Finite element versus limit equilibrium methods for analysis of lateral support

Jean Potgieter
Geotechnical Engineer
University of Pretoria
potgieter.jean@gmail.com

Prof S W Jacobsz Pr Eng
Department of Civil Engineering
University of Pretoria
sw.jacobsz@up.ac.za

INTRODUCTION

Soil-nails and anchors as means of lateral support in surface excavations require stability calculations during design. Generally, the acceptance criterion for such calculations is some arbitrary "factor of safety" (FoS).

Several methods of analyses are available to compute the FoS and hence the adequacy of these designs. At the top of the list lies the traditional limit equilibrium approach. It is simple to use and has been around for a long time. The SAICE (1989) Code of Practice for lateral support uses this approach for both soil-nails and anchors to calculate an appropriate FoS. However, in recent times, due to the advancement of computers, the finite element method has gained increasing popularity. Many geotechnical consultants are using finite elements as a standard form of analysis. Specifically, the strength reduction method is used to calculate the FoS. In this method the soil shear strength characteristics are appropriately reduced until failure occurs. The factor by which the shear strength was decreased at failure is taken as the FoS.

The issue of comparing limit equilibrium and finite element methods was brought up by construction company Terra Strata who funded this study. When designing lateral support, engineers often have conflicting views as to computing the critical FoS. This disparity is further amplified through using finite element versus limit equilibrium methods. It is often unclear why the FoS for the design is different using different methods of analysis. Geotechnical engineers sitting on opposite sides of the table will ask what friction angle and cohesion values were used without understanding the discrepancy in the method of analysis.

As part of a Master's dissertation (Potgieter 2016), the issue of comparing simplistic limit equilibrium methods to more complex finite element methods is rigorously addressed. This article is intended to be a summary of an envisaged SAICE journal article. The purpose of this summary is to guide readers to a better understanding of some of the advantages, disadvantages and pitfalls of finite element analysis and, in general, what has been found when geotechnical engineers want to compare the results of the two methods. To demonstrate the differences, an 8.5 m deep soil-nail excavation and a 17 m deep anchored excavation have been modelled with different methods. As a point of departure, the definition of FoS is discussed briefly.

DEFINITION OF FACTOR OF SAFETY

The term Factor of Safety is common to almost every engineering discipline. However, this is a rather arbitrary quantity that takes on different definitions in different scenarios. This provides the first challenge that is not well understood when comparing different methods.

In the past, within structural engineering, the maximum stress was evaluated at a certain point within a member, and this stress was then divided by an FoS to ensure that a certain margin of safety is maintained so that the calculated maximum stress is never reached. Within soil-nails and anchors, different definitions of FoS are formulated.

In the SAICE Code of Practice (1989), the FoS for soil-nails is specified in relation to the maximum capacity of the reinforcement elements, as shown in Equation 1.

$$FoS = \frac{T_{provided}}{T_{required}} \qquad (1)$$

$T_{required}$ can be calculated from a simple limit equilibrium Coulomb wedge, as demonstrated in Figure 1. Both the *activating force* (self-weight of the wedge W) and the *resisting force* (the tension from the reinforcement T and friction along the failure plane) have components parallel and perpendicular to the rupture plane. Equilibrium of forces parallel to the rupture plane is considered in stability calculations. Orthogonal components of the self-weight and reinforcement tension cause frictional resistance along the rupture plane, opposing sliding parallel to the rupture plane.

1. $T_{//}$ The *parallel component* of the *nail/anchor tension force*
2. T_{soil} The *normal component* of the *nail tension* multiplied by tanφ'
3. $W_{//}$ The *parallel component* of the *weight of the wedge*
4. W_{soil} The *normal component* of the *weight of the wedge* multiplied by tanφ'

In addition to these components, a surcharge could be included on the surface of the wedge. A cohesive strength component could also exist, resisting sliding on the rupture plane. Both are omitted for the sake of simplicity.

