University of Pretoria

Essays on Unconventional Monetary Policy

by

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A thesis submitted in fulfillment of the requirements for the degree of

Doctor of Philosophy (PhD)

in

Economics

Supervisor: Prof. Nicola Viegi

July 2019

Declaration of Authorship

I, Jacobus Cornelis Vermeulen, student number 27428959, declare that this thesis, titled 'Essays on Unconventional Monetary Policy' and the work presented in it are my own. I confirm that:

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"One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost."

Robert Lucas (1980:696)

Abstract

Following the Global Financial Crisis of 2007 – 2010, central banks around the world were forced into unprecedented policy interventions to stabilise asset markets and prevent the global financial system from collapsing. Because interest rates around the world were at historical lows, "conventional" interest rate policy was not an option. Central banks, led by the US Federal Reserve, resorted to "unconventional" monetary policies, first to stabilise markets during the height of the crisis, and then to support the economic recovery thereafter. The distinguishing characteristic of these unconventional policies was that they involved direct intervention by central banks in long-term fixed income markets, such as government bonds and agency debt.

This thesis considers the theoretical channels through which central bank purchases of long-term securities could impact (i) bond yields, (ii) other domestic asset markets, and (iii) spillovers to foreign countries. The theory is then tested and evaluated against the empirical evidence. Based on the empirical results, a simple closed-economy DSGE model is constructed. The model captures and illustrates the transmission from central bank asset purchase shocks to the aggregate economy. The asset purchase shock is subsequently converted to an endogenous balance sheet rule. Simulations show that combining this unconventional (balance sheet) rule with a conventional (short-term interest rate) rule yields a superior policy mix than under the conventional rule alone. Finally, the closed-economy model is extended to an open-economy framework, within which a similar balance sheet rule is evaluated in the context of international capital flows. Again, the combination of the balance sheet and interest rate policy is found to yield a superior outcome than interest rate policy alone.

The contribution of this thesis is twofold. It contributes to the understanding of the impact of central bank interventions in fixed income markets on long-term yields, as well as the externalities and spillovers to other asset markets. Furthermore, this thesis develops a robust and versatile framework, which is intuitively easy to grasp, within which various aspects of central bank balance sheet policy could be investigated.

This thesis' main conclusion is that *unconventional monetary policy could complement conventional policy* under normal market conditions, and that *unconventional policy need not be restricted to crisis times only.*

Acknowledgements

I am deeply grateful to Nicola Viegi for his immense knowledge and calm guidance and supervision. Nicola, your optimism, despite sometimes less-than-ideal conditions, was a major source of inspiration. I have learned so much from you and will always cherish our discussions, debates and coffees.

I am also grateful to Steve Koch and Sonja Laing for the invitation to spend time at the Department of Economics as a visiting researcher. This was an invaluable, and most enjoyable, part of the process.

There are authors and researchers whose blogs, lecture notes and public resources were immensely valuable but who are not necessarily explicitly cited below: Johannes Pfeifer, Michel Juillard, Eric Sims and Drago Bergholt. I specifically thank Matteo Falagiarda, whose work significantly influenced parts of this study, who generously shared his codes and insights. I also thank participants at the University of Surrey's 2018 DSGE Summer School, in particular Sandra Rhouma and Paul Levine, for valuable comments and suggestions.

I am grateful to my employer, Unisa, and specifically Johan Lötter, for granting me sabbatical leave and ample support otherwise to prioritise and complete this thesis. Johan, thank you for the positive environment you created and your unwavering support. I will always remember your passion and commitment towards "growing our own timber".

I thank my family, friends and colleagues for advice, encouragement, kind words and support.

My wife, Katinka, has been a pillar of strength throughout this journey. Thank you for your support and patience, and taking care of our young family while my time and attention was often focused elsewhere.

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Abbreviations

BoE	**B**ank **o**f **E**ngland
BoJ	**B**ank **o**f **J**apan
DJI	**D**ow **J**ones **I**ndustrial index
DSGE	**D**ynamic **S**tochastic **G**eneral **E**quilbrium
FDI	**F**oreign **D**irect **I**nvestment
FOMC	**F**ederal Reserve **O**pen-**M**arket **C**ommittee
ECB	**E**uropean **C**entral **B**ank
EME	**E**merging **M**arket **E**conomy(ies)
GFC	**G**lobal **F**inancial **C**risis
GSE	**G**overnment **S**ponsored **E**nterprise
IMF	**I**nternational **M**onetary **F**und
LSAPs	**L**arge **S**cale **A**sset **P**urchases
MEP	**M**aturity **E**xtension **P**rogramme
NCT	**N**et **C**urrent **T**ransfers
MBS	**M**ortgage **B**acked **S**ecurities
PBC	**P**ortfolio **B**alance **C**hannel
PBT	**P**ortfolio **B**alance **T**heory
QE	**Q**uantitative **E**asing
SOE	**S**mall **O**pen **E**conomy
TAF	**T**erm **A**uction **F**acility
TALF	**T**erm **A**sset-Backed Securities **L**oan **F**acility
TARP	**T**roubled **A**sset **R**elief **P**rogramme
UK	**U**nited **K**ingdom
US	**U**nited **S**tates
ZLB	**Z**ero **L**ower **B**ound

To Katinka, Ben and Carla, Mom and Dad

Chapter 1

Introduction

"Unconventional monetary policy" can be broadly defined as "the central bank [using] its balance sheet to affect asset prices and financial conditions beyond the short-term interest rate" (Borio and Disyatat, 2009:25). Following the Global Financial Crisis (GFC), central banks around the world were forced to stabilise asset markets and shortly thereafter stimulate their economies. However, because interest rates in much of the developed world were close to zero, there was very little scope for expansionary monetary policy by way of the traditional approach of lowering the short-term nominal interest rate. While some central banks had scope to cut policy rates when the crisis hit, aggressive rate cuts soon also left these banks without room for further manoeuvre. Such economies became known to be at, or close to, the zero lower bound (ZLB). Subsequently, central banks were forced to think outside the box, and started pursuing "unconventional" policies, predominantly large-scale asset purchases (LSAPs) and explicit forward guidance on the future path of short-term interest rates. Together these two measures colloqiually became known as "Quantitative Easing" (QE).

QE started out as focussed interventions in compromised asset markets, primarily real estate and financial markets in the US. Over time, however, QE became the norm for expansionary monetary policy aimed at boosting flagging inflation and broader economic activity. Its primary mechanism was through massive purchases of long-term government and agency debt instruments. The aim of such purchases was mainly to inject liquidity into financial markets (these purchases were financed by the creation of new reserves), while at the same time pushing down long-term interest rates by artificially boosting prices of long-term fixed income securities. By directly intervening on the long end of the yield curve – as opposed to the "conventional" approach of setting short rates – the central bank could therefore influence expectations, economic activity and inflation through alternative monetary policy transmission mechanisms.

The GFC originated in the United States (US) housing market, and spilled over to US financial markets and the banking sector, and ultimately to the rest of the world. The Fed's policy response to the crisis can be considered in two stages. First, the crisis had to be contained to minimise the spillovers and collateral damage. This involved stabilising US asset markets, specifically the housing market and sectors and institutions exposed to toxic mortgages and related financial instruments. This phase involved the Fed purchasing substantial levels of toxic instruments from financial institutions and transferring the risks to its own balance sheet, and included programmes such as the Troubled Asset Relief Programme (TARP), the Term Auction Facility (TAF) and Term Asset-Backed Securities Loan Facility (TALF), and other efforts to support general market liquidity. This allowed the credit crunch and spiral of falling asset prices to be contained. Then, once markets were stabilised, the broader economy needed to recover. The US economy was stuck at the ZLB, where interest rate policy alone was inadequate to boost economic activity. "Traditional" expansionary monetary policy by way of further lowering the policy rate was out of the question, and other measures were required. Many of the Fed's stabilisation programmes were subsequently expanded or continued, and were aimed at injecting liquidity into the banking sector and financial markets and keeping interest rates low in an attempt to support the economic recovery. These programmes are often lumped under the umbrella of "Quantitative Easing", and includes unconventional policy measures such as large-scale asset purchases (LSAPs) and explicit forward guidance on the short- to medium-term path of the policy interest rate.

As the Fed was combating domestic financial instability, cracks were already starting to appear in foreign and international financial and asset markets. Many economies were exposed to the US housing market and were exceedingly vulnerable to these toxic assets, as well as shifts in global asset prices and sentiment. The crisis consequently caused a significant international economic contraction. As interest rates in the United Kingdom (UK), mainland Europe and Japan were also at historic lows, central banks in these countries were therefore forced to also consider unconventional measures in stabilising financial markets and restoring economic activity. The Bank of England (BoE), European Central Bank (ECB) and the Bank of Japan (BoJ) therefore all adopted various unconventional monetary policies, ranging from outright purchases of long-term financial assets to supporting money markets by offering substantial and generous loans to the domestic banking sector.

Broadly, the impact of the Fed's unconventional monetary policy was twofold. First, it succeeded in stabilising US asset markets and boosting economic activity and inflation. Second, the massive liquidity boost inevitably spilled over to other economies. Such spillovers influenced asset markets and asset prices – and subsequently macroeconomic variables such as exchange rates, inflation and economic growth – in many foreign (both developed and emerging) economies.

This thesis is concerned with the domestic effects, as well as international spillovers, of unconventional monetary policies, and the role of central banks in this context. It can be divided into two broad sections: In Part I, the empirical evidence of the effect of such policies on various asset markets is investigated, and evaluated against the burgeoning theoretical literature. In Part II, two Dynamic Stochastic General Equilibrium (DSGE) models, both a closed-economy and open-economy model, are constructed so as to reflect the empirical evidence established in Part I. These models are then used to evaluate alternative monetary policy frameworks in the broader context of asset market volatility. The ultimate goal of the two models is to test whether "unconventional" monetary policies might be useful in "conventional" times. Specifically, the use of balance sheet policies, by way of central bank purchases or sales of long-term securities, are considered as a potential additional tool in the suite of monetary policy instruments.

Following this introduction and summary contained in chapter 1, chapter 2 provides a deep theoretical overview of unconventional monetary policy. It introduces the two main legs of unconventional policy, large-scale asset prices and explicit forward guidance. The theoretical literature is framed against the GFC of 2008 – 2010 and actions taken by central banks around the world to restore financial markets and economies following the worst economic crisis since the Great Depression. The majority of this chapter is dedicated to the impact of LSAPs, as practiced by the Fed and the BoE, in response to the crisis, and the theoretical considerations and motivation behind its undertaking.

One of the main motivations of the Fed's unconventional policy interventions was to provide support to asset markets such as housing, equities and bonds. Chapter 3 therefore investigates the impact of QE policies (concentrating on LSAPs) on asset markets from both a theoretical and empirical perspective. These theories can be applied to both domestic and foreign asset markets, and could also explain the international spillovers of the Fed's QE policies. Chapters 2 and 3 comprise Part I.

In Part II (chapters 4 and 5) the focus is shifted to a theoretical small open economy (SOE). An SOE is vulnerable to financial asset market volatility in the developed world, much like South Africa or other SOEs are vulnerable to developments in, for example, the US and UK housing and stock markets or European bond markets. It is argued that traditional interest rate policy is perhaps ill-suited to responding to international (exogenous) asset market developments, and explores alternative tools at the disposal of the monetary authority. To this end, a theoretical DSGE model in the New-Keynesian tradition is developed, which explores the use of the central bank's balance sheet (similar to LSAPs) as a potential alternative to traditional interest rate policy. Chapter 4 assumes a closed economy (focusing on domestic asset markets, the central bank and government) and constructs a DSGE model to simulate central bank asset market interventions. This simplification is an important first step in pinpointing alternative central bank rules which, when simulated, accurately mimics observed data and economic outcomes.

Chapter 5 utilises the theoretical framework developed in Chapter 4 and opens the economy to international capital flows and foreign trade. It tests the efficacy of balance sheet policies vis-a-vis traditional interest rate policy as a central bank response to exogenous capital flow and asset prices shocks.

Chapter 6 concludes.

Part I:
Theoretical analysis and empirical evidence

Chapter 2

Unconventional monetary policy: An overview

2.1 Introduction

The purpose of this chapter is to provide a thorough overview of unconventional monetary policy, contrasted against standard (conventional) contemporary policy making. It explores the nature of unconventional policies, which measures it might entail and how these measures can be implemented. It also discusses a number of theoretical motivations behind such policy measures and introduces the mechanics and various transmission mechanisms of unconventional policy.[1] Finally, it presents a timeline and detailed overview of the unconventional policy measures adopted by the Fed during and following the Global Financial Crisis, commonly referred to as the Fed's Quantitative Easing (QE).

Given the ambiguity often surrounding various interpretations of 'unconventional monetary policy' and the variety of terms colloquially used under this umbrella, it is necessary to first clearly define 'unconventional' policy and delineate it from 'conventional' monetary policy'. According to Borio and Disyatat (2009), the implementation of monetary policy at its most basic level consists of two core elements: (i) *signalling* of the desired policy stance, and (ii) *liquidity management operations* involving the use of the central bank's balance sheet to make this stance effective. Modern monetary policy making – before the crisis – broadly converged to the policy stance explicitly defined in terms of a short-term interest rate (the signal), with liquidity operations "designed exclusively to help make that interest rate effective" (Borio and Disyatat, 2009:2). Such liquidity operations were traditionally effected through the market for bank reserves. Given the

[1]These theoretical considerations are comprehensively treated in subsequent chapters.

central bank's monopoly over bank reserves, it can set both the "quantity and the terms on which it is supplied at the margin" (Borio and Disyatat, 2009:3), and therefore ensure the effectiveness of its policy stance. By engineering a shortage of bank reserves which forces banks to borrow – either from one another in the interbank market or from the central bank directly – and at the same time setting the policy rate which anchors the cost of borrowing, the central bank can to a large degree manage credit extension and money creation. *Conventional monetary policy* therefore refers to the central bank's control over the combination of short-term nominal interest rates and bank reserves. The central bank can effect changes in short-term interest rates by changing its policy or discount rate, thereby signalling its policy stance. Liquidity management operations, e.g. open-market operations, forex swaps, or other measures aimed at adjusting bank reserves, ensure that the policy rate becomes and remains the anchor for other short- and long-term interest rates. Consequently, through the traditional monetary policy transmission mechanisms (Mishkin, 2013), changes in the policy rate feed through to changes in the yield curve, which ultimately effect changes in aggregate economic activity, output and inflation.

The distinguishing feature of unconventional monetary policies is the fact that "the central bank actively uses its balance sheet to affect directly market prices and conditions beyond a short-term, typically overnight, interest rate" (Borio and Disyatat, 2009:1). While conventional monetary policy can be implemented "without calling for significant changes in the size of the central bank's balance sheet" (Borio and Disyatat, 2009:4), *unconventional monetary policy* on the other hand elevates "liquidity management operations from a passive to an active role" (Borio and Disyatat, 2009:5), where "the central bank uses its balance sheet to affect asset prices and financial conditions beyond the short-term interest rate" (Borio and Disyatat, 2009:25). Unconventional policy is therefore often referred to as 'balance sheet policy'. While this manner of unconventional monetary policy is nothing new – many central banks are, for example, known to intervene or have intervened in foreign exchange markets in an attempt to influence exchange rates over and above the impact of the policy rate – the recent crisis saw unconventional policies target a much wider range of interest rates and asset prices. Such policies included the unprecedented purchases of government and agency debt in order to provide liquidity to these markets while driving down yields and boosting asset prices, coupled with explicit forward guidance aimed at anchoring the future path of short-term interest rates. The two main pillars of unconventional policies employed by the Fed can therefore be categorised broadly as

1. *Forward guidance:* Communication about the likely future path of short-term interest rates in order to influence market expectations.

