An alternative quantitative approach to tactical asset allocation using the Kalman filter

by
Reder Evert van Rooyen

In partial fulfillment of the requirements of the degree
Master of Science in Financial Engineering

In the Faculty of Natural & Agricultural Sciences
University of Pretoria
Pretoria
26 October 2020

Abstract

Tactical asset allocation (TAA) is a dynamic investment strategy which seeks to actively adjust fund allocation to a variety of asset classes by systematically exploiting inefficiencies and temporary imbalances in equilibrium values. TAA adds value by underweighting fund allocation to those assets whose returns have been forecasted to underperform on a relative basis and overweighting those whose returns were forecasted to indicate outperformance. This approach contrasts with strategic asset allocation (SAA) in which a long-term investment view target allocation is established using a combination of target return and risk tolerance. Portfolio managers who employ TAA as an investment strategy aim to benefit from market timing, a non-trivial exercise involving the entry and exit of selected asset classes based on future performance.

TAA decisions are governed by three major considerations: valuation-based approaches, macroeconomic scenarios and technical/quantitative analyses. This work explores a quantitative analytical approach for TAA which adjusts portfolio weights based on forecasted returns of constituent asset classes. Asset returns are forecasted using the Capital Asset Pricing Model (CAPM), complemented with results obtained from the Kalman filter, a Bayesian forecasting tool whose recent application to time-dependent variable estimation has shown promising results. Using a decade of recent monthly return data, the performance of the TAA and SAA approaches are compared using a range of diagnostic metrics. The TAA approach outperforms its SAA counterpart for most of these metrics, even when the most recent returns (i.e. those affected by the coronavirus pandemic) are excluded.

Declaration

I declare that the dissertation/thesis, which I hereby submit for the Master of Science degree in Financial Engineering in the Department of Mathematics and Applied Mathematics at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

Signature:

Student name: Reder van Rooyen

Month Year: 26 October 2020

Ethics statement

The author, whose name appears on the title page of this dissertation/thesis, has obtained, for the research described in this work, the applicable research ethics approval. The author declares that he has observed the ethical standards required in terms of the University of Pretoria's Code of Ethics for Researchers and the Polict guidelines for responsible research.

Signature:

Student name: Reder van Rooyen

Month Year: 26 October 2020

Acknowledgments

Firstly, I would like to thank my parents and my brother for their ongoing support and for believing in me, oftentimes more than what I believe in myself.

Thank you to my supervisor, Prof. Gary van Vuuren for his valuable advice and support throughout this journey. I only ever saw *Gary* once in person, before he had agreed to take on the role of being my supervisor, yet I feel he has been right beside me this entire time. I have learnt so much from him, not only in terms of research and writing, but also, and perhaps more importantly, life in general.

Thank you to Verushen Coopoo for all the emotional support and words of encouragement he provided.

Lastly, thank you to MD.

Contents

List of Figures

List of Tables

Index

Alpha (α) - Measures the excess portfolio returns when the excess market return is equal to zero.

Asset allocation - Policy which describes the set of weights allocated to broad asset classes within an investment portfolio.

Beta (β) - Sensitivity of the portfolio returns to the overall market, calculated as the ratio of the covariance of portfolio returns and market returns to the variance of the market returns.

CAPM - Capital Asset Pricing Model.

CML - Capital Market Line.

Efficient frontier - A pillar of modern portfolio theory which describes a set of optimal portfolios that give the highest possible expected return for a given level of risk or, alternatively, the lowest risk for a desired expected return.

ETF - Exchange Traded Fund.

EWMA - Exponentially Weighted Moving Average.

Excess asset returns - The difference between the returns offered by an asset or portfolio of assets and the risk-free rate of return.

Index - Investment product which serves as a proxy for the performance of a particular segment of the market.

Kalman filter - A dynamic time series estimation algorithm, consisting of two main components: observable and unobservable variables.

Market premia - The excess returns the market offers above that of the risk-free rate of return.

Maximum diversification portfolio - The weighting technique among assets which produces the portfolio that maximises the diversification ratio.

Maximum drawdown - The overall maximum peak-to-trough fall in the value of an investment.

Maximum Sharpe ratio portfolio - The weighting technique among assets which produces the portfolio that maximises the Sharpe ratio.

Mean-variance portfolio optimisation - Popular framework used by portfolio managers to set weight allocations to different asset classes based on the optimal risk-return trade-off.

Measurement equation - The measurement equation relates unobservable variables to observable variables.

Minimum variance portfolio - The weighting technique among assets which produces the portfolio with the lowest possible risk, measured as volatility, based on historical asset returns.

QTAA - Quantitative Tactical Asset Allocation.

SFM - Single Factor Model.

Sharpe ratio - The ratio of portfolio returns less the risk-free rate of return to the volatility associated with the returns of the portfolio, commonly used to gauge portfolio performance.

SMA - Simple Moving Average.

Strategic asset allocation - Choice of target allocation based on a long-term investment view, established by considering a combination of target return and overall risk tolerance.

Systematic risk - Non-diversifiable risk inherent to the entire market or market segment.

Tactical asset allocation - Based on a set of objectives, it is a dynamic strategy which seeks to actively adjust the allocation of funds to a variety of asset classes.

Three-year rolling average returns - A performance metric which highlights the frequency and magnitude of a portfolio's stronger and poorer periods of performance, taking into account all the monthly returns over a specified rolling window.

TE - Tracking Error.

Transition equation - A model that governs the development of an unobservable variable over time.

Unobservable data - Variable or set of variables for which no historical data have been observed.

1 Introduction

Tactical asset allocation (TAA) is a dynamic strategy which seeks to actively adjust the allocation of funds to a variety of asset classes [62]. It aims to systematically exploit inefficiencies and/or temporary imbalances in equilibrium values among different asset classes. The debate around the use of TAA to complement strategic asset allocation (SAA) techniques has been an ongoing discussion for numerous years. TAA has the following objectives:

- increasing portfolio returns
- adapting to market conditions, and
- providing diversification.

There exists extensive literature on the use of various tools and methodologies which have been incorporated into portfolio strategies to deliver the best possible outcome for investors [1, 6, 12, 14, 49]. These have been tried and tested, but the jury is still out regarding their effectiveness to produce meaningful returns for tactically managed portfolios. This is especially true for multi-asset funds which aim to achieve an even greater level of diversification by investing in largely uncorrelated assets and hence income streams [50]. The decisions that must be made in any active management strategy are based on a variety of indicators, some of which function on a standalone basis while others are used in an integrated system, see for example [26]. These indicators are used to identify a changing economic environment where market inefficiencies possibly exist and to exploit these inefficiencies to best serve an investor's portfolio, whether it be retail investors or institutional investors.

The issue however is that these financial indicators have to be forecasted as accurately as possible to prevent making decisions which could be to the detriment of an investment portfolio. Forecasting of financial data has proven to be a significant challenge to market practitioners because financial data are often beset with noise [68]. They are not only tainted with noise, but incorrect data entries, missing data and spurious outliers are also common in financial data which renders the forecasting thereof a complex procedure. There therefore exists the need for a dynamic approach to financial forecasting which produces much more recent and hence relevant estimations of the nature of markets. The Kalman filter, which has its roots in engineering, has been proposed as an example of such a dynamic approach and attempts to improve on the performance of other methods of financial time series estimation such as simple linear regression and exponentially weighted moving averages. The Kalman filter is not completely new to the financial world and there already exists many examples of its application in financial engineering and the broader quantitative finance landscape [10, 20, 57]. In this research the Kalman filter algorithm will be used to estimate the time-varying, unobservable variables of the popular Capital Asset Pricing Model (CAPM). The CAPM is written as:

$$R_p - R_f = \alpha + \beta(R_m - R_f) + \varepsilon,$$

where R_p represents the return achieved by a portfolio of assets, R_f represents the risk-free rate of return, and R_m represents the return of the overall market. The CAPM was introduced by Sharpe (1964) [58] and follows directly from portfolio mean-variance analysis [47], the foundation of passive portfolio construction. It has become one of the most well-known asset pricing models in recent history and is used in various ways, such as estimating the cost of capital for firms and, more relevant to this study, evaluating the performance of managed portfolios [50]. In his work Sharpe (1964) showed that the excess returns of an asset or a portfolio of assets were a linear function of the excess market returns, where the returns of the overall market were represented by a market-capitalised equity index [58]. The CAPM falls into a broad category of single-factor models (SFM) used in the investment industry, although multi-factor models have become increasingly popular [22, 24].

The use of the Kalman filter in the CAPM framework will assist in the forecasting of asset returns. Based on these forecasts, a quantitative TAA framework will be developed and implemented with the intention of achieving the underpinning objectives of tactically weighted multi-asset portfolios.

The remainder of this dissertation is set out as follows: Chapter 2 provides a brief, yet detailed description of the Kalman filter algorithm, highlighting some of its current uses in practice. Given that TAA builds on strategic weights, it necessitates the revision of some of the key weighting techniques based on the historical returns of a range of assets. This review is discussed in Chapter 3, along with the introduction of the CAPM and some of its most important characteristics. These characteristics are key to understanding the subsequent sections. Chapter 4 then details the use of the CAPM in combination with the Kalman filter algorithm and the effect of its use on the determination of the values of α and β. The difference between the Kalman filter results and results from standard linear regression is illustrated and it is shown how monthly asset returns are forecasted using these results.

Chapter 5 provides some background information on the importance of asset allocation policies in practice and clearly defines both SAA, following on from modern portfolio theory, and TAA. Since TAA will form the basis of this study, some of the most prominent drivers of market timing are discussed in Chapter 6.

Chapter 7 describes the data and the methodology which were used in the research with results being displayed and analysed in Chapter 8. Chapter 9 presents the concluding remarks and suggestions for future research. The code which was implemented in a variety of platforms, together with a schematic representation of its flow, is provided in the Appendix. The Appendix also includes a discussion about the growth in alternative assets and the need to consider these assets in modern portfolio construction processes. Lastly, a comprehensive bibliography is presented.

2 The Kalman filter explained

The Kalman filter in its simplest form is a time series estimation algorithm which uses linear programming and variance reduction techniques to estimate hidden variables more accurately than single measurement approaches. It has been used to great effect in navigation systems, missile guidance systems and other aerospace applications in particular [29, 43, 44, 55]. Fitting then that it has received more attention in recent years (recent relative to its use in engineering) in financial engineering and quantitative finance in general. This chapter attempts to explain the most important aspects of this dynamic approach to time series estimation.

The first point to highlight is that there exist data which are unobservable, meaning that there are no historical data that have been observed for a specific variable or set of variables. Often these types of variables are required as input to other finance and economic models and there therefore exists the need to have a system with which to model these unobservable variables such that it is appropriate for use as input parameters. Previous applications of the Kalman filter, specifically in finance and economics, includes but are not limited to estimating; unobservable parameters and state variables in commodity futures prices [57], unobserved expected monthly inflation rates [10], unobservable stock betas [20], which is relevant to this study, and many more. The Kalman filter has many similarities with linear regression analysis, however it also has some unique characteristics which sets it apart as a superior method of time series analysis, especially when compared with standard linear regression.

2.1 Deriving the Kalman filter algorithm

The first step in understanding the Kalman filter is to describe its two main building blocks; (i) the measurement equation and the (ii) transition equation.

(i) Measurement equation

The measurement equation relates unobservable variables to observable variables in the form:

$$Y_t = m_t X + b_t + \varepsilon_t. \tag{1}$$

In Eq. (1), Y_t represents the observable variable and X_t the unobservable variable. Similar to the workings of [4], to simplify the measurement formula, assume that b_t is zero and m_t remains constant through time. Additionally, ε_t is assumed to have a mean of zero and a variance of r_t. Equation (1) then becomes:

$$Y_t = mX_t + \varepsilon_t. \tag{2}$$

(ii) Transition equation

The transition equation is based on a model that allows the unobservable variable to develop over time. It takes on the following general form:

$$X_{t+1} = a_t X_t + g_t + \theta_t. \tag{3}$$

Again, a simplification of Eq. (3) is adopted for illustrative purposes by assuming that g_t is zero, a_t remains constant and θ_t has a variance of q_t. Equation (3) then becomes:

$$X_{t+1} = aX_t + \theta_t. \tag{4}$$

Now that the groundwork has been laid, the iterative process that is the Kalman filter can be derived. Start by first inserting an initial value X_0 into Eq. (4), the simplified transition equation. X_0 has a mean value of μ_0 and a standard deviation of σ_0. Equation (4) then becomes:

$$X_{1P} = aX_0 + \theta_0, \tag{5}$$

where X_{1P} represents the predicted value for X_1 given the values for the other parameters in Eq. (5). Once this value has been determined it can be inserted into Eq. (2) to similarly calculate a predicted value for the observable variable:

$$Y_{1P} = m[aX_0 + \theta_0] + \varepsilon_t.$$

Given that Y in this context is an observable variable, an actual value for Y_1 is observed as and when it occurs. Therefore, there now exists both a predicted value as well as an observed value for the observable variable at t_1. The next step is to use this information and compute a variable which measures the error between the predicted and observed values as follows:

$$Y_E = Y_1 - Y_{1P}. \tag{6}$$

Previously, a prediction was made for X_1 namely X_{1P}, but given that Y_1 has been observed this prediction can be updated by incorporating the error term, Eq. (6). To do so, establish a new variable representing an adjusted prediction and label this variable at t_1, X_{1P-ADJ}. This follows the same naming convention used in [4]. Mathematically:

$$\begin{aligned} X_{1P-ADJ} &= X_{1P} + k_1 Y_E \qquad (7) \\ &= X_{1P} + k_1(Y_1 - Y_{1P}) \\ &= X_{1P} + k_1(Y_1 - mX_{1P} - \varepsilon_1) \\ &= X_{1P}(1 - mk_1) + k_1 Y_1 - k_1 \varepsilon_1, \end{aligned}$$

where k_1 is known as the Kalman gain. Determining the values for the Kalman gain requires the determination of the partial derivative of the variance of Eq. (7) with respect to k_1 to minimise the variance based on k_1. Set the partial derivative relative to $k_1 = 0$ and solve for the Kalman gain. If the variance of X_{1P} is p_1 (for ease of exposition) then the process will look as follows:

$$Var(X_{1P-ADJ}) = Var(X_{1P} + k_1 Y)$$

$$\therefore Var(X_{1P-ADJ}) = p_1(1 - mk_1)^2 + k_1^2 r_1, \tag{8}$$

and

$$\frac{\partial Var(X_{1P-ADJ})}{\partial k_1} = -2m(1 - mk_1)p_1 + 2k_1 r_1 = 0$$

$$\therefore k_1 = \frac{p_1 m}{(p_1 m^2 + r_1)} = \frac{Cov(X_{1P}, Y_{1P})}{Var(Y_{1P})}. \tag{9}$$

The idea is for the Kalman gain to be set to reduce variance of the adjusted predicted values for X_1. Once these steps have been implemented, one can use the adjusted predicted value in Eq. (4) and start the process again for subsequent time periods. At this stage it is also important to take note of the distinct advantages of using *adjusted* predictions rather than the initial predictions. If the value of k_1 in Eq. (9) is substituted back into Eq. (8) then:

$$Var(X_{1P-ADJ}) = p_1(1 - \frac{1}{(1 + \frac{r_1}{p_1 m^2})})^2 + k_1^2 r_1.$$

Earlier it was stated that the variance of X_{1P} equal to p_1. Now, as is shown in [4], $p_1 = (a\sigma_0)^2 + q_0$, where the bracketed term is < 1. This term is also squared which means that it is reduced even further because the quantity is < 1. Consequently the portion of the variance attributed to estimating X_1 has been significantly reduced through the use of X_{1P-ADJ} instead of X_{1P}.

The entire Kalman filter process is summarised in Table 2.1 [4].

Table 2.1: The Kalman filter process

Predict future unobserved variable (X_{t+1}) based on the current estimate of the unobserved variable, call it $X_{(t+1)P}$:	$X_{(t+1)P} = a_t * X_{tP-ADJ} + g_t + \theta_t$
Note: $X_{0P-ADJ} = X_0$ which is $N(\mu_0, \sigma_0^2)$	
Use the predicted unobserved variable to predict the future observable variable (Y_{t+1}), call it $Y_{(t+1)P}$:	$Y_{(t+1)P} = m_t * X_{(t+1)P} + b_{t+1} + \varepsilon_{t+1}$
When the future observable variable actually occurs, calculate the error in the prediction:	$Y_{(t+1)E} = Y_{(t+1)} - Y_{(t+1)P}$
Generate a better estimate of the unobserved variable at time (t + 1) and start the process over for time (t + 2):	$X_{(t+1)P-ADJ} = X_{(t+1)P} + k_{(t+1)} * Y_{(t+1)E}$
Note: k_{t+1} is the "Kalman gain" and is set to minimize the variance of $X_{(t+1)P-ADJ}$; p_{t+1} is the variance of $X_{(t+1)P}$:	$k_{t+1} = \dfrac{p_{t+1} * m}{\left(p_{t+1} * m^2 + r_t\right)} = \dfrac{Cov\left(X_{(1+1)P}, Y_{(t+1)P}\right)}{Var\left(Y_{(t+1)P}\right)}$

2.2 Estimating the remaining parameters

Special attention has been paid to the calculation of unobservable and observable variables, X_t and Y_t respectively, but the question now remains how the other parameters used in the Kalman filter algorithm are estimated. These unknown parameters include for example ε_t in the measurement equation and a and θ_t in the transition equation. A system which estimates these unknown parameters is required in order for it to feed into the Kalman filter algorithm which then produces the values for the unobservable variables as required. To describe this process, first take into account the distribution of the predicted observable variable at time t, Y_{tP}. If it is assumed that this variable is serially independent and normally distributed, and that the mean and variance are of the form:

$$E[Y_{tP}] = E[m(X_{tP-ADJ}) + \varepsilon_t] = mE[X_{tP-ADJ}]$$

and

$$Var(Y_{tP}) = Var(X_{tP-ADJ})m^2 + r_t$$

respectively, then a joint likelihood function emerges:

$$\prod_{t=1}^{t=T} \left\{ \left[\frac{1}{\sqrt{2\pi Var(Y_{tP})}} \right]^T e^{-\frac{\sum_{t=1}^{T}(Y_t - E[Y_{tP}])^2}{2Var(Y_{tP})}} \right\}. \tag{10}$$

Given that the observable data emerge from this jointly normal distribution, the remaining unknown parameters are chosen such that those values maximise Eq. (10). It is also common to, instead of using Eq. (10), use the natural logarithm of the likelihood function, otherwise known as the log-likelihood function:

$$-\frac{Tln(2\pi)}{2} - \frac{1}{2}\sum_{t=1}^{T} ln(Var(Y_{tP})) - \frac{1}{2}\sum_{t=1}^{T} \frac{(Y_t - E[Y_{tP}])^2}{Var(Y_{tP})}. \tag{11}$$

Similarly, the unknown parameters are chosen to maximise Eq. (11). Once the set of parameters is estimated the Kalman filter algorithm is applied, producing new time series for the variables Y_{tP} and X_{tP-ADJ} along with their respective distributions. Thereafter, the likelihood estimation will commence again producing a new set of maximum likelihood estimates (MLEs) which, as is clear by now, will be entered into Eq. (1) and Eq. (3) once again. This iterative process, which combines both the likelihood estimation and the Kalman filter algorithm, is known as the Expectation Maximisation algorithm [9].

This provides a simplified approach to understanding the main components of the Kalman filter. The link between the Kalman filter and its use within the context of this study will become clear in subsequent chapters after more of the groundwork has been laid in terms of modern portfolio theory in the Chapter 3.

3 Modern portfolio theory

In Chapter 2 the main components of the Kalman filter, a superior method of time-sensitive variable estimation, were discussed. This chapter firstly provides a revision of the main weighting techniques within mean-variance portfolio optimisation, part of the broader study of modern portfolio theory, and secondly, discusses the single-factor CAPM. The CAPM forms an integral part to understanding the application of the Kalman filter in this study.

3.1 Mean-variance portfolio optimisation

One of the most well-known methods of portfolio construction is the mean-variance selection process, introduced by Markowitz (1952) [47]. According to this framework an investor would select a portfolio by considering the expected returns and risk, measured as the volatility of returns, of a range of portfolios and select that portfolio which maximises the expected return for an investor's stated risk tolerance. These portfolios all differ from one another based on the percentage weight it allocates to each of the chosen asset classes. It therefore also naturally depends on an investor's objectives and overall risk tolerance. These factors will ultimately determine the optimal weighting allocation among the different asset classes to satisfy the investor's investment goals.

To apply this framework some assumptions must be made. Firstly, investors are assumed to be risk averse, i.e. for a given level of return an investor will prefer a portfolio with less risk associated with its returns. Secondly, investors seek to maximise their wealth, i.e. for a given level of risk or volatility an investor would choose a portfolio which offers the highest returns. If these assumptions are applied to the range of different percentage allocations mentioned previously, then one would observe the graph shown in Figure 3.1.

This is the efficient frontier which plots the different portfolios, constructed using the available risky assets and their respective return and risk profiles. Any individual asset or portfolio of these assets which lie to the right of the efficient frontier are inefficient.

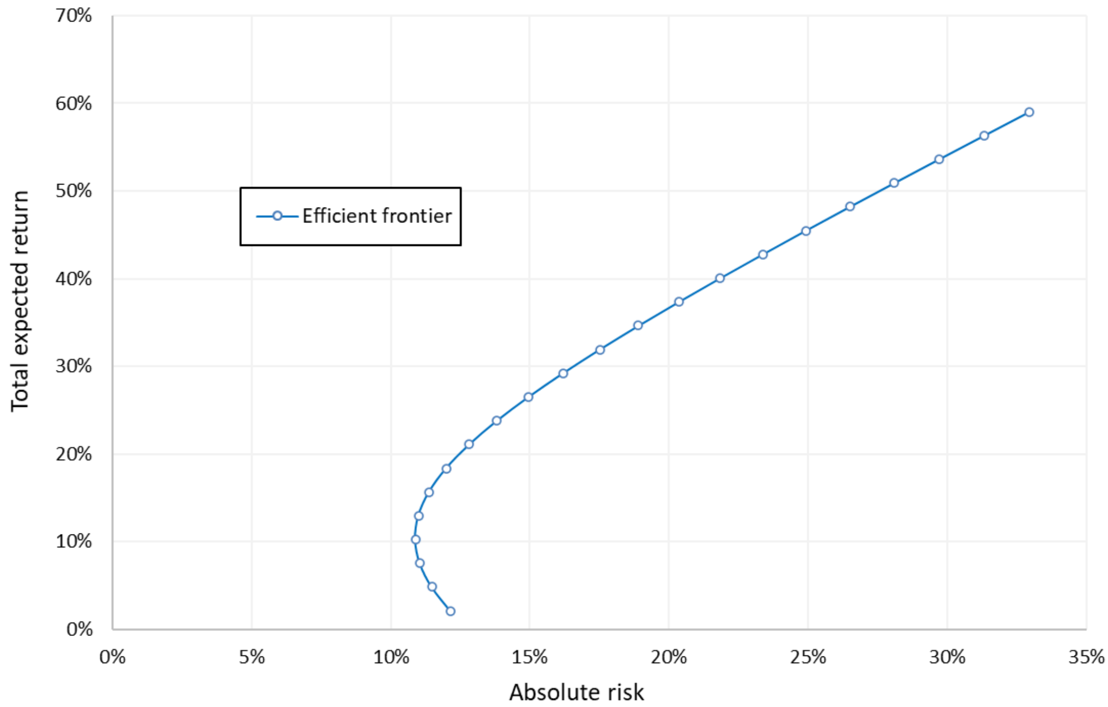


Figure 3.1: Efficient frontier

Below some of the most common types of portfolio construction techniques are briefly described. It is important that these weighting techniques are well understood when making investment decisions for longer time periods.