By doing either a closed vector diagram or some basic trigonometric

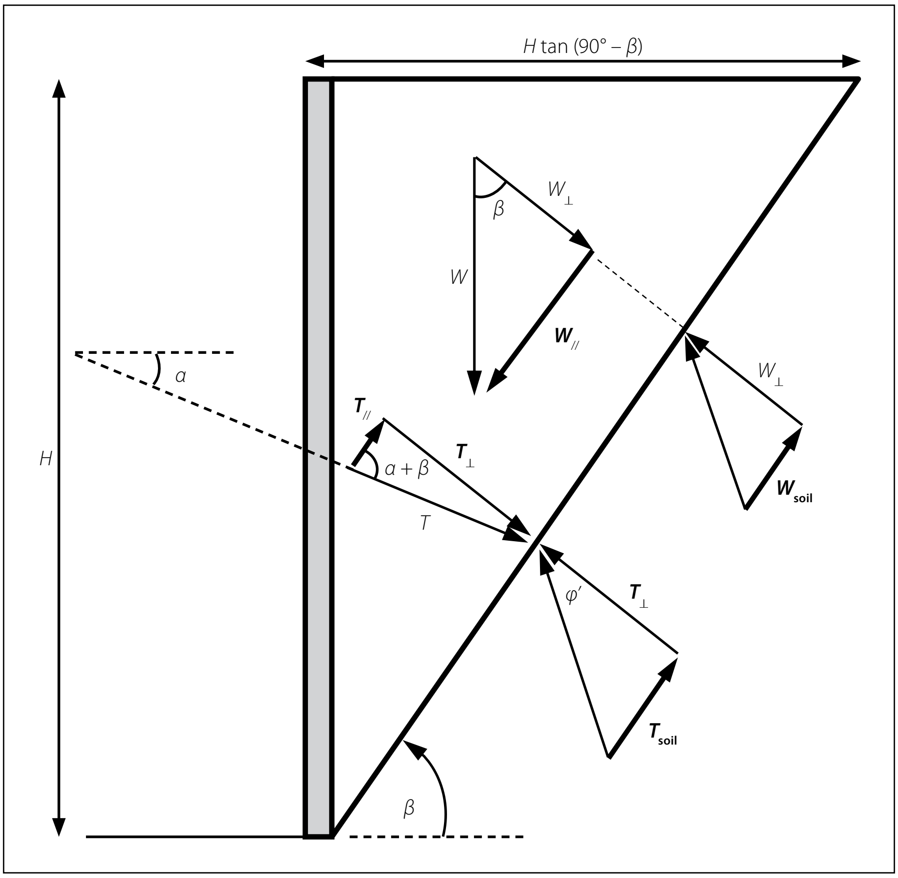

Figure 1 Forces considered on a trial wedge

expressions, the FoS can be rewritten in terms of the geometry of the trial failure wedge (H and β), soil shear strength characteristics (φ' and c') and the tension reinforcement provided (T).

$$\text{FoS} = \frac{T_{\text{provided}}[\cos(\beta + \alpha) + \sin(\beta + \alpha)\tan\varphi']}{W\sin\beta - \dfrac{cH}{\sin\beta} - W\cos\beta\tan\varphi'} \quad (2)$$

However, in the SAICE Code of Practice (1989), the FoS for anchors is specified considering the variability in soil shear strength characteristics as shown in Equation 3. In other words, the FoS is the number that the numerator has to be divided by in order to maintain exact equilibrium. Other codes around the world (such as FHWA 2003) use different definitions for the FoS.

$$\text{FoS} = \frac{\dfrac{cH}{\sin\beta} + W\cos\beta\tan\varphi' + \Sigma_{i=1}^{n}[T_i\sin(\beta + \alpha)\tan\varphi]}{W\sin\beta - \Sigma_{i=1}^{n}[T_i\cos(\beta + \alpha)]} \quad (3)$$

When using the strength reduction technique and finite elements, the software reduces the shear strength characteristics of the soil (c' and tanφ') to the point of failure, i.e. to an FoS of just below 1.0. This procedure is synonymous to the FoS defined in Equation (3), and only under this definition is the FoS from finite element and limit equilibrium programs comparable.

FAILURE MECHANISMS: FINITE ELEMENTS VERSUS LIMIT EQUILIBRIUM

One of the key differences in the FoS obtained with a "limit equilibrium wedge method", the "method of slices" and the "strength reduction technique" (finite elements) is the generation of the critical failure surface.

For each of the methods of analysis, the critical failure surface has to be found, i.e. the failure surface that yields the lowest FoS. The traditional trial wedge method typically assumes a straight line with the exit point specified at the toe of the wall. The method of slices generally assumes a circular failure surface. An advantage of the finite element method is the ability to determine a failure mechanism without making any *a priori* assumptions about its shape or position.

Early on in the history of the analysis of soil-nails, it was assumed that the inclusions would strengthen the reinforcement zone to such an extent that this zone could be analysed as a monolithic gravity retaining wall. Perhaps this led to the terminology of "internal" and "external stability" checks. In recent literature, owing to the better understating of individual soil-nail behaviour, viewing soil-nail lateral support systems as gravity retaining walls is no longer prevalent. It seems slightly unnecessary to separate internal and external stability checks, because there is a seamless transition between the two.