2. *Large-scale asset purchases:* Massive purchases of long-term (mostly) government debt or other financial assets, aimed at supporting various asset markets and stimulating the broader economy.

Conventional policies can therefore be neatly juxtaposed against unconventional policies in the sense that conventional policies are mainly concerned with short rates and bank reserves, while unconventional policies are concerned with long rates and asset prices.[2] While the ultimate goal of both categories of monetary policy is broadly similar, the channels and mechanisms at work to achieve this outcome are quite different.

Borio and Disyatat (2009) distinguish four broad categories of balance sheet policy based on (i) the impact on the private sector's balance sheet, and (ii) the market segment targeted. These categories are

1. *Exchange rate policy:* Targeted at affecting the exchange rate (levels and/or volatility) through operations in the foreign exchange market, for any given level of the policy rate.
2. *Quasi-debt management policy:* Targeted at altering the yields on government securities – and therefore asset prices and the cost of funding – by altering the composition of claims on the government (public) sector held by the private sector.
3. *Credit policy:* Targeted at altering financing conditions for the private sector. This can be achieved by targeting segments of the securities markets (including those that pertain to debt instruments) by altering the composition of private sector balance sheets.
4. *Bank reserves policy:* Setting a specific target for bank reserves (where any number of assets or asset classes on the central bank's balance sheet would be its counterpart).

Many of the unconventional policies adopted during the past decade is a combination of these. For example, quantitative easing, as practiced by the Fed, is a mixture of quasi-debt management policy (by way of purchasing long-term government and agency debt) and bank reserves policy (by financing these purchases by expanding bank reserves). Credit easing (Bernanke, 2009) is a mixture of credit policy (extending credit to private sector entities) and quasi-debt management policy (again by way of purchasing Treasuries and GSE debt).

[2] These two categories are not mutually exclusive. Conventional policies can certainly influence asset prices, just as bank reserves played an increasingly important role in recent unconventional policy programmes. This distinction is, however, useful in analysing the various channels through which these policies operate.

Finally, it is important to note that "a key feature of balance sheet policies is that they can be entirely decoupled from the level of interest rates" (Borio and Disyatat, 2009:1). By ensuring either that the market for bank reserves is fully insulated from such operations, or that any changes in bank reserves do not affect the reference market rate, "balance sheet policy can be implemented regardless of the prevailing interest rate level" (Borio and Disyatat, 2009:5). This is important for two reasons. First, it allows monetary stimulus by way of balance sheet policies even if short rates are stuck at the ZLB. Second, and on the other side of the spectrum, the speed and timing of the exit from such programmes can be navigated without the need to negotiate potential short-term interest rate disruptions.

For the remainder of this study we follow the classification favoured by Borio and Disyatat (2009:1) in referring to unconventional policies broadly as 'balance sheet policy', while referring to conventional policies as 'interest rate policy'.[3] Unconventional policy therefore includes measures such as large-scale asset purchases, targeted interventions in asset markets and explicit forward guidance. Conventional policy is concerned with setting and effecting the short-term interest rate.

2.2 Overview

Following sharp increases in credit spreads and subsequent tightening of credit conditions as a result of the global financial crisis, central banks around the world have had to apply aggressive monetary easing in an attempt to restore stability to financial markets. Part of these measures were conventional, in the form of significant cuts in short-term policy interest rates to levels close to zero. However, due to the magnitude and extreme consequences of the preceding crisis, this approach alone was inadequate, while interest rates in much of the developed world were already very low. As a result, "faced with the prospect of a deep economic downturn, and with short-term interest rates close to the zero lower bound, central banks judged that further monetary stimulus would be required to meet their objectives" (Bowdler and Radia, 2012:604). At the zero lower bound, however, "the central bank has no room to further reduce short-term interest rates" (Bowdler and Radia, 2012:606), and as a result central banks around the world have embarked upon a set of unconventional monetary policy measures. These policies are commonly referred to as 'Quantitative Easing' (QE). QE broadly involves large-scale purchases of financial assets, such as long-term government bonds or mortgage-backed securities (MBS), purchased mostly from the non-bank private sector. Such purchases

[3]This distinction is even more explicit in chapters 4 and 5, where monetary policy reaction functions are formally modelled as either an interest rate (Taylor) rule or a balance sheet rule, or some combination of the two.

are directly aimed at influencing "economic activity by altering the structure of private sector balance sheets" (Borio and Disyatat, 2009:8). These transactions are financed mainly by central bank money issuance. Since QE essentially expands "the central bank's balance sheet through asset purchases, financed by the creation of central bank money" (Joyce, McLaren, and Young, 2012:672), the mechanics of the process is best illustrated from the perspective of the balance sheets of the central bank, non-bank private sector and the banking sector (Table 2.1, adapted from Bowdler and Radia (2012)). In this example, the central bank purchases securities, e.g. long-term government bonds, from the non-bank private sector. The non-bank private sector's holdings of these assets falls, while the central bank's holdings increases. The transaction is financed through the central bank "issuing base money in the form of reserves held by commercial banks" (Bowdler and Radia, 2012:607). The banking sector's balance sheet therefore also expands by these newly created central bank reserves, which are matched against the increased deposits of the non-bank private sector.

TABLE 2.1: Balance sheet effects of QE

Non-bank private sector

Assets	Liabilities
– Securities	
+ Deposits	

Central bank

Assets	Liabilities
+ Securities	+ Reserves

Banking sector

Assets	Liabilities
+ Reserves	+ Deposits

Given the sheer volume of such transactions, QE policies "unusually increases the monetary base" (Fawley and Neely, 2013:52), and are designed to ultimately stimulate economic activity through massive injections of liquidity into the banking sector. A further objective of QE is that of "the central bank seeking to directly affect asset prices" (Bowdler and Radia, 2012:604), such as lowering the yields on longer term government bonds, in order to lower long-term borrowing costs and increase investment spending in other asset classes (e.g. equities). In times of financial distress, however, such as those necessitating unconventional policies such as QE, traditional monetary policy transmission channels might not be functioning properly. QE is theorised to circumvent these restrictions by working through seperate channels which might affect asset prices: *portfolio rebalancing* following the shock to the balance sheets of the non-bank private sector;

the *signalling* of future policy intentions by the central bank; and, the injection of *liquidity* into the financial system which should lower liquidity premiums. Higher asset prices would subsequently increase total wealth while at the same time lowering borrowing costs, which should translate into higher spending and investment.

Breedon, Chadha, and Waters (2012:704) liken QE to "just an extended open-market operation involving the unsterilized swap of central bank money for privately held assets". However, "the duration of the swap is intended to be both long term and of uncertain length" (Ibid.). A significant difference between QE and conventional monetary policy is therefore that, while conventional monetary policy seeks to affect short-term interest rates[4], QE aims to directly affect long-term interest rates. In the well-known liquidity premium theory of the term structure (Mishkin, 2013), long-term interest rates can be split into two components: (i) the expected average level of short-term interest rates expected over the asset's term to maturity, and (ii) the risk premium[5], the "additional return that investors demand for holding the risk associated with the longer term asset" (Gagnon, Raskin, Remache, and Sack, 2011a:42). According to Bhattarai and Neely (2016:4), the expected short rates are "a function of expected inflation, expected real activity and some judgement about the preferences of the central bank". The term premium is essentially the compensation demanded by investors for any additional risk they might incur over the lifetime of the security, is generally increasing over its maturity, and varies over time and across different bonds. Unconventional policy can influence both these components: forward guidance anchors expectations of future short rates, while asset purchases can reduce the risk premium (e.g. by the Fed providing liquidity in these markets or taking on some of the risk). It could also be as simple as the Fed's excess demand for such a security driving up its price and thereby reducing its interest rate.

It should be noted that not all central banks who apply QE do so in the same manner. Two distinct approaches can be discerned between the four large and notable QE-practicing central banks:

1. *Large-scale asset purchases (LSAPs)*: The Fed and BoE based their QE interventions on significant purchases of financial assets, such as government bonds, agency debt and MBS. Over the past ten years, the Fed purchased a combination of these securities totalling more than \$3 trillion (see Section 2.3 below). The BoE's asset purchases consisted mostly of UK government bonds (gilts) which were purchased in the secondary market, totalling around £375 billion (Joyce et al., 2012).

[4]In the conventional approach, the central bank sets the short-term policy rate which in turn influences longer-term interest rates through the yield curve.

[5]This is also referred to as the term or liquidity premium. Irrespective of the terminology employed, this premium represents compensation for uncertainty.

2. *Lending programmes*: In contrast to the approach followed by the Fed and BoE, the European Central Bank (ECB) and Bank of Japan (BoJ) focused on "supporting short-term money markets" (Martin and Milas, 2012:751) through providing substantial loans to their banking sectors.[6]

The essence of QE is that the central bank creates new money (in the form of reserves), which is used to purchase government bonds or other long-term financial assets in the secondary market, thereby "injecting broad money into the economy" (Bowdler and Radia, 2012:606). This is different from what is traditionally known as *printing money* or *monetizing debt*, where newly created central bank money is used to finance government spending or pay off existing government debt. Since the majority of these assets are purchased on the secondary market (and not, for example, new Treasuries issued by the state), QE should not be confused with the monetizing of government debt. Indeed, the Fed strongly states that "the Federal Reserve does not purchase new Treasury securities directly from the US Treasury, and Federal Reserve purchases of Treasury securities from the public are not a means of financing the federal deficit" (The Fed, 2015a).

2.2.1 Transmission channels of unconventional monetary policy

There is a burgeoning literature on the transmission channels of unconventional monetary policies in light of the recent crisis. Pioneering authors include Borio and Disyatat (2009), Krishnamurthy and Vissing-Jorgensen (2011a,b), Gagnon, Raskin, Remache, and Sack (2011a,b), Bowdler and Radia (2012) and Joyce, McLaren, and Young (2012). One of the earliest views on the transmission of balance sheet policy was that it alters "the composition of private sector balance sheets, exchanging claims that are imperfect substitutes for each other" (Borio and Disyatat, 2009:25). A critical difference between conventional and unconventional monetary policy is the mechanism through which long-term rates are affected. Under conventional policy, the central bank sets the short-term policy rate at its target level, and these adjustments filter through the yield curve to long-term rates. Under unconventional policy, however, with the short-term rate effectively stuck at the ZLB, the central bank cannot lower the policy rate further. Therefore, they attempt to influence long-term rates directly through the purchase of long-term (mostly debt) securities. In addition, through either implicit or explicit forward guidance, the central bank attempts to signal to the markets the future path of short-term rates. It is this difference between the two scenarios which raises the possibility that traditional monetary policy channels might not function in the same manner under unconventional

[6]The remainder of this study will concentrate on the Fed's LSAPs.

policy than it did under conventional policy. Indeed, it seems "quite reasonable to imagine that relationships across long-term interest rates and other asset prices may be quite different across the pre- and post-ZLB period" (Kiley, 2014:1058).

Monetary policy in general, and LSAPs specifically, can theoretically influence long-term interest rates through its effect on either, or a combination of, the expected path of future short-term rates or the term premium. The first broad distinction between channels through which LSAPs could influence asset prices is therefore between the *signaling* and *portfolio balance* channels (Bauer and Neely, 2014, Bauer and Rudebusch, 2014, Bhattarai and Neely, 2016, Gagnon et al., 2011b). The signaling channel plays an important role in expectations of short-term interest rate developments, while the portfolio balance channel is argued to work primarily through the term premium. While LSAPs are broadly "designed to affect the term-premium component of longer term interest rates" (D'Amico, English, Lopez-Salido, and Nelson, 2012:F416), this effect can be decomposed even further. Krishnamurthy and Vissing-Jorgensen (2011b:1) argue that "QE works through several channels that affect particular assets differently", breaking down the portfolio balance channel into five smaller channels, namely "duration risk, prepayment risk, default risk, degree of extreme safety, and liquidity (Ibid.). D'Amico et al. (2012) on the other hand condense a number of these channels, and focus their analysis on a scarcity (or local supply) channel and a duration channel (which together represent the portfolio balance channel) and a signaling channel. Bowdler and Radia (2012) and Joyce et al. (2012) distinguish between portfolio rebalancing, signaling and liquidity channels, while Weale and Wieladek (2016) suggest an expectations channel, through which LSAPs reduce economic uncertainty, in addition to the portfolio and signaling channels. Finally, Krishnamurthy and Vissing-Jorgensen (2013) distinguish between "narrow" and "broad" channels. A narrow channel would be one where spillovers to other assets are limited to assets which are extremely close substitutes in respect to the specific channel under consideration, whereas a broad channel generally sees spillover effects to weaker substitutes and thus to a larger range of other assets. The nature of the assets under consideration, as well as prevailing economic conditions, determine whether a channel is defined as narrow or broad.

2.2.1.1 Signaling channel

Changes in the expected path of short-term rates can be driven by "perceived new information that LSAP measures might relay about the state of the economy and the Federal Reserve's short-term interest-rate reaction function" (D'Amico et al., 2012:F424). According to Krishnamurthy and Vissing-Jorgensen (2013:59), the Fed's asset purchases "convey a signal that monetary policy is likely to be easier going forward, which reduces investors' expectations of the path of the federal funds rate and thereby has a broad impact on asset prices". Furthermore, following the drastic expansion of the Fed's balance sheet, the Fed is exposed to significant interest rate risk. If interest rates rise the value of the Fed's asset portfolio could plummet, which could raise fears of Fed insolvency (at least in an accounting sense), the burden which would be expected to be passed on to the taxpayer in some manner. The Fed therefore has a strong incentive to maintain low interest rates going forward.[7] Krishnamurthy and Vissing-Jorgensen (2011b) further find that QE announcements delayed an expected cycle of interest rate hikes by the Fed by just over a month on average. They interpret this as evidence of a signaling channel, where QE has the effect of keeping short rates lower for longer.

2.2.1.2 Portfolio balance channel

The second prominent channel in the literature is the influence of LSAPs on asset prices and yields through its effect on portfolio rebalancing. The portfolio balance channel has been described as follows:

"By purchasing a particular asset, the Fed reduces the amount of the security that the private sector holds, displacing some investors and reducing the holdings of others. In order for investors to be willing to make those adjustments, the expected return on the security has to fall. Put differently, the purchases bid up the price of the asset and hence lower its yield. These effects would be expected to spill over into other assets that are similar in nature, to the extent that investors are willing to substitute between the assets. These patterns describe what researchers often refer to as the portfolio balance channel." (Sack, 2009).