3.1.1 Minimum variance portfolio allocation

The efficient frontier depicts the trade-off between risk and reward for various weight allocations. As the additional risk is taken, an investor would want to be compensated for the additional risk in the form of increased returns. Even for the minimum amount of risk, there is a certain level of return that would be expected, otherwise, given the assumptions of investors operating in the market, an investor would simply invest all his wealth in risk-free assets.

On the efficient frontier of risky assets there exists a point beyond which the risk of any of the portfolios on the efficient frontier can not be decreased any further. This is the left-most point on the efficient frontier known as the Minimum Variance Portfolio (MVP). As the name suggests, this portfolio weighting technique aims to construct a portfolio, using the risky assets, that has the lowest possible amount of risk associated with it. As a result of this decreased level of risk, this portfolio is expected to offer the lowest return of *all* portfolios found on the efficient frontier.

3.1.2 Maximum diversification portfolio allocation

The maximum diversification weighting technique allocates funds among the different asset classes such that the diversification ratio is maximised. This approach seeks to take full advantage of the benefits of diversification within one's portfolio of assets [66]. Mathematically:

$$DR = \frac{w^T \sigma}{\sqrt{w^T \sum w}},$$

where σ is an n $\times$ 1 vector of individual asset volatilities σ_1, ..., σ_n and $\sum$ is the corresponding n $\times$ n variance covariance matrix of asset returns. The vector of asset weights, denoted by w, will be used in the diversification ratio optimisation [66].

3.1.3 Maximum Sharpe ratio/tangent portfolio allocation

Suppose a risk-free asset is included in the universe of investable assets. This inclusion has a very distinct effect on the shape of the efficient frontier as is illustrated in Figure 3.2.

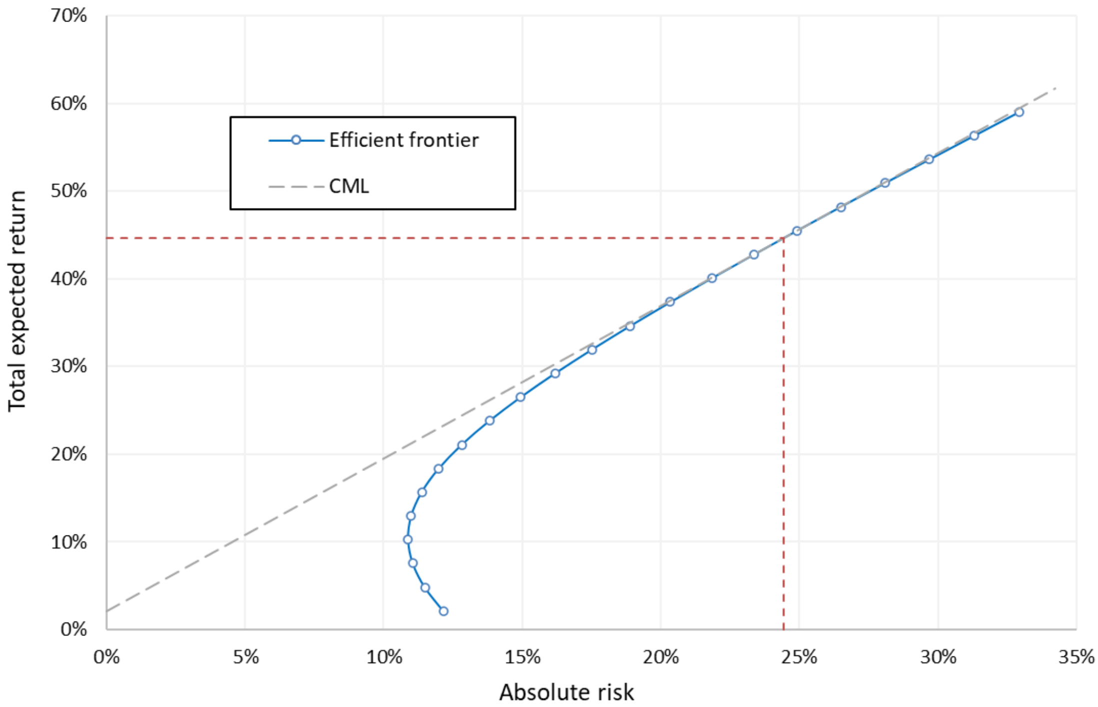


Figure 3.2: Efficient frontier with risk-free asset

The inclusion of a risk-free asset transforms the efficient frontier into a straight line with a y-intercept equal to the risk-free rate of return. This straight line is the Capital Market Line (CML) whose slope shows once again how the return of a portfolio on this line changes in response to the risk associated with the portfolio. The slope of this line, using the (x, y) coordinates of the risk-free asset and another point on the CML is expressed as follows:

$$\frac{R_p - R_f}{\sigma_p}, \tag{12}$$

where R_p is the return of a portfolio on the CML, σ_p is the risk associated with this portfolio represented by the volatility of portfolio returns, and R_f is the risk-free rate of return. This equation is of particular interest in the investment world. This is known as the *Sharpe ratio*, and measures the ratio of excess returns a portfolio produces compared to the risk-free rate of return to the volatility of portfolio returns, i.e. its risk. This ratio is a metric of the performance of the portfolio relative to a risk-free asset [58, 59, 60].

The intercept point of the CML to the efficient frontier of risky assets is called the tangent portfolio. This portfolio is thus the portfolio on the efficient frontier which maximises the Sharpe ratio and is therefore also referred to as the maximum Sharpe ratio portfolio. This portfolio will also have its own unique weighting scheme among the risky assets under consideration. This particular weighting technique will lead to a portfolio choice which will take on more risk relative to the minimum variance portfolio.

The MATLAB implementation of this optimisation problem can be viewed in the Appendix along with the other optimisation weighting techniques. Figure 3.3 provides a visual representation of where these portfolios lie relative to the hypothetical efficient frontier of four different risky assets and a risk-free asset which, in this instance, delivered an annual return fractionally below 2%.

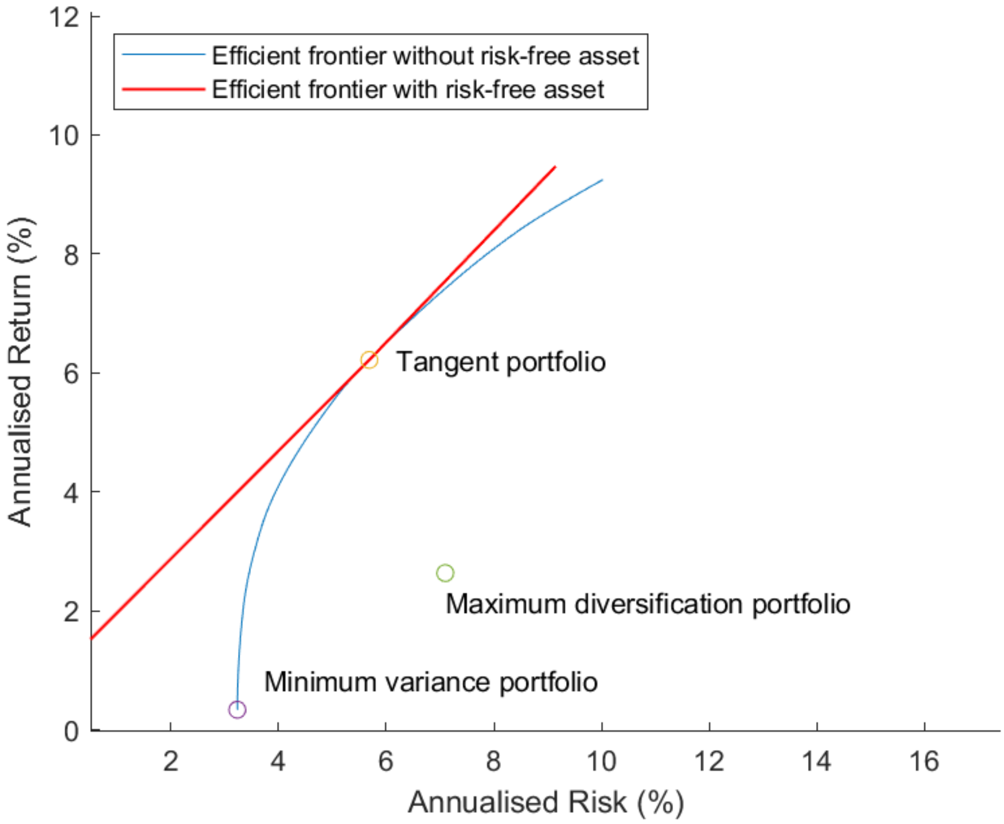


Figure 3.3: Visualising different weighting techniques

Figure 3.3 also illustrates an important point regarding the maximum diversification portfolio in that it is not guaranteed to be an efficient portfolio for a specific risk and return profile. The maximum diversification portfolio for this set of risky assets and risk-free asset is in fact inefficient given the historical returns which were used to determine this portfolio's weighting scheme.

3.2 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) was introduced by Sharpe (1964) [58] and follows directly from mean-variance analysis discussed previously. It has become one of the most well-known asset pricing models in recent history and is used in various ways, such as estimating the cost of capital for firms and, more relevant to this study, evaluating the performance of managed portfolios [38, 50, 59, 69]. The CAPM has many attractive features which explain in part why the model is still so widely used within the finance world. These features include the simplicity of the model and the ability of the model to produce intuitively pleasing predictions most individuals can understand.

The development of the CAPM in [58] was largely based on the framework defined by Markowitz (1952) [47] and it showed that the excess returns of an asset or a portfolio of assets were a linear function of the excess market returns represented by a market-capitalised index such as the S&P500 index or the MSCI All World Index. The CAPM is written as:

$$R_p - R_f = \alpha + \beta(R_m - R_f) + \varepsilon, \tag{13}$$

where R_p represents the return achieved by a portfolio of assets, R_f represents the risk-free rate of return, and R_m represents the return of the overall market. Equation (13) shows how excess portfolio returns are regressed on excess market returns, with the parameter β indicating the slope between these two quantities and α the intercept. Graphically this linear relationship can be seen in Figure 3.4 below.

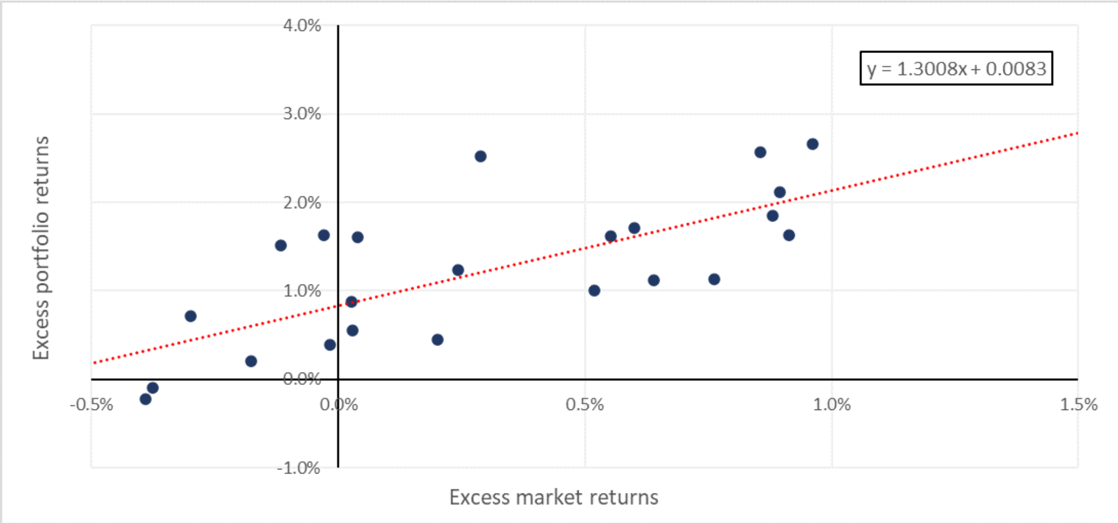


Figure 3.4: Excess return regression used to estimate relevant parameters for CAPM

The parameter β measures the sensitivity of the portfolio returns to the overall market, calculated as the ratio of the covariance of portfolio returns and market returns to the variance of market returns:

$$\beta = \frac{Cov(R_p, R_m)}{Var(R_m)}.$$

β thus measures the level of systematic risk faced by the portfolio. This type of risk is non-diversifiable and is inherent to any portfolio exposed to the various risk factors which naturally occur within the market.

The α parameter measures the excess portfolio returns when the excess market return is equal to zero, sometimes also referred to as Jensen's α [23, 38]. This value can be seen in Figure 3.4 as the intersection of the CAPM line with the excess portfolio returns axis and in this instance is 0.83%. Both these parameters are of great importance in the investment management industry as the ex-post analysis of the parameters play a significant role in the evaluation of the performance of portfolio managers.

Given only one risk factor, excess market returns or market premia, is taken into account, the CAPM essentially states that the amount by which an investor is rewarded in the form of excess portfolio returns, is directly proportional to the underlying market risk. Although the CAPM is easy to understand and makes sense within the risk-reward framework, many studies have used empirical evidence to prove that the theory underpinning this model is not valid [23, 33, 54]. There are numerous other multi-factor models, e.g the Fama-French factor models [22, 24], that have been proposed as a substitute for the CAPM, but the study of these models falls outside of the scope of this research.

The basic principles of the CAPM as discussed in this chapter, augmented with the use of the Kalman filter, will be used to support investment decisions by forecasting monthly asset returns, as will be shown in Chapter 4.

4 Using the Kalman filter and the CAPM to forecast asset returns

Forecasting financial data has proved to be an extremely challenging task for practitioners. One of the reasons for this is noise present in these data, making it difficult to produce accurate estimates of what the future holds [68]. This is particularly true when estimating asset returns. The traditional methodology employed in practice for predicting asset returns is the CAPM discussed in Chapter 3 [68]. The accuracy of the CAPM formulation requires robust estimates of α and β, usually estimated from market data using (e.g.) regression or principle component analysis. These parameters should reflect, as closely as possible, current market dynamics. This chapter aims to address this issue.

The question then naturally arises, what would be the most effective way of calculating values for α and β such that it meets this all important requirement of being a fair depiction of the current state of financial markets? The nature of the CAPM means that values for these parameters are determined by the relationship between excess portfolio or security returns and excess market returns. In this sense a decision has to be made in terms of the data for both portfolio or security excess returns and excess market returns which will be used to establish the behaviour of α and β. Traditionally more data are preferred over fewer. As [68] mentions, using return data over a three month period, for example, would not provide any valuable information. Using too many data on the other hand also has its own drawbacks, as certain data from say five years ago are then used within a regression analysis meant to be reflective of current market dynamics. So there already exists a question around the amount of historical data that must be used to capture this relationship between excess returns.

A possible solution to this dilemma is to use an approach which assigns higher weights to more recent data and lower weights to those data which occurred longer ago in the past via an exponential weighting scheme. This exponentially weighted moving average (EWMA) technique is a well known technique and has been applied in the determination of volatility, for example [40]. The volatility estimates produced by the EWMA technique are more responsive to market moves and do not give rise to the phenomenon of 'ghosting' [15], a scenario found in a normal moving average process, in which volatility shocks are abruptly incorporated into the volatility calculation and remain in the data present in the trailing window. Since all the data in the moving average calculation are equally weighted, these shocks contribute to the overall process in the same way as all the other, normal, data. As the trailing window moves with the passing of time, these shocks drop off resulting in inaccurate volatility estimations.

As attractive as the exponentially weighted moving average is due to the more recent and thus arguably more relevant estimates of α and β it provides, it has been shown that the effect is still insufficient [67]. A different technique has thus been required to provide the best estimates for these parameters of the CAPM.

4.1 Kalman filter α and β estimations

Following from the fundamentals highlighted in Chapter 2, it will now be shown how the Kalman filter can be used as an alternative technique to that of the exponentially weighted moving average in estimating the parameters α and β in the CAPM. The two main equations relating to the Kalman filter is the measurement equation and the transition equation. The measurement equation relates the unobservable variables to the observable variables. The transition equation on the other hand is based on a model that governs the development of unobservable variables over time.

In the context of the CAPM, the unobservable variables are α and β, while the observable variable is the excess portfolio return or $R_p - R_f$. The measurement equation in this instance will take on the form of the CAPM, i.e.

$$R_p - R_f = \alpha + \beta(R_m - R_f) + \varepsilon.$$

For ease of exposition, the subscripts in the CAPM formula will from now on be written as superscripts allowing for the use of subscripts to indicate the specific point in time. Furthermore, the excess returns, both that of the portfolio as well as the market will be written as single variables, $R_p - R_f$ will be written as R^p, and $R_m - R_f$ will be written as R^m. To this end the CAPM, and thus the measurement equation, will now take the form:

$$R_t^p = \alpha_t + \beta_t R_t^m + \varepsilon_t \tag{14}$$

at time t and the noise term $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

Since the transition equation allows for the development of variables over time, an assumption needs to be made in terms of the process which drives this development, which would then ultimately also determine the form of the equation. There are various options to choose from when deciding which underlying stochastic process would be most suitable for the time-varying unobservable variables α and β. The options consist of processes such as autoregressive models, mean-reverting models, random walk processes, different distributional assumptions, etc.

Faff et al. (2000) for example examined the ability of alternative models in capturing the time variation of systematic risk or β [20]. For their implementation of the Kalman filter three different stochastic processes were tested namely, a random walk, a first order autoregressive process with a constant mean and a random coefficient around a constant mean. The results suggested that the "Kalman filter algorithm, and in particular with the random walk parameterisation, consistently performs better than the simple market model beta" [20]. Therefore the random walk process is also incorporated in this Kalman filter implementation for both of the time-varying parameters in question.

According to the random walk process, current market exposure is assumed to be a normally distributed random variable with a mean equal to the mean of the previous period's exposure and with system noises also assumed to be normally distributed and uncorrelated [67]. State variables $x_t \epsilon \mathbb{R}^2$ are time-varying coefficients:

$$x_t = \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix}$$

at time t.

Given the assumption that these variables develop according to a random walk process, the transition equation is written, in matrix form, as:

$$\begin{bmatrix} \alpha_{t+1} \\ \beta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \gamma \\ \delta \end{bmatrix}, \tag{15}$$

where

$$\begin{bmatrix} \gamma \\ \delta \end{bmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_\gamma^2 & 0 \\ 0 & \sigma_\delta^2 \end{bmatrix} \right).$$

Given that Eq. (15) is written in matrix form, the same will be done for Eq. (14). Therefore the measurement equation can be re-written as:

$$R_t^p = \begin{bmatrix} 1 & R_t^m \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \varepsilon_t. \tag{16}$$

As mentioned in Chapter 2 there are some parameters which are not estimated during the Kalman filter process, but rather which feed into the algorithm. Maximum likelihood estimation is used to determine the remaining unknown variables, in this instance σ_γ^2 and σ_δ^2. The use of maximum likelihood estimation in conjunction with the Kalman filter algorithm is known as the Expectation Maximisation algorithm [9].

Comparing Kalman α and β with regression

As a precursor to how the Kalman filter will be used to forecast asset returns and therefore inform investment decisions, the estimates produced by the Kalman filter will be compared to that of a normal rolling 36-month unweighted linear regression, similar to what was done in [68]. This comparison will be done for the two variables respectively, applying both methods of estimation. For the purpose of this comparison a specific asset class has been chosen purely for illustration. The full list of asset classes which form part of this study is discussed in Chapter 7, but for now consider the evolution of both α and β through time for a portfolio of real estate investments from 2011. The results are depicted in Figures 4.1 and 4.2 below.

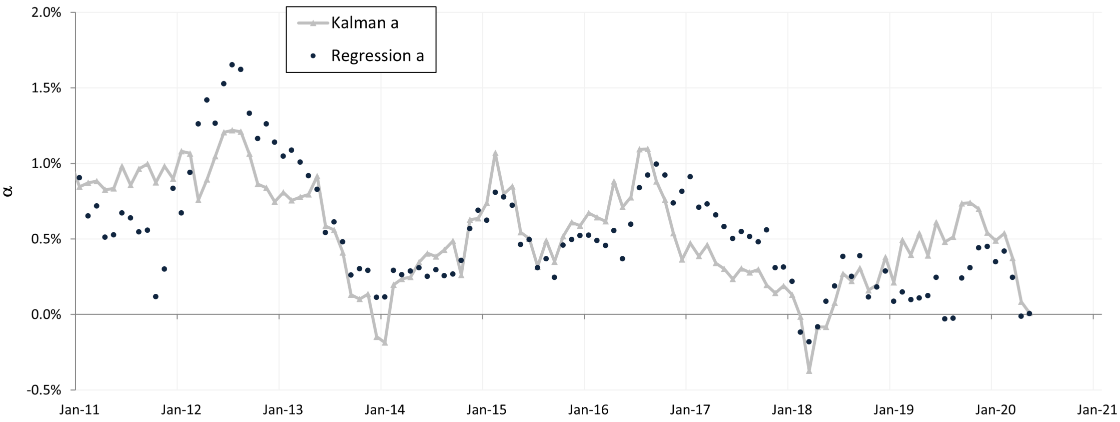


Figure 4.1: Real estate portfolio α

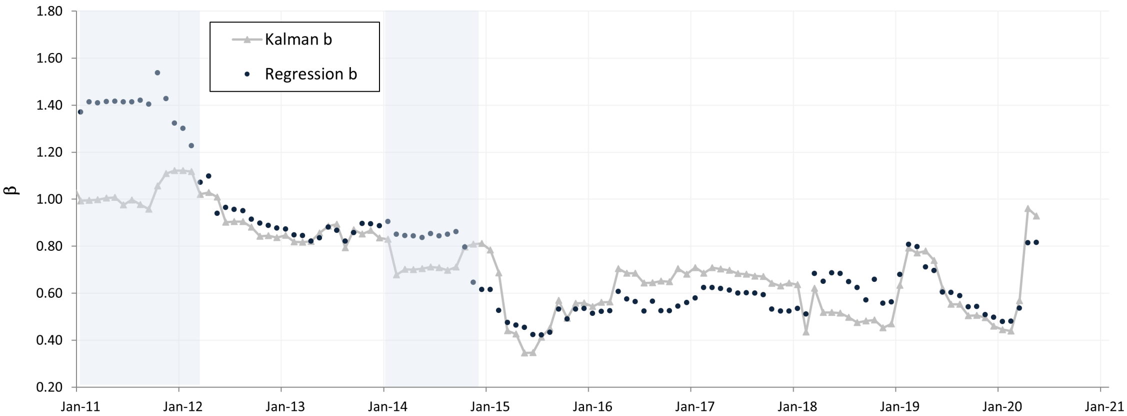


Figure 4.2: Real estate portfolio β

As can be seen from Figures 4.1 and 4.2 there are differences in the linear regression estimates and the estimates produced through the use of the Kalman filter, albeit subtle at times. There are however certain periods where these differences are quite noticeable, especially for the β estimates. Two periods are highlighted in Figure 4.2, the whole of 2011 where the linear regression and Kalman filter estimates showed the most significant differences, and during 2014 where the Kalman filter β estimates picked up declining systematic risk faced by this portfolio quicker than its linear regression counterpart. These differences in estimates could have played a crucial role in the outcome of the investment decisions made by portfolio managers during this time and illustrates the need for an approach such as the Kalman filter which provides more recent and thus relevant estimates for effective and timely decision making.

