The role of mathematical tasks in providing Grade 10 learners an opportunity to learn trigonometry

By

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MAGISTER EDUCATIONIS

Department of Science, Mathematics and Technology Education

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DECLARATION

I, Thandi Mahlangu, student number 17265577, declare that this dissertation entitled, "**The role of mathematical tasks in providing Grade 10 learners an opportunity to learn trigonometry**", which I hereby submit for the degree of Magister Educationis (MEd) at the University of Pretoria, is my own work and that all sources cited or quoted in this research paper have been indicated and acknowledged by means of a complete reference list. This work has not been submitted by me for a degree at this or any other Institution of Higher Learning.

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ETHICS STATEMENT

The author, whose name appears on the title page of this thesis, has obtained, for the research described in this work, the applicable research ethics approval. The author declares that she has observed the ethical standards required in terms of the University of Pretoria Code of ethics for research, and the Policy guidelines for responsible research.

Thandi Mahlangu

30 May 2021

DEDICATION

I dedicate my dissertation to God Almighty, the Creator of everything in the universe. I would like to thank God for his protection and encouragement and for giving me the strength and wisdom to complete this degree.

To the Mahlangu family – my gratitude for your support throughout my studies. Most importantly, my thanks to my brother, Kabelo, for always believing in me and for instilling in me the values of diligence and perseverance. You are my inspiration and my pillar of strength.

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ABSTRACT

Poor mathematics performance in South African schools is a major concern (Reddy et al., 2014) and learners' opportunity to learn is one of the concepts that needs to be explored in schools (Dowd, Friedlander & Guajardo, 2014). Several authors (Gür, 2009; Ebert, 2017; Rohimah & Prabawanto, 2019) state that learners believe that trigonometry is difficult and abstract compared with the other topics of mathematics. Opportunity to learn (OTL) is defined as the degree to which learners during instruction get exposed to the content of the mathematics intended curriculum (Reeves & Muller, 2005).

The term "mathematical tasks" refers to classwork problems, homework problems, projects, investigations and assignments. These tasks play a vital role in effective teaching and learning. Learners' OTL was explored according to the types and nature of tasks selected by the teacher, and the pedagogical approach and strategies used by teachers and the influence of these two aspects on the time spent on tasks and learner engagement. This study therefore aimed to answer the research question about the extent to which mathematics tasks provided Grade 10 learners an opportunity to learn trigonometry. A two-part conceptual framework was used: the first part focused on the task selection in terms of its nature and cognitive demand and the second part focused on the teacher-specific factors such as teachers' approaches and strategies. The influence of these two parts on the implementation of the tasks by the learners in terms of time-on-task and learner engagement was then described. A qualitative approach was followed, and a descriptive case study was conducted with two Grade 10 mathematics teachers from two formerly disadvantaged public schools in Gauteng Province. A qualitative research approach was used in which document analysis and classroom observations served as data collection techniques. A deductive analysis approach was implemented.

The study revealed that teachers mainly gave learners recall-type and routine procedure questions involving pure mathematics, which according to CAPS' cognitive demands, are classified as lower order thinking tasks. There was a lack of higher order mathematics tasks that could have provided the learners with and OTL trigonometry effectively. The study further revealed that both teachers' approaches were dominated by a teacher-centred approach where the focus was on the teachers and where teachers mainly used direct teaching as teaching strategy where learners were directed to learn through memorisation and recitation techniques.

Although, due to the small sample, the study's results cannot be generalised, I believe that the findings will contribute to pre- and in-service teacher training, where teachers come to realise the importance of appropriate mathematics tasks to contribute to learners' OTL. Moreover, the study's findings highlight the need to engage the four cognitive levels, namely knowledge (20%), routine procedures (35%), complex procedures (30%), and problem solving (15%) in the types of tasks. The value of teachers' choices regarding the teaching approaches and strategies used should never be under-estimated. Future research could possibly build on this study by examining the implementation of tasks to enhance learners' in-depth understanding of trigonometry.

Key words: Opportunity to learn; curriculum; mathematics tasks; cognitive demands; teaching strategies; teaching approaches; time-on-task; learner engagement.

LANGUAGE EDITOR

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22 July 2020

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TO WHOM IT MAY CONCERN

I hereby declare that I have edited Ms Thandi Mahlangu's dissertation,
The role of mathematical tasks in providing Grade 10 learners an opportunity to learn trigonometry,
which is to be submitted by her
in partial fulfilment of the requirements
for the degree Magister Educationis

My work included the following:

- Language editing
- Checking for logic and coherence
- Checking in-text academic referencing

Sincerely,

E H DE WAAL

LIST OF ABBREVIATIONS

ANA	Annual National Assessment
ATP	Annual Teaching Plan
CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
LOL	Low Order Thinking
FET	Further Education and Training
NCET	National Education Collaboration Trust
OTL	Opportunity to Learn
PCK	Pedagogical Content Knowledge
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SMK	Subject Matter Knowledge
TIMSS	Trends in International Mathematics and Science Study

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CHAPTER 1

INTRODUCTION AND CONTEXTUALISATION

1.1. Introduction

Mathematics plays a vital role in our everyday lives since it develops individual reasoning as well as analytical and problem-solving skills. It is present in all spheres of life, such as medicine, engineering, architecture, even in solutions to societal problems. Due to its perceived importance in the world, some form of mathematics is mandatory for all learners in South African schools. In South Africa, pure mathematics is compulsory from Grade 1 to Grade 9. At the end of Grade 9, learners must choose between mathematical literacy, technical mathematics, or pure mathematics in the Further Education and Training (FET) phase (Grades 10-12). As a result, learners who are unable to do pure mathematics continue with either mathematical literacy or technical mathematics.

Reports from both international and national studies, such as the Trends in International Mathematics and Science Study (TIMSS) and the Annual National Assessment (ANA) indicate that South African learners are still performing poorly in mathematics (McCarthy & Oliphant, 2013). It was thus clear that more attention should have been given to the issue of the poor performance of South African learners in mathematics (Shaughnessy, Moore, & Maree, 2013). Two years later, the findings of the latest TIMSS 2015 report (Reddy, Visser, Winnaar, Arends, Juan, Prinsloo & Isdale, 2016) indicated that South Africa’s Grade 9 mathematics learners still performed second worst from all participating countries.

The Department of Basic Education (DBE) and the Department of Higher Education and Training (DHET) are making efforts to improve mathematics performance in South Africa (DBE, 2016). The DBE is involved in several projects, which focus, amongst others, on improving teaching and learning of mathematics (DBE, 2018). The DBE is currently providing financial assistance to prospective teachers in the priority areas through the Funza Lushaka Bursary Scheme, which aims to develop teachers in mathematics education (Van der Berg, Taylor, Gustafsson, Spaull, & Armstrong, 2011). Also, institutions such as Old Mutual and the European Union play a major role in professional development programmes for mathematics teachers (Old Mutual, 2013; Gravett, Petersen & Ramsaroop, 2019). Furthermore, the Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ), together with DBE, are also developing effective teaching methods for teachers, especially those in the rural public schools (Spaull, 2013).

1.2. Problem statement

There are global attempts to improve mathematics education in schools (Pournara, Hodgen, & Pillay, 2015), and this may be because of poor learner performance outcomes globally. In South Africa, despite a number of projects addressing issues such as teacher development, curriculum implementation, and using teaching and learning materials effectively (Union, 2014), performance outcomes in mathematics are still relatively poor. The poor performance in mathematics among learners may be attributed several factors, firstly, a lack of teachers' knowledge and understanding of the curriculum; secondly, the lack of implementation of teaching strategies inside the classroom and, in particular, teachers' inability to select and implement meaningful tasks (Henningsen & Stein, 1997). Against this background this study will seek to explore the extent to which the nature and cognitive demand of mathematics tasks given to Grade 10 learners in some schools in Tshwane South provide opportunities for those learners to learn mathematics effectively.

1.3. Rationale

I follow the reports of scholars and the DBE, as well as the concerns of society regarding the shortcomings in mathematics achievement of the South African learners. Although mathematics is regarded as a priority subject in schools, learners' performance in this subject is far below that of learners in other developing countries (Spaull, 2013). Several studies (Tsanwani, Engelbrecht, Harding, & Maree, 2013; Arends, Winnaar, & Mosimege, 2017) have identified some factors that contribute to poor achievement and those studies are providing literature on ways to improve mathematics performance, such as teachers' professional development, curriculum implementation, and the use of instructional materials.

Moreover, I have observed that Grade 10 learners struggle with understanding basic mathematical concepts. I believe that possible reasons may be that learners are not actively engaged with mathematics during the lesson because a teacher-centred approach is mostly used where learners are excluded, and that there is a lack of competency in teachers to select relevant, valuable and graded mathematical tasks, the latter also mentioned by Galant (2013).

1.4. Purpose of the study

The manner in which mathematics tasks are planned by teachers and executed by learners plays a significant role in the expected outcomes of learners (Chapman, 2013; Zwahlen, 2014). This study's aim is to analyse daily mathematics tasks (classwork, homework, assignments, investigations and projects) given to Grade 10 learners, to determine the extent to which the tasks provide the learners an opportunity to learn.

The objectives of the study are:

- to determine what is prescribed and recommended by CAPS regarding the nature and cognitive demand of mathematics tasks;
- to investigate the mathematics tasks teachers present to learners in terms of their nature and cognitive demand, and compare their alignment with CAPS;
- to observe the teachers' lessons to determine if learners have sufficient time to carry out tasks;
- to assess classroom activities, homework problems, projects and assignments given to the learners by their teachers; and
- to observe teachers' approaches and strategies during instruction, as these play a role in how teachers involve learners in instruction.

The findings of this study could be used by the DBE and mathematics teachers to gain a deeper understanding of the value and role of meaningful mathematics tasks. These include task selection, task implementation, task evaluation and instructions in line with the above objectives.

1.5. Concept clarification

Based on literature, Table 1 provides a list of key concepts and a clarification of their meaning in this study.

Table 1. 1: Clarification of concepts used in this study

Concept	Clarification
Curriculum	This refers to a document that prescribes the knowledge and skills learners are expected to learn for the specific topics (Adu & Ngiba, 2014; Maimela, 2015).
Implementation	Execution; the process of solving tasks by learners
Engagement	Time spent on tasks, and individual, group discussion work during instruction
Intended curriculum	Set of objectives set at the beginning of mathematics curriculum (content and skills to be taught) (Kurz, 2011)
Learner engagement	Is defined as active involvement of the learner during learning activities (Ayçiçek & Yelken, 2018).
Types of mathematical tasks	Refer to learning tasks such as classwork, homework, assignments, projects and investigations done by learners.
Nature of mathematical tasks	Are contextual, numerical and problem-solving tasks.
Opportunity to Learn (OTL)	Elliott and Bartlett (2016) define OTL as a multi-dimensional construct with three core elements: instructional time, the content and instructional quality. Instructional time refers to the amount of time allocated to instruction with one component being the time spend on tasks. The content refers to the extent to which the content of instruction overlaps with the intended curriculum, while the instruction quality refers to teachers' uses of evidence-based instructional practices that can produce learner achievement.
Tasks design	Select, adapt already existing, or create tasks from scratch or the combination of these actions.

1.6. Research questions

The following primary and secondary questions guided the study:

Primary research question:

To what extent do mathematics tasks provide Grade 10 learners with an opportunity to learn?

Secondary research questions:

1. What are the requirements as stipulated by CAPS regarding mathematics tasks?
2. To what extent do mathematics tasks given by the teachers comply with the requirements in CAPS?
3. How can the engagement of tasks by the learners be described?
4. How do the teacher-specific factors influence the implementation of mathematics tasks by learners?

1.7. Working assumption

For the purpose of this study, I provide some working assumptions based on my knowledge of and beliefs about the role of mathematics tasks in the teaching and learning of mathematics. These assumptions may have an influence on how I will conduct the study, analyse the data, and make conclusions.

Assumption 1: When learners are exposed to mathematics tasks during instruction, they may get an opportunity to meaningfully engage with the content.

Assumption 2: Mathematics tasks are a central tool to teaching and learning; the fewer the tasks, the more the learners are disconnected from the content.

Assumption 3: Learners can make much more sense of mathematics through the process of thinking, communication, solving problems, reasoning, and justifying their answers, than merely by passively listening to the teacher and copying notes from the board.

Assumption 4: Mathematics tasks that are not aligned with the prescribed CAPS may contribute to a high failure rate among learners.

Assumption 5: Mathematics tasks that do not encompass all the cognitive levels illustrated in the Bloom's Taxonomy may hinder the learners' opportunity to engage with mathematics.

1.8. Conceptual framework

The current section will discuss the conceptual framework that applies to this study.

1.8.1. Introduction

Athanasou, Di Fabio, Elias, Ferreira, Gitchel, Jansen, and Mpofu (2012) state that a conceptual framework is an instrument that guides the data collection and its analysis. The purpose of this study is to determine the extent to which mathematics tasks provide Grade

10 learners with an opportunity to learn. Looking at the working definition of OTL as defined by Elliott and Bartlett (2016), OTL is a multi-dimensional construct with three core elements: instructional time, the content, and instructional quality. Instructional time refers to the amount of time allocated to instruction with one component being the time spend on tasks. The content refers to the extent to which the content of instruction overlaps with the intended curriculum, while the instruction quality refers to teachers' uses of evidence-based instructional practices that can produce learner achievement.

The conceptual framework for this study is based on the model of progression of mathematics tasks proposed by Henningsen and Stein (1997). This model explains the connections between three variables, namely the teacher, the learner and mathematics tasks. The conceptual framework therefore has two main pillars namely the mathematics tasks (in terms of its nature: contextual, numerical, and problem-solving tasks; and cognitive demands, and the teacher specific factors (in terms of teaching approaches and teaching strategies) and the way these two pillars have an influence on the time spend on tasks and learner engagement.

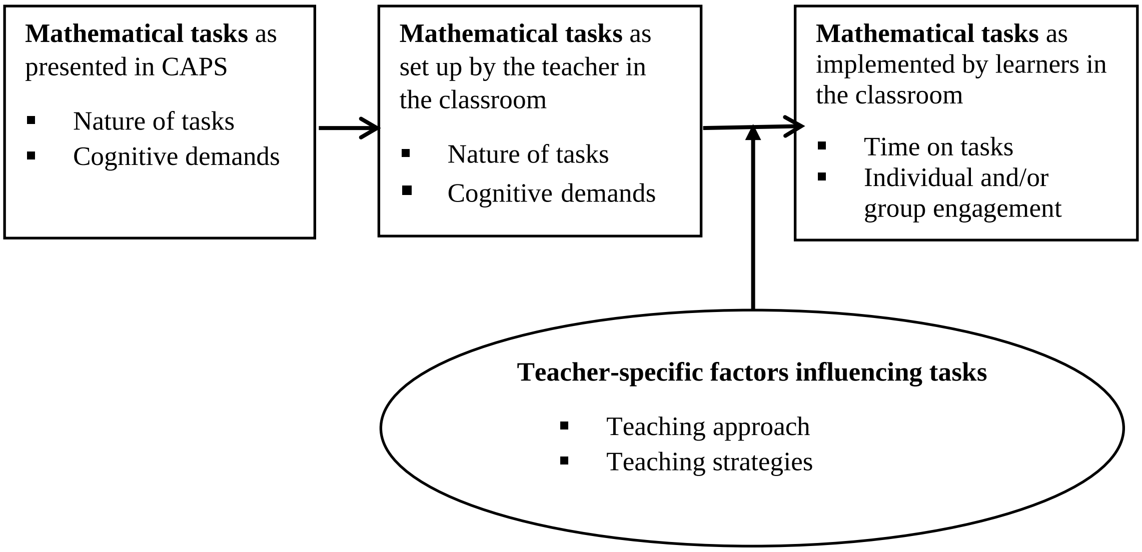


Figure 1. 1: Conceptual framework: Opportunity to learn trigonometry (Adapted from Henningsen, & Stein, 1997)

1.8.2. Mathematics tasks

- **The nature of the tasks**

According to Coles and Brown (2016), the nature of tasks has a significant impact on three vital elements in the teaching and learning of mathematics, namely the content taught; how learners learn; and how learners acquire knowledge and skills of mathematics (Coles & Brown, 2016). The tasks are in the form of classwork, homework, projects, assignments, and investigations. Furthermore, several authors such as Chapman (2013); Jackson, Shahan, Gibbons and Cobb (2012) and Spaull (2013) agree and further suggest that mathematics tasks help educators to enhance learners' conceptual understanding of mathematics in line with the curriculum objectives. Nevertheless, this researcher considers mathematics tasks as central to the learning of mathematics.

- **The cognitive demand of the tasks**

Henningsen and Stein (1997) affirm that the emphasis on mathematics tasks shapes the reason for students learning as they place a certain degree of cognitive demands on those learners. The term "cognitive demands" refers to the kind of thinking processes involved in solving the tasks (Henningsen, & Stein, 1997; Henhanffer, 2014). Awareness of these demands could enable teachers to prepare tasks that may engage learners' thinking processes and ultimately lead to learners' achievement.

1.8.3. Teacher-specific factors influencing tasks

- **Teaching approach**

The nature of teaching affects the learning outcomes (Ko, & Sammons, 2013). The way teachers choose to teach, whether using a teacher-directed or learner-directed approach or combination of the two in the classroom, is often a direct reflection of their beliefs about mathematics. By implication, teachers' pedagogical approaches strongly correlate to how they select and implement those tasks in the classroom. This implementation process ultimately has power to hamper or improve learners' understanding of mathematics.

- **Teaching strategies**

The emphasis on empowering learners to have an OTL mathematics requires effective teaching. Effective teaching is therefore not only about the action of the teacher, but more about the learning environment that a teacher creates in the classroom. (Saleem, Alimgeer, Saleem, Khurram & Saleem, 2013). The environment of the classroom is based on the teaching strategies used during instruction, such as cognitive guided instruction, discussion-based group work and, interactive lecture demonstration (Ismail, Shahrill, & Mundia, 2015). What learners understand about mathematics depends almost entirely on the experiences that the teacher creates daily in the classroom.

1.8.4. Time on tasks and learner's engagement

Stols (2013) explain that learning depends on effective use of time and some degree of learner engagement inside the classroom. Teachers should plan their lessons in such a way that sufficient time is given to learners to be actively engaged in solving these tasks, individually or in groups. CAPS specifies the number of hours (4.5 hours) of classroom time allocated to trigonometry in each week. These hours constitute 16% of total classroom time allocated to learners' engagement with tasks. CAPS (2011) clarifies that this time allocation should allow for a sufficient depth of engagement with the content as specified.

1.9. Research methodology

The paradigm that will underpin this study is based on social constructivism. This paradigm assumes that reality is not objectively determined, but is socially constructed (Nieuwenhuis, 2016). The ontological assumption is that the study is relative in nature, as the researcher is aware that knowledge is not absolute (Scotland, 2012). The underlying epistemological assumption is that by observing learners and teachers in their social context (classroom) and conducting a document analysis, I am subjectively involved in coming to understand the phenomenon (Scotland, 2012).

This study will employ a qualitative approach as it seeks to explore the mathematics tasks teachers use and how learners implement these mathematics tasks, providing them with an opportunity to learn mathematics efficiently. Maree (2016) defines qualitative research as a naturalistic approach because of its focus on the natural settings where interactions occur. The interaction in this study will be between the researcher as observer, and the teachers and learners as being observed, but also between the researcher and documents such as CAPS, the Annual Teaching Plan (ATP), and the learners' workbooks.

1.9.1. Research design

The research design adopted for this study will be a descriptive case study. Yin (2009) states that a case study is a method that provides the researcher with an opportunity to retain the holistic and meaningful characteristics of real-life events. According to Maree (2016) a descriptive case study describes a phenomenon in which a real-life context occurs.

1.9.2. Research site and sampling

This study has adopted non-probability sampling to gather information relevant to the phenomena. Morgan and Sklar (2012) posit that non-probability sampling is used frequently in qualitative research where the aim of the investigation is usually to create an in-depth description of the phenomenon, and not to generalise the findings (Nieuwenhuis, 2016). Both purposive and convenience sampling as types of non-probability sampling were used to recruit the schools and participants for this study. The participants of this study were two Grade 10 mathematics teachers and their learners. For the document analysis, CAPS, the ATP and learners' workbooks were used. The ATP breaks up the topics per school terms and weeks and is grade specific. The aim of the ATP is to ensure that all teachers and learners have a clear understanding of the topics that must be covered in Grade 10 mathematics content.

Maree (2016) emphasises that sampling involves pre-selected criteria relevant to the interest of the study. Hence, the selection of the participants will be made using specific criteria as indicated in Table 1.2 below:

Table 1. 2: Criteria for inclusion in, and exclusion from, the sample

	Criteria
Inclusion	• Two formerly disadvantaged public schools in Tshwane South District (Section 21) • Two Grade 10 mathematics teachers • Male and female teachers • Different race groups • Teacher with 10 or more years of experience in teaching mathematics
Exclusion	• Non-mathematics teachers • Private schools

1.9.3. Data collection methods

Observations and document analysis were used as methods to collect the qualitative data as indicated in Table 1.3 below. This data enabled me to answer the research questions.

Table 1. 3: The data collection methods

Method	Brief description	Research sub-questions
Observation **(Observation schedule)**	Is an essential data gathering method as it can provide the researcher with an inside perspective on the group dynamics and behaviours in different settings (Maree, 2016). The researcher will learn through personal experience and reflect on how the classroom settings are socially constructed in terms of power, communication lines, discourses and language (Nieuwenhuis, 2016).	ii) To what extent do mathematics tasks given by the teacher comply with the requirements in CAPS? iii) How can the implementation of tasks by the learners be described? iv) How do the teacher-specific factors influence the implementation of mathematics tasks by learners?
Document analysis **(Rubric)**	Documents, namely the FET Mathematics CAPS, ATP and learners' workbooks, will be studied and analysed as part of the data gathering process. Nieuwenhuis (2016) says that when using documents (textual data) as a data gathering method, the researcher will focus on all types of written communications such as mathematics problems, projects, assignments, classwork and homework that may shed light on the phenomenon under investigation.	i) What are the requirements as stipulated by CAPS regarding mathematics tasks? ii) To what extent do mathematics tasks given by the teachers comply with the requirements from CAPS?

1.9.4. Data analysis

Maimela (2015) defines data analysis as the procedure of organising data and arranging the organised data into meaningful patterns and categories. For the purpose of this study, I used deductive data analyses as the categories were pre-determined from literature and captured in the conceptual framework. The data from the observations will be transcribed verbatim. Once the data have been sorted and typed, I will familiarise myself with the contents, which means that I will read and re-read the text. Next, I will read my transcribed data word by word and allocate codes according to the conceptual framework's units.

Regarding the textual data from the documents, I will adopt an approach to content analysis as proposed by Hsieh and Shannon (2005) namely conventional content analysis. In conventional content analysis, coding categories are derived directly from the text data. "Content analysis" is defined as "systematic, replicable techniques for compressing many words of text into fewer content categories based on explicit rules of coding" (Nieuwenhuis, 2016, p. 111). Both these sets of data, from the observations and documents, will be organised and interpreted to report the results (Strydom, 2014).

1.9.5. Trustworthiness

For a study to be of high academic quality, it must be trustworthy. This implies that this study must be carried out in an ethical manner and that its findings represent as closely as possible what has been found from the selected secondary schools in the Gauteng Province. The following procedures as proposed by De Vos (2011) will be used to enhance trustworthiness of the study namely: prolonged engagement of the researcher as well as the participants in the field; multiple perspectives for collecting and contemplating data; and member validation of data.

1.9.6. Ethical considerations

The study was conducted in line with the University of Pretoria's (UP) Code of Ethics for Research. Ethical clearance was obtained from the Ethics Committee of UP before conducting the study. Furthermore, permission was obtained from the DBE. During fieldwork, letters of informed consent were given to the principals, teachers and parents in which their roles were explained. The teachers were informed that participation is voluntary and that they are at liberty to withdraw from this research at any stage (Strydom, 2014). Letters of assent were given to the learners to inform them of the purpose of the study, which their lessons will be observed and their workbooks analysed. This research will be available to all participants to review the collected data and findings of the study. Finally, the data will be stored for the period of 15 years in the University of Pretoria's archives.

1.10. Research structure

The chapters are outlined as follows:

Chapter 1: General orientation

In Chapter 1 a broad overview of the study is given. Chapter 1 starts with an introduction, rationale, and problem statement. This is followed by the purpose of the study, the clarification of concepts that emerge from the literature review, the research questions, and the working assumptions. A brief description is given of the conceptual framework and the research methodology. The chapter concludes by addressing the trustworthiness and the ethical considerations applicable to the study.

Chapter 2: Literature review

In Chapter 2, an in-depth literature review is given on international and national research that has been conducted with regard to OTL mathematics using mathematics tasks. This chapter includes the following headings: Mathematics as a priority subject; Curriculum assessment statement policy as a official guideline; Teaching and learning of trigonometry, Overview of OTL mathematics; mathematical tasks, Cognitive demands of mathematical tasks; Intended curriculum; The enacted curriculum; Teaching strategies and teaching approaches. This chapter also discusses the study's conceptual framework.

Chapter 3: Research methodology

Chapter 3 presents the qualitative investigation which forms part of my research design and methodology. The epistemological assumptions from which the research design was derived are discussed. Following the selection of participants, the data collection and the process of the data analysis and interpretation are discussed. Lastly, I discuss research trustworthiness, ethical considerations, and then conclude the chapter.

Chapter 4: Data analysis and interpretation of findings

The research findings are described in this chapter and are obtained through document analysis and classroom observations conducted with the mathematics teachers. The findings are also analysed, presented, and discussed based on the conceptual framework and the literature. Trends are identified and explained.

Chapter 5: Findings and recommendations

The dissertation ends with Chapter 5. This chapter provides a discussion of the main findings of the study and of the conclusions drawn in relation to the formulation of my research questions, the literature and the conceptual framework. I also discuss the limitations of the study and offer some recommendations for implementation. I then provide a summary of contributions as well as suggestions for future research.

After Chapter 5 a list of APA references as well as the appendices follow.

CHAPTER 2

LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1. Introduction

In this chapter, to clarify and justify this research endeavour, a literature review is conducted of different researchers' findings. This literature review is initially broadly reviewed and then narrowed down to the study. Scholarly articles were critically compared and contrasted in order to link the theory of OTL with the teaching and learning of trigonometry, and in particular the role of mathematical tasks in providing OTL. Setting the context of the study, an overview of literature pertaining to mathematics in general and trigonometry in particular, is provided. This is followed by a discussion of literature regarding OTL, being the focus of the study, and in particular, the role of tasks to provide this OTL. The cognitive demands of mathematics tasks and the Curriculum and Assessment Policy Statement (CAPS) is then analysed in terms of its nature as intended curriculum. As teacher-specific factors influence OTL, a discussion of teachers' knowledge regarding teaching approaches and strategies, regarding the subject matter, and the curriculum, follows. The literature review ends with a discussion of the conceptual framework that guided this study.

2.2. Mathematics as a priority subject

In this section a brief background of mathematics as a priority subject is provided, followed by a discussion of national and international assessments conducted on the subject. Next, I provide a status of mathematics in South Africa and lastly, I refer to specific guidelines for teaching and learning of mathematics.

Mathematics is considered an important discipline in different spheres of life such as the workplace and school (Ihendinihu, 2013). In the workplace, Mubeen and Saeed (2013) opine that mathematics plays a major role as it empowers individuals to solve complex problems. Botha, Maree and Stols (2013) concur with Mubeen and Saeed (2013) that in order for people to participate fully in society, and most importantly in the workplace, they must possess mathematical skills to apply that knowledge when solving problems. Mathematics is regarded as one of the priority subjects, and this is why the Department of Basic Education (DBE) has made some form of mathematics a compulsory subject up to Grade 12. Mathematics is compulsory from Grade 1 to Grade 9. At the end of Grade 9, learners must choose between mathematics, mathematics literacy and technical mathematics to be taken in the Further Education and Training (FET) phase (Grades 10-12). Mathematics in the FET phase covers ten main content areas. Each content area contributes towards the acquisition of specific skills. There has to be progression in terms of concepts and skills from Grade 10 to Grade 12, for all content areas are interdependent (DBE, 2014). Trigonometry is one of the content areas covered in the FET phase and is often introduced in Grade 9. This content area covers trigonometric concepts, processes and their applications to problem solving (Tuna, 2013).

2.2.1 International and national assessments

Public and private schools in South Africa have been participating in international and national assessment tests over the past few years. These assessment surveys in mathematics include international assessments from the Trends in International Mathematics and Science Study (TIMSS) for Grade 4 and Grade 8 learners (in SA Grade 5 and Grade 9 learners participated in the international survey), and from the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) for Grade 6 learners.

In South Africa, TIMSS was conducted in 1995, 1999, 2003, 2011, 2015, and 2019 under the support of the International Association for the Evaluation of Educational Achievement (IEA). Although South Africa performed poorly in relation to the other participating countries by either coming last or second last, there was an improvement from 2011 to 2015 with the mathematics score increasing by 20 points to 372 (Alex & Juan, 2017; Visser, Juan & Feza, 2015). Alex and Juan (2017) further show that, in the 2015 TIMSS results, learners achieved a high average of 372 out of a possible 1 000 points, compared to the previous result in 2011 of 352 – therefore an increase of 20 points. However, it remains a concern that, in spite of evidence of improvement in performance, South African learners are below the international benchmark of 500 points. Although we moved up from being last to being second last in the TIMSS, and although our performance increased by a few points, the increase is still not sufficient (Alex & Juan, 2017). South Africa is still stuck at the lower end of the rank when compared to other countries that participated in the TIMSS (Arends, Winnaar & Mosimege, 2017) despite efforts by the DBE, in collaboration with the National Collaboration Trust (NECT), to design a teaching programme to provide teachers with additional skills and methodologies required to teach FET Mathematics (ATP, 2015).

