Economic return quantity model for a multi-type empty container management with possible storage constraint and shared cost of shipping

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Containerisation has been said to be one of the most significant innovations of port management. Some ports usually tend to be more export driven and others import driven. This imbalance in the number of inbound and outbound containers necessitates movement of containers from locations or surplus to those of deficit. A port needs to cater for not only the level of containers required, but also the variety. Many factors affect the cost efficiency of container movement, and hence, that of port operations and these need to be managed in an integrated manner to optimise cost in a port.

We studied the management of a multi-type container system in a port where there can be savings from joint movement of containers, both empty return and procurement of replacement containers, and where there is limited storage capacity with storage cost. We developed a model to determine the optimal integrated return and purchase cycles and quantities. We illustrated the solution approach with a numerical example and performed sensitivity analysis. We believe this problem is rife and this model can guide such management of container return and replenishment in ports operations and management.

Keywords: empty container management, container repositioning, lot sizing, return management, economic order Quantity (EOQ)

1. Introduction

Globalisation has changed the nature of trade, production and transport in the world and containerisation has been a major driver of the modern trade flows. The Lloyd Marine intelligence report of 2009 stated that about 75 percent by volume and 60 percent by value of the global trade is done by sea while 52 percent of cargoes shipped by sea are carried through containers (Lee and Song, 2017). These container movements are said to be generally imbalance as some ports tend to send a lot more containers than they receive while some receive more than they send. This gap is said to be driven by production regionalisation and globalisation of trade (Yu et al, 2018), and this trend seems to be continuing into the near future. This creates a general gradient for flow of containers and there is usually the need to create a reverse flow for such imbalance. Song and Dong (2011) actually stated that about 20 percent of container movements is that of empties. This is a massive cost given the value of the global trade using containers. They stated that the drivers of the massive container imbalance include factors like trade imbalance, dynamic behaviour, uncertainty in demands/handling/transportation, types of equipment, blind spots in the transport chain, and a carrier's operational and strategic practice.

Trade imbalance is about the single most important of these factors, but planning issues are also contributory. Chen et al (2016) noted that there is currently no alternative to the long distance movement of goods across the globe. This makes it imperative for those in the transport industry to continue to find means to reduce the cost of managing this consequence of the dynamics of the global manufacturing and trading pattern which is not about to change any time soon. This has created the challenge of repositioning and replenishing containers in order to continue to meet the demand for its use. There is the need to continue to find data guided policies to minimise the cost of container repositioning and recovery because in a very competitive global industry, it is becoming more and more difficult to recoup the cost of Empty Container Repositioning (ECR) as cargoes move along the long global transport routes.

The challenge of managing ECR is not only limited to that between ports across many different countries, but also between the port and the hinterland. International shipping permits the specification of the term of shipment via the incoterm, and this can place different scopes of liability on the shipper and the carrier. This position is actually bolstered through the adoption of inter-modal transport, making it possible for the carrier to not only move containers hinterland, but also plan the possible return. The process is then said to involve carrier haulage (Lee and Song, 2017).

Other issues could complicate this problem further. One such issue is container attrition. This is because containers are usually lost or damaged or even converted to some other uses during its life cycle such that some of the containers that have been moved hinterland may not return to the port or may return in an unusable state. The shortage may also be dynamic in nature. This means even if there is a relative balance between the import and export demand rate for containers, there could be the challenge of alignment between the times of need such that the containers coming from import might not be available to be used for export at the appropriate time. These may necessitate the procurement of new containers to augment those in circulation while also repositioning empty containers in use.

Another complicating issue is guaranteeing the appropriate mix of containers available at the point of use. There are different types of containers: twenty-foot equivalent units (TEU) forty-foot equivalent units (FEU), other specialised container types like refrigerated containers, and possible combinations of these categories. This means it is not sufficient to have enough number of containers that could meet the aggregate demand, but also the appropriate mix to meet the demand for each type of container in appropriate quantities.

Empty Container (EC) repositioning is about moving containers around from areas of over-supply to areas of under supply. Such movement may include moving containers from the port to some locations hinterland and moving from hinterland to the port such that the container is being used and re-used. There may also be space constraint at the port. This may be because of growth in economy without a concomitant expansion of the port. The port, hence, charges some money for each day a container spends in the port, and the charges may also depend on the type of container stored.

The mode and contract of shipment of containers are other factors that might make container management and repositioning more difficult. While many private users may simply use trucks to move containers on road because of the relatively low number of usage and truck accessibility, it is usually not cheap to use such mode for mass movement of containers for repositioning. It pays to consolidate container shipment and move higher volume in order to reduce the cost of shipment of containers. For instance, a common carrier may provide a train head to drive a given number of containers at a fixed cost and charge some marginal cost for each of the container types and quantities moved by the head. If the shipper moves each type separately, s/he pays the fixed cost separately for each of the container types as opposed to the containers sharing this fixed cost.

The quantity and mix problem discussed are important for both the repositioned and newly procured containers. There is, hence, the need to determine the optimum number of containers to reposition and/or purchase per cycle as well as the length of the cycle and the mix of each type of container in each repositioning or replenishment cycle. This is a lot-sizing problem with integrated reverse logistics in which there are multiple types of containers, constraint on the storage capacity and economy of scale on the joint shipment. While there have been models of reverse logistics presented since

Schrady (1967), the complexities of the container repositioning discussed herein seems not well addressed, and hence the need for this work. A tactical decision to make during container repositioning is the determination of the shipment size and interval. Work has continued in this area and many of this draw from the field of repairable inventory items and reverse logistics.