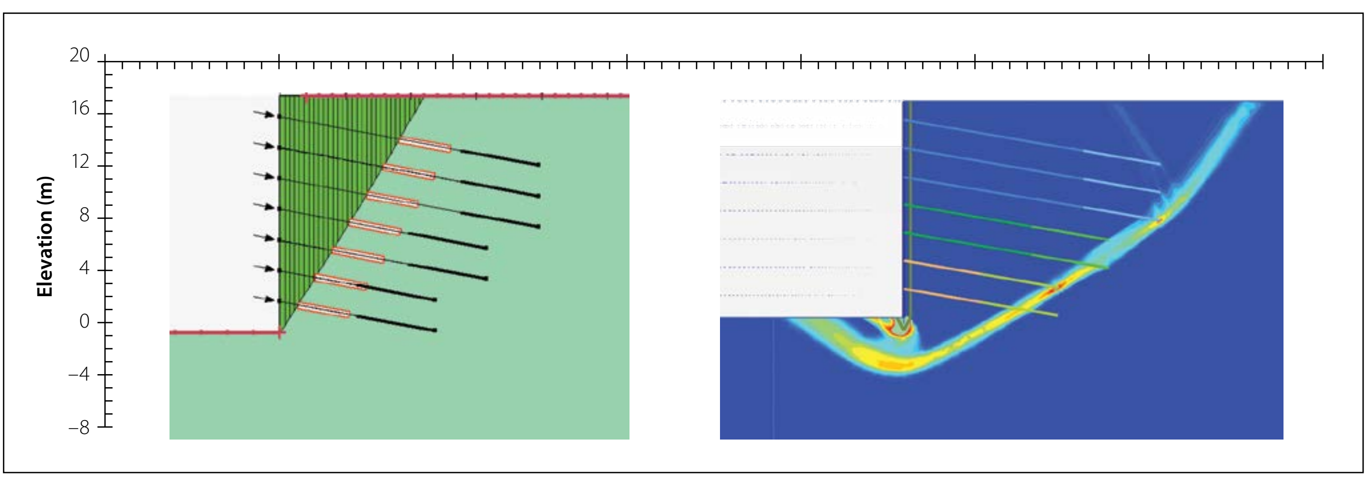

Figure 2 Internal failure found by method of slices (left), and global failure found by finite elements (right)

However, with soil-nails and anchors alike, it is useful to distinguish between a *global failure*, which extends behind soil-nails or anchors, and an *internal failure*, which occurs through soil-nails and anchors. Figure 2 shows an example of two models where internal and global failure can be seen.

If the critical failure plain, yielding the lowest FoS, extends around the outside of the anchors, this implies that the internal strength is of such a magnitude that the lowest FoS is found by avoiding the reinforcement elements altogether. In such a case, increasing the strength of the reinforcement elements will make no difference, but increasing the length will change the FoS.

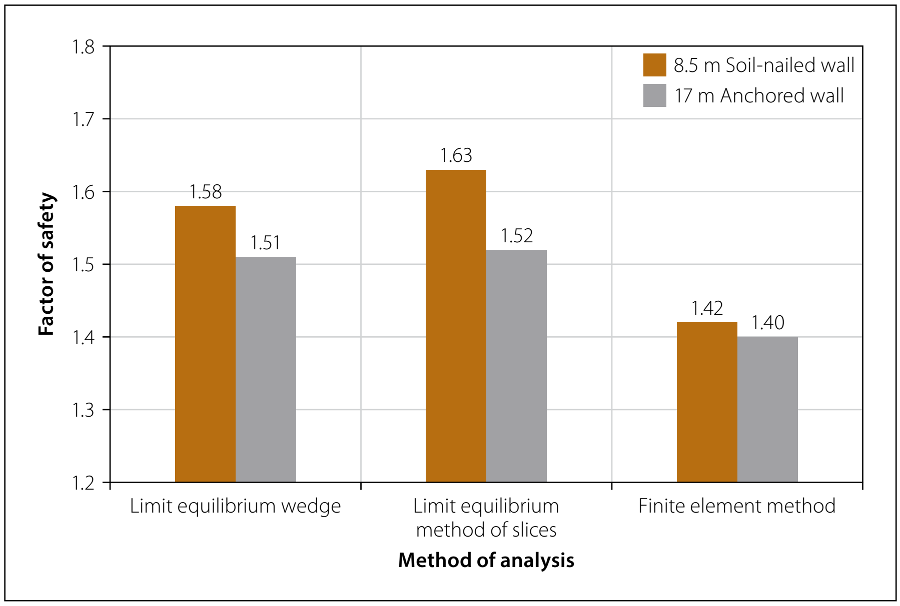

Figure 3 Factor of safety for soil-nails and anchors with different methods of analysis

Anchor strengths

Interesting to note is that the example in Figure 2 shows two methods (i.e. method of slices and finite elements) that were attempted to be modelled in the same way. However, the finite element method yields an increased FoS and the failure surface extends around the outside of the anchors. After careful inspection of the loads within the anchors at failure, it was found that the reinforcement elements behave differently to what was initially thought within the finite element models.

Within the limit equilibrium method of slices, the working load is specified as the tension force that exists within the anchor. This is in accordance with the SAICE Code of Practice (1989). The working load is factored by the allowable load on the anchor (80% of the ultimate strength) and the proof load testing (125–150% of the working load). Therefore, the modelled working load as experienced by the soil is between 53–64% of the ultimate anchor capacity, depending on whether the support is permanent or temporary.

