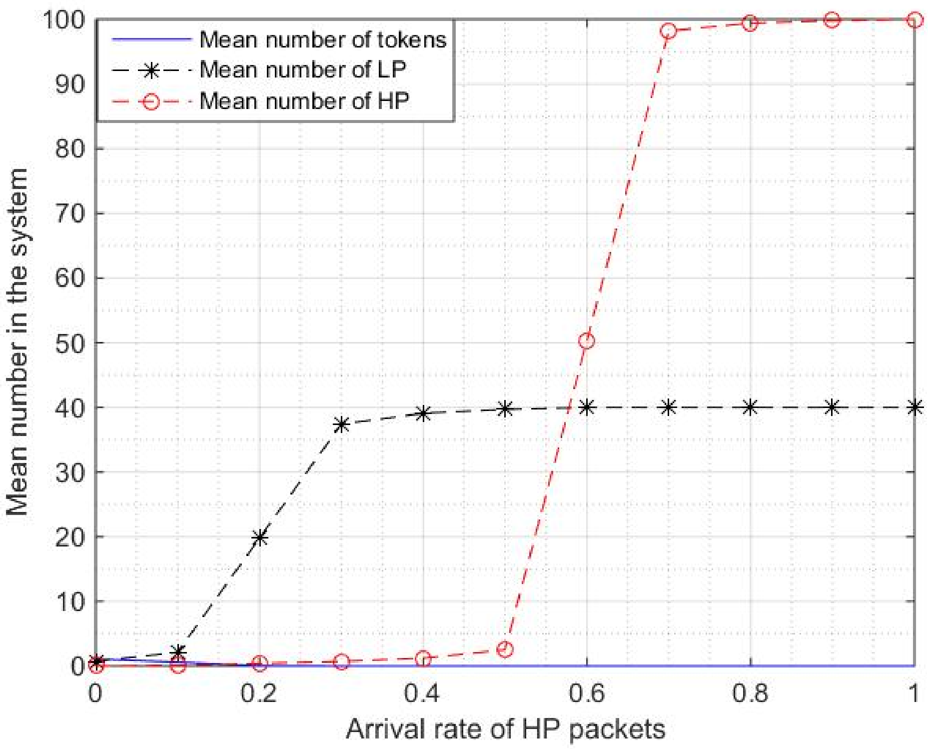

Figure 5.8. Effect of varying a_H. Here $a_L = 0.4$, $b = 0.6$, $\theta = 0.4$, $M = 100$, $N = 40$ and $K = 48$.

In Figure 5.9, the impact of the arrival rate of the energy tokens on the mean number of tokens, LP packets and HP packets in the system for the model including leakage and priority is investigated.

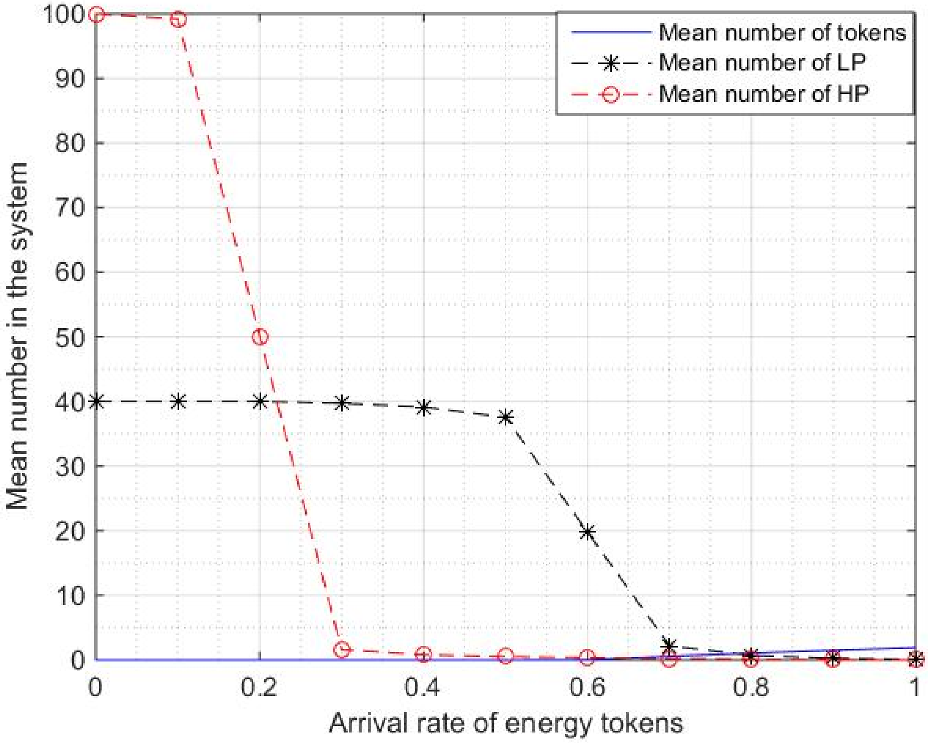

Figure 5.9. Effect of varying b. Here $a_H = 0.2$, $a_L = 0.4$, $\theta = 0.4$, $M = 100$, $N = 40$ and $K = 48$.

In Figure 5.10 the impact of varying the rate of the leakage on the mean number of tokens, LP packets and HP packets in the system is shown. As expected, the results show that the mean number of LP and HP packets increases with an increase in the rate of leakage. Leakage and usage by the HP and LP packets contribute to a decrease in the mean number of tokens in the system.

