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ABSTRACT

This paper utilises various recently developed econometric methods to obtain better approximations to the half-life for real exchange rates of ten South African Development Community (SADC) countries and to generate confidence intervals for half-life deviations from the purchasing power parity (PPP). The robust methods of Stock (1991), Elliott and Stock (2001), and Hansen (1999) imply that point estimates of less than 36 months exist, making them compatible with PPP. However, the results of ADF and ADF-GLS tests render the SADC real exchange rates as nonstationary processes, a result that is patently at odds with mean-reversion, implying at the same time the possibility of infinite half-lives. Therefore the empirical results appearing in this paper do not convincingly resolve the half-life version of the PPP puzzle and leaves room for future research in the directions of non-parametric methods and median unbiased confidence intervals.

JEL Codes: C12, C15, C23, F30, F31
Keywords: PPP, Half-life, Real Exchange Rates

Introduction

This paper presents evidence on the half-life version of the purchasing power parity (PPP) puzzle for ten of the fourteen Southern African Development Community (SADC) countries[1], using Rossi's (2005) methodology beased on local-to-unity asymptotic theory. An ordinary definition of absolute PPP is that the latter represents the exchange rate between two currencies multiplied by the relative national price levels. The relative form of this hypothesis is that PPP exists when the rate of depreciation of, say, the home currency relative to the foreign currency matches the difference in aggregate price inflation between the two countries in point (Sarno and Taylor, 2002).

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[1] Currently the SADC is constituted by Angola, Botswana, the Democratic Republic of Congo (DRC), Lesotho, Malawi, Madagascar, Mauritius, Mozambique, Namibia, South Africa, Swaziland, Tanzania, Zambia, and Zimbabwe. Owing to a lack of reliable data, four countries have been excluded in the analysis: the DRC, Lesotho, Namibia, and Zimbabwe.

The PPP hypothesis implies that the real exchange rate should be constant such that any deviations from equilibrium should be transitory. Yet most studies have found that real exchange rates exhibit a large degree of volatility and that their deviations from equilibrium are highly persistent. The half-life version[2] of the PPP puzzle is that a high degree of exchange rate volatility is generally associated with an implausibly slow speed of mean reversion.

According to the sticky price theories of international macroeconomics, the purchasing power parity (PPP) hypothesis is compatible with half-lives of real exchange rate of less than three years. However, economists are puzzled by the slow rate at which real exchange rates adjust to the PPP (Taylor and Taylor, 2004). This is an issue dealt with at length by Rogoff (1996), who points out that the high short-term volatility of real exchange rates is not compatible with the extremely slow rate at which shocks appear to die off.

Empirical analysis of the persistence of real exchange rate deviations from PPP is generally based on impulse response analysis. In this setting, the concept of a half-life is used to estimate how long it takes for the impulse response to a unit shock to dissipate by one half (Chortareas and Kapetanios, 2004). In this context, consider for instance that PPP holds continuously. This implies that the following relation should remain constant:

$$y_t = \ln\left[\frac{S_t P_t}{P^*_t}\right], \qquad (1)$$

where S_t represents the nominal exchange rate, P_t and P^*_t are the price levels in the domestic and foreign country, respectively. Following Rossi (2005), suppose that the deviations of the real exchange rate, y_t, from its long-run value y_0 follow a stationary autoregressive order one process such that

$$y_t = \varphi y_{t-1} + \varepsilon_t \, . \qquad (2)$$

Then the half-life of a process can be defined as the minimum value of H such that

$$E[y_{t+h} - y_0 \mid y_{t-s} - y_0] \leq \frac{1}{2}(y_t - y_0)\,, \qquad (3)$$

where E is the expectation operator and $s \leq 0$. Based on equation (2), estimates of H are obtained as follows:

$$\hat{h} = \ln(0.5)/\ln(\hat{\varphi})\,, \qquad (4)$$

where $\hat{\varphi}$ represents the estimate of φ. It is to be noted that for $\hat{\varphi} \geq 1$, the process has no half-life because it approaches infinity.

Kim, Silvapulle and Hyndman (2006) have identified the three main statistical properties of the half-life statistic as calculated using equation (4), namely, it has an

[2] For a detailed discussion of the mean-reversion version of the PPP, and their application to SADC countries, refer to Mokoena (2007) and Mokoena *et al.* (2008a). Also see Mokoena *et al.* (2008b) for an appraisal of the possibility that SADC country real exchange rates can be treated as long memory processes.

unknown and possibly intractable distribution; it may not possess finite sample moments since it takes extreme values as the coefficient approaches one; that it is biased in small samples; and that it is a nonlinear function of φ which is also biased downward.

