Volatility forecasting with bivariate multifractal models

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Abstract

This paper examines volatility linkages and forecasting for stock and foreign exchange markets from a novel perspective by utilizing a bivariate Markov-switching multifractal model that accounts for possible interactions between stock and foreign exchange markets. Examining daily data from major advanced and emerging nations, we show that generalized autoregressive conditional heteroskedasticity models generally offer superior volatility forecasts for short horizons, particularly for foreign exchange returns in advanced markets. Multifractal models, on the other hand, offer significant improvements for longer horizons, consistently across most markets. Finally, the bivariate multifractal model provides superior forecasts compared to the univariate alternative in most advanced markets and more consistently for currency returns, while its benefits are limited in the case of emerging markets.

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1 Introduction

Forecasting volatility in financial markets is not only critical for portfolio selection, risk management and the pricing of derivatives, but is also of high importance for market regulators as volatility shocks can have significant effects on asset prices. Consequently, a large strand of the literature has focused on modeling volatility dynamics in financial markets in the presence of statistical anomalies including fat tails, volatility jumps, and other nonlinearities in return series. Motivated by the renewed interest in understanding the nature of information transmissions across financial markets post the financial crisis, our study examines volatility linkages and forecasting for stock and currency markets from a novel perspective by utilizing multifractal models within a Markov-switching framework. This approach allows us to account for some of the well-documented statistical anomalies including persistence, long memory and structural changes in volatility, and thus provides a parsimonious framework for forecasting volatility dynamics in the presence of these statistical anomalies. The proposed model also accounts for possible causal effects across the stock and currency markets via a bivariate specification and provides a novel approach to volatility forecasting when compared to the models employed in the literature that are primarily based on univariate specifications.

Our empirical analysis focuses on the stock and currency markets as volatility in these markets, as witnessed during the recent "Great Recession", have wide repercussions on the economy as a whole via its effect on real economic activity and public confidence. Hence, volatility forecasting models geared towards currency and equity markets can provide signals regarding the vulnerability of the economy in general, and can, in turn, help policy makers design appropriate policies to mitigate possible negative effects of volatility shocks on the financial system. What is important to highlight at this stage is that movements in these two markets cannot be (rather should not be) viewed as independent of each other. In fact, the theoretical underpinnings linking the stock and currency markets can be derived from two widely used models: the flow oriented model (Dornbusch & Fischer, 1980) and the stock oriented model (Branson and Henderson 1985; Frankel 1983). Given these considerations, our forecasting application that is based on a bivariate specification focuses on the stock and currency market returns from major developed and emerging economies. Specifically, we analyze daily data from the equity and currency markets of the G6 (Canada, France, Germany, Italy, Japan and United Kingdom (UK)) and the BRICS (Brazil, Russia, India, China and South Africa) and examine the out-of-sample performance of univariate and multivariate volatility models across these advanced and emerging markets that dominate the market value and trading activity globally. The comparative analysis of these economies also allows us to examine whether the performance of alternative volatility models relates to the level of development in the underlying market.

This study offers two major contributions to the literature. Despite the multitude of studies on volatility forecasting, the literature utilizes primarily univariate specifications in forecasting exercises (Pilbeam and Langeland, 2015; Kourtis et al., 2016). However, given the overwhelming evidence (and theories) of causal relationships across the stock and currency markets, interestingly, the literature has largely ignored the ability of exchange rate volatility in forecasting stock market volatility out-of-sample and vice versa. Therefore, the first contribution of this study is to account for possible information spillovers across the stock and currency markets and integrate the information from both markets within a forecasting model that is based on a bivariate specification. If causality across these two market is indeed present, one can then argue that a multivariate specification should be able to outperform univariate counterparts. To that end, our bivariate approach allows to evaluate if exchange rate (stock market) volatility helps in forecasting the volatility of the stock market (exchange rate) and this paper provides the initial evidence in that regard.

As a second consideration, statistical anomalies in volatility dynamics including long memory, volatility clustering and structural changes in the volatility process have been well-documented in the literature. It can be argued that accounting for these statistical anomalies in forecasting exercises can improve the accuracy of forecasts. To that end, the Markov-switching multifractal (MSM) model employed in this study offers a parsimonious framework to address these statistical anomalies. In essence, the MSM is a model of asset returns that incorporates stochastic volatility components of heterogeneous durations. This model is thus able to capture outliers, long-memory, and volatility persistence. Therefore, the second contribution of this study is that it utilizes, for the first time, a bivariate MSM framework to model jointly, and then forecast, the exchange rate and stock market volatility of the G6 and BRICS countries. We then compare the performance of the bivariate MSM specification with univariate MSM and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) alternatives. Thus, our contribution is not only empirical but also methodological.

Our findings suggest that multifractal models can indeed offer significant improvements to volatility forecasting, however the improvement is not homogeneous across markets. We generally observe that the GARCH model offers superior out-of-sample forecasting performance for short horizons including 1, 5 and 10 days, particularly for advanced markets. The superior (short-horizon) forecasting performance of the GARCH model for developed markets is particularly evident in the case of currency markets that tend to exhibit stronger fluctuations relative to the stock markets. On the other hand, multifractal models generally offer potential improvements over the GARCH alternative in the case of long-term forecasts, consistently across most markets. We also find that the bivariate multifractal model that accounts for possible interactions between currency and stock markets can offer improvements over the univariate alternative. Our findings show that the bivariate model improves forecasting in most G6 countries and more consistently in their foreign exchange markets, while for stock markets, the advantage of the bivariate model is more limited. Similarly, we find that the bivariate model offers limited improvements for BRICS countries as well (with the exception of Brazil). Overall, our findings suggest that multifractal models can indeed offer significant improvement in volatility forecasting exercises, particularly for longer horizons, while the bivariate multifractal model can extend the superior performance to shorter horizons as well.

The remainder of the paper is organized as follows: Section 2 provides the background on stock and currency markets and describes the data used in our analysis. Section 3 presents the details of the MSM model, while Section 4 discusses the empirical findings. Finally, Section 5 concludes the paper.