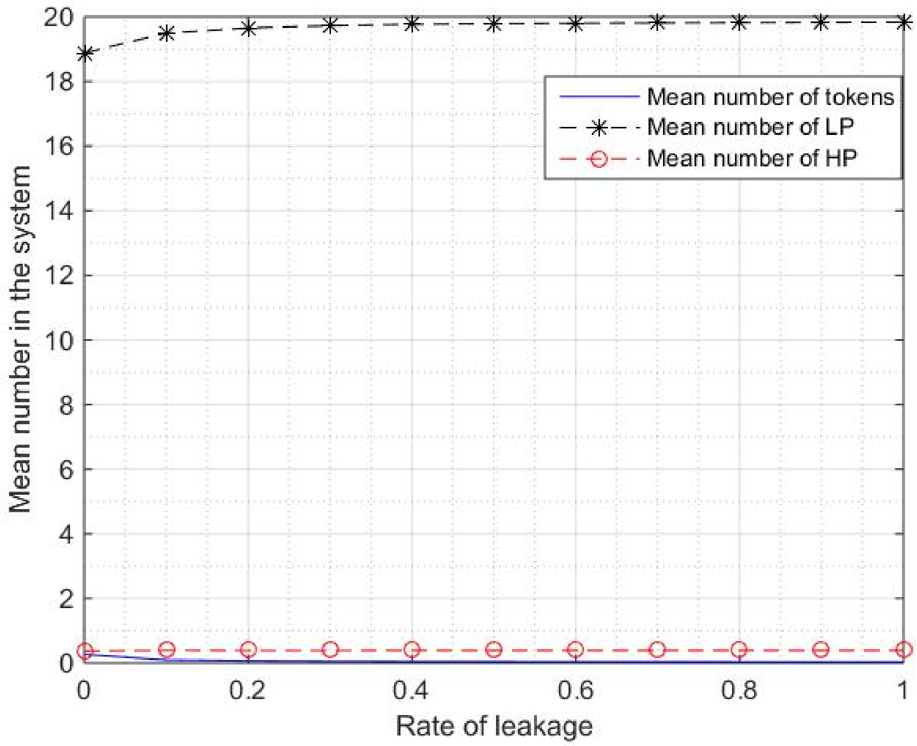

Figure 5.10. Effect of varying θ. Here $a_H = 0.2$, $a_L = 0.4$, $b = 0.6$, $M = 50$, $N = 20$ and $K = 24$.

5.5 MODEL WITH LEAKAGE, PRIORITY AND THRESHOLD INCORPORATED

In this section the results for the model with threshold incorporated are presented. The model was developed in Section 4.4.

In Figure 5.11 the impact of imposing a threshold and varying the HP packet arrival rate on the mean number of tokens, LP packets and HP packets in the system is investigated. As expected, the mean number of HP and LP packets in the system increases with an increase in the arrival rate of HP packets and the mean number of tokens decreases.

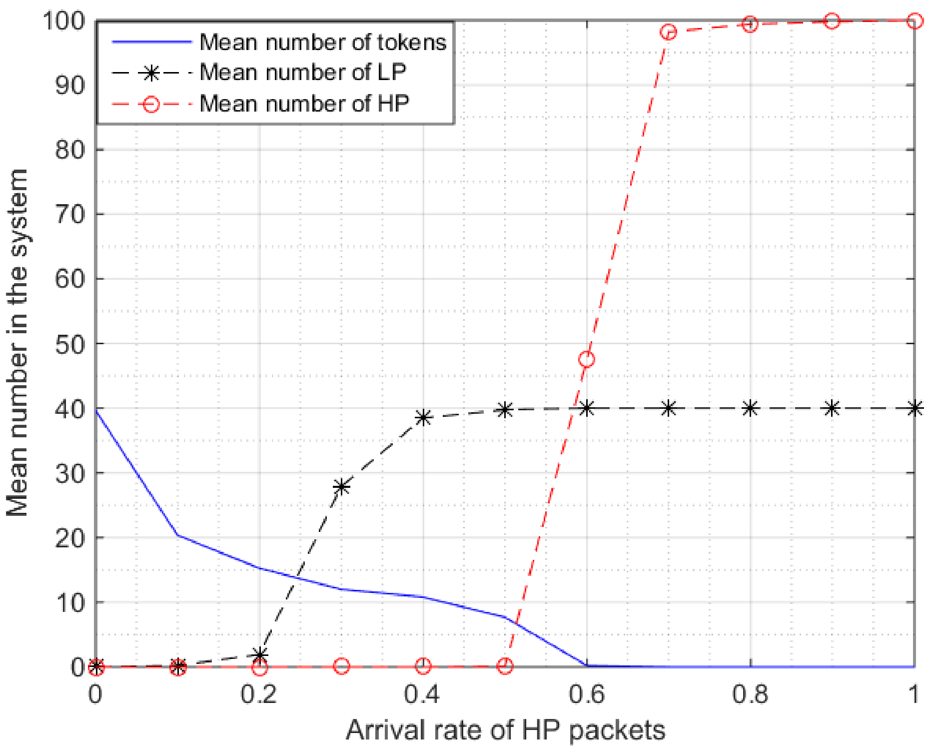

Figure 5.11. Effect of varying a_H. Here $a_L = 0.2$, $b = 0.6$, $\theta = 0.01$, $M = 100$, $N = 40$, $K = 48$ and threshold = 15.

In Figure 5.12 the effect of imposing a threshold and varying the tokens arrival rate on the mean number of tokens, LP packets and HP packets in the system is shown.

