Forecasting Volatility and Co-volatility of Crude Oil and Gold Futures: Effects of Leverage, Jumps, Spillovers, and Geopolitical Risks*

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Abstract

To forecast the covariance matrix for the returns of crude oil and gold futures, this paper examines the effects of leverage, jumps, spillovers, and geopolitical risks by using their respective realized covariance matrices. To guarantee the positive definiteness of the forecasts, we consider the full BEKK structure on the conditional Wishart model. By the specification, we can flexibly divide the direct and spillover effects of volatility feedback, negative returns, and jumps. The empirical analysis indicates the benefits of accommodating the spillover effects of negative returns, and the geopolitical risks indicator for modeling and forecasting the covariance matrix.

Keywords: Commodity Markets; Co-volatility; Forecasting; Geopolitical Risk; Jump; Leverage Effects; Spillover Effects; Realized Covariance.

JEL: C32, C33, C58, Q02.

1 Introduction

Our work is different form Asai, Gupta, and McAleer (2019), as Asai, Gupta, and McAleer (2019) exclude spillover effects and analysis of geopolitical indicator, which is the main purpose of the current paper.

Section 2 explains the technique of Koike (2016) for disentangling quadratic covarianton to continuous part and jump variation, and develop conditional Wishart models for guaranteeing positive definiteness of forecasts of covariance matrix. Section 2 also discusses the estimation of the models and tests for the effects of leverage, jump. spillover, and geopolitical risks. Section 3 shows the empirical results using the high frequency data of prices of crude oil and gold futures, and Section 4 gives some concluding remarks.

2 Econometric Methodology

2.1 Quadratic Covariation and Integrated Co-Volatility

Let X_s^* and Y_s^* denote latent log-prices at time s for two futures X and Y. Define $p^*(s) = (X_s^*, Y_s^*)'$, and let $W(s)$ and $Q(s)$ denote bivariate vectors of independent Brownian motions and counting processes, respectively. Let $K(s)$ be the 2×2 matrix process controlling the magnitude and transmission of jumps, such that $K(s)dQ(s)$ is the contribution of the jump process to the price diffusion. Under the assumption of a Brownian semimartingale with finite-activity jumps (BSMFAJ), $p^*(s)$ follows the stochastic differential equation:

$$dp^*(s) = \mu(s)ds + \sigma(s)dW(s) + K(s)dQ(s), \quad 0 \leq s \leq T \tag{1}$$

where $\mu(s)$ is a 2×1 vector of continuous and locally-bounded variation processes, and $\sigma(s)$ is the 2×2 matrix, such that $\Sigma(s) = \sigma(s)\sigma'(s)$ is positive definite.

Assume that the observable log-price process is the sum of the latent log-price process in equation (1) and the microstructure noise process. Denote the log-price process as $p(s) = (X_s, Y_s)'$. Consider non-synchronized trading times of the two assets, and let $\mathcal{T}$ and Θ be the set of transaction times of X and Y, respectively. Denote the counting process governing the number of observations traded in assets X and Y up to time T as n_T and m_T, respectively. By definition, the trades in X and Y occur at times $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_{n_T}\}$ and $\Theta = \{\theta_1, \theta_2, \ldots, \theta_{m_T}\}$, respectively. For convenience, the opening and closing times are set as $\tau_1 = \theta_1 = 0$ and $\tau_{n_T} = \theta_{m_T} = T$, respectively.

The observable log-price processes are given by:

$$X_{\tau_i} = X^*_{\tau_i} + \varepsilon^X_{\tau_i} \quad \text{and} \quad Y_{\theta_j} = Y^*_{\theta_j} + \varepsilon^Y_{\theta_j}, \tag{2}$$

where $\varepsilon^X \sim \text{iid}(0, \sigma^2_{\varepsilon X})$, $\varepsilon^Y \sim \text{iid}(0, \sigma^2_{\varepsilon Y})$, and $(\varepsilon^X, \varepsilon^Y)$ are independent of (X, Y).

Define the quadratic covariation (QCov) of the log-price process over $[0, T]$ as:

$$\text{QCov} = \underset{\Delta\to\infty}{\text{plim}} \sum_{i=1}^{\lfloor T/\Delta \rfloor} [p(i\Delta) - p((i-1)\Delta)]\, [p(i\Delta) - p((i-1)\Delta)]'\,. \tag{3}$$

Then we obtain:

$$\text{QCov} = \int_0^T \Sigma(s)ds + \sum_{0<s\leq T} K(s)K'(s). \tag{4}$$

The first term on the right-hand side of (4) is the integrated co-volatility (ICov) matrix over $[0, T]$, while the second term is the matrix of jump variability. We are interested in disentangling these two components from the estimates of QCov for the purpose of forecasting QCov.

There are several estimators for QCov and Icov (see the survey in Asai and McAleer (2017)). Among them, we use the estimators of Christensen, Kinnebrock, and Podolskij (2010) for QCov and Koike (2016) for ICov, respectively. The estimator of Koike (2016) is consistent under non-synchronized trading times, jumps and microstructure noise for the bivariate process in (2). Note that the realized kernel (RK) estimator of Barndorff-Nielsen et al. (2011) is positive (semi-)definite and robust to microstructure noise under non-synchronized trading times. However, the robustness to jumps is still an open and unresolved issue for the multivariate RK estimator.

As in Asai and McAleer (2017), we also apply thresholding of Bickel and Levina (2008) to guarantee the positive (semi-)definiteness of the estimators. Denote the estimators of QCov, ICov and jump component at day t as Ω_t, C_t and J_t, respectively. Note that J_t is close to $\Omega_t - C_t$, as it is obtained by thresholding the latter. Thus the thresholding produces a remaining error matrix, $E_t = \Omega_t - C_t - J_t$, for the realization of equation (4), where Ω_t, C_t and J_t are positive (semi-)definite. We exclude E_t in the empirical analysis.

In addition, we also disentangle the observed return series into continuous and jump components by applying the technique of Aït-Sahalia and Jacod (2012). For purposes of notation, define the return for X as $r^x_t = x_t - x_{t-1}$. Denote the continuous and jump components of the return as rc^x_t and rj^x_t, respectively. In the empirical analysis, we use returns for examining leverage and co-leverage effects on volatility and co-volatility, respectively. When the models include the ICov and jump components, we use the continuous return rather than the observed return itself.

2.2 Conditional Wishart Model

Let $\Omega_{t-h+1:t}$ denote the h-horizon average, defined by:

$$\Omega_{t-h+1:t} = \frac{1}{h}\left(\Omega_t + \cdots + \Omega_{t-h+1}\right),$$

where $h = 5$ and $h = 22$ give the weekly and monthly averages, respectively. In order to examine the effects of leverage, jump, and spillover effects, we consider the following structure for $\Omega_{t-h+1:t}$ $(h = 1, 5, 22)$:

$$\Omega_{t-h+1:t} = (1/\nu)H_t^{1/2}W_tH_t^{1/2}, \quad W_t \sim \text{iid } W_2(\nu, I_2), \tag{5}$$

where H_t is an $m \times m$ positive definite matrix, $H_t^{1/2}$ is a square root of a matrix defined by the eigenvalue decomposition, and $W_m(\nu, A)$ denotes the m-dimensional Wishart distribution with degrees-of-freedom parameter, ν, and $m \times m$ scale matrix, A. By the specification, $\Omega_{t-h+1:t}|H_t \sim W_2(\nu, (1/\nu)H_t)$, which yields $E(\Omega_{t-h+1:t}|H_t) = H_t$ and $V(\Omega_{ij,t-h+1:t}|H_t) = (1/\nu)(H_{ij,t}^2 + H_{ii,t}H_{jj,t})$ $(i, j = 1, 2)$.

