Online Appendix for "One Shape Fits All? A Comprehensive Examination of Cryptocurrency Return Distributions"

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Abstract

The Online Appendix provides additional tables for the study. Table A1 reports the descriptive statistics for cryptocurrency returns. Table A2 lists the return distributions considered in this research and the methods used to fit these hypothetical distributions. Table A3 reports the parameters and test statistics for the next best fitting distributions. Finally, Table A4 lists the respective probability density function, cumulative density functions and defines the location and scale parameters for the distributions fitted or identified as most likely.

Table A1. Summary statistics for cryptocurrency returns

The mean and median columns report the respective mean and median of the cryptocurrencies that comprise the sample. *Min* and *max* are the lowest and highest observations in the series. *S.D.* is the series standard deviation. *Ex. Kurt* is the excess kurtosis of each series and *Skew* is the skewness coefficient. *SW* is the Shapiro-Wilk test statistic. *Obs.* is the number of observations for each series. The length of each sample is reported in the last column. The asterisks, * and ** indicate statistical significance at the 5% and 1% levels of significance respectively.

#	Crypto.	Mean	Median	Min	Max	S.D.	Ex. Kurt	Skew	SW	Obs.	Dates
1	Bitcoin	0.00195	0.00194	-0.26620	0.35745	0.04330	7.77990	-0.17052	0.88600**	2254	28/04/2013-01/07/2019
2	Ethereum	0.00327	-0.00069	-1.30210	0.41234	0.07459	68.10900	-3.40560	0.79200**	1423	07/08/2015-01/07/2019
3	Ripple	0.00196	-0.00272	-0.61627	1.02740	0.07511	28.35700	2.03290	0.76500**	2157	04/08/2013-01/07/2019
4	Litecoin	0.00148	0.00000	-0.51393	0.82897	0.06599	25.04400	1.72110	0.78600**	2255	28/04/2013-01/07/2019
5	Bitcoin Cash	0.00001	-0.00435	-0.44604	0.43158	0.08600	6.27660	0.64174	0.89100**	708	23/07/2017-01/07/2019
6	EOS	0.00245	-0.00178	-0.38504	0.98700	0.08955	21.98900	2.20240	0.85100**	730	01/07/2019-01/07/2019
7	Binance	0.00813	0.00105	-0.40863	0.67517	0.08602	10.23400	1.27960	0.86300**	706	25/07/2019-01/07/2019
8	Cardano	0.00190	-0.00281	-0.28868	0.86154	0.08705	22.72300	2.80900	0.81300**	638	01/10/2017-01/07/2019
9	Stellar	0.00210	-0.00334	-0.36636	0.72310	0.07794	15.81400	1.96840	0.83700**	1791	05/08/2014-01/07/2019
10	Monero	0.00215	-0.00035	-0.37822	0.58464	0.07249	6.24490	0.64186	0.92900**	1866	21/04/2014-01/07/2019
11	Tron	0.00426	-0.00218	-0.38217	0.78667	0.10544	14.06700	2.05670	0.81600**	656	13/09/2017-01/07/2019
12	Dash	0.00307	-0.00178	-0.46757	1.27060	0.07869	43.59300	3.02040	0.78700**	1936	14/02/2014-01/07/2019
13	Chainlink	0.00454	-0.00286	-0.31783	0.48416	0.08857	3.32770	0.66293	0.95500**	649	20/09/2017-01/07/2019
14	NEO	0.00332	-0.00241	-0.52254	0.80117	0.09555	12.01100	1.10820	0.85600**	1025	09/09/2016-01/07/2019
15	IOTA	-0.00050	-0.00303	-0.37705	0.38396	0.08139	3.85420	0.15036	0.94500**	748	13/06/2017-01/07/2019

Table A2. List of distributions and the parameter estimation method applied

This table lists the hypothetical distributions that were fitted to each cryptocurrency return series with the aim of identifying the best fitting distributions. The method used to estimate the respective parameters for each distribution is stated in the second column. A detailed description of the distributions is available upon request.

