A plant-wide control framework for a grinding mill circuit†

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Abstract

This article proposes a generic plant-wide control framework which can be used to develop a hierarchical control structure (regulatory control, supervisory control, and optimisation) to operate a single-stage closed grinding mill circuit in an economically optimal manner.

An economic objective function is defined for the grinding mill circuit with reference to the economic objective of the larger mineral processing plant. A mineral processing plant in this study consists of a comminution and a separation circuit and excludes the extractive metallurgy at a metal refinery. The operational performance of a comminution circuit as represented by a single-stage grinding mill circuit, primarily depends on the performance of the grinding mill. Since grindcurves define the operational performance range of a mill, grindcurves are used to define the setpoints for the economic controlled variables for optimal steady-state operation. For a given metal

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price, processing cost, and transportation cost, the proposed structure can be used to define the optimal operating region of a grinding mill circuit for the best economic return of the mineral processing plant. Once the optimal operating condition is defined, the supervisory control aims to maintain the primary controlled variables at the optimal setpoints. A regulatory control layer ensures the stability of the plant in the presence of disturbances.

Introduction

Individual economic optimisation of the operational objectives of a comminution process and of a separation process may lead to sub-optimal operation of the larger mineral processing plant. A mineral processing plant in this study comprises only of comminution and separation, and excludes the extractive metallurgy at a metal refinery. The operational objective of the mineral processing plant is to maximise the economic value of the separator concentrate sold to the metal refinery by reducing the bulk of ore and increasing its contained value.[4] Although the concentrate from the separation process is the main generator of revenue for the mineral processing plant, the ability to extract the greatest benefit from the comminuted ore depends on the mineral liberation in the comminution product, i.e. effective separation of valuable material from gangue depends on the quality of the comminution product. Therefore, to achieve plant-wide optimal economic operation, the operation of the comminution circuit must be determined in reference to the operational aims of the separation circuit.[5]

The economic optimisation of mineral processing plants has been the topic of numerous studies. In the economic optimisation study of McIvor and Finch,[6] a size-by-size analysis of the recovery of minerals in a separation circuit is used to estimate the recoverable value of minerals distributed throughout the comminution circuit product size distribution. This is then used to define the steady-state target grind size for the comminution circuit to improve economic performance of the mineral processing plant. Schena et al.[7] optimise the profit of a copper plant based on the throughput of the plant. Simple empirical models are used to describe the grinding and flotation units and the influence of particle size on flotation efficiency. Sosa-Blanco et al.[8] provide a step-wise procedure to optimally tune a comminution circuit to maximise the economic efficiency of a separation circuit. An empirical relationship is used to describe the comminution circuit product ore size distribution to the separation mineral recovery. To address the effect of the dynamics of the comminution circuit on the separation circuit, Munoz and Cipriano[9] propose a predictive controller which optimises an economic objective function based on metallurgical performance indices. The economic objective function describes the income generated by the plant as a function of the comminution feed ore grade, the separator tailings grade, and the metal refinery recovery per tonne of ore processed. Wei and Craig[10] make use of an economic performance function based on the relation between the comminution product particle size and the separation concentrate recovery to compare the economic performance of non-linear Model Predictive Control (MPC) to single-loop Proportional-Integral-Derivative (PID) control for a comminution circuit. In all the cases mentioned above, a relationship between the comminution product particle size and the concentrate recovery and grade is used to define the revenue generated from selling the concentrate to a metal refinery (see also Matthews and Craig[11]).

Optimisation of the economic objective function for the comminution circuit depends on the operating range of the comminution circuit. The comminution product particle size which will optimise the economic objective function for different market conditions may not necessarily be achievable by the comminution circuit. The model used to describe the comminution circuit must capture the range of feasible operating conditions. The non-linear population balance models used in the optimisation studies above, especially related to the grinding mill, remain limited to a relatively small region of operation. The model parameters need to be updated for different feasible operating regions of the comminution circuit.[12,13]

The grindcurves of Van der Westhuizen and Powell[14] are quazi-static descriptions of the operable regions of the grinding mill in the comminution circuit. It relates the performance indicators of the mill - power draw, grind size, and discharge flow-rate - to the filling of the mill and its rotational speed. The grindcurves therefore provide the feasible operating conditions for the grinding mill to achieve a specific product specification. The grindcurves can be regarded as a mechanism for optimisation of the mill as a unit.[15] Consequently, it enables optimisation of the economic objective function over a larger range of mill operating conditions.

Once the operating condition is defined which optimises the economic objective function, a regulatory and supervisory control layer must maintain the comminution circuit at the desired operating condition. This raises additional questions, such as which variables should be controlled, which should be manipulated, which should be measured, what links should be made between them, and which set-points are appropriate for the controlled variables[16–18]? In other words, a realistic and systematic approach to plant-wide controller design is necessary.

From a review of the mathematically and process orientated approaches to plant-wide control,[19–22] Larsson and Skogestad[23] proposed a plant-wide control design framework using elements of both the mathematical and process orientated approaches. This framework, expanded by Skogestad,[24] distinguishes between economic control and regulatory control by dividing structural decisions into two parts: a top-down and a bottom-up analysis. The aim of the top-down analysis is to define an economic supervisory control structure that achieves close-to-optimal steady-state economic operation. The aim of the bottom-up analysis is to define a stable and robust regulatory control structure capable of operating under the conditions imposed by the economic supervisory layer. The plant-wide control design procedure is shown below. (Minasidis et al.[25] provide additional guidelines on decisions specific to each step.)

1. Top-down analysis (to address steady-state operation):

    (a) Define the operational economic objective.

    (b) Determine the steady-state degrees of freedom and the optimal steady-state operation.

(c) Select the primary controlled variables influencing the economic cost function.

(d) Select the main variable to manipulate the throughput.

2. Bottom-up analysis (to address dynamic operation):

   (a) Regulatory control.

   (b) Supervisory control.

   (c) Real-time optimisation.

The plant-wide control design framework of Skogestad[24] is applied by Downs and Skogestad[26] to industrial processes operated by the Eastman Chemical Company. The variety of frameworks available to address the plant-wide control problem illustrates the difficulty to find a unified approach. At least from the industrial perspective of J. Downs, one of the original proposers of the "Tennessee Eastman challenge problem",[27] the procedure outlined by Skogestad[24] is adequate to design a controller capable of optimising the process economics of a plant.[26]

The aim of this paper is to construct a control framework for a comminution circuit, specifically a single-stage closed grinding mill circuit, to achieve the operational goals which will produce the optimal economic operation of the larger mineral processing plant. In other words, the controller must specify the operating condition of the comminution circuit which will optimise the economic return of the mineral processing plant, and operate the plant at the specified operating condition. This paper is written in response to Hodouin et al.,[28] which concludes that "plant-wide optimisation is difficult to formulate on an economic basis and is obviously yet more complex to implement, although it is the only final goal of this industry".

The article is divided into four major sections: the mechanics of comminution, the economics of comminution, top-down control analysis, and bottom-up control analysis. The

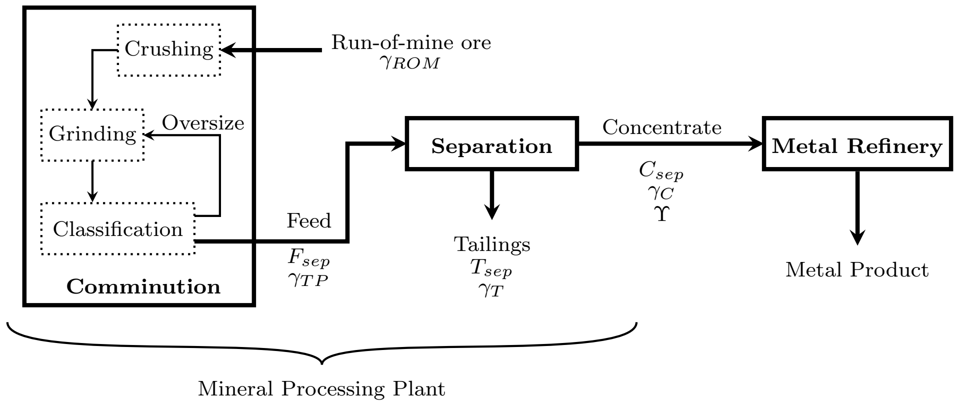

Figure 1: The chain of processes in a mineral processing plant. The mineral processing plant excludes the metal refinery.

top-down and bottom-up analyses are the main contributions of this article, but are only sensible in light of the discussion of the mechanics and economics of the comminution process. Readers familiar with control of comminution circuits may glance at the first two sections in passing before continuing with the plant-wide control analysis.

The mechanics of the comminution process

This section describes the comminution process and the related control challenges associated with the controlled variables. Fig. 1 illustrates the series of operations in a mineral processing plant to produce a valuable mineral concentrate product from mined ore. The first process in the mineral processing plant, comminution, consists of a sequence of crushing, grinding, and classification. A plant can have several stages of crushing and grinding, with numerous recycle streams. The functional aim of comminution is to convert run-of-mine ore into fine particles in order to liberate valuable minerals within the ore. Grinding mills are the main functional units to achieve this objective. The operational objective of the comminution section is to produce a product with consistent fineness at the maximum achievable throughput.[4,29,30] The operational objective of separation is to maintain a constant concentrate grade while maximising metal recovery.[31] The value of the concentrate sold to the metal refinery depends on the relationship between the recovery and grade of the concentrate.