4.2 Forecasting monthly asset returns

It has now been shown how the Kalman filter will be applied to estimate unobservable state space variables α and β. It will now be discussed how this study aims to use these estimates to forecast monthly asset returns. Recall from Eq. (13) that the CAPM is written as:

$$R_p - R_f = \alpha + \beta(R_m - R_f) + \varepsilon,$$

or as shown in Eq. (14) with the addition of the indicator of the time interval:

$$R_t^p = \alpha_t + \beta_t R_t^m + \varepsilon_t.$$

It can therefore be seen how the return of an asset or portfolio of assets consists of several components. The question now remains, how can these parts be used to forecast future returns? In answering this question, two assumptions are made regarding the risk-free rate of return, R_f, as well as the return of the overall market, R_m. These assumptions implicitly also determine the nature of the market premium, R_t^m, measured as the difference between the market return and the risk-free rate of return. Consider both R_f and R_m, which represents returns at t respectively. The expected returns at $t+1$ are set equal to the observed returns for R_f and R_m at t, i.e. next month's returns are expected to remain at current levels. This follows from the assumption that both the market return and the risk-free rate of return are Martingales and therefore the conditional expectation of the next value in the sequence of returns, given all prior values, is equal to the present value. The next element, which is ε_t, is assumed to have $\mu_t = 0$ and $var(\varepsilon_t) = \sigma_\varepsilon^2$, similar to Eq. (14).

The final elements for which assumptions need to be made are the corresponding values for α and β. It is here that the results from the Kalman filter are utilised. Given that market data have been observed at t, adjusted predictions for α and β can be determined, similarly to what is done in Eq. (7) and related to the CAPM in Eq. (16). The best possible estimates then for expected values for these parameters at $t+1$ are then assumed to be the adjusted predictions at t.

This provides all necessary information to forecast monthly asset returns at $t+1$. Mathematically, by decomposing excess returns into their individual components once again:

$$E[R^p_{t+1}] = E[R^f_{t+1} + \alpha_{t+1} + \beta_{t+1}(R^m_{t+1} - R^f_{t+1}) + \varepsilon_{t+1}]$$
$$E[R^p_{t+1}] = R^f_t + \alpha_t + \beta_t(R^m_t - R^f_t),$$

where α_t and β_t represents, as mentioned, adjusted predictions taking into account observed values at t.

This chapter showed how the Kalman filter is incorporated in the CAPM to produce estimates for the parameters α and β which are more representative of current market dynamics. It was seen how, in the case of β in particular, the Kalman filter estimates reflected a changing market environment much more efficiently than the linear regression estimates, supporting the fact that the Kalman filter is a superior metric for time-sensitive, noisy measurements. It was also shown how monthly asset returns are forecasted using these estimates. The next chapter describes the differences in asset allocation approaches and the objectives they aim to meet respectively.

5 The importance of asset allocation

Chapter 4 showed how the Kalman filter can be employed to forecast monthly asset returns. The aim is to use these forecasts to inform investment decisions and produce superior performing portfolios. Before this can be done potential approaches to asset allocation must be discussed.

Asset allocation describes the set of weights of broad asset classes within an investment portfolio. Asset allocation is the first step in developing an investment program. The manner in which these weights are allocated defines the behaviour of the overall portfolio and is ultimately setup to match the risk and return targets of the investor. As a simple example of how asset weights are determined, consider an investor who is much more concerned about the volatility of returns of the portfolio. Such an investor would allocate more funds towards money market funds, which have lower returns but considerably lower risk, than an investor who is more risk-tolerant.

There are a number of studies which have investigated more closely the importance of asset allocation policy [2, 5, 16, 34, 35, 61, 63]. Ibbotson (2010) gives an overview of some of these studies and the conclusions which can be drawn from the results. The consensus seems to be that asset allocation policy can, on average, describe upwards of 90% of the *variability* of portfolio returns [34]. Ibbotson and Kaplan (2000) also demonstrated that asset allocation, on average, explained close to 100% of long-term (five and 10 year horizons) portfolio *total returns* [35]. If an investor hires index managers, by definition asset allocation will determine majority of future returns. If an investor however hires highly concentrated managers, asset allocation may be overwhelmed, especially in the short run, by asset class manager performance. The relative importance of asset allocation compared to asset class manager performance thus depends on the nature of the portfolio in question and the time horizon. For example, with diversified equity and fixed-income investments, for the long term, asset allocation will always be of greater importance [35].

It was also determined that "relative performance between two multi-asset class funds is influenced by asset allocation as well" [35]. Relative returns however are influenced less than total returns because asset class returns have been removed to a large extent. In their study it was found that asset allocation only described approximately 40% of the *variability of annual relative returns* [35]. The rest of the variability had other factors driving it such as style, market timing, trading costs and security selection. Thus, asset allocation did play a role in describing some of the variability of returns, but that factors other than asset allocation policy were cumulatively more important.

There is thus no doubting the impact asset allocation techniques have on the returns of portfolios, albeit variability of portfolio returns, total portfolio returns or relative returns among portfolios. It is therefore critical to carefully consider the asset allocation technique used. There exists a variety of asset allocation techniques, all of which represent tools that investment professionals use to set optimal asset weights. The following subsections set out two of the most prominent types of asset allocation techniques and how these techniques are fundamentally different.

5.1 Strategic Asset Allocation

Strategic asset allocation (SAA) takes on a long-term investment view and establishes a target allocation among the assets of a defined opportunity set. These asset classes in general include equities, bonds and cash. In some instances real estate has also become more prevalent in the SAA decision making process. The weights allocated to these asset classes are largely determined by the portfolio objective, risk tolerance and time horizon. These factors are often grouped together and used to allocate funds according to an investment mandate. Investment mandates are put in place by the board of trustees to regulate the behaviour of portfolio managers and limit the amount of influence they have on investment decisions, particularly for investment products such as pension funds, endowments and foundations, which by their very nature, are long-term investment products. This is important because it prohibits to a certain extent the role a portfolio manager's own biases and motives have on the overall look and feel of an investor's portfolio, particularly institutional investors. It provides investors with a layer of protection, especially against reckless decisions from portfolio managers and their investment teams and aims to preserve capital in the long run [7].

Anson delves even further into the nuances of SAA stating that SAA "is designed to meet the goals of an organisation under normal market conditions over a full market cycle" [3]. The market cycle being referred to here can last anywhere from two to 10 years. During this period however an organisation will most probably conduct an allocation study to determine the appropriateness of its SAA "every three to five years" [3]. The primary risk associated with a fund or portfolio is largely attributable to its SAA policies. This is the reason why SAA is said to reflect the overall risk tolerance of investors, often large institutional investors, whose main objective is capital preservation.

It is also important at this stage to highlight the link between SAA and modern portfolio theory discussed in Chapter 3, in which various passive portfolio allocation techniques were presented. These portfolios form the foundation of achieving long-term investment objectives by assigning weights to different asset classes based on their historical returns. The strategic weights are in no way a reflection of what a portfolio manager perceives the market to look like in the near future. It takes into account observed performance and correlations between different income streams to best position a portfolio for acceptable future returns given its risk profile. These investors do not aim to add any more value to a portfolio other than the value which will be unlocked by staying the course and remaining invested for longer periods to best satisfy the needs of clients [3].

5.2 Tactical Asset Allocation

Tactical asset allocation (TAA), in contrast to SAA, is a dynamic strategy which seeks to actively adjust the allocation of funds to a variety of asset classes. TAA has the following objectives:

- increasing portfolio returns
- adapting to market conditions, and
- providing diversification.

A TAA strategy adjusts asset weights based on short-term forecasts of asset returns. It aims to systematically exploit inefficiencies and/or temporary imbalances in equilibrium values among different asset classes. TAA attempts to add value by underweighting asset classes which are forecasted to underperform on a relative basis and conversely overweighting those asset classes which show possible outperformance [45, 52, 62].

Investors who incorporate TAA in their investment strategies seek to gain from short term market movements, expect to change their asset weights in the near future and are in general not as concerned about the implications of the average weight invested in an asset class in the long run. These active allocation strategies are often also built around strategic weights, allowing for changes in asset weights, but limiting these shifts such that the tactical bets don't overwhelm the strategic allocation as set out in the investment mandate.

To effectively employ a TAA strategy is however non-trivial and it has been shown in the past how TAA decisions have had an inverse effect and consequently subtracted value from a given portfolio [3, 18]. It is therefore extremely important to have a system in place which allows for correct decisions to be made in a timely manner such that a portfolio can benefit as much as possible from these TAA decisions. An important distinction here is also the players involved in the TAA framework. SAA is often set out by a board of trustees whereas for TAA periodic changes in asset allocation are determined by the investment staff. It is however also true that the role of the board of trustees and that of the investment staff can sometimes become less clear with a lack of guiding principles [3].

This chapter aimed to provide greater insight into asset allocation policies and the importance thereof, clearly defining two different approaches to asset allocation. These approaches are different in terms of the goals they aim to achieve and the individuals who are involved in their respective decision making processes. The next chapter pays special attention to TAA and the tools and methodologies used to construct portfolios which aim to benefit from market-timing and, in doing so, deliver superior performance.

6 Key drivers of market timing

Chapter 5 highlighted the role that asset allocation policies play in portfolio and risk management. More importantly, it clearly distinguished between two approaches to asset allocation, strategic and tactical, and how these approaches have clearly defined and unique objectives. This chapter serves as a short literature review of some of the most well-known tools used for TAA purposes.

Portfolio managers who decide to employ TAA in their portfolios are said to attempt to benefit from "market timing" [8]. This is non-trivial and the implementation of market-timing strategies which allow one to exit and more importantly, re-enter a given market segment at exactly the right moment has proven to be a strenuous exercise for investment teams. There exists a plethora of literature on TAA drivers and the different tools which have been used in the past and are still being developed today to time one's position in the market the most efficiently. There are three main categories of TAA drivers; valuation-based methods, macroeconomic scenarios and more technical/quantitative analyses. These categories are discussed below, following a similar structure to [26].

6.1 Valuation-based

Valuation-based methods have in the past attempted to exploit various market regimes and identify possible changes in market regimes in order to time the market [26]. This method is based on the underlying principle of value investing whereby investors aim to buy into markets that are cheap and exit positions or avoid markets altogether that are deemed expensive. The market is described as cheap when equity data suggest it is undervalued relative to historical norms and expensive when overvalued relative to historical norms.

There are various indicators which are commonly used within value investing, some being used in combination with one another or that are layered in different filtering levels. The most popular of these indicators include the price-to-earnings or PE ratio, the price-to-book ratio and the price/earnings-to-growth ratio. An indicator which has recently received considerable attention from market practitioners is the cyclically adjusted price-earnings ratio or CAPE, which was introduced to asset allocation in [11]. The CAPE ratio is calculated using the current stock price of a company and dividing it by its long-term average earnings, adjusted for inflation. Importantly, this ratio assesses financial performance over a specified period, while isolating the impact of economic cycles. Given that it explicitly accounts for economic cycles, it allows analysts to evaluate a company's broader profitability over time by smoothing out any cyclical effects.

As is evident in this definition of the CAPE ratio, it will allow portfolio managers to make decisions regarding the equity portion of their investment portfolios by considering the overall financial stability of individual stocks, but also that of different sectors and industries. Using the information contained in this ratio, portfolio managers can decide to limit their exposure to the equity market, or certain portions thereof, if they foresee a drop in returns or perhaps increased volatility levels and instead move some of the funds to other asset classes such as cash. The changes to weights allocated to different asset classes based on the outlook one may have of certain segments of the market encapsulates the main motivation behind TAA [62].

6.2 Macroeconomic scenarios

Another method of establishing a changing market environment, which will ultimately also affect the nature of returns of various asset classes, uses macroeconomic indicators. In [12] the authors state how it has been shown that the "sensitivities of size-sorted stock portfolios to rates, industrial production, inflation, credit spreads, and consumption explain a significant portion of their relative performance over time".

Fama and French (1989) also conduct a study into the effect of macroeconomic variables in the returns of securities [21]. The first factor considered in this study is the so-called default spread, measured as the difference in yield between a market portfolio of corporate bonds and AAA rated bonds. The second factor investigated was the dividend yield, a factor commonly used to forecast stock returns. The final factor considered is the term spread, which indicates the spread in yield earned on AAA rated bonds and the one-month US treasury bill rate. Fama and French (1989) concluded that the default spread, a business-conditions variable, is high during periods of persistently poor performance by business evident, for example, during periods such as the Great Depression [21]. The default spread is low on the other hand in the presence of strong economic conditions. The dividend yield was found to be correlated to the default spread and moves in a similar fashion relative to long-term business conditions. Lastly, the term spread is related to shorter-term business cycles. It is high near troughs and low during peaks [21].

These findings are then briefly summarised in practical terms as is explained in the context of people's behaviour during different periods of business cycles, both in their capacity as consumers and also as investors. When business conditions are poor and income is low, people have less disposable income which they can use to invest, therefore expected returns on stocks and bonds in particular are high to incentivise investment with lower levels of income. On the other hand, when business is seemingly booming and there exists increased levels of income, the market is expected to clear at lower levels of return [21].

There exists even more evidence on the effect of macroeconomic factors on security returns and how this ultimately influences decision-making regarding asset allocation and the tactical tilting of weights across various asset classes. Many of these factors are listed in [12], which suggests that all these factors are indeed priced into markets, albeit to different extents.

6.3 Technical/quantitative analyses

The third of the three broad categories of drivers of market timing is in certain ways much simpler in its application, yet extremely useful in terms of the benefits it offers to portfolio managers. As mentioned in [26], these types of indicators have recently become common practice in many systematic trend-following trading strategies to tactically scale portfolio exposures to a range of asset classes. The indicators, even though it is easier to understand and implement, can also vary in terms of its complexity relative to other indicators in this same category. According to [19], the following criteria are necessary to implement a purely quantitative method to allocation:

1. It must have simple, purely mechanical logic.
2. The same model and parameters must be used across all the different asset classes being invested in.
3. It must be a price-based model.

By meeting this set of requirements it will ensure that these quantitative strategies are simple enough for investors to follow and mechanical enough to remove emotions and hence also subjective decision-making, oftentimes the downfall of many TAA approaches. This is especially true when economic variables need to be forecasted. Take, for example, a country's gross domestic product or GDP, which can also affect the behaviour of markets significantly. For one to make forecasts about this quantity, one requires knowledge of potential foreign exchange rates, unemployment rates, fiscal deficits, business innovations and changes in consumer behaviour, all of which will inherently include a certain level of subjectivity [67]. This means that any forecasts made using these variables can and probably will be influenced by the individual or team who are responsible for these forecasts. Therefore, any system which removes these issues regarding subjectivity, deserves just as much, if not more attention, specifically from portfolio managers in this instance who themselves might be confronted by their own perceptions and/or biases regarding current market and business cycle conditions.

The system proposed in [19] adheres to this criteria and uses one of the most well-known measures of trend, the 200-day simple moving average (SMA), together with one simple trading rule. The rule stated that when the equity index in question was trading above its 200-day SMA then the exposure to equities would remain unchanged and when the index was trading below its 200-day SMA then this exposure would be decreased, with the excess funds being allocated to cash or cash-like securities. This simple system was implemented, at first only considering two asset classes, equities and cash, and produced a portfolio which generated equity-like returns at levels of volatility often associated with bonds. These results were observed over an extended time-period and included many important historical events and encompassed multiple business cycles, essential for meaningful evaluation of TAA strategies.

A closer look however at more recent annual returns of this system revealed that it had underperformed equities for six of the eight years spanning from 2009 to 2016. This period saw equities gain substantial ground post the Great Financial Crisis of 2007/2008. As mentioned, this strategy only considered two asset classes and given the highlighted underperformance additional assets were incorporated. The inclusion of these asset classes naturally provided another source of diversification which was absent during the original investigation [19].

The updated system weighted all the new asset classes equally, i.e. 20% allocation across each of the five asset classes in question. Each of the asset classes were considered independently, with exposures remaining at the stated 20% level unless it was trading at levels below its own 200-day SMA, in which case the 20% allocation was moved to cash once again. This method, termed the quantitative tactical asset allocation (QTAA) portfolio was compared to a global tactical asset allocation (GTAA) portfolio which simply represented a buy-and-hold portfolio using an equal weighting scheme across the same asset classes. The results for both the in-sample and out-of-sample periods showed the same trends, with the QTAA portfolio providing higher returns and with lower levels of risk [19].

Other literature shows how another quantitative indicator, the VIX (CBOE Volatility Index), is used [46, 49, 51, 70]. The VIX measure was created to measure expected volatility in the equities market, specifically the volatility in the US equity market and is often referred to as the "fear index". It is based on the prices of options contracts written on the S&P 500 index and is calculated aggregating weighted asset prices of these options, both put and call options, over a range of strike prices.

When there exist elevated levels of uncertainty in the equities market, then the amount investors are willing to pay to purchase options contracts to hedge their positions also increase. The increase in the premiums for options is reflected in an increase in the VIX. Maggie and Copeland (1999) show how this indicator is statistically significant and how on days following increases in the VIX, “portfolios of large-capitalisation stocks outperform portfolios of small-capitalisation stocks” [46]. During this period value-based portfolios also outperform growth-based portfolios. This point is interesting given the current speculation around the effectiveness of value-based strategies which in recent years have underperformed their growth-based counterparts. On the days following a decrease in the VIX the opposite occurs. This once again gives portfolio managers a sense of the performance of markets in the short term, presenting them with an opportunity to add value to their portfolios if executed accurately.

Probabilistic momentum and implied volatility, which when used in combination can produce a four-state market classification [26], is another example among the literature on the use of quantitative indicators in TAA.

There are of course also arguments against the idea of market timing and hence TAA in general, as a profitable and sustainable method of portfolio construction. Figure 6.1 shows the impact on portfolios when missing some of the best trading days experienced in the equities market. This is relevant as most approaches to TAA, whether quantitative or not, inform decisions based on the underweighting or overweighting of the equities asset class and the moving of funds to and from other asset classes such as cash to accommodate these portfolio tilts. It therefore illustrates the impact of poor market timing, both in terms of exiting but also re-entering the equities market in particular. In this instance the S&P 500 Total Return Index was used to represent the overall equities market [42].

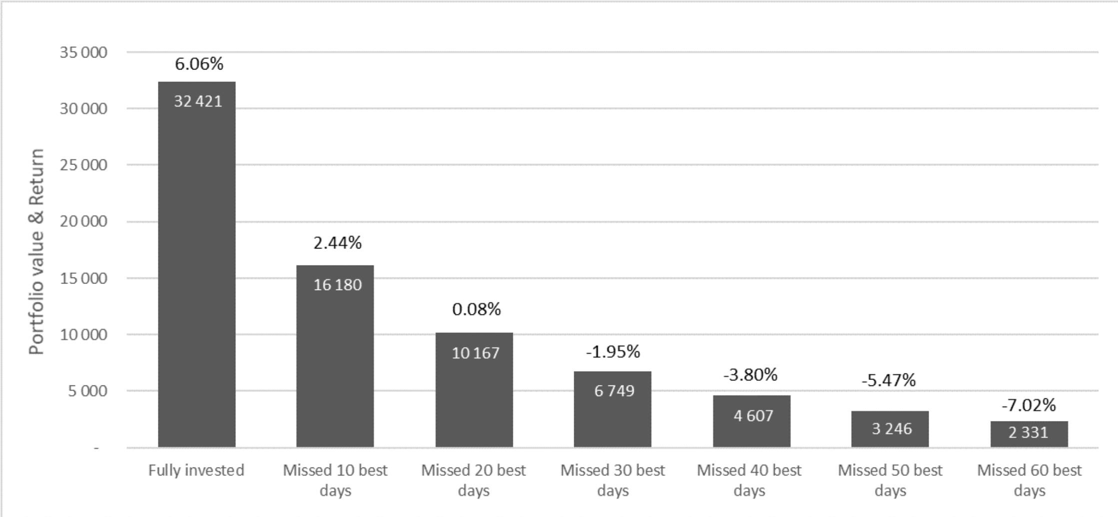


Figure 6.1: Impact of being out of the market (USD)

These sentiments are shared in [18] which explores the impact of Black Swan events[1] on investment portfolios. The study shows how, given 107 years worth of return data for the Dow Jones Industrial Average Index, missing "the best 10 days, out of 29 190 in the sample, resulted in portfolios 65% less valuable than a passive investment" [18]. It also shows the positive impact of avoiding the worst 10 trading days on portfolio values which just further emphasises the fact that the points of exiting and entering the market are equally important and both could have drastic effects on the overall portfolio value. Even more so when considering portfolios which are invested for longer time periods and thus being exposed to more Black Swan-like events along with the swings experienced by the market during seemingly normal economic conditions.

This chapter provided some necessary context regarding TAA and some key market-timing drivers used in practice. The next chapter gives a detailed explanation of the

[1]Taleb (2007) describes a Black Swan event as:
1) An outlier outside the realm of regular expectations because nothing in the past can convincingly point to its occurrence.
2) The event carries an extreme impact.
3) Explanations for the occurrence can be found after the fact, giving the impression that it can be explainable and predictable [65].

returns data that are used and the methodology which is implemented to make timely investment decisions. A quantitative TAA approach is adopted using the results from the application of the Kalman filter within the CAPM to forecast monthly asset returns.

7 Data and methodology

Chapter 6 explored the different categories of key market-timing drivers that are used to enhance portfolio performance through TAA. This chapter provides more insight into the proposed quantitative approach to TAA. This approach uses monthly returns data and incorporates the Kalman filter estimations for the time-varying variables α and β as used in the CAPM.