No.	Distribution Name	Parameter Estimation Method	No.	Distribution Name	Parameter Estimation Method
1	Beta	Maximum likelihood method	30	Laplace	Method of moments
2	Burr	Maximum likelihood method	31	Levy	Maximum likelihood method
3	Cauchy	Maximum likelihood method	32	Logistic	Method of moments
4	Chi-Squared (1 parameter)	Method of moments	33	Log-Gamma	Method of moments
5	Chi-Squared (2 parameters)	Maximum likelihood method	34	Log-Logistic (2 parameters)	Least squares method
6	Dagum	Maximum likelihood method	35	Log-Logistic (3 parameters)	Maximum likelihood method
7	Erlang (2 parameters)	Method of moments	36	Log-Pearson 3	Method of moments
8	Erlang (3 parameters)	Maximum likelihood method	37	Lognormal (2 parameters)	Maximum likelihood method
9	Error	Maximum likelihood method	38	Lognormal (3 parameters)	Maximum likelihood method
10	Error Function	Method of moments	39	Nakagami	Method of moments
11	Exponential (1 parameter)	Method of moments	40	Normal	Maximum likelihood method
12	Exponential (2 parameters)	Maximum likelihood method	41	Pareto	Maximum likelihood method
13	Fatigue Life (2 parameters)	Maximum likelihood method	42	Pareto 2	Maximum likelihood method
14	Fatigue Life (3 parameters)	Maximum likelihood method	43	Pearson 5	Maximum likelihood method
15	Frechet (2 parameters)	Least squares method	44	Pearson 6	Maximum likelihood method
16	Frechet (3 parameters)	Maximum likelihood method	45	Pert	Maximum likelihood method
17	Gamma (2 parameters)	Method of moments	46	Phased Bi-Exponential	Least squares method
18	Gamma (3 parameters)	Maximum likelihood method	47	Phased Bi-Weibull	Least squares method
19	Generalized Gamma	Maximum likelihood method	48	Power Function	Maximum likelihood method
20	Generalized Extreme Value	Method of L-moments	49	Rayleigh (1 parameter)	Method of moments
21	Generalized Logistic	Method of L-moments	50	Rayleigh (2 parameters)	Maximum likelihood method
22	Generalized Pareto	Method of L-moments	51	Reciprocal	Maximum likelihood method
23	Gumbel Max/Min	Method of moments	52	Rice	Maximum likelihood method
24	Hyperbolic Secant	Method of moments	53	Student's *t*	Method of moments
25	Inverse Gaussian (2 parameters)	Method of moments	54	Triangular	Maximum likelihood method
26	Inverse Gaussian (3 parameters)	Maximum likelihood method	55	Uniform	Method of moments
27	Johnson SB	Method of moments	56	Wakeby	Method of L-moments
28	Johnson SU	Method of moments	57	Weibull (2 parameters)	Least squares method
29	Kumaraswamy	Maximum likelihood method	58	Weibull (3 parameters)	Maximum likelihood method

Table A3. Unabridged results of goodness-of-fit tests for the next best fitting distributions

In column (2), the next best fitting distributions listed in column (5), (6) and (7) of Table 1 are reported, in no particular order. Column (3) reports the respective scale, location and shape parameters, where applicable in the latter case. The respective Kolmogorov-Smirnov (*KS*), Anderson-Darling (*AD*) and chi-squared (χ^2) test statistics are reported in column (4), (5) and (6). The values in brackets ( ) preceding the test statistics indicate the rankings of the corresponding distributions according to the respective tests. The asterisks * and ** indicate statistical significance at the respective 5% and 1% levels of significance. For a full list of the distributions tested and methods applied refer to Table A2 in the Online Appendix.

