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Inflation Aversion and the Growth-Inflation Relationship

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This paper re-examines the theoretical relationship between inflation and output growth by introducing inflation aversion in a standard overlapping generations monetary endogenous growth model with productive public expenditure. We show that when the time preference parameter is a negative function of expected money growth rate, then the theoretical growth-inflation relationship that emerges is "hump-shaped". This finding is consistent with recent empirical literature.

Key Words: Inflation; Inflation Aversion; Endogenous Growth.
JEL Classification Numbers: E31, O42.

1. INTRODUCTION

Recent empirical literature tends to suggest that growth is non-linearly related to inflation (Khan and Senhadji, 2001; Vaona and Schiavo 2007; Omay and Kan, 2010). More specifically, Omay and Kan (2010), López-Villavicencio and Mignon (2011), Vaona (2012) and Omay et al. (2018) detected a smooth inverted-U growth-inflation relationship. There are a number of endogenous growth models of money that attempt to explain the relationship between growth and inflation. The money-in-utility (MIU) model does not produce growth effects of inflation. In cash-in-advance (CIA) models, the same result holds with fixed labour supply, but effect is negative when labour-leisure choice is allowed. However, if there are two goods, one of which can be purchased using cash and while the other can be bought on credit, inflation could be growth-enhancing in a CIA model. In CIA models, the impact of inflation on growth is either positive or negative. The inverted-U-shaped relationship between growth and inflation is realized in the endogenous growth version of New Keynesian models, where results depend on elasticity of labour supply or between intermediate

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inputs (Vaona, 2012; Chen, 2015) and with the CIA constraint in a R&D-based model of endogenous growth with heterogeneous abilities (Arawatari et al. 2018).

In this paper, we re-examine the theoretical relationship between inflation and output growth by introducing inflation aversion in a standard overlapping generations monetary endogenous growth model with productive public expenditure. Our model, as in Wang and Zou (2011a), Wang and Zou (2011b) and Zou et al. (2011), assumes that the higher the expected inflation (captured by expected money growth rate in our case, given the endogeneity of the growth process in our model), the lower is the time preference (subjective discount) factor such that higher expected inflation leads the young-age consumer to be less patient, and hence attach lesser value to future consumption relative to the current. The theoretical growth-inflation relationship that is generated by our model is an inverted-U, that relates the impact of inflation on output growth as inflation becomes higher. This is a (an) novel (alternative) theoretical explanation to the extensively investigated relationship between growth and inflation. This result is consistent with recent empirical literature.

The remainder of the paper proceeds as follows: Section 2 outlines the model's economic setting, presents our theoretical model with the optimisation solutions and lays out the balanced growth path. Section 3 offers concluding remarks.

2. ECONOMIC SETTING

The economy is made up of four principal agents, namely two-period lived consumers, financial intermediaries (banks), firms and an infinitely-lived productive government (central bank). There is an infinite sequence of discrete time periods $t = 1, 2, \ldots$. At time t, there are two co-existing generations of young-age, born in time t and old-age, born in time $t-1$. At each time $t \geq 1$, N people are born and at $t = 1$, there exist N people, the initial old, who live for only one period. The population, N, is normalized to one.

There are four principal economic activities: (i) At time t, the young-age consumers supply their labour endowment to firms inelastically at a market wage income. Part of the wage income earned is consumed in time t while the rest is deposited into banks for future consumption, that is, in time $t + 1$. Consumers have access to one aggregate consumption good produced in time t and they derive utility from the consumption of part of this good in each of the two-periods of their lives. For the sake of simplicity and without any bearing on our theoretical results, we assume that the young-age consumer's wage income is not taxed by the government. Consumers are also averse to expected inflation, that is, they discount the future more strongly if they are expecting higher inflation during old-age; (ii) The financial intermediaries sector operates competitively by pooling resources in the form of deposits collected from consumers and lending out funds to firms after meeting the mandatory cash reserve requirements that are administered by the government; (iii) Firms are identical, infinitely-lived and use the same production technology to produce a single final good from the inelastically supplied labour, physical capital borrowed from banks and productive public expenditure. The representative firm maximizes its discounted streams of profit flows subject to the constraints it faces; (iv) Government meets its productive expenditure by generating seigniorage. Two main government policy instruments are: money growth rate and the cash reserve requirement. Government balances its budget on a period-by-period basis. Without loss of generality, we assume that there is a continuum of each type of economic agent with unit mass.