Nationally, the Annual National Assessment (ANA) in Mathematic for Grade 9 learners was an assessment instrument introduced by the DBE in 2011 to enable a systemic evaluation of education performance and thereby to enhance learner achievement (Howie, Combrinck, Tshele, Roux, McLeod Palane & Mokoena, 2012; McCarthy & Oliphant, 2013). However, the ANA was discontinued in 2015 because of administrative complaints put forward by teachers' unions, namely the South African Democratic Teachers Union (SADTU) and the National Professional Teachers' Organisation of South Africa (NAPTOSA). The reasons advanced by the unions were that time constraints prevented schools from consolidating and implementing the recommendations of ANA reports, and also that the ANA caused work overload on teachers in terms of administration. In addition, the results of the ANA were frequently used to name and shame the schools that performed poorly in the national assessment. Basic Education Minister, Mrs Angie Motshekga, announced on 24 May 2017 that the ANA would be replaced by another national mathematics assessment to be known as the National Integrated Assessment Framework (NIAF). This assessment would be conducted once every three years to a sample of learners at Grade 3, 6 and 9 level.

2.2.2. Status of mathematics in South Africa

Researchers in South Africa (Jantjies & Joy, 2015; Venkat & Spaull, 2015; Cai, Mok, Reddy & Stacey, 2016) have expressed concern regarding poor performance in, and a need to enhance, the learning of mathematics in the Further Education and Training (FET) band. Various published educational documents such as Chapter 9 of the National Development Plan (NDP) 2030, provided efforts to improve mathematics performance through policy innovations (Spaull, 2013), The documents also recommended teacher training workshops on curriculum issues, specifically on the use of instructional materials (Sinyosi, 2015). Yet research (Graven, 2014; Venkat & Spaull, 2015; Reddy, et al., 2016) across South Africa has revealed that majority of learners still perform poorly in mathematics, more specifically in public schools than in private schools (Heystek & Terhoven, 2015; Bosman & Schulze, 2018). The 2014 National Senior Certificate examination diagnostic report particularly indicated that Mathematics Paper 2 highlighted that performance in the trigonometry section was a cause for concern as candidates performed poorly in questions that tested basic knowledge (DBE, 2014). Kunene (2013) and Okitowamba (2016) also noted failure to understand trigonometry concepts on the part of the FET learners. This leaves us with the question as to what more should be done to provide optimal opportunities for learners to learn and thus improve their mathematics performance.

Marshman and Brown (2014) indicate that learners need to know and understand certain mathematics principles required for developing cognitive abilities. These principles include analytical thinking, critical reasoning, decision-making, and problem-solving skills. However, based on the review of the state of education in South Africa, the teaching of mathematics in schools does not necessarily emphasise the development of logical and critical reasoning together with higher-order thinking processes (Maharaj, 2007; Spaull & Kotze, 2015).

Various studies undertaken nationally have identified several factors that could be contributing to low performance in mathematics, especially among primary and lower secondary school learners. These factors include the following:

- the shortage of mathematics textbooks that are aligned and based on the prescribed curriculum (Bosman & Schulde, 2018);
- the way teachers give instructions in class (Engelbrecht, Nel, Nel & Tlale, 2015; Mutodi & Ngirande, 2014);
- how mathematics tasks are designed and used by teachers (Galant, 2013; Sinyosi, 2015);
- the time spent on those mathematics tasks (Stols, 2013);
- the alignment of tasks with the Mathematics CAPS; and learners' engagement in mathematics tasks (Berger, 2011);
- the classroom environment (Visser, Juan & Feza, 2015);
- insufficient opportunity to learn (Levy, Cameron, Hoadley & Naidoo, 2018) which encapsulates many of the above-mentioned factors, and as such form the core of this study.

2.2.3. The South African national Curriculum and Assessment Policy Statement (CAPS)

Since this study is based in a South African context, it was necessary to consult the official departmental documents teachers use to guide their teaching, and in particular, the type, nature, and cognitive demand of tasks used during instruction. The Mathematics Learning Programme (MLP) provides most of the planning required to teach FET mathematics. The programme provides a breakdown of the trigonometry topic into eight lessons (Introduction, Reciprocals, Calculator work, Special angles, Solving equations, Solving triangles, Cartesian plane and Pythagoras questions and Revision and consolidation). It gives a brief overview of every lesson and how to approach it to ensure that all aspects of its nature are covered in line with the CAPS. For example, according to CAPS (2011), the types of tasks such as classwork and homework are mandatory requirements in Grade 10. Both classwork and homework tasks can be problem-solving, contextual and/or numerical in nature (Wijaya et al., 2015). Teachers are advised to monitor the learners' progress to ensure their grasp of concepts DBE, 2016). The programme is aligned with CAPS in every aspect, including specifying the four cognitive levels required by CAPS. According to Mdladla (2017), the term "curriculum" refers to the specific subject content for teaching and learning inside the classroom. Researchers (Lizer, 2013; Adu & Ngibe, 2014; Maimela, 2015) define "curriculum" as the material resources intended to be utilised by teachers. Material resources include mathematics tasks as indicated in the curriculum and used to meet the stated objectives. Wium and Louw (2015) are considered experts in studying the South African National Curriculum. Maimela's (2015) view is consistent with the opinions held by Wium and Louw (2015) and claims that the purpose of the mathematics curriculum is to make the mathematics classroom an interesting, stimulating and challenging learning site where the teacher as well as the learners can share common resources and ideas.

In its most recent publication, the DBE (2011) has introduced the advanced curriculum for schools from Grade R-12, viz. the Curriculum and Assessment Policy Statement (CAPS). The CAPS is not a new curriculum, but a streamlined and strengthened National Curriculum Statement (NCS), introduced to our schools in 2012 for FET, beginning with Grade 10. The aim of the CAPS, with specific reference to mathematics, is to create an opportunity for learners to acquire relevant knowledge and mathematics skills (Mbatha, 2016). Further, the decision was made by policy makers together with the DBE to provide examples of mathematics tasks in the CAPS document to ensure that teachers are provided with guidance of appropriate tasks. Hence, the DBE policy makers and researchers highly recommend that mathematics teachers should use the intended curriculum as a guide to the topics to be covered and tasks to be used. What has been gathered from the literature concerning CAPS is that this curriculum is presented in three steps, namely the intended (prescribed) curriculum, the implemented (practised) curriculum, and the attained (achieved) curriculum (Khoza, 2015). CAPS as intended curriculum was used to determine to what extent teachers comply with the prescribed curriculum in terms of the nature and cognitive demand of tasks. The next section examines literature concerning the teaching and learning of trigonometry in line with the CAPS.

2.3. Teaching and learning of Trigonometry

Trigonometry is one of the topics of senior secondary school mathematics in South Africa and elsewhere in the world. It is one of the fundamental topics in the transition to FET mathematics within the South African context. The content of trigonometry in the Grade 10 mathematics curriculum includes trigonometric ratios of angles, inverse trigonometric functions, triangles, and angles; elevation, depression and special angles and solutions to simple trigonometric equations. It blends geometric, graphic, and algebraic reasoning that provides a space for making sense when solving problems involving triangles, trigonometric expressions, and graphs. Trigonometry in the curriculum is, therefore, a suitable ground for exploring, connecting and relating mathematical ideas, and for a meaningful combination of different scientific disciplines (Gur, 2009).

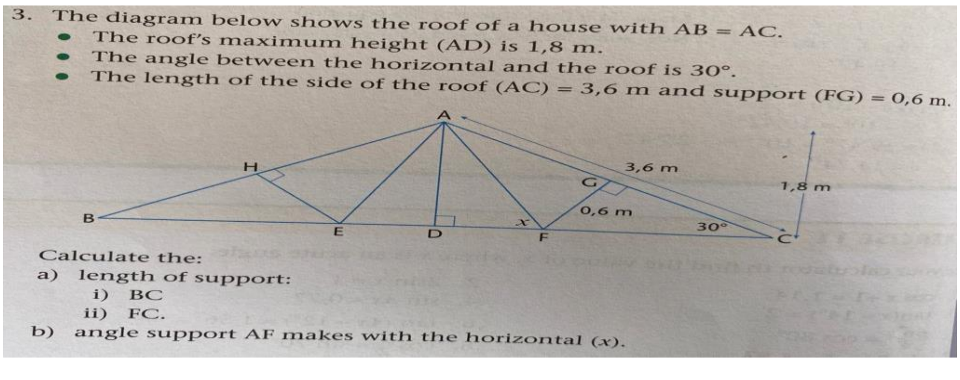
2.3.1. Teaching of Trigonometry

Understanding trigonometry provides a framework for learners to grasp and coordinate concepts (angles, measurement of angles and lengths, shapes and similarity). The study by Weber (2005) was critical of the approaches followed by the teachers during trigonometry instructions. Weber (2005) found that most approaches to trigonometry teach only procedural skills and do not allow students to fully understand trigonometry. Weber (2008) also found that much of trigonometry instruction focused on procedures and computations without an emphasis on applying the process. It is imperative that learners must understand concepts even if they understand a procedure; otherwise the students do not gain much from the procedure (Van Dyke & White, 2004).

In order to further elaborate on the need for understanding of the concepts, Weber (2005) conducted a study which compared two groups of learners taught in two different ways: the lecture-based and experimental-based instructional strategies. The results indicated that students taught in the lecture-based classroom developed limited understanding of trigonometry concepts while those who received the experimental-based instruction developed a much deeper understanding of those concepts. This deeper understanding of concepts may provide proficient OTL (Weber 2005). The document, Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding concurs with Weber (2005) and proposes the implementation of teaching where learners and teachers engage actively, discussing and experimenting with mathematics ideas (DBE, 2018). It should be noted however that various teaching strategies should be used in conjunction with each other as each has the potential of creating OTL. Although Weber (2005) found that the lecture-based strategy provided limited understanding of trigonometry concepts, Sharma (2019) found that it allowed teachers time to cover the syllabus, gives learners training in listening, provides an opportunity for better clarification of the concepts and stressing the significant ideas of trigonometry (Sharma, 2019). Both methods can therefore contribute in different ways to provide OTL trigonometry.

2.3.2. Learning of Trigonometry

The important goal in mathematics education is to involve learners in active learning. Brame and Biel (2015) and Kerrigan (2018) define active learning as instructional activities involving learners in doing things and thinking about and explaining what they are doing. This definition is broad and Bonwell and Eison (1991) recognise that a range of activities can fall within it, activities that according to Faus and Paulson (1998) and Edwards (2015) can range from simple to complex. A range of activities should therefore be introduced to the learners in class where the mathematics tasks should involve both lower and higher-order thinking. During the trigonometry class, a lower order thinking task may require learners to perform well-known procedures, apply simple applications and do calculations which might involve only a few steps. Higher-order thinking tasks may involve complex calculations and/or higher-order reasoning and the ability of learners to break the problem down into its constituent parts. An example of a contextual task involving lower and higher-order thinking is provided in an excerpt below:


3. The diagram below shows the roof of a house with AB = AC.
- The roof's maximum height (AD) is 1,8 m.
- The angle between the horizontal and the roof is 30°.
- The length of the side of the roof (AC) = 3,6 m and support (FG) = 0,6 m.

Calculate the:
a) length of support:
   i) BC
   ii) FC.
b) angle support AF makes with the horizontal (x).

The image above may be approached from either lower and higher-order thinking or both. In lower-order thinking, teachers may approach this task by asking learners to identify the triangle that has information and solve the unknown side (using ratios), learners will be guided to continue doing so until they solve the relevant side. In a higher-order thinking approach, teachers may ask learners to discuss roofing and how side and angles work, after that the teacher will allow learners to figure out how to solve for sides requested using different methods preferred by learners. Contextual tasks indicate that teachers should strive to use real-life settings to teach mathematics for conceptual understanding so as to enable comprehension of mathematical concepts.

Cognitive demands of mathematics tasks will be presented in further detail in the next two sections. Researchers (Stein & Smith, 1998; Son & Kim, 2015; Tekkumru-Kisa & Stein, 2015) affirm that the emphasis on mathematics tasks shape the reason for learning as it places a certain degree of cognitive demand on the learners. With the context of the study being set, the opportunity for learners to learn trigonometry in terms of tasks performed, being the focus of the study, is now discussed.

2.4. Overview of Opportunity to Learn (OTL) mathematics

Mathematics educators and researchers (McDonnel, 1995; Kurz, 2011; Stols, 2013; Wijaya, van den Heuvel-Panhuizen and Doorman, 2015; Mkhatshwa and Doerr, 2016; Walkowiak, Pinter & Berry, 2017; Yu & Singh, 2018) are acknowledging the concept of OTL as a topic worthy of scholarly consideration. Marshman and Brown (2014) added critical factors that are classroom specific, namely fiscal resources (teaching and learning support materials), types of tasks, time on tasks, learners' engagement, and instructional characteristics. Durksen, Way, Bobis, Anderson, Skilling and Martin (2017) extended the OTL conversation further by stating that one has to take into account how these factors work together (or not) in a classroom. The interaction between the listed factors as having the potential to provide OTL is discussed next.

With reference to resources, CAPS (2011) and the ATP (NECT, 2019) indicate that teachers need to select activities and exercises from the textbook, use additional teaching materials such as previous question papers, ensure the posters are displayed in the classroom, adopt FET mathematics training handouts from the DBE, and show the learners the recommended video clips at the end of the topic (NECT, 2019). The DBE (2011) and ATP (2015) advocate the use of additional resources and the participation of learners in tasks to make the mathematics classroom an engaging and stimulating learning place where teachers and learners can share resources and ideas during task implementation.

From the mentioned resources, the Mathematics Grade 10 pace setter specifies that only two types of tasks for Term 1 (Week 9-11) namely classwork and homework can be selected for learners (National Education Collaboration Trust, 2015). These tasks should be carefully chosen to adhere to the four cognitive levels, namely 20% Knowledge, 35% Routine Procedures, 30% Complex Procedures, and 15% Problem Solving.

Teachers should provide sufficient time for learners to be actively involved in solving the tasks, individually or in groups (CAPS, 2011). Stols (2013) concurs that learning depends on the degree of time and effort and warns that without efficient use of time on task, no learning is possible. The Gauteng Department of Education provides a detailed work schedule in which they suggest what topics to do, when to do them, and in what order to enable effective instruction (NECT, 2019).

The ATP (NECT, 2019) emphasises how learners should incorporate teachers' work to generate new knowledge. Solving tasks in pairs, working on the board or in the exercise books, and having small-group discussions are the factors learners are encouraged to implement. The DBE documents give prominence to the necessity for teachers to comply with CAPS specifications during instruction in relation to the nature and cognitive demands of those tasks. Teachers are expected to create an effective learning environment, so that they can make learning more interesting and engaging. Effective approaches and strategies are offered as guidelines in departmental and research documents such the CAPS (DBE, 2011), ATP (NECT, 2019) and the National Center for Educational Achievement (NCET, 2015).

This overview of OTL highlighted OTL as a list of a few critical factors necessary in developing an understanding of the OTL concept. The interaction between the factors are further operationalised in the next discussions.

2.4.1. Definitions of OTL

One of the critical variables in determining the learning of mathematics content is the concept of OTL. This concept has gained great momentum since 1995 until today, but theorists and researchers define OTL in different ways with different foci. McDonnel (1995), for example, refers to the concept of OTL as the process necessary for improving learners' performance in schools. Reeves and Muller (2005) define OTL as the degree of overlap between the information learners are taught and the information on which they were tested. Stols (2013), Wijaya, van den Heuvel-Panhuizen and Doorman (2015), and Yu and Singh (2018), describe

OTL as an occurrence that determines whether learners have had the opportunity to learn how to solve particular types of problems. Lastly, Elliott and Bartlett (2016) define OTL as a multi-dimensional construct with three core elements: instructional time, the content and instructional quality. Instructional time refers to the amount of time allocated to instruction with one component being the time spend on tasks. The content refers to the extent to which the content of instruction overlaps with the intended curriculum, while the instruction quality refers to teachers' uses of evidence-based instructional practices that can produce learner achievement. This definition provided by Elliott and Bartlett (2016) will serve as my working definition.

Wijaya (2017) also states that OTL entails the curriculum, and the use of meaningful and appropriate instructional strategies and resources – most importantly, the textbooks used by the teacher since all of the tasks given to learners are taken from the textbook. A study conducted by Mkhatshwa and Doerr (2016) and Henhaffer (2014) concur with Wijaya's (2017) view and said that the use of a textbook often determines what teachers will teach, how they will teach it, influencing how their learners acquire knowledge. The ATP (2015) advocates the use of additional resources and the participation of learners in tasks to make the mathematics classroom an engaging and stimulating learning place. To provide learners with an opportunity to learn, it is important for teachers to take cognisance of all other recommended additional and to align their tasks with the guidance stated in the CAPS documents. It can be concluded that mathematics tasks, especially those adopted from a textbook and other resources as a curriculum guide, play a crucial role in the teaching and learning of mathematics (Lee, Lee & Park, 2016). Fredericks (2005) asserts that teachers should consider other materials such as question papers and mathematics apps in line with the CAPS.

The discussions that follow on mathematics tasks, cognitive demands of mathematics tasks, CAPS, and teacher-specific factors contributing to OTL are considered key dimensions for this study and provide a balanced approach to understanding the concept of OTL as presented in the conceptual framework. The next section illuminates how OTL and mathematics task are interwoven and will therefore further point to aspects of OTL.

2.4.2. Mathematics tasks

In this section, a general description of mathematical tasks is provided, followed by the role of mathematics tasks in the teaching and learning of mathematics, the different types of tasks prescribed by the CAPS, the nature of tasks according to literature, and task design.

Bayazit (2013) holds an opinion that a task could be defined as a mathematics problem constructed by teachers for equipping learners with relevant knowledge, skills and abilities. Adams (2015) and Fredenberg (2015) state that mathematics tasks comprise activities presented to learners during and after instruction. The activities include problem solving, investigative assignments, project work, application, and problem posing task (Akayuure & Apawa, 2015). Such activities are largely recommended as supporting tools in realising learning objectives (Sullivan, Clarke & Clarke, 2012).

In their discussion, Pettersen and Nortvedt (2018), show how teachers are expected to provide OTL through promoting learners' ability to think, reason, and develop significant skills by working on tasks. Teachers promote leaners' ability to think through the use of effective activities that ultimately encourage active engagement with mathematics content. Therefore, it is the core responsibility of the teachers to select and develop appropriate and meaningful tasks that could promote learners' ability to think and reason effectively (Breen & O'Shea, 2010; Sullivan & Davidson, 2014; Nyman, 2016). Such meaningful tasks can create opportunities for learners to develop not only mathematics knowledge, but also skills to apply it (Zwahlen, 2014) Guberman and Leikin (2013) argue that in order to provide OTL, mathematics tasks should inform learners about the constructive nature of the learning process and the dynamic nature of mathematics problems as having different solution paths, and support learners' individual learning styles.

Shubashini and Sokkalingam (2011) concur with Zwahlen (2014) and further elaborate that OTL in trigonometry could be created when teachers consider some of the application tasks of trigonometric ratios that enable learners to connect procedural work to real-life situation. In Grade 10, learners begin the syllabus by relating diagrams of triangles to numerical relationships (similar triangles, basic trigonometric ratios 'SOH CAH TOA', special and reciprocals ratios and Pythagoras Theorem) and to manipulate the symbols involved in such relationships and later engage in working with practical examples (e.g., Angle of elevation and depression). Trigonometry task requires the use of all nature of tasks namely contextual, numerical and problem-solving in order to develop their mathematics understanding. Weber (2005) opines that in order to create OTL, the nature of tasks presented to learners should include both lower and higher-order cognitive demands as set out in the curriculum documents.

Both the theoretical and empirical evidence reviewed above show that tasks play a pivotal role in creating OTL trigonometry. Stein, Grover and Henningsen (1996) add that tasks are more than just content used to convey information through empirical discovery, but also guide learners to a particular theoretical concept that would ultimately enhance OTL. Breen and O'Shea (2010) put it as follows: "tasks with which learners engage go beyond driving what content the students learn and may determine how they come to think about, develop, use and make sense of mathematics" (p. 1). The role of mathematics tasks will further be discussed in the next section.

2.4.2.1. The role of mathematics tasks

As already mentioned, mathematics tasks play a major role in the teaching and learning of mathematics and are considered to be the central tool with reference to effective instruction (Sullivan, Clarke & Clarke, 2012; Moodley, 2013; Akayuure & Apawu, 2015). To show that mathematics tasks play a significant role during instruction, Lee, Lee and Park (2016) found that mathematics tasks connect teaching and learning inside the classroom and therefore become evidence of what learners actually 'do' in the classroom. From research (Zeringue, Spencer, Mark, Schwinden & Newton, 2010; Sullivan, Clarke & Clarke, 2012; Lee, Lee & Park, 2016), it can be summarised that mathematics tasks, designed and chosen by the teacher and executed by the learner, always play a crucial role in successfully strengthening the learning of mathematics. When the effective use of meaningful mathematics tasks is neglected, it can become an obstacle in learners' learning. This means that when learners do not get an opportunity to work on meaningful tasks, they cannot develop their problem-solving skills (Botson & Smith, 2009; Yesildere-Imre & Basturk-Sahin, 2016).

In view of the important role of mathematics tasks, teachers use mathematics tasks as a tool for transferring information to learners and engaging them with mathematics during and after instruction (Vale & Pimentel, 2011). It is through active engagement of learners in experimenting, discovering and solving problems, that learners are granted OTL mathematics (Chi & Wylie, 2014; Muijs & Reynolds, 2017; Watt, Carmichael & Callingham, 2017; Hilton, 2018). Shimizu, Kaur, Huang, and Clarke (2010) add that mathematics tasks determine the way in which leaners come to understand the content taught during instruction. These tasks, as prescribed and recommended by the CAPS document (DBE, 2011), are generally utilised in achieving the set of learning objectives (Wium & Louw, 2015; Mdladla 2017).

2.4.2.2. The types of tasks

The different tasks prescribed by the CAPS (DBE, 2011) are classwork, homework, projects, assignments, and investigation. These types of mathematics tasks serve different purposes and the integration of all of them during instruction can create a balanced way in which the learners acquire specific skills and knowledge. Several authors such as Chapman (2013), Jackson, Shahan, Gibbons and Cobb (2012) and Spaull (2013) agree that by using such a variety of different tasks, teachers can enhance learners' conceptual understanding of mathematics in line with the curriculum objectives.

The Grade 10 mathematics pace setter specifies that teachers should provide only classwork and homework trigonometry tasks during the first term of the academic year (DBE, 2011), and therefore no projects, assignments, and investigation were required during the time of the study. which follow in the next two sections The DBE (2011) emphasises that classwork activities in particular, should be done continually to enable the teacher to monitor learners' progress with the purpose of improving their learning. This also allows for time where learners can have interaction with the teacher and their peers. Hyde, Else-Quest, Alibali, Knuth and Romberg (2006) previously mentioned that the reason why learners do more of these classwork activities is to allow for practise and consolidation of work done in classroom and ultimately provide them with proficient OTL.

2.4.2.3. Nature of tasks

Current literature (Burkhardt & Swan, 2013; Cannon, 2016; Wang, Chen, Schweighardt, Zhang, Wells & Ennis, 2017) shows how the nature of tasks contribute to proficient teaching and learning. The nature of tasks has a significant impact on three vital elements in the teaching and learning of mathematics, namely the content taught; how learners learn; and how learners acquire knowledge and skills of mathematics (Coles & Brown, 2016). Henningsen and Stein (1997) and Viseu and Oliveira (2017) state that the nature of tasks can potentially impact and structure the way learners think. According to Henningsen and Stein (1997) and Viseu and Oliveira (2017), the nature of tasks can also serve to limit or to broaden learners' views of the subject matter with which they engage during and after instruction**.** There are learning and assessment tasks. The learning tasks refer to tasks used during the teaching and learning of mathematics, and have a formative nature (Schoenfeld, 2015). Assessment tasks can be formal or informal, and have a baseline, diagnostic, formative or summative nature (Yachina, Gorev & Nurgaliyeva, 2015). For the purpose of this study, I will only refer to learning tasks. Baskas

(2012) defines learning tasks as tasks that "determine what the learners will do to learn the content and accomplish achievement-based objectives" (p. 3). Learning tasks are set throughout the year and encourage learners to engage with mathematics. Shimizu, Kaur, Huang and Clarke (2010) agree with Baskas (2012) and elaborate further that learning tasks are also examples the teacher uses to teach learners a new concept or skill. Learning tasks involve activities such as classwork, homework, problems done on the board, assignments, projects and investigations. Wood (2017) states that all those activities provide information to be used to modify teaching and learning during and after instruction. In line with Baskas (2012) and Wood (2017), Blum (2015) holds the opinion that it is equally important that each task is meaningful and useful in ensuring that learners develop mathematical knowledge and skills.

Learning tasks are categorised and described in several ways. Tasks should be challenging (Nolte & Pamperien, 2017; Russo & Hopkins, 2017); rich in context (Bossé, Lynch-Davis, Adu-Gyamfi & Chandler, 2017; Pettersen & Nortvedt, 2018); and appropriate and worthwhile (Leikin & Sriraman, 2017; Yeo, 2017; Swars, Smith, Smith, Carothers & Myers, 2018). According to Ponte, Mata-Pereira, Henriques and Quaresma (2013) and Choy (2016), the appropriateness of tasks (worthwhile/non-worthwhile tasks) is determined by their degree of cognitive demand.

Authors (Sullivan, Zevenbergen & Mousley, 2003; Mkhwanazi & Bansilal, 2012; Hsu, 2013; Daroczy, Wolska, Meurers & Nuerk, 2015) have identified three types of tasks that can be used by both teachers and learners to solve mathematical problems. The tasks have an influence on the level of learner engagement (Nyman, 2016). Those tasks are contextual, numerical and problem-solving.

2.4.2.3.1. Contextual tasks

In recent years, there has been an increased emphasis on using real-life situations in mathematics so that learners can connect to the subject. In 2018, the DBE developed a document titled: The Mathematics Teaching and Learning Framework for South Africa. In this document, the DBE (2018) asserts that teachers should strive to use real-life settings to teach mathematics for conceptual understanding so as to enable comprehension of mathematical concepts, operations and relations. Umugiraneza, Bansilal and North (2017) conducted a study in the Department of Education in Kwa-Zulu Natal and found that mathematics teachers should be encouraged to use realistic contexts to make mathematics more meaningful and accessible for all learners. Thus, contextual tasks are described as the real or imagined situation in which a mathematics task is embedded (Clarke & Roche, 2018). Researchers (Sullivan, Zevenbergen, & Mousley, 2003; Gainsburg, 2008; Henhaffer, 2014; Wijaya, van den Heuvel-Panhuizen, & Doorman, 2015) seem to concur with Umugiraneza, Bansilal and North (2017) and the DBE (2018) that contexts can be used to motivate, to illustrate potential applications, and to anchor learner understanding.

2.4.2.3.2. Numerical tasks

Numerical tasks continue to be a critical section in the school mathematics curriculum and, indeed, a very important part of mathematics as such. The tasks are in the form of pure mathematics and can be separated into lower- and higher-order thinking numerical tasks. The lower order thinking numerical tasks fall under knowledge and routine procedure that requires minimal thinking and reasoning. Higher order thinking numerical tasks fall under complex and problem-solving procedure and these tasks require creative and critical thinking (Heong, Othman, Yunos, Kiong, Hassan & Mohamad, 2011). Tasks categorised under this level, do not have simplistic and routine procedure that can be used to solve the problem, but instant, learners need come up with their own way of finding solution, which tend to require utilization of high cognitive level calculations (Kalobo & Toit, 2015). For learners to engage with trigonometry, they need to be exposed to both lower and higher-order numerical tasks. The use of both lower and higher order numerical tasks can assist learners' to make connection between ideas and procedures when engaging in complex trigonometry tasks (Son & Kim, 2015).

2.4.2.3.3. Problem-solving tasks

Georgius (2014) found in a survey that problem-solving tasks require learners to make deep mathematical connections and develop conceptual understanding. Learners are encouraged to engage in reasoning and flexible problem solving (Georgius, 2014). Chapman (2013) perceived that problem-solving tasks can usually be represented in multiple ways, including by means of manipulative materials, diagrams and symbols as well as real-life situations. At the core of this, is the demand for learners to analyse task constraints that may limit possible solution strategies and to apply complex, non-algorithmic thinking (Champman, 2013). Teachers' design of mathematics tasks is arguably the most influential in determining the degree of learning (Durik, Shechter, Noh, Rozek & Harackiewicz, 2015; Nagle & Styers, 2015) as will be shown in the next section.

2.4.2.4. Mathematics task design

Nagle and Styers (2015) affirm that the nature of tasks also influences the degree of learning. Therefore, it seems logical to interrogate not only the role and nature of mathematics tasks, but also the process of designing various types of tasks.

According to Margolin (2013), design in mathematics involves designers such as professional mathematicians, teacher educators, teacher researchers, authors, publishers, learners, or a combination of these designers. Suzuka, Sleep, Ball, Bass, Lewis & Thames (2009) emphasise that the efforts to create opportunities for teaching and learning of mathematics content is centred on designing tasks. In this study, task design is regarded as entailing two crucial elements, namely the teacher and the learner. Teachers, as the one element, play an important role in making decisions when planning and designing mathematics tasks to be used during and after instruction (Sullivan, Clarke, Cheeseman, Mornane, Roche, Sawatzki & Walker, 2014). Task design is part of teachers' planning of all activities that take place every day in the classroom. The learner, as second element in task design, should be actively involved in the learning process by working on tasks given to them in the form of classwork, homework, assignments and projects. With reference to teachers, who are the focus of this study, Jones and Pepin (2016) and, later, Zwahlen (2014) mention that teachers could adopt three related processes in designing tasks. These processes are the selection of mathematics tasks from the available resources; the adaptation of already-existing tasks; and the creation of tasks from scratch; or even a combination of these three processes.