(1) Crypto.	(2) Distribution	(3) Parameters	(4) KS	(5) AD	(6) χ^2
Bitcoin	Laplace	λ=32.663, μ=0.00195	(2) 0.06832**	(2) 15.758**	(3) 134.22**
	Error	k=1.0 σ =0.0433 μ=0.00195	(3) 0.06832**	(3) 15.758**	(2) 134.22**
	Johnson SU	γ=0.04865 δ=1.2765, λ=0.03939 ξ=0.00399	(4) 0.07612**	(4) 19.543**	(4) 216.37**
	Log-Logistic (3P)	α=2.9022E+8 β=5.9336E+6 γ=-5.9336E+6	(5) 0.08098**	(6) 25.897**	(5) 270.7**
	Dagum (4P)	k =0.7079, α=75.029, β=1.3458, γ=-1.3343	(6) 0.08297**	(5) 24.695**	(6) 279.51**
Ethereum	Burr (4P)	k=0.70689, α=4.8888E+8	(2) 0.0607**	(2) 11.61**	(4) 95.89**
	Laplace	β=1.4037E+7, γ=-1.4037E+7, λ=18.959, μ=0.00327	(3) 0.0627**	(3) 12.416**	(3) 95.48**
	Error	k=1.0, σ=0.07459, μ=0.00327	(4) 0.0627**	(4) 12.416**	(2) 95.48**
	Log-Logistic	α=4.9652E+8 β=1.6590E+7 γ=-1.6590E+7	(5) 0.06487**	(5) 12.504**	(5) 128.11**
Ripple	Johnson SU	γ=-0.32166, δ=1.0678, λ=0.04567, ξ=-0.01969	(7) 0.10435**	(2) 29.197**	(2) 269.43**
	Log-Logistic (3P)	α=40.359, β=1.26, γ=-1.2615	(2) 0.08517**	(5) 35.255**	(7) 356.11**
	Dagum (4P)	k=0.8956, α=36.248, β=1.092, γ=-1.089	(3) 0.08675**	(4) 35.17**	(4) 346.56**
	Laplace	λ=18.83, μ=0.00196	(5) 0.10396**	(6) 47.391**	(6) 352.7**
	Burr (4P)	k=0.65044, α=182.22, β=4.7384, γ=-4.756	(4) 0.0887**	(3) 32.9**	(3) 342.25**
Litecoin	Log-Logistic (3P)	α=47.209, β=1.3311, γ=-1.3318	(2) 0.08407**	(5) 33.062**	(7) 354.94***
	Dagum (4P)	k=0.83862, α=37.702, β=1.0019, γ=-0.99617	(3) 0.08588**	(4) 32.656**	(6) 347.29**
	Burr (4P)	k=0.70045, α=232.78, β=5.6725, γ=-5.6857	(4) 0.08721**	(3) 31.718**	(5) 335.37**
	Laplace	λ=21.432, μ=0.00148	(5) 0.09399**	(6) 38.509**	(4) 293.48**
	Error	k=1.0, σ=0.06599, μ=0.00148	(6) 0.09399**	(7) 38.509**	(3) 293.48**
	Johnson SU	γ=-0.27736, δ=1.0773, λ=0.04156, ξ=-0.01516	(7) 0.09661**	(2) 26.59**	(2) 249.08**
Bitcoin Cash	Laplace	λ=16.444, μ=1.2223E-5	(2) 0.06649**	(2) 4.6116**	(3) 32.552**
	Error	k=1.0, σ=0.086, μ=1.2223E-5	(3) 0.06649**	(3) 4.6116**	(2) 32.552**
	Log-logistic (3P)	α=34.264, β=1.4005, γ=-1.4041	(4) 0.06942**	(7) 5.8334**	(7) 52.41**
	Dagum (4P)	k=0.88954, α=27.972, β=1.1025, γ=-1.0998	(5) 0.06997**	(6) 5.8052**	(6) 50.741**
	Johnson SU	γ=-0.23309, δ=1.3683, λ=0.08642, ξ=-0.01931	(7) 0.07126**	(4) 5.1926**	(5) 48.283**
	Burr (4P)	k=0.67078, α=2.2201E+8, β=7.7295E+6, γ=-7.7295E+6	(6) 0.07000**	(5) 5.3726**	(4) 45.256**
EOS	Burr (4P)	k=0.74541, α=45.642, β=1.6687, γ=-1.6863	(2) 0.05907*	(4) 5.1454**	(4) 55.59**
	Log-logistic (3P)	α=21.018, β=0.87071, γ=-0.87276	(3) 0.06259**	(6) 5.3781**	(5) 60.494**
	Dagum (4P)	k=0.79559, α=16.836, β=0.65265, γ=-0.64215	(4) 0.06309**	(5) 5.2191**	(6) 62.001**
	Laplace	λ=15.792, μ=0.00245	(5) 0.07039**	(2) 5.0744**	(3) 38.559**
	Error	k=1.0 σ=0.08955 μ=0.00245	(6) 0.07039**	(3) 5.0744**	(2) 38.559**

