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# Optimal tank sizing and operation of energy-water supply systems in residences

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# Abstract

Energy and water insecurities are global challenges, especially in arid and semi-arid regions. This paper proposes an optimal energy-water nexus management approach in residences using alternative energy and water resources, these alternatives are rainwater harvesting system, greywater recycling system, water storage, and gravity-fed distribution system. The Solving Constraint Integer Programming (SCIP) solver, in MATLAB's OPTI toolbox, is used to solve the multi-objective optimization problem. Simulation results show that the optimal tank sizing and system operations will minimize the capital and operational cost of the system. The solver produced a potable water savings of 20.5% and an energy savings of 62.54%.

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Keywords: Optimal tank sizing; energy-water nexus; energy efficiency; operational optimization.

# 1. Introduction

Energy generation is a water-intensive activity and vice versa [1], therefore energy-water nexus defines the mutual relationship between energy and water [2]. Household is the second largest energy consumer in South Africa after the mining and manufacturing industry, it accounts for 25% of the total energy consumed in the country [3]. Agriculture and households account for 80% of the total water consumption. Studies have shown that less than 15% of the precipitation is used for agricultural purposes and the rest is lost, this is a strong motivation for rainwater harvesting (RWH) system. In comparison with the RWH system, the greywater recycling (GWR) system is a more reliable non-

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potable water alternative that is limited by its high installation and operations cost. The RWHS is considered the most promising solution to water scarcity, however, its reliability can be improved by integrating it with multiple water sources. An integrated RWH and GWR system will complement the limitations of each system to produce a more reliable and resilient system that will ensure water supply all weather, minimise the use of utility water, wastewater disposal and the associated cost. This study combines the RWH, GWR and utility water supply systems to meet indoor and outdoor residential water demands.

The RWH and GWR systems have been investigated independently in [4, 5, 6, 7, 8, 9]. Rooftop rainwater harvesting is preferred to the micro-catchment and in-situ rainwater harvesting techniques because of poor water quality. In [10, 11, 12], greywater (GW) tanks are emptied daily to prevent the formation of bacteria, however, this is a waste of water resource in water stressed regions. The GW holding tank are sized using the daily water balance model, while the RW tank is most sized with monthly data for water balance simulation, reliability curve, dry period and Rippl method [13]. This study introduces an elevated and underground water storage tanks for holding grey and rain water. This is economical because it effectively combines the RW and GW collection, treatment, storage and distribution system.

The objectives of this study are to maximize financial savings by minimizing the size of water storage tanks, utility water consumption and operational cost of the system. To achieve these objectives, an optimal tank sizing and water management is proposed in this study.

# 2. Mathematical model formulation

# 2.1. Schematic model layout



Fig. 1. Schematic layout of the integrated RWH and GWR systems in residences.

Figure 1 shows the schematic diagram of the proposed system. It is made up of rainwater harvesting system, greywater recycling system, pumps, storage tanks, pipes, filters, and the water use-points. Pumps  $P_1$  pumps potable water (PW), while  $P_2$  pumps non-potable water (NPW) to the header tanks  $T_1$  and  $T_2$  respectively. Tank  $T_3$  holds treated NPW. Switches,  $u_1$  and  $u_2$ , control pumps  $P_1$  and  $P_2$ , while  $u_3$  is a unidirectional valve that controls the flow of PW from  $T_1$  to  $T_2$ .

# 2.2. Sub-models

#### 2.2.1. Potable water header tank

This is a cylindrical, uniform cross-sectional area tank that collects, stores and distributes PW to the various usepoints. Assuming a uniform cross-sectional area tank, the state dynamics of the PW in this tank is expressed in terms of the height (m) using water balance model [2], [7], [8]:

$$h_1(j) = h_1(0) + \frac{t_s}{A_1} \sum_{i=1}^{j} (Q_1 u_1(i) - Q_3 u_3(i) - D_{pw}(i)),$$
<sup>(1)</sup>

where  $h_1(j)$  is the instantaneous water level (m) in tank  $T_1$ ,  $A_1$  is the cross-sectional area of the tank  $(m^2)$  and  $A_1h_1$  is the volume of water in the tank  $(m^3)$ .  $h_1(0)$  is the water level at i = 0,  $t_s$  is the sampling period (s), j = 1, ..., N is sampling interval.  $Q_1$  and  $Q_3$  are flowrates  $(m^3/h)$ , and  $D_{pw}$  is the PW demand  $(m^3)$ .

