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Fixed Point Results for Fractal Generation in Extended Jungck–SP Orbit

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ABSTRACT In this paper, we extend Jungck–SP iteration with s–convexity in second sense and define its orbit. We prove the fixed point results for fractal generation via extended iteration and utilize these results to develop algorithms for fractal visualization. Moreover, we present some complex graphs of Julia and Mandelbrot sets in Jungck–SP orbit with s–convexity. We also present some examples to show the variation in images with involved parameters.

INDEX TERMS Jungck-SP, Complex graphics, Escape radius.

I. INTRODUCTION

If we study the historical background of fractals we observed that the word fractal is almost half-century-old. The field of fractals is also called fractal geometry. Fractals have many applications in our real life. In biology fractals are applied to estimate or observe many biologic or living procedures, phenomena or operations like the growth culture of micro-organism (i.e. bacteria, chlamydomonas and amoeba) and analyze the nerve fibre pattern, etc [1]. In physics fractals are used to determine and understand the turbulent flows in fluid Mechanics. In telecommunication fractals are used to manufacture antennae [2]. Computer networking, radar system and architectural models etc are the main applications of fractals [3]. In electronics and electrical engineering fractals are used to Manufacture radars, capacitors, security control system, radio, antennae for wireless system, transformer and lighting model etc [4]. Fractal geometry presented some beautiful and historic scene of nature, due to this reason fractal geometry achieved an historic importance from last five decades. Researcher developed some mathematical models and sketched beautiful aesthetic patterns of computer graphics via different mathematical techniques. In 1970's Mandelbrot in IBM visualized the complex graphs. He Observed a quite new pattern of graphs for a complex function $f(z) = z^2+c$ [5]. The obtained image was self similar and he named this complex graph a fractal. He visualized the work of G. Julia and discussed the properties of Julia sets [6]. He demonstrated that Julia sets had best richness of artistic patterns. After his work a series of research have been published on different types of fractals. The generalized form of Mandelbrot set was studied in [7]. Some rational, trigonometric, logarithmic and exponential function were studied in [8]. The quaternions, bi-complex and tri-complex function used in ([9], [10]) and in [11] to generate some generalized fractals (i.e. Julia and Mandelbrot sets). The fixed point theory gained the highest concentration when Rani et al. in [12] and [13] used some fixed point iterative methods in the visualization of fractals. They presented some superior fractals and discussed their characteristics. After studies of Rani et al. the use of various iteration procedures (used for an approximate finding of fixed points) in the generation of different types of fractals became very popular. Some types of fractals like Mandelbrot and Julia sets via different explicit iterations studied and generalized in [14], [15], [16], [17], [18] and [19], Iterated Function System fractals studied in [20] and [21], V-variable fractals and super-fractals demonstrated in [21] and [22], inversion fractals discussed in [23] and fractals arising from the root finding methods presented in [24], [25], [26] and

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[27]. The threshold escape radii for Jungck-Mann, Jungck-Ishikawa and Jungck-Noor iterations with the combination of s-convex function in second sense were proved in [28], [29]. The biological resembled images were demonstrated in [30], [31], [32], [33] and [34] and Modified results for Julia sets and Mandelbrot were established in [35]. Recently boundaries of filled Julia sets discussed in [36].

In this paper, we establish some new fixed point results in the generation of fractals (i.e. for Julia and Mandelbrot sets) by using Jungck–SP iteration with s–convexity. We define Jungck SP orbit with s–convexity and prove escape criterion for quadratic, cubic and nth degree complex polynomials. Furthermore, we develop algorithms to visualize Julia and Mandelbrot sets in Jungck–SP orbit with s–convexity. We also present some graphical examples to show the variation in images with involved parameters.

The next four sections of this paper are as follows: In section II we revise some basic definitions and implicit iterations. We prove results of threshold escape radius for Jungck-SP iteration with s-convexity in section III. Some complex graphs of Julia and Mandelbrot sets for different complex polynomials via proposed iteration present in section IV. In last section (i.e. In section V) we conclude this research.

II. PRELIMINARIES

Definition 1 (Julia set [37])**.** Let $f_c : \mathbb{C} \to \mathbb{C}$ be a complex polynomial function depends on $c \in \mathbb{C}$. The filled Julia set F_{f_c} of a function f_c is defined as

$$F_{f_c} = \{z \in \mathbb{C} : |f_c^p(z)| \nrightarrow \infty \text{ as } p \to \infty\}, \quad (1)$$

where $f_c^p(z)$ is the p-th iterate of the function f_c. The Julia set B_{f_c} of the complex polynomial function f_c is defined as the boundary of filled Julia set F_{f_c}, i.e. $B_{f_c} = \partial F_{f_c}$. The boundary of filled Julia set is called simply Julia set.

Definition 2 (Mandelbrot set [38])**.** The set consists of all parameters c, for which the filled Julia set of complex polynomial function $f_c : \mathbb{C} \to \mathbb{C}$ connected is called Mandelbrot set M, i.e.,

$$M = \{c \in \mathbb{C} : F_{f_c} \text{ is connected}\}, \quad (2)$$

or we can define Mandelbrot set equivalently as follows [39]:

$$M = \{c \in \mathbb{C} : |f_c^p(\theta)| \nrightarrow \infty \text{ as } p \to \infty\}, \quad (3)$$

where θ is any critical point of f (i.e., $f'(\theta) = 0$). Here, we choose θ as an initial point.

Definition 3 (Jungck iteration [40])**.** Let $P, Q : X \to X$ be the two maps such that P is one to one and Q is differentiable of degree greater or equal to 2. For any $x_0 \in X$ the Jungck iteration is defined in the following way

$$P(x_{k+1}) = Q(x_k), \quad (4)$$

where $k = 0, 1, \ldots$.