A single-stage closed grinding mill circuit as shown in Fig. 2, which is a common configuration in industry,[32] is used throughout this study to represent the comminution section in Fig. 1. The main elements in the circuit are a semi-autogenous (SAG) grinding mill, a sump, and a cyclone. The main focus of this article is SAG mills, compared to ball mills,[33] high-pressure grinding rollers,[13] or autogenous mills (AG).[34] Only balls contribute to the grinding media in ball mills, only ore for AG mills, whereas both balls and ore act as grinding media in SAG mills. SAG mills are generally larger with much lower ball fillings compared to ball mills. The motivations for using a SAG mill in a mineral processing plant are: low operating costs compared to conventional grinding, the increased demand to process large amounts of low-grade ore, reduced grinding media consumption, and the capability to handle larger sized ore which reduces preceding crushing requirements.[35]

This study limits its analysis to the single-stage closed grinding mill circuit shown in Fig. 2. It does not include a discussion of crushing circuits,[36,37] different circuit configurations,[38–41] or the control of stockpiles.[42] However, the plant-wide control design procedure followed in this study remains applicable and relevant to more complex circuit configurations and mill types with the appropriate changes in manipulated and controlled variables.

The nomenclature for Figs. 1 and 2 is described in Tables 1 and 2 respectively.

Process description

The SAG mill in Fig. 2 receives four streams: mined ore (MFO), water (MIW), underflow from the cyclone, and steel balls (MFB) to assist with the breakage of ore. The mill charge constitutes a mixture of grinding media and slurry. Grinding media refers to the steel balls and large rocks used for breaking the ore, and slurry refers to the mixture of fine ore material and water such that the mixture exhibits the same flow characteristics as water.

The mill is rotated along its longitudinal axis by a motor. As shown in Fig. 3, the charge in the mill is lifted by the inner liners on the walls of the mill to a certain height from where it cascades down, only to be lifted again by the liners through the rotating action of

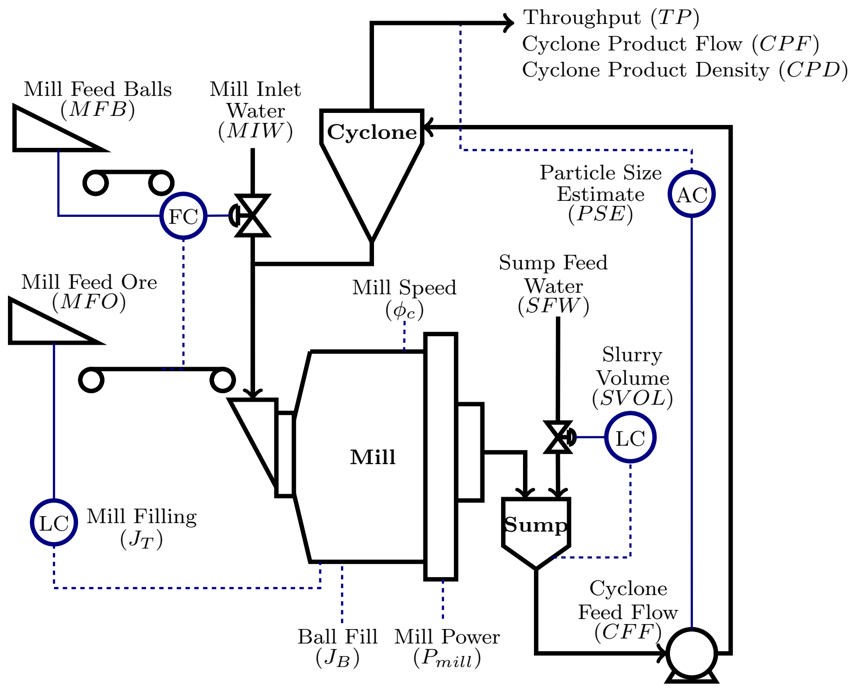

Figure 2: A single-stage closed grinding mill circuit with a regulatory control scheme.

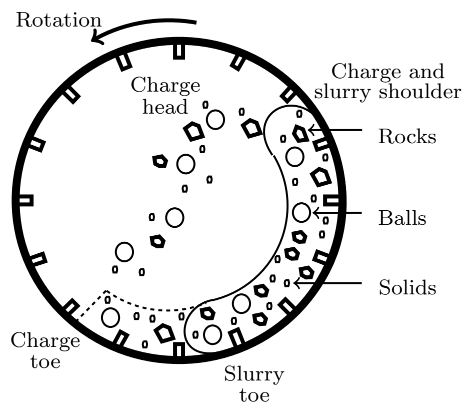

Figure 3: Cross-section of a SAG mill.

the mill. If the rotational speed is sufficiently fast the material in the charge will become airborne after reaching the top of its travel on the mill shell. The uppermost point where material leaves the mill shell is called the charge shoulder. The airborne particles follow a parabolic path reaching a maximum, called the charge head, making contact again with the mill charge at the bottom of the mill, called the charge toe. The cascading motion of the charge causes the ore to break through impact breakage, abrasion, and attrition. The ground ore in the mill mixes with the water to create a slurry. The slurry is discharged through an

Table 1: Nomenclature for mineral processing plant (see Fig. 1).

Variable	Unit	Description
C_{sep}	[t/h]	Separator concentrate mass flow-rate
T_{sep}	[t/h]	Separator tailings mass flow-rate
γ_{ROM}	[-]	Run-of-mine ore grade
γ_C	[-]	Separator concentrate grade
γ_T	[-]	Separator tailings grade
γ_{TP}	[-]	Comminution circuit throughput grade
Υ	[-]	Recovery
NSR	[\$/h]	Net smelter return
$P_{\$p}$	[\$/t]	Processing cost
$P_{\$t}$	[\$/t]	Transportation cost
$P_{\$s}$	[\$/t]	Cost of steel
$P_{\$v}$	[\$/t]	Metal Price
$P_{\$W}$	[\$/kWh]	Energy cost
κ_B	[kWh/t]	Energy required per tonne of steel balls consumed
κ_{1-6}	[-]	Constants

end-discharge grate where the aperture size of the end-discharge grate limits the particle size of the discharged slurry. It is assumed the in-mill slurry density is equal to the discharge slurry density (ρ_Q).

The ground ore in the mill mixes with the water to create a slurry. As shown in Fig. 3, the slurry in the mill begins to form at the shoulder of the charge. The toe of the slurry starts to grow downwards towards the toe of the charge as the slurry flow-rate through the mill increases. While the toe of the slurry is less than or equal to the toe of the charge, discharge occurs through the interstices in the grinding media. When the toe of the slurry exceeds the toe of the charge, a slurry pool forms at the bottom of the mill. Slurry discharge is then a combination of flow through the grinding media and the slurry pool.[43] Slurry pool conditions should be avoided as it decreases the mill power draw and breakage rate by cushioning material falling from the charge shoulder to the charge toe.

The discharged slurry is collected in a sump. The slurry in the sump is diluted with water (SFW) before it is pumped to the cyclone via a variable-speed pump. The cyclone is responsible for the classification of material discharged from the sump. The lighter and smaller particles in the slurry pass to the overflow of the cyclone, while the heavier and

Table 2: Nomenclature for single-stage closed grinding mill circuit (see Fig. 2).

Variable	Unit	Description
		Manipulated Variables
MFB	[t/h]	Feed-rate of steel balls to the mill
MFO	[t/h]	Feed-rate of ore to the mill
MIW	[m^3/h]	Flow-rate of water to the mill
SFW	[m^3/h]	Flow-rate of water to the sump
CFF	[m^3/h]	Flow-rate of slurry to the classifier
ϕ_c	[-]	Fraction of critical mill speed
		Controlled Variables
J_T	[-]	Fraction of mill volume filled by total charge
PSE	[-]	Particle size estimate (fraction of particles < 75 μm in cyclone overflow)
$SVOL$	[m^3]	Volume of slurry in sump
		Additional Circuit Variables
CFD	[t/m^3]	Cyclone feed density
CPD	[t/m^3]	Cyclone product density
CPF	[m^3/h]	Cyclone product flow-rate
J_B	[-]	Fraction of mill volume filled with balls
P_{mill}	[kW]	Mill power draw
Q	[m^3/h]	Mill discharge flow-rate
Q_S	[t/h]	Mill solids discharge flow-rate
ρ_Q	[t/m^3]	Mill slurry density
TP	[t/h]	Solids throughput, i.e. mass flow-rate of solids at cyclone overflow
ψ	[-]	Mill grind (fraction of particles in mill discharge < 75 μm)

larger particles pass to the underflow. The ‘cut-size’ of the cyclone is defined as the size which divides equally to overflow and underflow, and the efficiency of the cyclone refers to the sharpness of the cut. The underflow is passed to the mill for further grinding. The overflow is the cyclone product and is passed to the downstream separation process.

The solids in the cyclone overflow consists of a distribution of particle sizes. The aim of the circuit is to produce a product where a certain fraction of the particles are below a specification size, e.g. 60% of particles in the product must be below 75 μm. The particle size estimate (PSE) provides a measure of this fraction, and is a general indication of the quality of the product.

In this article, ‘rocks’ refer to ore too large to pass through the end-discharge grate, whereas ‘solids’ are ore small enough to discharge from the mill. The solids consist of the sum of 'fine' and 'coarse' ore, where fine ore is smaller than the product specification size and coarse ore is larger than the product specification size.