Studies (Guberman & Leikin, 2013; Sullivan, Askew, Cheeseman, Clarke, Mornane, Roche & Walker, 2015; Johnson, Severance, Penuel & Leary, 2016) find that the way teachers design mathematics tasks, and the way in which learners are asked to approach problems, determine the quality of mathematics in the classroom. Clarke and Roche (2018) argue that the decisions teachers make when designing tasks are critical and are based on their subject knowledge, experience of teaching, and how they think of ways in which learners can react on the tasks (Blömeke, Hoth, Döhrmann, Busse, Kaiser & König, 2015). To emphasise the importance of the process of designing mathematics tasks in teaching and learning, Clarke and Roche (2018) emphasise that learners' interests and engagement in the learning of mathematics, and trigonometry in particular, may be positively or negatively influenced by the nature of tasks. Geiger (2016) explains that the nature of tasks not only influence how the teacher and learner engage with trigonometry, but also the level of thinking required from the learners.

2.4.2.4.1. Select, adapt and create tasks

The implication created by some researchers (Venkat & Spaull, 2015; Nagnedrarao, 2017; Tchoshanov, Cruz, Huereca, Shakirova, Shakirova & Ibragimova, 2017) is that the relevance of content is what provides opportunities for learners to learn mathematics. Therefore, mathematics content is transferred to learners through the use of carefully selected, adapted and created mathematics tasks by using various quality textbooks. Some literature uses the words "select", "adapt" and "create" as integrated words in designing mathematics tasks. In this study, I will refer to these three concepts separately. The concept "select" could refer to taking a specific task (called "an exercise') from the textbook. "Adapt" alludes to making an exercise more realistic and relevant to a certain group of learners. Lastly, "create" points to generating an entirely new task.

- **Selection of tasks**

Wijaya, et al. (2015) believe that teachers' task-selection choices are influenced by available curriculum resources (e.g. textbooks). The DBE prescribes a textbook that provides teachers with ready-made materials and exemplars of the content to be covered in each grade (Galant, 2013). The aim of prescribing textbooks is to assist teachers in task selection and to save time during preparation and instruction. The recurrent theme of "lack of time" points to time as one of the main problems in classroom management (Assude, 2005). "There is not enough time for teaching mathematics" represents an excerpt from the study conducted by Assude (2005). Assude (2005) approaches the problem of task selection in mathematics classrooms by looking at strategies developed by teachers to manage the different kinds of time in a classroom in a French primary school. A similar study conducted in South Africa (Hurst, 2014) affirms that effective time management strategies can play an important role in the selection of mathematics tasks. Hence, the CAPS document for Grade 10 learners specifies the maximum of hours (i.e., 4.5 hours per week) to allow the teaching and learning of trigonometry (DBE, 2011). It is worthwhile mentioning that in addition of assisting teachers to manage time properly, the textbook also provides a structure, order and progression in the teaching and learning process, that is when the teacher uses a quality textbook. It provides a systematic progression of learning towards the acquisition of new knowledge (Stara, Chval & Stay, 2017).

- **Adaption of tasks**

The adaptation of tasks occurs when the teacher either modifies/redesigns/reconstructs existing tasks to create new tasks. Creating new tasks gives teachers an opportunity to assess learners' knowledge and skills regarding the specific learning outcomes of the lesson. Wijaya, et al. (2015) suggests that the teachers alter, adapt, or translate textbook offerings to make them appropriate for their learners. Henhaffer (2014) agrees and mentions that teachers should constantly adapt existing tasks to introduce learners to relevant content.

- **Creating tasks**

Creating a task occurs when a teacher creates a new task from scratch. Tasks created from scratch could give teachers an opportunity to assess all the cognitive levels specified in the CAPS document. DBE (2011) encourages teachers to use all the cognitive levels when creating tasks. The process allows teachers to assess the specific objectives of the lesson (Hopkins, 2017).

In conclusion, in selecting, adapting and creating tasks, a teacher can use a variety of textbooks, previous question papers, and curriculum documents (Son & Kim, 2015). Those mathematics problems are created according to specific cognitive demands (Fujii, 2016). Plastina (2015) states that well-created tasks reduce the gap between learners' present disposition and their expected goals. CAPS provide clear and succinct statements of the level of challenge of tasks as shown in the next section.

2.4.3. Cognitive demands of mathematics tasks

Cognitive demands of mathematics tasks refer to the kind of thinking processes involved in solving the tasks (Shimizu, Kaur, Huang & Clarke, 2010; Henhaffer, 2014; Johnson, Severance, Penuel & Leary, 2016). De Jager (2016) posits that cognitive demand is about creating and maintaining an intellectual challenge that encourages learners to improve mathematical skills and knowledge. Awareness and appropriate balance of these cognitive demands can enable teachers to prepare lessons that may engage learners' thinking processes and ultimately lead to better understanding of trigonometry content. The literature has identified two types of cognitive demands that should both be incorporated during instruction, namely lower- and higher-order thinking, that will now be discussed.

Lower-order thinking is characterised by the recall of information that requires learners to recall a fact, perform a simple operation, or solve familiar types of problems (Thompson, 2008). Stein and Smith (1996) state that lower order thinking tasks require learners to perform a memorised procedure in a routine manner. Martalyna, Isnarto and Asikin (2018) indicate that in selecting lower order thinking trigonometrical tasks teachers can encourage learners to develop skills such as understanding of basic trigonometry and connection to algebraic expressions.

With reference to higher-order thinking (HOT), helping all learners learn to think critically is an ambitious goal in mathematics education (National Research Council & Mathematics Learning Study Committee, 2001). Many education researchers (Henningsen & Stein, 1997; Pehmer, Gröschner & Seidel, 2015; Stevens, Lu, Baker, Ray, Eckert & Gamson, 2015; Kloppers & Vuuren, 2016; Palane, 2017) within the field of mathematics mention that HOT demands critical thinking and problem-solving skills. Empirical evidence gathered from sifting literature seems to conclude that higher order thinking, such as the ability to reason and think critically, is more likely to be realised when learners are given an opportunity to explore concepts prior to instruction.

Russo and Hopkins (2017) state that teachers need to pose both lower order thinking and higher-order thinking trigonometry tasks to reach all levels of learners' ability to understand concepts. Russo and Hopkins (2017) highlight various reasons for this need. Firstly, part of this could enable teachers to anticipate and address learner reactions when being challenged with both different levels of cognitive tasks. Secondly, teachers give an opportunity to learners to work on routine and procedural tasks as well as problem solving and conceptual understanding tasks. Literature has argued that learners are more engaged in learning when exposed to all cognitive levels (Tekkumru-Kisa, Stein, & Schunn 2015). The DBE has provided guidelines regarding cognitive levels of tasks that generate lower and higher levels of thinking. The levels are outlined in the CAPS document as the Bloom's Taxonomy.

2.4.3.1. Bloom's Taxonomy

Since the Bloom's Taxonomy was created in 1956, it has influenced mathematics teaching and assessment heavily throughout the world. It is still considered a significant guideline within the field of mathematics education (Adams, 2015). Testa, Toscano and Rosato (2018) state that Bloom's Taxonomy is a tool used by mathematics teachers to classify the levels of cognitive demands required in classroom situations. In the opinion of Adesoji (2018), and Khoza (2015) also, this taxonomy assists teachers in designing learning tasks and determining the simplicity or complexity of those tasks.

Bloom's Taxonomy classifies educational learning objectives according to six cognitive levels, namely: knowledge, comprehension, application, analysis, synthesis and evaluation. The levels are arranged from lowest to highest cognitive demand (Reeves, 2012; Assaly & Smadi, 2015). The knowledge level requires learners to only remember previously learned information while comprehension requires learners to demonstrate an understanding of the facts without relating it to any concept. Application involves using knowledge, as when a learner applies the knowledge to a new situation. On the other hand, the next three levels are higher-order cognitive levels where analysis, as the first higher cognitive level, requires learners to break down information into simpler parts and find evidence to support generalisations. Synthesis involves the ability to compile information in a different way or propose alternative solutions, while evaluation at the highest level requires learners to make and defend judgements about the value of ideas.

Based on Bloom's six cognitive levels in mathematics, the CAPS (DBE, 2011, p. 53) describes four cognitive levels according to which assessment has to be conducted. These four levels of cognitive demand are separated into two lower-level demands and two higher-level demands (Crompton, Burke & Lin, 2018). The two lower-level cognitive demands, namely knowledge and routine procedures, engage students in the task of memorisation and use of procedures without connection to meaning (Wu & Pei, 2018). The two higher-level cognitive demands, namely complex procedures and problem solving, involve learners in the process of using procedures with connection to meanings, and in the 'doing of mathematics' (Al Raqqad & Ismail, 2018). The percentage allocated to each category is: knowledge 20%, routine procedures 35%, complex procedures 30% and problem solving 15% (DBE, 2011).

2.4.4. Mathematics Curriculum and Assessment Policy Statement (CAPS) as prescribed policy

This section is divided into two parts, namely the intended curriculum and the enacted curriculum in creating OTL in classroom.

2.4.4.1. Intended curriculum

The intended curriculum is determined by the educational organisational system on a macro level (Phaeton & Stears, 2017). According to Seitz (2017), the intended curriculum refers to objectives set at the beginning of any curricular plan. Similarly, Kurz (2011) states that the intended curriculum establishes "the goal, the specific purposes, and the immediate objectives to be accomplished in mathematics education by both the teacher and the learner" (p. 5). The intended curriculum usually includes goals and expectations set by the curriculum policy makers and curriculum developers along with textbooks, official syllabi or curriculum standards set by a particular nation or organisation (Phaeton & Stears, 2017). The intended curriculum unfolds across three levels, namely the system level, the teacher level and the learner level in general education.

The system level refers to the collection of educational objectives according to grade, term, content and time needed to complete the topics. This general curriculum informs what content should be covered for a particular subject, grade, term, topics, sequences and instructional activities (Cai & Cirillo, 2014). Teachers are encouraged to use the curriculum every day in planning and organising their lessons and designing mathematics tasks. Hence, Botha (2011) states that teachers are free to plan and organise a system curriculum in the way it best meets the demands and needs of their learners.

In the second level, viz. teacher level, there are two curriculums, namely, the planned curriculum and the enacted curriculum. Both the planned and enacted curriculum occur in the classroom setting and are considered to be the teacher's responsibility. The planned curriculum represents a teacher's cumulative plan for covering the content prescribed in the system level (Matthew, Adams & Goos, 2016). In this planned curriculum teachers are believed to have the capacity to change the curriculum through their own interpretation of it (Brodie, Jina & Modau, 2009). Current literature provides evidence of several factors that influence teachers' planned curriculum. Firstly, personal pedagogical content knowledge (PCK) (Depaepe, Torbeyns, Vermeersch, Janssens, Janssen, Kelchtermans, Verschaffel & Van Dooren, 2015); secondly, teacher subject matter knowledge (SMK) (Mosabala, 2018) and lastly, familiarity with the general curriculum. Kurz (2011) says that teachers in their classroom, in their own interpretation of the curriculum, may deliberately plan to emphasise certain content domains and omit other content. The planned curriculum presents challenges for others, as they may be unable to plan for comprehensive coverage of the intended curriculum due to missing content expertise or inadequate professional development experiences.

2.4.4.2. The enacted curriculum

The enacted curriculum at the teacher's level comprises the content of the classroom instructions and its accompanying materials (e.g., textbooks) (Pepin, Gueudet & Lerman, 2018). As for textbooks, most teachers consider textbooks as the principal resource for curriculum enactment (Jayathirtha, 2018). The enacted curriculum plays a central role in the proposed concept of OTL since it is primarily through the teacher's enacted curriculum that learners access the system curriculum. On this level, both the teacher and the learner engage with mathematics content (Kaur, Tay, Toh, Leong & Lee, 2018). The teacher engages with learners through pedagogical practice, teacher instruction strategies and mathematics tasks designed/or given to learners.

At the learner's level, the engaged curriculum represents those portions of content coverage during which the learner is engaged in the teacher-enacted curriculum. On this level, learners only learn those portions of the enacted curriculum which they are actively engaged with. At the end of the intended curriculum chain, the model posits the displayed curriculum, which represents the content of the intended curriculum that a learner is able to demonstrate via classroom tasks, assignments and/or assessments. In the next section, teacher specific factors influencing OTL are discussed.

2.4.5. Teacher-specific factors influencing opportunity to learn

Over time, teachers manage to have operational ability to deliver content to learners. They develop ways to make the subject comprehensive. Studies (Henhanffer, 2014; Mdladla, 2017) show that teaches rely on a combination of factors to enable mastery of content by learners. Firstly, teacher's content knowledge has an important role to play because OTL rests on teachers' understanding the subject they are teaching. Next, the way they choose to teach (methods and strategies used) is often important in provide OTL. Lastly, familiarity with the curriculum is significant for the realisation of OTL because as a result of the curriculum document, teachers become aware of the topics needed to be covered per term, subject content to be taught and time allocation for each topic. All these factors are elaborated further in this section.

2.4.5.1. Teachers' knowledge

Artzt, Armour-Thomas, Curcio, & Gurl (2015) define teacher knowledge as an integrated system of internalised information acquired over time about pupils, content and pedagogy and beliefs are defined as an integrated system of internalised assumptions about the subject, the students, the learning, and teaching. They further believe that beliefs function as an interpretative filter for teachers' goals and knowledge and strongly affect classroom practice. Their views on knowledge and beliefs correspond with Gess-Newsome, Lederman & Gess-Newsome (2002) except that Artzt et al. (2015) also describe knowledge as organized into systems. Artzt et al. (2015) strongly believes that any discussion on a teacher's knowledge cannot be restricted to knowledge of mathematics and knowledge of teaching mathematics but needs to include a discussion on teacher's beliefs. He believes teachers' actions in the classroom are strongly guided by what they believe about mathematics and the teaching thereof.

2.4.5.1.1. Teachers' pedagogical content knowledge

The original definition represented by Shulman's (1986) (cited in Krauss, Brunner, Kunter, Baumert, Blum, Neubrand & Jordan, 2008) states that personal pedagogical content knowledge (PCK) comprises of knowledge on how best to teach mathematics and ways to formulate the subject to make it comprehensible to learners. Similarly, Botha (2011) described personal PCK in terms of knowledge of how to plan the mathematics content to be taught; how to teach the mathematical content effectively; how to use learners' prior knowledge in designing mathematics tasks; knowledge of how to teach difficult topics and how to rectify learners' misunderstandings. This implies that teachers have the ability to influence how learners learn, what they learn, and how much they learn as well as the manner in which they interact with mathematics activities (Echazarra, Salinas, Méndez, Denis & Rech, 2016).

Studies indicate that there are two classroom practices relevant to PCK, namely, teacher-directed and learner-centred practices. In a teachers-directed practice, learners put all their focus on the teacher as an important source of information (Henhanffer, 2014). A learner-centred approach does not eliminate the teacher. A learner-centred environment facilitates a more collaborative way for learners to learn. The teacher models instructions and acts as a facilitator, providing feedback and answering questions when needed (Mdladla, 2017). The instructional goal in learner-centred practice, based on the constructivist principle of learning, is to create teaching and learning where knowledge is constructed by the teacher and the learner (Garrett, 2008; Caro, Lenkeit & Kyriakides, 2016). However, some learners maintain that teacher-directed approach is the more effective strategy. In most cases, it is best for teachers to use a combination of approaches to ensure that all learners' needs are met (Resilient Educator, 2020).

Teachers' content knowledge has an important role to play because OTL rests on teachers' understanding the subject they are teaching, knowing the structure and the sequencing of concepts, developing factual knowledge essential to mathematics and trigonometry in particular, and guiding their learners (either through a teacher-directed or a learner-centred approach) into the different ways of mastering the content.

- **Teaching approaches**

The nature of teaching affects the learning outcomes (Ko & Sammons, 2013). The way teachers choose to teach when using a teacher-directed or learner-centred approach in the classroom is often a direct reflection of their content knowledge, abilities and beliefs about mathematics. In a teacher-directed approach, teachers play an important role in the learning process during instruction. The teacher selects and organises the relevant information and provides it in a form of a lesson, classwork, homework or project. In this approach, learners are expected to follow the instructions, listen, and copy what is written on the board. De la Sablonnière, Taylor and Sadykova (2009) state that this approach is often necessary at the beginning of the topic as teachers introduce new concepts and link them to prior knowledge. At the beginning of Grade 10 trigonometry topic, learners are required to disclose their understanding of what they had learned in Grades 8 and 9 to integrate the knowledge with the new information. The prior knowledge includes the geometry of 2D shapes, the construction of triangles and the Pythagoras theorem, as well as angles (NECT, 2019), while the new information includes defining the trigonometric ratios and extending these definitions to any angle (including special angles), definitions of the reciprocals of the trigonometric ratios, calculations in trigonometry and simple trigonometric equations (NECT, 2015). The teacher-directed approach can also be beneficial when teaching a large group of learners in a short period of time (De la Sablonnière et al., 2009). The teacher models instructions and acts as a facilitator, providing feedback, answering questions when needed and emphasising important sections (Mdladla, 2017). In this more teacher-centred view of mathematics education, assessment typically focuses on memorisation and routine procedure. Schwerdt and Wuppermann (2011) indicate that at times the approach does not produce a deep level of learner understanding as the teacher becomes the predominant source of information during instruction.

In a learner-centred approach, teachers facilitate learning and tend to encourage an active learner role (Mdladla, 2017). This approach is characterised by active learning and includes discussions and group work, such as cooperative learning, the project method, role-play, discovery, and experimentation (Jacobs, Vakalisa, Gawe, 2016). Learners are actively engaged in the processing of information and constructing of knowledge (Mykrä, 2015). Teachers who use a learner-centred approach "view knowledge through lenses of social and relational processes and therefore prioritise learners' individual processes of constructing personal knowledge and understanding" (Moate & Cox, 2015, p. 382).

Both teacher-directed and learner-centred approaches can be used effectively to create OTL trigonometry (Resilient Educator, 2020). When deciding what teaching method to use, a teacher needs to consider the topic, learner prior knowledge of the topic, learning objectives and the classroom environment.

- **Teaching strategies**

The emphasis on empowering learners to have an OTL trigonometry requires effective teaching. Effective teaching is not only about the actions of the teacher, but more about the learning environment that a teacher creates in the classroom (Saleem, Alimgeer, Saleem, Khurram, & Saleem, 2013). Effective teachers regard academic instruction as “their main classroom goal and have academic direction, creating an environment which is both social and task-oriented” (Kunene, 2013, p. 46). Academic instruction is based on appropriate teaching strategies used during instruction (Ismail, Shahrill, & Mundia, 2015), such as:

- Problem-based learning – Learners are encouraged to be involved in the process of teaching and learning by constructing and applying knowledge in social activities (Savery, 2015). This strategy is believed to promote learner development of critical thinking skills, problem-solving abilities and communication skills (Alrahlah, 2016).
- Cognitively-guided instruction – The teacher uses the learners’ existing knowledge gained from their answers to her questions, and builds her lesson on that (Berger, 2017).
- Inquiry-based learning (discovering) – Learners take primary accountability for their learning, particularly in exploring and solving mathematics tasks, which in turn lead them in the process of discovering (Trung, 2014).
- Active learning – This learning takes the form of group discussion, group work (pairs, mixed, cooperative) and experiments. During the process of active learning, learners are expected to contribute to the work by sharing their ideas and arguing intellectually in order to solve the problem (Hasan & Fraser, 2015).
- Differentiated teaching – This approach stresses that a single teaching style will not accommodate every student, especially when it is not matched with student needs (Suprayogi, Valcke, & Godwin, 2017). This type of instruction accommodates the diversity of students, adopting specific teaching strategies and monitoring individual student needs (Suprayogi, Valcke, & Godwin, 2017)

- Direct teaching – A teaching strategy where content, skills, and knowledge are transferred from the teacher to the learners. During this strategy, the teacher stands in front of a classroom and presents the information. Thus, learners are expected to follow instructions carefully (Dimitrios, Labros, Nikolaos, Maria & Athanasios, 2013).
- Interactive lecture demonstration – The teacher uses demonstrations (e.g. models, drawings, videos) to integrate content with real-life situations. The instruction process occurs while engaging in experiments by the teacher (Taufiq, Suhandi, & Liliawati, 2017).

What learners understand about trigonometry is almost entirely dependent on the experiences that the teacher creates daily in the classroom. Learners' experience of trigonometric concepts is based on the time spent on the subject and tasks. 'Time on tasks' is defined as the amount of time learners spend and actively engage in solving learning tasks (Kunene, 2011). The implication is that time spent on tasks allows learners to be actively involved, committed and attentive to the classroom activity. Time allocated and spent can be measured by several factors such as the teacher's classroom management, teaching approaches and teaching strategies.

2.4.5.1.2. Teachers' subject matter knowledge

In order to improve mathematics performance throughout the country, the DBE attempts to reform the subject matter knowledge of the teachers. The subject matter knowledge is only one component of the knowledge of a well-prepared teacher (Speer, King & Howell, 2015). Studies have provided evidence that teacher subject matter knowledge (SMK) is a foundational component during instruction (Nixon, Campbell & Luft, 2016). Rollnick and Mavhunga (2016) find that teachers' SMK influences their methods of presenting the subject matter (teachers' approaches) to learners, their design of mathematics tasks and choice of instructional strategies. The premise of the SMK is the teacher's role to help learners achieve understanding of the subject matter. In trigonometry, teachers themselves need to have solid knowledge of trigonometric ratios, inverse trigonometric ratios, triangles, angles, special angles, elevation and depression angles and solutions to simple and complex trigonometric equations (Rohimah & Prabawanto, 2019). A trigonometry teacher who has this knowledge has a high ability to help learners achieve a meaningful understanding of the subject matter and provide OTL. Hence, Labuschagne (2016) emphasises that teaching can not take place without solid SMK. Another important factor that studies show that it is influences teachers' planned curriculum relates to those teachers' familiarity with the general curriculum.

2.4.5.1.3. Familiarity with the curriculum

The last factor on a planned curriculum is familiarity with the general curriculum. Factors such as the teacher's familiarity with general curriculum also seems to influence how mathematics is being taught, how tasks are designed, and what method of instruction is used. The term 'familiarity' means the teacher needs to know the subject content as indicated in the curriculum and teach effectively, master the topics needed to be covered per term, and keep to the time allocation for each topic as stated in the CAPS document. CAPS specify the number of hours (4.5 hours) of classroom time allocated to trigonometry in each week. These hours constitute 16% of total classroom time allocated to learners' engagement with tasks. CAPS (2011) clarifies that this time allocation should allow for a sufficient depth of engagement with the content. The curriculum further provides guidelines on the different tasks and the cognitive levels required, providing examples. In an unpublished amendments document from the DBE, it is stated that the teachers experience a lack of guidance on the use of the prescribed cognitive levels. Troia and Graham (2016) believe that teachers who are familiar with the system curriculum tend to use curriculum material effectively during instruction.

The previous section dealt with the review of existing literature pertinent to mathematical tasks and the extent to which these tasks provide learners with an opportunity to comprehend content. The literature further provides some insight into the role of the teacher during instruction. In the next section, the conceptual framework is briefly provided as it provides the roadmap the study will follow.

2.5. Conceptual framework

According to Athanasou, Di Fabio, Elias, Ferreira, Gitchel, Jansen, and Mpofu (2012), a conceptual framework is defined as a constructed instrument used by the researcher when developing a model along which both information and knowledge are collected and analysed. The conceptual framework shows how elements of OTL (Stols, 2013; Kahn, 2014; Marshman & Brown, 2014; Walkowiak, Pinter & Berry, 2017; Yu & Singh, 2018) are integrated in the model of progression of mathematics tasks as proposed by Henningsen and Stein (1997). The elements of OTL are nature and cognitive demands of tasks, time on tasks, learner engagement, and teacher-specific factors. The model of progression describes three phases: determine through a document analysis how the national curriculum document, in this case CAPS, provides guidelines in terms of the nature and cognitive demands of trigonometry tasks for a specific grade. This is followed by observing teachers' task design in terms of selection, adaption, and creation of tasks, as used in the classroom, but also observing the teaching approaches and strategies used during instruction. Finally, it is observed how the choice of tasks as well as teacher-specific factors influence the time spend on tasks and the individual and/or group engagement in class. The conceptual framework therefore has two main pillars namely the mathematics tasks (in terms of its nature: contextual, numerical, and problem-solving tasks; and cognitive demands, and the teacher specific factors (in terms of teaching approaches and teaching strategies). The mathematics tasks comprise tasks used by the teachers as examples and given as classwork and homework.

The conceptual framework for this study is shown in Figure 2.1.

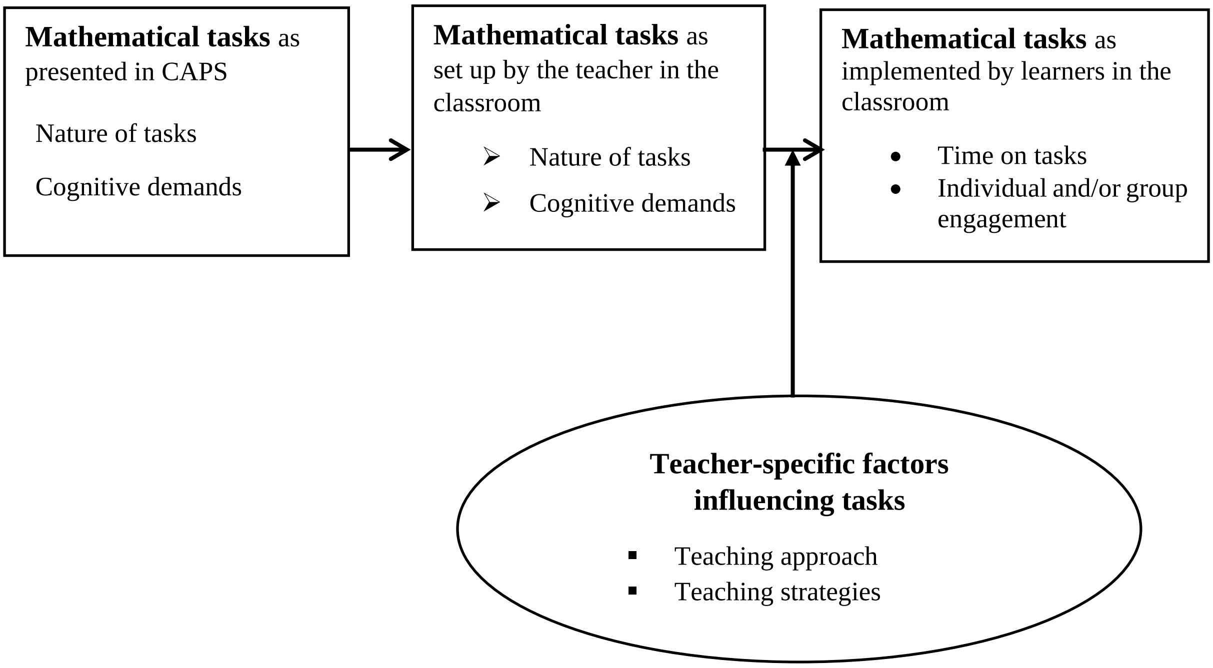


Figure 2. 1: Conceptual framework: Opportunity to learn trigonometry (Adapted from Henningsen, & Stein, 1997)

A discussion of the nature and cognitive demands of tasks, time on tasks, learner engagement, and teacher-specific factors as applicable to this study now follows.

2.5.1. The nature of the tasks

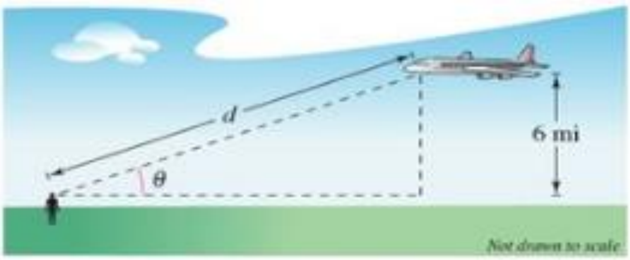
To distinguish between the natures of tasks, authors (Heong et al. 2011; Chapman, 2013; Wijaya et al. 2015) have identified contextual, numerical and problem-solving tasks. They are all clarified in examples given below.

2.5.1.1. Contextual tasks

Contextual tasks are defined as problems with experientially real contexts. The tasks play a significant role as a starting point of learners' learning to explore mathematical notions in a situation that is 'experientially real' for them (Widjaja, 2013).

The problem below is an example of a contextual task.

An airplane flying at an altitude of 6 miles is on a flight path that passes directly over an observer (see picture below).If θ is the angle of elevation from the observer to the plane, find the distance from the observer to the plane when $(a)\ \theta = 30^0, (b)\ \theta = 90^0, and\ (c)\ \theta = 120^0$.



According to Sullivan, Zevenbergen and Mousley (2013) the example above is designed around a real-life context that makes the problem 'becomes alive' for learners. The example presents the learners with a purpose in what they are studying and makes mathematics more engaging for them.

2.5.1.2. Numerical tasks

Table 2. 1 shows that the lower-order thinking numerical tasks consist of the mastery of skills and memorisation of procedures (Reys, Lindquist, Lambdin, & Smith, 2009). Learning in this type of task requires of learners to know how to use mathematical symbols, rules, and algorithms in order to solve mathematics problems (Hiebert, 2013). The exercises on lower-order thinking seem to provide learners with opportunity to engage in rote memorisation.