#### 2.2.2. Non-potable water header tank

This is a cylindrical, uniform cross-sectional area tank that collects, stores and distributes NPW to the toilet and lawn for toilet flush and irrigation. The state dynamics of the NPW in this tank is expressed in terms of the height using water balance model [2], [7], [8]:

$$h_{2}(j) = h_{2}(0) + \frac{t_{s}}{A_{2}} \sum_{i=1}^{J} (Q_{2}u_{2}(i) + Q_{3}u_{3}(i) - D_{npw}(i)),$$
<sup>(2)</sup>

where  $h_2(j)$  is the instantaneous water level (m) in tank  $T_2$ ,  $A_2$  is the cross-sectional area  $(m^2)$  of the tank and  $A_2h_2$  is the volume of water in the tank  $(m^3)$ .  $h_2(0)$  is the water level at i = 0,  $D_{npw}$  is the NPW demand  $(m^3)$ ,  $Q_2$  and  $Q_3$  are pump flowrates  $(m^3/h)$ .

#### 2.2.3. Underground NPW holding tank

This cylindrical, uniform cross-sectional area tank collects treated GW and RW from the residential wastewater points and rooftop respectively. Assuming a uniform cross-sectional area tank, the state dynamics of the NPW in this tank is expressed in terms of the height using water balance model [2], [7], [8]:

$$h_{3}(j) = h_{3}(0) + \frac{t_{s}}{A_{3}} \sum_{i=1}^{j} \left( S_{gw}(i) + S_{rw}(i) - D_{ov}(i) - Q_{2}u_{2}(i) \right),$$
(3)

where  $h_3(j)$  is the instantaneous water level (m) in the tank  $T_3$ ,  $A_3$  is the cross-sectional area  $(m^2)$  of the tank and  $A_3h_3$  is the volume of water in the tank  $(m^3)$ .  $h_3(0)$  is the water level at i = 0.  $S_{gw}$  and  $S_{rw}$  are the grey and rainwater supply  $(m^3/h)$ , respectively while  $D_{ov}$  is overflow  $(m^3)$ .  $S_{rw} = KIA_r/1000$ , where K, I and  $A_r$  are the run-off coefficient, rainfall intensity and the area of the roof [7].

#### 2.3. Objective function

The primary objective of this study is cost minimization, which will be achieved through optimal tank sizing and operations of the system. These objectives are formulated as a multi-objective optimization problem given by (4). The

first objective determines the optimal size of the tanks, the second objective minimizes the operational cost of pump  $P_1$ , the third objective simultaneously minimizes the operational cost of pump  $P_2$  and maximizes NPW consumption, while the last objective minimizes the switching frequency of the pumps to minimize maintenance cost due to wear and tear.

$$J = t_s \left\{ w_1 (A_1 H_1 + A_2 H_2) + w_2 \sum_{i=1}^{j} p_e(i) P_1 u_1(i) + w_3 \sum_{i=1}^{j} (p_e(i) P_2 - Q_2) u_2(i) + w_4 \sum_{i=1}^{j} (s_1(i) + s_2(i)) \right\},$$
(4)

where  $\sum_{i=1}^{n} w_i = 1$  is the weighting factors that determine the relative importance of each of the objectives. The optimization horizon is 24 hours and sampling time  $t_s = 1h$ . j = 1, ..., N is the sampling interval, where N is the total number of intervals. In general, the higher the switching frequency of a pump, the higher its maintenance cost. Therefore, this paper uses the Pretorian method to minimize the switching frequency of the pumps. This method introduces an auxiliary variable  $s_i$  represented by a value of 1 whenever the pump transitions from off to on state [11]. The auxiliary variable reduces the switching frequency of the pump by augmenting the adjacent on/off switches. The optimization problem formulated is subject to the following constraints:

$$h_{1}^{\min} \leq h_{1}(0) + \frac{t_{s}}{A_{1}} \sum_{i=1}^{j} \left( Q_{1}u_{1}(i) - Q_{3}u_{3}(i) - D_{pw}(i) \right) \leq H_{1},$$
(5)

$$h_{2}^{\min} \leq h_{2}(0) + \frac{t_{s}}{A_{2}} \sum_{i=1}^{j} \left( Q_{2}u_{2}(i) + Q_{3}u_{3}(i) - D_{npw}(i) \right) \leq H_{2},$$
(6)

$$h_{3}^{\min} \leq h_{3}(0) + \frac{t_{s}}{A_{3}} \sum_{i=1}^{j} \left( S_{gw}(i) + S_{rw}(i) - D_{ov}(i) - Q_{2}u_{2}(i) \right) \leq H_{3},$$
(7)

$$u_1(i) - s_1(i) \le 0,$$
 (8)

$$u_1(i) - u_1(i-1) - s_1(i) \le 0, \tag{9}$$

$$u_2(i) - s_2(i) \le 0, \tag{10}$$

$$u_2(i) - u_2(i-1) - s_2(i) \le 0, \tag{11}$$

$$H_{1,2}^{\min} \le H_1, H_2 \le H_{1,2}^{\max}, \tag{12}$$

 $u_m \in \{0,1\}$ , where m = 1, 2 and 3, (13)

$$s_1, s_2 \in \{0, 1\}.$$
 (14)