Definition 4 (Jungck-Mann iteration with s-convex combination in second sense [28])**.** Let $P, Q : \mathbb{C} \to \mathbb{C}$ be the two complex maps such that Q is a complex polynomial of degree greater or equal to 2, also differentiable and P is one to one. For any $x_0 \in \mathbb{C}$ the Jungck-Mann iteration with s-convex combination in second sense is defined as:

$$P(x_{k+1}) = (1-a)^s Q(x_k) + a^s P(x_k), \quad (5)$$

where $a, s \in (0, 1]$, $k = 0, 1, 2, \ldots$.

Definition 5 (Jungck-Ishikawa iteration with s-convex combination in second sense [28])**.** Let $P, Q : \mathbb{C} \to \mathbb{C}$ be the two complex maps such that Q is a complex polynomial of degree greater or equal to 2, also differentiable and P is one to one. For any $x_0 \in \mathbb{C}$ the Jungck-Ishikawa iteration is defined in the following way

$$\begin{cases} P(x_{k+1}) = (1-a)^s P(x_k) + a^s Q(y_k), \\ P(y_k) = (1-b)^s P(x_k) + b^s Q(x_k), \end{cases} \quad (6)$$

where $a, b, s \in (0, 1]$ and $k = 0, 1, 2, \ldots$.

Definition 6 (Jungck-Noor iteration with s-convex combination in second sense [16])**.** Let $P, Q : \mathbb{C} \to \mathbb{C}$ be the two complex maps such that Q is a complex polynomial of degree greater or equal to 2, also differentiable and P is one to one. For any $x_0 \in \mathbb{C}$ the Jungck-Ishikawa iteration is defined in the following way

$$\begin{cases} P(x_{k+1}) = (1-a)^s P(x_k) + a^s Q(y_k), \\ P(y_k) = (1-b)^s P(x_k) + b^s Q(u_k), \\ P(u_k) = (1-c)^s P(x_k) + c^s Q(x_k), \end{cases} \quad (7)$$

where $a, b, c, s \in (0, 1]$ and $k = 0, 1, 2, \ldots$.

III. ESCAPE CRITERION VIA JUNGCK-SP ITERATION WITH S-CONVEXITY

In this section we prove the threshold escape radius (i.e. escape criterion) for Jungck-SP iteration extended with s-convex combination in second sense for complex polynomial $f(x) = x^n - a_1 x + a_0$ where $n \geq 2$, $a_0, a_1 \in \mathbb{C}$. We abbreviated Jungck-SP orbit with s-convexity as JSPOs.

Definition 7 (Jungck-SP iteration with s-convex combination)**.** Let $P, Q : \mathbb{C} \to \mathbb{C}$ be the two complex maps such that Q is a complex polynomial of degree greater or equal to 2, also analytic and P is one to one. For any $x_0 \in \mathbb{C}$ the Jungck-SP iteration with s-convex combination in second sense is defined in the following way

$$\begin{cases} P(x_{k+1}) = (1-a)^s P(y_k) + a^s Q(y_k), \\ P(y_k) = (1-b)^s P(u_k) + b^s Q(u_k), \\ P(u_k) = (1-c)^s P(x_k) + c^s Q(x_k), \end{cases} \quad (8)$$

where $a, b, c, s \in (0, 1]$ and $k = 0, 1, 2, \ldots$.

Therefore, in proposed implicit iteration we deal with two different mappings, we break f into two mappings P and Q in such a way that $f = Q - P$ and P is one to one. This type of formation of f restraint us to adopt P as one to one mapping and Q as analytic mapping. Thus we derive new threshold escape radius and implement in our algorithms to visualize some kind of fractals.

Escape criteria play an important role in the generation of fractals. We use Jungck-SP iteration with s-convexity in second sense to prove desire results for the complex polynomial $f(x) = x^n - a_1 x + a_0$ where $n \geq 2$ to generate fractals in following way:

Theorem 1. *Assume that* $|x| \geq |a_0| > \alpha_1 = \left(\frac{2(1+|a_1|)}{sa}\right)^{\frac{1}{n-1}}$, $|x| \geq |a_0| > \alpha_2 = \left(\frac{2(1+|a_1|)}{sb}\right)^{\frac{1}{n-1}}$, $|x| \geq |a_1| > \alpha_3 = \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$ *where* $a, b, c, s \in (0, 1]$ *and* $a_0, a_1 \in \mathbb{C}$. *Define a sequence* $\{x_k\}_{k\in\mathbb{N}}$ *as follows:*

$$\begin{cases} P(x_{k+1}) = (1-a)^s P(y_k) + a^s Q(y_k), \\ P(y_k) = (1-b)^s P(u_k) + b^s Q(u_k), \\ P(u_k) = (1-c)^s P(x_k) + c^s Q(x_k), \end{cases} \tag{9}$$

where $s, a, b, c \in (0, 1]$ *and* $k = 0, 1, 2, \ldots$. *Then* $|x_k| \longrightarrow \infty$ *as* $k \longrightarrow \infty$.

