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ScienceDirect
IFAC PapersOnLine 52-16 (2019) 406–411

Dynamic State Feedback Decoupling of a DX A/C System

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Abstract: A temperature and humidity controller is designed for a direct expansion air conditioning (DX A/C) system making use of a state feedback decoupling approach of nonlinear control systems. It is shown that the nonlinear dynamics of a DX A/C system can be input-output decoupled by dynamic state feedback. The resulting decoupled system is of minimum phase. Thereafter, the decoupled model is used to design a pole placement controller with guaranteed stability. Unlike controllers based on approximate local linearization of the DX A/C model, the controller proposed is global in the sense that it can track temperature and humidity setpoints in the complete operating range of the DX A/C system. Effectiveness of the controller designed is demonstrated by simulation results.

Keywords: State feedback decoupling, DX A/C system, pole placement control, minimum phase system.

1. INTRODUCTION

Traditionally building HVAC systems control mainly deals with the problem of temperature and humidity control of conditioned space to provide comfort indoor environment and at the same time reduce energy consumption (Mei and Xia, 2017). Carbon dioxide (CO_2) concentration was also included in the control objectives in recent studies to further improve air quality (Lin et al., 2015; Kellett et al., 2019). Energy efficiency improvement of buildings can also be obtained by other schemes (Xia and Zhang, 2010; Sichilalu and Xia, 2015; Sichilalu et al., 2016; Fan and Xia, 2017).

Because the HVAC system model is highly nonlinear, local linearization around an operating point is often practiced to facilitate controller design. For instance, a linear-quadratic-Gaussian regulator (Qi and Deng, 2009) and a model predictive control (MPC) strategy (Razmara et al., 2015) were reported in the literature based on such a local linearization. Additionally, feedback linearization method was also adopted by (Ma et al., 2015) to control the temperature. In our previous work, a hierarchical strategy consisting of a top layer open loop controller to improve energy efficiency and a lower layer setpoint tracking MPC based on local linearization of the HVAC model was also proposed (Mei and Xia, 2017; Mei et al., 2018). However, the local linearization method inevitably results in a less accurate local control (Jimenez et al., 2017) and does not consider transients from one setpoint to the other.

A state feedback decoupling method is adopted in this study to tackle the setpoint tracking problem of a direct expansion (DX) A/C system in view of the coupling between temperature and humidity in an HVAC system. Moreover, the state feedback decoupling has the added benefit that the resulting system dynamics is input-output linear, which facilitates the design of a controller global in the sense of the singularity of meromorphic functions (Conte et al., 2007). The state feedback decoupling method for nonlinear multivariable systems was first studied by (Porter, 1970), which presented a decoupling approach for a class of nonlinear systems that can be decoupled with static state feedback (Semsar-Kazerooni et al., 2008). Later, a modified algorithm was proposed (Singh, 1981) that adds some integrators at appropriate states to achieve input-output decoupling of a nonlinear system that cannot be decoupled by the static state feedback method. Necessary and sufficient conditions under which this type of so-called dynamic state feedback decoupling can be used was studied and proved in the works of (Descusse and Moog, 1987; Isidori, 1995; Xia and Gao, 1993).

In particular, it was found that the DX A/C system model cannot be directly decoupled through static state feedback. Therefore, the dynamic state feedback decoupling method is employed. We show that the DX A/C system becomes input-output decouplable when two integrators are added to an input terminal, or the system is dynamic static feedback decouplable. The resulting decoupled model has seven state variables and a relative degree of six, which indicates that there is one internal dynamics that is rendered unobservable when the system outputs are forced to zero, or there is a one dimensional zero dynamics (Isidori and Moog, 1988). However, it was found out that the zero dynamics associated with this internal dynamics is asymptotically stable, indicating the decoupled DX A/C model is of minimum phase system (Isidori and Moog, 1988), which

10.1016/j.ifacol.2019.11.814

can be stabilised by a properly designed controller using the input-output model (Byrnes and Isidori, 1991; Wagner, 1991; Xia, 1993) under some mild regularity conditions. A pole placement controller is designed for this purpose.

The main contributions of this paper are: 1) The DX A/C problem studied presents a meaningful application for the dynamic state feedback decoupling method and the minimum phase systems theory in building energy systems. 2) The controller designed based on state feedback decoupling is a global controller that can control temperature and humidity in all operating modes of the DX A/C system because no local approximation is involved; 3) A pole placement controller is designed for the temperature and humidity set point tracking with guaranteed stability.