Controlled and manipulated variables

This section discusses the relationships between controlled and manipulated variables for the circuit shown in Fig. 2. It is a summary of the control aspects discussed in various textbooks.[4,44–46]

Fraction of the mill filled with charge (J_T)

When plants choose the set-point for the fraction of mill charge (J_T), the efficiency of power consumption is the primary factor, followed by the stability of the system.[32] If J_T is too high, the mill needs to be stopped so that the excess charge in the mill can be removed by manual labour. The stoppage interrupts production, and the additional human resources necessary to reduce the load increases operational cost. If J_T is too low in relation to the fraction of ball charge (J_B), the power applied to turn the mill (P_{mill}) is wasted on the energy transfer between ball-ball contact and ball-liner contact. This causes unnecessary liner damage,[47,48] increases ball abrasion, and reduces the mill solids discharge flow-rate (Q_S). Since no additional solids are added at the sump, the cyclone directly splits Q_S to produce the solids throughput (TP) at its overflow. Therefore, less Q_S at a lower J_T is significant as it results in a reduced TP.

A primary aim of any milling circuit controller is to stabilize the slow integrating action of the mill on its contents. To achieve this, industrial plants primarily manipulate J_T by adjusting MFO, otherwise by varying the mill speed (ϕ_c) if a variable speed drive is fitted, or as a last option by manipulating MIW.[32,49] Control of J_T by MFO is relatively straightforward as an increase or decrease in MFO results in a direct increase or decrease in J_T. In case MFO is not available as a manipulated variable, a decrease in ϕ_c will reduce the rate at which ore is broken and more ore is allowed to accumulate within the mill. On the other hand, by increasing ϕ_c the mill imparts more energy to the ore which causes the ore to break at a quicker rate. Since the ore breaks faster, it spends less time in the mill and J_T subsequently decreases. Finally, because water is the main transporting medium of ore through the mill, an increase in MIW will wash material out of the mill and reduce J_T. Conversely, a reduction in MIW increases the density of the slurry in the mill which reduces the fluidity of the slurry. The build-up of the thicker slurry increases J_T.

Sump slurry volume ($SVOL$)

The sump acts as a buffer between the mill and the cyclone. The only control requirement is to prevent the sump from overflowing or running dry. Reducing the SFW can counter an undesirable increase in the sump slurry volume ($SVOL$). If the increase in $SVOL$ is a result of a higher Q_S, a reduction in SFW will cause the cyclone feed density (CFD) to increase, the PSE to decrease, and the cyclone product density (CPD) to increase. The cyclone feed flow-rate (CFF) can be increased to counter an undesirable increase in $SVOL$, but this will cause PSE to increase. Therefore, the sump controller must manage the loop interactions between $SVOL$ at the sump and PSE at the cyclone. In industrial plants, SFW is used more frequently than CFF to control $SVOL$.[32] The area between overfill and underfill conditions allows $SVOL$ to vary to achieve a desired CFD and CFF such that the PSE is not affected. This requires a multi-variable controller capable of decoupling the dependencies between these variables.[50–52]

Cyclone feed density (CFD)

The downstream separation process requires CPD to be within a specific range for correct operation. The CPD can be corrected by CFD as they are directly proportional. However, CFD influences both CPD and PSE. A low CFD increases PSE as less coarse material reports to the overflow. A high CFD has the opposite effect. At a critically high CFD, the underflow at the cyclone apex will change from a wide 'umbrella' spray shape to a high density rope. The high density rope impedes flow to the underflow, i.e. the cyclone underflow chokes. The cyclone then functions as a T-pipe where CFF splits between the under- and overflow with the majority of CFF reporting to the cyclone overflow. This causes PSE to drop and CPD to rise beyond acceptable limits. Therefore, system stability, avoiding roping conditions, and the PSE and CPD set-points determine the operating range of CFD.

As mentioned above, SWF can easily manipulate CFD. A more indirect change in CFD occurs if MFO varies, as MFO changes the solids content in the circuit. Similarly, MIW can also indirectly alter CFD. The order in which the manipulated variables appear here - SFW, MFO, and MIW - is the order of preference for plants to control CFD.[32]

Cyclone product particle size estimate (PSE) and cyclone product throughput (TP)

There are two general philosophies to control a grinding mill circuit:

- Maintain PSE at setpoint, and maximize TP. (I.e. TP is allowed to vary as long as PSE deviates as little as possible from setpoint.)

- Maintain TP at setpoint, and push PSE towards an acceptable setpoint. (I.e. PSE is allowed to vary away from its setpoint as long as deviations in TP are minimised.)

For the second control philosophy above, the aim is not to maximise PSE. Grinding the ore too fine results in unnecessary losses of valuable material in the tailings as many separation circuits cannot separate too fine material from gangue. There is also a high energy cost to grind material very fine. A circuit may not necessarily achieve the desired PSE set-point at the specified TP. Since there is a limit to the degree of coarseness in the slurry for the separation circuit to operate efficiently, it may be necessary to sacrifice TP simply to maintain a minimum allowable PSE.

At steady-state TP is equal to MFO. The availability of ore from the mine provides a practical upper limit for TP and MFO. The physical structure of the mill limits the volume of ore the mill can grind efficiently and provides an additional constraint on TP. Alternatively, the capacity of the cyclone may limit TP. Since the manufacturing, maintenance and operating costs associated with a cyclone are very low compared to a mill, a well-designed circuit should not allow the cyclone to dictate the throughput capacity of the milling circuit.

The PSE is a function of both CFD and CFF. The effect of a higher CFF is to reduce the cyclone cut size and consequently increase PSE. The capacity of the cyclone provides an upper bound for CFF. The minimum cyclone inlet pressure necessary to keep the cyclone within its operable region provides the lower bound for CFF. It is important that the pump at the sump discharge is correctly sized to achieve the desired CFF.

Although CFF and CFD influence the cut-size of the cyclone, i.e. PSE, the cyclone does not break material. The achievable PSE depends on the mill grind (ψ), where ψ is the fraction of material in the discharge of the mill below the specification size. An increase in PSE is achieved through an improved ψ. Manipulating MFO will alter ψ, but the effect on J_T and TP should not be neglected. An increase in MFO increases J_T and the overall TP of the circuit, but it decreases ψ and therefore the achievable PSE. A lower J_T produced by a lower MFO is more conducive to producing a higher ψ and a better PSE, but it sacrifices TP and consequently reduces the mass of tradeable final concentrate product.

Additional circuit variables

The process measurements generally and readily available at industrial circuits should inform the control strategy. The survey of Wei and Craig[32] indicates that P_{mill}, J_T, and CFD are commonly measured variables, whereas CPD and the ore feed size distribution are less commonly measured. The variables CPF, TP, ψ, Q_S, ρ_Q, and J_B are not explicitly included as real-time measured variables for any of the plants surveyed.

Assuming steady-state operation, CPF should be equal to the sum of $\frac{MFO}{\rho_O}$, MIW, and SFW, where ρ_O (t/m^3) is the density of the ore. Also, TP equals MFO at steady-state by definition. Because a circuit rarely achieves steady-state, real-time measurements of CPF

and CPD are desirable to control the dynamics of the process. TP does not necessarily need to be measured in real-time as it can be inferred from measurements of CPF and CPD. Real-time measurements of CPF and CPD are difficult because of the amount of air in the cyclone overflow pipe.

Because of space restrictions at the discharge trommel of the mill, including flow, density and particle size instrumentation at the mill discharge is currently not viable for most plants. Through careful planning and design of greenfield comminution circuits, it should be possible to install existing flow, density, and particle size measurement instrumentation technologies at a mill discharge trommel. In the case where the mill discharges into a sump, both ρ_Q and Q_S can be back-calculated from a flow-balance around the sump if accurate measurements of $SVOL$, SFW, CFF, and CFD are available. If accurate measurements of Q_S and ρ_Q are available, it is possible to estimate the hold-up of water, solids, and grinding media within a mill.[53] In a similar manner, ψ can be back-calculated if a particle size measurement is made at either the outflow of the sump or at the overflow of the cyclone. However, the accuracy of the back-calculations are sensitive to errors in the measurements and in the modelling of the process units.

In general MIW is kept at a constant ratio of MFO to maintain ρ_Q within reasonable bounds. A very high density slurry will result in a non-flowing thick mud which reduces discharge flow-rate. On the other hand, a very high water content in the mill may cause a low-density slurry pool to form at the toe of the charge. The slurry pool absorbs the impact energy of falling material. This not only reduces P_{mill}, but also the rate at which fines are produced. If the discharge-grate conditions are such that no slurry pooling occurs, an increase in MIW will reduce the residence time of solids in the mill and increase discharge flow-rate. These discharge-grate conditions are typically a high fractional open area of the discharge grate, a high relative radial position of the open area, and a high relative radial position of the outermost grate aperture.[48]

SAG mills are usually designed with a constant J_B in mind. Because accurate real-time measurement of J_B is generally not available, J_B is difficult to include in control schemes to manipulate ψ and Q_S. J_B can be approximated inferentially using models and measurements of P_{mill} or J_T,[54] assuming the model parameters are correctly fitted to process data. In practice, a linear proportionality between the rock volume and the energy required per tonne of steel balls consumed (κ_B) is assumed. This assumption allows for the calculation of MFB to maintain an approximately constant J_B in terms of the ton of ore milled. At steady-state it implies that MFB is a constant fraction of MFO.

A more consistent ψ could be achieved through a very high J_B, but the heavy balls increase the power required to turn the mill and consequently increase the energy cost. κ_B depends on the ore characteristics, the mill liner type, the ball material, the ore grinding media hold-up, and J_B. A high J_B increases κ_B as there is more ball-ball and ball-liner contact rather than ball-ore contact. Although a low J_B reduces κ_B, it also reduces the grinding ability of the mill. For a very high mill rotational rate the balls may collide with exposed liners causing unnecessary liner wear[47,48] and a higher κ_B.