Proof. Because $f(x) = x^n - a_1 x + a_0$ with $n \geq 2$, where $a_0, a_1 \in \mathbb{C}$, $x_0 = x$, $y_0 = y$ and $u_0 = u$. We handle f as $f = Q - P$ with choice $Q(x) = x^n + a_0$ and $P(x) = a_1 x$, then

$$\begin{aligned} |P(u_0)| &= |(1-c)^s P(x) + c^s Q(x)| \\ &= |(1-c)^s a_1 x + (1-(1-c))^s (x^n + a_0)|. \end{aligned}$$

By expansion up to degree one of c and $1 - c$, and applying the facts $s \leq 1$, we obtain

$$\begin{aligned} |a_1 u_0| &\geq (1 - s(1-c))|x^n + a_0| - (1 - sc)|a_1 x| \\ &\geq (s - s(1-c))|x^n + a_0| - (1 - sc)|a_1 x|, \because s < 1. \end{aligned}$$

Since $|x| \geq |a_0|$ and $sc < 1$, we have

$$\begin{aligned} |a_1 u_0| &\geq sc\,|x|^n - sc\,|a_0| - |a_1 x| \\ &\geq sc\,|x|^n - |x| - |a_1 x| \\ &= |x| \left(sc\,|x|^{n-1} - (1 + |a_1|)\right). \end{aligned}$$

This provides

$$\begin{aligned} |u_0| &\geq |x| \left(\frac{sc\,|x|^{n-1}}{1+|a_1|} - 1\right) \\ |u_0| &\geq sc\,|x|. \end{aligned}$$

Because $|x| \geq |a_0| > \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$, this produced the situation $|x| \left(\frac{sc|x|^{n-1}}{1+|a_1|} - 1\right) > |x| \geq sc|x|$.
In second step of iteration, we have

$$\begin{aligned} |P(y_0)| &= |(1-b)^s P(u) + b^s Q(u)| \\ &= |(1-b)^s a_1 u + (1-(1-b))^s (u^n + a_0)|. \end{aligned}$$

By expansion up to degree one of b and $1 - b$, and applying facts $s \leq 1$, we obtain

$$\begin{aligned} |a_1 y_0| &\geq (1 - s(1-b))|u^n + a_0| - (1 - sb)|a_1 u| \\ &\geq (s - s(1-b))|u^n + a_0| - |a_1 u| \\ &\geq sb|u^n| - sb|a_0| - |a_1 u|. \end{aligned}$$

Since $|u_0| \geq |x| \left(\frac{sc|x|^{n-1}}{1+|a_1|} - 1\right) > |x| \geq |a_0|$ and $sb < 1$, we have

$$\begin{aligned} |a_1 y_0| &\geq sb|u^n| - |u| - |a_1 u| \\ &= sb|u^n| - (1 + |a_1|)|u| \\ &= |u| \left(sb|u|^{n-1} - (1 + |a_1|)\right) \\ &\geq |u| \left(\frac{sb|u|^{n-1}}{1+|a_1|} - 1\right). \\ &\geq |x| \left(\frac{s^2 bc|x|^{n-1}}{1+|a_1|} - 1\right). \end{aligned}$$

Because $|x| \geq |a_0| > \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$, which implies $|x|^n \left(\frac{sc|x|^{n-1}}{1+|a_1|} - 1\right)^n > |x|^n$. Hence $|u|^n > |x|^n \left(\frac{sc|x|^{n-1}}{1+|a_1|} - 1\right)^n > |x|^n \geq sc|x|^n$. Thus

$$|y_0| \geq s^2 bc\,|x|.$$

In last step of iteration, we have

$$\begin{aligned} |P(x_1)| &= |(1-a)^s P(y_0) + a^s Q(y_0)| \\ |a_1 x_1| &= |(1-a)^s a_1 y + a^s (y^n + a_0)|. \end{aligned}$$

This yields

$$\begin{aligned} |a_1 x_1| &\geq |y| \left(sa|y|^{n-1} - (1 + |a_1|)\right) \\ |x_1| &\geq |y| \left(\frac{sa|y|^{n-1}}{1+|a_1|} - 1\right). \end{aligned}$$

Since $|y| \geq |x|$ and $|x| \geq \left(\frac{2(1+|a_1|)}{sa}\right)^{\frac{1}{n-1}}$, thus $\frac{sa|y|^{n-1}}{1+|a_1|} \geq \frac{sa|x|^{n-1}}{1+|a_1|} > 2$. Hence $|y| \left(\frac{sa|y|^{n-1}}{1+|a_1|} - 1\right) \geq |x| \left(\frac{s^3 abc|x|^{n-1}}{1+|a_1|} - 1\right)$. Therefore

$$|x_1| \geq |x| \left(\frac{s^3 abc\,|x|^{n-1}}{1+|a_1|} - 1\right). \tag{10}$$

As $|x| > \left(\frac{2(1+|a_1|)}{sa}\right)^{\frac{1}{n-1}}$, $|x| > \left(\frac{2(1+|a_1|)}{sb}\right)^{\frac{1}{n-1}}$ and $|x| > \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$, then $|x| > \left(\frac{2(1+|a_1|)}{s^3 abc}\right)^{\frac{1}{n-1}}$ and this implies $\frac{s^3 abc|x|^{n-1}}{1+|a_1|} - 1 > 1$. Therefore there exists $\lambda > 0$ such that $\frac{s^3 abc|x|^{n-1}}{1+|a_1|} - 1 > 1 + \lambda$. As a result $|x_1| > (1 + \lambda)|x|$. Particularly $|x_1| > |x|$. Subsequently $|x_k| > (1 + \lambda)^k |x|$. Hence, the orbit of x tends to infinity and this completes the proof. □

Corollary 1. *Assume that*

$$|a_0| > \alpha_1, |a_0| > \alpha_2 \textit{and } |a_0| > \alpha_3,$$

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where $\alpha_1 = \left(\frac{2(1+|a_1|)}{sa}\right)^{\frac{1}{n-1}}, \alpha_2 = \left(\frac{2(1+|a_1|)}{sb}\right)^{\frac{1}{n-1}}$ *and* $\alpha_3 = \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$. *Then Jungck-SP orbit with s-convexity escapes to infinity.*

Corollary 2. *Assume that* $a, b, c, s \in (0, 1]$ *and*

$$|x| > \max\left[|a_0|, \alpha_1, \alpha_2, \alpha_3\right],$$

where $\alpha_1 = \left(\frac{2(1+|a_1|)}{sa}\right)^{\frac{1}{n-1}}, \alpha_2 = \left(\frac{2(1+|a_1|)}{sb}\right)^{\frac{1}{n-1}}$ *and* $\alpha_3 = \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$. *Then there exists* $\lambda > 0$ *such that* $|x_k| > (1+\lambda)^k|x|$ *and* $|x_k| \to \infty$ *as* $k \to \infty$.