Empirical evidence is generally mixed when it comes to point estimates of half-life deviations. For instance, Parsley and Wei (1995) found that the half-lives for the European Monetary System countries were 4.25 years. Other studies on real exchange rates, such as Frankel (1990), found that the half-life of the dollar-pound real exchange rate was 4.6 years. In addition, Lothian and Taylor (1996) estimated that the corresponding numbers were 2.8 for the franc-pound and 5.9 for the dollar-pound real exchange rate.

In the context of panel data analysis, the evidence on point estimates is also mixed. Frankel and Rose (1995) found that for 150 countries the half-life averaged 4 years. Moreover, Cheung and Lai (2000) estimated that the half-lives ranged between 2 and 5 years for industrial countries but under 3 years for developing economies. A study by Manzur (1990) assessing seven industrial countries found that the half-lives of their real exchange rates were 5 years, while Fung and Lo (1992) put the half-lives at 6.5 years for the sample of six industrial countries they consider.

The main contribution of this paper is that it follows Rossi's (2005) approach to generate point estimates and confidence intervals for the SADC in which deviations from PPP are in some cases compatible with nominal price and wage stickiness. To the best of our knowledge, except for the case of South Africa[3], no published article has ever produced these half-life confidence intervals for the SADC countries. The motivation for using Rossi's (2005) methodology is that she uses local-to-unity asymptotic theory in the presence, in most cases, of highly persistent data. As it is commonly observed, real exchange rates manifest themselves as processes with roots near-unity. This characteristic makes them provide no good small-sample approximation to the distribution of estimators and test statistics. In such cases econometricians use an alternative approach by modelling the dominant root of the autoregressive lag order polynomial as local-to-unity (Diebold, Killian, and Nerlove, 2008)**.** This approach leads to an alternative asymptotic approximation that provides a better small-sample approximation than imposing the order of integration.

The second reason for following Rossi (2005) is that she derives a measure of the half-life for a general $AR(p)$ process that allows for better asymptotic approximations in the presence of a root close to unity.

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[3] Recently, Mokoena (2007) presented the results of nonlinearity and stationarity tests in respect of the real exchange rates of the South African rand. The author found that the rand real exchange rate behaviour tends to be nonlinear and stationary in a majority of cases in the sample. This, in turn, suggested that, for the majority of the currencies in the sample, the real exchange rates of the rand were mean-reverting, implying that the purchasing power parity relation tended hold in a nonlinear manner for South Africa. This study, thus, extends the work of Mokoena (2007) by considering a larger number of countries.[3]

The rest of the paper is organised as follows: Section 2 discusses Rossi (2005) approach to measuring half-life deviations. Section 3 covers econometric issues and empirical evidence, and Section 4 concludes.

Rossi (2005) methodology for general $AR(p)$ processes

The approach followed is based on the factorization of the data generating process (DGP) of the following form:

$$y_t = d_t + u_t \qquad t = 1,2,..,T \tag{5}$$

$$u_t = \rho u_{t-1} + v_t \tag{6}$$

$$\rho = e^{c/T} \approx 1 + c/T \tag{7}$$

where d_t is a deterministic component, v_t is a zero mean, stationary and ergodic process, with finite autocovariances. Equation (7) represents local-to-unity asymptotics in the spirit of Stock (1991). The factorisation process produces:

$$(1-\lambda_1 L)(1-\lambda_2 L)...(1-\lambda_p L)(y_t - d_t) = \varepsilon_t$$

where λ_j are eigenvalues of an $AR(p)$ process. The half-life statistic for an $AR(p)$ process has been suggested by Rossi (2005) and takes the following form:

$$\hat{h} = Max\left\{\frac{\ln(0.5)b(1)}{\ln(\hat{\varphi})}, 0\right\}, \tag{8}$$

where $b(1) = (1-\lambda_2)(1-\lambda_3)...(1-\lambda_p)$ is the correction factor of an $AR(p)$ process, whereby p denotes the number of lags. Rossi (2005) treats a unit root process as having an infinite half-life. The author points out that the data generating process (5), can be rearranged to generate the following ADF regression:

$$y_t = \tilde{\mu}^o + \alpha(1)y_{t-1} + \sum_{j=1}^{p-1} \alpha *_{j-1} \Delta y_{t-j} + \varepsilon_t \tag{9}$$

where $\alpha(1) = 1 + \frac{c}{T} b(1)$, $\tilde{\mu}_0 = -\frac{c}{T}\mu_0 b(1)$, $\alpha^*{}_j = -\sum_{i=j+1}^{p-1} \alpha_j$ (10)

