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Rangan Gupta

University of Pretoria

Josine Uwilingiye

University of Pretoria

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Department of Economics  
University of Pretoria  
0002, Pretoria  
South Africa  
Tel: +27 12 420 2413  
Fax: +27 12 362 5207

# TIME AGGREGATION, LONG-RUN MONEY DEMAND AND THE WELFARE COST OF INFLATION

RANGAN GUPTA\* AND JOSINE UWILINGIYE\*\*

Abstract

Two recent studies have found markedly different measures of the welfare cost of inflation in South Africa, obtained through the estimation of long-run money demand relationships using cointegration and long-horizon approaches. Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income and that long-run properties of data are unaffected under alternative methods of time aggregation, we test for the robustness of the two estimation procedures under temporal aggregation and systematic sampling. Our results indicate that the long-horizon method is more robust to alternative methods of time aggregation, and, given this the welfare cost of inflation in South Africa for an inflation target band of 3 percent to 6 percent lies between 0.15 percent and 0.41 percent.

**Keywords:** Cointegration; Long-Horizon Regression; Money Demand; Time Aggregation; Welfare Cost of Inflation.

JEL Classification: C15; C32; C43; E31; E41; E52.

## 1. INTRODUCTION

Recently, two studies by Gupta and Uwilingiye (2008a, 2008b) estimated the long-run money demand relationship for South Africa, and then, in turn, went ahead and used the interest rate elasticity and semi-elasticity to obtain the size of the welfare cost of inflation for the economy. Using the same data set, but two different approaches to estimate the long-run money demand functions, namely the cointegration procedure outlined in Johansen (1991, 1995) and the long-horizon approach proposed by Fisher and Seater (1993) respectively, the authors ended up with markedly different measures of the welfare cost of inflation. Specifically speaking, Gupta and Uwilingiye (2008b), using the long-horizon methodology, found the value to fall by more than half as that obtained by the same authors in their previous study, where the estimations were obtained from the cointegration approach. The difference between the results, essentially emanated from the smaller sizes of the interest rate elasticity and semi-elasticity obtained under the long-horizon approach relative to the cointegration procedure.

At this stage, it is important to point out that such a finding is not an exception in the welfare cost literature. Besides, the importance of sample period, the money demand specifications, i.e., double-log (Meltzer, 1963) or semi-log (Cagan, 1956), and the versions

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\* To whom correspondence should be addressed. Contact Details: Associate Professor, Department of Economics, University of Pretoria, Pretoria, 0002, Phone: + 27 12 420 3460, Fax: + 27 12 362 5207, Email: Rangan.Gupta@up.ac.za.

\*\* Contact Details: Graduate Student, Department of Economics, University of Pretoria, Pretoria, 0002, Phone: + 27 12 420 5284, Fax: + 27 12 362 5207, Email: juwilingiye@yahoo.fr.

of the monetary aggregate, the importance of the estimation procedure, namely cointegration or long-horizon, have been noted by host of authors, with the latter producing the most drastic of differences in the measures of welfare costs within an economy over identical sample periods using same data sets.<sup>1</sup> To the best of our knowledge though, no study thus far has attempted to figure out which of the two methods is more robust and ideally suited in providing the estimates of interest elasticity and semi-elasticity, and, hence, the appropriate measure of the size of the distortionary effect of inflation in the money market. But, as pointed out by Gupta and Uwilingiye (2008a, 2008b), econometric methodologies, whether based on cointegration or the long-horizon approach, deriving welfare cost measures by estimating money demand relationships provide only the lower bounds to the welfare cost of inflation. Since, such welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates, and, hence takes a partial equilibrium approach. Given that, in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output, welfare cost estimates of inflation are likely to be much higher. Hence, the ideal approach to obtaining a welfare cost estimate of inflation would be to use a dynamic general equilibrium model. Nevertheless, this line of argument does not provide an answer to the controversy regarding the true size of the distortionary effect of inflation on the money market or in a partial equilibrium framework. In this paper, we make an attempt to resolve this issue by delving into the role of time aggregation on the long-run properties of the data, and, hence, the estimates of the welfare cost of inflation based on the money market.

