Forecasting Interest Rate Volatility of the United Kingdom: Evidence from over 150 Years of Data

Hossein Hassani[a,*], Mohammad Reza Yeganegi[b], Juncal Cuñado[c], Rangan Gupta[d]

[a] *The Statistical Research Centre, Bournemouth University, Bournemouth, UK*
[b] *Department of Accounting, Islamic Azad University Central Tehran Branch, Iran*
[c] *University of Navarra, School of Economics, Edificio Amigos, E-31080 Pamplona, Spain*
[d] *Department of Economics, University of Pretoria, Pretoria, 0002, South Africa*

Abstract

This study examines the very short, short, medium and long-term forecasting ability of different univariate GARCH models of United Kingdom (UK)'s interest rate volatility, using a long span monthly data from May 1836 to June 2018. The main results show the relevance of considering alternative error distributions to the normal distribution when estimating GARCH-type models. Thus, we obtain that the Asymmetric Power ARCH (A-PARCH) models with skew generalized error distribution are the most accurate models when forecasting UK interest rates, while for the short, medium and long-term term forecasting horizons (h=3 and h=6, h=12), GARCH models with generalized error distribution for the error term are the most accurate models in forecasting UK's interest rates.

Keywords: interest rates; volatility; GARCH models; forecasting; error distributions
JEL: C22; C53; G17.

1. Introduction

While UK interest rates were very stable during the 19th century and until World War I, the evolution of interest rates in the twentieth century showed periods of large increases and decreases, that is, periods of high volatility. For example, interest rates reached their highest level (17%) in 1979, after a decision of the conservative government to combat inflation after the oil price shock, and it also increased to 15% at the beginning of the 1990s to keep the value of the pound fixed in the European Exchange Rate Mechanism. These increases in interest rates were followed by the recessions occurred in UK in 1980/81 and 1991/92. On the other hand, in the context of the financial crisis, interest rates fell to their lowest levels in 300 years,

*Corresponding author
Email addresses: hhassani@bournemouth.ac.uk (Hossein Hassani), m.yeganegi@iauctb.ac.ir (Mohammad Reza Yeganegi), jcunado@unav.es (Juncal Cuñado), rangan.gupta@up.ac.za (Rangan Gupta)

reaching a level of 0.25% in 2016. Interest rates are not only a key tool of monetary policy, but they have also important implications on many economic variables, such as investment rate, economic growth, stock returns, or exchange rates, among others (Bernanke, 1983). Furthermore, interest rate volatility can be taken as an indicator of uncertainty and risk, and it is fundamental to price securities, which justifies the numerous attempts to model and forecast interest rates and their volatility. In this context, modelling and forecasting the interest rate volatility will be important from a monetary policy view (Walsh, 1984; Chadha and Nolan, 2001; Bartolini et al., 2002) and also for portfolio diversification (Barnhill and Maxwell, 2002), for risk managers and for hedging strategies (Carcano and Foresi, 1997; Chan et al., 2001).

An extensive number of papers on modelling interest rate volatility can be found in the literature. As it happens with most of the financial series, modelling interest rates requires to take into account the following characteristics usually observed in these variables, such as volatility clustering, excess kurtosis, asymmetric effects, non-linearities, time varying volatility and volatility clustering, long-memory or leverage effect (Franses and Dijk, 2000; Zumbach, 2013). In a seminal paper by Chan et al. (1992), for example, the authors assume that UK interest rate volatility is sensitive to interest rate levels -level effect-, while Brenner et al. (1996) and Koedijk et al. (1997) propose a model for the interest rate volatility that takes into account not only the level effect in Chan et al. (1992) but also the conditional heteroscedasticity effect of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) type models (Engle, 1982; Bollerslev, 1986), which explains the extensive use of these models to model interest rates (Longstaff and Schwartz, 1992). However, as conventional GARCH type models cannot account for asymmetries, new models such as Exponential GARCH (EGARCH) models (Nelson, 1991) or Asymmetric Power ARCH (APARCH) models (Ding et al., 1993) or GJR-GARCH models were introduced to model and forecast interest rates (Bali, 2000). Allowing for asymmetric volatility in these types of models implies that positive interest rate shocks increase volatility more than negative interest rate shocks. Furthermore, since the distribution of the innovations in these models is far from a normal (Drost and Klaasen, 1997), and usually unknown, semiparametric techniques can be used to model financial variables. Hou and Suardi (2011), for example, use a semiparametric smoothing technique to the GARCH model of short rate volatility to forecast US short-term interest rate volatility and obtain that this approach produces more accurate estimates of interest rate volatility than other parametric models. Tian and Hamori (2015) model the short-term interest rate in the euro-yen market using the Realized GARCH (RGARCH) model in order to capture the volatility clustering and the mean reversion effects of interest rate behaviour, and find that this model outperforms conventional GARCH-type models.

As such, the objective of the paper is to examine the short and long-term forecasting ability of UK interest volatility of different univariate Generalized Autoregressive Conditional Heteroscedasticiy (GARCH) models, using monthly data from May 1836 to June 2018. The contributions of the paper are the following. First, we analyse the forecasting accuracy of a wide number of GARCH models in order to take into account the time series characteristics of interest rates and their volatility. When using GARCH models, we try to capture the heavy tailed and asymmetric behaviour using alternative error term distributions, such as Normal, Students t, Generalized Error Distribution (GED), Skew Normal (SN), Skew Students (SSt), Skew Generalized Error Distribution (SGED), Inverse Gaussian (IG), Generalized Hyperbolic (GH) and Johnsons SU Distribution (JSU). Furthermore, the ARMA-Generalized Additive Semiparametric GARCH by Hou and Suardi (2011) and the Mixture Autoregressive Model by Wong and Li (2000) are also used to forecast UK interest rates. Finally, the analysis covers the time period from January 1833 to April 2018, a long period of time in which interest rates have shown a very heterogeneous behaviour, with stable interest rates at the beginning of the sample period and with more volatile interest rates from the second half of the 20th century to the end of the sample.

The remainder of this paper is organised as follows. Section 2 describes the different models that will be used to forecast UK interest rate volatility. Section 3 presents and discusses the empirical results. Finally, Section 4 summarizes and concludes this study.