The organization of the rest paper is as follows. In Section 2, the feedback decoupling method is described and the proposed controller is designed. The simulation results are given in Section 3. Finally, Section 4 conclude this paper.

2. PROBLEM FORMULATION

In this section, the nonlinear model for the air temperature and moisture content is established for a DX A/C system based on energy and mass balance. The simplified schematic diagram of the DX A/C system is shown in Fig. 1. The system is considered to be operating in cooling mode. The basic operating principles and assumptions of the system in the cooling mode are given below (Mei and Xia, 2017).

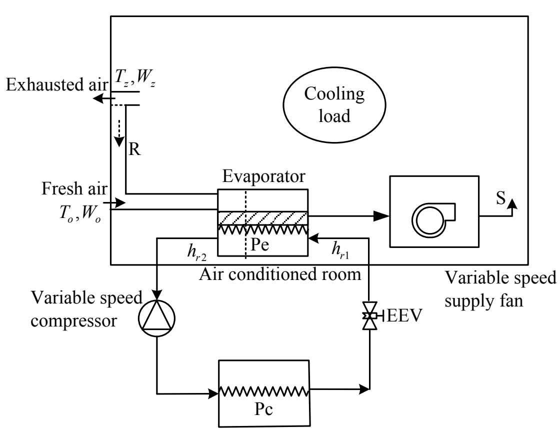

Fig. 1. Simplified schematic diagram of a DX A/C system

(1) A percentage (denoted by δ, $0 < \delta < 1$) of fresh air is allowed into the system and gets mixed with $1 - \delta$ percent of the recirculated air at the evaporator.
(2) Sufficient air mixing occurs inside the heat exchangers where it gets conditioned.
(3) Thermal losses in air ducts are negligible.
(4) The supply air enters the air-conditioned room to offset the cooling loads.
(5) The air in the conditioned room is drawn through a fan, $1 - \delta$ of this air gets recirculated and the rest is exhausted from the system by a fan.

Energy and mass conservation gives the following equation of the nonlinear DX A/C system (Mei and Xia, 2017):

$$
\begin{cases}
C_a \rho V \dot{T}_z = C_a \rho v_f (T_s - T_z) + Q_l, \\
\rho V \dot{W}_z = \rho v_f (W_s - W_z) + M_l, \\
C_a \rho V_{h1} \dot{T}_d = C_a \rho v_f (T_m - T_d) + \alpha_1 A_1 (T_w - \dfrac{T_m + T_d}{2}), \\
\rho V_{h2} (C_a \dot{T}_s + h_{fg} \dot{W}_s) = \rho v_f (C_a (T_d - T_s) + h_{fg} (W_m - W_s)) + \alpha_2 A_2 (T_w - \dfrac{T_d + T_s}{2}), \\
C_w \rho_w V_w \dot{T}_w = \alpha_1 A_1 (\dfrac{T_m + T_d}{2} - T_w) + \alpha_2 A_2 (\dfrac{T_d + T_s}{2} - T_w) - (h_{r2} - h_{r1}) m_r, \\
\dot{W}_s - \dfrac{2 \times 0.0198 T_s + 0.085}{1000} \dot{T}_s = 0.
\end{cases}
\tag{1}
$$

In the nonlinear system (1), T_s and W_s are the temperature and moisture content of the air leaving the DX cooling coil, respectively. T_z and W_z are the air temperature and moisture content in the conditioned space, respectively. v_f and m_f are the volumetric flow rate of supply air and refrigerant mass flow rate, respectively. T_d and T_w are the air temperature leaving the dry-cooling region on air side in the DX evaporator and the temperature of the DX evaporator wall, respectively. Q_l and M_l are the sensible heat and moisture loads in the conditioned space, respectively. T_0 and W_0 are the outdoor air temperature and moisture content, respectively. δ is the mixing ratio between the outside and recirculated air. $T_m = \delta T_0 + (1 - \delta) T_z$ and $W_m = \delta W_0 + (1 - \delta) W_z$ denote the mixed air temperature and moisture content before each DX evaporator cooling coil, respectively. C_a is the specific heat of air; V denotes the volume of the conditioned space; ρ represents the density of moist air; and h_{fg} is latent heat of vaporisation of water. V_{h1} and V_{h2} are the air side volumes in the dry-cooling and wet-cooling regions of the DX evaporator, respectively. α_1 and α_2 are the heat transfer coefficients between air side and the evaporator wall in the dry-cooling and wet-cooling regions of the DX evaporator, respectively. A_1 and A_2 are the heat transfer areas in the dry-cooling and wet-cooling regions of the DX evaporator, respectively. C_w, ρ_w and V_w denote the specific heat of air in the evaporator wall, the density of the evaporator wall and the volume of the evaporator wall, respectively. h_{r1} and h_{r2} are the enthalpies of refrigerants at evaporator inlet and outlet, respectively.