It must be realized that the data on the three critical variables, required in the estimation of a money demand function, namely, a monetary aggregate and measures of real income and the opportunity cost variable, are generally available at different frequencies. Specifically, the interest rate is available at the highest frequency of weeks, the monetary aggregates in monthly form, while, the real income, generally measured by real GDP, is available only at quarters. Given this, a quarterly money demand estimation would require one to convert the weekly and the monthly variables into their quarterly form. In this regard, two approaches that are generally used are either temporal aggregation or systematic sampling. Temporal aggregation simply means aggregating over the weeks (for the interest rate) or months (for the monetary aggregate) of a quarter and using the average value as the quarterly value. Systematic sampling, on the other hand, involves using a single observation from the sampling interval, such as the end of the interval observation, which in our case would be the last week or month of a specific quarter, depending on whether we are trying to convert the measure of the interest rate or the monetary aggregate,

The motivation to use the effect of time aggregation on the two methods of estimating long-run money demand relationships is derived from the recent work of Marcellino (1999). In this paper, the author indicated, theoretically and via an example, that aggregation, via temporal aggregation or systematic sampling, tends to affect only the

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<sup>1</sup> See for example Fischer (1981), Lucas (1981, 2000), Serletis and Yavari (2004, 2005, 2007), Serletis and Virk (2006) and Ireland (2007), amongst others.

short-run properties of the data, leaving the long-run aspects of the data unchanged.<sup>2</sup> Given this then, one would expect that within a specific econometric methodology, i.e., long-horizon or the cointegration approach, the estimates of the parameters in the long-run money demand relationships, log-log or semi-log, should not be significantly affected depending on whether the opportunity cost and the monetary aggregate variables were converted to their respective quarterly values based on temporal aggregation or systematic sampling. In other words, by using time aggregation, we expect to determine which of the estimates of the welfare cost of inflation via the money market, obtained through either the Johansen (1991, 1995) methodology or the Fisher and Seater (1993) approach is more robust, and, hence, should be taken more seriously. It is important to point out that, though Marcellino (1999) indicates that long-run properties of the data are virtually unchanged because of alternative sampling methods, the author does indicate the need to verify the theoretical claims with data relating to the specific question under consideration.

In this respect, we re-evaluate, based on the same data set, the results obtained by Gupta and Uwilingiye (2008a, 2008b) by using systematic sampling, instead of temporal aggregation used by the authors, to convert the measures of the monetary aggregate and the interest rate into their respective quarterly values. At this stage, it must be emphasized that we are not really trying to draw overwhelming conclusions regarding the robustness of these two alternative estimation methodologies, but, merely trying to deduce what is the appropriate size of the inflationary distortion on the welfare of the South African economy, via the money market. Given that South Africa has an inflation targeting band of 3 percent to 6 percent, the importance of knowing what is the true size of the welfare cost of inflation due to the distortion caused by the positive nominal interest rate on the money market, is of utmost importance. So, our study should not be evaluated in the light of an attempt to check for the credibility of the two methodologies under alternative sampling techniques, since the possibility of obtaining different conclusions based on a different set of variables, sample sizes and the economy(ies) concerned cannot be ignored. The remainder of the paper is organized as follows: Section 2 outlines the theoretical foundations involved in the estimation of the welfare cost of inflation based on the money market distortion. Section 3 discusses the data and the results, which includes the estimates of the parameters in the money demand functions, as well as the, measures of the welfare cost of inflation. Finally, Section 4 concludes.

## 2. THE THEORETICAL FOUNDATIONS

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the “*consumer’s surplus*”, that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if  $m(r)$  is the estimated function, and  $\psi(m)$  is the inverse function, then the welfare cost can be defined as:

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<sup>2</sup> Similar observations has also been made by Gupta and Komen (2008) while analyzing the causal relationship between the repo rate and the CPIX inflation in South Africa.

$$w(r) = \int_{m(r)}^{m(0)} \psi(x) dx = \int_0^r m(x) dx - rm(r) \quad (1)$$

where  $m$ , is the ratio of money balances to nominal income, and  $r$  measures the short-term nominal interest rate.

As seen from Equation (1), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue  $rm$  to deduce the deadweight loss.