The seminal work for lot sizing of repairable inventory item seems to have been Shrady (1967) and many lot-sizing models of reverse logistics seem to have extended this model. He developed a model of a system consisting of two inventory items: one in a Ready for Issue (RFI) state, and the other being in a Non Ready For Issue (NRFI) state. He discussed two repair policies called continuous supplement and substitution policies. He favoured the substitution policy in that it holds the NRFI items longer because they are much cheaper in overall cost. He ordered the cycles such that it uses the RFI items first, followed by the NRFI items and developed the optimal lot sizes for both items. He assumed that one manufacturing batch follows R recovery batches and that there would be no disposals from return items.

Mambini et al (1992) presented a multi item repairable inventory system with shared repair capacity and under a restriction on the maximum shortage period allowed. Richter (1996 a,b) extended Schrady's model solution into spaces where the search for solution considers multiples of repairs per procurement and multiple procurements per return, arguing it should produce better results than Shrady's. Teunter (2001) is another interesting work considering cases of return, repair and reuse where a certain number of the batch returned is discarded and formulated policies considering (1,R) and (R,1) policies and the optimum batch sizing under these conditions. He considered cases where R recovery batches follows M manufacturing batches as a generalisation of Schrady's as well. Other areas that have been explored in recoverable inventory include time varying demand, quality dependent recovery, and recovery with inspection and sorting, among others. Guide and Srivastava (1997) provided a good review of recoverable inventory items. They classified all models presented as single- or multi-echelon problems with various sub-classifications under each main category.

Modelling reverse logistics is an area that has gained pre-eminence because of the global concern for environmental degradation. Many governments proposed laws and institutions developed best practices for including environmental factors in logistics. Many lot-sizing models in this area have also referenced Schrady and its sequels. They considered lot-sizing problems incorporating various return strategies like remanufacturing, reuse, recycling and even disposal. Some such models incorporated costs for environmental degradation like Green House Gas (GHG) emission, environmental pollution and others. Fleischmann et al (1997) provides a comprehensive review of reverse logistics models up until about 1997 while Bazan, Jaber and Zanonib (2016) provides a review of subsequent papers. Thierry (1995) provides a comprehensive framework that many authors have used to classify all forms of recycling activities and has become a basis for classifying return logistics models. Among authors that have considered the effect of limited capacity storage on lot-sizing decisions is Ghosh, Chakar and Chaoudri (2015). They presented a model for deteriorating items with stock dependent demand and limited storage space.

2. Model formulation

We first derive the model with different holding costs for new and return items and then progress to a case where these are the same, after which we consider a model with economy via sharing of fixed shipping cost. Consider a container management system of a port where there are different types of containers used therein. Each of these types of containers is moved from a port to another (or hinterland). After use, the container is to be returned to the port. Not all the containers sent out return to the port for re-use because some of the containers become unusable (or lost) and need to be replaced and the lost (or damaged) containers are replaced through procurement.

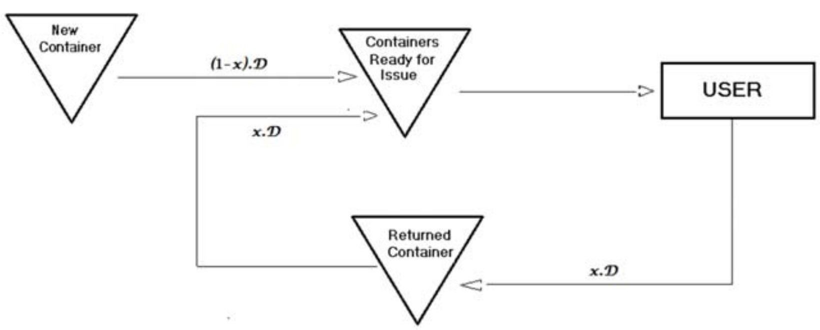

Figure 1: Schematics of a container return system

The ports authority may ship the containers (new or return) to the port in fixed batches. The port management can choose to ship different types of containers (return or procured) together or ship each type individually. If they ship a type of container, there is a fixed cost for each such shipment. If they ship different types together, there is a base fixed cost, and then marginal add-on for each type of containers shipped together (e.g. a case of a train head pulling many containers that could be of different types). This leads to possible economy of scale, and is partly an incentive from the carrier to procure high quantity of shipment. There is, however, also a restriction on the space available for storage of containers at the port. This space must be judiciously allocated among all the different types of containers to ensure that both the right aggregate quantities and varieties are available when needed.

In this study, we present a lot sizing model to determine the optimum number of containers to reposition for re-use, as well as the optimum number of new containers to order from vendor as a replacement. In addition, the optimum cycle time for collection and purchase as well as the integrated number of return and procurement cycles for such replenishments is determined. Most assumption would be considered obvious, and where necessary, such would be stated.

We define j as an index for types of containers managed at the port, $j = 1,2, \ldots n$, r as a subscript denoting returned containers, p as a subscript denoting purchased containers, D_j as the annual demand rate for container type j and x_j as the proportion of container type j returned to the port from points of use. For the fixed costs, K_{rj} is the fixed cost of returning a batch of useful container type j from point of use and K_{pj} the fixed cost of ordering a new batch of container type j to make up for lost or damaged containers.

The cost per unit per year of keeping a returned container type j is h_{rj}, and h_{pj}, the cost per unit per year of keeping a newly purchased container type j (this would later be modified during modelling). For cycle times, T_j is the return cycle time for container type j, T_j^* is the optimum return cycle time for container type j, T is a common return cycle time for all containers while T^* is the optimum common return cycle time for all containers. Also, m_j is the number of container return cycles per single cycle of procurement for a container type j (this assumption would later be modified during modelling) while C is the capacity of storage of all types of containers in the port terminal. The storage space requirement for a unit of container type j is defined as s_j and λ the Lagrange multiplier for a constraint function. Q_{rj}^* is the optimum return quantity for container type j while Q_{pj}^* is the optimum replenishment order quantity for container type j.