Before the mechanics of the strategy can be discussed, the return data of various asset classes available for investment by a portfolio manager, must be considered. Given the many types of underlying products found within each of these asset classes, decisions had to be made in terms of the indices used as proxies for their overall performance. The chosen indices are also products in which investors can take long or short positions and have over the years grown extensively in terms of notional amounts traded on a daily basis, so much so that it is deemed to provide the best picture of the overall state of various parts of the market.

7.1 Data

This section sets out the finer details relating to the data that are used for the purpose of this research. It also explains how the relevant data were sourced to allow for proper comparison between various investment approaches, both from a passive point of view, but also in portfolios which make use of tactical and timely tilts in asset weights.

7.1.1 Universe of investable assets

For the purpose of this study the available assets for SAA have been limited to equities, bonds, cash, real estate and commodities, similarly to what was done in [19]. These assets inherently also formed part of the TAA strategy's universe of investable assets, ensuring that both the SAA and TAA strategies had the same set of asset classes with which to construct portfolios. Table 7.1 shows these different asset classes and the indices used to represent each asset class's overall performance.

Table 7.1: Asset classes and their respective indices

Asset class	Index
Equities	MSCI ACWI
Bonds	FTSE WGBI
Cash	Three-month treasury bills
Real Estate	S&P Global REITs
Commodities	S&P GSCI

7.1.2 Description of indices

The *MSCI ACWI* is designed to "represent performance of the full opportunity set of large- and mid-cap stocks" [48]. It consists of stocks from as many as 23 developed and 26 emerging markets. The most recent research conducted established that this index covered approximately 85% of the total free float-adjusted market capitalisation in each market, ranging across 11 sectors and more than 3000 constituents [48]. The *FTSE WGBI* measures the performance of fixed-rate, local currency, investment-grade sovereign bonds. It currently includes sovereign debt from over 20 countries, denominated in a variety of currencies and has historical performance data spanning more than 30 years [27]. This index is also widely used as the benchmark of choice to evaluate several fixed-income investment products and thus made for an ideal candidate to use as an index for bonds in this study.

REITs or real estate investment trusts are investment vehicles which allow individuals to invest in large–scale, income producing real estate without having to physically own any commercial property. Majority of REITs do not develop property for the purpose of reselling, but rather to hold it as part of their investment portfolio. The index used to best represent the performance of REITs across the world is the S&P Global REITs index [64]. Lastly, the *S&P GSCI* was the first major investable commodity index in the world. "It is one of the most widely recognised benchmarks that is broad-based and production weighted to represent the global commodity market beta" [64]. This index is designed to be investable through the inclusion of some of the most liquid commodity futures, and provides additional diversification with low correlation to other asset classes.

It is clear that there exists some overlap between the different asset classes and underlying instruments using this chosen set of indices. This is a theme which is unavoidable in the context of this study, however all of these indices offer different products which investors can use and which are currently being traded in large volumes across the globe, and as such will also be considered for the TAA approach being investigated.

7.1.3 Sourcing the necessary data

Monthly price data were gathered for all of the selected indices. Majority of the price data were gathered using [71] as it provides a platform where data are easily and freely accessible. The data for the MSCI ACWI were collected directly from this institution's platform [48], which provides data for many investors who also use this index as one of the most important benchmarks in their investment performance and risk measurements. The Federal Reserve Bank of St Louis provided data for three-month US treasury bills, a proxy for cash returns [25].

As many data as possible were collected across the different asset classes. The earliest date for which there were data across all of the asset classes was selected as the starting point for this investigation. This date is set at 31 December 2007. The final date for which data were available at the time this study was conducted was 31 May 2020. This end date is quite significant given the market downturn experienced during the early months of 2020 at the height of the coronavirus pandemic. Special attention will be paid to this time period to evaluate the performance of the TAA approach compared to some of the other investment strategies employed during normal market conditions.

The price data sourced for the various asset classes' indices were then used to determine the logarithmic monthly returns according to:

$$R_i = ln(\frac{P_{t+1}}{P_t}).$$

These returns are used to compute excess security/portfolio returns, the observable variable in the Kalman filter algorithm which in turn is used to estimate time-varying, unobservable variables, α and β.

7.2 Methodology

According to [19], the following criteria are necessary to implement a purely quantitative method to asset allocation:

1. It must have simple, purely mechanical logic.
2. The same model and parameters must be used across all the different asset classes being invested in.
3. It must be a price-based model.

These requirements provided the guiding principles in the formulation of the proposed alternative quantitative approach to TAA. The strategic weights, which are set first on an annual basis, at the start of each new year, serve as the foundation on which the TAA framework will attempt to construct portfolios which aim to benefit from market-timing. When the strategic weights are set, they remain unchanged until the rebalancing of portfolios, i.e. from the start until the end of that specific calendar year. The strategic weights are set according to a selected reference portfolio. An investor has the option of choosing one of the following portfolios: minimum variance portfolio, maximum diversification portfolio or maximum Sharpe ratio portfolio. Then, in terms of time periods, an investor or portfolio manager acting on behalf of the investor, has the freedom to choose a lookback period and a desired evaluation period, both of which are in months.

The lookback period will determine how many months' worth of return data will be taken into consideration when setting the strategic weights for the selected reference portfolio. The lookback period has a significant effect on the weights and so careful consideration has to be given to this parameter. It is also important to remember that past returns are not always the best indicator of future performance and so it might be that the minimum variance portfolio, constructed based on historical monthly returns, is not necessarily the portfolio with the minimum variance of all portfolios on the efficient frontier when keeping these strategic weights constant over the evaluation period.

The evaluation period is the window over which the SAA and TAA frameworks will be compared. The window should ideally be 12 months or less to allow for the possible rebalancing of the appropriate reference portfolio. If the evaluation period is required to be longer than 12 months then it is suggested that the same process is followed over each year, rebalancing at the start of each new 12-month period. However, as mentioned previously, studies of the particular allocation and its appropriateness are usually conducted every three to five years [3]. Thus, there is a case for no rebalancing even when the evaluation period extends for more than 12 months.

At this stage the investor or portfolio manager has selected a lookback period, a reference portfolio for strategic weights and an evaluation period. The other two parameters that can be set are the maximum allocation to cash or cash-like securities and the portion of funds, expressed as a percentage of the weight allocated in equities, which is allowed to be invested in other asset classes. The maximum cash allocation for SAA has a strong bearing on the weights allocated towards the other asset classes, particularly equities, and can determine whether the fund sits at the high or low equity spectrum of multi-asset funds. In simpler terms, the lower the allocation in cash, the more funds are made available for investment in the other asset classes which form part of the opportunity set of investable assets.

This particular TAA strategy then allocates funds between equities, bonds, cash, real estate and commodities using either the min-max framework or the optimisation framework, with the strategic weights serving as the starting point for these portfolio tilts. Table 7.2 summarises the parameters which the user would be able to set depending on their preferences.

Table 7.2: Parameters for TAA approach

Parameter name	Example value
Rebalancing of SAA portfolio	Annual
Portion for TAA	50%
Maximum cash allocation	20%
Lookback period	60 months
Reference portfolio	Minimum variance
Initial investment (Rands)	100
Equity bounds	[0.4, 0.6]
Bond bounds	[0.4, 0.6]

7.2.1 Min-max framework

Next, it will be discussed how to decide whether to:

1. move funds from equities to either cash, real estate or commodities or
2. increase the holding in the equity asset class by taking a short position in either cash, real estate or commodities or
3. take two opposite and offsetting positions in the real estate or commodities asset classes.

To clearly set out the decision making process for the TAA strategy, different scenarios will be considered along with the corresponding actions that will be taken depending on what the following month's estimated asset returns look like. This TAA framework only looks at those asset classes which offer the *minimum* and *maximum* estimated returns as forecasted by the CAPM which incorporates the Kalman filter estimates of α and β. This process is described in Chapter 4. To this extent, various scenarios are highlighted below, followed by the action taken according to this TAA framework.

- *Equities estimated to offer minimum return and this return is positive*
  In this instance the strategic weight invested in equities is reduced and funds are allocated to either cash, real estate or commodities, depending on which is estimated to offer the maximum return, also positive in this instance, over the next month.
- *Equities estimated to offer minimum return and this return is negative, the maximum estimated return is however positive*

Here the strategic weight invested in equities is again reduced and funds are allocated to either cash, real estate or commodities, depending on which is estimated to offer the maximum return, also positive in this instance, over the next month.

- *Equities estimated to offer minimum return and this return along with the maximum estimated return are both negative*

  Here the strategic weight invested in equities is once again reduced and funds are allocated to either cash, real estate or commodities, depending on which is estimated to offer the maximum return, also positive in this instance, over the next month. This could entail a long position being taken in an asset class which is also estimated to offer negative returns to decrease the effect of the most negative returns offered by the equity asset class.

- *Real estate or commodities is estimated to produce the minimum return, but this return is positive*

  If the strategic weight is negative, i.e. a short position, this position will remain the same to prevent hurting portfolio returns as a result of changes to this position. If the strategic weight is positive, the weight will be reduced and excess weight will be allocated to the asset class which offers the best possible return, also positive, over the next month.

- *Real estate or commodities is estimated to produce the minimum return which is negative and the maximum estimated return across the remaining assets is positive*

  A short position is taken, thus resulting in a positive return, and a long position is taken in the asset which is estimated to offer the highest returns. If this asset class is either the equity asset class or cash, then its existing strategic weight will simply be increased.

- *Real estate or commodities is estimated to produce the minimum return and both this return and the estimated maximum return are negative*

Strategic weights are kept the same to avoid taking a potential long position for the upcoming month in an asset class which is estimated to produce a negative return.

- *Cash is estimated to produce the minimum return*
  Given how reluctant portfolio managers of multi-asset funds are to reduce their holding in cash, especially during extremely volatile market conditions, the strategic weights, which are already dependent on the maximum cash allocation parameter, will under no circumstances be lowered. It can be increased as was shown in the scenarios above.

This framework was manually built out using MATLAB code and can be viewed in the Appendix. Throughout this min-max TAA decision making process the weights are allocated in such a way that the sum of the weights invested across the different asset classes is still equal to one, with short selling being allowed as mentioned above. There is however no leverage which is used within this portfolio construction technique.

7.2.2 Optimisation framework

An alternative approach to the one mentioned previously was tested. Within the optimisation framework weights were allocated among the different asset classes, still adhering to bands set around the strategic weights, that resulted in the maximum portfolio return based on the forecasted returns of the individual asset classes. This approach was tested using the same parameters which were used in the min-max framework, e.g. maximum cash allocation, lookback period, evaluation period, etc. The results however illustrated how this type of approach was more heavily influenced by inaccurate forecasted returns. Here the ranking of the individual assets did not matter, the weights were purely set according to the particular combination which would maximise the return of the overall portfolio. Therefore, if majority of the forecasted returns were significantly different from realised returns then this weighting regime would provide extremely inefficient weight combinations.

It was thus decided that this approach did not show the necessary qualities to be investigated any further and the min-max approach was adopted as the TAA weighting technique of choice. The code for the optimisation framework can also be found in the Appendix.

An extremely detailed description of the different asset classes and their respective indices has now been provided. The two frameworks that were considered, the min-max framework and the optimisation framework, were also discussed, paying close attention to the rules that exist within each framework and the parameters that can be tailored to the needs of portfolio managers and their clients. The next chapter shows the results that were obtained using the min-max framework and compares the performance of common SAA approaches and their associated TAA approaches.

8 Results of TAA approach compared to common SAA approaches

In Chapter 7 the details regarding the implementation of the proposed quantitative TAA approach were discussed. This included information on the data that were used as well as an explanation of the two main methodologies of which the min-max framework was chosen as the most suitable system. This chapter illustrates the results of this implementation through the use of various tables, graphs and other visualisations which allow for a rigorous comparison between common SAA approaches and their TAA counterparts.

To do so, the necessary performance and risk metrics had to first be established. The following section describes these metrics and thereafter the results are shown, along with a detailed discussion.

8.1 Description of performance and risk metrics

8.1.1 Annual return and risk

Price data collected were monthly values. When portfolio managers or risk and performance analysts evaluate the risk and performance of a portfolio over a specific time horizon, annual values are preferred and are also much easier to explain to investors. Therefore, the monthly returns had to be converted into annual values. There exists two ways in which monthly return data are converted into annual return data. The first method is a very simple approach and purely applies linear scaling of monthly return data. For the purpose of this study however, the following method was adopted to convert monthly returns, R_m, into annual returns, R:

$$R = (1 + \bar{R_m})^{12} - 1, \tag{17}$$

where $\bar{R_m}$ represents the average monthly return over a specified period, e.g. 12, 24 or 36 months.

For the conversion of monthly volatility into annual volatility, the standard approach was used:

$$\sigma = \sigma_m \sqrt{12}.$$

This method is derived from the well-known square root of time rule which is based on the assumption that returns are normally distributed which leads to linear scaling with time of the return variance.

8.1.2 Sharpe ratio

The Sharpe ratio, Eq. (12), is commonly used to gauge the performance of a security or portfolio by adjusting for its risk [59, 60, 61]. It is the ratio of portfolio returns less the risk-free rate of return to the volatility associated with the returns of the portfolio. It subtracts the risk-free rate since this is the return investors expect to earn without taking on any risk, and so the Sharpe ratio considers those returns earned in excess of the risk-free rate per unit of risk - a proxy for which is volatility. Generally, the higher the Sharpe ratio the more attractive the risk-adjusted returns of the portfolio.

The Sharpe ratio is calculated using the returns produced by the TAA approach. These are compared to other SAA approaches' Sharpe ratios.

8.1.3 Tracking error

Tracking error is one of the most important ways to gauge the performance of a portfolio relative to a benchmark portfolio, and the ability of a portfolio manager to generate excessive returns and beat the chosen the benchmark, in line with the risks undertaken by portfolio managers [39]. Tracking error is defined as:

$$TE = \sqrt{Var(R_p - R_b)}, \tag{18}$$

where R_p represents the return achieved by a portfolio of assets and R_b represents the return achieved by the benchmark portfolio.

Lower tracking errors indicate that the performance of the portfolio closely resembles that of the benchmark portfolio, whereas high tracking errors reveal that there is a significant difference between the performance of the respective portfolios. A high tracking error is not negative per se, as [36] found that funds with a low tracking error show a higher beta, similar standard deviation and lower alpha compared to funds with high tracking error. Regardless, in practice restrictions are put in place for tracking error which active portfolio managers must adhere to [56].

8.1.4 Information ratio

"The information ratio is an important - perhaps the most important - measure of investment performance" [30]. Its calculation is based on the standard statistical formulas for the mean and standard deviation. In simpler terms, the information ratio is the average excess returns of a tracking portfolio beyond that of a benchmark portfolio, per unit of volatility in excess returns. This volatility in excess returns is calculated using Eq. (18). Mathematically the information ratio can then be calculated using:

$$IR = \frac{R_p - R_b}{TE}. \tag{19}$$

Equation 19 holds when computing the annual information ratio. There are however some nuances involved when computing the information ratio over a specific time period when returns are observed more frequently than on an annual basis. This study uses monthly returns produced by SAA and TAA portfolios. Therefore, one needs to be extremely careful when using these monthly returns to report an annual value for the information ratio. There are various ways in which to do this, but for this study it was decided that the first method discussed in [28] would be used. This method uses the arithmetic mean of excess returns, which then gets annualised using Eq. 17, in the numerator and applies the square root of time rule to convert the monthly tracking error to an annual tracking error in the denominator.

It is also important to note that the mean excess returns and volatility in excess returns or tracking error, are calculated over a specified time period, e.g. 12, 24 or 36 months.

Portfolio managers will prefer an investment strategy with the highest possible information ratio [30]. It is however important to keep in mind that the information ratio should be used to compare portfolios within the same style and universe of investable assets. With that being said, research conducted by [31] shows that a top quartile manager has an information ratio of 0.5 and an exceptional manager should achieve a value of 1.0 or above. Once again, these values depend on the universe of assets from which managers can choose and results vary across different fund categories [28].

8.1.5 Maximum drawdown

The maximum drawdown, or MDD, reflects the maximum fall in the value of an investment. It is measured as the difference between the value of the lowest trough and that of the highest peak before the trough [13]. Ideally the MDD is calculated over a long period of time to allow the value of an investment or portfolio to go through several so-called boom-bust cycles. The MDD is therefore used as another method of measuring the investment risk associated with a portfolio which consists of a basket of assets, such as the TAA portfolios used in this study. MDD is defined as:

$$MDD = \frac{(Trough\,Value - Peak\,Value)}{Peak\,Value}.$$

A low MDD value indicates slight fluctuations in the value of a portfolio and, therefore, a lesser degree of risk and vice versa. When comparing two different portfolios, an investor who wishes to receive the guarantee of more stable returns over a longer time horizon would more than likely choose the portfolio with a lower MDD. In line with normal risk-reward expectations, an investor who wishes to receive a higher return and therefore willing to take on additional risk would prefer a portfolio with a higher MDD.

8.2 Results

This section shows the results obtained by applying the rules-based TAA approach. It first focuses on the main metrics used to evaluate the performance of different investments and the risk associated with these investments. The parameters used are the same as the example set of parameters provided in Table 7.2. One could ultimately choose the exposure to equities one desires, but for the purpose of this section it was chosen to limit the exposure to equities to no more than 60% of the total portfolio and no less than 40% in terms of strategic weights set on an annual basis. This was done in an attempt to replicate a real-life example where the investment team could potentially be mandated to keep the average exposure to equities within certain thresholds in terms of SAA.

8.2.1 Performance and risk

Reference is made to the "market", which simply represents a 100% investment to the equity index chosen for this research, the MSCI ACWI. Table 8.1 shows the main performance metrics, average annual return, average annual volatility, Sharpe ratio and maximum drawdown. These metrics are measured from January 2011 to the end of May 2020.

Table 8.1: Descriptive statistics for market, SAA and TAA portfolios using data spanning January 2011 to May 2020

Minimum variance	Market	SAA	TAA
Annual return	5.64%	3.62%	6.29%
Annual volatility	13.57%	6.94%	7.27%
Sharpe ratio	0.37	0.43	0.78
Maximum drawdown	-22.69%	-11.31%	-7.59%
Maximum diversification			
Annual return	5.64%	1.99%	4.47%
Annual volatility	13.57%	9.48%	8.33%
Sharpe ratio	0.37	0.15	0.46
Maximum drawdown	-22.69%	-17.55%	-13.79%
Maximum Sharpe ratio			
Annual return	5.64%	6.80%	12.34%
Annual volatility	13.57%	8.75%	11.62%
Sharpe ratio	0.37	0.71	1.01
Maximum drawdown	-22.69%	-10.74%	-10.30%

For all three SAA weighting techniques, the addition of the TAA framework saw an improvement in the average annual return achieved by the portfolios. For the minimum variance weighting technique the average annual return was also increased from being below that offered by the market to exceeding market returns. This was done without a significant increase in the risk faced by the portfolio, measured as the average annual volatility of portfolio returns. In the case of the maximum diversification ratio weighting technique, the TAA portfolio showed decreased levels of risk when compared to its SAA counterpart. For the maximum Sharpe ratio portfolio there was a noticeable increase in the risk associated with the TAA portfolio.

In terms of the Sharpe ratio, and as would be evident from the preceding paragraph, the TAA portfolios all saw an improvement during the time period under observation. The Sharpe ratio for the maximum diversification TAA portfolio, for example, more than doubled the value achieved by the SAA portfolio. The maximum drawdowns faced by these portfolios were also less than its SAA counterparts, albeit to varying magnitudes across the three weighting techniques. This is promising for investors who place a great deal of emphasis on capital preservation, especially during negative market conditions.

Supplementary to the results illustrated in Table 8.1, Figures 8.1, 8.2 and 8.3 quantify characteristics of the returns produced by each of the portfolios in question. This is done in the form of boxplots and fitted normal distributions to the respective observed portfolio returns.

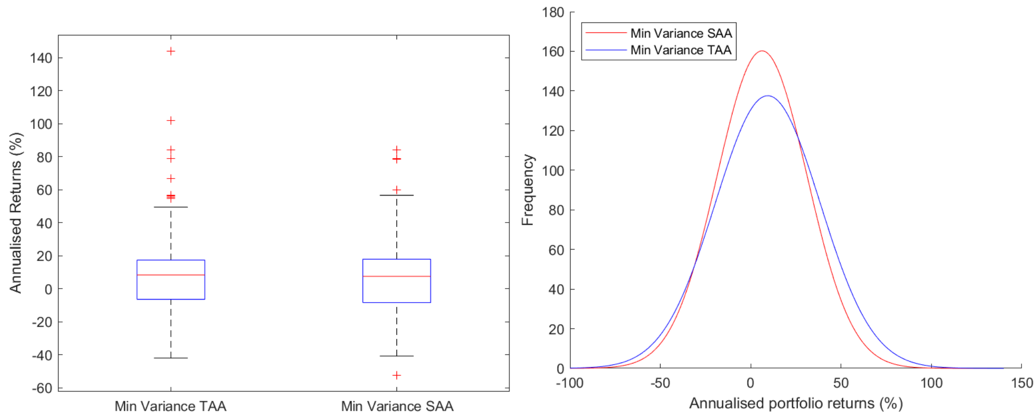


Figure 8.1: Boxplot and return distribution of minimum variance SAA and TAA portfolios

The boxplot in Figure 8.1 shows that the TAA portfolio was capable of producing significantly larger positive returns compared to the TAA portfolio. This can also be seen when looking at the respective return distributions, with a larger region under the TAA graph for higher positive annualised portfolio returns. This larger spread in positive portfolio returns also give the return distribution of the TAA portfolio a flatter appearance compared to the SAA portfolio. There is greater variability in the returns of the TAA portfolio which confirms the observation of a higher average annual volatility compared to the SAA portfolio.

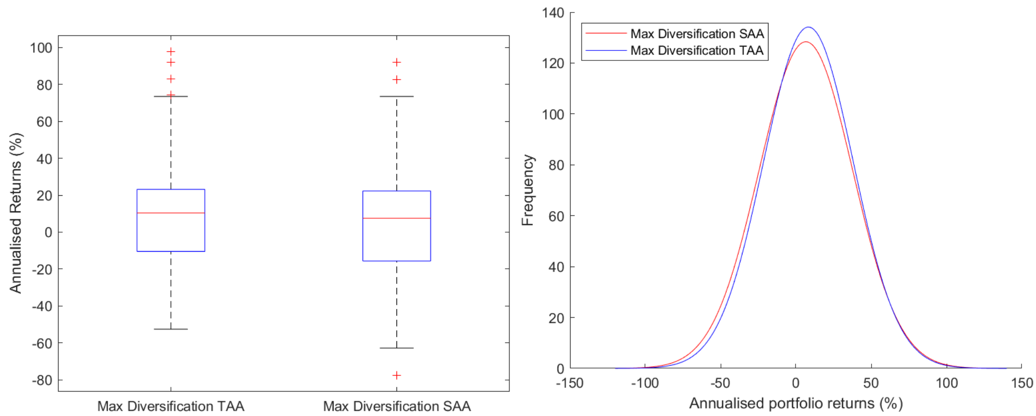


Figure 8.2: Boxplot and return distribution of maximum diversification SAA and TAA portfolios

For the maximum diversification weighting technique the differences between the TAA and SAA portfolios are more subtle. From the boxplot in Figure 8.2 the TAA portfolio once again was able to achieve higher positive returns and at the same time, provided some downside protection with smaller negative returns. In general, as can be seen from both the boxplot and return distribution, the returns of the TAA portfolio showed a lower level of dispersion, resulting in a higher average annual return and a lower average annual volatility of returns. These two characteristics are essential for portfolio managers as they aim to produce more consistent returns with less uncertainty.

