Quintile ranking of schools in South Africa and learners' achievement in probability

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ABSTRACT

There is some disparity in the quality of education among the various races and provinces in South Africa. Since the dawn of democracy in 1994, the government has tried to bridge the gap using quintile categorisation of public schools and its concomitant funding. The categorisation is based on the socioeconomic status of the community in which the schools are located. This study investigated the achievement of learners in the first four quintiles from one school district on the topic of probability. The study employed a quantitative research approach and used Bloom's taxonomy as the conceptual framework. A total of 490 Grade 12 learners from seven schools participated in the study. Results showed that learners in Quintile 4 had significantly higher achievement scores than learners in the lower quintiles at all levels of Bloom's taxonomy except synthesis. Counter intuitively, Quintile 1 students had higher achievement than those in Quintiles 2 and 3 at all cognitive levels of Bloom's taxonomy, with the exception of synthesis. The educational implications of the findings are discussed in relation to quintile ranking of schools and learner achievement.

Keywords: Statistics education research; Bloom’s taxonomy; Cognitive levels

1. Introduction

South Africa is a country with wide economic disparity between the rich and the poor (Spaull, 2015). This inequality, as posited by Graven (2014) and Letseka and Maile (2008), has contributed to unequal educational opportunities among learners from different socioeconomic backgrounds. Likewise, Spaull and Kotze (2015) have highlighted the fact that this educational inequality (in terms of access, performance, and resource availability) is particularly evident in mathematics, a subject in which South African learners have consistently fared poorly. To address the issue of socioeconomic status and disparity in access to education, the South African government has categorised the country’s public schools into five quintiles for the purpose of allocating financial resources (Dass & Rinquest, 2017; Graven, 2014). The categorisation is based on the socioeconomic status of a school and is determined by measures of average income, unemployment rates, and general literacy level in the school’s geographical area. The schools in the most economically disadvantaged (poorest) geographical areas are categorised as Quintile 1 schools and those in the most economically advantaged geographical areas (wealthiest) as Quintile 5 (Hall & Giese, 2008). Schools in Quintiles 1 to 3 are non-fee-paying schools and receive more funding per learner from the government than schools in Quintiles 4 and 5. The latter are fee-paying schools, on the assumption that parents can afford to pay fees, and require less governmental support than schools in lower quintiles. In general, it is assumed across the nine provinces that schools of the same quintile ranking should be of comparable socioeconomic status and standard.

Despite the introduction of the quintile classification of schools and its concomitant funding in education, one wonders whether the wide gap in the academic achievement of learners from different socioeconomic backgrounds is actually being bridged. Evidence abounds (e.g., Makwakwa, 2012;

McCarthy & Oliphant, 2013; Spaull & Kotze, 2015) that mathematics in general and some topics in particular, for example statistics and probability, seem to be difficult topics to learn for most learners in South Africa.

This study explores learners' achievement in probability in schools ranked in Quintiles 1 to 4 in a province in South Africa (there were no Quintile 5 schools in the study area). The research questions addressed are: 1) What are the achievement levels of learners in the different quintiles on probability according to the cognitive levels of Bloom's taxonomy? 2) Are there any significant differences in the achievement levels of learners in the different quintiles according to the cognitive levels of Bloom's taxonomy?

2. Background

The need for probability literacy has been recognised by educational authorities in many countries largely because of the importance associated with the learning of the concept (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016). These authors observed that this need has led to the inclusion of the topic in many curricula at different educational levels in a number of countries. In South Africa, for instance, probability was recently added to the topics in mathematics to be examined in the National Senior Certificate (NSC) examinations. The South African mathematics curriculum (Department of Basic Education (DBE), 2011), is structured to cover the following topics at the Further Education and Training level (Grades 10–12): comparison of the relative frequency of an experimental outcome with the theoretical probability of the outcome; mutually exclusive and complementary events; the identity for any two events A and B: P (A or B) = P (A) + P (B) – P (A and B); dependent and independent events; use of Venn diagrams, contingency tables, or tree diagrams to solve probability problems (where events are not necessarily independent); and probability problems using the fundamental counting principle.

At the international level, some studies have shown probability to be a problematic topic for both teachers and learners (Batanero & Díaz, 2012). According to these authors, learners find probability more difficult to understand than other mathematical topics because it requires greater interpretation and understanding of context. For example, Pratt (2011) investigated probability reasoning at the secondary school level in the United Kingdom and found that there are a multitude of problems associated with the teaching and learning of probability in that country. Similarly, in South Africa, since the introduction of statistics in the school curriculum, research has shown that teachers find the teaching of probability challenging (Makwakwa, 2012; Wessels & Nieuwoudt, 2011) and learners have not fared well on the topic in the National Senior Certificate (NSC) examinations (DBE, 2016). For this study, we were interested in investigating whether the quintile classification of schools has any bearing on learners' achievement in the topic of probability.