2 Background and Data Description

The foreign exchange market is the largest and most liquid financial market in the world. The Triennial Central Bank Survey of global foreign exchange markets performed by the Bank for International Settlements reports that trading in foreign exchange markets averaged 5.1 trillion U.S. dollars in April of 2016, with the turnover dominated by the U.S. dollars, followed by the Euro and Japanese Yen. Currency markets tend to be volatile and, with traders reacting to new information, exhibit periods of volatility clustering. Clearly, from a practical perspetive, accurate forecasting of exchange rate volatility is important to multinational firms, financial institutions and traders aiming to hedge currency risks (Balcilar et al., 2016). Volatility in the currency market is also closely followed by traders of foreign currency options who look to make profits by buying (selling) options if they expect volatility to rise above (fall below) what is implied by currency option premiums. On a macro scale, currencies have even become a barometer for investors' perception of political risks and economic development, particularly in emerging markets where currency fluctuations are closely related to geopolitical developments. Consequently, a large body of theoretical research has linked exchange rate volatility to trade and welfare (Clark et al., 2004; Rapach and Strauss, 2008). Similarly, a vast methodological and empirical literature exists around the development, assessment and application of exchange rate volatility forecasts.[1]

In the same vein, there also exists a large literature on forecasting equity market volatility.[2] Again, as with the currency market, as pointed out by Poon and Granger (2003) and Rapach et al. (2008), appropriate modeling and forecasting of volatility in stock markets is of importance due to several reasons: (i) when volatility is interpreted as uncertainty, it becomes a key input to investment de-

[1] See Pilbeam and Langeland (2015) and Balcilar et al. (2016) for detailed reviews.

[2] See for example, Kambouroudis et al. (2016) and Kourtis et al. (2016) for relevant reviews of the literature.

cisions and portfolio choices; (ii) volatility is the most important variable in the pricing of derivative securities; and (iii) financial risk management according to the Basle Accord as established in 1996 also requires modeling and forecasting of volatility as a compulsory input to risk-management for financial institutions around the world.

Several theories exist in the literature establishing a link between currency and stock markets. According to the traditional or flow-orientated theory, international trade (current account) is affected by exchange rate changes. As Dornbusch and Fischer (1980) note, changes in the exchange rate against other currencies will affect a country's international competitive advantage, real income, and output. The traditional theory postulates that depreciation of the domestic currency improves the competitiveness of local firms, leading to an increase in their exports and future cash flows, which consequently has a positive effect on stock market returns. From another perspective, as Gavin (1989) argues, shocks to the stock market can affect the aggregate demand through wealth and liquidity effects and influence money demand. Clearly, the domestic stock market plays an important role in influencing the capital in or outflows. For example, a decrease in the stock market index causes reduction in investors' wealth, leading to lower demand for money with lower interest rates, thus discouraging capital inflows. Consequently, such chain reactions can cause currency depreciation. Thus, it can be argued that a bidirectional relationship exists between the stock market and exchange rate movements.

The portfolio or stock-oriented approach, on the other hand, argues the presence of a negative relationship between stock prices and the foreign exchange rate. According to this approach, since the value of financial assets are determined by the present values of their future cash flows, expectation of relative currency values plays a significant role in their future cash flows. Thus, the stock price innovations may affect or be affected by exchange rate dynamics. The stock-oriented model argues that exchange rates are affected by stock price movements via the capital account (Frankel, 1987). This is the result of stock market movements leading to money flow into or out of the domestic economy, which affects the demand for money, hence leading to changes in interest rates and exchange rate movements.

Given these two theoretical models, a large empirical literature exists on the relationship between stock and currency market returns with applications to both developed and emerging markets (see for example, Adler and Dumas, 1984; Booth and Rotenberg, 1990; Jorion, 1990; Sercu and Vanhulle, 1992; Smith 1992; Bodnar and Gentry, 1993; Amihud, 1994 and Inci and Lee, 2014; Bahmani-Oskooee and Saha, 2016; Sui and Sun, 2016).[3] Similarly, there is also a vast literature on the dynamic linkages between equity and currency market volatilities (see for example, Jorion, 1990; Roll, 1992; Dumas and Solnik, 1995; Kanas, 2000; Caporale et al., 2002; Yang and Doong, 2004; Mishra et al., 2007; Walid et al., 2011; Kang and Yoon, 2013; Wang et al., 2013; Valls and Chuli, 2014; Ho and Huang, 2015; Fernndez-Rodrguez and Sosvilla-Rivero,

[3]The reader is referred to Bahmani-Oskooee and Saha (2015) for a detailed review.

2016).[4] While the evidence on the direction and sign of causality in returns and volatilities across these two markets is mixed, with the results contingent upon sample periods, countries chosen and methodologies employed; the literature provides robust evidence of spillovers for both returns and volatility across these two markets irrespective of whether one looks at advanced or developing economies.

Given the in-sample evidence of a causal link between stock and exchange rate returns, numerous studies have analysed with success the role of exchange rate returns in forecasting stock returns (Rapach et al., 2005; Gupta and Modise, 2013; Gupta et al., 2016; Sousa et al., 2016), and stock returns in forecasting exchange rate returns (Wright, 2008; Tortora, 2010; De Bruyn et al., 2015; Cenedese et al., 2016; Byrne et al., forthcoming) over out-of-sample periods for both developed and developing markets. However, despite the robust evidence of causality across stock and currency markets, volatility forecasting models utilized in the literature are primarily based on univariate specifications (e.g. Pilbeam and Langeland, 2015; Kourtis et al., 2016) that ignore important informational spillovers due to causality across the two markets. To the best of our knowledge, there are no studies that have looked into the ability of exchange rate volatility in forecasting stock market volatility out-of-sample and vice versa. Given the evidence of in-sample volatility spillovers across these two markets and the importance of volatility forecasts, we aim to fill this void in the literature. It is quite well-known that in-sample predictability does not necessarily translate into out-of-sample forecasting gains, and it is an out-of-sample exercise that provides the acid test for determining the predictive ability of variables and models (Campbell, 2008).