2. Model Description

2.1. Univariate GARCH Models

GARCH: The standard GARCH model formulates the conditional variance of a stochastic process $\{\varepsilon_t\}$ as follows (Bollerslev, 1986):

$$\begin{aligned} \varepsilon_t &= \sigma_t Z_t, \\ \sigma_t^2 &= Var(\varepsilon_t|\mathcal{F}_{\ t-1}) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \end{aligned} \tag{1}$$

where $E(\varepsilon_t|\mathcal{F}_{\ t-1}) = 0$ and $\mathcal{F}_{\ t-1}$ is the σ-field containing all the information available at time t. Z_t is a white noise, independent of $\mathcal{F}_{\ t-1}$ for all t with mean zero and variance 1. Bollerslev (1986) considers the conditional distribution of $\varepsilon_t|\mathcal{F}_{\ t-1}$ to be normal, however other distributions could be applied. In GARCH formulation, the $\{\varepsilon_t\}$ process has zero mean. In the case that process has time variant conditional mean, one may use an ARMA process to remove the conditional mean, and then apply the GARCH to model the conditional variance. In this case the model is called ARMA-GARCH and is formulated as follows:

$$y_t - \phi_0 - (\sum_{i=1}^{r} \phi_i y_{t-i}) - (\sum_{i=1}^{m} \theta_i \varepsilon_{t-i}) = \varepsilon_t = \sigma_t Z_t,$$

$$Var(\varepsilon_t|\mathcal{F}_{t-1}) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2.$$

A-PARCH: The general structure of Asymmetric Power $ARCH$ (A-PARCH) model is as follows (Ding et al., 1993):

$$\begin{aligned}
&\varepsilon_t = \sigma_t Z_t, \\
&\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^\delta,
\end{aligned}$$

where Z_t is a white noise, independent of $\mathcal{F}_{t-1}$ for all t with mean zero and variance 1, $\alpha_0 > 0$, $\alpha_i, \delta, \beta_i \geq 0$ and $-1 < \gamma_i < 1$. The model imposes a Box-Cox power transformation and the asymmetric absolute residuals to handle the asymmetric behavior and Taylor effect[1].

CGARCH: The Component GARCH (CGARCH) model (Engle and Lee, 1999) decompose the conditional variance to transitory and permanent components:

$$\begin{aligned}
&\varepsilon_t = \sigma_t Z_t, \\
&\sigma_t^2 = Var(\varepsilon_t|\mathcal{F}_{t-1}) = c_t + \sum_{i=1}^{q} \alpha_i \left(\varepsilon_{t-i}^2 - c_{t-i}\right) + \sum_{i=1}^{p} \beta_i \left(\sigma_{t-i}^2 - c_{t-i}\right), \\
&c_t = \alpha_0 + \rho c_{t-1} + \gamma(\varepsilon_{t-1}^2 - \sigma_{t-1}^2),
\end{aligned}$$

where Z_t is a white noise, independent of $\mathcal{F}_{t-1}$ for all t with mean zero and variance 1 and c_t is the permanent component. Using both permanent and transitory components, the CGARCH model has the ability to explain long term and short term movements in volatility.

EGARCH: The Exponential GARCH (EGARCH) model of Nelson (1991) is defined as follows:

$$\begin{aligned}
&\varepsilon_t = \sigma_t Z_t, \\
&\ln(\sigma_t^2) = \ln\left(Var(\varepsilon_t|\mathcal{F}_{t-1})\right) = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i z_{t-i} + \gamma_i \Big(|z_{t-i}| - E\left(|Z_{t-i}|\right)\Big)\right) \\
&\qquad + \sum_{i=1}^{p} \beta_i \ln\left(\sigma_{t-i}^2\right),
\end{aligned}$$

where Z_t is a white noise, independent of $\mathcal{F}_{t-1}$ for all t with mean zero and variance 1 and $E\left(|Z_t|\right)$ is the conditional expectation with respect to density function $f(z)$:

$$E\left(|Z_t|\right) = \int_{-\infty}^{\infty} |z| f(z) \mathrm{d}z.$$

[1] Taylor (1986) showed in some financial time series, the sample autocorrelation of absolute returns was larger than that of squared returns.

The EGARCH model captures the asymmetric behavior in time series trough the term $g(z_{t-i}) = \alpha_i z_{t-i} + \gamma_i\Big(|z_{t-i}| - E\left(|Z_{t-i}|\right)\Big)$. The ARCH effect term, $g(z_t)$ is linear in z_t with slop $\alpha + \gamma$ for $z_t > 0$ and $\alpha - \gamma$ for $z_t < 0$ (Nelson, 1991).

GJR-GARCH: Golsten et al. (1993) used an indicator function to explain the positive and negative shocks effect on volatility. Assuming the process has zero conditional mean, the Golsten, Jagannathan, Runkle GARCH (GJR-GARCH) model is formulated as follows:

$$\begin{aligned}
&\varepsilon_t = \sigma_t Z_t, \\
&\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\left(\alpha_i \varepsilon_{t-i}^2 - \gamma_i I_{t-i}\varepsilon_{t-i}^2\right) + \sum_{i=1}^{p}\beta_i \sigma_{t-i}^2,
\end{aligned}$$

where Z_t is a white noise, independent of $\mathcal{F}_{\ t-1}$ for all t with mean zero and variance 1 and the indicator function I_t takes on values 1 for positive values of ε_t and zero otherwise.

TGARCH: The Threshold GARCH (TGARCH) process is given by (Zakoian, 1994):

$$\begin{aligned}
&\varepsilon_t = \sigma_t Z_t, \\
&\sigma_t = \left(Var(\varepsilon_t|\mathcal{F}_{\ t-1})\right)^{1/2} = \alpha_0 + \sum_{i=1}^{q}\left(\alpha_i^+ \varepsilon_{t-i}^+ - \alpha_i^- \varepsilon_{t-i}^-\right) + \sum_{i=1}^{p}\beta_i \sigma_{t-i},
\end{aligned}$$

where Z_t is a white noise, independent of $\mathcal{F}_{\ t-1}$ for all t with mean zero and variance 1, $\varepsilon_t^+ = \max(\varepsilon_t, 0)$, $\varepsilon_t^- = \min(\varepsilon_t, 0)$ and α_i^+, α_i^- and β_i are real scalars. The TGARCH model uses different sets of coefficient to explain the rise and fall of the process. Using this feature, the TGARCH process has the ability to explain asymmetric behavior in financial time series.

2.2. Error Distributions in Univariate GARCH Models

As mentioned before, the common choice for error term distributions in variation of GARCH models are Normal and Student's t distributions. However, there are other choices which have the ability to capture the heavy tailed and asymmetric behavior in standardized residuals. The error distributions considered in this paper (provided by rugarch package) are Normal, Student's t, Generalized Error Distribution (GED), Skew Normal (SN), Skew Student's t (SSt), Skew Generalized Error Distribution (SGED), Inverse Gaussian (IG), Generalized Hyperbolic (GH) and Johnson's SU Distribution (JSU).