Sullivan et al. (2014) consider higher-order thinking numerical tasks as challenging to learners. Tasks categorised under this level require learners to engage in complex mental work in finding the solution to the problem (Armstrong, 2016). In addition, they require learners to solve problems by utilising high cognitive level calculations (Kalobo & Toit, 2015). Higher numerical tasks can be in the form of diagrams, pictures and symbols. Table 2. 1 above shows an example of a task requiring complex procedures. Examples of both lower- and higher-order thinking numerical tasks are given below:

Table 2. 1: Lower- and higher-order thinking tasks

Lower-order thinking numerical tasks	Higher-order thinking numerical tasks
Use the calculator to find the value of θ in the equations below: a) $\sin\theta = 0.78^0$ b) $4\ \cos\theta = 3\sin 41^0$ *Use your calculator to find the values of the trigonometric ratios.* a) $\cos(28^0 + 3^0)$ b) $\frac{5}{\cos 25^0}$	*Use the special angles to evaluate:* *a)* $\sin 90^0 - \cos 60^0 . \sec 60^0 + \csc 45^0 . \cos 45^0$ *b)* $1 + cot^2 60^0$ *Find the value of the following, without a calculator:* *a) If* $9\ \cos A - 7 = 0$, *find the value of* $csc^2 A$ *b)* $2\tan(2x\ + 12^0) - 3 = 1$ *c)* $\frac{\cos 60^0 . \sin 30^0 . \tan 60^0}{\sin 30^0 . \tan 30^0 . \cos 60^0}$

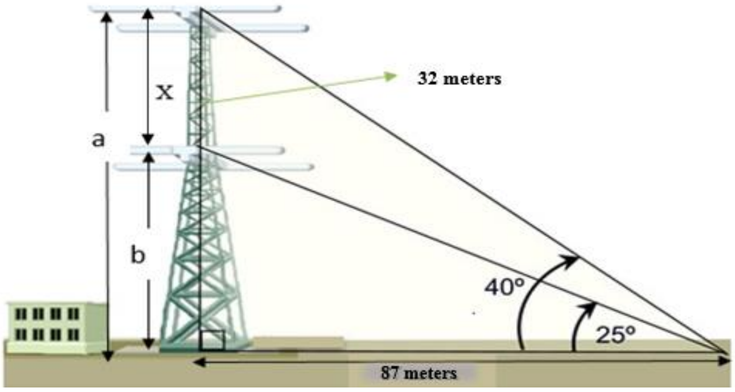
As shown above, tasks can vary not only with respect to mathematics content but also with regard to the cognitive processes involved in working on them (Shimizu, Kaur, Huang, & Clarke, 2010).

2.5.1.3. Problem-solving tasks

Problem-solving tasks allow learners to examine the question, find the key ideas and choose the appropriate strategy for solving the question. The tasks encourage learners to believe in their ability to think mathematically.

An example of a problem-solving task is given below:

A radio station tower was built in two sections. From a point 87 meters from the base of the tower, the angle of elevation to the top of the first section is 25º, and the angle of elevation to the top of the second section is 40º. To the nearest meter, what is the height of the top section of the tower?



From the example above, the key characteristic of problem-solving tasks is that it has the potential to engage learners with critically and analytically challenging tasks that enhance learners' mathematical understanding and mathematics skills development.

2.5.2. The cognitive demands of mathematics tasks

Cognitive demands put an emphasis on the kind of thinking processes involved in solving the mathematics tasks (Henhaffer, 2014). Henningsen and Stein (1997) concur that this emphasis on mathematics tasks shapes the reason for students' learning as they place a certain degree of cognitive demands on those learners. Awareness of these demands could enable teachers to prepare tasks that may engage learner's thinking processes. The four cognitive levels as stated in the CAPS are now discussed.

Knowledge, as the lowest level of cognitive demand, is referred to as "retention of specific, discrete pieces of information" (Adams, 2015, p. 152). These pieces of information are represented in terms of facts, definitions, identifying rules, and the direct use of formulae (Crompton, Burke & Lin, 2018). Tasks that fall under the knowledge level require learners to follow known procedures and test whether learners have gained specific information from the lesson. This level leads learners to lower-order thinking skills since learners are only required to reproduce previously learned facts with no connection to the meaning (Ubuz, Erbaş, Çetinkaya, & Özgeldi, 2010).

Routine procedure is described by researchers (Ubuz, Erbaş, Çetinkaya & Özgeldi, 2010; Long & Dunne, 2014; Tan, Ng & Shutler, 2017) as foundational cognitive learning. Tasks in this level are characterised by requiring familiar algorithms, well-known procedures, labelled diagrams and simple calculations (Berger, 2011; DBE, 2011; Morton & Colbert-Getz, 2017). In line with the mentioned characteristics, Wang (2018) opines that this type of task requires little effort of thinking and reasoning. Adesoji (2018) adds that learners only demonstrate understanding of facts and ideas by organising and comparing information when completing the task. Overall, routine procedures tasks require limited cognitive demand for successful completion of the tasks (Son & Kim, 2015).

In contrast to knowledge and routine procedures, complex procedures and problem solving involve higher-order thinking which requires creative and critical thinking skills (Heong, Othman, Yunos, Kiong, Hassan & Mohamad, 2011; Samo, 2017). The tasks in higher-order thinking highly require critical and non-algorithmic thinking (Kalobo & Toit, 2015). Such tasks are challenging, and they require mastery of mathematics concepts (Son & Kim, 2015). Kaweesi and Miiro (2016) conclude that higher-order thinking tasks require a critical thinking process directed at accomplishing classroom objectives with no routine solution method that is obvious to learners.

To conclude, all tasks mentioned in the four cognitive levels provide mathematics learners with OTL trigonometry. The opportunities enable learners to engage in lower to higher-order cognitive demanding tasks. Providing learners with sufficient OTL mathematics requires a sustained opportunity for learners to actively engage in both lower and higher cognitive thinking. However, a current study (Ni, Zhou, Cai, Li, Li, Q, & Sun, 2018) reveals that some teachers fail to engage learners in higher cognitive level tasks as there is not enough time for learners to engage in them.

2.6. Conclusion

Chapter 2 contains a critical analysis of national and international views on aspects such as the concept of OTL trigonometry, the nature and role of mathematics tasks and how the design of tasks and teacher-specific factors influence learner engagement. Furthermore, the CAPS is analysed in terms of its nature as intended curriculum. The cognitive grading of tasks in line with the Bloom’s Taxonomy is also provided. The chapter ends with a presentation of the conceptual framework that guided the study. The next chapter provides a discussion of the methodology used in this study.

CHAPTER 3
METHODOLOGY

3.1. Introduction

In this chapter, I give a description of the methodology used in this study, starting with the research paradigm underpinning the study. A brief discussion follows of the ontological and epistemological assumptions of the research study, followed by a discussion of the choice of qualitative research as research approach and descriptive case study as research design. Details of the research setting, techniques and procedures, and data analysis and interpretation, are provided. Finally, quality criteria in terms of trustworthiness and ethical considerations applicable to the study are discussed.

3.2. Research paradigm and paradigmatic assumptions

3.2.1. Research paradigm

The term “paradigm" represents a way of thinking adopted by a community of scientists in solving problems in their field. It has been defined and used in a number of related ways. Chilisa and Kawulich (2012) propose different versions of paradigms. The first version defines a paradigm as a shared world view that represents the beliefs and values in a discipline. The second version provides guidelines on how to solve problems within a particular discipline. Poni (2014) adds to the versions by stating that a paradigm is a framework containing all of the commonly accepted views about the direction research should take and how it should be performed. In my study, the underpinning paradigm is social constructivism (Creswell & Creswell, 2017).

According to Creswell and Poth (2017), constructivist theory involves how individuals construct knowledge in their real settings. In a school context, constructivism stresses the idea that learning is an active process in which learners learn from previous knowledge as well as from information acquired from teachers (Palincsar, 1998). This idea represents a movement from instructionist (objectivist) to a more subjective approach to learning, instruction and classroom management that emphasises the social and contextual nature of learning (Pitsoe, 2014).

In light of the social nature of learning, this study is rooted in social constructivism where meaning is socially constructed through the learners' involvement in the teaching and learning process. This theory assumes that understanding, significance and meaning are developed in coordination with other human beings (Amineh & Asl, 2015). Kim (2001) describes social constructivism based on three assumptions: reality, knowledge and learning. In terms of the first assumption, Ernest (2009) states that reality is constructed through different activities within a specific environment that an individual encounters and actively becomes involved in. The second assumption is knowledge, which is regarded as a human product of observed cultural and social activities (Kim, 2001). Learning, as the third assumption, refers to the idea that individuals, specifically learners, construct their own meaning during the learning process. Meaning is constructed through a combination of factors, namely instruction, types of tasks, and prior knowledge of the content (Hein, 1991). When considering these three assumptions, social constructivism views each learner as unique, with unique needs, knowledge and social background.

3.2.2. Paradigmatic assumptions

The nature of my study is based on two assumptions, namely ontological and epistemological assumptions. Ontological assumptions are concerned with what constitutes reality (Scotland, 2012). The ontological assumptions of this study responded to two questions, namely, what can be known within the classroom settings, and what is the nature of reality as experienced and constructed by Grade 10 teachers in public schools in Tshwane South District? The two questions indicate that I held a nominalist position when undertaking this study. The nominalist position enabled me to understand reality through words used by teachers, the experiences and practices of those teachers, and the product of their individual consciousness (Edirisingha, 2012). The epistemological assumption relates to the nature of knowledge created during the lesson (Holden & Lynch, 2004). The knowledge I gained during this study emanated from the experiences and insights of teachers. The underlying epistemological assumption is that by observing teachers in their social context (classroom) and conducting a document analysis of departmental documents and learners' books, I had an opportunity to become subjectively involved in coming to understand the research phenomenon (Scotland, 2012). The methodological framework complementing this proposed study will now be discussed.

3.3. Research methods

The research methods are described in terms of the research approach, research design, research site, population, sampling, data collection and data analysis.

3.3.1. Research approach

This study employed a qualitative research approach. Nieuwenhuis (2012) defines qualitative research as a naturalistic approach because its focus is on the natural settings where interactions occur. In this study the "natural setting" refers to the mathematics teachers' classroom environment in terms of their teaching approaches and strategies used to support the application value of mathematical tasks. The interactions occurred between the teachers, the learners, and the mathematics tasks. Through mathematics tasks, teachers provide learners an opportunity to learn mathematics effectively. The interactions allowed me to create an in-depth description of the phenomenon and not necessarily to generalise findings (Fouché, 2005). Silverman (2016) concurs that qualitative research is an approach for investigating the inner experiences of society and their understanding of their own reality. I investigated the various types of tasks (classwork, homework, assignments, investigations and projects) and cognitive levels of those tasks (knowledge, routine, complex procedures, and problem solving) as used by the teachers to provide their learners an OTL. This qualitative approach therefore allowed me to explain the phenomenon deeply and in detail (Queirós, Faria & Almeida, 2017).

3.3.2. Research design

Athanasou, Di Fabio, Elias, Ferreira, Gitchel, Jansen and Mpofu (2012) describe the research design as a strategy that guides the researcher to conduct the study. The design enabled me to choose the theoretical lens of the study and the method of collecting and analysing data (Mukherjee, 2017). Nieuwenhuis (2012, p. 70) defines a research design as "a plan of how one intends to accomplish a particular task, and in research this plan provides a structure that informs the researcher as to which theories, methods and instruments the study is based on".

The research design adopted for this study is a descriptive case study. Baxter and Jack (2008) define a case study as a method that provides the researcher with an opportunity to retain the holistic and meaningful characteristics of real-life events. It has been argued that the aim of the case study is not just to define a case for description's sake but to try and see the patterns, connections and dynamic that warrant inquiry (Henning, Van Rensburg & Smit, 2004;

Starman, 2013). A case does not necessarily refer to only a teacher or group of teachers, but focuses on a system of action (Nieuwenhuis, 2016). According to Maree (2016) a descriptive case study describes a phenomenon in which the real-life context occurs. I therefore adopted a descriptive case study because it allowed an in-depth exploration of how Grade 10 mathematics teachers design and use mathematics tasks in their natural context to provide learners with an OTL (Creswell & Garrett, 2008). One of the strengths of a case study is the use of multiple data collection instruments, and in this study observations, interviews and document analysis were used.

3.4. Research site, population and sampling

Neuman (1997) states that field researchers often draw diagrams and tables of a field site in order to orientate the readers of the research site(s) and population. Waure, Poscia, Virdis, Pietro and Ricciardi (2015) agree with Neuman (1997) and further emphasise that, in order to be able to interpret results from every kind of study and assess their generalisability, it is necessary to provide readers with sufficient information on the study population. Leedy and Ormrod (2005) define the population as predominantly a large collection of participants that forms the main focus of a scientific enquiry. It is for the benefit of the whole population that research projects are done. The population of this study consisted of Grade 10 mathematics teachers in South African secondary schools. Considering the relatively great sizes of populations, researchers often cannot test every individual in a population because it is often too costly and time-consuming, which was the case in this study (Waure et al., 2015). I therefore relied on specific sampling techniques as a scientific procedure to reduce the population size (Pickard, 2007).

This study adopted non-probability sampling to gather information relevant to the phenomenon. Morgan and Sklar (2012) posit that non-probability sampling is used frequently in qualitative research where the aim of the investigation is usually to create an in-depth description of the phenomenon, and not to generalise the findings (Nieuwenhuis, 2016). Both convenience and purposive sampling as types of non-probability sampling were used to recruit the schools and participants for this study.

To select the schools, convenience sampling was used because of the schools' accessibility and geographical proximity to me (Etikan, Musa & Alkassim, 2016). According to the Education Statistics in South Africa 2016 report from GDE (n.d.), there are 15 districts constituting the

Gauteng Department of Education. Tshwane South District, as one of 15 districts, was chosen. It consists of 10 regional offices from which I chose one district, namely Mamelodi. Two Section 21 public schools were subsequently chosen as research sites where qualitative data was collected. The South African School Act 84 of 1996 defines Section 21 schools as sponsored by, and governed by, government. Section 21 schools also manage their own grants received from the government (Roos, 2012). The sampled schools conformed to the inclusion and exclusion criteria specified in Table 3. 1.

The teachers, on the other hand, were chosen by using purposive sampling. Etikan, Musa and Alkassim (2016) define purposive sampling as the deliberate choice of a participant due to the qualities and the richness of the information the participant possesses. Palinkas, Horwitz, Green, Wisdom, Duan and Hoagwood (2015) mention that purposive sampling is a technique widely used in qualitative research studies. In this study I selected teachers who were knowledgeable and experienced in teaching Grade 10 mathematics as shown in the inclusion and exclusion criteria to be discussed later (Palinkas et al., 2015). Other aspects to consider when selecting participants are teachers' availability and preparedness (Palinkas et al., 2015), their interest in the study (Maree, 2016), and certain ethical issues (Moodley, 2013). Two teachers were selected using specific criteria as indicated in Table 3. 1 below. Teachers with different experiences were selected, not to compare them, but to gain insight into how individual teachers select mathematics tasks for the purpose of teaching and learning mathematics in their classrooms.

Table 3. 1: Criteria for inclusion and exclusion in the sample

	Criteria
Inclusion	• Two formerly disadvantaged public schools in Tshwane South District (Section 21) • Two Grade 10 mathematics teachers • Male and female teachers • Different race groups • Teacher with 10 or more years of experience in teaching mathematics
Exclusion	• Non-mathematics teachers • Private schools

3.5. Data collection

Qualitative data collection methods play an important role in providing rich and useful information to answer the research questions (Teddlie & Yu, 2007; Anyan, 2013). Observations and document analysis were used to collect data on tasks implemented by learners in two Grade 10 classes in a public school In Table 3. 2 below, a data collection timeline indicates when the data were collected in the two public schools located in Tshwane South district.

Table 3. 2: Data collection timeline

	Data collection instrument	Teacher A	Teacher B
1	Observation 1	05/03/2019	06/03/2019
	Document analysis	05/03/2019	06/03/2019
2	Observation 2	06/03/2019	12/03/2019
	Document analysis	06/03/2019	12/03/2019
3	Observation 3	07/03/2019	13/03/2019
	Document analysis	07/03/2019	13/03/2019
4	Observation 4	12/03/2019	03/04/2019
	Document analysis	12/03/2019	03/04/2019
5	Observation 5	14/03/2019	08/03/2019
	Document analysis	14/03/2019	08/03/2019
6	Observation 6	03/04/2019	
	Document analysis	03/04/2019	

3.5.1. Observation (Observation schedule)

Observation is an essential data gathering method as it can provide the researcher with an inside perspective on the group dynamics and behaviours in different settings (Maree, 2016). Kawulich (2012) defines observation as the "systematic description of the events, behaviours, and artifacts of a social setting" (p. 1). For the purpose of my study, I used a structured observation schedule during the classroom observations as a method of data collection. This schedule was prepared in advance based on the conceptual framework and research questions underpinning the study (Appendix H).

The purpose of choosing classroom observations had a dual purpose. Firstly, it enabled me to explore how the teachers create and utilise mathematics tasks. Secondly, it allowed me to consider how their teaching approaches and strategies influence the opportunities provided to develop mathematical understanding in their classrooms through specific mathematics tasks. The teachers were observed in their own natural settings, which provided me with an opportunity to gather information such as verbal and non-verbal communication, and their actions in delivering the content to learners (Barrett & Twycross, 2018). The advantage of observing teachers in their natural settings is that I gained insight into what actually happens in their classrooms (Kawulich, 2012). The lessons I observed ranged between 45 to 60 minutes. During the observations, I recorded the lessons by the means of a digital audio recording device. The purpose of recording the lessons was to get an accurate recall of what happened in class. Without disturbing the class, I took pictures of the tasks written and solved on the board and given to learners to do as classwork and homework, while I also completed the pre-determined observation schedule. After the lesson observation, I made field notes of the things I had encountered, which I thought might add to a description of the teacher-specific aspects and value to my findings. I was able to draw inferences that could not be obtained during audio recording, as there were other incidents that influenced the lesson (e.g. classroom destruction).

3.5.2. Document analysis

The document analysis was administered concurrently with the observations. Nieuwenhuis (2016) says that when using documents (textual data) as a data gathering method, the researcher needs to focus on all types of written communications that may shed light on the phenomenon under investigation. In this study, the written communications were in the forms of mathematics problems, projects, assignments, investigations, classwork and homework. According to Bowen (2009), such documents should be analysed and explored to elicit meaning, to gain understanding, and to develop empirical knowledge.

Guided by Nieuwenhuis (2016) and Bowen (2009), the main process of document analysis in my study was to produce an in-depth understanding of how the selected written texts (i.e., the completed classwork and homework tasks) provide the learners with an OTL mathematics content. This analysis enabled me to explore a specific objective of the study, namely whether the selected tasks complied with the mathematics CAPS document regarding the nature and cognitive demands of the tasks (see Chapter 1: Section 1.4).

The process of collecting textual data unfolded as follows:

- requested the annual teaching plan (ATP) from the teacher,
- made copies of tasks done by learners in their workbooks,
- requested evidence of projects or assignments done by learners, but the teachers did not give any,
- made observational field notes

3.6. Data analysis

Although there are various approaches to qualitative data analysis, I found the approaches made by Maimela (2015) and O'Reilly (2009) to be most appropriate for my study. Maimela (2015) states that the process of qualitative data analysis should aim to organise and arrange data into meaningful patterns and categories. To analyse the data of this study, I used deductive data analysis as I had pre-determined categories set out in the conceptual framework according to which I wanted to analyse and discuss the data. These pre-determined categories were; the nature and cognitive demands of tasks prescribed by the DBE and used by teachers, and the influence of that, together with teachers’ teaching approaches and strategies, on time spent on tasks in class as well as learner engagement.

3.6.1. Observation analysis

Creswell (2014) proposed an enterprise of six-step qualitative data analysis model. O'Reilly (2009) and later Maimela (2015) employed this model in their studies. In this study, I did not implement the steps entirely as Steps 4 (generating themes) and 5 (description and themes) are already determined in the conceptual framework.

The steps that were applicable to the observations phase of this study is shown below:

- Step 1: Organising and preparing the data for analysis

I began the process of analysing data by having the audio recordings of the observations transcribed. This qualitative data were analysed according to the pre-determined categories from the conceptual framework. The two pillars identified in the conceptual framework namely mathematics tasks and teacher-specific factors later became the two themes according to which data were presented and findings discussed. The pre-determined categories were called sub-themes. According to the conceptual framework, the two sub-themes under Theme 1 are nature of tasks and cognitive demands of tasks. The two sub-themes under Theme 2 are teachers’ teaching approaches and teaching strategies used during instruction.

- Step 2: Reading and reflecting on the data

The next step involved the process of data exploration where I read and reflected on the data to become familiar with it. Nieuwenhuis (2016) states that, after sorting out and typing the data, the researcher needs to know it inside out. This data was later divided into small parts containing some descriptive meaning.

- Step 3: Coding of the data

After reading and becoming familiar with the data, I began producing initial codes from the data. Coding involves breaking down data in such a manner as to specify practically and theoretically relevant patterns (Maguire & Delahunt, 2017). I identified relevant patterns related to how the teacher was delivering content. The way teachers choose to teach when the classroom was observed. Teachers could adopt either a teacher-centred or learner-centred approach or combination of both. I also observed how they selected and organised the relevant information as exercises from the textbook and provided it in a form of a lesson, classwork and homework. I then observed the method of delivery of content. The process involved the breaking down of information into patterns. Thereafter, coding began. The first two codes identified were teacher-centred and learner-centred approaches and those codes were assigned to Sub-theme 1: Teaching approaches under Theme 2: Teacher-specific factors. Next, other codes were developed based on evidence of strategies identified: Problem-based learning, active learning, direct teaching, cognitively guided instruction, interactive lecture demonstration and inquiry-based learning. The codes were relevant to a Sub-theme 2: Teaching strategies under Theme 2: Teaching-specific factors.

- Steps 4: Interpretation of the findings

The final step provided a final opportunity for analysis of the identified themes. This included the selection of extract examples from observations and relating the extracts back to the research objectives, theory, and literature. The overall product was a scholarly report of the findings as detailed in Chapter 4 of this study.

3.6.2. Document analysis (textual data)

The document analysis involved analyses of textual data through content analysis. I used a pre-determined analysis schedule according to which the tasks done in class by the teachers and learners, and tasks done at home by learners, were analysed. Regarding the textual data written on the board, I adopted approaches to content analysis as proposed by Hsieh and Shannon (2005) who define content analysis as the process of summarising and reporting the message retrieved from written data. Nieuwenhuis (2007) suggests that the researcher should follow a specific type of analysis, guided by rigor and certain procedures, to analyse texts and narratives. The content analysis procedure followed steps outlined by Cohen, Manion and Morrison (2002). Only four steps were considered applicable for this study:

- Step 1: Decide on the codes to be used in the analysis

I read through a small sample of text to determine the codes. Codes were identified through the process of discovering patterns with the data. The patterns included types of tasks given to learners namely classwork and homework and the levels of cognitive demands of those tasks. The next step involved identifying descriptive categories.

- Step 2: Conduct the categorising

I identified descriptive categories by analysing the nature and cognitive levels of the trigonometry content covered in the classwork and homework tasks. The descriptive categories imply that tasks (classwork and homework) were classified as contextual, numeric, and problem-solving tasks while cognitive levels were knowledge, routine procedures, complex procedures and problem solving.

In brief, numerical tasks were those classwork and homework tasks given to learners in the form of pure mathematics and they could be separated into lower and higher order thinking. Problem-solving tasks were regarded as those tasks that require learners to make deep mathematical connections and develop conceptual understanding. Those tasks demanded learners to analyse task constraints that may limit possible solution strategies and to apply complex, non-algorithmic thinking. Contextual tasks were those tasks that were based on a real-life situation or context. Teachers' explanations of content on the board was copied and analysed to determine whether it contained real-life examples in order to make mathematics meaningful to learners.

The lower-level cognitive demands are knowledge and routine procedures. Tasks that fall under the knowledge level were those that required learners to follow known procedures. Routine procedures require learners to apply familiar algorithms, well-known procedures, labelled diagrams and simple calculations. Lastly complex procedures and problem-solving tasks involved higher-order thinking. The higher-order thinking tasks demanded learners to apply critical and non-algorithmic thinking. I examined contents covered by teachers on the board to determine whether they were within lower and/or higher levels of cognitive demands.

- Step 3: Summarising

After the identification of descriptive categories, I allocated them to the applicable theme and sub-themes. The types of tasks (classwork and homework) were associated with Theme 1:

Mathematics Tasks and Sub-theme 1: Nature of tasks. The cognitive levels resorted under Sub-theme 2: Cognitive demands of tasks.

- Step 4: Making speculative inferences.

The interpretation of the data was grounded in theory.

3.7. Research trustworthiness

For a study to be of high academic quality, it has to be trustworthy. This implies that the findings of this study represented as closely as possible how the teachers from the selected secondary schools in the Gauteng Province designed and used mathematics tasks in order to create opportunities for learning. The following procedures as proposed by De Vos (2011) were used to enhance trustworthiness of the study, namely: continuing responsibility, timelines, observation and document analysis schedules, audit trails, availability of data, data analysis, as well as dependability of data.

Once a research study had been approved by the Research Ethics Committee in the Faculty of Education at the University of Pretoria, I had to adhere to and adopt the approved protocol and follow additional instructions from the committee. The continuing responsibilities that researchers have included, enrolling only those participants who met approved inclusion and exclusion criteria during the study process; properly obtaining and documenting informed consent; keeping accurate records and; increasingly being monitored by the research supervisor.

The time interval from data collection and analysis meets prescribed requirements. Neuman (2007) states that reporting accurate findings in a timely manner is one of the key indicators of a successful research. The authenticity of the study was ensured through the use of observation and document analysis schedule. Data collected can be made available when required on condition that all the ethical considerations guiding the release of data are met.

In analysing data, I was guided by the data analysis steps outlined by Creswell (2014) (Observations) and Cohen et al. (2002) (Document analysis). These steps ensured that the findings were a true reflection of the data obtained from the two participants. Keeping records of the raw data, field notes and transcripts helped me to systematise, relate, and cross-reference data, as well as facilitating the reporting of the research procedure. Again, I provided thick descriptions, so that those who seek to transfer the results to their own site can discern transferability. The research process is logical, traceable, and clearly documented.

However, the researcher may still encounter some challenges during the data collection process. The challenges are listed and briefly explained below:

- The Halo Effect occurs when the researcher is persuaded by cognitive bias of the overall impression, feelings and thoughts about the participants in the study (Cohen et al., 2011). As a professional, I was constantly aware of the positive and negative challenges I experienced in my field of work. The awareness of my feelings enabled me not to be unnecessarily impressed or disappointed by the conduct of participants. The field work and analysis of results were also conducted under constant supervision of my supervisor in order to minimise bias.
- Fear of victimisation takes place when teachers are reluctant to participate due to fear of being victimised by the school management. In order to address this challenge, I worked closely with the teachers by doing the following: I refrained from deceiving them, obtained their informed consent and assured them that no one would be able to trace their responses back to them. The responses were also kept anonymous, as the names of the schools and the participants were not captured on the rubric schedule.

3.8. Ethical considerations

The term “ethics” implies that researchers should conform to a set of rules and professional standards when undertaking a study (Strydom, 2014). A researcher must behave ethically by doing what is right, which includes treating participants fairly without any harm to anyone. The ethical principles involve showing respect to the participants and avoiding exposure of those participants to psychological harm (i.e., talking about matters unrelated to the topic, such as violence in schools) (Labuschagne, 2016).

The study was conducted in line with the University of Pretoria’s (UP) professional research code of ethics. Ethical clearance was obtained from the Ethics Committee of UP (Appendix A) before conducting the study, as well as from Gauteng Department of Education (Appendix B). During field work, letters of informed consent were signed by the principals, teachers and parents in which the purpose of the study and their roles as participants were explained. Teachers were also informed that participation was voluntary and that they were at liberty to withdraw from the research at any stage (Strydom, 2014). I confirmed the anonymity of the participants by using pseudonyms (De Vos, 2011). In the letters I made it clear that all the participants had been briefed about the purpose of the study and the methods to be used during data collection, and had been informed about confidentiality, anonymity and possible risk. Letters of assent were signed by the learners to inform them of the purpose of the study. They were also informed that I had to observe the teacher as s/he was presenting their mathematics lessons, and also had to look at the work they did in their workbooks (see the attached Appendices, C, D E and F). No photos of learners or teachers were taken, only of the mathematics tasks on the board and in learners' workbooks in order to analyse the data.

During the process of analysing the results, all the documents (e.g. rubric, learners' workbooks and audio recordings) which contained data from the participants were kept safe in a locked cupboard where I live. Only I had access to the data and, on request, my supervisor. Cohen et al. (2011) emphasises the importance of confidentiality, anonymity, and privacy as the main concerns in doing the research. This was accomplished by assigning a pseudonym to each participant and each school where they taught.

Finally, in communicating the research findings of this study, I did not disclose the real names of either the schools or the participants when reporting on the data and writing up the findings and therefore abided by the principles of honesty and transparency (Leedy, Ormrod & Johnson, 2019). By implication, these findings will be released in such a manner that utilisation by other scholars will be encouraged, since, according to Strydom (2014), this is the ultimate goal of any research project. This research will be available to all participants to review the collected data and findings of the study. Finally, data will be stored for the period of 15 years in the University of Pretoria's archives.

3.9. Conclusion

This chapter discusses in detail the research paradigm, assumptions, approach and design for this study. Following that, is a discussion of the research site, sampling procedure, data collections methods, and data analysis. Finally, the quality assurance criteria of the study and the ethical considerations that were taken into account are discussed. The next chapter presents the analysis and findings of the study.

CHAPTER 4

PRESENTATION AND DISCUSSION OF FINDINGS

4.1. Introduction

In Chapter 4, I report on the data collection process, the transcribing and the coding of the data based on the conceptual framework, the analysis of the data, and give information regarding the two Grade 10 teachers' biographical information and the trigonometry lessons presented. Each teacher is discussed according to the two themes identified from the conceptual framework (See Figure 4. 1 below), namely: 1) mathematics tasks and 2) teacher-specific factors. Arising from this discussion, the connection between the mathematics tasks, teacher-specific factors and the implementation of the tasks by the learners is discussed. The data collected from the two data collection instruments, namely document analysis and observations, are presented and discussed separately under each theme. In the discussion, the findings are related to the literature and other trends identified from the data, are discussed.