Some studies have shown that learners' achievement, especially in mathematics and science, seems to be associated with the socioeconomic status of their schools (Graven, 2014). For example, in a study on the effect of socioeconomic status on learner achievement, Van der Berg (2008) found that learners in the more affluent quintile schools significantly outperformed learners in the less affluent quintile schools. Similarly, Spaull (2011) found that the more affluent quintiles (Quintiles 4 and 5) outperformed the less affluent quintiles (Quintiles 1, 2 and 3) in academic achievement. Nonetheless, this author pointed out that learners' achievement does not improve evenly across the various socioeconomic levels (quintiles). Indeed, in the South African setting, Reddy et al. (2012) pointed out that, on average, Grade 9 learners in Quintiles 1 and 2 are three years behind in mathematics learning compared with Quintile 5 learners, which suggests that there is a considerable gap in mathematics achievement between the wealthier and poorer quintiles. In addition, Mpofu (2015) found that students from lower quintile schools had lower average scores in Grade 12, a higher dropout rate at university, and took more time to graduate compared to their counterparts from higher quintile schools. The implications of these findings are that the lower quintiles are underperforming academically. Njoroge and Nyabuto (2014), however, note that learners whose parents have a hands-on approach to their education, regardless of their socioeconomic status and background, are more likely to perform well academically.

It is worth noting that empirical evidence of the relationship between the quintile ranking of schools and student achievement in mathematics in South Africa is still quite limited. Whilst some of the past studies report a positive relationship between learner achievement and quintile ranking of schools, the studies did not compare the learners' achievements on specific aspects of the subject or at various cognitive levels, but instead used the overall achievement scores of the learners. There is a possibility that the disparity in learners' achievement in relation to school quintile may be only on some concepts or topics and/or at different cognitive levels. Hence, the consideration of learners' achievement based on specific concepts or topics and at various cognitive levels may paint a different picture of the situation and would perhaps provide more conclusive evidence on the association between quintile classification of schools and learner achievement. Thus, we aimed to assess learner achievement levels on the topic of probability according to Bloom's taxonomy in relation to the quintile ranking of the school.

The outcome of this study hopes to contribute to the debate on the impact of the availability, or otherwise, of school resources on educational outcomes in South Africa and elsewhere.

3. Conceptual framework

The study is underpinned by Bloom's (1956) taxonomy, initially described as a hierarchical model for the cognitive domain. The taxonomy provides classification of educational objectives to help teachers, administrators, and researchers assess curricular and evaluation problems with greater precision (Bloom, 1994). The taxonomy gives an indication of the type of cognitive process required to answer questions correctly. It consists of six cognitive skill levels: knowledge, comprehension, application, analysis, synthesis, and evaluation. These cognitive levels increase in complexity starting from the lowest level (knowledge) to the most complex (evaluation). The description of the taxonomy with sample keywords according to Huitt (2011) is presented below.

The knowledge cognitive level deals with recall or recognition of terms, ideas, procedures, theories, formulas, etc. For example, "What is the condition for two events, A and B, to be independent?" Some keywords used in this cognitive level are define, describe, identify, and list.

The comprehension cognitive level pertains to learners' abilities to grasp the meaning of previously learned material. This may be shown by translating material from one form to another, interpreting material (explaining or summarizing), or predicting consequences or effects. For example, "Give examples of mutually exclusive events." Examples of key words used under comprehension are comprehend, convert, distinguish, predict, summarize, and give examples.

The application cognitive level requires the ability to use learned material in new and concrete situations. It may include the application of rules, methods, concepts, principles, laws and theories, and equations. For example, the question might be "The probability that it will rain is 0.6 and the probability that it will not rain is 0.4. Show that these two events are mutually exclusive." Keywords that might be applicable here are complete, construct, demonstrate, discover, solve, and show.

The analysis cognitive level requires the ability to break down material into component parts so that its organisational structure may be understood. This exercise may include the identification of parts, analysis of the relationships between the parts, and the recognition of the organisational principles involved. For example, "In a class of 30 learners, 15 have travelled on an aeroplane and 17 have travelled on a ship. Each learner has travelled on at least one of the two. Draw a Venn diagram to illustrate the information." This question requires learners to be able to place values in the correct regions on the Venn diagram. The keywords used to assess the analysis level include break down, compare, contrast, outline, and distinguish.

The synthesis cognitive level requires the ability to put parts together to form a new whole; this may involve the production of a unique communication (thesis or speech) or a plan of operations (research proposal). For example, "Give an account of why the probability that it will rain today and the probability that it will not rain today are said to be mutually exclusive." Keywords used in framing synthesis questions include categorize, combine, compile, compose, create, modify, write, and tell.

The evaluation cognitive level requires learners to make a judgement on ideas. For example, "Given that $P(A) = 0.7$ and $P(B) = 0.5$, Is it possible that A and B are mutually exclusive? Explain." The keywords used in framing evaluation questions include compare, conclude, defend, explain, and support.