2.3. ARMA - Generalized Additive Semiparametric GARCH

The ARMA Generalized Additive Semiparametric GARCH (ARMA-GASGARCH) models the conditional mean and variance of a stochastic process as follows:

$$\begin{aligned}
&y_t - \phi_0 - (\sum_{i=1}^{r}\phi_i y_{t-i}) - (\sum_{i=1}^{m}\theta_i \varepsilon_{t-i}) = \varepsilon_t = \sigma_t Z_t, \\
&Var(\varepsilon_t|\mathcal{F}_{\ t-1}) = \sigma_t^2 = f_1(\varepsilon_{t-1}) + f_2(\sigma_{t-1}^2) + f_3(y_{t-1}).
\end{aligned}$$

where Z_t is white noise, independent of $\mathcal{F}_{t-1}$ for all t with mean zero and variance 1 and $f_i(.), (i = 1, 2, 3)$ are nonlinear univariate functions. The GASGARCH process given by Hou and Suardi (2011) considers the conditional mean to be fixed over the time and applies the nonparametric GARCH (Bühlmann and McNeil, 2002) model to estimate the nonlinear functions. In this paper, the same idea is developed to the models with ARMA conditional mean (which is the parametric component of the model). Following the Bühlmann and McNeil (2002) and Hou and Suardi (2011), an ARMA model is applied for to model the conditional mean and a nonlinear additive model is applied to build the conditional variance model. The nonlinear functions $f_i(.)$ are estimated using local linear kernel estimation (the bandwidth selection is carried out based on Li and Racine, 2004). Assuming, $Z_t(t = 1, \ldots, n)$ are iid random variables, one may use a kernel density estimation to estimate the innovations' distribution function. In this paper, nonparametric estimation are given based on Epanechnikov kernel function.

2.4. Mixture Autoregressive Model

The Mixture Autoregressive model (MAR) of Wong and Li (2000) formulates the conditional distribution of stochastic process $\{y_t\}$ as follows:

$$F(y_t|\mathcal{F}_{t-1}) = \sum_{k=1}^{K} \alpha_k \Phi \left(\frac{y_t - \phi_{k0} - \sum_{i=1}^{p_k} \phi_{ki} y_{t-i}}{\sigma_k} \right), \tag{2}$$

where $F(y_t|\mathcal{F}_{t-1})$ is the conditional distribution function of the y_t given past information, Φ denotes the Standardized Normal cumulative distribution function and $\mathcal{F}_{t-1}$ is the σ-field representing all available information trough time t. The mixing weights $\alpha_1, \ldots, \alpha_K$ satisfies following conditions:

$$\alpha_k > 0, k = 1, \ldots, K,$$
$$\sum_{k=1}^{K} \alpha_k = 1.$$

The formulation of MAR is a mixture of AR components with constant conditional variances. However, the MAR model as the flexibility to model time variant conditional variance as well as multi modal, heavy tail, asymmetry and changes in the shape of forecasting distribution. Based on conditional distribution (2), the conditional mean and variance of the model can be formulated as:

$$\begin{aligned} E(y_t|\mathcal{F}_{t-1}) &= \sum_{k=1}^{K} \alpha_k \left(\phi_{k0} + \sum_{i=1}^{p_k} \phi_{ki} y_{t-i} \right) = \sum_{k=1}^{K} \alpha_k \mu_{k,t} \quad , \\ Var(y_t|\mathcal{F}_{t-1}) &= \sum_{k=1}^{K} \alpha_k \sigma_k^2 + \sum_{k=1}^{K} \alpha_k \mu_{k,t}^2 - \left(\sum_{k=1}^{K} \alpha_k \mu_{k,t} \right)^2 . \end{aligned}$$

2.5. Forecasting Evaluation

Suppose $\hat{\sigma}_{t+h}$ is the h step ahead volatility forecast and the ε_{t+h} is the residual of the conditional mean model at time t:

$$\begin{aligned} \varepsilon_{t+h} &= y_{t+h} - E(y_{t+h}|\mathcal{F}_{\ t}), \\ \hat{\sigma}_{t+h} &= \sqrt{\widehat{Var}(y_{t+h}|\mathcal{F}_{\ t})} \end{aligned}$$

The root mean square error, RMSE, measures the accuracy of the forecast:

$$RMSE = \left(\frac{1}{n}\sum_{t=1}^{n}\eta_{t+h}\right)^{\frac{1}{2}},$$

where η_{t+h} is the h step ahead square error of volatility forecasting:

$$\eta_{t+h} = (|\varepsilon_{t+h}| - \hat{\sigma}_{t+h})^2$$

In order to compare the accuracy of forecasting models, one may use the Kolmogorov-Smirnov Predictive Accuracy, KSPA, test. The two sided KSPA tests the following hypothesis (Hassani and Silva, 2015):

$$\begin{cases} H_0 : F_{\eta^{(1)}_{i+h}}(z) = F_{\eta^{(2)}_{i+h}}(z) \\ H_1 : F_{\eta^{(1)}_{i+h}}(z) \neq F_{\eta^{(2)}_{i+h}}(z) \end{cases},$$

where $F_{\eta^{(k)}_{i+h}}(.)$ is the distribution function of square error corresponding to kth forecasting model. The rejection of the null hypothesis concludes that two models doesn't share the same forecasting accuracy.

2.6. Tests for Asymmetry in Volatility

The asymmetry in volatility is the situation in which, when underlying signal is going up and down, its volatility is different. One common statistical test for asymmetric volatility, is the Engle-Ng test (**?**).

Suppose ε_t is a zero mean stochastic process in (1) and S_t^- is a dummy variable which takes value of 1 if the ε_t is negative and zero otherwise. The Sign-Bias test, is defined as the t-ratio for the coefficient b in the following regression:

$$Z_t^2 = a + bS_t^- + \epsilon_t$$

where Z_t is standardized residual, i.e. $Z_t = \varepsilon_t/\sigma_t$. The Sign-Bias test only takes the sign of the signal into account and tests if the volatility is different in positive and negative signals. The Size-Sign-Bias test, however, tests the effect of the size of the signal on volatility, as well as its sign. The Size-Sign-Bias test is defined as the t-ratio for coefficients b_2 and b_3 of the following regression:

$$Z_t^2 = a + b_1 S_t^- + b_2 S_t^- \varepsilon^2 + b_3 S_t^+ \varepsilon^2 + \epsilon_t,$$

where $S_t^+ = 1 - S_t^+$.