In the finite element software, the working is modelled in the same way; however, the yield capacity is also specified. As deformation takes place by reducing the soil shear strength properties, the loads within the anchors increase, due to the loss of capacity within the soil. The anchor loads then increase until it reaches the yield capacity and because the elements are typically modelled as elastic plastic members; after the yield capacity is reached, infinite elongation occurs at the same yield capacity. Two problems arise. Firstly, for anchors, working loads in the limit equilibrium analysis are being compared to yield capacities in finite element software, which is incorrect. Secondly, the assumption is made that anchors behave sufficiently ductile so that the load can be spread to all anchors having reached their yield capacity – this is a recipe for disaster. According to the SAICE Code of Practice (1989), the working loads need to be modelled.

Most critical failure mechanisms

Leading on from the previous comment, the question arises: For anchors, what if the working load is used as the maximum tensile force in *both* the limit equilibrium and finite element models?

Figure 3 shows the FoS obtained from the different methods for both soil-nails and anchors if the working load is used. For both soil-nails and anchors the FoS

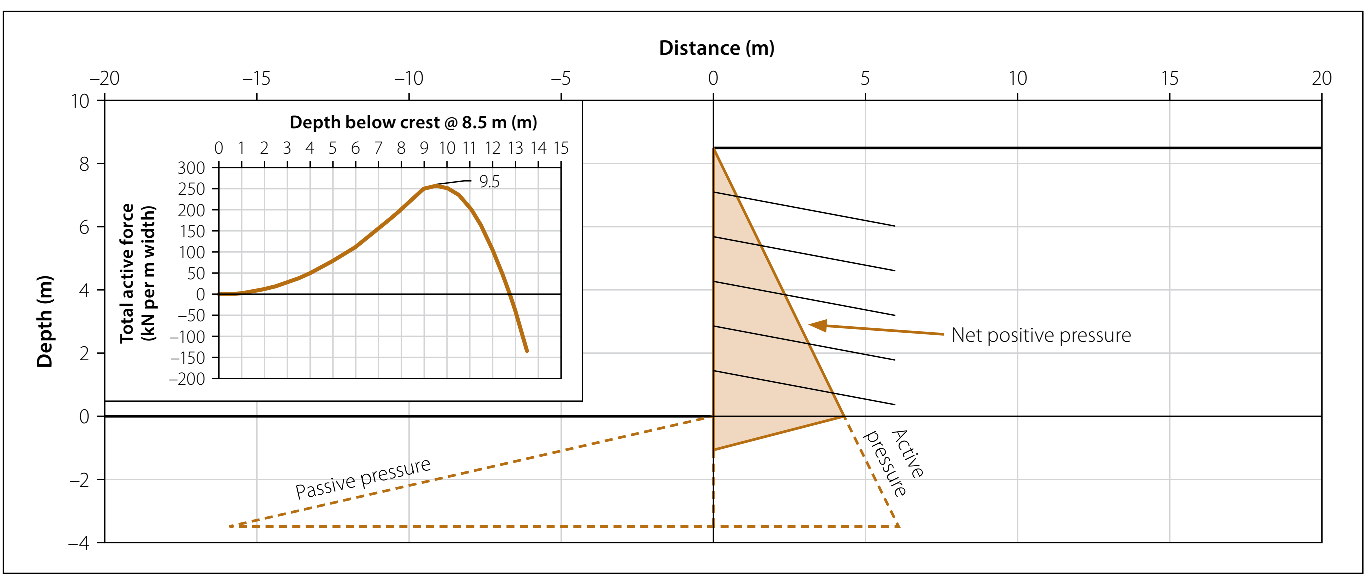

Figure 4 Active and passive pressures on a retaining wall cross section

derived from finite element analysis is somewhat lower.

The reason for this is that the finite element method has no assumptions regarding the slip surface, and therefore "naturally" finds the most optimal failure mechanism. All limit equilibrium methods have limitations around the shape, size and domain of the trial slip surface. When modelling a uniform soil for the entire excavation, the optimal slip surface extends below the toe of the support. To explain this, Figure 4 illustrates Rankine active and passive pressures, with the net positive pressure highlighted. For a depth increment immediately below the toe, the positive active pressure exceeds the resisting passive pressure, explaining the reduced FoS associated with mechanisms extending below the excavation toe.

Figure 5 shows the incremental shear strain distribution at failure for the soil-nailed excavation using the finite element method. At the exit side of the mechanism, a small passive wedge is seen extending approximately 1 m below the toe of the wall, which agrees well with the arguments made using Figure 4. On the entry side, the slip angle steepens behind the second soil-nail. Neither the planar wedge method nor the method of slices can capture this mechanism adequately. However, a limit equilibrium multiple wedge analysis, comprising a double wedge behind the excavation face with a passive resisting wedge at the toe, can model the same mechanism found by finite elements. Figure 6 shows a single wedge failure, a double wedge failure, and a compound failure mechanism with a passive wedge below the toe. A significant decrease in the FoS from 1.58 to 1.37 is observed for a failure extending beneath the toe of the excavation. When using a multiple wedge mechanism, the FoS calculated for a soil-nailed excavation agrees well with that calculated from the finite element strength reduction technique.