As another consideration in volatility forecasting exercises, the recent literature provides evidence of several statistical anomalies in volatility dynamics that may also affect the accuracy of forecasts, if ignored. For example, several recent studies document evidence of long memory in the conditional volatility of various financial and economic time series (see Ben Nasr et al., 2010; 2014; 2016 for detailed reviews). Similarly, other studies including Rapach and Strauss (2008), Rapach et al. (2008), Gil-Alana et al. (2014, 2016) and Yaya et al. (2015) report evidence of structural changes in the volatility process. Further, a related line of research on long memory and structural changes in volatility relates these phenomena to switching of regimes in the volatility process as first suggested by Diebold (1986) and Lamoureux and Lastrapes (1990). As Kellard et al. (2015), Lux et al. (2016), Segnon et al. (forthcoming) note, it could be very difficult to distinguish between true and spurious long memory processes. Therefore, a second contribution of our study is to address these statistical anomalies within a Markow-Switching multifractal model. This model is more parsimonious in parameterization than other regime-switching models and is well-known to give rise to apparent long memory over a bounded interval of lags (Calvet and Fisher, 2004). More importantly, it has limiting cases in which it converges to a "true" long memory process. In short, this study offers several contributions to the

[4]The reader is referred to Jebran and Iqbal (2016) for a detailed review.

volatility forecasting literature by addressing the causal relationships between stock and currency markets and the well-documented statistical anomalies that may affect the out-of-sample performance of forecasting models.

2.1 Data

Our empirical analysis focuses on G6 (Canada, France, Germany, Italy, Japan, UK) and BRICS (Brazil, Russia, India, China and South Africa) countries as their financial markets account for a significant percentage of market value and trading activity globally. Daily data for exchange rates to US dollars and stock market indices for these countries are obtained from Datastream. The sample size differs across countries due to data availability.[5]. We separate each time series into two subsets, i.e. in-sample data used for estimation and out-of-sample data for forecast assessment, as described in Table 1.

Daily returns are calculated as the log difference $r_t = 100 \times [\ln(p_t) - \ln(p_{t-1})]$, where p_t represents the daily values for the stock market indices or foreign exchange rates. Table 2 reports the descriptive statistics of stock market index and exchange rate returns. We observe higher mean returns and volatility for both the stock and currency markets in the case of emerging BRICS countries compared to the developed G6 countries. Within the BRICS group, Russia is the most volatile economy in both currency and stock market returns. In the case of the developed markets, the Canadian stock and currency markets experience the lowest mean returns and volatility. Examining the higher moments, we see that most countries in the sample, with the exception of Brazil and Russia, exhibit negative skewness in their returns suggesting greater likelihood of experiencing losses in these markets. Similarly, all of the countries have considerable positive kurtosis, in particular the currency returns for Russia and China exhibit extremely high kurtosis values. The high kurtosis values in all return series imply the presence of extreme movements in both the stock market and currency returns, providing hints for nonlinear return dynamics in these markets.

3 Multifractal Models

In this section, we provide a brief description of the mutltifractal (MF) model utilized in our volatility forecasting exercises. Mandelbrot et al. (1997) first introduced the multifractal apparatus into finance, adapting the approach of Mandelbrot (1974) to an asset-pricing framework. This multifractal model of asset returns (MMAR) assumes that asset returns r_t follow a compound process, in which an incremental fractional Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, the practical applicability of MMAR suffers from the non-causal nature of the time transformation and non-stationarity due to the inherent restriction to a bounded

[5]For European countries, their US exchange rates after 1999 were calculated according to Datastream synthetic Euro exchange rates.

interval. These limitations have been overcome by the development of an iterative version of the MF models, including the Markov-switching multifractal model (MSM), cf. Calvet and Fisher (2004) and Lux (2008). In this approach, asset returns are modeled as

$$r_t = \sigma \left(\prod_{i=1}^{k} M_t^{(i)} \right)^{1/2} \cdot u_t, \tag{1}$$

with u_t drawn from a standard Normal distribution $N(0,1)$ and instantaneous volatility being determined by the product of k volatility components or multipliers, $M_t^{(1)}$, $M_t^{(2)}$ ..., $M_t^{(k)}$, and a constant scale parameter σ. Each volatility component is renewed at time t with probability γ_i, depending on its rank within the hierarchy of multipliers, or remains unchanged with probability $1 - \gamma_i$. Calvet and Fisher (2004) propose to specify transition probabilities as

$$\gamma_i = 1 - (1 - \gamma_1)^{(b^{i-1})}, \tag{2}$$

with parameters $\gamma_1 \in (0,1)$ and $b \in (1, \infty)$; while Lux (2008) assumes $\gamma_i = 2^{(k-i)}$. Both specifications guarantee convergence of the discrete-time multifractal process to a limiting continuous-time version with random renewals of the multipliers.

This rather parsimonious approach allows us to preserve the hierarchical structure of MMAR while dispensing with its restriction to a bounded interval. While this model is asymptotically "well-behaved" (i.e. it shares all the convenient properties of Markov-switching processes) it is still capable of capturing several important properties of financial time series including volatility clustering and the power-law behavior of the autocovariance function of absolute moments

$$Cov(|r_t|^q, |r_{t+\tau}|^q) \propto \tau^{2d(q)-1}. \tag{3}$$

The Markov-switching MF model is rather characterized by only 'apparent' long-memory with an approximately hyperbolic decline of the autocorrelation of absolute powers over a finite horizon and exponential decline thereafter. In particular, approximately hyperbolic decline as expressed in eq. (3) holds only over an interval $1 \ll \tau \ll b^k$ with b the parameter of the transition probabilities of Eq. (2) and k the number of hierarchical levels.

In order to study the interactions and comovements among financial assets, the multifractal models can be easily extended to a multivatiate setting without imposing much restrictions such as a bivariate specification. For two financial return series $r_{n,t}$ (for $n = 1, 2$), and assuming that instantaneous volatility is composed of heterogeneous frequencies, the bivariate model of asset returns r_t can be specified as

$$r_t = \sigma. * [g(M_t)]^{1/2} . * u_t. \tag{4}$$

Here, r_t, σ, and u_t are all bivariate vectors: $r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix}$, $\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$, $u_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}$, and $.*$ denotes element by element multiplication. σ is the vector of constant scale parameters (the unconditional standard deviation); u_t is a 2×1 vector whose elements follow a bivariate standard Normal distribution with an unknown correlation parameter ρ, and $g(M_t)$ is the vector of the products of multifractal volatility components, i.e.