The survey by Wei and Craig[32] indicates most plants desire better measurement instruments rather than more actuators. However, the one actuator most plants desire is a variable speed drive to manipulate ϕ_c. Because this variable has a large impact on the operating region of a mill,[15] ϕ_c should be changed with great care. Viklund et al.[55] use ϕ_c to change the grinding efficiency of a mill processing different types of ore. Since ϕ_c and P_{mill} are approximately linearly related, the maximum available power provides the upper limit for ϕ_c.[14]

Disturbances

Variations in MFO, the ore feed size distribution, or the feed hardness perturbs the equilibrium of the mill. The effect of these variations on the behaviour of the grinding mill requires time to decay. Although it is desirable to control all three of these disturbances, only MFO can be controlled. Grinding mill circuits have to contend with the feed size distribution or the feed hardness as disturbances. These two disturbances vary both in the short and long term. If the disturbances are not effectively rejected by a control system, a lower recovery of valuable product in the downstream processes will result.

Variations in the feed size cause changes in the grinding media size, which affects the breakage characteristics in the mill. An increase in the size of the feed ore means more rocks are available to assist with breakage, but this reduces the critical sized material in the mill responsible for producing fines. Conversely, a reduction in the size of the feed ore reduces the availability of rocks for impact breakage.[56] A degree of feed size control is possible if ore is not run-of-mine, but is sourced from different parts of a stockpile.[42,57]

Variation of the hardness of the ore will cause P_{mill} to vary. If the hardness of the ore increases, more energy is required to break ore, and the mill subsequently produces fines at a slower rate. If changes in MFO do not alter the throughput, an increase in the hardness of the ore will increase J_T. The increase in J_T requires more power to rotate the charge at the desired speed. If the hardness decreases, the mill produces fines at a quicker rate and J_T will decrease. This decrease causes a reduction in P_{mill}. It should be noted that changes in MFO and feed hardness do not directly change P_{mill}. Rather, the feed hardness and MFO change J_T which consequently alters P_{mill}.

The economics of the comminution process

The aim of this section is to describe the relationship between comminution and separation, and to develop the economic objectives of the comminution circuit with reference to the mineral processing plant as a whole. This provides a mechanism to develop a control strategy for the comminution circuit to achieve the plant-wide economic objectives.

Efficient comminution is essential for efficient separation of valuable materials and gangue. The degree of liberation by comminution refers to the percentage of minerals occurring as free particles in the ore in relation to the total mineral content. The desired degree of liberation depends on the ore properties and the separation method. If there is a pronounced difference in density or magnetic susceptibility between the particles and the gangue, separation is possible through gravimetric and magnetic separation even when valuable minerals are completely locked inside gangue. In the case of chemical leaching, some part of the surface of a valuable component locked in gangue needs to be exposed to the reagent to achieve separation. For effective separation through froth flotation, almost all of the surface of the valuable particle needs to be exposed to the reagent. More grinding means more liberation and more exposure of the surface area of the valuable particle to the flotation reagent, but too much grinding means the particle is too small to float and it is lost to the tailings. Although the low degree of liberation sufficient for magnetic or gravimetric separation is less energy intensive, more gangue may possibly report to the concentrate. Less gangue may report to the concentrate in the case of froth flotation and leaching, but the higher degree of liberation required is more energy intensive.[4] Throughout the rest of this study it is assumed separation is achieved through froth flotation as this is the most common separation technique used.

The comminution circuit has only limited influence on the variables that determine the performance of a separation circuit. Laurila et al.[31] suggest a number of key variables to control the flotation process: product particle size (i.e. PSE), shape, and degree of mineral liberation; slurry feed-rate and density (i.e. CPF and CPD); mineral concentrations in the feed, concentrate, and tailings; mineralogical composition of ore; cell pulp levels and air inlet flow-rates; chemical reagents and their addition rate; electrochemical parameters; froth properties and wash water rate.

Variations in the ore mineralogy, CPF, CPD, and PSE are considered the main disturbances to the flotation circuit. Only the latter three disturbances can be influenced by the comminution circuit. If the comminution circuit is controlled efficiently, there should be minimal variations in CPF, CPD, and PSE.[58] Although this study assumes flotation is used for the separation section in Fig. 1, similar disturbances apply to a gold leaching plant.[5]

An overview of stabilizing flotation control is given in Laurila et al..[31] The review of Shean and Cilliers[58] considers current and future trends in the instrumentation, base level control, advanced control, and optimisation control of flotation circuits. The simulation studies of Desbiens et al.,[59] Bergh and Yianatos,[60] and Putz and Cipriano[61] use different types of process models to develop predictive controllers for flotation circuits. An example of an industrial application of a simple single-input/single-output generalized predictive controller is shown in Suichies et al..[62] As shown by Craig and Koch,[63] monetary benefits can be achieved by reducing the variation of the flotation level around a particular set-point. An economic performance comparison between a multi-variable controller and a single-loop proportional-integral (PI) controller at an industrial flotation circuit is given by Craig and Henning.[64] The experiment design must be correct for a statistically significant and valid comparison between controllers.[63]

The relationship between comminution and separation

Separator concentrate grade and recovery

The concentrate grade (γ_C) and recovery (Υ), depicted in Fig. 1, are accepted measures of the metallurgical performance of a mineral processing plant, but are not measures of the economic performance by themselves. Rather, the most profitable region on the grade-recovery curve determines the optimal economic operation of the separation process. As shown in Fig. 4, if the separation process produces a very high γ_C, a large quantity of valuable metals will report to the tailings which reduces Υ. If the separation produces a high Υ, more gangue may report to the final concentrate which reduces γ_C. The challenge is to improve both γ_C and Υ through process control.[4]

If only one metal is extracted from the separator concentrate, the material balance for the separation process at steady-state operation is

$$F_{sep} = C_{sep} + T_{sep}, \tag{1}$$

where F_{sep}, C_{sep}, and T_{sep} (t/h) are the separator feed, concentrate, and tailings respectively. The mass balance of valuable minerals/metals is

$$\gamma_F F_{sep} = \gamma_C C_{sep} + \gamma_T T_{sep}, \tag{2}$$

where γ_F and γ_T are the separator feed and tailing grades respectively. It is assumed the feed of the separation circuit is the product of the comminution circuit, i.e. $F_{sep} = TP$ and $\gamma_F = \gamma_{TP}$ where γ_{TP} is the grade of the comminution circuit product. Using (1) and (2) it is possible to express Υ as

$$\Upsilon = \frac{\gamma_C C_{sep}}{\gamma_{TP} TP} = \frac{\gamma_C \left(\gamma_{TP} - \gamma_T\right)}{\gamma_{TP} \left(\gamma_C - \gamma_T\right)}. \tag{3}$$

Effect of cyclone product flow-rate (CPF) and density (CPD) on recovery-grade curve

The throughput capacity of the flotation plant should be designed to match the throughput capacity of the milling circuit. Assuming the cyclone product of the milling circuit feeds directly into the flotation circuit, any variations in CPF will cause disturbances in the slurry levels of the flotation cells. The flotation circuit cell capacity therefore constrains CPF. Given adequate flotation control and no violation of the CPF constraint, it is assumed CPF has negligible effect on the recovery-grade relationship of the flotation circuit.

CPD describes the split between water and solids content in CPF. Increased dilution in the flotation feed lowers the mean residence time of particles in the flotation circuits. This disturbance can be handled if the capacity of the flotation circuit is large enough, but if the disturbance is too large it may require the use of a dewatering process prior to flotation.[8] If

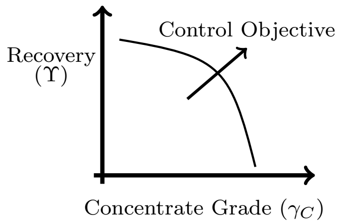

Figure 4: Typical concentrate grade (γ_C) and concentrate recovery (Υ) curve.

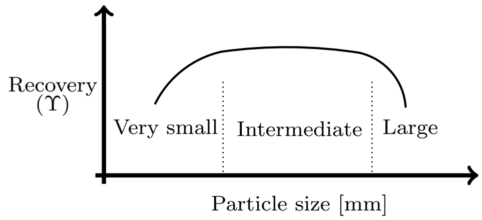

Figure 5: Typical concentrate recovery (Υ) of a separation circuit as a function of the flotation feed particle size distribution.

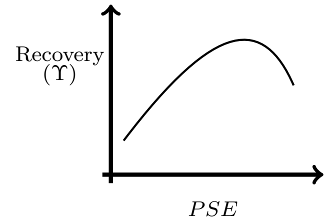

Figure 6: Concentrate recovery (Υ) as a function of the fraction of particles below 75 μm, i.e. PSE.

maintained within constraints, it is assumed CPD has negligible effect on the recovery-grade relationship if the flotation circuit is adequately controlled.

Effect of cyclone product particle size estimate (PSE) on recovery-grade curve

Fig. 5 shows Υ as a function of particle size distribution for a flotation circuit. The particle size distribution in the flotation feed is roughly divided into three groups: very small, intermediate and large. Because the very small particles have low momentum and a low rate of collision with bubbles, the probability of adhesion and subsequent flotation is low. With an increase in particle size, the degree of hydrophobicity necessary for a high level of flotation increases. For large particles, the rapid consumption of reagents by smaller particles leads to less complete surface coverage of the large particles and results in less floatable particles. The relationship between Υ and the ore particle size is relatively independent of moderate changes in PSE, CPD, and the mineral content.[65]

From Fig. 5 it is clear that the aim of a comminution circuit is to maintain a narrow distribution of intermediately sized particles to maximise valuables/gangue separation. Grinding material too fine not only reduces floatability, it also means unnecessary energy expenditure to achieve the high grind. Grinding material too coarse drastically reduces valuables/gangue separation. In the case where PSE deviates to above set-point, i.e. overgrind, the impact is to decrease γ_C and increase Υ. The opposite effect is seen for undergrind conditions. If the separation plant operates at the optimum recovery-grade ratio in terms of the refinery return, any deviation in PSE reduces the return from selling the concentrate to the smelter.[6]

As shown in Fig. 6, the relationship between Υ and PSE is a second order polynomial.[10] The corresponding γ_C can be determined from the grade-recovery relationship depicted in Fig. 4. Therefore, both Υ and γ_C can be empirically defined in terms of PSE as

$$\Upsilon(PSE) = \kappa_1 PSE^2 + \kappa_2 PSE + \kappa_3 \tag{4}$$

$$\gamma_C(PSE) = \frac{\kappa_4}{\Upsilon(PSE) - \kappa_5} + \kappa_6, \tag{5}$$

where the parameters $\kappa_{1,2,\ldots 6}$ are fitted to process data.

