Forecasting the Term Structure of Interest Rates of the BRICS: Evidence from a Nonparametric Functional Data Analysis*

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Abstract

In this paper, we develop a non-parametric functional data analysis (NP-FDA) model to forecast the term-structure of Brazil, Russia, India, China and South Africa (BRICS). We use daily data over the period of January 1, 2010 to December 31, 2016. We find that, while it is in general difficult to beat the random-walk model in the shorter-horizons, at longer-runs our proposed NP-FDA approach outperforms not only the random-walk model, but also other popular competitors used in term-structure forecasting literature. In addition, the NP-FDA model is also found to produce economic gains, besides statistical gains, over the random-walk model. Our results have important implications for both policymakers aiming to stabilize the economy, and for optimal portfolio allocation decisions of financial market agents.

Keywords: Functional data analysis, yield curve forecasting, performance evaluation, BRICS

JEL Codes: C53, E43, G17

1. Introduction

There exists a large number of studies that has highlighted the role played by the term-structure of interest rates as a leading indicator of economic recessions and inflation for both developed and emerging economies (see, Gupta *et al.* , 2013; Ozturk & Pereira, 2013; Hwang & Lee, 2016; Oliveira, 2016; Plakandaras *et al.* , 2017b,a; Gupta *et al.* , 2018; Aye *et al.* , 2019; Chang *et al.* , 2019; Plakandaras *et al.* , 2019; Pierdzioch & Gupta, 2019; Pönkä & Zheng, 2019, for example). At the same time, movements in the term-structure of interest rates serve as a valuable input for practitioners in finance to carry out bond portfolio management, derivatives pricing, and risk management (Caldeira *et al.* , 2016a). Hence, accurate forecasting of the term-structure of a yield curve is of paramount importance to both policy-makers and financial market agents in general, and

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*We would like to thank two anonymous referees and the subject editor, and former and current editors, Professor Ali M. Kutan and Professor Paresh K. Narayan respectively, for many helpful comments. However, any remaining errors are solely ours.

have understandably, resulted in a large literature (see, Caldeira *et al.* , 2016b; Byrne *et al.* , 2017; Caldeira *et al.* , 2018, for detailed reviews). In this regard, although forecasting of the term-structure has received considerable attention in most developed countries, and in particular the United States (US), there is dearth of predictive analysis conducted for emerging countries, barring the works of Luo *et al.* (2012), Vieira *et al.* (2017), Feng & Qian (2018), Shang & Zheng (2018), and Shu & Lo (2018) for Brazil, China and South Africa.

Against this backdrop, the objective of this paper is to provide a comprehensive forecasting exercise of the term structure of Brazil, Russia, India, China and South Africa (BRICS). The decision to choose these emerging bloc of countries is motivated by the fact that the BRICS have grown rapidly, and have become more integrated with the developed world in terms of trade and investment, with the group accounting for about a quarter of the world's Gross Domestic Product (GDP), which in turn, is expected to surpass that of the G7 countries by 2050 (Balcilar *et al.* , 2018; Plakandaras *et al.* , 2018). Note that, trade by these economies with the rest of the world has been growing at a fast rate, and based on the 2015 Global Energy Statistical Yearbook by Enerdata, the share of these countries in the total volume of world trade is about 18% (USD 7.7 trillion), which in fact is about 71% higher than what it was in 2008. Understandably, given the trade and financial dependence in the modern globalized world, possible slowdown of growth of the BRICS countries have important implications for the world economy through feedbacks via the trade-channel (Balli *et al.* , 2017). Hence, accurate prediction of the term-structure, which will in turn contain information about the future path of the GDP of the BRICS, and hence the global world, cannot be underestimated from the perspective of policy decisions. Moreover, post the recent global financial and European sovereign debt crises, emerging market bonds (just like equities) have become an integral component of portfolio decisions of agents in the world financial market searching for better returns (Sowmya *et al.* , 2016; Prasanna & Sowmya, 2017; Ahmad *et al.* , 2018; Stona & Caldeira, 2019), which these sovereign bond market of the BRICS have indeed produced, especially during the recent period of financial turmoil (Lozza & Petronio, 2018). In fact, the sovereign bond markets of China, Brazil, and India alone account for 50% of the outstanding local currency debt (Klingebiel, 2014). Again, in the process, making the precise forecast of the term-structure of interest rates of the BRICS important from the point of view of the global financial markets, and for investors who prefer to diversify their risk by investing a part of their portfolio in bonds issued by different countries rather than solely investing in their own country's government bonds.

In this sense, although our analysis is focused on statistical measures of predictive accuracy, it is important to evaluate the extent to which the apparent gains in predictive accuracy can be used in real time to improve investors' economic utility, that is, translate into better investment performance. Given that statistical significance does not necessarily imply economic significance, we follow what was done in Thornton & Valente (2012), Sarno *et al.* (2016), Caldeira *et al.* (2016c), and Gargano *et al.* (2019), among others, and assess the economic value of the predictive power of interest rates by investigating the utility gains accrued to investors who exploit the predictability of yield curve relative to a no-predictability alternative associated with the random-walk model.

Much progress has been made in forecasting yields for the advanced economies, as well as emerging markets in the limited number of studies that does exist, based on parametric models following the works of Diebold & Li (2006) and Christensen & Rudebusch (2011). Parametric models are indeed very useful if the "strong" assumptions behind these models hold for the data that is being investigated (Das, 2019). But this is not often the case when dealing with high-frequency data from the financial markets, with violations of the assumptions observed, in particular with those involving data generating processes (Doh, 2011; de Araújo & de Andrade, 2015; Feldhütter *et al.* , 2018, for detailed discussions). Given this, our aim is to compare, for the first time, a pure nonparametric functional model with parametric factor models and linear models widely considered in the term-structure literature, in forecasting the yield curve of the BRICS. In the process, following the work of Caldeira & Torrent (2017) on the United States (US), we interpret the yields of the BRICS countries as curves, since the term-structure of interest rates defines a relation between the yield of a bond and its maturity, i.e., we conduct a nonparametric functional data analysis (NP-FDA).

Note that, while there does exist quite a large number of studies on nonparametric estimation of the models of term-structure for both developed and developing bond markets (Florens-Zmirou, 1993; Ait-Sahalia, 1995, 1996; Stanton, 1997; Barzanti & Corradi, 1998; Jiang, 1998; Lin, 2002; de Andrés Sánchez & Gómez, 2004; Hagan & West, 2006; Andersen, 2007; Chiu *et al.* , 2008; Gómez-Valle & Martínez-Rodríguez, 2008; Gimeno & Nave, 2009; Laurini & Moura, 2010; Kaushanskiy & Lapshin, 2016; Sambasivan & Das, 2017, see for example,), the use of the NP-FDA models is rather new and restricted to the two studies mentioned above. While the the standard nonparametric term structure models can control for nonlinearity they do not take advantage of the additional information that could be implied by the smooth underlying function and the insights gained from exploring the derivatives of the curves (Levitin *et al.* , 2007). In this regard, Inoue & Rossi (2019) points out that studying the term structure in a functional manner and its associated shift, provides a more general approach to studying the impact of monetary policy shocks, associated with both conventional and unconventional measures, on the economy and financial markets. This is because, the notion of a scalar shock (like the exogenous movement in the short-term interest rate, forward guidance, and announcements of future asset purchases), can be extended to a monetary policy "event", summarized by the exogenous shift in the entire yield curve associated with unexpected monetary policy decisions. In other words, NP-FDA models are more informative than traditional parametric and nonparametric models associated with the term-structure. Given this, in our paper, we extend the existing parametric models-based works on Brazil, China and South Africa to the entire BRICS bloc, using not only a non-parametric approach that is unlikely to be misspecified due to nonlinearity, but adding to it a FDA structure, which contains additional information by relating the yield of a bond and its maturity via curves. In this regard, it must be pointed out that the only other study to have used a similar approach for the case of China is by Feng & Qian (2018), who in turn show that a functional principal component analysis model relative to popular alternatives in the out-of-sample forecasting.

