UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

South African Mathematics Challenge participation: developing problem-solving skills in Mathematically-gifted disadvantaged learners

by

Rebecca Anne Stones

Submitted in partial fulfilment of the requirements for the degree

MAGISTER EDUCATIONIS
(Learning Support, Guidance and Counselling)

Department of Educational Psychology
Faculty of Education
University of Pretoria

Supervisor:
Prof J. G. Maree

PRETORIA
November 2020

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

DECLARATION OF ORIGINALITY

I, Rebecca Anne Stones (student number 18284478), declare that the dissertation titled **"South African Mathematics Challenge participation: Developing problem-solving skills in mathematically-gifted disadvantaged learners"** which I hereby submit for the degree Magister Educationis in Educational Psychology at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

.... ……….………………………………

Rebecca Anne Stones **Date**

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

ETHICAL CLEARANCE CERTIFICATE

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE	CLEARANCE NUMBER: **EP 18/06/01**
DEGREE AND PROJECT	MEd South African Mathematics Challenge participation: developing problem-solving skills in Mathematically-gifted disadvantaged learners
INVESTIGATOR	Mrs Rebecca Anne Stones
DEPARTMENT	Educational Psychology
APPROVAL TO COMMENCE STUDY	01 October 2018
DATE OF CLEARANCE CERTIFICATE	20 August 2020

CHAIRPERSON OF ETHICS COMMITTEE: Prof Funke Omidire

CC Ms Bronwynne Swarts
Prof. J.G. Maree

This Ethics Clearance Certificate should be read in conjunction with the Integrated Declaration Form (D08) which specifies details regarding:

- Compliance with approved research protocol,
- No significant changes,
- Informed consent/assent,
- Adverse experience or undue risk,
- Registered title, and
- Data storage requirements.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

DECLARATION FROM LANGUAGE EDITOR

25 October 2020

DECLARATION OF LANGUAGE EDITING

I, Gabriel Germaine de Larch, hereby declare that I have edited the paper, **South African Mathematics Challenge participation: Developing problem-solving skills in mathematically-gifted disadvantaged learners**, for language and style.

If you have any queries, feel free to contact me by emailing
germainedelarch@gmail.com
I am a member of the South African Freelancers' Association and have a B. Hons Degree in English.

Sincerely,

Gabriel Germaine de Larch

ACKNOWLEDGEMENTS

This dissertation could not have been completed without the support of the following people:

- **Prof. Kobus Maree**, my supervisor, for his impeccable advice, attention to detail and unsurpassed work ethic.
- **The school leadership and teachers** who gave permission for the study and made me feel welcome at their schools.
- **The parents** who let their children take part in my study.
- **The participants** who so enthusiastically tried something new.
- **Mrs Joyce Jordaan**, from the Department of Statistics, for statistical analysis and her careful and clear explanations of why each test was chosen.
- **Gabriel de Larch and Mardeleen Müller**, for editing, including late nights and short notice.
- **Vivienne Naudé and Tebogo** Letshwene for being understanding about me taking time off to do something so different from my day job.
- **My mother** for her unfailing support, and **my late father** Rev. Dr Canon Bob Clarke, for his academic example.
- **Jackie Nakeng** for her amazing home team support over the years.
- **My children**, for inspiring me to study gifted children in the first place, and for being patient with my years of study.

ABSTRACT

South African Mathematics Challenge participation: Developing problem-solving skills in mathematically-gifted disadvantaged learners

The purpose of this study is to examine whether Olympiad participation can develop problem-solving skills in mathematically-gifted learners from disadvantaged schools. My methodological approach was QUAN→Qual, using a quasi-experimental design with a non-equivalent comparison group. I chose two schools from the same disadvantaged area, and identified the top 50 Grade 7 learners in each school by mathematics marks. The study consisted of a pre-test, three mathematics sessions and a post-test. The *Study Orientation in Mathematics Questionnaire (SOM)* (Maree, Prinsloo, & Claassen, 2011) was used as the pre- and post-test, and a focus group explored the learners' experience of the *SOM*. In the mathematics sessions, the intervention group worked through past papers of the SA Mathematics Challenge (South African Mathematics Foundation, 2018), and the alternative intervention group completed worksheets from a Department of Basic Education workbook.

My study revealed a positive relationship between success in traditional Mathematics and Study Attitude, Study Habits and overall Study Orientation, and an interaction between disadvantage and success in Mathematics. Participants were less disadvantaged than their surroundings would indicate, and had higher Mathematics anxiety than expected for their achievement level. The intervention did not increase problem-solving behaviour and both the quantitative and qualitative findings showed that the participants found the Olympiad type questions unfamiliar and difficult. This unfamiliarity is indicative of the limited enrichment opportunities for mathematically-gifted learners in disadvantaged areas of South Africa. Greater experience of Mathematics Olympiads is suggested to help mathematically-gifted disadvantaged learners live up to their problem-solving potential.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1 – INTRODUCTION

Chapter 1: Overview

Chapter 1 serves as an introduction to my thesis. Firstly, I cover the rationale and purpose for undertaking this topic, then detail the research questions and hypotheses that were used to direct my study, and define concepts particular to the study. Thereafter I situate the study within my conceptual framework of giftedness, and then I show the paradigmatic perspective from which I view the study, before describing the research methodology used. I conclude the chapter with an examination of the ethical issues that I considered.

1.1 INTRODUCTION AND RATIONALE FOR UNDERTAKING THE STUDY

1.1.1 Gifted disadvantaged children in South Africa

Gifted children are South Africa's future leaders, scientists and researchers and according to Lubinski and Benbow (2006, p. 316), "Those countries that flourish will be the ones most effective in developing their human capital and in nurturing individuals who will come up with the best ideas and innovations of tomorrow". Although in South Africa the term "gifted children from economically disadvantaged areas" is generally preferable, I have chosen to use "gifted disadvantaged children", for succinctness, and in line with international trends.

Gifted children in South Africa are typically "undervalued and under-served" (Van der Westhuizen, 2007, p. 1), particularly disadvantaged children, who do not have access to quality education. According to Statistics South Africa (2017), there were thirteen million children age 0-17 in South Africa (SA) living below the upper-bound poverty line (UBPL) in 2015. This figure represents 66.8% of SA's children. Definitions of giftedness vary from 2% to 5% of the population, which means that roughly 260 000 to 650 000 children in South Africa are untapped potential as both gifted and disadvantaged.

Education potentially facilitates an escape from poverty in SA: only 8.4% of adults with higher education are living below the UBPL compared to 79.2% of those with no education and 69.2% of those with only primary school education (Statistics South Africa, 2017). The effects of education are cumulative down generations: 70% of the black middle class send their children to former model C and private schools (Brown, 2016). Attending a Former Model C school confers significant advantage: only one in seven learners from former black schools gain an endorsement which allowed them entrance to university, but one in two learners from Former Model C schools achieve endorsements (Christie, Butler, & Potterton, 2007). If South

Africa could facilitate increased graduation of students from university, it would not only benefit the students and their immediate families, but benefit the wider community through "general effects on societal development including wealth, health, politics, science, ethics and culture" (Rindermann, Sailer, & Thompson, 2009, p. 20). Van Broekhuizen, van der Berg and Hofmeyr (2016, p. 66) found that "while attending a quintile 1-3 school largely precludes learners from gaining access to university, those who do make it into university tend to perform almost on par with their quintile 4 and 5 counterparts". This is an important finding, as it implies that investment in the skills of gifted disadvantaged learners at school level (resulting in more learners entering university) would pay off with more university passes.

1.1.2 Problem-solving skills in South Africa

For gifted disadvantaged individuals to develop into South Africa's leaders they need to develop skills that are valuable to the economy (Griesel & Parker, 2009). A study of employers' requirements of Higher Education in South Africa recognised the value of problem solving in the marketplace, and found that employers valued items such as "ability to recognize a problem situation", "ability to choose appropriate information address problems", and "a proactive approach to problem-solving" (Griesel & Parker, 2009, p.18). It is these kinds of problem-solving skills that the South African Mathematics Challenge emphasises (South African Mathematics Foundation, 2020b)

According to Ruf (2005, p. 135) "it takes an extremely high intellectual level to teach oneself reading, but it takes an even higher level to teach oneself math" so only a few gifted learners would be able to teach themselves mathematics, and the rest (the majority of gifted children) need to be overtly taught mathematical skills. This is borne out by my own experience. Over 10 sessions, I prepared a group of nine gifted children for entry in the SA Mathematics Challenge. The course included some overt teaching to assist children to recognise and handle particular common types of Olympiad questions, but the majority of the course consisted of the learners working through past papers from the SA Mathematics Challenge and SA Mathematics Olympiad, alone and in pairs. All nine gifted children qualified for the second round (a score of 50% or above). In contrast, at the end of teaching a module of Grade 8 geometry to gifted learners, I gave them ten geometry questions from the Grade 8 South African Mathematics Foundation (SAMF) Mathematics Olympiad. In contrast to the children who had undergone the Olympiad training course, these gifted learners only scored between 10% and 50%. I surmised that their lack of exposure to Olympiad-type questions put them at a disadvantage compared to the learners who had experienced many similar questions. This study aimed to examine the validity of this conjecture.

1.2 PURPOSE OF THE STUDY

The purpose of this study was to examine the possible effects of Olympiad participation on gifted disadvantaged children, particularly to explore whether Olympiad participation could develop problem-solving skills in mathematically-gifted learners from disadvantaged schools. Past papers to the SA Mathematics Challenge are freely available on the SAMF webpage (South African Mathematics Foundation, 2018), so this was a cost-effective intervention.

Mhlolo (2015, p. 166) identifies mathematical competence as “key to the welfare of a nation in the global economy” and warns of two groups that are most in danger of not realising their full potential: mathematically-gifted children, and economically disadvantaged children. Research in South Africa has largely neglected mathematically-gifted disadvantaged children. Although there is extensive research on Mathematics education in South Africa, Engelbrecht and Mwambakana (2016, p. 2) found that “little research has been done on the impact and efficiency of mathematics olympiads”. There was a gap at the intersection of these two areas of research, namely gifted disadvantaged children, and the impact of mathematics Olympiads. There have been no studies on the potential benefits of the SA Mathematics Challenge or Olympiad for gifted disadvantaged children. My research aimed to fill that gap.

1.3 RESEARCH QUESTIONS

Flowing from this gap in the body of knowledge, the primary research question was as follows: How valuable is participation in the SA Mathematics Challenge for developing problem-solving skills in mathematically-gifted disadvantaged learners? This gave rise to the following secondary research questions:

1. What are the essential aspects of current (group-based) programmes aimed at enhancing the problem-solving skills of mathematically-gifted learners in disadvantaged schools?
2. What is the impact of three hour-long facilitated sessions doing SA Mathematics Challenge past papers on mathematically-gifted disadvantaged learners’ study orientation in mathematics in general?

3. What is the impact of three hour-long facilitated sessions doing SA Mathematics Challenge past papers on mathematically-gifted disadvantaged learners' problem-solving skills in particular?

1.4 HYPOTHESES

The following two main null hypotheses that guided the study were:

1. There is no significant difference between the pre-test and post-test mean scores for the two groups.
2. There is no significant difference between the post-intervention scores of the two groups (intervention and alternative intervention).

My main alternative hypothesis was the following: There is a significant difference in the post-test mean scores of the intervention and the alternative intervention group.

1.5 CONCEPT CLARIFICATION

1.5.1 Mathematically gifted

Giftedness is defined in a variety of ways in research literature, ranging from having an IQ over 130, the top 3% to 5% of the population, or people with "extraordinary potential" (Streznewski, 1999, p. 5). For the purpose of this study, "mathematically-gifted learners" is defined as the top 50 learners in a grade, based on their mathematics year mark for the current year.

1.5.2 Disadvantaged

The term disadvantaged can be used to mean someone who is from the non-dominant culture (Lumadi, 1998). However, in this study, I have used the definition of Eriksson (1993, p. 107), that "the term 'disadvantaged' is viewed in a socio-economic perspective – such children may also come from different cultures, but are characterised by poverty and lack of adequate educational and social opportunity". The definition of disadvantage as being primarily an economic issue is widely used in South Africa (Howie et al., 2017; Jamieson, Berry, & Lake, 2017; Modisaotsile, 2012; Reddy et al., 2015; Van der Westhuizen, 2007; Xolo, 2007). This leads to a need for a definition of poverty.

1.5.3 The term "quintile schools" as a proxy for poverty

The Department of Basic Education categorises schools according to quintiles, or fifths of the population, based on the poverty level of the communities surrounding the schools, with quintile 1 being the poorest, and quintile 5 the richest (Murray, 2017). Quintile 1 to 3 schools are non-fee-paying. For the purpose of this study, a child attending a non-fee-paying school was defined as disadvantaged.

1.5.4 Learners

I chose to involve a sample of Grade 7 learners for the study, for several reasons. The SA Mathematics Challenge is written in May each year (South African Mathematics Foundation, 2017), which means that the Grade 7 paper is based mainly on work covered in the Intermediate Phase. The Mathematics syllabus covers the same five content areas each year from Grade 4 to Grade 6 (Department of Basic Education, 2011). Studying Grade 7 learners reduces the risk of basic mathematical skills acting as a confounding variable. Secondly, written problem-solving questions require reading skills (Maree & Erasmus, 2006). South African Grade 4 learners performed very poorly on the PIRLS Literacy Study, with 78% unable to "locate explicit information or reproduce information from a text" compared to 4% internationally (Howie et al., 2017, p. 73). The language level of the Mathematics Challenge seems similar for Grade 4 and Grade 7 (South African Mathematics Foundation, 2018), so it is preferable to select Grade 7 learners.

1.5.5 Problem-solving skills

Maker (2006, p. 38) defines problem solving as "the process of answering questions, resolving difficulties, creating solutions, and investigating perplexing situations". For this study, problem-solving ability was measured using the Problem-solving subsection of the Study Orientation for Mathematics (SOM) (Maree et al., 2011).

1.6 CONCEPTUAL FRAMEWORK

For this study, I developed my own conceptual framework of giftedness, called the UPPS framework of giftedness, based on my study of the literature. This framework is based on four concepts: a **unitary** intelligence, from which **potential** can be developed, assisted, and/or hindered by **precocity** (Piirto, 2004) and **socio-emotional** factors. I will describe this framework and its development in Chapter 2, but I offer a summary of it in the following paragraphs.

The first requirement of my conceptual framework of giftedness is that it refers to a **unitary** concept of intelligence, what Spearman (1904, p. 201) called "general intelligence" or *g*. This is supported by the research of many intelligence theorists since Spearman (Beaujean, 2015; Benson, Beaujean, McGill, & Dombrowski, 2018; Carroll, 1993; Lohman, 2001; Lubinski, 2016; McGrew & Evans, 2004; McGrew, 2009; Terman et al., 1926; Visser, Ashton, & Vernon, 2006).

Second, my theory of giftedness is not defined by achievement or eminence. Achievement is not routinely found in theories of giftedness, although Renzulli (1978, p. 182) does have an aspect of it in his circle of "task commitment". However, it is a commonly held view in the wider community, including amongst teachers. This is demonstrated by the way that teacher identification of gifted children tends to favour high-achieving children (Neber, 2004). An achievement-based definition of giftedness also denies the heritability of *g*, which has been established in the literature (Knowles, 2008; Sauce & Matzel, 2018). Instead of achievement, my conceptual framework is based on the **potential** of the gifted child. This definition is well-suited to the context of disadvantage, in that a child can have potential without having had a good education, or access to resources. It also enables "twice-exceptional" (both gifted and learning-disabled) children to be included (Wissing, 2012).

Even though this might seem contradictory, I find Piirto's inclusion of "**precocity**" (Sansom, Barnes, Carrizales, & Shaughnessy, 2018, p. 98) in her definition of giftedness useful, as it gives a practical framework for teacher interaction with gifted students without requiring experience of gifted education. She states that gifted children have much in common with older non-gifted learners, and that one can approach a young gifted learner as one would an older child (Sansom et al., 2018). This framework has the advantage of supporting acceleration, which is one of the most successful interventions for gifted children (Wai, 2015). A disadvantage of this component of the framework is that it ignores the emotional level of the young gifted child.

The last component of giftedness is the **socio-emotional** one, which I have included to ameliorate the shortcomings of the precocity component. I borrow from the theory of Dabrowski, specifically his five "over-excitabilities", which are psychomotor, sensual, intellectual, imaginational, and emotional (Daniels & Piechowski, 2009, p. 9). In my experience, mutual recognition of gifted people is often by recognition of over-excitabilities, for example recognising giftedness in a small child by their intense emotionality and excitement for intellectual questioning.

1.7 PARADIGMATIC PERSPECTIVES

Guba and Lincoln (1994, p. 107) define a paradigm as a "worldview" or "belief system" of the researcher, underpinning the study. Similarly, Chilisa and Kaluwich (2012, p. 1) use the word "view" to explain a paradigm. Sefotho (2015, p. 25) refines these definitions when he describes a paradigm as "a philosophical lens" and recommends overt statement of a paradigmatic perspective to a study because it "also helps the researcher to be congruent and consistent throughout the study" (Sefotho, 2015, p. 26). An additional advantage of explication of a paradigmatic perspective is that awareness of that perspective also affords both researcher and reader an opportunity to see, and thereby reduce the influence of any slant to that viewpoint. My chosen paradigm for this study is that of critical realism informed by pragmatism.

1.8 OVERVIEW OF RESEARCH METHODOLOGY

Chapter 3 contains a detailed description of the research methodology for the study. The following is a summary of the salient points.

1.8.1 Research approach

This study utilised a QUAN→qual research approach. This means that while the overarching approach was quantitative, one qualitative method was used in the study, namely a focus group. This matter will be explicated in more detail in Chapter 3.

1.8.2 Research design

I used a quasi-experimental design, which is similar to an experimental design, but either missing randomisation, or a control group (Keele, 2011). As with experimental designs, quasi-experimental designs facilitate prediction (Guba & Lincoln, 1994) and allows for findings to be generalised to the population from which the sample was drawn (Keele, 2011).

1.8.3 Sampling of participants

I used purposive sampling to choose the two schools to participate in my study, based on the following criteria: quintile of the school (quintile 1 or 2); sufficiently large to have a large number of Grade 7 learners in the school; similar in size to each other; and in the same area. Within each school, I selected the top 50 learners by mathematics year mark.

1.8.4 Data collection and documentation

My data collection method was testing within a Nonequivalent Comparison Group Design which is a quasi-experimental version of the Pretest-Posttest Comparison Group Design (Engel & Shutt, 2014). Additionally, as a result of ethical considerations rising from evaluation of learners' problem-solving skills using the *Study Orientation in Mathematics* (*SOM*) (Maree et al., 2011), I facilitated a focus group discussion by a subset of the learners to get feedback on their experience of the *SOM*.

1.8.5 Data analysis and interpretation

As this was primarily a quantitative study, most of the data analysis was completed using Statistical Package for the Social Sciences (SPSS)(IBM Corp., 2017). I will cover the details of the statistical analysis in Chapter 4.

1.9 ETHICAL CONSIDERATIONS

My study participants are considered "vulnerable" (World Health Organization, 2018) on two counts: firstly as minors, and secondly as disadvantaged. I obtained ethical clearance from the University of Pretoria and, because my research took place in schools, the Department of Education (Department of Basic Education, 2017b). In addition to ethical clearance, I was guided by the APA General Principles (Elias & Theron, 2012, pp. 150–152)

1.9.1 Beneficence and non-maleficence

The first principle of the APA General Principles means that the research should benefit and not harm participants in the study. As participation in the *SOM*, which was used as a pre- and post-test of problem-solving skills, could potentially be a negative experience for learners, I held a focus group to explore the experience of writing the *SOM*.

1.9.2 Fidelity and responsibility

To avoid conflicts of interest, I chose schools with which I had no prior contact with either the learners or the staff. To ensure that study participants and their parents, who most likely speak English as a second or third language, would be able to understand the intake letter, I wrote it in simple English and tested it on Grade 7 child and a second-language English speaker. Additionally, the letters were submitted to the University of Pretoria ethics committee, adding an extra check for ease of understanding and professionalism in my communication.

1.9.3 Integrity

Integrity refers to the promotion of "accuracy, honesty and truthfulness" (Elias & Theron, 2012, p. 151) in conducting research. I was self-reflexive in examination of all my written communication with participants, both written and verbal, to ensure that the use of simple English did not compromise the principle of integrity.

1.9.4 Justice

A study should be just in its extension of any services offered to the participants (Elias & Theron, 2012). As a result, I administered an alternative intervention to the "control" group.

1.9.5 Respect for people's rights and dignity

I respected for the participants' rights and dignity (Elias & Theron, 2012) by ensuring confidentiality of individual learners' test results. Participation in the study was optional, and subject to the signing of an informed assent form by the learners involved, and a consent form by their parents, as they were minors.

1.10 OUTLINE OF CHAPTERS

1.10.1 Chapter 1: Introduction

Chapter 1 introduces the dissertation, covering why I undertook the study, and the research questions that drove the study, and covering definitions of concepts used in the study. It gives a brief introduction to the research paradigm, design, and methodology. Lastly, it touches on ethical considerations.

1.10.2 Chapter 2: Literature study and conceptual framework

Chapter 2 examines the literature on giftedness worldwide, and in South Africa, paying particular heed to the special situation and needs of disadvantaged gifted learners. Out of this literature study comes my own theory of giftedness that drives the study, which is then explained. The first research question will be addressed in this chapter.

1.10.3 Chapter 3: Research methodology

Chapter 3 describes in detail my research paradigm, design, and methodology, and covers exactly how the study was implemented, to facilitate reproduction of the study.

1.10.4 Chapter 4: Data analysis and results

Chapter 4 covers the statistical data analysis process and the results arising from the data analysis.

1.10.5 Chapter 5: Discussion of findings

Chapter 5 presents the findings of my study and links them back to the literature to place my findings in context.

1.10.6 Chapter 6: Conclusion

Chapter 6 summarises the other chapters in the dissertation before returning to answer the research questions. It then discusses the strengths and limitations of the study, touches on the ethical considerations before making recommendations for improvements to the research project, and suggestions for further research.

1.11 SUMMARY OF CHAPTER 1

In this chapter, I discussed the value of studying gifted disadvantaged learners, both for their benefit, and for their country. I highlighted the paucity of studies into the benefit of participation in mathematical olympiads, and the gap in the literature where these two areas meet, namely the benefit of Mathematics Olympiad participation for gifted disadvantaged learners in South Africa.

I introduced the key concepts of mathematical giftedness, disadvantage, poverty, and problem solving as well as introducing the learners who were the participants in the study. I discussed the research questions and hypotheses that drove the study, and outlined the quasi-experimental design, quantitative methodology, and ethical considerations that inform the study.

Lastly, I situated my study within a context, firstly of my own conceptual framework of giftedness, which owes its roots to theories of Spearman (1904), Piirto (2004) and Dabrowski (Daniels & Piechowski, 2009). In the following chapter, I examine in greater detail the literature that constitutes the context of my research.

CHAPTER 2 – LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

Chapter 2: Overview

Chapter 2 examines worldwide definitions of giftedness, before surveying the literature on giftedness in South Africa and examining the special needs of disadvantaged gifted learners. Out of this literature study comes my theory of giftedness, which I explain. I then look at the literature on mathematics education and current group-based programmes aimed at enhancing the problem-solving skills of mathematically-gifted learners in disadvantaged schools in South Africa. Lastly, I look at Mathematics Olympiads and the Mathematics Challenge in particular.

2.1 DEFINING GIFTEDNESS

Despite intelligence tests having existed for over a century, since the first Binet Simon Test in 1905 (Binet & Simon, 1916), there is little agreement on a definition of giftedness. A literature review by Carman (2013, p. 2) assessed 74 studies that defined giftedness and found "a lack of consensus as to what qualifies a person to be defined as gifted for the purposes of research". However, certain themes come up repeatedly when looking at debates within the field of giftedness, and by extension, many common strands. Heuser and Wang (2017) outline a history of the debates within the field of intelligence, and identify four different axes of contention, namely, a definition of intelligence as cognitive vs. multiple intelligences; aptitude vs. achievement; nature vs. nurture; and individual vs. community.

2.1.1 Cognitive vs. multiple intelligences

The first major debate in intelligence research is whether giftedness can be ascribed to one general gifted factor or whether intelligence is made up of many factors. Spearman (1904) is known for his factor analysis of various intelligence factors. He concluded from the correlation between the different factors that there was an underlying general intelligence factor or *g*. Subsequent theorists have either supported or tried to refute the concept of *g*. According to Lubinski (2016, p. 901), in his paper covering the years from 1916 to 2016, "the first 50 years of research on precocious learners utilised selection procedures based on general intellectual ability, the past 50 years saw a movement to and an acceptance of the need for selection based on specific abilities".

Gardner's theory of Multiple Intelligences, which can be traced back to the seven-factor analysis of Thurstone in the 1920s and 30s (Zygmont, 2006) extends the definition of intelligence beyond just the intellectual to include types of intelligence such as "bodily-kinaesthetic" and "interpersonal" (Gardner & Hatch, 1989, p. 6). Gardener's theory of Multiple Intelligences (MI theory) has widespread support in the field of education. In South Africa, it is covered in the teacher training textbook used by the largest university in South Africa (Nieman & Monyai, 2006; University of South Africa, 2019), and is embedded in the teaching methodology of Radford House Primary (L. Breeds, personal communication, January 31, 2019), the first primary school for the gifted in South Africa (Kokot, 1999). However, MI theory has not had similar support amongst cognitive and differential psychologists (Lohman, 2001; Waterhouse, 2006). Gardner himself postulated that the greater popularity of MI theory with educators, as opposed to psychologists, is because educators "are much less wedded to disciplinary standards of evidence and acceptability" (Davis, Christodoulou, Seider, & Gardner, 2011, p. 5). Klein (1997) and Willingham (2004) object to the theoretical underpinnings of MI theory, with Willingham criticising the way of choosing the different intelligences, and suggesting a further five attributes that could be categorised as intelligences, using Gardner's own definitions of an intelligence. Visser (2006) takes a more practical approach, administering independent tests for each of Gardner's eight Multiple Intelligences, and finds significant correlation across the tests, with the exception of bodily-kinaesthetic, musical, and one of the intrapersonal tests, pointing towards the existence of an underlying general intelligence. Gardner rebuts both Visser (Gardner, 2006) and Waterhouse (Gardner & Moran, 2006), criticising the tests used to assess MI Theory as not testing the intelligence that they were intended to test. He suggests that the Explorama, a theme park consisting of 50 tests based on MI Theory could possibly be used to assess MI theory, but to date, no-one has done this testing. Gardner has created assessments for pre-schoolers based on MI Theory, called Project Spectrum, and subsequent factor analysis on Project Spectrum revealed that "Spectrum activities are not as separate from g as proposed by the defenders of multiple intelligences theory, nor as unitary as argued by the defenders of g factor models" (Castejon, Perez, & Gilar, 2010, p. 481).

Cattell and Horn divide *g* into just two factors, Gf (fluid intelligence) and Gc (crystallised intelligence) (Kaya, Stough, & Juntune, 2016). Carroll's factor analysis of Cattell and Horn's factors (Carroll, 1993) has been widely praised for its thoroughness and strong empirical basis (Beaujean, 2015; Benson et al., 2018; Lubinski, 2016). The Cattell-Horn Gf-Gc model and Carroll's Three-Stratum model are considered to be "consensus psychometric-based models for understanding the structure of human intelligence" (McGrew, 2009, p. 1), and are often referred to collectively as Cattell-Horn-Carroll or CHC theory (Beaujean, 2015; Benson et al., 2018; Castejon et al., 2010; Keith & Reynolds, 2010; Lubinski, 2016; Warne, 2016). CHC theory is widely used as a theoretical base for psychometric tests (Keith & Reynolds, 2010) and Warne (2016) recommends its use for guiding gifted education practitioners, demonstrating that its acceptance of *g* as a construct, as well as the division of intelligence into further strata, can guide educators on appropriate intervention for a gifted learner, including full-grade and single subject acceleration. Gross (2006, p. 425), in her longitudinal study of 60 young Australians over twenty years, strongly supports the concept of *g*. She says that when she was young, a highly gifted person was called a "whiz" in their area of talent, but the participants in her study are "*g* whizzes".

2.1.2 Aptitude vs. achievement

Another debate centres on whether intelligence should be considered as a measure of intellectual *potential*, or of *achievement*. Terman (1926, p. 20) did not consider achievement tests for identifying gifted children for his notable longitudinal study of 1000 gifted children as they are "inferior…as measures of native ability" but in contrast, Renzulli (1978, p. 182) included "task-commitment" in his three-ringed theory of giftedness, which points to a more achievement-oriented definition. The distinction is important when it comes to the identification of the gifted, as it has been shown that teacher identification of the gifted correlates to achievement rather than working memory, which can be seen as innate power of the brain (Kornmann, Zettler, Kammerer, Gerjets, & Trautwein, 2015). Interestingly, parents have been found to be better judges of giftedness than teachers (Dağlioğlu & Suveren, 2013; Gross, 1999), which implies that parents use different criteria for judging giftedness to teachers.

2.1.3 Nature vs. nurture

The concept of achievement leads to an examination of the heritability and malleability of intelligence. Ever since Spearman introduced the concept, the degree of heritability of *g* has been debated (Gladwell, 2008; Lubinski, 2016; Sauce & Matzel, 2018). Longitudinal studies by Terman et al. (1926), Gross (2006) and Kell, Lubinski and Benbow (2013) all show a much higher level of eminence amongst people identified as exceptionally or profoundly gifted as children than amongst the general population. At a heritability of 0.8, intelligence has a much higher heritability than most heritable psychological traits (Sauce & Matzel, 2018). When studying gifted children in disadvantaged areas, as my study does, it is important to consider the malleability of intelligence in impoverished communities. Sauce and Matzel (2018, p. 37) examine a number of twin studies and conclude that "differences in family background matter more when that background is relatively impoverished". This is important when considering enrichment of gifted children: those in disadvantaged areas are more likely to benefit from such intervention than those in affluent areas. Given that giftedness is not limited to one socio-economic group or race (Borland, 2004; Maree, 2018; Silverman, 2009), we know that there are gifted children living in disadvantaged areas of South Africa. Of the thirteen million children age 0-17 living below the upper-bound poverty line in South Africa (Statistics South Africa, 2017), over a quarter of a million would be defined as gifted. These children are being afforded the same poor education as their classmates, and not developing problem-solving skills or the malleable part of *g* to the same degree as their middle-class peers.

2.1.4 Community vs the individual

The Western view of giftedness being an individual matter is not the only viewpoint: in some cultures, giftedness is defined in relation to the community. According to McCann (2005), Australian Aboriginals define intelligence as thinking with the group, rather than being different to the group; and the Maori view of giftedness does include individual skill, but it needs to be used for the good of the community for it to count as giftedness. Maree (2018b, p. 133) maintains that there is no one African definition of giftedness, but says that factors such as "aptitude… respectfulness, obedience, trustworthiness and care for others" are often used when discussing giftedness. According to Ngara (2017), Shona-speakers in Zimbabwe believe that giftedness is a spiritual gift given for the benefit of the community, and words for giftedness in the Bantu group of languages (of which Shona is one) share a common root. Considering that all South African official languages, apart from English and Afrikaans, are Bantu languages (Jordan, 2015), the Shona view of giftedness is likely to be widespread in South Africa. Certainly Lumadi (1998) finds that among the Vhavenda, giftedness is encouraged primarily to benefit the community. Another interesting aspect of the Shona view of giftedness is the belief that giftedness is by definition striving against the odds, so it is more prevalent in poor communities (Ngara, 2017), but I have not been able to confirm if this view is shared by South African Bantu language speakers.

2.2 IDENTIFICATION OF THE GIFTED IN SOUTH AFRICA

Identification of the gifted in South Africa is a politically-charged topic, due to the difficulties associated with intelligence testing in a multilingual and multicultural country with widely varying access to education.

The locally-designed IQ tests, the *Junior South African Individual Scale* (*JSAIS*) (Madge, 1981) for children aged 3 years to 7 years 11 months, and the *Senior South African Individual Scale Revised* (*SSAIS-R*) (van Eeden, 1991) for children age 7 years to 16 years 11 months, are outdated, but are still widely used in South Africa (Laher & Cockcroft, 2013). Not only do they have outdated norms but are also based on an outmoded theoretical model, not being based on the Cattell-Horn-Cattell (CHC) framework (Laher & Cockcroft, 2013). The *JSAIS* was normed on white English and Afrikaans-speaking children in 1976, and adapted for English-speaking Indian children in 1981 (Te Nijenhuis, Murphy, & van Eeden, 2011). The same year, the *SSAIS-R* was normed on white, coloured and Indian children, with a home language or English or Afrikaans (Te Nijenhuis et al., 2011).

A previous version of the *JSAIS* and *SSAIS-R*, the New *South African Individual Scale (NSAIS)*(1962), was translated into isiXhosa in 1988, and from there into four other African languages (isiZulu, Southern Sotho, Northern Sotho, and Tswana) and normed on children from 9 to 19 years of age (Mayaba, 2016) and these translations of a 1962 test remain the only South African IQ tests available in these languages. The *JSAIS* was first translated into isiZulu and SeSotho in 2010 (Mawila, 2012) but improvements are ongoing (Bouwer, 2014) and translated versions are not commercially available (Health Professions Council of South Africa, 2017). Although *SSAIS-R* is only available in English and Afrikaans, van Eeden published studies in 1993 and 1997 on black children attending private and model C schools, who took the test in English. She concluded that norms for environmentally disadvantaged learners should be used when evaluating children whose home language is not that of the testing language (Cockcroft, 2013).

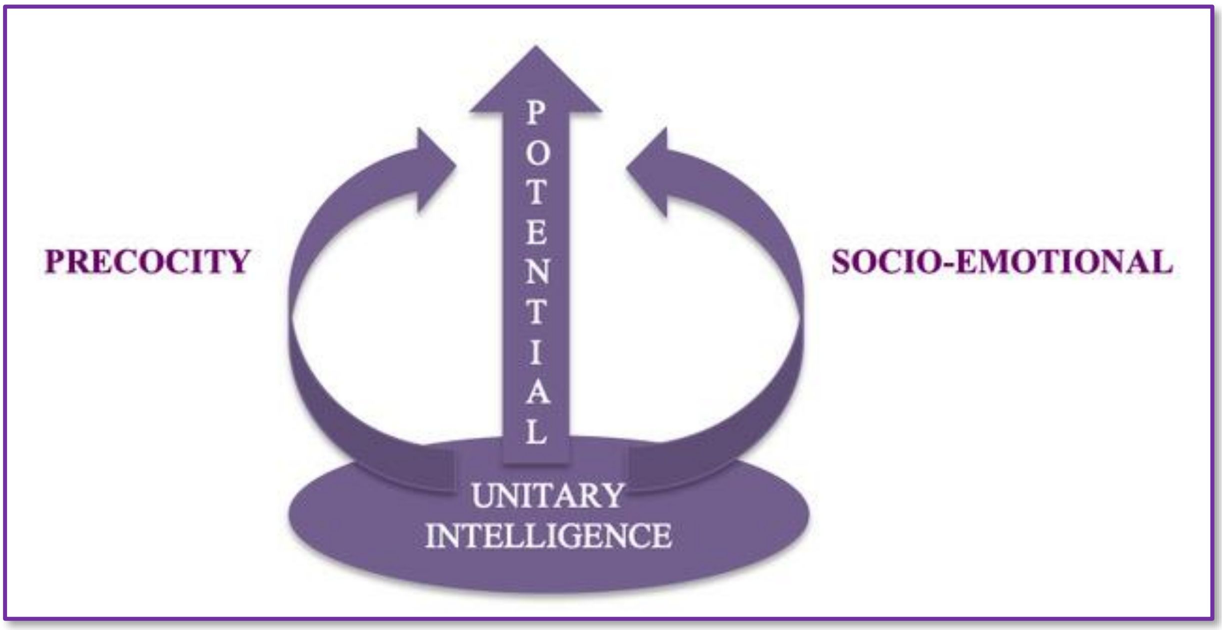
In recent years, the Wechsler tests, which originate from the United Kingdom, have been normed for the South African context. They are based on the CHC framework, so are based on up-to-date theory (Shuttleworth-Edwards, Garland, & Radloff, 2013).

Non-verbal tests, such as the *Raven's Standard Progressive Matrices* (*SPM*), the *Naglieri Non-Verbal Ability Test* (NNAT) and Form 6 of the *Cognitive Abilities Test* (COGAT) are often suggested as a culture or language-fair option for use in multicultural and multilingual societies such as South Africa (Laher & Cockcroft, 2017; Sarouphim, 2009). But non-verbal tests are not the panacea they might seem to be. Lohman and Gambrell (Lohman & Gambrell, 2012) compare the performance of Hispanic children with a first language of English with English-Language Learner (ELL) children, and find between 7.3 and 9.5 point difference in scores between first and second-language English speakers and that "reducing the language demands may actually increase the cultural loading of the test" (Lohman, 2013, p. 274). In South Africa, studies by Israel (2006) and Knowles (2008) find substantial language bias in the *Raven's Advanced Progressive Matrices* (RAPM) and RPM. Knowles (2008, p. 55) concludes that RPM should be used with "extreme caution" in South African high schools. Similar results have been found in Zambia and Kenya (Maree, 2018b). Mayaba (Mayaba, 2016) conjectures that the difference between IQ levels of urban and rural children in South Africa could be due to urban children having more experience of toys and computer games that contain patterns similar to the patterns in certain sub-tests of the RPM.

There is some hope in that in recent years researchers have become interested in improving instruments for assessing intelligence in South Africa (Bouwer, 2014; Laubscher & Olszewski-Kubilius, 1996; Mohlala, 2000). This includes an innovative idea of using career counselling to identify and support gifted learners in South Africa (Maree, 2018).

After examination of the four axes of contention in defining giftedness, and their application for identifying gifted children in disadvantaged areas in South Africa, I have combined what I consider pertinent into one conceptual framework that underpins my study.

2.3 UNITARY POTENTIAL PRECOCITY SOCIO-EMOTIONAL (UPPS) CONCEPTUAL FRAMEWORK OF GIFTEDNESS



For this study, I developed my own conceptual framework of giftedness, called the UPPS framework of giftedness, which is based on four concepts: a **unitary** intelligence, from which **potential** can be developed, assisted, and/or hindered by **precocity** (Piirto, 2004) and **socio-emotional** factors.

2.3.1 Unitary intelligence

The first requirement of my conceptual framework of giftedness is that it refers to a **unitary** concept of intelligence, what Spearman (1904, p. 201) called "general intelligence" or *g*. CHC theory, which has been widely accepted by intelligence theorists and test writers in recent years (Beaujean, 2015; Benson et al., 2018; Gross, 2006; Keith & Reynolds, 2010; Lubinski, 2016; McGrew, 2009; Warne, 2016) accepts an underpinning of *g*, although combined with higher-level strata. I will not be defining higher-level strata, but rather looking at other aspects that I consider important, in addition to *g*.

2.3.2 Potential

The second aspect of UPPS theory is **potential**, which is particularly important to consider when studying learners from disadvantaged areas. Although longitudinal studies show that people identified as gifted as children are many more times than the general population to attain PhDs, obtain patents, and show eminence in a variety of fields (Gross, 2006; Kell et al., 2013; Terman et al., 1926), the malleability of intelligence should be taken into account when studying learners from a background of poverty. Socio-economic status is a larger factor in achievement than *g* for those who are born into an economically disadvantaged area. Another reason to overtly state that my theory is potential-based, is that teachers have been found to identify giftedness more by achievement than the theory would support (Neber, 2004). As a researcher working in the area of education, it is important to counter teacher viewpoints that are popular but not borne out by research. A potential-based definition of giftedness also supports the heritability of *g*, which has been established in the literature (Knowles, 2008; Sauce & Matzel, 2018) and enables the inclusion of "twice-exceptional" children in the definition (Wissing, 2012). Unfortunately, in Chapter 3, I will show that my methodology had to diverge somewhat from this ideal concept of giftedness, when choosing participants for my study based on Mathematics marks, which is an achievement-based approach.

2.3.3 Precocity

Thirdly I borrow Piirto's concept of "**precocity**" (Sansom et al., 2018, p. 98) for my definition of giftedness, as it gives a practical framework for teacher interaction with gifted students. She states that gifted children have much in common with older non-gifted learners, and that a teacher can treat a young gifted learner as one would an older child (Sansom et al., 2018). This framework has the advantage of supporting acceleration, which is one of the most successful interventions for gifted children (Wai, 2015). A disadvantage of this component of the framework is that it ignores the emotional level of the young gifted child.

2.3.4 Social-emotional

The last component of the UPPS theory giftedness is the **socio-emotional** one, which I have included to counter the shortcomings of the precocity component. I refer particularly to Dabrowski's "over-excitabilities" (OEs) (Daniels & Piechowski, 2009, p. 9). Over-excitabilities (OEs) are used to describe overreaction to stimuli that is characteristic of gifted people. There are five OEs: psychomotor, sensual, intellectual, imaginational, and emotional (Mendaglio & Tillier, 2006, pp. 70–71). Psychomotor overexcitability refers to greater than normal movement and nervous energy caused by a mind that is running faster than average. It can often be mistaken for Attention deficit-hyperactivity disorder (ADHD) (Mullet & Rinn, 2015). Sensual overexcitability results in heightened awareness of input from the senses. Children with sensual overexcitability can show sensory-defensive behaviour, based on the excessive incoming stimuli. Conversely, a person can derive great pleasure from their sensual OE, appreciating a beautiful sunset or exquisite music on a deeper level than the average person. Intellectual overexcitability is the OE that is most easily linked to intelligence as it relates to an overwhelming urge to learn new things. Parents of children with intellectual OE are often accused of being pushy parents (Daniels & Piechowski, 2009), as other parents find it hard to understand how a child can be so driven to learn. Imaginational overexcitability results in imaginary friends, complicated and creative story-writing and accusations of not living in the real world. The last OE is emotional, which results in very small children with very big thoughts. For example, two of my gifted children asked numerous questions about death at a mere two-and-a-half-years-old, one showing particular empathy for the surviving family of the person who died.