For specifying H_t, we accommodate the effects of leverage, jumps, spillovers, and geopolitical risks. Define $\boldsymbol{h}_t = \text{vec}(H_t)$, $\boldsymbol{\omega}_{t-h:t-1} = \text{vec}(\Omega_{t-h:t-1})$, $\boldsymbol{c}_{t-h:t-1} = \text{vec}(C_{t-h:t-1})$ $(h = 1, 5, 22)$, and $\boldsymbol{j}_t = \text{vec}(J_t)$. For the structure of H_t, we consider six kinds of specifications:

$$\boldsymbol{h}_t = \boldsymbol{\kappa} + A_d^*\boldsymbol{\omega}_{t-1} + A_w^*\boldsymbol{\omega}_{t-5:t-1} + A_w^*\boldsymbol{\omega}_{t-22:t-1}, \tag{6}$$

$$\boldsymbol{h}_t = \boldsymbol{\kappa} + A_d^*\boldsymbol{\omega}_{t-1} + A_w^*\boldsymbol{\omega}_{t-5:t-1} + A_w^*\boldsymbol{\omega}_{t-22:t-1} + A_a^*\boldsymbol{\xi}_{t-1}, \tag{7}$$

$$\boldsymbol{h}_t = \boldsymbol{\kappa} + A_d^*\boldsymbol{\omega}_{t-1} + A_w^*\boldsymbol{\omega}_{t-5:t-1} + A_w^*\boldsymbol{\omega}_{t-22:t-1} + A_a^*\boldsymbol{\xi}_{t-1} + \boldsymbol{\lambda} g_{t-1}, \tag{8}$$

$$\boldsymbol{h}_t = \boldsymbol{\kappa} + A_d^*\boldsymbol{c}_{t-1} + A_w^*\boldsymbol{c}_{t-5:t-1} + A_w^*\boldsymbol{c}_{t-22:t-1} + A_j^*\boldsymbol{j}_{t-1}, \tag{9}$$

$$\boldsymbol{h}_t = \boldsymbol{\kappa} + A_d^*\boldsymbol{c}_{t-1} + A_w^*\boldsymbol{c}_{t-5:t-1} + A_w^*\boldsymbol{c}_{t-22:t-1} + A_j^*\boldsymbol{j}_{t-1} + A_a^*\boldsymbol{n}_{t-1}, \tag{10}$$

$$\boldsymbol{h}_t = \boldsymbol{\kappa} + A_d^*\boldsymbol{c}_{t-1} + A_w^*\boldsymbol{c}_{t-5:t-1} + A_w^*\boldsymbol{c}_{t-22:t-1} + A_j^*\boldsymbol{j}_{t-1} + A_a^*\boldsymbol{n}_{t-1} + \boldsymbol{\lambda} g_{t-1}, \tag{11}$$

where A_i^* $(i = d, w, m, j, a)$ are 4×4 matrices of parameters, $\boldsymbol{\kappa}$ and $\boldsymbol{\lambda}$ 4×1 vectors of parameters, g_t is a geopolitical risk indicator, $\boldsymbol{\xi}_t = \text{vec}(\zeta_t\zeta_t')$, $\zeta_t = (r_t^x\mathbf{1}(r_t^x < 0), r_t^y\mathbf{1}(r_t^y < 0))'$, $\boldsymbol{n}_t = \text{vec}(\eta_t\eta_t')$, $\eta_t = (rc_t^x\mathbf{1}(rc_t^x < 0), rc_t^y\mathbf{1}(rc_t^y < 0))'$, and $\mathbf{1}(z < 0)$ is the indicator function which takes one if $z < 0$, and zero otherwise.

For the parameters, we consider the BEKK (Baba, Engle, Kroner, and Kraft) specification (see Baba et al. (1985) and Engle and Kroner (1995)) in order to guarantee positive definiteness of H_t. We suppress the subscript i of A_i^* $(i = d, w, m, j, a)$. In the BEKK specification, A^* takes the following form:

$$A^* = \sum_{k=1}^{4}(A_k \otimes A_k), \quad A_1 = \begin{pmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 0 \\ a_{21,2} & a_{22,2} \end{pmatrix},$$
$$A_3 = \begin{pmatrix} 0 & a_{12,3} \\ 0 & a_{22,3} \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0 & 0 \\ 0 & a_{22,4} \end{pmatrix},$$

with $a_{22,k} > 0$. Proposition 2.3 of Engle and Kroner (1995) shows that there is no equivalent representation for the set of A_k matrices. For the remaining parameters:

$$\boldsymbol{\kappa} = \mathrm{vec}(KK'), \quad K = \begin{pmatrix} k_{11} & 0 \\ k_{21} & k_{22} \end{pmatrix}, \quad \boldsymbol{\lambda} = \mathrm{vec}(\Lambda), \quad \Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{21} \\ \lambda_{21} & \lambda_{22} \end{pmatrix},$$

with $k_{11} > 0$. By the specification, we obtain the alternative form of (11):

$$\begin{aligned} H_t =& KK' + A_d C_{t-1} A_d' + A_w C_{t-5:t-1} A_w' + A_m C_{t-22:t-1} A_m' \\ &+ A_j J_{t-1} A_j' + A_a \eta_{t-1} \eta_{t-1}' A_a' + \Lambda g_t. \end{aligned} \tag{12}$$

The BEKK structure (12) implies that H_t for (6), (7), (9), and (10) is always positive definite. When $g_t \geq 0$, the positive definiteness for (8) and (11) depends on the values of Λ. In order to reserve possibility for the (i, i)th element of Λ to take negative values, we do not impose restrictions such as $\Lambda = \check{\lambda}\check{\lambda}'$ with a 2×1 vector $\check{\lambda}$.

The models in (5) with (6), (9), and (10) are multivariate extensions of the volatility forecasting models of Andersen, Bollerslev, and Diebold (2007) and Corsi et al. (2010). The model (5) and (6) with $h = 1$ belongs to the conditional autoregressive Wishart (CAW) model of Golosnoy, Gribisch, and Liesenfeld (2012). As this specification uses the heterogeneous autoregression (HAR) introduced by Corsi (2009), we refer to equations (5) and (6) as the heterogeneous autoregressive conditional Wishart (HAR-CW) model. Equation (7) includes the spillovers of asymmetric effects, as in Kroner and Ng (1997), which we refer to as the HAR-A-CW model. Model (5) with (9) decomposes the past values of Ω_t into those of C_t and J_t, and accommodates the HAR terms of Ct and the spillover effects of jump variation and co-variation. The specification in (10) adds to (9) the spillovers of the leverage effects. We refer to equations (5) with (9) and (10) as the HAR-TCJ-CW and HAR-TCJA-CW models, respectively. Note that we use continuous returns for the HAR-TCJA-CW model, corresponding to the ICov in its specification. As (8) adds to (7) the effect of geopolitical risks, we refer to the model in (5) with (8) as the HAR-AG-CW model. In the same manner, we refer to the model in (5) and (10) as the HAR-TCJAG-CW model.