The half-life associated with the above regression is of the form:

$$H_a = \max\left\{\frac{\ln(0.5)}{\ln \alpha(1)}, 0\right\} \tag{11}$$

A conventional 95 per cent confidence interval associated with the above half-life statistic is of the following form:

$$\hat{H}_a \pm 1.96 \hat{\sigma}_{\hat{\alpha}(1)} \left[\frac{\ln(0.5)}{\hat{\alpha}(1)} [\ln(\hat{\alpha}(1))]^{-2}\right]. \tag{12}$$

To construct confidence intervals, this chapter follows Rossi (2005) by relying on Stock (1991), Elliott and Stock (2001), and Hansen (1999). The details of the strengths and weaknesses of these methods have been discussed at length by Rossi (2005). At this point it is worth pointing out that when the data are highly persistent, a bootstrap method that is valid is Hansen's (1999) grid-α bootstrap method, which has the range-preserving property. This method is supposed to ensure that the calculated half-life is nonnegative. In the latter context, negative half-lives are treated as invalid and cannot be interpreted meaningfully.

The biggest pitfall associated with the calculation of half-lives using Elliot and Stock (2001) and Stock (1991) is that the confidence intervals for half-lives are too wide and their upper bounds can approach infinity. The excessively wide confidence intervals are associated with a high degree of uncertainty in the magnitudes of point estimates. Thus, deviations from the parity condition may represent the absence of mean-reversion, calling to question the empirical validity of the PPP hypothesis in the case in point.

Empirical results

Table 1 presents the results of confidence intervals using standard asymptotic theory. MAIC was used to determine the lag length based on the demeaned data. According to the results, half-lives of less than 36 months would be compatible with the PPP hypothesis. Due to their lower power, as discussed in Mokoena (2007, 2008a, 2008b), the ADF and ADF-GLS results cannot be interpreted with high degree of confidence. It is better not to focus too much on them. According to the results appearing on Table 1, point estimates of half life deviations less than 36 months depend on the method used. Such cases include all countries except Tanzania, Zambia, and Malawi.

Table 2 presents the empirical results based on Stock (1991) approach. The main weakness of the approach is that is does not guarantee non-negative half lives. So, the Stock (1991) method can be seen as not reliable in respect of confidence intervals. It is noteworthy, however, that the median unbiased point estimates appear quite reasonable in the context of PPP.

In Table 2 the results associated with Mauritius results remain incomplete and those of Tanzania are invalid, while those associated with Zambia and Malawi have infinite half-lives. The reason for these problems is that the exchange rates of those countries may not be compatible with local-to-unity asymptotic theory used.

It is seen from Table 3 that Hansen's method is supposed to guarantee non-negative numbers. However, Tanzania is a serious problem. The Tanzania results are invalid owing to the negative numbers. The large numbers associated with Zambia under the Elliot and Stock (2001) method should be treated as infinity.

Taken together, the current approaches used in this analysis are not informative in terms of confidence intervals. However, point estimates in the case of Hansen's method may be biased.

INSERT TABLES 1 THROUGH 3

Conclusions

The objective of this paper has been to utilise various methods to obtain better approximations to the half-life for highly persistent processes in the presence of small samples and to generate confidence intervals for half-life deviations from PPP. The robust methods of Stock (1991), Elliott and Stock (2001), and Hansen (1999) imply that point estimates of less than 36 months exist, making them compatible with PPP. Specifically, the Elliot and Stock (2001) approach point to Mauritius, Mozambique, South Africa, and Swaziland as the main examples. However, the results of ADF and ADF-GLS tests render the SADC real exchange rates as nonstationary processes, a result that is patently at odds with mean-reversion, implying at the same time the possibility of infinite half-lives. Therefore the empirical results appearing in this paper do not convincingly resolve the half-life puzzle of the PPP hypothesis. It has been pointed out by Kim, Silvapulle and Hyndman (2006) that the half-life statistic has an unknown and possibly intractable distribution and that it may not possess finite sample moments since it takes extreme values as the coefficient $\hat{\rho}$ approaches one. Another implausible characteristic is that the half-life statistic it is biased in small samples. Moreover, the authors indicated that a tiny estimation error in $\hat{\rho}$ could result in extreme variability of $\hat{h}$, which makes it uncertain and unreliable.