Binance	Burr (4P)	k=0.63209 α=155.95 β=5.0849 γ=-5.104	(2) 0.05819*	(3) 4.4487**	(5) 50.148**
	Log-Logistic (3P)	α=25.031 β=0.99336 γ=-0.99008	(3) 0.06159**	(4) 5.1823**	(6) 55.927**
	Dagum (4P)	k=0.92304 α=22.83 β=0.8853 γ=-0.87774	(4) 0.06231**	(5) 5.2091**	(7) 57.202**
	Johnson SU	γ=-0.38813 δ=1.2798 λ=0.07424 ξ=-0.0229	(5) 0.06861**	(2) 4.3397**	(2) 41.139**
	Error	k=1.0 σ=0.08602 μ=0.00813	(7) 0.07298**	(7) 6.419**	(3) 42.037**
	Laplace	λ=16.441 μ=0.00813	(6) 0.07298**	(6) 6.419**	(4) 42.037**
Cardano	Log-logistic (3P)	α=14.769, β=0.57539, γ=-0.57917	(2) 0.05101	(2) 2.4379	(3) 21.282*
	Dagum (4P)	k=0.76682, α=12.567, β=0.45671, γ=-0.44661	(3) 0.05107	(1) 2.201	(1) 15.297
	Cauchy	σ=0.03362, μ=-0.00384	(4) 0.05365	(4) 3.7271*	(4) 24.212**
	Laplace	λ=16.245, μ=0.0019	(5) 0.05738*	(5) 4.7353**	(6) 30.227**
	Burr (4P)	k=0.79473, α=22.139, β=0.77855, γ=-0.79404	(1) 0.04803	(3) 2.4485	(2) 2.4485*
	Error	k=1.0 σ=0.08705 μ=0.0019	(6) 0.05738*	(6) 4.7353**	(5) 30.227**
Stellar	Burr (4P)	k=0.68958, α=54.512, β=1.6335, γ=-1.6522	(2) 0.05752**	(2) 12.343**	(2) 101.82**
	Log-Logistic (3P)	α=21.096, β=0.74267, γ=-0.74499	(3) 0.05806**	(4) 13.461**	(7) 117.91**
	Dagum (4P)	k=0.85039, α=18.309, β=0.6153, γ=-0.61002	(4) 0.05998**	(3) 13.343**	(5) 109.91**
	Laplace	λ=18.146, μ=0.0021	(5) 0.06779**	(6) 17.195**	(4) 107.72**
	Error	k=1.0, σ=0.07794, μ=0.0021	(6) 0.06779**	(7) 17.195**	(3) 107.72**
	Johnson SU	γ=-0.54608, δ=1.2246, λ=0.05826, ξ=-0.03537	(7) 0.08301**	(5) 16.284**	(6) 116.01**
Monero	Laplace	λ=19.509, μ=0.00215	(2) 0.03335*	(1) 2.8561*	(2) 32.735**
	Johnson SU	γ=-0.23412, δ=1.3701, λ=0.07299, ξ=-0.01421	(3) 0.04302**	(3) 4.011**	(3) 46.422**
	Burr (4P)	k=0.74198, α=4.3157E+7, β=1.3926E+6, γ=-1.3926E	(4) 0.04392**	(4) 6.4933**	(5) 71.168**
	Log-Logistic (3P)	α=37.833, β=1.3723, γ=-1.3722	(5) 0.04571**	(6) 7.0555**	(7) 80.187**
	Error	k=1.0, σ=0.07249, μ=0.00215	(1) 0.03335*	(2) 2.8561*	(1) 32.735**
	Dagum (4P)	k=0.82585, α=24.885, β=0.85039, γ=-0.84104	(6) 0.0461**	(5) 6.8757**	(6) 77.322**
	Cauchy	σ=0.03111, μ=-0.00121	(7) 0.04967**	(8) 9.9949**	(4) 57.283**
Tron	Log-logistic (3P)	α=18.944, β=0.87653, γ=-0.87877	(2) 0.07368**	(4) 6.4715**	(3) 52.876**
	Burr (4P)	k=0.69276, α=43.278, β=1.7054, γ=-1.7286	(3) 0.07521**	(2) 6.1651**	(2) 49.278**
	Dagum (4P)	k=0.81019, α=16.171, β=0.70478, γ=-0.69426	(4) 0.07541**	(3) 6.3448**	(4) 54.218**
	Laplace	λ=13.413, μ=0.00426	(5) 0.09139**	(5) 8.8597**	(6) 61.905**
	Error	k=1.0, σ=0.10544, μ=0.00426	(6) 0.09139**	(6) 8.8597**	(5) 61.905**
Dash	Burr (4P)	k=0.68588 α=82.07 β=2.3778 γ=-2.3945	(2) 0.05988**	(2) 15.219**	(3) 123.28**
	Log-logistic (3P)	α=29.256 β=0.99537 γ=-0.9960	(3) 0.06113**	(4) 16.862**	(5) 136.82**
	Dagum (4P)	k=0.87476 α=25.711 β=0.83968 γ=-0.834	(4) 0.06258**	(5) 16.868**	(4) 134.47**
	Laplace	λ=17.973 μ=0.00307	(5) 0.0738**	(6) 24.728**	(7) 152.23**
	Johnson SU	γ=-0.44068 δ=1.031 λ=0.04171 ξ=-0.02635	(7) 0.08202**	(3) 16.064**	(2) 94.576**
Chainlink	Log-logistic (3P)	α=20.231, β=0.92849, γ=-0.92847	(2) 0.02708	(3) 0.82823	(3) 11.098
	Dagum (4P)	k=0.89545, α=16.892, β=0.75275, γ=-0.74576	(3) 0.02728	(4) 0.83559	(4) 11.243
	Johnson SU	γ=-0.43444, δ=1.662, λ=0.11679, ξ=-0.03247	(4) 0.0287	(2) 0.69379	(2) 7.5697
	Hypersecant	σ=0.08857, μ=0.00454	(5) 0.04994	(5) 1.7029	(5) 11.486