Based on Markowitz (1952) and Sharpe's (1964) seminal theories of portfolio selection and asset pricing, rational investors would adjust their portfolios in response to changes in risk and returns in a certain asset or asset class. Their views were subsequently expanded by Tobin (1969:26), who argues that "when the supply of any asset is increased, the structure of rates of return, on this and other assets, must change in a way that induces the public to hold the new supply". Extending this argument, a change in the supply of one asset would affect both the yield on that specific asset, as well as the spread between returns on that asset and alternative assets (Andrés, López-Salido, and Nelson, 2004). This view has come to be known as the "portfolio balance theory" (PBT). In the context of QE, the PBT suggests that LSAPs, by removing e.g. longer-term government bonds from the secondary market, reduces the supply of these bonds.

[7] This also has the effect of making the Fed's forward guidance statements more credible by establishing that the Fed has "skin in the game". It is therefore in the Fed's best interests to maintain a low interest rate in order to protect the value of its balance sheet.

As a result, the private sector "is left holding money in the form of bank deposits rather than gilts"[8] (Bowdler and Radia, 2012:609). Since long-term government bonds are generally higher-yielding instruments than money, money cannot be viewed as a close substitute for bonds. Therefore, "changes in relative holdings of the two will induce portfolio rebalancing and movements in asset prices (Ibid.). The PBT in this context is neatly summarised by Gagnon et al. (2011a:43), who note that "investors view different assets as substitutes and, in response to changes in the relative rates of return, will attempt to buy more of the assets with higher relative returns". Developments in one asset market are therefore sure to influence demand and subsequently prices in other asset markets through a substitution effect.

Based on the PBT, QE purchases change investors' portfolios by essentially exchanging long-term bonds with (short-term) money holdings. Investors now have to rebalance their portfolios by investing these increased money holdings elsewhere. To the extent that they could regard other assets as closer substitutes for bonds than money, they would subsequently "reduce their increased money holdings resulting from QE purchases and buy those other assets" (Joyce et al., 2012:694), which would put upward pressure on the prices of those assets. Investors might even be willing to "acquire slightly more risky assets that are now relatively cheaper in comparison to domestic government bonds" (Bowdler and Radia, 2012:610), such as investment-grade corporate bonds or blue-chip equities. While the impact of QE on bond markets is relatively easy to predict, its impact on other asset prices are less obvious. Changes in the yields and prices of government bonds resulting from QE would "affect the rate at which investors discount future cash flows" (Joyce et al., 2012:694), but it is not immediately clear how this will influence other asset markets. While "QE should eventually push up riskier asset prices, the impact at the time announcements are made could be ambiguous" (Joyce et al., 2012:693). However, according to Martin and Milas (2012:757), "the impact of QE programmes on government bond rates is only part of a chain of causation that connects government bond rates, returns on other assets, aggregate demand, and then output and inflation".

2.2.1.3 Other channels

A number of other channels are also proposed in the literature. While they can be interpreted as special instances of the portfolio balance channel, as they work mostly through the term premium (that is, they are not suggested to change the expected path of short-term interest rates), and a number of them are applicable only to very specific

[8]Colloquially government bonds are known as 'Treasuries' in the US, and 'gilts' in the UK.

assets, they provide valuable insight breaking down substitution and price effects of LSAPs.

1. *Scarcity*[9] *channel:* This channel assumes that investors have a preference for securities of a particular maturity. Such a 'preferred habitat' arises because of investors' specific preferences of mainly risk and liquidity characteristics. According to D'Amico et al. (2012:F425), "in segmented-market models featuring imperfect asset substitution, a reduction in the stock of securities of a particular maturity in the hands of private investors creates a shortage of those assets that cannot be wholly relieved, at existing asset prices, by substitution into other securities." LSAPs has the effect of increasing the scarcity of longer term Treasuries, creating a state of excess demand for these instruments. Subsequently "the market for long-term securities clears at a lower equilibrium quantity and a higher price" (D'Amico et al., 2012:F425), generating downward pressure on long-term Treasury yields. Furthermore, Treasury bonds are viewed as "high quality collateral and a long-term extremely safe (in nominal terms) store of value" (Krishnamurthy and Vissing-Jorgensen, 2013:59). Investors have a special demand for such safe assets; therefore, if the supply of long-term safe assets is reduced by Fed purchases (i.e. such assets become scarcer), the safety premium on those long-term safe bonds remaining in the market will increase. These bonds are now valued more highly, increasing its price through the safety premium and thereby lowering its yield. The closest substitute to Treasuries in this context could perhaps be high-quality (Aaa-rated) corporate bonds (Krishnamurthy and Vissing-Jorgensen, 2013). Since virtually no securities are regarded to be as safe as Treasuries, and the private sector has limited ability (relative to the government) to produce such long-term safe assets, this is an extremely narrow channel and the effect on other asset classes can be expected to be limited. The "price of the scarce asset will be inflated relative to other benchmarks, or equivalently, its yield will be lower than benchmarks" (Krishnamurthy and Vissing-Jorgensen, 2013:79), in line with the flight-to-safety effects often associated with Treasuries.

2. *Liquidity channel:* A liquidity premium is reflected in the "expected ease of trading in the given bond" (Bhattarai and Neely, 2016:4). If the central bank is viewed as a consistent buyer of assets (almost a market-maker), other investors could be encouraged to participate in the market, which could lead to higher prices. Because of severe liquidity constraints this channel was likely more significant during the early stages of the crisis. On the other hand, QE purchases of long-term securities are generally financed by reserve issuance, which has the effect

[9] The scarcity channel is also referred to as the safety premium or preferred-habitat channel.

of increasing the liquidity in the hands of investors. "Treasury bonds carry a liquidity price premium" (Krishnamurthy and Vissing-Jorgensen, 2011b:6), which should fall in response to higher overall liquidity. Furthermore, the Fed's purchases diminish the private sector's holdings of the relevant securities, which could lead to "a thinner market in the future with fewer opportunities to sell the security to other private market participants" (Hancock and Passmore, 2015:858). The liquidity channel therefore paradoxically also suggests that prices of Treasury bonds will *fall* and yields will *rise*. Which of the two effects dominate would be largely determined by current liquidity conditions in the markets. For example, when market liquidity is constrained, the former effect could be expected to dominate as investors are desperate for liquidity and therefore places a high premium on Treasuries. On the other hand, in an environment of improved liquidity, there are presumably more close substitutes to Treasury bonds available to investors. The latter effect could then result in the desirability of Treasuries, as derived from their relative liquidity, to fall.

3. *Risk pricing and duration risk channel:* According to Krishnamurthy and Vissing-Jorgensen (2011b:5), LSAPs "can reduce the duration risk in the hands of investors and thereby alter the yield curve, particularly reducing long-maturity bond yields relative to short-maturity yields". The duration risk premium is generated by the assumption that there is a subset of investors who have a preferred habitat (similar to the scarcity premium channel above), while a second subset of investors are arbitrageurs who trade across maturities (Vayanos and Vila, 2009). The latter group therefore becomes the "marginal investors for pricing duration risk" (Krishnamurthy and Vissing-Jorgensen, 2011b:5). This is echoed by D'Amico et al. (2012:F416), who suggest that LSAPs have the effect of removing "aggregate duration from the outstanding stock of Treasury debt", thereby reducing term premiums on securities across all maturities. With less aggregate duration risk to hold, investors "should require a lower premium to hold that risk" (Gagnon et al., 2011b:7). Consequently, "when the Fed removes duration from the portfolios of investors, the investors substitute by purchasing other long-duration assets to make up for the lost duration" (Krishnamurthy and Vissing-Jorgensen, 2011b:37). In contrast to the suggestion that premiums across all maturities should fall, Krishnamurthy and Vissing-Jorgensen (2013) find that this is a quite narrow channel, in that duration risk appears to only apply to the specific assets purchased. Finally, QE could also contribute to the mispricing of risk. Ellis (2015:7) argues that the purchase of safe assets by the central bank could cause investors searching for yield elsewhere, which could lead to "under-pricing of risk in financial markets more generally".

4. *Prepayment risk premium channel:* Prepayment risk is the risk that mortgages are paid off sooner than the original maturity of the contract, which would be undesirable from an investor's perspective as it creates the possibility of an uneven future cash flow in any mortgage-backed security (MBS).[10] The duration of MBS are expected to fall as (long-term) interest rates decline, as the incentive to prepay inreases (Gagnon et al., 2011b). Because this channel is very specific to MBS it is often included under the safety channel, with higher prepayment risk representing a lower safety premium (see e.g. Krishnamurthy and Vissing-Jorgensen (2011b)). Similar to the safety or scarcity premium, if the Fed purchases MBS as part of its LSAP operations, it reduces prepayment risk in the hands of the private sector. When this risk is removed from the market, the premium required to hold on to the remaining risk increases. Subsequently MBS prices rise and yields fall.

5. *Default risk channel:* In the standard asset pricing model, "for bonds that have very low default, the bond price rises as a function of the safety[11] of the bond" (Krishnamurthy and Vissing-Jorgensen, 2011b:7). As the economy recovers, corporate default risk will fall, implying a lower default risk premium (Krishnamurthy and Vissing-Jorgensen, 2011b). To the extent that LSAPs raise expectations of an economic recovery, risk premiums on corporate bonds, especially those on lower grade corporate bonds, will fall, implying higher corporate and private bond prices and lower yields.

6. *Inflation risk channel:* LSAPs are generally expected to have a stimulatory impact on economic activity and therefore on inflation, especially as the economy recovers from the crisis and is expected to grow stronger in the near future (Krishnamurthy and Vissing-Jorgensen, 2011b). Higher inflation erodes the gains from discounted nominal payments, such as bond coupon payments. Therefore, if expected inflation increases, "the expected return on bonds relative to real assets falls" (Mishkin, 2013:142). This could lead to lower demand for bonds exposed to inflation risk, and a fall in the value of these assets. On the other hand, assets which are not subject to inflation risk would see their desirability increase, leading to an increase in their demand and subsequently their price. Higher expected inflation could also raise expectations of an increase in nominal rates to prevent inflation from exceeding its target level, which creates an interesting tension between the goal of containing inflation and the Fed's incentive to keep short rates low in order to protect the value of its asset portfolio. It would therefore appear that higher

[10] Krishnamurthy and Vissing-Jorgensen (2011b) note that the duration on the 30-year MBS is only around 7 years.

[11] While this terminology also suggests a type of "safety" premium, it is distinct from the scarcity/safety premium discussed above. The scarcity premium is derived from investors' preferences for assets of a specific maturity, while the safety premium is derived exclusively from the risk of default attached to a given asset.

inflation expectations could exert two contradictory effects on yields for certain asset classes (e.g. Treasuries and corporate bonds). Through lowering expected returns of these assets it lowers its prices and increases its yields. On the other hand, the mechanical reduction of long-term real interest rates might lead to an increase in nominal or coupon rates to maintain the same level of real returns.

7. *Discount rates:* Government bond yields are often viewed as the benchmark rate, or discount rate, for other assets. Changes in Treasury yields therefore "may also affect the rate at which investors discount future cash flows" (Joyce et al., 2012:693). A fall in these yields imply a fall in the discount rate. This would lead to a higher present value of expected future cash flows and profits, increasing current asset prices. Since LSAPs are generally observed to have lowered the yields on longer-term government bonds, LSAPs are therefore associated with a fall in the discount rate and a mechanical increase in other asset prices.

8. *Capital constraints channel:* This channel operates where risk premiums (i.e. expected returns) are high, assets are complex (e.g. MBS), and capital constraints and segmentation are high (Krishnamurthy and Vissing-Jorgensen, 2013).[12] LSAPs will lower these risk premiums, and would therefore have its largest effect in markets where risk premiums are highest. It is viewed as a narrow channel because of the degree of segmentation and specialisation required to trade in these complex assets, so its spillovers to other asset markets are expected to be limited (Krishnamurthy and Vissing-Jorgensen, 2013). LSAPs could, however, have a broader effect by "shoring up the balance sheets of financial intermediaries" (Bhattarai and Neely, 2016:17), thereby relaxing capital constraints. According to Krishnamurthy and Vissing-Jorgensen (2013:107), the capital constraints channel can influence substitute assets "since today's asset price rises, investor balance sheets are strengthened today, which relaxes capital constraints". As a result, "assets concentrated in the portfolios of [these] specialized investors will also rise in price" (2013:67).

In conclusion, Gagnon et al. (2011b:8) argue that "lower prospective returns on agency debt, agency MBS, and Treasury securities should cause investors to seek to shift some of their portfolios into other assets such as corporate bonds and equities and thus should bid up their prices." However, while "QE should eventually push up riskier asset prices, the impact at the time announcements are made could be ambiguous" (Joyce et al.,

[12]These criteria also imply that this channel does not apply to Treasuries, as the Treasury market does not reflect a risk premium over other asset classes, is not illiquid, nor has high barriers to entry.

2012:693). While the aggregate impact of LSAPs on government bond markets is relatively easy to predict, its impact on other asset prices are therefore less obvious, and, for some channels, even ambiguous.

2.3 Adoption and implementation of US QE: 2008–2014

Because "bond markets play a relatively more important role than banks in the US and UK economies" (Fawley and Neely, 2013:56), the Fed and BoE's QE policies were concentrated on large-scale asset purchases (LSAPs), rather than direct lending to banks as was the approach of the ECB and BoJ. In the US, this involved the Fed purchasing substantial quantities of long-term debt securities such as Treasuries (government bonds) and mortgage-backed securities. These purchases "reduced the available supply of securities in the market, leading to an increase in the prices of those securities and a reduction in their yields" (The Fed, 2015b). These operations were initially aimed at supporting mortgage lending and housing markets[13], but was subsequently extended to promoting the economic recovery in general through the reduction in yields of a wide range of longer-term securities. These interventions were broadly aimed at boosting private sector borrowing and spending and were ultimately "designed to ease financial conditions and to support a sustained economic recovery" (Gagnon et al., 2011a:41).

2.3.1 Timeline of the Fed's QE announcements

- November 25, 2008: The Fed announces plans to purchase $100 billion in government-sponsored enterprise (GSE) debt[14] (also known as agency debt) and $500 billion in mortgage-backed securities (MBS) issued by those GSEs.
- March 18, 2009: The Fed announces additional purchases of $100 billion in GSE debt, $750 billion in MBS, and $300 billion in long-term Treasury securities.
- August 10, 2010: In response to a "worrisome disinflationary trend" (Fawley and Neely, 2013:72), the Fed signals its intentions of continuing, or even expanding, QE by announcing that it would maintain the size of its balance sheet by reinvesting principal payments from agency debt and MBS into longer-term Treasury securities. These purchases will be concentrated "in the 2- to 10-year sector of the nominal Treasury curve" (Federal Reserve Bank of New York, 2010).

[13] By purchasing significant amounts of debt in these sectors the Fed transferred risk unto themselves and away from households and other financial institutions.

[14] Including Fannie Mae, Freddie Mac, Ginnie Mae, and the Federal Home Loan Bank.

- November 3, 2010: The Fed announces that it will purchase an additional \$600 billion in longer-term Treasury securities in order to "promote a stronger pace of economic recovery and to help ensure that inflation, over time, is at levels consistent with its mandate" (11/03/2010 FOMC statement, Table D.1).

- September 21, 2011: The Fed announces the Maturity Extension Programme (MEP), which involves an additional \$400 billion purchase of long-term Treasuries (with maturities of between 6 and 30 years), while at the same time selling the equivalent in short-term Treasuries (with maturities of less than 3 years).

- June 20, 2012: The Fed announces the extension of the MEP, which would see the Fed continue to purchase long-term Treasuries and selling short-term Treasuries totalling \$267 billion.