Corollary 3. *Assume that*

$$|x_m| > |x| > \max\left[|a_0|, \alpha_1, \alpha_2, \alpha_3\right],$$

where $\alpha_1 = \left(\frac{2(1+|a_1|)}{sa}\right)^{\frac{1}{n-1}}, \alpha_2 = \left(\frac{2(1+|a_1|)}{sb}\right)^{\frac{1}{n-1}}$ *and* $\alpha_3 = \left(\frac{2(1+|a_1|)}{sc}\right)^{\frac{1}{n-1}}$ *and for some* $m \geq 0$. *Thus there exists* $\lambda > 0$ *such that* $|x_{m+k}| > (1+\lambda)^k|x_m|$ *and* $|x_k| \to \infty$ *as* $k \to \infty$.

IV. APPLICATION TO FRACTALS

In literature authors used different approaches to generate fractals. Some popular algorithms to visualize the fractals are, distance estimator [41], escape time [42] and potential function algorithms [43]. To generate Julia and Mandelbrot sets, we use escape time algorithms. The escape time algorithm iterate the function upto the desire number of iterations. The algorithm generate two sets, one is consists of points for which the JSPOs does not escape to infinity (i.e. Julia or Mandelbrot set) and the second set consists of points for which the JSPOs escape to infinity (i.e. Fatou domains). In section we present Julia and Mandelbrot sets at different values of n and input parameters. We fixed maximum number of iterations at $K = 20$ for all Julia and Mandelbrot sets.

In next two subsections we generate Julia and Mandelbrot sets in JSPOs by using algorithms 1 and 2.

A. JULIA SETS IN JSPOS

We know that a very small change in any input parameter cause drastic change in Julia set. So the comparison of Julia sets generated by proposed iteration with classical Julia sets is not possible. As an example we generate a classical quadratic Julia set for $f(x) = x^2 + a_0$ with $a_0 = -123 + 0.807\mathbf{i}$ in figure 1. This image is also called Douady rabbit. But at the same value of a_0 the JSPOs does not converge. We generate quadratic Julia sets of complex polynomial $f(x) = x^2 - a_1x + a_0$ where $a_0.a_1 \in \mathbb{C}$ in JSPOs at different value of a_0. We observe that quadratic Julia sets in figures (2–4) resemble with classical quadratic Julia set in figure 1 (i.e. Douady rabbit). In figures (2–4) we fix $f(x)$ along with a, b, c and change the value of s. The image in figure 2 is like a fat Douady rabbit, image in figure 3 is smart and image in figure 4 is smartest Douady rabbits respectively.

Algorithm 1: Julia set visualization

Input: $f(x) = x^n - a_1x + a_0$–a complex polynomial, A–area for image, K–fixed number of iterations, $colourscale[0..h-1]$ colourscale with h colours.
Output: Julia set in area A.

```
for x_0 ∈ A do
    R–threshold escape radius proved in Theorem 1
    k = 0
    while k ≤ K do
        P(x_{k+1}) = (1 − a)^s P(y_k) + a^s Q(y_k),
        P(y_k) = (1 − b)^s P(u_k) + b^s Q(u_k),
        P(u_k) = (1 − c)^s P(x_k) + c^s Q(x_k)
        if |x_{k+1}| > R then
            break
        k = k + 1
    i = ⌊(h − 1) k/K⌋
    colour x_0 with colourscale[i]
```

Algorithm 2: Mandelbrot set visualization

Input: $f(x) = x^n - a_1x + a_0$–a complex polynomial, A–area for image, K–fixed number of iterations, $colourscale[0..h-1]$ colourscale with h colours.
Output: Mandelbrot set in area A.

```
for a_0 ∈ A do
    R–threshold escape radius proved in Theorem 1
    k = 0
    x_0 = any one critical point of f
    while k ≤ K do
        P(x_{k+1}) = (1 − a)^s P(y_k) + a^s Q(y_k),
        P(y_k) = (1 − b)^s P(u_k) + b^s Q(u_k),
        P(u_k) = (1 − c)^s P(x_k) + c^s Q(x_k)
        if |x_{k+1}| > R then
            break
        k = k + 1
    i = ⌊(h − 1) k/K⌋
    colour a_0 with colourscale[i]
```

In figures 5–8 we fix $f(x)$ along with s and the values of a, b, c. In figures 9 and 10 we change the value of a_1 also. We observe rotational symmetry in figures 5–10. The swirls are oriented clockwise in all graphs of figures 5–10. The input parameters were as follows:

- Fig. 1: $a_0 = -0.123 + 0.807\mathbf{i}, A = [-1.5, 1.5] \times [-1.5, 1.5]$,
- Fig. 2: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a, b, c = 0.5$,$s = 0.9$, $A = [-4, 4] \times [-3.5, 3.5]$,
- Fig. 3: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a, b, c = 0.5$,$s = 0.8$, $A = [-4, 4] \times [-3.5, 3.5]$,
- Fig. 4: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a, b, c = 0.5$,$s = 0.75$, $A = [-4, 4] \times [-3.5, 3]$,

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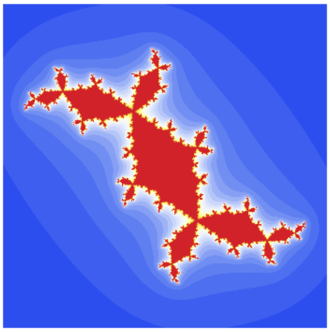

FIGURE 1. Quadratic-Julia set in JSPOs.

