

SUPPLEMENTARY DATA

Thorley, J. and T. Clutton-Brock. 2019. A unified-models analysis of the development of sexual size dimorphism in Damaraland mole-rats *Fukomys damarensis*

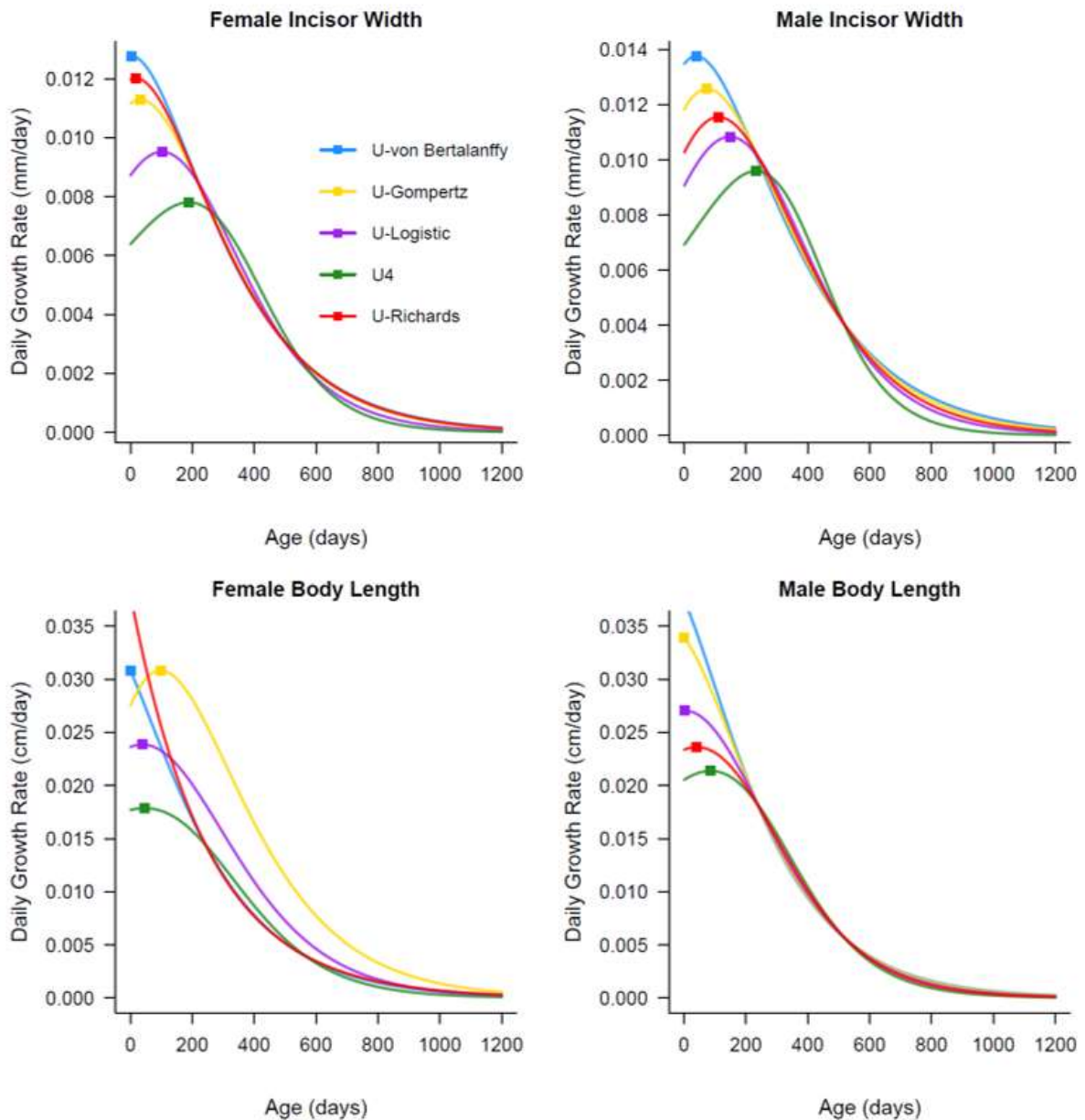
Supplementary Data S1. Methods for the literature search of the use of different sigmoid growth formulations in the mammal literature.