Instead of the above specifications, we may consider a regression model for $\acute{\boldsymbol{\omega}}_{t-h+1:t} = \text{vech}(\Omega_{t-h+1:t})$ as:

$$\acute{\boldsymbol{\omega}}_{t-h+1:t} = \boldsymbol{\kappa}^{\dagger} + A_d^{\dagger}\acute{\boldsymbol{c}}_{t-1} + A_w^{\dagger}\acute{\boldsymbol{c}}_{t-5:t-1} + A_w^{\dagger}\acute{\boldsymbol{c}}_{t-22:t-1} + A_j^{\dagger}\acute{\boldsymbol{j}}_{t-1} + A_a^{\dagger}\acute{\boldsymbol{n}}_{t-1} + e_t, \tag{13}$$

where e_t is the 3×1 vector of disturbance, $\acute{\boldsymbol{c}}_{t-h:t-1} = \text{vech}(C_{t-h:t-1})$ $(h = 1, 5, 22)$, $\acute{\boldsymbol{j}}_t = \text{vech}(J_t)$, $\acute{\boldsymbol{n}}_t = \text{vech}(\eta_t \eta_t')$, $\boldsymbol{\kappa}^{\dagger}$ is the 3×1 vector, and $A_i^{\dagger}$ $(i = d, w, m, j, a)$ are 3×3 matrices. The number of parameters in (13) is the same as the number of free parameters in (10). In other words, the BEKK structure produces a positive definite matrix using no additional parameters.

2.3 Estimation of CW Models and Tests for Effects of Spillover and Geopolitical Risks

Define $\boldsymbol{\theta} = (\nu, \text{vech}(K)', \text{vec}(A_d)', \text{vec}(A_w)', \text{vec}(A_m)', \text{vec}(A_j)', \text{vec}(A_a)')'$ for the HAR-TCJA-CW model. The log-likelihood function is given by:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{T} l_t, \tag{14}$$

where

$$\begin{aligned} l_t = &-\log \Gamma_m(\nu/2) + \frac{m\nu}{2}\log(\nu/2) - \frac{\nu}{2}\log|H_t| \\ &+ \frac{\nu - m - 1}{2}\log|\Omega_{t-h+1:t}| - \frac{\nu}{2}\text{tr}\left(H_t^{-1}\Omega_{t-h+1:t}\right) \end{aligned}$$

with $m = 2$, and $\Gamma_m(z)$ is the multivariate gamma function defined by:

$$\Gamma_m(z) = \pi^{m(m-1)/4} \prod_{j=1}^{m} \Gamma\left(z + (1-j)/2\right).$$

We obtain the maximum likelihood estimator, $\hat{\boldsymbol{\theta}}$, by maximizing the log-likelihood function (14).

We can show the asymptotic normality of the maximum likelihood estimator for the conditional Wishart model, by simplifying the results of Zhou, Zhu, and Li (2018) for the matrix-F distribution. Note that we are unable to use the approach of McAleer et al. (2008) based on the vector random coefficient (VRC) process for the BEKK-GARCH model, as it is hard to derive VRC representation for the conditional Wishart model (see also discussions of McAleer (2019)).

For examining the effects of leverage, jumps, spillovers, and a geopolitical indicator, we use equations (6)-(11). Define the null hypotheses as $H_0^{kl,v}$: $a^*_{k^2l^2,d} = a^*_{k^2l^2,w} = a^*_{k^2l^2,m} = 0$, $H_0^{kl,j}$: $a^*_{k^2l^2,j} = 0$, $H_0^{kl,a}$: $a^*_{k^2l^2,a} = 0$, and $H_0^{kl,g}$: $\lambda_{kl} = 0$ for $k, l = 1, 2$. We divide $H_0^{kl,v}$, $H_0^{kl,j}$, and

$H_0^{kl,a}$ $(k, l = 1, 2)$ into two categories: one is the test for the 'direct effect' with $k = l$, while the other is the test for 'spillover effects' with $k \neq l$. In each category, we test the effects of volatility feedback, jumps, and negative returns. For the geopolitical risk, $H_0^{kl,g}$ indicates that there is no effect on the (k, l)th element of H_t.

Table 1 shows the null hypotheses to be tested in our empirical analysis. We carry out Wald tests for the hypotheses $H_0^{kl,v}$ $(k, l = 1, 2)$, while we use t tests for the remaining hypotheses. The Wald statistics have the asymptotic $\chi^2(3)$ distribution under the respective null hypotheses.

3 Empirical Analysis

We consider the effects of jumps, leverage, spillovers, and geopolitical risk for two futures contracts traded on the New York Mercantile Exchange (NYMEX), namely West Texas Intermediate (WTI) Crude Oil and Gold. The trades at NYMESX cover 24 hours with the CME Globex system. Using the future prices every 1-minute, we calculate Ω_t, C_t, and J_t by the approach of Koike (2016), as the estimates of the matrices of quadratic co-variation, integrated co-volatility, and the matrix of jump co-variations, respectively. We also calculate the corresponding open-close returns and their continuous components, r_t and rc_t, respectively, for the two futures.

The sample period covers September 27, 2009 to May 25, 2017, giving 1978 observations. Table 2 indicates the descriptive statistics of r_t and C_t. The empirical distribution of the returns is highly leptokurtic, while that of volatility is skewed to the right, with heavy tails. Figures 1 and 2 display the times series plots of returns and the estimates of quadratic variation, integrated volatility, and jump variation. The values jump variations are relatively small for most days, but there exist obvious non-negligible variations. Figure 3 illustrates the product of two returns, the estimates of quadratic co-variation, integrated co-volatility, and jump co-variability. Figure 3 implies the time series dependence on QCov and ICov can be found, especially for the year 2011.

For the geopolitical risk, we use the daily share of the geopolitical risk indicator suggested by Caldara and Iacviello (2018). Figure 4 shows the time series plot of the geopolitical risk indicator.

In the following, we consider two kinds of periods for estimation and forecasting. Period 1 starts on October 27, 2009 and ends on August 18, 2015, with 1500 observations, while Period 2 covers October 4, 2011 to May 25, 2017, with 1456 observations. The first 1000 observations for each period are used for estimation, while the remaining observations are retained for forecasting. We treat crude oil futures as the first variable, and gold futures as the second.

Table 3 reports AIC and BIC for six models with respect to six kinds of covariance matrices over 2 periods. For the daily covariance model, AIC chose the HAR-TCJA (HAR-TCJAG) model for Period 1 (Period 2), while the HAR-A-CW model has the smallest BIC. Regarding the weekly covariance model, AIC selected HAR-A-CW, and HAR-CW has the smallest BIC for both periods. For the monthly covariance model, the HAR-A-CW has the smallest AIC and BIC for the both periods. The results indicate that including leverage effects often improves the information criterion, and that accommodating the geopolitical risk indicator can improve the daily covariance model.

Among the hypotheses listed in Table 1, we first examine the direct effects on volatility from its past volatility, jumps, and negative returns. The null hypotheses are $H_0^{kk,v}: a^*_{k^2k^2,d} = a^*_{k^2k^2,w} = a^*_{k^2k^2,m} = 0$, $H_0^{kk,j}: a^*_{k^2k^2,j} = 0$ and $H_0^{kk,a}: a^*_{k^2k^2,a} = 0$ $(k = 1, 2)$, with the parameters defined in equations (6)-(11). Table 4 shows the results for the direct tests for the daily covariance model. While the direct effects from past volatility and returns are significant at the five percent level, the effects of past jumps are insignificant. The result is against the findings by Asai, Gupta, and McAleer (2019), and might be caused by the structure for guaranteeing positive definiteness of the covariance matrix. The sign of the t statistics for testing $H_0^{kk,a}: a^*_{k^2k^2,a} = 0$ $(k = 1, 2)$ indicates that a negative return increases future volatility, showing the existence of leverage effects.