2.1. Consumers

Consumers have the same preferences, hence in each period there is a representative agent. The representative young-age consumer supplies its endowment of time inelastically, n_t, to earn a real wage, w_t. In period t, part of the wage income is consumed, c_t, while the rest is saved and deposited with the bank, d_t. In period $t+1$, the period t young-age consumer is old, retired and consumes c_{t+1} from the total investment of young-age savings. Formally, the representative young-age consumer's problem is to [1]:

$$\max\left[\log(c_t) + \rho(E_t(\mu_{t+1}))\log(c_{t+1})\right] \tag{1}$$

subject to:

$$p_t c_t + p_t d_t = p_t w_t \tag{2}$$

$$p_{t+1} c_{t+1} = (1 + i_{dt+1})\, p_t d_t \tag{3}$$

where ρ is the discount factor or time preference parameter that indicates how much the young-age consumer discounts old-age consumption relative to young-age consumption. Since our model investigates the growth-inflation relationship in the face of inflation aversion, we postulate that ρ is a function of conditionally expected, at t, gross money growth rate (μ) at time $t+1$, that is, $\rho = \rho(E_t(\mu_{t+1}))$, with $\rho'(\mu) < 0$ and $\rho''(\mu) > 0$. This relationship between inflation aversion and the discount factor is also followed in Wang and Zou (2011a), Wang and Zou (2011b) and Zou et al. (2011). d_t are real deposits and $1 + i_{dt+1}$ is the gross nominal interest rate on bank deposits in period $t+1$. p_t and p_{t+1} are the price levels in

[1] Optimisation solutions for the different economic agents are fully set out in the Appendix.

periods t and $t+1$, respectively. The feasibility constraint is presented by (2) (first-period budget constraint) and is for the young-age consumer, and (3) is the budget constraint of the old-age consumer.

2.2. Banks

The economy is populated by a finite number of banks operating in a competitive environment. Banks are, however, subjected to a mandatory cash reserve requirement, γ_t, which is set and superintended over by government. The cost of operating a bank is assumed to be zero and that bank deposits are one-period contracts. These assumptions guarantee that all banks levy the same nominal loan rate, i_{lt}, and that the depositor earns the same nominal deposit rate, i_{dt}. Banks maximize profits by pooling deposits and then deciding on the level of loans to extend and the required cash reserves to hold in order to meet the reserve requirements. Banks receive interest income, $i_{lt}L_t$, from loans to consumers and subsequently meet their deposit rate obligations, $i_{dt}D_t$, to consumers. The constraint to the balance sheet is the obligatory reserve requirement, $(1-\gamma_t)\,D_t = L_t$. Specifically, all banks attempt to:

$$\max \Pi_{Bt} = i_{lt}L_t - i_{dt}D_t \tag{4}$$

s.t:

$$M_t + L_t \ \le \ D_t \tag{5}$$
$$M_t \ \ge \ \gamma_t D_t \tag{6}$$

with $\prod_{Bt}$ the bank's net profit function; L_t are nominal loans extended to consumers while D_t are nominal bank deposits. M_t represents cash reserves held by banks to meet the reserve requirement. (5) and (6) are the feasibility and reserve requirement constraints. Given that (5) and (6) are binding, the solution to the bank's problem is expressed as:

$$i_{lt} = \frac{i_{dt}}{1-\gamma_t} \tag{7}$$

Going by (7), it is clear that the cash reserve requirement induces a wedge between i_{lt} and i_{dt}. M_t is rate-of-return dominated by loans, hence (5) will be binding as banks will hold just enough real money balances to satisfy the mandatory reserve requirements.