2.3.5 Applying the UPPS framework to gifted disadvantaged learners

The four aspects of the UPPS framework can be applied to children from all socio-economic backgrounds. The concept of *g* is supportive of disadvantaged learners as it acknowledges that learners from all walks of life can be gifted, even if they have not yet developed up to their potential. Precocity and the socio-emotional aspect of the UPPS framework given practical tools for approaching and understanding gifted learners, whatever their background. Acceleration, which is acknowledging precocity in an educational space, is a successful and inexpensive way of handling gifted learners (Wai, 2015), and one that I found was fairly well used in disadvantaged schools in South Africa, as evidenced by the number of 11-year-olds among my study participants (the normal age for a Grade 7 is 12 turning 13). Gifted underachievement has been linked to socio-emotional difficulties (Blaas, 2014): Dabrowski's OEs could be used by teachers to empathise with gifted learners who are intense, rather than rejecting them as badly-behaved, and to identify learners who are underachieving. Worldwide, it has been found that gifted children are vulnerable to dropping out of school (Matthews, 2009; Zabloski, 2010). As gifted people are "a precious human capital resource" (Kell et al., 2013), gifted children not living up to their true potential is challenging, especially in developing country contexts.

2.4 EDUCATION OF GIFTED DISADVANTAGED CHILDREN IN SOUTH AFRICA

2.4.1 Historical provision for gifted children

Given the history of an elitist education for whites and a sub-standard Bantu Education for blacks under Apartheid, it is not surprising that post-1994, government policy has favoured inclusive education and gifted education has taken a back seat (Department of Basic Education, n.d.; Oswald & de Villiers, 2013; Rabie, 2013). According to Reddy (2014), under the Apartheid government, gifted programmes existed for white children only. This statement appears not to be totally accurate, as Eriksson (1987) describes the Schmerenbeck Educational Centre as open to all races, and Dewar (1986) quotes minutes from the First National Workshop on the Education of the Gifted Child in 1978, where management of the centre voted against applying for governmental assistance as it would interfere with the multiracial policy of the centre. However, the picture accompanying Eriksson's article only shows white children, so it is likely that the majority of the 1000 children attending the centre were white. The after-school gifted education centres were disbanded in the 1990s (Van der Westhuizen, 2007), which is a waste of infrastructure that could have been made inclusive of disadvantaged learners.

2.4.2 Current provision for gifted children

The Education White Paper on Special Needs Education does not mention gifted learners (Department of Education, 2001). This implies that gifted learners do not have special needs, and will succeed without any assistance. However, gifted learners do need support to flourish.

This assertion is evidenced by, for instance, a study in Chile. It showed that economically disadvantaged students (who received insufficient support) were less likely to qualify for prestigious universities (Gomez-Arizaga & Conejeros-Solar, 2014). Moreover, some eminent black South Africans (who received more sufficient support) acknowledged the power of a mentor in their own lives (Maree, 2007; Xolo, 2007).

In post-1994 South Africa, the Department of Education has stressed Inclusive Education as an ideal (Mhlolo, 2017a). However, teachers feel ill-equipped to cater to the needs of gifted children (Oswald & de Villiers, 2013) and usually no provisions are made for them in the inclusive education classroom (Marumo & Mhlolo, 2017; Oswald & de Villiers, 2013). Mhlolo (2014b) says there are no schools for the gifted in Sub-Saharan Africa, and there is no teacher training centred around the needs of the gifted, but I found that there are a number of schools that do provide for gifted learners in South Africa. In 2019 there were two schools specifically for the gifted in South Africa: Radford House and the Gifted and Advanced Learning Academy of South Africa (GALASA), both private schools in the northern suburbs of Johannesburg. GALASA closed down at the end of 2019 (D. Silman, personal communication, December 9, 2019), leaving Radford, which caters for learners from Grade 000 to 12 in 2020 as the sole dedicated school for the gifted in South Africa. PE Montessori in Port Elizabeth says the school is suitable for gifted learners (PE Montessori, 2017), and Parkview Junior School, a government primary school in Johannesburg, has a one hour a week programme for gifted learners in grades 2 and 3 (Parkview Junior School, 2018). Centurus Colleges, which consist of the three private schools Pecanwood College, Southdowns College, and Tyger Valley College, also offer enrichment for gifted learners (Centurus Colleges, 2019).

Low-fee or free private schools and scholarships to existing private schools offer enriched opportunities to high achieving disadvantaged learners. Admission to the Oprah Winfrey Leadership Academy for Girls (OWLAG) is on the basis of academic and leadership potential, and financial need, and no fees are charged (Oprah Winfrey Leadership Academy for Girls, 2019). The African School for Excellence in Tsakane, a township in Gauteng, uses an innovative teaching concept, using teaching assistants and online resources such as Khan Academy to reduce direct teaching time by qualified teachers, keeping costs to R7000 per learner per year, most of which is covered by sponsors, leaving only R200 a month for parents to pay. The school has problem solving at its core and uses the Cambridge education system (Fairbanks, 2014). The Royal Bafokeng Trust oversees 46 rural schools (Royal Bafokeng Nation, 2019) and is headed by Ian McLachlan, who was previously head of the prestigious private school, St Stithians College (D. Silman, personal communication, December 9, 2019).

The Royal Bafokeng schools are not specifically targeted at gifted learners but do afford some gifted disadvantaged learners access to quality education. Horizon Education Trust Star College offers scholarships to learners who win the Horizon Mathematics Competition or the Star College entrance examination (Star College Boys High School, 2017). Some other private schools have entrance examinations that disadvantaged learners can use to access private school education (Eden Schools, 2019; Epworth School, 2019; Roedean School (SA), 2019; St Cyprian's School, 2019). Similarly, the Student Sponsorship Programme (SSP) and the Allan Gray Orbis Foundation offer scholarships to a variety of private schools for disadvantaged learners (Allan Gray Orbis Foundation, 2019; Student Sponsorship Programme, 2019). Young Engineers and Scientists of Africa (YESA) offers project-based enrichment to disadvantaged learners that would suit the gifted, but currently there are no Mathematics projects (Young Engineers and Scientists of Africa, 2019). I will discuss provisions for mathematically-gifted learners in the next section.

2.4.3 Mathematics education in South Africa

Mhlolo identifies mathematical competence as "key to the welfare of a nation in the global economy" (Mhlolo, 2015, p. 166) and warns of two groups that are most in danger of not realising their full potential: mathematically-gifted children, and economically disadvantaged children. My study looks at those who are at the intersection of the two groups.

Teacher training in South Africa focuses on mathematical procedural methods rather than problem solving and creative thinking (Engelbrecht & Mwambakana, 2016), and consequently learners, even at good schools, prefer method over understanding (Long & Wendt, 2017). Mhlolo (2017b) recounts an exchange between a gifted learner and a teacher who insisted on using a particular methodology and rejected a creative alternative method that the learner used. The response of another gifted child in the room showed that even the observer found that situation demoralising, let alone the child taken to task for using the "wrong" method. In addition, gifted disadvantaged children in South Africa are likely to be taught by mathematics teachers whose own mathematical reasoning is substandard. Du Plessis (2015, p. 5) observed that "South African Mathematics teachers' competencies compare poorly to those of their counterparts in other Eastern and Southern African countries". Venkat and Spaull (2015) found that 79% of South African Grade 6 mathematics teachers were classified as having content knowledge levels below Grade 6.

2.4.4 Current provision for mathematically-gifted children

Opportunities for mathematics enrichment do exist in South Africa and can be divided into those that are available to learners in more affluent areas, and those available in disadvantaged areas.

Two primary schools in East London offer Mathematics extension: Stirling Primary School has a “gifted mathematicians” group (Stirling Primary School, 2019), and Hudson Park Primary pulls the top 10-12 Mathematics learners from Grade 3 to 7 for extension once a week (Hudson Park Primary School, 2019). These two schools are historically advantaged, in that they were both reserved for white learners under Apartheid, which meant that more resources were spent on them. In the early 1990s these schools became Model C schools, which were partially state-funded but run by a school governing body, which could set fees and control admissions (Christie & McKinney, 2017). Under the new dispensation these schools are still considered advantaged, as the surrounding residents are in the 5th (top) quintile of earning in the country (Province of the Eastern Cape, 2018). In high school, the Independent Examination Board (IEB) offers an Advanced Placement (AP) Mathematics grade 12 course, which covers much of the first semester syllabus at university. Problem solving and critical thinking are required in AP Mathematics. Learners who do AP Mathematics in school, do better in the National Senior Certificate (NSC) grade 12 final Mathematics examination and in university Mathematics than similarly gifted learners who do not do AP Mathematics (Du Plessis, 2015). The IEB is mostly used by private schools, although it is possible for state schools to offer AP Mathematics through the IEB (Independent Examination Board, 2019). This should be considered by schools in disadvantaged areas to extend mathematically-gifted learners and give them a head start on university Mathematics.

There are several options open to learners in disadvantaged areas, including Maths and Science focus schools in the Western Cape, Schools of Specialisation in Gauteng, LEAP Schools, and the University of the Witwatersrand Talent Target Programme (TTP). Both the Maths and Science Focus Schools and the Schools of Specialisation have their roots in the national Dinaledi Schools Programme, which started in 2001. Dinaledi schools were given support and facilities, on condition that 60% of their Grade 10-12 learners were enrolled in Mathematics. By 2015 there were 500 Dinaledi schools countrywide. According to David Silman, former head of the Dinaledi Unit that oversaw the Dinaledi Programme, at this point the programme was extended to include 300 technical high schools and 200 primary schools, which diluted the programme (D. Silman, personal communication, December 9, 2019). The draft norms and standards for focus schools (Department of Basic Education, 2016) state that Mathematics is a compulsory subject at both Mathematics and Science Focus Schools.

Several schools in the Western Cape call themselves Maths and Science Focus Schools, including the Centre for Science and Technology (Lemmon, 2017), the Cape Academy of Maths, Science and Technology (The Cape Academy of Maths, Science and Technology, 2019), and Claremont High School (Claremont High School, 2019). The Gauteng Schools of Specialisation include a variety of different vocational specialisations including STEM (Mthethwa, 2019).

The LEAP schools are no-fee private schools funded by donors. There are currently six LEAP schools, operating in Langa and Gugulethu/Crossroads in Cape Town; Alexandra, Diepsloot, and Ga-Rankuwa in Gauteng, and lastly Jane Furse in Limpopo Province, the only extension opportunity for mathematically-gifted learners in rural areas.

The three specialist school types have common attributes: Mathematics and Science are compulsory, and the schools have lower learner:educator ratios than ordinary schools (Claremont High School, 2019; LEAP Science and Maths Schools, 2019; Mthethwa, 2019; The Cape Academy of Mathematics, 2019). The Western Cape schools and the LEAP schools also have a longer school day. The learners at the Gauteng Schools of Specialisation are selected by an entrance examination written at neighbouring government schools and teachers have been chosen for their higher education levels (Mthethwa, 2019). The LEAP schools provide outreach to nearby ordinary schools in the form of after-school centres and camps staffed by volunteers, including refugees from other parts of Africa, local professionals, and senior learners from the LEAP schools (LEAP Science and Maths Schools, 2019).

The Targeting Talent Programme (TTP), run by the University of the Witwatersrand (Wits) and funded by Goldman Sachs and the Telkom Foundation, identifies learners with high potential in Mathematics and Science before they choose their subjects for the FET phase (Grade 10-12), gives enrichment in mathematics, science, and language, and supports learners with various programmes from then until they are at university (University of Witwatersrand, 2019a).

There are many online opportunities for Mathematics extension, such as IXL, DragonBox, Dreambox, Khan Academy, and The Art of Problem-Solving. Although 22.5 million South Africans have Internet access via cell phones (Department of Basic Education, 2018b), the cost of data is still a barrier to frequent use in the home: out of 230 nations worldwide, South Africa was only the 134th cheapest in terms of data prices per Gb (Howdle, 2019) so these opportunities need to be offered to learners in schools to ensure take-up. Unfortunately, although progress has been made in ICT coverage at South African schools, by 2018 there were still 15448 schools without a computer lab and 14682 schools without internet connectivity (Business Tech, 2019).

Table 1: Summary of extension available to mathematically-gifted learners in South Africa

Extension	School type	Selection	Maths compulsory	Longer day?	Small classes	Outreach	Number of schools	Province
Maths pull-out	Fee-paying State	Top 10-12 Grade 3-7	Yes	N/A	Yes	No	2	E/Cape
Gifted pull-out	Private & State	Yes	No	N/A	Yes	No	4	Gauteng
Radford	Private	IQ test, assessment	Yes	No	Yes	No	1	Gauteng
IEB AP Maths	Private & State	None	Yes	No	Yes	No	2676 Gr. 12 (2018)	All
Dinaledi School	State	Entrance exam	60% of Grade 10-12 must do Maths	No	No	No	500 (2015)	All
Maths and Science Focus School	State	Entrance exam	Yes	Yes	Yes	No	9+ (2019)	W/Cape
School of Specialisation	State	Entrance exam	Yes	No	Yes	No	6 Maths (2018)	Gauteng
LEAP School	No-fee private	Entrance exam	Yes	Yes	Yes	Yes	6 (2019)	W/Cape Gauteng Limpopo
Oprah Winfrey	No-fee private	Entrance exam	No	No	Yes	Yes	1	Gauteng
African School for Excellence	Low-fee private	Interview	Yes	Yes	Yes	Yes	1	Gauteng
Wits TTP Extension	State	65% in Maths, Science & English Gr. 9	Yes	n/a	Yes	No	41 (2019)	W/Cape Gauteng Limpopo
ACE (self-paced)	Private	None	No	No	Yes	No	250+ (2019)	All
Scholarships	Private	Entrance exam	No	No	Yes	No	Unknown	All
Online programmes	Internet access	None	No	No	N/A	N/A	N/A	All

2.5 PROBLEM-SOLVING SKILLS

2.5.1 The value of problem-solving skills

Problem-solving skills are higher-level, creative skills. These skills are beneficial at university and in the workplace (Griesel & Parker, 2009). Matheson (2012) introduced a problem-solving environment in a Grade 10 class. After only three weeks, the students were insisting that they not just be told the answer but also how someone else got to the answer. The process of problem solving, and full understanding of how and why a person got to a specific answer, was valued, not just the end product. This example would, most likely, not work in the type of South African schools (such as the vast majority of township and rural schools (Venkat & Spaull, 2015)) that most urgently need mathematical intervention, as it was dependent on the skill of the teacher, who had a Masters in mathematics, as even the Maths and Science Specialisation Schools teachers have an Honours level education, not Masters.

2.5.2 Current level of problem-solving skills

According to Chirove and Mogari (2014), South African learners are lacking in problem-solving strategies and skills, and mathematics textbooks used in South African schools use routine rather than non-routine problems. The general level of problem-solving skills at school level in South Africa is demonstrated in South Africa's performance in the Trends in International Mathematics and Science Study (TIMSS), which "assesses a range of problem-solving situations within Mathematics, with about two-thirds of the items requiring students to use applying and reasoning skills" (Grønmo, Lindquist, Arora, & Mullis, 2015). South Africa came second-last in Mathematics for Grade 4 and Grade 8, although Grade 5 learners wrote the Grade 4 test and Grade 9 learners wrote the Grade 8 test (Business Tech, 2016). In contrast to South Africa's procedural-based curricula, countries that scored highly on the TIMSS focus on concepts, connections, and problem solving (Mhlolo, 2011). Maree and Erasmus (2006) stress the need for informal Mathematics learning to develop problem-solving skills. This is hard to achieve in a country where parents work long hours and have low levels of mathematics education themselves. Even South Africa's best performers in the TIMSS did poorly in problem solving (Long & Wendt, 2017).

According to Chirove and Mogari (2014), South African teachers cannot do non-routine problems themselves. Govender (2014b) did a study where 14 second-year students studying Mathematics education wrote the Grade 7 SA Mathematics Challenge first round paper. The SA Mathematics Challenge is a Mathematics Olympiad for Grade 4 to 7 learners, which aims "to promote problem solving in Mathematics education" and emphasises participation as an important way to develop such skills (Association for Mathematics Education of South Africa, 2018). Only one student in Govender's study had participated in a Mathematics Olympiad as a learner. Only 28.6% of the student teachers scored a high enough mark to qualify for the second round of the Grade 7 Mathematics Challenge. After the intervention, which consisted of assisting with marking 900 Grade 4-7 Mathematics Challenge papers, participating in a discussion on the paper, and working in groups to categorise the types of questions, their average score improved from 45.72% to 75.4%. These results show two things; firstly, that South African teachers and learners find Mathematics Olympiad type problem-solving questions unfamiliar and difficult, and secondly that relatively limited exposure to such questions can radically improve problem-solving skills.

2.5.3 Current problem-solving programmes for mathematically-gifted learners

Good problem-solvers have meta-cognition developed through problem-solving experience (Nieuwoudt, 2015). Mathematics competitions expose learners to problem solving (Engelbrecht & Mwambakana, 2016). Various mathematics competitions are open to learners in South Africa, including the Horizon Maths Challenge (Grades 5-7), University of Pretoria (Grades 6-11), University of the Witwatersrand (Grade 6-university level), BRICS (Grades 1-12), Nelson Mandela University (Grades 8-12), the UCT Mathematics Competition (Grades 8-12), the South African Mathematics Foundation (SAMF) Mathematics Challenge (Grades 4-7) and the SAMF Mathematics Olympiad (Grades 8-12). Most of these competitions are free, or have free entry for learners from no-fee schools (South African Mathematics Foundation, 2020b; University of Cape Town, 2019; University of Pretoria, 2019; University of Witwatersrand, 2019b). Conquesta, although widely referred to as an Olympiad, uses routine questions such as learners would find in school mathematics textbooks, rather than non-routine problems (Conquesta Olympiads, 2019), so I have excluded it from this list. There are also several free Olympiad training programmes, provided by SAMF, namely the Siyanqoba training and SAMF Olympiad training (Grade 7-12) (South African Mathematics Foundation, 2020a).

2.5.4 The SA Mathematics Challenge

The South African Mathematics Foundation (SAMF) is considered to be the premier Mathematics Olympiad in South Africa (Long, Engelbrecht, Scherman, & Dunne, 2016), with a path from Grade 4 to the International Mathematics Olympiad. The SA Mathematics Challenge is the Grade 4-7 version of this Olympiad, with separate papers for each grade.

Approximately 100 000 children take part in the SAMF Mathematics Olympiad for Grade 8-12 learners annually and 80 000 in the SA Mathematics Challenge (South African Mathematics Foundation, 2020b). This pre-eminence is one reason that I chose to utilise the SA Mathematics Challenge in my study. The other reasons are Govender's study on in-service teachers, and my own experience as a teacher with the SAMF Olympiads. The SAMF Olympiads are well-known, and are the only Olympiads recommended by name in the Mathematics Teaching and Learning Framework (Department of Basic Education, 2018b). The learning framework, developed by the Mathematics Ministerial Task Team, a group of mathematics educators from schools and universities, details how educators should develop conceptual understanding in learners, and includes many examples of non-standard problem-solving questions, in contrast with earlier Department-provided workbooks, like *Mathematics in English Grade 7* (Department of Basic Education, 2018a), which I used as the alternative intervention in my study. Lastly, the SAMF Olympiad past papers and answer sheets for many years are available easily online (South African Mathematics Foundation, 2018), which means that they are available to teachers and learners across the country, as long as they have access to the internet (and preferably a printer).

2.5.5 Assessment of problem-solving skills

There are a variety of tests of problem solving, but a dearth of those normed on South African learners. The *Study Orientation in Mathematics Questionnaire* (*SOM*) (Maree et al., 2011) is normed on South African Grade 7-12 learners with a variety of home languages, including those from disadvantaged areas so is suitable for the schools in my study. The questionnaire as administered to Grade 7 learners consists of 76 questions, answered with a Likert scale, which are then assigned to one of five categories, namely Study Attitude (SA), Mathematics Anxiety (MA), Study Habits in Mathematics (SH), Problem-Solving Behaviour in Mathematics (PSB) and Study Milieu in Mathematics (SM). Study attitude measures the learner's attitude to mathematics and learning mathematics. Mathematics Anxiety measures the level of panic and doubt a learner has about mathematics. Study Habits refers to the learner's time management, focus, and consistent Mathematics practice. Problem-Solving Behaviour measures the learner's level of metacognition in Mathematics, which as mentioned before is important in the development of Mathematical problem-solving skills. Lastly, Study Milieu refers to the learner's home and school environment and barriers to learning Mathematics. This is particularly relevant when studying disadvantaged learners. A composite score is also calculated, which is considered to be "a measure of a learner's study orientation" (Maree, 2020).

2.6 SUMMARY OF CHAPTER 2

In this chapter I discussed the four axes of contention when defining giftedness: cognitive vs. multiple intelligences, aptitude vs. achievement, nature vs. nurture, and community vs. the individual. I then looked at identification of the gifted in South Africa, the tests that are available, and their pitfalls. This led to a description of my own conceptual framework of giftedness, which is based on a **unitary** intelligence, from which **potential** can be developed, assisted, and/or hindered by **precocity** and **socio-emotional** factors.

I looked at the education of gifted children in South Africa, both historically and currently, and in particular the offerings for mathematically-gifted children, noting which options were available for the cohort of my study, mathematically-gifted children in disadvantaged areas. Lastly, I examined problem-solving skills, starting with the value of problem solving for gifted disadvantaged learners and the community as a whole. I then investigated the current level of problem-solving skills in South Africa and the problem-solving programmes available to gifted disadvantaged children in South Africa, before looking at the SA Mathematics Challenge in particular. I ended the chapter by describing the assessment of problem-solving skills, and the *Study Orientation in Mathematics*, that I have chosen to use to evaluate the development of problem-solving skills in the participants in my study. In the following chapter I detail the research methodology for my study, and the accompanying ethical considerations.

CHAPTER 3 – RESEARCH METHODOLOGY

Chapter 3: Overview

In this chapter, I start by anchoring my study in my own epistemology, and then narrow my focus to the particular methodology and design chosen for this study, explaining why they are suitable for the study. Next, I discuss the selection of the participating schools and learners for the study, and the rationale behind the selection. I then describe what I did in each session of the study, to enable replication of the study. Lastly, I go into the ethical considerations of doing such a study.

3.1 EPISTEMOLOGY OF THE STUDY

My chosen paradigm for this study is that of critical realism informed by pragmatism. According to Cruickshank (2011), critical realism is a type of post-positivism. According to Chilisa and Kaluwich (2012, p. 8) "post-positivists, like positivists, believe that there is a reality independent of our thinking that can be studied". However, critical realism "recognises that knowledge is fallible and thus open to revision and replacement" (Cruickshank, 2011, p. 4). Similarly to critical realism, pragmatism accepts the fallibility of knowledge, but in addition it emphasises practicality (Ormerod, 2006; Reason, 2003; Sefotho, 2015).

I chose to view this study from the viewpoint of critical realism, firstly because it resonates with my own worldview that reality exists as a concept, but it is not immutable, at least not from the human perspective, where we are continually updating our views of reality as research adds new viewpoints and value (Guba & Lincoln, 1994). In a multicultural society, it would be inappropriate to think that only the beliefs and views that I grew up with are valuable, and as I am exposed to new people, and their cultures, I am able to adapt and refine my views. In a similar way, research in a multicultural society needs to adapt from a Eurocentric view to see from multiple cultural stances (Stones, Maree, & Jordaan, 2021). The advantage of the pragmatic prism to my lens is the emphasis on the practical: research does not have to reach an ideal that might be unattainable to contribute to the body of knowledge on gifted education in South Africa, or to benefit gifted disadvantaged children.

One potential disadvantage of using any post-positivist paradigm with social science is that post-positivism is concerned with prediction (Guba & Lincoln, 1994) and prediction is more difficult in social science as the latter involves people (Barnes et al., 2012). To mediate this shortcoming of the paradigm, first, I made the research questions as simple as possible, to avoid confusion. Second, my extra perspective of pragmatism means that any shortcomings of critical realism can be weighed against the benefits of the paradigm, as pragmatists take value where they find it (Ormerod, 2006).

3.2 METHODOLOGICAL APPROACH

The methodological approach I chose was QUAN→qual. This means that it was primarily a quantitative study, but with one qualitative methodological instrument used. The arrow indicates that the phases of the research were carried out sequentially; first quantitative and then qualitative (Johnson & Onwuegbuzie, 2004).

3.2.1 Quantitative methodology

Quantitative methodology is defined by Keele (2011, p. 35) as a “formal, objective, deductive approach to problem solving”. The methods used include experimental, quasi-experimental, correlational, and descriptive (Keele, 2011). It is usually evaluated using statistical analysis (Barnes et al., 2012; Keele, 2011; Slevitch, 2011; Thomas, 2010).

Some might see the choice of quantitative research methodology as only suited for the positivist or post-positivist paradigm, but there are others who eschew limiting methodology choice very narrowly depending on the paradigm. Guba and Lincoln (1994, p. 105), say that “both qualitative and quantitative methods may be used appropriately with any research paradigm” and this is supported by other theorists (Knox, 2004).

The benefits of the quantitative methodology vest predominantly in the ability to make predictions, and to generalise the findings to the wider community (Guba & Lincoln, 1994). Prediction is important in education where there are hundreds of thousands of children who are both gifted and disadvantaged in South Africa. The aim of my research is to, hopefully, ultimately, have an impact on the lives of many other children, not just the ones who participated in my study.

One criticism of quantitative research is that it ignores the uniqueness of individuals and differences in their experience (Keele, 2011). To counter this, I employed both quantitative and qualitative methods. First, using quantitative methodology, I involved a large sample size and chose schools from the bottom two quintiles, to maximise the percentage of disadvantaged children in my sample. Secondly, using qualitative methodology, after each session of the *Study Orientation in Mathematics* (Maree et al., 2011), I conducted a focus group to examine the learners’ experience of using this instrument. This was also important from the perspective of ethics, and I will cover this in more detail later in this chapter. Another potential disadvantage of using a quantitative methodology is that it does not get to the why of the problem, only the what (Thomas, 2010). I do not claim to have solved this issue in my study, but accept the limitations of this type of research in favour of its strengths.

3.2.2 Qualitative methodology

Qualitative and quantitative methodologies can complement each other's shortcomings (Erasmos, 2013). The aim of qualitative methodology is to look from the perspective of the participants of a study. It does not attempt generalisability or objectivity, and sample size is unimportant (Slevitch, 2011). The advantage of qualitative research methods is that they can delve deeper into the personal experience of participants. The pragmatic viewpoint of my epistemology enabled me to use a focus group in a study that was largely quantitative, because it was of practical benefit to my study and study participants.

3.3 RESEARCH DESIGN

The following table summarises my research methodology and design:

Table 2: Summary of Research Methodology for my Study

Epistemology	Critical realism with pragmatism			
Approach	QUAN→qual			
Design	Quasi-experimental			
Research question	How valuable is participation in the SA Mathematics Challenge for developing problem-solving skills in mathematically-gifted disadvantaged learners (MGDL)?			
Secondary research questions	What are the essential aspects of current (group-based) programmes aimed at enhancing the problem-solving skills of MGDL?	What is the SA Mathematics Challenge?	What is the impact of three hour-long facilitated sessions doing SA Mathematics Challenge past papers on MGDL's study orientation in mathematics in general?	What is the impact of three hour-long facilitated sessions doing SA Mathematics Challenge past papers on MGDL's problem-solving skills in particular?
Null hypotheses	There are no significant differences between the pre-test and post-test mean scores for the test group		There are no significant differences between the post-intervention scores of the two groups	
Participant schools	<u>School 1:</u> Quintile 2 Urban township		<u>School 2:</u> Quintile 2 Urban township	
Participant learners	Grade 7 learners Top 50 out of 355 (selected by Grade 6 Mathematics mark)		Grade 7 learners Top 50 out of 340 (selected by Grade 6 Mathematics mark)	

Note. Adapted from *https://repository.up.ac.za/handle/2263/28984*. Copyright (2011) by J.J. Botha.

3.3.1 Experimental design

Experimental design is known as the "gold standard" of quantitative research design, because it "provides the most convincing evidence to support the value of a treatment" (Keele, 2011, p. 41). An experimental design has a control group that does not receive the treatment or intervention, and sampling is randomised (Keele, 2011). According to Muijs (2011), an experimental research design is best at determining a causal link between two variables. To determine causality between two variables, one needs a relationship between the variables, time order (which is controlled by the researcher in an experiment) and to eliminate confounding variables (which is better controlled in experimental than non-experimental research). This is very useful in educational research, where potential educational practices
are put to the test in advance of rolling them out in the classroom.

3.3.2 Quasi-experimental design

Educational research takes place in the real world, where it is hard to control all variables or organise a truly randomised trial. In addition, it is unethical to withhold an intervention from a control group. As a result, educational research often makes use of a quasi-experimental design (Muijs, 2011). A quasi-experimental design is similar to an experimental design, but either missing randomisation, or a control group (Keele, 2011). I used a quasi-experimental design, without true randomisation. This is because of the logistical difficulties associated with implementing true randomisation of the sample in a school environment where children are grouped into classes, and the choice of the school itself is not truly random.

As with experimental designs, quasi-experimental designs facilitate prediction – albeit to a limited extent only (Guba & Lincoln, 1994) and allow for findings to be generalised to the population from which the sample was drawn (Keele, 2011). It may not be possible to convincingly demonstrate a causal link between the treatment condition and observed *outcomes*. My study aimed for generalisation of findings to gifted disadvantaged learners across the region from where the learners in my sample came so this means a quasi-experimental research design was appropriate for the study. A disadvantage of a quasi-experimental design, compared to a true experimental design, is that the causal link is not definitely proved, but rather inferred. This is due to the lack of randomisation (Keele, 2011). I attempted to make the selection of participants as random as possible, as described in the next section. Another problem with experimental and quasi-experimental designs is that a researcher can introduce personal bias into the study, but "bias does not limit an ability to be reflective" (Barnes et al., 2012). I attempted to overcome personal bias by recording what I expected to happen, and by careful consideration of how to introduce the study to participating schools, teachers and learners, to minimise passing on my own bias to the participants in the study.

3.3.3 Non-equivalent comparison group design

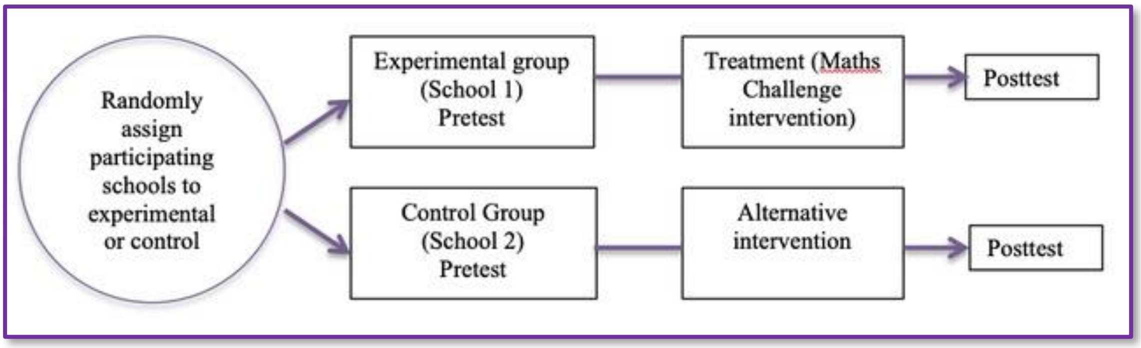


Figure 1: Non-equivalent Comparison Group Design.

Adapted from "The quantitative research process" by K. Maree and J. Pietersen, 2016, First steps in research, p. 168. Copyright 2016 by K. Maree and J. Pietersen.

I used a Non-equivalent Comparison Group Design, which is a quasi-experimental version of the Pretest-Posttest Comparison Group Design (Engel & Shutt, 2014). There are two groups in a Comparison Group Design, one of which receives the treatment or intervention and one that receives a different intervention. The disadvantage compared to using a traditional control group is that both groups receive some sort of intervention, so it is a comparison of interventions rather than comparing what would happen if there were no intervention. However, it is generally accepted that it is only right and ethical to offer both groups some benefit in the study. The word "non-equivalent" is included in the design name because the two groups are not randomly assigned, it is not known whether the groups are truly equivalent (Engel & Shutt, 2014). I tried to approximate equivalence, as described below.

3.4 SAMPLING OF PARTICIPANTS

3.4.1 Selection of schools

I used a two-step approach to sampling. Firstly, I utilised convenience sampling, choosing schools in my home province, and open to participating in the study. This means that the sample was not truly random. The disadvantage of non-probable sampling is that the findings cannot be generalised to the general population of school learners. However, from my literature study, I have found that non-probable sampling is a common method in education research (Bickell, 2016; Du Plessis, 2015; Jenkins, 2004; Lombard & Grosser, 2008). I also attempted to ameliorate the disadvantage of convenience sampling by using purposive sampling to choose the two schools to participate in my study, aiming to match them as closely as possible to each other, since for practical reasons I would be administering the intervention at one school and the alternative intervention at the other school, rather than assigning learners randomly to the intervention and "control" group. I looked at the list of Gauteng schools (Department of Basic Education, 2017a) and chose two large quintile 2 schools in the same township. Quintile 2 schools are in communities where the residents are in the bottom 40% of South Africa economically. All schools in quintiles 1 to 3 are no-fee schools, so children attending them would be considered to be disadvantaged. School 1, where I administered the intervention, had 355 learners in grade 6 at the end of 2018, and School 2, where I administered the alternative intervention, had 340 learners in grade 6 at the end of 2018.

3.4.2 Selection of learners within the chosen schools

Giftedness and mathematical giftedness are defined in a variety of ways (Mhlolo, 2015; Semakane, 1994; Zaram, 2016). I would have liked to have used the cut-off that the high-IQ society, Mensa, uses for selection, which is the top 2% (Mensa South Africa, 2018). However, this would either have meant that the sample from each school would have been only seven learners, which would have been too small to make statistical inferences, or I would have had to populate the experimental group from multiple schools, which would have introduced logistical complications. I decided to choose 50 learners from each school, because "sample size is critical in quantitative research. A large sample ensures better representativeness and generalisability of findings as well as proper use of statistical tools" (Slevitch, 2011, p. 76).

I did not use an IQ test for selection. Firstly, as discussed in Chapter 2, IQ testing is a highly contested subject in South Africa, due to norming difficulties in a multicultural and multilingual environment (Bouwer, 2014; Erasmos, 2013; Knowles, 2008; Maree, 2018b, 2018a; Mawila, 2012; Zygmont, 2006). Even non-verbal tests such as a Raven's Progressive Matrices (RPM) and the Naglieri Nonverbal Ability Test (NNAT) have shortcomings for use with English second-language speakers (Lohman, Korb, & Lakin, 2008). Secondly, IQ testing is costly and time-consuming. An alternative would have been teacher identification, but giftedness is given little emphasis in teacher training in South Africa (Van der Westhuizen & Maree, 2006). Parent identification of the gifted is generally better than teacher identification (Dağlioğlu & Suveren, 2013; Gross, 1999) but contact with parents in a quantitative study with large numbers of learners would have been impractical.

A requirement of my study was that the learners have a sufficient grasp of the basic concepts of mathematics for their grade, as it is impossible to access higher-level learning such as problem solving without a basic understanding of concepts (Johnson & Schmidt, 2006).

Taking the issue of gifted identification together with the requirement for basic mastery of mathematical concepts, I decided to use the learners' mathematics marks to identify mathematically-gifted learners. Therefore, for the purposes of this study, the definition of mathematically gifted was the top 50 of the grade by mathematics marks at the end of Grade 6, which worked out to the top 14.1% of the grade in School 1 and the top 14.7% of the grade in School 2. I asked both schools to provide me with a list of the top 50 Mathematics learners in Grade 6 in 2018. The Mathematics marks for the top 50 in School 1 ranged from 51% to 90% and in School 2, the range was from 58% to 84%. Consent and assent forms were distributed to the chosen learners by each school, and in the case of School 2, a parent information evening was held by the deputy principal, to ensure buy-in by the parents and learners.

3.5 DESCRIPTION OF THE STUDY

3.5.1 The schools and the timetable of the study

Table 3: Timetable of the Study

	School 1			School 2		
Week	**Date**	**Activity**	**Learners**	**Date**	**Activity**	**Learners**
1	29 Jan. 2019	*SOM* Focus group	45 8	11 Feb. 2019	*SOM* Focus group	44 10
2	5 Feb. 2019	SA Maths Challenge 2013	43	18 Feb. 2019	DBE worksheets 1-3	46
3	12 Feb. 2019	SA Maths Challenge 2014	40	25 Feb. 2019	DBE worksheets 4,5	45
4	19 Feb. 2019	SA Maths Challenge 2018	28	4 Mar. 2019	DBE worksheet 6	40
5	26 Feb. 2019	*SOM* Focus group	27 8	11 Mar. 2019	*SOM* Focus group	44 10

The study took place at two quintile 2 urban schools in Gauteng, in the first term of 2019, and the participants were Grade 7 learners. Each session consisted of an hour after school. At School 1, I had an initial meeting with the Head, then the Head of the Mathematics department assisted me with selection of the participants, and thereafter one of the Mathematics teachers took me to the classroom before each session. At School 2, I spoke to the Deputy Head, who organised a parent information evening, although I was only informed about this after it had taken place. As a result of the parent information evening, I was requested to provide participation certificates for the children who took part in the study, which I agreed to do, and these were handed out at the end of the fifth and final session. Other requests were made that could not be entertained, due to university policy. School 1 did not ask for participation certificates, so I did not provide them. The differences of the parent information meeting and participation certificates may have influenced attendance at the sessions, which you can see from the above table, dropped off in the later sessions for School 1 but not for School 2.

3.5.2 Problem-solving skills assessment: Study Orientation for Mathematics (SOM)

I used the *Study Orientation for Mathematics* (*SOM*) (Maree et al., 2011), as the pre-test and post-test. I scored the pre- and post-tests according to the Scoring Key, and recorded the results in my spreadsheet.

3.5.2.1 Psychometric properties of the SOM

The *SOM* is designed for Grade 7-12 learners (Maree et al., 2011). Advantages of the *SOM* include that it was normed on learners from different language and socio-economic groups in South Africa (Maree, Van der Walt, & Ellis, 2009) and it is quick to administer, and does not require a psychologist to administer (Maree, 2020). The overall aim of the instrument is to identify learners with a negative study orientation in Mathematics, and to gain a greater understanding of learners who are not achieving in the subject (Maree et al., 2011). To this end, it is not an exact match with my study, which was targeting mathematically-gifted learners. However, the fourth sub-test is in line with the requirements of my study, namely to assess problem-solving behaviour in Mathematics.

The sub-tests of the *SOM* for Grade 7-9 learners consist of Study Attitude, Mathematics Anxiety, Study Habits, Problem-Solving Behaviour and Study Milieu. Study Attitude (14 questions), covers the feelings and attitudes that learners have towards mathematics. Mathematics Anxiety (14 questions) covers the degree to which the learners exhibit anxious behaviours such as sweating, nail-biting, and/or frequent trips to the toilet. Such anxiety gets in the way of rational thought, and inhibits learners from asking questions and taking risks, which in turn hampers success in Mathematics. Study Habits (17 questions) refers to consistent study habits such as practising examples, learning theorems, and doing assigned work diligently. Problem-Solving Behaviour (18 questions), which is the sub-test that most relates to my study, includes the "cognitive and meta-cognitive learning strategies in Mathematics" (Maree et al., 2011, p. 11), or the act of self-reflection when approaching problem solving in Mathematics. Study Milieu (13 questions) highlights the impact of socio-economic situation and home language vs. language of learning on learners. Milieu issues include whether learners have space, light, and facilities to do homework at night, and whether the language of Mathematics is confusing to them. Information Processing (16 questions) is only assessed in learners from Grade 10-12, and covers "general and specific learning, summarising and reading strategies, critical thinking and understanding strategies (like the optimum use of sketches, tables, diagrams)" (Maree et al., 2011, p. 12). In the following sections, I will cover the standardisation, validity, and reliability of the *SOM*.

3.5.2.2 Standardisation of the SOM

The *SOM* was normed on 3013 Grade 8-11 learners at high schools across South Africa, with the expectation that the norm table for Grades 8 and 9 could be used for Grade 7 learners, and the norm table for Grades 10 and 11 could be used for Grade 12 learners. The samples in the initial study by Maree et al. (2011) were chosen randomly on three levels: the education department of the learner (which until only a few years previously had been racially segregated, and so could be used as a proxy for race), language of instruction, and area (urban or rural). This sampling resulted in a spread of race and language group reflecting the general high school population, including black learners from disadvantaged urban schools such as the participants in my study.

Maree et al. (2011) also analysed the results by language group, with the following groups identified:

- African language speakers completing the English questionnaire
- English speakers completing the English questionnaire (i.e. home language)
- Afrikaans speakers completing the Afrikaans questionnaire (i.e. home language).

The learners in my study all fell into the first category, with two possible exceptions, one of whom wrote down both English and Zulu for home language, and the other of whom wrote down English, Xhosa, and Zulu.

A comparison of the averages and standard deviations for boys and girls in the *SOM* show statistical differences by gender for the Mathematics Anxiety and Study Milieu and for the questionnaire as a whole in Grades 8 and 9, where girls are more anxious about Mathematics, and their Study Milieu is less than that of boys. The situation is reversed in Grade

10, when the study sample only includes learners who have chosen to do Mathematics as a subject for their final three years of school. In Grades 10 and 11 girls outperformed boys on all fields except Information Processing (Maree et al., 2011).

3.5.2.3 Validity of the SOM

i. Content validity

The content validity of a test refers to whether the individual test items cover the correct content (Mertens, 2015; Muijs, 2004). In the case of the *SOM*, this would mean analysing whether the questions measure Study Attitude, Mathematics Anxiety, Study Habits, Problem-Solving Behaviour, Study Milieu, and Information Processing. For example, a question about the learners' home environment would be appropriate to measure Study Milieu, but not to measure Mathematics Anxiety. Content validity can be judged by a literature search, assessment by experts in the fields, and even by asking people from the target group of the test (Muijs, 2004). The authors of the *SOM* tried to ensure the content validity of the *SOM* by reviewing the literature on the subject, getting experts to check the ordering and wording of questions, checking the item field correlations, and checking with experts whether all the important aspects of each item were included (Maree et al., 2011).

ii. Construct validity

Construct validity refers to whether the instrument as a whole measures the theoretical constructs that it purports to measure (Maree et al., 2011; Muijs, 2004). To determine construct validity in the original study, Maree et al. (2011) examine the inter-correlations between the test items. The five items assessed for Grade 7-9 learners are Study Attitude, Mathematics Anxiety, Study Habits, Problem-Solving Behaviour, and Study Milieu. Low correlations would be expected between the fields since they are considered to be discrete aspects of Study Orientation. However, some fields have high correlations. Study Attitude correlates with Study Habits; Study Attitude correlates with Problem-Solving Behaviour; Study Habits correlate with Problem-Solving Behaviour, and Mathematics Anxiety correlates with Study Milieu (Maree et al., 2011). Additionally, there is a low correlation between two distinct groupings of items. Study Habits, Study Attitude, and Problem-Solving Behaviour combine to measure "academic behaviour in Mathematics" and Mathematics Anxiety and Study Milieu combine to measure "helplessness, anxiety and lack of control… in Mathematics" (Maree et al., 2011, p. 45).

iii. Criterion-related validity: Concurrent validity

Concurrent or simultaneous validity refers to the extent to which an instrument reflects current behaviour (Mertens, 2015). To determine the concurrent validity of the *SOM*, it was compared to two existing tests, the *Diagnostic Tests in Mathematical Language (DTML)* and the *Achievement Test in Mathematics (ATM)*. All the items except Problem-Solving Behaviour correlated at the 1% level. Maree et al. (2011) speculate that the lack of correlation in this sub-test is due to the questions in the *DTML* and the *ATM* not requiring problem-solving skills to answer successfully, which, far from being a draw-back of the *SOM*, show its unique benefit.

iv. Criterion-related validity: Predictive validity

Predictive validity refers to the extent to which a certain instrument can predict what a person will do in the future (Mertens, 2015). A study of the *SOM* in relation to school mathematics results in the Northern Cape found that for "both genders and across all three race groups, the set of study orientation scales contributed significantly (at the 1% level) to the explanation of variance in mathematics achievement for Grade 9 learners" (Moodaley, Grobler, & Lens, 2006, p. 652), demonstrating clear predictive validity for the *SOM*, at least for white, black and so-called "coloured" (mixed-race) learners in ex-model C schools.