Tables 5 and 6 report the results for the direct tests for the weekly and monthly covariance models, respectively. All test statistics reject the null hypotheses of no effects at the five percent level, except for $a^*_{44,a} = 0$ for the second period of the monthly covariance models with jumps. Note that, even for this case, $a^*_{44,a}$ is significant for the models without jumps. The results indicate that the effects of jumps and negative returns are positive and significant in explaining future volatility.

Second, we examine the spillover effects. Tables 7-9 show the results for the tests for spillover effects for the daily, weekly, and monthly covariance models, respectively. For most of the cases, the test statistics are insignificant at the five percent level. The exceptions are found in several cases for the null hypothesis $H_0^{12,a}: a^*_{14,a} = 0$. The result that $a^*_{14,a} > 0$ shows that a negative return of gold futures increases the one-step-ahead volatility of crude oil futures. When the fluctuations in returns of the gold futures are high, the negative returns may affect the volatility of crude oil futures.

Third, we investigate the effects of geopolitical risks based on the HAR-AG-CW and HAR-

TCJAG-CW models. Table 9 shows the t-test statistics and P-values. For the daily covariance model, λ_{11} is significant for three of four cases, but the sign is indeterminate. Regarding the weekly covariance model, λ_{11} is significant for the HAR-AG-CW model. For the monthly covariance model, γ_{11} is significant for three of four cases, and λ_{12} is significant in one case. In Table 9, λ_{22} is insignificant in all cases. The result $\lambda_{11} \neq 0$ indicates that the geopolitical risk tends to affect the future volatility of crude oil.

For in-sample estimation, there is no spillover effects from volatility and jumps. Instead, we often found spillovers from negative returns of gold futures to volatility of crude oil futures. Regarding the geopolitical risk indicator, the empirical results show that part of the variation of the future volatility of crude oil futures can be explained by the indicator.

We compare out-of-sample forecasts of six kinds of CW models. We estimate each model using the first 1000 observations, and obtain a forecast, $\hat{\Omega}^{f}_{1001}$ We re-estimate each model fixing the sample size at 1000, and obtain new forecasts based on the updated parameter estimates. For comparing the out-of-sample forecasts, we extend the idea of Patton (2011) for univariate volatility models. Patton (2011) examined the functional form of the loss function for comparing volatility forecasts using imperfect volatility proxies, such that the forecasts are robust to the presence of noise in the proxies.

As an extension of Patton (2011), we state that a loss function is "robust" if the ranking of any two forecasts of the co-volatility matrix, $\hat{\Omega}^{(1)}_{T+j}$ and $\hat{\Omega}^{(2)}_{T+j}$, by expected loss is the same whether the ranking is performed using the true covariance matrix or an unbiased volatility proxy, $\acute{\Omega}_t$. In the univariate case, Patton (2011) showed that squared forecast error and quasi-likelihood type loss functions are robust to the forecast error and the standardized forecast error, respectively. We consider their multivariate counterparts, as follows:

$$\text{MSFE} : L(\acute{\Omega}_t, \hat{\Omega}^{(i)}_{T+j}) = \text{tr}\left[\left(\hat{\Omega}^{(i)}_{T+j} - \acute{\Omega}_t\right)^2\right], \tag{15}$$

$$\text{QLIKE} : L(\acute{\Omega}_t, \hat{\Omega}^{(i)}_{T+j}) = \text{tr}\left(\acute{\Omega}_t^{-1}\hat{\Omega}^{(i)}_{T+j}\right) - \log\left|\acute{\Omega}_t^{-1}\hat{\Omega}^{(i)}_{T+j}\right| - m, \tag{16}$$

which are expected to be robust to the forecast error, $\hat{\Omega}^{(i)}_{T+j} - \acute{\Omega}_t$, and the standardized forecast error, $\acute{\Omega}_t^{-1}\hat{\Omega}^{(i)}_{T+j}$, respectively.

Table 11 shows the results of MSFE and QLIKE for the six models for 2 periods and the total period. For forecasting the future covariance matrices, MSFE and QLIKE selected the models without jumps. For forecasts of the daily and weekly covariance models, the HAR-A-CW model often has the smallest MSFE and QLIKE. Regarding the monthly covariance model, the HAR-CW and HAR-A-CW models are competitive, but the differences are negligible.

4 Concluding Remarks

In this paper, we investigated the effects of leverage, jumps, spillovers, and geopolitical risks on forecasting the covariance matrix for the returns of crude oil and gold futures. For this purpose, we considered the Conditional Wishart (CW) model with a full BEKK specification to guarantee positive definiteness of the covariance matrix and flexibility of the parameters simultaneously. The specification enables us to distinguish the direct and spillover effects of volatility feedbacks, negative returns, and jumps. In the empirical analysis, we used five-minute data of crude oil and gold futures to estimate the quadratic covariation, the continuous covariance matrix, and the matrix of variations of jumps.

It was found that: (i) there is no spillover effects from volatility and jumps; (ii) negative returns of gold increase the future volatility of crude oil, (iii) it is better to use previous values of the quadratic covariation than its continuous and jump components for forecasting the covariance matrix; and (iv) accommodating the geopolitical risk indicator often improves the forecasts of the future volatility of crude oil. The CW model can be improved by combining the structures of the full BEKK model and the diagonal GARCH model of Ding and Engle (2001). In other words, we may use the structure of the diagonal GARCH model for volatility feedbacks (and jumps), while we consider the full BEKK specification for the spillover effects arising from the negative returns. This remains a topic for future research.

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Table 1: Null Hypotheses for Testing the Direct, Spillover, and Geopolitical Risk Effects

$H_0^{11,v}$	No direct effects from volatility of X to volatility of X
$H_0^{22,v}$	No direct effects from volatility of Y to volatility of Y
$H_0^{11,j}$	No direct effects from jump variation of X to volatility of X
$H_0^{22,j}$	No direct effects from jump variation of Y to volatility of Y
$H_0^{11,a}$	No direct effects from negative return of X to volatility of X
$H_0^{22,a}$	No direct effects from negative return of Y to volatility of Y
$H_0^{12,v}$	No spillover effects from volatility of Y to volatility of X
$H_0^{21,v}$	No spillover effects from volatility of X to volatility of Y
$H_0^{12,j}$	No spillover effects from jump variation of Y to volatility of X
$H_0^{21,j}$	No spillover effects from jump variation of X to volatility of Y
$H_0^{12,a}$	No spillover effects from negative return of Y to volatility of X
$H_0^{21,a}$	No spillover effects from negative return of X to volatility of Y
$H_0^{11,g}$	No effect from geopolitical risk index to volatility of X
$H_0^{22,g}$	No effect from geopolitical risk index to volatility of Y
$H_0^{12,g}$	No effect from geopolitical risk index to co-volatility of (X, Y)

Table 2: Descriptive Statistics of Returns, Volatility and Co-Volatility

Stock	Mean	Std.Dev.	Skew.	Kurt.
Return				
Crude Oil	0.0256	0.6824	−0.4784	8.0378
Gold	0.0185	1.4887	−0.0573	5.0225
Volatility				
Crude Oil	0.4238	0.5070	7.2211	90.688
Gold	1.5152	1.6510	3.3689	19.071
Co-Volatility				
(Crude Oil, Gold)	0.1368	0.2844	−0.7851	37.359

Note: The sample period is from September 27, 2009 to May 25, 2017.