Mineral processing plant revenue

A mineral processing plant generates revenue by selling the separator concentrate to a smelter. The revenue should cover the cost of comminution and separation for the plant to be economically viable. The revenue is approximated as

$$\text{Revenue} = \text{Net Smelter Return} - (\text{Comminution Cost} + \text{Separation Cost})\,. \tag{6}$$

Net smelter return

The price a smelter will pay for a concentrate depends on metal recovery, metal prices, operating costs, capacity constraints, deleterious elements, transportation costs, metal premiums and discounts, types of flotation concentrates, processing capital costs etc. A thorough comprehension and appreciation of market conditions, the characteristics of the concentrate, and the abilities of potential buyers to process the concentrate is required to negotiate the best possible contract between the smelter and the mineral processing plant.[66] Although simplified, a basic contract may stipulate the items listed below:

- Valuation $P_{\$v}$ (\$/t): pay the lowest market price per ton of metal recoverable from the concentrate
- Transport $P_{\$t}$ (\$/t): cost for transport per ton of concentrate between mineral processing plant and refinery
- Process $P_{\$p}$ (\$/t): cost per ton of concentrate processed by the smelter

The net smelter return (NSR) (\$/h) can therefore be expressed as

$$\begin{aligned}\text{NSR} &= \text{Concentrate Metal Value} && -(\text{Transport cost} + \text{Process cost}) \\ &= P_{\$v}\,(\text{Metal in Concentrate}) && -(P_{\$t} + P_{\$p})\,(\text{Total Concentrate}) \\ &= P_{\$v} C_{sep} \gamma_C && -(P_{\$t} + P_{\$p})\,C_{sep}. \end{aligned} \tag{7}$$

It is assumed the metal content of the ore fed to the separation process is equal to the metal content of the unprocessed run-of-mine ore fed to the comminution circuit. In other words, the grade of the run-of-mine ore fed to the comminution circuit (γ_{ROM}), also known as the mill head-grade, is equal to the grade of the separator feed (γ_{TP}). Using (3) to (5), NSR in (7) can be expressed in terms of Υ, γ_{ROM}, γ_C and PSE as

$$\text{NSR} = P_{\$v} \Upsilon(PSE) \gamma_{ROM} TP - (P_{\$t} + P_{\$p})\,\Upsilon(PSE) \frac{\gamma_{ROM}}{\gamma_C(PSE)} TP. \tag{8}$$

(The NSR in (7) is relatively basic and can be altered based on the more specific details of the contract between a mineral processing plant and a refinery.)

Given possible improvements in the separation of gangue from valuable minerals, a company may decide to reprocess tailings. Because the size of the ore has already been sufficiently reduced, there is no energy cost if the particle size distribution is already within specification.[4] Therefore, the economic value of the tailings is given by

$$\text{Tailing Value} = P_{\$v}\,(1 - \Upsilon(PSE))\,\gamma_{ROM} TP. \tag{9}$$

The NSR depends on the trade-off between Υ and γ_C as illustrated by Fig. 4.[67] A high γ_C is less costly to smelt, but the resulting lower Υ may yield a lower return on the final product. A low γ_C with high Υ is costly to smelt but returns more final product. If the metal price is high, a lower γ_C can be targeted by the separation circuit to improve Υ and increase return. To achieve a low γ_C and high Υ, the comminution circuit may need to run at a constant and high TP at the cost of a consistently correct ψ. If the metal price decreases, the separation circuit may target a higher γ_C at the cost of Υ. In this case the comminution circuit may need to focus on achieving a correct and constant ψ while sacrificing a consistently high TP. In the case of a high grade ore, the comminution circuit may aim to maintain a consistently correct ψ while maximising TP to extract the maximum value. If a low grade ore is processed, the aim may be to operate at a constantly high TP while attempting to maintain ψ as close as possible to its desired value.

Comminution and separation cost

The cost of comminution accounts for approximately 50% of the operational cost of the mineral processing plant, followed by the cost of separation at less than 20% of the operational cost.[4,68] The high comminution cost is primarily due to the high power required to turn the grinding mill. According to some estimates, comminution accounts for approximately 2-3% of the electricity usage of the world.[68–70] The consumption of steel balls as grinding media is the second largest operational cost of comminution, but is only a small fraction of the milling power cost. The pumping energy cost in the comminution circuit itself is negligible compared to the power used by the mill.[71]

The main operating cost for the separation circuit is the cost of reagents added to separate gangue and minerals. A constant separation process cost is assumed as the economics of the separation circuit falls outside the scope of this study. These costs can be included if a good model of the flotation process is available.[72]

If $P_{\$W}$ (\$/kWh) is the cost of energy, and $P_{\$s}$ (\$/t) is the cost of steel balls, the comminution circuit operating cost ($/h) can be approximated as[9]

$$\text{Comminution cost} = P_{mill}\left(P_{\$W} + P_{\$s}/\kappa_B\right). \tag{10}$$

Top-down analysis

The previous section formulated the economic impact of the product of a comminution circuit on the mineral processing plant revenue. This provides a mechanism to define the operational goals of the comminution circuit in terms of the economic objectives of the mineral processing plant. The revenue of the mineral processing plant is defined in terms of the output variables of the grinding mill circuit by relating the grade (γ_C) and recovery (Υ) of the separator concentrate to the product quality (PSE) and quantity (TP) of the comminution circuit. It is assumed the control of the separation circuit is such that a constant functional description relates the output of the comminution circuit to the output of the separation circuit. Once the operational goals of the comminution circuit are quantified, a sufficient control structure must be constructed for the comminution circuit to achieve these goals. Such a control structure is proposed in this section based on the plant-wide control design procedure of Skogestad.[24]

Operational economic objective

The questions to answer in this step are: what is the scalar cost function which defines the economic objective of operation, and what are the available dynamic and steady-state degrees of freedom to achieve this economic objective? Given the mineral processing plant economic objective defined in (6), the comminution process economic scalar cost function is defined in terms of (8) and (10) as

$$J_{comm} = \Upsilon(PSE)\gamma_{ROM}TP\left(P_{\$v} - \frac{P_{\$t} + P_{\$p}}{\gamma_C(PSE)}\right) - P_{mill}\left(P_{\$W} + \frac{P_{\$s}}{\kappa_B}\right). \tag{11}$$

Plant operating limits and downstream operation requirements constrain the cost function in (11).

The steady-state behaviour of the plant primarily determines the economics of the plant. Generally the steady-state degrees of freedom are the same as the economic degrees of freedom. Identifying the dynamic degrees of freedom is generally easier than identifying the economic (steady-state) degrees of freedom. However, it is the number of economic degrees of freedom (N_{SS}) and not the variables themselves which is important to determine. N_{SS} gives the number of controlled variables to be selected in the third step of the top-down analysis.[73]

N_{SS} is determined by subtracting the number of manipulated and controlled variables with no economic steady-state effect (N_0) from the number of dynamic degrees of freedom (N_D). Since there are six manipulated variables, it means $N_D = 6$:

- Mill ore feed-rate (MFO)
- Mill inlet water flow-rate (MIW)
- Mill ball feed-rate (MFB)
- Sump feed water flow-rate (SFW)
- Cyclone feed flow-rate (CFF)
- Mill rotational speed (ϕ_c)

Levels in tanks generally form part of the variables with no economic steady-state effect. There are four levels throughout the circuit to consider:

- Total charge filling in the mill (J_T)
- Ball filling in the mill (J_B)
- Mill slurry volume

- Sump slurry volume ($SVOL$)

As discussed earlier, J_T, J_B and the mill slurry volume can all affect TP, PSE and P_{mill}. Therefore, these three levels in the mill remain crucial steady-state degrees of freedom to define. At the sump, $SVOL$ has no economic steady-state effect and is controlled through either SFW or CFF. This means $N_0 = 1$. Consequently, there are five steady-state degrees of freedom: $N_{SS} = N_D - N_0 = 6 - 1 = 5$.

Optimal steady-state operation: Grindcurves

Once the cost function is identified, the question is what are the operational conditions for optimal steady-state operation? In other words, how should the set-points be chosen for optimal circuit operation? The operating performance of the mill primarily determines the optimal steady-state of operation for the single-stage grinding mill circuit. The main mill performance indicators are P_{mill}, Q_S and ψ. The quazi-static curves of P_{mill}, Q_S and ψ as functions of J_T and ϕ_c are called grindcurves and can be used to determine the optimal steady-state operating region of a mill.[14,15]

Van der Westhuizen and Powell[14] developed grindcurves of an industrial open-circuit SAG mill with a ball filling of 7.9%. These grindcurves were developed assuming J_B and ρ_Q remained constant for all equilibrium conditions produced by different combinations of J_T and ϕ_c. It is possible to maintain J_B relatively constant by setting MFB at a constant ratio of MFO. Similarly, MIW is generally set at a constant ratio of MFO to maintain a constant ρ_Q. Grindcurves should not necessarily be defined only in terms of two degrees of freedom: J_T and ϕ_c. Adequate manipulation of J_B and ρ_Q within system constraints can allow the grindcurves to be defined in terms of four degrees of freedom.