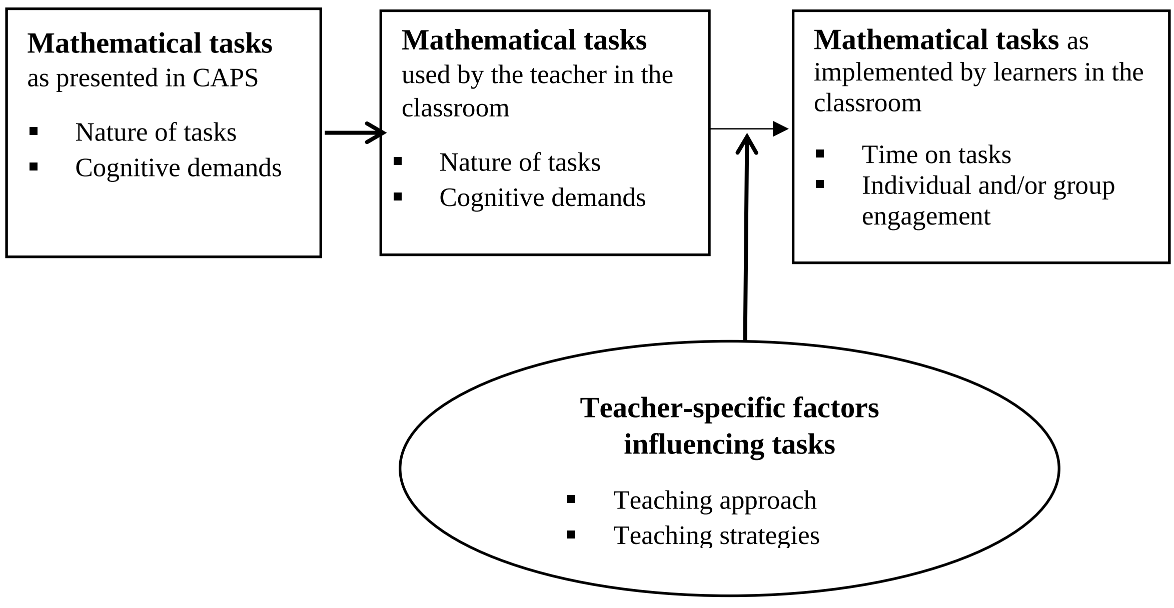


Figure 4. 1: Conceptual framework: Opportunity to learn trigonometry (Adapted from Henningsen, & Stein, 1997)

The research questions were asserted as follows:

Primary research question**:**

To what extent do mathematics tasks provide Grade 10 learners with an opportunity to learn?

Secondary research questions**:**

1) What are the requirements as stipulated by CAPS regarding mathematics tasks?
2) To what extent do mathematics tasks given by the teachers comply with the requirements from CAPS?
3) How can the implementation of tasks by the learners be described?
4) How do the teacher-specific factors influence the implementation of mathematics tasks by learners?

4.2. Data collection process

The process of data collection took place at two Section 21 secondary schools in the Tshwane South district in Pretoria. This process of collecting qualitative data was undertaken between March and April 2019 and covered a four-week period. Data was collected in line with the descriptive case study design adopted for this study. This design allowed me to collect two sets of data (document analysis and observations) simultaneously, during a single data collection phase (Creswell, 2007). Data was collected from two different classes for each of the two teachers who volunteered to participate in the study. Data was collected from five observations per teacher, with document analysis. The duration of the five observations and accompanying document analysis was approximately an hour each and was conducted during the trigonometry lessons. These sets of data were based on the teachers' delivery of their lessons.

4.3. Data analysis process

The study's deductive data analysis approach is discussed in detail in Chapter 3. In this section I only discuss the transcripts and coding of the data.

4.3.1. Transcribing the data

The digital audio-recorded data was transcribed verbatim and translated into English if the data was captured in either Sepedi or Tsonga. Transcripts of the observations were checked several times to ensure their validity. All information, including the Annual Teaching Plan (ATP), was loaded into a software programme called ATLAS.ti 8 for the purpose of analysis. ATLAS.ti 8 is a software programme for analysing qualitative textual, visual and audio data (Friese, 2019). This programme offered support to me during my data analysis process in which texts were analysed and interpreted using coding and annotating activities.

4.3.2. Coding of the data

In the process of coding the data, I adopted a deductive approach rooted in my conceptual framework. In line with my conceptual framework two themes, namely 1) mathematics tasks and 2) teacher-specific factors, were identified. From both themes sub-themes were generated. Several codes were identified and each was then assigned to its relevant sub-theme. The process of assigning codes was consistent with how the raw data was analysed. I used the software program ATLAS.ti 8 to code the transcripts according to a set of sub-themes and to provide the description of associated codes as tabulated in Tables 4.1 and 4.2 below.

4.3.2.1: Theme 1: Mathematics tasks

According to the conceptual framework, there are two sub-themes under Theme 1, namely Nature of tasks (TN) and Cognitive demands of tasks (TCD). The first column in Table 4.1 below indicates the two sub-themes, including the associated codes as stated in the second column. The third column provides the description of codes.

Table 4. 1: Sub-themes and description of codes for Theme 1

Sub-themes (content analysis and observations)	**Codes**	**Description of Codes**
Nature of Tasks (TN)	TNCW: Classwork	Classwork includes contextual, numeric, and problem-solving tasks.
	TNHW: Homework	Homework includes contextual, numeric, and problem-solving tasks.
	TNA: Assignment	An assignment can involve collections of previous examination papers that can expose learners to the structure of the papers and the content. An assignment includes problem-based and representational tasks.
	TNP: Project	The project should have clear specifications. The focus should generally be on problem-based tasks in real-life situations.
	TNI: Investigation	An investigation includes mathematics application,

		contextual and open questions
Tasks Cognitive Demands (TCD)	TCDK: Knowledge	• Straight recall • Identification of correct formula (no changing of the subject) • Use of mathematics • Appropriate use of mathematics vocabulary
	TCDR: Routine procedures	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved. • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class
	TCDC: Complex procedure	• Problems involve complex calculations and/or higher-order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real-world context • Could involve making significant connections between different representations • Requires conceptual understanding
	TCDP: Problem solving	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts

4.3.2.2: Theme 2: Teacher-specific factors

The two sub-themes from the conceptual framework under Theme 2 are the teaching approach (TA) and teaching strategies (TS). The columns in Table 4. 2 below indicate the two sub-themes, instruments used to collect data, codes, and the description of the codes respectively.

Table 4. 2: Sub-themes and description of codes for Theme 2

Sub-themes (observations)	Codes	Description of codes

Teaching Approach (TA)	TAT: Teacher-centred	TAT1: Instruction was about lecturing. TAT2: Learners are required to listen, duplicate, memorise, drill, calculate, and take notes. TAT3: Teacher follows prescribed procedure in the textbook when explaining/illustrating a mathematics concept, process or relationship. TAT4: Limited time for questions. TAT5: Limited time for engagement with tasks in their exercise books.
	TAL: Learner-centred	TAC1: Teacher consistently asks academically relevant questions that provide opportunities for learners to elaborate and explain their mathematical thinking. TAC2: Lesson is connected to learners' prior knowledge. TAC3: Use of meaningful real-world applications. TAC4: Posing challenging and interesting questions Teachers need to pose a variety of levels and types of questions using appropriate wait times that elicit, engage and challenge learners' thinking. TAC5: Encourages learners to disclose their own understanding of what they have learned. TAC6: The teacher explains and illustrates the content with appropriate diagram and gives concrete examples.
Teaching Strategies (TS)	TS: Teaching strategies	TS1: Problem-based. TS2: Active learning: small-groups, whole-class interactive work and cooperative TS3: Direct teaching. TS4: Cognitively guided instruction. TS5: Interactive lecture demonstration. TS6: Inquiry based learning.

4.4. Information regarding the two Grade 10 Participants

This section presents biographical information regarding the two participants. Both participants taught the first part of a trigonometric topic which focused on trigonometric ratios, reciprocals, solving two-dimensional problems, solving trigonometry equations and using diagrams to determine the numerical values of ratios for three weeks. I have used pseudonyms to protect the identity of the participants and name them Teacher A and Teacher B. Both teachers were teaching at a Section 21 (public school) in the Tshwane South district.

4.4.1. Teacher A

Teacher A is a 35-year-old male with 11 years' teaching experience as mathematics teacher in a public school. He completed a BEd (FET) Natural Science degree at Tshwane University of

Technology. He is teaching at a Section 21 (public school) in Mamelodi with 1 605 learners where most of the learners are black Africans. He is responsible for two Grade 10 mathematics classes and there are approximately 54 learners in each class. In eleven years of teaching, Teacher A has taught various subjects such as Physical Science and Mathematics Literacy. He is now in his fourth year of teaching Grade 10 Mathematics.

4.4.2. Teacher B

Teacher B is a 42-year-old male with 13 years' teaching experience as mathematics teacher in a public school. He obtained a Higher Education Diploma (HED) and BEd (Hons) Education Management at the University of South Africa. He is teaching at a Section 21 (public school) in Mamelodi with 1 220 learners where most of the learners are black Africans. He is responsible for one Grade 10 mathematics class and there are 49 learners in this class. He is the Head of Department (HoD) of Mathematics. In thirteen years of teaching, Teacher B has taught only Mathematics and Physical Science in Grade 10.

In the following two sections, Section 4.5 and 4.6, I present and discuss the findings from Teacher A and Teacher B according to the two main themes. The findings from the observations and document analysis are integrated to present an in-depth and meaningful view of each teacher's use of mathematics tasks. It should also be noted that the language of all direct quotes under Theme 2 from both participants have not been edited.

4.5. Teacher A

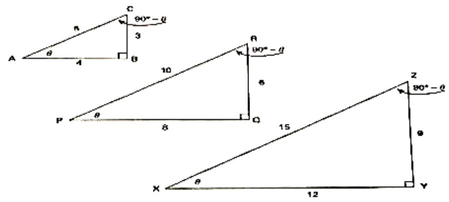
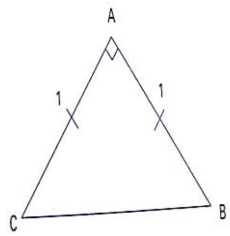
4.5.1. Theme 1: Mathematics tasks

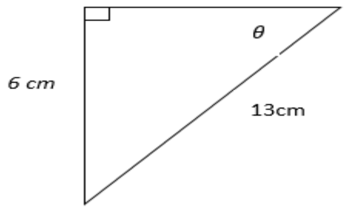
According to the conceptual framework the nature and cognitive demand of tasks are discussed. All discussions are structured in line with the defined order of the description of codes as depicted in Table 4. 1. In each of the five lessons, Teacher A provided learners with two different types of tasks, namely classwork and homework. During the time of data collection, tasks such as assignments, projects and investigations were not given to learners because, according to the Grade 10 mathematics pace setter, teachers were expected only to provide classwork and homework during the time of data collection as the method of assessment (DBE, 2011).

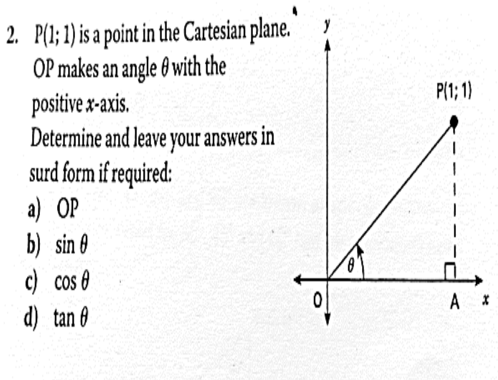
4.5.1.1. Classwork tasks done by Teacher A on the board

In each of the five lessons, one task was selected from the textbook. These tasks were used by the teacher during instruction to explain the trigonometry content. Learners were engaged by using their calculators with the focus on providing accurate answers. Table 4. 3 below shows the five tasks and a discussion of each task according to its nature and cognitive demand.

Table 4. 3: Classwork tasks explained by Teacher A

Classwork tasks		**Nature of tasks**	**Cognitive demands**
A	Exercise 5.2: no. 1 1. Use the triangles and complete the statements:  a) In △ABC, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ b) In △PQR, $\cos\theta = \frac{\square}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ c) In △XYZ, $\cos\theta = \frac{\text{adjacent}}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ d) In △ABC, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ e) In △PQR, $\tan\theta = \frac{\square}{\text{adjacent}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ f) In △XYZ, $\tan\theta = \frac{\text{opposite}}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$	Numerical	The task is on Level 1: Knowledge. The task required recalling and naming of trigonometric ratios which directed learners to use mathematical facts and to recall the ratios such as $\sin\theta$, $\cos\theta$ and $\tan\theta$. This task encouraged memorisation, rote and superficial learning. It also gave learners minimal opportunity to fully engage in trigonometry as it involved constant repetition of information.
B	Exercise 5.2: no. 8 8. △ABC is an isosceles triangle with $\hat{A} = 90°$ and AB = AC = 1 unit. a) Calculate BC in surd form. b) Calculate $\hat{B}$ and $\hat{C}$. c) Determine: (i) sin 45° (ii) cos 45° (iii) tan 45° 	Numerical	The task is on Level 2: Routine procedure. The question requires the use of the Pythagoras theorem to find the length of BC. Number b requires the application of properties of an isosceles triangle. In Number c, the task required little effort of thinking and reasoning, which was the identification and direct use of simple formulas, such as

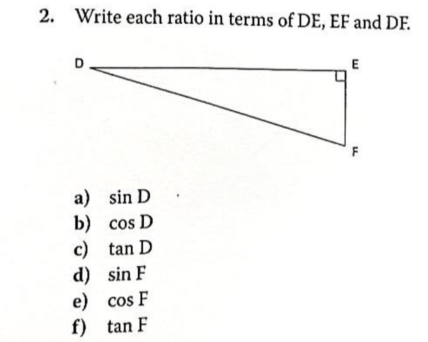
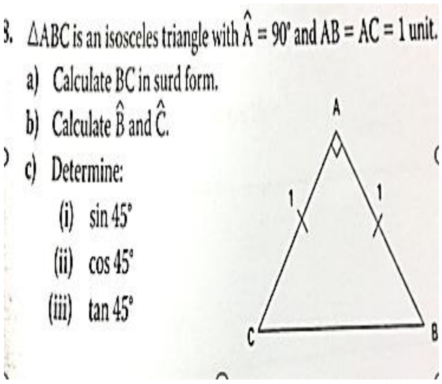
			$\sin\theta = \frac{opp}{adj}$. The task involved reproducing previously learned work.
C	Exercise 5.7: no. a 	Numerical	The task is on Level 2: Routine procedures. In this lesson, learners are taught to perform well-known procedures that are generally similar to those encountered in the everyday lesson such as solving for an unknown variable. Routine calculations involve: • applying a well-known algorithm in a familiar context (e.g. Pythagoras) • retrieving information from a simple diagram (triangle) • following simple steps to aid in the development of a solution. For example, using sine, cosine or tangent to find the value of θ as shown in the diagram (Task C).
D	Exercise 5.4: no. a-g 1. Use your calculator and solve for θ (correct to one decimal place) with the restriction that $0° < \theta < 90°$: a) $\sin\theta = 0{,}5$ b) $\cos\theta = 0{,}922$ c) $\tan\theta = 2{,}8765$ d) $\sin\theta = \frac{1{,}9}{2{,}6}$ e) $2\tan\theta = 1$ f) $3\cos\theta = 0{,}5$ g) $\frac{\sin\theta}{3} = 0{,}13$	Numerical	The task is on Level 2: Routine procedure. In terms of cognitive demands, the task requires lower-order thinking. The questions focus on solving for θ. Learners need to use their calculators to find the value of θ. This task requires the application of basic mathematics operations. It is

			not thought-provoking and does not stimulate interest or elicit mathematical thinking.
E	Exercise 5.8: no. 2  2. P(1; 1) is a point in the Cartesian plane. OP makes an angle θ with the positive x-axis. Determine and leave your answers in surd form if required: a) OP b) $\sin\theta$ c) $\cos\theta$ d) $\tan\theta$	Numerical	The task is on Level 2: Routine Procedure. The task requires lower-order thinking as not only recalling, but reasoning of basic trigonometric ratios in the Cartesian plane are required. For example, during instruction the teacher solves the following questions: finding trigonometric ratios ($\sin\theta$, $\cos\theta$ & $\tan\theta$) and the length of the hypotenuse (OP). The task does not encourage critical thinking as the teacher applies SOH CAH TOA as the only approach to solve the problem. SOH CAH TOA is a helpful mnemonic for remembering the definitions of the trigonometric functions sine, cosine, and tangent i.e., sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse, and tangent equals opposite over adjacent.

4.5.1.2. Classwork tasks done by learners in their exercise books

In each of the five lessons, two tasks were selected from the textbook and given to learners to do during the course of the lesson. One of the tasks was used by the teacher to explain the topic of the day. Table 4. 4 below shows the two tasks and a discussion of each task according to its nature and cognitive demand.

Table 4. 4: Tasks done by the learners as classwork

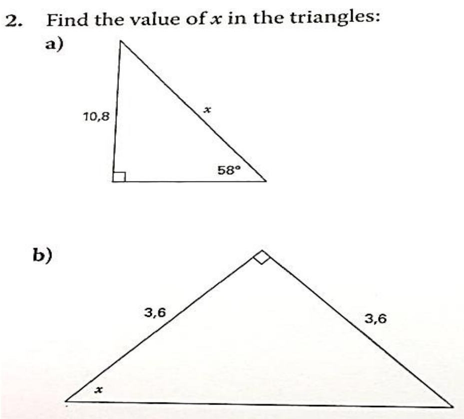
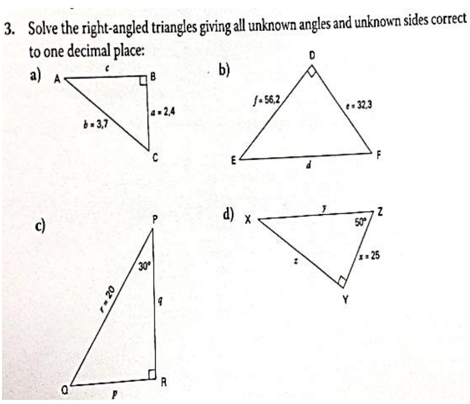
Classwork tasks		Nature of tasks	Cognitive demands
A	Exercise 5.2: no. 2  	Numerical	The task is on Level 1: Knowledge. This question requires lower-order thinking. The task involves simple calculations where there is an obvious route to the solution, using basic trigonometry ratios to find the different angles ($\widehat{D}$ and $\widehat{F}$). Learners need to recall the ratios of sin θ, cos θ and tan θ .
B	Exercise: 5.2 no: 8  	Numerical	The task is on Level 2: Routine procedure. This is the same task solved by the teacher during instruction when he explains the concept of triangles and special angles. Learners have to solve a well-known problem, use known procedures and straight recall, which are available from their class notes. The question requires the use of the Pythagorean theorem to calculate the length of the hypotenuse and use the basic trigonometry ratios.

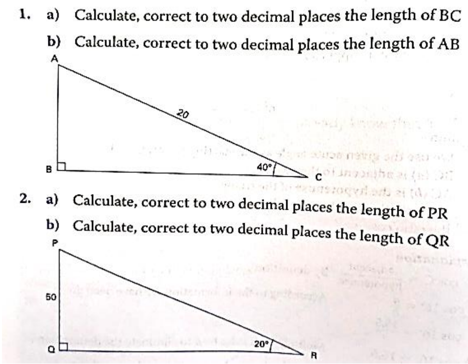
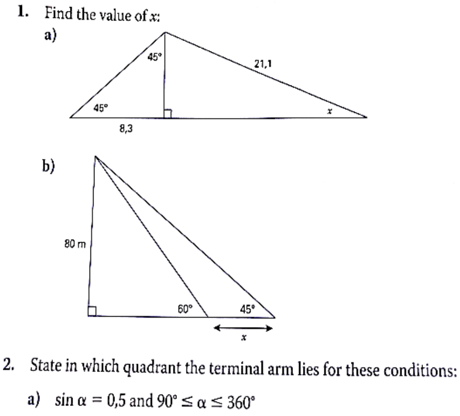
4.5.1.3. Homework tasks done by learners in their exercise books

In each of the five lessons, one task was selected from the textbook. These tasks were given to learners to do as homework. These tasks were solved by the teacher the following day as part of the corrections while learners spent time using their calculators to provide the teacher with an accurate answer. Table 4. 5 below shows the five tasks and a discussion of each task according to its nature and cognitive demand.

Table 4. 5: Tasks done by learners as homework

Homework tasks	Nature of tasks	Cognitive demands

A	Exercise: 5.2 no: 8 	Numerical	The task is on Level 2: Routine procedure. In terms of cognitive demand, the task requires lower-order thinking. In Question 2a, a right-angled triangle with one side and one angle are given, requiring the learners to calculate the hypotenuse side. Number 2b has right-angled triangle with two sides given, requiring learners to calculate one of the unknown angles (x). The questions require the use of basic trigonometric ratios (e.g. $\sin\theta$, $\cos\theta$ and $\tan\theta$).
B	Exercise: 5.4 no: i-p **i)** $4\tan(\theta - 30°) = 1$ **j)** $\frac{2\sin(\theta - 10°)}{3} = 0{,}102$ **k)** $4\cos 2\theta = 3{,}5$ **l)** $\frac{\tan 2\theta}{2} = 5{,}102$ **m)** $\frac{\sin(3\theta - 35°)}{3} = 0{,}17$ **n)** $2\tan(5\theta + 20°) = 24{,}9$ **o)** $\cos\theta = \sin 20°$ **p)** $\sin\theta = \tan 40°$	Numerical	The task is on Level 2: Routine procedure. The question is: “Use your calculator to solve for θ (correct to one decimal place) with the restriction of $0^0 < \theta < 90^0$.” This task requires the application of basic mathematics operations. In solving the task, learners need to apply arithmetic and algebraic knowledge and also use their calculators to find the final answer.
C	Exercise 5.11: no. 3 (a-d) 	Numerical	The task is on Level 2: Routine procedure. This task requires analysing and identifying given information in a right-angled triangle and solving the unknown side. Learners need to use the Pythagoras theorem and choose the correct ratios in attempting to solve the problem. The question is about solving unknown sides and angles in right-angled triangles.

D	Exercise 5.6: no. 1-2  1. a) Calculate, correct to two decimal places the length of BC b) Calculate, correct to two decimal places the length of AB 2. a) Calculate, correct to two decimal places the length of PR b) Calculate, correct to two decimal places the length of QR 	Numerical	The task is on Level 2: Routine procedure. The task is about solving the unknown sides where one side and one other angle are given in a right-angled triangle. Question 1(a) requires simple calculations such as choosing the correct trigonometry ratio which involves a few steps. Question 1(b) requires the use of the Pythagoras theorem.
E	Exercise 5.12: no. 1-2  1. Find the value of x: a) b) 2. State in which quadrant the terminal arm lies for these conditions: a) $\sin \alpha = 0{,}5$ and $90° \leq \alpha \leq 360°$	Numerical	The task is on Level 3: Complex procedure. For learners to solve for angle x in Questions 1 (a) and 1(b), they need to analyse and make connections between triangles. The problem requires not just a straightforward single answer, but multiple steps. Learners need to have the following knowledge: knowledge of arithmetic structure, and knowledge of algebraic notation. Question 2 (a) requires basic knowledge of the Cartesian plane and learners have to decide in which quadrant sin is positive in which the terminal arm will lie.

4.5.2. Theme 2: Teacher-specific factors

According to the conceptual framework, there are two sub-themes under this theme, namely teaching approaches and teaching strategies. All discussions on the sub-themes and codes are structured strictly in line with the order as they appear in Table 4. 2.

4.5.2.1. Teaching approaches

Teacher A often began his lessons with an introduction to the topic dealing with the notations and terminology used in trigonometry and definitions of trigonometric ratios. For example, he introduced new terms by saying: *"trigonometry is based on the concept of similar triangles,* *each ratio indicated is constant for the particular angle θ and each ratio has a name. This name is based on the position of the sides in relation to the angle".* He continued by explaining that *"the ratio that involves the opposite side and the hypotenuse is the ratio sin θ, the ratio that involves the adjacent side and the hypotenuse is the ratio cos θ, and the ratio that involves opposite side and adjacent side is the ratio tan θ."* After this brief introduction, he continued with the lesson. During the lesson, he discussed, explained, and demonstrated mathematical concepts that the learners must familiarise themselves with, such as similar triangles, Pythagoras' theorem, right-angled triangles, and notation for the sides of a triangle. In each of the five lessons, Teacher A used exercises from the prescribed textbook, as he was guided by the lesson plan stipulated by the DBE (TAT3). He used a number of examples from the textbook in order to reinforce learners' understanding. However, he worked out the examples by himself on the whiteboard while also providing the steps and algorithms required to solve the problems (TAT1). While he did this, the learners copied the notes into their exercise books (TAT2). Thus, no opportunity was given to the learners to work with the teachers by solving the problems on the whiteboard so that they could generate their own understanding (TAT5). Overall, the teaching approach involved lecture and demonstration as shown below:

Teacher: *In ∆ABC, given two sides (side AB and AC) and finding the unknown side (CD), we use Pythagoras' theorems. Are we together*?

Learners: *Yes.*

Teacher: *You know Pythagoras' theorem from previous grade, it the square of the hypotenuse and is equal to the sum of the squares of the other two sides*. (Demonstrating on the board)

Teacher: *In this case, this CD side is the hypotenuse, this one* (Pointing side AB) *is adjacent and the other one* (Pointing side AC) *is opposite.*

Learners (take notes)

The teacher then starts solving the problem by himself:

Writing: $BC^2 = AB^2 + AC^2$

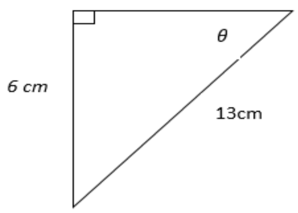
$= 1^2 + 1^2$

$= \sqrt{2}$

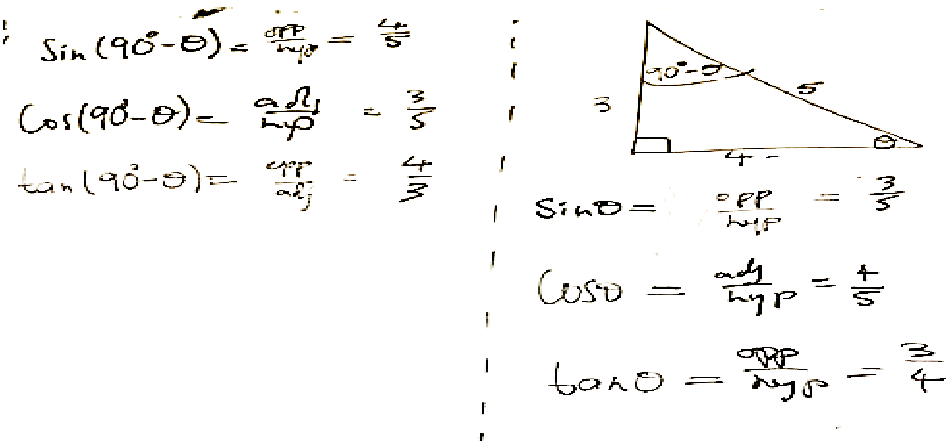
The above shows that learners did not make their own contributions; instead, they only passively listened to the teacher and took notes (TAT2). They appeared to accept whatever the teacher presented to them. Thus, a power differential between the teacher and learners was not diffused. The power differential is the inherently (in terms of the profession) greater power and influence that teachers possess over the learners. This power was depicted as learners continued to readily agree with content presented to them without posing questions. Three minutes prior to the end of the period, the teacher gave learners classwork to do which they never had time to work on as shown below (TAT5).

Teacher: *Ladies and gentlemen, do Exercise 5 (number 7, 8, and 9). For homework do Exercise 6 (number 1 and 2). I will see you all tomorrow*.

Before beginning instruction, Teacher A explored learners' background knowledge of trigonometry by asking them closed questions to allow them to integrate the new material into their existing knowledge. In almost every lesson, much of the time spent on assessing background knowledge focused on basic mathematical skills as shown in the example below:



The example above shows that Teacher A assessed background knowledge by asking the learners to name the different sides of a right-angled triangle. Some of these questions were adapted from the Grade 9 Mathematics textbook. The above content was fully covered in Grade 9 work (TAC2). All five lessons were based on pure mathematics and not on applying them to real-life situations in which learners could have had the opportunity to do problem solving (TAC3). The teacher explained and illustrated the content with appropriate diagrams from the textbook and gave abstract information to learners (TAC6). The diagram used from the textbook is shown below:



After talking, explaining and illustrating, Teacher A posed questions throughout the lessons which were largely closed questions, not challenging and not of interest to the learners (TAC4). He asked closed questions such as, "*Do you understand?", "Is my explanation of this length right*?", to which learners replied, "*Yes*". His questioning technique also turned the learners into mostly passive listeners, as shown in the following example:

Teacher: (After drawing on whiteboard) *What is this?* (Referring to triangle)

Learner: *Triangle*.

Teacher: *And this?* (Referring to right-angled triangle)

Learner: (Quiet)

Teacher: *It is right-angled triangle. Right-angled triangle has 90 degrees. With this right-angled triangle, I would like to introduce you to chapter called Trigonometry*.

Teacher: *Do you know what is Trigonometry?*

Learners: (Quiet)

Teacher: *Trigonometry is one of the chapters we cover from Grade 10-12. Trigonometry is the study of the relationship between the sides and angles of triangle.* (Teacher starts reading from the textbook): *In trigonometry we deal with angles of any size that also involves the study of angles turning in a circle (such as the hand of clock), waves (such as sound and light waves) and oscillation (such as a pendulum swinging). Yah, that's Trigonometry*.

Learners: (Quiet while listening to the teacher and writing the definition in the exercise books).