3.5.2.4 Reliability of the SOM

A highly reliable test instrument has low measurement error. This is measured by comparing test and retest scores and coming up with a correlation or reliability coefficient. A coefficient of 0.7 is considered acceptable for research purposes (Muijs, 2004), such as the situation where I used the *SOM*. In the original study by Maree et al. (2011), the reliability coefficients for the different fields for African language learners who did the test in English (the same demographic as the learners in my study) range from 0.67 to 0.77, with overall reliability of all the fields together at 0.89. This is somewhat lower than for the learners who did the test in their home language, as can be seen from Table 4.

Table 4: Reliability coefficients for the different fields for Grades 8 and 9 by language group

Fields	**African languages (N=955)**	**English (N=119)**	**Afrikaans (N=167)**
1	0.73	0.86	0.80
2	0.72	0.84	0.87
3	0.77	0.88	0.87
4	0.67	0.82	0.82
5	0.69	0.74	0.83
SOM total	0.89	0.95	0.95

Note. Adapted from *Manual for the Study Orientation Questionnaire in Mathematics* (Maree et al., 2011, p. 40).

3.5.3 Focus group

As the learners were at the younger end of the spectrum of the norming of the *SOM*, and doing the assessment in English, rather than their mother tongues, I did all that I could to assist with clear understanding of the vocabulary used in the instrument. At the beginning of each administration of the instrument, I explained that if there were any words that they did not understand, the learners should put up their hands to ask me for the meaning. Each time a learner asked the meaning of a word, I thanked the learner for helping their classmates, and wrote the word, and an easier equivalent, on the board, and gave a brief verbal explanation of the word to the whole group. Such assistance with English vocabulary is allowed in the test instructions, which state that testers "may answer questions on the instructions or meaning of words, provided that they can do this without influencing learners' answers" (Maree et al., 2011, p. 17). The same words came up in all four sessions of the *SOM*, and several were also mentioned in the focus groups. After each administration of the *SOM* I ran a focus group to find out how the learners viewed their experience of taking part in the *SOM*, asking the following questions:

1. **Have you seen a questionnaire like the *SOM* before?**
2. **What did you think of the *SOM*?**
3. **Did you understand all the questions in the *SOM*? Which didn't you understand? What did you not understand about each?**
4. **Would you have preferred to answer the *SOM* in another language? Which?**
5. **Were there any questions you particularly liked answering? Why?**
6. **Were there any questions you didn't like answering? Why?**
7. **Do you have anything else you would like to share with the group?**

3.5.4 Intervention

3.5.4.1 Summary of the intervention

Table 5: Summary of the intervention

The intervention consisted of three hour-long facilitated sessions where the learners worked through past papers of the SA Mathematics Challenge for Grade 7. Learners from School 1 participated in the experimental intervention. The sessions took place once a week on a Tuesday, straight after school so that learners did not miss regular classes. The learners were seated in double desks, and were given one past paper to share between two learners at a desk. There were enough desks that a few learners could sit alone. I encouraged the learners to work in pairs, and to discuss their answers, but also said that they could work alone if they preferred. Most learners chose to work in pairs.

Week	1	2	3
Activity	SA Maths Challenge 2013 First round	SA Maths Challenge 2014 First round	SA Maths Challenge 2018 First round
Attendance	43	40	28
Working together or alone	Learner choice – mostly pairs		
Teacher input	Minimal, tried to get learners to think through problems themselves		
Homework	Six learners took worksheet home	Six learners took worksheet home	Two learners took worksheet home
Feedback on previous week	N/A	Answer sheet and went over common errors	Answer sheet and went over common errors

At the first session, I gave the learners the Grade 7 SA Mathematics Challenge first round paper for 2013; at the second session, they were given the first-round paper for 2014, and at the final session they received the first-round paper for 2018. The first-round paper was chosen, as it is easier than the second-round paper, which tends to have the later questions on a similar level to the grade above.

After each session, I marked the past paper done by each learner, noting in my spreadsheet how many questions were completed, and how many were correct. This was to explore the possible effect of more or less practice. Learners were encouraged to take home the past papers to complete at home. Six learners did this after each of the first two sessions, and two after the last session. After seeing how poorly the learners did on the Olympiad type questions, I decided not to return papers with a mark on the top of the sheet, because I felt it would be bad for morale and would also put an emphasis on scores rather than on the process of learning through making mistakes and trying again. Instead, the following week I gave each learner an answer sheet with answers to the SA Mathematics Challenge questions. These are provided on the SA Mathematics Challenge website, and give both the correct answer and a brief explanation. I also explained a selection of the answers to the learners, choosing questions where many people had made the same type of mistake.

3.5.4.2 Example intervention questions

Below are examples of the type of question asked in the SA Mathematics Challenge (South African Mathematics Foundation, 2018). All the papers used can be found in the annexures.

6. How many whole numbers between 1 and 1 000 are divisible by 5 and 6?

6. Hoeveel heelgetalle tussen 1 en 1 000 is deelbaar deur 5 en 6?

(A) 31 (B) 32 (C) 33 (D) 34 (E) 35

7. The median of five numbers is 5 and the mode is 6. What is the largest possible sum of the five numbers?

7. Die mediaan van vyf getalle is 5 en hul modus is 6. Wat is die grootste moontlike som van die vyf getalle?

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

8. In which column is the number 856 in this table?

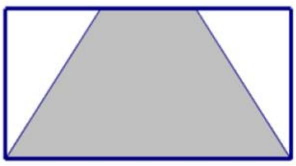
8. In watter kolom is die getal 856 in hierdie tabel?

A	B	C	D	E	F	G
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
⋮	⋮	⋮	⋮	⋮	⋮	⋮

(A) B (B) C (C) D (D) E (E) F

9. In this rectangle, the top side has been divided into thirds. What fraction of the rectangle has been shaded?

9. In hierdie reghoek is die boonste sy in derdes verdeel. Watter breuk van die reghoek is verdonker?



3.5.5 Alternative intervention

3.5.5.1 Summary of the alternative intervention

Table 6*:* Summary of the Alternative intervention

Week	1	2	3
Activity	Maths in English Book 1 Worksheets 1-3	Maths in English Book 1 Worksheets 4-5	Maths in English Book 1 Worksheet 6
Attendance	46	45	40
Working together or alone	Learner choice – mostly alone		
Teacher input	Minimal, tried to get learners to think through problems themselves		
Homework	Five learners took worksheet home	Six learners took worksheet home	No learners took worksheet home
Feedback on previous week	N/A	Answer sheet & went over common errors	Answer sheet & went over common errors

For the alternative intervention, I followed the same format as the intervention, so there were also three weekly sessions, each lasting an hour, taking place straight after school. The alternative intervention sessions ran on a Monday at School 2, whereas the intervention sessions ran on a Tuesday at School 1. Both sets of sessions took place in the first term, although School 1 started and finished two weeks earlier than School 2. My aim was to make the only difference between the two schools be the worksheets that the learners were given. As at School 1, the learners were seated in double desks, and were given one worksheet to share between two learners at a desk. I encouraged the learners to work in pairs, and to discuss their answers, but said that they could work alone if they preferred. In contrast to the intervention group, most learners chose to work alone.

In the alternative intervention, the worksheets were taken from *Mathematics in English Book 1: Grade 7 book 1 terms 1 & 2* (Department of Basic Education, 2018a). In the first session I gave them worksheet *1: Commutative property of addition and multiplication*, worksheet *2: Associative property of addition and multiplication*, and worksheet *3: Distributive property of multiplication over addition*. In session 2 I gave them worksheet *4: Zero as the identity of addition, one as the identity of multiplication and other properties of numbers*, and worksheet *5: Multiples*, and in the third session I gave them worksheet *6:*

Divisibility and factors. As with the intervention, learners were encouraged to take worksheets home to complete for homework. Five learners did this after the first session, and six after the second session. One learner who completed not only worksheets 4 and 5 but also worksheet 6 for homework after the second session was given worksheet *14a: Square and cube numbers* to do during the third session.

After each session I marked the papers and recorded how many questions each learner completed, and how many were correct. There were far more questions in the alternative intervention than the SA Mathematics Challenge, because the questions were simple drill questions, rather than complex problem-solving questions. As with the intervention, I gave feedback at the start of the next session, in the form of an answer sheet and going over a few common errors on the board. I drew up the answer sheets myself as I was not able to find answer books for *Mathematics in English Book 1: Grade 7 book 1 terms 1 & 2* (Department of Basic Education, 2018a). Even though the marks were better than in the intervention, for the purposes of uniformity, I did for School 2 as School 1 and I did not give learners back their papers or give them their marks.

3.5.5.2 Example of alternative intervention questions

Below are examples of the type of question asked in the *Mathematics in English Book 1:*

1. Use zero as the identity of addition, or one as the identity of multiplication to write a sum for the following:

		Zero as the identity of addition	One as the identity of multiplication
a.	5	5 + 0 = 5	5 × 1 = 5
b.	7		
c.	9		
d.	100		
e.	34		
f.	2,5		
g.	0,1		

2. Use zero as the identity of addition, or one as the identity of multiplication to solve the following:

a. $b + 0 = \square$ b. $d \times \square = d$ c. $e \times 1 = \square$

$b \times 1 = \square$ $d + \square = d$ $e + 0 = \square$

3. Choose the correct property of number to write and equivalent statement to complete the equation.

a. 4 + 5 = 4 + 5 = 5 + 4
b. 2(3 + 9) =
c. 3 + (4 + 8) =
d. 5(9 – 8) =
e. 9 + 12 =
f. (2 × 5) × 11 =

3.6 DATA ANALYSIS

According to Maree (2016), post-positivist-orientated researchers favour a deductive data analysis strategy, and interpretivists favour an inductive strategy. In line with the pragmatic approach, which takes what is useful from any approach, data analysis in my study was conducted both deductively and inductively.

3.6.2 Quantitative data analysis

As my study was mostly quantitative, the primary data analysis used statistical analysis. This was done with the assistance of the Statistics department at the University of Pretoria and will be covered in more detail in Chapter 4.

3.6.3 Qualitative data analysis

The amount of qualitative data generated was minimal, arising from the focus groups held after the administration of the *SOM*. This was analysed thematically, both deductively and inductively. The focus group questions were designed firstly to evaluate whether the experience of participating in the *SOM* was in any way detrimental to the young participants, and secondly to check on the effect of being assessed in English as opposed to their home languages. The answers were also assessed to look for themes that I had not anticipated.

3.7 QUALITY ASSURANCE

Quality assurance can be defined as the process of evaluating to what extent results are both consistent over time and reliable in comparison to real world experience (Maree, 2016).

3.7.1 Quantitative quality assurance

In quantitative research, validity and reliability need to be assessed to determine the quality of the research (Mertens, 2015).

3.7.1.1 Internal validity of quantitative data

There are a number of threats to internal validity of quantitative data (Engel & Shutt, 2014; Maree & Pietersen, 2016; Mertens, 2015), which are detailed in the following sections.

i. History

History refers to the external events that take place during the time frame of the study but could influence the participants in the study (Maree & Pietersen, 2016; Mertens, 2015). I attempted to counter this threat by running my intervention and alternative intervention as close to each other as possible, and both on weekdays after school. In the end, they ran 13 days apart, but still during the first term of the school year. One event that I was not able to anticipate was that the organising teacher at School 2 arranged a parent-teacher meeting prior to the study, where the parents requested participation certificates for the learners. This may have served to artificially reduce attrition rates at School 2.

ii. Maturation

Maturation refers to the natural growth of the participants in terms of skills and age during the study (Mertens, 2015). This is unlikely to have influenced my study much, as it was only five weeks in duration.

iii. Instrumentation

Instrumentation is said to have been a problem if the pre- and post-tests differ (Mertens, 2015), which they did not in my study, or the instrument used is not reliable itself. The *SOM* is highly reliable (Maree et al., 2011).

iv. Testing

A definite threat to my study was that the pre- and post-tests were administered only four weeks apart. This could result in learners remembering what they answered the previous time, and answering in the same way in the post-test. A mitigating factor is that this effect would have had the same influence on both the intervention and alternative intervention groups, as they both wrote the pre- and post-test four weeks apart.

v. Statistical regression

Statistical regression refers to the statistically noted effect that if a person gets an outlying result in the first assessment with an instrument, subsequent scores tend to move towards the mean (Maree & Pietersen, 2016). In Chapter 4 I will explore whether this happened.

vi. Selection bias

Selection bias happens if the two groups being compared differ (Maree & Pietersen, 2016; Mertens, 2015). The way to counter this is to use entirely random selection. Due to practicalities I was not able to use randomisation, so I attempted to match the schools as closely as possible. The range of Grade 6 Mathematics marks did differ between the two groups, with School 1's marks ranging from 51% to 90% and School 2's marks from 58% to 84%. In Chapter 4 I will examine whether the difference between these ranges is significant.

vii. Mortality

Mortality refers to changes in the groups during the study due to participants to leaving the study for whatever reason (Maree & Pietersen, 2016; Mertens, 2015). In my study, more participants dropped out of the intervention group than the alternative intervention group.

viii. Contamination

Contamination is when there is contact between the control and experimental groups (Engel & Shutt, 2014; Mertens, 2015). I guarded against this by never revealing to either school which was the other school where the study was taking place. Although the schools were in the same overall area, it is unlikely that learners from the two schools had contact with each other, as the schools were approximately a twenty-minute drive apart.

ix. Treatment misidentification

Treatment misidentification is when the participants experience something other than what the researcher intended (Engel & Shutt, 2014). Variations include compensatory equalisation of treatment, where the researcher favours the control group, and the placebo effect. As a researcher, I did feel that the alternative group were being short-changed in not getting the intervention, but I resisted the temptation to deviate from how I had interacted with the intervention group. The placebo effect was also a real risk, but it applied to both groups. The participants, parents, and teachers expressed enthusiasm about the study and appeared to be grateful that their school had been chosen for the study.

3.7.1.2 External validity of quantitative data

External validity refers to the extent to which a study can be generalised (Mertens, 2015). According to Mertens (2015, p. 189) "tension always exists between internal and external validity", with the highest internal validity achieved in a laboratory, and the highest external validity in the real world.

i. Time

Because a study takes place at a specific time, it cannot be generalised to other historical times (Creswell, 2013). I have accepted that my study is anchored in the time that it was done, and as time passes, the results could become less relevant.

ii. Selection

Selection validity refers to the degree to which the participants in a study represent the greater population (Creswell, 2013). Because I selected only two schools for my study, from the same area in Gauteng, it is not possible to generalise beyond this area, without replication of the study in other disadvantaged areas of the country.

iii. Setting

The characteristics of the setting of the experiment, such as the personal characteristics of the researcher interacting with the participants, can affect the extent to which the study can be generalised to other settings (Creswell, 2013). Only replication in other settings can truly counter this threat to external validity, and it was not possible to do this in the time available for my study.

3.7.1.3 Reliability of quantitative data

The reliability of quantitative data is the extent to which research findings or results from a test instrument can be replicated (Maree & Pietersen, 2016). In my study two aspects could be scrutinised for reliability: firstly the reliability of the *SOM* as an instrument, and secondly the reliability of my own results. The reliability of the *SOM* has been covered earlier in this chapter, and the reliability of my research findings will be covered in Chapter 4.

3.7.2 Qualitative quality assurance

Although the qualitative part of my study was small, and supporting the main quantitative study, it was important to check both trustworthiness of the data generated from the focus group. Trustworthiness can be divided into credibility, transferability, dependability, and confirmability (Mertens, 2015; Nieuwenhuis, 2016)

3.7.2.1 Credibility of qualitative data

In qualitative research, various procedures are used to improve credibility or validity, including crystallisation, member checks, long term observation, peer examination, collaborative research, presenting discrepant information, and avoiding researcher bias (Creswell, 2013; Maree, 2016). Below are listed the techniques that I used in my data analysis.

i. Crystallisation

In analysing the focus group responses, I identified themes and looked for patterns.

ii. Discrepant information

Not all results fit into the identified themes, so I noted opinions expressed in the focus group that ran counter to the majority view.

iii. Long-term observation

Although my study was only five weeks long at each of the two schools, four separate focus groups took place during the study, which enabled me to analyse more data.

iv. Researcher bias

I was aware of which group was the control group and which was the alternative intervention group, so there could have been researcher bias at play. I tried to minimise this by self-reflection, and by having the same set of questions for all four focus group events.

3.7.2.2 Transferability of qualitative data

Transferability parallels external validity in quantitative research, and is achieved by thick description and multiple cases (Mertens, 2015). Because my focus group questions were limited, and the learners gave short answers, the description that I wrote down could not be described as "thick". The transferability was somewhat increased by the number of separate focus groups that took place, using the same questions (four in total).

3.7.2.3 Dependability of qualitative data

Dependability is considered to be the qualitative equivalent of reliability in quantitative research (Mertens, 2015). However, unlike reliability, which aims for no change over time, dependability assumes that there is change over time, but that it is documented. In the case of my study, only four weeks passed between the first and second focus group at each school, so it is not surprising that the children said very similar things in the two focus groups.

3.7.2.4 Confirmability of qualitative data

Confirmability refers to the degree of neutrality maintained by the researcher in qualitative research (Nieuwenhuis, 2016). I ensured this by maintaining a professional distance between the participants and myself as researcher. This was helped by the roles that we were playing in the research, of teacher and learners, where there usually is a professional distance. Secondly, documentation that can be perused by following researchers is important for confirmability (Nieuwenhuis, 2016). All my research notes are available for future researchers.

3.8 ETHICAL CONSIDERATIONS

In my study, my ethics were guided by the American Psychological Association (APA) General Principles, which are A) beneficence and non-maleficence, B) fidelity and responsibility, C) integrity, D) justice, and E) respect for people's rights. Additionally, I was also guided by the APA requirements for Research and Publication and Assessment (American Psychological Association, 2017). Lastly, I was required to gain ethical clearance from the University of Pretoria and the Department of Basic Education before embarking on my study.

3.8.1 APA General Principles

3.8.1.1 Beneficence and nonmaleficence

According to the APA General Principle of beneficence and nonmaleficence, participation in a study should benefit and not harm participants, and be just in its extension of any services offered to the participants (Elias & Theron, 2012). As a result, I administered an alternative intervention to the "control" group.

3.8.1.2 Fidelity and responsibility

To keep to the APA principle of fidelity and responsibility (Elias & Theron, 2012), I remained professional in all my dealings with schools, teachers, and learners throughout the study. This was also important because I was representing the University of Pretoria, and the wider research community. As a result, I dressed and conducted myself in a formal yet friendly manner, was honest, and carefully documented all interactions. Additionally, I sought ethical clearance from the University of Pretoria and the Department of Basic Education, which included the study design, and all letters sent to schools. These letters can be seen in the Appendices.

3.8.1.3 Integrity

The APA principle of integrity requires that psychologists are honest and "avoid unwise or unclear commitments" (American Psychological Association, 2017, p. 4). To this end, I was very careful not to promise improvement in mathematical, problem solving or other skills for the participants in my study.

3.8.1.4 Respect for participants' rights and dignity

Lastly I endeavoured to respect the participants' rights and dignity (Elias & Theron, 2012). The participants in my study would be considered "vulnerable" (World Health Organization, 2018) on two counts: firstly because they are minors, and secondly because they are disadvantaged. Bearing this in mind, I ensured confidentiality of individual learners' scores on the *SOM* and on the worksheets they completed as part of the intervention or alternative intervention, especially bearing in the mind that these tests could influence the way a teacher, or other students interact with a child were the results to be made public.

3.8.2 Ethical standards in research and publication

3.8.2.1 Institutional approval

I gave accurate information to the university and the Department of Education when obtaining ethical approval.

3.8.2.2 Informed consent to research

I obtained informed consent from the school principals of the two schools involved in my study, and both consent from the parents of the participants and assent from the participants themselves, as they were minors. I checked the language level of my parent and learner letters with second-language and young learners to ensure that the level was appropriate for my intended audience.

3.8.2.3 Informed consent to recording voices or images in research

No recordings or photographs of participants were made in my research. I did take photographs of the board to assist with noting which words /questions the learners had found difficult in the *SOM*.

3.8.2.4 Client/Patient, student, and subordinate research participants

I chose schools with which I had no prior relationships, and none of the participants, teachers or school management was previously known to me.

3.8.2.5 Dispensing with informed consent

I did not at any time dispense with informed consent.

3.8.2.6 Offering inducement for research participation

No inducement was offered to participants in the research. The parents and management of School 2 did request participation certificates. After consultation with my supervisor, I agreed to this request, although I was concerned that it could jeopardise the anonymity of the study.

3.8.2.7 Deception in research

The APA ethical standard on deception in research requires that psychologists only use deception if it is necessary, does not cause pain to participants, and that it is explained as soon as possible (American Psychological Association, 2017). There was no deception involved in my research, as the consent forms clearly stated which intervention the participants would be participating in.

3.8.2.8 Debriefing

The APA requires psychologists to share appropriate information with participants about the study (American Psychological Association, 2017). I will send copies of my published dissertation to both schools involved in the study, and ask the schools to pass the information along to the parents, whose children will have completed primary school by the time the study is published.

3.8.2.9 Humane care and use of animals in research

No animals were used in my research.

3.8.2.10 Reporting research results

The APA ethics standards on reporting research results state that psychologists should not invent data, and if they find significant errors in their data after publication, they will correct them (American Psychological Association, 2017). I declare that my data is not fabricated, and I have checked and rechecked my data in an effort to maintain accuracy in transcription.

3.8.2.11 Plagiarism

My study was entirely my own, and all references in my dissertation are acknowledged.

3.8.2.12 Publication credit

I take responsibility for this study as my own work.

3.8.2.13 Duplicate publication of data

None of the data in my study has been previously published.

3.8.2.14 Sharing research data for verification

I will make my research data available to later researchers who would like to verify my study. To this end, I have included as much information in the body of this dissertation as is practical.

3.8.3 Ethical standards in assessment

I used an assessment instrument in my study, namely the *Study Orientation in Mathematics* (Maree et al., 2011). As a result, I consider myself bound by the APA ethical standards in assessment (American Psychological Association, 2017).

3.8.3.1 Bases for assessments

The APA ethical standard on bases for assessment states that psychologists should base their opinions on "information and techniques sufficient to substantiate their findings" (American Psychological Association, 2017, p. 13). To this end, I used an existing instrument, and will limit my findings to what can be substantiated.

3.8.3.2 Use of assessments

The *SOM* has been normed on Grade 8-11 South African learners, including disadvantaged learners. The original norm sample consisted of 3013 learners. Schools were selected randomly within three sub-populations, namely education department, medium of instruction at the school (English or Afrikaans), and area (urban or rural). Twenty schools were chosen randomly, and within each school thirty learners were chosen, also randomly. The Education Department was a fairly good proxy for race group at the time of the original study. Of the 3013 learners selected, 1741 were in grade 8 or 9, of which 1241 were chosen for the proportionate sample. Out of this proportionate sample, 1004 learners (76.8%) were tested in a language that was not their home language, and 995 had an African language as their home language, like the learners in my study. The percentage of learners tested in not their home language correlates reasonably well with the 79.8% of Grade 8 and 9 learners in South Africa who are educated in a language that is not their home language (Maree et al., 2011).

Maree et al. (2011) extrapolated norms for Grade 7 and 12 learners, which enabled me to use the *SOM* for my study's participants, who were Grade 7 learners at schools in a disadvantaged area. I used the English language version of the *SOM*, as English was the language of learning and teaching in both participating schools. Although I was not able to administer the *SOM* in the home languages of the participants, due to a large number of different home languages, and my own language limitations, I tried to minimise the effects of second-language administration by explaining meanings to any words the learners did not understand.

3.8.3.3 Informed consent in assessments

The *SOM* was mentioned in the informed consent and assent forms, so participants were aware that it would be used. In addition, I included a focus group after each administration of the *SOM*, to understand how the learners experienced the assessment. If any serious concerns had arisen from the focus groups, I would have been able to address them timeously.

3.8.3.4 Release of test data

I chose not to routinely release test data to learners or their parents, due to practical issues with large numbers of participants and a language barrier. However, I would release the information to individuals who requested such information.

3.8.3.5 Test construction

I used an assessment that has been standardised by professionals, rather than creating my own instrument.

3.8.3.6 Interpreting assessment results

In Chapter 4, I will bear in mind the APA ethical standard of interpreting test results, which is the purpose of the assessment as well as the characteristics of the individual being assessed (American Psychological Association, 2017).

3.8.3.7 Assessment by unqualified persons

The APA does not condone the administration of assessments by unqualified persons. As I am not a psychologist, I chose an instrument that can be administered by a qualified Mathematics teacher, which I am.

3.8.3.8 Obsolete tests and outdated test results

The instrument used in my study is not obsolete.

3.8.3.9 Test scoring and interpretation services

I scored the *SOM* myself so the ethical standard on test scoring and interpretation services is not relevant.

3.8.3.10 Explaining assessment results

I did not release results to participants so I did not explain assessment results, except the group-level explanation that will follow in Chapter 4 of this dissertation.

3.8.3.11 Maintaining test security

I have kept the test materials safe from public scrutiny, as published materials and required by the APA Standard 9.10 on maintaining test security (American Psychological Association, 2017).

3.9 SUMMARY OF CHAPTER 3

In Chapter 3, I started with the theoretical underpinnings of my study: the paradigm of critical realism, tinged with pragmatism; then the QUAN → Qual methodological approach and the quasi-experimental research design. I then discussed my selection criteria for both schools and learners within those schools. Next, I described the study in detail, covering the schools and the timetable of the study; the assessment instrument used, the focus group, the intervention, and the alternative intervention. Lastly, I situated my study ethically, taking into account the APA general ethical principles, and the ethical standards for research and assessment. This leads to the next chapter, where I describe the results of the study.

CHAPTER 4 – DATA ANALYSIS AND RESULTS

Chapter 4: Overview

In this chapter, I start by examining the internal reliability of the data. After that, I compare the two schools that were used in the study, in terms of demographics, and then I compare the pre-tests of the *Study Orientation in Mathematics Questionnaire (SOM)* (Maree et al., 2011). Lastly, I discuss the results of the study. In this discussion I compare pre- and post-test differences for both schools, firstly using the full sample, and then using a subset of the data, in case my initial definition of mathematical giftedness was not rigorous enough.

4.1 QUANTITATIVE DATA ANALYSIS

4.1.1 Data reliability

The reliability of the *SOM* was discussed in Chapter 3. To check the internal consistency or reliability of the dataset from my study, a Cronbach Alpha test was run on the pre-test dataset (Tavakol & Dennick, 2011). This consisted of all the participants who completed both the pre- and the post-test, which were 27 from School 1 (the intervention group[1]) and 40 from School 2 (the alternative intervention group[2]). In Table 7 I compare my study sample to the African language learners in the original sample used for norming the *SOM*. The reliability of my sample compared well with the original sample. The Problem-Solving Behaviour sub-test had a reliability of 0.78, which is above the acceptable cut-off of 0.7 (Muijs, 2004).

Table 7: Reliability coefficients for the different fields for the pre-test

Fields	African languages (N=955)	My study (N=67)
1. Study Attitude	0.73	0.68
2. Maths Anxiety	0.72	0.65
3. Study Habits	0.77	0.73
4. Problem-Solving Behaviour	0.67	0.78
5. Study Milieu	0.69	0.72
SOM total	0.89	0.87

.

Note. Adapted from Manual for the Study Orientation Questionnaire in Mathematics (Maree et al., 2011, p. 40)

[1] The intervention group answered SA Mathematics Challenge past papers in weeks 2 to 4 of the study.
[2] The alternative intervention group completed worksheets from the Department of Basic Education in weeks 2 to 4 of the study.

4.1.2 Demographic comparison of the two schools

Secondly, I compared the demographics of the two schools, to see if my samples from the two schools could be considered to be equivalent. This was necessary because I did not randomly assign learners to the intervention and the alternative intervention, rather choosing to run the intervention at School 1 and the alternative intervention at School 2.

4.1.2.1 Gender

The Pearson's chi-square test on the cross-tabulation of gender by school showed that gender distribution did not differ significantly between the two schools (p-value = 0.34). The Pearson's chi-squared test was chosen because both gender and school are categories, and do not imply an order (Mat Roni, Merga, & Morris, 2020). In both schools, the number of girls far outweighed the number of boys in the top 50 in the grade. See Table 8 for a breakdown of the gender distribution of the groups.

Table 8: Gender comparison of intervention and alternative intervention groups

Gender	School 1 (Intervention)	School 2 (Alternative intervention)	Total
Female	24 (88.9%)	32 (80.0%)	56 (83.6%)
Male	3 (11.1%)	8 (20.0%)	11 (16.4%)

4.1.2.2 Age

The age range was from 11 to 14 years in the intervention group, and 11 to 13 years in the alternative intervention group. The median age for learners from both schools was 12.00 years, and the mean for the intervention group is 12.30 years with a standard deviation of 10.9 months, whereas the mean for the alternative intervention group was 12.08 years, with a standard deviation of 7.4 months. Because age was not normally distributed, the non-parametric Mann-Whitney U test was used to analyse the data (Mat Roni, Merga, & Morris, 2020; Pietersen & Maree, 2016a). The p-value on these tests was 0.35, which means that there was no significant difference between the two school samples in terms of age.

4.1.2.3 Home language

The number of home languages spoken by the participants was extensive, with all eleven official languages, plus "other", represented between the two schools. One participant did not choose a home language on either the pre- or post-test. Two participants at each school listed more than one home language. There was some difference in terms of the spread of language groups at the different schools. Table 9 shows that the intervention group was dominated by Sotho-Tswana languages (Southern Sotho, Northern Sotho, and Tswana), and the alternative intervention group dominated by Nguni languages (Xhosa, Zulu, Swati, and Ndebele) (Jordan, 2015). However, both groups could be described as African language speakers taking the *SOM* in English. Two participants listed English as a home language, but alongside Xhosa, or Xhosa and Zulu, as other home languages, so this could just indicate that the language of learning and teaching at school was also used at home.

Table 9: Home language comparison of intervention and alternative intervention groups

Language group	School 1 (Intervention)	School 2 (Alternative intervention)
English	0	2 (4.5%)
Sotho-Tswana languages	16 (55.2%)	10 (22.7%)
Nguni languages	6 (20.7%)	23 (52.3%)
Other languages	6 (20.7%)	9 (20.5%)
Not given	1 (3.4%)	0

4.1.2.4 Grade 6 Mathematics marks

The selection for learners from both schools was done according to their Mathematics marks from the end of Grade 6 the previous year. School 1 had 355 Grade 6 learners in 2018, and School 2 had 340 Grade 6 learners. In each case the top 50 learners were selected for my study. I analysed the Grade 6 marks for the participants who completed the study, participating in both the pre- and post-tests. The Mann-Whitney U test, which is a non-parametric test used in place of an independent t-test for small or non-normal samples (Pietersen & Maree, 2016a) was used to compare the Grade 6 marks for the two schools. The sample from School 1 had a much broader range of marks (51% to 90%) than that of School 2 (58% to 84%), but the median of School 1's sample (72%) was considerably higher than School 2's sample (65%). The mean for School 1 was 71.26% with a SD of 10.97 and the mean for School 2 was 66.28%, with a SD of 6.46. A p-value of 0.04 was obtained from the Independent Samples Mann-Whitney U test which is below the 5% significance chosen for this study, so it can be said that the distribution of grade 6 marks was not the same across schools.

4.1.3 Comparison of pre-tests at the two schools

I compared the pre-test of the *SOM* at School 1 to the pre-test at School 2. For this comparison, first descriptive statistics were computed across the two schools, and then a non-parametric test (Mann-Whitney U) was performed. The aim of these tests was to find out if the schools could be considered to be equivalent in terms of problem-solving skills prior to my interventions. As shown in, p-values greater than 0.05 were obtained for all three tests, across all fields of the *SOM*. This shows that prior to my intervention, the two groups can be considered to be on a par in terms of all sub-tests of the *SOM*, and in terms of overall Mathematics study orientation.

Table 10: Significance of statistical tests comparing pre-tests at School 1 and School 2

Fields	Levene's Test for Equality of Variances	T-test for Equality of Means (2-tailed)	Mann-Whitney U
1. Study Attitude	0.33	0.97	0.81
2. Maths Anxiety	0.70	0.59	0.77
3. Study Habits	0.92	0.89	0.82
4. Problem-Solving Behaviour	0.47	0.72	0.52
5. Study Milieu	0.50	0.41	0.34
SOM total	1.00	0.79	0.75

4.2 QUALITATIVE DATA ANALYSIS

4.2.1 Focus groups

In addition to statistical analysis of the *SOM*, I also conducted a focus group each time after the *SOM* was administered. This was to assess how participants viewed their experience of the *SOM*, especially since they were at the younger end of the age spectrum of the norming of the test, and nearly all of them were being assessed in a language that was not their home language.

The participants for the focus group were partially chosen by choosing learners from the top, middle, and bottom of the group, based on their Grade 6 marks, and partially from learners asking if they could join the group. School 1's focus group consisted of six girls and two boys. The two boys dropped out of the study before the second focus group session. At School 2 the focus group consisted of seven girls and three boys, who were all present at both focus group sessions.

4.2.1.1 Focus group after the pre-test of the SOM

The first focus groups were held at each school straight after the first assessment with the *SOM*. The questions were designed to gauge the experience of being assessed with the *SOM*.

1. **Have you seen a questionnaire like the *SOM* before?**
2. **What did you think of the *SOM*?**
3. **Did you understand all the questions in the *SOM*? Which didn't you understand? What did you not understand about each?**
4. **Would you have preferred to answer the *SOM* in another language? Which?**
5. **Were there any questions you particularly liked answering? Why?**
6. **Were there any questions you didn't like answering? Why?**
7. **Do you have anything else you would like to share with the group?**

Answers to the pre-test questions were similar for both schools. Neither had ever seen anything like the *SOM* before. Question 2 was greeted by an awkward silence by School 1. One participant in School 2 said that the *SOM* was "easy" but no one else volunteered anything. I then suggested several words, and half the participants chose "interesting" as the best word to describe what they thought of the *SOM*. In answer to question 3, both groups listed some of the words that they had asked me for meanings. At this point in both focus groups, the groups warmed up a bit. The words that both groups listed as difficult to understand were "anxious", "convey", "enthusiastically", "theorems" and "perspire" (Maree et al., 2011). In addition, the School 2 focus group mentioned "geometry" and "memorisation" (Maree et al., 2011). Most of the School 1 group were happy doing the *SOM* in English, but one participant would have preferred to do it in Sotho. School 2 was more enthusiastic about this question and listed Sepedi (2 learners), Swati, Xhosa and Zulu as preferred languages to do the *SOM*.

Questions 5 and 6 elicited more passionate responses than the other questions, especially question 6. As shown in Table 11, the most popular questions in the SOM were positively phrased questions from the Study Attitude section.

Table 11: Questions of the SOM which at least one participant liked answering

Question	Subsection	School
I enjoy solving Maths problems	Study Attitude	1, 2
I believe that I can do well in Maths	Study Attitude	1, 2
I believe it is important to use Maths to help make the world a better place	Study Attitude	1
I test myself in writing as well as orally on Maths that I learn	Study Habits	2
I keep my Maths homework up to date by completing every day's work	Study Habits	2
I explain Maths to my friends, parents or other persons	Problem-Solving Behaviour	1

I found that participants interpreted the question "were there any questions you didn't like answering?" differently to my expectations. I thought they might choose questions that were too personal, or that they had been embarrassed to answer in the affirmative. However, it seems that as relatively young learners, they chose questions as "disliked" because they disagreed with the statement made in the question. The question chosen by the most participants (two from School 1 and one from School 2) was "I postpone my Maths homework". Participants responded to the question with wide eyes and open mouths, saying, "it's not true!" They had a similar response to four of the Study Milieu questions, for example "it is my parents' or teachers' fault that I do not work in Maths", which one participant said "didn't feel right". Other Study Milieu questions that mentioned problems at home, and personal problems were said to be "untrue for me" or "not true". They also listed four Maths Anxiety questions in the list of questions that they didn't like answering, but it was more that the questions (such as "I move my feet when my Maths teacher asks me a question" and "in the Maths class I find I have to visit the toilet") were outside their experience. Their responses were accompanied by giggling and one reason given was "I've never done that in my life". Considering that all the participants were chosen as the top 50 in their grade in Mathematics, it is not surprising that Maths Anxiety was outside of most of the participants' experience. In addition, because I allowed extra participants to join the focus groups over and above my initial selection of participants with low, middle, and high marks, it is possible that the focus group sample was skewed to participants who were high-achieving and at lower risk of Maths Anxiety than the overall study selection. Looking at the *SOM* results for focus group members, there were some whose scores indicated Maths Anxiety, but they were in the minority, and perhaps were quiet during discussion of those questions.

4.2.1.2 Focus group after the post-test of the SOM

The post-test focus group included questions about the experience of participating in the study, as well as the experience of participating in the *SOM* for a second time, as can be seen from the questions listed below:

1. **What have you seen that was like the worksheets we did?**
2. **What did you think of the sums we did?**
3. **How do you feel about the level of difficulty (or easiness) of the sums?**
4. **What did you like about the sessions we had?**
5. **What didn't you like about the sessions we had?**
6. **What did you learn from participating in this study?**
7. **What was it like answering the *SOM* again?**
8. **In which ways did you answer the same as the first time or different from the first time?**
9. **What else would you like to share with the group?**
10. **Were there any questions you didn't like answering? Why?**
11. **Do you have anything else you would like to share with the group?**

The two groups answered question 1 differently, but that is to be expected, as the sums they did were different. The SA Mathematics Challenge group mostly said that they had not seen anything like that before, with one participant mentioning the Social Science Challenge. The participants at School 2 all said they had seen sums like the Department of Basic Education worksheets.

The attitude of the participants to their experience, as expressed by questions 2 to 6 was also different. The alternative intervention focus group gave shorter and more general answers like "fine" and "easy", whereas the intervention group balanced positive statements with statements that acknowledged the effort involved. Positive statements included that the sums were "almost equal to normal Maths", "great because I was learning something new", "some sums were so challenging, but also so nice". Even the answer to question 7 showed this difference with a participant from the intervention group saying "I think it was so we could have a second chance".

The only question that resulted in the same answer between the two groups was question 8. In both groups one participant said that they answered the same as before, and the others all said that they answered differently, and in one group, the participants were quite disbelieving of the dissenting learner.

4.3 RESULTS

The main null hypotheses for my study were:

1. There is no significant difference between the pre-test and post-test mean scores for the two groups.
2. There is no significant difference between the post-intervention scores of the two groups (intervention and alternative intervention).

The alternative hypothesis was that there is a significant difference in the post-test mean scores of the intervention and alternative intervention groups. To evaluate these hypotheses, I examined the change in the Problem-Solving Behaviour sub-test of the *SOM* from the pre-test to the post-test in both schools.

4.3.1 Comparing pre- to post-test: Intervention group

A Related-Samples Wilcoxon Signed Rank Test was chosen as a non-parametric test to investigate whether there was a significant change from the pre- to the post-test in the intervention group (Maree & Pietersen, 2016). I looked at the results for the Problem-Solving Behaviour sub-test of the *SOM* in particular, as this was being used to assess whether the SA Mathematics Challenge intervention had improved the participants' problem-solving skills. The null hypothesis investigated was "the median of differences between *SOM* PSB post-test and *SOM* PSB pre-test equals 0". The significance of this test was 0.21, which is above 0.05, so the null hypothesis was not rejected; in other words, there was no significant improvement to problem-solving skills from the SA Mathematics Challenge intervention, which was against my expectations.

4.3.2 Comparing pre- to post-test: Alternative intervention

When investigating whether the alternative intervention had had any effect on the problem-solving skills of the participants, I used the same null hypothesis as for School 1. My expectation was that the SA Mathematics Challenge group would have had an improvement to their problem-solving skills, and that the alternative intervention group would have had a smaller or no improvement in their problem-solving skills. The p-value of the Related-Samples Wilcoxon Signed Rank Test on the alternative group was 0.07, which is above 0.05, so the null hypothesis was also retained. In other words, as with the SA Mathematics Challenge group, there was also no significant improvement to problem-solving skills from the alternative (Department of Basic Education worksheets) intervention.

4.4 DISCUSSION OF RESULTS

There are multiple possible reasons for the lack of significant improvement in the problem-solving skills of the intervention group. These relate to the pre-test results, the length of the study, and whether the participants were actually gifted and had sufficient basic Mathematical skills to cope with higher-level Mathematical thinking.

4.4.1 Pre-test equivalence

The first possibility is that the groups were not actually equivalent, as the participants were not chosen randomly. However, this seems unlikely, as the demographics showed no significant differences between the two samples in terms of gender balance, age demographics, or pre-test scores. Only the Grade 6 marks showed a significant difference between the two groups.

4.4.2 Length of the study

The study was noticeably short, with only three hour-long sessions dedicated to the intervention, especially considering that this was the first exposure that participants had to the SA Mathematics Challenge or similar Olympiad-style mathematics problems. In contrast, the course that I ran on the SA Mathematics Challenge and Mathematics Olympiad, which partly inspired this study, consisted of ten sessions, which ran for over 1.5 hours for the majority of sessions, and included overt teaching of skills and ways to approach such problems. The study by Govender (2014b), which was the other inspiration for this study, was short, consisting of two sessions, but did also include a step where the 14 in-training teachers marked 900 SA Mathematics Challenge scripts and categorised the questions into types, which could have conferred vicarious experience on how to answer such questions. The participants in that study also were second-year education students, who had all done Mathematics to at least first year university level, so the underlying Mathematical concepts required by the Grade 7 SA Mathematics Challenge paper would have been well embedded. It is possible that repeating the study over a longer period, and perhaps developing or using lessons on the types of Olympiad-type questions, such as provided by the South African Mathematics Foundation (SAMF), might give different results.