Whilst the Engle-Ng's size and sign bias test is common for testing asymmetry in volatility, its results depends on the volatility model used to calculate the σ_t and Z_t. In addition to Engle-Ng's, we used two sample Kolmogorov-Smirnov (K-S) test to compare the volatility when the interest rate is going up and down (first difference of interest rate is positive and negative). In order to test the asymmetry using K-S test, we define the positive and negative subclasses as follows:

$$\begin{aligned} C^+ &= \{\varepsilon_t^2 | y_t - y_{t-1} > 0\} \\ C^- &= \{\varepsilon_t^2 | y_t - y_{t-1} < 0\} \end{aligned} \tag{3}$$

where y_t is the origianl time series and ε_t is the residual of the optimum ARIMA model fitted to y_t. The K-S test is performed to copmare the positive and negative subclasses.

3. Empirical Results

Our variable of interest is the monetary policy instrument of the Bank of England (BoE), called the Bank Rate. Our sample covers the period of May 1836 to June 2018. Note that even though the monthly data on the Bank Rate (BR) is available from November 1694, the data remained virtually constant until the beginning of our sample period, and hence, we decided to ignore the early part of the available data. The data is sourced from "A Millennium of Macroeconomic Data for the UK" - a database maintained by the BoE at: `https://www.bankofengland.co.uk/statistics/research-datasets`. Figure 1 shows the bank rate from May 1836 to June 2018 and its Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). Since the ACF shows nonstationary behavior, the stationarity of the time series is tested using Augmented Dickey-Fuller Test (MacKinnon, 1996).[2] Table 1 shows the result of the test for the original time series and its first-difference. According to the Table 1, the BR data is non-stationary (the null hypothesis of the test is retained at the 5% level of significance), however, the first-difference of the BR is stationary. Figure 2 plots the first-differenced version of the BR (dBR) variable and the corresponding ACF and PACF. The LM-test for testing ARCH effect is applied to both series (after removing the conditional mean). The LM-test P-values are presented in Table 1. According

[2]The test is applied using the "fUnitRoots" package in R.

to the LM-test results, there exist ARCH effect in both series. The P-value of K-S test for asymmetry in volatility (based on the subclasses defined in 3) is 1.459e-13, which shows the asymmetry in volatility of BR. The RMSE for out-of-sample volatility forecasting of the

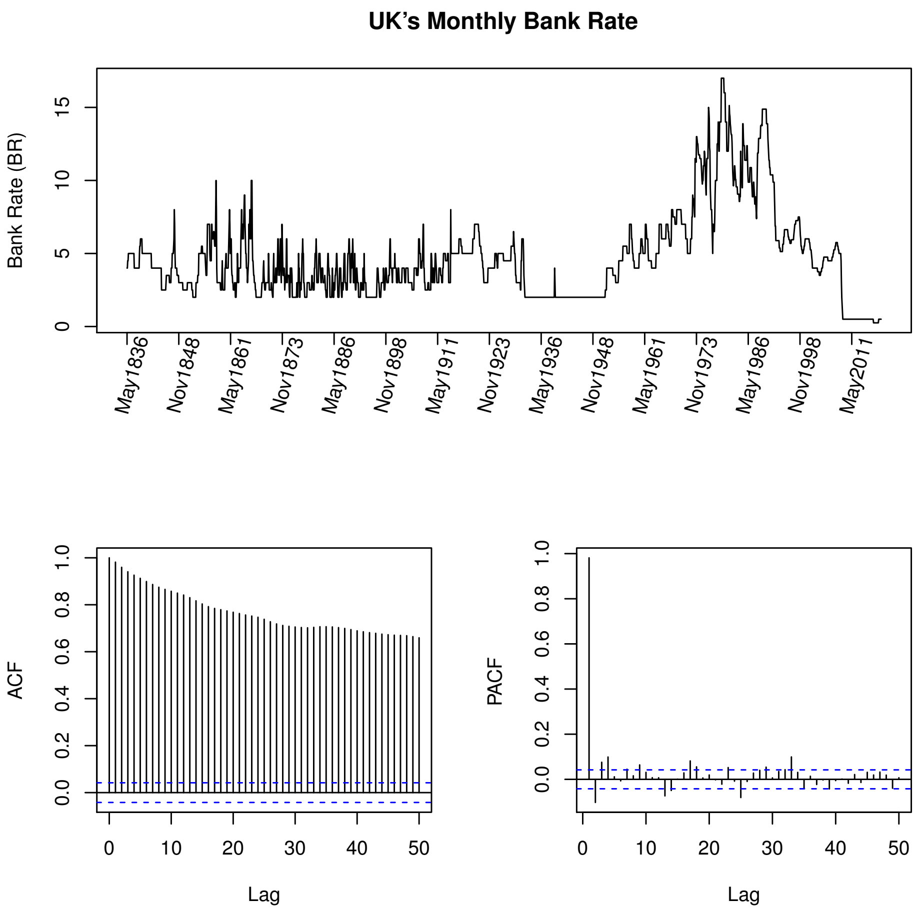

Figure 1: BR time series and its ACF and PACF.

Table 1: The results of Augmented Dickey-Fuller and LM Test

Series' Name	Time Series' Variance	Parameter (Lag order)	Augmented DF Test Statistic	Augmented DF Test P-value	ML Test Statistic	ML Test P-value
BR	9.0425	20	-1.6682	0.0902	174.4383	0.0000
BR's First Difference (dBR)	0.3263	20	-12.1211	$< 2.2e\text{-}16$	174.246	0.0000

different models are presented in detail in Tables 2 and 3. Table 4 summarizes the models with minimum RMSE for very short- (h=1), short- (h=3), medium- (h=6) and long-terms (h=12) out-of-sample horizons associated with volatility forecasting. The ARMA-A-PARCH model with SGED error distribution gives the most accurate volatility forecasts for the very short-term horizon and the GARCH model with GED error distribution for short-, medium-