We used Web of Science to search for studies that have employed sigmoid or sigmoid-like functions to characterise growth in mammals since 1980. We focussed on four journals: *Journal of Mammalogy*, *Mammal Study*, and *Marine Mammal Science* and *Mammal Review*, returning all studies where the topic (Title, Abstract, Keywords) = “growth”. This search was carried out on the 15th February 2019, returning 696 studies. An initial screening of the titles and abstracts meant that we could exclude many studies that were unrelated to individual growth modelling (e.g. population growth, phylogeography etc.). After this initial screening step, we then examined each paper fully to determine whether the study had modelled the complete post-natal development of known-age individuals using traditional two-, three- or four-parameter growth functions. We found 48 studies that matched this criterion. We then classified the studies into three categories. 1) Those that employed the [flexible] Richards function (irrespective of whether they also tested other growth functions), 2) Studies that fitted a single two- or three- parameter growth function*, 3) Studies that compared the fit of multiple two- or three- parameter growth function*. The complete list of studies is provided as an additional file alongside other data used in this paper.

*Two- or three-parameter curves include the logistic, Gompertz, von Bertalanffy, monomolecular, and two-parameter versions of these functions where a single parameter is fixed, in addition to quadratic functions, and the Brody function.

Supplementary Data S2. Instantaneous growth rate equations for unified models of sigmoid growth. Equations taken from Svagelj et al. 2019. They represent the first derivatives of growth models. $Y(t)$ is the daily growth rate at age t . A , k , T_i and d represent the upper asymptote, maximum relative growth rate, age at the inflection point and shape parameter, respectively.

Model	Instantaneous rate of growth
von Bertalanffy	$Y(t) = \frac{9}{4} \cdot A \cdot k \cdot e^{-\frac{9}{4}k \cdot (t-T_i)} \cdot \left(1 - \left(\frac{1}{3} \cdot e^{-\frac{9}{4}k \cdot (t-T_i)}\right)^2\right)$
Gompertz	$Y(t) = e \cdot A \cdot k \cdot e^{-e \cdot k \cdot (t-T_i)} \cdot e^{-e^{-e \cdot k \cdot (t-T_i)}}$
Logistic	$Y(t) = 4 \cdot A \cdot k \cdot \frac{e^{-4 \cdot k \cdot (t-T_i)}}{(1 + e^{-4 \cdot k \cdot (t-T_i)})^2}$
U4	$Y(t) = 4^{\frac{4}{3}} \cdot A \cdot k \cdot e^{-4^{\frac{4}{3}}k \cdot (t-T_i)} \cdot \left(1 + \left(3 \cdot e^{-4^{\frac{4}{3}}k \cdot (t-T_i)}\right)\right)^{-\frac{4}{3}}$
Richards	$Y(t) = \frac{1-d}{(1-d) \cdot d^{\frac{1}{1-d}}} \cdot A \cdot k \cdot e^{\frac{-k \cdot (t-T_i)}{d^{\frac{1}{1-d}}}} \cdot \left(1 + \left((d-1) \cdot e^{\frac{-k \cdot (t-T_i)}{d^{\frac{1}{1-d}}}}\right)\right)^{\frac{d}{1-d}}$



Supplementary Data S3. Instantaneous growth rates (growth velocity) for the Unified growth models applied to female incisor width (a), male incisor width (b), female body length (c), and male body length (d). Instantaneous growth rates represent the first derivative of the best-fitting non-linear mixed effects model in each case. Points on each curve highlight the estimated point of maximum growth in either model. All models were fitted to data on known-age, captive individuals.

Supplementary Data S4. The best fitting random effects structure for each form of Unified growth model fitted to skeletal traits. In all cases, random effects were specified at the level of the individual, and it was assumed that random effect were uncorrelated with one another. The best fitting model was identified through AIC comparisons and likelihood ratio testing.