Inequalities (5), (6) and (7) are the state constraints for the water levels in tanks  $T_1$ ,  $T_2$  and  $T_3$  respectively. Inequalities (8) and (10) initialize the auxiliary variables,  $s_1$  and  $s_2$ , as the initial statuses of  $u_1$  and  $u_2$ . Inequalities (9) and (11) minimises the switching frequency of the pumps, equation (13) is the boundary constraint of the tanks, while equation (13) and (14) are the constraints for the binary control variables,  $u_1, u_2, u_3, s_1$  and  $s_2$ , respectively.

# 2.4. Case study

The case study is a family of four residing in a residence located in Durban, KwaZulu Natal province of South Africa. Durban is in the south-eastern part of South Africa, it has a humid sub-tropical climate and an annual rainfall of 1,009mm. The residence has a roof area of  $100m^2$ , irrigation area of  $150m^2$  and electricity consumption subject to ESKOM's home-flex time-of-use (TOU) tariff structure. The properties of the pumps are  $P_1 = P_2 = 700 W$  and flowrates,  $Q_1 = Q_2 = 750 m^3/h$ . The PW, NPW demand and GW supply ( $S_{gw}$ ) profile of the residential consumer is shown in figure 2.

ESKOM's time-of-use tariff is given in [2]:

$$p_e(t) = \begin{cases} p_{off} = 0.6281 \, R/Kwh, \, if \, t \in [0, 6] \cup [10, 18] \cup [20, 24]; \\ p_{peak} = 1.9935 \, R/Kwh, \, if \, t \in [7, 10] \cup [18, 20]. \end{cases}$$
(15)



Fig. 2. Hourly water demand profile of the residence.

# 3. Simulation results and discussion

The linear optimization problem is solved with the Solving Constraint Integer Programming (SCIP) solver, which is currently one of the fastest non-commercial solvers found in MATLAB's OPTI toolbox [2].

#### 3.1. Optimal switching operation of the pumps and control valves

Figure 3 shows the relationship between the optimal operation of the pumps and the water level in the tanks. It also shows the optimal size of tanks  $T_1$  and  $T_2$ . Figure 3(a) shows the mode of operation of the switches, it is seen that the solver uses the information from the time-of-use tariff to optimize the operation of the switches by operating them at off-peak periods.  $u_1$  is operated from 03:00 to 07:00,  $u_2$  is operated from 16:00 to 17:00 and  $u_3$  is operated from 03:00 to 04:00.

Figure 3(b) shows the tank sizes and water levels in tank  $T_1$ ,  $T_2$  and  $T_3$ . The water level in these tanks change with the operation of the switches.  $h_1$  increases when  $u_1$  is active and decreases when  $u_2$  and/ or  $u_3$  is switched on. Similarly,  $h_2$  increases when  $u_2$  and/ or  $u_3$  is switched on and decreases when there is NPW demand.  $h_3$  rises when treated RW and GW are collected into tank  $T_3$ , however it starts decreasing when  $u_2$  is switched on. Figure 3(b) shows that the optimal size of the tanks set by the solver are  $H_1 = 0.5022 m$  and  $H_2 = 0.2490 m$ .

Table 1. Floperties of the header talks	Table 1.	Properties	of the	header	tanks
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	Radius (m)	$h_i^{min}$ (m)	Optimal height (m)	Capacity (Liters)
Tank $T_1$	0.4	0.1	0.5022	252.5
Tank $T_2$	0.4	0.1	0.2490	125



Fig. 3. The relationship between the (a) switching of the pumps and (b) the water levels in the storage tanks.

# 4. Conclusion

This paper presents an optimal energy-water control strategy in residences. The simulation produced a PW savings of 20.5% and an energy cost savings of 62.54%. The simulation results show that effective energy and water utilization in residences can be achieved with optimal tank sizing and operational optimization of the combined system. Future work will include energy savings and the economic analysis of the proposed system.

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