Although Bloom's taxonomy has been revised, with changes made to the original names, the language of the old version of the taxonomy has been adapted here because it is well known and universally accepted across disciplines and national borders, as argued by Karaali (2011). Bloom's taxonomy has been used in numerous studies. For example, Assaly and Smadi (2015) used the taxonomy to evaluate the cognitive levels of masterclass textbook questions in Israel. They used a checklist based on Bloom's taxonomy of the cognitive domain to record and tally the cognitive levels of the questions collected for mastering the reading section of the masterclass textbook. Liman and Isma'il (2015) used the taxonomy to study the relationship between three elements of the cognitive domain: remembering, understanding, and application in mathematical achievement among secondary school learners in Nigeria. Karaali (2011) utilised the sixth level of Bloom's taxonomy (evaluation cognitive level) in calculus classroom. She said that most textbooks rarely give examples of activities that involve the evaluation cognitive levels. However, based on her study, she concluded that evaluative tasks have a place in the mathematics classroom and that teachers should incorporate such in teaching the subject.

4. Research method

To address the research questions, a cross-sectional survey research design was used as the mode of inquiry for this study. A cross-sectional survey entails collecting data at one point in time or over a short period in order to make inferences about a population (Creswell, 2015). The study was conducted in an education circuit in one province in South Africa. In South Africa, each provincial education department is divided into district education departments, which are further divided into circuits. So, an education circuit is composed of schools in the same geographical area. There were 12 secondary schools in the circuit where the study was conducted: among them were four Quintile 1 schools, four Quintile 2 schools, three Quintile 3 schools and one Quintile 4 school. Stratified random sampling (Creswell, 2015) based on quintiles was used to select two schools from each of Quintiles 1 to 3 and one school (the only school) from Quintile 4. There were no Quintile 5 schools in the study area. All Grade 12 mathematics learners in the seven schools participated in the study with the exception of those who were absent on the day of the test. There were 490 participants (280 girls and 210 boys). There were 143 Quintile 1 learners, 107 learners from Quintile 2, 92 Quintile 3 learners and 148 Quintile 4 learners.

Teachers in the circuit meet at least once a month to plan collectively and prepare for lessons and examinations. Because the Quintile 4 school had boarding facilities, some of its learners resided on the school premises. Schools in Quintiles 1 to 3 had no boarding facilities, but because some learners travelled long distances to school the common practice among these schools was that Grade 12 learners were encouraged to find accommodation closer to the school. The teachers in these schools had comparable qualifications and years of teaching experience: all teachers had taught the subject for a minimum of four years. Teachers in the circuit normally meet twice every term to discuss questions and moderate examination papers after marking. Content workshops are organised by the Department of Basic Education at the beginning of every year to enhance/refresh teachers' content knowledge and the teaching methodology.

4.1. Data collection

The instrument for data collection was a pen-and-paper achievement test developed by the researchers, as we did not find an existing instrument that covered all the sub-topics and concepts of probability in the curriculum. The questions required open-ended responses, with some questions requiring short answers and others long answers. The marks allocated to each question depended on the complexity of the question and the steps required to solve the problem. The construction of the test was guided by Bloom's taxonomy and the Grades 10–12 mathematics curriculum assessment guidelines (DBA, 2011). The test covered all subtopics of probability in the curriculum, namely mutually exclusive and complementary events; the identity for any two events; dependent and independent events; use of Venn diagrams, contingency tables and tree diagrams to solve probability problems; and fundamental counting principles (see Appendix). There was no time constraint for the test in order to eliminate undue strain on learners and the possibility of errors being made because of time pressure; this strategy ensured that learners could perform to the best of their ability.

To ensure the validity of the test and the marking guide, three experts in the field of mathematics education reviewed the documents in relation to the curriculum and assessment guidelines. One of the experts was a mathematics subject advisor and the other two were senior mathematics educators and markers. These experts evaluated the mark allocation of each question, the language used, and the content covered, as well as the classification of the questions according to the cognitive levels of Bloom's taxonomy. They made recommendations regarding the diction of the questions and the mark allocation. They also judged the level of alignment of each question against the curriculum by using a 3-point rating scale (1 = not aligned; 2 = fairly aligned; 3 = much aligned). All questions were retained because they all had an average rating of 2.5. The instrument was pilot-tested in a school that did not participate in the main study. The results from the pilot test were used to compute the test's reliability. The test-retest method was used to ascertain the reliability of the instrument and a reliability coefficient of 0.723 was obtained. According to Hof (2012), an acceptable reliability coefficient should lie between 0.70 and 0.90.

4.2. Data analysis

We performed descriptive and inferential analysis using SPSS 23. The descriptive statistics involved the mean and standard deviations of the learners' scores at each level of Bloom's taxonomy. These gave a picture of overall performance of the quintiles at each level of Bloom's taxonomy. The inferential analysis explored whether any difference in the achievement of the quintiles was statistically significant.

5. Findings

The mean and standard deviations of the learners' scores at the various levels of Bloom's taxonomy are presented in Table 1.