$$g(M_t) = \begin{bmatrix} g(M_{1,t}) \\ g(M_{2,t}) \end{bmatrix} \tag{5}$$

where each $g(M_{q,t})$ is defined, as in the univariate case, as the product of the volatility components for series n

$$g(M_{q,t}) = \prod_{i=1}^{k} M_{n,t}^{(i)}, \tag{6}$$

with $M_{n,t}^{(i)}$ denoting the volatility component at frequency i of series n

$$M_t^{(i)} = \begin{bmatrix} M_{1,t}^{(i)} \\ M_{2,t}^{(i)} \end{bmatrix}. \tag{7}$$

In this specification, $M_t^{(i)}$ are drawn from a bivariate Binomial distribution $M = (M_1, \;\; M_2)'$, with M_1 taking values $m_1 \in (1, 2)$ and $2 - m_1$, and M_2 taking values $m_2 \in (1, 2)$ and $2 - m_2$. While the framework by Calvet et al. (2006) allows for variation of the correlation (ρ_m) between components M_1 and M_2, they report that a correlation ρ_m equal to one is never rejected in their empirical applications. We, therefore, restrict this parameter to unity to economize on the number of parameters to be estimated.

Finally, whether or not certain volatility components (new arrivals) are updated for the individual MF processes is governed by the transition probabilities γ_i, which are specified as in the univariate version, cf. Eq. (2). The correlation of arrivals between the two series is characterized by a parameter $\lambda \in [0, 1]$, i.e., the probability of a new arrival at hierarchy level i for one time series given a new arrival in the other time series is $(1-\lambda)\gamma_i + \lambda$. New arrivals are independent if $\lambda = 0$ and simultaneous if $\lambda = 1$.

4 Empirical Findings

For our empirical study, we focus on the out-of-sample assessment of volatility forecasts and conduct comparisons of GARCH and univariate and bivariate multifractal models. Table 3 presents the in-sample GARCH(1, 1) estimates for the stock indices (st) and exchange rates (ex); and Table 4 reports the in-sample univariate and bivariate multifractal model estimates for the stock indices (st) and exchange rates (ex). Note that the vector of multifractal model parameters consists of $\{m_{1,st}, m_{1,ex}, \sigma_{st}, \sigma_{ex}, \rho, \lambda\}$.

We adopt the two-stage procedure proposed by Calvet et al. (2006), which combines an ML estimator for the first group of parameters $\{m_{1,st}, m_{1,ex}, \sigma_{st}, \sigma_{ex}\}$ with an SML estimator for the second group $\{\rho$ and $\lambda\}$. The latter are obtained through a particle filter approach, keeping the first set of parameters at their ML-estimated values. The two-stage approach allows to reduce computation time compared to the complete SML approach. The first four of these parameters could be identified by an estimator for a univariate multifractal model, while the remaining ones require the complete bivariate data set.

We observe that in terms of fractality of volatility as measured by the parameters $m_{1,st}$ and $m_{1,ex}$ reported in Table 4, the foreign exchange markets exhibit somewhat stronger fluctuations than the stock markets for most countries (except for Canada, France and Germany). In terms of correlation of innovations ρ, most countries exhibit negative and somewhat weak correlations between their stock and foreign exchange markets, implying that these market segments are not necessarily driven by common fundamental uncertainties. The only exception is France, Germany and Italy for which we observe a relatively more pronounced degree of co-movement between the stock and currency markets. Considering that these three countries are the only G6 countries in the sample that use the Euro as their currency, it is possible that being in the Eurozone contributes to a stronger causal relationship between currency and stock markets. We also observe that the high correlation across the stock and currency markets often pertains to volatility arrivals λ, which appear quite plausible.

4.1 Out-of-sample analysis

Tables 5 to Table 7 present the performance metrics for out-of-sample forecasts from the GARCH,[6] univariate and bivariate MF models, respectively. We consider alternative time horizons ranging from 1 day to 100 days. We report in the tables the relative mean square error (MSE) and the relative mean absolute error (MAE) computed by dividing the MSE and MAE estimates by the pertinent MSE and MAE of the naive volatility predictor (using historical volatility). For ease of comparison, in Table 6, † indicates an improvement of the univariate MF model against the GARCH model at 5% level. Similarly, in Table 7, ♯ indicates an improvement of the bivariate MF model against the GARCH model at 5% level and * indicates an improvement of the bivariate MF model against the univariate MF model at 5% level.[7]

[6] We have also conducted out-of-sample forecasts based on the bivariate DCC-GARCH, we did not report the results, which are very similar with the univariate GARCH ones. The results are available upon request.

[7] Comparisons are based on the test statistics of Diebold and Mariano (1995).

Our results are not entirely homogeneous, but are quite encouraging. Let us first compare the forecast performances based on the GARCH and the univariate multifractal models reported in Tables 5 and Table 6, respectively. Examining the MSE values, we observe that the GARCH model generally produces better forecasts in most short-term horizons, i.e. 1, 5 and 10 days, which is particularly evident in the case of the developed countries. The superior short-horizon forecast performance of the GARCH model for developed markets is consistent particularly in the case of currency markets that tend to exhibit stronger fluctuations relative to the stock markets. Although the same observation generally holds for emerging markets as well, we see that the multifractal model can provide improvement in forecasts across both the short and long horizons for several emerging markets including India, China and South Africa. Overall, based on the MSE criteria, we conclude that the multifractal models generally offer potential improvement over the GARCH alternative in the case of long-term forecasts. On the other hand, examining the findings based on the MAE criterion, we observe that the Diebold and Mariano (1995) test for the differences in predictive accuracy rejects the GARCH model in favor of the univariate multifractal model at most forecast horizons and for almost all countries in the sample. The only exception to this is currency markets for France, Italy, UK, Brazil and Russia where the GARCH model shows superior forecasting performance consistently at all horizons. In short, the comparison of the univariate MF model with the GARCH alternative yields quite encouraging results, suggesting that multifractal models can provide improvement over volatility forecasts, particularly in longer horizons.

Next, we turn our attention to the bivariate multifractal model which accounts for possible interactions between currency and stock markets. Table 7 reports the relative MSE and MAE values for the volatility forecasts based on the bivariate multifractal model. As mentioned earlier, $^\sharp$ indicates an improvement of the bivariate MF model against the GARCH model at 5% level and * indicates an improvement of the bivariate MF model against the univariate model at 5% level. We generally confirm the earlier observations from Table 6 that the GARCH model provides relatively better forecasts at shorter horizons, but multifractal models offer more accurate predictions for longer horizons according to the MSE criterion. Once again, consistent with the earlier results, we observe that multifractal models outperform the GARCH model in most cases according to MAE across all forecast horizons.