Table 3: Model Selection via Information Criteria

Model	Daily Cov. AIC	Daily Cov. BIC	Weekly Cov. AIC	Weekly Cov. BIC	Monthly Cov. AIC	Monthly Cov. BIC
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	−355.78	10.49	−7437.03	−7070.75†	−14182.25	−13815.97
HAR-A-CW	−481.63	−9.01†	−7531.33†	−7058.71	−14388.47†	−13915.85†
HAR-AG-CW	−475.56	32.51	−7526.20	−7018.13	−14382.64	−13874.57
HAR-TCJ-CW	−418.77	53.85	−6280.54	−5807.92	−10136.50	−9663.88
HAR-TCJA-CW	−521.07†	57.89	−6312.06	−5733.10	−10161.00	−9582.04
HAR-TCJAG-CW	−515.07	99.33	−6306.92	−5692.51	−10156.13	−9541.72
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	−1467.93	−1101.65	−8299.37	−7933.09†	−15325.34	−14959.06
HAR-A-CW	−1582.16	−1109.54†	−8376.23†	−7903.61	−15556.15†	−15083.53†
HAR-AG-CW	−1589.47	−1081.40	−8374.82	−7866.75	−15551.21	−15043.14
HAR-TCJ-CW	−1524.57	−1051.95	−7177.73	−6705.11	−10899.95	−10427.33
HAR-TCJA-CW	−1614.63	−1035.67	−7202.43	−6623.47	−10921.92	−10342.96
HAR-TCJAG-CW	−1621.66†	−1007.25	−7199.68	−6585.27	−10919.24	−10304.83

Note: '†' denotes the model selected by AIC and BIC among the six models.

Table 4: Tests of Direct Effects on the Daily Covariance Models

Model	$H_0^{11,v}$	$H_0^{22,v}$	$H_0^{11,j}$	$H_0^{22,j}$	$H_0^{11,a}$	$H_0^{22,a}$
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	1244.6	598.80				
	[0.0000]	[0.0000]				
HAR-A-CW	976.96	486.47			6.5807	5.0878
	[0.0000]	[0.0000]			[0.0000]	[0.0000]
HAR-AG-CW	973.80	472.28			6.6953	5.0695
	[0.0000]	[0.0000]			[0.0000]	[0.0000]
HAR-TCJ-CW	890.37	469.33	0.1037	1.1754		
	[0.0000]	[0.0000]	[0.9173]	[0.2398]		
HAR-TCJA-CW	108.05	406.33	0.0874	1.1021	5.6778	5.0523
	[0.0000]	[0.0000]	[0.9304]	[0.2704]	[0.0000]	[0.0000]
HAR-TCJAG-CW	701.14	396.18	0.0785	1.0578	5.6843	5.1483
	[0.0000]	[0.0000]	[0.9374]	[0.2901]	[0.0000]	[0.0000]
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	1308.3	809.83				
	[0.0000]	[0.0000]				
HAR-A-CW	994.18	712.48			6.7084	3.1948
	[0.0000]	[0.0000]			[0.0000]	[0.0014]
HAR-AG-CW	992.24	697.83			6.7504	3.1691
	[0.0000]	[0.0000]			[0.0000]	[0.0015]
HAR-A-CW	947.57	577.37	0.1068	1.3661		
	[0.0000]	[0.0000]	[0.9149]	[0.1719]		
HAR-TCJ-CW	788.02	484.08	0.0874	1.4859	6.1482	3.0904
	[0.0000]	[0.0000]	[0.9303]	[0.1373]	[0.0000]	[0.0020]
HAR-TCJA-CW	789.73	480.60	0.0005	1.5404	6.2133	3.0167
	[0.0000]	[0.0000]	[0.9996]	[0.1235]	[0.0000]	[0.0026]

Note: We perform Wald tests for the hypotheses $H_0^{11,v}$ and $H_0^{22,v}$, while we use t tests for remaining hypotheses. The entries show the test statistics. P-values are given in brackets.

Table 5: Tests of Direct Effects on the Weekly Covariance Models

Model	$H_0^{11,v}$	$H_0^{22,v}$	$H_0^{11,j}$	$H_0^{22,j}$	$H_0^{11,a}$	$H_0^{22,a}$
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	15126	6935.9				
	[0.0000]	[0.0000]				
HAR-A-CW	14204	6336.3			6.9584	4.9890
	[0.0000]	[0.0000]			[0.0000]	[0.0000]
HAR-AG-CW	14844	6374.4			7.0026	4.9485
	[0.0000]	[0.0000]			[0.0000]	[0.0000]
HAR-TCJ-CW	6530.8	3788.2	12.458	5.8031		
	[0.0000]	[0.0000]	[0.0000]	[0.0000]		
HAR-TCJA-CW	6162.2	3478.1	9.1037	5.7926	4.8802	3.3982
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0007]
HAR-TCJAG-CW	5935.0	3376.1	8.9475	5.7807	4.8987	3.0912
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0020]
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	15758	6513.5				
	[0.0000]	[0.0000]				
HAR-A-CW	14206	6160.3			8.2330	2.7083
	[0.0000]	[0.0000]			[0.0000]	[0.0068]
HAR-AG-CW	14233	6159.3			8.3836	2.7029
	[0.0000]	[0.0000]			[0.0000]	[0.0069]
HAR-TCJ-CW	8379.4	3371.1	12.463	5.2151		
	[0.0000]	[0.0000]	[0.0000]	[0.0000]		
HAR-TCJA-CW	7523.8	2825.2	10.659	5.1418	6.6795	2.0230
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0431]
HAR-TCJAG-CW	7594.6	2944.0	10.583	5.1519	6.2988	2.0000
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0455]

Note: We perform Wald tests for the hypotheses $H_0^{11,v}$ and $H_0^{22,v}$, while we use t tests for remaining hypotheses. The entries show the test statistics. P-values are given in brackets.

Table 6: Tests of Direct Effects on the Monthly Covariance Models

Model	$H_0^{11,v}$	$H_0^{22,v}$	$H_0^{11,j}$	$H_0^{22,j}$	$H_0^{11,a}$	$H_0^{22,a}$
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	97492	53456				
	[0.0000]	[0.0000]				
HAR-A-CW	98506	56199			10.168	5.8479
	[0.0000]	[0.0000]			[0.0000]	[0.0000]
HAR-AG-CW	98289	54744			10.225	5.8021
	[0.0000]	[0.0000]			[0.0000]	[0.0000]
HAR-TCJ-CW	18805	10757	13.124	3.6226		
	[0.0000]	[0.0000]	[0.0000]	[0.0003]		
HAR-TCJA-CW	17301	10450	9.8033	3.7172	5.0165	2.6162
	[0.0000]	[0.0000]	[0.0000]	[0.0002]	[0.0000]	[0.0089]
HAR-TCJAG-CW	17547	10105	9.5076	3.8152	4.3201	2.4460
	[0.0000]	[0.0000]	[0.0000]	[0.0001]	[0.0000]	[0.0144]
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	105989	53583				
	[0.0000]	[0.0000]				
HAR-A-CW	114102	60018			10.299	2.3873
	[0.0000]	[0.0000]			[0.0000]	[0.0170]
HAR-AG-CW	114587	60043			10.235	2.3892
	[0.0000]	[0.0000]			[0.0000]	[0.0170]
HAR-TCJ-CW	26702	7637.8	12.906	2.4480		
	[0.0000]	[0.0000]	[0.0000]	[0.0144]		
HAR-TCJA-CW	26840	6730.2	11.090	2.5625	3.1839	1.2732
	[0.0000]	[0.0000]	[0.0000]	[0.0104]	[0.0015]	[0.2029]
HAR-TCJAG-CW	27427	7512.0	11.128	2.5828	3.1186	1.3508
	[0.0000]	[0.0000]	[0.0000]	[0.0098]	[0.0018]	[0.1768]

Note: We perform Wald tests for the hypotheses $H_0^{11,v}$ and $H_0^{22,v}$, while we use t tests for remaining hypotheses. The entries show the test statistics. P-values are given in brackets.