Note that, unlike the majority of the existing studies on emerging (and developed) bond markets based on monthly data, including the most closely-related study to our work by Feng & Qian (2018) in terms of methodology, we rely on daily data covering the period of January 1, 2010 to December 31, 2016. Further, we use information contained in the term-structure of the interest rates only, rather than macroeconomic variables. With the term-structure being a leading indicator for the macroeconomy of the BRICS (as can be deduced from the studies cited above), high-frequency (in our case, daily) forecasts are likely to be more valuable to policymakers to predict the future path of lower-frequency (monthly or quarterly) macroeconomic variables (like measures of output and inflation), using methods of nowcasting (Breitung & Roling, 2015; Knotek II & Zaman, 2019; Ghysels *et al.* , 2018). In addition, with the term-structure providing leading information for the macroeconomy, then ideally, one should be forecasting the bond yields based on their own (lagged equilibrium) information content, instead of relying on macroeconomic predictors. Hence, we believe our study is more reliable compared to existing work on term-structure forecasting of emerging markets, as we base our forecasts on high-frequency nonparametric functional models without incorporating macroeconomic information.

So, to sum up the contributions of our study are in multiple directions. First, from an econometric perspsctive, we use the NP-FDA models in our analysis, which are more informative than traditional parametric and nonparametric models associated with the term-structure, since these models contain additional information by relating the yield of a bond and its maturity via curves, besides ensuring no misspecification due to nonlinearity. Second, we look at the entire BRICS bloc together, instead of existing single-country analysis within this group. This allows us to provide a multi-country comparative perspective to the performance of the NP-FDA paper at the same time. Studing this bloc simultaneously is important since, the accurate prediction of the term-structure, which will in turn contain information about the future path of the GDP of the BRICS, and hence the global world, is of tremendous value to policymakers. Third, unlike existing studies we use high-frequency data to forecast the term-structure, daily), since daily forecasts of this leading indicator are likely to be more important to policy authorities to predict the future path of lower-frequency (monthly or quarterly) macroeconomic variables. Finally, realizing that while daily forecasts of the term-structure is important from the perspective of nowcasting macroeconomic variables, our approach of forecasting the bond yields based on their own (lagged equilibrium) information content, instead of relying on macroeconomic predictors, is a relaitively more relevant method, since with the term-structure providing leading information for the macroeconomy, then ideally, one should not feed macroeconomic information into these models. The remainder of the paper is organized as follows: Section 2 outlines the basics of the functional data methodology and the associated nonparametric estimation, while Section 3 presents the alternative forecasting models. Section 4 discusses the data and results, with Section 5 concluding the paper.

2. Functional Data Methodology and Nonparametric Estimation

In this section, which follows closely the discussion in Caldeira & Torrent (2017), of we present the functional data analysis methodology used in the paper, and we explain how the methodology aims to address the problem we are interested in.[2] First we define that a random variable $\mathbf{Y}$ is said to be a functional variable, if it takes values in an infinite dimensional space, say E. An observation of $\mathbf{Y}$ is said to be a functional data and it is denoted here by Y. In this paper, the yield curve is viewed as a function (curve) that links maturities to yields. More precisely, when $\mathbf{Y}$ (respectively Y) denotes a random curve (respectively observation), one would be identifying $\mathbf{Y} = \{\mathbf{y}(\tau); \tau \in M\}$ (respectively $Y = \{y(\tau); \tau \in M\}$), where $M \subset \mathbb{N}$ stands for the set of arbitrary maturities.

Nonparametric Functional Estimation. We now describe the proposed estimator and explain how it is used to forecast the BRICS yield for a given maturity[3]. Consider that one is interested in forecast of a dependent scalar variable Y as a function of a functional regressor $\mathbf{Y}$. Let $r(Y) = E(y|\mathbf{Y} = Y)$ be the nonlinear regression operator, where $\mathbf{Y}$ is a functional variable taking values in a semi-metric space (E, d). y is a real random variable and Y is a fixed element in E. Specific to our problem, the regressand is the yield at some specific maturity, while the regressor is the yield curve. Now, let $(\mathbf{Y}_t)_{t=1,...,T}$ be a sample of T random curves. The kernel estimator for $r(Y)$ used for our problem is

$$\hat{r}_h^\tau(Y_s) = \frac{\sum_{t=1}^{T-h} y_{t+h}(\tau) K(b^{-1} d(Y_s, \mathbf{Y}_t))}{\sum_{t=1}^{T-h} K(b^{-1} d(Y_s, \mathbf{Y}_t))}, \tag{1}$$

where d is a suitable semi-metric (as described below), and since d is non-negative, K is a density function with positive support (i.e., $\{v \in \mathbb{R}$ such that $K(v) > 0\} \subseteq \mathbb{R}^+$), $b > 0$ is a bandwidth, and h is the forecast horizon. Moreover, $y_{t+h}(\tau)$ represents the yield for maturity τ at time $t + h$. Y_s is an observed curve where one might want to evaluate the

[2]For further details about the method, see Ferraty & Vieu (2006) and Ramsay & Silverman (2005).
[3]For other estimators of this type see Caldeira & Torrent (2017)

conditional expectation. For instance, $\hat{r}_h^{\tau}(Y_T)$ gives the h-steps ahead forecast from T. Therefore, we are working in the state-domain. In particular, we are considering a time series $\{y_t(\tau)\}_{t=1}^{T}$, and each $y_{t+h}(\tau)$ is predicted based on curves $\{Y_j\}_{j=1}^{t}$ for a given horizon h.

Next we present some details about the estimation procedure. As in Caldeira & Torrent (2017), we consider a typical kernel-type estimator with the bandwidth $b > 0$ acting as a scalar parameter which determines the degree of smoothness in estimation. We denote this estimator simply as KER.