Figure 2 is the quantity-time graph of the inventory level of containers in an inventory management system for a single type of containers. We have adopted the modelling paradigm of Koh et al (2002) wherein there is simultaneous consumption of both the purchased and repaired containers as opposed to Schrady's complementary or substitution policies because the container consumption process seems more like Koh's. The thick line is the graph of the returned containers position and the thin line, that of the new containers. In this example, there are m cycles of returned containers for every new container cycle. Also, the norm is that the return rate of the container is usually close to 1, and this is without any loss of generality for the model development. In general, it would be assumed initially in the model derivation that each container type, j, may have a separate number of return cycle, m_j, per single procurement cycle. This would later be modified for all types of containers to have a common cycle, m, per return so we can take advantage of economy of scale due to shared fixed cost in purchase and/or return.

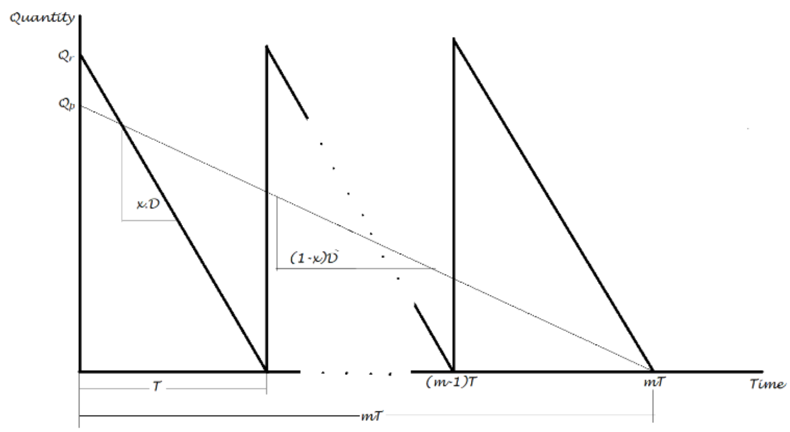

Figure 2: Quantity-time graph for a single container system with different cycles

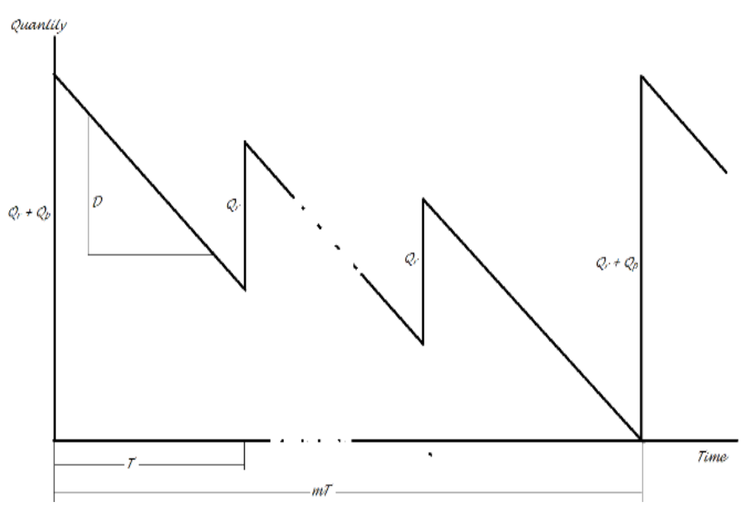

Figure 3: Aggregate inventory level for a single container type

Figure 3 shows the aggregate inventory level position for a single container type with consumption occurring simultaneously from both the returned and procured stocks. This can be generalised for a multi-container system. To derive the model, we start by considering the single item case of Figure 2 for the return and replenishment containers. We progress to derive the cost function for a system with n different types of containers in which there is a constraint on the storage space available. In deriving the model, a general case in which all items are ordered separately, return cycles per procurement cycle are different and the holding costs for new and recycled containers are different is first presented, from which particular cases of shared ordering cost and constant holding cost rates are then derived.

Consider a container management system in which there is a container type j with both return and procurement cycles. The total cost of this system consists of the costs of managing the returned containers and those of managing the procurement of the new containers. The cycle cost of managing returned containers (excluding purchase cost since it is independent of Q) is shown in 1 while the cycle cost or managing new containers (excluding purchase cost) is shown in 2.

$$\frac{x_j D_j K_{rj}}{Q_{rj}} + \frac{Q_{rj} h_{rj}}{2} \tag{1}$$

$$\frac{(1 - x_j) D_j K_{pj}}{Q_{pj}} + \frac{Q_{pj} h_{pj}}{2} \tag{2}$$

In order to be able to integrate the cycles for the return and procurement sub-systems, we chose to work on their cycle times. We would then retrieve the equivalent optimal lot-sizes (quantities) for the return and procurement systems from these optimum cycle times since T and Q are jointly determined. The quantities, Q, in the cost functions for the return, the procurement and the total system costs can be written in terms of their cycle times as follows:

$$Q_{rj} = x_j D_j T_j \tag{3}$$

$$Q_{pj} = m_j \left(1 - x_j\right) D_j T_j \tag{4}$$

$$Q_j = [x_j + m_j (1 - x)] D_j T_j \tag{5}$$

Substituting 3 and 4 into 1 and 2 respectively, we have 6 and 7. The total cost of managing n-type container system is shown in 8. Considering there is possible space limitation and it may be impossible to store all containers if the combined return quantity and order quantity is too large, the space constraint for all containers managed can be written as 9, and from 5 and 9, we have 10.