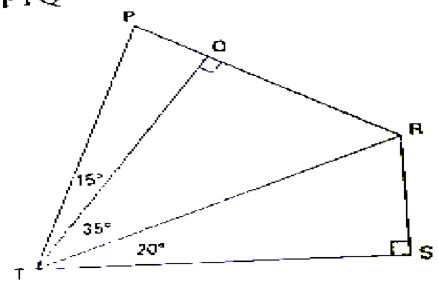
Teacher A also engaged learners by asking questions that required of them to give short answers instead of asking (TAC4) questions that could provide opportunities for learners to elaborate and explain their mathematics vocabulary and thinking (TAC1). He asked closed questions such as, "*Are we together?"; "Am I right*?"; "C*orrect*?"; "*Is that clear*?" to which learners replied, "*Yes*". This type of teaching indicates that he did not place learning at the centre of the classroom environment, as neither teacher nor learners shared responsibility for creating meaningful real-world applications (TAC3). He allowed learners to adopt a more peripheral, as opposed to active, role during instruction.

All the examples above, with regard to teaching approaches, show that Teacher A did not attempt to use constructivist teaching and learning approaches, as he focused more on solving tasks by himself. Furthermore, he intermittently asked closed questions in his lesson as he used the "chalk and talk" method. This implies that he adopted a teacher-centred teaching approach, where he dominated the classroom by talking, explaining, illustrating and solving tasks on the board. The learners listened passively and wrote down notes (TAC5).

4.5.2.2. Teaching strategies

In each of the five lessons, Teacher A selected the tasks from the textbook. Not one of the tasks were problem-based (TS1) as shown in the example below:

5. In the diagram:
$PT = 72$ cm, $TQ \perp PR$, $RS \perp TS$
$P\hat{T}Q = 15°$; $Q\hat{T}R = 35°$ and $R\hat{T}S = 20°$



Determine:

a) QT
b) PQ
c) TR
d) QR
e) RS
f) TS

In the diagram the following information was given: $PT = 72\ cm, TQ \perp \text{PR}, \text{RS} \perp$ TS: $P\hat{T}Q\ 15^0; Q\hat{T}\,\text{R} = 35^0$ and $R\hat{T}\,\text{S} = 20^0$. The question required learners to determine the length of the triangles. It is evident from the above diagram that the task requires pure mathematics calculations (algebraic). As a result of the types of tasks that were posed to learners, there was no opportunity for them to discuss the subject content themselves in small groups (TS2), as is shown in the following extract from one of his lessons.

Teacher: *What is the answer, who wants to talk and tell us the correct answer?*

Learner: *Sir, I got 20.01*.

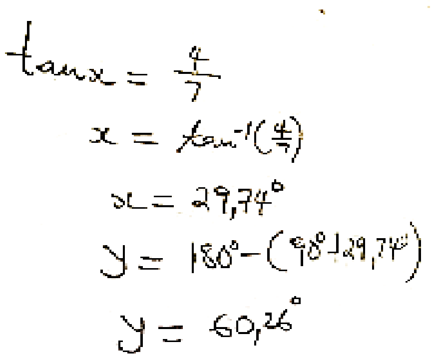
Teacher: *What is 20.01*?

Learners: (Quiet)

Teacher: *Who has the correct answer?*

Learner*: I think x = 29.7 and y = 60.26*

Teacher: (Without saying anything he turns to the whiteboard and solves the problem)



(He then looks at them and says) *Therefore x = 16.99 and y = 12.80*.

Learners (several): *Yes*.

Proceeding with the lesson, Teacher A applied direct teaching as strategy, by motivating learners to ask questions after he concluded his explanation of content (TS5). The interaction was stifled by the fact that the method was not consistently applied throughout the lesson. Hence, some learners did not ask questions or add to the discussion, to which he responded by saying, *"Your silence says to me that either you are not listening to me or you do not understand the content*." To enhance learners' understanding, he summarised the content by using the same words he used earlier (in a form of direct telling) (TS3). This strategy is called a rehearsal teaching strategy where the teacher attempt to help learners to memorise the information by repeating it over and over in the same manner without using different representations or wording.

4.6. Teacher B

4.6.1. Theme 1: Mathematics tasks

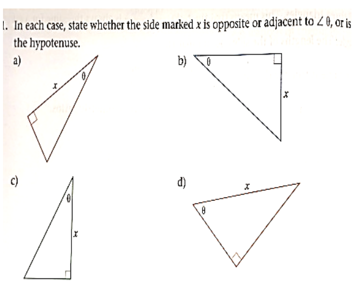
According to the conceptual framework, there are two sub-themes under this theme, namely the nature and cognitive demand of tasks. All discussions on the sub-themes and codes are structured strictly in line with the order as they appear in Table 4. 1.

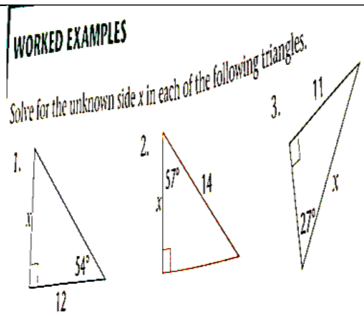
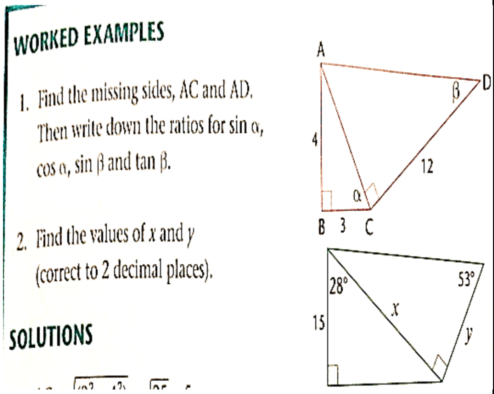
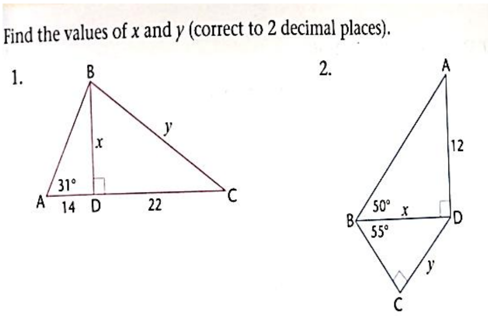
In each of the five lessons, Teacher B provided learners with two different types of tasks, namely classwork and homework. During the time of data collection, tasks such as assignments, projects and investigations were not given to learners because, according to the Grade 10 mathematics pace setter, teachers were expected only to provide classwork and homework during time of data collection as the method of assessment (DBE, 2011). In the next section, the tasks done by teacher on the board, tasks given to learners to do as classwork and homework are discussed.

4.6.1.1. Classwork tasks done by Teacher B on the board

In each of the five lessons, tasks used by the teacher were selected from the textbook. The teacher used them during instruction to teach (explain) the trigonometry content. In Table 4. 6 below the five tasks are discussed according to their nature and cognitive demands.

Table 4. 6: Classwork tasks explained by Teacher B

Classwork tasks		**Nature of tasks**	**Cognitive demands**
A	Exercise 1: no. 1 	Numerical	The task is on Level 1: Knowledge, which falls under lower order thinking. The question of the task is “in each case, state whether side marked ‘x’ is opposite, adjacent to angle θ or hypotenuse”. This task involves naming a right-angled triangle’s sides. Learners just need to demonstrate the knowledge of facts.
B	Unit 3: worked example	Numerical	The task is on Level 1: Knowledge. The question is to find the unknown side *x* in right-angled triangles. Learners must understand the basic trigonometric ratios

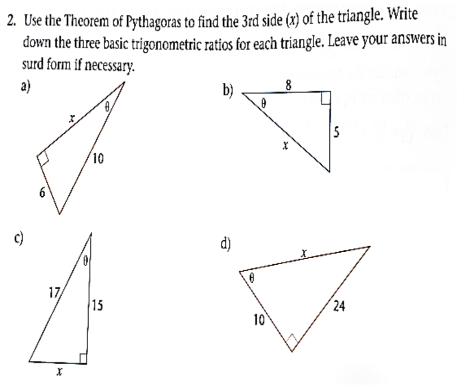
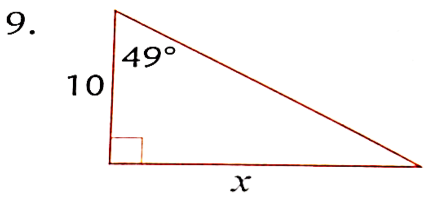
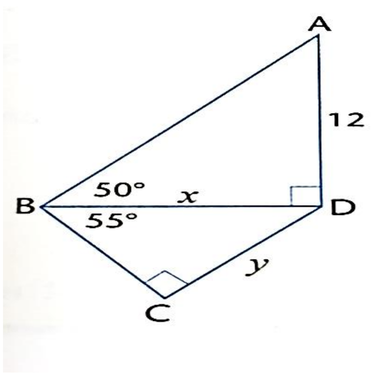
			and how to apply them in mathematics operation. All the questions are similar in that a side and an angle of a right-angled triangle are given, and one side is asked. In all problems, learners still need to use the same procedure for identifying the correct ratio to find the solution.
C	Unit 3: Worked example 	Numerical	The task with two questions and both are on Level 2: Routine procedure. Question 1(a) reads: “Find the missing sides, AC and AD, then write down the ratios for sin θ, cos α, sin β and tan β." Question 1(b) reads: “Find the value of x and y (correct to two decimal places).” In solving the task, one needs to interpret the given data in the diagram to determine in which triangle to start working. Once one has identified the triangle, one needs to apply knowledge of the three basic trigonometry ratios to find the unknown side and then proceed to the other triangle to solve the unknown side in the same way. In this task one needs to perform well-known procedures.
D	Exercise 6: no. 1-2 	Numerical	The task is on Level 2: Routine procedure. In this task, in both Questions 1 and 2, the learner’s task is to find the unknown sides (x and y) working with more than one triangle. Tasks C and D are similar and only differ in shape and values given. Therefore, to find the solution, one must analyse the tasks and check the relationship between information given and unknown information. Here one needs to perform well-known procedures, simple applications and calculations, which

			might involve many steps.
E	Exercise 11: no. 1, 3, 5, 7, & 9 Use your calculator to find the val 1. $\cos x + 1 = 1{,}14$ 3. $\tan(x - 14°) = 2$ 5. $\frac{\sin x}{4} = \cos 80°$ 7. $\sin 2x + 5 = 5{,}83$ 9. $2\tan(2x + 12°) - 3 = 1$	Numerical	The task is on Level 2: Routine procedure. The question is: "Use your calculator to find the value of *x*, where *x* is an acute angle." The task integrates with algebraic equations. The purpose in solving this equation is to find the value of angle *x* that will make the equation true. The question requires simple algebraic calculations as one must perform well-known procedures in solving equations to calculate the numerical answer. Calculators allow learners to make quick, accurate mathematic calculations when used correctly.

4.6.1.2. Classwork tasks done by learners in their exercise books

In each of the five lessons, three tasks were selected from the textbook and given to learners to do during instruction as classwork. These classwork tasks were always given 5-10 minutes before the lesson ends. Learners did not receive enough time to do this classwork during the period. They ended up doing the same classwork tasks as part of their homework. Table 4. 7 below show the two tasks and a discussion of each task according to its nature and cognitive demand.

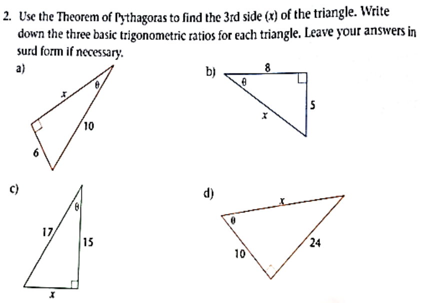
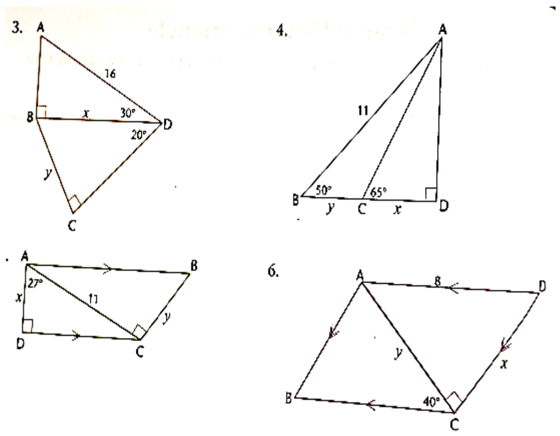
Table 4. 7: Tasks done by the learners as Classwork

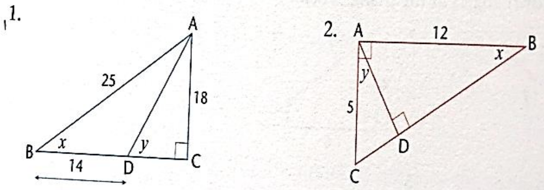
Classwork tasks		Nature of tasks	Cognitive demands
A	Exercise 1: no. a-d  	Numerical	The task is on Level 2: Routine procedure. The question reads: "Use the Theorem of Pythagoras to find the 3rd side (x) of the triangle. Write down the three basic trigonometric ratios for each triangle. Leave your answer in a surd form if necessary." Tasks involve use of theorem and ratios. Learners are required to perform well-known calculations where there is an obvious route to the solution. They just need to use the formula, substitute given values correctly, and write ratios
B	Exercise 5.2: no. 8  	Numerical	The task is on Level 2: Routine procedure. Learners need to perform a well-known procedure, and straight recall from classroom notes. The question requires the identification and use of correct ratio.
C	Exercise 6: no. 2  	Numerical	The task is on Level 2: Routine procedure. The questions focuses on solving for sides x and y. This question requires the use of ratios and Pythagoras' theorem and has multiples ways of finding the solutions. One needs to have knowledge of the theorem. One also needs to see the relationship between the triangles in term of sides and angles.

4.6.1.3. Homework tasks done by learners in their exercise books

In each of the five lessons, one task was selected from the mathematics textbook and given to learners to do as homework. Table 4. 8 below shows the five tasks and a discussion of each task according to its nature and cognitive demand.

Table 4. 8: Tasks done by learners as homework

Homework tasks		**Nature of tasks**	**Cognitive demands**
A	Exercise 5.2: no. 2  	Numerical	The task is on Level 1: Knowledge. This is the same task done by the teacher during instruction. However, the question is for the learner to determine the three ratios in each question (2a-d). The task demand presupposes retrieving learned work, which is well-known procedures.
B	Exercise 5.2: no. 3-6 	Numerical	The task is on Level 2: Routine procedure. The tasks appear under the lower cognitive level. They are similar to those used during instructions. The question is to find the unknown side (*x and y)* in more than one triangle. In this task there is no obvious route to the solution, one needs to apply knowledge gained about trigonometry. Questions 5 and 6 integrate with geometry where learners need to apply the knowledge of parallel lines.
C	Exercise 10: no.1-2	Numerical	The task is on Level 2: Routine procedure. The question is to find unknown angles (*x* and *y*) in more than one right-angled triangle.

	Find the unknown angles x and/or y in the diagrams below.  		One needs to find the relationship between information given and unknown, and after comparison use formulas such as trigonometry ratios and Pythagoras'' theorem.
D	Exercise 8: no. 1-9 Use a calculator to find the value of θ in the equations below: 1. $\sin\theta = 0{,}78$ 2. $\cos\theta = 0{,}213$ 3. $\tan\theta = \cos 21°$ 4. $\tan\theta = 3\sin 41°$ 5. $3\sin\theta = 2$ 6. $4\cos\theta = 0{,}865$ 7. $\sin\theta = \tan 12° + \cos 72°$ 8. $\cos\theta = 0{,}213$ 9. $\tan\theta = \frac{\sin 21°}{\cos 21°}$	Numerical	The task is on Level 1: Knowledge. The question reads: "Use a calculator to find the value of θ in the equations below." Learners must apply knowledge of algebraic expressions to solve trigonometry equations. The task consists of simple problems that require calculators to find the solution.
E	Exercise 11: no. 2-8 x, where x is an acute angle. 2. $2\sin x = 1$ 4. $\sin 3x = 0{,}77$ 6. $\tan(4x - 12°) = 1{,}56$ 8. $\cos\frac{x}{2} = \sin 70°$	Numerical	The task is on level 2: Routine procedure. The question is: Use your calculator to find the value of x, where x is an acute angle. To solve the task, one should have a basic knowledge of algebra. Learners must use their calculators to find accurate answers. Calculators help learners to simplify answers quickly, as they follow a well-known algorithm.

4.6.2. Theme 2: Teacher-specific factors

According to the conceptual framework, there are two sub-themes under this theme, namely teaching approaches and teaching strategies. All discussions on the sub-themes and codes are structured strictly in line with the order as they appear in Table 4. 2.

4.6.2.1. Teaching approach

In each of the five lessons observed, Teacher B introduced his lesson by letting the learners know about the topics for the week. The topics were separated into units which were covered within the period of four weeks. The units involved the following: similar triangles and trigonometric definitions, special angles and reciprocal ratios, solving right-angled triangle problems and solving simple trigonometric equations. In each of the five lessons, the teacher discussed, explained and solved pure mathematics problems. The problems included, among others, investigating the ratios between the sides of a 90^0 triangle with a fixed angle, creating the link between the trigonometric definitions and similar triangles, solving special angles and reciprocal ratios, and angle of depression and elevation. He did not allow ample time for learners to engage with these problems either on the board or when classwork was done in their exercise books. Throughout the lessons, learners were merely required to listen while answering short questions and copying notes, as evident in the following extract.

Teacher: *Today we are going to discuss about angles of elevation and depression.* (While moving around)

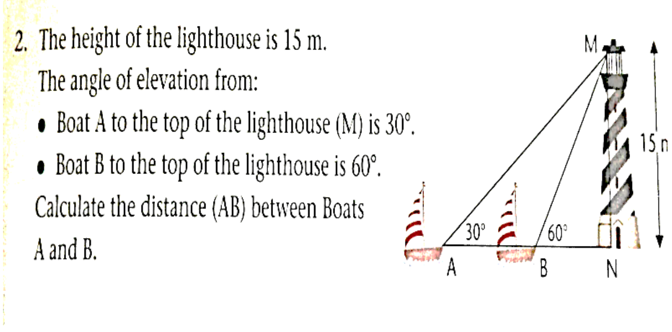
Learners: (Quiet)

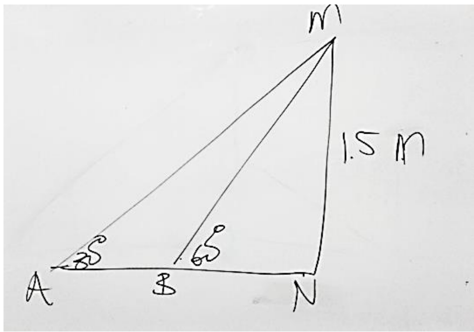
Teacher: *This is the application of trigonometry as I will be solving problems from page 95, exercise 7 number 1.*

Teacher: (Reads the question) *The height of the lighthouse is 15 m. The angle of elevation from: Boat A to the top of the lighthouse (M) is* 30^0*, Boat B to the top of the lighthouse is* 60^0. *Calculate the distance (AB) between Boats A and B.*

Learners: (Listening and taking notes)

The teacher then continues by simplifying a real-life task (problem-based) to a basic numerical task (Lesson number 4) as indicated below:

<table>
<tr><td>Task from the textbook (A)</td><td>Task drawn on the whiteboard (B)</td></tr>
<tr><td>

2. The height of the lighthouse is 15 m.
The angle of elevation from:
• Boat A to the top of the lighthouse (M) is 30°.
• Boat B to the top of the lighthouse is 60°.
Calculate the distance (AB) between Boats A and B.

</td><td>


</td></tr>
</table>

Teacher: *Look at the picture (*Diagram B*); do you see we have right-angled triangles?*

Learners: *Yes*

Teacher: *We have ΔMAN and ΔMBN.*"

(After the explanation and solution, as shown in diagram A, he then continues)

Teacher: *You move from one triangle to another. How do we choose a triangle? We choose the triangle that has more information, which triangle?*

Learners (several): *That has more information*.

Teacher: *Sharp, we are looking at this triangle (*Diagram B; triangle MBN*) because it has more information, okay*.

Learners: *Yes, Sir.*

Teacher: *We are given the opposite side and the angle. Now tell me which ratio should we apply?*

Learners (several): *Tan.*

Teacher: *Tan. Tan will be?*

Learners (several): *4/3.*

Teacher: (nodding) *Yes.*

Learner: *Sir they made a mistake, side MN is 15 not 1.5*

Teacher: *Ooh! The answer will be different, when you practice use 15m not 1.5m.*

The situation revealed that the tasks were more on problem solving where learners were going to get a chance to engage in discussion in the process of trying to make sense of the information. Instead, the teacher promptly shifted the role of the tasks from higher- to lower-order thinking. He drew a right-angled triangle using the information from the task scenario. The task scenario is shown in Diagram A above. This task scenario contained real-life problems. However, in his drawing (Diagram B), Teacher B did not integrate the real-life information from Diagram A into his drawing; hence there was a shift in the cognitive level. This explains that Teacher B did not assess the learners' understanding of the elevation and depression content as required from Diagram A, but instead proceeded with the lesson by solving the task himself on the whiteboard.

4.6.2.2. Teaching strategies

In each of the five lessons, Teacher B selected the tasks from the textbook. From all the tasks chosen by the teacher as classwork and homework, the nature of only one task was problem-based. The learners unfortunately did not get an opportunity to solve this task as the teacher ignored the real-life context and reduced the complexity by re-drawing two triangles to solve it, as with all other tasks he showed on the board. As a result of the types of tasks that were posed to learners (classwork and homework), there was no opportunity for them to discuss the subject content themselves in small groups, as is evident in the following extract from one of his lessons:

Teacher: *What is the answer, what is the distance AB? who wants to talk and tell us the correct answer?*

Learner: *Sir, I got 20.01*.

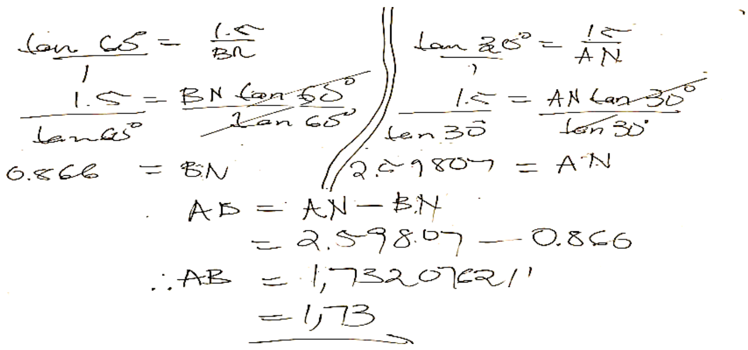
Teacher: *What is 20.01*?

Learners: (Quiet)

Teacher: *Who has the correct answer?*

Learner: *"I think AB = 1.73 m"*

Teacher: (Without saying anything, he turns to the whiteboard and solves the problem.)



(He then looks at them and says) *Therefore AB = 1.73.*

Learners (writing notes): *Yes*.

He then proceeds:

Teacher B selects other tasks to solve on the board so he can explain and show the learners the procedure for solving those tasks. Learners copy that into their exercise books. Throughout the lesson, learners appear to accept whatever the teacher presents to them. Learners do not ask questions, even when they teacher says, "*Any question before I continue?*" Learners keep quiet and continue to copy the work from the whiteboard. Two or three minutes before the bell rings, the teacher gives classwork and homework to do.

Teacher: *Students, your homework is on page 93 exercise 6, number 1 and 2. I will see you all tomorrow*.

Learners: *Yes, Sir*.

In conclusion, all five lessons of Teacher B were characterised by direct teaching. The teacher dominated classroom instruction, thus learners' participation in the learning was minimal. The teacher discussed, explained, and demonstrated mathematics concepts to the learners, while the learners had no chance to contribute anything to their own learning. The teacher did not give learners group work to do; it was an individual work mostly. He did not give learners the opportunity to engage in learning activities that could develop their critical and analytical thinking skills. No sufficient time was allowed for learners to solve tasks by themselves.

4.7. Summary

Tables 4. 9 and 4. 10 below provide a snapshot of the two teachers' background information and the findings from the study.

Table 4. 9: Theme 1: Snapshot of backgrounds of Teachers A and B, and mathematics tasks used

Participants	Teacher A	Teacher B
Background information		
Qualifications and experience	BEd (FET) degree in Natural Science. Experienced teacher with 11 years' experience of teaching Physical Science and Mathematics Literacy. Included in these 11 years is four years of teaching Grade 10 Mathematics.	Higher Education Diploma (HED) and BEd (Hons) in Educational Management. 13 years' experience of teaching Physical Science and Mathematics (Grade 10).
Mathematics tasks		
Nature of tasks	• All tasks were classwork and homework. • All tasks done by teacher and learners were numerical in nature.	• All tasks were classwork and homework. • All tasks done by the teacher and learners were numerical in nature.
Cognitive demands of tasks	• **Classwork tasks done by Teacher A**: A total of five tasks were done on the whiteboard, 1 task was on higher order thinking (Level 3: Complex procedure) while the remaining 4 tasks were on lower order thinking (Level 2: Routine procedures). • **Classwork tasks done by learners:** 2 tasks were done by learners during the course of the lesson, and both were on lower order thinking (Level 1 & 2: Knowledge & Routine procedures). • **Homework tasks done by learners:** 5 tasks were given to learners to do at home, and 4 tasks were on lower order thinking (Level 2: Routine procedures) and 1 task was on higher order thinking (Level 3: Complex procedures).	• **Classwork tasks done by Teacher B**: A total of 5 tasks were done on the white board and all were on lower order thinking (2 tasks where on Level 1 and three tasks were on Level 2). • **Classwork tasks done by learners:** 3 tasks were done, and all were on lower order thinking (Routine procedures). Two tasks fall under Level 1 (Knowledge) and three tasks fall under Level 2 (Routine procedures). • **Homework tasks done by learners**: 5 tasks were handed out to learners and all were on lower- order thinking level (Level 2: Routine procedures).

Table 4. 10: Theme 2: Snapshot of teaching approaches of Teachers A and B, and their strategies

	Teacher A	**Teacher B**
Teacher-specific factors		
Teaching approaches	• He began by explaining, discussing and demonstrating mathematical concepts. • He selected tasks from the prescribed textbook. • He then worked out the examples by himself on the whiteboard. • Learners passively listened to the teacher and took notes. • He did not give learners sufficient time to work on classwork tasks. • He tried to encourage participation by asking learners closed questions. • He did not assess learners' understanding. • In each of the five lessons, he did not afford learners time to work on classwork tasks.	• He began by explaining, discussing and demonstrating mathematics concepts. • He selected tasks from the prescribed textbook. • He worked out the examples by himself on the whiteboard. • He asked learners straightforward questions. • Decline in the level of difficulty only on the complex task. • Learners' notable participation involved listening and copying notes from the whiteboard. • He did not assess learners' understanding. • In each of the five lessons, he did not allow adequate time for completion of classwork tasks.
Teaching strategies	• All lessons were direct teaching based • No group work or whole-class interactive work.	• Teaching involved information-giving session (Direct teaching). • No cooperative work (group work).

4.8. Discussion

4.8.1. Theme 1: Mathematics tasks

4.8.1.1. Nature of tasks

Findings from the study indicate that the types of the tasks given by both teachers during instruction were classwork and homework. Both teachers selected these tasks from the one textbook they use every day, which is in accordance with the findings from the study conducted by Wijaya, Heuvel-Panhuizen and Doorman (2015). One of the most predominant findings of this study is that teachers gave an average of three and five classwork and homework activities respectively to the learners to do on their own. The DBE's (2011) guidelines indicate that continuous assessment in the form of classwork activities should be done continually to monitor learners' progress with the purpose of improving their learning. Sullivan et al. (2013) concur that learners should be provided with opportunities to engage with different types of tasks, namely: homework, classwork, assignments, projects, and investigations. It therefore stands to reason that the fact that few classwork and homework exercises were given to learners deprived them of the opportunity to work on the tasks.

Findings from my study indicate that Teacher A and Teacher B handed out an average of one homework task per week, consisting of five exercises, contrary to the recommendation by Cooper (2001) that Grade 10 learners should be spending more time on both classwork and homework tasks. The tasks allow for practising, extending and consolidating work done in classroom. The reason why learners do more of these activities is to provide them with proficient OTL (Hyde et al., 2006).

This study also shows that the tasks that were chosen by both teachers were numerical in nature. The tasks were in the form of pure mathematics. Numerical tasks, when used in isolation from other kinds of tasks, such as procedural, problem-solving and contextual, may fail to encourage effective learner development of mathematical competency, and instead cause learners to do rote learning by following rules and procedures. The findings are strongly consistent with Hsus' (2013) study, who finds that students are indeed learning how to use simple facts and calculation procedures but are not learning how to find creative solutions to lower-order as well as higher-order questions.

4.8.1.2. Cognitive demands of tasks

The CAPS (DBE, 2011) prescribes and recommends various levels of cognitive demands of tasks that learners should do in mathematics, namely Level 1: Knowledge (20%); Level 2: Routine Procedures (35%); Level 3: Complex Procedures (30%); and Level 4: Problem Solving (15%). Cognitive demand is about creating and maintaining an intellectual challenge that encourages learners to improve mathematics skills and knowledge. The findings relating to levels of cognitive demands used by both teachers to facilitate learning in their instructional practices are discussed in this section. Three of the four levels of cognitive demand were identified in the study, namely: Knowledge, Routine Procedures and Complex Procedures. The finding indicates that only two tasks from classwork given by both teachers were on Level 1 (Knowledge). Knowledge, as the lowest level of cognitive demand, involves retention of information that is presented in terms of facts, definitions, identifying rules and direct use of formulae. Familiar algorithms, well-known procedures, label diagrams, and simple calculations (Ubuz, Erbas¸ Cetinkaya & Özgeldi, 2010) characterise Level 1 tasks. The given Level 1 tasks required of learners to reproduce previously learned facts with no connection to the actual meaning of the questions. For example, teachers reminded learners of facts from the previous grade(s), such as the Pythagoras theorem that states that the square of the hypotenuse is equal to the sum of the squares of the other two sides ($r^2 = x^2 + y^2$). The finding is consistent with Ubuz et al. (2010) who find that the majority of learners were involved in tasks that only required of them to reproduce facts and memorise content.