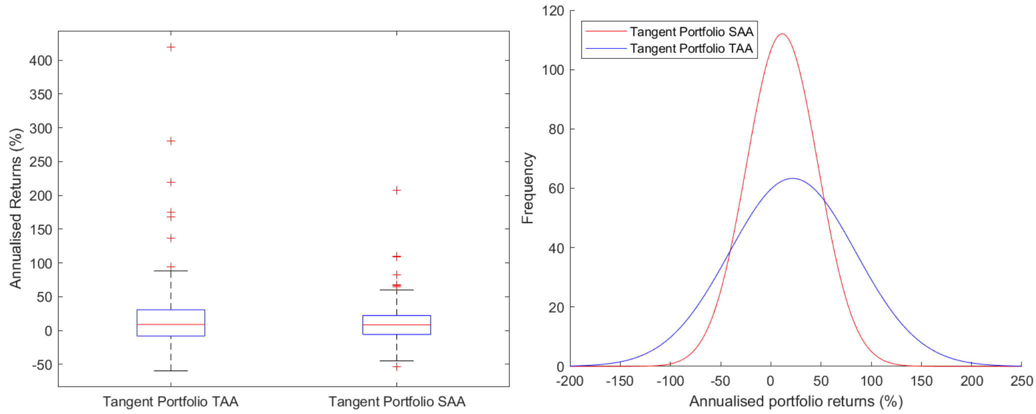


Figure 8.3: Boxplot and return distribution of maximum Sharpe ratio SAA and TAA portfolios

Despite the maximum Sharpe ratio TAA portfolio's ability to produce significantly higher positive returns and achieving a higher average annual return over the time period being considered, it came at the cost of much higher volatility of portfolio returns, evident from Figure 8.3. This confirms the results observed in Table 8.1 regarding the average annual volatility of the TAA portfolio returns compared to its SAA counterpart. As a result, the TAA portfolio does show potential to deliver large positive returns, but it does also leave the portfolio vulnerable to a higher probability of excessive negative returns compared to the SAA portfolio, which will undoubtedly hurt the portfolio performance over longer periods, potentially suffering from large capital losses.

A different perspective would be to look at the total return, shown as the cumulative value of a portfolio over the period in question, of the various SAA reference portfolios as well as its associated TAA portfolios. The final values of the portfolios were measured at the end of May 2020 and is shown in Figures 8.4 and 8.5. For illustrative purposes the cumulative return of the market is also shown in both the figures.

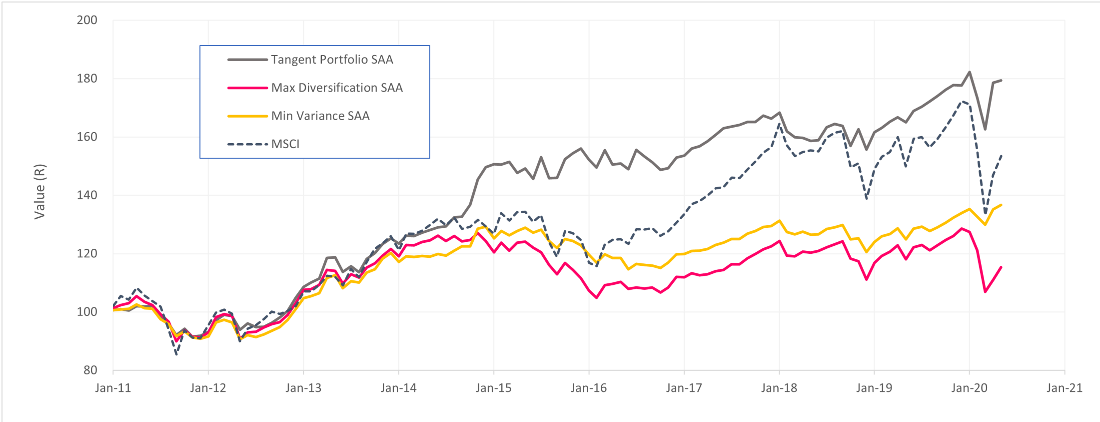


Figure 8.4: Cumulative performance of various SAA portfolios using data spanning January 2011 to May 2020

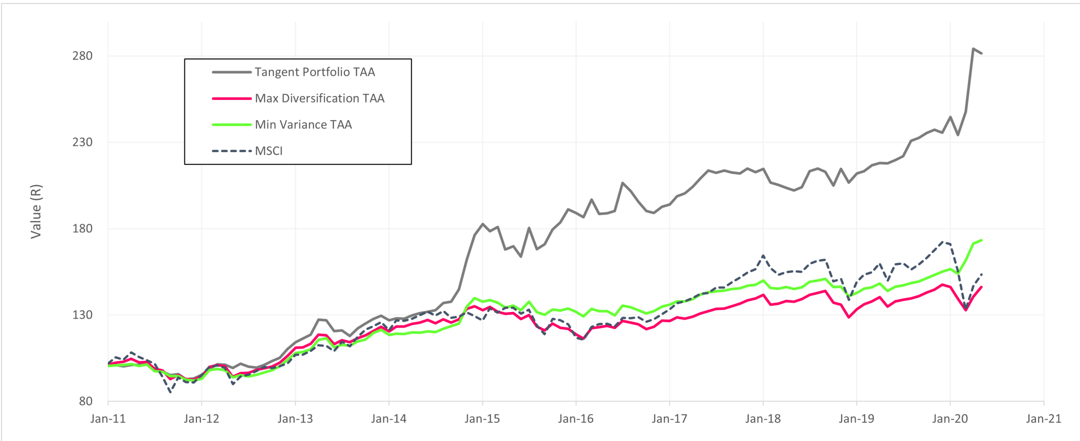


Figure 8.5: Cumulative performance of various TAA portfolios using data spanning January 2011 to May 2020

When looking at the SAA portfolios, only the maximum Sharpe ratio portfolio managed to outperform the market over this time period. There were however two TAA portfolios which managed to outperform the market, the minimum variance and maximum Sharpe ratio portfolios, whereas the cumulative performance of the maximum diversification portfolio came in only just below that of the market.

Instead of plotting all the SAA portfolios on one graph and all TAA portfolios on another, Figures 8.6, 8.9 and 8.12 plot each individual SAA portfolio against its TAA counterpart to more easily observe the difference in cumulative performance between the portfolios. Figures 8.7, 8.10 and 8.13 show the exponentially weighted moving average volatilities of these portfolios, while Figures 8.8, 8.11 and 8.14 show the evolution of the annualised tracking error and information ratio of the respective TAA portfolios over the specified time period. Figures 8.8, 8.11 and 8.14 also include shaded areas which represent "good" information ratios as discussed in Section 8.1 [31].

For the purpose of calculating tracking error and information ratio, 36 months of return data were used and thus these values are shown from January 2014, where a sufficient amount of data had been observed. The results obtained are shown for each of the weighting techniques investigated in this study.

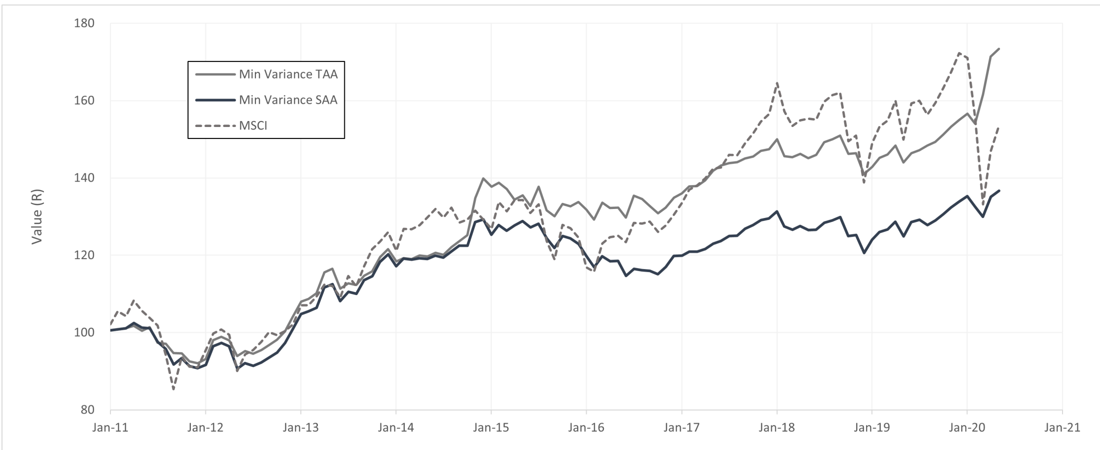


Figure 8.6: Cumulative performance of minimum variance SAA and TAA portfolios using data spanning January 2011 to May 2020

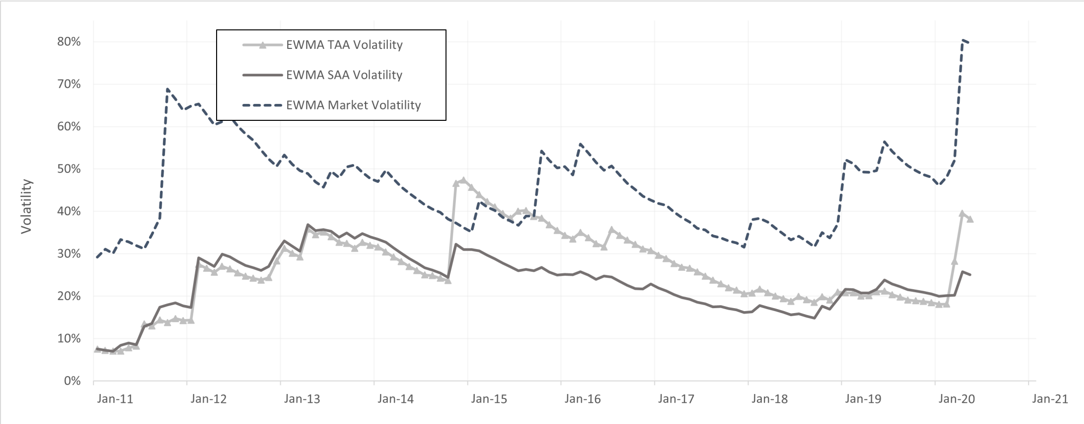


Figure 8.7: EWMA volatilities of minimum variance SAA and TAA portfolios

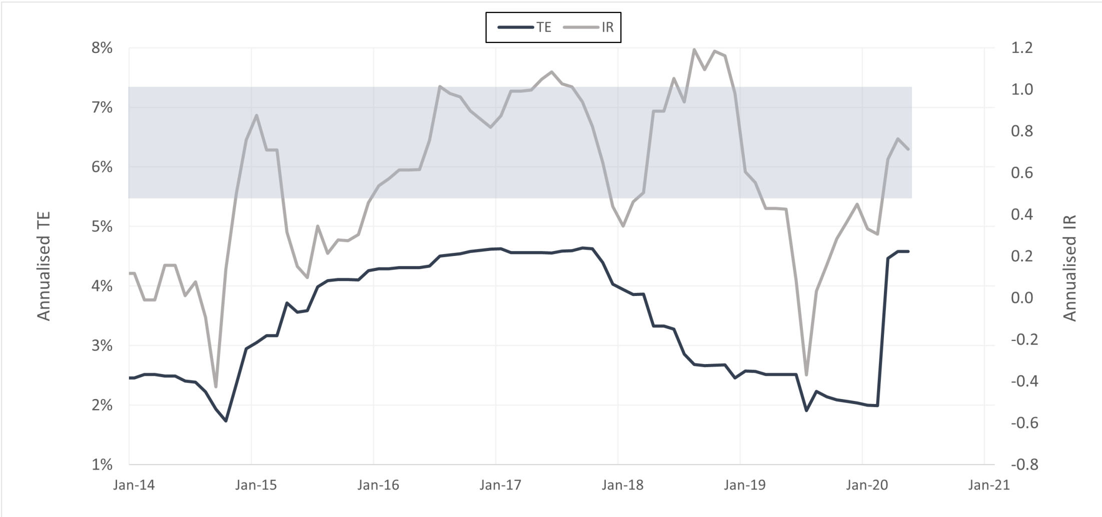


Figure 8.8: Annualised tracking error and information ratio for minimum variance TAA portfolio

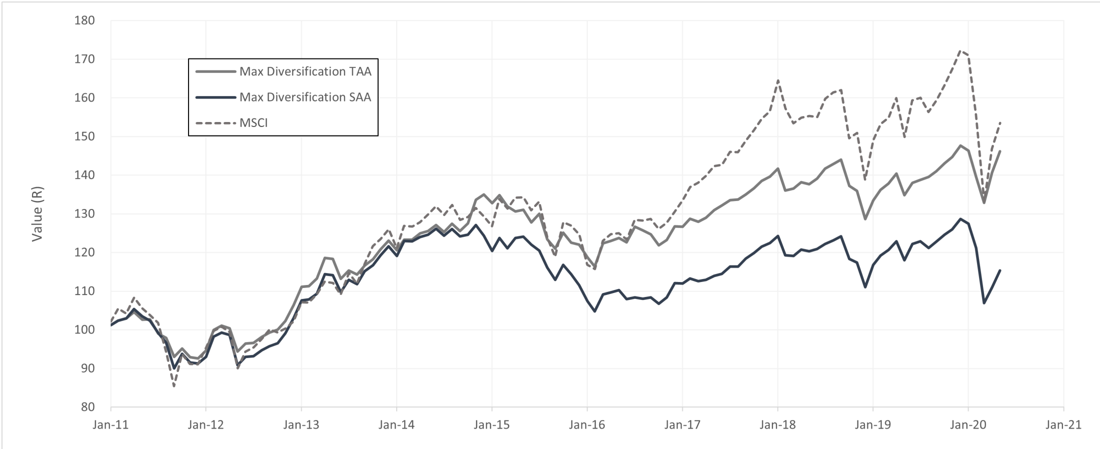


Figure 8.9: Cumulative performance of maximum diversification SAA and TAA portfolios using data spanning January 2011 to May 2020

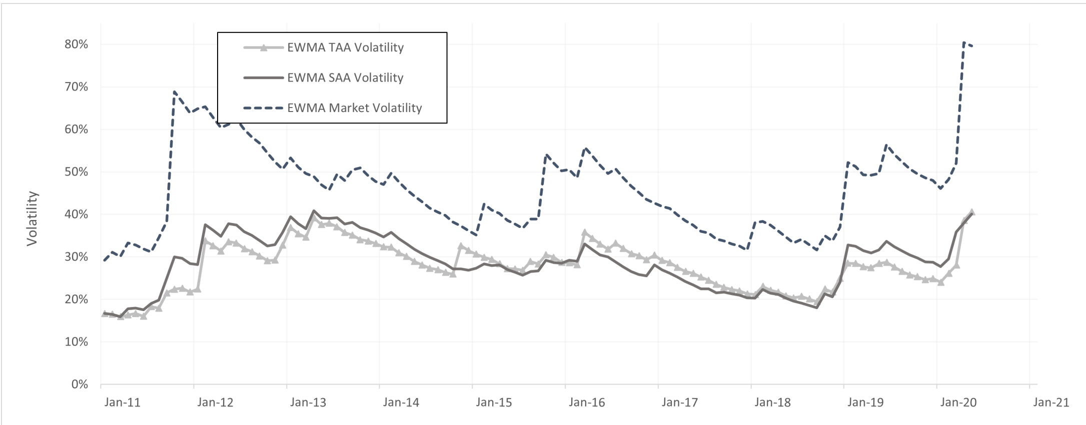


Figure 8.10: EWMA volatilities of maximum diversification SAA and TAA portfolios

Figure 8.11: Annualised tracking error and information ratio for maximum diversification TAA portfolio

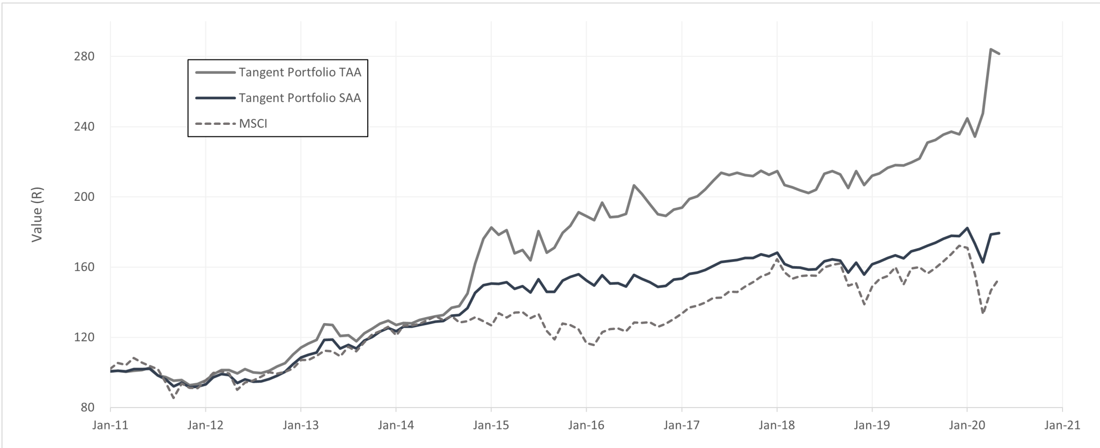


Figure 8.12: Cumulative performance of maximum Sharpe ratio SAA and TAA portfolios using data spanning January 2011 to May 2020

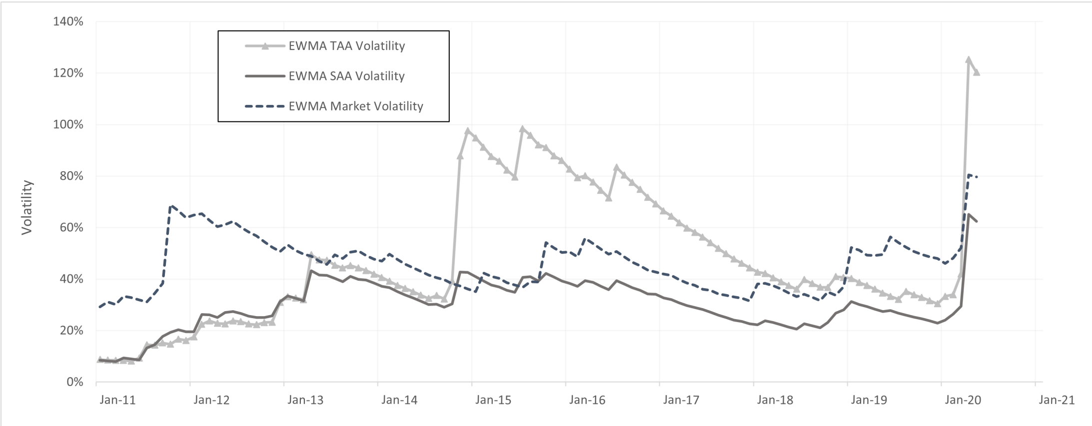


Figure 8.13: EWMA volatilities of maximum Sharpe ratio SAA and TAA portfolios

Figure 8.14: Annualised tracking error and information ratio for maximum Sharpe ratio TAA portfolio

The results show that each of the TAA portfolios produced a higher portfolio value than the associated SAA portfolio at the end of May 2020. The maximum diversification TAA portfolio did so without any noticeable differences in EWMA volatility over the entire period. The minimum variance TAA portfolio showed similar levels of EWMA volatility compared to the SAA portfolio until the end of 2014, at which point its volatility was somewhat elevated. It did however return to levels similar to that of the SAA portfolio before a brief spike during 2020.

The EWMA volatility of the maximum Sharpe ratio TAA portfolio would be worrying for a portfolio manager seeing that not only was it elevated compared to its SAA counterpart for certain time periods, but also that it was significantly higher than the EWMA volatility of market returns. The significantly higher portfolio value thus came at some price in terms of the risk faced by the portfolio and the overall risk tolerance of the investor would ultimately determine whether such a TAA strategy would be acceptable. This highlights perhaps one of the shortcomings of the proposed TAA framework, as it is evident in this instance, that it does not take into consideration excess risk the decisions regarding portfolio tilts imposes on the portfolio of assets.

When looking at tracking error from 2014 onward, and using the SAA portfolios as the benchmark portfolios, both the minimum variance and maximum diversification TAA portfolios stay within 1%-5%. For the minimum variance TAA portfolio in particular, the tracking error remains below 4% majority of the time. The maximum Sharpe ratio TAA portfolio shows the highest levels of tracking error, meaning greater dispersion of excess returns around the mean excess return delivered by the portfolio relative to the SAA portfolio. The higher tracking error observed for the maximum Sharpe ratio TAA portfolio could be attributable to the lack of constraints regarding the size of the tactical deviations from the strategic benchmark weights, commonly referred to as "tactical ranges" [13]. With the other two weighting techniques, the allocation towards equities is oftentimes less than what would be observed with the maximum Sharpe ratio portfolio, and since the size of portfolio tilts are dependent on the portion of the SAA portfolio allocated towards equities, the tactical deviation from the strategic weights are less extreme.

Lastly, for the given time period, all three TAA portfolios delivered information ratios which were predominantly positive, indicating the ability of the portfolios to produce excess returns over the benchmark portfolios when taking into consideration 36 months' worth of return data. The information ratios across all three weighting techniques also entered and remained within the 0.5-1.0 shaded area for large parts, which is indicative of outperformance over a sustained period of time, a promising result for this TAA framework [56]. There were indeed some periods during which the TAA portfolios underperformed their SAA counterparts per unit volatility of excess returns, i.e. a negative information ratio.

It is however important to highlight that this study doesn't factor in fees of any kind, which would be expected to erode some of the positive values of the information ratio these portfolios managed to achieve.

From the above it is clear that the amount which is allowed to be invested for TAA purposes, and consequently tilting the weights allocated to the different asset classes, could possibly have a significant effect on the behaviour of the TAA portfolios relative to its SAA counterparts. This became visible when observing the results of the maximum Sharpe ratio TAA portfolio, where the percentage weight allocation for TAA purposes is greater than the other weighting techniques given the large portion of the overall portfolios invested in equities. Figures 8.15, 8.16, 8.17 and 8.18 show the same figures as before, but for various percentage weight allocations, specifically for the maximum Sharpe ratio TAA portfolio.