Finally, comparing the performance of the univariate and bivariate multifractal models in Table 7, based on the MSE criterion, we observe that the bivariate model improves forecasting in most G6 countries (Canada, France Germany and Japan) and more consistently in their currency markets, while for stock markets, improvements are observed only for France, Germany and UK at relatively short horizons. On the other hand, the bivariate model appears to offer limited improvements for BRICS countries with the exception of Brazil. Similarly, examining the forecasting performance of the bivariate MF model according to the MAE criterion, we observe similar mixed results. Generally, the bivariate model provides better predictions than the univariate alternative particularly in the case of Japan and Brazil across both the stock and foreign exchange markets, while a mixed pattern of improvements is observed in either the stock market or currency returns depending on the market of focus. Overall, our findings suggest that multifractal models can offer significant improvement in out-of-sample volatility forecasts over the GARCH specification, particularly for longer horizons, while the bivariate multifractal model can extend the superior performance to shorter horizons as well.

5 Conclusion

Volatility forecasting has significant implications for portfolio diversification, risk management and the valuation of derivatives. Consequently, a large literature exists on the performance of various volatility forecasting models with applications to numerous financial markets and asset classes. This paper contributes to the literature by utilizing, for the first time, a bivariate Markov-switching multifractal specification to model jointly, and then forecast, the exchange rate and stock market volatility in the G6 (Canada, France, Germany, Italy, Japan, UK) and BRICS (Brazil, Russia, India, China and South Africa) countries. The Markov-switching multifractal model employed in this study offers a parsimonious framework to address the possible interactions between stock and foreign exchange markets and also accounts for some of the well-known statistical anomalies in volatility dynamics including long memory, volatility clustering and structural changes in the volatility process. By doing so, this paper contributes to the literature from both empirical and methodological perspectives.

The analysis of the out-of-sample performance of alternative volatility models shows that the GARCH model generally offers superior volatility forecasts for short horizons, particularly for currency returns in advanced markets. Multifractal models, on the other hand, offer significant improvements for longer forecast horizons, consistently across most markets. Comparing the performance of the univariate and bivariate multifractal models, we observe that the bivariate MF model provides superior forecasts compared to the univariate alternative in most G6 countries and more consistently for currency returns, while its benefits are limited in the case of emerging markets. Overall, our findings suggest that multifractal models can indeed offer significant improvement in out-of-sample volatility forecasts, particularly for longer horizons. However, the benefits from multifractal models over the univariate counterparts are not uniform across different markets and asset classes.

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Tables and Figures

Table 1: Sample periods for G6 and BRICS stock and currency markets.

	in sample	out of sample
S&P/TSX 60 index CANADIAN $ to US$	31/12/1993 - 31/12/2006	1/01/2007 - 19/08/2016
France CAC 40 index FRENCH FRANC to US$	31/12/1993 - 31/12/2006	1/01/2007 - 19/08/2016
German DAX GERMAN MARK to US$	31/12/1993 - 31/12/2006	1/01/2007 - 19/08/2016
Italy FTSE MIB ITALIAN LIRA TO US$	1/01/1998 - 31/05/2009	1/06/2009 - 19/08/2016
Japan Nikkie Japanese Yen to US$	1/03/1973 - 31/12/1996	1/01/1997 - 21/07/2016
UK FTSE100 British Pound to US$	3/01/1984 - 31/12/2000	1/01/2001- 21/07/2016
Brazil Bovespa Index BRAZILIAN REAL TO US$	1/07/1994 - 28/02/2007	1/03/2007 - 19/08/2016
Russian MICEX RUSSIAN ROUBLE TO US$	22/09/1997 - 31/12/2007	1/01/2008 - 19/08/2016
Indian (BSE) - Sensex INDIAN RUPEE TO US$	31/12/1993 - 31/12/2006	1/01/2007 - 19/08/2016
China SSE Composite Index CHINESE YUAN TO US$	1/07/1996 - 31/05/2007	1/06/2007 - 19/08/2016
South Africa FTSE/JSE SOUTH AFRICA RAND TO US$	30/06/1995 - 31/05/2007	1/06/2007 -19/08/2016

Table 2: Descriptive statistics

	Canada		France		Germany		Italy		Japan		UK	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
Mean	0.021	0.000	0.011	0.000	0.026	0.000	-0.008	-0.001	0.011	-0.009	0.022	-0.001
S.D	1.049	0.527	1.420	0.609	1.465	0.616	1.574	0.62	1.338	0.668	1.094	0.632
Skewness	-0.709	-0.126	-0.055	-0.210	-0.148	-0.192	-0.201	-0.182	-0.348	-0.407	-0.476	-0.369
Kurtosis	12.989	9.419	7.562	5.452	7.386	5.413	7.572	5.436	12.160	8.667	12.479	10.219
Min	-9.788	-5.046	-9.472	-4.617	-8.875	-4.616	-13.331	-4.617	-16.135	-5.630	-13.029	-8.169
Max	9.37	4.338	10.595	3.844	10.797	3.841	10.877	3.844	13.235	6.256	9.384	4.589

	Brazil		Russia		India		China		South Africa	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
Mean	0.048	0.02	0.060	0.049	0.038	0.013	0.031	-0.004	0.041	0.022
S.D	2.123	0.975	2.615	1.661	1.513	0.364	1.432	0.088	1.225	1.004
Skewness	0.433	0.363	0.105	5.14	-0.222	0.459	-0.109	-0.796	-0.460	0.253
Kurtosis	16.450	20.514	19.149	303.357	9.417	13.900	9.321	107.797	9.219	9.11
Min	-17.229	-11.778	-23.336	-35.808	-11.936	-3.064	-12.975	-2.031	-12.690	-8.523
Max	28.818	10.801	27.501	48.25	15.49	3.512	13.471	1.81	7.423	9.808

Note: This table reports the descriptive statistics for G6 and BRICS countries foreign exchange rate and stock market index returns. ST and EX denote stock indices and foreign exchange rates, respectively.