2.3. Firms

The economy is characterized by infinitely-lived identical firms that each produce a single final good, y_t, using the same Barro (1990)-type production technology. The representative firm employs physical capital, k_t, labour, n_t, and economy-wide average capital, g_t, to produce the single good, such that:

$$y_t = Ak_t^{\alpha}\left(n_t g_t\right)^{1-\alpha} \tag{8}$$

where $A > 0$ is a technology parameter, $0 < \alpha(1-\alpha) < 1$ is the elasticity of output with respect to capital and labour or productive government expenditure, respectively. At time t, the final good can either be allocated for consumption, c_t, or stored. The constraint to the firms' investment in physical capital, i_{k_t}, is the availability of funding to the firms, in the form of bank loans. This emanates from our assumption that firms have the capacity to convert bank loans, L_t, into fixed capital formation such that $p_t i_{k_t} = L_t$. The representative firm therefore maximizes its discounted stream of net profits subject to the evolution of two constraints: capital and bank loans.

Formally, the firm's problem is to

$$\max_{k_{t+1},n_t} \sum_{i=0}^{\infty} \beta^i \left[p_t y_t - p_t w_t n_t - (1+i_{lt})L_t\right] \tag{9}$$

subject to

$$k_{t+1} \leq (1-\delta_k)k_t + i_{kt} \tag{10}$$

$$p_t i_{kt} \leq L_t \tag{11}$$

$$L_t \leq (1-\gamma_t)D_t \tag{12}$$

where β is the firm owners' (constant) discount rate and δ_k is the (constant) rate of capital depreciation. The firm solves the following recursive problem in order to determine the demand for labour and investment:

$$V(k_t) = \max\left[p_t Ak_t^{\alpha}\left(n_t g_t\right)^{1-\alpha} - p_t w_t n_t - p_t(1+i_{lt})(k_{t+1}-(1-\delta_k)k_t) + \beta V(k_{t+1})\right] \tag{13}$$

yielding the following first order conditions:

$$n_t : w_t = (1-\alpha)A\left(\frac{k_t}{n_t}\right)^{\alpha} g_t^{1-\alpha} \tag{14}$$

(14) represents the firm's optimal hiring decision. The firm will hire labour up to a point whereby the marginal product of labour is equal to the real wage.

$$k_{t+1} : (1+i_{lt}) = \beta\left(\frac{p_{t+1}}{p_t}\right)\left[\alpha A\left(\frac{n_{t+1}g_{t+1}}{k_{t+1}}\right)^{1-\alpha} + (1+i_{lt+1})(1-\delta_k)\right] \tag{15}$$

(15) is an efficiency condition that provides for the optimal investment decisions of the firm. The firm compares the cost of increasing investment in the current period with the future stream of benefits generated from the additional capital invested in the current period. We assume full depreciation of capital between periods such that $\delta_k = 1$, hence, without any loss of generality, (15) simplifies to

$$(1 + i_{lt}) = \beta \left(\frac{p_{t+1}}{p_t}\right) \left[\alpha A \left(\frac{n_{t+1} g_{t+1}}{k_{t+1}}\right)^{1-\alpha}\right] \tag{16}$$

2.4. Government

The consolidated government, assumed to be infinitely-lived, purchases g_t units of consumption goods. Government expenditure is assumed to be a productive factor in the firms' production function. Government's productive consumption expenditure is wholly financed by seigniorage (inflation tax). Formally, the government's budget constraint at time t, in real (per-capita) terms, is:

$$g_t = \frac{M_t - M_{t-1}}{p_t} \tag{17}$$

with $M_t = \mu_t M_{t-1}$, where $\mu > 1$ is the gross growth rate of money. The government coordinates operations of treasury and the central bank, both of which serve the government's interests. The consolidated government has two tools of monetary policy: the mandatory reserve requirements and the growth of money supply. Government, through the central bank, raises revenue through seigniorage and manages its expenditures through treasury. Given that $M_t = \mu_t M_{t-1}$ and $m_t = \gamma_t d_t$ from (6), the government's budget constraint, in real terms, can be expressed as

$$g_t = \gamma_t d_t \left(1 - \frac{1}{\mu_t}\right) \tag{18}$$

2.5. Equilibrium

This economy's competitive equilibrium is characterised as a sequence of prices $\{p_t, i_{lt}, i_{dt}\}_{t=0}^{\infty}$, allocations $\{c_t, c_{t+1}, n_t, i_{kt}\}_{t=0}^{\infty}$, stocks of financial assets $\{m_t, d_t\}_{t=0}^{\infty}$, and policy variables $\{\gamma_t, \mu_t, g_t\}_{t=0}^{\infty}$ such that:

- The consumer maximizes utility given by (1) subject to (2) and (3);
- Banks maximize profits, taking i_{lt}, i_{dt} and γ_t as given such that (7) holds;
- The real allocations solve the firm's date t profit maximization problem, given prices and policy variables, such that (14) and (15) holds;

• The money market equilibrium conditions: $m_t = \gamma_t d_t$ is satisfied for all $t \geqslant 0$, givent that $1 + i_{lt+1} > \frac{p_t}{p_{t+1}}$ $(= \frac{1}{\Pi_{t+1}})$;

• The loanable funds market equilibrium condition: $i_{lt} = \frac{i_{dt}}{(1-\gamma_t)}$ where the total supply of loans $l_t = (1 - \gamma_t)d_t$ is satisfied for all $t \geqslant 0$;

• The goods market equilibrium condition: $c_t + i_{kt} + g_t = Ak_t^{\alpha}(n_t g_t)^{1-\alpha}$ is satisfied for all $t > 0$;

• The labour market equilibrium condition: $(n_t)^d = 1$ for all $t > 0$;

• The government budget constraint in (17) is balanced on a period-by-period basis;

• d_t, p_t, i_{lt}, i_{dt} and A are positive for all $t > 0$.

2.6. The Balanced Growth Path

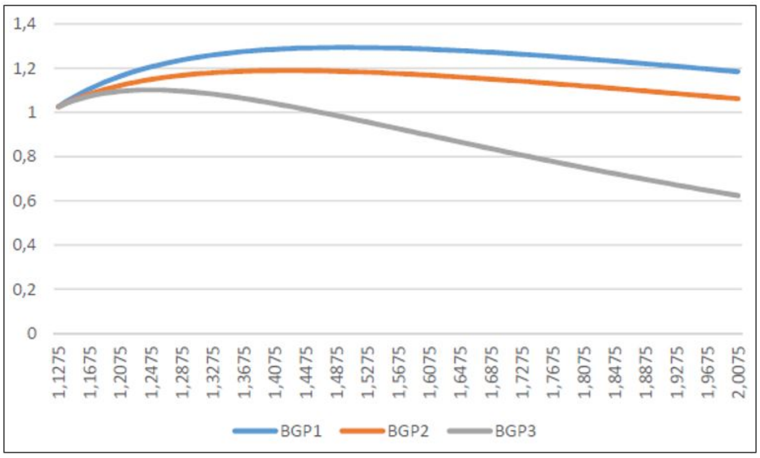
Without any loss of generality, setting $\delta_k = 1$, and constant policy parameters of $\mu_t = \mu$ and $\gamma_t = \gamma$ set by the government for all t, the gross balanced growth rate, Ω, under a perfect foresight equilibrium, i.e., $E_t(\mu_{t+1}) = \mu$, is expressed as:

$$\Omega = A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}(1-\gamma)^{\frac{1}{\alpha}}\gamma^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho(\mu)}{1+\rho(\mu)}\right)^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu}\right)^{\frac{1-\alpha}{\alpha}} \tag{19}$$

As predicted by (19), there are two competing effects on growth. On one hand, growth in money supply is associated with an increase in productive public expenditure, whose impact on growth is positive. On the other hand, the growth in money supply, which proxies inflation since the gross growth rate of money is tied with the gross rate of inflation ($\Pi_{t+1} = \frac{p_{t+1}}{p_t}$) in the steady-state given the gross growth rate of the economy (i.e., $\Omega\Pi = \mu$), reduces bank deposits through a higher rate of discounting of (lower weight on) future consumption, and hence, negatively impacts the growth of the economy. The positive effect dominates initially, but as the money growth rate becomes higher and crosses a threshold, the negative impact kicks-in, producing the hump-shaped relationship between growth and inflation. In this model, the threshold effect depends upon the degree of inflation aversion and the weight of public expenditure in the production function. Understandably, the more inflation averse the agent and the lower the elasticity of output with respect to the government expenditure, the sooner the threshold will be reached. We quantitatively indicate these points in Figure 1, based on a numerical simulation.