- September 13, 2012: The Fed announces that it will purchase additional agency MBS at the rate of \$40 billion per month as long as "the outlook for the labor market does not improve substantially ...in a context of price stability" (9/13/2012 FOMC statement, Table D.1).

- December 12, 2012: The Fed announces that it will continue purchasing long-term Treasuries under the MEP of \$45 billion per month; however, these purchases would no longer be sterilised through the sale of short-term Treasuries.

- December 18, 2013: The Fed announces, in light of stronger economic performance, that it will slow down the pace of its purchases in agency MBS to \$35 billion per month and longer-term Treasury securities to \$40 billion per month.

- October 29, 2014: The Fed announces the conclusion of its asset purchase program, thus signalling the exit from QE. The Fed also indicates that it will maintain its portfolio at its current size, which should help support the current accommodating financial conditions.

The Fed's November 2008 and March 2009 asset purchase programmes are commonly referred to as 'QE1'. According to the FOMC, the goal of QE1 was to "reduce the cost and increase the availability of credit for the purchase of houses, which in turn should support housing markets and foster improved conditions in financial markets more generally" (11/25/2008 FOMC statement, Table D.1). However, in spite of the prevailing turmoil in financial markets at the time, these announcements came as somewhat of a surprise, with the effect that "10-year constant maturity Treasury yields fell by a cumulative total of 94 basis points" (Neely, 2012:9) over this time. These LSAPs, however, thus succeeded in lowering real long-term interest rates through its effect on term premia (Gagnon et al., 2011a) and substantially increased bank reserves (Fawley and Neely, 2013). According to Gagnon et al. (2011a:44), the Fed's purchases between December 2008 and March 2010, totalling around \$1.7 trillion, represented about 22 percent of the market at the time. These purchases were deliberately large relative to their markets in order to have a noticeable impact on their yields.

While financial market disorder had mostly receded by the second half of 2010, "real activity remained sluggish" (Fawley and Neely, 2013:72), which, coupled with the worrying disinflationary trend, warranted further intervention by the Fed. 'QE2', the November 2010 announcement, was widely expected by the markets following the cautionary August 10 announcement, as well as subsequent comments by then-Chairman Ben Bernanke and the FOMC regarding the subdued levels of inflation. This meant that "asset prices had already adjusted to these expectations and did not change much when the announcement finally came" (Fawley and Neely, 2013:73).

The Fed's September 2011 announcement was aimed at reducing long-term interest rates relative to short-term interest rates, and was subsequently nicknamed 'Operation Twist' after the similar programme of the 1960s. Since the long-term asset purchases were entirely funded by sales of short-term assets, the operation did not expand the monetary base or impact the size[15] of the Fed's balance sheet.

'QE3', introduced in September 2012, differed from previous asset purchases in that the Fed committed to a steady pace of purchases for as long as deemed necessary, instead of a predetermined, lump sum, total quantity. These purchases, totalling \$85 billion per month, continued through 2013. In September 2013, the Fed indicated that it might consider adjusting the pace of these purchases in the near future in light of stronger economic performance. This stance was reiterated in October 2013.

In December 2013, the Fed started slowing down the pace of their monthly asset purchases and kept reducing it throughout 2014. QE was finally halted in October 2014 as the Fed deemed financial markets sufficiently recovered, citing "sufficient underlying strength in the broader economy" 10/29/2014 FOMC statement, Table D.1). This brought unconventional monetary policy in the form of LSAPs to a close.

Finally, two categories of QE interventions can be discerned. If the FOMC announces a new asset purchase programme, or expansion of an existing programme, it could be viewed as an expansionary or accommodative policy stance. Conversely, if the FOMC announces termination of, or reduction in, an asset purchase programme, it could be viewed as a contractionary or tightening policy stance. Applying this broad grouping to the FOMC statements yields Table 2.2. Eight out of ten announcements reflect

[15] It did, of course, affect the maturity mix.

an accommodative monetary policy stance, while the final two could be classified as contractionary.

Table 2.2: Monetary policy stance throughout QE announcements

Accommodative	**Contractionary**
11/25/2008	
3/18/2009	
8/10/2010	
3/11/2010	
9/21/2011	
6/20/2012	
9/13/2012	
12/12/2012	
	12/18/2013
	10/29/2014

This is hardly surprising, even stating the obvious, since the whole point of QE and LSAPs was to support the economic recovery following the crisis, and as such the majority of policy interventions should be expansionary. However, this formal distinction is useful in subsequent analyses of the reaction of bond market yields to various QE announcements and programmes, and will be considered in greater detail in chapter 3 below.

2.4 Conclusion

This chapter provided a detailed theoretical overview of unconventional monetary policy. It provided a working definition of 'unconventional monetary policy', and elucidated the distinction between conventional and unconventional policies. Various transmission channels of unconventional policy were considered. It was argued that the two main channels are the signalling channel, which relays information on the expected future path of short-term interest rates, and the portfolio balance channel (PBC), which impacts the term or liquidity premium. Together, these two components (expected path of short-term interest rates and the term premium) make up the long-term interest rate; therefore, changes to any of these components would ultimately influence long-term interest rates.

The two pillars of unconventional policy, as applied by the Fed, are forward guidance and large-scale asset purchases (LSAPs). Forward guidance is argued to work mainly through the signalling channel, thereby influencing the expectations component of long rates, whereas LSAPs influence mainly the term premium through the PBC. The PBC could, in turn, be broken down into several smaller channels. These include a scarcity channel, liquidity channel, duration risk channel, prepayment risk channel, default risk channel, inflation risk channel, discount rate channel and capital constraints channel. The relative strength of these channels are, however, dependent on the specific securities under consideration; subsequently many of these channels could be considered as special cases of the PBC.

Finally, a detailed timeline of the Fed's QE interventions were provided. All FOMC announcements spanning 2008–2014 were considered, and all statements which referenced large-scale asset purchases were discussed. The dates, direction and magnitudes of these unconventional policy interventions will be important inputs in the empirical investigations below.

Chapter 3

QE and domestic asset markets

3.1 Introduction

One of the aims of the Fed's Quantitative Easing (QE) interventions was that of lowering long-term bond yields. Indeed, between the Fed's first announcement in November 2008, detailing the extent of their large-scale asset purchases (LSAPs), and the conclusion of various QE programmes in October 2014, long-term yields had fallen on aggregate. LSAPs lie at the core of the Fed's QE interventions and involves the Fed purchasing substantial amounts of securities such as longer-term Treasuries (US government bonds), asset-backed securities or other long-term debt from the private sector. While these interventions were originally designed to support the housing market, following the subprime lending collapse and subsequent global financial crisis, LSAPs are argued to have had other significant effects on the US economy. The increased demand for long-term debt securities drives up their prices and lowers its yields, while the liquidity injected to finance these purchases is expected to filter through the banking sector and financial markets and boost the broader economy. Falling yields due to LSAPs have also supported other asset markets, such as equities and corporate bonds, in their recovery after the crisis, while the substantial increase in market liquidity boosted spending and investment behaviour, preventing deflation.

Bond markets are argued to be efficient and forward-looking and that expectations matter. Political events or market developments are often instantly priced in by bond markets, just as expectations about e.g. a forthcoming economic data release or election results are. In the context of QE, the forward-looking nature of the bond market manifests in the fact that "all the effects on yields occur when market participants update their expectations and not when actual purchases take place" (Gagnon, Raskin, Remache, and Sack, 2011a:48). Therefore, it can be expected that long-term bond yields should fall when the Fed announces an expansionary LSAP programme as market participants update their expectations. Yields could also be expected to fall in the months following such an announcement in response to the actual purchases being carried out in the bond market, as changes in relative supply and demand impact prices and yields. However, closer scrutiny of the behaviour of the long-term bond market relative to QE and LSAPs shows that this theory is not always supported by practice. As will be illustrated below, not all LSAPs are associated with falling bond yields. While some QE programmes are, consistent with the theories discussed in Chapter 2, associated with a fall in yields shortly after their announcements, these programmes then see yields rising as the transactions (actual trades) are implemented in the market. Other QE programmes see a contradictory response in bond yields relative to the nature of the announcement, consistent with some of the caveats discussed in Chapter 2.

This chapter investigates the response of domestic asset markets to QE announcements and LSAP transactions in the US. While Chapter 2 considered the channels theoretically linking LSAPs to bond market yields, this chapter extends the theoretical analysis to the link between bond market yields and other asset markets. It then explores US long-term financial data, and aims to highlight behaviour and identify trends coinciding with QE announcements and various QE programmes. The empirical analysis concludes with an event study approach, in which financial market responses immediately after a QE announcement are analysed using high-frequency (daily) data. The chapter is structured as follows: Section 3.2 revisits and extends some of the theory discussed in chapter 2 specifically related to the bond market. Bond market data is interrogated in section 3.3 against the backdrop of the Fed's various QE programmes and announcements, while the stock market and corporate bond markets are also considered. This section also expands the analysis to a high-frequency event study approach, and attempts to quantify market response to QE by calculating time-varying elasticities for each QE programme. Finally, Section 3.4 proposes some arguments to support the empirical observations from the preceding analyses. Section 3.5 concludes.

3.2 Theoretical overview

3.2.1 Bond market theory

When studying the role of asset prices in the economy and their impact on economic activity and financial fragility, the focus is generally on equities and real estate (and, to a lesser degree, the exchange rate). According to the IMF (2000:77), this is because of their "overwhelming role in the composition of private sector portfolios". However, the importance of bond markets should not be underestimated. Bond markets play a critical role in economies through raising funds for governments and firms across the world. These funds are often used to finance long-term investment or infrastructure projects and public works programmes. Bonds are also very popular assets for institutional investors, as they are often associated with low risk and constant and predictable cash flows, while there exists a substantial secondary market for bonds. Finally, since LSAPs work directly on the bond market, and then spills to the other asset markets, the bond market is a necessary foundation from which to analyse the full transmission of unconventional monetary policy to asset markets and the aggregate economy. The bond market is therefore the starting point to the theoretical discussions.

3.2.1.1 The relationship between yield and price

There is generally an inverse relationship between bond yields and bond prices. Consider a simple coupon bond with a face value of \$1,000 and a coupon rate of 5%, paid annually. An investor in the primary market will initially purchase this bond for \$1,000 and will receive \$50 per year until the bond's maturity, whereupon the coupon payment will cease and the face value of the bond will be repaid to the investor. The yield on a bond is derived from the discounted stream of expected future cash flows from holding this bond;[1] therefore, if the bond is held to maturity, the yield will be equal to the coupon rate. However, the primary investor can also sell this bond in the secondary market. The price at which he can sell the bond in the secondary market is not, however, necessarily equal to the face value of the bond. For any of a myriad reasons there might be a higher or lower demand for this specific class of bond which would impact the value and price of the bond in the secondary market. Crucially, however, even though the bond may trade at either a premium (higher price than its par or issue value) or discount (lower price than par), the coupon rate and payment on the bond is fixed. Assume then that this bond is in high demand, and therefore trades for \$1,050 (i.e. at a premium) in the secondary market. Ignoring the maturity and face value for simplicity's sake, the \$50 coupon payment now represents a yield of 4.76% on the purchase price of \$1,050. Since the coupon payments and face value at maturity are fixed, whoever purchases this bond in the secondary market is therefore essentially paying a higher price than what the primary investor paid to purchase the same future cash flow. As a result, the return and yield on this investment will necessarily be lower. Conversely, if the bond should

[1] Technically, the *yield to maturity* is the interest rate i which satisfies the following generalised formula: $PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + ... + \frac{C+F}{(1+i)^n}$, where C is the coupon payment, F is the face value repaid on maturity and n is the number of years to maturity. PV represents the present value or current price of the bond. To calculate the yield on a zero-coupon or discount bond, such as Treasury bills, the formula $PV = \frac{F}{(1+i)^n}$ can be used.

trade for \$950 (i.e. at a discount) in the secondary market, its yield increases to 5.26% in this simplified illustration.

3.2.1.2 Demand for bonds

Mishkin (2013) identifies four factors which could influence the demand for a financial asset: wealth, expected return, risk and liquidity. Higher demand would bid up asset prices, whereas a fall in demand would push the asset's price down. Higher yields, *ceteris paribus*, represents a higher expected return on an asset. Extending this to the bond market it is evident that at lower prices (tantamount to higher interest rates or yields) the quantity of bonds demanded would be higher and *vice versa*. Since bond prices and yields work in opposite directions, whether an investor would prefer high prices or high yields depends significantly on his initial position. High prices will be desirable for a current (existing) bond holder, whose interest rate (yield) is already locked in but who can still benefit from a capital appreciation in his bond portfolio. High yields would be desirable for a prospective bond buyer who is hoping for strong cash flows relative to the purchase price or the opportunity to purchase bonds at a discount. In Mishkin's (2013) framework, the demand for bonds would further be positively influenced by an increase in wealth and an increase in liquidity, and negatively influenced by an increase in its perceived riskiness. A fall in the demand for bonds is associated with a fall in bond prices and increase in bond yields, while higher demand for bonds would have the opposite effect, *ceteris paribus*. Expected inflation and expected future interest rates also influence bond demand in this framework. Both higher expected inflation and interest rates lower the expected return on bonds, leading to a fall in demand for bonds. These factors could also spill over to other asset markets, where a fall in the demand for bonds might, for example, increase the demand for near substitute investment goods such as equities. This will be discussed in more detail in subsequent sections.

One of the primary determinants of bond prices is the level of prevailing interest rates in the market. Lower market interest rates increase the desirability and consequently price (value) of existing bonds (which were likely issued with relatively higher coupon rates tied to earlier market rates). This lowers the yield on these older bonds, bringing them more in line with the rates on newly issued instruments. Furthermore, lower interest rates lower the discount rate (i) on existing bonds and their related cash flows, mechanically increasing its prices according to the PV formula above. For example, expansionary monetary policy, such as a reduction in the discount or federal funds rate, is associated with an increase in bond prices and commensurate fall in bond yields. Similarly, massive purchases of bonds through various QE programmes will increase demand for bonds, bidding up their prices and lowering their yields. Monetary policy therefore plays an important role in determining bond prices, which will be reflected on balance sheets of households and firms. Either an increase in the demand for bonds or a decrease in prevailing market interest rates could lead to a capital appreciation in bonds. Given the role of the balance sheet and net worth and its effect on spending and investment, bond prices could therefore have a significant impact on aggregate demand.

It is not only the demand for bonds that influences its price and return, but also its supply. According to Mishkin (2013), expected profitability of investment opportunities, expected inflation and the government budget deficit are all factors which influence the supply of bonds. However, as the focus of this chapter is on QE policies, which in the US were conducted in the secondary market and for the most part did not involve the issue (supply) of new bonds, this analysis concentrates mainly on the demand side, and changes in the stock of existing bonds on the secondary market.

3.2.2 Expected impact of unconventional policy on domestic asset markets

Chapter 2 discussed the impact of large-scale asset purchases (LSAPs) on yields via the term premium. The following section considers the impact of LSAPs on bond yields and prices from a different perspective, while considering the spillovers to other asset markets.