Table 3: GARCH(1,1) model estimates

	Canada		France		Germany		Italy		Japan		UK	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
ω	0.008	0.001	0.011	0.003	0.019	0.004	0.017	0.002	0.019	0.002	0.022	0.004
	(0.002)	(0.000)	(0.003)	(0.001)	(0.004)	(0.001)	(0.004)	(0.001)	(0.003)	(0.000)	(0.005)	(0.001)
β	0.905	0.956	0.931	0.964)	0.903)	0.960	0.892	0.966	0.830	0.950	0.901	0.946
	(0.010)	(0.005)	(0.007)	(0.006)	(0.009)	(0.007)	(0.009)	(0.004)	(0.012)	(0.005)	(0.013)	(0.007)
α	0.088	0.041	0.062	0.027	0.087	0.027	0.102	0.029	0.169	0.048	0.075	0.046
	(0.010)	(0.005)	(0.007)	(0.004)	(0.009)	(0.004)	(0.009)	(0.003)	(0.013)	(0.005)	(0.010)	(0.006)
μ	0.056	0.002	0.056	-0.008	0.072	-0.007	0.040	-0.018	0.051	-0.002	0.034	0.004
	(0.012)	(0.005)	(0.017)	(0.009)	(0.017)	(0.009)	(0.0169)	(0.0096)	(0.009)	(0.007)	(0.012)	(0.008)

	Brazil		Russian		India		China		South Africa	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
ω	0.140	0.018	0.136	0.001	0.054	0.001	0.018	0.001	0.018	0.005
	(0.032)	(0.002)	(0.026)	(0.000)	(0.012)	(0.000)	(0.006)	(0.000)	(0.005)	(0.001)
β	0.852	0.788	0.857	0.809	0.863	0.731	0.946	0.660	0.891	0.879
	(0.018)	(0.019)	(0.015)	(0.012)	(0.015)	(0.018)	(0.009)	(0.062)	(0.013)	(0.009)
α	0.121	0.196	0.131	0.189	0.118	0.267	0.044	0.195	0.100	0.120
	(0.014)	(0.021)	(0.015)	(0.012)	(0.013)	(0.021)	(0.007)	(0.047)	(0.012)	(0.011)
μ	0.149	0.011	0.182	0.007	0.069	-0.002	0.083	-0.002	0.079	0.016
	(0.030)	(0.008)	(0.038)	(0.003)	(0.020)	(0.002)	(0.022)	(0.001)	(0.016)	(0.009)

Note: This table reports the GARCH(1, 1) estimates for foreign exchange rate (EX) and stock index (ST) returns for G6 and BRICS countries.

Table 4: Multifractal models estimates

	Canada		France		Germany		Italy		Japan		UK	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
m_1	1.288	1.249	1.245	1.194	1.277	1.187	1.304	1.531	1.353	1.568	1.248	1.277
	(0.013)	(0.012)	(0.011)	(0.014)	(0.011)	(0.014)	(0.013)	(0.0171)	(0.010)	(0.021)	(0.005)	(0.013)
σ	0.912	0.378	1.331	0.563	1.432	0.587	1.736	0.292	1.106	1.296	1.077	0.648
	(0.041)	(0.015)	(0.049)	(0.023)	(0.056)	(0.024)	(0.019)	(0.031)	(0.046)	(0.055)	(0.013)	(0.030)
ρ	-0.078		0.302		0.275		-0.303		-0.027		-0.115	
	(0.011)		(0.013)		(0.008)		(0.013)		(0.008)		(0.016)	
λ	0.028		0.262		0.287		0.129		0.074		0.146	
	(0.007)		(0.011)		(0.014)		(0.013)		(0.010)		(0.006)	

	Brazil		Russian		India		China		South Africa	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
m_1	1.317	1.607	1.295	1.548	1.316	1.637	1.339	1.621	1.308	1.553
	(0.012)	(0.012)	(0.026)	(0.017)	(0.022)	(0.020)	(0.016)	(0.021)	(0.035)	(0.021)
σ	1.493	0.196	1.637	0.212	1.535	0.160	1.493	0.047	1.568	0.025
	(0.118)	(0.034)	(0.025)	(0.016)	(0.025)	(0.018)	(0.019)	(0.012)	(0.033)	(0.011)
ρ	-0.090		-0.163		-0.087		-0.016		-0.102	
	(0.012)		(0.010)		(0.014)		(0.013)		(0.018)	
λ	0.075		0.088		0.106		0.118		0.105	
	(0.010)		(0.014)		(0.009)		(0.013)		(0.007)	

Note: This table reports the in-sample univariate and bivariate multifractal model estimates for foreign exchange rate (EX) and stock index (ST) returns for G6 and BRICS countries. The estimation is based on the two-step procedure of Calvet et al. (2006). In the first step, m_1 and σ estimates for stock indices and exchange rates are obtained by maximizing the univariate likelihood and in the second step, ρ and λ estimates are obtained by maximizing the simulated bivariate likelihood given the estimates from the first step. Standard errors for the second step estimates are computed as in Calvet et al. (2006, Appendix A.4).

Table 5: Volatility forecasts: GARCH model

		Canada		FRANCE		Germany		Italy		Japan		UK	
		ST	EX	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
	MSE												
1		0.729	0.833	0.855	0.917	0.859	0.920	0.952	0.941	0.796	0.938	0.815	0.937
5		0.731	0.841	0.883	0.924	0.883	0.927	0.983	0.943	0.892	0.954	0.824	0.945
10		0.808	0.843	0.910	0.924	0.907	0.927	1.001	0.945	0.951	0.965	0.862	0.952
20		0.850	0.895	0.964	0.932	0.967	0.936	1.008	0.944	1.120	0.982	0.921	0.957
50		1.009	0.961	1.034	0.957	1.034	0.961	1.016	0.956	1.220	0.990	0.982	0.971
100		1.014	0.980	1.041	1.011	1.058	1.018	1.033	0.996	1.232	1.022	0.992	0.993
	MAE												
1		0.971	1.131	1.008	0.990	0.917	0.976	1.052	0.900	1.130	1.074	0.982	0.868
5		0.985	1.129	1.022	0.991	0.926	0.977	1.073	0.906	1.174	1.086	0.994	0.871
10		1.028	1.128	1.044	0.989	0.947	0.975	1.098	0.909	1.208	1.101	1.011	0.873
20		1.044	1.145	1.069	0.994	0.971	0.981	1.099	0.913	1.311	1.122	1.018	0.886
50		1.129	1.148	1.093	1.001	1.008	0.989	1.127	0.932	1.475	1.166	1.018	0.910
100		1.164	1.152	1.065	1.014	1.002	1.002	1.160	0.974	1.663	1.237	1.021	0.951