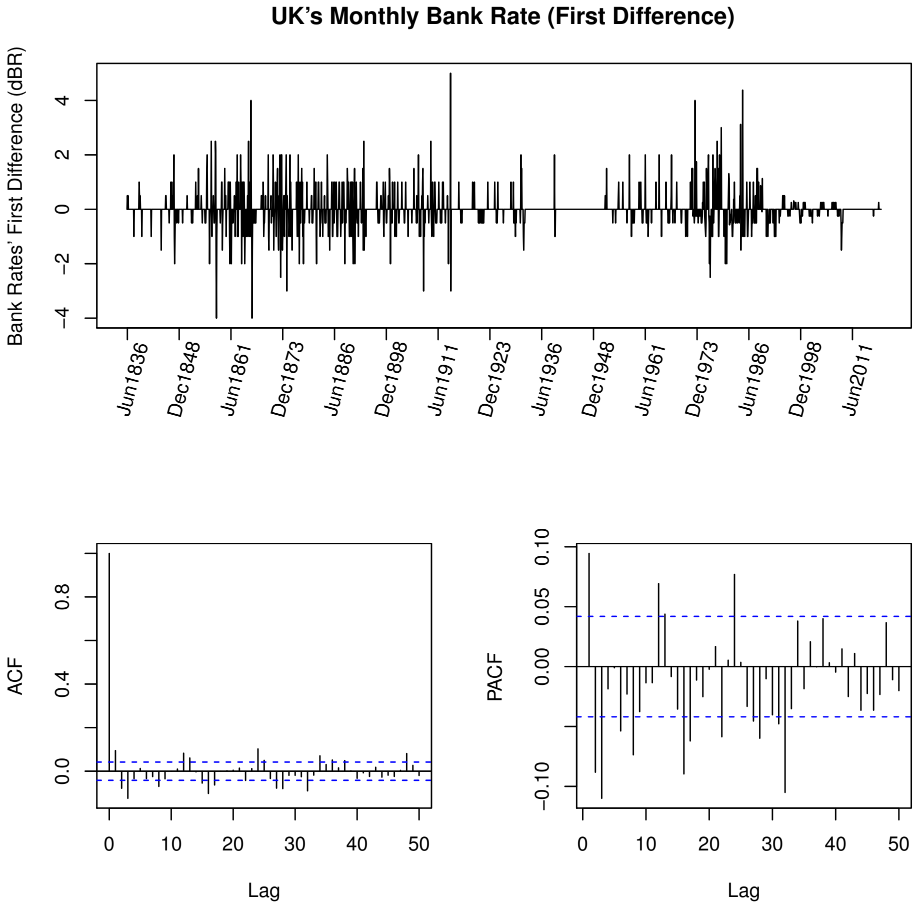

Figure 2: First-difference of the BR (dBR) time series and its ACF and PACF.

and long-term horizons, in terms of minimum RMSE. As it can be seen in Table 4, the best model in very short-term forecasting is an asymmetric one, with both sign and size bias; which is in line with the K-S test results in Table 1. In other forecasting horizons, despite the Engle-Ng's test results showing the significant size-bias in volatility, the minimum RMSE model is a symmetric one with symmetric error distribution. In other words, the asymmetric behavior in BR data shows its effect only in very short-term forecasting. In other forecasting horizons, although there is evidence of asymmetry in past, the out-of-sample volatility forecasting won't get affected by asymmetry.

The KSPA test is applied to compare accuracy of different models with the minimum RMSE model (Table 4). The p-values of the test are given in Table 5 and 6. The models for which the KSPA test's null hypothesis is retained under $\alpha = 0.05$ significance level, (i.e. the models and predictors with same accuracy as the minimum RMSE model) are marked in Tables 5 and 6. The results show that at the very-short forecasting horizon, the most accurate models either have asymmetric behavior (A-PARCH) or asymmetric distribution (SGED and JSU). At other forecasting horizons, however, the performance of the symmetric

Table 2: Out-of-sample volatility forecasting RMSE of dBR

Model	h = 1	h = 3	h = 6	h = 12
GARCH (Normal)[a]	0.5030	0.4940	0.5010	0.5240
GARCH (SN)[a]	0.4464	0.4615	0.4792	0.5099
GARCH (Student's t)[a]	0.6475	0.9610	1.9800	8.7328
GARCH (SSt)[a]	0.5188	0.5148	0.5083	0.5039
GARCH (GED)[a]	0.4077	0.4039	0.3994	0.3941
GARCH (SGED)[a]	0.4181	0.4171	0.4149	0.4113
GARCH (IG)[a]	0.4226	0.4334	0.4460	0.4550
GARCH (GH)[a]	0.4337	0.4394	0.4190	0.4090
GARCH (JSU)[a]	0.4065	0.4061	0.4037	0.4000
TGARCH (Normal)[a]	0.5331	0.5401	0.5520	0.5750
TGARCH (SN)[a]	0.5306	0.5384	0.5640	0.6179
TGARCH (Student's t)[a]	0.7599	0.7579	0.4729	0.4579
TGARCH (SSt)[a]	0.5284	0.5252	0.4590	0.4604
TGARCH (GED)[a]	1.3561	1.3523	1.3477	1.3388
TGARCH (SGED)[a]	6.1216	6.1216	6.1216	6.1216
TGARCH (IG)[a]	0.4464	0.4653	0.4410	0.4520
TGARCH (GH)[a]	6.1216	6.1216	6.1200	6.1200
TGARCH (JSU)[a]	3.3909	2.0135	0.4577	0.4597
EGARCH (Normal)[a]	.[b]	.[b]	8.04E+13	1.38E+12
EGARCH (SN)[a]	0.5530	0.5386	0.5146	0.5614
EGARCH (Student's t)[a]	.[b]	.[b]	.[b]	.[b]
EGARCH (SSt)[a]	540.1583	8.9836	0.4559	0.4590
EGARCH (GED)[a]	.[b]	.[b]	.[b]	.[b]
EGARCH (SGED)[a]	.[b]	.[b]	.[b]	.[b]
EGARCH (IG)[a]	.[b]	.[b]	2.19E+18	4.35E+16
EGARCH (GH)[a]	.[b]	.[b]	5.75E+26	8.58E+33
EGARCH (JSU)[a]	0.8292	0.7893	0.7737	0.7589

[a]. The conditional mean model is ARMA.

[b]. The value is computationally infinite.

Table 3: Out-of-sample volatility forecasting RMSE for dBR (Continued)