THE KERNEL ESTIMATOR (KER) is defined by

$$R^{KER}(Y_s) = \frac{\sum_{t=1}^{T-h} y_{t+h}(\tau) K(d_q(Y_s, \mathbf{Y}_t)/b_{opt})}{\sum_{t=1}^{T-h} K(d_q(Y_s, \mathbf{Y}_t)/b_{opt})}, \tag{2}$$

where $(\mathbf{Y}_t, Y_t)_{t=1,\dots,T}$ are observed pairs and b_{opt} is a bandwidth which is selected by the following cross-validation type procedure

$$b_{opt} = \arg\min_b GCV(b), \tag{3}$$

where

$$GCV(b) = \sum_{t=1}^{T-h} \left(y_{t+h}(\tau) - R_{(-t)}^{KER}(Y_t) \right)^2, \tag{4}$$

with

$$R_{(-t)}^{KER}(Y_s) = \frac{\sum_{j=1, j\neq t}^{T-h} y_{j+h}(\tau) K(d_q(Y_s, \mathbf{Y}_j)/b)}{\sum_{j=1, j\neq t}^{T-h} K(d_q(Y_s, \mathbf{Y}_j)/b)}. \tag{5}$$

It is important to note that a curve is generally described as having domain in an interval of real numbers. Nevertheless the functional methodology for data analysis allows us to work with a finite set of observed points for each curve. In this regard, the statistical analysis of the discretized curve requires a careful choice of a measure of distance. In the next paragraph, we define the notion of distance in the functional methodology, and we explain which specific measure of distance is convenient for the data we are analyzing.

Semi-metric Spaces. It turns out that there is no equivalence between norms when considering infinite dimensional spaces. Therefore, the choice of a preliminary norm is crucial. In fact, semi-metric spaces are more suitable to problems like the ones we are dealing with here[4]. A semi-metric d may be defined as a metric but such that $d(x, y) = 0 \nRightarrow x = y$, where $x, y \in E$. There are many semi-metrics available in literature, with each one being appropriate for a particular problem, depending on the characteristics presented by the

[4]For details, see sub-section 3.1 of Ferraty & Vieu (2006).

data at hand. Regarding our problem associated with the BRICS yield curve, we observe a discretized curve evaluated on a small number of points (usually less than 20 points). A suitable semi-metric for this case is the principal component analysis (PCA) semi-metric, which is based on the idea of dimension-reduction. More precisely, the PCA semi-metric and its empirical version as stated in Ferraty *et al.* (2005), could be viewed under the assumption that $E\int \mathbf{Y}^2(s)ds < \infty$. Hence, the functional random variable $\mathbf{Y}$ can be expanded in the following way

$$\mathbf{Y} = \sum_{k=1}^{\infty} \left(\int \mathbf{Y}(s) v_k(s) ds \right) v_k, \tag{6}$$

where $v_1, v_2, ...$ are orthonormal eigenfunctions of the covariance operator $\Gamma_{\mathbf{Y}}(t,s) = E\left(\mathbf{Y}(t)\mathbf{Y}(s)\right)$ associated with the eigenvalues. Now let

$$\tilde{\mathbf{Y}}^{(q)} = \sum_{k=1}^{q} \left(\int \mathbf{Y}(s) v_k(s) ds \right) v_k, \tag{7}$$

be a truncated version of Y (eq.(6)). Therefore, for all $(Y_1, Y_2) \in E^2$ we have the following parameterized family of semi-metrics

$$\sqrt{\sum_{k=1}^{q} \left(\int (Y_1(s) - Y_2(s)) v_k(s) ds \right)^2}. \tag{8}$$

Since Γ_Y is not observable, its empirical version is set to be $\Gamma_Y^n(t,s) = n^{-1} \sum_{i=1}^{n} Y_i(t) Y_i(s)$. As a result, the empirical version of the semi-metric used in the estimator is defined for all $(Y_1, Y_2) \in E$ as

$$d_q(Y_1, Y_2) = \sqrt{\sum_{k=1}^{q} \left(\int (Y_1(s) - Y_2(s)) v_{k,n}(s) ds \right)^2}. \tag{9}$$

In practice we need to estimate q. One possibility is choosing q via a cross-validation type procedure.

Estimation details. It is well known that yield curves are non-stationary in the mean, which is an explanation why the random-walk model is so hard to beat in forecasting exercises. In order to overcome this difficulty, we calculate the mean of each curve, that is, we construct a time series $m_t = \frac{1}{n_\tau} \sum_{j=1}^{n_\tau} Y_t^j$, $t = 1, \cdots, T$, where n_τ is the number of maturities, and Y_t^j the yield for maturity j at time t. Then we calculate the demeaned curves $\ddot{Y}_t = Y_t - m_t 1'$, $t = 1, \cdots, T$, where $1'$ is a $1 \times n_\tau$ vector of ones. The proposed forecast of Y is then the sum of the NP-FDA forecast of $\ddot{Y}$ plus a random-walk forecast of m.

The PCA semi-metric parameter, q, was set equal to one in all estimations. Caldeira

& Torrent (2017) argues that for yield curve forecasts, q should be a small number like 1, 2 or 3. We considered these three values for q, and the results obtained were similar, with them being available upon request from the authors. We present the results for $q = 1$. For bandwidth selection, we used GCV as in equation (4). We also considered an asymmetrical Gaussian kernel for K, which in this case is a density of a Gaussian random variable truncated to the range $(0, \infty)$.

3. Competing Models

3.1. Random-Walk model

The main benchmark model adopted in the paper is the random-walk (RW), and for which the $t + h$-step-ahead forecasts for an yield of maturity τ, are given by:

$$y_{t+h}(\tau) = y_t(\tau) + \varepsilon_t(\tau), \qquad \varepsilon_t(\tau) \sim \mathcal{N}\left(0, \sigma^2(\tau)\right). \tag{10}$$

In the RW, a h-step-ahead forecast, denoted $\widehat{y}_{t+h}(\tau)$, is simply equal to the most recently observed value $y_t(\tau)$. In practice, it is difficult to beat the RW in terms of out-of-sample forecasting accuracy, since yields are usually nonstationary or nearly nonstationary, and hence is a good benchmark for judging the relative prediction power of other models. Many other studies that consider interest rate forecasting have shown that consistently outperforming the random-walk is difficult (see, for example, Duffee, 2002; Moench, 2008).

3.2. Univariate autoregressive model

It is possible to generalize the RW model and, forecast the maturity-τ yield based on a first-order univariate autoregressive model (AR) estimated using the available data for that maturity:

$$y_t(\tau) = \alpha + \beta y_{t-1}(\tau) + \varepsilon_t. \tag{11}$$

The 1-step ahead forecast is produced as $\hat{y}_{t+1}(\tau) = \hat{\alpha} + \hat{\beta} y_{t-1}(\tau)$. The forecasts for h-step ahead horizon are obtained as:

$$\hat{y}_{t+h|t}(\tau) = \left(1 + \hat{\beta} + \hat{\beta}^2 + \ldots + \hat{\beta}^{h-1}\right) \hat{\alpha} + \hat{\beta}^h y_t(\tau).$$

3.3. Vector autoregressive model

The fact that the yield curve can be considered a vector process composed of yields of different maturities, implies that the cross-section information might be important in understanding yield curve movements. However, neither the RW nor the AR models exploit this information to produce the forecasts. Thus, a first-order unrestricted vector autoregressive model (VAR) for yields is a natural extension of the univariate AR model. The estimated model is:

$$y_t = A + B y_{t-1} + \varepsilon_t, \tag{12}$$

where $y_t = (y_t(\tau_1), y_t(\tau_2), \ldots, y_t(\tau_N))'$. The 1-step ahead forecast is produced as $\hat{y}_t = \hat{A} + \hat{B} y_{t-1}$, while the h-step ahead forecasts are obtained as:

$$\hat{y}_{t+h|t} = \left(I + \hat{B} + \hat{B}^2 + \ldots + \hat{B}^{h-1}\right) \hat{A} + \hat{B}^h y_t. \tag{13}$$

3.4. Dynamic Nelson-Siegel (DNS) model

Diebold & Li (2006) have introduced dynamics into the original Nelson & Siegel (1987) model, and showed that the resulting model has good forecasting power. The Nelson & Siegel (1987) model was originally designed to describe the cross-sectional aspects of the yield curve by imposing a parsimonious three-factor structure on the links between yields of different maturities. For this model, the structure of the factor loadings in the DNS depend on a single loading parameter λ. This ensures the interpretation of the factors as level, slope and curvature.