Neo	Cauchy	σ=0.03564, μ=-0.00336	(2) 0.05475**	(2) 5.0467**	(2) 44.858**
	Dagum (4P)	k=0.94131, α=30.731, β=1.3267, γ=-1.3243	(3) 0.05749**	(5) 6.7734**	(6) 56.874**
	Log-logistic (3P)	α=33.62, β=1.4782, γ=-1.4795	(4) 0.05819**	(4) 6.7278**	(7) 57.068**
	Burr (4P)	k=0.63498, α=1029.5, β=37.437, γ=-37.463	(5) 0.05843**	(3) 5.6344**	(5) 48.783**
	Error	k=1.0, σ=0.09555, μ=0.00332	(7) 0.06576**	(7) 8.0336**	(3) 47.794**
	Laplace	λ=14.801, μ=0.00332	(6) 0.06576**	(6) 8.0336**	(4) 47.794**
Iota	Laplace	λ=17.376, μ=-5.0180E-4	(2) 0.02846	(1) 0.44619	(2) 6.2261
	Johnson SU	γ=-0.07223, δ=1.5072, λ=0.09673, ξ=-0.00628	(3) 0.03837	(3) 1.641	(3) 21.57*
	Burr (4P)	k=0.81856, α=2.2473E+7, β=8.6130E+5, γ=-8.6130E+5	(4) 0.04006	(4) 2.0695	(4) 22.427**
	Dagum (4P)	k=0.80889, α=35.925, β=1.3871, γ=-1.3768	(5) 0.04078	(5) 2.1694	(5) 22.558**
	Error	k=1.0, σ=0.08139, μ=-5.0180E-4	(1) 0.02846	(2) 0.44619	(1) 6.2261