The same can be shown for anchors. Figure 7 shows good agreement between the FoS when a multiple wedge failure and the finite element method are compared. In this scenario, the anchor yield capacity was specified in both methods of analysis. Figure 8 demonstrates the same point – the working load was used in both methods and material below the toe was specified as rock, forcing the failure through the toe of the excavation. Close

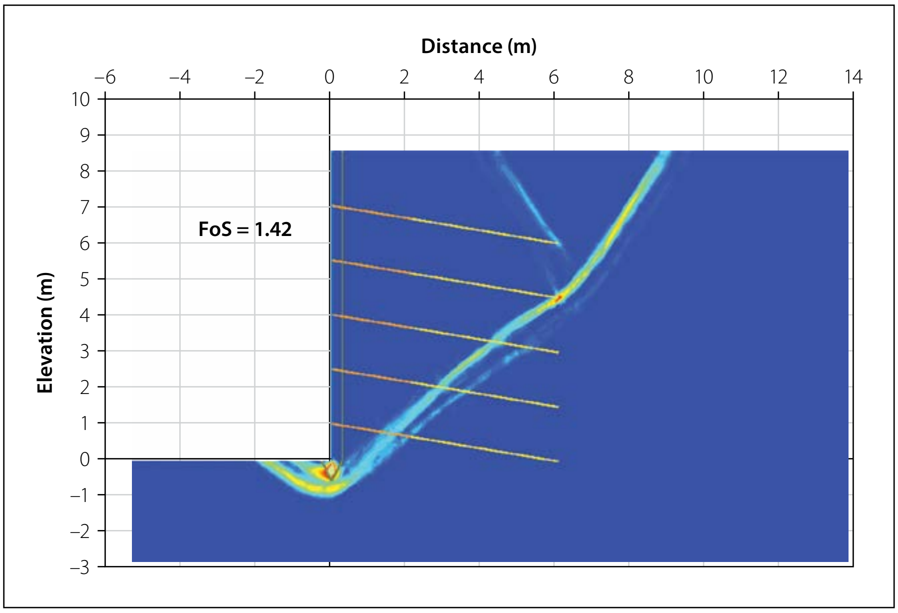

Figure 5 FE (SRF) method showing shadings of incremental shear strain at failure for soil-nailed excavation

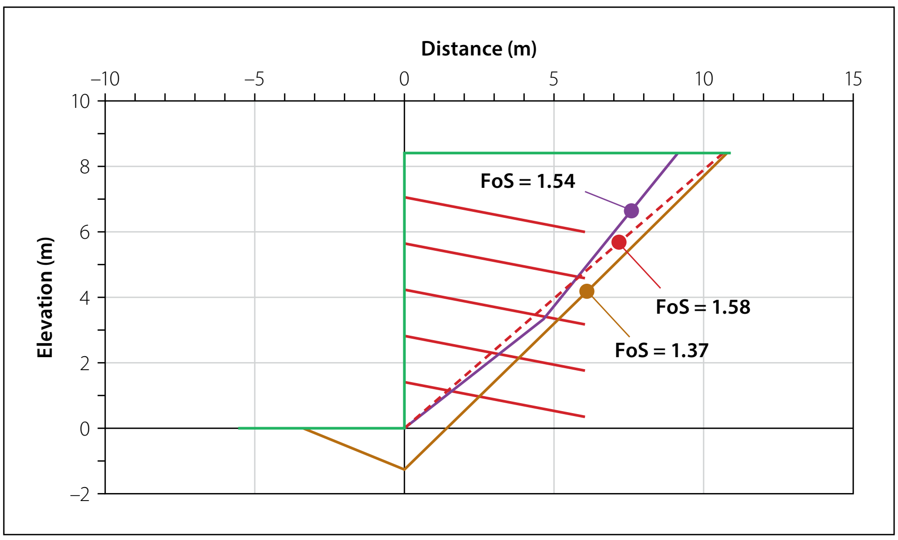

Figure 6 Various failure mechanisms and FoS including planar, double and passive wedges