4.4.3 Giftedness and basic mathematical skills of the participants

The third limitation of the study is the sampling. Selecting participants by their Grade 6 Mathematics marks was always only a proxy for identifying giftedness. In School 1, the top 50 was 14.1% of the Grade, and in School 2, the top 50 was 14.7% of the Grade. I was surprised at how low the Mathematics marks went in the selected groups, down to 51% in the intervention group, and 60% in the alternative intervention group. This might have resulted in a lack of basic mathematical skills to tackle higher-level questions such as posed in the SA Mathematics Challenge.

4.4.3.1 Dropout rate and relative difficulty of the interventions

The dropout rate was much higher in the intervention group, with 23 out of 50 participants (46%) dropping out before the end of the study, compared to only 3 out of 44 participants (7%) dropping out of the alternative intervention group. This is a significant difference, and it is worth exploring the potential reasons for it.

The higher dropout rate at School 1 could be due to the intervention being more difficult for the participants than the alternative intervention. An examination of the marks the participants gained in the different worksheets shows that the intervention was considerably more difficult for the participants than the alternative intervention. Although I decided not to share marks with the participants, to emphasise growth and learning over getting sums correct, participants were supplied with answer sheets after each session so they would have been aware of how they were doing. On average, participants in the intervention group answered 42 questions, and only 21% of these were correct. In contrast, the questions in the alternative intervention were much shorter, with participants answering on average 105 questions, and getting 82% correct.

Another possible contributing reason for the differing dropout rates at the two schools is the different way the two schools approached my study. The deputy head of School 2 arranged a meeting with the parents of the potential participants, but no such meeting was arranged for School 1. I was only informed about the meeting after it had taken place, when the deputy head made several requests in exchange for participation. Several of the requests were not possible, but I did agree to provide the participants with participation certificates on completion of the study. It is possible that the knowledge that they would receive participation certificates (and that their parents knew about the certificates) might have encouraged some School 2 participants, who might otherwise have dropped out, to stay with the study to the end.

4.4.3.2 Selection of a subset of the participants

To investigate whether low mathematics skills were a limit to developing problem-solving skills, I examined the top stratum of my study participants. To do this, I went back to the initial sample, and selected the top 5% of the grade in each school. Definitions of giftedness range from the top 2% to 5% of the population, and the sample size for 5% was still workable, statistically. This resulted in a sample of 17 learners from each school. From there, I excluded learners that did not complete both the sessions of the *SOM*. This resulted in a sample of 12 learners (2 boys and 10 girls) from School 1 and 14 learners (2 boys and 12 girls) from School 2. The lowest Grade 6 mark in this sample was 74% at School 1 and 68% at School 2. This sample was less affected by the dropout rate at School 1 than the larger sample. At School 1, five participants (30%) of the smaller sample dropped out, compared to 3 participants (18%) at School 2.

4.5 DATA ANALYSIS OF THE 5% SAMPLE

4.5.1 Demographic comparison of the two schools

I compared the demographics of the 5% sample of each school in the same way that I had compared the overall study groups.

4.5.1.1 Gender

As with the larger sample, the gender distribution was similar between the two groups. Girls constituted 83.3% of the intervention group sample and 85.7% of the alternative intervention sample. The p-value from the Pearson chi-squared test was 0.43 for the subset, compared to 0.17 for the larger groups, once again showing that the groups were comparable in gender distribution. See Table 12 for the gender breakdown of the 5% sample at the two schools.

Table 12: Gender comparison of intervention and alternative intervention 5% sample

Gender	Intervention 5% sample	Alternative intervention 5% sample	Total
Female	10 (83.3%)	12 (85.7%)	22 (84.6%)
Male	2 (16.7%)	2 (14.3%)	4 (15.4%)

4.5.1.2 Age

The age ranges for the two sub-groups were the same as for the full study group at each school: 11 to 14 years for the intervention sub-group, and 11 to 13 years for the alternative intervention sub-group. The median was 12.00 years for both sub-groups. The mean for the intervention sub-group was 12.25 years with a standard deviation of 11.6 months, and 12.00 years (SD=8.1 months) for the alternative intervention sub-group. Once again age was compared using a non-parametric test, due to the small sample size (Mat Roni et al., 2020; Pietersen & Maree, 2016a). The p-value for the Mann-Whitney U test was 0.53, compared to 0.35 on the larger group, showing that there was no significant difference between the two 5% samples in terms of age. Table 13 shows the age distribution of the 5% groups at both schools.

Table 13: Age comparison of intervention and intervention 5% sample

Age	Intervention 5% sample	Alternative intervention 5% sample	Total
11	3 (25.0%)	3 (21.4%)	6 (23.1%)
12	4 (33.3%)	8 (57.1%)	12 (46.2%)
13	4 (33.3%)	3 (21.4%)	7 (26.9%)
14	1 (8.3%)	0	1 (3.8%)
Total	12	14	26

4.5.1.3 Home language

As with the larger samples, both the sub-groups consisted mainly of participants whose home language was not English. There was just one participant who listed English alongside Xhosa and Zulu as home languages. As can be seen in Table 14, similar to the larger groups, the intervention sub-group was dominated by Sotho-Tswana languages (69.2%) and the largest language group in the alternative sub-group was Nguni (50.0%).

Table 14: Home language comparison of intervention and alternative intervention 5% groups

Language group	Intervention 5% group	Alternative intervention 5% group
English	0	1 (6.3%)
Sotho-Tswana languages	9 (69.2%)	5 (31.3%)
Nguni languages	2 (15.4%)	8 (50.0%)
Other languages	2 (15.4%)	2 (12.5%)

4.5.1.4 Grade 6 Mathematics marks

The Mann-Whitney U test was used to compare the Grade 6 marks for the two sub-groups. The intervention sub-group ranged from 74% to 90%, with a mean of 81.17% and the alternative intervention sub-group ranged from 68% to 84%, with a mean of 73.50%. A p-value of 0.04 was obtained, so like with the larger samples, the distribution of Grade 6 marks differed across schools. See Table 15 for the comparison of the distribution of Grade 6 marks across the two schools for both the 5% samples and the larger samples.

Table 15: Grade 6 marks: Comparison of intervention and alternative intervention 5% groups

	Intervention group (N=27)	**Alternative intervention group (N=39)**	**Intervention 5% sample (N=12)**	**Alternative intervention 5% sample (N=14)**
Minimum	51	58	74	68
Maximum	90	84	90	84
Mean	71.26	66.28	81.17	73.50
Median	72.00	65.00	80.00	72.50
Standard deviation	10.97	6.46	5.10	5.79

4.5.2 Comparison of pre-tests at the two schools

A Mann-Whitney U non-parametric test was used to compare the pre-test results between the intervention sub-group and the alternative intervention sub-group. As can be seen from Table 16, like with the larger samples, the p-values obtained were all greater than 0.05, which means that in all cases, the null hypothesis was not rejected. In other words, the two sub-groups can be considered to be equivalent in terms of overall Study Orientation and all sub-tests of the *SOM* prior to the interventions.

Table 16: Two-sided p-values of Mann-Whitney tests comparing pre-tests between schools

Fields	**Full sample**	**5% sample**
1. Study Attitude	0.81	0.90
2. Maths Anxiety	0.77	0.16
3. Study Habits	0.82	0.49
4. Problem-Solving Behaviour	0.52	0.94
5. Study Milieu	0.34	0.30
SOM total	0.75	0.53

4.6 RESULTS FOR THE 5% SAMPLE

The raw score results of the *SOM* pre-tests and post-tests for the 5% sample can be seen in Table 17, and the results for the full sample can be found in Addendum C.

Table 17: Pre and post-test results of the SOM for the 5% sample at both schools by grade 6 marks

	Gr. 6 Maths	Pre-test						Post-test					
School	mark	SA	MA	SH	PSB	SM	Total	SA	MA	SH	PSB	SM	Total
1	90	48	35	38	36	45	202	52	40	41	38	42	213
1	87	40	42	42	51	37	212	48	28	52	47	30	205
1	86	50	42	54	55	44	245	52	39	59	57	40	247
1	85	45	37	55	51	30	218	40	36	43	36	37	192
1	84	55	43	51	50	42	241	53	42	55	62	48	260
1	81	47	49	50	26	41	213	34	32	37	28	46	177
1	79	51	38	52	65	39	245	53	43	56	46	44	242
1	78	43	38	43	31	37	192	43	45	43	45	40	216
1	77	54	46	54	46	49	249	44	28	35	50	31	188
1	77	46	48	45	49	34	222	52	37	53	50	44	236
1	76	43	32	48	48	36	207	35	40	37	43	37	192
1	74	36	28	38	41	34	177	33	42	33	32	35	175
2	84	49	49	45	46	37	226	43	43	51	34	44	215
2	84	48	45	42	26	42	203	42	50	48	40	46	226
2	78	50	43	55	47	40	235	47	45	46	47	39	224
2	78	43	39	55	52	49	238	45	46	63	66	47	267
2	75	45	54	39	37	51	226	47	43	49	45	28	212
2	75	50	40	45	38	33	206	36	31	38	41	32	178
2	74	52	34	58	48	41	233	56	43	57	47	38	241
2	71	46	53	55	55	49	258	51	56	55	54	51	267
2	70	38	26	33	44	28	169	49	33	50	34	44	210
2	68	55	48	61	56	45	265	51	51	57	59	46	264
2	68	46	48	57	53	39	243	54	51	59	57	49	270
2	68	51	37	51	62	43	244	50	29	47	50	35	211
2	68	44	44	50	46	39	223	41	48	51	46	39	225
2	68	35	49	33	24	42	183	39	35	47	40	38	199

4.6.1 Comparing pre- to post-test: intervention sub-group

As with the larger sample, a Related Samples Wilcoxon Signed Rank Test was used to analyse the *SOM* Problem-Solving Behaviour sub-test scores, comparing pre- and post-test scores for the intervention group. My hypothesis was that the participants in the 5% sample would have benefited from the intervention, and improved their problem-solving skills. As can be seen in Table 18, the one-sided p-value obtained was 0.36, which is greater than 0.05, so the intervention did not result in a significant difference in problem-solving skills, even for the 5% sample.

Table 18: One-sided p-values of Related Samples Wilcoxon Signed Rank test comparing pre- and post-tests of 5% samples for both schools

Fields	Intervention 5% sample	Alternative intervention 5% sample
1. Study Attitude	0.22	0.49
2. Maths Anxiety	0.28	0.43
3. Study Habits	0.20	0.12
4. Problem-Solving Behaviour	0.36	0.17
5. Study Milieu	0.24	0.49
SOM total	0.23	0.28

4.6.2 Comparing pre- to post-test: Alternative intervention sub-group

As with the intervention 5% sample, a Related-Samples Wilcoxon Signed Rank Test was used to compare the pre- and post-test results for the Problem-Solving Behaviour sub-test of the *SOM* for the alternative group. The p-value obtained was 0.17 (see Table 18), which although lower than the intervention group, was still above 0.05, meaning that there was no statistically significant improvement to problem-solving skills after participation in the alternative intervention.

4.7 DISCUSSION OF RESULTS FOR THE 5% SAMPLE

4.7.1 Pre-test equivalence

As with the larger sample, the demographics of the two 5% samples were equivalent in terms of gender, age and pre-test *SOM* scores. As with the larger sample, the grade 6 marks showed a significant difference between the two groups.

4.7.2 Definitions of giftedness

The 5% sample was drawn specifically to address one limitation of the study groups, namely that the top 50 samples at the two schools consisted of a rather broad percentage of the Grade for a study of giftedness (14.1% of the grade at School 1 and 14.7% of the grade at School 2). The top 5% is quite commonly used as a definition of giftedness (Lohman, 2013; Maree, 2018b; Sternberg & Kaufman, 2018; Tourón & Freeman, 2018; Yakavets, 2014), and the numbers in my sample were not too small for statistical analysis.

Unfortunately, using the 5% sample did not solve the other disadvantage of sampling that Grade 6 Mathematics marks were used as a proxy for giftedness and basic mathematical skills. This is borne out by a study of the correlations between Grade 6 marks at the two schools, and the sub-tests of the *SOM* pre-tests.

4.7.3 Correlation between Grade 6 marks and SOM pre-tests

Pearson correlation coefficients were calculated to examine the correlation between Grade 6 marks at the schools and the sub-test results on the *SOM* pre-test. The Pearson coefficient measures the linear relationship between two variables (Pietersen & Maree, 2016a) and can be used where both variables are continuous (Muijs, 2004), as was the case with Grade 6 tests and the *SOM* pre-test scores. The Pearson correlation coefficient also represents the strength, or effect size, of a relationship. A Pearson coefficient close to 1 indicates a strong positive relationship between the variables, a coefficient close to -1 indicates a strong negative relationship and a coefficient close to zero indicates a weak relationship between the two variables.

As can be seen from Table 19, Grade 6 marks correlated weakly with the Problem-Solving Behaviour pre-test at both schools. This was not surprising as true problem solving such as found in the SA Mathematics Challenge is not routinely found in mathematics textbooks in South Africa (Chirove & Mogari, 2014). However, it was problematic in that Grade 6 marks were used as a proxy for giftedness when choosing the sample for my study. It is possible that if I had used the pre-test score on the Problem-Solving Behaviour sub-test on the *SOM* as a proxy for giftedness, or actual IQ test results, I might have had different results. This strategy is recommended for future studies on the topic.

Table 19: Pearson correlation coefficients between Grade 6 marks and SOM pre-test

	Pearson correlation (*r*)	**N**	**p (2-tailed)**	**Pearson correlation (*r*)**	**N**	**p (2-tailed)**
Intervention				5% sample		
Study Attitude	0.12	27	0.54	0.24	12	0.45
Maths Anxiety	0.24	27	0.24	0.15	12	0.65
Study Habits	-0.16	27	0.43	0.02	12	0.95
Problem-Solving Behaviour	-0.20	27	0.33	0.06	12	0.85
Study Milieu	0.09	27	0.65	0.25	12	0.43
SOM total	-0.02	27	0.94	0.19	12	0.55
Alternative intervention				5% sample		
Study Attitude	0.19	40	0.24	0.21	14	0.46
Maths Anxiety	0.40*	40	0.01**	0.08	14	0.78
Study Habits	-0.05	40	0.76	-0.11	14	0.71
Problem-Solving Behaviour	-0.14	40	0.40	-0.34	14	0.24
Study Milieu	0.36*	40	0.02**	0.03	14	0.93
SOM total	0.17	40	0.29	-0.10	14	0.73

Note: *p significant at 5% level or less, **p significant at 1% level or less

*r>0.2 (small effect), **r>0.5 (medium effect), ***r>0.8 (large effect)

As can be seen from Table 19, there was no significant correlation between Grade 6 marks and any of the sub-tests for the 5% sample. However, when looking at the larger group, there was a correlation between grade 6 marks at the alternative intervention school and two sub-tests of the *SOM*. For the alternative intervention group, the p-value for Grade 6 marks correlated to Maths Anxiety was 0.01, and the p-value for Grade 6 marks correlated to Study Milieu was 0.02, and both sub-tests had a small effect size. This means that at School 2 (the alternative intervention group), children with higher Grade 6 marks were less Maths anxious than children with lower marks, but the same could not be said of the children at School 1 (the intervention group). Similarly, at School 2 poor Grade 6 marks correlated with a poor study environment at home, which is what one would expect, but at School 1, the correlation was not statistically significant. This is in line with the finding that the distribution of grade 6 marks was not the same across the two schools.

4.7.4 Determining similarity of the sample groups prior to the intervention

Effect size is a way of measuring the strength of a relationship between two variables (Hoy & Adams, 2016; Muijs, 2004). Statistical significance indicates whether an effect is likely to be due to the intervention, or just due to chance, but effect size tells us the relative success of the intervention and can be compared across studies (Muijs, 2004). Usually effect size is calculated on statistically significant data, because even a large effect size could still be due to chance (Pietersen & Maree, 2016b). However, it is sometimes useful to look at effect size where results are not statistically significant, especially in the case of small samples (Muijs, 2004; Pietersen & Maree, 2016b). I used Cohen's *d* for the effect size, because it is well-known and easy to compute (Muijs, 2011).

As mentioned earlier in this chapter, the Grade 6 marks of the two study groups were found to be statistically different, both for the original sample and for the 5% sample. I looked at the effect size, to see how large the difference was. As can be seen from Table 20, the effect size when comparing the Grade 6 marks of the Intervention group with those of the Alternative intervention group was 0.57, which is considered a medium effect size (Pietersen & Maree, 2016b). The effect size with the 5% sample was 1.41, which is large (Pietersen & Maree, 2016b).

Table 20 also shows that the 5% samples at the two schools were more different to each other than the original samples at the two schools. In the original sample, most effect sizes were minimal, with only Study Milieu showing a small effect size of 0.21. With the 5% sample, Mathematics Anxiety had a medium effect size (0.52) and Study Milieu and overall Study Orientation in Mathematics also showed a small effect size, as can be seen from Table 20.

Table 20: Effect sizes of SOM pre-test results and Grade 6 marks between the intervention group and alternative intervention group

	Intervention mean (SD)	**Alternative intervention mean (SD)**	**Effect size (*d*)**
Full sample			
Study Attitude	44.78 (7.88)	44.70 (7.29)	0.01
Maths Anxiety	38.48 (7.80)	39.53 (7.85)	0.13
Study Habits	47.85 (9.43)	47.53 (9.74)	0.03
Problem-Solving Behaviour	46.48 (12.70)	45.45 (10.66)	0.09
Study Milieu	38.74 (7.66)	37.13 (7.92)	0.21*
SOM total	216.33 (29.44)	214.33 (31.49)	0.07
Grade 6 marks	71.26 (10.97)	66.28 (6.46)	0.57**
5% sample			
Study Attitude	46.50 (5.58)	46.57 (5.42)	0.01
Maths Anxiety	39.83 (6.38)	43.50 (7.72)	0.52**
Study Habits	47.50 (6.16)	48.50 (9.16)	0.13
Problem-Solving Behaviour	45.75 (10.75)	45.29 (10.93)	0.04
Study Milieu	39.00 (5.41)	41.29 (6.24)	0.39*
SOM total	218.58 (22.87)	225.14 (26.97)	0.26*
Grade 6 marks	81.17 (5.10)	73.50 (5.79)	1.41***

Note: **d*>0.2 (small effect), ***d*>0.5 (medium effect), ****d*>0.8 (large effect)

Based on the smaller effect sizes for the pre-tests of the *SOM* and the grade marks, I conclude that it would be better to look at the results from the full sample rather than the 5% sample.

4.7.5 Mathematics Anxiety change after the intervention

Participants found the intervention questions considerably harder than the alternative intervention questions, with only 21% of intervention questions answered correctly, compared to 82% of the alternative intervention. Because of this, I explored Mathematics Anxiety on the *SOM* post-test, to see if the SA Mathematics Challenge intervention increased Mathematics Anxiety in participants. On the *SOM*, higher scores indicate a more positive Study Orientation to Mathematics, so higher scores on the MA field indicate greater confidence, and lower scores indicate more anxiety about Mathematics. As shown in Table 21, among the intervention group, participants were slightly more anxious about Mathematics after the SA Mathematics

Challenge intervention, but not significantly so. In contrast, learners were more confident in their Mathematics ability after participation in the alternative intervention, but also not significantly so. In both cases the effect size was negligible. From this, I can conclude that the difference in difficulty of interventions did not increase Mathematics anxiety in the intervention group.

Table 21: Paired t-test results showing differences between pre- and post-tests of the SOM per group

	Pre-mean (SD)	Post-mean (SD)	t	df	p	Effect size (*d*)
Intervention						
Study Attitude	44.78 (7.88)	43.85 (7.88)	-0.77	26	0.23	0.12
Maths Anxiety	38.48 (7.80)	38.00 (7.93)	-0.25	26	0.40	0.06
Study Habits	47.85 (9.43)	46.85 (9.93)	-0.73	26	0.24	0.10
Problem-Solving Behaviour	46.48 (12.70)	44.67 (11.28)	-1.08	26	0.14	0.15
Study Milieu	38.74 (7.66)	38.14 (6.23)	-0.47	26	0.32	0.08
SOM total	216.33 (29.44)	211.52 (30.07)	-1.12	26	0.14	0.16
Alternative intervention						
Study Attitude	44.70 (7.29)	44.95 (6.68)	0.22	39	0.41	0.04
Maths Anxiety	39.53 (7.85)	40.92 (8.20)	1.09	39	0.14	0.17
Study Habits	47.53 (9.74)	50.25 (8.89)	2.08	39	0.02*	0.29*
Problem-Solving Behaviour	45.45 (10.66)	47.68 (10.15)	1.44	39	0.08	0.21*
Study Milieu	37.13 (7.92)	38.08 (7.51)	0.87	39	0.20	0.12
SOM total	214.33 (31.49)	221.88 (32.83)	1.70	39	0.05*	0.23*

Note: *p significant at the 5% level or less

*d>0.2 (small effect), **d>0.5 (medium effect), ***d>0.8 (large effect)

4.8 SYNOPSIS OF RESULTS

The main null hypotheses for my study were:

1. There is no significant difference between the pre-test and post-test mean scores for the two groups.
2. There is no significant difference between the post-intervention scores of the two groups (intervention and alternative intervention).

The alternative hypothesis was that there is a significant difference in the post-test mean scores of the intervention and alternative intervention groups. The null hypotheses were not rejected in either of the instances. I will examine the possible reasons for these results when I discuss my findings in Chapter 5.

4.9 SUMMARY OF CHAPTER 4

In Chapter 4, I analysed the quantitative data, looking at data reliability, then compared the pre-test results of the two schools where I administered the intervention and the alternative intervention, in terms of gender, age, home language, and Grade 6 Mathematics marks, and lastly compared the *SOM* pre-test results of the two schools. Then I analysed the qualitative data resulting from the focus groups that were held before each administration of the *SOM*. In the Results section, I compared the pre- and post-test results of both schools. As part of my discussion of the results, I examined possible reasons for no statistically significant improvement in the intervention group's problem-schooling skills, and examined the top 5% of the grade at each school to check whether the definition of giftedness had been too broad. The same statistical tests were done on the 5% sample as the larger group, and I discussed pre-test equivalence of the two groups, definitions of giftedness, and correlations between the grade 6 Mathematics marks at the two schools and the sub-tests of the *SOM*. Lastly, I examined effect sizes for some of the results and summarised the results.

In the following chapter, I relate my findings to the findings of others. In relating my findings to current literature (evaluating the findings of other researchers), I focus on objectively assessing and relating, first, the quantitative outcomes of my study and, second, the qualitative findings by bearing in mind the following four questions:

(1) Do previous findings concur with the findings of my study?

(2) Which of the findings do not concur with previous findings?

(3) Are there findings in my study that have never been reported before?

(4) Did specific trends emerge from the findings of my study?

CHAPTER 5 – DISCUSSION OF FINDINGS

Chapter 5: Overview

In this chapter, I start by discussing the purpose of research, and how new research fits into the existing body of research. I then situate my own research findings within the existing body of research, first considering the quantitative findings and then the qualitative findings. Lastly, I consider my overall findings, situating them in the context of prior research into the education of the gifted disadvantaged learner, both internationally and in South Africa in terms of Mathematics education, problem solving, and Mathematics Olympiads.

5.1 WHAT IS RESEARCH?

5.1.1 Research as systematic enquiry

According to Mertens (2015, p. 50) research is a "process of *systematic inquiry* that is designed to collect, analyse, interpret and use *data* … to understand, describe, predict, or control an educational or psychological phenomenon or to empower individuals*"*. This systematic approach assists researchers in the pursuit of objectivity (Hoy & Adams, 2016). Journals that publish scientific findings value this systematic approach (Editage Insights, 2013; Garg, Das, & Jain, 2015; Public Library of Science, 2020).

5.1.2 The role of significance

What about significance? *PLOS One* lists its criteria for publication as "scientific validity, strong methodology, and high ethical standards – not perceived significance" (Public Library of Science, 2020, para. 2). The American Statistical Association's *ASA Statement on Statistical Significance and P-values* warns against misuse of the *p*-value (Wasserstein & Lazar, 2016, pp. 131–132) and lists the following six principles:

1. *P*-values can indicate how incompatible the data is with a specific statistical model.
2. *P*-values do not measure the probability that the studied hypothesis is true, or the probability that the data was produced by random chance alone.
3. Scientific conclusions and business or policy decision should not be based only on whether a *p*-value passes a specific threshold.
4. Proper inference requires full reporting and transparency.
5. A *p*-value, or statistical significance, does not measure the size of an effect or the importance of a result.

6. By itself, a *p*-value does not provide a good measure of evidence regarding a model or hypothesis.

As the reader can see from the above principles, the ASA values the entire research process, and encourages description of this process in papers, to show the nuances of research rather than binary thinking that one side of the significance divide a finding is true, and on the other side it is false. Similarly, studies of why journals reject articles for publications do not mention significance as a criterion for acceptance (Celik, Gedik, Karaman, Demirel, & Goktas, 2014; Editage Insights, 2013; Garg et al., 2015).

5.1.3 What is 'good' research?

Editage Insights (2013) mentions inadequate preparation, design flaws, poor writing, and a lack of originality as reasons for rejection by journals. Garg et al. (2015) did a study of 1000 consecutive articles submitted to the *Journal of Clinical and Diagnostic Research*, of which 522 were rejected. The most frequent reason for rejection was "commonality" or lack of originality, which accounted for 44.6% of rejections, followed by non-compliance (17.8%) and plagiarism (11.1%). Celik et al. (2014) administered a questionnaire to 232 editors and referees of Turkish education journals, and found that the most common mistake overall was unoriginality, and the mistake most likely to result in rejection was the presence or suspicion of ethical violations such as plagiarism or falsification. The section with the largest number of mistakes, and consequently the greatest effect on rejection, was the discussion, conclusion, and suggestions section. In this section, the most frequent mistake was "not discussing the topic with reference to the relevant literature (parallel and opposing views) and/or the discussion is not based on the research questions and findings" (Celik et al., 2014, p. 1850). This shows the importance of situating a study within the research field as well as how it has grown the understanding of the research community. To this end, throughout this chapter I will relate my findings to other research, bearing in mind the following questions:

- Do previous findings concur with the findings of my study?
- Which of the findings do not concur with previous findings?
- Are there findings in my study that have never been reported before?
- Did specific trends emerge from the findings of my study?

5.2 QUANTITATIVE FINDINGS

The quantitative data was generated by the pre- and post-tests using the *Study Orientation in Mathematics Questionnaire* (*SOM*) (Maree et al., 2011), as well as by marking the answers of the participants in both the SA Mathematics Challenge intervention and the alternative intervention, which used worksheets from the workbook *Mathematics in English: Grade 7 book 1 terms 1 & 2* (Department of Basic Education, 2018a).

5.2.1 The *SOM*

The *SOM* was designed to assess learners' attitudes to Mathematics that could influence their success in Mathematics. It was designed with various aims in mind, namely to be used as a diagnostic test, to allow teachers to help learners to improve in Mathematics, to give study guidelines, and for research in education (Maree et al., 2011). It acknowledges that ideally learners will learn Mathematics through problem solving, and has a specific problem-solving sub-test, which is why I chose it to assess problem solving before and after my intervention. It has also been used in numerous studies on disadvantaged learners in South Africa (Jagals, 2013; Maree, Pretorius, & Eiselen, 2003; Molepo, Owen, Ehlers, & Maree, 2005; Moodaley et al., 2006).

5.2.2 SA Mathematics Challenge intervention

The intervention consisted of participants working through past papers of the SA Mathematics Challenge for Grade 7 learners (South African Mathematics Foundation, 2018), based on Govender's (2014b) study of 14 pre-service teachers. The student teachers found the Olympiad style questions difficult and only 28.6% of them would have qualified for the second round. Given the poor performance of the average potential Mathematics teacher in their first attempt at the Olympiad and their hugely improved scores after Govender's intervention, I wanted to explore whether mathematically-gifted Grade 7 learners in a disadvantaged school would be able to teach themselves problem-solving skills by working through SA Mathematics Challenge questions with minimal support.

5.2.2.1 Problem-solving behaviour

Learners at South African schools are generally poor at problem-solving behaviour. Long and Wendt (2017) studied the top 24% of Grade 9 South African learners in the TIMSS and compared them to similar samples in England and Australia. They found that although South Africa's mathematical high achievers were equivalent in many sections to the other countries, they lagged behind in problem solving. Similarly, Chirove and Mogari (2014) studied learners at a school in Gauteng and found that 85.6% of learners performed poorly in a test of non-routine mathematical problems. This is perhaps why the Department of Basic Education Mathematics teaching and learning framework highlights problem solving as a desirable skill and specifically mentions the SA Mathematics Challenge and the Mathematics Olympiad (Department of Basic Education, 2018b).

Studies of problem solving in Mathematics vary in their approach on a continuum from overtly teaching problem-solving strategies to pure experience of problem solving. On the overt teaching end of the spectrum, Kūma (2015) recommends teaching “various methods of solutions and reasoning, as well as training in problem solving” as preparation for Mathematics Olympiads. In contrast, Matheson (2012) studied a teacher who was implementing problem solving in two high school classes. The learners worked in groups to solve problems, and teaching was only done to consolidate concepts after learners had discovered them for themselves. It was successful in changing the attitude of learners from waiting for input from the teacher to taking responsibility for their own learning. Na, Han, Lee, and Song (2007) and Chirinda (2013) combined experience of problem solving with self-reflection by learners, similar to the categorisation used by Govender (2014b). In Chirinda’s study participants improved in problem solving after the 10-week (45 hour) intervention. Yazgan (2015, p. 1807) found that learners who use strategies such as “*make a drawing*, *look for a pattern*, *guess and check*, *make a systematic list*, *simplify the problem*, and *work backward*” were more successful in problem solving.

My study was on the experiential end of the spectrum, with no overt teaching of strategies, and learners were not asked to formally categorise their problem-solving methods. They received feedback in the form of the answer sheet, which not only gave the correct answer, but also how it was reached, and when facilitating I would prompt learners to think if they had come across similar problems in previous weeks. The results in the problem-solving sub-test of the *SOM* showed a very slight decrease in problem-solving behaviour from the pre-test to the post-test, with a negligible effect size. The participants also did not improve their average marks from the first to the third session of the SA Mathematics Challenge past papers. This is different to Govender’s result, where the in-service teachers improved their marks on the SA Mathematics Challenge after his intervention (Govender, 2014b). The learners in my SA Mathematics Challenge intervention completed on average 42 Olympiad style questions, which was more than the 25 questions that the teachers were exposed to in Govender’s study, but they did not have to categorise the questions like the teachers did, and they had only completed Grade 6, not first year university like the in-service teachers.

The results for the top 5% of the grade at the intervention school were similar to the larger sample at that school. There was a slight decrease in problem-solving behaviour from the pre- to the post-test, and once again a negligible effect size, so the greater selectivity of sample did not make a difference to the results in terms of problem-solving skills. The pattern of correct answers to the SA Mathematics Challenge questions was also similar to the larger group, with most correct answers in the second session, although the percentage of correct answers overall was slightly higher, 25% compared to 21% for the larger group. One factor confounding analysis of the correct answers is that the SA Mathematics Challenge poses multiple choice questions. Often learners did not show their reasoning. In the cases where they did, I occasionally noticed faulty reasoning combined with a correct answer, which casts aspersions on other correct answers given without reasoning.

5.2.2.2 Other sub-tests of the SOM

Although my study was centred on problem-solving skills, all sub-tests of the *SOM* were administered, and the results analysed to see if any other aspects of study orientation in Mathematics were affected by the interventions. Other studies have found a positive correlation between each of the sub-tests of the *SOM* and success in Mathematics (Maree et al., 2003, 2011; Moodaley et al., 2006).

i. Study Attitude

The Study Attitude item on the *SOM* is designed to measure learners' feelings and attitudes towards Mathematics (Maree et al., 2011). As in the Problem-Solving Behaviour sub-test, scores for the intervention group decreased slightly from a mean of 44.78 in the pre-test to 43.85 in the post-test, with a negligible effect size. The Study Attitude of the learners was still high, with both the pre- and post-test mean of the group higher than the 80th percentile compared to the Grade 8 and 9 learners in the norm group (Maree et al., 2011). Study Attitude scores also decreased slightly for the 5% sample, from a mean of 46.50 in the pre-test to a mean of 44.92 for the post-test, with a small effect size. My finding that the Grade 7 learners who were above the 86th percentile of their class by Mathematics achievement scored at the 80th percentile in Study Attitude compared to Grade 8 and 9 learners can be considered to be in line with the finding of Maree et al. (2011) that Study Attitude increases with age and the findings that Study Attitude correlates positively with success in Mathematics (Maree et al., 2003, 2011;

Moodaley et al., 2006). Additionally, the learners in my study who were in the top 5% of the class scored higher in Study Attitude than those in the larger sample (top 14.1% of the class), which is also in line with the finding that disadvantaged learners' attitude to school correlates positively with achievement at school (Palomar-Lever & Victorio-Estrada, 2017) and that Study Attitude correlates positively with achievement in Mathematics (Maree et al., 2003, 2011; Moodaley et al., 2006).

ii. Mathematics Anxiety

According to Pajares (1996), people are more likely to persist with a task if they believe they can succeed at it. Anxiety interferes with this self-belief. In the *SOM*, a high score in the Mathematics Anxiety sub-test indicates a high confidence in Mathematical ability, or low anxiety about Mathematics. The scores of the intervention group decreased slightly between the pre- and post-test, indicating a slight increase in anxiety levels, with a negligible effect size. The mean Mathematics Anxiety score for both the pre- and post-test was at the 60th percentile, which is quite low for learners who ranged from the 86th to 99th percentile in terms of Mathematics achievement. The vast majority of the participants in both interventions were girls, and the literature has shown that gifted girls to tend to underestimate their mathematical ability (Pajares, 1996).

The results for the 5% sample were similar to the larger group, with the Mathematics Anxiety score dropping from the pre- to the post-test, indicating a slight increase in anxiety. The Mathematics Anxiety pre-test score of the 5% sample was slightly higher in the pre-test than the larger group, and slightly lower than the larger group in the post-test, and the effect size was small. Once again, the anxiety was more severe than would have been expected for learners who were succeeding in Mathematics. The learners in the 5% sample were by definition achieving in Mathematics at the 95th percentile or above in their grade, but their confidence levels were around the 62nd percentile in the pre-test and at the 55th percentile in the post-test. This relatively high level of anxiety in high-performing students contradicts the findings of Hart et al. (2016) and Lindskog, Winman, and Poom (2017), who found that Mathematics anxiety is inversely related to success in Mathematics, but supports the findings that educationally disadvantaged high school and university students in a rural area all experienced Mathematics anxiety to some degree (Hlalele, 2012, 2019).

iii. Study Habits

Study habits have been correlated with academic success (Maree, 2015; Maree & Ebersöhn, 2002; Moodaley et al., 2006; Sikhwari, 2016). Maree and Ebersöhn (2002) describe an interesting case of a high school learner in South Africa who was doing more subjects than normal and obtaining A symbols in all but one subject, despite an IQ of 84. Conversely, gifted learners can underachieve, due to not learning study skills in earlier grades because up till then the work has been easy for them (Gross & van Vliet, 2005; Hertzog, 2003).

The SA Mathematics Challenge intervention did not result in improved study habits for the participants. Scores in Study Habits decreased slightly from the pre- to the post-test, with a negligible effect size. For the 5% sample, there was also a small decrease, with a small effect size. The test scores for Study Habits on the pre-test were just below the 75th percentile of the norm group (Maree et al., 2011). This is lower than one would expect from groups selected by achievement at the 86th and 95th percentile respectively, so would appear to contradict the findings of Maree et al. (2011). However, the participants in my study were somewhat younger than the norm group, and study habits increase with age (Maree et al., 2009). Taking the younger age of the participants in my study into account, my findings could be said to agree with the findings of Maree et al. (2011).

iv. Study Milieu

Study Milieu refers to the learners' environment. Maree et al. (2011, p. 12) mention "non-stimulating learning and study environments", language difficulties and not relating to the social background depicted in word problems, and physical disability as potential Study Milieu factors that could hinder children in their Mathematics achievement.

The participants in the SA Mathematics Challenge intervention attended a quintile 2 school, which means that the area from which the learners were drawn is at the 20th to 40th percentile in terms of income in the country. The mean Study Milieu score for the study group was at the 65th percentile compared to the norm group (Maree et al., 2011), which is higher than the 20th to 40th percentile that one would expect taking into account the socio-economic area that the school was situated. This contradicts the findings that poverty is directly related to the standard of education (Nortje, 2017) and that there is a positive relationship between both school resources and academic success (Lemmon, 2017; Letsoalo, Masha, & Maoto, 2019) and family resources and academic success (Gaillard, 2019; Uleanya & Bunmi Omoniyi, 2019). However, various factors have been shown to protect disadvantaged children from the poverty trap, including being identified as gifted (Bolland, Besnoy, Tomek, & Bolland, 2019), good child rearing methods (Lipina, 2016), a positive relationship with a teacher and support from family or the community (Williams, Bryan, Morrison, & Scott, 2017), a positive attitude to school (Palomar-Lever & Victorio-Estrada, 2017), and having a goal on which to focus (Kotzé & Niemann, 2013). The learners from my study had been identified as gifted by their selection to the study, had above-average scores in Study Attitude, and may have had some of the other protective factors despite the poverty of their community. Taking the protective factors into account, my findings could be said to be in line with both the findings that there is a positive relationship between school and home resources and academic success (Gaillard, 2019; Lemmon, 2017; Letsoalo et al., 2019; Uleanya & Bunmi Omoniyi, 2019) and the findings on protective factors mitigating the effects of poverty (Bolland et al., 2019; Palomar-Lever & Victorio-Estrada, 2017).

I did not expect Study Milieu to change from the pre- to the post-test, and results for the intervention group were in line with this expectation. There was a very slight decrease in score for the larger group, and a very small increase for the 5% sample, both with negligible effect size. The above-mentioned studies (Letsoalo et al., 2019; Uleanya & Bunmi Omoniyi, 2019) on study milieu examined differences in study milieu between groups of learners, rather than changes in study milieu for individual learners, so I was unable to relate my findings in regard to change in study milieu to their research.

5.2.2.3 Overall study orientation in Mathematics

Overall study orientation in Mathematics predicts success in Mathematics (Maree et al., 2011, 2009; Moodaley et al., 2006), Engineering (Maree et al., 2003), and Natural sciences (Maree, 2015). The *SOM* was specifically designed to have a “problem-centred approach to study orientation” (Maree et al., 2011, p. 9), which fits with the aims of my study.

The SA Mathematics Challenge intervention did not improve overall study orientation in the participants. There was a small decrease in Study Orientation from the pre- to the post-test, with a small effect size, for both the larger group, and the 5% sample. The 5% sample had a slightly higher mean score (218.58) compared to the larger group (216.33) on the pre-test, but they had very similar scores on the post-test (211.92 for the 5% sample and 211.52 for the larger sample). The above-mentioned studies on study orientation looked at the correlation between study orientation and academic success, rather than improvement in study orientation brought about by an intervention (Maree et al., 2003, 2011, 2009; Moodaley et al., 2006), so it was not possible to relate my findings to the findings of these studies.

5.2.3 Alternative intervention group

The alternative intervention was provided to fulfil the APA ethical requirement of justice, which states that a study should be just in extension of services to participants (Elias & Theron, 2012). The alternative intervention consisted of worksheets from *Mathematics in English Book 1: Grade 7 book 1 terms 1 & 2* (Department of Basic Education, 2018a), which are straightforward mathematical questions, such as learners would have been exposed to in class, rather than Olympiad type questions.

5.2.3.1 Problem-Solving Behaviour

Learners at South African schools perform poorly in Mathematics compared to other countries. South Africa came 47th out of 48 countries in the TIMSS 2015 for Grade 4 learners, and 37th out of 38 countries for Grade 8 learners, even though South Africa chose to assess Grade 5 and Grade 9 learners due to syllabus alignment (Mullis, Martin, Foy, & Hooper, 2016).

Worksheets such as used in the alternative intervention have been categorised as "drill and kill", unsuited for gifted learners (Baldwin, 2006; Singer, Sheffield, Freiman, & Brandl, 2016; Treffinger & Isaksen, 2005). In contrast, Mhlolo (2014a, p. 1590), while a strong advocate for Olympiad style problem solving for mathematically-gifted learners, warns against dismissing practice worksheets, saying that "understanding of both procedural and conceptual knowledge should be the ultimate goal and priority of all Mathematics learning as it refers to an integrated and functional grasp of mathematical ideas".

The methodology of the alternative intervention was essentially experiential like the SA Mathematics Challenge intervention, in that I did not overtly teach the learners, just provided worksheets, encouragement to keep trying, and (the following week) answers to the worksheets. The one difference between the SA Mathematics Challenge intervention and the alternative intervention was that the alternative intervention worksheets were routine questions on topics from term 1 of Grade 7, so the participants had likely received overt instruction in the concepts from their school Mathematics teachers. The results from the first two weeks of the intervention were noticeably better than those from week 3. The mean percentage of correct questions answered in the first week was 95%; in the second week it was 85%, and in the last week 61%. It could be that the learners scored lower in the third week due to not having had overt instruction in class on the last topic. However, if this were the case, the learners would have been behind in the syllabus as it was the second last week of the term by the time they did worksheet 6, and the workbook contains 16 revision worksheets and 29 worksheets for term 1. This method of working with peers without a teacher was favoured by some of the gifted learners in a study by Diezmann and Watters (2002), although with more challenging questions than contained in the alternative intervention worksheets.

Problem-solving behaviour improved slightly from the pre- to the post-test for both the full alternative intervention group and the 5% sample, with a small effect size for the larger group and a negligible effect size for the 5% sample. This slight improvement in skills in a three hour intervention correlates positively with the findings by Reder, Gauly, and Lechner (2020), Wang et al. (2017), and Gladwell (2008), who found that practice improves skills, but practice needs to be long term to have a significant effect.