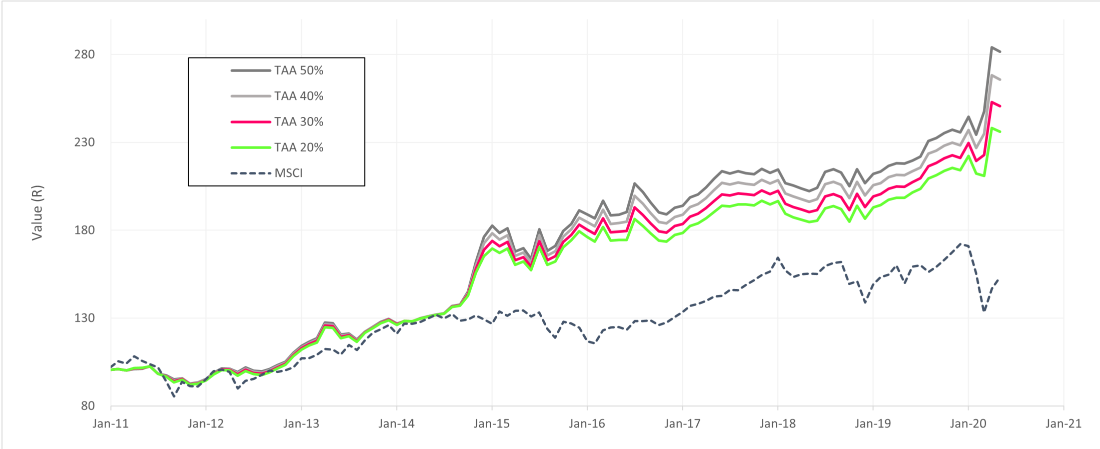


Figure 8.15: Cumulative performance of various maximum Sharpe ratio TAA portfolios using data spanning January 2011 to May 2020

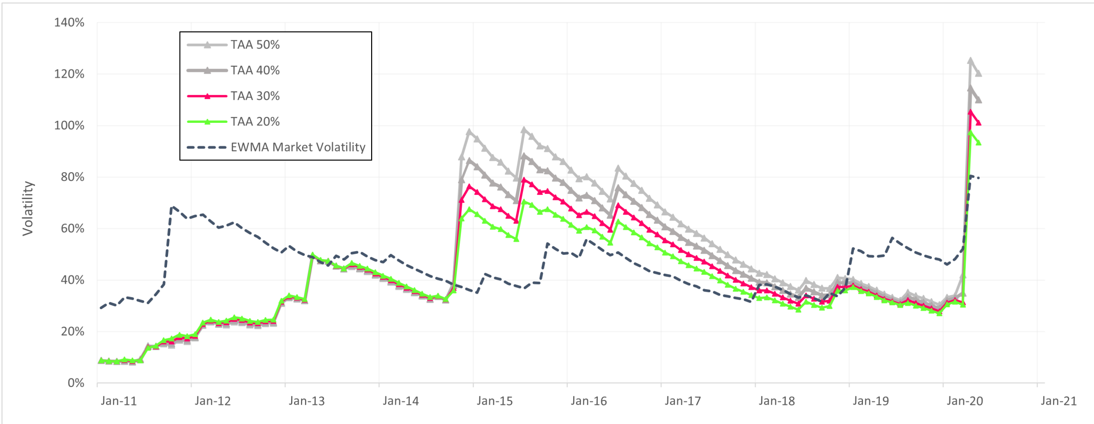
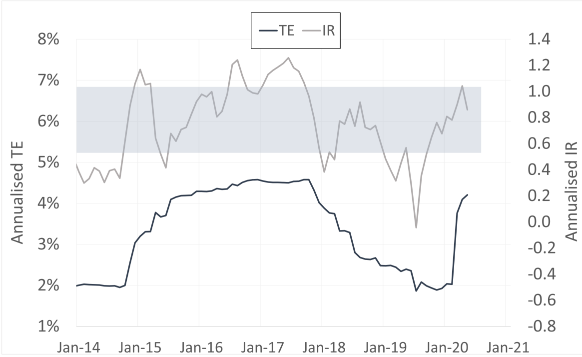
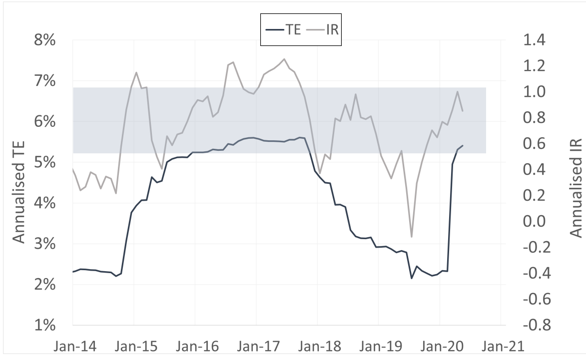


Figure 8.16: EWMA volatilities of various maximum Sharpe ratio TAA portfolios

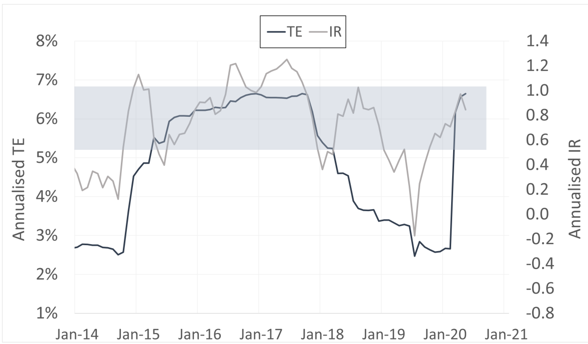


(a) TAA 20%

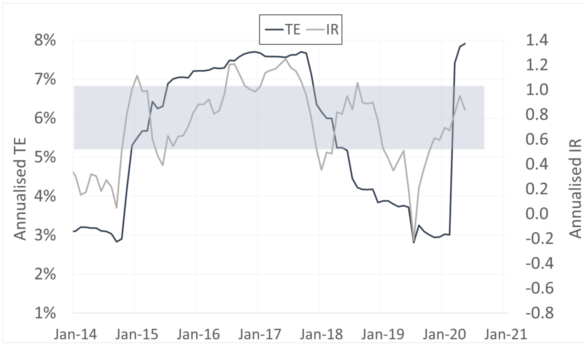


(b) TAA 30%

Figure 8.17: Annualised tracking error and information ratio for various maximum Sharpe ratio TAA portfolios (20% and 30%)



(a) TAA 40%



(b) TAA 50%

Figure 8.18: Annualised tracking error and information ratio for various maximum Sharpe ratio TAA portfolios (40% and 50%)

Figure 8.15 suggests that by decreasing the percentage weight allocation and thus making the tactical ranges narrower, the cumulative performance of the maximum Sharpe ratio TAA portfolio is reduced. Despite the weaker performance, the EWMA volatility of the portfolio, shown in Figure 8.16, is however reduced the as the percentage allocation is decreased. The narrower tactical ranges also reduced the tracking error, meaning that the behaviour of the TAA portfolio more closely resembled that of the SAA portfolio. The 20% weight allocation for example produced a maximum Sharpe ratio TAA portfolio with a tracking error which did not exceed 5%, similar to the other two weighting techniques.

Despite this, the information ratio remains at similar levels. This suggests that the mean excess returns achieved by the TAA portfolios, once again over a 36 month period, decreased in line with the decrease in tracking error, leaving the information ratio largely unaffected.

8.2.2 Portfolio composition

To provide a clearer picture with regards to the entering and exiting of positions within the different asset classes, Figure 8.19 illustrates how, over the course of five years in this instance, the weight allocation to the *commodities* asset class changed on a monthly basis using the minimum variance portfolio as the reference SAA portfolio. Figure 8.20 shows the same positions, but this time using the maximum diversification portfolio as the reference SAA portfolio. Recall from Chapter 7 that the amount available for use for tactical allocation purposes is dependent on the portion of the portfolio invested in equities, i.e. the higher the amount allocated towards equities, the higher the amount for TAA and vice versa.

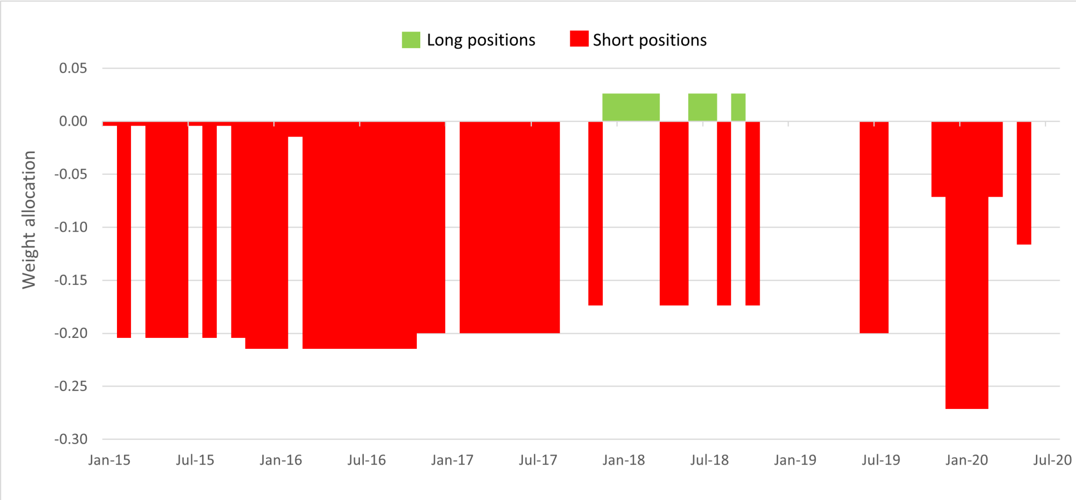


Figure 8.19: Long/short positions over the last five years for *commodities* using minimum variance portfolio as reference portfolio

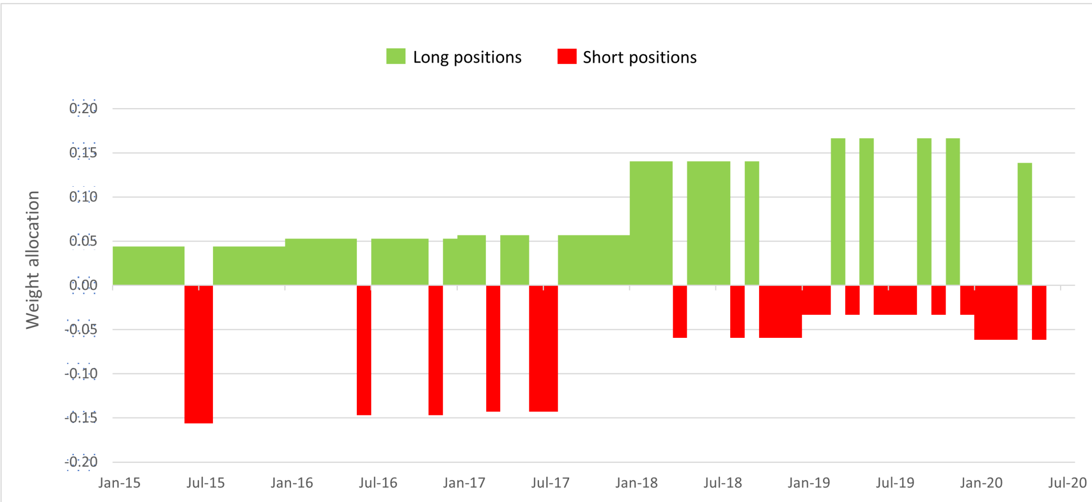


Figure 8.20: Long/short positions over the last five years for *commodities* using the maximum diversification portfolio as reference portfolio

These periodic changes in weights were as a direct result of commodites' forecasted returns relative to equities and the other asset classes as described by the min-max framework in Chapter 7. This process is repeated on a monthly basis, when the TAA framework is active, resulting in different weights being invested in different asset classes. This could be in the form of long positions as shown by the green bars in Figures 8.19

and 8.20, or in the form of short positions, i.e. the red bars. These graphs would look different for the various asset classes considered in this study and the commodities asset class was used purely for illustrative purposes.

As mentioned in Chapter 5, portfolio managers who employ tactical investment decisions are not as concerned about the implications of the average weight invested in an asset class in the long run. Figures 8.21, 8.22 and 8.23 however illustrates the effect the proposed TAA framework had on the average weight allocated to each asset class. Absolute weights were used in the construction of these graphs but in reality, for the minimum variance SAA and TAA portfolios, the weight allocated to real estate and commodities were both negative, whereas for the maximum Sharpe ratio SAA and TAA portfolios only the allocation to commodities was negative. The maximum diversification portfolios, although it did employ short selling, had overall long positions for all the asset classes.

Figure 8.21: Average weight allocation across asset classes for minimum variance SAA and TAA portfolios

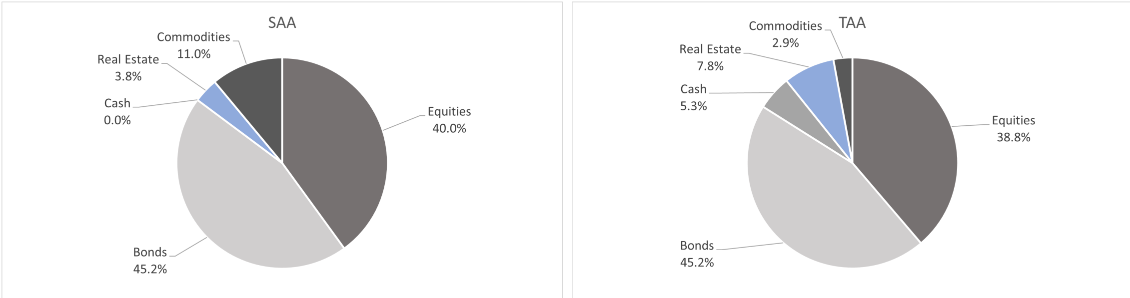


Figure 8.22: Average weight allocation across asset classes for maximum diversification SAA and TAA portfolios

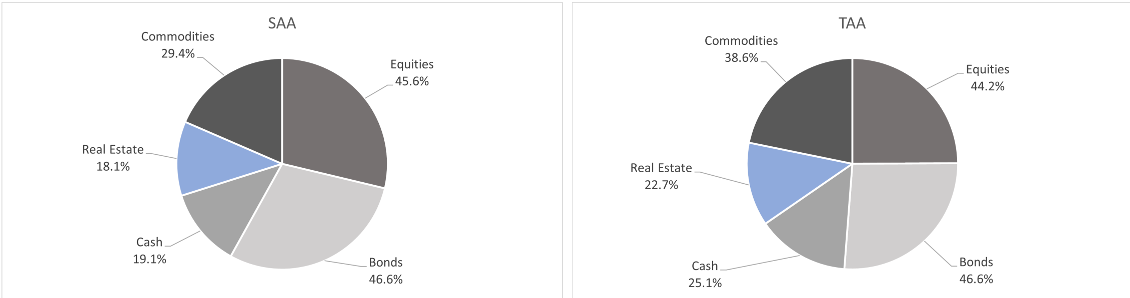


Figure 8.23: Average weight allocation across asset classes for maximum Sharpe ratio SAA and TAA portfolios

Figure 8.23 once again illustrates the impact of a lack of constraints on deviations from strategic weights which resulted in holdings in commodities (short) and real estate (long) which would undoubtedly cause some concerns for many investors. This issue was further exacerbated through opposite and offsetting positions being taken in these two asset classes simultaneously based on their forecasted returns. Similar to what was done for Figures 8.15-8.18, the results can be compared when varying percentage weight allocations are considered for TAA purposes.

This chapter focused on the results that were obtained and the use of these results for comparison between SAA and TAA approaches. This was done using clearly defined and widely used performance and risk metrics. Both portfolio performance and portfolio composition were illustrated to understand the behaviour of the TAA approach relative to its SAA reference portfolio. The next chapter provides some concluding remarks based on these observations and points to be considered for future research and development of a more robust TAA framework.

9 Conclusion and suggestions for future research

An alternative TAA approach was investigated to establish whether it would improve the performance of various SAA investment portfolios. This approach is quantitative in nature and uses estimations of α and β parameters as they appear in the CAPM to forecast asset returns. These estimations are produced using the Kalman filter, a time series estimation algorithm which has seen increasing popularity in quantitative finance. This algorithm aims to produce estimations of unobservable data which are more recent, relative and thus a much more accurate representation of current market conditions as measured by α and β.

These estimations and subsequent return forecasts where used to test two separate TAA frameworks; the min-max and the opitmisation frameworks. The approach of choice was the min-max framework which ranked the return potential of various assets according to estimated best and worst performers. Based on these rankings, asset weights were tactically adjusted on a monthly basis to improve on purely strategic strategies. The three SAA weighting techniques explored were the minimum variance, maximum diversification, and maximum Sharpe ratio portfolios. The TAA approach used the strategic weights set by these techniques and tilted holdings of different asset classes in an attempt to improve portfolio performance measured by a set of metrics commonly employed in the investment industry.

For the period 2011 to May 2020 it was seen, for all three SAA weighting techniques, the addition of the TAA framework saw an improvement in the average annual return achieved by the portfolios. For the minimum variance weighting technique the average annual return was also increased from being below that offered by the market to exceeding market returns. This was done without a significant increase in the risk faced by the portfolio, measured as the average annual volatility of portfolio returns. In the case of the maximum diversification ratio weighting technique, the TAA portfolio showed decreased levels of risk when compared to its SAA counterpart. For the maximum Sharpe ratio portfolio there was a noticeable increase in the risk associated with the TAA portfolio. These trends were also observed when considering the EWMA volatilities of portfolio returns.

In terms of the Sharpe ratio, the TAA portfolios all saw an improvement during the time period under observation. The Sharpe ratio for the maximum diversification TAA portfolio, for example, more than doubled the value achieved by the SAA portfolio. The maximum drawdown faced by these portfolios were also less than its SAA counterparts, albeit to varying magnitudes across the three weighting techniques.

In terms of cumulative performance, only the maximum Sharpe ratio SAA portfolio managed to outperform the market over this time period. There were however two TAA portfolios which managed to outperform the market, the minimum variance and maximum Sharpe ratio portfolios. All the TAA portfolios showed higher portfolio values over the period, but as has been mentioned already, this improved cumulative performance was accompanied by an increase in risk, particularly for the minimum variance TAA and maximum Sharpe ratio TAA portfolios.

When looking at tracking error from 2014 onward, using 36 months of return data and the SAA portfolios as the benchmark portfolios, both the minimum variance and maximum diversification TAA portfolios stayed within a 1%-5% range. For the minimum variance TAA portfolio in particular, the tracking error remained below 4% majority of the time. The maximum Sharpe ratio TAA portfolio showed the highest levels of tracking error, meaning greater dispersion of excess returns around the mean excess return delivered by the portfolio relative to the SAA portfolio. All three TAA portfolios delivered information ratios which were predominantly positive, indicating the ability of the portfolios to produce excess returns over the benchmark portfolios, and regularly were at levels greater than 0.5. There were however some periods during which negative information ratios were observed for these portfolios.

An overall cause for concern of this approach was the elevated levels of volatility and tracking error for the maximum Sharpe ratio TAA portfolio in particular, given the higher amount available to execute the proposed TAA framework. The min-max framework allows for varying percentage weight allocations to tilt portfolio holdings, but it does not take into account the overall risk of the portfolio and residual risk compared to the benchmark portfolios which in this instance were the associated SAA portfolios. The results for cumulative performance, EWMA volatility of portfolio returns, tracking error and information ratio for the maximum Sharpe ratio TAA portfolio were computed again for different weight allocation percentages. The EWMA volatilities decreased with a decrease in amount allowed for TAA purposes along with a decline in cumulative performance. The tracking error also decreased with lower percentage values, but interestingly this occurred without any impact on the levels of the observed information ratio.

This same process could be repeated for the other weighting techniques to establish whether similar trends would be observed. In general, the proposed TAA framework could be adjusted to explicitly and dynamically account for risk faced by the portfolio and the residual risk when compared to the benchmark portfolios. Furthermore, more sophisticated methods could be used to determine the optimal percentage or amount made available in order to perform the necessary portfolio tilts and consequently improve the performance of the TAA portfolios for any weighting technique or investment strategy.

The performance of the TAA portfolios also would have been largely aided by the ability to take short positions in the commodities market in response to declining oil prices and to reduce the equity portion of the portfolio to escape some of the negative effects of the coronavirus on the market. Both of these events occurred during 2020 and to perhaps provide a different perspective of the performance of the TAA portfolios, one might be inclined to exclude the returns of 2020 and focus on a time period with no exogenous shocks and more stable market conditions. Table 9.1 shows pertinent performance metrics for the time period 2015 to the end of 2019.

Table 9.1: Descriptive statistics for various portfolios using data spanning January 2015 to December 2019

Minimum variance	Market	SAA	TAA
Annual return	6.63%	0.87%	2.26%
Annual volatility	11.73%	5.85%	6.07%
Sharpe ratio	0.47	-0.04	0.19
Maximum drawdown	-15.59%	-11.01%	-6.62%
Maximum diversification			
Annual return	6.63%	0.98%	2.09%
Annual volatility	11.73%	7.76%	7.45%
Sharpe ratio	0.47	-0.01	0.14
Maximum drawdown	-15.59%	-15.56%	-13.65%
Maximum Sharpe ratio			
Annual return	6.63%	3.76%	6.57%
Annual volatility	11.73%	7.28%	10.70%
Sharpe ratio	0.47	0.37	0.51
Maximum drawdown	-15.59%	-7.49%	-10.30%

Some of the same patterns mentioned in Chapter 8 can be observed again from 2015 to 2019. It can be seen how the TAA framework managed to improve the average annual return across all three weighting techniques. The average annual volatility of portfolio returns for the minimum variance TAA portfolio increased slightly compared to its SAA counterpart. For the maximum diversification weighting technique on the other hand, the TAA framework managed to decrease the average annual volatility slightly. The maximum Sharpe ratio TAA portfolio saw a significant increase in average annual volatility.

The maximum drawdowns experienced by the TAA portfolios were all less than that of the market. Only one TAA portfolio, the maximum Sharpe ratio portfolio showed a worse maximum darwdown than its SAA counterpart. Sharpe ratios in general saw an improvement across all three weighting techniques, with two of the TAA portfolios, the minimum variance and maximum diversification TAA portfolios, changing the negative Sharpe ratios which would have been achieved by the SAA portfolios into positive values. Once again it should be highlighted that these results do not take into account additional fees which would have been associated with the increased level of activity inherent in managing these TAA portfolios.