An increase in the Fed's demand for bonds, due to their LSAPs, drives up bond prices and lowers bond yields. In addition, massive purchases of government securities, while artificially increasing demand for them, drains them from the secondary market, consequently reducing their supply.[2] Krishnamurthy and Vissing-Jorgensen (2011b:90) argue that "asset prices are a function of the expected stock of assets . . . held by the private sector", and that "LSAPs affect prices through changes in the expected amount of this stock". D'Amico and King (2013:426) call this a "local supply effect", arguing that "the yield on a given security [would fall] in response to purchases of that security and securities of similar maturity". LSAPs reduce the supply of long-term assets (e.g. government bonds) and increase the supply of short-term assets (bank reserves or money). However, since these long- and short-term assets are not perfect substitutes, this "reduction in supply of the riskier longer term assets reduces the risk premiums required to hold them and thus reduces their yields" (Gagnon et al., 2011a:42). An additional impact of the reduction in supply is derived from the observation that "investors have a unique demand for low-default-risk assets of particular maturities" (Krishnamurthy

[2]Note that this refers only to the supply (stock) available for trade on the secondary market. It does not mean that the government supplies (issues) fewer in the primary bond market.

and Vissing-Jorgensen, 2011b:36). Therefore, when the Fed purchases, and thereby removes from the market, a large quantity of such assets, "investors bid up the price on the remaining low-default-risk assets, decreasing the yields on these assets" (Ibid.). QE actions therefore work through both the supply and demand channels, and have the dual effect of (a) increasing bond prices and lowering their yields, and (b) reducing the volume of bonds available to trade in the secondary market.

In addition to its direct effects on the bond market, "central bank asset purchases in themselves will push up the prices of the assets bought and ... the prices of other assets will also rise" (Joyce et al., 2012:674) through the process of portfolio rebalancing. Central bank asset purchases, occurring almost exclusively in secondary markets, increases the money holdings of both the banks and non-bank private sector (asset sellers) in the form of bank deposits (see Table 2.1). These investors' portfolios have now seen a reduction in its long-term assets (the sale of longer-term government bonds to the central bank), in exchange for short-term money holdings (deposit money created by the central bank to finance its asset purchases). As a result, investors will seek to rebalance their portfolios by exchanging these money holdings for other assets which might be substitutes for those assets sold to the central bank. Investors will therefore "use the money to rebalance their portfolios with a larger share of these other assets" (Ibid.). This view is supported by Krishnamurthy and Vissing-Jorgensen (2011b:36), who argue that central bank asset purchases can be "expected to spill over into other assets that are similar in nature, to the extent that investors are willing to substitute between the assets". Furthermore, QE, through lowering bond yields, also makes these bonds less attractive to yield-seeking investors relative to other assets, e.g. equities. This is amplified by the fall in the volume of bonds available on the secondary market.

Finally, a fall in bond yields represents an increase in bond prices. This increases the collateral base and balance sheets of firms and households, leading to an increase in investment and consumption spending and, ultimately, output.[3]

The theoretical impact of QE on asset markets could therefore be summarised as follows:

1. QE lowers long-term bond yields, while pushing up bond prices.
2. QE increases the money holdings of the private sector in exchange for (reducing the private sector's holdings of) longer-term assets.

[3]Detken and Smets (2004:7) discuss the reverse, where "asset price crashes have often been associated with sharp declines in economic activity and financial instability" through its impact on collateral values. This point was also made by Bordo and Jeanne (2002).

3. This increases the demand for – and subsequently prices of – other assets, such as corporate bonds or equities, insofar as these assets could be deemed substitutes or near-substitutes for those longer-term assets.

4. QE is therefore also aimed at boosting equity prices and rallying stock markets.

In summary, lower yields would lower borrowing costs for firms and households, while higher asset prices would increase the net worth (balance sheets) of asset holders. The combination of these effects should stimulate spending and raise demand overall, and thereby boost broader economic activity

3.3 Exploring the data

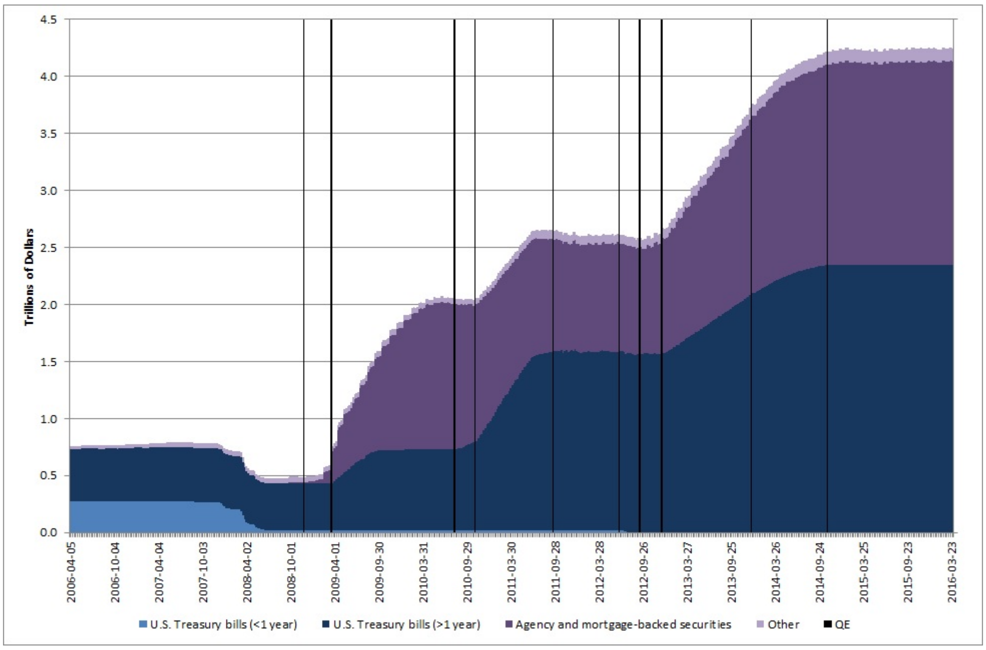
3.3.1 US bond markets

Earlier studies found that QE has led to a general reduction in bond yields in the US. Meaning and Zhu (2011:73) find that "the lasting reduction in bond supply via central bank asset purchases lowered government bond yields significantly", a result echoed by Gagnon et al. (2011a), Krishnamurthy and Vissing-Jorgensen (2011b) and Fawley and Neely (2013). Even though "yields fell significantly over the course of each programme" (Meaning and Zhu, 2011:74)[4], the initial rounds of QE (in both the US and UK) were somewhat more effective in lowering bond yields, whereas later QE operations had a more subdued effect.

Figure 3.1 illustrates how drastically the Fed's holdings of agency debt, MBS and long-term Treasury securities increased over the course of its various QE programmes. Throughout, these purchases were financed by "new reserve issuance" (Fawley and Neely, 2013:78), except for the $667 billion of long-term Treasuries purchased during Operation Twist which were financed by sales of short-term securities. The vertical lines represent major QE announcements or events (see Section 2.3). The explosion in the Fed's balance sheet following the crisis is evident. During late-2008, the Fed's balance sheet totalled just under $500 billion, with almost 80% made up of longer-term US Treasury securities. Between QE1 and QE2, however, the Fed's balance sheet quadrupled, mostly through purchases of agency debt and MBS. Following the first QE2 announcement in November 2008, the Fed aggressively purchased long-term Treasuries, while gradually continuing their purchases of other long-term debt. By the time that QE was formally halted (October 2014), the Fed had accumulated assets of close to $4 trillion, eight times more than it had when the crisis struck.

[4]Their results were published in December 2011, by which time only QE1, QE2 and Operation Twist were launched.

FIGURE 3.1: Federal Reserve Bank assets



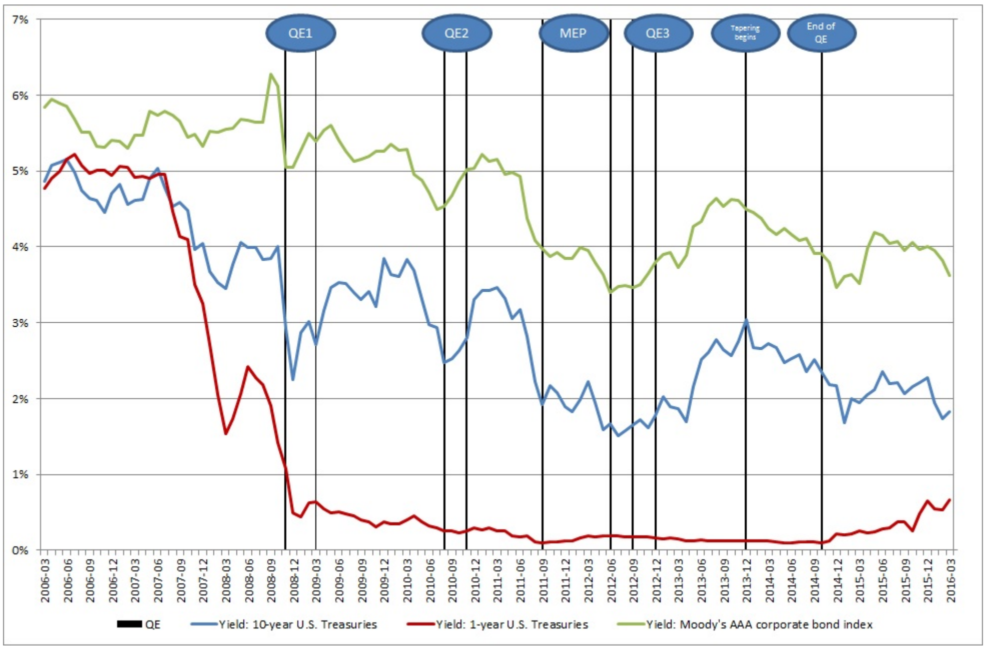
Source: Board of Governors of the Federal Reserve System. Retrieved from http://www.federalreserve.gov/.

Figure 3.2 illustrates the response of various US bond market yields to QE announcements. It shows yields on 10-year and 3-month US Treasuries over the past 10 years, representative of short-term and long-term Treasury yields, respectively, as well as the yields on an index of investment-grade corporate bonds, providing a representative view of a full spectrum of bond market yields.

Long-term and corporate bond yields fell on aggregate since the start of QE. Short-term bond yields remained close to the zero lower bound since the onset of the crisis, and have only recently started to increase. While there is a marked fall in long-term bond yields around QE1, the pattern is reversed around QE2. Bond yields were already falling by the time QE1 was formally announced and, as Martin and Milas (2012:754) point out, such a decline "may also reflect anticipation of a QE programme". Because of the forward-looking nature of financial markets, the main impact of LSAPs on yields is likely to "occur when expectations of purchases are formed rather than when the purchases are actually made" (Joyce et al., 2012:679). In the spirit of this argument, market expectations of a second round of QE might have led to the fall in long-term yields observed in the months leading up to QE2, although – paradoxically – yields increased somewhat during QE2.

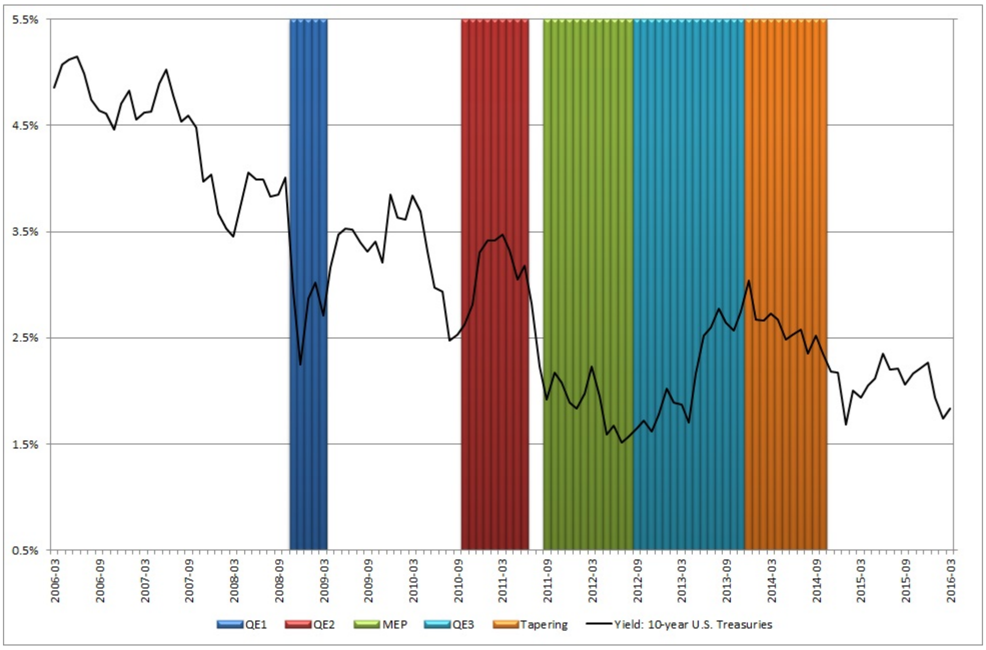
Figure 3.3 is an alternative representation of the relationship between QE programmes and long-term yield movements. Each individual QE programme is highlighted and plotted against the yield on 10-year US Treasuries.

FIGURE 3.2: Selected bond yields in the US (March 2006 – March 2016)



Source: Board of Governors of the Federal Reserve System. Retrieved from FRED Database, Federal Reserve Bank of St. Louis.

FIGURE 3.3: Bond yields over QE programmes in the US (March 2006 – March 2016)



Source: Own calculations: Board of Governors of the Federal Reserve System. Retrieved from FRED Database, Federal Reserve Bank of St. Louis.

In the months preceding QE1, QE2 and the MEP bond yields fell sharply. Throughout the duration (implementation) of QE1 and QE2, however, yields trended upward, albeit not returning to the peaks preceding the respective programmes. Conversely, QE3 was not preceded by a sharp fall in yields; throughout its duration, however, yields climbed steadily, consistent with yield behaviour during QE1 and QE2. The MEP clearly succeeded in reducing long-term yields, although its positive impact on short-term yields was muted (see Figure 3.2). During Tapering yields decreased, falling sharply briefly after the exit announcement.

Over the course of QE as a whole, long-term bond yields clearly fell on aggregate. This was also one of the main aims of the Fed's LSAPs. However, the behaviour of yields around the time of QE announcements is notable. While many QE announcements are associated with a quick and sharp decrease in long-term yields, the fact that yields often enter an upward phase shortly after a QE announcement is of particular interest. These data also support Martin and Milas (2012:753)'s finding that "initial large-scale asset purchases were effective in reducing government bond rates in the US and UK", but a "lack of impact from later initiatives" was evident in both countries. This could also be attributed to the possibility that some "law of diminishing returns seems to have set in once the surprise factor associated with the original purchases waned" (Borio and Disyatat, 2009:22).