The Dynamic Nelson-Siegel model (DNS) is given by:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda_t \tau} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) + \epsilon_t(\tau), \tag{14}$$

where β_1 can be interpreted as the level of the yield curve, β_2 as the slope, and β_3 as the curvature. The parameter λ determines the exponential decay of β_2 and of β_3. The vector of time-varying coefficients β_t follows a VAR process. Once forecasts of the factors are available, the corresponding forecasts of the yields can be retrieved simply by exploiting again the cross-sectional dimension of the system. The DNS can be interpreted as a dynamic factor model, and the Kalman filter can be used to obtain the likelihood function via the decomposition of the prediction error (Jungbacker & Koopman, 2015).

4. Data and Results

4.1. Data

This paper focuses on the daily zero coupon government bond yields of maturities of 12, 24, 36, 48, and 60 months, for the BRICS: Brazil, China, India, Russia, and South Africa. The daily yields are sourced from the Datastream database of Thomson Reuters, with the sample period covering January 1, 2010 to December 31, 2016, yielding a total of $T = 1488$ daily observations. Note that the start and end dates of the data set is purely driven by data availability on the bonds across the BRICS, and ensures continuous data availability on the same set of maturities covering 1- to 5-year across the 5 emerging markets. This is understandably important, since we want to compare the results across the 5 emerging markets, and would need homogeneity in terms of the maturities of the bonds. It must be noted that our models only use data information on the bond yields at daily frequency, unlike low-frequency Affine term-structure models

(Vasicek, 1977; Cox *et al.* , 2005; Duffie & Kan, 1996; Dai & Singleton, 2002), which also incorporate macroeconomic and financial variables into the econometric framework.

At this stage, it is important to emphasize on the basic characteristics of the bond markets of these 5 countries. In terms of Govt debt to GDP ratio, an indicator of maturity of debt market, barring Russia, other BRICS nations have a relatively deep sovereign debt market, with this ratio ranging from 66% in Brazil and India to 18% in Russia (Gandhi, 2016). China's bond market has seen a rapid growth in recent times making it the third largest bond market of the world after the US and Japan. Unlike the US, where the corporate debt market is bigger than the government debt market, India and China have bigger sovereign debt segment with its ratios to corporate debt being at 2.7 and 2.1 respectively (Gandhi, 2016). At the same time, the South African government bond market is highly liquid and has been ranked sixth internationally in terms of liquidity, as measured by the turnover ratio, and is known to provide attractive fixed income yields relative to the developed markets (Hassan *et al.* , 2013). In terms of local currency denomination of these bonds, Arslanalp & Tsuda (2014), Lu & Yakovlev (2017), and Shu & Lo (2018) find that the sovereign debt holdings for China, Brazil, India, South Africa and Russia equates to 1340.27 billion USD, 859.48 billion USD, 680.49 billion USD, 181.0 billion USD, and 132 billion USD respectively. Clearly then, we can conclude that among the BRICS countries, while government bond markets have been traditionally strong in case of Brazil, India and South Africa, there has been tremendous growth in the case of China, with Russia generally lagging behind in this bloc.

4.2. Results

The forecasting analysis is performed based on a pseudo real-time exercise, i.e., we never use information which is not available at the time the forecast is made. For computing our results we use a rolling estimation window of 500 daily observations (i.e., 2 years).[5] We produce forecasts for 1-week, 1-month, 3-month, 6-month, and 12-month-ahead.

To compare the performance of out-of-sample forecasts, we compute the root mean square forecast error (RMSFE). Moreover, the Diebold & Mariano (1995) test (DM-test) is used assess whether each of the model outperforms the RW. Tables 1 and 2 report statistical measures of the out-of-sample forecasting performance at various horizons derived from the alternative models for the BRICS. Following the existing literature, all RMSFE are calculated relative to those of the RW model. The first row of entries in each panel of the tables report the value of RMSFE (expressed in basis points) for the random-walk model (RW), while all other rows report statistic relative to the RW. Asterisks (**10%, *5%) on the right indicate the level of significance for the forecast comparison

[5] We have also estimated the models using an expanding window. However, the results obtained were qualitatively similar to those presented here, and are available upon request from thee authors.

test (i.e., a model has a lower RMSFE than the RW in the statistical sense). The drawback of using RMSFE is that it is an aggregate forecast error measure. Although some models, which are very different in formulation, have approximately the same average squared forecast error, it is possible that the information contained in the forecast series for each model is different. Therefore, to further investigate whether one forecast contains different information from the RW forecast, the forecast encompassing test of Fair & Shiller (1990) are also performed (hereafter referred to as FS-test).[6] The results from application of the Fair & Shiller (1990) encompassing test are given in Table 3 for all competing models relative to RW forecasts. A “checkmark" indicates that the model contain independent information, relative to RW, that is useful in predicting future yields at a significance level of 5%. That is, the coefficient β_m (Equation 17) is statistically different from zero at the significance level of 5%. The following model abbreviations are used in Tables 1-3: AR(1) for the first-order univariate autoregressive model, VAR(1) for the first-order vector autoregressive model, DNS for the dynamic Nelson-Siegel model with a VAR specification for the factors, and NP-FDA stand for non-parametric functional data analysis, respectively.

We start the model evaluation by investigating the performances of the interest rates forecasts at 1-week horizon (across all models and datasets). First, as documented in the literature, the RW is found to be a very competitive benchmark in forecasting the term-structure of bond yields, especially at short horizons. RW outperforms almost all other competing models. One important exception involves Russia. In this case, the NP-FDA is able to beat RW for all maturities considered. The differences are statistically significant according to DM-test. According to the FS-test, the NP-FDA method is the only one that contains additional information regarding the RW model for all countries except South Africa. In addition, with respect to Brazil and Russia, the test rejects H_0 for four out of the five maturities considered.

We now summarize in detail the out-of-sample country-specific forecasting performance for all maturities across all forecast horizons, except the 1-week-ahead case, which has already been discussed above. The reason for undertaking this type of a presentation is primarily due to the fact that there are more cases, where alternative models outperform the RW, and hence, needs detailed discussion.

- For the Brazilian yield curve, the VAR, the DNS-VAR and the NP-FDA are are able to outperform the RW, as reflected by the relative RMSFE. The DNS-VAR shows the best performance, beating RW at almost all maturities and for all forecast horizons. Furthermore, such gains are statistically significant in many cases. Regarding FS-test the same three models are the only ones that contain significant different information from the RW forecast, barring the VAR at 12-month ahead forecasting

[6] See the appendix for details on the FS-test.

horizon.