When the results of this paper are compared with those appearing in Mokoena et al. (2008a), in which PPP was found to hold in four of the ten SADC countries, based on the same data set and sample periods, one is inclined to believe that a different and improved framework with regard to the calculation of half-lives is necessary to achieve consistent results that include tighter confidence intervals. For future research, the most promising approach seems to be in the spirit of Kim, Silvapulle, and Hyndman (2006). The authors propose a non-parametric method for point and interval estimation of half-life. This method relies on the use of the bias-corrected bootstrap to approximate the sampling distribution of the half-life. It estimates the kernel density of the above bootstrap distribution, by adopting the transformed kernel density method and the data-based bandwidth selection method. It uses the highest density region method for point and interval estimation of half-life. Besides the latter work, recently Pesavento and Rossi (2006) have developed a method that provides median unbiased confidence intervals with accurate coverage properties regardless of whether the root is unity or close to unity. The results of this paper should, thus, be taken as suggestive and not definite, because of the problems mentioned above.

References

Cheung, Y. W. and Lai, K. S. (2000). On the purchasing power parity puzzle. J*ournal of International Economics*, 52:321-330.

Chortareas, G. and Kapetanios, G. (2004). How puzzling is the PPP puzzle? An alternative half-life measure of convergence to PPP, *Queen Mary Working Paper number 522*, University of London.

Diebold, F.X., Killian, L. and Nerlove, M. (2008). Time Series Analysis forthcoming in Durlauf, S. and Blume, L. (eds), *The New Palgrave Dictionary of Economics*, 2nd ed., London: Macmillan.

Elliott, G. and Stock, J. (2001). Confidence intervals for autoregressive coefficients near one. *Journal of Econometrics*, 103: 155-181.

Frankel, J. A. (1990).Zen and the Art of Modern Macroeconomics: A Commentary, in Haraf, W. S. and Willett, T. D.(eds), *Monetary Policy for a Volatile Global Economy*, American Enterprise Institute for Public Policy Research.

Frankel, J. and Rose, A. (1995). A Survey of empirical research on nominal exchange Rates, in Grossman, E. and Rogoff, K. (eds), *The Handbook of International Economics*, Vol. 3, Amsterdam: North Holland.

Fung, H. G. and Lo, W. C. (1992). Deviations from purchasing power parity. *Financial Review*, pp. 553-570.

Hansen, B. (1999). Bootstrapping the autoregressive model. *Review of Economics and Statistics*, 81(4):594-607.

Kim, J. H., Silvapulle, P. and Hyndman, R.J. (2007). Half-life estimation based on the bias-corrected bootstrap: A highest density region approach. *Working paper number 11/06*, Monash University.

Lothian, J. R. and Taylor, M. P. (1996).Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries. *Journal of Political Economy*, 104:488-509.

Manzur, M. (1990). An international comparison of prices and exchange rates: a new test of purchasing power parity. *Journal of International Money and Finance*, 9(1): 75-91.

Mokoena, T. M. (2007). Taking the puzzle out of the purchasing power parity puzzle: an application in respect of South Africa. *South African Journal of Economics*, 75:22-27.

Mokoena, T. M., Gupta, R., and Van Eyden, R. (2008a). Testing for PPP in SADC Real Exchange Rates. *Working Paper, No. 200822*, Department of Economics, University of Pretoria.

Mokoena, T. M., Gupta, R. and Van Eyden, R. (2008b). Testing for Fractional Integration in SADC Real Exchange Rates. *Working Paper, No. 200811*, Department of Economics, University of Pretoria.

Parsley, D. C. and Wei, C. J. (1995).Purchasing Power Disparity During the Floating Rate Period: Exchange Rate Volatility, Trade Barriers and Other Culprits. *Working Paper No. 5032*, NBER.

Pesavento, E .and Rossi, B. (2006). Small sample confidence intervals for multivariate IRFs at long horizons. *Journal of Applied Econometric*s, 21(8): 1135-1155.

Rogoff, K. (1996).The purchasing power parity puzzle. *Journal of Economic Literature*, 34: 647-668.

Rossi, B. (2005).Confidence intervals for half-life deviations from the purchasing power parity. *Journal of Business and Economic Statistics*, 23(4):*432-442.*
Sarno, L. and Taylor, M. P. (2002). Purchasing power parity and the real exchange rate. *International Monetary Fund Staff Paper*s, 49:65-105.
Stock, J. (1991). Confidence intervals for the largest autoregressive root in U.S. macroeconomic time series. *Journal of Monetary Economics*, 28: 435-459.
Taylor, A. M. and Taylor, M. P. (2004). The Purchasing Power Parity Debate. *NBER Working Papers 10607*, National Bureau of Economic Research, Inc.