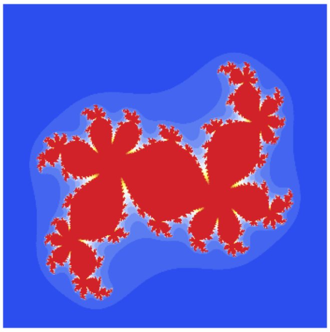

FIGURE 2. Quadratic-Julia set in JSPOs.

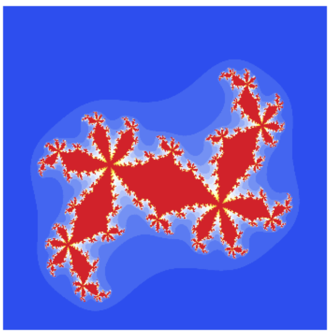

FIGURE 3. Quadratic-Julia set in JSPOs.

- Fig. 5: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a, b = 0.1$,$c = 0.8$,$s = 0.5$, $A = [-6, 6] \times [-9.5, 4]$,
- Fig. 6: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a, c = 0.4$,$b = 0.6$,$s = 0.5$, $A = [-6, 6] \times [-5.5, 3.5]$,
- Fig. 7: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a = 0.7$,$b = 0.45$,$c = 0.55$,$s = 0.5$, $A = [-4.5, 4.5] \times [-3.5, 2.5]$,
- Fig. 8: $a_0 = 2 - 2.5\mathbf{i}$, $a_1 = \sqrt{5}$,$a, b, c, s = 0.5$, $A = [-4.5, 4.5] \times [-4.5, 3]$,
- Fig. 9: $a_0 = -0.5 - 0.3\mathbf{i}$, $a_1 = \mathbf{e}^{\frac{1}{\mathbf{i}}}$,$a, b, c, s = 0.9$, $A = [-1.7, 1.7] \times [-1.5, 1.5]$,
- Fig. 10:$a_0 = 2 - 2.5\mathbf{i}$, $a_1 = 1 + \mathbf{i}$, $a, b, c, s = 0.1$, $A = [-3.5, 2.8] \times [-3.5, 2.8]$.

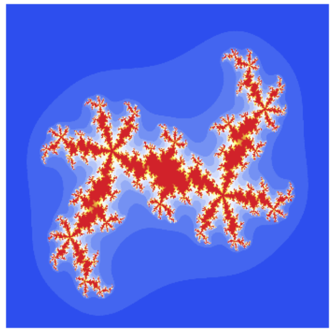

FIGURE 4. Quadratic-Julia set in JSPOs.

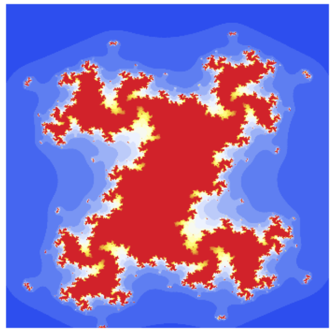

FIGURE 5. Quadratic-Julia set in JSPOs.

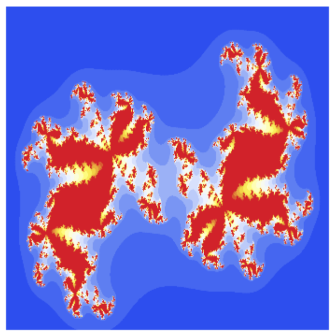

FIGURE 6. Quadratic-Julia set in JSPOs.

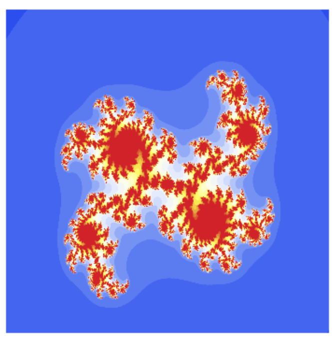

FIGURE 7. Quadratic-Julia set in JSPOs.

In second example of Julia sets, we present some cubic Julia sets of complex polynomial $f(x) = x^3 - a_1x + a_0$ where $a_0, a_1 \in \mathbb{C}$ in JSPOs. We notice that the cubic Julia set

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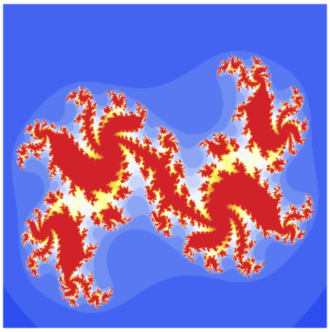

FIGURE 8. Quadratic-Julia set in JSPOs.

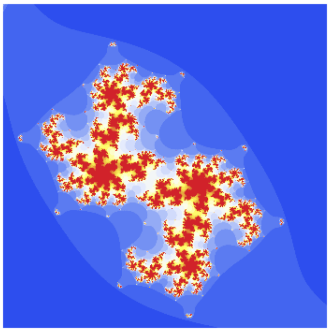

FIGURE 9. Quadratic-Julia set in JSPOs.

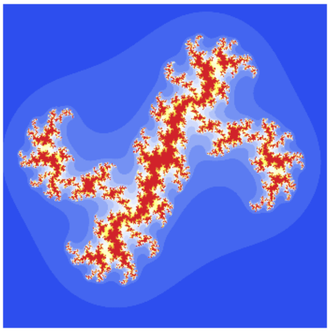

FIGURE 10. Quadratic-Julia set in JSPOs.

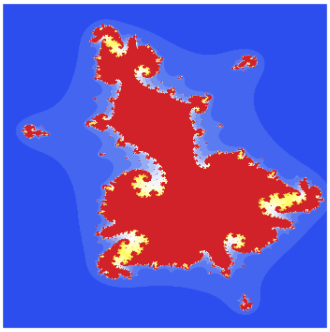

FIGURE 11. Cubic-Julia set in JSPOs.