Since the function  $m$  has the dimensions of a ratio to income, so does the function  $w$ . The value of  $w(r)$ , represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at  $r$  or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by:  $\ln(m) = \ln(A) - \eta \ln(r)$  or is  $m(r) = Ar^{-\eta}$ , the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A \left( \frac{\eta}{1-\eta} \right) r^{1-\eta} \quad (2)$$

where  $A > 0$  is a constant and  $\eta > 0$  measures the absolute value of the interest elasticity of money demand.

While, for a semi-log money demand specification i.e.,  $\ln(m) = \ln(B) - \xi r$  or  $m(r) = Be^{-\xi r}$ ,  $w(r)$  is obtained by the following formula:

$$w(r) = \frac{B}{\xi} \left[ 1 - (1 + \xi r) e^{-\xi r} \right] \quad (3)$$

where  $B > 0$  is a constant and  $\xi > 0$  measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

As can be seen from (2) and (3), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

Besides providing the theoretical general equilibrium justifications for Bailey's consumer surplus approach, Lucas (2000), also takes a compensating variation approach in estimating the welfare cost of inflation. To start off, Lucas (2000) uses Brock's (1974) perfect foresight version of Sidrauski's (1967) Money-in-the-Utility (MIU) model, and defines the welfare cost of a nominal interest rate  $r$ ,  $w(r)$ , to be the income compensation needed to leave the household indifferent between living in a steady-state with an interest rate constant at  $r$  and an otherwise identical steady-state with the interest rate of zero. With,  $w(r)$  being obtained from the solution to the following equation:

$$u[1 + w(r))y, \phi(r)y] = u[y, \phi(0)y] \quad (4)$$

Assuming a homothetic current period utility function  $(u(c,m) = \frac{1}{1-\sigma} \left[ \text{cf} \left( \frac{m}{c} \right) \right]^{1-\sigma})$ ;  $\sigma \neq 1$  and setting up the dynamic programming problem (see Lucas (2000) for details), Lucas obtains a differential equation in  $w(r)$  of the following form:

$$w'(r) = \psi \left( \frac{\phi(r)}{1 + w(r)} \right) \phi'(r) \quad (5)$$

For any given money demand function, Equation (5) can be solved numerically for an exact welfare cost function  $w(r)$ . In fact, with  $m(r) = Ar^{-\eta}$ , equation (5) can be written as:

$$w'(r) = \eta Ar^{(-\eta)} (1 + w(r))^{\frac{1}{\eta}} \quad (6)$$

yielding a solution for log-log specification

$$w(r) = -1 + \left( 1 - Ar^{1-\eta} \right)^{\frac{\eta}{\eta-1}} \quad (7)$$

While, for the semi-log model (5) yields

$$w'(r) = \left[ \xi B e^{-\xi r} \left( r + \frac{1}{\xi} \log(1 + w(r)) \right) \right] \approx \left[ \xi B e^{\xi r} \left( r + \frac{1}{\xi} w(r) \right) \right] \quad (8)$$

with a solution

$$w(r) = -e^{-\frac{B e^{-\xi r}}{\xi}} \left\{ e^{\frac{B e^{-\xi r}}{\xi}} - Ei \left[ \frac{B}{\xi} \right] + Ei \left[ \frac{B e^{-\xi r}}{\xi} \right] \right\} \quad (9)$$

and where  $Ei(x) = -\int_x^\infty \frac{e^{-t}}{t} dt$ , and one uses the principal value of the integral.

Note to calculate  $w(r)$  under Bailey's (1956) and Lucas'(2000) approaches, we use the estimates of  $\eta$  and  $\xi$  obtained from both the cointegration and long-horizon regression. While,  $A$  and  $B$  are obtained directly from the cointegrating relationships, the values of the same, under the long-horizon regression, is derived to ensure that they

match the geometric means of the data for the log-log and the semi-log specifications respectively, i.e.,  $A = \bar{m}/\left(\bar{r}\right)^{-\eta}$ ,  $B = \bar{m}/\left(e^{-\xi\bar{r}}\right)$  with  $\bar{m}$  and  $\bar{r}$  being respectively the geometric means of  $m$  and  $r$  respectively.<sup>3</sup>