APPENDIX

Table 1. Confidence intervals based on standard asymptotic and ADF tests

Countries	$\hat{\alpha}(1)$	ADF	ADF-GLS	$\hat{h}_a$	$\left(\hat{h}_a{}^l, \hat{h}^u{}_a\right)$	$\hat{h}^*{}_a$	$\hat{\gamma}$
Angola	0.972	-2.37	-0.53	24.10	3.84; 44.30	24.10	0.10
Botswana	0.961	-2.29	-1.18	17.40	2.18; 32.60	16.20	0.06
Madagascar	0.966	-1.99	-1.16	20.20	0;40.40	18.10	0.01
Malawi	0.991	-0.79	0.01	81.00	0; 284.00	86.00	0.00
Mauritius	0.861	-3.95	-2.61	4.63	1.99; 7.30	4.91	0.20
Mozambique	0.949	-2.13	-0.48	13.30	0.76; 25.90	14.10	0.20
SA	0.964	-1.72	-0.68	19.00	0; 41.10	20.10	0.10
Swaziland	0.882	-2.73	-2.13	5.51	1.29; 9.73	4.66	0.01
Tanzania	0.96	-1.40	0.53	16.90	0, 40.90	-28.80	-0.30
Zambia	0.99	-1.29	1.51	70.50	0, 178.00	70.50	0.03

Note: For each bilateral real exchange rate the chapter reports the estimated test statistic of the demeaned ADF regression, the estimated coefficient of the lagged regressor $\hat{\alpha}(1)$ as defined in (9) and the DF-GLS test proposed by Elliott, Rothemberg and Stock, 1996 (ADF-GLS). The lag lengths are selected by the modified AIC criterion based on the OLS and on the GLS detrending methods proposed by Ng and Perron (2001).

Table 2. Confidence intervals based on Stock (1991)

Countries	$\hat{c}_l ; \hat{c}_u$	$c_l^{0.05}$	$h^a{}_{0.05}$	$h^a{}_{median}$	$\hat{b}(1)$	$h^*{}_{0.05}$	$h^*{}_{median}$
Angola	-19.0; 2.62	4.79	5.29	11.43	1.00	4.7	5.29
Botswana	-18.0; 2.77	19.02	21.07	47.75	0.4	17.69	19.60
Madagascar	-14.5; 3.31	20.71	23.35	70.74	0.37	18.59	20.96
Malawi	-4.15; 4.62	45.89	67.26	∞	0.72	48.71	71.38
Mauritius	*	*	*	*	*	*	*
Mozambique	-16.10;3.02	8.25	9.21	23.48	0.81	8.71	9.72
South Africa	-11.6 ;3.78	15.05	17.25	128.83	0.79	15.94	18.26
Swaziland	-23.8;1.64	4.47	4.86	9.04	1.25	3.77	4.11
Tanzania	-8.53;4.18	3.79	4.49	∞	2.98	6.49	-7.68
Zambia	-7.65;4.28	17.86	21.61	∞	0.99	17.87	21.62

Note. As in the previous Table, for each real exchange rate we ran a demeaned ADF regression. The two-sided and one-sided median unbiased confidence intervals for c, denoted respectively by $(\hat{c}_l ; \hat{c}_u)$ and $\hat{c}_{0.05}$ were obtained as discussed in Rossi (2005, Table 3). We report one-sided lower bounds for the median unbiased confidence intervals for the half-life (h) with 95 per cent coverage. Upper bounds were infinity for all currencies. h_{median} is the median unbiased estimate of the half-life (based on the median unbiased estimate of c).

Table 3. Confidence intervals based on Elliot and Stock (2001) and Hansen (1999)

Countries	Elliot and Stock (2001)		Hansen (1999)	
	$h^{a}_{0.05}$	$h^{*}_{0.05}$	$h^{a}_{0.05}$	$h^{*}_{0.05}$
Angola	89.45	68.28	6.07	6.79
Botswana	42	17.54	6.44	7.19
Madagascar	46.44	31.8	5.45	6.09
Malawi	117.01	94.79	6.45	7.2
Mauritius	1.53	1.16	6.44	7.2
Mozambique	22.31	17.2	6.42	7.17
South Africa	34.39	19.14	6.43	6.61
Swaziland	4.25	2.74	6.43	5.74
Tanzania	95.04	-124.14	-10.4	-11.61
Zambia	2092.56	1598.29	6.43	6.79

Note: We report the one-sided confidence intervals for the alternative half-lives (as described in Table 1) with 95 per cent coverage. Upper bounds are not reported because they diverted to infinity.