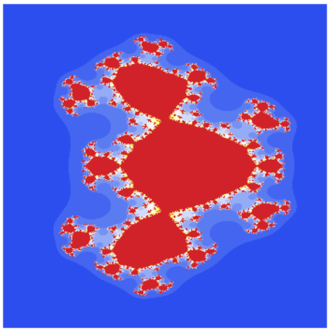

FIGURE 12. Cubic-Julia set in JSPOs.

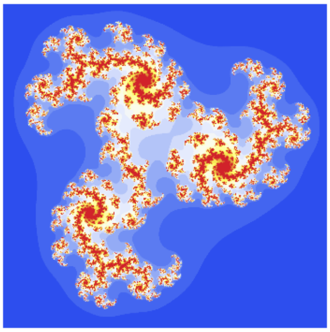

FIGURE 13. Cubic-Julia set in JSPOs.

changes with change of input parameters. The figures (11, 12 and 13) present some variations in shapes of cubic Julia set. The input parameter for cubic Julia set were as follows:

- Fig. 11: $a_0 = -0.6 - 0.045\mathbf{i}$, $a_1 = \frac{1}{\mathbf{i}}$,$a, b, c, s = 0.9$, $A = [-1.2, 1.4] \times [-1.4, 1.2]$,
- Fig. 12: $a_0 = -0.629$, $a_1 = \mathbf{i}$,$a, b, c, s = 0.5$, $A = [-1.4, 1.4] \times [-2, 2]$,
- Fig. 13:$a_0 = -0.06 + 0.02\mathbf{i}$, $a_1 = \sqrt{\mathbf{i}}$, $a, b, c, s = 0.1$, $A = [-1.7, 1.5] \times [-2, 2]$.

The next example of Julia sets, we demonstrate some quadric Julia sets of complex polynomial $f(x) = x^4 - a_1x + a_0$ where $a_0, a_1 \in \mathbb{C}$ in JSPOs. The images of quadric Julia sets change with different values of input parameters. The figures (14, 15 and 16) present the variations in shapes of quadric Julia set. The input parameter for quadric Julia sets were as follows:

- Fig. 14: $a_0 = -0.85 + 1.52\mathbf{i}$, $a_1 = 1 - \mathbf{i}$,$a, b, c, s = 0.9$, $A = [-1.4, 1.4] \times [-1.4, 1.4]$,
- Fig. 15: $a_0 = -0.95$, $a_1 = 1$,$a = 0.5, b, c = 0.9, s = 0.4$, $A = [-1.5, 1.4] \times [-1.4, 1.4]$,
- Fig. 16:$a_0 = -0.06 + 0.02\mathbf{i}$, $a_1 = \sqrt{\mathbf{i}}$, $a, b, c, s = 0.1$, $A = [-1.7, 1.5] \times [-2, 2]$.

B. MANDELBROT SETS IN JSPOS

Here we visualize some complex graphs of Mandelbrot sets for a polynomial $f(x) = x^n - a_1x + a_0$ where $a_0, a_1 \in \mathbb{C}$

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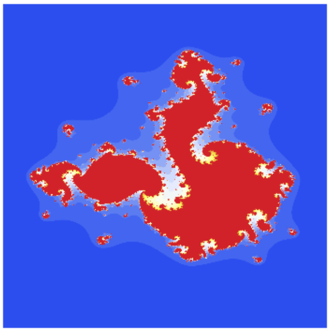

FIGURE 14. Quadric-Julia set in JSPOs.

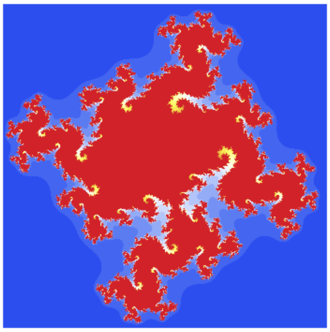

FIGURE 15. Quadric-Julia set in JSPOs.

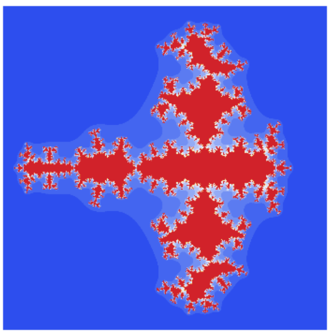

FIGURE 16. Quadric-Julia set in JSPOs.

in JSPOs and compare them with classical Mandelbrot sets. We discuss our results in the form of some examples. The first example presents some quadratic Mandelbrot sets. The graph in figure 17 is the classical quadratic Mandelbrot set for $f(x) = x^2 + a_0$, whereas the figures 18–24 show the variety of quadratic Mandelbrot sets for different input parameters in JSPOs. The classical Mandelbrot set contains a large cardioid, a largest bulb on left side of main cardioid on real axis and infinite many small bulbs around the perimeter of cardioid and largest bulb. Furthermore each bulb has its own antennas. When we magnify the Mandelbrot set closer to its antennas we can see the copies of itself. In figures 18–20 we fix $f(x)$ along with a, b, c and change the value of s, we observe that the graphs are resemble with classical quadratic Mandelbrot set but the shape of largest bulb and cardioid

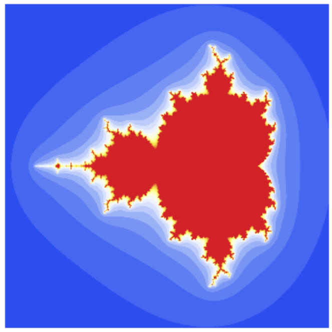

FIGURE 17. Classical quadratic-Mandelbrot set.