Table 1. Means and standard deviations of the learners' scores at Bloom's taxonomy levels according to quintile

Quintile	Quintile 1 (n = 143)		Quintile 2 (n = 107)		Quintile 3 (n = 92)		Quintile 4 (n = 148)		Total (n = 490)	
Bloom's cognitive level	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Knowledge	66.39	19.33	60.55	23.12	51.32	15.56	91.17	8.30	69.78	22.69
Comprehension	37.34	19.66	36.76	17.75	18.12	10.64	60.84	14.72	40.73	22.18
Application	50.11	21.27	35.24	28.79	17.84	19.57	51.67	21.06	41.28	26.12
Analysis	37.49	12.15	27.52	13.36	11.51	11.35	50.76	11.25	34.44	18.34
Synthesis	17.65	20.60	22.43	25.91	14.4	28.76	22.47	16.69	19.54	22.71
Evaluation	9.44	20.70	8.56	17.79	5.43	18.01	21.74	28.48	12.21	23.21

The average achievement of Quintile 4 learners was higher than all the other quintiles at all levels of Bloom's taxonomy. Following Quintile 4 was Quintile 1, then Quintile 2 and Quintile 3 respectively. The box plots of the distribution of scores across the levels of Bloom's taxonomy for the quintiles are shown in Figure 1.

Excluding the outliers, the same trend of achievements was observed among the quintiles across the levels of Bloom's taxonomy, especially the first four cognitive levels. At the synthesis level of Bloom's taxonomy, the minimum, the first quartile, the second quartile, and the upper quartile scores of Quintile 3 are identical, meaning that over 75% of learners scored zero (and hence the box and the whiskers are not visible). Similarly, the minimum, the first quartile, the second quartile, and the upper quartile scores of Quintile 4 are identical (about 22%). The same observation (the invisible box and whiskers) could be made for Quintiles 1 and 3 at the evaluation level of Bloom's taxonomy; over 75% of the learners in Quintiles 1 and 3 scored zero at the evaluation level of Bloom's taxonomy.

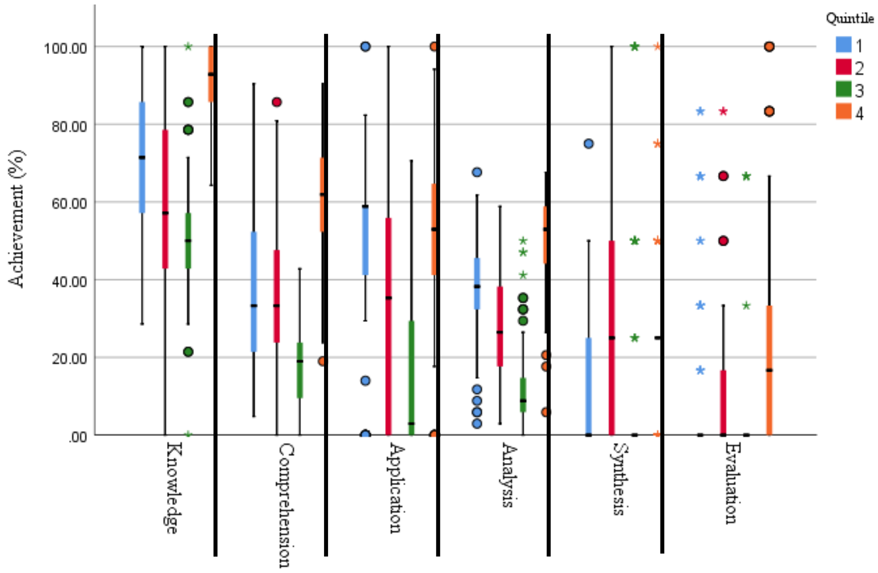

Figure 1. Quintile scores according to levels of Bloom's taxonomy

Welch's ANOVA (Welch, 1951) was used to examine whether the learners' achievement scores on the test significantly differed by quintiles. This was used because the Levene test for homogeneity of variance was significant for all levels of Bloom's taxonomy except for the analysis level (see Table 2), hence one-way ANOVA could not be used. Welch's ANOVA is a good approach for performing an ANOVA when the homogeneity of variances' assumption is not met (Jan & Shieh, 2014).

Table 2. Levene test of homogeneity of the variances of learners' achievement

	Levene statistic	df_1	df_2	*p*-value
Knowledge	36.825	3	486	< 0.001
Comprehension	17.430	3	486	< 0.001
Application	8.682	3	486	< 0.001
Analysis	2.391	3	486	0.068
Synthesis	21.306	3	486	< 0.001
Evaluation	23.928	3	486	< 0.001

The independent variable represents school quintile and the dependent variable is the learners' score.

The Welch test of the learners' achievement score (see Table 3) reveals a statistically significant difference among the mean achievement scores of the quintiles at all levels with the exception of the synthesis level of Bloom's taxonomy.