Lastly, the statistics shown in Table 8.1 and those statistics mentioned throughout the study, are based on data from the end of 2007 to May 2020. To draw more informed conclusions regarding the results and hence validity of this type of approach, its behaviour and performance will have to be examined over a longer time frame. Some of the indices used in this study were only formed much later in the piece given new developments in the market and the type of products on offer to investors. Given that some of these indices are very much still in its infancy stages, there were no available data which stretched further back than 2007. Therefore, as time goes by, newly realised returns should be incorporated such that there are returns for multiple asset classes and their representative indices across numerous business cycles. By exposing this approach to a longer time frame, one would inevitably encounter more market crashes, recessions and possibly even depressions. It would be worth taking note of the performance of portfolios during extreme events and prolonged market trends.

The objective of this TAA approach, which assesses forecated returns produced by the

CAPM using Kalman filter estimates of α and β, and adjusts weights accordingly on a monthly basis, was to improve the overall performance and risk characteristics of portfolios which would otherwise employ a strategic and thus more static weighting allocation. These improvements would be as a result of an effective quantitative market-timing mechanism which allows a portfolio manager to enter and exit certain market segments in such a way that it could exploit inefficiencies and temporary imbalances in equilibrium values.

Definite changes in the overall behaviour of the TAA portfolios were observed compared to their SAA counterparts. This is evident when calculating key performance and risk metrics used in industry using a little less than 10 years of monthly portfolio returns. The TAA portfolios showed a definite improvement in terms of average annual return, Sharpe ratio and cumulative performance. Results varied when considering the average annual volatility in portfolio returns and observing EWMA volatilities over this specific time period, as some TAA portfolios increased the amount of volatility in returns and thus risk faced by the portfolio, whereas others managed to limit this risk. The maximum drawdown of the TAA portfolios also showed noticeable improvements, albeit at different levels. These trends were also observable when looking at a shorter time period which did not include an exogenous shock such as the coronavirus pandemic of 2020.

Despite a lack of explicit constraints relating to the behaviour of the TAA portfolios relative to its SAA counterparts, the tracking error of two of the TAA portfolios never exceeded 4%, whilst still producing average to above average information ratios for sustained periods of time. It was shown how, in the case of elevated tracking error values, a decrease in the percentage weight allocation for TAA purposes can address concerns investors and portfolio managers may have without negatively affecting the corresponding information ratios.

10 Appendix

10.1 Growth in alternative assets

There has been a dramatic change in the alternative asset landscape in the aftermath of the financial crisis during which equity returns in particular were devastated [37]. This change has been evident in terms of both notional amounts invested and liquidity in markets for these assets as more investors look towards alternative assets because of their low correlation with "classical financial assets" [32]. This has prompted portfolio managers to actively consider including these asset classes in their clients' portfolios. This consideration is important for any TAA approach which aims to take advantage of market inefficiencies. Two reports, from JP Morgan and PwC respectively, echo these sentiments [41, 53].

JP Morgan suggests that: "As investor sophistication increases, and plans are less constrained in their view of asset class boundaries and the management of α and β - alternatives are playing an even more important role in enhancing portfolio risk/return characteristics" [41]. The possible inclusion of alternative assets in a portfolio not only expands the opportunity set, but also allows investors flexibility in deciding when and where these assets should be incorporated. Figure 10.1 shows some early results of how the allocation of funds among the different asset classes has changed.

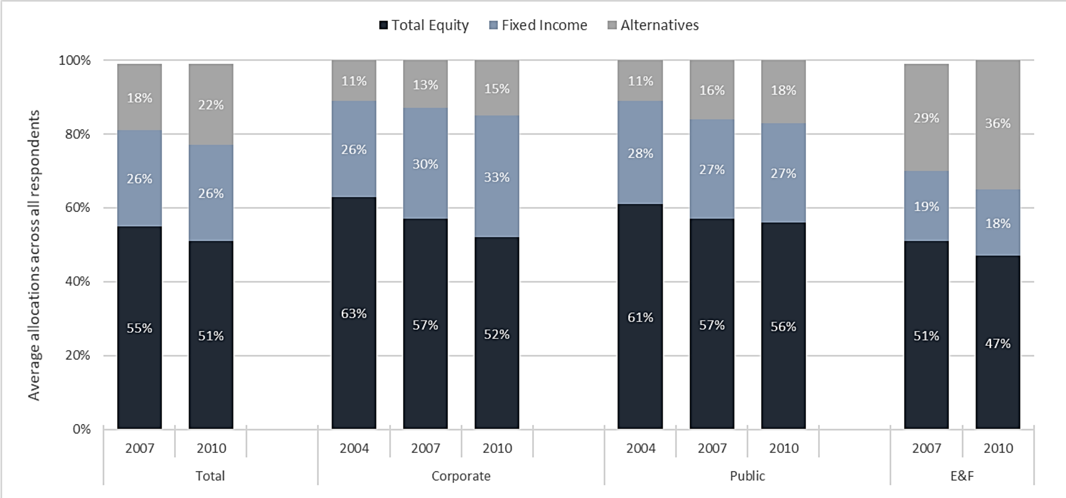


Figure 10.1: Shift in allocation from traditional to alternative assets

PwC states: "Alternative firms, with their emphasis on investment outcomes rather than products, and specialisation rather than commoditisation, will increasingly attract investors seeking customisation, diversification and genuine long-term alpha" [53]. More recent data regarding the growth in alternative assets as can be seen from Figure 10.2 [53].

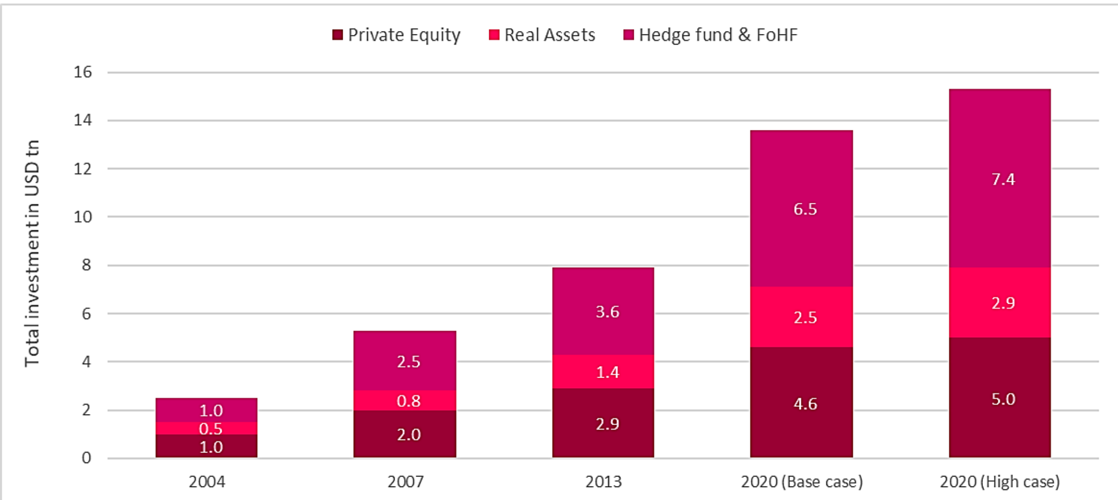


Figure 10.2: Increasing investments in alternative assets

Taking all of the above data into consideration it is clear that any future study of asset allocation would have to ultimately include various assets which do not form part of the more traditional asset classes. These alternative assets could potentially be added to the opportunity set of asset classes for SAA and TAA strategies as more return data become available and more thorough research on the return characteristics of these assets can be conducted by analysts and portfolio managers.

10.2 Support VBA code to Kalman filter

The following code shows, in detail, the implementation of the Kalman filter algorithm in VBA. The various steps that form part of this time series estimation method are discussed in Chapter 2, whereas its specific application to the CAPM and subsequent forecasting of asset returns are discussed in Chapter 4.

```
Dim j As Integer

Application.ScreenUpdating = False

    Calculate
    Range("U2:U151").Copy
    Range("B2").Select
    Selection.PasteSpecial Paste:=xlPasteValues

    Range("L7:R8").Select
    Selection.ClearContents
    Range("C2:D151").Select
    Selection.ClearContents

Dim i As Integer

'   Initialise
    Range("L2:L3").Copy
    Range("C2").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Transpose:=True

'   Initialise first r_m value
    Range("V2").Copy
    Range("J2").Select
    Selection.PasteSpecial Paste:=xlPasteValues

    Range("P2:P3").Copy
    Range("L7").Select
    Selection.PasteSpecial Paste:=xlPasteValues
    Range("Q2:R3").Copy
    Range("Q7").Select
    Selection.PasteSpecial Paste:=xlPasteValues

    j = Range("T1").Value
```

```
For i = 1 To j

'   New y value
    Cells(2 + i, 2).Copy
    Range("M7").Select
    Selection.PasteSpecial Paste:=xlPasteValues

'   Update r_m
    Cells(2 + i, 22).Copy
    Range("J2").Select
    Selection.PasteSpecial Paste:=xlPasteValues

'   Update x
    Range("L12:L13").Copy
    Cells(2 + i, 3).Select
    Selection.PasteSpecial Paste:=xlPasteValues, Transpose:=True

    Range("P12:P13").Copy
    Range("L7").Select
    Selection.PasteSpecial Paste:=xlPasteValues

'   Update p
    Range("Q12:R13").Copy
    Range("Q7").Select
    Selection.PasteSpecial Paste:=xlPasteValues
Next i
    Range("L4").Select
    Selection.ClearContents

Application.ScreenUpdating = True
End Sub
```

10.3 Schematic representation of the functionality and programme flow of the TAA strategy

Figure 10.3 shows the different steps and components of the method used for this study. It graphically illustrates Sections 10.4 to 10.9 from the parameterisation of the investment problem at hand to the calculation of various performance and risk metrics.

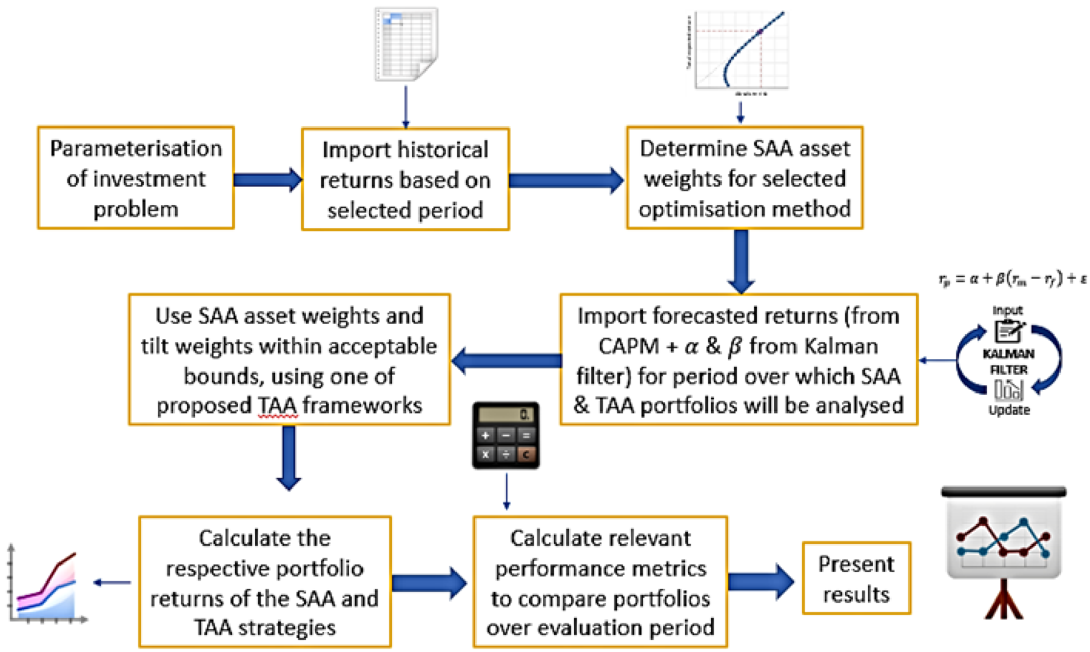


Figure 10.3: Schematic representation of TAA strategy

10.4 Parameterisation of investment problem

The code below shows how a user of this strategy can, at their own discretion, select various parameters which will be used to build and execute the applicable TAA strategy.

```
% Parameterization
mySelection = Block ; % Select "All" or "Block"
evaluationStartString = 31-Dec-2010 ; % User input
evaluationStartDate = datetime(evaluationStartString, 'InputFormat', 'dd-MMM-yyyy');
% Date when portfolio weights are set
lookBackPeriod = 60 ; % years worth of historical data considered
evaluationPeriodEnd = 12 ;
% How many months strategy will be reviewed for
TAAMethod = MinMax ; % Optimization or MinMax
TAAPortion = 0.5 ;
CashAllocation = 0.2 ; % Maximum portion allowed
% to be invested in cash-like securties
ReferenceSAA = MinVariance ; % Max Sharpe, Min Variance or Max Diversification
```

10.5 Setting up the evaluation and lookback periods based on dates of available data

The amount of return data used forms an integral part to the TAA strategy setup. Here it is shown how, based on the time frame specified by the user, only certain portions of the return data are selected from all available historical data. It is also shown how the amount of time over which the strategy will be implemented and evaluated is selected.

```
% Make provisions such that we stay within the timeframe of available data
% both for lookback and evaluation period
if dateshift(evaluationStartDate, 'end', "month", evaluationPeriodEnd) <= ...
        returnData.Dates(end, 1)
    evaluationPeriod = [evaluationStartDate, ...
        dateshift(evaluationStartDate, 'end', "month", evaluationPeriodEnd)];
else
    evaluationPeriod = [evaluationStartDate, returnData.Dates(end, 1)];
end

if dateshift(evaluationStartDate, 'end', "month", -lookBackPeriod) >= ...
        returnData.Dates(1, end)
    historicalLookbackPeriod = [dateshift(evaluationStartDate, 'end', "month", ...
        -lookBackPeriod), evaluationStartDate];
else
    historicalLookbackPeriod = [returnData.Dates(1, end), evaluationStartDate];
end
```

```
if mySelection == "Block"
    % does the necessary calculations for the given block of timeframe
    d1 = find(returnData.Dates == historicalLookbackPeriod(1, 1), 1, "first");
    d2 = find(returnData.Dates == historicalLookbackPeriod(1, 2), 1, "first");
% elseif mySelection == "All" % uses entire dataset
%     d1 = 1;
%     d2 = find(returnData.Dates == lastPossibleDate);
elseif mySelection == "All"
    % Starts at chosen start date and runs till very last date of dataset
    d1 = 1;
    d2 = find(returnData.Dates == historicalLookbackPeriod(1, 2), 1, "first");
end
```

10.6 Implementation of SAA approaches according to investment mandate using only equities, bonds and cash

The following code illustrates how the different SAA weighting allocations are determined depending on the choice of optimisation method. It also includes the lines of code used to construct the efficient frontier.

```
% Mean variance optimization including all of the
% from start of lookback period to current evaluation date
meanvariancePortfolioAll = Portfolio("AssetList", assetClasses(1, :), "AssetMean", ...
    meanStore{1, :}', "RiskFreeRate", ...
    mean(returnData{d1:d2, 'RiskFreeRate'}), "AssetCovar", ...
    cov([returnData{d1:d2, assetClasses(1, :)}]), ...
"InitPort", [0.5, 0.5, zeros(1, size(alternativeAssets, 2))]);
% initial portfolio allocates 50% to equities and 50% to bonds
% and zero to the other assets
meanvariancePortfolioAll = setBudget(meanvariancePortfolioAll, (1 - CashAllocation), 1);
meanvariancePortfolioAll = setBounds(meanvariancePortfolioAll, ...
    [0.4, 0.4, -alternativeAssetsWeights],...
    [0.6, 0.6, alternativeAssetsWeights]); % Short selling allowed
pwgtAll = estimateFrontier(meanvariancePortfolioAll, 200);
% use 200 different portfolios just for illustration
[meanvariancePortfolioRiskAll, meanvariancePortfolioReturnAll] = ...
    estimatePortMoments(meanvariancePortfolioAll, pwgtAll);
```

```
% Minimum variance portfolio including all asset classes fot TAA
pwgtMinVarAll = pwgtAll(:, find(min(meanvariancePortfolioRiskAll) == ...
    meanvariancePortfolioRiskAll, 1));
[minVarianceRiskAll, minVarianceReturnAll] = estimatePortMoments(meanvariancePortfolioAll,
    pwgtMinVarAll);

pwgtSharpeAll = estimateMaxSharpeRatio(meanvariancePortfolioAll);
[sharpeRiskAll, sharpeReturnAll] = estimatePortMoments(meanvariancePortfolioAll, ...
    pwgtSharpeAll);
```

```
% Consider diversification ratio, using the weightings of the initial
% portfolio setup and then finding weigths which maximize this ratio to
% form the maximum diversification portfolio
assetList1 = [assetClasses(1, :), 'RiskFreeRate'];
objFun = @(MDweightsAll) -calculateDiversificationRatio(MDweightsAll, ...
    returnData, assetList1, d1, d2);
x0_1 = [meanvariancePortfolioAll.InitPort; CashAllocation];
options_1 = optimset('TolX', 1.0e-8);
[maxDiversificationWeightsAll, maxDiversificationRatioAll, exitflag] = fmincon(objFun, ...
    x0_1, [], [], ones(1, numel(assetList1(1, :))), ...
    1, [0.4; 0.4; -alternativeAssetsWeights'; 0], ...
    [0.6; 0.6; alternativeAssetsWeights'; CashAllocation], [], options_1);
maxDiversificationRatioAll = -maxDiversificationRatioAll;
maxDivRiskAll = std([maxDiversificationWeightsAll]' * ...
    [returnData{d1:d2, [assetClasses(1, :), 'RiskFreeRate']}]');
maxDivReturnAll = [maxDiversificationWeightsAll]' * ...
    [meanStore{1, assetClasses(1, :)}, mean(returnData{d1:d2, 'RiskFreeRate'})]';
```

10.7 Visualising the various SAA approaches with the efficient frontier

The code below is used to plot the portfolios of the various SAA weighting techniques relative to the efficient frontier.

```
% Plot historical efficient frontier with different weighting techniques
gradientValue = (sharpeReturnAll - (mean(returnData{d1:d2, 'RiskFreeRate'})))/...
    sharpeRiskAll;
chosenX = sharpeRiskAll + 0.01;
extendedY = gradientValue*chosenX + mean(returnData{d1:d2, 'RiskFreeRate'});
```

```
figure
hold on
plot(meanvariancePortfolioRiskAll.*sqrt(12).*100, ...
    (((1 + meanvariancePortfolioReturnAll).^12)-1).*100);
plot([0, sharpeRiskAll*sqrt(12)*100, chosenX*sqrt(12)*100], ...
    [((1 + mean(returnData{d1:d2, 'RiskFreeRate'}))^12 - 1)*100, ...
    ((1 + sharpeReturnAll)^12 - 1)*100, ((1 + extendedY)^12 - 1)*100], ...
    "Color",'r',"LineWidth",1);
scatter(sharpeRiskAll*sqrt(12)*100, ((1 + sharpeReturnAll)^12 - 1)*100);
scatter(minVarianceRiskAll*sqrt(12)*100, ((1 + minVarianceReturnAll)^12 - 1)*100);
scatter(maxDivRiskAll*sqrt(12)*100, ((1+maxDivReturnAll)^12 - 1)*100);
text(sharpeRiskAll*sqrt(12)*100 + 0.5, ((1 + sharpeReturnAll)^12 - 1)*100, ...
    "Tangent portfolio");
text(minVarianceRiskAll*sqrt(12)*100 + 0.5, ...
    ((1 + minVarianceReturnAll)^12 - 1)*100 + 0.5, "Minimum variance portfolio")
text(maxDivRiskAll*sqrt(12)*100, ((1+maxDivReturnAll)^12 - 1)*100 - 0.5, ...
    "Maximum diversification portfolio");
xlabel('Annualised Risk (%)');
ylabel('Annualised Return (%)');
legend({'Efficient frontier without risk-free asset', ...
    'Efficient frontier with risk-free asset'}, "Location","northwest");
title('Efficient Frontier');
xlim([0 18]);
ylim([0 24]);
hold off
```

10.8 TAA problem setup

The two TAA frameworks that were tested are the min-max framework and the optimisation framework. Here these two frameworks are implemented in MATLAB. As can be seen, the code is divided into different blocks. For the min-max framework a loop is run to calculate the minimum and maximum return vectors based on the forecasted monthly returns of the universe of assets. It then does the weighting allocation based on several checks which replicate the scenarios mentioned in Chapter 7.

The "elseif" statement shows where the optimisation framework starts. It does so by first establishing the upper and lower weight limits of each asset class and performs the optimisation process which determines weights, within the prescribed limits, which produces the portfolio with the highest possible return based on the forecasted monthly asset returns.