Since the moisture content and temperature at the evaporator outlet has the following relationship (Qi and Deng (2008)): $W_s = \frac{0.0198 T_s^2 + 0.085 T_s + 4.4984}{1000}$, (1) can be reduced to

$$
\begin{cases}
\dot{T}_z = (C_a \rho v_f (T_s - T_z) + Q_l)/(C_a \rho V), \\
\dot{W}_z = \dfrac{\rho v_f}{\rho V} (r_t - W_z) + \dfrac{M_l}{\rho V}, \\
\dot{T}_d = \dfrac{C_a \rho v_f (T_m - T_d)}{C_a \rho V_{h1}} + \dfrac{\alpha_1 A_1}{C_a \rho V_{h1}} (T_w - \dfrac{T_m + T_d}{2}), \\
\dot{T}_s = \dfrac{C_a \rho v_f (T_d - T_s)}{q_t} + (\rho v_f h_{fg} (W_m - r_t) + \alpha_2 A_2 (T_w - \dfrac{T_d + T_s}{2}))/q_t, \\
\dot{T}_w = \dfrac{\alpha_1 A_1 (\frac{T_m + T_d}{2} - T_w) + \alpha_2 A_2 (\frac{T_d + T_s}{2} - T_w)}{C_w \rho_w V_w} - \dfrac{(h_{r2} - h_{r1}) m_r}{C_w \rho_w V_w},
\end{cases}
\tag{2}
$$

where $r_t = \frac{0.0198T_s^2+0.085T_s+4.4984}{1000}$, $q_t = C_a\rho V_{h2} + \rho V_{h2} h_{fg}\frac{0.0396T_s+0.085}{1000}$.

Remark 1. In the previous work (Semsar-Kazerooni et al. (2008)), a static decoupling method was employed for the HVAC system to study the indoor air temperature and humidity control problem. In the HVAC system model, an internal physical dynamic process of AC system was not considered and the intrinsic relation between the supply air temperature and relative humidity was also not considered. In this paper, we build a physical model that reflects the intrinsical dynamic process of the HVAC system.

2.1 State feedback decoupling of the DX A/C System

Consider a nonlinear system

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{3}$$

where input $u \in R^m$ is the control input, $y \in R^m$ is the output and $x \in R^n$ is the state.

Definition 1: Using the notation for the system described in (3), the Lie derivative of $h(x)$ with respect to the vector field $f(x)$ is written as $L_f h(x)$, which is defined by

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) = \frac{\partial h(x)}{\partial x} f(x).$$

When we repeat the calculation of the Lie derivative, the following notation can be used:

$$L_f^k h(x) = L_f L_f^{k-1} h(x) = \frac{\partial L_f^{k-1} h(x)}{\partial x} f(x),$$

for any integer $k \geq 1$ with $L_f^0 h(x) = h(x)$.

Definition 2: (Isidori (1995)) The nonlinear system (3) is said to be of vector relative degree $\rho = \{\rho_1, \ldots, \rho_m\}$ if the following conditions are satisfied for all $x \in R^n$:

(a) for all $1 \leq j \leq m,\ 1 \leq i \leq m,\ k < \rho_i - 1$,

$$L_{g_j} L_f^k h_i(x) = 0;$$

(b) the $m \times m$ Falb-Wolovich matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{\rho_1-1} h_1(x) & \cdots & L_{g_m} L_f^{\rho_1-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{\rho_m-1} h_m(x) & \cdots & L_{g_m} L_f^{\rho_m-1} h_m(x) \end{bmatrix} \tag{4}$$

is nonsingular.