Table 3. Welch test of equality of means

	F-statistic	df_1	df_2	*p*-value
Knowledge	229.669	3	218.999	< 0.001
Comprehension	225.313	3	259.882	< 0.001
Application	69.069	3	249.835	< 0.001
Analysis	239.596	3	251.304	< 0.001
Synthesis	2.753	3	234.173	0.043
Evaluation	11.629	3	262.323	< 0.001

The Games-Howell post-hoc test for multiple comparisons was used to determine which pairs of the four groups' means significantly differed at the Bloom's taxonomy levels of knowledge, comprehension, application, analysis, and evaluation. The results are shown in Table 4.

Table 4. Games-Howell post-hoc multiple comparisons test

Dependent variable	Quintile *I*	Quintile *J*	Diff in Means (*I* – *J*)	*SE*	*p*-value	95% CI
Knowledge	1	2	9.311*	2.782	0.005	(2.11, 16.52)
		3	18.544*	2.315	< 0.001	(12.55, 24.54)
		4	–21.416*	1.792	< 0.001	(–26.06, –16.77)
	2	1	–9.311*	2.782	0.005	(–16.52, –2.11)
		3	9.233*	2.756	0.005	(2.09, 16.38)
		4	–30.727*	2.335	< 0.001	(–36.81, –24.65)
	3	1	–18.544*	2.315	< 0.001	(–24.54, –12.55)
		2	–9.233*	2.756	0.005	(–16.38, –2.09)
		4	–39.960*	1.752	< 0.001	(–44.52, –35.40)
	4	1	21.416*	1.792	< 0.001	(16.77, 26.06)
		2	30.727*	2.335	< 0.001	(24.65, 36.81)
		3	39.960*	1.752	< 0.001	(35.40, 44.52)
Comprehension	1	2	1.122	2.426	0.967	(–5.15, 7.40)
		3	19.803*	2.040	< 0.001	(14.52, 25.08)
		4	–22.860*	2.099	< 0.001	(–28.29, –17.43)
	2	1	–1.122	2.426	0.967	(–7.40, 5.15)
		3	18.681*	2.039	< 0.001	(13.39, 23.97)
		4	–23.982*	2.099	< 0.001	(–29.42, –18.55)
	3	1	–19.803*	2.040	< 0.001	(–25.08, –14.52)
		2	–18.681*	2.039	< 0.001	(–23.97, –13.39)
		4	–42.663*	1.637	< 0.001	(–46.90, –38.43)
	4	1	22.860*	2.099	< 0.001	(17.43, 28.29)
		2	23.982*	2.099	< 0.001	(18.55, 29.42)
		3	42.663*	1.637	< 0.001	(38.43, 46.9)
Application	1	2	16.442*	3.315	< 0.001	(7.85, 25.04)
		3	33.923*	2.714	< 0.001	(26.89, 40.95)
		4	–0.046	2.499	1.000	(–6.51, 6.41)
	2	1	–16.442*	3.315	< 0.001	(–25.04, –7.85)
		3	17.480*	3.447	< 0.001	(8.54, 26.42)
		4	–16.489*	3.281	< 0.001	(–25.00, –7.98)
	3	1	–33.923*	2.714	< 0.001	(–40.95, –26.89)
		2	–17.480*	3.447	< 0.001	(–26.42, –8.54)
		4	–33.969*	2.673	< 0.001	(–40.89, –27.05)
	4	1	0.046	2.499	1.000	(–6.41, 6.51)
		2	16.489*	3.281	< 0.001	(7.98, 25.00)
		3	33.969*	2.673	< 0.001	(27.05, 40.89)
Analysis	1	2	10.943*	1.647	< 0.001	(6.68, 15.21)
		3	26.828*	1.571	< 0.001	(22.76, 30.90)
		4	–12.343*	1.400	< 0.001	(–15.96, –8.73)
	2	1	–10.943*	1.647	< 0.001	(–15.21, –6.68)
		3	15.885*	1.739	< 0.001	(11.38, 20.39)
		4	–23.286*	1.585	< 0.001	(–27.39, –19.18)
	3	1	–26.828*	1.571	< 0.001	(–30.90, –22.76)
		2	–15.885*	1.739	< 0.001	(–20.39, –11.38)
		4	–39.171*	1.506	< 0.001	(–43.07, –35.27)
	4	1	12.343*	1.400	< 0.001	(8.73, 15.96)
		2	23.286*	1.585	< 0.001	(19.18, 27.39)
		3	39.171*	1.506	< 0.001	(35.27, 43.07)
Evaluation	1	2	1.099	2.500	0.972	(–5.37, 7.56)
		3	4.222	2.618	0.374	(–2.56, 11.00)
		4	–14.166*	3.163	< 0.001	(–22.34, –5.99)
	2	1	–1.099	2.500	0.972	(–7.56, 5.37)
		3	3.123	2.550	0.612	(–3.49, 9.73)
		4	–15.265*	3.107	< 0.001	(–23.30, –7.23)
	3	1	–4.222	2.618	0.374	(–11.00, 2.56)
		2	–3.123	2.550	0.612	(–9.73, 3.49)
		4	–18.388*	3.203	< 0.001	(–26.68, –10.10)
	4	1	14.166*	3.163	< 0.001	(5.99, 22.340)
		2	15.265*	3.107	< 0.001	(7.23, 23.30)
		3	18.388*	3.203	< 0.001	(10.10, 26.68)