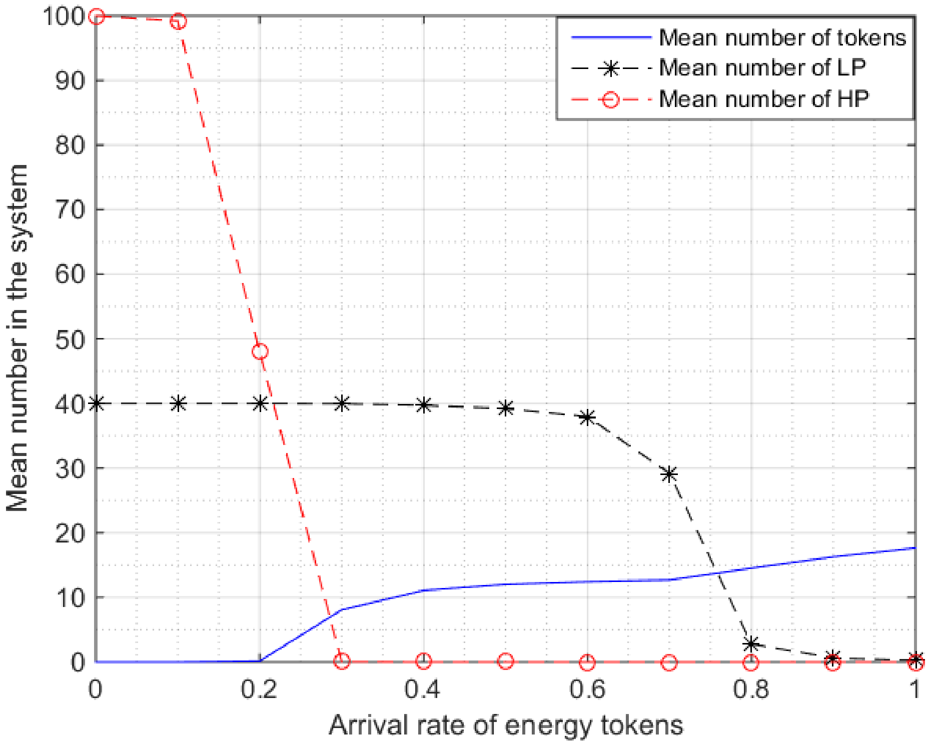

Figure 5.12. Effect of varying b. Here $a_H = 0.2$, $a_L = 0.4$, $\theta = 0.01$, $M = 100$, $N = 40$, $K = 48$ and threshold = 15.

In Figure 5.13 the effect of imposing a threshold and varying the leakage rate on the mean number of tokens, LP packets and HP packets in the system is shown. As expected, an increment in the rate of leakage results in a decrease in the mean number of tokens and an increment in the mean number of HP and LP packets. The decrease in the mean number of tokens in the system is attributed to a combination of usage by the packets and leakage.

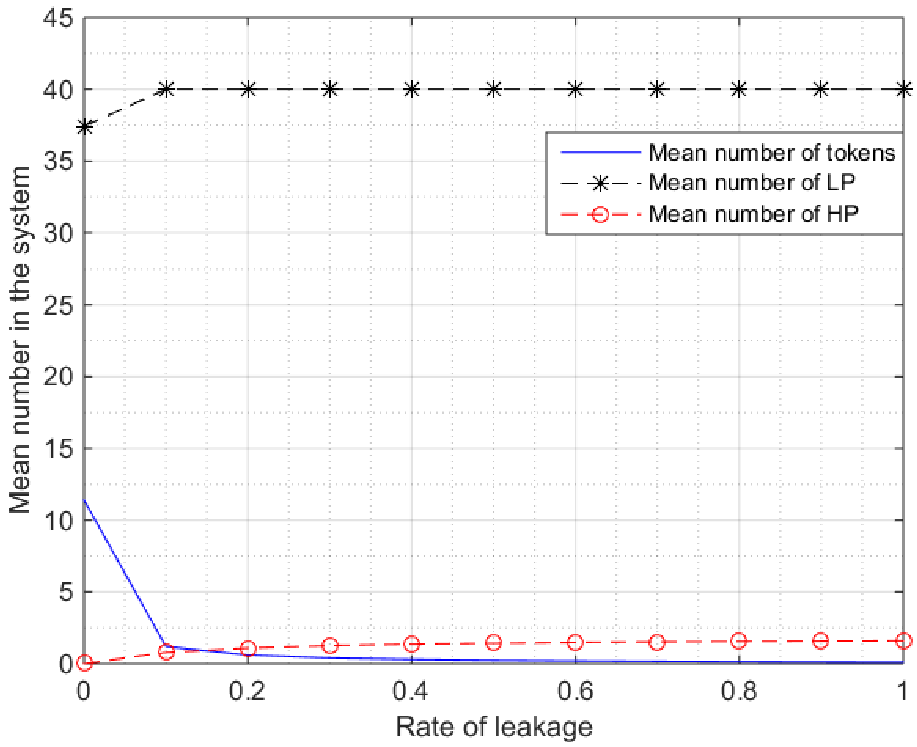

Figure 5.13. Effect of varying θ. Here $a_H = 0.4$, $a_L = 0.2$, $b = 0.52$, $M = 100$, $N = 40$, $K = 48$ and threshold = 15.

In Figure 5.14 and Figure 5.15, the effect of increasing the threshold is shown. At a threshold of 15 with a rate of leakage of 0.01, hardly any LP packets are transmitted and the graph showing the mean number of LP packets in the system appears to be constant. A decrease in threshold with the same rate of leakage will imply that there are tokens available for transmission of LP packets, as shown in Figure 5.15.

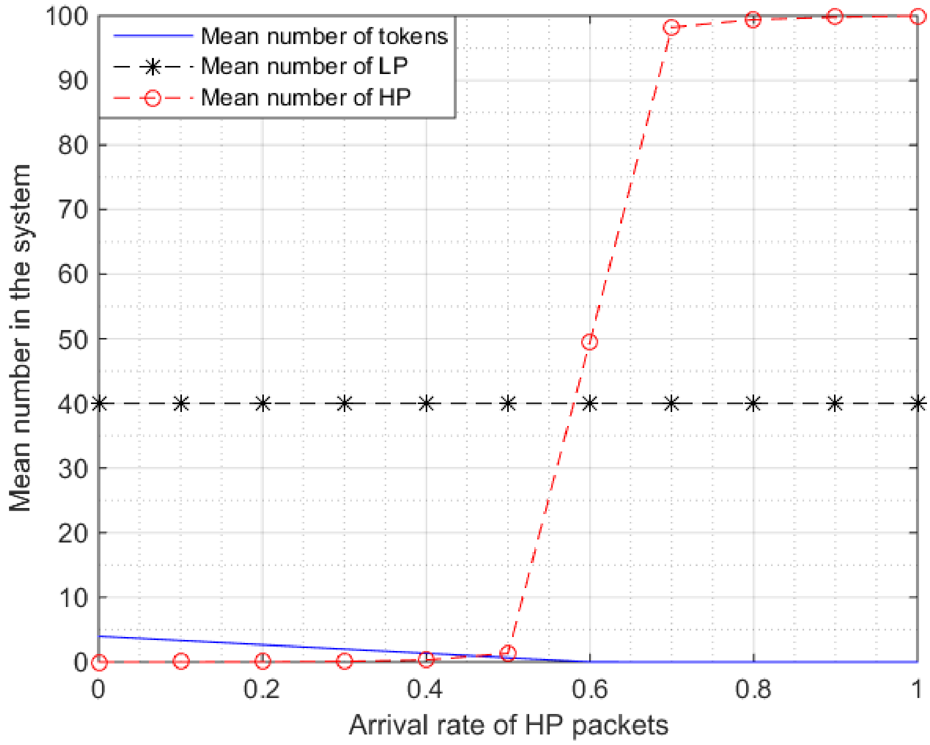

Figure 5.14. Effect of varying a_H. Here $a_L = 0.2$, $b = 0.6$, $\theta = 0.15$, $M = 100$, $N = 40$, $K = 48$ and threshold = 15.