5.2.3.2 Other sub-tests of the SOM

i. Study Attitude

Study Attitude for the alternative intervention group was almost unchanged from the pre- to the post-test, with a mean of 44.70 on the pre-test and 44.95 on the post-test for the larger group, and a mean of 46.57 on the pre-test and 46.50 on the post-test for the 5% sample, with a negligible effect size in both cases. The pre-test scores were very similar to those of the intervention group, with a high Study Attitude compared to the norm group (Maree et al., 2011). My findings of high Study Attitude scores in learners in the top 14.7% of the Grade, and even higher scores for those in the top 5% of the Grade is in line with other research, which correlate academic success positively with Study Attitude (Goodman et al., 2011; Heuser & Wang, 2017; Maree et al., 2003, 2011; Moodaley et al., 2006; Palomar-Lever & Victorio-Estrada, 2017).

ii. Mathematics Anxiety

The literature shows a variety of origins for Mathematics anxiety. These include the influence of adults who have Mathematics anxiety (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015), lower skills in Mathematics (Maree et al., 2011; Ramirez, Shaw, & Maloney, 2018), gender stereotypes (Luttenberger, Wimmer, & Paechter, 2018; Ramirez et al., 2018) and teaching method in the Mathematics classroom (Newstead, 1998).

The mean score on the Mathematics Anxiety sub-test of the *SOM* increased (i.e. indicating an improvement in confidence in Mathematics) slightly from 39.53 in the pre-test to 40.92 in the post-test for the alternative intervention participants, with a negligible effect size. This finding contrasts the finding by Newstead (1998) that discovery teaching methods result in higher levels of Mathematics confidence. The worksheet content is traditional, and although the way that it was presented to the learners with minimal input from a teacher could be described as experiential, it was less experiential than the SA Mathematics Challenge intervention, as most learners chose to work alone.

For the 5% sample, the score decreased from 43.50 to 43.14, with a negligible effect size. The levels of confidence in mathematics were relatively low for the top 14.7% and top 5% of the grade. Even after the intervention, the mean score of the larger group was at the 65th percentile of the norm (Maree et al., 2011), and the smaller group at the 82nd percentile. This finding that even high-achieving learners in a poverty setting exhibit Mathematics anxiety concurs with the finding by Hlalele (2019) that rural university access program students all exhibited Mathematics anxiety in an academic setting. Although the students in Hlalele's study were rural, and mine urban, both groups were disadvantaged and the top cohort of their grade, as only the most successful learners even get into university from disadvantaged schools in South Africa (Van der Berg, 2015).

iii. Study Habits

It is important for gifted learners to develop good study habits as they have been linked to academic success (Maree, 2015; Maree & Ebersöhn, 2002; Moodaley et al., 2006; Onoshakpokaiye E, 2015; Sikhwari, 2016). There was a statistically significant improvement in the Study Habits scores from the pre- to the post-test for the alternative intervention participants, with a small effect size. There was also an increase in the Study Habits score for the 5% sample, although it was not statistically significant, with a small effect size. The mentioned studies (Maree, 2015; Maree & Ebersöhn, 2002; Moodaley et al., 2006; Onoshakpokaiye E, 2015; Sikhwari, 2016) try to show that better study habits result in success, and I was not able to find other studies that showed that mathematical practice of routine worksheets improved study habits, but it could possibly be part of the reason why practising mathematical skills in a structured way improves academic success, as has been shown by the Kumon method (Usmadi, Agita, & Ergusni, 2020). The other reason would be practising a particular algorithm to develop automaticity (Department of Basic Education, 2018b).

iv. Study Milieu

Like the participants in the SA Mathematics Challenge intervention, the participants in the alternative intervention were from a quintile 2 school, i.e. the 20th to 40th percentile in terms of income in the country. The Study Milieu pre-test score for the alternative intervention group was only at the 60th percentile of the norm group, and the 5% sample was somewhat higher, at the 75th percentile. This finding that the Study Milieu score was higher for higher achieving learners is in line with findings that Study Milieu relates directly to academic achievement (Gaillard, 2019; Lemmon, 2017; Letsoalo et al., 2019; Lipina, 2016; Serrano Corkin, Coleman, & Ekmekci, 2019; Uleanya & Bunmi Omoniyi, 2019) and to achievement in Mathematics in particular (Maree & Erasmus, 2006; Maree et al., 2011).

I did not expect Study Milieu to change from the pre- to the post-test, and results for the 5% group were in line with this expectation. There was a slight increase in score for the larger group, both with negligible effect size.

5.2.3.3 Overall study orientation in Mathematics

The alternative intervention resulted in a statistically significant improvement in overall study orientation, from 214.33 to 221.88, with a small effect size. The improvement in the 5% sample was smaller (225.14 to 229.21) with a negligible effect size, though starting from a higher base. The means of both groups were a little lower than the norms of the original norm group (Maree et al., 2011), with the top 14.7% scoring just below the 80th percentile on the pre-test and at the 85th percentile on the post-test, and the top 5% mean scores for the pre- and post-test both being between the 85th and 90th percentile. As I said in the section about overall study orientation for the SA Mathematics Challenge intervention, the available studies on study orientation did not investigate improvement in study orientation brought about by an intervention, but my findings did concur with the prior findings that a positive study orientation in Mathematics is correlated with academic achievement (Goodman et al., 2011; Maree et al., 2003, 2011, 2009; Moodaley et al., 2006; Palomar-Lever & Victorio-Estrada, 2017).

5.3 QUALITATIVE FINDINGS

5.3.1 Experience of the *SOM*

5.3.1.1 Newness of the experience

The *SOM* was a new experience for both the intervention group and the alternative intervention group, and the participants found it hard to express their feelings about it. Participants would have been exposed to mass testing in the form of school tests and exams, and the Annual National Assessment (ANA), but had never experienced anything similar to the *SOM*. Other studies that use the *SOM* (Erasmus, 2014; Festus & Seraphina, 2015; Jagals, 2013; Kotze, 2006; Maree & Erasmus, 2006; Maree et al., 2003; Moodaley et al., 2006; van Schalkwyk, 2014) did not report the participants' experience of taking the *SOM*, so these findings are new.

5.3.1.2 Language

The *SOM* was administered in English, which was the language of teaching and learning at both schools, and not the exclusive home language of any of the participants (the only participants who listed English as a home language also listed at least one African language in addition to English). The participants were split on whether it was better to do the test in English or their home language: at the intervention school only one participant wanted to do the *SOM* in another language, but at the alternative intervention school there were many suggestions of alternative languages. Even though *SOM* norms for Grade 8 and 9 learners can be used for Grade 7 learners (Maree et al., 2011), many words used in the *SOM* were unfamiliar to the participants. The words that the participants found difficult were largely the same at both schools. This difficulty with doing the assessment in English was considered by the authors of the *SOM*, who gave norms for learners doing the test in a second language (Maree et al., 2011). This finding is also in line with the later development of a primary school version of the *SOM*, the *Study Orientation Questionnaire in Maths (Primary)* (Maree, van der Walt, & Ellis, 2020) for learners in Grades 4 to 7, with simplified vocabulary and the option of an isiZulu questionnaire (Maree et al., 2009) and the development of a study orientation questionnaire for Setswana-speaking learners in Grades 3 to 5, in English, but normed on learners with a home language of Setswana and educated through the medium of English (Maree & Erasmus, 2006). The finding that it is difficult as a second-language learner to do Mathematics and be assessed in English is also in line with other more general research in Mathematics education (Erasmus, 2014; Mochesela, 2007; Pillay, 2010; Sepeng, 2013).

5.3.1.3 Popular and unpopular questions

The most popular questions for participants in the *SOM* were the positively phrased ones, particularly from the Study Attitude and Study Habits items of the *SOM.* The most unpopular question was a negatively phrased question from the Study Habits item “I postpone my Mathematics homework”. Other disliked questions were Study Milieu and Mathematics Anxiety items to which the participants in the focus group did not relate. This finding may have been skewed by my selection for the focus groups, which started with me choosing two learners from the top, middle, and bottom of the group by marks, but I ended up allowing other learners who wanted to participate to join the groups. As no other studies have reported on the experience of writing the *SOM*, these findings are new.

5.3.2 Experience of the SA Mathematics Challenge

5.3.2.1 Newness of the experience

The participants in the intervention had never seen the SA Mathematics Challenge before, despite entry to the SA Mathematics Challenge being free for the first 100 learners from no-fee schools (South African Mathematics Foundation, 2020b), and past papers being available on the internet (South African Mathematics Foundation, 2018). This lack of participation in Mathematics Olympiads is in line with statistics on participation in the SA Mathematics Challenge. In 2015 there were 3 852 957 senior primary learners in South Africa (South African Market Insights, 2020), and only 110 000 of them (South African Mathematics Foundation, 2020b), or 3%, participated in the SA Mathematics Challenge. The participation percentage is likely to be lower today, as approximately 80 000 learners participated in the 2019 SA Mathematics Challenge (South African Mathematics Foundation, 2020b). Govender's (2014a) study of why learners participate in Mathematics Olympiads showed that successful participants in Olympiads attended schools that had a long tradition of participating in multiple Mathematics Olympiads, and learners generally started participating in Grade 3 or 4. My finding that the SA Mathematics Challenge was new to learners in the disadvantaged schools where my intervention took place concurs with the experience of Govender (2014a, p. 2), who observed that "learners who are successful in these competitions are usually from the more affluent or advantaged schools" and with the findings from Govender's (2014b) study on pre-service Mathematics teachers, where only one out of the 14 student teachers in the study had participated in a Mathematics Olympiad as a learner. The Zenex Foundation (2020) has called for more research into the learners and schools that participate in the SA Mathematics Challenge.

5.3.2.2 Difficulty level

The participants in the SA Mathematics Challenge intervention found the Olympiad-style questions difficult. Unlike the alternative intervention group, the SA Mathematics Challenge participants emphasised the newness of the experience, and balanced the effort involved against the enjoyment with phrases such as "so challenging but also so nice" and contrasting the sums to "normal maths". These statements correlate positively with the experience of the in-service teachers in Govender's (2014b, p. 7) study, who also described the Grade 7 SA Mathematics Challenge first round paper as "challenging" and "difficult" to do, even as trainee Mathematics teachers. These observations are also in line with the research by Mochesela (2007) who used a problem-based approach with Grade 9 learners in a South African township school. She found that learners often guessed answers because they did not understand the question, and 91.1% of learners needed to be shown a method before knowing how to approach a problem. I did not have questions about language in the focus group after the intervention, which might have been illuminating, given the answers about language in the first focus group.

5.3.3 Experience of the alternative intervention

5.3.3.1 Newness of the experience

The alternative intervention was not intended to be a new Mathematics experience for the participants. The qualitative results showed that the learners found the sums they did familiar, with learners agreeing that they had seen problems like the ones they did in the worksheets before. The answers to the questions about whether they liked the sums were also different to the intervention group, who emphasised how different the sums were, and that they were challenging. In contrast, the alternative intervention group just said that the sums were "fine" and "easy" and "help us with other Maths", recognising that they were practice of methods they had learnt in class. The familiarity with the type of sum in the alternative intervention worksheets correlates positively with findings that South African schools generally use routine problems rather than problem solving in their classrooms (Chirove & Mogari, 2014; Engelbrecht & Mwambakana, 2016; Mochesela, 2007), despite guidance from the Department of Basic Education that "All learners, not only gifted learners, need to develop the thinking skills needed to solve non-standard problems" (Department of Basic Education, 2018b, p. 18). This finding that routine problems are more familiar to learners than problem-solving questions also correlates positively with international findings (Schoevers & Kroesbergen, 2017).

5.3.3.2 Difficulty level

In contrast to the SA Mathematics Challenge intervention, the participants in the alternative intervention did not find the worksheets difficult. The learners used the word "easy" to describe the sums in both question 2 "What did you think of the sums we did?" and question 3 "How do feel about the level of difficulty (or easiness) of the sums?" and unlike in the SA Mathematics Challenge intervention focus group, the learners made no mention of effort or challenge. This finding that the alternative intervention worksheets were easy for the participants contrasts with findings on how South African learners in general struggle with Mathematics (Modisaotsile, 2012; Mullis et al., 2016), but correlates with the participants' success in class Mathematics, as evidenced by their selection for the study, which was on the basis of their Grade 6 Mathematics marks. A possible explanation for the difference can be found in the gap between internal school assessments at disadvantaged schools in South Africa and external assessments such as the Annual National Assessment (ANA), TIMSS, and Southern Africa Consortium for Measuring Education Quality (SACMEQ) (Chetty, 2016).

5.4 INTEGRATION OF QUANTITATIVE AND QUALITATIVE FINDINGS

5.4.1 Success in traditional Mathematics

The participants in my study were chosen by their Mathematics marks at the end of Grade 6. Mathematics is generally taught procedurally in South African schools (Engelbrecht & Mwambakana, 2016), because teachers are unfamiliar with problem solving themselves (Govender, 2014b) and generally have a poor level of Mathematics understanding (Venkat & Spaull, 2015). The quantitative part of my study found a positive relationship between success in traditional Mathematics (as defined by Mathematics marks) and Study Attitude, Study Habits, and overall Study Orientation, which are in line with research on these aspects of study orientation (Heuser & Wang, 2017; Maree, 2015; Maree & Ebersöhn, 2002; Maree et al., 2003, 2011; Moodaley et al., 2006; Onoshakpokaiye E, 2015; Palomar-Lever & Victorio-Estrada, 2017; Sikhwari, 2016). The qualitative part of the study supported the quantitative findings, with the most liked questions from the *SOM* being the positively-phrased questions from the Study Attitude, and Study Habits items of the *SOM*.

5.4.2 The influence of poverty

By selection, the participants in my study can be considered to be disadvantaged, by their attendance at quintile 2 schools, where parents are not required to pay school fees due to the poverty of the surrounding area. My study found that the participants in the study were less disadvantaged by the poverty of their surroundings than would be expected, scoring at the 65th percentile in Study Milieu when their school was in a quintile 2 (20th to 40th percentile) area in terms of socio-economic status (SES). These Study Milieu scores contradict studies that show a positive relationship between study milieu and success in Mathematics (Gaillard, 2019; Lemmon, 2017; Letsoalo et al., 2019; Serrano Corkin et al., 2019; Uleanya & Bunmi Omoniyi, 2019; Van der Berg, 2015). However, these results are in line with studies that certain factors protect against poverty, including being identified as gifted (Bolland et al., 2019), study attitude (Palomar-Lever & Victorio-Estrada, 2017) and study habits (Kotzé & Niemann, 2013; Williams et al., 2017).

The interaction between disadvantage and success in Mathematics was also evident in the Mathematics anxiety findings from my study. Studies have found that Mathematics anxiety is negatively correlated with success in Mathematics (Maree et al., 2011), but the participants in my study, who were chosen for their success in Mathematics, still presented with high Mathematics anxiety. This finding of Mathematics anxiety in high-achieving disadvantaged learners correlates positively with the study by Hlalele (Hlalele, 2012, 2019) who found that Mathematics anxiety is high for disadvantaged learners, even gifted ones.

5.4.3 Problem solving

Both the quantitative and qualitative findings from my study showed that Olympiad type questions were unfamiliar to the participants, and that the SA Mathematics Challenge intervention was more difficult for the participants than the alternative intervention. The finding that the SA Mathematics Challenge was new to the participants in my study was in line with other research into problem solving in South Africa (Engelbrecht & Mwambakana, 2016; Govender, 2014a; Mochesela, 2007) and abroad (Schoevers & Kroesbergen, 2017), and can be linked to findings that teachers in South Africa are themselves unfamiliar with problem solving and Olympiad type questions (Govender, 2014b). The 21% average score for the problem-solving questions in the SA Mathematics Challenge intervention compared to the 82% average score for the routine problems in the alternative intervention showed exactly how unfamiliar and difficult the participants found the SA Mathematics Challenge intervention, despite the participants' past success in traditional school Mathematics. The qualitative findings emphasised the newness and difficulty of the SA Mathematics Challenge questions for the participants, and were in line with previous findings that even high-achieving South African learners find problem-solving questions difficult (Chirove & Mogari, 2014; Long & Wendt, 2017).

The qualitative findings on the *SOM* showed that the participants in my study had difficulty with English. Difficulties with problem solving can be exacerbated by language difficulties (Mochesela, 2007; Sepeng, 2013; Sepeng & Webb, 2012; Tambychik & Meerah, 2010) so it would be worth investigating this angle of difficulty with problem solving in future studies.

5.5 SUMMARY OF CHAPTER 5

In this chapter, I started by discussing what research is, the role of significance and whether it is necessary, and what determines quality in research. I then discussed my quantitative findings and how they relate to the findings of others, and my qualitative findings and how they relate to other research. Lastly, I integrated my quantitative and qualitative findings, also placing them in the context of other research. In my final chapter, I will take these findings and assess them in the light of my research questions, draw conclusions and make recommendations for future research.

CHAPTER 6 – FINDINGS AND CONCLUSION

Chapter 6: Overview

I start this chapter with a summary of the previous chapters of this dissertation. Then I return to the research questions, to answer them. Then I cover the strengths and limitations of the study, pay attention to ethical considerations and finally end with recommendations for further research.

6.1 SUMMARY OF CHAPTERS

6.1.1 Chapter 1: Introduction

I started Chapter 1 by explaining the rationale behind my study. I chose to study gifted disadvantaged learners because they are South Africa's future leaders, scientists and researchers, and I chose to investigate problem-solving skills because they are important skills that are valued in the workplace, and require overt instruction, even for the gifted. I chose to use the SA Mathematics Challenge due to my personal experience with teaching gifted children Mathematics.

In the rest of the Chapter I covered some common definitions used in my study, briefly outlined my conceptual framework, paradigmatic perspective, research methodology and touched on ethical considerations before summarising the coming chapters.

6.1.2 Chapter 2: Literature study and conceptual framework

Chapter 2 was a detailed literature study. I started with worldwide definitions of giftedness, examining four debates in the field: one unitary gifted factor or *g* versus multiple intelligences; aptitude vs. achievement; nature vs. nurture; and lastly, which is relevant in Africa, community vs. the individual. I then examined identification of the gifted in South Africa, which is fraught with difficulties due to South Africa being a multilingual and multicultural country, and a paucity of tests in many home languages, as well as differences between the urban and rural experience.

I then described my own UPPS conceptual framework, based on a **unitary** concept of intelligence, based on CHC theory (Beaujean, 2015; Benson et al., 2018; Gross, 2006; Keith & Reynolds, 2010; Lubinski, 2016; McGrew, 2009; Warne, 2016), but based on **potential** to grow and learn rather than achievement, which is important when studying disadvantaged children. The third aspect is **precocity** (Piirto, 2004), which gives a simple practical framework for teachers to approach gifted learners, by treating them as if they were older children, but it is balanced with a socio-emotional component, which takes into account the emotional response to the world of gifted children, as detailed by Dabrowski (Daniels & Piechowski, 2009).

I examined the opportunities available to mathematically-gifted learners in South Africa, in terms of specialised schools, after-school programmes, scholarships, and online programmes, before looking at problem-solving mathematical programmes, and the SA Mathematics Challenge in particular. Lastly, I looked at how to assess problem-solving skills in South African children, and the *Study Orientation in Mathematics Questionnaire (SOM)* (Maree et al., 2011), which I chose to use for my pre- and post-testing.

6.1.3 Chapter 3: Research methodology

Chapter 3 placed my study in the context of my own epistemology, and then detailed the research methodology used in the study. My paradigm for the study was critical realism with pragmatism. My research methodology was QUAN→ Qual, as my study used a quasi-experimental design with pre- and post-tests of the *SOM*, but I also held focus groups after each administration of the *SOM*, to examine the participants' experience of the *SOM* and the study.

I also explained my sampling method, how I selected the two quintile 2 schools in the same township, and chose learners within those schools by their Grade 6 Mathematics marks as a proxy for giftedness. I then described both the intervention and the alternative intervention in detail, and showed why the *SOM* was a good instrument for assessing the participants. Lastly, I explained how I had taken the APA general ethical principles into consideration in my study design.

6.1.4 Chapter 4: Data analysis and results

Chapter 4 started with examining the internal reliability of the SOM as an instrument, before assessing the quantitative data. I started with a demographic comparison of the two schools where my study took place, as well as comparing the two schools in terms of Grade 6 Mathematics marks and pre-tests of the *SOM*. They were equivalent in terms of demographics and the *SOM* pre-tests, but there was a statistically significant difference between the two schools in terms of Grade 6 marks.

Secondly, I studied the qualitative data. The focus groups after the first administration of the *SOM* showed similar results in both schools, but after the interventions the answers were subtly different, with both groups giving positive answers, but the intervention group acknowledging the effort involved in the intervention.

The results of the pre- and post-tests of the Problem-Solving Behaviour sub-tests of the *SOM* were not significant for either the intervention or the alternative group. In my discussion, I conjectured that this could have been because my selection of participants was rather broad for a gifted study, so I redid the statistical analysis on a sample of the top 5% of each school, though still chosen by Grade 6 Mathematics marks. The 5% sample also did not have an improvement in the pre- and post-test for either the intervention or the alternative intervention group. I reverted to the larger sample for the rest of my analysis, because the difference between the Grade 6 marks at the two schools was more marked with the 5% sample and there was less correlation between grade 6 marks and pre-tests of the *SOM* than with the larger group.

6.1.5 Chapter 5: Discussion of findings

Chapter 5 started with a discussion of what constitutes good research, and how the research process is more important than significant results. I then discussed the quantitative findings from my study, exploring the results for the intervention group and the alternative intervention group separately, examining the results in the various sub-tests of the *SOM* before and after the interventions. In each case I noted where findings concurred with previous research, where findings differed from previous research, and where my findings were new. Next, I discussed the qualitative findings, recounting the participants' experience of the *SOM*, and the experience of the two interventions. Once again, I compared my findings to those of previous researchers, noting where my findings concurred with previous findings, differed from previous findings, and were entirely new.

6.2 ANSWERING THE RESEARCH QUESTIONS

The primary research question was "How valuable is participation in the SA Mathematics Challenge for developing problem-solving skills in mathematically-gifted disadvantaged learners?" I will first examine the secondary research questions before answering the main research question.

6.2.1 What are the essential aspects of current (group-based) programmes aimed at enhancing the problem-solving skills of mathematically-gifted learners in disadvantaged schools

The opportunities available for mathematically-gifted learners in disadvantaged areas of South Africa are limited. Table 1 in Chapter 2 details the options, such as entrance to Dinaledi, Maths and Science Focus Schools, and Schools of Specialisation to learners who live near to such schools; no-fee or low-fee private schools such as LEAP Schools and Oprah Winfrey Leadership Academy and African School for Excellence; and mentorship programmes such as the Wits TTP Extension programme. Mathematics competitions such the SA Mathematics Challenge, SAMF Mathematics Olympiad, and various university-run mathematics competitions are open to poor schools for free (South African Mathematics Foundation, 2020b; University of Cape Town, 2019; University of Pretoria, 2019; University of Witwatersrand, 2019b), but if you look at the learners who qualified for the third round of the SA Mathematics Challenge, they were nearly all from fee-paying schools (South African Mathematics Foundation, 2019). Lastly, there are two free Olympiad training programmes, provided by SAMF, the Siyanqoba regional training for high school learners, and the online SAMF Olympiad training for Grades 7-12 (South African Mathematics Foundation, 2020a).

6.2.2 What is the impact of three hour-long facilitated sessions doing SA Mathematics Challenge past papers on mathematically-gifted disadvantaged learners' study orientation in mathematics in general?

The participants in both the SA Mathematics Challenge intervention and the alternative intervention groups started the study with a high level of study orientation in Mathematics, showing a positive relationship between success in traditional Mathematics (which was the selection criterion for the study) and Study Attitude, Study Habits and overall Study Orientation. The qualitative findings supported the quantitative findings, with the most liked questions from the *SOM* being the positively-phrased questions from the Study Attitude and Study Habits items of the *SOM*, and both were in line with findings on the positive relationship between academic success and Study Attitude, Study Habits and overall Study Orientation (Heuser & Wang, 2017; Maree, 2015; Maree & Ebersöhn, 2002; Maree et al., 2003, 2011; Moodaley et al., 2006; Onoshakpokaiye E, 2015; Palomar-Lever & Victorio-Estrada, 2017; Sikhwari, 2016).

The interaction between disadvantage and success in Mathematics on Study Orientation was evident in the Study Milieu and Mathematics Anxiety scores on the *SOM*. The participants in my study scored higher in Social Milieu than their surroundings predicted, and they were more anxious about Mathematics than one would expect for participants chosen for their success in Mathematics, which supports the findings of Hlalele (Hlalele, 2012, 2019) that even high-achieving disadvantaged learners have high Mathematics anxiety.

The SA Mathematics Challenge intervention resulted in a small decrease in overall Study Orientation from the pre- to the post-test, with a small effect size. This contrasted with the statistically significant improvement in Study Orientation, with a small effect size, for the alternative intervention group. The key to this difference could lie in the relative familiarity and difficulty of the two interventions, as evidenced by the average percentage of correct answers in the two interventions (21% for the intervention versus 82% for the alternative intervention), as well as the qualitative findings. The qualitative findings revealed how the participants in the SA Mathematics Challenge intervention had never experienced anything like that before, and although they were positive about learning something new, they also used words indicating the effort involved. In contrast, the participants in the alternative intervention saw it as practice of similar sums to those they had learnt in class. This practice led to a statistically significant improvement in Study Habits for the alternative intervention group, which did not happen for the SA Mathematics Challenge participants.

6.2.3 What is the impact of three hour-long facilitated sessions doing SA Mathematics Challenge past papers on mathematically-gifted disadvantaged learners' problem-solving skills in particular?

Previous studies show that South African learners are generally unfamiliar with problem-solving questions (Govender, 2014a, 2014b) and consequently poor at problem solving (Chirove & Mogari, 2014; Long & Wendt, 2017). The qualitative findings for the SA Mathematics Challenge intervention were in line with previous findings, highlighting the unfamiliarity of problem-solving type questions to the participants. Both the SA Mathematics Challenge and the alternative intervention groups scored above the 80th percentile for problem solving compared to the norm group (Maree et al., 2011) on the pre-test of the *SOM*, but this is below the level that might be expected for learners in the top 14% or top 5% of the grade.

There was a slight decrease in Problem-Solving Behaviour from the pre-test to the post-test for the SA Mathematics Challenge group, with a negligible effect size, compared to a slight increase in Problem-Solving Behaviour with a small effect size for the alternative intervention groups. The participants in both interventions experienced difficulty with the English language in the administration of the *SOM*, so it is possible that the SA Mathematics Challenge group also experienced language difficulties with the SA Mathematics Challenge intervention, where the questions were more language-heavy than the alternative intervention.

6.2.4 Main research question: how valuable is participation in the SA Mathematics Challenge for developing problem-solving skills in mathematically-gifted disadvantaged learners?

The quantitative data analysis showed that the SA Mathematics Challenge intervention did not result in an improvement to problem-solving behaviour in participants. The qualitative findings showed that the participants found the SA Mathematics Challenge problem-solving questions unfamiliar and difficult. I will go into the possible reasons for these results when discussing the limitations of the study.

6.3 STRENGTHS OF THE STUDY

6.3.1 Selection of schools

Choosing two schools matched by size and quintile in the same township resulted in schools that were well matched in terms of demographics (gender, age, home language) as well as in results on the pre-tests of the *SOM*.

6.3.2 Instrument for assessment

The *SOM*, used for assessment of problem-solving skills was designed for Grade 7-12 learners from South Africa, and was normed on disadvantaged learners, and learners writing the English version of the test in their home language. It is a valid and reliable scientific instrument, which can be administered not only by psychometrists, but also by Mathematics teachers. It has also been used in a number of studies in South Africa (Jagals, 2013; Molepo et al., 2005; Moodaley et al., 2006).

6.3.3 Focus groups

The study was mainly a quantitative study, but the qualitative part of the study gave useful insight into some of the quantitative results, such as the relative familiarity and difficulty of the interventions.

6.4 LIMITATIONS OF THE STUDY

There were various limitations to the study, which could have affected the results.

6.4.1 Sample size and area

Although the sample size was sufficient for statistical purposes, the sample size was not large (27 learners at the intervention school and 40 learners at the alternative intervention completed the study), and the two schools chosen were from the same township in Gauteng, so results cannot be generalised beyond the area where the study took place.

6.4.2 Selection by Grade 6 marks

Due to the difficulties of using IQ tests to assess disadvantaged learners and practical considerations of time, expense, and personnel, I chose not to use an IQ test to assess the learners for giftedness, and decided to use Grade 6 marks for selection. In conjunction with this, also for practical reasons, I did the intervention at one school and the alternative intervention at the other school, which meant that I could not randomly assign the learners to the intervention or alternative intervention group. School marks at the end of Grade 6 should be equivalent from one school to another, but there is no guarantee, as in-school assessments are not standardised across schools at this educational stage. In the case of these two schools, the marks were notably different. Secondly, Grade 6 marks did not correlate with the *SOM* pre-test results, with the marks from the intervention group more different to the *SOM* pre-tests than the marks from the alternative intervention group. This means that the *SOM* pre-test for problem-solving behaviour, which was the main item being assessed, did not correlate with the criterion for selection. I was not able to remedy this situation at the data collection stage by analysing the top 5% by results in the Problem-solving behaviour sub-test of the *SOM* as I did not administer the *SOM* to the entire grade at each school, only to those selected by grade 6 marks.

6.4.3 Length of study and lack of overt teaching

I chose to have five sessions for my study, as that would allow me to arrange with the schools and complete the study in the same term. I was concerned that if I made the study too long, the schools might not want to have the study at their school, or there might be significant attrition from the groups. Because the administration of the *SOM* took two sessions, this left only three hour-long sessions for the intervention, which was very limited in terms of practising a new skill. In contrast, the Maths Olympiad course I ran for gifted learners mentioned in Chapter 1 ran for 10 sessions, most of which ran for much longer than an hour.

Secondly, because I wanted the study to be replicable by teachers with limited skills in Mathematics Olympiad type questions, such as evidenced by Govender's study of trainee teachers (Govender, 2014b), I chose to not teach problem-solving skills to the participants, but just encouraged them to keep trying, and explained the answers to the previous week's questions on the board at the start of the next session, using the answer sheets provided by the SAMF (South African Mathematics Foundation, 2018). Actual overt skills building such as offered by the Siyanqoba regional training for high school learners or the online SAMF Olympiad training for Grades 7-12 (South African Mathematics Foundation, 2020a), in conjunction with practice of past papers, might have resulted in better skills acquisition.

6.4.4 Relative difficulty of interventions

The intervention and the alternative intervention were not equivalent in terms of difficulty. Participants in the intervention only got 21% of their answers correct, and some of those will have been due to luck, as the SA Mathematics Challenge is multiple choice. In contrast, participants in the alternative intervention were doing work that was very familiar to them, and they got 82% of the questions correct. Even though I did not give the learners marks on papers, I did give them answer sheets, and it would be hard for learners not to notice how they were doing. The qualitative part of the study backs this up: participants in the alternative intervention described the sums they did as "easy" and "fine", whereas the participants from the intervention qualified their positive statements with statements about overcoming difficulty.

The relative difficulty of the interventions is likely the major reason for the dropout rate in the SA Mathematics Challenge intervention, but due to other confounding variables (the greater involvement of parents at the alternative intervention school, and the promise of participation certificates for staying to the end), one cannot be totally sure.

6.5 ETHICAL CONSIDERATIONS

My ethics were guided by the APA General Principles of beneficence and non-maleficence; fidelity and responsibility; integrity; justice; and respect for people's rights (Elias & Theron, 2012). I also conformed to the APA requirements for Research and Publication and Assessment (American Psychological Association, 2017) as listed in Chapter 3. These requirements included institutional approval from the Department of Basic Education, the University of Pretoria and the schools concerned (see Annexure A for the school participation letter), informed consent and assent (see Annexure B), and use of an instrument that is reliable and valid, and normed on learners similar to those in my study.

6.6 RECOMMENDATIONS

6.6.1 Improvements to this research project

If I were to repeat this study, I would recommend the following changes:

- Selection by IQ test, Grade 6 standardised Annual National Assessment (ANA) results, or the Problem-Solving Behaviour sub-test of the *SOM*
- 10 or more sessions of SA Mathematics Challenge practice rather than just three
- Overt teaching on how to approach various types of Mathematics Olympiad questions.

6.6.2 Further research

I suggest the following further research to increase understanding of gifted disadvantaged learners and Mathematics Olympiad participation:

- Olympiad preparation techniques used by no-fee schools that are successful in the SA Mathematics Challenge or Mathematics Olympiad
- How successful are the SAMF Olympiad training programmes for gifted disadvantaged learners?
- A comparative study of *SOM* problem-solving behaviour scores of learners at two disadvantaged schools, one that is successful in the SA Mathematics Challenge, and one that is not.

6.7 SUMMARY OF CHAPTER 6

Gifted disadvantaged learners are important to the future development of our country, and should be nurtured (Lubinski & Benbow, 2006). The UPPS framework can be used as a guideline on how to support gifted disadvantaged learners. A unitary concept of giftedness acknowledges that gifted children are to be found in all socio-economic groups, while potential acknowledges that even gifted learners need support to flourish, and is inclusive as it allows twice-exceptional learners to be included in the definition of giftedness. Precocity gives a simple and cheap option for handling gifted learners: acceleration, which was well used in the schools that I studied, with several under-age learners in my study. Lastly, the socio-emotional aspect of the UPPS framework reminds teachers to take into account the emotional dimension when teaching gifted learners, nurturing passion rather than shutting it down.

My study found a positive relationship between success in traditional Mathematics and Study Attitude, Study Habits, and overall Study Orientation. Poverty and giftedness were shown to interact: the gifted disadvantaged learners in my study were less disadvantaged by their surrounding than one would expect, and conversely had higher Mathematics anxiety than expected for their achievement level.

While I have listed the opportunities for mathematically-gifted learners in disadvantaged areas of South Africa in Chapter 2, much more could be done. The participants in my study found the problem-solving questions in the SA Mathematics Challenge unfamiliar and difficult. Greater experience of Mathematics Olympiads, possibly coupled with teaching problem-solving techniques, is recommended to help mathematically-gifted disadvantaged learners live up to their potential as South Africa's problem-solvers.

LIST OF REFERENCES

Allan Gray Orbis Foundation. (2019). Scholarship. Retrieved November 16, 2019, from Allan Gray Orbis Foundation website: https://www.allangrayorbis.org/entrepreneurship-development-programmes/scholarship/

American Psychological Association. (2017). Ethical principles of psychologists and code of conduct. https://doi.org/10.1093/med:psych/9780199845491.003.0103

Association for Mathematics Education of South Africa. (2018). About the Challenge. Retrieved January 15, 2018, from http://www.amesa.org.za/Challenge/Preface.html

Baldwin, A. (2006). Diversity: Perceptions of pre-service teachers – a cause for concern for gifted minorities. In G. I. Eriksson & B. Wallace (Eds.), *Diversity in gifted education: International perspectives on global issues* (pp. 272–282). Oxford, United Kingdom: Routledge.

Barnes, L., Hauser, J., Heikes, L., Hernandez, A. J., Richard, P. T., Ross, K., … Palmquist, M. (2012). Experimental and quasi-experimental research. Retrieved February 2, 2018, from Writing@CSU, Colorado State University website: https://writing.colostate.edu/guides/guide.cfm? guideid=64

Beaujean, A. A. (2015). John Carroll's views on intelligence: Bi-Factor vs. higher-order models. *Journal of Intelligence*, *3*(4), 121–136. https://doi.org/10.3390/jintelligence3040121

Benson, N. F., Beaujean, A. A., McGill, R. J., & Dombrowski, S. C. (2018). Revisiting Carroll's survey of factor-analytic studies: Implications for the clinical assessment of intelligence. *Psychological Assessment*, *30*(8), 1028–1037. https://doi.org/10.1037/pas0000556

Bickell, A. (2016). *WISC-IV test performance of grade 3 Xhosa-speaking children* (Master's thesis, University of Fort Hare, East London, South Africa). Retrieved from http://hdl.handle.net/20.500.11837/581

Binet, A., & Simon, T. (1916). *The development of intelligence in children (the Binet-Simon scale)*. Baltimore, MD: William & Wilkins Company.

Blaas, S. (2014). The relationship between social-emotional difficulties and underachievement of gifted students. *Australian Journal of Guidance and Counselling*, *24*(2), 243–255. https://doi.org/10.1017/jgc.2014.1

Bolland, A. C., Besnoy, K. D., Tomek, S., & Bolland, J. M. (2019). The effects of academic giftedness and gender on developmental trajectories of hopelessness among students living in economically disadvantaged neighborhoods. *Gifted Child Quarterly*, *63*(4), 225–242. https://doi.org/10.1177/0016986219839205

Borland, J. H. (2004). Issues and practices in the identification and education of gifted students from under-represented groups. *National Research Center on the Gifted and Talented, University of Connecticut*, (May), 1–46.

Botha, J. J. (2011). *Exploring mathematical literacy: The relationship between teachers' knowledge and beliefs and their instructional practices* (Master's thesis, University of Pretoria, Pretoria, South Africa). Retrieved from https://repository.up.ac.za/handle/2263/28984

Bouwer, L. (2014). *An improvement of the quality of the translated Sesotho Junior South African Individual Scale (GIQ-8) test items* (Master's thesis, University of Johannesburg, Johannesburg, South Africa). Retrieved from http://hdl.handle.net/10210/12240

Brown, J. (2016, August 26). Black middle class floating the economy. *City Press*. Retrieved from http://city-press.news24.com/Business/black-middle-class-floating-the-economy-20160826

Business Tech. (2016). Black vs white middle class in South Africa. Retrieved from Business Tech website: https://businesstech.co.za/news/business/134749/black-vs-white-middle-class-in-south-africa/

Business Tech. (2019). Here's how many South African schools don't have the internet or a computer lab – and what it will cost to fix the problem. Retrieved November 19, 2019, from Business Tech website: https://businesstech.co.za/news/internet/259171/heres-how-many-south-african-schools-dont-have-the-internet-or-a-computer-lab-and-what-it-will-cost-to-fix-the-problem/

Carman, C. A. (2013). Comparing apples and oranges. *Journal of Advanced Academics*, *24*(1), 52–70. https://doi.org/10.1177/1932202X12472602

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytics studies*. https://doi.org/10.1103/PhysRevD.72.042004

Castejon, J. L., Perez, A. M., & Gilar, R. (2010). Confirmatory factor analysis of Project Spectrum activities. A second-order g factor or multiple intelligences? *Intelligence*, *38*(5), 481–496. https://doi.org/10.1016/j.intell.2010.07.002

Celik, E., Gedik, N., Karaman, G., Demirel, T., & Goktas, Y. (2014). Mistakes encountered in manuscripts on education and their effects on journal rejections. *Scientometrics*, *98*(3), 1837–1853. https://doi.org/10.1007/s11192-013-1137-y

Centurus Colleges. (2019). Academic enrichment centre. Retrieved December 1, 2019, from Centurus Colleges website: http://www.centuruscolleges.co.za/academic-enrichment-centre/

Chetty, M. (2016). *Comparing school based assessments with standardised national assessments in South Africa* (Doctoral thesis, University of the Witwatersrand, Johannesburg, South Africa). Retrieved from http://hdl.handle.net/10539/20787

Chilisa, B., & Kawulich, B. (2012). Selecting a research approach: Paradigm, methodology and methods. In C. Wagner, B. Kawulich, & Mark Garner (Eds.), *Doing social research: A global context* (1st ed.). London, United Kingdom: McGraw Hill Higher Education.

Chirinda, B. (2013). *The development of mathematical problem solving skills of grade 8 learners in a problem-centred teaching and learning environment at a secondary school in Gauteng* (Master's thesis, University of South Africa, Pretoria, South Africa). Retrieved from http://hdl.handle.net/10500/11876

Chirove, M., & Mogari, D. (2014). Relationship between learners' mathematics-related belief systems and their approaches to non-routine mathematical problem solving: A case study of three high schools in Tshwane North District (D3), South Africa. *2014 ISTE International Conference on Mathematics, Science and Technology Education*, (June), 119–130. Retrieved from http://hdl.handle.net/10500/22916

Christie, P., Butler, D., & Potterton, M. (2007). *Report to the Minister of Education: Ministerial committee on schools that work*. Retrieved from http://www.education.uct.ac.za/sites/default/files/image_tool/images/104/schoolsthatwork.pdf

Christie, P., & McKinney, C. (2017). Decoloniality and "Model C" schools: Ethos, language and the protests of 2016. *Education as Change, 21*(3), 1–21. https://doi.org/10.17159/1947-9417/2017/2332

Claremont High School. (2019). Frequently asked questions. Retrieved November 17, 2019, from https://claremonthigh.co.za/index.php/frequently-asked-questions/

Cockcroft, K. (2013). The Senior South African Individual Scales – Revised: A review. In S. Laher & K. Cockcroft (Eds.), *Psychological assessment in South Africa: Research and applications* (pp. 48–59). https://doi.org/10.18772/22013015782

Conquesta Olympiads. (2019). Conquesta Olympiads. Retrieved November 16, 2019, from Conquesta Olympiads website: http://www.conquestaolympiads.com/

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). https://doi.org/10.1016/S0953-7562(10)80014-0

Cruickshank, J. (2011). The positive and the negative: Assessing critical realism and social constructionism as post-positivist approaches to empirical research in the social sciences. In *The IMI Working Papers Series* (No. 42). Oxford, United Kingdom.

Dağlioğlu, H. E., & Suveren, S. (2013). The role of teacher and family opinions in identifying gifted kindergarten children and the consistence of these views with children's actual performance. *Kuram ve Uygulamada Egitim Bilimleri*, *13*(1), 444–453.

Daniels, S., & Piechowski, M. M. (2009). Embracing intensity: Overexcitability, sensitivity, and the developmental potential of the gifted. In S. Daniels & M. M. Piechowski (Eds.), *Living with intensity* (pp. 3–17). Tucson, AZ: Great Potential Press.

Davis, K., Christodoulou, J., Seider, S., & Gardner, H. (2011). The theory of multiple intelligences. In R. J. Sternberg & S. B. Kaufman (Eds.), *Cambridge handbook of intelligence* (pp. 485–503). Cambridge, United Kingdom: Cambridge University Press.

Department of Basic Education. (n.d.). National Curriculum Statement (NCS) Grades R to 12. Retrieved January 30, 2018, from Department of Basic Education website: https://www.education.gov.za/Curriculum/NationalCurriculumStatementsGradesR-12.aspx

Department of Basic Education. (2011). *Curriculum and assessment policy statement Grades 4 to 6: Mathematics*. Pretoria, South Africa: Department of Basic Education.

Department of Basic Education. (2016). *Norms and standards for public schools that provide education with a specialised focus (focus schools)*. Pretoria, South Africa.

Department of Basic Education. (2017a). Quarter 4 of 2016: March 2017: Gauteng. Retrieved April 7, 2018, from Schools Masterlist Data website: https://www.education.gov.za/LinkClick.aspx?fileticket=E0TOnuayAFI%3D&tabid=466&portalid=0&mid=7070

Department of Basic Education. (2017b). *The consolidated Department of Basic Education research protocols*. Pretoria, South Africa: Department of Basic Education.