$$\frac{K_{rj}}{T_j} + \frac{T_j x_j D_j h_{rj}}{2} \tag{6}$$

$$\frac{K_{pj}}{m_j T_j} + \frac{m_j T_j (1 - x_j) D_j h_{pj}}{2} \tag{7}$$

$$\sum_{j=1}^{n} \left[\frac{K_{rj}}{T_j} + \frac{T_j x_j D_j h_{rj}}{2} + \frac{K_{pj}}{m_j T_j} + \frac{m_j T_j (1 - x_j) D_j h_{pj}}{2} \right] \tag{8}$$

$$\textstyle\sum_{j=1}^{n} Q_j s_j \leq C \tag{9}$$

$$\sum_{j=1}^{n} T_j D_j s_j \left[x_j + m_j \left(1 - x_j\right) \right] \leq C \tag{10}$$

The problem becomes minimising 8 subject to 10. To solve this problem, we observe that it is either 10 is binding or not. If 10 is not binding, then we may simply drop it and solve 8. This is illustrated in Figure 4 showing the total cost function for a single container case, i.e. $n = 1$, with space constraint. If the constraint is binding, then the capacity limit, C, is to the left of the optimum quantity, Q_j, as shown, rendering the optimum quantity, Q_j, infeasible and hence, making C the best quantity to select since the cost function is convex in Q.

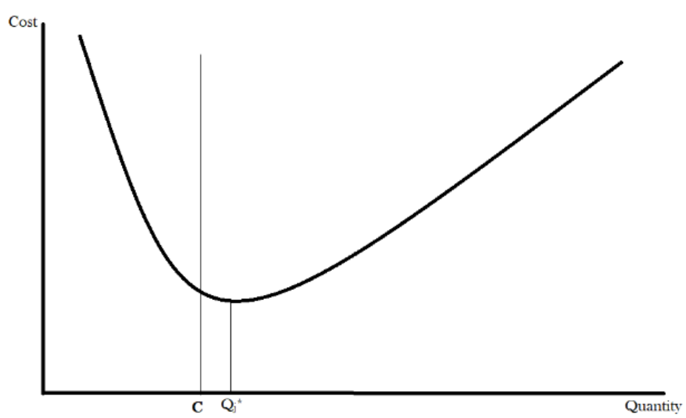

Figure 4: Behaviour of optimal cost under storage capacity constraint

In a multi-type container system where the constraint is not binding, we partially differentiate with respect to T_j to obtain the optimal cycle time, T_j, for each item, j, we have 10. If the returned container is considered as good as new and their holding cost is the same, then 11 becomes 12.

$$T_j^* = \sqrt{\frac{2\sum_{j=1}^{n}(K_{rj} + \frac{K_{pj}}{m_j})}{\sum_{j=1}^{n} D_j(x_j h_{rj} + m_j(1-x_j)h_{pj})}} \qquad (11)$$

$$T_j^* = \sqrt{\frac{2\sum_{j=1}^{n}(K_{rj} + \frac{K_{pj}}{m_j})}{\sum_{j=1}^{n} D_j h_j[x_j + m_j(1-x_j)]}} \qquad (12)$$

We may obtain the optimum lot size of containers to return or purchase from 3 and 4 respectively. Equation 11 (or 12 as appropriate) yields n equations. Since each m_j is also an unknown, each of these n equations of T_j can be solved iteratively for the optimum m_j and $T_j$value for each j. In a case where the constraint is binding, we may transform the inequality in 10 into an equation using the Lagrange multiplier, λ, and rewrite equations 8 with 10 as the Lagrangean function

$$\sum_{j=1}^{n}\frac{1}{T_j}(K_{rj} + \frac{K_{pj}}{m_j}) + \left[\sum_{j=1}^{n}\frac{D_j T_j}{2}\left[x_j\left(h_{rj} + 2\lambda s_j\right) + m_j(1-x_j)(h_{pj} + 2\lambda s_j)\right]\right] - \lambda C \qquad (13)$$

Partially optimising 13 with respect to T_j for each j yields 14. Again, if the returned container is considered as good as new and their holding costs taken as the same, equation 14 simplifies to 15.

$$T_j^* = \sqrt{\frac{2\sum_{j=1}^{n}(K_{rj} + \frac{K_{pj}}{m_j})}{\sum_{j=1}^{n} D_j\left[x_j\left(h_{rj} + 2\lambda s_j\right) + m_j(1-x_j)(h_{pj} + 2\lambda s_j)\right]}} \qquad (14)$$

$$T_j = \sqrt{\frac{2\sum_{j=1}^{n}(K_{rj} + \frac{K_{pj}}{m_j})}{\sum_{j=1}^{n} D_j\left[\left(h_j + 2\lambda s_j\right)[x_j + m_j(1-x_j)]\right]}} \qquad (15)$$

2.1. Ordering/Collection cost economy of scale consideration

If there is economy of scale for joint return of different types of container, then we seek to have a common optimal cycle, m, for all containers such that $m_j = m \; \forall \; j$. Let us say each time an order is placed, there is a fixed portion of ordering cost to which some marginal cost of order is added for each container type collected (or purchased).

There will be cost savings because of shared fixed order cost when multiple containers are moved together (return or purchase cycle).