- For the case of China, the NP-FDA is the only method that statistically outperforms RW at 3- and 6-months ahead horizon, according to DM-test. For short-term forecasts no model outperforms the RW. Some models beat the NP-FDA for 12-step ahead horizon. In terms of the FS-test, the NP-FDA is the most important forecasting model, containing significant additional information over RW in all cases considered.

- Based on the DM-test, the results for the Indian yield curve are very similar to those found for China. Indeed the NP-FDA method outperforms the RW for almost all maturities and forecast horizons. In many cases the results are statistically significant. Also, the DNS-VAR and AR models perform well for long-term forecasts. Looking at Table 3, we see that the NP-FDA is the only method that contain significant additional information over RW at 1-month ahead forecast. For longer than 1-month horizons AR, DNS-VAR and NP-FDA present valuable contributions.

- In the case of Russian yield curve the results are slightly different, the NP-FDA approach performs well in short- and medium-term forecasts but is outperformed by the RW for longer forecasts. The DNS-VAR presents lower RMSFE than RW for the 12-month-ahead forecast, but performs much worse than RW at all other horizons. In this case no model systematically outperforms the RW. Nevertheless, based on the FS-test, NP-FDA is the most consistent method, bringing relevant forecasting information for all horizons considered.

- Finally, for the South Africa yield curve, the NP-FDA shows the best performance among all models in terms of RMSFE, with few exceptions. Based on the FS-test, the NP-FDA is the best model in the sense that no other model has more H_0 rejections than NP-FDA. It is worth noting that NP-FDA contain significantly different information from the RW forecasts at all maturities at 1- and 3- month-ahead horizons.

To summarize, while there is some heterogeneity across the countries in terms of the performances of the alternative models relative to the RW, especially at horizons beyond one-week, in general, the NP-FDA is the only method that is able to systematically outperform the RW at all maturities and all datasets for medium- and long forecast horizons. The gains are over 10% at some maturities at the 3-month ahead horizon, and reaches to 8% for the 6-month- and 12-month-ahead horizons. The VAR and DNS-VAR models beat NP-FDA for a few selected datasets and long forecast horizons but they perform much worse than the NP-FDA in the remaining cases. Specifically, DNS-VAR model does a good job for the Brazilian yield curve at long-horizon forecast (i.e., 3-, 6-, and 12-month-ahead) and for Indian yields at 12-month-ahead forecasts. But the scale tilts in favor of the NP-FDA approach decisively, when we observe from Table 3 that NP-FDA approach overcomes its competitors in terms of the additional information content in almost all datasets and forecast horizons, with the model able to outperform the RW mainly for medium- and long-term horizons in a statistically significant manner.

Our results are in line with Caldeira & Torrent (2017) obtained for the US, in the sense that the NP-FDA method performs better than the other alternative models considered at medium to long forecast horizons, but unlike Caldeira & Torrent (2017), we are able to produce gains for all maturities, and not just the short- and medium-term maturities. Note that, given the non-stationarity of yields, and hence suggesting market efficiency, makes it difficult to beat the RW model. The fact that the NP-FDA model performs well even at shorter-horizons for Russia, could be tied to the bond market being least developed and hence having lowest efficiency. However, the superior performance of the NP-FDA approach especially at medium to longer-horizons within the BRICS bloc is an indication that allowing for a nonparametric functional approach brings in additional information beyond what the yields already contain, and hence leads to better forecasting results, but this is primarily true beyond the immediate short-run.

Now the importance of forecasting the term structure of interest rates accurately is of tremedous importance to the policymakers, since the term spread provides a signal of recession, especially if there is an inversion of the yield curve, i.e., if short-term rates are expected or forecasted to become lower than long-term rates. This is because, investor will be unwilling to make long-term investment in capital. At the same time, if long-term rates are expected to be exceptionally high than the short-term rates, then the economy could end up getting overheated due to higher economic activity, i.e., aggregate demand, and could produce inflationary impact, which is also not reasonable especially for inflation targeting countries like Brazil, Russia, India and South Africa in our case. In other words, the monetary authority does indeed face a tradeoff in terms of ensuring the adjustment of the policy rate to affect the short-end of the yield curve based on how the path of the forecasts of the bond yilds of various maturities look like. But with unemployment being a major concern in many emerging economies, including the BRICS, the central banks would likely to be more worried if an inversion of the yield curve is forecasted, but would require to ensure that lowering of the short-term interes rate as a response also does not end-up overheating the economy. Naturally, the policymaker would want to know precisely the future values of the yield curve. As we show, while RW model tends to do a good job in predicting the yields at the immediate horizon, information from the NP-FDA curve needs to be utilized to make accurate long-term predictions of the yield curve. It must be realized though, that for the policymaker, it is of paramount importance to see if the inversion of the yield curve is actually likely to be persistent, since if the inversion is transitory and does not last long in the future, then there could be no recession or even if it takes place it might not be too deep. Hence, the long-term forecasts of the yields are likely to drive the decision of the policy authority to change the policy rate to affect the short-end of the yield curve and prevent the recession. We must emphasize that policy rates cannot be changed frequently (if forecasts end-up being consistently inaccurate) to ensure lack of monetary policy uncertainty (which can also cause recessions, asi indicated by Istrefi & Mouabbi (2018)), and thus the focus on accurate long-term forecasts cannot be underestimated. And herein comes the importance of our NP-FDA approach, given its ability to outperform the standard competitors used in the literature to predict the yield curve at longer-horizons. In sum, our results tend to highlight that the central banker must realize importance of the NP-FDA framework for producing accurate forecast of the term structure, and must include it in its suit of models.

4.3. Economic-based forecast evaluation

Although our analysis is focused on statistical measures of predictive accuracy, it is important to evaluate the extent to which the apparent gains in predictive accuracy can be used in real time to improve investors' economic utility, that is, translate into better investment performance. Given that statistical significance does not necessarily imply economic significance (Thornton & Valente, 2012; Sarno *et al.* , 2016; Caldeira *et al.* , 2016c; Gargano *et al.* , 2019), we assess the economic value of the predictive power of interest rates by investigating the utility gains accrued to investors who exploit the predictability of yield curve relative to a no-predictability alternative associated with the random-walk model.

We consider a mean-variance investor with quadratic utility and relative risk aversion γ who allocates her portfolio between between a risk-free rate versus a bond with 1-5 years maturity (Rapach & Zhou, 2013). At the end of t, the investor allocates the following share of her portfolio to bond with maturity τ_i during $t+1$:

$$w_{i,t} = \left(\frac{1}{\gamma}\right)\left(\frac{\hat{r}_{t+h}^{(\tau_i)}}{\hat{\sigma}_{t+h}^{2,(\tau_i)}}\right) \tag{15}$$

where $\hat{r}_{t+h}^{(\tau_i)} = \tau_i y_t^{\tau_i} - (\tau_i - h)\hat{y}_{t+h}^{\tau_i - h}$ is a return forecast for the bond with maturity τ_i in time t and $\hat{\sigma}_i^{2}$[7] is a forecast of the variance of bond returns. Over the forecast evaluation period, the investor realizes the average utility,

$$\hat{\nu}_i = \hat{\mu}_i - 0.5\gamma\hat{\sigma}_i^2, \tag{16}$$

where $\hat{\mu}_i$ ($\hat{\sigma}_i^2$) is the sample mean (variance) of the portfolio formed on the basis of $\hat{r}_{t+h}^{(\tau_i)}$

[7]We follow the strategy of Rapach & Zhou (2013) and estimate the variance of bond returns using the sample variance computed from a one-year (252-obs) rolling window of historical returns.

and $\hat{\sigma}_i^2$ over the forecast evaluation period. The difference between utility (16) with model m and random-walk represents the utility gain accruing to using the competitors models forecast of the bond yields in place of the random-walk forecast in the asset allocation decision. This utility gain (certainty equivalent return) can be interpreted as the portfolio management fee that an investor would be willing to pay to have access to the information in the model forecast relative to the information in the random-walk.