### 3. DATA AND RESULTS

As in Gupta and Uwilingiye (2008a, 2008b), we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this study are the money balances ratio ( $rm3$ ), generated by dividing the broad measure of money supply (M3)<sup>4</sup> by the nominal income (nominal GDP), and short term interest rate, in our case, proxied by the 91 days Treasury bill rate ( $tbr$ ).<sup>5</sup> All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by  $lrm3$  and  $ltbr$ , respectively. Note, given that weekly values of the 91 days Treasury Bill rate is only available from the beginning of 1981, and to keep our data set identical to the one used by Gupta and Uwilingiye (2008a, 2008b), we use monthly data on both M3 and the interest rate measure to convert them into quarterly figures via systematic sampling, unlike temporal aggregation used by Gupta and Uwilingiye (2008a, 2008b).

#### INSERT TABLE 1 HERE

After obtaining all the series in their quarterly forms, as is standard in time series analysis, we start off by studying the univariate characteristics of the systematically sampled series. In this regard, we performed tests of stationarity on our variables ( $lrm3$ ,  $ltbr$  and  $tbr$ ) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips-Perron (PP) test. As in Gupta and Uwilingiye (2008a), all the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected. As all the variables were found to be non-stationary, to avoid obtaining estimates for the interest rate elasticity and semi-elasticity based on spurious regressions, the Johansen (1991, 1995)

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<sup>3</sup> For details regarding the estimation methodologies refer to Gupta and Uwilingiye (2008a, 2008b).

<sup>4</sup> See Gupta and Uwilingiye (2008) for details regarding the reasons behind the choice of M3 as the appropriate monetary aggregate for South Africa, over narrower aggregates generally used in literature. Basically, the authors indicate that the ratio of M3 to GDP is less volatile when compared to the corresponding ratios of M1 and M2 to GDP, and also M3 was used to account for the financial innovations that have taken place in the South African economy over the sample period being used of our concern.

<sup>5</sup> We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

cointegration method and the long-horizon regression proposed by Fisher and Seater (1993) was used to obtain the long-run relationships.

Before deriving the long-run money demand relationships using the Johansen (1991, 1995) methodology, a test for the stability of the VAR model, including a constant as an exogenous variable was performed. Given that no roots were found to lie outside the unit circle for the estimated VAR based on 2 lags<sup>6</sup> for both specification of money demand, we conclude that the VARs are stable and suitable for further analysis. Once the issues of stability and the optimal lag length were settled, we tested for the cointegrating relationship based on the Johansen (1991, 1995) approach. For this purpose, we included two lags in the VAR, and allowed the level data to have linear trends, but the cointegrating equations to have only intercepts. Based on the Pantula Principle, the Maximum Eigen Value tests, showed that there is one stationary relationship in the data ( $r = 1$ ) at 5 percent level of significance for both the log-log and the semi-log specifications. The results have been reported in Tables 2 and 3.<sup>7</sup> Interestingly, unlike with the temporally aggregated data used by Gupta and Uwilingiye (2008a), the trace test failed to detect any cointegrating relationship. Thus immediately, we get to see the differences in the results obtained under the two alternative sampling techniques, even though Marcellino (1999) claims that alternative forms of aggregation do not tend to affect the long-run properties of the data.

### INSERT TABLES 2 AND 3 HERE

As we are more interested in the relationship between the money balance ratio and interest rate, for both specifications,  $lrm3$  was restricted to unity. Given that we have only one cointegrating vector, the normalizing restriction on  $lrm3$  is enough to exactly identify the long-run relationship. However, as in Gupta and Uwilingiye (2008a), we encountered two serious econometric problems with this restriction. First, the restriction was not binding. Secondly, the adjustment coefficient of  $lrm3$  was insignificant under both the specifications. Imposing an additional zero restriction on the adjustment coefficient of  $lrm3$  did ensure binding restrictions, but at the cost of suggesting that the ratio of real balance to income was in fact exogenous and we should not be normalizing on  $lrm3$ . Given this, we decided to normalize on the interest rate variable, i.e.,  $lbr$  for the log-log specification and  $tbr$  for the semi-log specification. Further, with the adjustment coefficients on  $lrm3$  still being insignificant in both the models, we restricted them to