Fig. 7 illustrates the general parabolic shape of the grindcurves in Van der Westhuizen and Powell[14] for J_T in the range of 0.18 to 0.45 and ϕ_c in the range of 0.60 to 0.75. Certain conclusions can be drawn from the grindcurves:

- The peaks of Q_S and P_{mill} do not coincide.

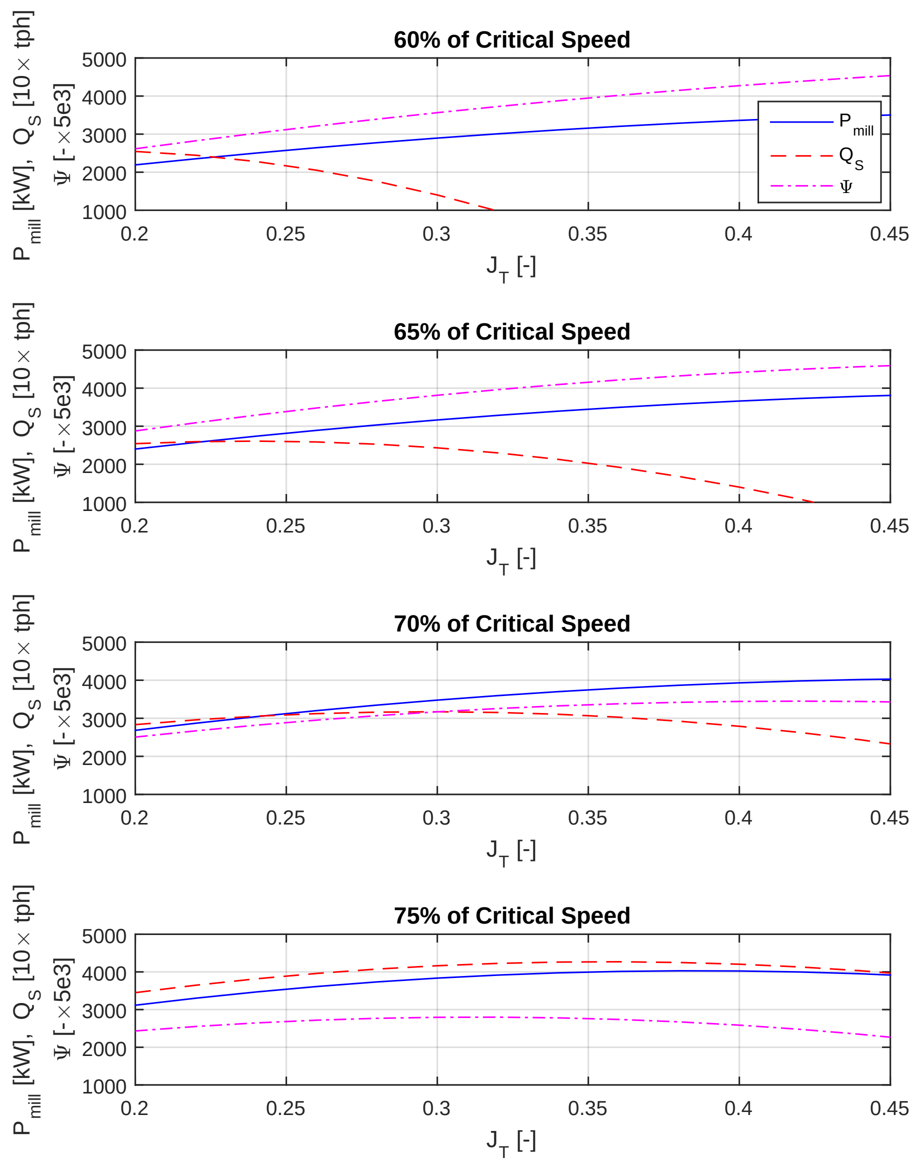

Figure 7: Mill grindcurves for an open circuit SAG mill. (Re-created from data in Van der Westhuizen and Powell (2006)[14] with permission.)

- Q_S increases with ϕ_c.
- The curve shapes are dramatically changed by ϕ_c.
- The peak values change according to changes in ϕ_c.
- Q_S becomes coarser as ϕ_c increases, implying ψ reduces as ϕ_c increases.

The grindcurves in Van der Westhuizen and Powell[14] are for an open-circuit grinding

Table 3: Data of grindcurve peaks from Van der Westhuizen and Powell (2006).[14] (* - indicates peaks that are extrapolated to above 50% mill filling.)

Mill Speed	Power Peak		Discharge Peak		Grind Peak	
ϕ_c	J_T	P_{mill}	J_T	Q_S	J_T	ψ
0.60	0.54*	3603*	0.18	261	0.63*	1.00*
0.65	0.55*	3931*	0.23	252	0.58*	0.94*
0.70	0.47	4028	0.31	326	0.37	0.69
0.75	0.39	4037	0.35	424	0.33	0.56

mill. If the circuit is closed, the grindcurve peaks move slightly relative to the open-circuit case, but the curve shapes do not change.[15] Although not labelled as such, Craig et al.[74] developed grindcurves for an industrial closed-circuit single-stage grinding mill at a single fixed speed. The circuit was controlled by the multi-variable Inverse Nyquist Array controller of Hulbert et al..[50] The steady-state data showed a clear parabolic relationship between TP and J_T. The peak of TP occurred at a lower J_T compared to the peak of P_{mill}. Therefore, the results of the open-circuit grindcurves can be extrapolated to the closed-circuit case.[75]

Powell and Mainza[75] indicate how a comprehensive grindcurve of a production SAG mill (or ball mill) can be established within one day. Powell et al.[15] illustrate various practical calibration issues for industrial mills, indicate how to fit the grindcurve in Table 3 (obtained from Van der Westhuizen and Powell[14]) using simple first and second order polynomials, and discuss the practical value of the grindcurves to industrial plants. However, further research is required to track the movement of grindcurves in real-time as the characteristics of the ore changes.

It is simplistically assumed that for the closed-circuit case the cyclone maintains a consistent cut-size and water recovery to underflow, and that the variations in CFF to maintain a consistent cut-size and water recovery has a negligible effect on the grindcurves. In this case, the following relations apply

$$TP = \kappa_{TP} Q_S \tag{12a}$$

$$PSE = \kappa_{PSE} \psi, \tag{12b}$$

where κ_{TP} and κ_{PSE} are constants. Alternatively, (12) can be replaced by more complex cyclone models.[76–79] Thus, the grindcurves of Q_S and ψ in terms of J_T and ϕ_c can be used to define the performance of the circuit in terms of PSE and TP. In the case where the circuit is closed by a screen rather than a cyclone, the vibrating screen models of Meyer and Craig[80] may be used to establish these relations.

Power (P_{mill}) for different mill fillings (J_T) and speeds (ϕ_c)

Since P_{mill} is an indication of the rate of kinetic and potential energy imparted to the charge, P_{mill} is determined by J_T. An increase in J_T means an increase in mass in the mill which allows more charge to absorb the energy available from the rotating mill. This increase in J_T results in a higher P_{mill}. At very high J_T the movement of the centre of mass of the charge towards the centre of the mill dominates the increase in mass, which reduces the torque of the mill. At the same time, the increase in J_T lifts the toe of the charge which reduces the potential energy imparted to the charge. Therefore, P_{mill} peaks somewhere between low and high J_T.

As ϕ_c increases, more kinetic and potential energy is imparted to the charge and results in a higher P_{mill}. A second order polynomial passing through the origin can be used to describe the relationship between P_{mill} and J_T. For higher ϕ_c, the curve between P_{mill} and J_T is steeper and the peaks are more pronounced. The peaks move from higher to lower J_T as ϕ_c is increased.[14]

A generally held belief by the mineral processing industry is that ψ or Q_S is maximised when P_{mill} is at its maximum.[81] The rationale is that since P_{mill} is an indication of the rate of energy imparted to the charge, the maximum breakage of ore occurs at maximum P_{mill}. Plants may therefore decide to operate at the point where maximum P_{mill} occurs. However, as seen from the grindcurves, maximum ψ and Q_S does not necessarily coincide with maximum P_{mill}. In most cases P_{mill} reaches its maximum only after Q_S and ψ pass their peaks.[82] If the difference between the peaks of Q_S and P_{mill} in terms of J_T is very small, a power-peak-seeking controller can be used as a relatively simple solution to increase circuit throughput.[74] Otherwise, if the difference is significant, operating at maximum P_{mill} may not be the best operating strategy.

Solids discharge rate (Q_S) for different mill fillings (J_T) and speeds (ϕ_c)

If there are no discharge limitations or slurry pooling, the rate of breakage of coarse sizes to below the discharge grate size determines Q_S. The breakage rate of coarse material ($>$ 10 mm) decreases as the mill filling J_T increases. Because the coarser material is broken by impact breakage, the increase in J_T lifts the toe of the charge and reduces the drop height from shoulder to toe. However, the mass breakage rate, which is the total mass of coarse ore multiplied by the coarse ore breakage rate parameter, determines the value of Q_S. As J_T is increased from a low level, the coarse ore breakage rate parameter will decrease slower than the mass of coarse ore will increase. The cumulative effect is therefore a higher Q_S. At high J_T the coarse ore breakage rate constant will decrease faster than the mass of coarse ore will increase and the consequent effect is a reduced Q_S. A peak for Q_S occurs between these two effects.[14]

For J_T in the range 20%-40% in Fig. 7, moving from $\phi_c = 65\%$ to $\phi_c = 75\%$ results in a dramatically higher Q_S. Changes in ϕ_c below 65% or above 75% do not have the same large influence. The Q_S peak shifts to a higher J_T as ϕ_c increases. This is opposite to the P_{mill} peaks which move to a lower J_T as ϕ_c increases. Similar to power curves, the Q_S curves become steeper as ϕ_c increases. Mill control at higher ϕ_c is difficult as a small change in J_T can have a large impact on Q_S.