The gradual winding down of QE ("tapering") during 2014 is associated with a gradual drop in longer-term bond yields. However, it could have been expected that this tapering would have reduced demand in the bond markets (due to the Fed now playing a gradually smaller role), leading to lower bond prices and higher yields. Given that one of the Fed's initial goals with QE was to lower bond yields and interest rates through LSAPs, the reverse could have been expected to occur once these purchases stopped. However, since the Fed is maintaining its accumulated holdings, and not actively selling these securities back into the secondary market, it would not appear to have put upward pressure on bond yields. A concern at the height of global QE programmes was the prospect of unwinding QE and the reversal of the Fed's asset purchases. It was feared that "large-scale sales of government bonds might threaten the stability of an already fragile market and lead to rapid increases in bond rates" (Martin and Milas, 2012:763). However, Figures 3.2 and 3.3 suggest that QE tapering did not result in drastic increases in bond rates. This can at least partly be ascribed to the Fed not flooding the markets with government bonds but rather maintaining its portfolio as it ceased its LSAPs. The gradual fall in long-term bond yields might point to lower term premia, reflecting an improved economic outlook relative to the years during and shortly after the crisis.

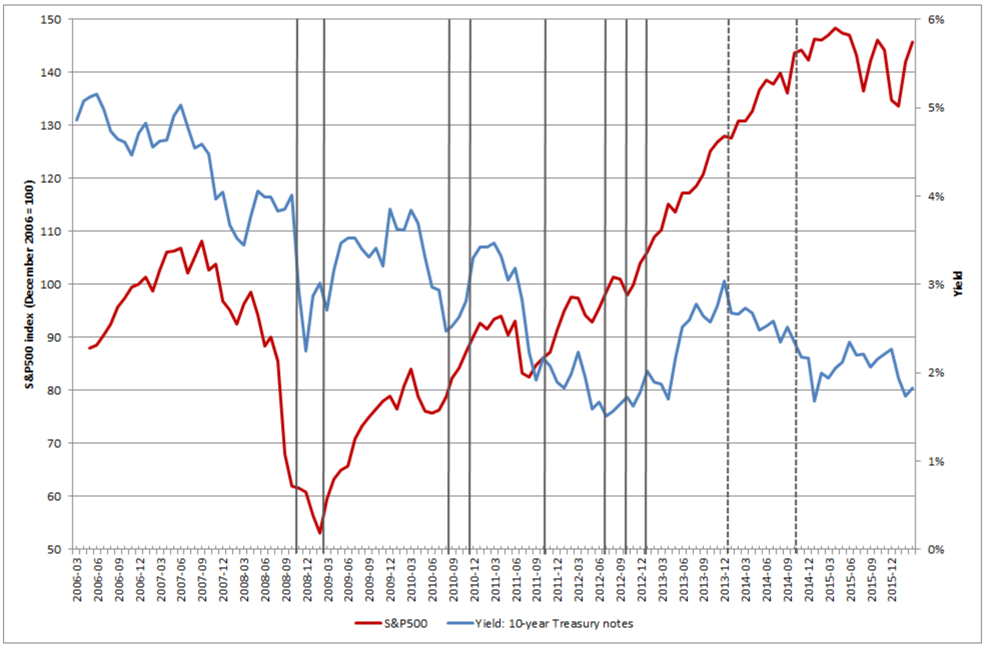
3.3.2 Other domestic asset markets

3.3.2.1 Equities

The return on a stock is positively related to its value (price). If stock prices increase, an investor could sell his initial holdings (which he presumably purchased at a lower price earlier) and realise a profit (positive return). In both the generalized dividend model and the Gordon growth model (Mishkin, 2013:185–186), the current stock price is a positive function of expected dividends as well as its growth rate, and a negative function of the return required by investors, k_e. k_e is a positive function of the uncertainty, or risk, involved in investing in equities; that is, a higher perceived risk of investing in equities will require a higher return to compensate investors for this uncertainty. Consequently, higher perceived riskiness of stocks should translate to lower stock prices. As was predicted in section 3.2 above, changes in the relative risk characteristics among asset classes would also influence asset prices. Spillover effects from LSAPs to stock prices could therefore be present insofar as the channels discussed in section 2.2 induce relative changes in these characteristics between bonds and equities. An increase in the federal funds rate, for example, increases the discount rate used to calculate the present value of expected future cash flows attached to a stock, leading to a fall in its current price, *ceteris paribus*.

Between February 2009 and April 2015, the S&P500 index has risen by about 179 percent (even though its growth from the pre-crisis high in September 2007 until the same date has been a much more modest 37 percent). The same pattern is present in the Dow Jones industrial index (DJI). The DJI bottomed out in February 2009, from where it embarked on an extended bull run until February 2015. Over this period, the DJI has risen by about 158 percent, while its growth from its pre-crisis high was a similarly modest 30 percent. While it would therefore appear that, on the whole, QE might have succeeded in rallying the stock market, the response of the stock market to individual QE announcements was much less pronounced than the response of bond markets, perhaps reflecting the relatively longer time it takes for financial market developments to be incorporated into stock prices. This is illustrated in Figure 3.4, which shows movements in equities in response to both the Fed's QE announcements and the bond market. Interestingly, QE announcements on a number of occasions are associated with the start of a mini-rally on the stock market (e.g. March 2009, September 2011, June 2012). Of further interest is the behaviour of the stock market relative to the benchmark 10-year Treasury yield. Up until around September 2011, movements in the S&P500 mirrored movements in 10-year bond *yields*. Thereafter, yields continued their general downward trend (albeit less sharply), while equity prices continued to climb. Thus, after September

FIGURE 3.4: Equities, bonds and QE announcements in the US (March 2006 – March 2016)



Source: Own calculations: US Federal Housing Finance Agency; Chicago Board Options Exchange; Board of Governors of the Federal Reserve System. Retrieved from FRED Database, Federal Reserve Bank of St. Louis.

2011, equity prices mirrored bond *prices*, and no longer bond *yields*. The first half of the sample therefore suggests that there was a positive relationship between equity prices and bond yields, which appears to be a partial contradiction of the suggestion that QE, through lowering long-term bond yields, should boost equity prices. This could perhaps be explained by an underlying substitution effect which might therefore be present, insofar as rising bond prices incentivises investors to move to the relative safety of the bond market, perhaps in the interest of shoring up their balance sheets, but at the expense of higher-yielding opportunities elsewhere. On the other hand, the extreme risk aversion at the height of the crisis and the massive losses racked up by large international firms would have caused stock prices to plummet, likely irrespective of developments in bond markets.

Of course there are numerous factors which could influence stock prices, so to attribute these movements solely to developments in the bond market or QE announcements would be unwise. Some of the notable sharp falls in the S&P500 index under the QE period include:

- March 2010 – May 2010: Concerns about European sovereign debt crisis, culminating in Greek government's bailout request (April 23); "Flash Crash" (May 6).

- June 2011 – August 2011: Continuing concerns about the European sovereign debt crisis and the US debt ceiling crisis and eventual Standard&Poor's debt rating downgrade (August 6) precipitated a massive flight to safety.[5]
- March 2012 – May 2012: Yet more concerns about Europe's debt crisis, with Spain and Greece at the forefront.
- August 2012 – October 2012: European recession, Chinese economic slowdown.

The strong performance of the stock market fits one of the theoretical motivations of the Fed in undertaking QE, where a fall in long-term bond yields would transfer resources of yield-seeking investors out of bonds and into substitutes such as the stock market. However, the above-mentioned falls in equities during periods of falling yields in the bond market could suggest that decreasing long-term bond yields pushed (at least some) investors out of equities and perhaps back to the bond market. This could be explained by the observation made in Section 3.2.1.1, that an investor's behaviour in response to falling bond yields would depend upon his initial position. Falling yields are desirable for a current (existing) bond holder, whose interest rate (yield) is already locked in but who can still benefit from capital appreciation due to higher bond prices. Investors in this category would therefore see an improvement in their balance sheet or net worth. High or increasing yields would be desirable for a yield-seeking investor, who will be hoping for strong cash flows relative to the purchase price or the opportunity to purchase bonds at a discount. In a period of falling yields, these investors are more likely to seek higher returns elsewhere, e.g. the stock market. The first category of investor could therefore be said to favour capital appreciation and might be generally more risk-averse, while the latter could prefer taking on more risk in search of yield. The observed dips in equities highlighted above might therefore represent an over-supply of equities due to some investors shifting their funds to the bond market in search of capital appreciation instead of higher (and riskier) yields.

3.3.2.2 Corporate bonds

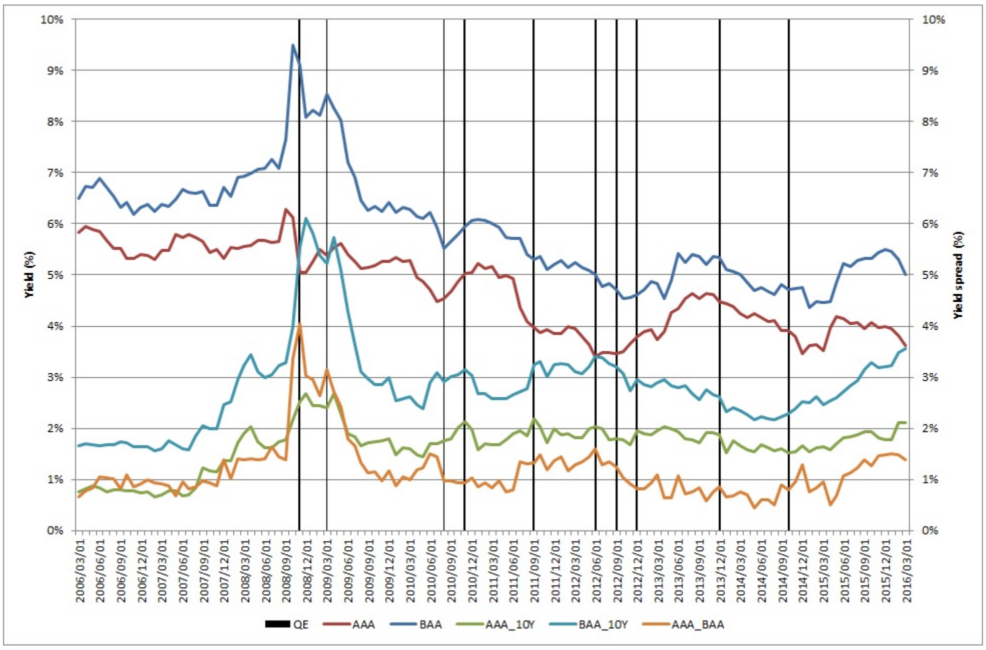
Yields on corporate bonds followed movements in longer-term Treasuries closelyfootnoteA number of spreads are shown here. AAA_10Y and BAA_10Y represent the difference between 10-year Treasury yields and, respectively, AAA and BAA rated corporate bonds, while AAA_BAA represents the spread between AAA and BAA rated corporate bonds. While the Fed's LSAPs significantly lowered the yields on Treasuries, it "has

[5] This flight to safety could to a large degree explain the sustained fall in Treasury yields due to higher demand for safe assets throughout 2011.

had limited spillover effects for private sector bond yields” (Krishnamurthy and Vissing-Jorgensen, 2013:58). The spreads between investment-grade corporate bond yields and Treasury yields could reflect both a risk premium and a liquidity premium. Treasuries are viewed as free of default risk, while at the same time being highly liquid. The sharp spikes in AAA_10Y and BAA_10Y spreads around December 2008 could involve both of these factors: Given the prevailing uncertainty in financial markets shortly after the onset of the crisis, the risk premium on corporate bonds increased substantially, while the widespread credit crunch also impaired the liquidity of corporate bonds relative to Treasuries.

While investment-grade yields generally track long-term Treasury yields, spreads remain relatively stable around a narrow band. Apart from the sharp increase in yield spreads during the crisis, reflecting a substantial risk premium for corporate bonds, not a lot else could be read in them. Corporate bond yields generally trend downward for a sustained period following a QE announcement while yield spreads generally narrow in response to QE announcements (Figure 3.5), perhaps indicating a fall in the risk premium associated with the Fed’s support for the markets. This is consistent with the *default risk channel* (see Section 2.2.1).

FIGURE 3.5: Corporate bond yields, spreads and QE announcements in the US (March 2006 – March 2016)



Source: Moodys; Board of Governors of the Federal Reserve System. Retrieved from FRED Database, Federal Reserve Bank of St. Louis.

Finally, according to Krishnamurthy and Vissing-Jorgensen (2011b:6), “when there are less long-term Treasuries . . . the spread between Baa and Aaa bonds rises”. An increase in this spread is therefore viewed as evidence of the safety premium, or scarcity, channel discussed in Section 2.2.1 above.

3.3.3 High-frequency data analysis

The previous section analysed yield behaviour using low-frequency (monthly) data. Investigating yield movements directly around QE announcements, using high-frequency (daily) bond market data is therefore a logical next step. Several of the studies mentioned earlier drew their conclusions based on event studies, where they analysed market reaction immediately following QE announcements. Implicit in the event-study approach is the belief that bond markets are efficient, "in the sense that all the effects on yields occur when market participants update their expectations and not when actual purchases take place" (Gagnon et al., 2011a:48). In other words, even though the actual asset purchases might only occur some time after an announcement, the markets immediately following the announcement price in the effect these future purchases are expected to have. One of the notable advantages of such a high-frequency approach is its ability to "address the fundamental identification issue of distinguishing the exogenous effect of LSAPs from other contemporaneous effects . . . During such short intervals of time, economic conditions do not change and if one detects changes in yields, this can indeed be attributed to the policy announcement" (Buraschi and Whelan, 2015:24-25).

Digging deeper into each notable[6] QE announcement reveals some interesting responses by the bond market (see also Figures 3.2 and 3.3).

- **QE1**: On 11/25/2008 the Fed announces purchases of \$100 billion in GSE debt and \$500 billion in MBS. Bond yields had already been falling sharply since the collapse of Lehman brothers in September 2008 heralded the start of the global financial crisis, falling even further after the announcement. Shortly after the announcement, however, yields increased again, with longer-term yields increasing substantially more than short-term yields. On 3/18/2009 the Fed announces further purchases of \$100 billion in GSE debt and \$750 billion in MBS, as well as \$300 billion in long-term Treasuries. This caused a brief fall in 10-year Treasury yields, while yields on 20- and 30-year Treasuries appear to have increased. After the announcement long-term yields continued their upward trajectory.
- **QE2**: On 8/10/2010 the Fed announces that principal payments from previous LSAPs will be reinvested in longer-term Treasuries. In the 4 months leading

[6] Following Gagnon et al. (2011a), an announcement is deemed "notable" when an FOMC statement or minutes contains "new information concerning the potential or actual expansion of the size, composition, and/or timing of LSAPs" (2011a:48). These are exactly the FOMC announcements highlighted in Section 2.3 above.

up to this announcement bond yields had been steadily falling. Shortly after this announcement, yields started climbing. On 11/3/2010 the Fed announces purchases of a further $600 billion in Treasuries, with the majority of purchases intended to be in the 2- to 10-year maturity range. The general upward trend in yields continued unabated.

- **MEP**: On 9/21/2011 the MEP ("Operation Twist") is announced, in which the Fed will purchase $400 billion of longer-term Treasuries, financed by selling an equal amount of short-term Treasuries, up until the end of June 2012. In addition, principal payments from MBS and GSE debt will be reinvested in MBS and no longer in long-term Treasuries. This announcement saw a sharp fall in longer-term yields with a brief, but temporary, increase in short-term yields. On 6/20/2012 the MEP was extended until the end of 2012 by the Fed purchasing long-term Treasuries, financed by short-term Treasuries, at a pace of roughly $45 billion per month (totalling $267 billion over six months). This announcement, similar to the first QE2 announcement of 8/10/2010, was followed by climbing yields.