We define K'_{rj} as the marginal increase in fixed cost of returning a batch of container type j from point of use to the port and K'_{pj} the marginal increase in fixed cost of ordering a new batch of container type j to make up for lost or damaged containers. K_r^f is the fixed portion of the cost of returning a batch of useful container type j from point of use to the port, whether single or multiple types of containers are involved. K_p^f is the fixed portion of the cost of ordering a new batch of container type j to make up for lost or damaged containers whether single or multiple types are involved. We can re-write the ordering costs of return and purchase for each container type in the form 16 and 17. If each of the items are returned and purchased independently, the total ordering cost per cycle return for all items would be 18. If we take advantage of the economy of joint return and purchase, then $n = 1$ for the fixed ordering cost portion and 18 becomes 19.

$$K_{rj} = K_r^f + K'_{rj} \tag{16}$$

$$K_{pj} = K_p^f + K'_{pj} \tag{17}$$

$$\sum_{1}^{n}(K_{rj} + \frac{K_{pj}}{m}) = n\left(K_r^f + K_p^f\right) + \sum\nolimits_{j=1}^{n}\left(K'_{rj} + K'_{pj}\right) \tag{18}$$

$$\sum_{1}^{n}(K_{rj} + \frac{K_{pj}}{m}) = \left(K_r^f + K_p^f\right) + \sum\nolimits_{j=1}^{n}\left(K'_{rj} + K'_{pj}\right) \tag{19}$$

It can be seen that 19 would always be less than 18 for any $n > 1$. We would, however, need to check that this saving is not outweighed by the cost of storing the containers, especially because this can be aggravated by the effect of storage constraint. This is because joint shipment may lead to a higher maximum inventory level (I_{max}) in the system since all containers arrive at the same time. This makes it quite easy to sub-optimise consequent to the increased cost of holding container inventory, especially as the constraint becomes binding. This means the savings in joint collection and purchase needs to be weighed against this possible additional cost of holding stock.

From 11, the optimal common cycle time (h now assumed equal) becomes 20. If the constraint is also binding with the economy of scale present, and since 10 also holds true and now becomes an equality, we may make T the subject from 10 as in 21.

$$T^* = \sqrt{\frac{2\left[\left(K_r^f + K_p^f\right) + \sum_{j=1}^{n}\left(K'_{rj} + K'_{pj}\right)\right]}{\sum_{j=1}^{n} D_j h_j\left[x_j + m_j\left(1 - x_j\right)\right]}} \tag{20}$$

$$T_j = T = \frac{C}{\sum D_j s_j\left[x_j + m_j\left(1 - x_j\right)\right]} \tag{21}$$

Solving 21 and 14 together for any j yields the equations to calculate the value of λ as in 22. When the holding cost is identical, it becomes 23. It can be seen from 22 and 23 that we cannot factorise λ completely, hence, we have to solve for λ iteratively. We may also use the optimal value(s) of m ($m_j{}'s$) obtained from solving the unconstrained problem to find the appropriate value of λ.

$$\frac{2}{C^2}\sum_{1}^{n}\left(K_{rj} + \frac{K_{pj}}{m_j}\right) = \frac{\sum_{j=1}^{n} D_j\left[x_j h_{rj} + m_j\left(1 - x_j\right)h_{pj} + 2\lambda s_j(x_j + m_j(1 - x_j))\right]}{\left(\sum D_j s_j\left[x_j + m_j\left(1 - x_j\right)\right]\right)^2} \tag{22}$$

$$\frac{2}{C^2}\sum_{1}^{n}\left(K_{rj} + \frac{K_{pj}}{m_j}\right) = \frac{\sum_{j=1}^{n} D_j\left[(h_j + 2\lambda s_j)\left(x_j + m_j\left(1 - x_j\right)\right)\right]}{\left(\sum D_j s_j\left[x_j + m_j\left(1 - x_j\right)\right]\right)^2} \tag{23}$$

2.2. Proof of convexity

To check that 11 and 14 actually give the minimum, it suffices to find the second derivative of 8 (or 13) with respect to T (or T_j) and confirm that they are positive (semi) definite, which both give 24. This function, 24, is positive definite since all the input parameters are non-negative and non-zero, and hence 11 and 14 (and their variants presented) are minima for the cost functions in 8 and 13 respectively.

$$\frac{2}{T^3\sum\left(K_{rj} + \frac{K_{pj}}{m}\right)} \tag{24}$$

3. Solution procedure and algorithm

To determine what the best cost would be, it is pertinent to answer two questions. The first is if there is savings as a result of joint purchase and/or collection of containers. The second is if the capacity constraint is violated or not when the optimal quantities are determined from the optimal cycle times estimated. The process of determining the optimal cost by answering these two questions has been used to formulate an algorithm and is presented together with the solution procedure flow chart (Figure 5).