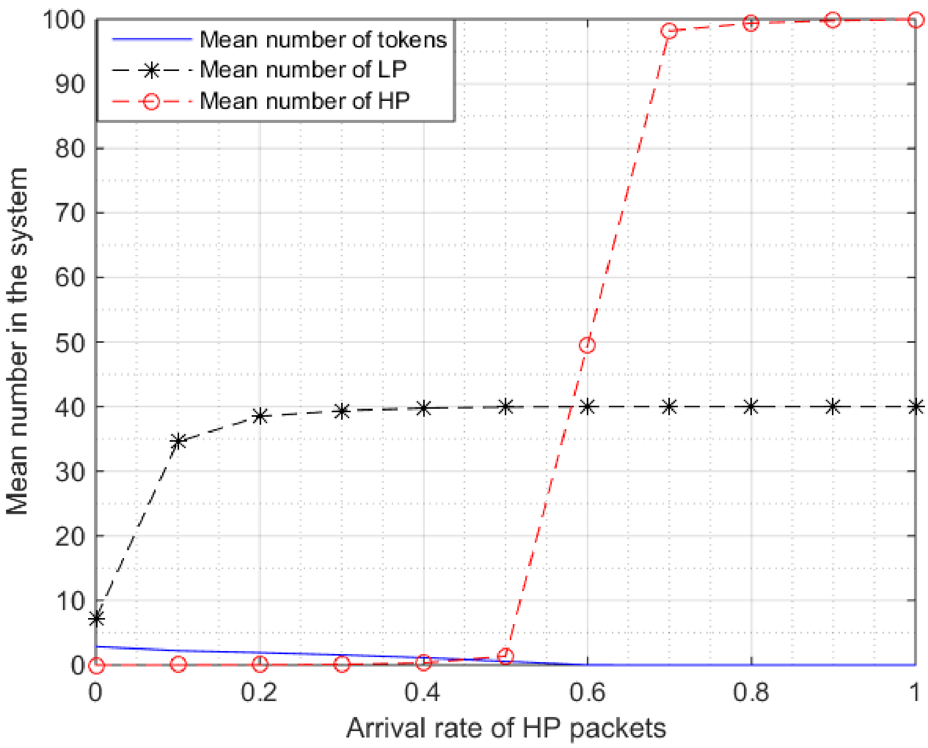

Figure 5.15. Effect of varying a_H. Here $a_L = 0.2$, $b = 0.6$, $\theta = 0.15$, $M = 100$, $N = 40$, $K = 48$ and threshold = 5.

When the threshold is kept at 5 and the rate of leakage is reduced to 0.05, the results in Figure 5.16 are obtained. As observed, a decrease in leakage results in a decrease in the mean number of LP packets in the system. The mean number of tokens at 0 is also higher than in Figure 5.15.

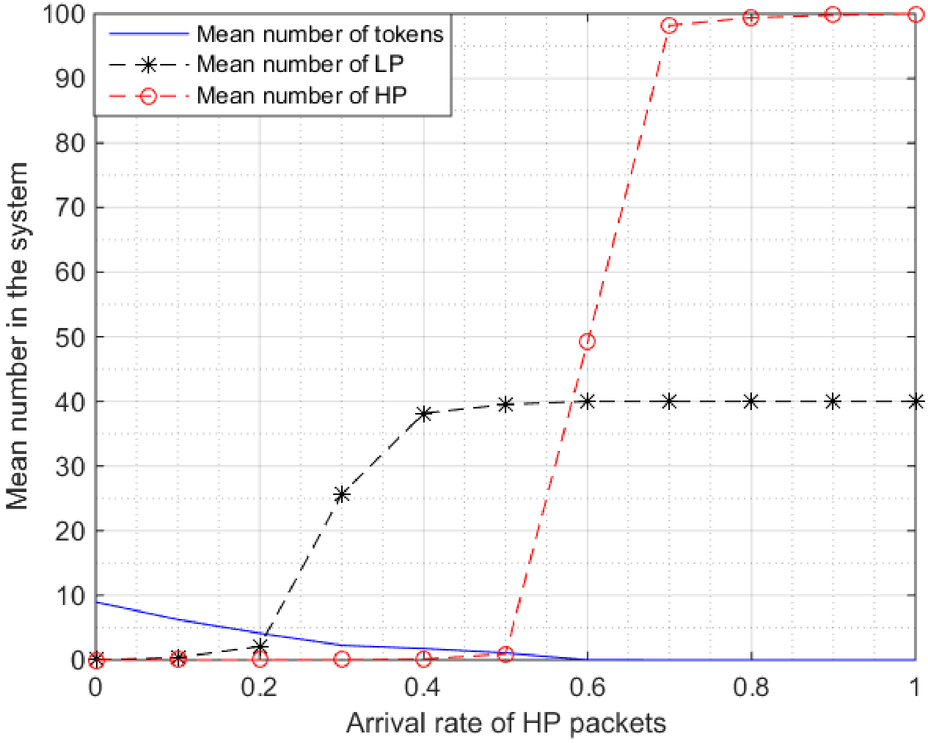

Figure 5.16. Effect of varying a_H. Here $a_L = 0.2$, $b = 0.6$, $\theta = 0.05$, $M = 100$, $N = 40$, $K = 48$ and threshold = 5.