If the relative degree $\rho = \sum_{i=1}^{m} \rho_i$ of the nonlinear system (3) is less than the number of states n, the system inevitably exists internal dynamics (Isidori and Moog, 1988). The model (3) can be transformed to a normal form by a coordinate transformation $\phi(x) : x \longrightarrow (\xi, \eta)$, where ξ, η are the external dynamics and internal dynamics, respectively. The dynamic equations of the normal form depend on the control variables and can be expressed by

$$\dot{\xi}_\rho = b(\xi, \eta) + A(\xi, \eta)u, \tag{5}$$

where $\xi_\rho \in R^m,\ \forall \xi_\rho = [\xi_{\rho_1}^1, \cdots, \xi_{\rho_m}^m]^T$. When the matrix $A(\xi, \eta)$ is nonsingular, substituting (6)

$$u = A^{-1}(\xi, \eta)[-b(\xi, \eta) + v], \tag{6}$$

into the nonlinear system (3) achieves the decoupling and yields

$$\begin{cases} \dot{\xi}_j^i = \xi_{j+1}^i, \\ \dot{\xi}_{\rho_i}^i = v_i, \\ \dot{\eta} = q(\xi, \eta) + p(\xi, \eta)v, \\ y_i = \xi_1^i,\ j = 1, \ldots, \rho_i - 1, i = 1, \ldots, m, \end{cases} \tag{7}$$

where v_i is the ith control input.

Definition 3: (Isidori and Moog, 1988) In the normal system (7), dynamics

$$\dot{\eta} = q(0, \eta) \tag{8}$$

is the zero dynamics of the system (3). If the zero dynamics (8) is (locally) asymptotical stability, the system (3) is then of (locally) minimum phase system.

If the rank of the matrix $A(x)$ is less than m, the system has no relative degree, then the nonlinear system (3) cannot be decoupled by static state feedback.

To see the decouplability of the DX A/C system using state feedback, the system dynamics (2) is first put in the form of (3) with $x = [T_s, T_z, T_d, T_w, W_z]^T$ being the state vector, $u = [v_f, m_r]^T$ the input, $y = [W_z, T_z]^T$ the output, and the functions $f(x), g(x)$ and $h(x)$ defined by

$$f(x) = \begin{bmatrix} \dfrac{\alpha_2 A_2(T_w - \frac{T_d+T_s}{2})}{q_t} \\ \dfrac{Q_l}{C_a\rho V} \\ \dfrac{\alpha_1 A_1}{C_a\rho V_{h1}}(T_w - \dfrac{T_m + T_d}{2}) \\ \dfrac{\alpha_1 A_1(\frac{T_m+T_d}{2} - T_w) + \alpha_2 A_2(\frac{T_d+T_s}{2} - T_w)}{C_w\rho_w V_w} \\ \dfrac{M_l}{\rho V} \end{bmatrix}, \tag{9}$$

$$g(x) = \begin{bmatrix} \dfrac{C_a\rho(T_d - T_s) + \rho h_{fg} F_w}{q_t} & 0 \\ \dfrac{C_a\rho(T_s - T_z)}{C_a\rho V} & 0 \\ \dfrac{C_a\rho(T_m - T_d)}{C_a\rho V_{h1}} & 0 \\ 0 & -\dfrac{h_{r2} - h_{r1}}{C_w\rho_w V_w} \\ \dfrac{\rho}{\rho V}(r_t - W_z) & 0 \end{bmatrix}, \tag{10}$$

where $F_w = W_m - r_t$.

It can be calculated that the DX A/C system has the following Falb-Wolovich matrix with $\rho_1 = 1, \rho_2 = 1$

$$A(x) = \begin{bmatrix} \dfrac{r_t - W_z}{V} & 0 \\ \dfrac{T_s - T_z}{V} & 0 \end{bmatrix}. \tag{11}$$

Therefore, the static decoupling method cannot be applied to the DX A/C system, because the matrix is singular. For a system with a singular Falb-wolovich matrix, it was found out that it may still be decoupled by a dynamic state feedback (Singh, 1981). In fact, a necessary and sufficient condition, and a systematic design method based on the so-called dynamic extension algorithm (Xia, 1993) were discovered for the dynamic decoupling of nonlinear systems. For example, if we add two integrators at v_f

$$v_f = \zeta_1,\quad \dot{\zeta}_1 = \zeta_2,\quad \dot{\zeta}_2 = v_1, \tag{12}$$

where ζ_1 and ζ_2 are the new states and v_1 is the new control variable. The DX A/C system can now be transformed into

$$\begin{cases} \dot{\widetilde{x}} = \widetilde{f}(\widetilde{x}) + \widetilde{g}(\widetilde{x})v, \\ y = h(\widetilde{x}), \end{cases} \tag{13}$$