Table 4 reports certainty equivalent (average utility gains in annualized percent return) for a mean-variance investor with $\gamma = 5$ who allocates among 1 to 5 years bonds and risk-free bills using forecasts based on competitors models in place of random-walk forecasts. The results indicate that the NP predictive approach is the only one that provide economic value relative to the RW benchmark for all considered dataset and forecast horizon (1- and 3-month ahead). Specifically, NP approach provide annualized gain above 0.5% for twenty of the fifty cases considered, meaning in this cases that that the investor would be willing to pay more than 0.50% to have access to the information in the NP compared to the random-walk forecasts. The best performance of the NP approach is found for the Brazilian market, in which the utility gains are greater than 100 basis points for all considered maturities at 3-month ahead forecast. For the Indian market the NP approach is the only one that provides utility gains, all other models presents negative equivalent uncertainty, reflecting the fact that none of the alternative models are economically superior to the random-walk benchmark. Similar picture are found for Russian bond yields. Overall, the average utility gains provide support for NP approach to forecast yield curve in emerging markets, emphasizing the need to supplement standard statistical criteria with more direct economic-based measures when analyzing out-of-sample bond return predictability.

There are several studies such as Thornton & Valente (2012) and Sarno *et al.* (2016) which argue that the statistical evidence on bond return predictability does not translate into economic gains. Our results are in line with Gargano *et al.* (2019), who have found evidence that reconciling the statistical and economic predictability.

5. Final remarks

Given the importance of bond markets of emerging economies for the purpose of portfolio diversification, in this paper, we develop a non-parametric functional data model to forecast the term-structure of BRICS countries. Our results show that while it is in general difficult to beat the random-walk model in the short-run (one-week- and one-month-ahead), at longer-horizons (3-month-, 6-month-, and 12-month-ahead) our proposed approach outperforms not only the random-walk model, but also other popular competitors used in this literature. Having said this, even in the short-run, there are instances when our proposed model outperforms the random-walk. As term-spread is known to predict recessions and inflation, our results also have important implications for the policymakers. In particular, if the forecasts indicate that the short-term yields are going to increase (decreases) at a faster (slower) rate than the long-term yields on government bonds, and hence implies an inversion of the yield curve, then there is an imminent recession, which in turn would require the monetary authority to reduce the policy rate accordingly to ensure that the negative impact on the economy is avoided, as the lowering of the monetary policy instrument, will lower the short-term bond yields. But, for this to happen, policy authorities would need to rely on a nonlinear functional data-based approach (rather than a random-walk model) to produce accurate forecasts of the yield curve particularly at the long-horizon, to devise appropriate policies to ensure stable growth and inflation. For shorter-horizon forecasts of the term-structure, the central banks in the BRICS can use the RW model, but it needs to be realized that policy decisions are likely to be based on long-term forecasts, and herein is the importance of our NP-FDA approach from the perspective of policy-making.

A possible limitation of our work is that the NP-FDA methodology, requires demeaning of the data to ensure non-stationarity at the curve-level. Future econometric development would be to investigate different strategies for separating the level and shape of the curve, by taking into account the forecasting performance of the sum of the level forecast with the NP-FDA (curve-shape) prediction.

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Table 1: **Relative Root Mean Square Forecast Errors for Yields of Brazil, China, and India**

Note: In these tables we present the forecasting performance of the various models for selected maturities. The Table reports the Root Mean Squared Forecast Errors relative to the Random-Walk (RW) model obtained by using individual yield models for the horizons 5-, 21-, 63-, 126, and 252-step-ahead. The evaluation sample is 2011:1 to 2016:12 ($\approx$ 1000 forecasts). The first line in each panel of the table reports the value of RMSFE (expressed in basis points) for the RW, while all other lines reports statistics relative to the RW. The following model abbreviations are used in the table: RW stands for the Random-Walk, (V)AR for the first-order (Vector) Autoregressive Model, DNS for dynamic Nelson-Siegel model with a VAR specification for the factors, and NP stand for the non-parametric functional data analysis, respectively. Numbers smaller than one indicate that models outperform the random-walk, whereas numbers larger than one indicate underperformance. The stars on the right of the cell entries signal the level at which the Diebold and Mariano (1995)'s test rejects the null of equal forecasting accuracy (*, and ** mean respectively rejection at 5%, and 10% level).