Almost all the tasks used for classwork and homework were on Level 2 (Routine Procedures). Learners were required to solve well-known problems, use known procedures and straight recall, all available from their class notes. These types of tasks required little effort or thinking and reasoning. In a strikingly similar finding, Stols (2013) notices the absence of higher-order (Level 3) questions in tasks during instruction as learners spent their time practising Level 2 (Routine Procedures). Stols' (2013) findings are also supported by Son and Kim (2015) who find that majority of tasks, presented in the form of diagrams, were on Level 2, which required minimal thinking and reasoning. Mdladla (2017) finds that teachers in Grades 9 and 11 selected and used only lower-level (Levels 1 & 2) cognitive demand tasks.

The findings of this study further revealed that tasks involving Level 3 (Complex Procedures) occurred sparingly during instructions and in homework. Only Teacher B introduced a Level 3 task but did not present the task as such as he preferred to simplify it himself, reducing it to a Level 2 (Routine Procedure) task. The results of this study are also strongly consistent with findings from the study conducted by Mdladla (2017) who discovered that teachers were inclined to decrease the complexity level of tasks. Stols (2013) emphasises that in order to develop a deep understanding of mathematics, a sufficient number of tasks should be done on all levels.

4.8.2. Theme 2: Teacher-specific factors

4.8.2.1. Teaching approaches

Findings from the study indicate that both teachers adopted a teacher-centred approach of delivering content. The teacher-centred approach is characterised by teachers giving instructions; lecturing; following prescribed procedures from the textbook; giving limited time for learners to engage with tasks; while learners are required to listen, duplicate, memorise, calculate, and take notes.

Both teachers showed preference for only one method of delivering instruction. They used a chalk, board-and-talk method commonly known as lecturing. The results are very much consistent with Venkat and Spaulls' (2015) finding that a teacher-centered approach is often adopted within the public sector schools where teachers become the primary controllers in the classroom. In all of the observed lessons in this study, both teachers required learners to copy, calculate, and take notes. In a closely related finding, I discovered that the recurring theme in the study conducted in a public secondary school was that learners remained passive by merely listening and copying the content presented to them. Teachers A and B consulted the prescribed textbook as the only resource for the curriculum enactment. Thus, learners only learned content of the curriculum that the teachers actively engaged them with through the textbook. The finding is consistent with Herlinda's (2014) assertion that teachers over-rely on the prescribed textbooks and do not consider using other textbooks or materials in the classroom.

The study revealed limited **e**ngagement between the teachers, learners and the mathematics content. Engagement is divided into three categories, namely: teacher-learner engagement, peer engagement and learner-task engagement. In the first category, both teachers tried to engage learners, by asking questions that require short answers instead of asking questions that could provide opportunities for learners to expand and explain their mathematical vocabulary and thinking. In the second category it was clear that learners did not communicate their mathematics reasoning with their peers during implementation of tasks. Regarding the last category, the findings revealed that teachers created limited active learner-task engagement. Both teachers solved all the work by themselves on the whiteboard. Gutierrez (2014) also discovered that learners are becoming more and more disengaged during task implementation. Teachers also need to create an engaging learning environment that can make learners feel able to participate freely and ask questions as part of the learning process. Carlson (2005) concurs that learners want more autonomy to engage in and construct their own learning.

This study also gives an overview of time allocation for task implementation. The study shows that tasks were always given only a few minutes before the end of the lesson. Thus, learners did not receive enough time to work on or solve those tasks in class, contrary to the stipulation in CAPS that learners must complete five to eight tasks daily in the classroom (DBE, 2011). It is in this consolidation phase that the teacher can engage with the learners and address their misunderstandings and errors. This lack of ability to create time for tasks is the reason why both teachers focused more on teaching than learning. Similarly, when enquiring about time spent on tasks, Jones and Pepin (2016) find that teachers generally spend the majority of their class time on explanations (through examples or other means). Teachers do not allow enough time for the learners to spend on meaningful tasks. It is important for teachers to manage their time properly, in line with the CAPS guidelines.

4.8.2.2. Teaching strategies

Both teachers used only the direct teaching strategy. Although Teacher B had the opportunity to use the problem-based strategy, he instead transformed the task from a real-life problem into a numerical problem. By using direct teaching, active learning as required for developing conceptual understanding, is absent. With direct teaching, teachers attempt to transfer content, skills and knowledge from the teacher to the learners. Both teachers were standing in front of their classrooms and presented the information. Thus, learners were expected to listen and follow instructions carefully. A similar finding was found in the study by Dimitrios, Labros, Nikolaos, Koutiva, and Athanasios (2013), who state that direct teaching dominates the teaching and learning process.

In only one lesson, Teacher B chose Exercise 7 from the textbook that dealt with the application of trigonometry. This task (Exercise 7) was challenging and demanded mastery of mathematics concepts and for learners to apply them in a real-life situation. However, during the process of instruction he declined the level of difficulty of that task. A possible reason may be that he thought learners would not be able to manage, or that it would take too long to guide them to understanding the problem. The finding of this study is strongly consistent with other researchers such as Stein et al. (2000) and Staple (2007) who note that some of the tasks that teachers select are of high cognitive demand, but that they reduce the difficulty level of those tasks at implementation, in other words, as Teacher B did. Similarly, Mdladla (2017) reports that the teachers in his study, like Teacher B, solved the difficult problems on behalf of the learners by demonstrating the procedures for finding the solutions. Reducing the complexity level of that task during implementation transformed the problem into a procedural task, and the opportunity for connecting mathematics with a real-life context was lost. Wijaya et al. (2015) are of the opinion that teachers can promote deeper levels of understanding by consistently probing learners' understanding of tasks in class rather than simplifying or solving the task on their behalf.

Findings also revealed that learners were not engaged in active learning. Active learning is generally regarded as any instructional strategy that engages learners in the learning process. It also embraces the use of cooperative learning groups. The study reveals that both teachers did not give learners an opportunity to share ideas and argue with peers intellectually in order to collaboratively solve mathematics problems. They did not allow enough time for the learners to spend on meaningful tasks. The findings are consistent with the study by Aksit, Niemi and Nevgi (2016) who state that most participants in their study did not implement active learning. On the other hand, Savery (2015) find that learners learned through actively constructing their own knowledge and connecting new ideas in groups in order to enhance their understanding. The finding of Savery (2015) is also shared by Hasan and Fraser (2015), whose study explores the effectiveness of teaching strategies with specific reference to active learning in mathematics.

4.9. Findings, trends and explanations

From the analysis of the data and discussion of the findings according to Theme 1 and Theme 2, the following findings, trends and explanations are delineated.

Alignment between prescribed and implemented tasks

Anderson (2002) explains "alignment" in the context of classroom instruction, as the agreement between the objectives, as stated in the departmental documents, and activities that are mutually supportive in a classroom setting. In this study, "alignment" refers to the degree of agreement between the type and nature of tasks as well as the cognitive levels of tasks selected, and tasks recommended in the CAPS document. To provide learners with an opportunity to learn, it is important for teachers to take cognisance of departmental documents and to align their tasks with the requirements stated in these documents. These various mathematics tasks serve different purposes and integrating all of them during instruction can create a balance for certain skills and knowledge that learners should acquire. Anderson (2002) provides a model that can be adopted to determine whether the tasks are aligned to CAPS. According to the model, complete alignment is evident when there is a balance between types, nature and the level of cognitive demands of the tasks that are in line with the CAPS requirements. Partial alignment exists when not all tasks are implemented as stipulated in CAPS. In the study I have noted that complete alignment occurs when all types of tasks prescribed for trigonometry are selected, namely classwork and homework. All the tasks were numerical in nature, however, which implies that there was partial alignment. Partial alignment was also noticed in terms of cognitive levels, as all of tasks were on lower levels (Levels 1 & 2). The nature of tasks can be procedural, problem solving, contextual, numerical, and representational. Numerical tasks are in the form of pure mathematics and can be separated into lower and higher order thinking (DBE, 2014). In this study, numerical tasks used during instruction fall under lower-order thinking (Levels 1 and 2), which require low cognitive demands. These tasks require learners to know how to use mathematical symbols, rules and algorithms to solve mathematics problems (Zohar & Dori, 2003). The exercises on lower-order thinking (Levels 1 and 2) do not provide learners with ample opportunity to engage actively in mathematics activities, as they require rote memorisation. The larger part of partial alignment found in this study could emanate from two factors, namely the teachers' overreliance on the textbook and their avoidance of dealing with higher-level questions (Level 3 and 4) involving real-life tasks.

Anderson (2002) states that alignment between the ATP, learning materials (textbook, math apps etc.), and tasks selected, is important for active learning. The study revealed that teachers were aware of the CAPS document during the implementation of tasks, as shown by their consistent adherence to the ATP document. The ATP provides detailed information regarding the day-to-day teaching of the subject. Moodley (2013) opines that this provision by the DBE makes the work of the teachers much lighter and provides them with clarity regarding the content to be taught, the time frames in which to achieve this, and the resources to be used. Although both teachers followed the ATP, the selected tasks' content was mostly on lower levels and some were even reduced in terms of their cognitive demands. This was because the tasks were selected from only one textbook in spite of the guidelines from the ATP encouraging teachers to use multiple teaching resources. Polikoff (2015) concludes that choosing teaching and learning material from only one textbook is in disagreement with the objectives of CAPS and, ultimately, the ATP. The finding is strongly supported by Fredericks' (2005) assertion that teachers over-rely on textbooks and do not consider other materials such as previous question papers and mathematics apps for the classroom to stimulate critical thinking. Tasks in mathematics apps integrate the curriculum with various contextual problems, where the problem situation is challenging and interesting to learners. The aim is to gain greater insight into the mathematics tasks that learners and teachers encounter on a daily basis, to attain some understanding of the teaching and learning taking place in classrooms. Therefore, by using such tasks, teachers will adhere to the requirements set out in CAPS (DBE, 2011), stating that the tasks should cover all the cognitive levels, including challenging and real-life problems.

Real-life problems and critical thinking

In recent years, there has been an increased emphasis on using real-life problems in the mathematics classroom so that learners can connect theory with practice. The DBE (2018) asserts that teachers should strive to use real-life settings to teach mathematics for conceptual understanding to enable comprehension of mathematical concepts, operations and relations. However, Teacher A spent time solving numerical routine tasks on the board. During the data collection, he marked the previous homework and classwork in the form of solving problems on the whiteboard by himself. Teacher A never created opportunities for challenging learners with real-life problems and critical thinking while teaching and explaining the content. Teacher B, on the other hand, had a great opportunity to challenge the learners to think critically by applying a real-life problem when he asked them to solve Exercise 7 (Real-life problem: calculate the distance between two boats). Unfortunately, he reduced the complexity of the task by explaining and drawing the triangle with information from the scenario.

Engaging in real-life scenarios requires time. Hence, I experienced that both teachers did not create time for learners to spend on challenging tasks that require critical thinking and reasoning skills, namely Level 3 and 4 tasks. In terms of CAPS (DBE, 2011), an effective classroom is when teachers pose tasks that require learners to develop their capacity for critical thinking and reasoning skills. Critical thinking and reasoning skills are crucial for exploring new content and design, for developing possible solutions to problems, and for evaluating the success or efficiency of the implemented solutions. Thus, the tasks allocated by both teachers did not provide learners with an opportunity to learn trigonometry with the focus on conceptual understanding and on applying their knowledge to real-life contexts.

Trigonometry

Trigonometry is an important topic in the school mathematics curriculum. It is one of the fundamental topics introduced at the beginning of the FET phase and is based on pre-knowledge such as space and shapes (Geometry of 2D shapes) and Pythagoras' theorem from the previous General Education and Training phase. Therefore, a solid understanding of trigonometry is essential for learners in Grade 10 since it lays the foundation for the next two years of the FET phase. Trigonometry is a content area that combines algebraic, graphic and geometric reasoning (Moore, 2010). In Grade 10, learners need to relate diagrams of triangles to numerical relationships and to manipulate the symbols involved in such relationships. This multifaceted nature of trigonometry challenges learners' understanding. Blackett and Tall (1991) point out that many learners have difficulty understanding the topic because they lack the ability to link trigonometric representations with algebraic and geometric reasoning and real-life problems.

Trigonometry requires the use of appropriate tasks (from numerical to problem-based) on all four prescribed levels of cognitive demands, including learners' higher-order thinking, in order to develop their mathematics understanding. The use of appropriate and high cognitive tasks will improve learners' ability to make connections between the mathematics content and real-life situations. Therefore, the nature of tasks presented to learners and given to them as classwork and homework should correspond to the categories and include higher-order cognitive demands as set out in the curriculum documents. The tasks should encourage learners to justify their own ideas. Weber's (2005) study emphasises that trigonometry instruction should focus on non-routine problems based on real-life contexts.

To teach trigonometry effectively, a learner-centred approach will be valuable where the teacher creates opportunities for meaningful learning, allowing learners to actively construct their own mathematical knowledge through appropriate task. Learners will have an opportunity to engage in the trigonometry and, through critical thinking and reasoning, do problem-solving. Similar to this study, a study by Zengin, Furkan, and Kutluca (2012) finds that many teachers used the teacher-centred approach to teach trigonometry, stressing procedural skills and not allowing learners to understand the topic. What learners understand about mathematics is almost entirely dependent on the experiences the teacher creates daily in the classroom. For learners to experience mathematics, sufficient time should be spent on meaningful activities requiring discussion, reasoning and reflection on their thoughts and calculations. It is best for teachers to adopt teaching strategies such as problem-based, active learning and cognitive guided instruction. These strategies will assist the teacher to create opportunities for collaborative learning, where learners can work in groups exchanging ideas with one another.

4.10. Conclusions

In this chapter, I present the findings and discussions of this case study. The aim of the study is to analyse the types, nature, and cognitive demand of daily mathematics tasks used by teachers and given to learners. I further explore teachers' teaching approaches and strategies, including the time spent by learners engaging in tasks during instruction. The chapter starts with a description of the biographical information of the two teachers. This is followed by the presentation of the results of the qualitative data collected concurrently from a document analysis and classroom observations. Next, I discuss the presented data. From this analysis, I find that the mathematics tasks selected by teachers were classwork and homework, as prescribed during the period of data collection, but the nature of the tasks were mainly numerical and fell within Levels 1 and 2 of cognitive demands. In the discussion, the findings are related to literature to find similarities to, and contradictions of, this study's findings. Lastly, findings, trends and explanations emanating from this chapter were provided.

In the next chapter, the research questions are answered. I reflect on my research study and draw conclusions from it. I also discuss the limitations and significance of the study and make recommendations for further research.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

5.1. Introduction

In this chapter, the research questions that guided the study are answered according to the findings from the analysed data in light with the findings and the relevant literature. Next, I make provision for the fact that I may have been wrong in my interpretation of the tasks used by teachers and assigned to learners during instruction. This is followed by the conclusions, recommendations and limitations of the study. Lastly, I offer my own reflections of this study.

5.2. Discussion of the research questions

This study aimed to analyse daily mathematics tasks (classwork, homework, assignments, projects and investigations) given to Grade 10 learners, to determine the extent to which the tasks provide the learners an opportunity to learn. The nature and cognitive demands of tasks used during instruction were observed and analysed according to the departmental documents. Furthermore, teachers' teaching styles and teaching strategies contributing to the selection and implementation of the mathematics task

s were investigated. The four secondary research questions that guided the study are now discussed and are used to answer the primary research question.

5.2.1. Sub-question 1: What are the requirements as stipulated by CAPS regarding mathematics tasks?

This question was purposefully chosen as the first question, to set an initial benchmark to be used in my discussion later of how the two teachers' instructional practices comply with this set of requirements. As part of the working definition of OTL, it was stated that the content refers to the extent to which the content of instruction overlaps with the intended curriculum. In order to answer this question, I performed a document analysis on the CAPS and ATP documents for mathematics (DBE, 2011) as well as a literature review of studies conducted on this topic. I used the CAPS and ATP documents to determine what those documents prescribe and recommend regarding the types, nature and cognitive demands (depth), content coverage (breadth), and time frame required in order to complete the sub-topics. The focus of this study was only on the Grade 10 trigonometry topic and I realised that both these documents (CAPS and ATP) provide clear and succinct statements of what needs to be taught and learnt in order to provide learners' an OTL trigonometry. In line with that ATP, trigonometry is the last of five topics in Term 1 in Grade 10 and covers the following sub-topics: angles, special angles, reciprocals, calculator work, trigonometric equations, solving triangles and Cartesian planes as well as Pythagoras questions (CAPS, 2011).

According to CAPS (2011), the various types of tasks such as classwork, homework, assignments, projects and investigations are mandatory requirements in Grade 10. Both classwork and homework tasks can be problem-solving, contextual and/or numerical in nature (Wijaya et al., 2015; Georgius, 2014; Yeo, 2017). Assignments, on the other hand, could involve a collection of previous examination papers that may expose learners to the structure of the papers and the content. Assignments can also be problem-based and/or contextual in nature. Projects should have clear specifications and should generally focus on problem-based tasks situated in real-life contexts. Lastly, an investigation could include mathematics applications, contextual and open questions. Son and Kim (2015) indicate that these tasks should place a certain degree of cognitive demand on the learners.

In the CAPS (DBE, 2011), four levels of cognitive demands, based on Bloom's Taxonomy, are prescribed when implementing tasks. These four levels of cognitive demands (Knowledge 20%, Routine Procedures 35%, Complex Procedures 30%, and Problem Solving 15%) are separated into two lower-level demands and two higher-level demands. The lower-level cognitive demands, namely knowledge and routine procedures, engage students in the task of memorisation and use of procedures without connection to meaning (Wu & Pei, 2018). The higher-level cognitive demands, namely complex procedures and problem solving, involve learners in the process of using procedures with connection to meanings and in the "doing of mathematics" (Al Raqqad & Ismail, 2018). Awareness of lower- and higher-level cognitive demands could enable teachers to prepare lessons that may engage learners' thinking processes and ultimately lead to better understanding of mathematics content. Thus, teachers need to carefully plan the tasks and time needed to adhere to these prescriptions and to provide learners the opportunity to be engaged with both lower- and higher-level tasks.

The area repeatedly highlighted in the DoE (2009), is that of finding a balance between coverage (breadth) and depth (cognitive level) in the curricula. Stols (2013) agrees that the coverage and the cognitive level play a major role in creating OTL. In terms of coverage, more detail is provided in the CAPS on the exact scope of the content to be taught and assessed (DBE, 2009). The emphasis is placed on the topics and sub-topics, the expectations for learning, the time teachers allocate to particular sub-topics, the types and nature of tasks they should pose, the kinds of questions and discussions they could lead, and the responses they may expect. All these are part of content coverage and all influence the opportunities learners have to learn (Stols, 2013).

In terms of time, CAPS specifies the number of hours (4.5 hours) of classroom time allocated to trigonometry in each week. These hours constitute 16% of total classroom time allocated to learners' engagement with tasks. CAPS (2011) clarifies that this time allocation should allow for a sufficient depth of engagement with the content as specified. A teacher is responsible for the effective use of the allocated time inside the classroom. Stols (2013) concurs that learning depends on the degree of time and effort, and warns that without efficient use of time on task, no learning is possible.

Teachers need to plan when they will get learners to practise the concepts learned by completing the exercises from the textbook. They also need to work through the lesson plan and decide where (either as classroom activity or homework) they will involve learners to actively do the exercises. The textbook used in the classroom needs to be taken into consideration since all of the tasks given to learners are taken from the textbook. Despite its importance, Henhaffer (2014) states that teachers should use the textbook and additional materials as and when needed during instruction. CAPS explains fluency in computation skills, as the ability to use web links and video clips as additional materials (DBE, 2013).

To conclude; the official departmental documents prescribe five types of tasks, namely classwork, homework, assignments, projects and investigations that should be of numerical, contextual, and problem-solving nature. These tasks should be carefully chosen to adhere to the four cognitive levels, namely 20% Knowledge, 35% Routine Procedures, 30% Complex Procedures, and 15% Problem Solving. Teachers should also plan their lessons in such a way that sufficient time is given to learners to be actively involved in solving these tasks, individually or in groups. In adhering to these prescriptions and recommendations, learners will be given sufficient OTL trigonometry.

5.2.2. Sub-question 2: To what extent do mathematics tasks given by the teachers comply with the requirements from CAPS?

This question weighed the degree in which mathematics tasks observed and analysed in terms of types, nature, cognitive demands, resources used, and the time frames implemented by teachers conform to the CAPS requirements. In terms of types of tasks, the Mathematics Grade 10 pace setter specifies that learners need to perform only two types of tasks for Term 1 (week 9-11) namely classwork and homework learning tasks (CAPS). These tasks need to be allocated frequently (continually) so as to enable the learners to grasp the content (National Education Collaboration Trust, 2015). Hyde et al. (2006) agree that learners should be more involved in doing classwork and homework because active involvement in solving problems provide learners with OTL. During this term, I conducted a document analysis to determine whether teachers complied with the CAPS specifications with reference to the types and frequency of tasks. In each of the five lessons, both Teachers A and B provided learners with the required types of tasks in keeping with the CAPS. However, in terms of frequency, few classwork and homework activities were given to learners by the two teachers.

To ensure CAPS compliance in terms of the content, teachers need to complete all the sub-topics stated in the ATP and lesson plans (NECT, 2015). The content covered in this topic included the following sub-topics: defining the trigonometric ratios and extending these definitions to any angle (including special angles); definitions of the reciprocals of the trigonometric ratios; calculations in trigonometry; simple trigonometric equations. In addition, solving right-angled triangles and applying the theorem of Pythagoras were also covered (NECT, 2019). But prior to this information, teachers are required by CAPS to determine whether learners have an understanding of similar triangles and the theorem of Pythagoras (DBE, 2014), since this information was covered in the previous grades (Grade 8 & 9) (NECT, 2019). The findings indicate that both teachers complied with CAPS as they addressed all the sub-topics during teaching. While examining the content coverage from the learners' exercise books, the content required learners to name trigonometric ratios, thus directing learners to use mathematical facts and to recall ratios such as $\sin\theta$, $\cos\theta$ and $\tan\theta$. Some of the problems, in addition, required the use of the Pythagoras theorem to find the lengths of the sides of a triangle.

Both teachers expected learners to use their calculators. Overall, there was compliance to CAPS in terms of content coverage by both teachers.

According to CAPS (2011), care needs to be taken to ask questions on all natures of tasks and all four cognitive levels. The natures of tasks include problem solving, contextual, and numerical tasks, while cognitive levels include Level 1: Knowledge (20%); Level 2: Routine Procedures (35%); Level 3: Complex Procedures (30%); and Level 4: Problem Solving (15%). (CAPS, 2011). An OTL is provided when teachers comply with the guidelines from the prescribed curriculum in terms of nature and cognitive demands of tasks. Classwork activities done by both teachers and learners as well as homework tasks given to learners were analysed to determine whether they complied with the mentioned nature and cognitive level requirements. In terms of the nature of tasks, Teachers A and B selected only pure mathematical content for both classwork and homework (numeric) and thus did not comply with CAPS. With reference to the cognitive demands of tasks, I found that all of the tasks selected by both teachers were on a lower level (Knowledge and Routine Procedures). Out of a total of five tasks done on the whiteboard by Teacher A, only one was on higher-order thinking while the remaining four were on lower-order thinking. All five tasks done by Teacher B were on lower-order thinking. A similar scenario was found in tasks approached by learners (classwork and homework). The tasks were all on the lower-order thinking level (Routine Procedures). Overall, the higher-order cognitive skills, namely critical thinking, problem solving, and decision making were omitted by both teachers, contrary to CAPS' expectations. Stols (2013) emphasises that in order to develop a deeper understanding of mathematics, enough work must be done on all levels.

With reference to resources, CAPS (2011) and the ATP (NECT, 2019) indicate that teachers need to select activities and exercises from the textbook, use additional teaching materials such as previous question papers, ensure the posters are displayed in the classroom, adopt FET mathematics training handouts from the DBE, and show the learners the recommended video clips at the end of the topic (NECT, 2019). Through observation, I noticed that both teachers did not comply with the CAPS (2011) guidelines as they over-relied on the use of textbooks and did not pay attention to all the other recommended teaching materials. In all the lessons, all classroom activities, including classroom and homework tasks, were selected from the prescribed mathematics textbook without supplementing them with other learning materials.

Similar to CAPS (2011), researchers (Polikoff, 2015; Wijaya et al., 2015) encourage the use of the textbook as a resource for learners, but not as the only resource.

In terms of the time, CAPS (2011) indicates that sufficient time needs to be allocated to teachers and learners to work through the topics. A data collection instrument (document analysis) was designed to collect the data from the learners' workbooks. Time-on-task was measured by firstly considering the number of days the learners actually spent working on the trigonometry topic compared to the number of days suggested by the DBE. The Gauteng Department of Education provides a detailed work schedule in which they suggest what topics to do, when to do them, and in what order (NECT, 2019). When observing the work schedule and the learners' workbook, I was able to conclude that time-on-tasks did not comply with departmental work schedule as the trigonometry topic was started by both teachers a week later than required. Again, time-on-task may have not been sufficient as both teachers had already started with their term tests, and at times they did not come to class to teach the sub-topics. The implication is that more teaching work may have been done by both teachers over a short period of time. Stols (2013) finds that when insufficient time is allocated to a topic, the consequences of this are that fewer days are spent on a topic, fewer exercises on the topic can be done and, ultimately, in all sub-topics a limited number of exercises are done on a higher level of cognitive demand. Stols' (2013) findings on time are strongly consistently with my findings that time (in terms of number of days) allocated to the topic and sub-topics did not comply with CAPS requirements.

Next, time-on-task was measured by time allocated for classwork activities in terms of teaching and classwork tasks. I noticed that both teachers spent time working on teaching and allocated the last few minutes to classwork tasks. Thus, learners did not receive enough time to do the classwork during the period and ended up with incomplete classwork activities. Both teachers advised learners to take the incomplete classwork activities home and complete them as homework. Therefore, adequate time for learning through classwork and homework tasks for learners was not provided. Kahn (2014) states that learners cannot learn to think critically, analyse information, formulate logical arguments, and work as part of a team unless they receive enough time to be involved in mathematics content.

To conclude, the DBE documents give prominence to the necessity for teachers to comply with CAPS specifications during instruction in relation to the types, nature, cognitive demands, , and a time frame. Observations and document analyses as data collection instruments were employed to determine whether Teachers A and B complied with the CAPS guidelines. Two types of tasks selected for Term 1 were compliant with CAPS. There was failure to act in accordance with the CAPS guidelines with regard to the nature of those tasks, as all the tasks were numerical. With regard to the content coverage, both teachers completed all the sub-topics as necessitated by CAPS. However, all of those tasks were on lower-order thinking levels and were, for this reason, non-compliant with the CAPS guidelines in terms of cognitive demand. Finally, both teachers did not comply with CAPS with regard to allocation of time-on-tasks.

5.2.3. Sub-question 3: How can the implementation of tasks by the learners be described?

Research Questions 1 and 2 specify the requirements of the CAPS and the extent to which teachers complied with those requirements. In this section, the focus shifts from teachers to learners. The implementation of tasks by the learners is represented and described in the CAPS (DBE, 2011) and ATP (NECT, 2019) documents. In terms of these documents, the implementation of the tasks by learners encompasses connecting lessons to prior knowledge, experiences, active listening, engagement and familiarity with prescribed books and other materials when working on a mathematical concept, process or relationship. Learners will need to work together to discuss the content.

Observations and document analyses were used to collect data on tasks implemented by learners in two Grade 10 classes in a public school. During the course of instruction, tasks were placed on the board in order to provide teaching and learning. Teachers introduced the topic of trigonometry by writing problems on the board and asking short questions. At this stage, learners were required to disclose their understanding of what they had learned in Grades 8 and 9 to integrate the knowledge with the new information. The prior knowledge included the geometry of 2D shapes, the construction of triangles and the Pythagoras theorem, as well as angles (NECT, 2019), while the new information included defining the trigonometric ratios and extending these definitions to any angle (including special angles), definitions of the reciprocals of the trigonometric ratios, calculations in trigonometry and simple trigonometric equations (NECT, 2015).

Learners' active participation and reflective listening are considered important at this stage to demonstrate that they are still mindful of prior knowledge and also keen to learn the new information. The goal of active listening is that learners will understand the concepts and communicate that understanding among themselves and back to the teachers to confirm the accuracy of the content. Although learners from both classrooms listened attentively, they did not make their contributions when the teachers explored prior knowledge (i.e., Pythagoras' theorem). They only gave chorus responses to concepts that they were familiar with. When working on new content, learners in both classrooms listened passively to the teachers and only took notes. Learners did not work cooperatively in pairs or small groups to grasp the concepts. It is logical that the acquisition of new knowledge demands of learners to actively engage with mathematical activities and show commitment (through active listening and/or working cooperatively) to learn the mathematical content (Carlson, 2005).

Learners have had an opportunity to implement both classwork and homework. Elements suggested by the ATP (NECT, 2019) emphasise how learners should incorporate teachers' work to generate new knowledge. Solving tasks in pairs, working on the board or in the exercise books, and having small-group discussions are the elements learners are encouraged to implement. In Teacher A's classroom, learners engaged in two classwork tasks. Due to a lack of time to complete these tasks, learners were instructed to complete those tasks as homework. In the class of Teacher B, the learners performed three classwork tasks. In the same way, they ended up doing the same classwork tasks as part of their homework. As observed, all the tasks (classwork and homework) were found to be on the lower levels (Levels 1 and 2**).** Classwork and homework tasks undertaken by the learners were therefore fewer than required by CAPS and ATP and did not demand critical thinking.