where $\widetilde{x} = [x, \zeta_1, \zeta_2]^T$, the new control variable is $v = [v_1, v_2] = [v_1, m_r]^T$, $\widetilde{g}_1(\widetilde{x}) = [0,0,0,0,0,0,1]^T$, $\widetilde{g}_2(\widetilde{x}) = [0,0,0,-\frac{h_{r2}-h_{r1}}{C_w\rho_w V_w},0,0,0]^T$ and

$$\widetilde{f}(\widetilde{x}) = \begin{bmatrix} \frac{C_a\rho\zeta_1(T_d-T_s)+\rho\zeta_1 h_{fg}F_w}{q_t} + \\ \frac{\alpha_2 A_2(T_w - \frac{T_d+T_s}{2})}{q_t} \\ \frac{(T_s-T_z)\zeta_1}{V} + \frac{Q_l}{C_a\rho V} \\ \frac{C_a\rho(T_m-T_d)\zeta_1 + \alpha_1 A_1(T_w - \frac{T_m+T_d}{2})}{C_a\rho V_{h1}} \\ \frac{\alpha_1 A_1(\frac{T_m+T_d}{2} - T_w) + \alpha_2 A_2(\frac{T_d+T_s}{2} - T_w)}{C_w\rho_w V_w} \\ \frac{\zeta_1(r_t - W_z)}{V} + \frac{M_l}{\rho V} \\ \zeta_2 \\ 0 \end{bmatrix}.$$

The system (13) is now decouplable by static state feedback, since we calculate

$$\begin{cases} \dot{W}_z = \alpha_{11}(\widetilde{x}) = L_{\widetilde{f}}h_1(\widetilde{x}), \\ \ddot{W}_z = \alpha_{12}(\widetilde{x}) = L^2_{\widetilde{f}}h_1(\widetilde{x}), \\ W_z^{(3)} = \alpha_{13}(\widetilde{x}) + \beta_{11}(\widetilde{x})v_1 + \beta_{12}(\widetilde{x})v_2 \\ \quad = L^3_{\widetilde{f}}h_1(\widetilde{x}) + L_{\widetilde{g}_1}L^2_{\widetilde{f}}h_1(\widetilde{x}) + L_{\widetilde{g}_2}L^2_{\widetilde{f}}h_1(\widetilde{x}) \\ \quad = w_1, \\ \dot{T}_z = \alpha_{21}(\widetilde{x}) = L_{\widetilde{f}}h_2(\widetilde{x}), \\ \ddot{T}_z = \alpha_{22}(\widetilde{x}) = L^2_{\widetilde{f}}h_2(\widetilde{x}), \\ T_z^{(3)} = \alpha_{23}(\widetilde{x}) + \beta_{21}(\widetilde{x}) + \beta_{22}(\widetilde{x}) \\ \quad = L^3_{\widetilde{f}}h_2(\widetilde{x}) + L_{\widetilde{g}_1}L^2_{\widetilde{f}}h_2(\widetilde{x}) + L_{\widetilde{g}_2}L^2_{\widetilde{f}}h_2(\widetilde{x}) \\ \quad = w_2. \end{cases} \tag{14}$$

In (14), w_1 and w_2 are the new control variables and other parameters are listed in Appendix A.

The Falb-wolovich matrix of (13) now becomes

$$A(\tilde{x}) = \begin{bmatrix} \frac{r_t - W_z}{V} & \beta_{12}(\widetilde{x}) \\ \frac{T_s - T_z}{V} & -\frac{\alpha_2 A_2(h_{r2}-h_{r1})\zeta_1}{C_w\rho_w V_w V q_t} \end{bmatrix}, \tag{15}$$

where $\frac{r_t^0 - W_z^0}{V} < 0, \beta_{12}(\widetilde{x}^0) < 0, \frac{T_s^0 - T_z^0}{V} < 0, \frac{-\alpha_2 A_2(h_{r2}-h_{r1})}{C_w\rho_w V_w V q_t^0/\zeta_1^0} < 0$, and

$$|A(\widetilde{x}^0)| = \frac{\alpha_2 A_2(h_{r2}-h_{r1})\zeta_1^0}{C_w\rho_w V_w V^2 q_t^0}(W_z^0 - r_t^0 + \frac{0.0396T_s^0 + 0.085}{1000}(T_s^0 - T_z^0)) \neq 0.$$

The $A(\tilde{x})$ is nonsingular for the extended system state vector reference $\widetilde{x}^0$, indicating the extended system is static state feedback decouplable, or the original system is decouplable by dynamic state feedback.