Department of Basic Education. (2018a). *Mathematics in English: Grade 7 book 1 terms 1 & 2* (8th ed.). Pretoria, South Africa: Department of Basic Education.

Department of Basic Education. (2018b). *Mathematics teaching and learning framework for South Africa: Teaching Mathematics for understanding*. Pretoria, South Africa: Government printer.

Department of Education. (2001). *Education white paper 6, special needs education: Building an inclusive education and training system*. Pretoria, South Africa: Government printer.

Dewar, M. (1986). *Gifted Education and Ideology: The growth of the gifted education movement in South Africa*. (Master's thesis, University of Cape Town, Cape Town, South Africa). Retrieved from http://hdl.handle.net/11427/14570.

Diezmann, C. M., & Watters, J. J. (2002). The importance of challenging tasks for mathematically gifted students. *Gifted and Talented International*, *17*(2), 76–84.

Du Plessis, H. (2015). *Evaluating the effectiveness of Advanced Programme Mathematics in preparing learners for university Mathematics* (Master's thesis, Stellenbosch University, Stellenbosch, South Africa). Retrieved from http://hdl.handle.net/10019.1/97080

Eden Schools. (2019). Scholarships and bursaries. Retrieved November 16, 2019, from http://edenschools.co.za/durban-process/durban-scholarships-and-bursaries/

Editage Insights. (2013). Most common reasons for journal rejection. https://doi.org/https://doi.org/10.34193/EI-A-6091

Elias, M. J., & Theron, L. C. (2012). Linking purpose and ethics in thesis writing: South African illustrations of an international perspective. In J. G. Maree (Ed.), *Complete your thesis or dissertation successfully: Practical guidelines* (pp. 145–160). Cape Town, South Africa: Juta.

Engel, R. J. E., & Shutt, R. K. (2014). *Fundamentals of social work research*. Thousand Oaks, CA: SAGE Publications.

Engelbrecht, J., & Mwambakana, J. (2016). Validity and diagnostic attributes of a mathematics olympiad for junior high school contestants. *African Journal of Research in Mathematics, Science and Technology Education*, *20*(2), 175–188. https://doi.org/10.1080/18117295.2016.1190211

Epworth School. (2019). Grade 8 Entrance / Scholarship Examination. Retrieved November 16, 2019, from https://www.epworth.co.za/join-us/grade-8-scholarship-entrance-examination/

Erasmos, C. (2013). *The impact of enrichment programs on the performance of gifted Science learners* (Master's thesis, University of South Africa, Pretoria, South Africa). Retrieved from http://hdl.handle.net/10500/14261

Erasmus, P. (2014). Assessment strategy in Mathematics for second language learners. *Literacy Information and Computer Education Journal*, *5*(4), 1655–1660. https://doi.org/10.20533/licej.2040.2589.2014.0221

Eriksson, G. I. (1987). An education centre for growth. *Gifted Child Today*, *10*(4), 25–28.

Eriksson, G. I. (1993). The global village beyond 2000: Networking with the gifted disadvantaged. In B. Wallace & H. B. Adams (Eds.), *Worldwide perspectives on the gifted disadvantaged* (pp. 100–132). Oxford, United Kingdom: AB Academic Publishers.

Fairbanks, E. (2014, November 28). How to build an elite school for all. *Mail & Guardian*. Retrieved from https://mg.co.za/article/2014-11-28-00-how-to-build-an-elite-school-for-all

Festus, A. B., & Seraphina, K. M. (2015). Effects of emotional intelligence skills acquisition on students' achievement in Senior Secondary School geometry in Keffi Education Zone, Nasarawa State, Nigeria. *Asian Journal of Education and E-Learning*, *03*(04), 243–257.

Gaillard, C. (2019). Finding the missing variables: A systematic review of mathematics improvement strategies for South African public schools. *South African Journal of Education*, *39*(3), 1–9. https://doi.org/10.15700/saje.v39n3a1582

Gardner, H. (2006). On failing to grasp the core of MI theory: A response to Visser et al. *Intelligence*, *34*(5), 503–505. https://doi.org/10.1016/j.intell.2006.04.002

Gardner, H., & Hatch, T. (1989). Multiple intelligences go to school. *Educational Research*, *18*(8), 4–10. https://doi.org/10.3102/0013189X018008004

Gardner, H., & Moran, S. (2006). The science of Multiple Intelligences theory: A response to Lynn Waterhouse. *Educational Psychologist*, *41*(4), 227–232. https://doi.org/10.1207/s15326985ep4104

Garg, A., Das, S., & Jain, H. (2015). Why we say no! A look through the editor's eye. *Journal of Clinical and Diagnostic Research*, *9*(10), JB01–JB05. https://doi.org/10.7860/JCDR/2015/17160.6699

Gladwell, M. (2008). *Outliers*. London, United Kingdom: Penguin Books.

Gomez-Arizaga, M. P., & Conejeros-Solar, L. (2014). Gifted students' readiness for college. *Gifted Education International*, *30*(3), 212–227. https://doi.org/10.1177/0261429413486573

Goodman, S., Jaffer, T., Keresztesi, M., Mamdani, F., Mokgatle, D., Musariri, M., … Schlechter, A. (2011). An investigation of the relationship between students' motivation and academic performance as mediated by effort. *South African Journal of Psychology*, *41*(3), 373–385. https://doi.org/10.1177/008124631104100311

Govender, V. G. (2014a). Factors contributing to the popularity of Mathematics Olympiads and competitions in some schools: An interrogation of learners' and teachers' views. In M. Lebitso & A. Maclean (Eds.), *20th Annual National Congress of the Association for Mathematics of South Africa* (pp. 70–86). Retrieved from http://www.amesa.org.za/AMESA2014/Proceedings/index.html

Govender, V. G. (2014b). Using the South African mathematics challenge to develop pre-service mathematics teachers' problem-solving abilities. In M. Lebitso & A. Maclean (Eds.), *20th Annual National Congress of the Association for Mathematics of South Africa* (pp. 87–104). Retrieved from http://www.amesa.org.za/AMESA2014/Proceedings/index.html

Griesel, H., & Parker, B. (2009). *Graduate attributes: A baseline study on South African graduates from the perspective of employers*. Pretoria, South Africa: Higher Education South Africa & the South African Qualifications Authority.

Grønmo, L. S., Lindquist, M., Arora, A., & Mullis, I. V. S. (2015). TIMSS 2015 Mathematics framework. In *TIMSS 2015 Assessment Frameworks* (pp. 11–27). Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Gross, M. U. M. (1999). Small poppies: Highly gifted children in the early years. *Roeper Review*, *21*(3), 207–214.

Gross, M. U. M. (2006). Exceptionally gifted children: Long-term outcomes of academic acceleration and nonacceleration. *Journal for the Education of the Gifted*, *29*(4), 404–429. https://doi.org/10.4219/jeg-2006-247

Gross, M. U. M., & van Vliet, H. E. (2005). Radical acceleration and early entry to college: A review of the research. *Gifted Child Quarterly*, *49*(2), 154–171. https://doi.org/10.1177/001698620504900205

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105–117). Thousand Oaks, CA: SAGE Publications.

Hart, S. A., Logan, J. A. R., Thompson, L., Kovas, Y., McLoughlin, G., & Petrill, S. A. (2016). A latent profile analysis of math achievement, numerosity, and math anxiety in twins. *Journal of Educational Psychology*, *108*(2), 181–193. https://doi.org/10.1037/edu0000045

Health Professions Council of South Africa. (2017). Tests that have been classified and reviewed. Retrieved October 25, 2020, from Government Gazette website: https://www.hpcsa.co.za/Uploads/PSB_2019/List_of_Classified_tests_Board_Notice_155_of_2017.pdf

Hertzog, N. B. (2003). Impact of gifted programs from the students' perspectives. *Gifted Child Quarterly*, *47*(2), 131–143. https://doi.org/10.1177/001698620304700204

Heuser, B. L., & Wang, K. (2017). Global dimensions of gifted and talented education: The influence of national perceptions on policies and practices. *Global Education Review*, *4*(1), 4–21.

Hlalele, D. (2012). Exploring rural high school learners' experience of Mathematics anxiety in academic settings. *South African Journal of Education*, *32*(3), 267–278. https://doi.org/10.15700/saje.v32n3a623

Hlalele, D. (2019). Exploring rural university access programme students' experience of Mathematics anxiety in academic settings. *Africa Education Review*, *16*(1), 40–57. https://doi.org/10.1080/18146627.2016.1224580

Howdle, D. (2019). Worldwide mobile data pricing: The cost of 1GB of mobile data in 230 countries. Retrieved November 16, 2019, from Cable website: https://www.cable.co.uk/mobiles/worldwide-data-pricing/#resources

Howie, S., Combrinck, C., Roux, K., Tshele, M., Mokoena, G., & Palane, N. M. (2017). *PIRLS Literacy 2016 Progress in International Reading Literacy Study 2016: South African children's reading literacy achievement*. Pretoria.

Hoy, W. K., & Adams, C. M. (2016). *Quantitative research in education: A primer* (2nd ed.). London, United Kingdom: SAGE Publications.

Hudson Park Primary School. (2019). Mathematics FP. Retrieved November 16, 2019, from Hudson Park Primary School website: http://www.hphs.co.za/primary/FPMathematics

IBM Corp. (2017). *IBM SPSS Statistics for Windows*. Armonk, NY: IBM Corp.

Independent Examination Board. (2019). Advanced programmes. Retrieved November 16, 2019, from Independent Examination Board website: https://www.ieb.co.za/pages/advancedprogrammes

Israel, N. (2006). *Raven's Advanced Progressive Matrices within a South African context* (Master's thesis, University of the Witwatersrand, Johannesburg, South Africa). Retrieved from http://hdl.handle.net/10539/1703

Jagals, D. (2013). *An exploration of reflection and Mathematics confidence during problem solving in senior phase Mathematics* (Master's thesis, North-West University, Potchefstroom, South Africa). Retrieved from http://hdl.handle.net/10394/9067

Jamieson, L., Berry, L., & Lake, L. (Eds.). (2017). *South African child gauge 2017*. Retrieved from http://www.ci.uct.ac.za/ci/child-gauge/2017

Jenkins, D. J. (2004). *The predictive validity of the General Scholastic Aptitude Test (GSAT) for first year students in Information Technology* (Master's thesis, University of Zululand, Richard's Bay, South Africa). Retrieved from http://hdl.handle.net/10530/57

Johnson, K., & Schmidt, A. (2006). The effects of teaching problem solving strategies to low achieving students. *Action Research Projects, 62*. Retrieved from http://digitalcommons.unl.edu/mathmidactionresearch/62

Johnson, R. B., & Onwuegbuzie, A. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, *33*(14), 14–26. https://doi.org/10.3102/0013189X033007014

Jordan, S.-C. (2015). The languages of South Africa. Retrieved October 16, 2019, from Alpha Omega Translations website: https://alphaomegatranslations.com/foreign-language/the-languages-of-south-africa/

Kaya, F., Stough, L. M., & Juntune, J. (2016). Verbal and nonverbal intelligence scores within the context of poverty. *Gifted Education International*, (1966). https://doi.org/10.1177/0261429416640332

Keele, R. (2011). *Nursing research and evidence-based practice: Ten steps to success*. Sudbury, MA: Jones & Bartlett Learning.

Keith, T. Z., & Reynolds, M. R. (2010). Cattell-Horn-Caroll abilities and cognitive tests: What we've learned from 20 years of research. *Psychology in the Schools*, *47*(7), 635–650.

Kell, H. J., Lubinski, D., & Benbow, C. P. (2013). Who rises to the top? Early indicators. *Psychological Science*, *24*(5), 648–659. https://doi.org/10.1177/0956797612457784

Klein, P. D. (1997). Multiplying the problems of intelligence by eight: A critique of Gardner's theory. *Canadian Journal of Education / Revue Canadienne de l'éducation*, *22*(4), 377. https://doi.org/10.2307/1585790

Knowles, J. (2008). *Assessment of non-verbal intelligence in South African schools: Do language and gender bias performance on the Raven's Standard Progressive Matrices?* (Master's thesis, University of the Witwatersrand, Johannesburg, South Africa). Retrieved from http://hdl.handle.net/10539/8238

Knox, K. (2004). A Researcher's dilemma - philosophical and methodological pluralism. *Electronic Journal of Business Research Methods*, *2*(2), 119–128. https://doi.org/10.1080/03085140500465899

Kokot, S. J. (1999). Discovery learning: Founding a school for gifted children. *Gifted Education International*, *13*, 269–282.

Kornmann, J., Zettler, I., Kammerer, Y., Gerjets, P., & Trautwein, U. (2015). What characterizes children nominated as gifted by teachers? A closer consideration of working memory and intelligence. *High Ability Studies*, *26*(1). https://doi.org/10.1080/13598139.2015.1033513

Kotze, J. (2006). *The effect of a dynamic technological learning environment on the geometry conceptualisation of pre-service Mathematics teachers* (Master's thesis, North-West University, Potchefstroom, South Africa). Retrieved from http://hdl.handle.net/10394/1359

Kotzé, M., & Niemann, R. (2013). Psychological resources as predictors of academic performance of first-year students in higher education. *Acta Academica*, *45*(2), 85–121.

Kūma, D. (2015). Novelties in Math Olympiads in Latvia. In F. M. Singer, F. Toader, & C. Voica (Eds.), *9th International Mathematical Creativity & Giftedness (MCG) Conference* (pp. 54–59). Sinaia, Romania.

Laher, S., & Cockcroft, K. (2013). Contextualising psychological assessment in South Africa. In S. Laher & K. Cockcroft (Eds.), *Psychological assessment in South Africa: Research and applications* (pp. 1–14). https://doi.org/10.18772/22013015782.6

Laher, S., & Cockcroft, K. (2017). Moving from culturally biased to culturally responsive assessment practices in low-resource, multicultural settings. *Professional Psychology: Research and Practice*, *48*(2), 115–121. https://doi.org/10.1037/pro0000102

Laubscher, L., & Olszewski-Kubilius, P. (1996). The impact of a college counseling program on economically disadvantaged gifted students and their subsequent college adjustment. *Roeper Review*, *18*(3), 202–208. https://doi.org/10.1080/02783199609553735

LEAP Science and Maths Schools. (2019). LEAP Science & Maths Schools. Retrieved November 17, 2019, from LEAP Science and Maths Schools website: http://leapschool.org.za/

Lemmon, D. K. (2017). *An outcome evaluation of the Centre of Science and Technology*. (Master's thesis, University of Cape Town, Cape Town, South Africa). Retrieved from http://hdl.handle.net/11427/25363.

Letsoalo, M. E., Masha, J. K., & Maoto, R. S. (2019). The overall performance of Grade 12 Mathematics and Physical Science learners in South Africa's Gauteng province. *Journal of Gender, Information and Development in Africa*, *8*(1), 9–42. https://doi.org/10.31920/2050-4284/2019/8n1a1

Lindskog, M., Winman, A., & Poom, L. (2017). Individual differences in nonverbal number skills predict Math anxiety. *Cognition*, *159*, 156–162. https://doi.org/10.1016/j.cognition.2016.11.014

Lipina, S. J. (2016). The biological side of social determinants: Neural costs of childhood poverty. *Prospects*, *46*(2), 265–280. https://doi.org/10.1007/s11125-017-9390-0

Lohman, D. F. (2001). Fluid intelligence, inductive reasoning, and working memory: Where the theory of Multiple Intelligences falls short. In N. Colangelo & S. Assouline (Eds.), *Talent Development IV: Proceedings from the 1998 Henry B. and Jocelyn Wallace National Research Symposium on talent development* (pp. 219–228). Scottsdale, AZ: Gifted Psychology Press.

Lohman, D. F. (2013). Identifying gifted students: Nontraditional uses of traditional measures. In C. M. Callahan & H. Hertberg-Davis (Eds.), *Fundamentals of gifted education*. New York, NY: Taylor & Francis/Routledge.

Lohman, D. F., & Gambrell, J. L. (2012). Using nonverbal tests to help identify academically talented children. *Journal of Psychoeducational Assessment*, *30*(1), 25–44. https://doi.org/10.1177/0734282911428194

Lohman, D. F., Korb, K. A., & Lakin, J. M. (2008). Identifying academically gifted English-language learners using nonverbal tests: A comparison of the Raven, NNAT, and CogAT. *Gifted Child Quarterly*, *52*(4), 275–296. https://doi.org/10.1177/0016986208321808

Lombard, K., & Grosser, M. (2008). Critical thinking: Are the ideals of OBE failing us or are we failing the ideals of OBE? *South African Journal of Education*, *28*(4), 561–579.

Long, C., Engelbrecht, J., Scherman, V., & Dunne, T. (2016). Investigating the treatment of missing data in an Olympiad-type test - the case of the selection validity in the South African Mathematics Olympiad. *Pythagoras*, *37*(1), 1–14. https://doi.org/10.4102/pythagoras.v37i1.333

Long, C., & Wendt, H. (2017). A comparative investigation of South Africa's high-performing learners on selected TIMSS items comprising multiplicative concepts. *African Journal of Research in Mathematics, Science and Technology Education*, *1*(2), 1–8. https://doi.org/10.1080/02699931.2011.628301

Lubinski, D. (2016). From Terman to today: A century of findings on intellectual precocity. *Review of Educational Psychology*, *86*(4), 900–944. https://doi.org/10.3102/0034654316675476

Lubinski, D., & Benbow, C. P. (2006). Special section: Doing psychological science study of mathematically precocious youth after 35 years. *Perspectives on Psychological Science*, *1*(4), 316–345. https://doi.org/10.1111/j.1745-6916.2006.00019.x

Lumadi, T. E. (1998). *Sociocultural factors in the family that are significant for the development of giftedness in Vhavenda children* (University of South Africa). Retrieved from http://hdl.handle.net/10500/17261

Luttenberger, S., Wimmer, S., & Paechter, M. (2018). Spotlight on Math anxiety. *Psychology Research and Behavior Management*, *11*, 311–322. https://doi.org/10.2147/PRBM.S141421

Madge, E. M. (1981). *Manual for the Junior South African Individual Scale (JSAIS)*. Pretoria, South Africa: Human Sciences Research Council.

Maker, C. J. (2006). Creativity, intelligence, problem-solving and diversity. In G. Eriksson & B. Wallace (Eds.), *Diversity in gifted education: International perspectives on global issues* (pp. 28–45). Oxford, United Kingdom: Routledge.

Maloney, E. A., Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2015). Intergenerational effects of parents' math anxiety on children's math achievement and anxiety. *Psychological Science*, *26*(9), 1480–1488. https://doi.org/10.1177/0956797615592630

Maree, J. G. (2015). Barriers to access to and success in higher education: Intervention guidelines. *South African Journal of Higher Education*, *29*(1), 390–411.

Maree, J. G. (2018a). Career-life counselling for the gifted in sub-Saharan Africa. In B. Wallace, D. A. Sisk, & J. Senior (Eds.), *The SAGE Handbook of Gifted and Talented Education* (pp. 373–389). London, United Kingdom: SAGE Publications.

Maree, J. G. (2018b). Gifted education in Africa. In S. I. Pfeiffer, E. Shaunessy-Dedrick, & Megan Foley-Nicpon (Eds.), *APA handbook of giftedness and talent* (1st ed.). https://doi.org/https://doi.org/10.1037/0000038-000

Maree, J. G. (2020). Study Orientation in Mathematics. Retrieved September 23, 2020, from https://jvrafricagroup.co.za/catalogue/som

Maree, J. G., & Ebersöhn, L. (2002). Emotional intelligence and achievement: Redefining giftedness? *Gifted Education International*, *16*(3), 261–273. https://doi.org/10.1177/026142940201600309

Maree, J. G., & Erasmus, C. P. (2006). Mathematics skills of Tswana-speaking learners in the North West Province of South Africa. *Early Child Development and Care*, *176*(1), 1–18. https://doi.org/10.1080/03004430500209696

Maree, J. G., Pretorius, A., & Eiselen, R. J. (2003). Predicting success among first-year engineering students at the Rand Afrikaans University. *Psychological Reports*, *93*(2), 399–409. https://doi.org/10.2466/pr0.2003.93.2.399

Maree, J. G., Prinsloo, W. B. J., & Claassen, N. C. W. (2011). *Manual for the Study Orientation Questionnaire in Maths (S.O.M.)*. Johannesburg, South Africa: JvR Psychometrics.

Maree, J. G., van der Walt, M. S., & Ellis, S. M. (2020). TriMaths. Retrieved September 23, 2020, from https://jvrafricagroup.co.za/catalogue/tri-maths

Maree, J. G., Van der Walt, M. S., & Ellis, S. M. (2009). Developing a Study Orientation Questionnaire in Mathematics for primary school students. *Psychological Reports*, *104*(2), 425–438. https://doi.org/10.2466/PR0.104.2.425-438

Maree, K. (2007). A reflective conversation with Archbishop Desmond Tutu. *Gifted Education International*, *23*(2), 188–192. https://doi.org/10.1177/026142940702300208

Maree, K. (2016). Planning a research proposal. In K. Maree (Ed.), *First steps in research* (2nd ed.). Pretoria, South Africa: Van Schaik.

Maree, K., & Pietersen, J. (2016). The quantitative research process. In K. Maree (Ed.), *First steps in research* (2nd ed., pp. 162–170). Pretoria, South Africa: Van Schaik.

Marumo, J. M., & Mhlolo, M. K. (2017). Strategies used by teachers for supporting mathematically gifted learners in Bloemfontein high schools. In D. Pitta-Pantazi (Ed.), *The 10th Mathematical Creativity and Giftedness International Conference* (pp. 57–60). Nicosia, Cyprus: Department of Education, University of Cyprus, Cyprus.

Mat Roni, S., Merga, M. K., & Morris, J. E. (2020). *Conducting quantitative research in education*. https://doi.org/10.1007/978-981-13-9132-3

Matheson, D. (2012). *Teaching through problem solving: Bridging the gap between vision and practice* (Master's thesis, Simon Fraser University, Burnaby, British Columbia, Canada). Retrieved from http://summit.sfu.ca/system/files/iritems1/12405/etd7370_DMatheson.pdf

Matthews, M. S. (2009). Gifted learners who drop out: Prevalence and prevention. In *International handbook on giftedness* (pp. 527–528). https://doi.org/10.1007/978-1-4020-6162-2

Mawila, D. (2012). *An explorative investigation of the quality of items of the performance scales on the translated Sesotho version of the Junior South African Individual Scales JSAIS (GIQ-8)* (Master's thesis, University of Johannesburg, Johannesburg, South Africa). Retrieved from http://hdl.handle.net/10210/8695

Mayaba, P. L. (2016). *The cultural and linguistic appropriateness of the Individual Scale for Zulu-speaking pupils: A Bakhtinian analysis* (Doctoral thesis, University of KwaZulu-Natal, South Africa). Retrieved from http://hdl.handle.net/10413/14903

McCann, M. (2005). International perspectives on giftedness: Experimental and cultural observations of IQ and creativity with implications for curriculum and policy design. *International Education Journal*, *6*(2), 125–135.

McGrew, K., & Evans, J. J. (2004). Internal and external factorial extensions to the Cattell-Horn-Carroll (CHC) theory of cognitive abilities: A review of factor analytic research since Carroll's seminal 1993 treatise. *Carroll Human Cognitive Abilities (HCA) Project Research Report #2*, *2*.

McGrew, K. S. (2009). CHC theory and the human cognitive abilities project: Standing on the shoulders of the giants of psychometric intelligence research. *Intelligence*, *37*(1), 1–10. https://doi.org/10.1016/j.intell.2008.08.004

Mendaglio, S., & Tillier, W. (2006). Dabrowski's theory of positive disintegration and giftedness: Overexcitability research findings. *Journal for the Education of the Gifted*, *30*(1), 68–87. https://doi.org/10.1177/016235320603000104

Mensa South Africa. (2018). Mensa South Africa. Retrieved March 10, 2018, from Mensa South Africa website: http://mensa.org.za/

Mertens, D. M. (2015). *Research and evaluation in education and psychology* (4th ed.). London, United Kingdom: SAGE Publications.

Mhlolo, M. K. (2011). *From coherence in theory to coherence in practice: A stock-take of the written, tested and taught National Curriculum Statement for Mathematics (NCSM) at Further Education and Training (FET) level in South Africa* (Doctoral thesis, University of the Witwatersrand, Johannesburg, South Africa). Retrieved from http://hdl.handle.net/10539/11274

Mhlolo, M. K. (2014a). Is rote learning of number concepts 'inherently rotten' or is it just a blame and shame game that vitiates principles of natural progression? *Mediterranean Journal of Social Sciences*, *5*(27), 1581–1591. https://doi.org/10.5901/mjss.2014.v5n27p1581

Mhlolo, M. K. (2014b). Opening up conversations of the plight of the mathematically talented students in sub-Saharan African countries. In G. Howell, L. Sheffield, & R. Leikin (Eds.), *The 8th Conference of MCG International Group for Mathematical Creativity and Giftedness* (pp. 77–81). Denver, CO: University of Denver.

Mhlolo, M. K. (2015). Examining covert impediments to inclusive education for the mathematically gifted learners in South Africa. In F. M. Singer, F. Toader, & C. Voica (Eds.), *9th International Mathematical Creativity & Giftedness (MCG) Conference* (pp. 166–171). Retrieved from http://www.mcg-9.net/

Mhlolo, M. K. (2017a). Re-examining the educational provision for mathematically gifted students across poverty strata in South Africa. In D. Pitta-Pantazi (Ed.), *The 10th Mathematical Creativity and Giftedness International Conference* (pp. 67–71). Nicosia, Cyprus: Department of Education, University of Cyprus, Cyprus.

Mhlolo, M. K. (2017b). Regular classroom teachers' recognition and support of the creative potential of mildly gifted mathematics learners. *ZDM - Mathematics Education*, *49*(1), 81–94. https://doi.org/10.1007/s11858-016-0824-6

Mochesela, P. R. (2007). *The role of the problem-based approach in the performance of Grade 9 learners in solving word problems* (Master's thesis, University of South Africa, Pretoria, South Africa). Retrieved from http://hdl.handle.net/10500/559

Modisaotsile, B. M. (2012). The failing standard of Basic Education in South Africa. *AISA POLICY Brief*, (72), 1–8. https://doi.org/http://www.section27.org.za/wp-content/uploads/2013/10/Spaull-2013-CDE-report-South-Africas-Education-Crisis.pdf

Mohlala, S. C. (2000). *The identification of gifted children in an under-resourced rural area* (Master's thesis, University of South Africa, Pretoria, South Africa). Retrieved from http://hdl.handle.net/10500/17476

Molepo, J., Owen, J., Ehlers, R., & Maree, J. (2005). Probleemgebaseerde benadering tot wiskunde in graad 9 en 11 in die Limpopo-provinsie [Problem-based approach to mathematics in Grade 9 and Grade 11 in the Limpopo Province]. *SA Tydskrif Vir Natuurwetenskap En Tegnologie*, *24*(4), 124–133.

Moodaley, R. R., Grobler, A. A., & Lens, W. (2006). Study orientation and causal attribution in Mathematics achievement. *South African Journal of Psychology*, *36*(3), 634–655. https://doi.org/10.1177/008124630603600312

Mthethwa, A. (2019, March 26). A new breed of schools opens the doors of science and technology for township pupils. *Daily Maverick*. Retrieved from https://www.dailymaverick.co.za/article/2019-03-26-a-new-breed-of-schools-opens-the-doors-of-science-and-technology-for-township-pupils/

Muijs, D. (2004). *Doing quantitative research in education with SPSS*. https://doi.org/10.4324/9781315248264-8

Muijs, D. (2011). *Doing quantitative research in education with SPSS* (2nd ed.). https://doi.org/10.4135/9781849203241

Mullet, D. R., & Rinn, A. N. (2015). Giftedness and ADHD: Identification, misdiagnosis, and dual diagnosis. *Roeper Review*, *37*(4). https://doi.org/10.1080/02783193.2015.1077910

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 international results in Mathematics*. Boston, MA: IEA.

Murray, M. (2017). How does the grade obtained at school for English and Mathematics affect the probability of graduation at a university? *Pythagoras*, *38*(1), 1–7. https://doi.org/10.4102/pythagoras.v38i1.335

Na, G., Han, D., Lee, K., & Song, S. (2007). Mathematically gifted students' problem solving approaches on conditional probability. *PME31*, *4*, 1–8.

Neber, H. (2004). Teacher identification of students for gifted programs: Nominations to a summer school for highly-gifted students. *Psychology Science*, *46*(3), 348–362.

Newstead, K. (1998). Aspects of children's Mathematics anxiety. *Educational Studies in Mathematics*, *36*(1), 53–71. https://doi.org/10.1023/A:1003177809664

Ngara, C. (2017). Gifted education in Zimbabwe. *Cogent Education*, *4*(1), 1–13. https://doi.org/10.1080/2331186X.2017.1332840

Nieman, M. M., & Monyai, R. B. (Eds.). (2006). *The educator as mediator of learning*. Pretoria, South Africa: Van Schaik.

Nieuwenhuis, J. (2016). Analysing qualitative data. In K. Maree (Ed.), *First steps in research* (2nd ed.). Pretoria, South Africa: Van Schaik.

Nieuwoudt, S. (2015). Developing a model for problem-solving in a Grade 4 Mathematics classroom. *Pythagoras*, *36*(2), 1–7. https://doi.org/10.4102/pythagoras.v36i2.275

Nortje, J. M. (2017). The effect of poverty on education in South Africa. *Educor Multidisciplinary Journal*, *1*(December), 47–62.

Onoshakpokaiye E, O. (2015). Relationship of study habits with Mathematics achievement. *Journal of Education and Practice*, *6*(10), 168–171.

Oprah Winfrey Leadership Academy for Girls. (2019). Admissions. Retrieved December 14, 2019, from Oprah Winfrey Leadership Academy for Girls website: https://owlag.co.za/admissions

Ormerod, R. (2006). The history and ideas of pragmatism. *Journal of the Operational Research Society*, *57*(8), 892–909. https://doi.org/10.1057/palgrave.jors.2602065

Oswald, M., & de Villiers, J.-M. (2013). Including the gifted learner: Perceptions of South African teachers and principals. *South African Journal of Education*, *33*(1), 1–21. https://doi.org/10.15700/saje.v33n1a603

Pajares, F. (1996). Self-efficacy beliefs and mathematical problem-solving of gifted students. *Contemporary Educational Psychology*, *21*(4), 325–344. https://doi.org/10.1006/ceps.1996.0025

Palomar-Lever, J., & Victorio-Estrada, A. (2017). Academic success of adolescents in poverty. *Social Psychology of Education*, *20*(3), 669–691. https://doi.org/10.1007/s11218-017-9389-7

Parkview Junior School. (2018). Parkview Junior School: Information booklet. Retrieved November 16, 2019, from http://www.parkviewjunior.co.za/information-booklet/

PE Montessori. (2017). FAQs about Montessori education. Retrieved November 16, 2019, from https://www.pemontessori.co.za/education/faqs-about-montessori-education

Pietersen, J., & Maree, K. (2016a). Overview of some of the most popular statistical techniques. In K. Maree (Ed.), *First steps in research* (2nd ed., pp. 249–304). Pretoria, South Africa: Van Schaik.

Pietersen, J., & Maree, K. (2016b). Statistical analysis II: Inferential statistics. In K. Maree (Ed.), *First steps in research* (2nd ed., pp. 219–236). Pretoria, South Africa: Van Schaik.

Piirto, J. (2004). *Understanding creativity*. Tucson, AZ: Great Potential Press.

Pillay, P. (2010). *Managing the teaching of critical thinking skills in English Home Language to second language speakers in the Further Education and Training phase* (Masters thesis, North-West University, Vanderbijlpark, South Africa). Retrieved from http://hdl.handle.net/10394/7176

Province of the Eastern Cape. (2018). Publication of additional allocations by provincial departments: Education (vote 6). *Provincial Gazette*, *25*(4153), 116–288. Retrieved from http://www.gpwonline.co.za

Public Library of Science. (2020). PLOS One. Retrieved September 4, 2020, from PLOS ONE website: https://journals.plos.org/plosone/static/publish

Rabie, E. H. (2013). *The lived experiences of grade 11 learners considered academically gifted* (Master's thesis, Stellenbosch University, Stellenbosch). Retrieved from https://scholar.sun.ac.za/handle/10019.1/85711

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, *53*(3), 145–164. https://doi.org/10.1080/00461520.2018.1447384

Reason, P. (2003). Pragmatist philosophy and action research. *Action Research*, *1*(1), 103–123. https://doi.org/10.1177/14767503030011007

Reddy, L. (2014). *Evaluating the first year roll-out of the Imibala gifted and talented enrichment programme of the Imibala Trust in the Western Cape*. (Master's thesis, University of Cape Town, Cape Town, South Africa). Retrieved from http://hdl.handle.net/11427/21796.

Reddy, V., Zuze, T., Visser, M., Winnaar, L., Juan, A., Prinsloo, C., … Rogers, S. (2015). *Beyond benchmarks: What twenty years of TIMSS data tell us about South African education*. Cape Town, South Africa: Human Sciences Research Council.

Reder, S., Gauly, B., & Lechner, C. (2020). Practice makes perfect: Practice engagement theory and the development of adult literacy and numeracy proficiency. *International Review of Education*, *66*(2–3), 267–288. https://doi.org/10.1007/s11159-020-09830-5

Renzulli, J. S. (1978). What makes giftedness? Reexamining a definition. *The Phi Delta Kappan*, *60*(3), 180–184.

Rindermann, H., Sailer, M., & Thompson, J. (2009). The impact of smart fractions, cognitive ability of politicians and average competence of peoples on social development. *Talent Development and Excellence*, *1*(1), 3–25.

Roedean School (SA). (2019). Scholarships & bursaries. Retrieved November 16, 2019, from Roedean School (SA) website: http://www.roedeanschool.co.za/about-roedean-school/scholarship-bursaries/

Royal Bafokeng Nation. (2019). Who are the Royal Bafokeng Nation? Retrieved December 14, 2019, from Royal Bafokeng Nation website: https://www.rbnoperationsroom.com/home/static/en_US/id/6/title/who+are+the+royal+bafokeng+nation.html

Ruf, D. (2005). *Losing our minds: gifted children left behind*. Tucson, AZ: Great Potential Press.

Sansom, S., Barnes, B., Carrizales, J., & Shaughnessy, M. F. (2018). A reflective conversation with Jane Piirto. *Gifted Education International*, *34*(1), 96–111. https://doi.org/10.1177/0261429416650950

Sarouphim, K. M. (2009). The use of a performance assessment for identifying gifted Lebanese students: Is DISCOVER effective? Status of education of the gifted in Lebanon. *Journal for the Education of the Gifted*, *33*(2), 275–295.

Sauce, B., & Matzel, L. D. (2018). The paradox of intelligence: heritability and malleability coexist in hidden gene-environment interplay. *Psychological Bulletin*, *144*(1), 26–47. https://doi.org/10.1037/bul0000131

Schoevers, E. M., & Kroesbergen, E. H. (2017). Enhancing creative problem solving in an integrated visual art and geometry program: A pilot study. In D. Pitta-Pantazi (Ed.), *The 10th Mathematical Creativity and Giftedness International Conference* (pp. 27–32). Nicosia, Cyprus: Department of Education, University of Cyprus, Cyprus.

Sefotho, M. M. (2015). A researcher's dilemma: Philosophy in crafting dissertations and theses. *J Soc Sci*, *42*(12), 23–36.

Semakane, S. K. (1994). *The academic self-efficacy beliefs of disadvantaged gifted black middle school students in the North West province of South Africa* (Master's thesis, Potchefstroomse Universiteit vir Christelike Hoër Onderwys, Potchefstroom, South Africa). Retrieved from http://hdl.handle.net/10394/8140

Sepeng, P. (2013). Exploring issues of the use of language as a pedagogical tool in the learning of mathematics. *2013 ISTE International Conference on Mathematics, Science and Technology Education*, 123–132.

Sepeng, P., & Webb, P. (2012). Exploring mathematical discussion in word problem-solving. *Pythagoras*, *33*(1), 1–8. https://doi.org/10.4102/pythagoras.v33i1.60

Serrano Corkin, D., Coleman, S. L., & Ekmekci, A. (2019). Navigating the challenges of student-centered Mathematics teaching in an urban context. *Urban Review*, *51*(3), 370–403. https://doi.org/10.1007/s11256-018-0485-6

Shuttleworth-Edwards, A. B., Garland, E. K., & Radloff, S. E. R. (2013). WAIS-III test performance in the South African context: Extension of a prior cross-cultural normative database. In S. Laher & K. Cockcroft (Eds.), *Psychological assessment in South Africa: Research and applications* (pp. 17–32). https://doi.org/10.18772/22013015782.7

Sikhwari, T. D. (2016). Study habits, attitudes and academic achievement: Comparing Grade 12 learners between two secondary schools. *Journal of Educational Studies*, *15*(2), 43–61.

Silverman, L. K. (1997). *What we have learned about gifted children 1979-1997*. Retrieved from http://www.gifteddevelopment.com/articles/what-we-have-learned-about-gifted-children

Singer, F. M., Sheffield, L. J., Freiman, V., & Brandl, M. (2016). Research on and activities for mathematically gifted students. In G. Kaiser (Ed.), *ICME-13 Topical Surveys*. https://doi.org/10.1007/978-3-319-39450-3

Slevitch, L. (2011). Qualitative and quantitative methodologies compared: Ontological and epistemological perspectives. *Journal of Quality Assurance in Hospitality and Tourism*, *12*(1), 73–81. https://doi.org/10.1080/1528008X.2011.541810

South African Market Insights. (2020). South Africa's education statistics. Retrieved September 24, 2020, from South African Market Insights website: https://www.southafricanmi.com/education-statistics.html

South African Mathematics Foundation. (2017). SA Mathematics Challenge: 2017 South African Mathematics Challenge. Retrieved from South African Mathematics Foundation website: http://www.samf.ac.za/sa-mathematics-challenge

South African Mathematics Foundation. (2018). Challenge questions & solutions. Retrieved June 13, 2018, from South African Mathematics Foundation website: https://www.samf.ac.za/en/sa-maths-challenge-past-question-papers-solutions

South African Mathematics Foundation. (2019). 2019 South African Mathematics Challenge: Final round qualifiers. https://doi.org/10.1017/CBO9781107415324.004

South African Mathematics Foundation. (2020a). Learner development. Retrieved August 30, 2020, from South African Mathematics Foundation website: https://www.samf.ac.za/en/mathematics-learner-development

South African Mathematics Foundation. (2020b). South African Mathematics Challenge. Retrieved August 30, 2020, from South African Mathematics Foundation website: http://www.samf.ac.za/en/sa-mathematics-challenge

Spearman, C. (1904). "General intelligence", objectively determined and measured. *American Journal of Psychology*, *15*, 201–293. Retrieved from http://psychclassics.yorku.ca/Spearman/

St Cyprian's School. (2019). Scholarships. Retrieved November 16, 2019, from St Cyprian's School website: https://www.stcyprians.co.za/admissions/scholarship-bursaries/

Star College Boys High School. (2017). 2017 scholarship policy. Retrieved November 16, 2019, from Star College Boys High School website: http://starboyshigh.co.za/wp-content/uploads/2017/05/scholarship-policy_2017.pdf

Statistics South Africa. (2017). Poverty trends in South Africa: An examination of absolute poverty between 2006 and 2011. In *Statistics South Africa*. Pretoria, South Africa: Statistics South Africa.

Sternberg, R. J., & Kaufman, S. B. (2018). Theories and conceptions of giftedness. In S. I. Pfei (Ed.), *Handbook of giftedness in children* (2nd ed.). https://doi.org/10.1007/978-3-319-77004-8

Stirling Primary School. (2019). Academic focus. Retrieved November 16, 2019, from Stirling Primary School website: http://www.stirlingps.co.za/academic.htm

Stones, B., Maree, K., & Jordaan, J. (2021). Mathematics Olympiad participation: Developing problem-solving skills in mathematically-gifted disadvantaged learners. *Roeper Review*.

Streznewski, M. (1999). *Gifted grownups: The mixed blessings of extraordinary potential*. New York, NY: John Wiley and Sons.

Student Sponsorship Programme. (2019). Scholar recruitment. Retrieved November 16, 2019, from Student Sponsorship Programme website: http://ssp.org.za/scholarships/

Tambychik, T., & Meerah, T. S. M. (2010). Students' difficulties in mathematics problem-solving: What do they say? *Procedia - Social and Behavioral Sciences*, *8*, 142–151. https://doi.org/10.1016/j.sbspro.2010.12.020

Tavakol, M., & Dennick, R. (2011). Making sense of Cronbach's alpha. *International Journal of Medical Education*, *2*, 53–55. https://doi.org/10.5116/ijme.4dfb.8dfd

Te Nijenhuis, J., Murphy, R., & van Eeden, R. (2011). The Flynn effect in South Africa. *Intelligence*, *39*(6), 456–467. https://doi.org/10.1016/j.intell.2011.08.003

Terman, L. M., Baldwin, B. T., Bronson, E., DeVoss, J. C., Fuller, F., Kelley, T. L., … Yates, D. H. (1926). *Genetic studies of genius volume 1: Mental and physical traits of a thousand gifted children* (2nd ed.; L. M. Terman, Ed.). Stanford, CA: Stanford University Press.

The Cape Academy of Mathematics, S. and T. (2019). The Cape Academy of Maths, Science & Technology: Centre of Excellence! Retrieved November 17, 2019, from The Cape Academy of Mathematics, Science and Technology website: http://www.camst.co.za/

Thomas, P. Y. (2010). *Towards developing a web-based blended learning environment at the University of Botswana* (Doctoral thesis, University of South Africa, Pretoria, South Africa). Retrieved from http://hdl.handle.net/10500/4245

Tourón, J., & Freeman, J. (2018). Gifted education in Europe: Implications for policymakers and educators. *APA Handbook on Giftedness and Talent*, 55–70.