Lastly, the actual return of the TAA portfolios is calculated using the above mentioned weightings by multiplying it with the respective observed monthly asset returns.

```
% Import forecast data
forecastData = readtable(...
    ['C:\Users\reder\Desktop\TAA\' ...
    'FINAL SUBMISSION\Resubmission\Kalman - time varying beta NEW v5 extended NEW.xlsm'],
    "Sheet", "Returns forecast", "Range", 'D2:I152', "ReadVariableNames",true);
forecastData = removevars(forecastData, 'Var4');

% Exclude bonds from TAA, since divestment can only be made from equities
% at this stage and add all the alternative asset classes
forecastDataExclBonds = removevars(forecastData, 'Bonds');

assetClassesTAA = forecastDataExclBonds.Properties.VariableNames;

% Append the necessary dates
forecastDataExclBonds = [table(returnData.Dates), forecastDataExclBonds];
% forecastDataExclBondsInclCash = removevars(forecastDataExclBonds, 'RiskFreeRate');
forecastDataExclBondsInclCash = forecastDataExclBonds;
```

```
% Run a loop to calculate the maximum and minimum return vectors based on forecasted
% returns, but also create a vector for these names and observed data
for iTAA = 1 : length(forecastDataExclBondsInclCash{:, 1})
    maximumReturn(iTAA, 1) = max(forecastDataExclBondsInclCash{iTAA, ...
        assetClassesTAA(1, :)});
    maxAssetNameLogical(iTAA, :) = ...
        maximumReturn(iTAA,1) == forecastDataExclBondsInclCash{iTAA, ...
        assetClassesTAA(1, :)};
    maxAssetName(iTAA, 1) = assetClassesTAA(1, maxAssetNameLogical(iTAA, :));

    minimumReturn(iTAA, 1) = min(forecastDataExclBondsInclCash{iTAA, ...
        assetClassesTAA(1, :)});
    minAssetNameLogical(iTAA, :) = minimumReturn(iTAA, 1) == ...
        forecastDataExclBondsInclCash{iTAA, assetClassesTAA(1, :)};
    minAssetName(iTAA, 1) = assetClassesTAA(1, minAssetNameLogical(iTAA, :));
end
```

```
if ReferenceSAA == "Max Sharpe"
    targetRiskWeightsTAA = pwgtSharpeAll(strcmp('MSCIWorld', assetClasses(1, :)));
    targetRiskWeightsTAA = [targetRiskWeightsTAA; 1 - sum(pwgtSharpeAll); ...
        pwgtSharpeAll(end - 1: end, 1)];
    bondAllocation = pwgtSharpeAll(strcmp('Bonds', assetClasses(1, :)));

    % Use this code if your reference portfolio is the minimum variance
    % portfolio
elseif ReferenceSAA == "Min Variance"
    targetRiskWeightsTAA = pwgtMinVarAll(strcmp('MSCIWorld', assetClasses(1, :)));
    targetRiskWeightsTAA = [targetRiskWeightsTAA; 1 - sum(pwgtMinVarAll); ...
        pwgtMinVarAll(end - 1: end, 1)];
    bondAllocation = pwgtMinVarAll(strcmp('Bonds', assetClasses(1, :)));

    % Use this code if your reference portfolio is the maximum diversification
    % portfolio
elseif ReferenceSAA == "Max Diversification"
    targetRiskWeightsTAA = maxDiversificationWeightsAll(strcmp('MSCIWorld', ...
        assetClasses(1, :)));
    targetRiskWeightsTAA = [targetRiskWeightsTAA; maxDiversificationWeightsAll(end, 1); ..
        maxDiversificationWeightsAll(end - 2: end - 1, 1)];
    bondAllocation = maxDiversificationWeightsAll(strcmp('Bonds', assetClasses(1, :)));
end
 % Looks for asset being shorted
ShortIndex = find(targetRiskWeightsTAA < 0);

% Logical indexing for max asset class for which weight needs to be adjusted
% maxAssetName

maximumScenarioTAAReturn = maximumReturn(d3:d4);
minimumScenarioTAAReturn = minimumReturn(d3:d4);

% Setup the TAA framework, starting with new universe of assets, which
% includes equities, bonds (excluded previously) and whichever asset
% class delivers the highest return for that next month as per the forecasts
newUniverseOfAssetsReturnsForecast = forecastDataExclBonds{d3:d4, assetClassesTAA(1, :)};

TAAFactorSingle = targetRiskWeightsTAA(1) .* TAAPortion;
TAAFactor = repmat(TAAFactorSingle, [length(targetRiskWeightsTAA), 1]);
 if TAAMethod == "MinMax"
    % Use this section when not running optimization for weightings

    logicalAssetNamesTAA = maxAssetNameLogical' - minAssetNameLogical';
    minAssetNameLogical = minAssetNameLogical';
    maxAssetNameLogical = maxAssetNameLogical';

    for iMin = d3 : size(logicalAssetNamesTAA, 2)
        positionOfMinAsset(iMin, 1) = find(minAssetNameLogical(:, iMin), 1, 'first');
        positionOfMaxAsset(iMin, 1) = find(maxAssetNameLogical(:, iMin), 1, 'first');
        reductionFactor(iMin, 1) = TAAFactor(positionOfMinAsset(iMin));
        shortAsset = assetClassesTAA(1, ShortIndex);
```

```
        if ~isempty(shortAsset)
            if ismember(positionOfMinAsset(iMin, 1), ShortIndex) % Here the minimum
                %  return is that of an asset which is being shorted
                if length(shortAsset) == 2
                    if forecastDataExclBonds{iMin, ...
                            shortAsset{positionOfMinAsset(iMin) - 2}} < 0
                        logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = ...
                            logicalAssetNamesTAA(:, iMin) .* reductionFactor(iMin);
                    else
                        logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = 0;
                        % No additional leverage
                    end
                else
                    if forecastDataExclBonds{iMin, shortAsset{1}} < 0
                        logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = ...
                            logicalAssetNamesTAA(:, iMin) .* reductionFactor(iMin);
                    else
                        logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = 0;
                        % No additional leverage
                    end
                end
            elseif positionOfMinAsset(iMin, 1) == 2
                logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = 0;
            else
                logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = ...
                    logicalAssetNamesTAA(:, iMin) .* reductionFactor(iMin);
            end
        else
            % Min asset is cash which most probably has positive return and
            % positive weight allocation
            if positionOfMinAsset(iMin, 1) == 2
                logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = 0;
                % Other asset classes has extra layer of filtering
            elseif forecastDataExclBondsInclCash{iMin, positionOfMinAsset(iMin, 1) + 1} ...
                    > 0
                logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = 0;
            else
                logicalAssetNamesAndRF(1:size(assetClassesTAA, 2), iMin) = ...
                    logicalAssetNamesTAA(:, iMin) .* reductionFactor(iMin);
            end
        end
        TAAweightAll(1:size(assetClassesTAA, 2), iMin) = ...
            targetRiskWeightsTAA + logicalAssetNamesAndRF(:, iMin);

    end
```

```
    TAAweight = TAAweightAll(1:size(assetClassesTAA, 2), d3:d4);
    TAAweightTranspose = TAAweight';

elseif TAAMethod == "Optimization"
    % Use this section when running optimization
    targetRiskWeightsTAAUpper = targetRiskWeightsTAA + TAAFactor;
    targetRiskWeightsTAALower = targetRiskWeightsTAA - TAAFactor;

    upperShortTarget = targetRiskWeightsTAALower(ShortIndex, 1);
    lowerShortTarget = targetRiskWeightsTAAUpper(ShortIndex, 1);

    % Adjust for short sales
    targetRiskWeightsTAAUpper(ShortIndex, 1) = upperShortTarget;
    targetRiskWeightsTAALower(ShortIndex, 1) = lowerShortTarget;
    for iTAAweights = 1 : length(newUniverseOfAssetsReturnsForecast(:, 1))
        objFunTAA = @(TAAweights) -maximizeTAAReturns(TAAweights, ...
            newUniverseOfAssetsReturnsForecast(iTAAweights, :));
        x0_TAA = targetRiskWeightsTAA(:, 1);
        options_TAA = optimset('TolX', 1.0e-8);
        [TAAweight(:, iTAAweights), ~, exitflagTAA(iTAAweights)] = ...
            fmincon(objFunTAA, x0_TAA, [], [], ...
            ones(1, size(targetRiskWeightsTAA, 1)), sum(targetRiskWeightsTAA), ...
            targetRiskWeightsTAALower, targetRiskWeightsTAAUpper,...
            [], options_TAA);
    end
end
```

10.9 Calculating the necessary performance and risk metrics and some visualisations

This snippet of code briefly illustrates how some of the main performance and risk metrics were determined. This includes portfolio returns, cumulative returns, annual returns and risk, the calculation of Sharpe ratios, etc.

Graphs are constructed to show the distribution of monthly portfolio returns for both the SAA and TAA approaches. Boxplots are also determined for these returns by leveraging on the statistical and visualisation functionalities of MATLAB.

```
% Now use weights which were determined using the lookback
% period to calculate returns generated by those portfolios during the
% evaluation period, to compare with TAA
scale = 12; % months in a year

annualTAAReturn = (1 + meanTAA)^scale -1;
annualTAARisk = stdDevTAA*sqrt(scale);

minimumVarianceAllPortfolioReturns = sum(returnData{d3:d4, [assetClasses(1, :), ...
    'RiskFreeRate']} .* [pwgtMinVarAll; 1 - sum(pwgtMinVarAll)]', 2);
minimumVarianceAllPortfolioAnnualReturn = ...
    (1 + mean(minimumVarianceAllPortfolioReturns))^scale - 1;
minimumVarianceAllPortfolioAnnualRisk = ...
    std(minimumVarianceAllPortfolioReturns)*sqrt(scale);

tangentPortfolioReturn = sum(returnData{d3:d4, ...
    [assetClasses(1, :), 'RiskFreeRate']} .* [pwgtSharpeAll; 1 - sum(pwgtSharpeAll)]', 2);
tangentPortfolioAnnualReturn = (1 + mean(tangentPortfolioReturn))^scale - 1;
tangentPortfolioAnnualRisk = std(tangentPortfolioReturn)*sqrt(scale);
```

```
maximumDiversificationPortfolioReturns = returnData{d3:d4, [assetClasses(1, :), ...
    'RiskFreeRate']} * maxDiversificationWeightsAll;
maxDivPortfolioAnnualReturn = ...
    (1 + mean(maximumDiversificationPortfolioReturns))^scale - 1;
maxDivPortfolioAnnuallRisk = std(maximumDiversificationPortfolioReturns)*sqrt(scale);

% Sharpe ratio
minimumVariancePortfolioSharpe = ...
    sharpe(minimumVarianceAllPortfolioReturns, returnData{d3:d4, 'RiskFreeRate'});
maxDivSharpe = ...
    sharpe(maximumDiversificationPortfolioReturns, returnData{d3:d4, 'RiskFreeRate'});
tangentSharpe = sharpe(tangentPortfolioReturn, returnData{d3:d4, 'RiskFreeRate'});
TAASharpe = sharpe(TAAPortfolioReturns, returnData{d3:d4, 'RiskFreeRate'});
```

```
minimumVarianceAlpha = portalpha(minimumVarianceAllPortfolioReturns, ...
    returnData{d3:d4, 'MSCIWorld'}, returnData{d3:d4, 'RiskFreeRate'});
maxDiversificationAlpha = portalpha(maximumDiversificationPortfolioReturns, ...
    returnData{d3:d4, 'MSCIWorld'}, ...
    returnData{d3:d4, 'RiskFreeRate'});
tangentAlpha = portalpha(tangentPortfolioReturn, returnData{d3:d4, 'MSCIWorld'},...
    returnData{d3:d4, 'RiskFreeRate'});
TAAAlpha = portalpha(TAAPortfolioReturns, returnData{d3:d4, 'MSCIWorld'}, ...
    returnData{d3:d4, 'RiskFreeRate'});
```

```
figure
hold on
plot(returnData.Dates(d3:d4), (cumprod(1 + minimumVarianceAllPortfolioReturns) - 1)...
    *100, 'Color','red');
plot(returnData.Dates(d3:d4), (cumprod(1 + maximumDiversificationPortfolioReturns) - 1)...
    *100, 'Color', 'cyan');
plot(returnData.Dates(d3:d4), (cumprod(1 + tangentPortfolioReturn) - 1)*100, ...
    'Color', 'green');
plot(returnData.Dates(d3:d4), (cumprod(1 + TAAPortfolioReturns) - 1)*100, ...
    'Color','black');
title("Cumulative monthly returns for different asset allocation strategies")
legend({'Minimum variance', 'Minimum variance all asset included', 'TAA'}, ...
    'Location','southoutside');
xlabel('Dates')
ylabel('Cumulative monthly returns')
hold off
hold off
```

```
figure
hold on
boxplot(((1 + returnSetupBox).^12 - 1).*100, portfolioOrigin);
ylabel('Annualised Returns (%)')
hold off

figure
hold on
histfit(TAAreturns);
xlabel('Monthly TAA Medium Equity returns');
ylabel('Frequency');
hold off

figure
hold on
histfit(SAAreturns);
xlabel('Monthly SAA Medium Equity returns');
ylabel('Frequency')
hold off
```

References

[1] Ammann, M. and Zimmermann, H., 2001. Tracking error and tactical asset allocation. *Financial Analysts Journal*, 57(2), pp.32-43.

[2] Ang, A. and Bekaert, G., 2004. How regimes affect asset allocation. *Financial Analysts Journal*, 60(2), pp.86-99.

[3] Anson, M.J., 2004. Strategic versus tactical asset allocation. *The Journal of Portfolio Management*, 30(2), pp.8-22.

[4] Arnold, T., Bertus, M.J. and Godbey, J., 2008. A simplified approach to understanding the Kalman filter technique. *The Engineering Economist*, 53(2), pp.140-155.

[5] Assoé, K., L'Her, J.F. and Plante, J.F., 2006. The relative importance of asset allocation and security selection. *The Journal of Portfolio Management*, 33(1), pp.46-55.

[6] Blitz, D.C. and van Vliet, P., 2008. Global tactical cross-asset allocation: applying value and momentum across asset classes. *The Journal of Portfolio Management*, 35(1), pp.23-38.

[7] Brennan, M.J., Schwartz, E.S. and Lagnado, R., 1997. Strategic asset allocation. *Journal of Economic Dynamics and Control*, 21(8-9), pp.1377-1403.

[8] Brinson, G.P., Hood, L.R. and Beebower, G.L., 1986. Determinants of portfolio performance. *Financial Analysts Journal*, 42(4), pp.39-44.

[9] Brockwell, P.J., Davis, R.A. and Calder, M.V., 2002. *Introduction to time series and forecasting,* 2, pp. 3118-3121. New York: Springer.

[10] Burmeister, E., Wall, K.D. and Hamilton, J.D., 1986. Estimation of unobserved expected monthly inflation using Kalman filtering. *Journal of Business & Economic Statistics*, 4(2), pp.147-160.

[11] Campbell, J.Y. and Shiller, R.J., 1988. Stock prices, earnings, and expected dividends. *The Journal of Finance*, 43(3), pp.661-676.

[12] Clewell, D., Faulkner-Macdonagh, C., Giroux, D., Page, S. and Shriver, C., 2017. Macroeconomic dashboards for tactical asset allocation. *The Journal of Portfolio Management*, 44(2), pp.50-61.

[13] Corporate Finance Institute, 2020. Maximum drawdown: A measure of the maximum fall in the value of the investment. [Online] Available at: https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/maximum-drawdown/ [Accessed July 15 2020].

[14] Dahlquist, M. and Harvey, C. R., 2005. Global tactical asset allocation. [Online] Available at: https://ssrn.com/abstract=795376 [Accessed 16 June 2020].

[15] Dowd, K., 2003. *An introduction to market risk measurement*. John Wiley & Sons..

[16] Drobetz, W. and Köhler, F., 2002. The contribution of asset allocation policy to portfolio performance. *Financial Markets and Portfolio Management*, 16(2), pp.219-233.

[17] EDHEC-Risk Institute. 2020. Indices for investment solutions. [Online] Available at: https://risk.edhec.edu/indices-investment-solutions#tab_374 [Accessed 6 June 2020].

[18] Estrada, J., 2009. Black swans, market timing and the Dow. *Applied Economics Letters*, 16(11), pp.1117-1121.

[19] Faber, M., 2017. A quantitative approach to tactical asset allocation revisited 10 years later. *The Journal of Portfolio Management*, 44(2), pp.156-167.

[20] Faff, R.W., Hillier, D. and Hillier, J., 2000. Time varying beta risk: An analysis of alternative modeling techniques. *Journal of Business Finance & Accounting*, 27(5-6), pp.523-554.

[21] Fama, E.F. and French, K.R., 1989. Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics*, 25(1), pp.23-49.

[22] Fama, E. F. and French, K. R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), pp. 3-56.

[23] Fama, E.F. and French, K.R., 2004. The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives*, 18(3), pp.25-46.

[24] Fama, E.F. and French, K.R., 2015. A five-factor asset pricing model. *Journal of Financial Economics*, 116(1), pp.1-22.

[25] Federal Reserve Bank of St Louis: Economic Data, 2020. 3-Month treasury bill: Secondary market rate. [Online] Available at: https://fred.stlouisfed.org/series/TB3MS [Accessed 6 June 2020].

[26] Flint, E. and Maré, E., 2019. Regime-based tactical allocation for equity factors and balanced portfolios. *South African Actuarial Journal*, 19(1), pp.27-52.

[27] FTSE Russell, 2020. Global fixed income indexes. [Online] Available at: https://www.ftserussell.com/products/indices/global-fixed-income-indexes [Accessed 2020 July 15]

[28] Goodwin, T.H., 1998. The information ratio. *Financial Analysts Journal*, 54(4), pp.34-43.

[29] Grewal, M.S., Henderson, V.D. and Miyasako, R.S., 1990. Application of Kalman filtering to the calibration and alignment of inertial navigation systems. *29th IEEE Conference on Decision and Control*, pp. 3325-3334.

[30] Grinold, R.C., 1989. The fundamental law of active management. *The Journal of Portfolio Management*, 15(3), pp.30-37.

[31] Grinold, R. C. and Kahn, R. N., 2000. *Active portfolio management: a quantitative approach for providing superior returns and controlling risk*. 2nd edn. New York: McGraw-Hill.

[32] Hafner, C. M., 2020. Alternative assets and cryptocurrencies. *Journal of Risk and Financial Management*, 13, pp.1-7.

[33] Hasler, M. and Martineau, C., 2019. Does the CAPM Predict Returns?. [Online] Available at: https://ssrn.com/abstract=3368264 [Accessed 30 May 2020].

[34] Ibbotson, R.G., 2010. The importance of asset allocation. *Financial Analysts Journal*, 66(2), pp.18-20.

[35] Ibbotson, R.G. and Kaplan, P.D., 2000. Does asset allocation policy explain 40, 90, or 100 percent of performance?. *Financial Analysts Journal*, 56(1), pp.26-33.

[36] Israelsen, C.L. and Cogswell, G.F., 2007. The error of tracking error. *Journal of Asset Management*, 7(6), pp.419-424.

[37] Jackwerth, J.C. and Slavutskaya, A., 2016. The total benefit of alternative assets to pension fund portfolios. *Journal of Financial Markets*, 31, pp.25-42.

[38] Jensen, M.C., 1968. The performance of mutual funds in the period 1945-1964. *The Journal of Finance*, 23(2), pp.389-416.

[39] Jorion, P., 2003. Portfolio optimization with tracking-error constraints. *Financial Analysts Journal*, 59(5), pp.70-82.

[40] JP Morgan, 1996. RiskMetrics technical document. [Online] Available at: https://www.msci.com/documents/10199/5915b101-4206-4ba0-aee2-3449d5c7e95a [Accessed 19 July 2020].

[41] JP Morgan Asset Management, 2010. Next generation alternative investing. [Online] Available at: https://www.jpmorgan.com/cm/BlobServer/Next_Generation_Alternative_Investing.pdf. [Accessed 19 June 2020].

[42] JP Morgan Asset Management, 2020. Impact of being out of the market. [Online] Available at: https://am.jpmorgan.com/us/en/asset-

management/gim/adv/impact-of-being-out-of-the-market [Accessed 1 August 2020].

[43] Kirubarajan, T., Bar-Shalom, Y. and Wang, Y., 2001. Passive ranging of a low observable ballistic missile in a gravitational field. *IEEE transactions on Aerospace and Electronic Systems*, 37(2), pp.481-494.

[44] Krakiwsky, E.J., Harris, C.B. and Wong, R.V., 1988. A Kalman filter for integrating dead reckoning, map matching and GPS positioning. *IEEE PLANS'88., Position Location and Navigation Symposium, Record.'Navigation into the 21st Century'*, pp.39-46.

[45] Lee, W., 2000. *Theory and methodology of tactical asset allocation*, 65. John Wiley & Sons.

[46] Maggie M. and Copeland, T. E., 1999. Market timing: Style and size rotation using the VIX. *Financial Analysts Journal*, 55(2), pp.73–81.

[47] Markowitz H. M., 1952a. Portfolio selection. *Journal of Finance*, 7(1), pp.77–91.

[48] MSCI, 2020. End of day index data search. [Online] Available at: https://www.msci.com/end-of-day-data-search [Accessed 6 June 2020].

[49] Nystrup, P., Hansen, B.W., Madsen, H. and Lindström, E., 2016. Detecting change points in VIX and S&P 500: A new approach to dynamic asset allocation. *Journal of Asset Management*, 17(5), pp.361-374.

[50] Perold, A.F., 2004. The capital asset pricing model. *Journal of Economic Perspectives*, 18(3), pp.3-24.

[51] Persson, C. and Williams, R., 2020. Quantitative tactical asset allocation: Using the VIX to exploit bull and bear market movements in a mean-variance portfolio. [Online] Available at: https://gupea.ub.gu.se/bitstream/2077/65567/1/gupea_2077_65567_1.pdf [Accessed 6 June 2020].

[52] Philips, T.K., Rogers, G.T. and Capaldi, R.E., 1996. Tactical asset allocation: 1977-1994. *Journal of Portfolio Management*, 23(1), p.57.

[53] PricewaterhouseCoopers, 2020. Alternative asset management 2020: Fast forward to centre stage. [Online] Available at: https://www.pwc.com/gx/en/industries/financial-services/asset-management/publications/alternative-asset-management-2020.html [Accessed 20 June 2020].

[54] Rossi, M., 2016. The capital asset pricing model: A critical literature review. *Global Business and Economics Review*, 18(5), pp.604-617.

[55] Schmidt, S.F., 1981. The Kalman filter - Its recognition and development for aerospace applications. *Journal of Guidance and Control*, 4(1), pp.4-7.

[56] Schneider, C., 2009. *How Useful is the Information Ratio to Evaluate the Performance of Portfolio Managers?* Diplomica Verlag.

[57] Schwartz, E.S., 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. *The Journal of Finance*, 52(3), pp.923-973.

[58] Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3), pp.425-442.

[59] Sharpe, W.F., 1966. Mutual fund performance. *The Journal of Business*, 39(1), pp.119-138.

[60] Sharpe, W.F., 1975. Adjusting for risk in portfolio performance measurement. *The Journal of Portfolio Management*, 1(2), pp.29-34.

[61] Sharpe, W.F., 1992. Asset allocation: Management style and performance measurement. *Journal of Portfolio Management*, 18(2), pp.7-19.

[62] Shtekman, A. and Stockton, K., 2010. A primer on tactical asset allocation strategy evaluation. [Online] Available at: https://www.vanguard.co.uk/documents/adv/literature/tactical-asset-allocation.pdf [Accessed June 15 June 2020].

[63] Statman, M., 2001. How important is asset allocation?. *Journal of Asset Management*, 2(2), pp.128-135.

[64] S&P Dow Jones Indices, 2020. Indices overview. [Online] Available at: https://www.spglobal.com/spdji/en/indices/commodities/sp-gsci/#overview [Accessed 21 June 2020].

[65] Taleb, N.N., 2007. *The black swan: The impact of the highly improbable*, 2. Random house.

[66] Theron, L. and van Vuuren, G., 2018. The maximum diversification investment strategy: A portfolio performance comparison. *Cogent Economics & Finance*, 6(1), pp.1-16.

[67] Thomson, D.B., 2016. Hedge fund return attribution and performance forecasting using the Kalman filter (Masters dissertation, North-West University (South Africa), Potchefstroom Campus).

[68] Thomson, D. and van Vuuren, G., 2018. Attribution of hedge fund returns using a Kalman filter. *Applied Economics*, 50(9), pp.1043-1058.

[69] Treynor, J., 1965. How to rate the performance of investment funds. *Harvard Business Review*, 41, pp.63-75.

[70] Vorlow, C.E., 2017. Simple tactical asset allocation strategies on the S&P 500 and the impact of VIX fluctuations. *Handbook of Investors' Behavior During Financial Crises*, pp.383-400. Academic Press.

[71] Yahoo Finance, 2020. [Online] Available at: https://finance.yahoo.com/ [Accessed 7 June 2020].