*$p < 0.05$

The post-hoc result showed that at the knowledge level, Quintile 4 achieved significantly better than the other quintiles. It was followed by Quintile 1 that achieved significantly better than Quintiles 2 and 3. The achievement of Quintile 2 was also found to be significantly better than that of Quintile 3. Similarly, at the comprehension level, Quintile 4 achieved significantly better than the other quintiles, and Quintiles 1 and 2 achieved significantly better than Quintile 3.

At the application level, Quintiles 4, 1, and 2 all had statistically significant better achievement than Quintile 3. There was no statistically significant difference between the achievements of Quintiles 1 and 4, however, both quintiles achieved significantly better than Quintile 2, and Quintile 2 significantly achieved better than Quintile 3. At the analysis level, as was found at the other levels of Bloom's taxonomy, Quintile 4 achieved significantly better than the other quintiles, followed by Quintile 1 that achieved significantly better than Quintiles 2 and 3, and Quintile 2 that was significantly better than Quintile 3. At the evaluation level, Quintile 4 achieved significantly better than the other quintiles and no statistically significant difference was found between the achievements of Quintiles 1, 2, and 3.

6. Discussion of findings

The findings reveal that students in Quintile 4 overall achieved higher average mean scores than students in all the other quintiles at all cognitive levels of Bloom's taxonomy: knowledge, comprehension, application, analysis, synthesis, and evaluation. Students in Quintile 1 followed, showing higher achievement than students in Quintiles 2 and 3 at all cognitive levels except for synthesi,s where the Quintile 1 learners achieved slightly lower than Quintile 2 learners. Quintile 3 learners had the lowest average achievement scores at all cognitive levels. The Games-Howell post-hoc test for multiple comparisons showed that there were statistically significant differences among the mean scores of the quintile achievements at knowledge, comprehension, application, analysis, and evaluation cognitive levels. We also note that the mean scores of students from all the quintiles decreased substantially from the knowledge cognitive level to the evaluation cognitive level.

The higher achievement of students from Quintile 4 (fee-paying schools) compared to Quintiles 1, 2 and 3 (non-fee-paying schools) at all cognitive levels of Bloom's taxonomy agrees with the assertions of Spaull (2011) and Van der Berg (2008) that learners in more affluent schools outperform those in less affluent schools in academic achievement scores. This could occur because fee-paying schools, despite receiving less funding and support from the government than non-fee-paying schools, supplement the government funding with school fees; hence the possibility is that they are able to acquire additional teaching and learning resources to aid teaching and enhance students' learning (Mestry & Ndhlovu, 2014; Wilmot & Dube, 2015). Mestry and Ndhlovu (2014) noted that some fee-paying schools use fees to hire additional teachers on contract. This helps the schools to reduce learner-teacher ratio thus enhancing the quality of teaching. This may not be the case in most of the non-fee-paying schools, where teachers are burdened with an extensive workload as a result of the high learner-teacher ratio as well as a shortage of resources to make teaching easier. Many studies (e.g. Jana, Arui, Dutta, & Sar, 2016; Ogbonnaya, Mji, & Mohapi, 2016; Skipton & Cooper, 2014) have shown that a high learner-teacher ratio has a negative impact on teachers' effectiveness and learners' achievement.

There is the possibility of the influence of parental support and socioeconomic status of the learners' parents on the findings of this study. One's educational achievements have a strong relationship with the socioeconomic status of one's parents (Perera, 2014). Larocque, Kleiman, and Darling (2011) observed that parental support or involvement in their children's education positively affects the children's learning. Parents have the responsibility to ensure that their children attend school regularly and do their homework. Educated parents, as is likely the case of many parents of the Quintile 4 learners, are often gainfully employed and able to provide for the educational needs of their children—they serve as mentors to their children and often provide additional teaching at home (Lee & Burkham, 2002). This support gives their children an edge over those whose parents do not provide such support. This view is in line with the finding of Visser, Juan, and Feza (2015), and Taylor and Yu (2009) that parental education is one component of socioeconomic status that directly benefits children's educational achievement in South Africa. Taylor and Yu also argue that better educated parents are more likely to get involved in their children's school community thereby increasing the teachers' sense of accountability towards the parents and concomitantly the school quality.

It is also possible that the differences found between the achievement of learners in the Quintile 4 school and the achievement of learners in the Quintiles 1 to 3 schools might have been because the Quintile 4 school was a boarding school. A boarding school might offer a stable and structured learning environment (Lawrence, 2005) and through supervised study sessions, a boarding school might give learners out-of-class access to teachers who might provide extra academic support to them after normal school hours (Martin, Papworth, Ginns, & Liem, 2014). These might have led to better achievement of Quintile 4 learners in the study.