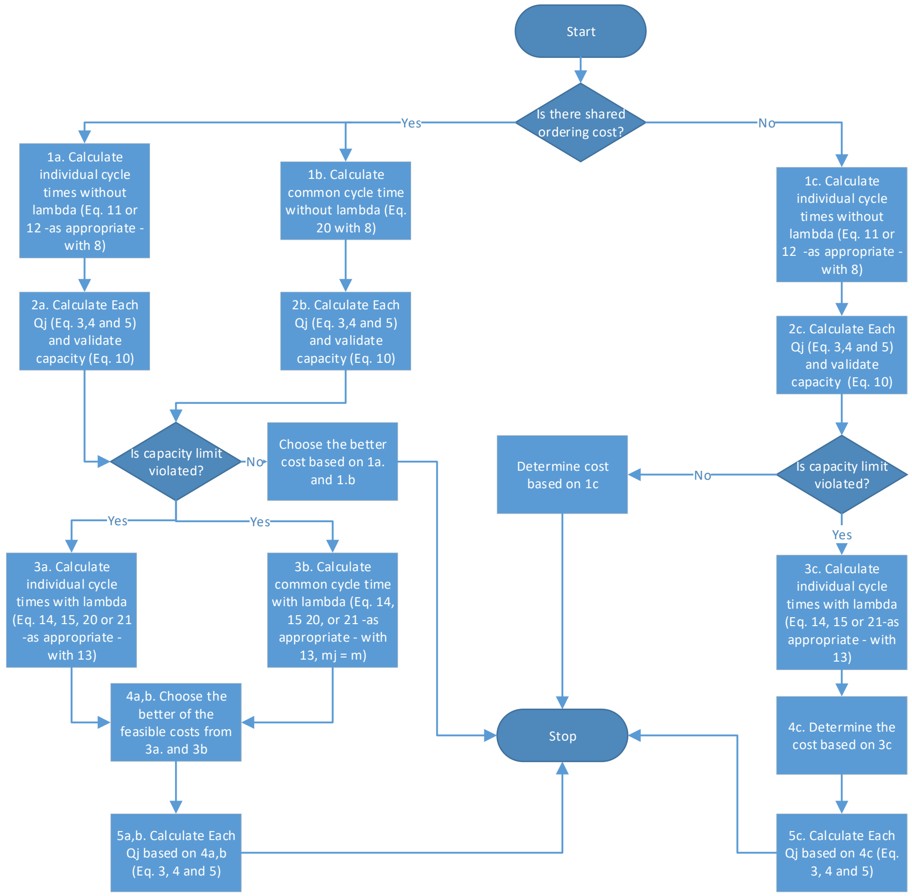

Figure 5: Solution procedure flow chart

3.1. Solution algorithm

1. Establish if there is economy of scale in ordering cost in order to choose the path to adopt for the solution. If there is opportunity for savings due to joint return or procurement ordering cost, follow paths a and b (the left of the flow chart) else, follow path c (the right).
2. Using 11 (or 12 as appropriate for common h) iteratively with 8, solve for the optimal T_j's for the individual items (containers) and compute the optimal cost
   a. Check feasibility of solution using 10
3. Adopting the economy of scale in the ordering cost for joint collection and/or purchase and using 20, solve for T_j's
   a. Check feasibility of the solution using 10
4. If both are feasible, choose the minimum of steps 2 and 3
   a. Adopting the better of the feasible costs, using 3 and 4, calculate Q_j's,

i. Stop

b. Else, proceed to step 5

5. Adopt the m_j values obtained from step 2 or 3. Using 22 (or 23 as appropriate for common h), solve for lambda
6. Using 14 (or 15), solve for T_j's if individually determined
7. For joint ordering with economy of scale, using 21, determine T. Adopt equation 19 into 14 (or 15) for order cost, solving iteratively with varying m values.
8. Adopting T_j's from steps 5 and 6, and adopting lambda from step 4 or 5 as appropriate, using 13, calculate the costs.
9. Choose the minimum from step 8
10. End

4. Numerical example and sensitivity analysis

Two numerical examples were solved using the proposed solution procedure, $n = 3$. For the first example (hereinafter Example 1), the storage capacity constraint is not binding while it is binding in the second example (hereinafter Example 2). The following input parameters apply to both examples:

$D_1 = 15\,000\ containers$, $D_2 = 20\,000\ containers$, $D_3 = 25\,000\ containers$, $s_1 = 20\ ft^3/container$, $s_2 = 15\ ft^3/container$, $s_3 = 10\ ft^3/container$, $x_1 = 0.95$, $x_2 = 0.9$, $x_3 = 0.8$, $K_{r1} = \$10\,000$, $K_{r2} = \$8\,000$, $K_{r3} = \$7\,000$, $K_{p1} = \$15\,000$, $K_{p2} = \$12\,000$, $K_{p3} = \$10\,500$, $K_r^f = \$7\,000$, $K_p^f = \$10\,000$, $K'_{r1} = \$3\,000$, $K'_{r2} = \$2\,400$, $K'_{r3} = \$2\,100$, $K'_{p1} = \$6\,000$, $K'_{p2} = \$4\,800$, $K'_{p3} = \$4\,200$, $h_{r1} = \$40/year$, $h_{r2} = \$30/year$, $h_{r3} = \$20/year$, $h_{p1} = \$60/year$, $h_{p2} = \$50/year$, $h_{p3} = \$40/year$.

The storage capacity for all container types in Example 1 is given by $C = 200\,000\ ft^3$, and for Example 2, $C = 100\,000\ ft^3$.

Four possible scenarios can result depending on the presence of the binding storage capacity constraint and the economies of scales achieved by joint ordering. Tables 1 and 2 present the results from the two examples, with the results in the former Table corresponding to a case where the constraint is not binding and in the latter Table, the constraint is binding.