- **QE3**: On 9/13/2012 the Fed announces purchases of $40 billion of MBS per month. On 12/12/2012 the Fed announces that it will continue to purchase $45 billion of long-term Treasuries per month, but it will no longer be financed by selling short-term Treasuries. Shortly after the first announcements long-term yields briefly decreased, whereafter it continued its upward trend. Following the second announcement long-term yields continued to grow, briefly dipping a few months later, and then increasing sharply over the next 6 months.

- **Tapering**: From 12/18/2013 the Fed gradually slows down the pace of monthly purchases in both MBS and long-term Treasuries. Shortly after this announcement long-term yields started steadily falling over the next 12 months. Short-term yields then slowly started climbing as the Fed indicated its intention to cease LSAPs during late-2014.

- **Exit**: On 10/29/2014 the Fed announces the conclusion of QE. The Fed will no longer purchase additional securities, but it will maintain its portfolio at its current size by not selling these securities back into the market. Long-term yields steadily decreased after the exit announcement, and then temporarily fell sharply two months later.

3.3.3.1 Event studies

The remainder of this section will employ an event-study approach, based on the market reaction to significant QE announcements within two narrow event windows. A one-day window (i.e. changes in closing yields between the day prior to the announcement and the day of the announcement) is investigated in line with Gagnon et al. (2011a), who argue that a one-day window allows "sufficient time for revised expectations to become fully incorporated in asset prices" while "keeping the window narrow enough to make it unlikely to contain the release of other important information" (2011a:50). In addition, a two-day window (changes in closing yields from the day prior to the announcement and the day *after* the announcement) is also analysed. This is the window selected by Krishnamurthy and Vissing-Jorgensen (2011b), who argue that "during a period of low liquidity, the prices of such assets may react slowly in response to an announcement" (2011b:11). Since initial rounds of QE played out following a severe credit crunch and in an environment of persistent poor liquidity, theirs is a valid motivation for the larger window. Related to this "slow-response" argument, Gagnon et al. (2011a) also investigate a two-day window, arguing that this allows for "lagged reactions to the news by some market participants" (2011a:51).

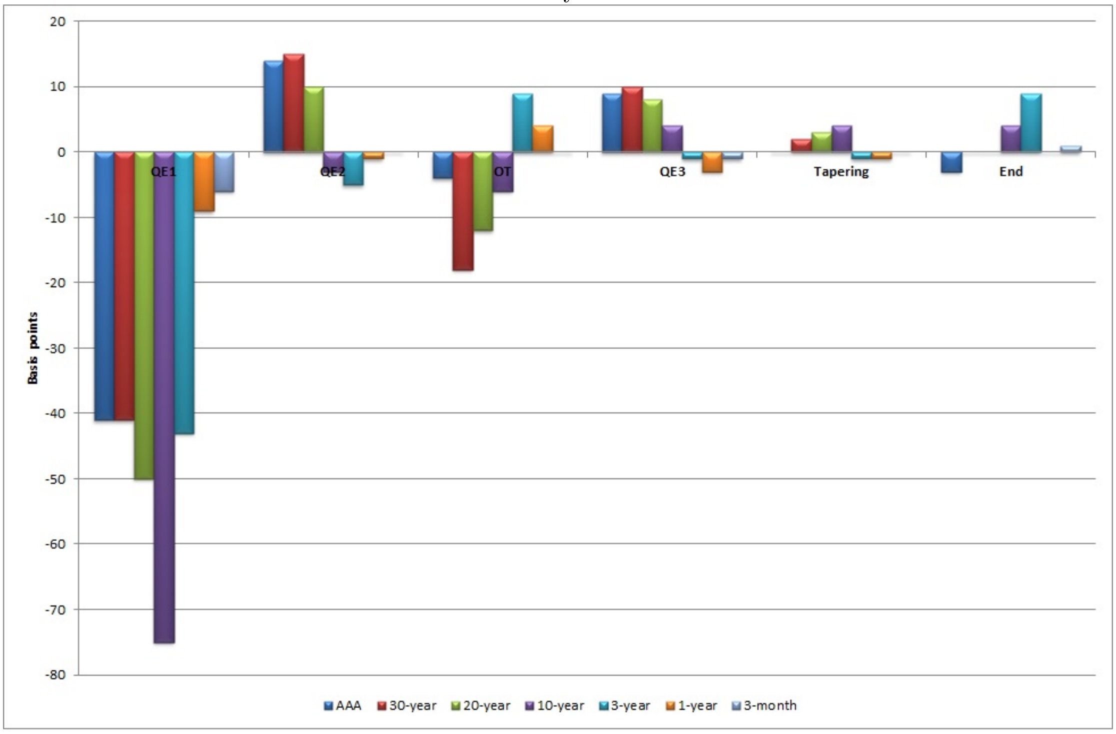
Figure 3.6 illustrates the the cumulative yield changes on a range of bonds following all these QE announcements. The numbers reported in the graphs are the total change in various bond yields following all announcements related to a specific QE programme. For example, 10-year US Treasury yields fell by 24 and 51 basis points, respectively, following the two QE1 announcements of November 2008 and March 2009. The total yield change is 75 basis points, and is reported in the first panel of Figure 3.6.

Echoing the findings in the literature, yield changes are generally larger over the two-day window than the one-day window. While yields mostly change in the same direction over the two windows, a notable difference arises during QE2. While longer-term yields have decreased over the two-day window, in line with QE1 and QE3 movements, they have actually *increased* over the one-day window. This is consistent with the findings of Meaning and Zhu (2011), who also detect an increase in longer-term yields associated with QE2 over a one-day event window. However, while the behaviour of 20- and 30-year Treasury yields around QE2 are inconsistent between the two event windows, 10-year yields fell across both windows. Krishnamurthy and Vissing-Jorgensen (2011b) report identical numbers, with the cumulative change in 30-year and 10-year Treasury yields over the two QE2 announcements equal to 3bp and -24bp, respectively.

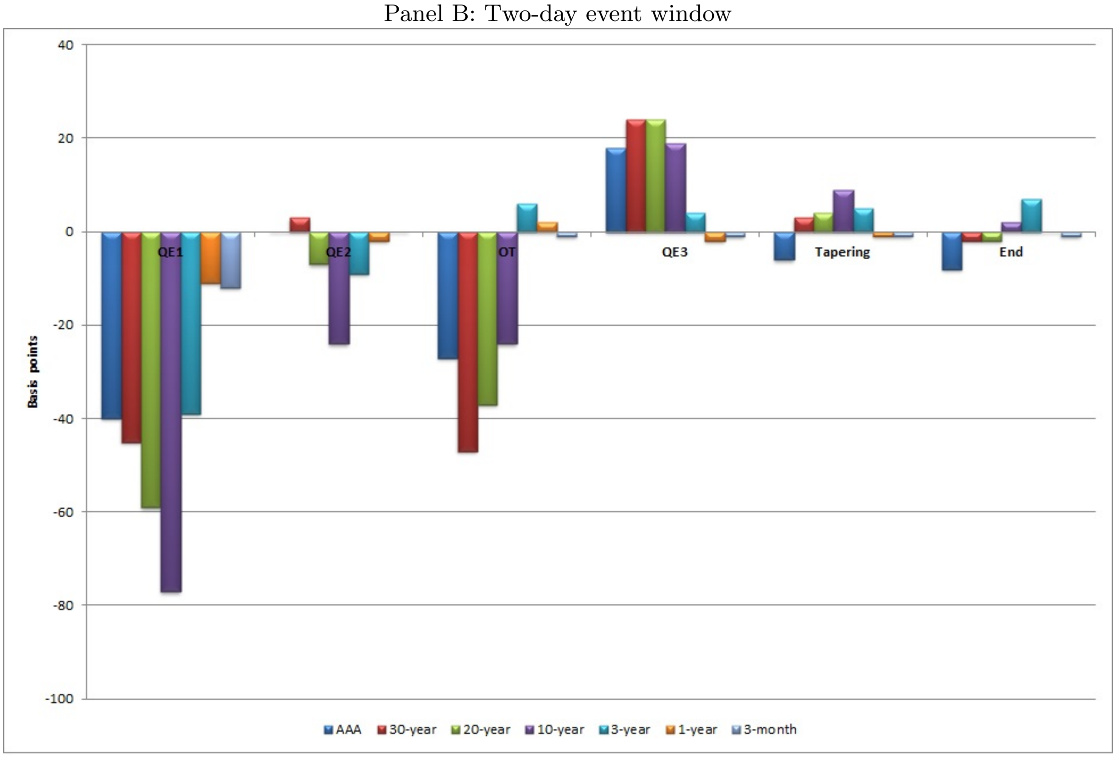
The results from the two-day window support the findings of previous authors who studied the effects of QE during 2011/2012. QE1, QE2 and Operation Twist all lowered long-term yields. The effect on short-term yields was extremely small, indicating a flattening of the yield curve and therefore a reduction in the term premium, echoing Gagnon et al. (2011a)'s findings. The largest single effect was the 77bp drop in 10-year

Figure 3.6: Cumulative yield changes in US bond markets following QE announcements

Panel A: One-day event window



Panel B: Two-day event window



Source: Own calculations: Board of Governors of the Federal Reserve System. Retrieved from FRED Database, Federal Reserve Bank of St. Louis.

Treasury yields during QE1. These findings also suggest a somewhat inconsistent response by bond markets to QE announcements. QE1 involved total asset purchases of around \$1.75 trillion in agency debt, MBS and long-term Treasuries, and is associated with a cumulative 77bp fall in 10-year Treasury yields. QE2 involved asset purchases of only \$600 billion in long-term Treasuries, and the impact on 10-year yields was a commensurately smaller 24bp fall. Operation Twist, which essentially involved the Fed swopping short-term Treasuries for long-term Treasuries, saw 10-year yields fall by 23bp, while 1-year and 3-year yields rose by only 2 and 6 basis points, respectively, flattening the yield curve somewhat. This could imply that the larger the magnitude of the asset purchase the larger the effect on bond yields, that QE ran into diminishing returns (Goodhart and Ashworth, 2012), or a combination of both. QE3, however, actually saw an increase in long-term bond yields and commensurate steepening of the yield curve. This might point to the possibility that the announced asset purchases were smaller than the markets expected (Krishnamurthy and Vissing-Jorgensen, 2011b). Market expectations thus exceeded the expected price effect, and the increases in yields following these announcements perhaps reflect adjustments back to equilibrium. Yields were already trending upwards in the days before the QE3, tapering and QE exit announcements, which could mean that markets expected the slowing down of asset purchases. The fact that yields continued to increase even after these announcements could indicate that the slowdown effect was larger than markets anticipated, leading to further price and yield corrections. This point is also made by Gagnon et al. (2011a:50), who link falls in yields to "greater-than-expected LSAP purchases" and *vice versa*. This is also consistent with Krishnamurthy and Vissing-Jorgensen (2011b)'s "overshooting" hypothesis, where, if the markets anticipate the size of QE programmes incorrectly, a yield adjustment follows after the announcement, either immediately or gradually thereafter, evident in, for example, the increase in longer-term yields following the November 2010 announcement.

The impact of LSAPs on the term premium could also be derived from the reaction of long-term yields relative to short-term yields. During QE1 over the one-day window, for example, 10-year yields fell by 75bp and 1-year yields fell by 9bp. This implies an effective narrowing of the term premium by 66bp and a substantial flattening of the yield curve. QE1, QE2 and the MEP therefore flattened the yield curve, while QE3 and Tapering are associated with a steepening of the yield curve.

Finally, QE had a substantial impact on corporate bond yields[7], indicating that its effects were not limited to the assets that were actually purchased and could point to investors rebalancing their bond portfolios following central bank actions (Meaning and Zhu, 2011). Such portfolio rebalancing would involve exchanging the excess money

[7] The response of corporate bond yields following QE announcements is highly correlated to the responses of long-term Treasury yields (see Figure 3.6).

holdings (created by the Fed purchasing long-term Treasuries from investors) for the closest substitute to long-term Treasuries, in this case possibly investment-grade corporate bonds.

It should be noted, however, that not all QE announcements involved the same scope, nor did they all point to "easing" or an accommodating policy stance (see the classification in section 2.3). The December 2013 Tapering announcement, for example, indicated that asset purchases would be slowing down, which points to a gradual reversal of accommodating monetary policy. The October 2014 announcement similarly signalled the end of asset purchases and the exit of QE, indicating that bond markets will no longer be supported by "artificial" demand created by the Fed. Just as earlier QE announcements and accompanying LSAPs, designed to support financial markets, were expected to lower bond yields, both of these "exit" announcements could therefore be expected to increase bond yields. Indeed, according to Figure 3.6, these announcements had a positive, albeit quite small, impact on Treasury yields. Crucially, the Fed indicated that it would be maintaining the size of their bond portfolio rather than attempting to sell these assets back into the market. An oversupply of long-term securities which would have resulted from the Fed liquidating attempting to liquidate its portfolio would surely have driven bond prices down and yields significantly higher.

Finally, even though these event studies point to yields generally falling within a day or two after the announcement, these falls appear not to be persistent. After almost every such fall in yields, the bond market enters a phase of increasing yields.

3.3.3.2 Elasticities

Diminishing returns of QE, as measured by the impact of QE announcements on bond yields, was briefly alluded to above. Table 3.1 attempts to quantify the impact of various QE programmes by calculating elasticities of 10-year Treasury yields relative to the size of the relevant QE programme. The elasticities are calculated as the fraction of the change in yield to the size of the programme, which roughly indicates by how many basis points yields change for each billion dollars of assets purchased.

TABLE 3.1: Yield/QE elasticities

Programme	Size of intervention	Cumulative yield response	Elasticity
ONE-DAY WINDOW:			
QE 1	\$1.75 trillion	-75bp	-0.043
QE 2	\$600 billion	-3bp	-0.005
MEP	\$667 billion	-6bp	-0.009
QE 3	\$1.14 trillion	4bp	0.004
Tapering	\$825 billion	4bp	0.005
TWO-DAY WINDOW:			
QE 1	\$1.75 trillion	-77bp	-0.044
QE 2	\$600 billion	-24bp	-0.040
MEP	\$667 billion	-24bp	-0.036
QE 3	\$1.14 trillion	19bp	0.017
Tapering	\$825 billion	9bp	0.011

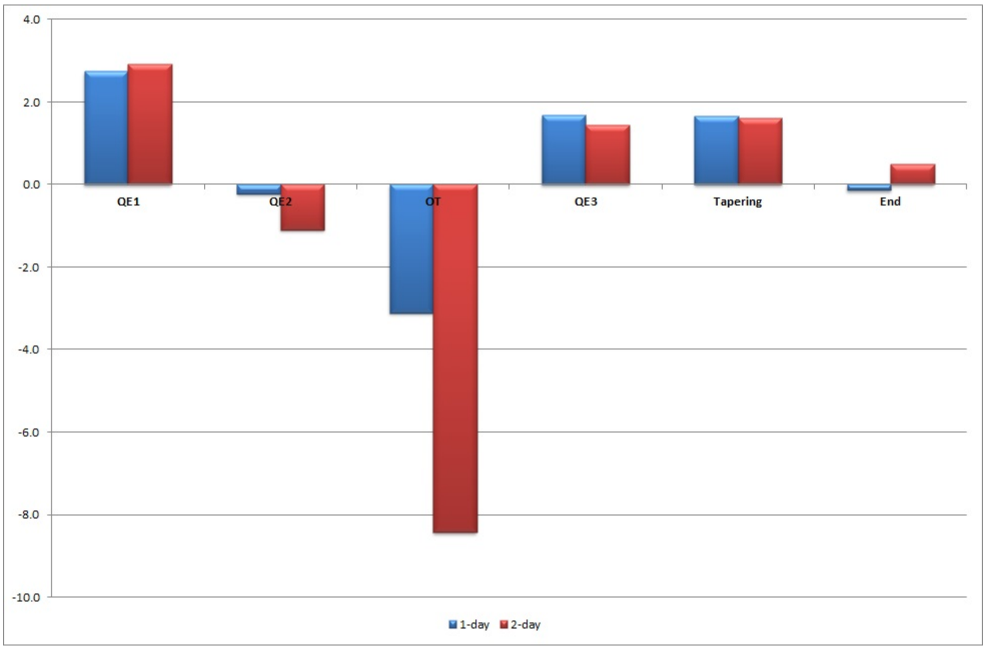
Source: Own calculations: Board of Governors of the Federal Reserve System. Retrieved from FRED Database, Federal Reserve Bank of St. Louis.