Grind (ψ) for different mill fillings (J_T) and speeds (ϕ_c)

The production of fines is influenced by the breakage rate constant of fines and the slurry content of the mill. Theoretically, for an increased J_T the breakage rate of fines and the solids content in the slurry both increase. Therefore, the total mass breakage rate, which is the product of the slurry solids content and the fines breakage rate, increases as J_T increases.

Although a higher ϕ_c results in an increased Q_S and P_{mill}, a higher ϕ_c also results in a coarser product. Reducing ϕ_c will improve ψ, but this will sacrifice Q_S. Whereas the P_{mill} and Q_S curves are steeper for a higher ϕ_c, the ψ becomes less influenced by J_T as ϕ_c increases. The ψ is expected to increase as P_{mill} increases and Q_S reduces. As seen in Fig. 7, the larger the difference between the P_{mill} and Q_S curves at a high J_T, the finer the product will be.

The findings above may suggest that for a finer product the mill must be run at a lower ϕ_c. However, because of the consequent reduction in Q_S, a lower ϕ_c may not increase the discharge rate of fines. The discharge rate of fines is highest at a high ϕ_c even though ψ is lower. (In other words, at high ϕ_c the percentage of material passing 75 μm may be lower, but the tonnes per hour of material passing 75 μm is higher.) The ϕ_c should thus be chosen depending on whether Q_S or ψ is a priority as determined by the economic optimisation strategy.[14]

Primary (economic) controlled variables

Once optimal operation is defined, the question is which five economic (steady-state) degrees of freedom should be controlled to maintain optimal operation? Given that the grindcurves are used to define the optimal circuit operation, the five variables to maintain optimal economic operation are:

- Total charge filling in the mill (J_T)
- Ball filling in the mill (J_B)
- Mill slurry density (ρ_Q)
- Particle size estimate (PSE)
- Throughput (TP)

In accordance with the assumptions made in the development of the grindcurves, J_B, ρ_Q, and the cut for the cyclone are to be kept constant. Consequently, it is suggested to control J_T as this determines Q_S and ψ which in turn determine TP and PSE (see (12)). However, Q_S and ψ are also dependent on ϕ_c. The cost function in (11) should therefore be maximised with respect to J_T and ϕ_c with the grindcurves as equality constraints.

The prominent economic controlled variable is therefore J_T. This is confirmed by the study of Borell et al.[82] where a heuristic on/off supervisory controller to maximize the throughput of an industrial open-circuit AG mill was applied. In their case, J_T was adjusted using MFO to optimise the mill power usage for improved Q_S.

The maximum of the cost function in (11) gives the desired PSE as functions of J_T and ϕ_c. If the mill is disturbed through ore hardness or feed size distribution variations, it will affect PSE. These disturbances can be rejected to some extent by adjusting CFF as long as the sump does not run dry or overflow.[52] This is in accordance with the demand to keep the cut-size of the cyclone constant for the grindcurves to be applicable.

Because TP passes through a maximum as given by the grindcurves, it cannot be selected as a controlled variable. Setting TP at a too high value may lead to an infeasible operating condition. Rather, a manipulator of TP should be selected and the operating condition that produces the maximum of the cost function in (11) should specify its value.

Location of throughput manipulator

The location of the manipulator of TP is a dynamic issue, but it has significant economic implications. There are three options to manipulate throughput: MFO, CFF, or ϕ_c. Since TP has a maximum as given by the grindcurves, using MFO as manipulator for TP may produce unfeasible operation. Skogestad[24] suggests locating the throughput manipulator close to the bottleneck of the process, and in the case of a recycle stream, the throughput manipulator should be located within the recycle loop. Within the loop, either the mill capacity or the cyclone capacity can constrain TP. Ideally the constraint should be determined by the mill as the capacity of cyclones are relatively inexpensive and easy to adapt. Therefore, the mill is the bottle-neck of the process. For this reason, ϕ_c and not CFF is chosen as the variable responsible for manipulating the throughput. Also, the influence of ϕ_c on the production of solids in the mill is much greater than that of CFF.

The cost function in (11) suggests the overall profit can be increased if TP increases. However, the inverse relationship between TP and PSE may negate any potential benefits from the increase in TP.[83] The use of ϕ_c as manipulated variable provides a degree of leverage to increase TP without sacrificing PSE.[52,55] As seen from the grindcurves, as ϕ_c increases the peak of Q_S (or equivalently the peak of TP) increases, and the peak of ψ (or equivalently the peak of PSE) reduces. Thus, optimisation of (11) with respect to ϕ_c with the grindcurves as constraint equations already considers the inverse relationship between PSE and TP.

Summary of top-down analysis

In summary, the cost function in (11) indicates the economic objective of the grinding circuit with reference to the final product produced by the mineral processing plant. The cost function is defined in terms of the product quality (i.e. PSE) and quantity (i.e. TP) of the circuit. Both PSE and TP, performance measures of the circuit, can be related to the performance measures of the mill, Q_S and ψ, as discussed earlier. Since grindcurves describe Q_S and ψ as functions of J_T and ϕ_c, the revenue of the plant can be expressed as a function of J_T and ϕ_c. The primary economic controlled variable is therefore J_T, with ϕ_c the manipulator of TP.

Bottom-up analysis

The generalized control loop shown in Fig. 8 provides a structural separation between the various aspects to be considered to achieve a specific control objective of a process plant.[5] It indicates the flow of data between the process, the controller, and various peripheral control tools. There are three issues to resolve within this loop:[84–86]

1. The realistic approach to *system analysis and synthesis* for complex systems.

2. The development of *advanced control strategies* which can be implemented with relative ease.

3. The design of accurate *control-relevant process models* to be used for model-based control and observer design.

To develop economic objectives and to incorporate them within a steady-state optimiser (Block 1) addresses the first issue of *system analysis and synthesis*. This requires correct identification of variables required for optimisation (Block 8) and variables required for control (Block 7). Once the desired steady-state set-points are identified, the second issue of *advanced controller design* (Block 2) can be addressed. If the advanced controller is model-based, the third issue of developing accurate *control-relevant process models* needs to be addressed. These models allow for observer design for state and parameter feedback (Block 6) from the relevant process measurements (Block 4).

The top-down analysis can be seen as a response to the need for an adequate approach for *system analysis and synthesis*. In reference to Fig. 8, the top-down analysis quantifies the economic objectives (Block 1), defines a reasonable model to describe the optimal operating point of the plant, and specifies the main economic steady-state variables (Block 8).

The bottom-up analysis provides a framework for an implementable *advanced control strategy*. In reference to Fig. 8, it defines a regulatory controller and a supervisory control layer (Block 2) based on the available manipulated, controlled and measured variables (Blocks 4, 5, 6 and 7). Although the bottom-up analysis does not provide a procedure to design accurate *control-relevant process models*, it aids to identify where modelling is necessary to relate various variables to each other. The meeting point between the bottom-up and top-down is the real-time optimiser which optimises the objective function defined in

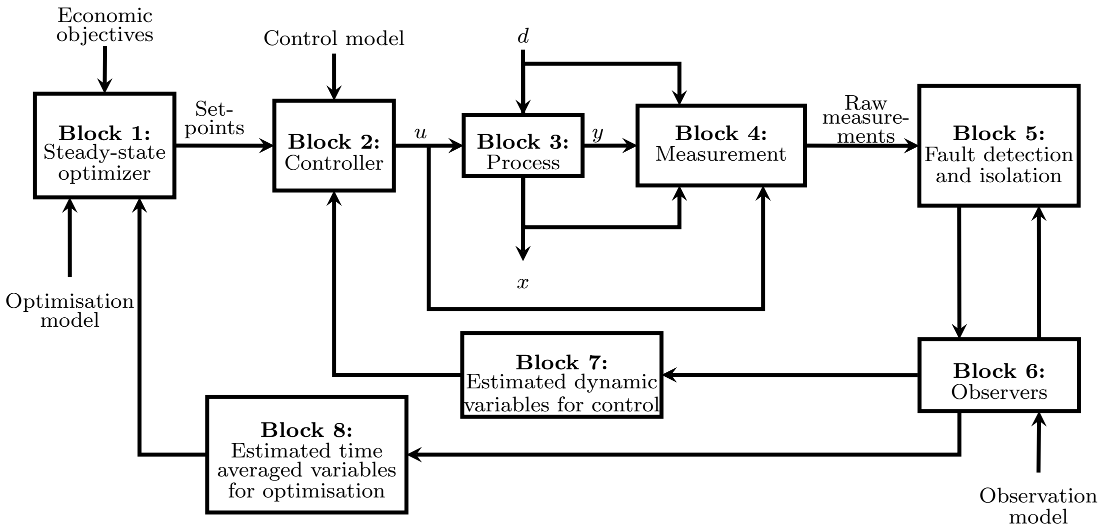

Figure 8: Generalised control loop for mineral processing. (Adapted from Hodouin (2011)[5] with permission.)

the top-down procedure. It should be noted that there is a time-scale difference between the layers, where regulatory control is generally performed in seconds, supervisory control in minutes, and optimisation in hours or days.