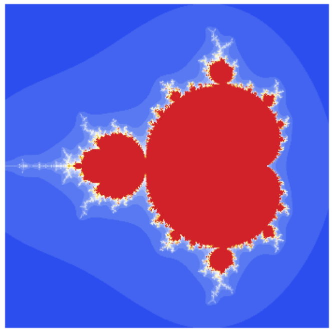

FIGURE 18. Quadratic-Mandelbrot set in JSPOs.

slightly change in each figure. We again fix $f(x)$ along with s and change the values of a, b, c, same changes observe as we discuss earlier (See in figures 21 and 22). We analyze that at different values of input parameters the behavior of quadratic Mandelbrot set is different and area of each image also different. The images in figures 23 and figure 24 have different directions from images in figures 18–22. The image in figure 24 is completely different from the images in figures 18–23. The input parameters were as follows:

- Fig. 17: $A = [-2.3, 1] \times [-1.5, 1.5]$,
- Fig. 18: $a_1 = 2$, $a_2 = 10$,$a, b, c, s = 0.9$, $A = [-9, 6] \times [-6, 6]$,
- Fig. 19: $a_1 = 2$, $a_2 = 10$,$a, b, c = 0.9$,$s = 0.7$, $A = [-11, 3] \times [-6, 6]$,
- Fig. 20: $a_1 = 2$, $a_2 = 10$,$a, b, c = 0.9$,$s = 0.5$, $A = [-9.5, 3] \times [-6, 6]$,
- Fig. 21: $a_1 = 2$, $a_2 = 10$,$a = 0.7$,$b = 0.8$,$c = 0.6$,$s = 0.9$, $A = [-15, 3] \times [-8, 8]$,
- Fig. 22: $a_1 = 2$, $a_2 = 10$,$a = 0.8$,$b = 0.7$,$c = 0.8$,$s = 0.9$, $A = [-15, 3] \times [-8, 8]$,
- Fig. 23: $a_1 = \frac{\mathbf{i}}{3}$, $a, b, c, s = 0.5$, $A = [-0.1, 0.6] \times [-0.3, 0.3]$,
- Fig. 24: $a_1 = \mathbf{i}$, $a, b, c, s = 0.1$, $A = [-1.1, 9.6] \times [-3.3, 3.3]$.

In next example, we test the variations in cubic Mandelbrot set for a complex polynomial $f(x) = x^3 - a_1 x + a_0$ where $a_0, a_1 \in \mathbb{C}$ in JSPOs and compare them with classical cubic Mandelbrot set. The image in figure 25 is classical cubic

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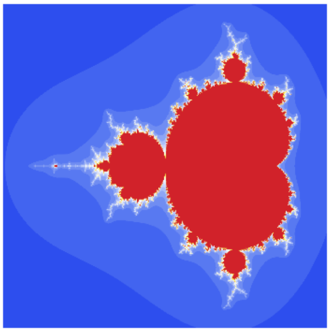

FIGURE 19. Quadratic-Mandelbrot set in JSPOs.

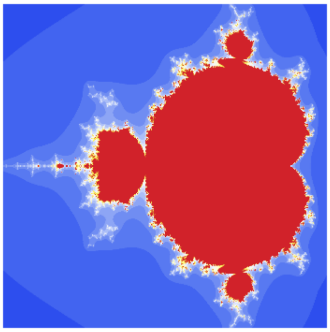

FIGURE 20. Quadratic-Mandelbrot set in JSPOs.

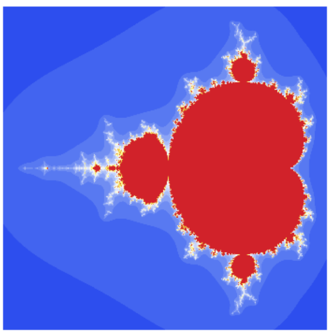

FIGURE 21. Quadratic-Mandelbrot set in JSPOs.

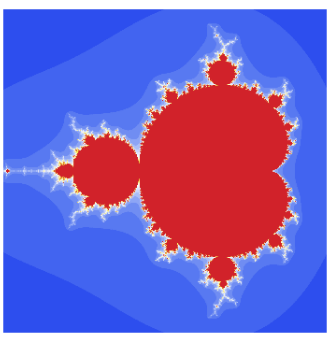

FIGURE 22. Quadratic-Mandelbrot set in JSPOs.

Mandelbrot set for $f(x) = x^3 + a_0$. The classical cubic

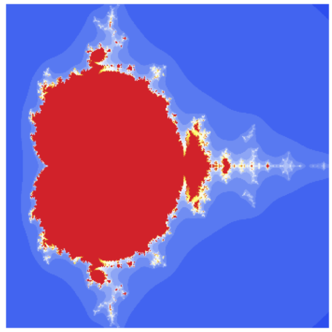

FIGURE 23. Quadratic-Mandelbrot set in JSPOs.

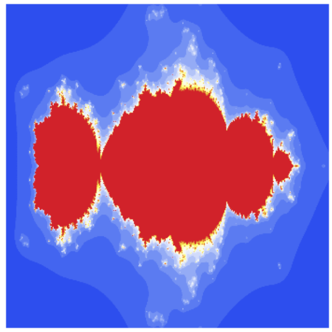

FIGURE 24. Quadratic-Mandelbrot set in JSPOs.

Mandelbrot has two large cardioid and two largest bulbs. The cardioids are also called the primary Mandelbrot set and bulbs around the perimeter of cardioids are called secondary Mandelbrot sets. The figures 26–28 are present cubic Mandelbrot sets at different input parameters in JSPOs. The images in figures 26 and 28 have same main cardioids resembled with classical cubic Mandelbrot set but the secondary Mandelbrot sets is different. The image in figure 27 have a chain of Mandlebulbs in diagonal form. The Mandlebulbs in this figure originate from origin. The input parameters were as follows:

- Fig. 25: $A = [-1, 1] \times [-1.7, 1.7]$,
- Fig. 26: $a_1 = 1, a, b, c, s = 0.9$, $A = [-1, 1] \times [-2, 2]$,
- Fig. 27: $a_1 = \frac{\mathbf{1}}{\mathbf{2}}$, $a, b = 0.5, c = 0.1, s = 0.9$, $A = [-0.8, 0.8] \times [-1.8, 1.8]$,
- Fig. 28: $a_1 = \frac{\mathbf{1}}{\mathbf{i}}$, $a, b, c, s = 0.1$, $A = [-2, 2] \times [-2, 2]$.