Models	*Brazil*					*China*					*India*				
	1-Year	2-Years	3-Years	4-Years	5-Years	1-Year	2-Years	3-Years	4-Years	5-Years	1-Year	2-Years	3-Years	4-Years	5-Years
Horizon = 1-week ahead															
RW	0.210	0.293	0.323	0.328	0.338	0.114	0.076	0.077	0.078	0.078	0.123	0.125	0.125	0.124	0.122
AR	1.101	1.091	1.090	1.091	1.092	1.233	1.571	1.487	1.463	1.436	1.064	1.063	1.063	1.065	1.069
VAR	1.115	1.085	1.099	1.099	1.167	1.142	1.227	1.209	1.223	1.258	1.080	1.075	1.073	1.074	1.075
DNS-VAR	1.131	1.039	1.040	1.049	1.094	2.124	1.125	1.150	1.189	1.234	1.114	1.086	1.074	1.066	1.062
NP-FDA	1.278	1.083	0.968**	0.980	1.095	1.220	1.078	1.058	1.042	1.031	1.097	1.020	0.987	0.985	0.999
Horizon = 1-month ahead															
RW	0.465	0.599	0.652	0.663	0.669	0.265	0.161	0.161	0.162	0.162	0.245	0.246	0.245	0.244	0.243
AR	1.068	1.055	1.049	1.047	1.044	1.320	1.918	1.815	1.778	1.735	1.072	1.084	1.095	1.103	1.108
VAR	1.072	0.988	0.986	1.007	1.064	1.121	1.231	1.278	1.326	1.374	1.131	1.116	1.104	1.093	1.082
DNS-VAR	1.010	0.938**	0.925**	0.916*	0.922*	1.443	1.185	1.241	1.311	1.380	1.312	1.263	1.235	1.218	1.207
NP-FDA	1.110	1.021	0.973	0.980	0.993	1.024	1.003	1.007	1.011	1.012	1.048	1.003	0.970**	0.950**	0.941**
Horizon = 3-months ahead															
RW	0.909	1.174	1.293	1.316	1.311	0.464	0.311	0.313	0.315	0.315	0.445	0.453	0.459	0.465	0.471
AR	1.126	1.099	1.086	1.079	1.077	1.310	1.617	1.554	1.522	1.493	1.051	1.061	1.067	1.067	1.064
VAR	1.159	0.983	0.924**	0.920**	0.946**	1.085	1.186	1.203	1.222	1.244	1.180	1.156	1.132	1.109	1.088
DNS-VAR	0.903**	0.831	0.792	0.784	0.781	1.210	1.206	1.197	1.202	1.215	1.221	1.177	1.153	1.138	1.127
NP-FDA	0.921**	0.886	0.879	0.884	0.894	1.017	0.982	0.963**	0.950**	0.941**	1.005	0.987	0.970**	0.955**	0.942**
Horizon = 6-months ahead															
RW	1.523	1.871	2.06	2.091	2.088	0.588	0.486	0.487	0.487	0.485	0.584	0.611	0.632	0.646	0.656
AR	1.215	1.168	1.143	1.126	1.117	1.250	1.192	1.168	1.156	1.148	1.005	0.993	0.982	0.972	0.963**
VAR	1.294	1.074	0.970**	0.947	0.944	1.052	1.043	1.047	1.054	1.065	1.116	1.079	1.053	1.035	1.022
DNS-VAR	0.797*	0.701*	0.664*	0.660*	0.653*	1.081	1.002	0.993	0.994	1.001	1.076	1.040	1.018	1.004	0.994
NP-FDA	1.081	1.005	0.971	0.975	0.980	0.919**	0.971**	0.952**	0.940**	0.933	0.975	0.961**	0.953**	0.951**	0.951**
Horizon = 12-months ahead															
RW	2.569	2.636	2.658	2.611	2.572	0.813	0.718	0.712	0.705	0.698	0.757	0.786	0.808	0.822	0.831
AR	1.467	1.380	1.332	1.278	1.224	1.016	0.855*	0.851*	0.850*	0.852*	0.897*	0.864*	0.846*	0.836*	0.831*
VAR	1.391	1.273	1.193	1.170	1.1380	0.980	0.866*	0.867*	0.871*	0.877*	1.066	1.004	0.964**	0.938**	0.920**
DNS-VAR	0.785*	0.743*	0.716*	0.706*	0.690*	1.050	0.834*	0.821*	0.816*	0.812*	0.937**	0.918**	0.905**	0.893*	0.882*
NP-FDA	1.074	0.992	0.934**	0.920**	0.899*	1.007	0.974	0.964**	0.951**	0.959**	0.991	0.974	0.963**	0.956**	0.953**

Table 2: **Relative Root Mean Square Forecast Errors for Yields of Russia and South Africa**

Note: In these tables we present the forecasting performance of the various models for selected maturities. The Table reports the Root Mean Squared Forecast Errors relative to the Random-Walk (RW) model obtained by using individual yield models for the horizons 5-, 21-, 63-, 126, and 252-step-ahead. The evaluation sample is 2011:1 to 2016:12 ($\approx$ 1000 forecasts). The first line in each panel of the table reports the value of RMSFE (expressed in basis points) for the RW, while all other lines reports statistics relative to the RW. The following model abbreviations are used in the table: RW stands for the Random-Walk, (V)AR for the first-order (Vector) Autoregressive Model, DNS for dynamic Nelson-Siegel model with a VAR specification for the factors, and NP stand for the non-parametric functional data analysis, respectively. Numbers smaller than one indicate that models outperform the random-walk, whereas numbers larger than one indicate underperformance. The stars on the right of the cell entries signal the level at which the Diebold and Mariano (1995)'s test rejects the null of equal forecasting accuracy (*, and ** mean respectively rejection at 5%, and 10% level).

Models	***Russia***					***South Africa***				
	1-Year	2-Years	3-Years	4-Years	5-Years	1-Year	2-Years	3-Years	4-Years	5-Years
Horizon = 1-week ahead										
RW	0.383	0.349	0.356	0.357	0.356	0.175	0.188	0.197	0.201	0.204
AR	1.203	1.344	1.278	1.197	1.153	1.096	1.098	1.098	1.096	1.093
VAR	1.180	1.233	1.228	1.194	1.165	1.129	1.127	1.124	1.116	1.108
DNS-VAR	1.431	1.389	1.288	1.236	1.206	1.041	1.005	1.005	1.006	1.007
NP-FDA	0.903**	0.831*	0.893*	0.903**	0.891*	1.800	1.605	1.582	1.542	1.520
Horizon = 1-month ahead										
RW	0.857	0.876	0.884	0.871	0.852	0.341	0.367	0.383	0.391	0.393
AR	1.284	1.401	1.331	1.233	1.151	1.067	1.055	1.047	1.041	1.035
VAR	1.281	1.255	1.230	1.197	1.167	1.114	1.099	1.083	1.064	1.049
DNS-VAR	1.223	1.123	1.085	1.090	1.133	1.023	1.016	1.018	1.018	1.018
NP-FDA	0.947**	0.977	0.991	0.984	0.971**	0.784*	0.754*	0.729*	0.706*	0.695*
Horizon = 3-months ahead										
RW	1.498	1.514	1.502	1.464	1.419	0.556	0.587	0.603	0.607	0.603
AR	1.410	1.444	1.374	1.302	1.227	1.118	1.110	1.103	1.093	1.085
VAR	1.372	1.298	1.253	1.216	1.189	1.116	1.109	1.087	1.066	1.047
DNS-VAR	1.326	1.265	1.249	1.266	1.307	1.031	1.041	1.050	1.053	1.052
NP-FDA	0.893*	0.946**	0.972	0.977	0.975	0.907**	0.917**	0.907**	0.896*	0.892*
Horizon = 6-months ahead										
RW	1.897	1.956	1.951	1.905	1.846	0.754	0.808	0.843	0.858	0.859
AR	1.471	1.351	1.261	1.202	1.156	1.200	1.178	1.152	1.128	1.108
VAR	1.458	1.318	1.25	1.209	1.184	1.151	1.116	1.063	1.016	0.980
DNS-VAR	1.376	1.253	1.213	1.221	1.268	1.093	1.119	1.128	1.128	1.126
NP-FDA	1.036	1.055	1.067	1.071	1.071	1.023	1.016	0.976	0.937**	0.907**
Horizon = 12-months ahead										
RW	2.718	2.812	2.804	2.730	2.633	1.096	1.166	1.209	1.225	1.219
AR	1.273	1.112	1.027	0.982	0.963**	1.185	1.164	1.124	1.088	1.055
VAR	1.282	1.154	1.094	1.065	1.052	1.135	1.076	0.996	0.927**	0.874*
DNS-VAR	1.060	0.971**	0.938**	0.940**	0.944**	1.267	1.284	1.282	1.277	1.276
NP-FDA	1.025	0.988	0.999	1.035	1.073	1.074	1.033	0.979	0.939**	0.914**

Table 3: **Comparisons of the relative (to random-walk) information content for out-of-sample forecasts based on Fair-Shiller regression**

Note: The Table reports the Fair-Shiller pairwise forecast comparisons for the competitors models relative to random-walk. The individual models are the following: AR refers to autorregressive model of order 1; VAR refers to vector autorregressive of order 1, DNS refers to Dynamic Nelson-Siegel model, and NP-FDA stands for non-parametric functional data analysis. All figures are based on out-of-sample observations. Checkmark indicate that a given model contain independent information that is useful in predicting future yields at a significance level of 5%.