Table 1: Results from Example 1

Scenario	**Common (or individual) cycle times)** ($\boldsymbol{years}$)	**Number of return cycles per procurement cycle**	**Total cost** ($\$/\boldsymbol{year}$)	**Space requirements** ($\boldsymbol{ft^3}$)
1	$T_1 = 0.1808$ $T_2 = 0.1690$ $T_3 = 0.1750$	$m_1 = 5$ $m_2 = 3$ $m_3 = 2$	$TC_1 = 143\,770.65$ $TC_2 = 141\,985.92$ $TC_3 = 140\,000.00$ $\boldsymbol{TC = 425\,756.57}$	$C_1 = 65\,104$ $C_2 = 60\,851$ $C_3 = 52\,500$ $\boldsymbol{C = 178\,455}$
3	$\boldsymbol{T = 0.1895}$	$\boldsymbol{m = 2}$	$\boldsymbol{TC = 357\,525.56}$	$\boldsymbol{C = 179\,075}$

From the data and for the two examples, we can see that that joint ordering results in cost total reductions when compared to equivalent individual ordering policies. In Example 1, the optimal solution is achieved when jointly ordering (for all container types) every $T = 0.1270$ years and having $m = 2$ returns cycles for all container purchase cycles. Under this optimal policy, the total cost is $\$357\,525.56$ per year. When the storage capacity constraint is binding (as is the case in Example 2), the optimal cycle time and the number of return cycles per purchase cycle remain the same but the total cost increases to $\$362\,151.68$ per year.

It can also be verified that the value of lambda (the Lagrange multiplier) for the first example is negative ($\lambda = -0.8748$) if used in the computation of the optimal cycle time or quantity as indicated in 14 of 15 instead of 11 or 12 when the constraint is not binding. This is expected because negative lambda indicates that the capacity was not yet exhausted, and is, therefore, not necessary given the computational effort involved compared to the simpler equations (11 or 12).

The capacity constraint in the second example justifies the use of 14 of 15 instead of 11 or 12, and it can be verified that lambda in that case is positive, $\lambda = 0.0816$. This factor is necessary to adjust the optimal cycle time (and hence optimal order quantity) for all items j in order to be within the capacity limit. It can also be seen that with an appropriate choice of lambda, the capacity just got fully utilised as indicated in Figure 4, and the container mix was appropriately allocated.

Table 2: Results from Example 2

Scenario	Common cycle time (or individual cycle times) ($years$)	Number of return cycles per procurement cycle	Total cost ($\$/year$)	Feasibility	Space requirements (ft^3)
1	$T_1 = 0.1808$ $T_2 = 0.1690$ $T_3 = 0.1750$	$m_1 = 5$ $m_2 = 3$ $m_3 = 2$	$TC_1 = 143\ 770.65$ $TC_2 = 141\ 985.92$ $TC_3 = 140\ 000.00$ $\boldsymbol{TC = 425\ 756.57}$	Storage capacity constraint violated (i.e. Not feasible)	$C_1 = 65\ 104$ $C_2 = 60\ 851$ $C_3 = 52\ 500$ $\boldsymbol{C = 178\ 455}$
2	$T_1 = 0.0926$ $T_2 = 0.0926$ $T_3 = 0.1112$	$m_1 = 5$ $m_2 = 3$ $m_3 = 2$	$TC_1 = 177\ 228.30$ $TC_2 = 168\ 562.77$ $TC_3 = 154\ 626.98$ $\boldsymbol{TC = 500\ 418.05}$	Feasible	$C_1 = 33\ 326$ $C_2 = 33\ 306$ $C_3 = 33\ 368$ $\boldsymbol{C = 100\ 000}$
3	$T = 0.1895$	$m = 2$	357 525.56	Storage capacity constraint violated (i.e. Not feasible)	$\boldsymbol{C = 179\ 075}$
4	$\boldsymbol{T = 0.1058}$	$\boldsymbol{m = 2}$	$\boldsymbol{TC = 383\ 364.92}$	**Feasible**	$\boldsymbol{C = 100\ 000}$

4.1. Sensitivity Analysis

Sensitivity analysis was done for the case where the constraint is not binding. The following observations were made from the results of the sensitivity analysis as presented in Table 3.

- Changes to the storage capacity for all container types (i.e. C) affected the optimal total cost and the cycle time but not the number of return cycle per procurement. In general, as the storage capacity decreases the cycle time decreases as well. A 50% decrease in capacity results in a 44% decrease in the cycle time. Increasing the capacity also had no effect on the cycle time since it was non-binding. This is because the cycle time and quantity are jointly determined and the optimum quantity would not change until when the capacity becomes binding.

- Changes to the cost of holding returned containers (i.e. h_{rj}) were found to have significant impacts on the total cost and the cycle time but not the optimal number of returns per purchase cycle. Case in point, a 50% increase in the holding cost for the returned containers resulted in a 14% decrease in the cycle time and a 16% increase in the total costs. A similar percentage decrease resulted in an increase of 12% in the cycle time and a decrease of 20% in the total cost. Despite these sizable changes, the number of return cycles per procurement cycle remained flat at two for all percentage decreases and increases tested.

- Changes to the holding costs of purchased containers (i.e. h_{pj}) were found to have similar effects on the objective function and decision variables as changes to the holding costs of returned container. However, the effects on the total cost and cycle time were not as severe as those caused by the holding costs of the returned containers. For example, a 50% decrease in the retuned containers' holding costs resulted in 12% decrease in the total cost while a similar change to the purchased containers' costs resulted in a decrease of 9%. This may be attributed to the assumption that the fraction of returned containers is close to unity, making the complementary function multiplying h_{pj}, $(1 - x_j)$, close to zero. This makes that the effect of the returned containers on the various cost components more dominant.

- The cost of returning a batch of useful containers (i.e. K_{rj}) did not affect the optimal solution for every case of joint ordering. Only the solution of individual ordering was only affected when K_{rj} was decreased by up to 50%. It is also interesting to note that this happened only in this particular case where the solution obtained from individual ordering policy resulted in lower total costs than a joint ordering policy. When this cost was separated into a fixed portion and a variable portion (i.e. K_{rj}^f and K'_{rj}), the number of return cycles per procurement cycle was still not affected by any of the changes but the cycle time and the total cost showed some movement, with the most notable ones being decreases of 10% to the cycle time and 7% to the total cost as a result of a 50% decrease in the variable cost of returning a batch of useful containers.