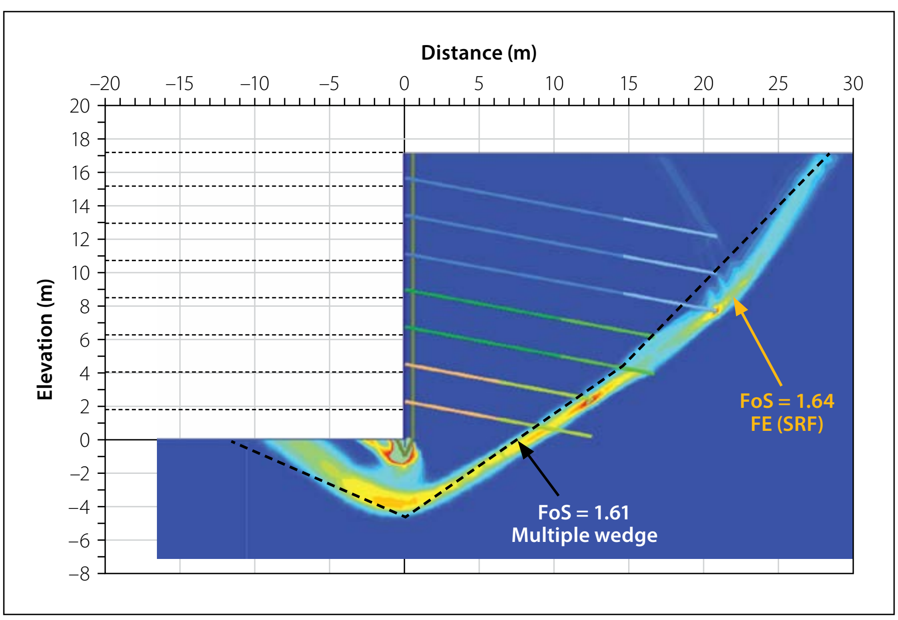

Figure 7 Failure mechanisms and FoS for finite element method and multiple wedge analysis using anchor yield capacity

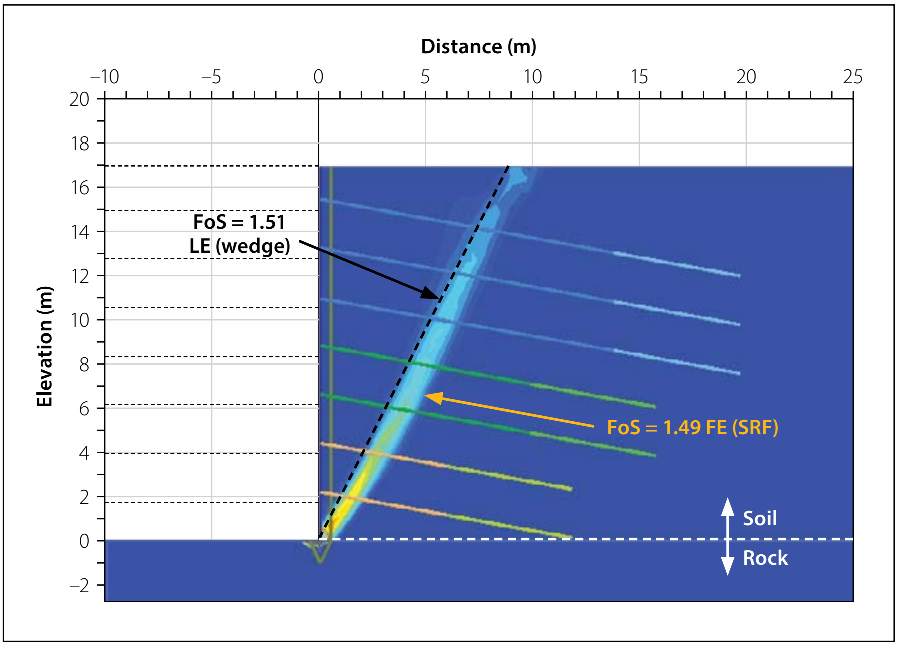

Figure 8 Failure mechanisms and FoS for finite element method and wedge method for rock below toe using anchor working loads

agreement is seen between the FoS from both methods.

WHEN USING FINITE ELEMENTS

Software structural features

All analysis software has built-in structural features. The purpose of these features is to model real-life elements such as shotcrete walls, soil-nails, anchors, piles, etc. However, many analysts will use these program features without a proper understanding of how the structural element behaves and its intended purpose. Seldom will the program have structural elements called "soil-nail" or "ground anchor" and even if the software does, the elements should not be used blindly without a comprehensive understanding of the underlying behaviour.

All analyses in geotechnical engineering are simplified models of what actually occurs. We have to accept that soils are non-linear complex in-situ materials that behave far from our theoretical models, and that soil-structure interaction is even more complex!

Perhaps an illustration would work the best here to demonstrate the point: A finite element program might require a user to input the hole diameter and the stiffness of the bonded portion of an anchor. An inexperienced designer might blindly enter the values from site, say 102 mm as the hole diameter and 30 GPa as the concrete stiffness. Another designer might argue that, because there are high strength steel cables combined with concrete, an equivalent stiffness needs to be used. Both designers would be incorrect. The purpose of these values is to determine the axial stiffness of the bonded portion of the anchor. However, this portion will be in tension, causing cracking of the concrete, and therefore the axial rigidity (area x stiffness) of the steel cables has to be used.

It is imperative to understand the role each input parameter plays in the modelling of soil–structure interaction. With the guidance of literature and past examples, analysts can successfully use structural elements to model real-life problems. However, it is important to understand the underlying effects of elements.