Treffinger, D. J., & Isaksen, S. G. (2005). Creative problem solving: the history, development, and implications for gifted education and talent development. *Gifted Child Quarterly*, *49*(4), 342–353. https://doi.org/10.1177/001698620504900407

Uleanya, C., & Bunmi Omoniyi, I. (2019). Effects of household poverty trap on learners' academic performances: A case of rural high schools in Nongoma Circuit of South Africa. *Affrika: Journal of Politics, Economics and Society*, *9*(1), 139–165. https://doi.org/10.31920/2075-6534/2019/9n1a7

University of Cape Town. (2019). UCT Mathematics Competition. Retrieved November 16, 2019, from University of Cape Town website: http://www.uctmathscompetition.org.za/

University of Pretoria. (2019). UP Mathematics Competition. Retrieved November 16, 2019, from University of Pretoria website: https://www.up.ac.za/mathematics-and-applied-mathematics/article/47663/up-mathematics-competition

University of South Africa. (2019). Prescribed Materials for 2019. Retrieved October 12, 2019, from https://www.unisa.ac.za/sites/myunisa/default/Books/Prescribed-Books

University of Witwatersrand. (2019a). Targeting Talent Programme (TTP). Retrieved December 1, 2019, from University of Witwatersrand website: https://www.wits.ac.za/setmu/programmes/current-programmes/

University of Witwatersrand. (2019b). Wits Mathematics Competition. Retrieved November 16, 2019, from http://wmc.ms.wits.ac.za/

Usmadi, Agita, A., & Ergusni. (2020). The effect of application Kumon Learning Method in learning Mathematics of ability troubleshooting Mathematics of students. *Journal of Physics: Conference Series, 1429*(1). https://doi.org/10.1088/1742-6596/1429/1/012005

Van Broekhuizen, H., Van der Berg, S., & Hofmeyr, H. (2016). Higher education access and outcomes for the 2008 national matric cohort. In *Stellenbosch Working Paper Series No. WP16/2016*. https://doi.org/10.2139/ssrn.2973723

Van der Berg, S. (2015). What the Annual National Assessments can tell us about learning deficits over the education system and the school career. *South African Journal of Childhood Education*, *5*(2), 16. https://doi.org/10.4102/sajce.v5i2.389

Van der Westhuizen, C. (2007). Undervalued and under-served: The gifted disadvantaged. *Gifted Education International*, *23*(2), 138–148.

Van der Westhuizen, C., & Maree, J. G. (2006). Some thoughts on the training of teachers of gifted learners. *Gifted Education International*, *21*, 201–217.

van Eeden, R. (1991). *Manual for the Senior South African Individual Scale — Revised (SSAIS-R)*. Pretoria, South Africa: Human Sciences Research Council.

van Schalkwyk, F. M. (2014). *Constructing a modelling-based learning environment for the enhancement of learner performance in Grade 6 Mathematics classrooms: A design study* (Doctoral thesis, North-West University, Potchefstroom, South Africa). Retrieved from http://hdl.handle.net/10394/12210

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development*, *41*, 121–130. https://doi.org/10.1016/j.ijedudev.2015.02.002

Visser, B. A., Ashton, M. C., & Vernon, P. A. (2006). g and the measurement of Multiple Intelligences: a response to Gardner. *Intelligence*, *34*(5), 507–510. https://doi.org/10.1016/j.intell.2006.04.006

Wai, J. (2015). Long-term effects of educational acceleration. In S. G. Assouline, N. Colangelo, J. Vantassel-Baska, & A. Lupkowski-Shoplik (Eds.), *A nation empowered: Evidence trumps the excuses holding back America's brightest students* (Vol. 2, pp. 73–84). Washington, DC: The Connie Belin & Jacqueline N. Blank International Center for Gifted Education and Talent Development.

Wang, C., Weng, J., Yao, Y., Dong, S., Liu, Y., & Chen, F. (2017). Effect of abacus training on executive function development and underlying neural correlates in Chinese children. *Human Brain Mapping*, *38*(10), 5234–5249. https://doi.org/10.1002/hbm.23728

Warne, R. T. (2016). Five reasons to put the g back Into giftedness: An argument for applying the Cattell–Horn–Carroll theory of intelligence to gifted education research and practice. *Gifted Child Quarterly*, *60*(1), 3–15. https://doi.org/10.1177/0016986215605360

Wasserstein, R. L., & Lazar, N. A. (2016). The ASA's statement on p-values: Context, process, and purpose. *American Statistician*, *70*(2), 129–133. https://doi.org/10.1080/00031305.2016.1154108

Waterhouse, L. (2006). Multiple Intelligences, the Mozart Effect, and emotional intelligence: A critical review. *Educational Psychologist*, *41*(4), 207–225. https://doi.org/10.1207/s15326985ep4104

Williams, J. M., Bryan, J., Morrison, S., & Scott, T. R. (2017). Protective factors and processes contributing to the academic success of students living in poverty: Implications for counselors. *Journal of Multicultural Counseling and Development*, *45*(3), 183–200. https://doi.org/10.1002/jmcd.12073

Willingham, D. T. (2004). Reframing the mind. *Education Next*, *4*(3), 19–24. Retrieved from http://educationnext.org/files/ednext20043_18.pdf

Wissing, A. (2012). *The role of the grade four teacher in providing support for the cognitively gifted English Second Language (ESL) underachiever* (Master's thesis, University of South Africa, Pretoria, South Africa). Retrieved from http://hdl.handle.net/10500/6549

World Health Organization. (2018). Environmental health in emergencies. Retrieved March 5, 2018, from World Health Organization website: http://www.who.int/environmental_health_emergencies/vulnerable_groups/en/

Xolo, S. (2007). Developing the potential of the gifted disadvantaged in South Africa. *Gifted Education International*, *23*, 201–206. https://doi.org/10.1177/026142940702300211

Yakavets, N. (2014). Reforming society through education for gifted children: The case of Kazakhstan. *Research Papers in Education*, *29*(5), 513–533. https://doi.org/10.1080/02671522.2013.825311

Yazgan, Y. (2015). Sixth graders and non-routine problems: Which strategies are decisive for success? *Educational Research and Reviews*, *10*(13), 1807–1816. https://doi.org/10.5897/ERR2015.2230

Young Engineers and Scientists of Africa. (2019). Young engineers and scientists of Africa. Retrieved November 16, 2019, from Young Engineers and Scientists of Africa website: http://www.yesa.org.za/

Zabloski, J. (2010). *Gifted dropouts: A phenomenological study* (Doctoral thesis, Liberty University, Lynchburg, VA). Retrieved from https://digitalcommons.liberty.edu/doctoral/337

Zaram, G. N. (2016). *An experimental study of self-regulated learning with mathematically gifted pupils in Nigerian primary schools* (Doctoral thesis, Nelson Mandela Metropolitan University, Port Elizabeth, South Africa). Retrieved from http://hdl.handle.net/10948/13381

Zenex Foundation. (2020). Zenex: Terms of reference - evaluation of the South African Mathematics Challenge. Retrieved September 24, 2020, from Sangonet Pulse website: http://ngopulse.org/opportunity/2020/08/17/terms-reference-evaluation-south-african-mathematics-challenge

Zygmont, C. S. (2006). *An exploratory factor analysis of the Junior South African Individual Scales (JSAIS)* (Master's thesis, Stellenbosch University, Stellenbosch, South Africa). Retrieved from http://hdl.handle.net/20.500.11892/31920

ANNEXURE A: SCHOOL PARTICIPATION LETTER

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

REQUEST FOR PARTICIPATION AND INFORMED ASSENT/CONSENT SCHOOL PRINCIPAL

Dear Sir/Madam

I am currently busy with my MEd in Educational Psychology at the University of Pretoria on the following topic: "The value of Mathematics Olympiad participation for developing problem-solving skills in gifted disadvantaged learners". I would like to ask your permission to conduct a part of this research at your school.

Mathematically-gifted learners in disadvantaged areas have potential but need assistance in developing that potential into skills that are valuable to the economy. Both universities and employers value problem-solving skills. The SA Mathematics Challenge is a Mathematics Olympiad for primary school learners that aims to enhance problem-solving skills. I will explore whether this claim is valid for mathematically-gifted Grade 7 learners, by assessing their problem-solving skills before and after an intervention.

I will need the school's assistance in selecting the top 50 learners in Grade 7 by their 2018 mathematics year marks, and distributing informed assent/consent forms to the selected learners and their parents. Only learners who have returned an assent/consent form that is signed by both the learner and a parent/guardian may take part in the study.

The study will consist of five sessions of one hour, after school. In the first and last session the learners will complete a questionnaire called the Study Orientation in Mathematics, and some will take part in a focus group on this experience. The other sessions will consist of mathematics problem-solving exercises, either from the SA Mathematics Challenge, or from materials provided by the Department of Basic Education. I will facilitate all the sessions.

Participation is voluntary and can be withdrawn at any time. Only my supervisor and I will know which schools were used in the research. Pseudonyms will be used for your school and learners during data collection, analysis and in the published research. During the study only my supervisor and I will have access to the data collected. After completion of the study, the material will be stored at the university's Educational Psychology Department according to the policy requirements.

If you agree to allow me to conduct this research in your school, please fill in the consent form provided below. If you have any questions, do not hesitate to contact my supervisor or me.

Thank you for your consideration of this request.

____________________________ ____________________________

Mrs R.A. Stones Prof J.G. Maree (Supervisor)

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

INFORMED CONSENT

SCHOOL PRINCIPAL

Title of research project: The value of Mathematics Olympiad participation for developing problem-solving skills in gifted disadvantaged learners

I confirm that I have been informed about the nature of this research.
I understand that learners may withdraw from this study at any stage, without prejudice.

__________________________ Signature: ________________ Date: ____________
(School principal's name)

Mrs R.A. Stones (Researcher) Signature: ________________ Date: ____________
Contact number:

ANNEXURE B: INFORMED CONSENT LETTER

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

REQUEST FOR PARTICIPATION AND INFORMED CONSENT
PARENT/GUARDIAN

Dear Grade 7 parent/guardian

Your child/ward is invited to participate in a study of mathematically-gifted children from disadvantaged schools. The results of the study will be published. You may request a copy of the study from me. No names of participants or schools will be included in the final publication.

The study will consist of five sessions of one hour, after school. In the first and last sessions your child will complete a questionnaire called the Study Orientation in Mathematics and could talk about this experience in a small group. In the other sessions your child/ward will do mathematics problem-solving with other Grade 7 learners. I will facilitate all the sessions.

Your child/ward may choose not to participate in the study and may stop participating at any time without stating reasons. No information will be kept about participants who choose to leave the study.

If you are willing for your child to participate in this study, please complete the form below. Thank you for your consideration of this request.

______________________________ ______________________________

Mrs R.A. Stones Prof J.G. Maree (Supervisor)

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

REQUEST FOR PARTICIPATION AND INFORMED ASSENT

LEARNER

Dear Grade 7 learner

You are invited to participate in a study of mathematically-gifted children from disadvantaged schools. The results of the study will be published. You may request a copy of the study from me. No names of participants or schools will be included in the final publication.

The study will consist of five sessions of one hour, after school. In the first and last sessions you will complete a questionnaire called the Study Orientation in Mathematics and could talk about this experience in a small group. In the other sessions you will do mathematics problem-solving with other Grade 7 learners. I will facilitate all the sessions.

You may choose not to participate in the study and may stop participating at any time without stating reasons. No information will be kept about participants who choose to leave the study.

If you are willing to participate in this study, please complete the form below. Thank you for your consideration of this request.

____________________________ ____________________________

Mrs R.A. Stones Prof J.G. Maree (Supervisor)

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

INFORMED ASSENT/CONSENT
LEARNER AND PARENT/GUARDIAN

Title of research project: The value of Mathematics Olympiad participation for developing problem-solving skills in gifted disadvantaged learners

I confirm that I have been informed about the nature of this research.

I understand that learners may withdraw from this study at any stage, without prejudice.

__________________________ Signature: ________________ Date: ____________
(Learner's name)

__________________________ Signature: ________________ Date: ____________
(Parent/guardian's name)

Mrs R.A. Stones (Researcher) Signature: ________________ Date: ____________
Contact number:

ANNEXURE C: PRE- AND POST-TEST RESULTS

Table 22: Pre- and post-test results of the SOM for the intervention group

Gr. 6 Maths mark	Pre-test						Post-test					
	SA	MA	SH	PSB	SM	Total	SA	MA	SH	PSB	SM	Total
90	48	35	38	36	45	202	52	40	41	38	42	213
87	40	42	42	51	37	212	48	28	52	47	30	205
86	50	42	54	55	44	245	52	39	59	57	40	247
85	45	37	55	51	30	218	40	36	43	36	37	192
84	55	43	51	50	42	241	53	42	55	62	48	260
81	47	49	50	26	41	213	34	32	37	28	46	177
79	51	38	52	65	39	245	53	43	56	46	44	242
78	43	38	43	31	37	192	43	45	43	45	40	216
77	54	46	54	46	49	249	44	28	35	50	31	188
77	46	48	45	49	34	222	52	37	53	50	44	236
76	43	32	48	48	36	207	35	40	37	43	37	192
74	36	28	38	41	34	177	33	42	33	32	35	175
72	52	48	44	56	49	249	44	37	43	51	42	217
72	37	36	44	29	41	187	39	35	44	34	30	182
72	24	48	20	12	35	139	29	48	24	20	41	162
71	53	39	55	54	32	233	52	46	62	59	36	255
69	40	38	38	38	40	194	36	39	45	41	39	200
68	42	45	53	42	45	227	45	55	47	35	48	230
66	49	32	62	52	48	243	55	41	60	54	45	255
62	49	30	61	64	44	248	48	46	55	54	38	241
61	30	40	47	49	28	194	42	30	50	39	33	194
60	50	27	56	63	27	223	41	31	55	55	26	208
60	44	17	43	48	16	168	46	16	48	52	25	187
57	45	41	45	48	43	222	34	37	37	26	40	174
55	53	30	64	62	45	254	56	44	62	60	42	264
54	31	43	36	32	46	188	32	41	35	36	38	182
51	52	47	54	57	39	249	46	28	54	56	33	217

SA: Study Attitude in Mathematics
MA: Mathematics Anxiety
SH: Study Habits in Mathematics
PSB: Problem-Solving Behaviour in Mathematics
SM: Study Milieu in Mathematics
Total: Overall Study Orientation in Mathematics

Table 23: Pre- and post-test results of the SOM for the alternative intervention group

Gr. 6 Maths mark	Pre-test						Post-test					
	SA	MA	SH	PSB	SM	Total	SA	MA	SH	PSB	SM	Total
84	49	49	45	46	37	226	43	43	51	34	44	215
84	48	45	42	26	42	203	42	50	48	40	46	226
78	50	43	55	47	40	235	47	45	46	47	39	224
78	43	39	55	52	49	238	45	46	63	66	47	267
75	45	54	39	37	51	226	47	43	49	45	28	212
75	50	40	45	38	33	206	36	31	38	41	32	178
74	52	34	58	48	41	233	56	43	57	47	38	241
71	46	53	55	55	49	258	51	56	55	54	51	267
70	38	26	33	44	28	169	49	33	50	34	44	210
68	55	48	61	56	45	265	51	51	57	59	46	264
68	46	48	57	53	39	243	54	51	59	57	49	270
68	51	37	51	62	43	244	50	29	47	50	35	211
68	44	44	50	46	39	223	41	48	51	46	39	225
68	35	49	33	24	42	183	39	35	47	40	38	199
67	52	33	49	45	21	200	46	39	41	43	31	200
66	42	48	33	44	50	217	47	42	41	42	45	217
66	43	27	39	43	29	181	44	35	46	45	35	205
66	24	28	25	30	32	139	38	37	22	40	28	165
65	50	52	56	59	46	263	55	50	58	66	47	276
65	47	48	42	41	46	224	54	48	54	52	51	259
65	44	36	55	52	33	220	42	44	60	51	30	227
65	46	36	48	34	29	193	37	43	48	38	28	194
65	44	38	49	33	34	198	50	47	51	38	40	226
63	50	45	46	49	44	234	47	46	47	38	44	222
63	47	36	59	50	30	222	35	21	38	37	27	158
63	44	38	54	40	43	219	48	52	61	57	47	265
62	52	28	58	64	35	237	56	39	61	62	34	252
62	52	43	52	53	39	239	55	42	54	60	39	250
62	43	41	47	55	46	232	44	51	59	64	48	266
62	45	39	55	43	35	217	52	36	49	54	32	223
62	38	38	33	39	31	179	32	42	40	30	27	171
61	54	47	59	46	42	248	55	43	61	55	42	256
61	50	35	48	57	34	224	33	22	41	36	30	162
61	47	27	59	61	26	220	51	36	63	66	34	250
60	42	36	53	64	30	225	38	28	62	49	30	207
60	47	45	54	39	36	221	39	31	57	46	37	210
60	45	34	47	41	23	190	45	44	52	60	40	241
60	38	31	46	35	36	186	38	42	40	43	34	197
60	17	30	19	22	24	112	36	40	46	38	37	197
58	39	29	44	41	30	183	43	32	40	36	26	177
Unknown	46	38	51	52	44	231	43	44	57	48	42	234

SA: Study Attitude in Mathematics
SH: Study Habits in Mathematics
SM: Study Milieu in Mathematics
MA: Mathematics Anxiety
PSB: Problem-Solving Behaviour in Mathematics
Total: Overall Study Orientation in Mathematics

ANNEXURE D: FOCUS GROUP QUESTIONS

Focus group questions after the pre-test of the *SOM*

1. **Have you seen a questionnaire like the *SOM* before?**
2. **What did you think of the *SOM*?**
3. **Did you understand all the questions in the *SOM*? Which didn't you understand? What did you not understand about each?**
4. **Would you have preferred to answer the *SOM* in another language? Which?**
5. **Were there any questions you particularly liked answering? Why?**
6. **Were there any questions you didn't like answering? Why?**
7. **Do you have anything else you would like to share with the group?**

Focus group questions after the post-test of the *SOM*

1. **What have you seen that was like the worksheets we did?**
2. **What did you think of the sums we did?**
3. **How do you feel about the level of difficulty (or easiness) of the sums?**
4. **What did you like about the sessions we had?**
5. **What didn't you like about the sessions we had?**
6. **What did you learn from participating in this study?**
7. **What was it like answering the *SOM* again?**
8. **In which ways did you answer the same as the first time or different from the first time?**
9. **What else would you like to share with the group?**
10. **Were there any questions you didn't like answering? Why?**
11. **Do you have anything else you would like to share with the group?**

ANNEXURE E: INTERVENTION WORKSHEETS

SA Mathematics Challenge 2013
GRADE 7 FIRST ROUND

NOTE:
- Answer the questions according to the instructions on the answer sheet.
- You may use a calculator.
- The questions test insight. Complex calculations will therefore not be necessary.
- We hope you enjoy it!

SA Wiskunde-uitdaging 2013
Graad 7 Eerste Ronde

LET OP:
- Beantwoord die vrae volgens die instruksies op die antwoordblad.
- Jy mag 'n sakrekenaar gebruik.
- Die vrae toets insig. Omslagtige berekeninge is dus onnodig en tydrowend.
- Ons hoop jy geniet dit!

---

1. What number is exactly halfway between 5,6 and 5,65?

1. Watter getal is presies halfpad tussen 5,6 en 5,65?

(A) 5,025 (B) 5,625 (C) 5,62 (D) 5,605 (E) 5,635

---

2. Which one of these is *not* true?

2. Watter een hiervan is *nie* waar nie?

(A) $1\times1\div1\times1 = 1$ (B) $2\div2+2\div2 = 2$ (C) $3\times3-3+3 = 3$ (D) $(4-4)\div4+4 = 4$ (E) $5+5\times(5-5) = 5$

---

3. What is the 83rd number in the following pattern?
1; 3; 5; 7;….

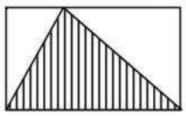
3. Wat is die 83ste getal in die volgende patroon?
1; 3; 5; 7;….

(A) 85 (B) 165 (C) 62 (D) 97 (E) 102

---

4. The sketch shows a 6 cm by 4 cm rectangle.
What area is shaded?

4. Die skets toon 'n 6 cm by 4 cm reghoek.
Watter oppervlakte is verdonker?

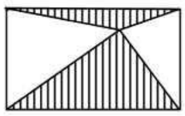


(A) 12 cm^2 (B) 10 cm^2 (C) 9 cm^2 (D) 8 cm^2 (E) 7 cm^2

---

5. The sketch shows a 6 cm by 4 cm rectangle.
What area is shaded?

5. Die skets toon 'n 6 cm by 4 cm reghoek.
Watter oppervlakte is verdonker?



(A) 12 cm^2 (B) 10 cm^2 (C) 9 cm^2 (D) 8 cm^2 (E) 7 cm^2

---

6. The average of eleven numbers is 8. If a twelfth number is added to these numbers, the average of all twelve numbers is now 11. What is the twelfth number added?

6. Die gemiddelde van elf getalle is 8. As 'n twaalfde getal by hierdie getalle getel word, is die gemiddelde van al twaalf getalle nou 11. Wat is die twaalfde getal wat bygetel is?

(A) 11 (B) 12 (C) 33 (D) 44 (E) 22

SAMF
SOUTH AFRICAN MATHEMATICS FOUNDATION

sasol
reaching new frontiers

AMESA
The Association for Mathematics Education of South Africa

7. Calculate the value of

$$1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}$$

7. Bereken die waarde van

$$1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}$$

(A) $1\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $1\frac{2}{3}$ (D) $3\frac{1}{2}$ (E) $5\frac{1}{2}$

8. A small rectangular garden measuring 3,75 m by 2,5 m is to be paved with equal square tiles so that the area is covered exactly. Tiles may not be cut. What is the minimum number of square tiles that could be used?

8. 'n Klein reghoekige tuin met afmetings 3,75 m by 2,5 m moet plavei word met ewe-groot vierkantige teëls sodat die area presies bedek word. Teëls mag nie gesny word nie. Wat is die minimum getal vierkantige teëls wat gebruik kan word?

(A) 12 (B) 10 (C) 8 (D) 6 (E) 4

9. The notation a ℧ b means "the remainder when a is divided by b". What is the value of 123 ℧ (45 ℧ 6)?

9. Die notasie a ℧ b beteken "die res as a gedeel word deur b". Wat is die waarde van 123 ℧ (45 ℧ 6)?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

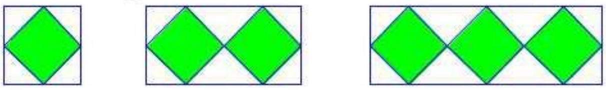
10. When a number is multiplied by itself, the result is a *square number*. For example, $3 \times 3 = 9$ and $6 \times 6 = 36$ are square numbers. How many square numbers are there from 1 to 1000?

10. As 'n getal met homself vermenigvuldig word, is die resultaat 'n volkome *vierkant*. Byvoorbeeld, $3 \times 3 = 9$ en $6 \times 6 = 36$ is vierkante. Hoeveel vierkante is daar van 1 tot 1000?

(A) 31 (B) 961 (C) 20 (D) 21 (E) 22

11. In the pattern below, the diagram with two squares has six triangles. If the pattern continues to grow, how many triangles are there in a diagram with six squares?

11. In die patroon hieronder: Die diagram met twee vierkante het ses driehoeke. As die patroon voortgesit word, hoeveel driehoeke is daar in 'n diagram met ses vierkante?



(A) 12 (B) 14 (C) 16 (D) 18 (E) 20

12. In question 11, how many triangles are there in a diagram with 60 squares?

12. In vraag 11: Hoeveel driehoeke is daar in 'n diagram met 60 vierkante?

(A) 120 (B) 122 (C) 140 (D) 160 (E) 142

13. In question 11, how many squares are there in a diagram with 60 triangles?

13. In vraag 11: Hoeveel vierkante is daar in 'n diagram met 60 driehoeke?

(A) 30 (B) 32 (C) 28 (D) 29 (E) 31

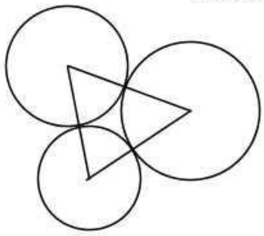
14. With one digit you can form one number, e.g. 9. With two digits (e.g. 6 and 8) you can form two numbers, namely 68 and 86. How many different four-digit numbers can be formed with four different digits?

14. Met een syfer kan jy een getal vorm, bv. 9. Met twee syfers (bv 6 en 8) kan jy twee getalle vorm, nl. 68 en 86. Hoeveel verskillende viersyfer-getalle kan met vier verskillende syfers gevorm word?

(A) 8 (B) 10 (C) 16 (D) 24 (E) 32

15. Three circles with radii 7 cm, 8 cm and 9 cm touch each other externally without overlapping. What is the perimeter of the triangle formed by joining the three centres of the circles?

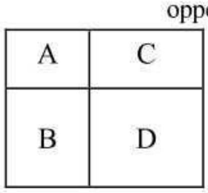
15. Drie sirkels met radiusse van 7 cm, 8 cm en 9 cm raak mekaar uitwendig. Wat is die omtrek van die driehoek wat gevorm word deur die drie middelpunte van die sirkels te verbind?



(A) 30 cm (B) 24 cm (C) 48 cm (D) 12 cm (E) $7\pi + 8\pi + 9\pi$

16. If the areas of rectangles A, B and C below are 12 cm^2, 21 cm^2 and 20 cm^2 respectively. What is the area of rectangle D?

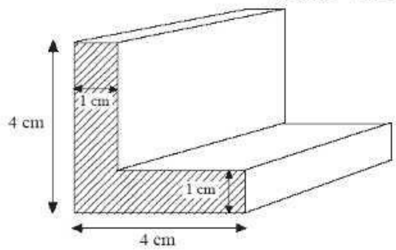
16. Die oppervlakte van reghoeke A, B en C hieronder is onderskeidelik 12 cm^2, 21 cm^2 en 20 cm^2. Wat is die oppervlakte van reghoek D?



(A) 32 cm^2 (B) 35 cm^2 (C) 55 cm^2 (D) 56 cm^2 (E) 88 cm^2

17. The figure below shows a metal bar which is 4 cm high, 4 cm wide, 1 cm thick and 12 cm long. What is the volume of the bar?

17. Die skets hieronder wys 'n metaalstaaf wat 4 cm hoog, 4 cm breed, 1 cm dik en 12 cm lank is. Wat is die volume van die staaf?



(A) 96 cm^3 (B) 7 cm^2 (C) 16 cm^2 (D) 192 cm^3 (E) 84 cm^3

18. How many two-digit numbers are there with both digits even?

18. Hoeveel tweesyfer-getalle is daar met beide syfers ewe?

(A) 20 (B) 25 (C) 45 (D) 50 (E) 30

19. Numbers are arranged in the following pattern:

1	2	3	4	5	6	**row 1**
7	8	9	10	11	12	**row 2**
13	14	15	16	17	18	**row 3**
…	…	…	…	…	…	**row 4**

What will the third number in row 81 be?

19. Getalle word in die volgende patroon rangskik:

1	2	3	4	5	6	**ry 1**
7	8	9	10	11	12	**ry 2**
13	14	15	16	17	18	**ry 3**
…	…	…	…	…	…	**ry 4**

Wat sal die derde getal in ry 81 wees?

(A) 480 (B) 486 (C) 483 (D) 485 (E) 241

20. Tom, Fred and Rhoda put their apples into a bag. Tom and Fred together had 17 more apples than Rhoda. Tom had 7 apples. Rhoda had 5 apples. How many apples did Fred have?

20. Tom, Fred en Rhoda sit hul appels in 'n sak. Tom en Fred het saam 17 meer appels as Rhoda. Tom het 7 appels gehad. Rhoda het 5 appels gehad. Hoeveel appels het Fred gehad?

(A) 15 (B) 10 (C) 12 (D) 22 (E) 5

21. a, b, c and d are four adjacent dates in a calendar as shown. Which statement is NOT true for *any* calendar?

21. a, b, c en d is vier aangrensende datums in 'n kalender soos hieronder. Watter bewering is NIE waar vir *enige* kalender nie?

Mon	Tues	Wed	Thu	Fri	Sat	Sun
		a	b			
		c	d			

(A) $c - a = d - b$ (B) $c = a + 7$ (C) $d = a + 8$ (D) $a + c = b + d$ (E) $a + d = c + b$

22. In the above calendar, $a + b + c + d = 52$. What is $a + b$?

22. In die kalender hierbo is $a + b + c + d = 52$. Wat is $a + b$?

(A) 19 (B) 25 (C) 26 (D) 27 (E) One cannot say / Mens kan nie sê nie

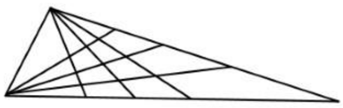
23. a, b, c and d are *any* four consecutive numbers, for example 2, 3, 4, 5 or 14, 15, 16, 17. Which statement is NOT true for *any* such four numbers?

23. a, b, c en d is *enige* vier opeenvolgende getalle, byvoorbeeld 2, 3, 4, 5 of 14, 15, 16, 17. Watter bewering is NIE waar vir *enige* sulke vier getalle nie?

(A) $c - a = d - b$ (B) $c = a + 2$ (C) $d = a + 3$ (D) $a + c = b + d$ (E) $a + d = c + b$

24. In the triangle three lines are drawn from two corners to the opposite sides of the triangle. This divides the triangle into 16 non-overlapping sections. If 10 lines from two corners are drawn in the same way, how many non-overlapping sections will the triangle have?

24. In die driehoek word drie lyne vanaf twee hoeke na die teenoorstaande sye van die driehoek getrek. Dit verdeel die driehoek in 16 dele wat mekaar nie oorvleuel nie. As 10 lyne op dieselfde manier van twee hoeke getrek word, hoeveel nie-oorvleuelende dele sal daar wees?



(A) 100 (B) 121 (C) 20 (D) 107 (E) 54

25. Six pencils and four pens cost R62. However, four pencils and six pens cost R84. How much do five pencils and five pens cost?

25. Ses potlode en vier penne kos saam R62. Maar vier potlode en ses penne kos R84. Hoeveel kos vyf potlode en vyf penne?

(A) R31 (B) R73 (C) R56 (D) R96 (D) R62

SA Mathematics Challenge 2014

Grade 7 First Round

SA Wiskunde-uitdaging 2014

Graad 7 Eerste Ronde

NOTE:
- Answer the questions according to the instructions on the answer sheet.
- You may use a calculator.
- The questions test insight. Complex calculations will therefore not be necessary.
- We hope you enjoy it!

LET OP:
- Beantwoord die vrae volgens die instruksies op die antwoordblad.
- Jy mag 'n sakrekenaar gebruik.
- Die vrae toets insig. Omslagtige berekeninge is dus onnodig en tydrowend.
- Ons hoop jy geniet dit!

---

1. Which statement is *not* true?

(A) $1 + 1 - 1 \times 1 = 1$
(B) $1 - 1 \times 1 + 1 = 1$
(C) $2 - 2 \div 2 + 2 = 2$
(D) $3 - 3 + 3 \times 3 = 9$
(E) $4 - 4 \div 4 \times 4 = 0$

1. Watter bewering is *nie* waar nie?

(A) $1 + 1 - 1 \times 1 = 1$
(B) $1 - 1 \times 1 + 1 = 1$
(C) $2 - 2 \div 2 + 2 = 2$
(D) $3 - 3 + 3 \times 3 = 9$
(E) $4 - 4 \div 4 \times 4 = 0$

(A) A (B) B (C) C (D) D (E) E

---

2. Part of a calendar is shown below. The sum of the numbers in the first row (from Monday to Thursday) is 26. What is the date of the Monday in the first row?

2. 'n Deel van 'n kalender word hieronder getoon. Die som van die getalle in die eerste ry (van Maandag tot Donderdag) is 26. Wat is die datum van die Maandag in die eerste ry?

Mon	Tues	Wed	Thurs

(A) 23 (B) 26 (C) 10 (D) 8 (E) 5

---

3. What is the angle between the hour hand and the minute hand on an analogue clock at 08:00?

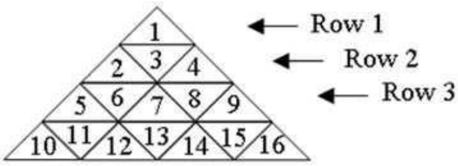
3. Wat is die hoek tussen die uur- en minuutwysers op 'n analooghorlosie om 08:00?

(A) 20° (B) 120° (C) 130° (D) 150° (E) 200°

---

4. Numbers are arranged in a triangle as shown. There are three numbers in Row 2 and five numbers in Row 3. If the pattern is continued, how many numbers are there in Row 20?

4. Getalle word in 'n driehoek rangskik soos getoon. Daar is drie getalle in Ry 2 en vyf getalle in Ry 3. As die patroon voortgesit word, hoeveel getalle is in Ry 20?



(A) 19 (B) 21 (C) 33 (D) 39 (E) 41

---

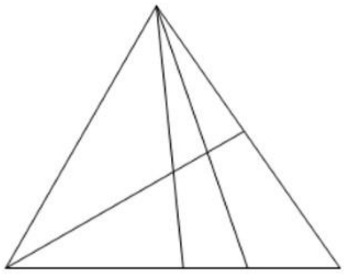
5. Refer to the previous question. What is the first (left) number in Row 20?

5. Verwys na die vorige vraag. Wat is die eerste (linker) getal in Ry 20?

(A) 362 (B) 400 (C) 401 (D) 324 (E) 200

6. How many different triangles (of all sizes) are in this figure?

6. Hoeveel verskillende driehoeke (van alle groottes) is daar in hierdie figuur?



(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

7. Calculate the value of

7. Bereken die waarde van

$$1+\frac{1}{1+\frac{1}{3}}$$

(A) $2\frac{1}{3}$ (B) $2\frac{3}{4}$ (C) $2\frac{1}{4}$ (D) $1\frac{2}{3}$ (E) $1\frac{3}{4}$

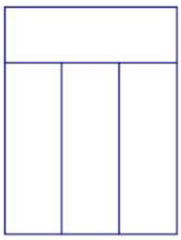
8. A car is travelling at 60 km/h. How many metres does it cover in 12 seconds?

8. 'n Motor beweeg teen 60 km/h. Hoeveel meter beweeg dit in 12 sekondes?

(A) 200 (B) 240 (C) 720 (D) 500 (E) 600

9. In the figure below, there are four equal rectangles. The longer side of each rectangle is 9 cm long. What is the perimeter of the figure?

9. Die figuur hieronder bestaan uit vier ewe-groot reghoeke. Die langste sy van elke reghoek is 9 cm lank. Wat is die omtrek van die figuur?



(A) 36 cm (B) 42 cm (C) 45 cm (D) 48 cm (E) 54 cm

10. X567Y is a five-digit number which is divisible by 3. What is the largest possible value of Y?

10. X567Y is 'n vyf-syfer getal wat deelbaar is deur 3. Wat is die grootste moontlike waarde van Y?

(A) 3 (B) 6 (C) 7 (D) 8 (E) 9

11. In a test consisting of 20 multiple choice questions, 6 points are awarded for each correct answer and 2 points are deducted for each wrong answer. David answered all the questions and scored 88. How many questions did David answer correctly?

11. In 'n toets bestaande uit 20 veelkeuse vrae, verdien elke korrekte antwoord 6 punte, en vir elke verkeerde antwoord word 2 punte afgetrek. David het al die vrae beantwoord en 'n telling van 88 punte behaal. Hoeveel vrae het David korrek beantwoord?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

12. To find the number in a box in the diagram below, we apply the following rule to the two numbers immediately below the box:

"Multiply the number on the left by 3 and then subtract the number on the right".

For example, $A = 3 \times 5 - 6 = 9$.

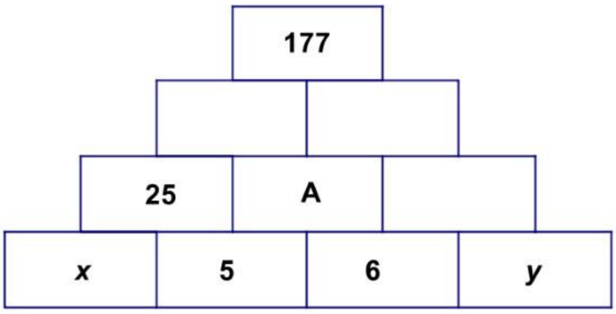
What is the value of x?

12. Om die getal in 'n boks in die diagram hieronder te bereken, pas ons die volgende reël toe op die twee getalle direk onder die boks:

"Vermenigvuldig die getal links onder met 3 en trek dan die getal regs af".

Byvoorbeeld, $A = 3 \times 5 - 6 = 9$.

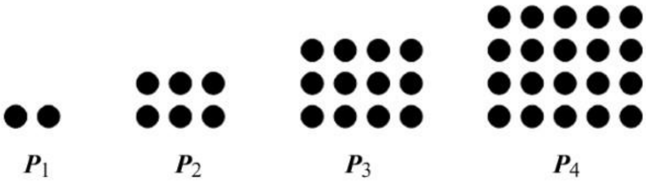
Wat is die waarde van x?



(A) 10 (B) 20 (C) 15 (D) 70 (E) $6\frac{2}{3}$

13. Sipho uses dots to build patterns as shown below. How many dots will he use for P_{50}?

13. Sipho bou patrone met kolletjies soos hieronder. Hoeveel kolletjies sal hy gebruik vir P_{50}?



(A) 2601 (B) 1275 (C) 2550 (D) 2500 (E) 2600

14. When a bucket is half full of water, it has a mass of 12 kg. When the bucket is one-third full of water it has a mass of 10 kg. What is the mass of the empty bucket?

14. Wanneer 'n emmer halfvol water is, het dit 'n massa van 12 kg. Wanneer die emmer een-derde vol water is, het dit 'n massa van 10 kg. Wat is die massa van die leë emmer?

(A) 2 kg (B) 5 kg (C) 6 kg (D) 4 kg (E) 8 kg

15. The average of five numbers is 60. If the smallest number is replaced by 80, the average is 65. What number was replaced?

15. Die gemiddelde van vyf getalle is 60. As die kleinste getal vervang word met 80, is die gemiddelde 65. Watter getal is vervang?

(A) 60 (B) 55 (C) 50 (D) 48 (E) 45

16. If $3! = 3 \times 2 \times 1$ and $4! = 4 \times 3 \times 2 \times 1$, what is the value of $\frac{20!}{19!}$?

16. As $3! = 3 \times 2 \times 1$ en $4! = 4 \times 3 \times 2 \times 1$, wat is die waarde van $\frac{20!}{19!}$?

(A) 20 (B) 19 (C) 39 (D) 380 (E) 400

17. All the counting numbers are arranged in columns as shown below. In which column is 2014?

17. Al die natuurlike getalle word in kolomme rangskik soos hieronder. In watter kolom is 2014?

A	B	C	D	E	F	G
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
⋮	⋮	⋮	⋮	⋮	⋮	⋮

(A) C (B) D (C) E (D) F (E) G

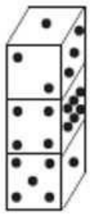
18. Four chocolates and two cooldrinks cost R35, while two chocolates and four cooldrinks cost R43. What does one chocolate and one cooldrink cost?

18. Vier sjokolades en twee koeldranke kos saam R35, terwyl twee sjokolades en vier koeldranke R43 kos. Hoeveel kos een sjokolade en een koeldrank?

(A) R10 (B) R11 (C) R12 (D) R13 (E) R16

19. Three dice with faces numbered 1 to 6 are stacked as shown. Seven of the 18 faces are visible, and 11 faces hidden (side, back, bottom, between). How many dots are *not* visible?

19. Drie dobbelstene met hul sye genommer van 1 tot 6 word gepak soos getoon. Sewe van die 18 sye is sigbaar en 11 sye is nie sigbaar nie (links, agter, onder, tussen). Hoeveel kolletjies is altesaam *nie* sigbaar nie?



(A) 21 (B) 22 (C) 31 (D) 41 (E) 51

20. In a village, in one month, one-tenth of the people are sick and nine-tenths are well. In the next month, seven-tenths of those who were sick are now well, while three-tenths of the people who were well are now sick. What fraction of the people is sick at the end of the second month?

20. In 'n dorpie is daar in 'n sekere maand een-tiende van die mense siek en nege-tiendes is gesond. In die volgende maand is sewe-tiendes van die mense wat siek was nou gesond, terwyl drie-tiendes van die mense wat gesond was nou siek. Watter breuk van die mense is aan die einde van die tweede maand siek?

(A) $\frac{1}{10}$ (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{2}{5}$ (E) $\frac{1}{2}$

SA Mathematics Challenge 2018
GRADE 7 FIRST ROUND

SA Wiskunde-uitdaging 2018
GRAAD 7 EERSTE RONDE

NOTE:
- Answer the questions according to the instructions on the answer sheet.
- You may use a calculator.
- The questions test insight. Complex calculations will therefore not be necessary.
- We hope you enjoy it!

LET OP:
- Beantwoord die vrae volgens die instruksies op die antwoordblad.
- Jy mag 'n sakrekenaar gebruik.
- Die vrae toets insig. Omslagtige berekeninge is dus onnodig en tydrowend.
- Ons hoop jy geniet dit!

---

1. Which of the following statements is incorrect?

(A) $2 + 0 \times 1 + 8 = 10$
(B) $2 - 0 + 1 + 8 = 11$
(C) $2 \times 0 \div 1 + 8 = 8$
(D) $2 + 0 \div 1 \times 8 = 2$
(E) $2 + 0 \times 1 \times 8 = 16$

1. Watter van hierdie bewerings is onwaar?

(A) $2 + 0 \times 1 + 8 = 10$
(B) $2 - 0 + 1 + 8 = 11$
(C) $2 \times 0 \div 1 + 8 = 8$
(D) $2 + 0 \div 1 \times 8 = 2$
(E) $2 + 0 \times 1 \times 8 = 16$

(A) A (B) B (C) C (D) D (E) E

---

2. What is the remainder when 123 456 789 is divided by 100?

2. Wat is die res as 123 456 789 gedeel word deur 100?

(A) 9 (B) 89 (C) 11 (D) 19 (E) 189

---

3. Some numbers read the same when written forwards and backwards, for example 121. How many such numbers are there between 10 000 and 11 000?

3. Sommige getalle lees dieselfde van voor en van agter, byvoorbeeld 121. Hoeveel sulke getalle is daar tussen 10 000 en 11 000?

(A) 1 (B) 25 (C) 10 (D) 9 (E) more than 25

---

4. In this diagram, the row and column totals are given in the shaded blocks. What is the value of T?

4. In hierdie diagram word die ry- en kolomtotale in die verdonkerde blokkies gegee. Wat is die waarde van T?

a	b	59
d	c	T
64	61	

(A) 61 (B) 64 (C) 65 (D) 66 (E) 68

---

5. If all the whole numbers from 1 to 100 are written down, how many times would the digit 4 be written?

5. As al die heelgetalle van 1 tot 100 neergeskryf word, hoeveel keer word die syfer 4 geskryf?

(A) 10 (B) 11 (C) 19 (D) 20 (E) 21

---

6. How many whole numbers between 1 and 1 000 are divisible by 5 and 6?

6. Hoeveel heelgetalle tussen 1 en 1 000 is deelbaar deur 5 en 6?

(A) 31 (B) 32 (C) 33 (D) 34 (E) 35

7. The median of five numbers is 5 and the mode is 6. What is the largest possible sum of the five numbers?

7. Die mediaan van vyf getalle is 5 en hul modus is 6. Wat is die grootste moontlike som van die vyf getalle?

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

8. In which column is the number 856 in this table?

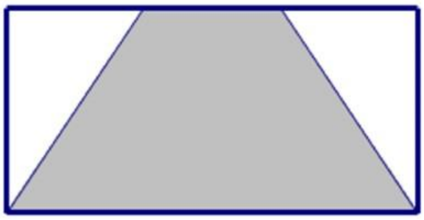
8. In watter kolom is die getal 856 in hierdie tabel?