In Table 5.5, the arrival rate of HP packets is varied at two different thresholds. It is observed that an increase in the threshold results in an increase in the mean number of LP packets in the system. Since the HP packets are always transmitted, the effect of the threshold on the mean number of the HP packets is hardly noticeable.

Increasing the threshold imposed on the token buffer, will result in fewer LP packets being transmitted. Since the HP packets are always transmitted, an increase in the threshold will have little to no effect on the mean number of HP packets in the system, as shown in the Table 5.5.

Table 5.5. Effect of varying a_H. Here $a_L = 0.2$, $b = 0.6$, $\theta = 0.05$, $M = 100$, $N = 40$, $K = 48$, threshold = 5, threshold = 10 and threshold = 15.

	threshold = 5		*threshold* = 10		*threshold* = 15	
Arrival rate of HP packets	Mean number of HP packets	Mean number of LP packets	Mean number of HP packets	Mean number LP packets	Mean number of HP packets	Mean number LP packets
0	0	0	0	0	0	0
0.1	0	0.0732	0	0.1216	0	0.2358
0.2	0.0004	0.3413	0	0.7199	0	1.9162
0.3	0.0078	2.0039	0.0001	7.9085	0	27.9068
0.4	0.0884	29.1149	0.0056	36.6219	0.0009	38.5045
0.5	0.6731	38.7473	0.1971	39.413	0.1073	39.7742
0.6	48.7302	39.9841	47.6821	39.9955	47.456	39.9994
0.7	98.2	40	98.2	40	98.2	40
0.8	99.4	40	99.4	40	99.4	40
0.9	99.8	40	99.8	40	99.8	40
1	100	40	100	40	100	40

The results in Table 5.6 are obtained by keeping all the system parameters constant and varying the threshold. From the results obtained it is observed that increasing the threshold beyond 10 has no impact on the mean number of HP packets in the system, because HP packets are in constant transmission. On the other hand, at a threshold of 15, the mean number of LP packets in the system is at full capacity, as the tokens in the system either leak or are used by the HP packets.

Table 5.6. Effect of varying the threshold. Here, $a_L = 0.2$, $a_H = 0.4$, $b = 0.6$, $\theta = 0.15$, $M = 100$, $N = 40$, $K = 48$.

Threshold	Mean number of HP packets	Mean number of LP packets
0	1.1695	19.5443
5	0.3474	39.7652
10	0.3269	39.9999
15	0.3269	40
20	0.3269	40
25	0.3269	40
30	0.3269	40

The effect of varying the rate of leakage and threshold is shown in the results presented in Table 5.7 and Table 5.8.

Table 5.7. Effect of varying the threshold. Here, $a_L = 0.2$, $a_H = 0.4$, $b = 0.6$, $\theta = 0.05$, $M = 100$, $N = 40$, $K = 48$.

Threshold	Mean number of HP packets	Mean number of LP packets
5	0.1681	38.1741
10	0.0825	39.9015
15	0.0791	39.9988
20	0.0791	40
25	0.0791	40
30	0.0791	40

Table 5.8. Effect of varying the threshold. Here, $a_L = 0.2$, $a_H = 0.4$, $b = 0.6$, $\theta = 0.01$, $M = 100$, $N = 40$, $K = 48$.

Threshold	Mean number of HP packets	Mean number of LP packets
5	0.0884	29.1149
10	0.0056	36.6219
15	0.0009	38.5046
20	0.29×10^{-3}	39.3144
25	0.1632×10^{-3}	39.7270
30	0.122×10^{-3}	39.9845

The results presented in Table 5.9 and Table 5.10 show the effect of changing the constant parameters (the arrival rates of the HP and LP packets).

Table 5.9. Effect of varying the threshold. Here, $a_L = 0.2$, $a_H = 0.4$, $b = 0.4$, $\theta = 0.01$, $M = 100$, $N = 40$, $K = 48$.

Threshold	Mean number of HP packets	Mean number of LP packets
5	48.7302	39.841
10	47.6821	39.9955
15	47.4560	39.9994
20	47.4317	40
25	47.4303	40
30	47.4302	40

Table 5.10. Effect of varying the threshold. Here, $a_L = 0.6$, $a_H = 0.4$, $b = 0.4$, $\theta = 0.01$, $M = 100$, $N = 40$, $K = 48$.

Threshold	Mean number of HP packets	Mean number of LP packets
5	48.7302	39.9982
10	47.6821	39.9994
15	47.4560	39.9999
20	47.417	40
25	47.4303	40
30	47.4302	40

5.6 COMPARISON

To illustrate the effect of imposing a threshold on the energy buffer, the results obtained from the basic model with leakage and priority incorporated and the basic model with leakage, priority and threshold incorporated are given in Table 5.11 and Table 5.12.

Table 5.11. Effect of varying a_H. Here $a_L = 0.2$, $b = 0.6$, $\theta = 0.15$, $M = 100$, $N = 40$, $K = 48$ and threshold = 5.

	Model with no threshold incorporated		Model with threshold incorporated	
Arrival rate of HP packets	Mean number of HP packets	Mean number of LP packets	Mean number of HP packets	Mean number LP packets
0	0.0092	0.0727	0	7.2348
0.1	0.3165	0.4606	0.006	34.5768
0.2	0.39	0.7159	0.0309	38.505
0.3	0.5618	1.7686	0.1068	39.3992
0.4	1.1695	19.5443	0.3474	39.7652
0.5	2.5	38.2542	1.3468	39.9324
0.6	50.2988	39.9775	49.559	39.9992
0.7	98.2	40	98.2	40
0.8	99.4	40	99.4	40
0.9	99.8	40	99.8	40
1	100	40	100	40

Table 5.11 and Table 5.12 show that imposing a threshold on the model has an impact on the mean number of packets in the system and one is therefore able to exert some control over the system. This means that the HP packets, which are the emergency packets, will always be transmitted.

Table 5.12. Effect of varying b. Here $a_H = 0.2$, $a_L = 0.4$, $\theta = 0.15$, $M = 100$, $N = 40$, $K = 48$ and threshold=5.