We calibrate the model to produce a steady-state growth rate of 2.5% (in line with World Bank figures) for an inflation rate of 10%, i.e., for $\mu = 1.1275$, with $\gamma = 0.25$, $\alpha = 0.70$, $\rho = \frac{1}{\mu}$, which requires a value of $A = 25.72$. This Balanced Growth Path, i.e., BGP1 has been plotted in

FIG. 1. Relationship between Growth and Inflation



Note: Vertical Axis: Gross Growth Rate (Ω); Horizontal Axis: Gross Money Growth Rate (μ); BGP2 is balanced growth path (BGP) with relatively higher weight on productive public expenditure than BGP1; BGP3 is BGP with relatively higher inflation aversion than BGP1.

Figure 1 as the value of μ increases, and as can be seen, BGP1 is hump-shaped in line with our theoretical prediction. In BGP2, we increase α to 0.75, so now $(1-\alpha) = 0.25$, i.e., the weight on public expenditure in the production function is reduced. Note that to produce a growth rate of 2.5% again for $\mu = 1.1275$, we require $A = 26.23$, given all the other parameter settings. As can be observed from Figure 1, higher money growth rate produces a lower BGP, i.e., BGP2, with the turning-point of the inverted u-shaped curve reached at a lower value of μ (approximately at $\mu^* = 1.43$) compared to that under the BGP1 (which was approximately at $\mu^* = 1.50$). Understandably, a relatively lower value of the elasticity of substitution of public expenditure with respect to output, causes the negative impact of inflation aversion to take over earlier. Finally, we increase the degree of inflation aversion, so that $\rho = \frac{1}{\mu^2}$, which in turn, given all the original other parameter settings under BGP1, requires a value of $A = 27.46$ to produce a growth rate of 2.5% for an inflation rate of 10%. As shown in Figure 1 by BGP3, the strong inflation aversion sharply reduces the growth rate as money growth rate increases, with BGP3 falling below BGP1, and reaching a turning point much earlier (at an approximate value of $\mu^* = 1.29$). This is understandable, given that the parameter defining the degree of inflation aversion in the discount factor has doubled.

The numerical experiments conducted in Figure 1, confirm that the relationship between growth and inflation is a smooth non-linear function, which initially increases and then falls beyond a certain threshold value of money growth rate. In addition, this optimal value is contingent on the elasticity of substitution of public expenditure with respect to output and the degree of inflation aversion, with a decrease in the former and an increase in the latter resulting in a lower inflection point in terms of the money growth rate. At this stage we must emphasize that the objective of this numerical exercise is to convey the theoretical predictions of our model graphically, without making any effort to match long-run averages of real data corresponding to a country or countries.

3. CONCLUSION

We re-examine the theoretical inflation-growth nexus by incorporating inflation aversion into the representative agent's time preference parameter in a standard overlapping generations monetary endogenous growth model with productive public expenditure. By assuming that the time preference (subjective discount) factor is a negative function of expected inflation, the higher is the expected money growth rate, the more impatient the young agent is, which in turn leads to a lesser weight on future consumption relative to the current. In the process, inflation aversion, in the wake of increased money growth rate, reduces savings and hence growth. But at the same time higher money growth rates increases economic growth rate via an increase in productive public expenditures financed through seigniorage. Given this, the theoretical growth-inflation relationship that is generated by our model is an inverted-U due to the two competing effects. As a result, our paper provides a novel way of producing an empirically consistent hump-shaped growth-inflation relationship, via the incorporation of inflation aversion in an otherwise standard monetary endogenous growth model with productive public expenditure.