- With regards to the ordering cost for purchased containers (i.e. K_{pj}), its movement also affects the optimal cost and cycle time, but not as significantly as K_{rj}. This makes sense because while K_{pj} is divided by m_j, K_{rj} is not. Hence, since $m_j \geq 1 \; \forall \, j$, the effect gets more significant as m increases.

- While decreasing the storage space requirement for each container type (i.e. s_j) by 25% and 50% did not affect the optimal solution at all, increasing it by the same amounts resulted in changes to the optimal solution. This is explainable because the effect only kicks in when the storage capacity constraint becomes binding, hence, for both the 25% and 50% increases the cycle time increased by 11.1% in both cases while resulting increases to the total cost were smaller at 2.5% while reducing s_j only increases idle capacity.

Table 3: Results from the sensitivity analysis

Parameters		Common cycle time (or individual cycle times if optimal)		Number of common return cycles per procurement cycle (or individual cycles)		Total cost	
Base example		$\mathbf{0.1895}$		$\mathbf{2}$		$\mathbf{357\,525.56}$	
C	-50	0.1058	-44.2	2	0	383 364.62	$+7.2$
	-25	0.1585	-16.2	2	0	352 578.17	-1.4
	$+25$	0.1895	0	2	0	357 525.56	0
	$+50$	0.1895	0	2	0	357 525.56	0
K_{rj}	-50	$T_1 = 0.1419$ $T_2 = 0.1380$ $T_3 = 0.1479$	-25.1 -27.2 -22.0	$m_1 = 5$ $m_2 = 3$ $m_3 = 2$	$+150$ $+50$ 0	347 035.59	-2.9
	-25	0.1895	0	2	0	357 525.56	0
	$+25$	0.1895	0	2	0	357 525.56	0
	$+50$	0.1895	0	2	0	357 525.56	$+5.9$
K_{pj}	-50	0.1895	0	2	0	357 525.56	$+5.9$
	-25	0.1895	0	2	0	357 525.56	$+5.9$
	$+25$	0.1895	0	2	0	357 525.56	$+5.9$
	$+50$	0.1895	0	2	0	357 525.56	$+5.9$
K_{rj}^{f}	-50	0.1809	-4.5	2	0	335 808.26	-6.1
	-25	0.1853	-2.2	2	0	346 825.48	-3.0
	$+25$	0.1936	$+2.2$	2	0	367 933.42	$+4.8$
	$+50$	0.1977	$+4.3$	2	0	378 070.95	$+5.7$
K_{pj}^{f}	-50	0.1771	-6.5	2	0	326 090.98	-8.8
	-25	0.1834	-3.2	2	0	342 144.22	-4.3
	$+25$	0.1954	$+3.1$	2	0	372 309.99	$+4.1$
	$+50$	0.2011	$+6.1$	2	0	386 559.84	$+8.1$
K_{rj}'	-50	0.1803	-4.9	2	0	344 607.50	-3.6
	-25	0.1849	-2.4	2	0	351 118.45	-1.8

	+25	0.1934	+2.3	2	0	363 832.34	+1.8
	+50	0.1983	+4.6	2	0	370 042.32	+3.5
K'_{pj}	−50	0.1706	−10.0	2	0	331 260.61	−7.3
	−25	0.1803	−4.9	2	0	344 607.50	−3.6
	+25	0.1983	+4.6	2	0	370 042.32	+3.5
	+50	0.2067	+9.1	2	0	382 185.76	+6.9
h_{rj}	−50	0.2116	+11.7	2	0	286 391.30	−19.9
	−25	0.2082	+9.9	2	0	325 408.97	−9.0
	+25	0.1751	−7.6	2	0	386 985.85	+8.2
	+50	0.1635	−13.7	2	0	414 356.82	+15.9
h_{pj}	−50	0.2064	+8.9	2	0	328 297.60	−8.2
	−25	0.1974	+4.2	2	0	343 222.84	−4.0
	+25	0.1825	−3.7	2	0	371 277.70	+3.8
	+50	0.1762	−7.0	2	0	384 538.34	+7.6
s_j	−50	0.1895	0	2	0	357 525.56	0
	−25	0.1895	0	2	0	357 525.56	0
	+25	0.2116	+11.7	2	0	366 285.48	+2.5
	+50	0.2116	+11.7	2	0	366 285.48	+2.5

5. Conclusion

Container return management, including repositioning, is an important part of ports management activities. Repositioning is usually necessary when there is a gap between demand and supply levels for containers in ports, and there is usually the need to move such around. These movements can affect the cost of port operation significantly, and hence, the need to plan them appropriately, not only in terms of meeting the aggregate objectives, but also the mix of containers needed, given that shortages could be deemed to have occurred even when there are containers, but not the types needed for the transaction in time.

A model that could be used to determine the optimum lot size to move in a multi-item container management system has been presented in an environment where there could be storage capacity constraint and significant savings in moving different types of containers together as a batch. There is also the need to manage the top up containers for lost or damaged ones in an integrated manner. This scenario is common in container return management, and is deserving of attention.

We derived the optimal lot sizes for the container movements and repositioning for both the procured and returned containers under different scenarios, formulated the appropriate solution algorithm to determine the optimal quantity to reposition and procure and also presented two numerical examples to illustrate some of the important scenarios. We also showed the sensitivity of the solutions derived to changes in different parameters. We believe this model would be useful for most port managers.

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