Therefore, the control design approach of Skogestad[24] and the control flow-diagram of Hodouin[5] can be regarded as complementary processes to develop a plant-wide controller.

An industrial example of the bottom-up process for a grinding mill circuit can be seen in Craig et al.,[74] where grindcurves are established to determine the optimal operating point of the mill, the controller is developed to maintain the mill at the optimal operating point, and the economic impact of the controller evaluated based on the variation in recovery in terms of PSE.

Regulatory control

The aim of the regulatory layer is to stabilize the plant. This is generally achieved by means of simple single-loop PID controllers. The regulatory layer should ensure the plant does not drift too far away from its nominal operating point. In light of the top-down analysis in the previous section, the following variable-pairing is proposed to stabilise the circuit:

- ϕ_c is set by the operator to achieve the desired TP.
- MFO is used to control J_T.
- MIW is used to maintain a constant ρ_Q.
- MFB is used to maintain a constant J_B.
- SFW is used to control $SVOL$ and to reject disturbances in PSE.
- CFF is used to control PSE.

The slow integrating effect of the mill on J_T is managed through MFO. However, MFO can only adjust the ore content. The water and ball content also needs to be stabilised. As mentioned previously, if a linear proportionality between the rock volume and ball breakage rate is assumed, a relatively constant J_B can be maintained by setting MFB as a constant fraction of MFO. If ρ_Q is measured, MIW can be used to control ρ_Q. Since ρ_Q is generally not measured, MIW is set at a constant ratio of MFO to maintain a relatively constant ρ_Q.

The fast integrating action of the sump on its in and outflows requires regulatory control of $SVOL$ through either SFW or CFF. As shown in Fig. 2, since SFW is closer to $SVOL$ and CFF is closer to PSE, the natural pairing is $SFW \leftrightarrow SVOL$ and $CFF \leftrightarrow PSE$, which is in accordance with the guidelines in Minasidis et al.[25] for regulatory control. However, the transfer function model developed in Craig and MacLeod[87] from step-tests at an industrial circuit indicates SFW has a quicker and larger response on PSE compared to CFF. The mechanism by which SFW affects PSE is by changing CFD. The survey of Wei and Craig[32] confirms that plants use SFW to influence PSE, or use SFW to control $SVOL$. Because of the coupling between the variables, a multi-variable controller capable of managing the available volume in the sump is recommended to manipulate SFW and CFF to keep $SVOL$ between extreme limits and reduce the variability in PSE.[50–52]

The regulatory control scheme is illustrated in Fig. 2. The level controller (LC) for J_T manipulates MFO. The flow controllers (FC) manipulate MIW and MFB which are set as constant ratios of MFO. The level controller (LC) for slurry volume manipulates SFW. The analyser controller (AC) for PSE manipulates CFF. No distinct connection is shown in the figure between TP and ϕ_c, as this should be set by the operator. A decoupled multi-variable design approach is recommended given the strong interaction between variables.[50,88,89] Examples of industrial applications of multi-variable control to grinding mill circuits is seen in Hulbert et al.[50] Craig et al.,[74] Craig and MacLeod,[90] Bouche et al.,[91] and Chen et al..[92]

Supervisory control

The supervisory control layer aims to control the primary economic controlled variables using the set-points to the regulatory layer as manipulated variables. The supervisory layer should preferably avoid saturation of manipulated variables used for regulatory control. For grinding mill circuits, the regulatory and supervisory layers are often combined into one layer. The two general alternatives are advanced single loop control or multi-variable control.[50,88]

The majority of industrial mineral processing plants make use of SISO PID controllers to achieve their control objectives even though the success of advanced process controllers in other process industries are well documented.[32,86] Model-based controllers, such as MPC, provide significant advantages over PID when applied to grinding mill circuits.[33,39,93–95] The economic performance of PID control compared to non-linear MPC when applied to a grinding mill circuit was evaluated by Wei and Craig[10] and results showed that non-linear MPC can improve performance with respect to recovered mineral value in downstream flotation circuits.[52] The ability of MPC to negotiate strong coupling between variables, long time delays, variable constraints, and plant non-linearities, qualifies it as a good candidate for a regulatory and supervisory controller.[96]

An impediment to implementing model-based control in grinding mill circuits, is the lack of sufficient real-time measurements to estimate the necessary model states and parameters for state-feedback.[28,29] The number of available real-time measurements on industrial circuits are generally far less than the size of the state vector.[32] Therefore, the peripheral tools of the control loop, such as inferential measurements, observers and fault-detection and isolation schemes, become as important as the controller itself.[48,53,97–99]

As seen earlier, the primary economic controlled variables are J_T and ϕ_c. The set-points of J_T and ϕ_c are specified by the optimisation layer. These set-points define the desired PSE and TP. The manipulator to achieve the desired TP is ϕ_c. For a specific J_T and ϕ_c, disturbances to PSE can be managed through changes to CFF and SFW.

Assuming MPC is used for regulatory and supervisory control, the MPC controller must perform the tasks delineated for the regulatory controller and the supervisory controller. If MFB and MIW are constant ratios of MFO, the controller can make use MFO, SFW, and CFF as manipulated variables to control the primary economic controlled variable J_T, as well as the variables $SVOL$ and PSE. Control of $SVOL$ is required to ensure the sump does not run dry or overflows, and control of PSE is required to maintain a constant cut-size at the cyclone amid plant disturbances. Since each of the manipulated variables impact each of the controlled variables, an adequate plant model is required for the controller to decouple the variable interaction.[13,48,76] Because the sump acts as a buffer between the mill and the cyclone, it can assist to reduce the effect of feed ore hardness and size distribution disturbances on PSE. The degree of disturbance rejection depends on the size of the sump and the ability of the MPC controller to allow $SVOL$ to drift between its minimum and maximum value.

Optimisation

The task of the optimiser is to determine the setpoints of the primary controlled variables and to detect changes in the active constraint regions. A good example of this is seen in the demand side management simulation study of Matthews and Craig,[11] where an optimiser varies the mill power draw depending on the cost of electricity in certain time periods. Over a one week period, the controller is able to maintain the total required TP for the week and maintain the specified PSE while reducing the cost of grinding for the week. The increase in profit is not a result of increased TP, but rather the reduction in power consumption. Lestage et al.[100] provides an example of real-time optimisation using linear programming to maximise TP of a grinding mill circuit.

For this study, the optimisation layer maximises the economic cost function in (11) in terms of the equality constraints defined by the grindcurves. This provides a set-point for the primary economic controller variables J_T and ϕ_c. The setpoints for the variables J_T and ϕ_c need to be updated whenever there are significant changes in the cost of electricity, the grade of the ore from the mine, the market price for concentrator product, or the processing and transportation cost of the concentrator product.

In other words, in terms of the overall hierarchical controller structure, the optimisation layer uses the grindcurves to determine the optimal steady-state at which the supervisory controller (such as an MPC) should maintain the plant. The grindcurves do not necessarily form the model for the supervisory layer. In the case of MPC as supervisory controller, the model for the MPC can be established around the optimal steady-state defined by the optimisation layer. The MPC aims to maintain the plant at steady-state amid disturbances[51] and model-plant mismatches.[99,101,102] Given the time-scale difference between the optimisation and supervisory layers, an MPC may optimise the process at the local optimum over short time periods,[52] while the optimisation layer establishes the global optimum over a long time period.[7–9]

Conclusion

This paper formulated a control structure capable of achieving the operational economic goals of the comminution circuit. The operational economic objective is defined by the cost function in (11). The cost function is defined in terms of the performance indicators of the milling circuit as a whole: PSE and TP. The circuit performance indicators can be related to the mill performance indicators - Q_S and ψ - through (12), if it is assumed the cyclone maintains a constant cut and the variations in CFF to maintain a consistent cut has a negligible effect on the grindcurves. The grindcurves define the mill performance indicators in terms of J_T and ϕ_c, if it is assumed J_B and ρ_Q remains constant for all ranges of J_T and ϕ_c. The optimisation layer maximises (11) in terms of J_T and ϕ_c, the primary economic controlled variables, to optimise the revenue which the mineral processing plant can generate. The grindcurves act as equality constraints for the maximisation of (11).

The regulatory and supervisory layer aims to control the plant at the operating condition specified by the optimisation layer. Because the process contains non-linearities, long time delays and strong interaction between variables, a multi-variable controller such as MPC is well-suited to control the grinding mill circuit.[92,95,103,104] For a target J_T at a specific ϕ_c, the MPC controller can make use of MFO, SFW, and CFF to regulate J_T, reject disturbances in PSE, and maintain $SVOL$ within operation bounds. The other two manipulated variables, MFB and MIW, are set as constant fractions of MFO to maintain a constant J_B and ρ_Q in the mill. The capacity of the circuit, i.e. the final TP, is manipulated by ϕ_c as defined by the optimisation layer.

For future work, the operation of the separation circuit, which is assumed to be a flotation circuit, should be included in the plant-wide strategy. Given the large number of variables available to influence the performance of the flotation circuit, and the complex non-linear interactions of the process, modelling the dynamics of the process remains challenging.[72,105] Therefore, including the flotation circuit in a plant-wide control strategy of a mineral processing plant is not a trivial task.[58,60] In addition, the grindcurves tend to shift as the operation of the circuit continues. As seen from the grindcurves in Craig et al.[74] and Van der Westhuizen and Powell[14] fitted to industrial data, there is an inherent uncertainty and variance in the grindcurves. Future work involves online estimation of the shape of the grindcurves to continually track optimal operating condition of the circuit.

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