APPENDIX A

A.1. OPTIMISATION SOLUTIONS FOR ECONOMIC AGENTS

Note that from the solution to the consumer's problem, we have:

$$d_t = \frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))} w_t \tag{A.1}$$

The bank's solution follows directly from its net profit function, and the fact that (5) and (6) holds. We also obtain, from substituting (6) into (5), the following

$$l_t = (1-\gamma_t)d_t \tag{A.2}$$

Recall the firm's optimisation problem, in recursive form:

$$V(k_t) = \max\left[p_t A k_t^{\alpha}\left(n_t g_t\right)^{1-\alpha} - p_t w_t n_t - p_t(1+i_{lt})(k_{t+1}-(1-\delta_k)k_t) + \rho V(k_{t+1})\right] \tag{A.3}$$

which yields the following first order conditions (FOC):

$$n_t : w_t = (1-\alpha)A\left(\frac{k_t}{n_t}\right)g_t^{1-\alpha} \tag{A.4}$$

$$k_t : p_t(1+i_{it}) = \rho V'(k_{t+1}) \tag{A.5}$$

with the solution to the FOC for k_{t+1} found in the derivative of the value function with respect to k_t, updated for one period. Formally:

$$V'(k_{t+1}) = p_{t+1}\alpha A\left(\frac{n_{t+1}g_{t+1}}{k_{t+1}}\right)^{1-\alpha} + (1+i_{lt+1})(1-\delta_k) \tag{A.6}$$

which results in (15). Substituting $\delta_k = 1$ and n_{t+1} into (15), yields

$$(1+i_{lt}) = \rho\left(\frac{p_{t+1}}{p_t}\right)\left[\alpha A\left(\frac{n_{t+1}g_{t+1}}{k_{t+1}}\right)^{1-\alpha}\right] \tag{A.7}$$

A.2. DERIVATION OF THE BALANCED GROWTH PATH (BGP) OF GROSS GROWTH RATE

Note that from the solution to the consumer's problem, we have, in real terms:

$$d_t = \frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}w_t \tag{A.8}$$

and from the solution of the banks' problem, we have

$$l_t = (1-\gamma_t)d_t \tag{A.9}$$

From (11), we have

$$l_t = i_{kt} \tag{A.10}$$

Given the assumption that capital fully depreciates between periods such that $\delta = 1$, then (10) reduces to

$$k_{t+1} = i_{kt} \tag{A.11}$$

such that (A.10) can then be expressed as

$$l_t = k_{t+1} \tag{A.12}$$

We can also express (A.12) as

$$k_{t+1} = (1 - \gamma_t)d_t \tag{A.13}$$

Given that $d_t = \frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))} w_t$, we have

$$k_{t+1} = (1 - \gamma_t)\frac{\rho(E_t(\mu_{t+1}))}{1 + \rho(E_t(\mu_{t+1}))} w_t \tag{A.14}$$

From (14), we have $w_t = (1 - \alpha)A\left(\frac{k_t}{n_t}\right)^{\alpha} g_t^{1-\alpha}$ and in equilibrium, $n_t = 1$ such that

$$w_t = (1 - \alpha)Ak_t^{\alpha} g_t^{1-\alpha} \tag{A.15}$$

Thus, (A.14) can then be expressed as:

$$k_{t+1} = (1 - \gamma_t)\frac{\rho(E_t(\mu_{t+1}))}{1 + \rho(E_t(\mu_{t+1}))}(1 - \alpha)Ak_t^{\alpha} g_t^{1-\alpha} \tag{A.16}$$

From the government's budget constraint in (17), and that in equilibrium, $\delta_k = 1$ and $n_t = 1$ we have:

$$g_t = \frac{M_t - M_{t-1}}{p_t} \tag{A.17}$$

We have that $M_t = \mu_t M_{t-1}$ such that (A.17) can be expressed as

$$\begin{aligned} &= m_t - \frac{M_{t-1}}{M_t}\frac{M_t}{p_t} \\ &= m_t - \frac{1}{\mu_t} m_t \\ &= m_t \left(1 - \frac{1}{\mu_t}\right) \end{aligned}$$

From (6), we have $m_t = \gamma_t d_t$ such that

$$g_t = \gamma_t d_t \left(1 - \frac{1}{\mu_t}\right) \tag{A.18}$$

and given that from (A.1) we have $d_t = \frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))} w_t$ and using (14) twice and that in equilibrium, $n_t = 1$ we can express (A.18) as

$$g_t = \gamma_t(1-\alpha)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))} A k_t^{\alpha} g_t^{1-\alpha}\left(1-\frac{1}{\mu_t}\right) \tag{A.19}$$

which simplifies to

$$g_t = \left(\gamma_t(1-\alpha)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))} A k_t^{\alpha}\left(1-\frac{1}{\mu_t}\right)\right)^{\frac{1}{\alpha}} \tag{A.20}$$