A	B	C	D	E	F	G
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
⋮	⋮	⋮	⋮	⋮	⋮	⋮

(A) B (B) C (C) D (D) E (E) F

9. In this rectangle, the top side has been divided into thirds. What fraction of the rectangle has been shaded?

9. In hierdie reghoek is die boonste sy in derdes verdeel. Watter breuk van die reghoek is verdonker?



(A) $\frac{2}{5}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (E) $\frac{7}{10}$

10. The number 2A36B is a five-digit odd number which is divisible by 15. How many possible different values can A have?

10. Die getal 2A36B is ’n vyf-syfer onewe getal wat deelbaar is deur 15. Hoeveel verskillende moontlike waardes kan A hê?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

11. Which number lies on the number line $\frac{1}{4}$ of the way from $\frac{1}{8}$ to $\frac{1}{4}$?

11. Watter getal lê op die getallelyn $\frac{1}{4}$ van die afstand van $\frac{1}{8}$ na $\frac{1}{4}$?

(A) $\frac{1}{32}$ (B) $\frac{3}{16}$ (C) $\frac{5}{16}$ (D) $\frac{7}{48}$ (E) $\frac{5}{32}$

12. What is the value of

$$1-2+3-4+5-6+\cdots+99?$$

12. Wat is die waarde van

$$1-2+3-4+5-6+\cdots+99?$$

(A) 99 (B) 100 (C) 49 (D) 36 (E) 50

13. Which of these numbers is divisible by 3?

13. Watter van hierdie getalle is deelbaar deur 3?

(A) $10^{2018}+3$ (B) $10^{2018}+4$ (C) $10^{2018}+5$ (D) $10^{2018}+6$ (E) $10^{2018}+7$

14. In a hockey match, the final score was 2–1. How many different half-time scores were possible?

14. In 'n hokkiewedstryd was die eindtelling 2–1. Hoeveel verskillende rustydtellings was moontlik?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

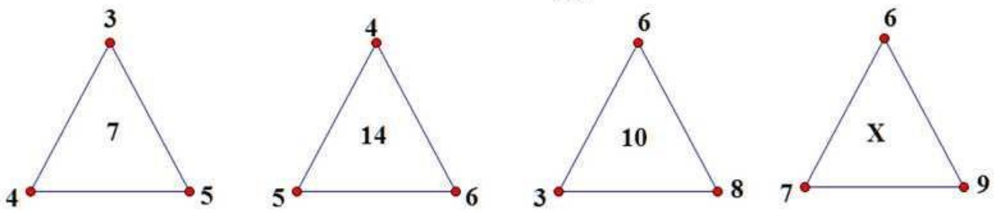
15. Four candidates are standing for election as School President. If 999 votes are cast, what is the smallest number of votes that any candidate could obtain and still win the election?

15. Vier kandidate neem deel aan 'n verkiesing vir skoolpresident. As 999 stemme uitgebring word, wat is die minste getal stemme wat enige kandidaat kon trek en steeds die verkiesing wen?

(A) 251 (B) 250 (C) 249 (D) 500 (E) 499

16. In each triangle below, the same calculation is performed with the three numbers at the vertices to get the number inside the triangle. What is the value of X?

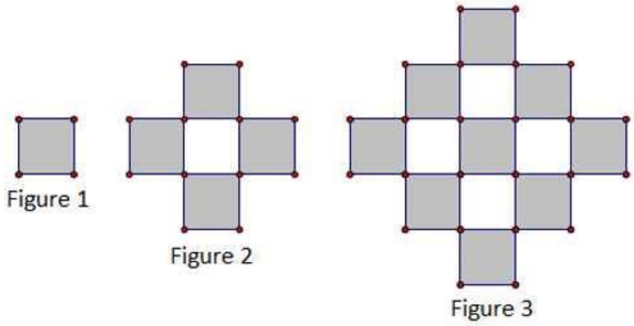
16. In elke driehoek hieronder word dieselfde berekening uitgevoer met die drie getalle by die hoekpunte om die getal binne die driehoek te gee. Wat is die waarde van X?



(A) 33 (B) 34 (C) 32 (D) 35 (E) 36

17. Figures are made with black and white tiles as below. How many black tiles are there in Figure 10?

17. Figure word met swart en wit teëls gemaak, soos hieronder. Hoeveel swart teëls is daar in Figuur 10?



(A) 96 (B) 98 (C) 100 (D) 102 (E) 104

18. A bag contains 4 blue, 5 green and 11 red marbles. How many green marbles must be added to the bag so that 75 percent of the marbles are green?

18. 'n Sak bevat 4 blou, 5 groen en 11 rooi albasters. Hoeveel groen albasters moet by die sak bygevoeg word sodat 75 persent van die albasters groen is?

(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

19. If 6 chickens lay 36 eggs in 4 days, how many eggs would 8 chickens lay in 2 days, laying at the same rate?

19. As 6 hoenders 36 eiers in 4 dae lê, hoeveel eiers sal 8 hoenders in 2 dae lê, as hulle teen dieselfde pas eiers lê?

(A) 24 (B) 18 (C) 12 (D) 30 (E) 42

20. Michael correctly adds the lengths of three sides of a rectangle, and gets 70 cm. Bianca also adds together the lengths of three sides of the same rectangle and correctly gets 59 cm. What is the perimeter of the rectangle?

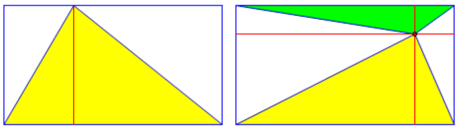
20. Michael bepaal die som van die lengtes van drie sye van 'n reghoek korrek as 70 cm. Bianca bepaal die som van die lengtes van drie sye van dieselfde reghoek korrek as 59 cm. Wat is die omtrek van die reghoek?

(A) 129 cm (B) 86 cm (C) 80 cm (D) 90 cm (E) 96 cm

ANNEXURE F: INTERVENTION ANSWER SHEETS

GRADE 7(1)

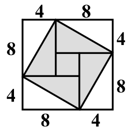
2. $3\times3 - 3 + 3 = 9 - 3 + 3 = 6 + 3 = 9$
3. nth number $= 2\times n - 1$, so 83rd number $= 2 \times 83 - 1 = 165$
4. & 5.



6. We know: $\dfrac{\text{Sum of numbers}}{11} = 8$, so Sum of numbers $= 11 \times 8 = 88$

   If the new number is x, then $\dfrac{88 + x}{12} = 11$. So $x = 12 \times 11 - 88 = 44$

7. $1 + \dfrac{1}{1 + \frac{2}{3}} = 1 + \dfrac{1}{\frac{5}{3}} = 1 + \frac{3}{5}$
8. First fit the tiles in the width: If one tile is used in the width, it has a side of 2,5 m, and cannot cover the length exactly. If 2 tiles are used in the width the tiles have a length of 1,25 m, and then 3 of them can fit into the length. So $2\times3 = 6$ tiles are the minimum number of tiles.
9. 45 ℧ 6: $45\div6 = 7$ res 3, dus is 45 ℧ 6 = 3. Dan vir 123 ℧ 3: $123\div3 = 41$ res 0, dus is 123 ℧ 3 = 0
10. The largest, by guess-and-improvement is $31 \times 31 = 961$. So there are 31 squares smaller than 1000
11. # *Triangles* = 2×*squares* + 2, or 2×(*squares* + 1). So *Triangles (6)* = 2×6 + 2 = 14

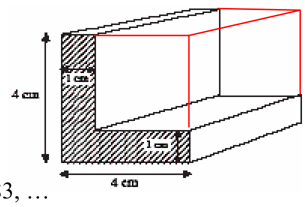


12. *Triangles (60)* = 2×60 + 2 = 122
13. $2\times x + 2 = 60$, so $x = 29$
14. Make a list, varying the numbers systematically. If the digits are a, b, c and d:
    abcd, abdc, acbd, acdb, adbc, adcb and similarly if the first digit is b, c, and d. So $6 \times 4 = 24$
15. $2 \times (7 + 8 + 9) = 2 \times 24$
16.

	c	d
a	12	20
b	21	D

Using a representation like this, Area D = b × d
We know a × c = 12, b × c = 21, a × d = 20
Multiply them all together: $a^2 \times c^2 \times b\times d = 12 \times 20 \times 21$
But $a\times c = 12$, so $a^2\times c^2 = 144$, so $b\times d = 12\times20\times 21 \div 144 = 35$

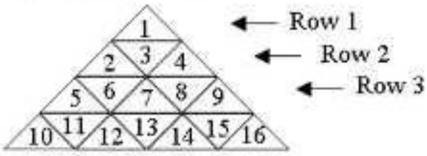
17. Volume = area of base × length = 7 cm^2 × 12 cm = 84 cm^3
    Or think of cutting out a rectangular prism:
    Volume $= 4\times4 \times 12 - 3\times3 \times 12 = 7 \times 12$



18. The first digit can be 2, 4, 6 or 8. The second digit can be 0, 2, 4, 6 or 8, which gives $4 \times 5 = 20$ possible combinations
19. The 6th column are multiples of 6, with formula $6 \times$ row n.
    So the last number in row 80 is $6 \times 80 = 480$. Then row 81 is 481, 482, 483, …
20. T+F = R+17
    So 7+F=5+17
    So F=15
21. Fill in numbers in the calendar, and test each statement with the numbers.
22. Test specific cases, e.g. if $a = 5$, then $b = 6$, $c = 12$ and $d = 13$, the a+b+c+d = 36, which is not correct. Choose a better value for a …
    Or: We know $a + d = c + b$, so $a + b + c + d = a + d + c + b = 2\times(a + d) = 52$.
    So $a + d = 26$, so $a + (a + 8) = 26$, so $a = 9$
23. Choose different consecutive numbers, and test each statement with the numbers.
24. 3 lines from two corners divide the triangle in 4 × 4 sections
    10 lines from two corners will divide the triangle in 11 × 11 sections = 121
25. **6** pencils and **4** pens cost R62
    4 pencils and **6** pens cost R84
    Add them:
    10 pencils and **10** pens cost R146
    Divide by 2:
    5 pencils and **5** pens cost R73

GRADE 7(1)

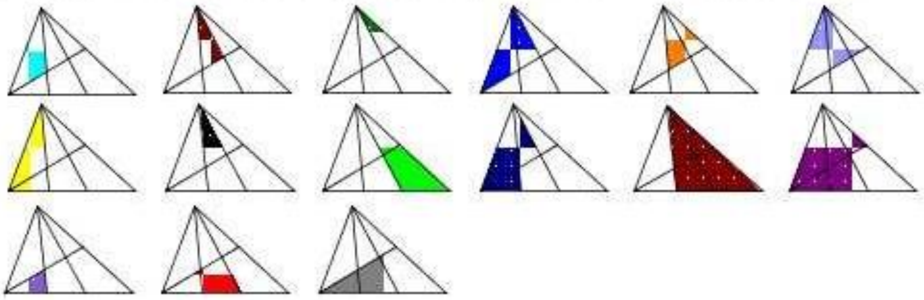
1. C. $2 - 2 \div 2 + 2 = 2 - 1 + 2 = 3$
2. E. Sum of 4 consecutive numbers is 26, i.e. 5+6+7+8. Therefore, Mon is the 5[t]
3. B. Each hour is equivalent to 30°, thus the four hours between 08:00 and 12:00 is equivalent to $4 \times 30°$
4. D. Investigate the *structure* by finding a pattern in special cases:



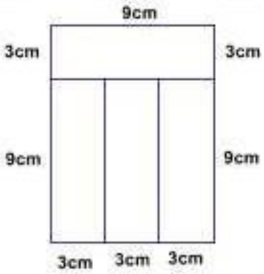
Row number	1	2	3	4		n
Number of numbers	1	3	5	7		$2\times n - 1$

Therefore, there will be 2x20 − 1=39 numbers in Row 20

5. A. The last number in each Row is the sequence 1, 4, 9, 16, ………n^2
The last number in Row 19 is 19×19. So the first number in Row 20 is 19×19 + 1 = 362
6. E.



7. E. $1 + \frac{1}{1+\frac{1}{3}} = 1 + \frac{1}{\frac{4}{3}} = 1 + \frac{3}{4} = 1\frac{3}{4}$
8. A. $\frac{60\ km}{1\ h} = \frac{60\ 000\ m}{3600\ sec} = \frac{600\ m}{36\ sec} = \frac{200\ m}{12\ sec}$
9. B.



10. E. X567Y is a multiple of 3, and 5+6+7 = 18 is a multiple of 3, so X+Y must be a multiple of 3. So the largest value of Y is 9, e.g. 3+9, 6+9, 9+9.
11. B. Use trial and improvement:
$20 \times 6 - 0 \times 2 = 120 \neq 88$ *He did not have all correct*
$19 \times 6 - 1 \times 2 = 112 \neq 88$ *He did not have 19 correct*
$17 \times 6 - 3 \times 2 = 96 \neq 88$ *He did not have 17 correct*
$16 \times 6 - 4 \times 2 = 88$ *He had 16 correct!*
12. A. $3x - 5 = 25$, so $x = 10$
13. C. Structure!

P_1	P_2	P_3	P_4	…	P_{50}
1×2	2×3	3×4	4×5	…	?

14. C. Difference in mass of water when half full and one third full:
$\frac{1}{2} - \frac{1}{3} \equiv 12 - 10$ kg, so $\frac{1}{6} \equiv 2$ kg
Thus, when half full, the water will be 6 kg, which means that the bucket has a mass of 6 kg.
15. B. The sum of the five numbers is $a + b + c + d + e = 60 \times 5 = 300$
The new sum is $80 + b + c + d + e = 65 \times 5 = 325$
So $80 - a = 25$, so $a = 55$
16. A. $\frac{20!}{19!} = \frac{20\times19\times18\times......\times1}{19\times18\times.........\times1} = 20$
17. C. Note that the numbers is column G are multiples of 7, in column F are all one less than a multiple of 7, etc. 2014 is 2 less than a multiple of 7 (2014 = 287 × 7 + 2), so 2014 will be in column E, which are all 2 less than a multiple of 7 (5, 12, 19…)

18. D. 4 choc + 2 cool = R35
2 choc + 4 cool = R43
6 choc + 6 cool = R78
1 choc + 1 cool = R13

19. D. The total number of dots that are not visible = total dots − visible dots
The total of the numbers on one die = 1+2+3+4+5+6 = 21, so the total on the three dice is 63.
Numbers 1, 1, 2, 3, 4, 5, 6 are visible, and these total 22.
So the total dots *not* visible = 63 − 22 = 41

20. C. Suppose there are 100 people

	Sick	Well
Month 1	10	90
Month 2	3+27=30	7+63=70

So $\frac{30}{100}$ of the people is sick

GRADE 7(1)

1. E $2 + 0 \times 1 \times 8 = 2 + 0 = 2$

2. B

3. C 10001, 10101, 10201, 10301, 10401, 10501, 10601, 10701, 10801, 10901

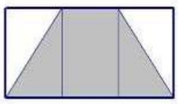
4. D Column totals must be the same as the row totals, ie, 59 + T = 64 + 61

5. D 4, 14, 24, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 64, 74, 84, 94

6. C Multiples of 30: 30, 60, 90, …, 990 (33 ×30)

7. A The five numbers are 3, 4, 5, 6, 6

8. A The remainder when 856 is divided by 7 is 2, which is true for all the numbers in column B



9. C

10. D If a number is divisible by 15 it is divisible by 3 and 5, which means that B is 5 (it is odd). Thus, 2 + A + 3 + 6 + 5 =A + 16 must be a multiple of 3, which means that A could be 2 or 5 or 8

11. E The gap from $\frac{1}{8}$ to $\frac{1}{4}$ is $\frac{1}{8}$. So the number is $\frac{1}{8} + \frac{1}{4} \times \frac{1}{8} = \frac{5}{32}$.

12. E $(1-2) + (3-4) + (5-6) + \cdots (97-98) + 99 = (-1) \times 49 + 99$

13. C $10^{2018} + 5 = 10000 \ldots 5, \therefore$ the sum of the digits is 6

14. A The possible scores are 0-0, 0-1, 1-0, 1-1, 2-0, 2-1

15. A The votes could be 251, 250, 249, 249

16. A The calculation is $6 \times 7 - 9$

17. C The black tiles are 1×1, 2×2, 3×3, 4×4, …, 10×10

18. D Use the answers to test. Total number of marbles is 20. Add 40 to number of green and total. Thus, there will be 45 green marbles and a total of 60.

19. A $6C, 36E, 4D \rightarrow 1C, 6E, 4D \rightarrow 8C, 48E, 4D \rightarrow 8C, 24E, 2D$

20. B $2L + B = 70\ and\ l + 2B = 59, so\ 3L + 3B = 129 \rightarrow L + B = 43 \rightarrow 2L + 2B = 86$

ANNEXURE G: ALTERNATIVE INTERVENTION WORKSHEETS

1

Commutative property of addition and multiplication

Term 1

Commutative property of addition and multiplication

Are the following true or false?

- 3 + 4 = 4 + 3
- 3 × 4 = 4 × 3
- 20 + 5 = 5 + 20
- 20 × 5 = 5 × 20

What do you notice?

The **commutative property of addition and multiplication** says that you can swap numbers around and still get the same answer when you add or multiply. The order in which you move the numbers around does not matter.

An **equation** says that two things are the same using an equal sign (=), e.g. 7 + 4 = 12 – 1

1. Use the commutative property of addition or multiplication to make the equations true.

Example: 5 + 1 = 1 + 5 (addition) and 5 × 1 = 1 × 5 (multiplication)

a. 13 + 2 =
13 + 2 = 2 + 13

b. 62 + 31 =

c. 4 × 5 =

d. 7 × 9 =

e. = 8 × 9

f. = 15 × 12

g. Make your own equations using the commutative property of addition and multiplication.

2. Use the commutative property of addition or multiplication to make the equations true.

Example: $f + e = e + f$ (addition) and $f \times e = e \times f$ (multiplication)

a. $a + b =$

b. $c \times d =$

c. $m \times n =$

d. $= g + h$

e. $= p \times 2$

f. $s \times t =$

g. Make your own equations using the commutative property of addition and multiplication.

3. Show that the given equation are equal when you substitute $a = 2$, $b = 5$ and $c = 3$.

Example: $a + b = b + a$ (addition)
$a + b = 2 + 5$ and $b + a = 5 + 2$
$= 7$ $= 7$
$a + b = b + a$

$a \times b = b \times a$ (multiplication)
$a \times b = 2 \times 5$ and $b \times a = 5 \times 2$
$= 10$ $= 10$
$a \times b = b \times a$

a. $c + a = c + a$

b. $c \times a = c \times a$

c. $b \times a = a \times b$

d. $b + a = a + b$

e. $b \times c = c \times b$

f. $b + c = c + b$

4. Write an equation to show how each diagram illustrates the commutative property of multiplication.

a. 

b. 

c. 

d. 

Problem solving

If $a = 20$ and $b = 15$, write an **associative property of addition and multiplication** statement and solve it.

2 Associative property of addition and multiplication

Term 1

Are the following true or false?

5 + (3 + 2) = (5 + 3) + 2

9 × (2 × 3) = (2 × 3) × 9

(12 + 14) + 13 = 12 + (14 + 13)

(11 × 2) × 4 = 11 × (2 × 4)

What do you notice?

The **associative property of addition and multiplication** says that it doesn't matter how you group numbers when you add or multiply.

1. Use the associative property of addition or multiplication to make the statements true.

Example: (5 + 1) + 3 = 5 + (1 + 3) (addition)

(5 × 1) × 3 = 5 × (1 × 3) (multiplication)

a. (6 + 2) + 4 =

(6 + 2) + 4 = 6 + (2 + 4) | Solve it: 12 = 12

b. (7 + 3) + 1 =

c. 8 × (10 × 4) =

d. 4 × (5 × 2) =

e. (11 × 3) × 2 =

f. (12 × 2) × 4 =

2. Use the associative property of addition or multiplication to make the statements true.

Example: $f + (g + h) = (f + g) + h$ (addition)

$f \times (g \times h) = (f \times g) \times h$ (multiplication)

a. $(a + b) + c =$

$a + (b + c)$

b. $(m + n) + c =$

c. $(g \times h) \times i =$

d. $(c \times d) \times f =$

e. $(k \times z) \times d =$

f. $(a + d) + v =$

g. $(a \times c) \times d =$

h. $(k \times l) \times m =$

i. $(v + c) + r =$

3. Solve if $a = 2$, $b = 4$ and $c = 3$. Show that the associative properties hold and calculate the answers.

Examples:

$a + (b + c) = (a + b) + c$	$a \times (b \times c) = (a \times b) \times c$
$2 + (4 + 3) = (2 + 4) + 3$	$2 \times (4 \times 3) = (2 \times 4) \times 3$
$2 + 7 = 6 + 3$	$2 \times 12 = 8 \times 3$
$9 = 9$	$24 = 24$
$a + (b + c) = (a + b) + c$	$a \times (b \times c) = (a \times b) \times c$

a. $(c + a) + b = c + (a + b)$

b. $(b \times a) \times c = a \times (b \times c)$

c. $b \times (c \times a) = c \times (b \times a)$

d. $b + (c + a) = (b + c) + a$

4. If $m = 1$, $n = 7$ and $q = 2$, show that the expressions are equal.

a. $(q + m) + n = q + (m + n)$

b. $(n \times m) \times q = m \times (n \times q)$

c. $n \times (q \times m) = q \times (n \times m)$

d. $n + (q + m) = (n + q) + m$

Problem solving

If $a = 25$, $b = 30$ and $c = 10$, write an **associative property of addition and multiplication** statement and solve it.

3 Distributive property of multiplication over addition

2(3)	3(9)	4(100)

4(6)	7(8)

What do the brackets mean?
Look at this statement:
2(3 + 2).
How do you think I will calculate this?

The distributive property lets you multiply a single number and each of two or more numbers between brackets (the products of which you then add together).
You will get the same answer when you multiply a group of numbers added together as when you do each multiplication separately and then add them together.

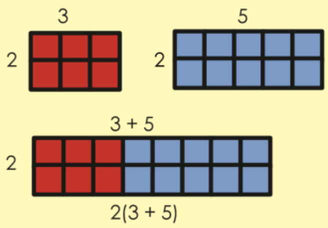
$2(3 + 2) = 2(5) = 10$

$2(3 + 2) = (2 \times 3) + (2 \times 2) = 6 + 4 = 10$

Usually we follow the rule that anything in brackets must be done first. In this example it would have been very easy to do this, 2(3+2) = 2(5) = 10. But the distributive property becomes very useful when what is inside the brackets is more complicated.

1. **Use the distributive property to write a sum for each diagram so that you can calculate the total number of blocks in each drawing.**

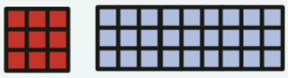
Example:

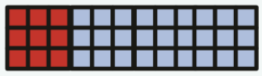


$2 \times 3 + 2 \times 5$

$2(3 + 5)$

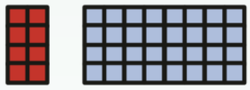
a.





b.

c.



d. Draw a diagram for:

i. $5(2 + 3)$

ii. $6(1 + 4)$

2. Use the distributive property of multiplication to make these statements true.

Example: 4(5 + 9) = 4 × 5 + 4 × 9 (4 × 5) + (4 × 9)

a. 3(4 + 2) = ______ Calculate it:
3 | 4 | 2
12 + 6 = 18

b. 10(2 + 3) = ______
+ =

c. 5(3 + 1) = ______
+ =

3. Use the distributive property of multiplication to make these statements true.

Example: 4 × 5 + 4 × 3 = (4 × 5) + (4 × 3) = 4(5 + 3)

a. 3 × 2 + 3 × 5 = ______ Calculate it:
3 | 2 | 5
6 + 15 = 21

b. 6 × 1 + 6 × 4 = ______
+ =

c. 3 × 2 – 3 × 1= ______
+ =

4. If a = 3, b = 2 and c = 4, calculate the following:

Example:
$a(b + c) = a \times b + a \times c$
3(2 + 4) = 3 × 2 + 3 × 4
3(6) = 6 + 12
18 = 18

a. $b(a + c)$

b. $c(b + a)$

c. $a(c + b)$

Problem solving

If a = 5, b = 9 and c = 11, write a distributive property statement and calculate the answer.

4 Zero as the identity of addition, one as the identity of multiplication, and other properties of numbers

Term 1

What do you notice?

3 + 0 = 5 + 0 = 100 + 0 =
0 + 16 = 0 + 250 = 72 + 0 =

3 × 1 = 5 × 1 = 100 × 1 =
1 × 16 = 1 × 250 = 1 × 72 =

DEFINITION

Zero as the identity of addition:
The sum of zero and any number is the number itself. The answer will always be the number that **zero** is **added** to.

DEFINITION

One as the identify of multiplication:
The product of 1 and any number is always the number itself. The answer will always be the number that **one** is multiplied **by**.

1. Use zero as the identity of addition, or one as the identity of multiplication to write a sum for the following:

		Zero as the identity of addition	One as the identity of multiplication
a.	5	5 + 0 = 5	5 × 1 = 5
b.	7		
c.	9		
d.	100		
e.	34		
f.	2,5		
g.	0,1		

2. Use zero as the identity of addition, or one as the identity of multiplication to solve the following:

a. $b + 0 =$ ☐ b. $d \times$ ☐ $= d$ c. $e \times 1 =$ ☐

$b \times 1 =$ ☐ $d +$ ☐ $= d$ $e + 0 =$ ☐

3. Choose the correct property of number to write and equivalent statement to complete the equation.

a. 4 + 5 =
4 + 5 = 5 + 4

b. 2(3 + 9) =

c. 3 + (4 + 8) =

d. 5(9 – 8) =

e. 9 + 12 =

f. (2 × 5) × 11 =

4. Say whether the following are true or false. If it is false, explain why it is false.

a. $9 + 2 = 2 + 9$

b. $5 - 4 = 4 - 5$

c. $4(2 + 1) = 4 \times 2 + 4 \times 1$

d. $3 + 0 = 3$

e. $8 - (3 - 2) = (8 - 3) - 2$

f. $2(5 - 4) = 2 \times 5 - 2 \times 4$

5. If $a = 2$, $b = 5$, $c = 8$, solve the following:

Example:

$b + a = a + b$

$5 + 2 = 2 + 5$

$7 = 7$

a. $a + c = c + a$

b. $b + (c + a) = (b + c) + a$

c. $a + 0 =$

d. $b(a + c)$

e. $a(c - b)$

f. $b \times 1 =$

6. Match column A with column B

Column A	Column B
Associative property of numbers	$a \times 1 = a$
Commutative property of numbers	$(a + b) + c = a + (b + c)$
Distributive property of numbers	$a + 0 = a$
Zero as the identity of addition	$a + b = b + a$
One as the identity of multiplication	$a(b + c) = a \times b + a \times c$

Problem solving

- What should I add to a number so that the answer will be the same as the number?
- By what should I multiply a number so that the answer will be the same as the number?
- Write five statements that are true using the properties of number.
- Write five statements that are false using the properties of number. Explain your answer.

5 Multiples

Term 1

How fast can you give me the first 12 multiples of 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, and 10s?

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

1. Use the number board to complete the following:

Example: The multiples of 6 are 6, 12, 18, … 72, or

We can write it as: multiples of 6: {6,12,18, 24, 30, 36, 42, 48, 54, 60, 66, 72}

a. Multiples of 4: {____}

b. Multiples of 7: {____}

c. Multiples of 5: {____}

d. Multiples of 8: {____}

e. Multiples of 2: {____}

f. Multiples of 9: {____}

2. Write down the first 12 multiples of the numbers below. Circle all the common multiples and identify the lowest common multiple (LCM).

Example: Multiples of 2: 2, (4), 6, (8), 10, (12), 14, (16), 18, (20), 22, (24)

Multiples of 4: (4), (8), (12), (16), (20), (24), 28, 32, 36, 40, 44, 48

The LCM is 4.

a. Multiples of 5: {____}
Multiples of 10: {____}
LCM? __________

b. Multiples of 5: {____}
Multiples of 6: {____}
LCM? __________

c. Multiples of 90: {____}
Multiples of 20: {____}
LCM? __________

3. What is the LCM for the following?

Example: Multiples of 4 and multiples of 7
Multiples of 4: { 4, 8, 12, 16, 20, 24, (28) }
Multiples of 7: {7, 14, 21, (28) }

a. Multiples of 2 and multiples of 8

b. Multiples of 3 and multiples of 6

c. Multiples of 5 and multiples of 3

d. Multiples of 4 and multiples of 8

e. Multiples of 70 and multiples of 60

f. Multiples of 100 and multiples of 125

Problem solving

In our homes there are various things that come in multiples. Give five examples of multiples from your home.

6 Divisibility and factors

Term 1

Your little brother messed up your notes. Find the missing information.

A number is divisible by [illegible] if the number formed by the last three digits is divisible by 8.
A number is divisible by 3 if the sum of the digits is divisible by 3.
A number is divisible by 10 if the last digit is [illegible]
A number is divisible by [illegible] if the last digit is either 0 or 5.
A number is divisible by 4 if the number formed by the last two digits is divisible by [illegible]
A number is divisible by 9 if the sum of the digits is divisible by 9.
A number is divisible by [illegible] if the last digit is 0, 2, 4, 6 or 8.
A number is divisible by 6 if it is divisible by 2 and it is divisible by 3.

1. Tick whether the numbers are divisible by 2, 3, 4, 5 or 10. You can have more than one answer.

	2	3	4	5	10
a. 376	✔				
b. 7 232					
c. 9 050					
d. 6 312					
e. 2 355					

2. The following numbers are divisible by?

Example: 6 is divisible by 1, 2, 3 and 6.

a. 12 b. 36 c. 42 d. 24 e. 64

3. Which two numbers, when multiplied, give you this number?

Example: 6 = 2 × 3, 6 = 1 × 6

a. 12 b. 36 c. 42 d. 24 e. 64

4. What do you notice if you compare the answers to questions 2 and 3?

5. For each of the numbers given below, write down:

(i) All the possible multiplication sums using only two numbers that will give you this number. (ii) All the numbers used in these multiplication sums, in ascending order (but do not repeat a number). (iii) Complete the sentence: "These are the factors of__________." (iv) Complete the sentence: "Factors of__________ = {_________}. "

Example: i. 12: 1 × 12, 2 × 6, 3 × 4
ii. 1, 2, 3, 4, 6, 12
iii. These are the factors of 12.
iv. Factors of 12 = {1, 2, 3, 4, 6, 12}

a.	b.	c.
i. **18:** ____________________	i. **25:** ____________________	i. **36:** ____________________
ii. ____________________	ii. ____________________	ii. ____________________
iii. ____________________	iii. ____________________	iii. ____________________
iv. Factors of __ = {__________}	iv. F__ = {__________}	iv. F__ = {__________}

6. Complete the following, using the example to guide you.

Example: i. Factors of 12 are 1, 2, 3, 4, 6 and 12
Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30
ii. The common factors are: 1, 2, 3, 6
iii. The highest common factor is 6.

The abbreviation for highest common factor is HCF.

a. Factors of 8: {....}
Factors of 16: {....}

(i) ______
(ii) ______
(iii) ______

b. Factors of 3: {....}
Factors of 12: {....}

(i) ______
(ii) ______
(iii) ______

c. Factors of 3: {....}
Factors of 9: {....}

(i) ______
(ii) ______
(iii) ______

7. Complete the table.

	Words	Factors	Common factors	HFC
a. 4 and 8	Factors of 4 and Factors of 8	1, 2, 4, 1, 2, 4, 8	1, 2, 4,	4
b. 9 and 12				
c. 4 and 28				
d. 12 and 36				

Find out!

When in everyday life do we use HCF?

ANNEXURE H: ALTERNATIVE INTERVENTION ANSWER SHEETS

Worksheet 1

1. True, true, true, true
b) 62+31=31+62
c) 4x5=5x4 d) 7x9=9x7
e) 9x8=8x9 f) 12x15=15x12

2. a) a+b=b+a b) cxd=dxc
c) mxn=nxm d) h+g=g+h
e) 2xp=px2 f) sxt=txs

3. a) c+a=c+a b) cxa=cxa
3+2=3+2 3x2=3x2
5=5 6=6

c) bxa=axb d) b+a=a+b
5x2=2x5 5+2=2+5
10=10 7=7

e) bxc=cxb f) b+c=c+b
5x3=3x5 5+3=3+5
15=15 8=8

4. a) 3x4=4x3 b) 6x6=6x6
c) 2x6=6x2 d) 1x4=4x1

Worksheet 2

1. True, true, true, true
b) (7+3)+1 = 7+(3+1)
10+1 = 7+4
11 = 11

c) 8x(10x4) = (8x10)x4 d) 4x(5x2) = (4x5)x2
8 x 40 = 80 x 4 4 x 10 = 20 x 2
320 = 320 40 = 40

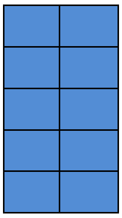
e) (11x3)x2 = 11x(3x2) f) (12x2)x4 = 12x(2x4)
33x2 = 11x6 24x4 = 12x8
66 = 66 96 = 96

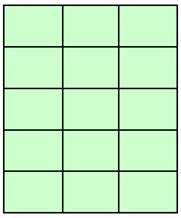
2 b) (m+n)+c = m+(n+c) c) (gxh)xi = gx(hxi)
d) (cxd)xf = cx(dxf) e) (kxz)xd = kx(zxd)
f) (a+d)+v = a+(d+v) g) (axc)xd = ax(cxd)
h) (kxl)xm = kx(lxm) i) (v+c)+r = v+(c+r)

3. a) (c+a)+b = c+(a+b) b) (bxa)xc = ax(bxc)
(3+2)+4 = 3+(2+4) (4x2)x3 = 2x(4x3)
5+4 = 3+6 8x3 = 2x12
9 = 9 24 = 24

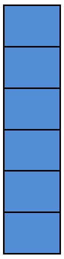
c) bx(cxa) = cx(bxa) d) b+(c+a) = (b+c)+a
4x(3x2) = 3x(4x2) 4+(3+2) = (4+3)+2
4 x 6 = 3 x 8 4 + 5 = 7 + 2
24 = 24 9 = 9

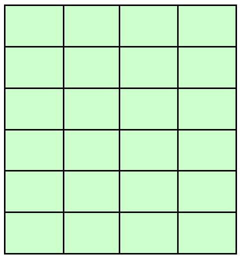
4. a) (q+m)+n = q+(m+n) b) (nxm)xq = mx(nxq)
(2+1)+7 = 2+(1+7) (7x1)x2 = 1x(7x2)
3 +7 =
2+8
7 x2 = 1x14
10





= 10
14





= 14

c) nx(qxm) =
qx(nxm) d) n+(q+m) = (n+q)+m
7x(2x1) = 2x(7x1) 7+(2+1) = (7+2)+1 7 x 2 = 2 x 7
7 + 3 = 9 + 1
14 = 14 10 = 10

<u>Worksheet 3</u>

1. a) 3x3+3x9 b) 2x4+2x6 c) 4x2+4x8
3(3+9) 2(4+6) 4(2+8)

d)i. ii.

2. a) 3(4+2) = 3x4+3x2 = (3x4)+(3x2) = 12 + 6 = 18
b) 10(2+3) = 10x2+10x3 = (10x2)+(10x3) = 20 + 30 = 50
c) 5(3+1) = 5x3+5x1 = (5x3)+(5x1) = 15 + 5 = 20

3. a) 3x2+3x5 = (3x2)+(3x5) = 3(2+5) = 6 + 15 = 21
b) 6x1+6x4 = (6x1)+(6x4) = 6(1+4) = 6 + 24 = 30
c) 3x2-3x1 = (3x2)-(3x1) = 3(2-1) = 6 – 3 = 3

4.

a) b(a+c) = bxa+bxc	b) c(b+a) = cxb+cxa
2(3+4) = 2x3+2x4	4(2+3) = 4x2+4x3
2(7) = 6+8	4(5) = 8 + 12
14 = 14	20 = 20

c) a(c+b) = axc+axb
3(4+2) = 3x4+3x2
3(6) = 12+6
18 = 18

<u>Worksheet 4</u>

3+0=3	5+0=5	100+0=100	3x1=3	5x1=5	100x1=100
0+16=16	0+250=250	72+0=72	1x16=16	1x250=250	1x72=72

Anything + 0 = the same number
Anything x 1 = the same number

1.

a) 5+0=5	5x1=5
b) 7+0=7	7x1=7
c) 9+0=9	9x1=9
d) 100+0=100	100x1=100
e) 34+0=34	34x1=34
f) 2,5+0=2,5	2,5x1=2,5
g) 0,1+0=0,1	0,1x1=0,1

2.

a) b+0=b	bx1=b	b+0=bx1
b) dx1=d	d+0=d	dx1=d+0
c) ex1=e	e+0=e	ex1=e+0

3.

	b) 2(3+9) = 2x3+2x9	c) 3+(4+8) = (3+4)+8
d) 5(9-8) = 5x9-5x8	e) 9+12 = 12+9	f) (2x5)x11 = 2x(5x11)

4. a) true
b) false because 5-4=1 and 4-5=-1
c) true
d) true
e) false because 8-(3-2) = 8-1 = 7 and (8-3)-2= 5-2= 3
f) true

5.

a) a+c = c+a	b) b+(c+a)=(b+c)+a	c) a+0
2+8 = 8+2	5+(8+2)=(5+8)+2	= a

10 = 10 5+10 = 13+2 =2
15 = 15

d) b(a+c) e) a(c-b) f) bx1
= bxa+bxc = axc-axb = b
= 5x2+5x8 = 2x8-2x5 = 5
= 10+40 = 16-10
= 50 = 6

6.
Associative property of numbers (a+b)+c=a+(b+c)
Commutative property of numbers a+b=b+a
Distributive property of numbers a(b+c)=axb+axc
Zero as the identity of addition a+0=a
One as the identity of multiplication ax1=a

Worksheet 5

1. a) 4,8,12,16,20,24,28,32,36,40,44,48
b) 7,14,21,28,35,42,49,56,63,70,77,84
c) 5,10,15,20,25,30,35,40,45,50,55,60
d) 8,16,32,40,48,56,64,72,80,88,96
e) 2,4,6,8,10,12,14,16,18,20,22,24
f) 9,18,27,36,45,54,63,72,81,90,99,108

2. a) 5,10,15,20,25,30,35,40,45,50,55,60
10,20,30,40,50,60,70,80,90,100,110,120
LCM = 10

b) 5,10,15,20,25,30,35,40,45,50,55,60
6,12,18,24,30,36,42,48,54,60,66,72
LCM = 30

c) 90,180,270,360,450,540,630,720,810,900,990,1080
20,40,60,80,100,120,140,160,180,200,220,240
LCM = 180

3. a) 2,4,6,8
8
LCM=8

b) 6,12,18
3,6
LCM=6

c) 3,9,12,15
5,10,15
LCM=15

d) 4,8
8
LCM=8

e) 70,140,210,280,350,420

60, 120,180,240,300,360,420
LCM=420

f) 100,200,300,400,500
125,250,375,500
LCM=500

Worksheet 6

A number is:
divisible by 8 if the number formed by the last three digits is divisible by 8
divisible by 3 if the sum of the digits is divisible by 3
divisible by 10 if the last digit is 0
divisible by 5 if the last digit is either 0 or 5
divisible by 4 the number formed by the last three digits is divisible by 4
divisible by 2 if the last digit is 0,2,4,6 or 8
divisible by 6 if it is divisible by 2 and it is divisible by 3

1.

		2	3	4	5	10
a)	376	▯		▯		
b)	7232	▯		▯		
c)	9050	▯			▯	▯
d)	6312	▯	▯	▯		
e)	2355		▯		▯	

2. a) 12 is divisible by 1,2,3,4,6 and 12
b) 36 is divisible by 1,2,3,4,6,9,12 and 36
c) 42 is divisible by 1,2,3,6,7,14,21 and 42
d) 24 is divisible by 1,2,3,4,6,8,12 and 24
e) 64 is divisible by 1,2,4,8,16,32 and 64

3. a) 12 = 1x12, 12=2x6, 12=3x4
b) 36 = 1x36, 36=2x18, 36=3x12, 36=4x9, 36=6x6
c) 42 = 1x42, 42=2x21 42=3x14 42=6x7
d) 24 = 1x24, 24=2x12 24=3x8, 24=4x6
e) 64 = 1x64, 64=2x32 64=4x16 64=8x8

4. The biggest factor times the smallest factor makes the number, the second biggest factor times the second smallest factor makes the number and so on.

5. a)
i) 18: 1x18, 2x9, 3x6
ii) 1,2,3,6,9,18
iii) these are the factors of 18
iv) Factors of 18 = {1,2,3,6,9}

b)
i) 25: 1x25, 5x5
ii) 1,5,25
iii) these are the factors of 25
iv) Factors of 18 = {1,5,25}

c) i) 36: 1x36, 2x18, 3x12, 4x9, 6x6
ii) 1,2,3,4,6,9,12,18,36
iii) these are the factors of 36
iv) Factors of 36 = {1,2,3,4,6,9,12,18,36}

6. a) i) Factors of 8 are 1,2,4 and 8
Factors of 16 are 1,2,4,8 and 16
ii) The common factors are: 1,2,4 and 8

iii) The highest common factor is 8

b) i) Factors of 3 are 1,3
Factors of 12 are 1,2,3,4 and 12
ii) The common factors are: 1 and 3
iii) The highest common factor is 3

c) i) Factors of 3 are 1,3
Factors of 9 are 1,3 and 9
ii) The common factors are: 1 and 3
iii) The highest common factor is 3

7.

	Words	Factors	Common factors	HCF
b) 9 and 12	factors of 9 and factors of 12	1,3,9 1,2,3,4,6,12	1,3	3
c) 4 and 28	factors of 4 and factors of 28	1,2,4 1,2,4,7,14,28	1,2,4	4
d) 12 and 36	factors of 12 and factors of 36	1,2,3,4,6,12 1,2,3,4,6,9,12, 18,36	1,2,3,4,6,12	12