In addition, the language of teaching and learning might have had an influence on the findings of this study. Language plays a key role in the learners' processing of mathematical texts and their interpretation of mathematical problems. Fluency enables them to ask questions and to discuss their answers with others (Hoosain, 1991). Mathematics is taught in English, which is at least a second language, and in some instances a third language, to most of the learners in South Africa, especially those of low socioeconomic status (Setati, Molefe, & Lange, 2008). These learners have little access to the English language outside of schooling (Gravin, 2014). To achieve a high score on the test, learners needed to master the probability concepts, understand the questions, and be able to communicate their understanding in English when they answer the questions. Where learners experience difficulties in grasping concepts and terminology in a subject because of a language barrier, it will be difficult for them to excel in it. This is in consonance with the finding of Visser et al. (2015) that speaking the language of teaching and learning at home is strongly related to learners' achievement in mathematics in South Africa, and also with the view of Durkin (1991) that "mathematics education begins with language and stumbles because of language" (p. 3). This suggests that the poor achievement of learners in Quintiles 1 to 3 on the test may be connected to language barrier.

Turning the focus to the non-fee-paying schools (Quintiles 1 to 3), this study shows that students at Quintile 3 schools achieved the lowest scores at all levels of Bloom's taxonomy whereas students at Quintile 1 schools, the schools presumed to be least resourced according to the quintile ranking, achieved above Quintiles 2 and 3 schools at all levels of the taxonomy, except at the synthesis level, where they achieved slightly lower than Quintile 2. This is not in agreement with the assertion by the Department of Basic Education (as cited by Hall & Giese, 2008) that higher quintile schools perform better than lower quintile schools in terms of achievement scores. Although the exact reason for this is unclear, one may not rule out the possibility of ranking error in the quintile ranking of schools (Collingridge, 2013). There is also the possibility that any factor or a combination of factors such as school leadership, teachers' dedication and classroom teaching practices, and learner discipline contributed to the higher achievement of learners in the Quintile 1 schools than in the Quintiles 2 and 3 schools.

One should also not rule out the effect of teacher knowledge and instructional practice on the findings of this study, especially as it pertains to the non-fee-paying quintiles. Despite the fact that all the teachers whose learners participated in the study were of comparable academic qualifications and years of teaching experience, there could be differences in their knowledge of the subject matter and pedagogical knowledge that influenced their classroom practices and consequently the achievements of their learners. The teachers may have qualified through participation in differing academic programmes, which implies that the content, and perhaps the quality, of their training may also differ.

The results revealed that the mean score of the learners in all quintiles dropped, moving from the lower levels to the higher cognitive levels of Bloom's taxonomy. The low-level questions test knowledge and comprehension; they ask for definitions requiring rote memorization and paraphrasing of information given by the teacher or in the textbooks. High-level questions require application, analysis, synthesis, or evaluation of information or concepts. Addressing a high-level question requires critical thinking as a consequence of conceptual understanding of the subject matter. Learners' poor achievement on the high-level questions suggests that they lack conceptual understanding of the topic, which suggests that the teaching of the topic in all the quintiles focused more on memorisation of concepts and procedures than on conceptual understanding. This could have occurred because the teachers lack in-depth knowledge of the topic, which limited their abilities to teach in ways that could help learners to think critically in order to address the high-level questions.

7. Conclusion and recommendations

In conclusion, learners in Quintile 4 (fee-paying quintile) overall achieved a higher mean score in probability than learners in Quintiles 1 to 3 (non-fee-paying quintiles), supporting the view that fee-paying quintiles tend to achieve higher academic scores than non-fee-paying quintiles. Among the non-fee-paying quintiles (Quintiles 1 to 3), there was a decline in achievement from Quintile 1 to Quintile 3, This finding is counterintuitive and makes a case for an in-depth investigation into the teaching and learning of mathematics in the non-fee paying schools and the quintile classifications of the schools.

Overall, the findings of this study support the claim by Mpofu (2015) that quintile ranking of schools in South Africa is a useful tool but not a perfect means of categorisation to help improve learner achievement. This is particularly true for topics (for example probability) that have been newly introduced into the South African school curriculum.

The findings of this study provide evidence with practical implications. First, we recommend regular content-specific professional development for all teachers, especially on newly introduced topics. This would help teachers to be more effective in teaching the content of the subject, especially at higher cognitive levels. Secondly, because school leadership plays a significant role on learners' achievement (Dutta & Sahney, 2016; Leithwood, Harris, & Hopkins, 2008), the leadership capabilities of the low quintile schools should be investigated with the view of providing necessary support to these schools. Third, the findings of this study make a case for in-depth study of teachers' classroom practices in teaching probability in schools across the different quintiles. The study provides a picture of the teachers' difficulties in teaching probability, as well as the educational challenges experienced by learners when approaching this topic.