The next example demonstrate some graphs of quadric Mandelbrot sets for a complex polynomial $f(x) = x^4 - a_1 x + a_0$, where $a_0, a_1 \in \mathbb{C}$ in JSPOs. The figure 29 is classical quadric Mandelbrot set for $f(x) = x^4 + a_0$. Interesting images visualize in figures 30–32 at different values of input parameters in JSPOs. The images in figures 30–32 totally different from classical quadric Mandelbrot set in shapes. The input parameters were as follows:

- Fig. 29: $A = [-1.5, 1.5] \times [-1.5, 1.5]$,
- Fig. 30: $a_1 = \mathbf{i}$,$a, b, c, s = 0.9$, $A = [-1.6, 1] \times [-1, 1]$,
- Fig. 31: $a_1 = 2 + \mathbf{i}$, $a, b = 0.5, c = 0.1, s = 0.9$, $A = [-3.5, 3.5] \times [-3.5, 3.5]$,

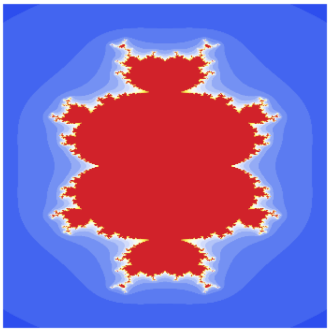

FIGURE 25. Classical cubic-Mandelbrot set.

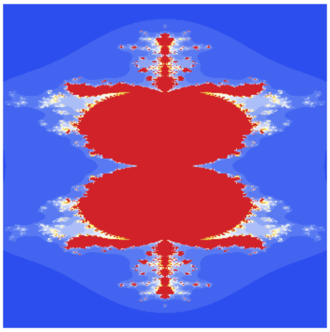

FIGURE 26. Cubic-Mandelbrot set in JSPOs.

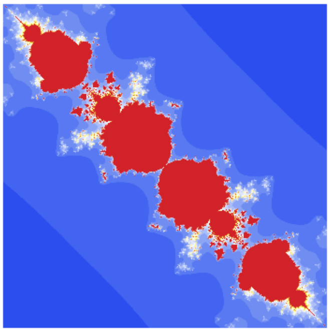

FIGURE 27. Cubic-Mandelbrot set in JSPOs.

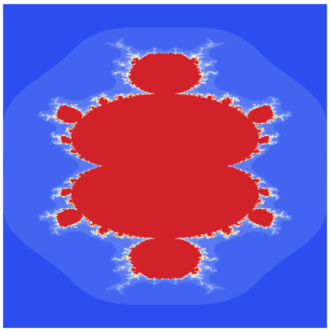

FIGURE 28. Cubic-Mandelbrot set in JSPOs.

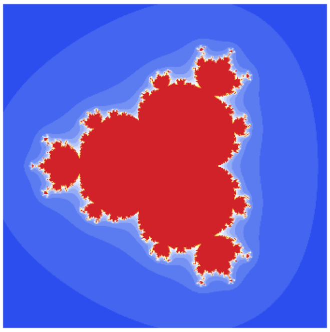

FIGURE 29. Classical quadric-Mandelbrot set.

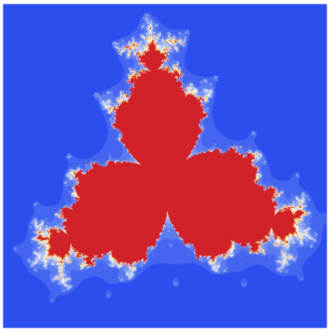

FIGURE 30. Quadric-Mandelbrot set in JSPOs.

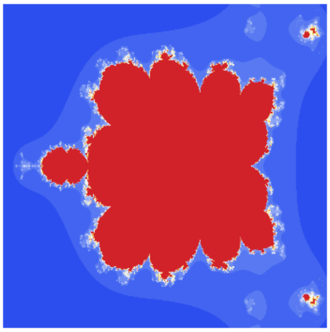

FIGURE 31. Quadric-Mandelbrot set in JSPOs.

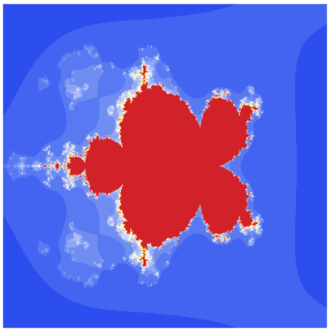

FIGURE 32. Quadric-Mandelbrot set in JSPOs.

- Fig. 32: $a_1 = 1$, $a = 0.5, b, c = 0.9, s = 0.4$, $A = [-1.5, 1] \times [-1, 1]$.

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V. CONCLUSIONS

We studied Jungck-SP iteration as an application of fractal geometry. We extended Jungck–SP iteration with s-convexity and defined its orbit (JSPOs). We proved some fixed point results for complex polynomial $f(x) = x^n - a_1x + a_0$ (where $n \geq 2$) to generate fractals in JSPOs. We used the established results in algorithms to visualize Julia and Mandelbrot sets. We showed in examples that the images of Julia and Mandelbrot sets changed with change in involved parameters.

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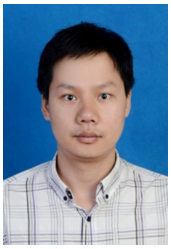

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