	Model with no threshold incorporated		Model with threshold incorporated	
Arrival rate of energy tokens	Mean number of HP packets	Mean number of LP packets	Mean number of HP packets	Mean number LP packets
0	100	40	100	40
0.1	99.2	40	99.2	40
0.2	50.0992	39.9965	49.6711	39.9999
0.3	1.6	39.7135	0.907	39.9848
0.4	0.8	39.1576	0.2456	39.9389
0.5	0.5333	37.5313	0.0846	39.8477
0.6	0.3912	19.5903	0.0309	39.6762
0.7	0.2682	1.9691	0.0111	39.3791
0.8	0.2564	0.7099	0.0036	38.7974
0.9	0.2596	0.4053	8.4532×10^{-4}	37.2292
1	0.003	0.0196	0	25.1935

5.7 DISCUSSION

During the development of the models, assumptions of some parameter values were made, namely for transmission of a packet a token is required, a packet is transmitted before a token leaks and finally the transmission of an HP packet takes priority transmission over that of an LP packet.

For the model with a threshold imposed on the buffer, the following assumptions are considered: When the tokens are equal to or more than the threshold, both HP packets and LP packets are transmitted. Below the specified threshold no LP packets are transmitted. The existing tokens are either reserved for HP packets that may arrive or are lost owing to leakage. The models were developed in discrete time. The arrival and service processes were modelled as geometric.

5.7.1 Basic model

The basic model is the foundation of all the models. As the name states, it has only tokens and packets. The results are presented in Section 5.2. The results for both the queue and system are presented. The mean number of tokens and packets in the system is higher than the mean number of tokens and packets in the queue by one. Further in the analysis of the models, only the results for the mean number of tokens and packets in the system are presented.

The simulated results of the basic model are as expected. The mean number of data packets in the system increases with a decrease in the mean number of tokens in the system as the arrival rate of data packets increases from 0 to 1, as depicted in Figure 5.1. The tokens are essential in transmitting the packets. The reverse is true for when the arrival rate of tokens is varied from 0 to 1. The results for this model are given in Figure 5.1, Figure 5.2 and Figure 5.3.

In Table 5.1 and Table 5.2 the results of the mean number of tokens and packets are presented. The results presented in Table 5.1 are obtained by varying the arrival rate of data packets and using two arrival rates of tokens. A higher arrival rate of tokens results in a reduction in the mean number of packets in the system. An arrival rate of 0.6 of data packets with an arrival rate of 0.4 for the packets results in a mean number of packets of 17.2, while an arrival rate of 0.6 of data packets with an arrival rate of 0.6 for the packets results in a mean number of packets of 4.75.

5.7.2 Model with leakage incorporated

To ensure that a practical system is captured in the model, leakage was incorporated in the basic model. The results of this model are given in Figure 5.4, Figure 5.5, Figure 5.6 and Figure 5.7.

Introducing leakage to the system significantly affects the mean number of tokens in the system. This is mainly because the tokens that are not used by the packets leak. This is depicted in Figure 5.5.

As expected, the mean number of tokens decreases with an increase in the rate of leakage. Usage by the data packets also contributes to the decrease in the mean number of tokens, as depicted in Figure 5.6. However, the packet buffer is not at maximum capacity when the rate of leakage is 1. This is because the mean number of tokens increases exponentially. The probability of the packet buffer being full increases with the probability of leakage.

5.7.3 Model with leakage and priority incorporated

In addition to leakage, priority was incorporated in the model. Priority caters for emergency data received by the system that has to be transmitted immediately. The results for this model are given in Figure 5.8, Figure 5.9 and Figure 5.10. Only the results for varying the arrival rate of HP packets are presented, as the model is a Geo/Geo/1 pre-emptive system and therefore, transmission of HP packets is the major focus.

An increase in the arrival rate of HP packets results in an increase in the mean number of both the HP packets and the tokens in the system. Since this is a pre-emptive system, the HP packets are given priority when transmitting data. In the absence of HP packets, LP packets are transmitted. However, an increase in the arrival rate of HP packets implies that there are more HP packets in the system and therefore transmission of LP packets is halted. This is shown in Figure 5.8.

On the other hand, an increase in the arrival rate of tokens implies that there are more tokens in the system available for transmission of both packets. The mean number of both HP packets and LP packets decreases. In this case, the rate of leakage is kept constant. This is depicted in Figure 5.9.

Varying the rate of leakage in the system significantly affects the mean number of tokens in the system, as shown in Figure 5.10. As observed, the LP packet buffer is almost at full capacity, while the HP packet buffer is not. Keeping in mind that a packet is transmitted before a token leaks, one observes that there is a very slight increase in the mean number of HP packets in the system.

5.7.4 Model with leakage, priority and threshold incorporated

To ensure that the researcher has some level of control, a threshold is imposed on the system. This is the final model and is a combination of the model with leakage and priority. The results of this model are given in Figure 5.11, Figure 5.12 and Figure 5.13.

In Figure 5.11 the effect of imposing a threshold while varying the arrival rate of HP packets is shown. The LP packet reaches maximum capacity at a faster rate after the threshold is imposed. The mean number of LP packets dramatically increases to 35 when the arrival rate of HP packets is increased from 0.2 to 0.4. In Figure 5.12 the effect of varying the arrival rate of tokens is shown. These results are the inverse of the results presented in Figure 5.11.

In Figure 5.13, there is a sharp increase in the mean number of LP packets in the system and the LP packet buffer quickly reaches maximum capacity. This is expected, as the available tokens are reserved for the HP packets. This shows that the threshold imposed grants some level of control with regard to transmission of LP packets. The HP packet buffer never attains full capacity as the HP packets are constantly transmitted.

In order to observe the effect of the threshold imposed on the token buffer, the rate of leakage of tokens is kept small. A high probability of leakage implies that there will be fewer tokens in the system and therefore the threshold value will be reached quickly, resulting in an insignificant transmission of LP packets. This is illustrated in Figure 5.14, Figure 5.15 and Figure 5.16. A decrease in leakage will result in more tokens being available in the system and therefore more LP packets will be transmitted. Figure 5.14 can be compared to Figure 5.11. One observes that an increase in the rate of leakage results in fewer LP packets being transmitted. In Figure 5.15, the LP packet buffer reaches full capacity much faster than in Figure 5.16.