Because the criteria for the quintile ranking of schools are identical across the provinces, the findings of this study may be generalised to other provinces in the country. The findings, especially those relating to the drop in learners' achievement from the lower to the higher cognitive levels of Bloom's taxonomy, might also apply to other countries given the problems learners have in learning the topic of probability in many countries. Future studies should look into school leadership and teacher classroom instructional practices in the quintiles and their possible impact on leaners' achievement in probability.

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Appendix: Test questions

Question number	Question	Bloom's cognitive level
Q1	Identify with reasons the portions of figure that exhibits the properties listed below.	

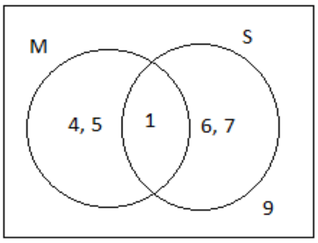

Figure 1

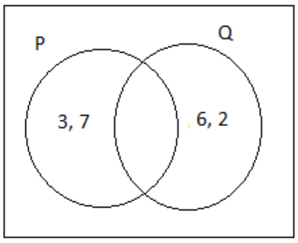

Figure 2

Question number	Question	Bloom's cognitive level
1.1.1	Mutually exclusive	Comprehension
1.1.2	Inclusive	Comprehension
1.1.3	Complementary	Comprehension
1.1.4	Exhaustive	Comprehension
	Write down a mathematical expression for P(A or B)	
1.2.1	If A and B are mutually exclusive	Knowledge
1.2.2	If A and B are inclusive	Knowledge
Q2	Given that $P(A) = 0.6$ $P(B) = 0.5$ $P(A \cap B) = 0.2$	
2.1	Illustrate the information on the Venn diagram	Analysis
2.2	Find $P(A \cup B)$	Comprehension
2.3	Find $P(A \cup B)'$	Comprehension
2.4	Show that events A and B are not independent	Application
2.5	Are the events A and B complementary?	Knowledge
2.6	Give two reasons to support your answer in question 2.5	Comprehension
Q3	Thandeka has a bag containing 5 green balls and 7 red balls. Two balls are picked at random from the bag one after the other. Illustrate the information on a tree diagram if	
3.1.1	The first ball was replaced before the second ball was picked	Analysis
3.1.2	The first ball was not replaced and second ball was picked	Analysis
3.2	Find the probability that the balls selected were of different colours	Comprehension
3.3	Find the probability that the two balls picked were of the same colour	Comprehension
3.4	Find the probability that at least one of the of the balls picked was green	Comprehension
3.5	Beside the use of the tree diagram, provide any other way of obtaining the sample space to the question assuming that the probability of picking a ball is independent.	Synthesis
3.6	For a number of experiments provide any two ways by which one can determine whether a tree diagram drawn is correct or wrong	Knowledge
Q4	Each of the 200 employees of a company wrote a competency test. The results are shown in the table below.	

	Pass	Fail	Total
Male	*a*	32	*d*
Female	72	50	122
Total	118	*b*	*c*

Question number	Question	Bloom's cognitive level
4.1a	Find the values of *a*, *b*, *c*, and *d*	Comprehension
4.1b	Are the events Pass and Fail mutually exclusive? Explain your answer	Comprehension
4.2	Show that the competency test is independent of gender	Application
4.3	Give any alternate solution to question 4.2	Synthesis
4.4	Calculate the probability that a learner selected at random was a male who passed or a female.	Application

Q5	Consider the word MAN. Write down all the three-letter arrangements that can be made if the	
5.1.1	Letters can be repeated	Comprehension
5.1.2	Letters cannot repeated	Comprehension
	Write down all the two-letter arrangements that can be made if the	
5.2.1	Letters can be repeated	Comprehension
5.2.2	Letters cannot be repeated	Comprehension
5.3	How many three-letter arrangements are possible in 5.1.1	Knowledge
5.4	How many three-letter arrangements are possible in 5.1.2	Knowledge
5.6	How many two-letter arrangements are possible in 5.2.2	Knowledge
	From the above investigation provide a formula that can be used to arrive at the same answer:	
5.7.1	5.1.1	Evaluation
5.7.2	5.1.2	Evaluation
5.7.3	5.2.1	Evaluation
5.7.4	5.2.2	Evaluation
Q6	Three vacant places are to be filled by 5 people.	
6.1.1	In how many ways can the first place be filled?	Comprehension
6.1.2	In how many ways can the second place be filled?	Comprehension
6.1.3	In how many ways can the third place be filled?	Comprehension
6.1.4	In how many ways can the 5 people fill the 3 places?	Comprehension
	Compute the following	
6.2.1	5!	Comprehension
6.2.2	(5 – 3)!	Comprehension
6.2.3	$\frac{5!}{(5-3)!}$	Comprehension
6.2.4	Compare your answers in 6.1.3 with the result in 6.2.3	Evaluation
6.2.5	Hence, for *n* items occupying *r* positions at a time predict an appropriate formula to assist in calculation of the number of ways to do this.	Evaluation