Model	Brazil					China					India					Russia					South Africa				
	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$	$\tau=5$
Horizon = 1-*week ahead*																									
AR													✓	✓											
VAR												✓	✓								✓				
DNS		✓	✓	✓									✓	✓											
NP-FDA		✓	✓	✓	✓				✓	✓			✓	✓	✓	✓	✓	✓		✓					
Horizon = 1-*month ahead*																									
AR																								✓	✓
VAR		✓	✓																						
DNS		✓	✓	✓	✓											✓						✓			
NP-FDA		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓			✓	✓	✓	✓	✓	✓
Horizon = 3-*months ahead*																									
AR																									
VAR		✓	✓	✓	✓																				✓
DNS	✓	✓	✓	✓	✓			✓	✓	✓		✓	✓	✓	✓						✓				
NP-FDA	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Horizon = 6-*months ahead*																									
AR													✓	✓	✓										
VAR			✓	✓	✓																			✓	✓
DNS	✓	✓	✓	✓	✓			✓	✓						✓								✓	✓	✓
NP-FDA		✓	✓	✓	✓	✓	✓	✓	✓	✓			✓		✓	✓					✓			✓	✓
Horizon = 12-*months ahead*																									
AR							✓		✓	✓		✓		✓	✓			✓	✓	✓					
VAR							✓	✓	✓	✓				✓	✓					✓			✓	✓	✓
DNS	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓					
NP-FDA		✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓					✓	✓	✓

Table 4: **Performance of economic evaluation of yield curve out-of-sample forecasting**

Note: This table reports average utility gain (Δ) is the portfolio management fee (in annualized percent return) that an investor with mean-variance preferences and risk aversion coefficient of five would be willing to pay to have access to the forecasting method relative to the Random-Walk benchmark forecast. The forecasts are obtained using individual yield models for horizons 1- and 3-month-ahead. The following model abbreviations are used in the table: (V)AR stands for the first-order (Vector) Autoregressive Model, DNS for dynamic Nelson-Siegel model with a VAR specification for the factors, and NP stand for the non-parametric functional data analysis, respectively. Positive numbers indicate that models outperform the random-walk, whereas a negative entry indicate underperformance.

Models	1-year	2-years	3-years	4-years	5-years	1-year	2-years	3-years	4-years	5-years
Brazil	Forecast horizon = 1-month ahead					Forecast horizon = 3-month ahead				
AR(1)	−0.049	0.228	0.417	0.609	0.866	0.547	−0.439	−2.074	−3.315	−4.107
VAR(1)	0.044	0.258	0.768	1.470	3.963	1.706	4.865	8.733	11.04	10.12
DNS-VAR	0.082	0.134	0.086	0.407	0.152	3.01	9.78	16.87	20.42	19.18
NP-FDA	0.137	0.533	1.359	2.093	2.560	4.384	14.90	20.32	23.49	23.48
China	Forecast horizon = 1-month ahead					Forecast horizon = 3-month ahead				
AR(1)	−0.004	−0.023	0.006	0.080	0.107	−0.017	−0.0578	−0.063	0.024	0.074
VAR(1)	0.023	0.065	0.084	0.066	0.022	0.020	0.081	0.082	0.015	−0.071
DNS-VAR	−0.003	0.047	0.062	0.032	0.011	−0.146	−0.149	−0.117	−0.077	−0.034
NP-FDA	0.158	0.302	0.425	0.534	0.630	0.092	0.272	0.463	0.673	0.881
India	Forecast horizon = 1-month ahead					Forecast horizon = 3-month ahead				
AR(1)	−0.072	−0.153	−0.257	−0.401	−0.556	−0.126	−0.292	−0.431	−0.618	−0.861
VAR(1)	−0.046	−0.219	−0.454	−0.645	−0.740	−0.157	−0.470	−0.866	−1.263	−1.608
DNS-VAR	−0.283	−0.268	−0.214	−0.139	−0.054	−0.326	−0.649	−0.660	−0.525	−0.318
NP-FDA	0.070	0.349	0.615	0.848	1.091	0.180	0.675	1.137	1.528	1.889
Russian	Forecast horizon = 1-month ahead					Forecast horizon = 3-month ahead				
AR(1)	−0.012	−0.027	−0.035	−0.039	−0.042	0.047	−0.089	−0.146	−0.167	−0.175
VAR(1)	0.011	−0.005	−0.014	−0.029	−0.044	0.081	0.018	−0.042	−0.084	−0.121
DNS-VAR	−0.014	−0.02	−0.028	−0.056	−0.075	0.079	0.076	0.071	0.058	−0.031
NP-FDA	0.001	−0.024	−0.037	−0.044	−0.036	0.108	0.080	0.054	0.027	0.006
South Africa	Forecast horizon = 1-month ahead					Forecast horizon = 3-month ahead				
AR(1)	0.037	0.027	0.040	0.067	0.094	0.152	0.064	0.043	0.090	0.130
VAR(1)	0.029	0.057	0.076	0.084	0.085	0.067	0.110	0.105	0.043	−0.061
DNS-VAR	−0.017	−0.019	−0.017	−0.021	−0.032	−0.064	−0.128	−0.183	−0.254	−0.348
NP-FDA	0.080	0.185	0.290	0.370	0.422	0.064	0.142	0.197	0.200	0.152

Appendix: The Fair-Shiller comparisons of the relative information content

The Section 4 has highlighted which of the models have relatively low or high values of the RMSFE, which is an aggregate forecast error measure. But although some models, which are very different in formulation, have approximately the same average squared forecast error, it is possible that the information contained in the forecast series for each model is different, which is likely to become obscure in the error aggregation process (Brooks, 1998).

To further investigate whether one forecast contains different information from the RW forecast, the forecast encompassing tests of Fair & Shiller (1990) are performed. As noted by Fair & Shiller (1990), simply comparing out-of-sample forecasts based on RMSFE has limitations. Further insights into the nature of the different forecasting models can be obtained by regressing mean square residuals of the two alternative out-of-sample forecasts:

$$y_t^{(\tau)} - y_{t-h}^{(\tau)} = \alpha + \beta_m \left(\widehat{y}_{t,m}^{(\tau)} - y_{t-h}^{(\tau)} \right) + \epsilon_t. \tag{17}$$

where $y_t^{(\tau)}$ is the yield for maturity τ in time t, $y_{t-h}^{(\tau)}$ is the forecast of $y_t^{(\tau)}$ by the random-walk model, $\widehat{y}_{t,m}^{(\tau)}$ is the forecast for $y_t^{(\tau)}$ by model m, α and β are parameters to be estimated, and ϵ_t represents the forecast error.

If model m contain independent information that has power in predicting the yields (relative to the RW), then β_m should be significantly different from zero. If, however, the information contained in one forecast is simply a subset of that contained in the other(s), then the coefficient on the former should be insignificant. As pointed out by Fair & Shiller (1990), this test avoids the inherent ambiguity of RMSFE comparisons.