Table 7: Tests of Spillover Effects on the Daily Covariance Models

Model	$H_0^{21,v}$	$H_0^{12,v}$	$H_0^{21,j}$	$H_0^{12,j}$	$H_0^{21,a}$	$H_0^{12,a}$
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	0.2681	0.7538				
	[0.9659]	[0.8605]				
HAR-A-CW	0.0009	0.6086			1.4674	5.5389
	[1.0000]	[0.8945]			[0.1423]	[0.0000]
HAR-AG-CW	0.0003	0.3501			1.4585	5.0694
	[1.0000]	[0.9504]			[0.1447]	[0.0000]
HAR-TCJ-CW	0.0132	0.3111	0.2061	0.1892		
	[0.9996]	[0.9579]	[0.8367]	[0.8499]		
HAR-TCJA-CW	0.0008	0.4262	0.0874	0.0798	0.7934	3.3141
	[1.0000]	[0.9348]	[0.9364]	[0.8450]	[0.4275]	[0.0009]
HAR-TCJAG-CW	0.0008	0.3532	0.0785	0.0721	0.8914	3.2252
	[1.0000]	[0.9497]	[0.9426]	[0.8602]	[0.3727]	[0.0013]
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	0.0026	0.20520				
	[1.0000]	[0.5617]				
HAR-A-CW	0.0059	0.9392			0.1416	1.0107
	[0.9999]	[0.8160]			[0.8874]	[0.3122]
HAR-AG-CW	0.0065	0.2656			0.1551	0.9495
	[0.9999]	[0.9664]			[0.8767]	[0.3423]
HAR-TCJ-CW	0.0003	3.0456	0.0488	0.0441		
	[1.0000]	[0.3846]	[0.9611]	[0.9648]		
HAR-TCJA-CW	0.0018	0.7205	0.0324	0.3588	0.0970	0.7887
	[1.0000]	[0.8684]	[0.9742]	[0.7197]	[0.9228]	[0.4303]
HAR-TCJAG-CW	0.0021	0.3652	0.0081	0.1053	0.0886	0.7227
	[1.0000]	[0.9473]	[0.9935]	[0.9161]	[0.9294]	[0.4699]

Note: We perform Wald tests for the hypotheses $H_0^{12,v}$ and $H_0^{21,v}$, while we use t tests for remaining hypotheses. The entries show the test statistics. P-values are given in brackets.

Table 8: Tests of Spillover Effects on the Weekly Covariance Models

Model	$H_0^{21,v}$	$H_0^{12,v}$	$H_0^{21,j}$	$H_0^{12,j}$	$H_0^{21,a}$	$H_0^{12,a}$
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	0.1869	0.3117				
	[0.9797]	[0.9578]				
HAR-A-CW	0.0018	0.7844			0.1133	1.8145
	[1.0000]	[0.8532]			[0.9098]	[0.0696]
HAR-AG-CW	0.0028	0.3619			0.1108	1.8895
	[1.0000]	[0.9480]			[0.9117]	[0.0588]
HAR-TCJ-CW	0.0016	0.0034	0.0128	0.0722		
	[1.0000]	[0.9999]	[0.9898]	[0.9424]		
HAR-TCJA-CW	0.0011	0.0084	0.0035	0.1386	0.0497	1.5286
	[1.0000]	[0.9998]	[0.9972]	[0.8959]	[0.9604]	[0.1264]
HAR-TCJAG-CW	0.0011	0.0075	0.0015	0.1684	0.0064	1.5488
	[1.0000]	[0.9998]	[0.9988]	[0.8663]	[0.9949]	[0.1214]
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	0.0477	3.8799				
	[0.9973]	[0.2747]				
HAR-A-CW	0.0038	1.6697			0.0556	1.1598
	[0.9999]	[0.6437]			[0.9557]	[0.2461]
HAR-AG-CW	0.0078	0.6179			0.0505	1.1107
	[0.9998]	[0.8923]			[0.9597]	[0.2685]
HAR-TCJ-CW	0.0005	1.0131	0.0226	0.0152		
	[1.0000]	[0.7981]	[0.9820]	[0.9878]		
HAR-TCJA-CW	0.0021	0.3899	0.0094	0.0344	0.0091	1.3441
	[1.0000]	[0.9423]	[0.9925]	[0.9725]	[0.9927]	[0.1789]
HAR-TCJAG-CW	0.0017	0.3589	0.0083	0.0801	0.0060	1.3533
	[1.0000]	[0.9486]	[0.9934]	[0.9361]	[0.9952]	[0.1760]

Note: We perform Wald tests for the hypotheses $H_0^{12,v}$ and $H_0^{21,v}$, while we use t tests for remaining hypotheses. The entries show the test statistics. P-values are given in brackets.

Table 9: Tests of Spillover Effects on the Monthly Covariance Models

Model	$H_0^{21,v}$	$H_0^{12,v}$	$H_0^{21,j}$	$H_0^{12,j}$	$H_0^{21,a}$	$H_0^{12,a}$
Period 1: 10/27/2009 – 09/10/2013						
HAR-CW	0.0956	0.9043				
	[0.9924]	[0.8244]				
HAR-A-CW	0.0236	6.1819			0.0912	0.9857
	[0.9990]	[0.1031]			[0.9273]	[0.3243]
HAR-AG-CW	0.0330	5.4193			0.0948	1.0266
	[0.9984]	[0.1435]			[0.9245]	[0.3046]
HAR-TCJ-CW	0.0277	0.0011	0.0069	0.9459		
	[0.9988]	[1.0000]	[0.9945]	[0.3442]		
HAR-TCJA-CW	0.0091	0.0032	0.0000	1.0269	0.0514	1.1392
	[0.9998]	[1.0000]	[1.0000]	[0.3044]	[0.9590]	[0.2546]
HAR-TCJAG-CW	0.0062	0.0015	0.0007	1.0266	0.0095	1.1370
	[0.9999]	[1.0000]	[0.9995]	[0.3046]	[0.9924]	[0.2555]
Period 2: 10/04/2011 – 08/15/2015						
HAR-CW	0.0062	0.3542				
	[0.9999]	[0.9495]				
HAR-A-CW	0.0320	0.0052			0.1126	2.5827
	[0.9985]	[0.9999]			[0.9104]	[0.0098]
HAR-AG-CW	0.0297	0.0046			0.1167	2.4502
	[0.9986]	[0.9999]			[0.9071]	[0.0143]
HAR-TCJ-CW	0.0053	0.0403	0.0059	0.0649		
	[0.9999]	[0.9979]	[0.9695]	[0.5160]		
HAR-TCJA-CW	0.0073	0.0272	0.0005	0.6696	0.0052	1.1204
	[0.9998]	[0.9988]	[1.0000]	[0.5031]	[0.9958]	[0.2626]
HAR-TCJAG-CW	0.0087	0.0339	0.0002	0.8350	0.0519	1.2969
	[0.9998]	[0.9984]	[0.9998]	[0.4037]	[0.9586]	[0.1947]

Note: We perform Wald tests for the hypotheses $H_0^{12,v}$ and $H_0^{21,v}$, while we use t tests for remaining hypotheses. The entries show the test statistics. P-values are given in brackets.