Model	h = 1	h = 3	h = 6	h = 12
CGARCH (Normal)[a]	0.4174	0.4484	0.4560	0.4660
CGARCH (SN)[a]	0.4402	0.4556	0.4721	0.4871
CGARCH (Student's t)[a]	0.4820	0.4365	0.4272	0.4383
CGARCH (SSt)[a]	0.4557	0.4369	0.4423	0.4595
CGARCH (GED)[a]	0.4348	0.4306	0.4287	0.4253
CGARCH (SGED)[a]	0.4410	0.4290	0.4224	0.4117
CGARCH (IG)[a]	0.4392	0.4480	0.4550	0.4560
CGARCH (GH)[a]	0.4332	0.4148	0.4150	0.4270
CGARCH (JSU)[a]	0.5738	0.5747	0.5755	0.5770
A-PARCH (Normal)[a]	0.4694	0.4767	0.4950	0.5170
A-PARCH (SN)[a]	0.4636	0.4751	0.4910	0.5328
A-PARCH (Student's t)[a]	0.4235	0.4271	0.4259	0.4322
A-PARCH (SSt)[a]	0.9808	0.6871	0.4836	0.4464
A-PARCH (GED)[a]	1.7951	1.6933	1.5227	1.2562
A-PARCH (SGED)[a]	0.4051	0.4043	0.4020	0.3992
A-PARCH (IG)[a]	0.4187	0.4200	0.4230	0.4300
A-PARCH (GH)[a]	0.4099	0.4119	0.4170	0.4370
A-PARCH (JSU)[a]	0.4062	0.4058	0.4024	0.3981
GJR-GARCH (Normal)[a]	0.4665	0.4680	0.4720	0.4880
GJR-GARCH (SN)[a]	0.5026	0.4956	0.4976	0.5214
GJR-GARCH (Student's t)[a]	0.5389	0.4647	0.4432	0.4491
GJR-GARCH (SSt)[a]	0.5459	0.5229	0.5001	0.4692
GJR-GARCH (GED)[a]	0.4306	0.4314	0.4304	0.4315
GJR-GARCH (SGED)[a]	0.4161	0.4144	0.4113	0.4076
GJR-GARCH (IG)[a]	0.5143	0.4445	0.4330	0.4450
GJR-GARCH (GH)[a]	0.4387	0.4138	0.4050	0.4020
GJR-GARCH (JSU)[a]	0.6323	0.6388	0.6491	0.6801
GASGARCH [a]	10.3293	24.1132	4.26E+13	2.2347
MAR	0.5838	0.5877	0.5944	0.6223

[a]. The conditional mean model is ARMA.

Table 4: Minimum RMSE models for forecasting dBR volatility

	h=1	h=3	h=6	h=12
Model	A-PARCH	GARCH	GARCH	GARCH
Error distribution	SGED	GED	GED	GED
RMSE value	0.4051	0.4039	0.3994	0.3941
Engle-Ng Sign-Bias test P-Vlue	3.524e-14	0.3007	0.3007	0.3007

GARCH model with symmetric distribution is not statistically different from the asymmetric models with asymmetric distributions. As can be seen, none of the models with accuracy same as the minimum RMSE models has normal distributions, which in turn shows the importance of heavy-tailed behavior at all forecasting horizons.

3.1. Simulation study

The reliability of the results is investigated using a simulation study. In this study, 1000 sample path of length 2000 is simulated from minimum RMSE models. The performance of competing models in out-of-sample forecasting is measured using average out-of-sample RMSE.

In first simulation the sample paths are simulated from A-PARCH(SGED) (A-PARCH model with SGED distribution, minimum RMSE model for h=1 in Table 4, with parameter estimated based on the real data) and the average one step ahead forecasting RMSE is calculated for models with similar forecasting accuracy as A-PARCH(SGED) (models with similar forecasting accuracy as A-PARCH(SGED) are marked in Tables 5 and 6). In second simulation, each sample path is generated from GARCH(GED) (GARCH model with GED distribution, the minimum RMSE model for h = 3, 6 and 12 in Table 4, with parameter estimated based on the real data). The average h step ahead forecasting RMSE is calculated for models with similar forecasting accuracy as GARCH(GED) in h = 3, 6 and 12 forecasting horizons (models with similar forecasting accuracy as GARCH(GED) are marked in Tables 5 and 6). The average RMSEs are presented in Table 7.

As it can be seen in Table 7, the minimum RMSE models in real data has the minimum RMSE in simulations as well for short and medium -term forecasting (h = 1,3, 6). In long-term forecasting (h =12), however, the minimum RMSE model in real data is $GARCH$(GED), whilst in simulation $GJR-GARCH$(GH) model has minimum average RMSE which is not surprising since two models have very close RMSE in real data (see Tables 2 and 3) and $GJR-GARCH$(GH) model has highest P-value (most similarity with minimum RMSE model) in KSPA test (see Table 6), for long-term forecasting. In order to verify the asymmetry test results (Table 4) in long-term forecasting, the Engle-NG test is

Table 5: $KSPA$ test p-values (two-tailed) for comparing the out-of-sample forecasts to minimum RMSE of dBR volatility forecast

	h = 1	h = 3	h = 6	h = 12
Minimum RMSE model → Comparing to ↓	A-PARCH (SGED)[a]	GARCH (GED)[a]	GARCH (GED)[a]	GARCH (GED)[a]
GARCH(Normal)[a]	0.0000	0.0000	0.0000	0.0000
GARCH(SN)[a]	0.0000	0.0000	0.0000	0.0000
GARCH(Student's t)[a]	0.0000	0.0000	0.0000	0.0000
GARCH(SSt)[a]	0.0000	0.0001	0.0000	0.0000
GARCH(GED)[a]	0.0153			
GARCH(SGED)[a]	0.0017	**0.0828**[*]	**0.0921**[*]	0.0023
GARCH(IG)[a]	0.0291	0.0199	0.0226	0.0000
GARCH(GH)[a]	0.0066	**0.1023**[*]	0.0419	0.0199
GARCH(JSU)[a]	**0.0665**[*]	**0.5235**[*]	**0.7015**[*]	**0.1530**[*]
TGARCH(Normal)[a]	0.0000	0.0000	0.0000	0.0000
TGARCH(SN)[a]	0.0000	0.0000	0.0000	0.0000
TGARCH(Student's t)[a]	0.0000	0.0000	0.0000	0.0000
TGARCH(SSt)[a]	0.0000	0.0001	0.0001	0.0000
TGARCH(GED)[a]	0.0000	0.0000	0.0000	0.0000
TGARCH(SGED)[a]	0.0000	0.0010	0.0001	0.0000
TGARCH(IG)[a]	0.0001	0.0076	0.0291	0.0001
TGARCH(GH)[a]	0.0000	0.0010	0.0001	0.0000
TGARCH(JSU)[a]	0.0000	0.0000	0.0001	0.0000
EGARCH(Normal)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(SN)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(Student's t)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(SSt)[a]	0.0000	0.0007	0.0001	0.0000
EGARCH(GED)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(SGED)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(IG)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(GH)[a]	0.0000	0.0000	0.0000	0.0000
EGARCH(JSU)[a]	0.0000	0.0000	0.0000	0.0000

[a]. The conditional mean model is ARMA.

[*]. Difference between Min. RMSE model and Alternative models is insignificant at test level $\alpha = 0.05$ (The marked models has similar forecasting accuracy as the Min. RMSE models, according to KSPA test results).