One of the critical assumptions made with regard to soil-nails is that, at normal angles of installation, soil-nails act in tension only. Software might incorporate shear strength into the soil-nail which will increase the FoS, but this is incorrect. This example illustrates the importance of a modeller understanding the established theory (i.e. soil-nails act in tension), so that modelling can be done accordingly.

Mohr Coulomb model

According to ICE (2012), an elastic-plastic soil model with the Mohr Coulomb yield criterion is still the most widely used soil model in geotechnical practice. Its simplicity makes it easy to understand. Although it falls short in terms of modelling movement and deformation, due to its linear elastic perfectly plastic nature, the strength criterion still proves to be acceptable. Limit equilibrium methods use the Mohr Coulomb strength criterion.

Finite element models – the consequence of perfectly plastic materials

Implicit within linear elastic perfectly plastic models is the assumption that infinite ductility exists. If both the soil and reinforcement elements are modelled using elastoplastic materials, at failure, the yield strength of both materials will be reached. The yield strength defines the onset of the perfect plastic behaviour.

Figure 9 represents hypothetical loads mobilised against the back of a retained excavation face as a function of face movement. The net driving force from the soil and the resisting force from the reinforcement elements (suppose soil-nails) are plotted on the vertical axis. The nett driving force from the soil is expressed in terms of the mobilised horizontal earth pressure coefficient.

Suppose the soil-nails were "wished into place" and that the initial in-situ horizontal soil stresses are defined by K_0. Initially (at point *a*), no force is mobilised in the nails, as no movement of the excavation face had occurred. As the excavation face is allowed to move, tensile load will mobilise within the nails and, at the same time, the horizontal earth pressure coefficient within the soil will begin to reduce from K_0. As the face is allowed to move further, an equilibrium point will be reached where the driving force from the soil and the restraining force from the nails will match (point *b*). The point of equilibrium, and hence the amount of movement to reach this state, depend on the stiffnesses of the soil and nails, as well as the initial in-situ stress.

The soil is not yet in a state of failure. Now, at point *b*, the strength reduction procedure reduces the soil shear strength properties (c' and tanφ') by a constant factor. As the soil "weakens", the force exerted on the wall increases. The reduction in strength of the soil is taken up by the capacity within the reinforcement elements in order to maintain equilibrium. Equilibrium will be maintained up to the point where the soil shear strength has been reduced by such an extent that the nails reach their capacity. At this point, both the soil and reinforcement have reached their plastic capacity, and beyond this point equilibrium can no longer be maintained and failure occurs (point *c*).

The magnitude of the SRF at failure depends only on the reduction in soil shear strength required to transition between the initial active state (SRF = 1.0 in Figure 9) and a state that will cause the nails to fail (SRF = 1.42 in Figure 9). Note that the magnitude of the SRF at failure does not depend on the location of the point of equilibrium (point *b*).

Therefore, when using the Mohr Coulomb soil model and elastoplastic reinforcement models, the FoS from finite elements is independent of:

- the initial stress ratio K_0
- the stiffness parameters E' and ν'
- the modelling of the construction sequence.

However, these variables are likely to alter the stress state at working conditions (point *b*), hence the amount of movement.

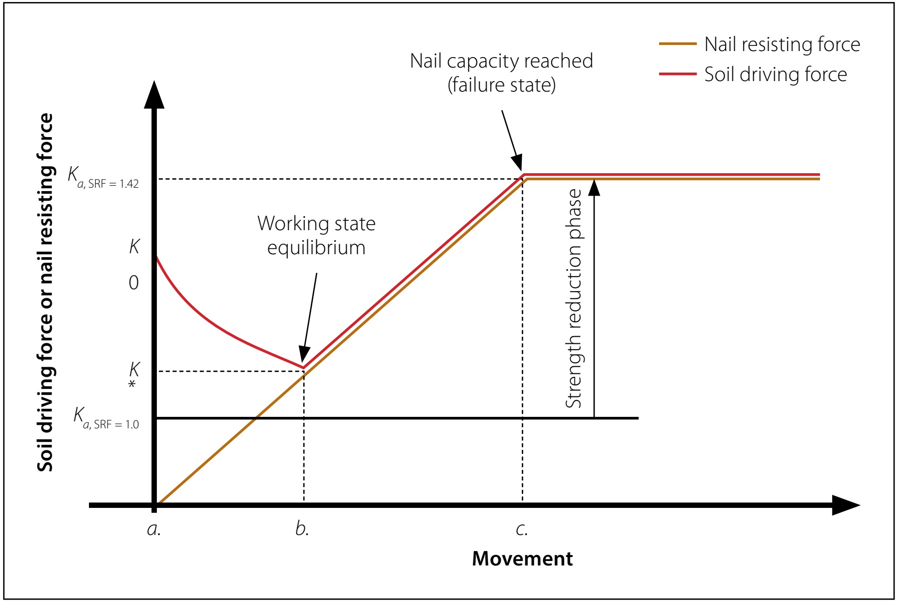

Figure 9 Hypothetical loading path of soil and nails from in-situ stress to failure

Choice of angle of dilation

When using finite element models, an angle of dilation has to be specified. The angle of dilation has a significant effect on the strength of soil when the problem is constrained. For example, in a bearing capacity problem, the angle of dilation has been found to significantly increase the bearing capacity of the soil. In contrast, in a slope stability problem, the angle of dilation does not have a significant impact, due to the low level of kinematic constraint of the failure.