Plugging this expression for g_t back into (A.16), we have

$$\begin{aligned} k_{t+1} &= (1-\gamma_t)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}(1-\alpha)Ak_t^{\alpha}\left[\left(\gamma_t(1-\alpha)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}Ak_t^{\alpha}\left(1-\frac{1}{\mu_t}\right)\right)^{\frac{1}{\alpha}}\right]^{1-\alpha} \\ &= (1-\gamma_t)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}(1-\alpha)Ak_t^{\alpha}\left[\gamma_t(1-\alpha)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}Ak_t^{\alpha}\left(1-\frac{1}{\mu_t}\right)\right]^{\frac{1-\alpha}{\alpha}} \\ &= (1-\gamma_t)\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}(1-\alpha)Ak_t^{\alpha}\gamma_t^{\frac{1-\alpha}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}\right)^{\frac{1-\alpha}{\alpha}} \\ &\times A^{\frac{1-\alpha}{\alpha}}k_t^{1-\alpha}\left(1-\frac{1}{\mu_t}\right)^{\frac{1-\alpha}{\alpha}} \end{aligned} \tag{A.21}$$

This simplifies to

$$k_{t+1} = A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}(1-\gamma_t)\gamma_t^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}\right)^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu_t}\right)^{\frac{1-\alpha}{\alpha}} k_t \tag{A.22}$$

and dividing both sides of (A.22) by k_t, we have a growth path given by

$$\Omega_{t+1} = A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}(1-\gamma_t)\gamma_t^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho(E_t(\mu_{t+1}))}{1+\rho(E_t(\mu_{t+1}))}\right)^{\frac{1}{\alpha}}\left(1-\frac{1}{\mu_t}\right)^{\frac{1-\alpha}{\alpha}} \tag{A.23}$$

REFERENCES

Arawatari, R., T. Hori, and K. Mino, 2018. On the nonlinear relationship between inflation and growth: A theoretical exposition. *Journal of Monetary Economics* **94**, 79–93.

Barro, R. J., 1990. Government spending in a simple model of endogeneous growth. *Journal of political economy* **98(5, Part 2)**, S103–S125.

Chen, S.-H., 2015. Fiscal and monetary policies in a transactions-based endogenous growth model with imperfect competition. *The Japanese Economic Review* **66(1)**, 89–111.

Khan, M. S. and A. S. Senhadji, 2001. Threshold effects in the relationship between inflation and growth. *IMF Staff papers* **48(1)**, 1–21.

López-Villavicencio, A. and V. Mignon, 2011. On the impact of inflation on output growth: Does the level of inflation matter? *Journal of macroeconomics* **33(3)**, 455–464.

Omay, T. and E. Kan, 2010. Re-examining the threshold effects in the inflation–growth nexus with cross-sectionally dependent non-linear panel: Evidence from six industrialized economies. *Economic Modelling* **27(5)**, 996–1005.

Omay, T., R. Van Eyden, and R. Gupta, 2018. Inflation–growth nexus: evidence from a pooled cce multiple-regime panel smooth transition model. *Empirical Economics* **54(3)**, 913–944.

Vaona, A., 2012. Inflation and growth in the long run: A new keynesian theory and further semiparametric evidence. *Macroeconomic Dynamics* **16(1)**, 94–132.

Vaona, A. and S. Schiavo, 2007. Nonparametric and semiparametric evidence on the long-run effects of inflation on growth. *Economics Letters* **94(3)**, 452–458.

Wang, G. and H. F. Zou, 2011a. Inflation aversion and macroeconomic policy in a perfect foresight monetary model. *Economic Modelling* **28(4)**, 1802–1807.

Wang, G. and H. F. Zou, 2011b. Inflation aversion and the optimal inflation tax. *Annals of Economics and Finance* **12(1)**, 13–30.

Zou, H. F., L. Gong, and X. Zeng, 2011. Inflation aversion. *Annals of Economics and Finance* **12(1)**, 1–11.