		Brazil		Russian		India		China		South Africa	
		ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
	MSE										
1		0.754	0.794	0.848	0.748	0.905	0.869	0.899	0.960	0.779	0.890
5		0.742	0.896	0.858	0.943	0.918	0.907	0.943	1.011	0.761	0.931
10		0.789	0.942	0.826	0.891	0.936	1.053	0.962	0.994	0.801	0.993
20		0.860	1.002	0.922	1.000	0.976	1.059	0.979	0.993	0.874	1.028
50		0.956	1.042	1.010	1.040	0.985	1.060	1.000	0.994	0.977	1.094
100		0.977	1.060	1.019	1.023	0.995	1.194	0.993	0.994	0.998	1.124
	MAE										
1		0.678	0.964	0.582	0.305	0.863	1.212	0.892	1.116	0.919	1.074
5		0.697	0.989	0.605	0.320	0.877	1.265	0.903	1.093	0.925	1.095
10		0.731	1.033	0.638	0.318	0.913	1.330	0.917	1.043	0.952	1.120
20		0.788	1.074	0.721	0.343	0.967	1.382	0.930	1.024	0.997	1.143
50		0.886	1.136	0.904	0.374	1.028	1.483	0.955	1.025	1.095	1.230
100		0.936	1.156	1.061	0.377	1.081	1.753	0.948	1.026	1.164	1.337

Note: This table reports the relative MSE and MAE values for the volatility forecasts based on the GARCH(1,1) model of foreign exchange rate (EX) and stock index (ST) returns for G6 and BRICS countries.

Table 6: Volatility forecasts: univariate multifractal model

	Canada		FRANCE		Germany		Italy		Japan		UK	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
MSE												
1	0.842	0.926	0.903	0.982	0.894	0.941	0.926†	0.943	0.932	1.029	0.881	0.951
5	0.864	0.936	0.922	0.974	0.912	0.944	0.944†	0.947	0.978	1.027	0.903	0.938
10	0.883	0.934	0.933	0.976	0.926	0.946	0.961†	0.949	0.988	1.036	0.914	0.954
20	0.898	0.941	0.952†	0.980	0.951†	0.946	0.973†	0.946	1.023†	1.040	0.929	0.958
50	0.934†	0.955†	0.988†	0.984	1.009†	0.952†	0.998†	0.954	1.035†	1.053	0.971†	0.978
100	0.977†	0.961†	1.01†	1.061	1.025†	0.989†	1.02†	0.993	1.015†	1.088	0.984†	1.004
MAE												
1	0.951†	1.016†	0.967†	1.153	0.899†	0.975	1.024†	0.909	0.879†	0.999†	0.953†	0.946
5	0.945†	1.01†	0.963†	1.171	0.895†	0.970	1.034†	0.913	0.901†	1.029†	0.962†	0.946
10	0.966†	1.005†	0.979†	1.161	0.909†	0.967†	1.055†	0.914	0.901†	1.034†	0.971†	0.944
20	0.972†	1.013†	0.997†	1.174	0.925†	0.971†	1.055†	0.914	0.932†	1.044†	0.985†	0.956
50	1.005†	1.019†	1.027†	1.181	0.965†	0.978†	1.088†	0.923†	0.932†	1.053†	1.019	0.968
100	1.045†	1.025†	1.036†	1.240	0.976†	1.009	1.113†	0.949†	0.929†	1.067†	1.033	1.005

	Brazil		Russian		India		China		South Africa	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
MSE										
1	0.786	0.863	0.866	0.606†	0.904	0.873	0.864	0.988	0.834	0.899†
5	0.769	0.893	0.867	0.848†	0.91†	0.878†	0.948	0.994	0.827	0.92†
10	0.790	0.916†	0.869	0.793†	0.914†	0.905†	0.961	0.994	0.837	0.94†
20	0.839†	0.971†	0.906†	0.893†	0.943†	0.905†	0.97†	0.994	0.879	0.971†
50	0.958	1.081	0.978†	0.948†	0.968†	0.906†	0.995	0.994	0.942†	0.993†
100	1.012	1.121	1.007†	0.948†	0.971†	0.954†	0.996	0.992	0.994†	1.013†
MAE										
1	0.695	1.140	0.572†	0.362	0.844†	1.119†	0.879†	0.976†	0.916	1.110
5	0.695	1.151	0.58†	0.418	0.845†	1.123†	0.897†	0.984†	0.911†	1.079†
10	0.705†	1.169	0.587†	0.415	0.85†	1.142†	0.908†	0.987†	0.921†	1.082†
20	0.722†	1.203	0.608†	0.439	0.873†	1.152†	0.92†	0.988†	0.942†	1.09†
50	0.765†	1.262	0.648†	0.472	0.897†	1.151†	0.946†	0.997†	0.986†	1.114†
100	0.798†	1.296	0.665†	0.474	0.908†	1.187†	0.948†	1.007†	1.018†	1.131†

Note: This table reports the relative MSE and MAE values for the volatility forecasts based on the univariate multifractal model of foreign exchange rate (EX) and stock index (ST) returns for G6 and BRICS countries. † indicates an improvement of the univariate MF model against the GARCH model at 5% level. Comparisons are based on the test statistics of Diebold and Mariano (1995).