From (14), it can be seen that the extended system (13) has a vector relative degree of $\{\widetilde{\rho}_1, \widetilde{\rho}_2\} = \{3,3\}$, which is less than the extended system's dimension, indicating the existence of internal dynamics (Isidori, 1995). To this regard, T_s cannot be a internal dynamics because it is contained in the result of differentiating $\ddot{T}_z$ once more. Therefore, only $T_d = \eta$ can be chosen as the internal dynamics.

To study the zero dynamics of (13), one can calculate that

$$\begin{aligned} \dot{\eta} =& \frac{C_a\rho(T_m-\eta)\zeta_1 + \alpha_1 A_1(T_w - \frac{T_m+\eta}{2})}{C_a\rho V_{h1}} \\ =& \frac{-\alpha_1 A_1\eta}{2C_a\rho V_{h1}} + (\frac{-\zeta_1\eta}{V_{h1}} + \frac{T_m\zeta_1}{V_{h1}} + \frac{\alpha_1 A_1(T_w - \frac{T_m}{2})}{C_a\rho V_{h1}}) \\ =& -\frac{VM_t}{V_{h1}(T_s-T_z)}\eta - \frac{\alpha_1 A_1 M_t}{\alpha_2 A_2 V_{h1}(T_s-T_z)}\eta + \\ & \frac{\alpha_1 A_1 T_s}{\alpha_2 A_2 V_{h1}(T_s-T_z)} + \frac{VT_m M_t}{V_{h1}(T_s-T_z)} - \\ & \frac{\alpha_1 A_1(T_m - T_s)}{2C_a\rho V_{h1}} + \frac{\alpha_1 A_1 h_{fg} V M_t F_w}{\alpha_2 A_2 C_a V_{h1}(T_s-T_z)} \\ & - \frac{\alpha_1 A_1 q_t}{\alpha_2 A_2(\frac{N_t}{M_t} - \frac{0.0396T_s+0.085}{1000})} \\ & (\frac{(T_s-T_z)(N_t T_z^{(2)} - M_t W_z^{(2)})}{M_t^2} + \dot{W}_z(\frac{N_t}{M_t} - 1)). \end{aligned} \tag{16}$$

Denote $\xi = [W_z, \dot{W}_z, \ddot{W}_z, T_z, \dot{T}_z, \ddot{T}_z]^T$ and let the zero dynamics $(\xi, \eta) = (\xi^0, \eta)$, where ξ^0 is the new state vector reference, then (16) becomes

$$\begin{aligned} \dot{\eta} =& (\frac{Q_l}{C_a\rho V_{h1}(T_s^0 - T_z^0)} + \frac{\alpha_1 A_1 Q_l}{\alpha_2 A_2 V_{h1} C_a\rho V(T_s^0 - T_z^0)})\eta + \\ & \frac{\alpha_1 A_1 T_s^0}{\alpha_2 A_2 V_{h1}(T_s^0 - T_z^0)} - \frac{Q_l T_m^0}{C_a\rho V_{h1}(T_s^0 - T_z^0)} - \\ & \frac{\alpha_1 A_1(T_m^0 - T_s^0)}{2C_a\rho V_{h1}} - \frac{\alpha_1 A_1 h_{fg} Q_l F_w^0}{\alpha_2 A_2 C_a^2\rho^2 V_{h1}(T_s^0 - T_z^0)}. \end{aligned} \tag{17}$$

Since $F_w^0, T_s^0, T_m^0 = \delta T_0 + (1-\delta)T_z^0$ are constant and Q_l is positive due to the indoor sensible load. (17) is then asymptotically stable when $T_z^0 > T_s^0$. In this study, we assume that the DX A/C system is operating in the cooling mode which ensures $T_z^0 > T_s^0$. Therefore, (13) is of minimum phase system according to Definition 3.