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<sup>6</sup> The choice of two lags was based on the unanimity of the Schwarz Information Criterion (SC) Hannan-Quinn (HQ) Information Criterion. Note the optimal lag length used by Gupta and Uwilingiye (2008a) based on temporally aggregated data was four. However, it must be noted that although there existed overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen tests with two lags. The authors, thus, had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship of the semi-log form

<sup>7</sup> As in Ireland (2007), we also used the Phillips-Ouliaris (1990) test for cointegration. However, unlike Ireland (2007), the test could not detect any cointegrating relationship between the chosen variables. Hence, the results of the test have been suppressed to save space. They are, however, available upon request.



zero, and obtained binding restrictions.<sup>8</sup> Note with *lrm3* now treated as the right-hand side variable, weak exogeneity of the same is what should be expected. The adjustment coefficients of *ltbr* and *lbr* were negative and significant, with them correcting for 7.1 percent and 8.4 percent of the disequilibrium in the next period, respectively. Based on the above restrictions, the interest elasticity of money demand is found to be equal to 0.2316, while, 2.4794 was the obtained value for the interest semi-elasticity of money demand.<sup>9</sup> The values of A and B, based on cointegrating relationships are, respectively, 0.3187 and 0.7153.<sup>10</sup> Note in Gupta and Uwilingiye (2008a), the estimates of the intercept and slope coefficient based on temporally aggregated data implied values of  $\mathcal{A} = 0.3323$  and that of  $\eta = 0.2088$ , while for the semi-log specification  $B = 0.6862$  and  $\xi = 2.1991$ . So, as can be seen, systematic sampling increases the values of the elasticity and semi-elasticity. However, the value of the intercepts increases for the semi-log model and falls for the log-log model. Note, we can use a t-test<sup>11</sup> to deduce whether the value of the inverse of the interest elasticity and inverse of the semi-elasticity are significantly different from its own value obtained via a different sampling technique. We find that they are significantly different at one percent level of significance. The results have been reported in Table 4.

**INSERT TABLE 4 HERE**

<sup>8</sup> Note the value of the LR test statistics for binding restrictions, both long- and short-run, for the log-log and the semi-log specifications respectively, were  $\chi^2(1) = 0.5578$  (0.4551) and  $\chi^2(1) = 0.0587$  (0.8085), where the numbers in the parenthesis indicates the probability of committing a Type I error.

<sup>9</sup> The obtained cointegrating relationships are:

(i) Log-Log:  $ltbr = -4.9388 - 4.3186(lrm3)$ , and;  
[-3.1490]

(ii) Semi-Log:  $lbr = -0.1352 - 0.4033(lrm3)$ .  
[-3.0974]

<sup>10</sup> See Gupta and Uwilingiye (2008a) for a discussion on how the values for the parameters of the money demand functions was obtained out of the estimated inverse versions of the same. The obtained cointegrating relationships were:

(i) Log-log:  $ltbr = -5.2760 - 4.7898(lrm3)$ , and;  
[-3.8797]

(ii) Semi-Log:  $lbr = -0.1713 - 0.4547(lrm3)$ .  
[-3.8888]

$$t_{n_{temp} + n_{sys} - 2} = \frac{\beta_{temp} - \beta_{sys}}{S_p \sqrt{\frac{1}{n_{temp}} + \frac{1}{n_{sys}}}}; S_p = \frac{(n_{temp} - 1)s_1^2 + (n_{sys} - 1)s_2^2}{n_{temp} + n_{sys} - 2}, \quad \text{with}$$

$\beta_i, i = temp, sys, \beta = \frac{1}{\eta}, \frac{1}{\xi}$  and  $n_i =$  number of observations in  $i$ th model. Note in our

case:  $n_{temp} = n_{sys}$ .