Table 10: Tests for Effects of Geopolitical Risks

Model	Period 1: 10/27/2009 – 09/10/2013			Period 2: 10/04/2011 – 08/15/2015		
	λ_{11}	λ_{12}	λ_{22}	λ_{11}	λ_{12}	λ_{22}
HAR-AG-CW						
Daily Cov.	0.2569	0.3636	−0.1652	3.3611∗	−0.6431	0.0625
	[0.7973]	[0.7162]	[0.8688]	[0.0008]	[0.5202]	[0.9501]
Weekly Cov.	−0.7679	0.0202	−0.1481	2.0644∗	−0.1869	0.0161
	[0.4425]	[0.9838]	[0.8823]	[0.0390]	[0.8518]	[0.9872]
Monthly Cov.	−0.1878	0.1751	−0.1118	0.4956	−0.3039	0.0950
	[0.8510]	[0.8610]	[0.9110]	[0.6202]	[0.7612]	[0.9243]
HAR-TCJAG-CW						
Daily Cov.	0.0553	0.0235	0.0156	3.0660∗	−0.7036	0.1839
	[0.9559]	[0.9812]	[0.9876]	[0.0022]	[0.4817]	[0.8541]
Weekly Cov.	−0.5571	−0.3855	0.1332	0.8548	−0.6374	0.2211
	[0.5774]	[0.6998]	[0.8941]	[0.3926]	[0.5238]	[0.8250]
Monthly Cov.	−0.1617	−0.4181	0.4538	0.0895	−0.6652	−0.1907
	[0.8715]	[0.6758]	[0.6500]	[0.9287]	[0.5059]	[0.8488]

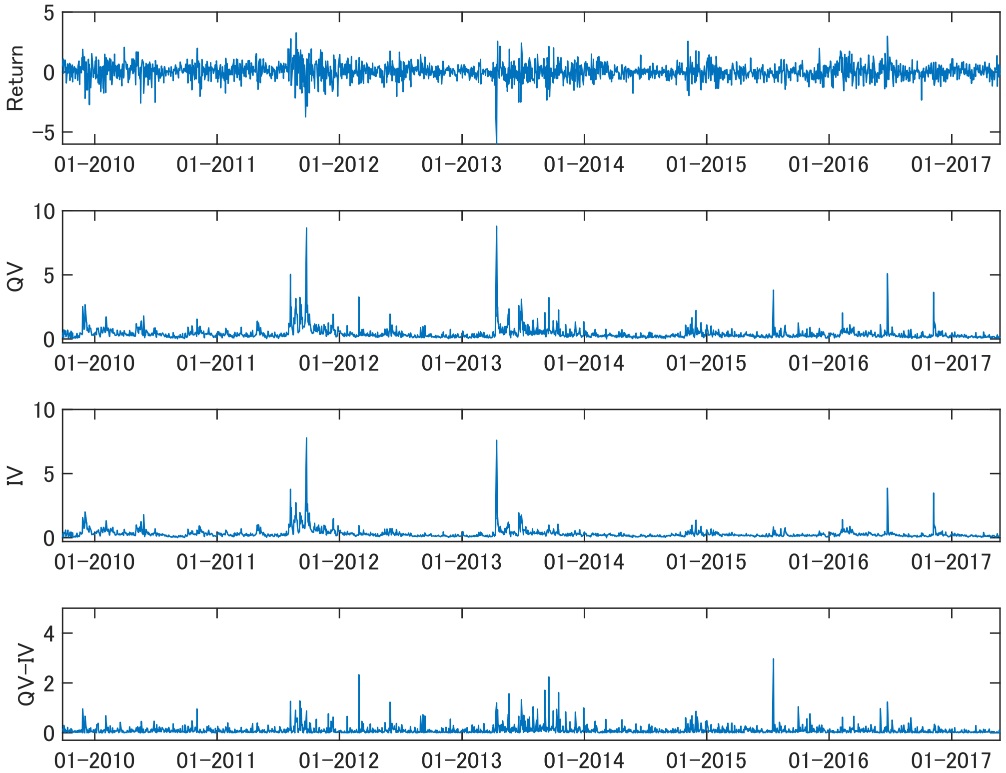
Note: The entries show the t test statistics. P-values are given in brackets. '∗' denotes significance at the 5% level.

Table 11: Out-of-Sample Forecast Evaluation

Model	MSFE			QLIKE		
	Period 1	Period 2	Total	Period 1	Period 2	Total
Daily Covarinace Models						
HAR-CW	1.4880	0.8856*	2.1486	0.4592	0.5484	0.3614
HAR-A-CW	1.4457	0.8914	2.0535*	0.4382	0.5145	0.3545
HAR-AG-CW	1.4455*	0.8906	2.0539	0.4348*	0.5084*	0.3541*
HAR-TCJ-CW	1.8906	1.0748	2.7851	0.7607	0.8739	0.6366
HAR-TCJA-CW	1.7518	1.0202	2.5540	0.7114	0.8076	0.6058
HAR-TCJAG-CW	1.7501	1.0170	2.5539	0.7063	0.7965	0.6074
Weekly Covarinace Models						
HAR-CW	0.0970	0.0569	0.1410	0.0253*	0.0231*	0.0279*
HAR-A-CW	0.0939	0.0561	0.1354	0.0257	0.0234	0.0281
HAR-AG-CW	0.0938*	0.0560*	0.1352*	0.0257	0.0234	0.0282
HAR-TCJ-CW	0.4489	0.2390	0.6789	0.1250	0.1150	0.1360
HAR-TCJA-CW	0.4317	0.2294	0.6536	0.1232	0.1133	0.1340
HAR-TCJAG-CW	0.4317	0.2287	0.6543	0.1231	0.1132	0.1339
Monthly Covarinace Models						
HAR-CW	0.0069	0.0045*	0.0095	0.0022*	0.0019	0.0025*
HAR-A-CW	0.0068*	0.0046	0.0093*	0.0022	0.0019*	0.0026
HAR-AG-CW	0.0068	0.0046	0.0093	0.0022	0.0019	0.0027
HAR-TCJ-CW	0.3028	0.1571	0.4626	0.0922	0.0833	0.1019
HAR-TCJA-CW	0.3013	0.1556	0.4610	0.0916	0.0825	0.1016
HAR-TCJAG-CW	0.3000	0.1541	0.4600	0.0913	0.0820	0.1015