A lateral support problem is closer to that of a slope stability problem, and the choice of the angle of dilation has a noticeable (but not major) impact on the FoS for lateral support problems. For the analysis of an 8.5 m soil-nail wall and a 17 m excavation, the FoS is increased by up to 0.08 by increasing the dilation from 0° to φ'. However, there is negligible change in the calculated FoS when changing the angle of dilation from 0.5φ' to φ'. It was found that low angles of dilation can cause numerical problems, especially when using soil-nails, because soil-nails rely partially on some sort of lateral expansion to mobilise the loads. Taking the angle of dilation as half the friction angle proves satisfactory.

In-situ stress definition

When carrying out finite element computations, the initial in-situ stress state requires specification. Several options exist depending on the problem under consideration and the software used. Commonly, a gravity turn-on procedure is used where the weight of the soil is "switched on". Vertical stresses are calculated from self-weight. For an elastic plane-strain problem with a horizontal soil surface, the horizontal stresses are a function of Poisson's ratio, with the coefficient of lateral earth pressure at-rest defined as:

$$K_0 = \frac{\nu}{1 - \nu} \tag{4}$$

Another common way to specify the in-situ horizontal stresses is by using Jaky's (1944) empirical solution:

$$K_0 = 1 - \sin\phi' \tag{5}$$

Some software by default uses Equation 4 to define the in-situ stress state, while others use Equation 5. Most software also allows the user to manually specify the in-situ stress state.

Using the Mohr Coulomb model, the yield criterion is a function of the soil shear strength parameters, c' and φ'. It is therefore possible to specify the in-situ stress state using Equation 4 so that K_0 is less than the active pressure coefficient (K_a). This violates the yield criterion. Poisson's ratio must therefore not be defined indiscriminately without considering this aspect.

Shotcrete wall end bearing

Extending the shotcrete wall to the toe of the excavation has an influence on the FoS for the finite element method, as some end bearing is mobilised below the wall. Stopping the installation of the shotcrete wall 0.5 m above the toe decreases the FoS without altering the failure mechanism. The FoS decreases by 0.06 to 0.08 for an 8.5 m soil-nail and 17 m anchor excavation respectively.

The difference in FoS shown in Figures 5 and 6 can be partially attributed to this artefact.

Guidance for routine finite element modelling

The current SAICE Code of Practice (1989) has served us well. The fact that it is still being used routinely within geotechnical practice in South Africa after almost 30 years means that the code is a robust document that was indeed well ahead of its time. However, there is most likely a consensus within practice that an updated new code of practice is in order. At the time of writing, there have been several discussions on producing a new code of practice.

Although fundamentalists in the geotechnical industry tend to shy away from the use of finite elements, we do not believe that finite elements will be used any less in practice. It would therefore be worthwhile to include a chapter on "Good practice in finite element modelling of soil-nails and anchors" in the new code. And at the core of this should be the statement that reads, "The finite element method is both a powerful and complex tool for the analysis of geotechnical problems. Due to the complex nature of the software, the FoS produced by finite elements should *always* be cross-referenced with a well-established limit equilibrium method."

CONCLUDING REMARKS

The provision, by geotechnical engineers, of lateral support for vertical and steep excavations will continue to be a major requirement in infrastructure development, especially in urban areas. It is the job of an engineer to provide adequate design recommendations using soil-nails, anchors or some other form of lateral support. In judging the adequacy of such a design, the FoS continues to be the governing quantity to satisfy technical personnel, clients and legal obligations. When analysing the FoS, engineers need to be aware of different definitions stipulated by codes of practice. In addition, the use of finite elements provides further depth and complexity to understanding the FoS.

When using a Mohr Coulomb failure criterion, some of the parameters exclusive to the finite element methods, such as Young's modulus, Poisson's ratio and in-situ stress, make little to no difference in calculating the FoS.

The first major difference between the finite element method and the limit equilibrium method lies in the former finding a more optimum failure surface. Should the same failure surface be used, the same approximate FoS is obtained. The second significant difference lies in the underlying behaviour of the structural elements used with finite elements.

The finite element can no doubt add value to analysing lateral support problems. Unique failure mechanisms and movements are major reasons for using a finite element package. However, due to the complexities involved in the failure mechanism, the underlying structural behaviour of elements, and material properties, it is recommended that a finite element analysis should always be counterchecked with a more simplistic limit equilibrium method.

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