After having estimated the money demand relationships via the Johansen (1991, 1995) cointegration approach, we resorted to the long-horizon approach of Fisher and Seater (1993) to obtain the estimates of  $A$  and  $\eta$ , and  $B$  and  $\xi$ . Our estimate of the interest rate elasticity,  $\eta$ , yields a value of 0.1160, and that of interest semi-elasticity,  $\xi$ , equal to 1.1027.<sup>12</sup> Once we obtain the estimated values for  $\eta$  and  $\xi$  using the long-horizon regression, we then calculate the values of  $A$  and  $B$  such that the curves obtained pass through the geometric means of the data. This gives us values of  $A = 0.4221$  and  $B = 0.6166$ . Note, the corresponding values of  $A$  and  $\eta$ , and  $B$  and  $\xi$  obtained by Gupta and Uwilingiye (2008b) were 0.4255, 0.1073, 0.6035 and 1.001 respectively. As with the Johansen (1991, 1995) approach based on the systematic sampling, the values of the elasticity and semi-elasticity increases, when compared to those obtained by Gupta and Uwilingiye (2008b) under temporal aggregation. While, as above, the value of the intercepts increases for the semi-log model and falls for the log-log model. Again as with the cointegration approach, under the long-horizon approach, the t-tests on the interest elasticity and semi-elasticity across the two models under alternative sampling techniques reveal that they are statistically different at one percent level of significance. The results have been presented in Table 4. So clearly, unlike as suggested by the theoretical results of Marcellino (1999), long-run elasticities of money demand are significantly affected by alternative sampling techniques. Nevertheless, given the theoretical results of Marcellino (1999), the important aspect that needs to be determined here, would be the robustness of the welfare cost estimates based on the alternative values of the interest elasticity and semi-elasticity of the money demand functions. In other words, we want to know, which of the two estimation methods produces the least changes across the alternative sampling methods.

Having obtained the estimates for  $\eta$  and  $\xi$  and the values for  $A$  and  $B$ , both from the Johansen (1991, 1995) approach and the long-horizon regression, we are now in a position to obtain the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach and Lucas' (2000) compensating variation method. The results have been reported in Table 4. Note for the sake of comparison, in Table 5, we also present the welfare cost estimates, based on the values of  $\eta$ ,  $\xi$ ,  $A$  and  $B$ , obtained by Gupta and Uwilingiye (2008a, 2008b) using both of the above mentioned estimation methodologies. Plugging these values into the corresponding formula for the welfare cost measures, given by equations (2), (3), (7) and (9), and using the fact that the average real rate of interest<sup>13</sup> over this period was equal to 7.70 percent, so that a zero rate of inflation

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<sup>12</sup> Given that  $lm3$ ,  $lbr$  and  $tbr$  are all  $I(1)$ , the interest elasticity and semi-elasticity are obtained from an OLS estimation of the following equation:  $m_t - m_{t-k-1} = a_k + b_k [r_t - r_{t-k-1}] + e_{kt}$ , where  $m$  is the log of the ratio of money balance, while  $r$  is the log of the nominal interest rate in the log-log specification and is specified in levels for the semi-log version of the money demand. Following Serletis and Yavari (2004 and 2005),  $K$  is set equal to 30.

<sup>13</sup> Note, as in Ireland (2007), we define the real rate of return to be equal to the difference between the nominal interest rate and the inflation rate, where the inflation rate is obtained as the percentage change in the seasonally adjusted series of the CPI. In addition, the real rate of interest

would also imply a nominal rate of interest equal to 7.70 percent,, we obtain the baseline value of  $w$  under price stability. Naturally then, a value of  $r = 10.70$  corresponds to a three percent rate of inflation, while, when  $r = 13.70$ , the economy experiences a six percent inflation, and so on. So the welfare costs of inflation are evaluated by subtracting the value of  $w$  at an inflation equal to zero from the value of the same at a positive rate of inflation. Based on Tables 5, 6<sup>14</sup> and 7 the following conclusions can be drawn:

- (i) Except for the welfare costs evaluated under the compensating variation method for the double log model estimated with the Johansen (1991, 1995) cointegration approach,<sup>15</sup> systematic sampling tends to increase the welfare cost of inflation in all the other cases;
- (ii) Under the long-horizon approach, irrespective of whether we use systematic sampling or temporal aggregation and the compensating variation or the consumer surplus approach, the pattern of movement of the welfare cost of inflation as we increase the interest rate stays the same. In other words, the welfare cost estimates from the semi-log model tends to be higher than the log-log version at higher interest rates across both methods of aggregation. Further, under both systematic sampling and temporal aggregation, the compensating variation approach produces slightly higher welfare cost estimates for both type of money demand functions;
- (iii) With the cointegration approach, except for the log-log model under compensating variation with systematic sampling, welfare costs are always lower under the consumer surplus method across both sampling technique and econometric models. Again, as with the long-horizon, the semi-log version of the model tends to yield higher costs of welfare at higher interest rate across the sampling techniques;
- (iv) Over all, when we compare the two methodologies based on the percentage difference in the welfare cost estimates across the two sampling techniques, the long-horizon approach tends to produce more robust estimates of the welfare cost of inflation via the money market. In other words, for 3 percent, 6 percent, 10 percent and the 15 percent levels of inflation, the percentage change in the welfare cost of inflation for moving from temporal aggregation to systematic sampling is consistently lower under the Fischer and Seater (1993) approach in comparison to the Johansen (1991, 1995) cointegration methodology. Based on this criteria solely, we would want to conclude that the widest range of the welfare cost estimates for a target band of 3 percent to 6 percent rate of inflation, falls between 0.15 percent (obtained from the semi-log model estimated with temporal aggregation) to 0.41 percent (obtained from the log-log model estimated with systematic sampling) These numbers, in turn, are much lower than the range of 0.34 percent (obtained from the log-log and semi-log model estimated with temporal aggregation)

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was found to be stationary based on the ADF, the DF-GLS, the KPSS and the PP tests of unit roots.

<sup>14</sup> Note, we have replicated Table 1 from Gupta and Uwilingiye (2008b) as Table 6 in this paper.

<sup>15</sup> The exception arises due to the fact that even though the interest elasticity increases, the size of the fall in  $\mathcal{A}$  is such that it tends to reduce the welfare cost estimates, based on equation (7), under the Johansen (1991, 1995) approach for the log-log model, when compared to the same model estimated with temporally aggregated data.

to 0.90 percent (obtained from the log-log model estimated with systematic sampling) based on the cointegration approach under alternative methods of sampling.

#### INSERT TABLES 5, 6 AND 7 HERE

#### 4. CONCLUSIONS

Two recent studies by Gupta and Uwilingiye (2008a, 2008b) have found markedly different measures of the welfare cost of inflation in South Africa, obtained through the estimation of long-run money demand relationships using cointegration and long-horizon approaches, respectively. Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income, and that long-run properties of data are unaffected under alternative methods of time aggregation (Marcellino, 1999), in this paper, we tested for the robustness of the two estimation procedures under temporal aggregation and systematic sampling. Our results indicate that the long-horizon method is more robust, in terms of lower percentage change in the welfare cost measures across the two alternative methods of time aggregation, And, given this the welfare cost of inflation in South Africa for an inflation target band of 3 percent to 6 percent lies between 0.15 percent and 0.41 percent. Based on these set of results, we can, thus, conclude that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least when compared to a Friedman (1969)-type deflationary rule of zero nominal rate of interest.

It is, however, important to point out that, in this study, we are only looking at welfare cost of inflation using a partial equilibrium approach. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Hence, the path ahead should involve obtaining the size of the welfare cost of inflation using a dynamic general equilibrium endogenous growth model. Then only, we will be able to deduce whether there are possibly larger gains of reducing the inflation target below 3 percent.

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Table 1: Unit Root Tests (Systematic Sampling)

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t \tau_\mu$	$\tau_t \tau_\mu$	
LRM3	$\tau_t$	-0.04	2.57	-0.03	0.31	-0.30	Non- stationary
	$\tau_\mu$	-0.22	0.05	-0.27	0.32***	-0.62	
	$\tau$	-0.69		-0.69			
D(LRM3)	$\tau_t$	-13.71***	94.04***	-13.71***	0.09***	-13.58***	Stationary
	$\tau_\mu$	-13.31***	177.13***	-13.30***	0.56*	-13.35***	
	$\tau$	-13.31		-13.31***			
LTBR	$\tau_t$	-2.61	11.89***	-2.29	0.29	-2.52	Non- stationary
	$\tau_\mu$	-2.45	17.21***	-2.28	0.90	-1.35	
	$\tau$	-0.76	-2.28	-0.82			
D(LTBR)	$\tau_t$	-8.60***	37.01	-8.61***	0.03***	-8.52***	Stationary
	$\tau_\mu$	-8.60***	73.93	-8.60***	0.09***	-7.84***	
	$\tau$	-8.62***		-8.62***			
TBR	$\tau_t$	-2.50	7.02***	-2.32	0.27	-2.47	Non- stationary
	$\tau_\mu$	-2.45	10.25***	-2.30	0.73*	-1.63*	
	$\tau$	-0.86		-0.78			
D(TBR)	$\tau_t$	-9.60***	46.04***	-9.62***	0.03***	-9.65***	Stationary
	$\tau_\mu$	-9.60***	92.08***	-9.62***	0.08***	-9.48***	
	$\tau$	-9.62		-9.65***			