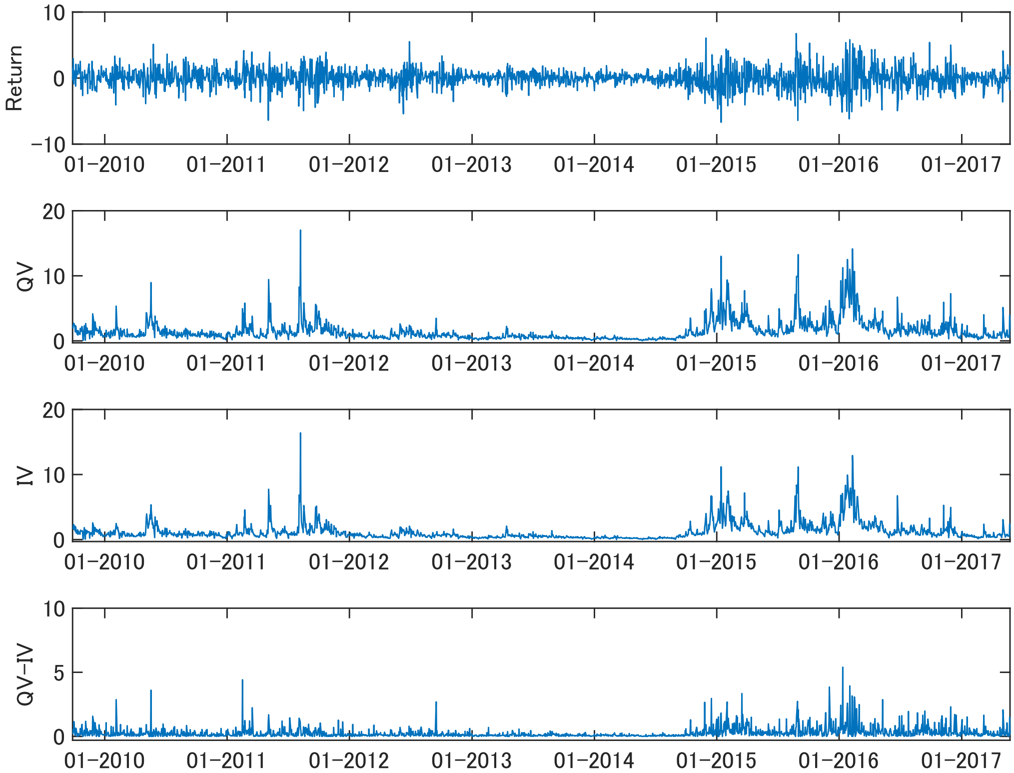
Note: The table reports the mean squared forecast error (MFSE) and quasi-likelihood-based measure (QLIKE), defined by (15) and (16), respectively. The values in Total are not necessarily the same as the sums of those of Periods 1 and 2 due to rounding errors. Period 1 indicates 09/11/2013 – 08/18/2015, while Period 2 is 08/19/2015 – 05/25/2017. '$*$' denotes the model which has the smallest value of the six models in the corresponding period.

Figure 1: Return and Estimates of Quadratic Variation, Integrated Volatility, and Jump Variability for Crude Oil Futures



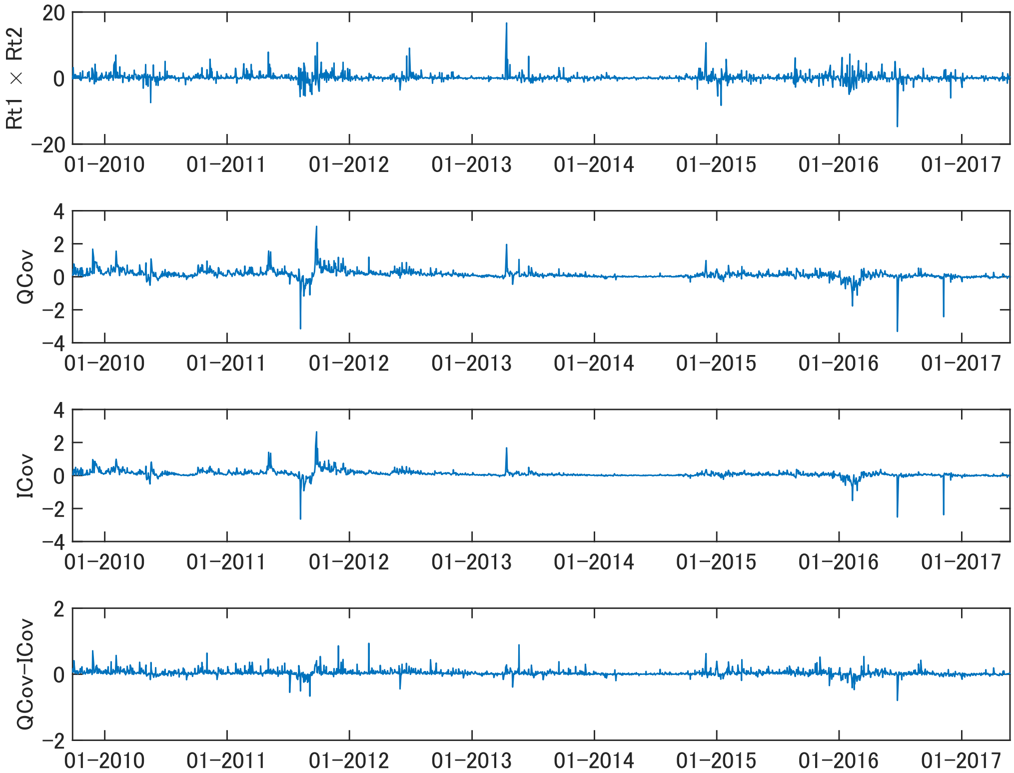
Note: Figure 1 shows the (1, 1)-elements of Ω_t, C_t, and J_t.

Figure 2: Return and Estimates of Quadratic Variation, Integrated Volatility, and Jump Variability for Gold Futures



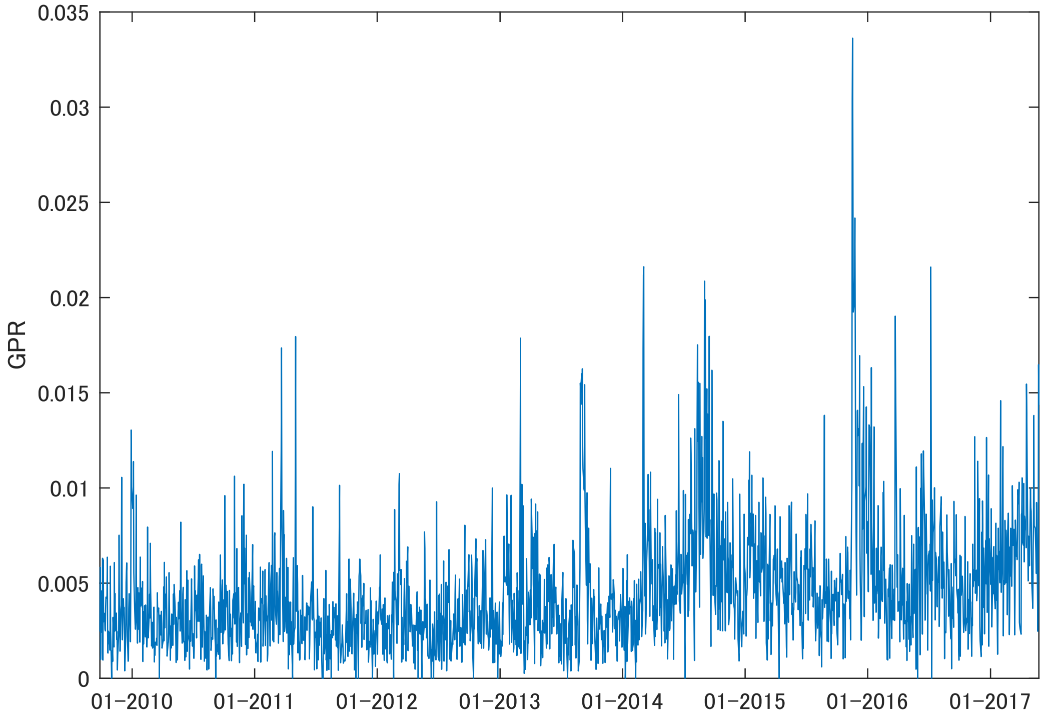
Note: Figure 2 shows the $(2,2)$-elements of Ω_t, C_t, and J_t.

Figure 3: Product of Returns and Estimates of Quadratic Covariation, Integrated Covolatility, and Jump Co-variability for Crude Oil and Gold Futures



Note: Figure 3 shows the (2, 1)-elements of Ω_t, C_t, and J_t.

Figure 4: Geopolitical Risk Indicator



Note: Figure 4 shows the share of the daily geopolitical risk indicator suggested by Carldara and Iacoviello (2018).