Introducing the coordinate transformation $T(\widetilde{x}) = [W_z, \dot{W}_z, \ddot{W}_z, T_z, \dot{T}_z, \ddot{T}_z, T_d]^T$, (13) can be transformed into a normal form in the z-coordination as follows:

$$\begin{aligned} \dot{\xi} &= A\xi + Bw, \\ \dot{\eta} &= \gamma(\xi,\eta) + \delta(\xi,\eta)w, \\ y &= C\xi, \end{aligned} \tag{18}$$

where $z = [\xi, \eta]^T = [z_1, z_2, z_3, z_4, z_5, z_6, z_7]^T$ is the state vector and $w = [w_1, w_2]^T$ is the new control vector. The system matrices are

$$A = \begin{bmatrix} 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&0 \end{bmatrix}, B = \begin{bmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&0 \\ 0&0 \\ 0&1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Now, one can design the control input w for the decoupled system, after which the control input for the original system can be obtained by

$$u = a^{-1}(z)(w - b(z)). \quad (19)$$

2.2 Pole placement controller design

In this paper, a pole placement controller is designed to stabilize (18).

For the decoupled two-input-two-output system represented by (18), a separate controller is designed for each input-output pair. For the first pair, denote $\xi_w = [z_1, z_2, z_3]^T$ as the state vector, and w_1 is the control input. With the desired output and state vector denoted by y_1^0 and $\xi_w^0 = [z_1^0, z_2^0, z_3^0]^T$ respectively, the error vector $e_w = [e_1, e_2, e_3]^T$ can be defined as

$$e_w = \xi_w - \xi_w^0 = [z_1 - z_1^0, z_2 - z_2^0, z_3 - z_3^0]^T. \quad (20)$$

By using the first equation of (18), which represents the linearized system, a classical pole placement control is applied and can be expressed as follows:

$$w_1 = (W_z^0)^{(3)} - c_1 e_1 - c_2 e_2 - c_3 e_3, \quad (21)$$

where the control parameters c_1, c_2 and c_3 are nonnegative real constants to be chosen.

For the second input-output part, denote $\xi_t = [z_4, z_5, z_6]^T$ as the state vector, and w_2 is the control input. The desired output command is y_2^0 and the desired state vector is defined as $\xi_t^0 = [z_4^0, z_5^0, z_6^0]^T$, then the error vector $e_t = [e_4, e_5, e_6]^T$ can be written as

$$e_t = \xi_t - \xi_t^0 = [z_4 - z_4^0, z_5 - z_5^0, z_6 - z_6^0]. \quad (22)$$

Here, the pole placement control approach is also employed to solve the above control problem of the minimum phase subsystem (18). The pole placement controller for the minimum phase system (18) can be designed by

$$w_2 = (T_z^0)^{(3)} - c_4 e_4 - c_5 e_5 - c_6 e_6, \quad (23)$$

where the control parameters c_4, c_5, c_6 are nonnegative real constants.

A simulation test is carried out to demonstrate that the designed controllers (21) and (23) can track the indoor air temperature and humidity setpoints in the next section.

3. SIMULATION RESULTS

In this section, numerical simulation results are provided to demonstrate the effectiveness and applicability of the proposed pole placement controller through state feedback decoupling. The volume of DX conditioned space is 77 m^3. The parameters of the DX A/C system are listed in table 1.

Choosing the system output commands: z_1^0 =12.3/1000 kg/kg, z_4^0 = 24 °C. The control parameters in (21) and (23) are taken as: $c_1 = 0.2, c_2 = 0.5, c_3 = 0.5, c_4 = 0.2, c_5 = 0.5, c_6 = 0.5$.

The tracking results are shown in Figs. 2 and 3, which illustrate that the air temperature and relative humidity can track the given references over a 24-hour period, respectively.

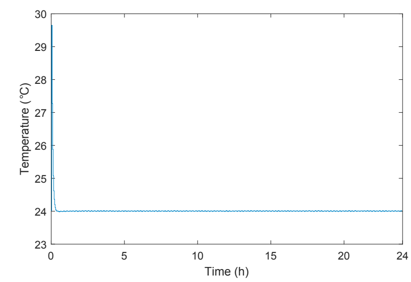

Fig. 2. Response of the air temperature

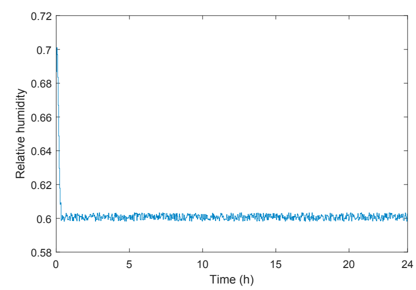

Fig. 3. Response of the moisture content

4. CONCLUSION

This paper reports the findings that the nonlinear dynamics of a DX A/C system can be decoupled by dynamic state feedback. Moreover, the resulting decoupled system was shown to be minimal phase with a stable zero. A pole placement controller is designed to control the indoor air temperature and humidity based on the decoupled input-output linear model. Effectiveness of the controller designed is demonstrated by simulation results. Further, this study provides a practical application of the state feedback decoupling method for nonlinear systems and minimum phase theory developed from the early 1970s to 1990s, during which these theories found very few applications in building energy systems.