Table A4. Fitted and most likely distributions

This table sets out the probability density functions (PDFs), the cumulative density functions (CDFs) and the respective location, scale and shape parameters corresponding to the distributions fitted or the most likely distributions identified in column (2) of Table 1. A list for all distributions, including those not fitted or not identified as most likely, is available upon request.

Distribution	PDF	CDF	Parameters
Cauchy	$f(x) = \left(\pi\sigma\left(1 + \frac{x-\mu}{\sigma}\right)^2\right)^{-1}$	$F(x) = \frac{1}{\pi} arctan\left(\frac{x-\mu}{\sigma}\right) + 0.5$	σ - scale parameter (σ >0) μ - continuous location parameter
Burr (4P)	$f(x) = \frac{ak\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}}$	$F(x) = 1 - \left(1 + \left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-k}$	k - continuous shape parameter (k >0) α - continuous shape parameter (α >0) β - continuous scale parameter (β >0) γ - continuous location parameter
Dagum (4P)	$f(x) = \frac{ak\left(\frac{x-\gamma}{\beta}\right)^{\alpha k-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}}$	$F(x) = \left(1 + \left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-k}$	k - continuous shape parameter (k >0) α - continuous shape parameter (α >0) β - continuous scale parameter (β >0) γ - continuous location parameter
Error	$f(x) = c_1\sigma^{-1} exp\left(-\lvert c_0 z\rvert^k\right)$	$F(x) = \begin{pmatrix} 0.5\left(1 + \frac{\Gamma_{\lvert c_0 z\rvert^k}(1/k)}{\Gamma(1/k)}\right) x \geq \mu \\ 0.5\left(1 - \frac{\Gamma_{\lvert c_0 z\rvert^k}(1/k)}{\Gamma(1/k)}\right) x < \mu \end{pmatrix}$ Where $c_0 = \left(\frac{\Gamma(3/K)}{\Gamma(1/K)}\right)^{1/2}$ and $z \equiv \frac{x-\mu}{\sigma}$	k - continuous shape parameter (k >0) σ - scale parameter (σ >0) μ - continuous location parameter
Laplace	$f(x) = \frac{\lambda}{2} exp\left(-\lambda\lvert x-\mu\rvert\right)$	$F(x) = \begin{pmatrix} 0.5 exp(-\lambda\lvert x-\mu\rvert)) x \leq \mu \\ 1 - 0.5 exp(-\lambda\lvert x-\mu\rvert) x > \mu \end{pmatrix}$	λ - continuous inverse scale parameter (λ >0) μ - continuous location parameter
Johnson SU	$f(x) = \frac{\delta}{\lambda\sqrt{2\pi}\sqrt{z^2+1}} exp\left(-0.5(\gamma + \delta \ln (z + \sqrt{z^2+1}))^2\right)$	$F(x) = \phi\left(\gamma + \delta \ln (z + \sqrt{z^2+1})\right)$ where $z \equiv \frac{x-\xi}{\lambda}$, $z \equiv \frac{x-\xi}{\lambda}$	γ - continuous shape parameter δ - continuous shape parameter (δ >0) λ - continuous inverse scale parameter (λ >0) ξ - continuous location parameter

and ϕ is the Laplace Integral.