Table 7: Volatility forecasts: bivariate multifractal model

	Canada		FRANCE		Germany		Italy		Japan		UK	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
MSE												
1	0.845	0.906*	0.891*	0.936*	0.882*	0.935*	0.929$^{\sharp}$	0.945	0.929	0.975*	0.868*	0.943*
5	0.867	0.917*	0.906*	0.939*	0.895*	0.939*	0.947$^{\sharp}$	0.951	0.981	0.98*	0.885*	0.938
10	0.886	0.917*	0.922*	0.938*	0.915*	0.937*	0.962$^{\sharp}$	0.960	0.989	0.986*	0.902*	0.950
20	0.899	0.926*	0.946*$^{\sharp}$	0.942*	0.945*$^{\sharp}$	0.941*	0.975$^{\sharp}$	0.953	1.028$^{\sharp}$	0.993*	0.918*	0.956
50	0.936$^{\sharp}$	0.946*$^{\sharp}$	0.994$^{\sharp}$	0.947*$^{\sharp}$	1.027$^{\sharp}$	0.947*$^{\sharp}$	1.002$^{\sharp}$	0.963	1.039$^{\sharp}$	1.004*	0.973	0.973*
100	0.976$^{\sharp}$	0.954*$^{\sharp}$	1.023$^{\sharp}$	0.992*$^{\sharp}$	1.05$^{\sharp}$	0.993$^{\sharp}$	1.019$^{\sharp}$	1.020	1.019$^{\sharp}$	1.032*	0.989	0.993*
MAE												
1	0.936*$^{\sharp}$	1.015$^{\sharp}$	1.008	1.006*	0.950	0.992	1.006*$^{\sharp}$	0.928	0.86*$^{\sharp}$	0.884*$^{\sharp}$	0.952$^{\sharp}$	0.884*
5	0.93*$^{\sharp}$	1.009$^{\sharp}$	1.003$^{\sharp}$	0.999*	0.937	0.987	1.017*$^{\sharp}$	0.934	0.887*$^{\sharp}$	0.891*$^{\sharp}$	0.961$^{\sharp}$	0.875*
10	0.949*$^{\sharp}$	1.006$^{\sharp}$	1.02$^{\sharp}$	0.996*	0.954	0.984	1.036*$^{\sharp}$	0.937	0.886*$^{\sharp}$	0.892*$^{\sharp}$	0.969$^{\sharp}$	0.872*
20	0.956*$^{\sharp}$	1.016$^{\sharp}$	1.041$^{\sharp}$	1.001*	0.977	0.989	1.037*$^{\sharp}$	0.933	0.919*$^{\sharp}$	0.902*$^{\sharp}$	0.985$^{\sharp}$	0.884*
50	0.988*$^{\sharp}$	1.029$^{\sharp}$	1.077$^{\sharp}$	1.008*	1.025	0.996	1.07*$^{\sharp}$	0.941	0.919*$^{\sharp}$	0.912*$^{\sharp}$	0.993*$^{\sharp}$	0.894*$^{\sharp}$
100	1.026*$^{\sharp}$	1.035$^{\sharp}$	1.093	1.043*	1.042	1.031	1.094*$^{\sharp}$	0.973	0.914*$^{\sharp}$	0.922*$^{\sharp}$	1.011*$^{\sharp}$	0.925*$^{\sharp}$

	Brazil		Russian		India		China		South Africa	
	ST	EX	ST	EX	ST	EX	ST	EX	ST	EX
MSE										
1	0.754*	0.817*	0.861*	0.760	0.902	0.868*	0.863	0.990	0.817*	0.888*$^{\sharp}$
5	0.762*	0.87*$^{\sharp}$	0.862*	0.816$^{\sharp}$	0.909$^{\sharp}$	0.857*$^{\sharp}$	0.945	0.993$^{\sharp}$	0.804*	0.911*$^{\sharp}$
10	0.799	0.899*$^{\sharp}$	0.859*	0.813$^{\sharp}$	0.917$^{\sharp}$	0.903$^{\sharp}$	0.960	0.993$^{\sharp}$	0.825*	0.936*$^{\sharp}$
20	0.854	0.953*$^{\sharp}$	0.902$^{\sharp}$	0.841*$^{\sharp}$	0.943$^{\sharp}$	0.905$^{\sharp}$	0.974	0.992$^{\sharp}$	0.873*$^{\sharp}$	0.973$^{\sharp}$
50	0.967	1.046*	0.979$^{\sharp}$	0.862*$^{\sharp}$	0.971$^{\sharp}$	0.91$^{\sharp}$	0.994	0.993$^{\sharp}$	0.941$^{\sharp}$	0.996$^{\sharp}$
100	1.019	1.088*	1.008$^{\sharp}$	0.868*$^{\sharp}$	0.973$^{\sharp}$	0.952$^{\sharp}$	0.997	0.992$^{\sharp}$	0.996$^{\sharp}$	1.015$^{\sharp}$
MAE										
1	0.684*	1.003*	0.565*	0.447	0.843$^{\sharp}$	1.118$^{\sharp}$	0.881	0.977$^{\sharp}$	0.918	1.088*
5	0.688*$^{\sharp}$	1.014*	0.575*	0.473	0.844$^{\sharp}$	1.122$^{\sharp}$	0.892$^{\sharp}$	0.986$^{\sharp}$	0.912$^{\sharp}$	1.07*$^{\sharp}$
10	0.699*$^{\sharp}$	1.031*	0.584	0.474	0.854$^{\sharp}$	1.143$^{\sharp}$	0.905$^{\sharp}$	0.989$^{\sharp}$	0.922$^{\sharp}$	1.076*$^{\sharp}$
20	0.711*$^{\sharp}$	1.065*$^{\sharp}$	0.605	0.483	0.874$^{\sharp}$	1.149$^{\sharp}$	0.921$^{\sharp}$	0.991$^{\sharp}$	0.948$^{\sharp}$	1.087$^{\sharp}$
50	0.759*$^{\sharp}$	1.118*$^{\sharp}$	0.647	0.497	0.901$^{\sharp}$	1.151$^{\sharp}$	0.949$^{\sharp}$	1.001$^{\sharp}$	1.001$^{\sharp}$	1.112$^{\sharp}$
100	0.792*$^{\sharp}$	1.157*	0.667	0.501	0.91$^{\sharp}$	1.184$^{\sharp}$	0.947	1.012$^{\sharp}$	1.036$^{\sharp}$	1.132$^{\sharp}$

Note: This table reports the relative MSE and MAE values for the volatility forecasts based on the bivariate multifractal model of foreign exchange rate (EX) and stock index (ST) returns for G6 and BRICS countries. $^{\sharp}$ indicates an improvement of the bivariate MF model against the GARCH model at 5% level; * indicates an improvement of the bivariate MF model against the univariate model at 5% level. All comparisons are based on the test statistics of Diebold and Mariano (1995).