To illustrate the effect of changing the threshold, the results in Table 5.5 are presented. The threshold is imposed to ensure that there are always tokens available in the system for HP packet transmission. Therefore, the parameters affected by the threshold are the mean number of HP and LP packets in the system. An increase in the threshold implies that there will be fewer tokens available in the system for the transmission of LP packets. This contributes to the increase in the mean number of LP packets in the system when the threshold is increased, as shown in Table 5.5.

The results in Table 5.6, Table 5.7 and Table 5.8 show the effect of changing the rate of leakage on the threshold. In Table 5.6 the leakage is 0.15, while in Table 5.7, the leakage is 0.05 and all the parameters are kept constant. At a threshold of 10 and a rate of leakage of 0.05, one observes that the mean number of HP packets in the system is 0.0791, which is much lower than in the system with a threshold of 10 and a rate of leakage of 0.15. In Table 5.8, the leakage is reduced further to a value of 0.01. Here both the effect of leakage and threshold are observed on the mean number of HP and LP packets in the system. The mean number of both packets in the system is much lower than in Table 5.7.

To observe the effect of changing the constant parameters (the arrival rates of the HP and LP packets), the results in Table 5.9 and Table 5.10 are presented. An increase in the arrival rate of LP packets shows no significant change in the mean number of HP packets and only a slight increase in the mean number of LP packets in the system. This is expected, as this is a pre-emptive system.

To illustrate the effect of the threshold on the model, the model with leakage and priority incorporated and the model with leakage, priority and threshold incorporated are simulated and the results presented in Table 5.11 and Table 5.12. The parameters used in the simulation of both models are the same and a threshold of 5 is imposed on the basic model with leakage, priority and threshold incorporated. As expected, when a threshold is imposed and the arrival rate of the HP packets is varied, the mean number of HP packets increases at a slower rate in comparison to the basic model with leakage and priority incorporated. The mean number of LP packets, on the other hand, is much higher in the model with threshold imposed.

From the results obtained, it is observed that the rate of leakage significantly affects the basic model with leakage, priority and threshold incorporated. If the rate of leakage is low, the mean number of both HP and LP packets in the system decreases. This is because there are more tokens in the system for transmission of both HP and LP packets.

The threshold imposed on the token buffer also has a significant impact on the model. A higher threshold will mean that there are more tokens in the system for the transmission of HP packets but fewer for transmission of LP packets.

The results obtained are compared to the existing models developed in [56]. The sensor nodes with energy harvesting capability are modelled with two queues, one for energy and the other for data. The models presented are Markovian with energy and data arriving as Poisson arrivals. The queueing system was described by a QBD and the performance measures calculated using the matrix-geometric methods. In [56] it is observed that an increase in the arrival rate of energy results in a decrease in the mean data delay. The results in [56] are used to validate the results in this research. The numerical results reveal that energy harvesting is important in the performance of WSNs. Imposing a threshold ensures that the HP packets in the system are always transmitted.

CHAPTER 6 CONCLUSION AND FUTURE WORK

6.1 CHAPTER OVERVIEW

In this research, analysis and management of energy harvesting in wireless sensor networks is done.

- In Chapter 2, an extensive literature survey on energy harvesting in WSNs is presented. Background on WSNs is provided, including their applications, structures and challenges. Energy-harvesting is also discussed, including the different energy sources, models and energy harvesting techniques. Queueing theory, which is an essential component of this research, is discussed. Finally, threshold is introduced.
- In Chapters 3 and 4, the research methodology is presented and the assumptions made are specified. In Chapter 3, the research method of the basic model is presented. In Chapter 4, the research method of the basic model with leakage incorporated, the basic model with leakage and priority incorporated is presented and the basic model with leakage, priority and threshold incorporated are presented. The transition matrices are developed and presented.
- In Chapter 5, the simulated results of the models developed in Chapter 3 and 4 are presented. The results presented are in the form of the system performance parameters. An overall discussion is also given in this chapter. The simulated results are discussed and reasons provided to support the results. An increase in threshold will result in more tokens being available for transmission of HP packets.

The models presented are developed to represent the following conditions:

- An ideal system where data packets use the available tokens for transmission.
- A system with leakage of tokens incorporated, the data packets use the tokens and the unused tokens leak.
- A system with both leakage of tokens and priority incorporated; The HP data packets are given priority during transmission. After the HP packets are transmitted, the LP packets are transmitted and the unused tokens leak.
- A system with leakage of tokens, priority and threshold incorporated. A threshold is included to control the model to some extent. It ensures that there are always tokens in the system for transmission of high priority packets.

6.2 FUTURE WORK

The research has provided a solution that enabled us to analyse and manage the harvested energy in WSNs. However, future work is possible and suggestions are offered below.

6.2.1 Service process of the system

In developing the system model, other queueing models can be employed. In this research, the service of the system is assumed to be geometric owing to the nature of the queue developed. However, it could be modelled as a PH/PH/k system with a phase type distribution that is essentially a matrix inversion of the geometric distribution.

6.2.2 Low-priority packet transmission

The proposed model ensures that the HP data packets are always transmitted, this could lead to loss of LP packets due to an overflow of the LP packet buffer. A delay can be incorporated in the model to ensure that the HP packets are transmitted for a certain time period, and thereafter the LP data packets are transmitted.

The priority of the system could also be designed as a non-preemptive system. This priority is described as follows; if a LP data packet is receiving service and an HP packet arrives, service to the LP data packet is not interrupted.

6.2.3 Factors affecting the battery

In addition to leakage, other factors affecting the battery can be included in the model. Some of these factors are:

- Temperature
- Charging cycle and discharging capacity
- State of capacity and charging voltage. The state of capacity is defined as the ratio of the battery's current capacity to the maximum ampere-hours that it can discharge under the specified conditions.
- Age of battery.