Trait	Model	Random Effect			n
		<i>A</i>	<i>k</i>	<i>Ti</i>	
Female Incisor Width	von Bertalanffy	+	+		1762
	Gompertz	+	+		
	Logistic	+	+	+	
	U4	+	+	+	
	Richards	+	+		
Female Body Length	von Bertalanffy	+	+		1689
	Gompertz	+	+		
	Logistic	+	+	+	
	U4	+	+	+	
	Richards	+	+		
Male Incisor Width	von Bertalanffy	+	+	+	1709
	Gompertz	+	+	+	
	Logistic	+	+	+	
	U4	+	+	+	
	Richards	+	+	+	
Male Body Length	von Bertalanffy	+	+		1762
	Gompertz	+	+		
	Logistic	+	+		
	U4	+	+	+	
	Richards	+	+		

Supplementary Data S5. Estimates for random effects, temporal autocorrelation, and power of variance covariate in models of skeletal growth on captive Damaraland mole-rats. Models in bold denote the best-fitting model within each comparison.

		Random Effects Standard deviation			Temporal autocorrelation	Power of variance covariate
		<i>A</i>	<i>k</i>	<i>T_i</i>	ϕ	
Females						
Incisor Width	U-von Bertalanffy	0.435	0.00024	-	0.394	-0.312
	U-Gompertz	0.426	0.00025	-	0.393	-0.351
	U-Logistic	0.424	0.00029	22.62	0.296	-0.242
	U4	0.423	0.00031	24.80	0.316	-0.413
	U-Richards	0.431	0.00024	-	0.393	-0.332
Males						
Incisor Width	U-von Bertalanffy	0.376	0.00027	12.76	0.483	0.290
	U-Gompertz	0.372	0.00028	16.75	0.439	0.290
	U-Logistic	0.394	0.00028	22.81	0.376	0.255
	U4	0.434	0.00024	27.01	0.384	0.122
	U-Richards	0.380	0.00289	20.14	0.399	0.279
Females						
Body Length	U-von Bertalanffy	0.968	0.00019	-	0.547	-1.21
	U-Gompertz	0.961	0.00018	-	0.542	-1.24
	U-Logistic	0.947	0.00018	16.25	0.517	-1.22
	U4	0.840	0.00027	30.33	0.452	-1.22
	U-Richards	0.971	0.00020	-	0.550	-1.20
Males						
Body Length	U-von Bertalanffy	0.903	0.00023	-	0.594	-1.08
	U-Gompertz	0.898	0.00022	-	0.580	-1.13
	U-Logistic	0.871	0.00023	-	0.551	-1.27
	U4	0.828	0.00027	19.61	0.470	-1.31
	U-Richards	0.832	0.00025	-	0.542	-1.34

Supplementary Data S6. Parameter estimates and model comparison for ‘interval equation’ models of growth in wild Damaraland mole-rats. n refers to the total sample size of each analysis. Here, sample size refers to the number of recapture events (i.e. each row of data includes the two successive captures on an individual).

Trait	n	Model	<i>A</i>	<i>k</i>	AIC	ΔAIC
Female Incisor Width	211	von Bertalanffy	6.05 (0.06)	0.0040 (0.0002)	84.54	0.00
		Logistic	6.03 (0.05)	0.0055 (0.0003)	123.96	39.42
Male Incisor Width	224	von Bertalanffy	6.79 (0.06)	0.0039 (0.0002)	139.56	0.00
		Logistic	6.68 (0.05)	0.0060 (0.0002)	164.80	25.24
Female Body Length	224	von Bertalanffy	18.82 (0.15)	0.0034 (0.0002)	529.98	0.00
		Logistic	18.53 (0.12)	0.0049 (0.0003)	543.42	13.44
Male Body Length	225	von Bertalanffy	19.60 (0.10)	0.0044 (0.0002)	391.63	0.00
		Logistic	19.42 (0.09)	0.0060 (0.0002)	408.18	16.55