\*\*\*] indicates statistical significance at 10(5)[1] percent level.

Table 2: Estimation and Determination of Rank (Log-Log)

Null hypothesis	Alternative hypothesis	Test statistic	0.05 critical value	Prob. **
<i>Trace Statistic</i>				
$r=0$	$r=1$	15.15050	15.49471	0.0563
$r=1$	$r=2$	0.157021	3.841466	0.6919
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
<i>Maximum Eigenvalue Statistic</i>				
$r=0$	$r=1$	14.99348	14.26460	0.0383
$r=1$	$r=2$	0.157021	3.841466	0.6919
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Table 3: Estimation and Determination of Rank (Semi-Log)

Null hypothesis	Alternative hypothesis	Test statistic	0.05 critical value	Prob. **
<i>Trace Statistic</i>				
$r=0$	$r=1$	14.88209	15.49471	0.0617
$r=1$	$r=2$	0.115014	3.841466	0.7345
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
<i>Maximum Eigenvalue Statistic</i>				
$r=0$	$r=1$	14.76707	14.26460	0.0416
$r=1$	$r=2$	0.115014	3.841466	0.7345
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Table 4: Across Model Test on the Interest Elasticity and Semi-Elasticity

Johansen (1991, 1995)			
Model Specification	Test	t-Statistic	Conclusion
Log-log	$1/\eta^{temp} = 1/\eta^{syst}$	-2.53657***	Reject Ho
Semi-log	$1/\zeta^{temp} = 1/\zeta^{syst}$	-513.498***	Reject Ho
Fisher and Seater (1993)			
Log-log	$1/\eta^{temp} = 1/\eta^{syst}$	282.5805556***	Reject Ho
Semi-log	$1/\zeta^{temp} = 1/\zeta^{syst}$	31.0631031***	Reject Ho

\*\*\* indicates statistical significance at the 1 percent level.

Critical value of  $t_{334}$  at 1 percent level of significance is 2.32.

Table 5: Welfare cost estimates (Systematic sampling).

Inflation Rate	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0039	0.0039	0.0019	0.0017	0.0023	0.0040	0.0021	0.0017
6	0.0075	0.0087	0.0038	0.0039	0.0046	0.0090	0.0041	0.0040
10	0.0119	0.0162	0.0062	0.0075	0.0077	0.0168	0.0068	0.0077
15	0.0173	0.0270	0.0092	0.0129	0.0116	0.0283	0.0101	0.0135

Table 6: Welfare cost estimates (Temporal aggregation).

Inflation Rate	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0034	0.0034	0.0018	0.0015	0.0037	0.0035	0.0019	0.0016
6	0.0067	0.0076	0.0035	0.0035	0.0072	0.0079	0.0037	0.0036
10	0.0108	0.0143	0.0057	0.0068	0.0117	0.0149	0.0062	0.0070
15	0.0156	0.0241	0.0084	0.0118	0.0172	0.0251	0.0092	0.0123

Source: Table 1, Gupta and Unwilingye (2008b).

Table 7: Percentage Change in Welfare Cost Estimate Under Temporal Aggregation and Systematic Sampling

Inflation rate	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	14.71	14.71	5.56	13.33	37.84	14.29	10.53	6.25
6	11.94	14.47	8.57	11.43	36.11	13.92	10.81	11.11
10	10.19	13.29	8.77	10.29	34.19	12.75	9.68	10.00
15	10.90	12.03	9.52	9.32	32.56	12.75	9.78	9.76

Values Computed Using: 
$$\left( \frac{(w^{syst} - w^{temp})}{w^{temp}} \right) \times 100$$