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Table 1. System parameters

Notations	Values	Notations	Values
C_a	1.005 kJ kg^{-1} $°C^{-1}$	A_1	4.414 m^2
ρ	1.2 kg/m^3	A_2	17.656 m^2
h_{fg}	2450 kJ/kg	V_{h1}	0.04 m^3
α_1	0.089 kW m^{-2} $°C^{-1}$	V_{h2}	0.16 m^3
α_2	0.103 kW m^{-2} $°C^{-1}$		

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Appendix A

The system parameters of (14) are listed below:

$$\alpha(\tilde{x}) = \Big[(C_a\rho V(T_d - T_s) + \rho h_{fg}F_w\zeta_1 - \frac{\alpha_2 A_2}{2}(T_d + T_s))' \\ q_t + \frac{\alpha_2 A_2 q_t}{C_w\rho_w V_w}\cdot(\alpha_1 A_1(\frac{T_m + T_d}{2} - T_w) + \alpha_2 A_2 \\ (\frac{T_s + T_d}{2} - T_w)) - \frac{1}{2}(q_t^2)'\Big]/q_t^2;$$

$$\alpha_{11}(\tilde{x}) = \frac{\zeta_1(r_t - W_z)}{V} + \frac{M_l}{\rho V};$$

$$\alpha_{12}(\tilde{x}) = \frac{\zeta_2(r_t - W_z)}{V} + \frac{\zeta_1(\dot{r}_t - \dot{W}_z)}{V};$$

$$\alpha_{13}(\tilde{x}) = \frac{2\zeta_2(\dot{r}_t - \dot{W}_z) - \zeta_1\ddot{W}_z}{V} + \frac{0.0396\dot{T}_s}{1000}\dot{T}_s + \\ \frac{(0.0396T_s + 0.085)\zeta_1}{1000V}\cdot\alpha(\tilde{x});$$

$$\beta_{11}(\tilde{x}) = \frac{r_t - W_z}{V};$$

$$\beta_{12}(\tilde{x}) = -\frac{\alpha_2 A_2(h_{r2} - h_{r1})\zeta_1}{C_w\rho_w V_w V q_t}\cdot\frac{0.0396T_s + 0.085}{1000};$$

$$\alpha_{21}(\tilde{x}) = \frac{(T_s - T_z)\zeta_1}{V} + \frac{Q_l}{C_a\rho V};$$

$$\alpha_{22}(\tilde{x}) = \frac{(\dot{T}_s - \dot{T}_z)\zeta_1}{V} + \frac{(T_s - T_z)\zeta_2}{V};$$

$$\beta_{21}(\tilde{x}) = \frac{T_s - T_z}{V},\ \beta_{22}(\tilde{x}) = -\frac{\alpha_2 A_2(h_{r2} - h_{r1})\zeta_1}{C_w\rho_w V_w V q_t};$$

$$\alpha_{23}(\tilde{x}) = \frac{2\zeta_2(\dot{T}_s - \dot{T}_z) - \zeta_1\ddot{T}_z}{V} + \frac{\zeta_1}{V}\cdot\alpha(\tilde{x});$$

$$M_t = z_5 - \frac{Q_l}{C_a\rho V},\ N_t = z_2 - \frac{M_l}{\rho V};$$

$$T_s = \{-(0.085M_t - 1000N_t) - [(0.085M_t - 1000N_t)^2 - \\ 0.0792M_t((4.4984 - 1000z_1)M_t + 1000z_4N_t)]^{\frac{1}{2}} \\ \}/(0.0396M_t);$$

$$T_w = \frac{z_7 + T_s}{2} - \frac{M_t(C_a\rho V(z_7 - T_s) + \rho h_{fg}VF_w)}{\alpha_2 A_2(T_s - z_4)} + \\ \frac{q_t/\alpha_2 A_2}{(\frac{N_t}{M_t} - \frac{0.0396T_s + 0.085}{1000})}\Big[\frac{(T_s - z_4)(N_t z_6 - M_t z_3)}{M_t^2} \\ + z_2(\frac{N_t}{M_t} - 1)\Big].$$