Table 6: $KSPA$ test p-values (continued)

	h = 1	h = 3	h = 6	h = 12
Minimum RMSE model → Comparing to ↓	A-PARCH (SGED)[a]	GARCH (GED)[a]	GARCH (GED)[a]	GARCH (GED)[a]
CGARCH(Normal)[a]	0.0000	0.0000	0.0000	0.0000
CGARCH(SN)[a]	0.0000	0.0000	0.0000	0.0000
CGARCH(Student's t)[a]	0.0000	**0.0921***	**0.1256***	0.0116
CGARCH(SSt)[a]	0.0002	**0.0830***	0.0362	0.0001
CGARCH(GED)[a]	0.0000	0.0199	0.0014	0.0002
CGARCH(SGED)[a]	0.0000	**0.0742***	**0.0594***	0.0199
CGARCH(IG)[a]	0.0014	0.0199	0.0005	0.0000
CGARCH(GH)[a]	0.0057	0.0007	0.0000	0.0004
CGARCH(JSU)[a]	0.0000	0.0000	0.0000	0.0000
A-PARCH(Normal)[a]	0.0000	0.0000	0.0000	0.0000
A-PARCH(SN)[a]	0.0000	0.0000	0.0000	0.0000
A-PARCH(Student's t)[a]	0.0006	**0.0665***	0.0372	0.0007
A-PARCH(SSt)[a]	0.0000	0.0002	0.0027	0.0001
A-PARCH(GED)[a]	0.0000	0.0000	0.0000	0.0000
A-PARCH(SGED)[a]		**0.0828***	**0.0665***	**0.1850***
A-PARCH(IG)[a]	0.0199	0.0012	0.0101	0.0257
A-PARCH(GH)[a]	0.0020	0.0004	**0.0594***	0.0003
A-PARCH(JSU)[a]	0.0076	0.0101	0.0057	**0.1135***
GJR-GARCH(Normal)[a]	0.0000	0.0000	0.0000	0.0000
GJR-GARCH(SN)[a]	0.0000	0.0000	0.0000	0.0000
GJR-GARCH(Student's t)[a]	0.0000	0.0027	0.0043	0.0001
GJR-GARCH(SSt)[a]	0.0000	0.0020	0.0012	0.0001
GJR-GARCH(GED)[a]	0.0004	**0.0594***	0.0291	0.0002
GJR-GARCH(SGED)[a]	0.0023	**0.3813***	**0.0549***	0.0488
GJR-GARCH(IG)[a]	0.0000	0.0257	0.0372	0.0031
GJR-GARCH(GH)[a]	0.0000	**0.1758***	**0.2532***	**0.3261***
GJR-GARCH(JSU)[a]	0.0000	0.0000	0.0000	0.0000
GASGARCH [a]	0.0000	0.0000	0.0000	0.0000
MAR	0.0000	0.0000	0.0000	0.0000

[a]. The conditional mean model is ARMA.

*. Difference between Min. RMSE model and Alternative models is insignificant at test level $\alpha = 0.05$.

Table 7: Average RMSE of models similar to minimum RMSE models based on 1000 simulations (for h =1, series are simulated from A-PARCH(SGED) and for h = 3,6,12 the series are simulated from GARCH(GED)).

Forecasting Model	h = 1	h = 3	h = 6	h = 12
$GARCH$(GED)[a]	.[b]	0.0156	0.0218	0.0291
$GARCH$(SGED)[a]	.[b]	0.0246	0.0269	.[b]
$GARCH$(GH)[a]	.[b]	0.0286	.[b]	.[b]
$GARCH$(JSU)[a]	0.0308	0.0191	0.0236	0.0293
$CGARCH$(Student's-t)[a]	.[b]	0.0688	0.0694	.[b]
$CGARCH$(SSt)[a]	.[b]	0.0692	.[b]	.[b]
$CGARCH$(SGED)[a]	.[b]	0.0407	0.0398	.[b]
$A-PARCH$(Student's-t)[a]	.[b]	0.0712	.[b]	.[b]
$A-PARCH$(SGED)[a]	0.0011	0.0328	0.0345	0.0365
$A-PARCH$(GH)[a]	.[b]	.[b]	0.0347	.[b]
$A-PARCH$(JSU)[a]	.[b]	.[b]	.[b]	0.0728
$GJR-GARCH$(GED)[a]	.[b]	0.0280	.[b]	.[b]
$GJR-GARCH$(SGED)[a]	.[b]	0.0226	0.0292	.[b]
$GJR-GARCH$(GH)[a]	.[b]	0.0248	0.0249	0.0250

[a]. The conditional mean model is ARMA.

[b]. Doesn't have the same accuracy as the minimum RMSE model in this forecasting horizon (according to Tables 5 and 6).

applied to $GJR-GARCH$(GH) model, fitted to real data. The Engle-NG Sing-Bias test's P-value for $GJR-GARCH$(GH) is 0.1531 which retain the null hypothesis of the test, under which there is no Sign Bias (or asymmetry) in the model.

4. Conclusion

The main objective of this paper is to evaluate the very short, medium and long-term forecasting ability of different univariate GARCH models of UK interest rate volatility, using a long span monthly data from May 1836 to June 2018. With this purpose, we calculate the forecasting ability of different univariate GARCH- type models (GARCH, Threshold GARCH, Exponential GARCH, Component GARCH, and Asymmetric Power ARCH). Furthermore, when using these GARCH-type models, we try to capture the heavy tailed and asymmetric behavior using alternative error term distributions, such as Normal, Students t, Generalized Error Distribution (GED), Skew Normal (SN), Skew Students (SSt), Skew Generalized Error Distribution (SGED), Inverse Gaussian (IG), Generalized Hyperbolic (GH) and Johnsons SU Distribution (JSU), ARMA-Generalized Additive Semiparametric GARCH by Hou and Suardi (2011) and the Mixture Autoregressive Model by Wong and Li (2000) are also used to forecast UK interest rates.

Although the results suggest that the forecasting accuracy of each of the models depends on the forecasting horizon, they clearly show the relevance of considering alternative error distributions to the normal distribution when estimating GARCH-type models. For example, for the very short-term forecasting horizon (h=1), A-PARCH models with skew generalized error distribution is the most accurate models when forecasting UK interest rates, while for short, medium and long-term term forecasting horizons (h=3 and h=6, h=12), GARCH models with generalized error distribution for the error term are the most accurate models. A simulation study is employed to verify the accuracy of the forecasting models. Simulation results approve accuracy of A-PARCH models with skew generalized error distribution in short-term forecasting, $GARCH$ models with generalized error distribution for the error term for medium-term forecasting whilst in long-term forecasting show the $GJR-GARCH$ with Generalized Hyperbolic error distribution has the highest accuracy. Whilst this study considered a broad family of volatility models, with different error distributions, developing volatility models with more flexible error distributions (e.g. the Scale Mixture of Skew Normal family of distributions (Ferreira et al., 2011)) could increase forecasting accuracy.