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ADDENDUM A DEVELOPING TRANSITION MATRICES

The transition matrices developed are in the form

$$P = \begin{bmatrix} B & C & & & \\ E & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & \ddots & \ddots & \ddots \\ & & & A_2 & A_1 + A_0 \end{bmatrix}, \tag{A.1}$$

Matrices B and C have only tokens in the current state. For the model with leakage incorporated developed in Chapter 4, only the B matrix is affected by leakage.

A.1 MODEL WITH LEAKAGE AND PRIORITY INCORPORATED

For a current state 000, there is no leakage of tokens but the number of tokens increases as shown in Table A.1.

Table A.1. No leakage of tokens.

(CS) 000	
$a_{00}b$	001
$a_{00}\bar{b}$	000
$a_{01}b$	000
$a_{01}\bar{b}$	010
$a_{10}b$	000
$a_{10}\bar{b}$	100
$a_{11}b$	010
$a_{11}\bar{b}$	110

When there is one token or more in the current state there is leakage and the next states have a probability of leakage. The final states are given in brackets in the Table A.2 and Table A.3.

Table A.2. A maximum of one token leaking.

(CS) 001	l_{10}	l_{11}
$a_{00}b$	002	001
$a_{00}\bar{b}$	001	000
$a_{01}b$	012 (001)	011(000)
$a_{01}\bar{b}$	011 (000)	010
$a_{10}b$	102 (001)	101(000)
$a_{10}\bar{b}$	101(000)	100
$a_{11}b$	112 (011)	111(010)
$a_{11}\bar{b}$	111(010)	110

Table A.3. A maximum of two tokens leaking.

002	l_{20}	l_{21}	l_{22}
$a_{00}b$	003	002	001
$a_{00}\bar{b}$	002	001	000
$a_{01}b$	013 (002)	012(001)	011(000)
$a_{01}\bar{b}$	012(001)	011(000)	010
$a_{10}b$	103(002)	102 (001)	101(000)
$a_{10}\bar{b}$	102(001)	101(000)	100
$a_{11}b$	113(012)	112 (011)	111(010)
$a_{11}\bar{b}$	112(011)	111(010)	110

The boundary conditions impose a restriction. This means that the tokens and packets cannot go to a higher state. The buffer sizes of both the tokens and packets should not be exceeded.

A.2 MODEL WITH LEAKAGE, PRIORITY AND THRESHOLD INCORPORATED

For a threshold of three, the model was developed as shown in Table A.4, Table A.5, Table A.6 and Table A.7. The table gives the details of the state space. Imposing a threshold of three means that when there are fewer than three tokens, no LP packets are transmitted. For illustration the tokens in the system are assumed to be four. The states in brackets indicate the final state after an HP packet is transmitted. In the table CS is the current state.

Table A.4. Threshold of three with a maximum of one token leaking.

(CS) 001	l_{10}	l_{11}
$a_{00}b$	002	001
$a_{00}\bar{b}$	001	000
$a_{01}b$	012	011
$a_{01}\bar{b}$	011	010
$a_{10}b$	102 (001)	101(000)
$a_{10}\bar{b}$	101(000)	100
$a_{11}b$	112 (011)	111(010)
$a_{11}\bar{b}$	111(010)	110

Table A.5. Threshold of three with a maximum of two tokens leaking.

002	l_{20}	l_{21}	l_{22}
$a_{00}b$	003	002	001
$a_{00}\bar{b}$	002	001	000
$a_{01}b$	013 (002)	012	011
$a_{01}\bar{b}$	012	011	010
$a_{10}b$	103(002)	102 (001)	101(000)
$a_{10}\bar{b}$	102(001)	101(000)	100
$a_{11}b$	113(012)	112 (011)	111(010)
$a_{11}\bar{b}$	112(011)	111(010)	110

Table A.6. Threshold of three with a maximum of three tokens leaking.

003	l_{30}	l_{31}	l_{32}	l_{33}
$a_{00}b$	004	003	002	001
$a_{00}\bar{b}$	003	002	001	000
$a_{01}b$	014 (003)	013 (002)	012	011
$a_{01}\bar{b}$	013 (002)	012	011	010
$a_{10}b$	104(003)	103(002)	102 (001)	101(000)
$a_{10}\bar{b}$	103(002)	102(001)	101(000)	100
$a_{11}b$	114(013)	113(012)	112 (011)	111(010)
$a_{11}\bar{b}$	113(012)	112(011)	111(010)	110

Table A.7. Threshold of three with a maximum of four tokens leaking.

004	l_{40}	l_{41}	l_{42}	l_{43}	l_{44}
$a_{00}b$	005	004	003	002	001
$a_{00}\bar{b}$	003	003	002	001	000
$a_{01}b$	015(004)	014 (003)	013 (002)	012	011
$a_{01}\bar{b}$	014(003)	013 (002)	012	011	010
$a_{10}b$	105(004)	104(003)	103(002)	102 (001)	101(000)
$a_{10}\bar{b}$	104(003)	103(002)	102(001)	101(000)	100
$a_{11}b$	115(014)	114(013)	113(012)	112 (011)	111(010)
$a_{11}\bar{b}$	114(013)	113(012)	112(011)	111(010)	110

For the state with both LP packets and tokens, the next state is shown in Table A.8. The method of obtaining the state space applies to all the current states that have LP packets and tokens, for example 021, 022 to $0NK$, with N as the LP packet buffer and K as the token buffer.

Table A.8. Threshold of three with a maximum of one token leaking with LP packets in the current state.

(CS) 011	l_{10}	l_{11}
$a_{00}b$	012	011
$a_{00}\bar{b}$	011	010
$a_{01}b$	022	021
$a_{01}\bar{b}$	021	020
$a_{10}b$	112 (011)	111(010)
$a_{10}\bar{b}$	111(010)	110
$a_{11}b$	122 (021)	121(020)
$a_{11}\bar{b}$	121(020)	120

For the boundary, the state space is obtained in the same manner but care has to be taken that the number of tokens and packets does not exceed the specified buffer size. The boundary conditions have to be obtained with caution.