In conclusion, the implementation of tasks by learners takes place during classroom instructions where learners are required to engage actively in order to combine existing knowledge with new information. In my study, the degree of engagement was passive, with learners only taking notes and responding to questions as a chorus. Learners also carried out lower-level tasks and worked only individually during classroom instructions. Therefore, the implementation of the tasks by the learners contradicted the description given by both CAPS and ATP.

5.2.4. Sub-question 4: How do the teacher-specific factors influence the implementation of mathematics tasks by learners?

The question explored how teacher-specific factors such as approaches and strategies influence the implementation of tasks by learners. Teachers are expected to create an effective learning environment, so that they can make learning more interesting and engaging. Effective approaches and strategies are offered as guidelines in departmental and research documents such the CAPS (DBE, 2011), ATP (NECT, 2019), the National Centre for Educational Achievement (NCET, 2015), and Henningsen and Stein (1997). The word "influence" in this question is inspired by the idea that competence in mathematical process skills such as critical thinking, problem solving, investigating, generalising and proving, is important. I used a structured observations schedule to collect information from the teachers on how these teacher-specific factors deepened content mastery by their learners. The influence on learners' implementation of tasks was assessed in terms of a teacher- and/or learner-centred approach and various teaching strategies such as active and direct teaching.

During instruction, both teachers spent time explaining, asking simple questions and solving tasks on the board. There was some evidence of teacher-learner engagement as both teachers tried to engage learners by asking questions that mostly required short answers instead of asking questions that could provide opportunities for learners to elaborate and explain their mathematical thinking and reasoning. There was no evidence of peer engagement, and the findings revealed that teachers did not encourage active learner-task engagement, as limited time was allowed for learners to engage with tasks. If learners have more opportunity to practise mathematics tasks in class, teachers have the opportunity to walk around the classroom and support learners where necessary, an opportunity lost in this study. Learners had limited time to get involved in learning. Teachers A and B assessed prior knowledge, but both teachers posed questions that prompted learners to give short answers. The two teachers did not provide an opportunity for learners to develop critical thinking and explain their mathematical vocabulary and thinking.

Teachers A and B relied heavily on the prescribed textbook to support their class instruction. All tasks used by the teacher and assigned as classwork and homework came directly from the textbook. The teachers worked out the tasks on the whiteboard themselves. The DBE (2011) and ATP (2015) advocate the use of additional resources and the participation of learners in tasks to make the mathematics classroom an engaging and stimulating learning place where teachers and learners can share resources and ideas during task implementation. The two teachers could not go beyond driving the content the learners had to do because they did not exploit other resources. The discussion on the findings so far indicates that both teachers adopted a teacher-centred approach of delivering content.

According to the DBE requirements, teachers are expected to pose challenging tasks, connect tasks to real-world applications and provide ample time-on-task (NECT, 2019). All of the tasks used on the board and given in the form of classwork and homework were on a lower level (knowledge and routine procedures) and contained simple problems that required calculators to find the solutions. The situation emerged in Teacher B's class where the learners should have had a chance to work on a problem-solving (Level 4) task and connect it to a real-life situation. Teacher B had, however, simplified the real-life task (problem-based) to a basic numerical lower-level task. Russo and Hopkins (2017) affirm that teachers are frequently reluctant to pose challenging tasks to learners and to apply problem-based learning as teaching strategy.

Both Teachers A and B used direct teaching as a teaching strategy . The term "direct teaching" refers to instructional strategies that are structured, carefully sequenced and directed by teachers (Dimitrios et al., 2013). This strategy can be very useful an valuable when learners are involved. In this study the teachers stood in front of the classrooms and discussed, explained, and demonstrated mathematical concepts to the learners, but the learners did not pose questions or add to the teachers' discourse. Learners only listened to instructions, answered short questions and copied notes. Furthermore, there was also no opportunity for learners to explore the subject content in small groups or pairs (active learning). Active learning allows learners to share ideas and intellectually argue for a solution to the problem (Hasan & Fraser, 2015). Using a direct teaching strategy alone did not influence learners to engage actively in mathematics and share mathematical ideas. At the end of the day, the teaching strategies followed by Teachers A and B did not influence learners to think critically, reason logically and creatively solve problems while implementing tasks.

In conclusion, concepts such as prior learning, a high level of engagement, higher-order thinking and practical real-world applications influence learners in the development of the problem-solving skills needed to learn mathematics. These concepts reflect a shift from a teacher-centred to learner-centred teaching approach. Approaches and strategies disseminated in theoretical and philosophical contexts, backed by both the literature and the departmental documents, support this shift. The observations indicated that Teachers A and B had embraced teaching approaches and strategies irreconcilable with the shift. Ultimately, during the execution of tasks, the learners were not inspired to become competent in mathematical process skills.

5.2.5. Primary research question

To what extent do mathematics tasks provide Grade 10 learners with an opportunity to learn trigonometry?

In answering the primary research question, it can be said that the mathematics tasks selected by the teachers and implemented by the learners did not provide sufficient OTL. Two main reasons were that the teachers did not grant learners meaningful opportunities to actively engage and take part in meaningful discussions on solving a broad range of tasks. The tasks chosen by the teachers did not encourage critical thinking, problem solving, investigation, generalisation and proving.

Although the teachers used classwork and homework as prescribed in the official documents, the classwork did not provide sufficient OTL because the learners were not actively involved in solving problems during class time. The nature of the tasks, except for one or two tasks, were all of a numerical nature, which also did not provide sufficient OTL trigonometry as they encouraged memorisation, rote and superficial learning of basic mathematical operations. Moreover, the tasks were not thought-provoking and did not stimulate the learners' interest or elicit mathematical thinking. As meaningful interaction with the content is needed, it is through contextual and problem-solving tasks that learners can demonstrate a conceptual understanding of trigonometry and its application value in everyday life.

A set of measures show that care had not been taken by both teachers to ask questions on all the four levels of cognitive demand. The whole range of tasks were based primarily on lower-level cognitive demands (Levels 1 & 2) in the trigonometry lessons. In so doing, the tasks did not offer sufficient OTL, as the learners could not demonstrate a deep understanding of the content and apply complex and non-algorithmic thinking. The teacher-centred approaches and direct teaching strategies used by both teachers did not provide a space for learners to actively engage in content. Teachers' practices, characterised by these approaches and strategies, involve information-giving sessions with no opportunity for group and whole-class interactive work. Learners merely listened to the teachers and copied solutions from the board, and were not given time to explain their mathematical vocabulary and thinking, as the discourse in the classroom was clouded by short chorus-answers. Learners were not given the autonomy to engage in and improve their learning, which is a key element for OTL to be realised.

To summarise, OTL has been conceived as the degree to which the tasks used by teachers and implemented by learners motivate the learners to develop mathematical skills such as critical thinking, logical reasoning and creative problem solving when performing tasks. The findings suggest that the tasks selected by the two teachers did not provide the learners with OTL trigonometry. Some factors have been associated with this lack of provision of the OTL, such as the nature and the low cognitive level of the tasks, but also the use by teachers of the teacher-centred approaches and direct teaching strategies. Overall, the mathematics tasks did not allow Grade 10 learners an opportunity to learn trigonometry.

5.3. Providing for errors in my conclusion

Intentionally or not, I might have been inaccurate in some of my conclusions made in this study. However, I endeavoured to improve the credibility and trustworthiness of my findings by employing data triangulation. The findings from the document analysis and observations were set side by side (compared) to uncover similarities and differences. Next, I verified the results of the study with information drawn from the literature.

The Hawthorne effect could have led to errors that might have occurred while gathering data. This refers to the situation where teachers modify their behaviour since they are aware of being studied (Payne & Payne, 2004). I tried to overcome the Hawthorne effect through establishing rapport and building a trusting relationship with the participants. This was done by clearly explaining the purpose of the study to the participants and guaranteeing them the confidentiality of the study. I also emphasised to the teachers that the investigation only focused on the uniqueness of an individual participant, that they would not be judged, and that the findings would not be discussed with the HOD.

5.4. Conclusions

Some conclusions regarding how the nature and cognitive demands of mathematics tasks and teachers' approaches and strategies create OTL for Grade 10 learners appear below.

Nature and cognitive demands of the tasks

- A closer look at how to implement higher-level tasks needs to be explored, as the cognitive and difficulty levels were lowered when teachers demonstrated and talked of mathematics as a practice that focuses mainly on procedures and drilling.
- Teachers should not only select and implement lower-level tasks. Many of the tasks selected were easy Level 2 procedures without practical application.
- The following elements of mathematics lessons need to be pointed out during teacher training and development: mathematics tasks should not be too easy; learners' understanding should be closely observed; small-groups and peer interactions should be implemented; there should be variety in levels and types of oral questioning during instruction.
- The teachers should not avoid the integration of real-world problems with critical thinking.
- Grade 10 teachers should select an adequate number of tasks for learners.
- Teachers should allow ample time for learners to complete tasks.

Mathematics teachers' instructional practices

Teachers' approaches and strategies

- Mathematics teachers' instructional practices should be mainly learner-centred.
- The instructional practices should include the use of active learning instructional strategies such as cooperative learning and discussions
- Teachers should spend a considerable amount of time encouraging, through open questions, active participation of learners. A focus on classroom activities showed that both teachers spent a significant amount of lesson time talking with inactive learners. Whole-class teaching was the single pedagogic approach favoured by both teachers.
- Mathematics teachers should not only follow the prescribed textbook, but also, as stated in the ATP and CAPS documents, consult additional teaching resources.
- Mathematics teachers need to attain an adequate sense of cognitively guided instruction and inquiry-based learning.
- Learners' ideas and ways of thinking should be considered and acknowledged.

5.5. Recommendations for further research

Several aspects of teaching and learning mathematics require further research in order to select and implement tasks that provide learners with OTL. These include investigation into:

- The teachers' pedagogical ability to engage learners in such a manner as to explore the depth of their prior knowledge during teaching.
- The nature and level of teachers' content knowledge required to teach trigonometry effectively.
- Time-on-tasks as a teaching strategy that accelerates learning.
- Mathematics teachers' familiarity with the CAPS documents while selecting tasks, including their understanding of the task design process and how to engage the content while solving problems.
- Identification of relevant tasks that not only relate to learners' daily lives but also to how such tasks can be applied effectively to the required lesson content.
- The development of effective questioning techniques to fully engage learners not only in lower-order, but also higher-order cognitive thinking.
- The training of teachers in the use of various instructional strategies in order to achieve the teaching and learning objectives. Instructional strategies could include cognitively guided instruction and small-group discussions.
- How a learner-centred teaching approach, encompassing an active learning instructional strategy, can enhance learner understanding and performance.
- Mathematics teachers' grasp of the concept of critical thinking and the associated problem-solving skills.
- Continuous support of mathematics teachers in their teaching practices as part of teacher a development programme (CPD).

5.6. Limitations of the study

Under this sub-section, it must be pointed out that there were limitations in the process of sampling, gathering data, and conclusions. The sample size was the first limitation of this study as a very small number of mathematics teachers participated. Furthermore, all the participants were males and black and taught in two public schools. The results could have been different if female teachers or people from different racial backgrounds or even if private schools had been included. Therefore, generalisations cannot be made that the findings represent all Grade

10 mathematics teachers in all schools in the Gauteng district. Nevertheless, generalisation was not an aim of the study.

The second limitation noted during the data collection process was that I was the only observer, which could have weakened the reliability of the collected data. Reliability reflects consistency and replicability of results over time (Heale & Twycross, 2015). One cannot therefore assume that a similar study would inevitably yield the same results if it were to be repeated using the same methodology. More reliable data can be obtained from the observation technique if the study includes more than one observer applying the same document analysis and observation processes. I also realised that I had missed valuable communication between the teacher and the learners during classroom observations as I sat at the back of the class so as not to intrude by moving around in the classroom with the audio recorder. I could have used a better recorder to tape the dialogue between the teacher and learners, since some of the audio clips were of poor quality and could not be transcribed. This study was also bound by time and previous personal experiences as a mathematics teacher within the FET band, and could be susceptible to subjective assumptions. Al-Natour (2011) argues that personal dimensions can impact research projects in unpredictable ways. Although my conclusions were scrutinised by my supervisor, the possibility that subjectivity may have influenced my findings cannot be ruled out.

5.7. Reflection

From the time when I decided to study for a master's degree, I was committed to explore a field that appealed to me. It was my interest in this field that kept me focused on my studies. I have discovered a lot about myself and have grown in a number of ways – personally, in my work as a mathematics teacher, and academically. I was determined to become an effective and reflective teacher to my learners and felt the desire to improve myself. Hence, I enrolled for postgraduate studies believing that it would enable me to research and read more about opportunities to teach mathematics effectively through the use of tasks.

On a much broader scale, public schools in South Africa have been getting more attention from the DBE, politics and media. As a mathematics teacher, I started to realise the importance of this subject and its impact in our society. It is indeed my wish that the findings from this study will assist the Department of Education, as well as mathematics teachers, to enhance teachers' knowledge and skills of different teaching styles and teaching strategies, since these have an impact in selecting, adapting and creating tasks as well as implementing them in class.

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APPENDICES

Appendix A:

Requesting permission: Letter to Gauteng Department of Education

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Miss T. Mahlangu
Science, Mathematics and Technology
Groenkloof campus
University of Pretoria

Mahlanguthandi.980@gmail.com

Cell: 078 449 3943

20 February 2019

Dear Sir/Madam

Letter of consent to the Gauteng Department of Education (GDE) Tshwane South

I hereby request permission to use two formally disadvantaged public schools (Section 21) in the Tshwane South district for my research. I would like to invite two Grade 10 Mathematics teachers to participate in this research aimed at investigating the role of mathematical tasks in providing Grade 10 learners an opportunity to learn. This research will be reported upon in my Master's dissertation at the University of Pretoria.

If consent can be obtained from GDE, the data will be collected by means of document analysis and observations. The observations will be audio recorded while I will also be completing an observation schedule during class time to allow for a clear and accurate record of the teachers' classroom practices. The observations will be done during the normal school program and will not disrupt the classroom timetable.

All participation is voluntary and participating teachers may withdraw from this study at any time. Pseudonyms will be used for all the parties (schools and participants) involved to guarantee confidentiality and anonymity. Only my supervisor and I will have access to the audio recordings which will be password protected. The study will be conducted in English and there will be no incentives for the participating schools or teachers.

After the successful completion of my Master's degree, I will give feedback to the GDE in the form of a written report and if the GDE is willing, I would like to do a PowerPoint presentation of my findings to the mathematics subject facilitators.

Yours sincerely

20 /02/2019

__________________________ __________________________

Researcher: Miss T Mahlangu Date

20/02/2019

__________________________ __________________________

Supervisor: Dr JJ Botha Date

Natural Science Building 4-1, Groenkloof Campus, UP

E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Miss T Mahlangu to conduct her research in Tshwane South District schools for her Master's research study. I hereby also grand consent to Miss T Mahlangu to audio record the lessons and make copies of the learners' workbooks.

District official for Tshwane South's name: ____________________

District official of Tshwane South's signature: ___________________

Date: __________________________

Email address: ______________________________

Contact number: ___________________________

Appendix B:

Gauteng Department of Education research approval letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**14 January 2019**
Validity of Research Approval:	**04 February 2019 – 30 September 2019** **2019/09**
Name of Researcher:	**Mahlangu T**
Address of Researcher:	**583 Tuksdorp Res**
	Giosvenor Street
	Hatfield Pretoria 0083
Telephone Number:	**078 449 3943**
Email address:	**Mahlanguthandi.980@gmail.com**
Research Topic:	**The role of Mathematical Tasks in providing Grade 10 Learners an opportunity to learn (OTL).**
Type of qualification	**Masters**
Number and type of schools:	**Two Secondary Schools.**
District/s/HO	**Tshwane South.**

<u>Re: Approval in Respect of Request to Conduct Research</u>

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

15/01/2019

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix C

Requesting permission: Letter to Principal

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Miss T Mahlangu

Science, Mathematics and Technology

Groenkloof campus
University of Pretoria

mahlanguthandi.980@gmail.com

Cell: 078 449 3943

20 February 2019

Dear Sir/Madam

Letter of consent to the Principal

I hereby request permission to use your school for my research. I would like to invite a Grade 10 Mathematics teacher to participate in this research aimed at investigating the role of mathematical tasks in providing Grade 10 learners an opportunity to learn. This research will be reported upon in my Master's dissertation at the University of Pretoria.

The data collection process will be as follows: I will observe the lessons taught by the Grade 10 mathematics teacher. The duration of these observations will be determined by the time needed to complete the topic. The observations will be done during the normal school program and will not disrupt the teacher's timetable. I would like to be granted permission to make copies of all mathematical tasks assigned to learners. The observations will be digitally audio recorded and I will complete an observation schedule during class time. This will allow for a clear and accurate record of the teacher's classroom practice.

All participation is voluntary and once committed to the research, teachers may still withdraw at any time. Confidentiality and anonymity are guaranteed at all times by using pseudonyms for the school and the teacher. The school and the teacher will therefore not be identifiable during the research study or in the findings of my research. However, only my supervisor and I will have access to the digital audio recordings that will be password protected. The study will be conducted in English and there will be no incentives for the participating schools or teachers.

After the successful completion of my Master's degree, I will give feedback to the school in the form of a written report and if the school is willing, I would like to do a presentation of my findings to all mathematics teachers at that school.

For any questions before or during the research, please feel free to contact me. If you allow me to conduct this study in your school, please sign this letter as a declaration of your consent.

Yours sincerely

___________________________	20/02/2019 ___________________________
Researcher: Miss T Mahlangu	Date
___________________________	20/02/2019 ___________________________
Supervisor: Dr JJ Botha	Date

Natural Science Building 4-1, Groenkloof Campus, UP

E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Miss T Mahlangu to conduct her research in this school for her Master's degree research. I also grant consent to Miss T Mahlangu to access and make copies of the learners' workbooks, projects and assignments to be analysed and to audio record the lessons.

School principal's name: ____________________

School principal's signature: ___________________

Date: ___________________________

Email address: ______________________________

Contact number: ____________________________

Appendix D:

Letter of consent to the mathematics teacher

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Miss T Mahlangu
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mahlanguthandi.980@gmail.com
Cell: 078 4493943

20 February 2019

Dear Sir/Madam

Letter of consent to the mathematics teacher

You are invited to participate in a study for my research project aimed at investigating the role of mathematical tasks in providing Grade 10 learners an opportunity to learn. The research will be reported upon in my Master's dissertation at the University of Pretoria. It is proposed that you form part of this study's data collection phase by being observed and to provide me access to your learners' workbooks. The duration of my observations will depend on the length of the topic. I will be making copies of the tasks (homework, classwork, projects and assignments) you have assigned to learners including your annual teaching plan. During my observations, I would like to audio record and take field notes of the lessons to ensure that I capture accurate information of your classroom practice.

Should you declare yourself willing to participate in this research, you will be one of two teachers that form part of my research project. Your participation is voluntary and confidentiality and anonymity will be guaranteed at all times. This will be done by allocating pseudonyms to you and the school during all phases of the research process. Only my supervisor and I will have access to the audio recordings which will be password protected.

The study will be conducted in English and there will be no incentives for the participating schools or teachers.

After the successful completion of my Master's degree, I will give feedback of my findings to the school in the form of a written report and if the school is willing, I would like to do a presentation of my findings to all mathematics teachers at your school.

If you are willing to participate in this research study, please sign this letter as a declaration of your consent, i.e. that you participate willingly and that you understand that you may withdraw at any time.

Yours sincerely

20/02/2019

___________________________ ____________________________

Researcher: Miss T Mahlangu Date

20/02/201

___________________ ____________________________

Supervisor: Dr JJ Botha Date

Natural Science Building 4-1, Groenkloof Campus, UP

E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Miss T Mahlangu to observe one of my Grade 10 mathematics classes, have access to and make copies of the learners' workbooks, projects and assignments for her Master's degree research study. I also grant consent to Miss T Mahlangu to analyse the mathematical tasks and audio record the lessons.

Teacher's name: ____________________

Teacher's signature: ___________________

Date: ___________________________

Email address and contact number: ______________________________

Appendix E:

Letter of consent to the parents/guardians

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Miss T Mahlangu
Science, Mathematics and Technology
Groenkloof campus
University of Pretoria
mahlanguthandi.980@gmail.com

Cell: 078 449 3943

20 February 2019

Dear Sir/Madam

Letter of consent to the parents/guardians

I am currently enrolled for a Master's degree at the University of Pretoria. My research is aimed at investigating the role of mathematical tasks in providing Grade 10 learners an opportunity to learn. The research will be reported upon in my Master's dissertation at the University of Pretoria. In order to do the research, I will observe your child's mathematics teacher. I would like to audio record this lesson as it will help me to have an accurate record of the teacher's classroom practice. I need permission to conduct the study in the classroom that your child attends.

During the course of this study, I will be focusing on how the teacher implements mathematical tasks and the strategies used during instruction. The interaction will be audio recorded to ensure that I capture accurate information of the lessons. The audio recordings will be taken from the front of the class and I will, as far as possible, only record the teacher. I will however be seated at the back of the class. All recordings will be password protected and will only be used for my Master's degree. All children's confidentiality and anonymity will be protected at all times and only my supervisor and I will have access to the recordings and tasks.

By signing this letter you will be granting me permission to be present in the class where your child is being taught.

Yours sincerely

20/02/2019

_________________________ _______________________

Researcher: Miss T Mahlangu Date

20/02/2019

_________________________ ________________________

Supervisor: Dr JJ Botha Date

Natural Science Building 4-1, Groenkloof Campus, UP

E-mail: hanlie.botha@up.ac.za

I the undersigned hereby grant consent to Miss T Mahlangu to audio record the lessons where my child will be present and to make copies of my child's workbook to analyse the mathematical tasks done. I am aware that my child will remain anonymous and that the findings of this research will be used to promote teaching and learning in the mathematics classroom.

Parent's/guardian's name: ____________________

Parent's/guardian's signature: ___________________

Date: __________________________

Child's name: _______________________________

Grade (e.g. 10 C): ___________________________

Appendix F:

Letter of assent to the learners

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Miss T Mahlangu
Science, Mathematics and Technology
Groenkloof campus
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mahlanguthandi.980@gmail.com

Cell: 074 449 3943

20 February 2019

Dear learner

Letter of assent to the learners

I am enrolled for a Master's degree at the University of Pretoria and my research project aims at investigating the role of mathematical tasks in providing Grade 10 learners an opportunity to learn. I will be sitting in class observing your mathematics teacher while taking field notes and doing audio recording. I will not be recording you but the teacher. I will also make some copies of the mathematical tasks in your workbooks, such as homework, classwork, projects and assignment that have been assigned to you by the teacher. That will be the only way you will be involved in the research and you do not have to do anything except what your teacher expects from you. If you have any questions you may contact me at any time.

Yours sincerely

20/02/2019

Researcher: Mss T Mahlangu Date

20/02/2019

Supervisor: Dr JJ Botha Date

Natural Science Building 4-1, Groenkloof Campus, UP

E-mail: hanlie.botha@up.ac.za

I hereby grant permission to Ms Mahlangu to be present in the mathematics class where the lesson will be audio recorded.Miss Mahlangu may also make copies of tasks assigned to you by the teacher.

Learner's name: ____________________

Learner's signature: ___________________

Date: __________________________

Grade (e.g 10): ___________________________

Appendix G:

Document analysis

Document analysis

Brief overview: Research shows that the cognitive demand of mathematical tasks assigned to learners contribute to learners' development in mathematics. Hence, the purpose of this study is to determine the extent to which the tasks provide the learners an opportunity to learn (OTL). The collected data will be used to answer research questions 1, 2, and 3. The data will be collected from the learners' exercise book, learners' scripts, including all mathematical problems that teachers used to explain the content during instruction in line with the CAPS objectives.

SECTION A - Cognitive levels of tasks (Research Question 1 and 2)

The tasks will be analysed according to the following cognitive levels form the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011, p. 322).

Cognitive levels	Description of skills to be demonstrated	✓	Comments
Knowledge 20%	• Estimation and appropriate rounding off numbers		
	• Straight recall		
	• Identification and direct use of correct formulas		
	• Use of mathematics facts		
	• Appropriate use of mathematical		
	• Vocabulary		
Routine Procedures 35%	• Perform well-known procedures		
	• Simple applications and calculation, which might involve many steps		

	• Derivation from given information may be involved		
	• Identification and use (after changing the subject) of correct formula generally similar those encountered in class		
Complex Procedures 30%	• Problems involving complex calculations and/or higher order reasoning		
	• Investigations to describe rules and relationships - there is often not an obvious route to the solution		
	• Problems not based on a real world context - could involve making significant connections between different representations		
	• Conceptual understanding		
Problem Solving 15%	• Unseen, non-routine problems (which are not necessarily difficult)		
	• Higher order understanding and processes are often involved		

	• Might require the ability to break the problem down into its constituent parts		

Cognitive demands of mathematical tasks (Research Question 2)

Cognitive demands: Codes: 0 = extremely, 1 = very, 2 = moderate, 3 = slightly, 4 = not at all

	Cognitive demands of mathematical tasks						
	Mathematical tasks	**0**	**1**	**2**	**3**	**4**	**Comments**
Cognitive Demands	Task offers an appropriate level of challenges						
	Challenging problems for which students' reasoning is evident in their work on the task						
	Task requires consideration cognitive efforts and may involve some level of anxiety for the learners to the predictable nature of the solution process required						
	Task requires learners to access relevant knowledge and experiences and make appropriate use of them in working through the tasks						

	Task involves either reproducing previously learned facts, rules, formula or definitions to memory						

Appropriate Learning tasks (Research Question 2)

Codes: 0 = extremely, 1 = very, 2 = moderate, 3 = slightly, 4 = not at all

	Sub-topic : ________						**Date:___________**
	Mathematical tasks	**0**	**1**	**2**	**3**	**4**	**Comments**
Problem solving	Learners build new mathematical knowledge through problem solving						
	Learners have to apply a variety of appropriate strategies to solve problems						
Reasoning and proofs	Used various types of reasoning and methods of proofs						
	Learners get an opportunity to develop an argument when completing the tasks						

Communications	Language of mathematics is used to express mathematical ideas precisely						
	Learners communicate their mathematical reasoning during implementation of the tasks						
	Encourages multiple perspectives						
Representations	Use presentations such as pictures, sketches, models and instruments to communicate mathematical skills						
	Use presentations to model and interpret physical, societal and mathematical phenomena						

SECTION B: Implementation of the tasks by learners (Research Question 3)

Name of a learner ______________________	Number of problems ________________
Type of task	Problem and solution
Classwork	

Homework	
Assignment	
Project	

Appendix H:

Observation Schedule

Observation schedule

Brief overview: Mathematical tasks are considered to be a tool used by both the teacher and the learner during and after instruction. Hence, the purpose of this study is to determine the extent to which the tasks provide the learners an opportunity to learn (OTL). The collected data will be used to answer research questions 2 and 4.

<table>
<tr><td>Research topic</td><td colspan="9">The role of mathematical tasks in providing Grade 10 learners an opportunity to learn</td></tr>
<tr><td>Researcher</td><td colspan="9">Thandi Mahlangu</td></tr>
<tr><td>Grade</td><td colspan="9">10</td></tr>
<tr><td>Date of observation</td><td colspan="9"></td></tr>
<tr><td>Observed lesson number</td><td colspan="9"></td></tr>
<tr><td>Lesson topic</td><td colspan="9"></td></tr>
<tr><td>Duration of lesson</td><td colspan="9"></td></tr>
<tr><td>Time of lesson</td><td colspan="9"></td></tr>
<tr><td>Audio recording of the lesson</td><td>YES</td><td></td><td>NO</td><td></td><td>Copies of mathematical tasks</td><td>YES</td><td></td><td>NO</td><td></td></tr>
</table>

SECTION A: Tasks that the teacher uses as part of the instruction to explain the concept. (Research Question 2)

Tasks done in class			
Number	Task	Teacher's solution	Cognitive level

SECTION B: Teacher's actions and learners' actions (Research Question 4)

(*Capture observations by placing codes in the appropriate blocks at each time interval. Each code outlines a category. If the categories provided are insufficient, list the category that best captures the observation.*)

Codes: **W** = writing on the board; **E** = explaining/demonstrating; **D** = disciplining;

Q = questions and answer; **I** = individual work and **G** = group work/discussion

Time	**Teacher's action**	**Learners' actions**
5 min		
10 min		
15 min		
20 min		
25 min		
30 min		
35 min		
40 min		
45 min		
50 min		

Adapted from Mdladla (2017)

Teacher's approaches and strategies

Rating Scale: 0 = does not meet the characteristic; 1 = partially meets the characteristic and

2 = fully meets the characteristic

	0	1	2
Teaching approach			
Teacher centred approach			
▪ Instruction was about lecturing.			
▪ Learners are required to listen, duplicate, memorise, drill, calculate, and take notes.			
▪ Teacher follows prescribe procedure on the textbook in explaining/illustrating a mathematical concept, process or			

▪ No time for questions.			
▪ No time for engagement with tasks either on the board or in their exercise books.			
Learner-centred approach			
▪ Teacher consistently asks academically relevant questions that provide opportunities for learners to elaborate and explain their mathematical work and thinking.			
▪ Lesson is connected to students' prior knowledge to make meaningful real-world applications.			
▪ Posing challenging and interesting questions.			
▪ Encourages learners to disclose their own understanding of what they have learned.			
▪ The teacher explains and illustrates the content with appropriate diagrams, and gives concrete examples.			
Comments			
Teaching strategies			
▪ Is problem-based, authentic, and interesting.			
▪ Encouraging learners' participation during instruction.			
▪ Active learning: Small-groups, whole-class interactive work and co-operative learning.			
▪ Direct teaching.			
▪ Cognitively-guided instruction.			
▪ Interactive lecture demonstration.			
▪ Inquiry based learning.			
Comments			