The Engle-Ng Sign-Bias test for asymmetry, shows the asymmetric behavior of minimum RMSE model for short-term ($h = 1$) forecasting and symmetric behavior in medium and long -term forecasting. The dBR data, on the other hand, has asymmetric volatility it's self. According to these results, even though the UK's monthly Bank Rate has asymmetric volatility, the asymmetry is only evident in short-term forecasting horizon (h = 1). In medium and long -term forecasting horizons, symmetric volatility models have highest accuracy. In other words, whilst the asymmetric nature of the data is important in short-term forecasting, it doesn't improve the accuracy of longer forecasting horizons. According to these results we could conclude that the interest rate volatility is more sensitive to past positive innovations than to past negative innovations, which explains why volatility tends to be higher when interest rates are high (Bali, 2000). Furthermore, based on our results, this asymmetric behavior of interest rate volatility is more pronounced or evident when short-term forecasting horizons are considered, while its relevance decreases when longer forecasting horizons are analyzed. Therefore, and as stated above, the forecasting ability of the models will depend on the forecasting horizon we are interested in, and thus, it will differ depending on the objective of our forecasting exercise.

The results on the forecasting ability of these models should be taken into account when forecasting interest rates for portfolio diversification, investment or hedging strategies, for pricing different financial securities, for risk managers, or for forecasting different macroeconomic variables dependent on interest rates. Furthermore, these results suggest, when the time series has asymmetric volatility, the use of asymmetric models is not necessary for achieving the highest out-of-sample forecasting accuracy.

Financial Acknowledgement

Juncal Cuñado gratefully acknowledges financial support from the Ministerio de Economía y Competitividad (ECO2017-83183-R).

References

Bernanke, B.S. 1983. Irreversibility, Uncertainty and Cyclical Investment. Quarterly Journal of Economics 97, 85-106.

Walsh, C.E. 1984. Interest rate volatility and monetary policy. Journal of Money, Credit and Banking 16, 133-150.

Chadha, J. and Nolan, C. 2001. Inflation Targeting, Transparency and Interest Rate Volatility: Ditching Monetary Mystique in the UK. Journal of Macroeconomics 23, 349-366.

Bartolini, L, Bertola, G. and Prati, A. 2002. Day-to-day monetary policy and the volatility of the Federal Funds Interest Rate. Journal of Money, Credit and Banking 34, 137-159.

Barnhill, T.M. and Maxwell, W.F. 2002. Modeling correlated market and credit risk in fixed income portfolios. Journal of Banking and Finance 26, 347-374.

Carcano, N. and Foresi, S. 1997. Hedging against interest rate risk: Reconsidering volatility-adjusted immunization. Journal of Banking and Finance 21, 127-141.

Chan, K.C., Karolyi, G.A., Longstaff, F.A. and Sanders, A., 2001. An Empirical Comparison of Alternative Models of Short-Term Interest Rates. The Journal of Finance 52, 209-227.

Franses, P. and van Dijk, D., 2000. Non-linear time series models in empirical finance. Cambridge University Press.

Zumbach, G., 2013. Discrete Time Series, Processes and Applications in Finance. Springer Finance Series. Springer, Berlin.

Chan, K.C., Karolyi, G.A., Longstaff, F.A. and Sanders, A.B., 1992. An empirical comparison of alternative models of the short-term interest rate. The Journal of Finance 47, 1209-1227.

Brenner, R.J., Harjes, R.H. and Kroner, K. 1995. Another look at alternative models of the short-term interest rate. Journal of Financial and Quantitative Analysis 1, 85-107.

Koedijk, K.G., Nissen, F., Schotman, P. and Wolff, C. 1997. The Dynamis of Short-Term Interest Rate Volatility Reconsidered. European Finance Review 1, 105-130.

Engle, R., 1982. Autoregressive Conditional Heteroskedasticity with estimate of variance of UK inflation. Econometrica 50, 987-1008.

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics 31, 307-328.

Longstaff, F.A. and Schwartz, E.S., 1992. Interest Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model. Journal of Finance 47, 1259-1282.

Nelson, D.B., 1991. Conditional Autoregressive Conditional Heteroskedasticity in assets returns: A new approach. Econometrica 59, 347-370.

Ding, Z., Granger, C. and Engle, R. 1993. A long memory property of stock returns and new model. Journal of Empirical Finance 1, 83-106.

Bali, T.G., 2000. Testing the empirical performance of stochastic volatility models of the short-term interest rate. Journal of Financial and Quantitative Analysis 35, 191-215.

Drost, F.C. and Klaassen, C., 1997. Efficient estimation in semiparametric GARCH models. Journal of Econometrics 81, 193-221.

Hou, A.J. and Suardi, S., 2011. Modelling and forecasting short-term interest rate volatility: A semiparametric approach. Journal of Empirical Finance 18, 692-710.

Tian, S. and Hamori, S., 2015. Modeling interest rate volatility: A realized GARCH approach. Journal of Banking and Finance 61, 158-171.

Wong, C.S. and Li, W.K., 2000. On a Mixture Autoregressive Model. Journal of the Royal Statistical Society. Series B 62, 95-115.

Taylor, S.J., 1986. Modelling Financial time series, Wiley: New York.

Engle, R.F. and Lee, G., 1999. A permanent and transitory component model of stock return volatility, In: Cointegration, Causality and Forecasting: A Festschrift in Honor of Clive W.J. Granger, Oxford University Press, New York.

Golsten, L.R., Jagannathan, R. and Runkle D., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks, The Journal of Finance, 48(5), 1779-1801.

Zakoian J.M., 1994. Threshold heteroskedastic models, Journal of Economic Dynamics and Control, 18, 931-955.

Bühlmann, P. and McNeil, A., 2002. An algorithm for nonparametric GARCH modelling, Computational Statistics and Data Analysis, 40, 665-683.

Li, Q. and Racine, J.S., 2004. Cross-validated local linear nonparametric regression, Statistica Sinica, 14, 485-512.

Hassani, H. and Silva, E.S., 2015. A Kolmogorov-Smirnov based test for comparing the predictive accuracy of two sets of forecasts, Econometrics, 3, 590-609.

MacKinnon, J.G., 1996. Numerical distribution functions for unit root and cointegration tests, Journal of Applied Econometrics, 11, 601618.

Ferreira C. S., Bolfarine, H., Lachos, V. H. 2011. Skew scale mixture of normal distributions: properties and estimation, *Statistical Methodology*, 8, 154-171.