Interpreting these numbers are not straightforward, as certain QE programmes involved once-off bulk asset purchases (QE1 and QE2), while QE3 involved continuous small purchases, but with – crucially – the exact time horizon unknown at the time of the announcement. The total size of the QE3 programme was therefore only known *ex post*, and not *ex ante* like QE1, QE2 and the MEP. The latter three announcements contained information on the exact size and timeframe of the interventions, whereas the QE3 announcements were left open-ended. Despite this caveat, however, the numbers confirm that yields were less responsive to later rounds of QE. While there is some discrepancy in the magnitude of the elasticities between the one- and two-day windows, the general trend and direction of responses are consistent over both windows.

3.3.4 Equities

Figure 3.7 below illustrates the immediate response by the stock market (proxied here by the S&P 500 index) to QE announcements. The one-day and two-day windows measure, respectively, the percentage change in the closing value of the index between the day before and the day of, and the day before and the day after, the announcements. Where

Figure 3.7: S&P 500 response to QE announcements



Source: Own calculations: Board of Governors of the Federal Reserve System. Retrieved from Federal Reserve Bank of St. Louis, FRED Database. Monthly data.

QE programmes stretched over more than one announcement the cumulative changes over the days of the FOMC statements are reported.

Quite clearly, the stock market rallied on the news of QE1. Over both the one- and two-day windows, the S&P 500 index jumped by close to 3%. Of interest, however, is the different reaction of the stock market to the two announcements which together comprise QE1: Over the one-day window, equities grew by 0.66% on November 25, 2008, and 2.09% on March 1, 2009. Over the two-day window, however, equities grew by 4.21% but contracted by 1.30% on those same respective dates. Comparing Figure 3.7 to Figure 3.6 shows an inverse relationship between bond markets and equities, at least for the first two QE programmes. This is consistent with the theory discussed earlier which suggested that a fall in bond market yields could boost stock prices. During later QE programmes, however, this relationship does not appear to hold as clearly.

3.4 Discussion of results

The evidence presented above raise an interesting paradox. QE and LSAPs are aimed at lowering long-term bond yields. While, in the long run, this aim appears to have been achieved, the dynamics around and some time after a QE announcement sometimes indicate the opposite effect. It is observed that long-term bond yields increase during a number of expansionary QE programmes (QE1, QE2 and QE3), while they decrease during the one (and only) contractionary programme (Tapering) (see figure 3.3). This is in contrast to the immediate response of bond yields to these announcements, where event studies show falling 10-year Treasury yields in response to QE1 and QE2, and increasing yields in response to QE3 and Tapering (figure 3.6). Table 3.2 summarises these observations across the two analyses.

TABLE 3.2: 10-year Treasury yield response to QE announcements

Programme	**Event study**	**Broad trend**
QE 1	Decrease	Increasing
QE 2	Decrease	Increasing
MEP	Decrease	Decreasing
QE 3	Increase	Increasing
Tapering	Increase	Decreasing

QE1, QE2 and QE3 are followed by periods of increasing bond yields, while Tapering is associated with falling yields (Figure 3.3). While the instant market response to QE1 and QE2 are consistent with the Fed's stated aims (i.e. lowering long-term yields), this does not appear to have had a long-lasting impact on the markets. QE3 is even more contradictory, as yields increase both instantaneously and gradually after the announcement, in spite of the programme being classified as expansionary. The MEP is, in fact, the only programme where both the immediate and subsequent bond market responses are consistent with the aim of lowering long-term yields.

3.4.1 Explaining the paradox

3.4.1.1 Overshooting

Because the bond market is forward-looking, expectations about future developments are often priced in before the fact. For example, once the market expects a round of LSAPs to be announced in the near future the expected impact of these LSAPs are incorporated in bond pricing and yields. However, market expectations are not necessarily entirely accurate, nor does the market know the exact size of the LSAP before it is announced. This implies that the "expectations effect" that is priced into bonds might easily be too large, but just as well too small. To explain the observation that bond yields often start increasing shortly after an expansionary QE announcement, Krishnamurthy and

Vissing-Jorgensen (2011b) propose an overshooting[8] hypothesis. With reference to QE1, they argue that "markets may have priced in more than a $600bn QE announcement" (Krishnamurthy and Vissing-Jorgensen, 2011b:24), which implies that yields had – by the time of the announcement – fallen further than what the actual size of the LSAPs would justify. This explains the correction of increasing yields shortly thereafter.

This reasoning could be extended to QE2 and QE3. Both these announcements were preceded by falling yields, perhaps reflecting markets pricing in the expected announcements, but were followed by a period of increasing yields (see Table 3.2). Markets might have been expecting a larger asset purchase programme but, after prematurely pricing in this erroneous expectation, had to correct this overshooting through increasing yields, following the realisation of the actual (less-than-expected) size of the programme.

3.4.1.2 Risk premia and portfolio composition

The Fed's LSAPs, which drives bond prices up, could make bonds desirable for certain investors. Especially in a time of risk aversion and falling net worth, one of the safest ways to shore up balance sheets is to purchase bonds which is expected to generate capital appreciation.[9] When the market expects a next round of LSAPs, it expects that bond prices will shoot up after the announcement. One way to shore up balance sheets, while also lowering portfolio risk, is to go long in Treasuries. If a critical mass of market participants follow this strategy, it might increase bond demand and subsequently prices leading up to a QE announcement, causing falling yields over the same period. Following the announcements, these investors might want to cash out their positions in order to realise their capital gains. Supply of bonds increases, pushing down prices and increasing yields.

Furthermore, when confidence is high(er), investors are more willing to take risks. This might lead to a fall in demand for government bonds, relative to riskier assets such as corporate bonds or equities. Coupled with the relatively lower demand for long-term Treasuries (later LSAP programmes are generally smaller in magnitude than QE1), this lowers bond prices and drives up bond yields.

[8] Romer (2006:235) defines overshooting as "a situation where the initial reaction of a variable to a shock is greater than its long-run response". Even though Romer (2006) discusses overshooting in the context of Dornbusch (1976)'s famous exchange rate overshooting hypothesis, his definition is equally applicable here.

[9] This explanation would be highly relevant for earlier rounds of QE, where, close to the crisis, capital appreciation (strong balance sheets) and risk aversion were highly prized.

3.4.2 Decreasing elasticities

It was argued earlier that, because financial markets are forward-looking, the impact of a QE transaction would have been priced in when market expectations were formed, and not only once the announcement was made. Therefore, the impact of QE on bond yields would arguably be biggest when the surprise to the markets was largest. Considering the turmoil in financial markets leading up to QE1, financial markets could well have expected some kind of intervention by the Fed. However, the Fed's intention of purchasing financial assets on unprecedented scale most likely surprised markets, leading to a sharp reaction in bond yields. Conversely, by the time QE2 and QE3 were implemented, markets were already cognisant of the Fed's unconventional monetary policy strategy. It was therefore easier for markets to predict the Fed's actions, which would have led markets to price in the expected impact of future Fed LSAPs somewhat before the fact. The result was that, when the interventions were formally announced, the impact on bond yields were rather muted.

3.5 Conclusion

This chapter evaluated the empirical evidence from US bond and equities markets against the theoretical considerations put forward in Chapter 2. It extended the theoretical considerations around the impact of LSAPs on bond market yields to the spillovers to other US asset markets resulting from such policy interventions. The change in the composition and size of the Fed's balance sheet over the course of the QE interventions was highlighted, and the behaviour of yields relative to QE announcements and the implementation of the various programmes were analysed. These analyses also included event studies, where the impact of FOMC announcements pertaining to QE on bond market yields and equity prices were investigated over narrow windows in an attempt to isolate its effects.

It was found that QE1, QE2 and Operation Twist are all associated with a fall in long-term bond yields over the two-day window, while QE2 saw an increase in yields over the one-day window. QE3 and Tapering are associated with an increase in long-term yields. The event studies echoed the results from previous studies, where it was established that accommodative QE announcements are associated with a sharp initial fall in bond yields. After this initial shock the bond market enters a trend of increasing yields and falling prices. This paradox could be explained by the overshooting hypothesis, where there is a discrepancy between the market's expectation and the actual size of the Fed's QE intervention. This leads to a sharp reaction of yields around the announcements as the expectation of Fed intervention is priced in. However, to the extent that market expectations overshot the actual size of the QE programme, the bond market would then gradually move back to some equilibrium, leading to a cycle of increasing yields.

Finally, elasticities were estimated for each QE programme, where the cumulative yield responses were compared to the total size of each intervention. The signs (directions) of these elasticities were found to be consistent with the results from the event studies.

Part II:
Dynamic Stochastic General Equilibrium modelling

Chapter 4

Monetary policy rules in a closed economy DSGE model

4.1 Introduction

Unconventional monetary policy, such as large-scale asset purchases, played a prominent role in the economic recovery following the global financial crisis. Because interest rates in much of the developed world were stuck at the ZLB, central banks could not apply expansionary monetary policy through traditional interest rate policy, and subsequently unconventional policies became the order of the day. However, such measures need not be limited to crisis times. Unconventional policies could potentially complement conventional policy making even when there is scope to maneuver the short-term interest rate. That is, there may be room for unconventional policies even during conventional times. Furthermore, different monetary policy instruments could be more suited to specific targets, which would allow the central bank to improve its optimal policy mix. To this end, this chapter develops a DSGE model[1] to investigate the possibility of utilising balance sheet policies to complement traditional interest rate policy.

In the canonical NK DSGE literature the primary monetary policy instrument is the short-term nominal interest rate. The goal of this chapter is to test whether balance sheet policies could be effective even if interest rate policy is operational (i.e. not at the ZLB), and an economy is not facing a crisis (hence the qualification of "conventional" times). A secondary aim is to test how different macroeconomic variables respond to either interest rate or balance sheet policy, and – subsequently – which policy could be more suited to specific variable(s). Section 4.2 constructs a detailed closed economy DSGE model, attempting to reflect the empirical evidence found in Chapter 3, and which

[1] I thank Matteo Falagardia of the ECB for generously sharing his codes and insights.

would enable the simulation of alternative monetary policy instruments in response to various shocks. The baseline model is calibrated and simulated in Section 4.3. In Section 4.4, the baseline model is extended to incorporate a balance sheet rule, which functions as an alternative – potentially complementary – policy instrument to the short-term interest rate. Sensitivity analyses are performed in Section 4.5. Section 4.6 concludes.

The novel contribution of this chapter is the result that central bank balance sheet policies could function as a complementary monetary policy instrument (relative to the traditional short-term interest rate instrument), even in situations where nominal interest rates are not stuck at the ZLB.

4.2 Closed economy DSGE model

The model presented here is a richer representation of the workhorse New Keynesian DSGE model described in Galí (2015). This model includes nominal rigidities in the form of staggered price setting by firms à la Calvo (1983), as well as portfolio frictions in the form of short- and long-term bonds and a preferred-habitat setting for households' holdings of bonds of various maturities. In addition, the central bank's ability to manipulate its balance sheet is introduced as a policy response to complement a traditional Taylor interest rate rule. Such balance sheet policies, in the form of asset sales or purchases by the central bank, coupled with the imperfect substitutability between households' holdings of short- and long-term bonds[2], provides an alternative channel through which monetary policy can affect aggregate demand. It draws heavily from Falagiarda (2014), Harrison (2012) and Andrés, López-Salido, and Nelson (2004).

Firms are monopolistically competitive, and hire labour to produce a homogenous final good. Households consume this good and supply labour inputs to firms. There is a government sector which conducts fiscal policy, and a central bank which conducts monetary policy. Monetary policy is conducted by a choice of two separate policy reaction functions: (i) a traditional short-term interest rate (Taylor) rule, and (ii) balance sheet policies by way of sales or purchases of long-term government bonds. The government conducts fiscal policy through raising taxes on the household, while issuing and servicing both short- and long-term government bonds. There is no explicit banking sector, and capital and investment in capital are ignored to keep the model as tractable as possible. Finally, the foreign sector is excluded here in order to first concentrate on rigorously deriving and modeling the central bank's policy reaction functions. This restriction will subsequently be relaxed in chapter 5, where the economy is opened up to investigate the central bank's use of the policy tools proposed here in response to asset market

[2]This imperfect substitutability has its roots in Tobin's (1969) seminal work.

volatility arising from international capital flows. There are thus five markets in this model economy: labour, goods, money, short-term bonds and long-term bonds.

4.2.1 Households

The economy is populated by a representative household who derives utility from consumption C_t and real money holdings M_t/P_t, and supplies labour N_t to the firm. The household's optimisation problem is an extension of the canonical DSGE framework and closely follows Falagiarda (2014), with the following notable differences: capital, investment (and consequently rental income) and government spending are excluded, while habit formation in consumption is also ignored.[3] The household therefore maximises utility according to

$$\max_{C_t, \frac{M_t}{P_t}, N_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \phi_t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{1}{1-\delta} \left(\frac{M_t}{P_t} \right)^{1-\delta} - \chi \frac{N_t^{1+\phi}}{1+\phi} \right] \tag{4.1}$$

where β is the discount rate and ϕ_t is a preference shock which will serve as the demand shock in the simulations below. σ, δ, ϕ and χ are parameters which respectively represent the coefficient of relative risk aversion (CRRA, equal to the inverse of the elasticity of intertemporal substitution), elasticity of money demand, the inverse of the Frisch elasticity of labour supply, and a shift parameter which scales the steady state labour supply to realistic values.[4] The household's nominal budget constraint is given by

$$\begin{aligned} P_t C_t + P_t T_t + M_t + \frac{B_t}{R_t} + \frac{B_{L,t}^H}{R_{L,t}}(1 + AC_t^L) \\ = W_t N_t + M_{t-1} + B_{t-1} + \frac{B_{L,t-1}^H}{R_t} \end{aligned} \tag{4.2}$$

which in real terms becomes

$$\begin{aligned} C_t + T_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1 + AC_t) \\ = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}^H}{P_t R_t} \end{aligned} \tag{4.3}$$

[3] While the Falagiarda (2014) model is quite rich, the aim here is simply to isolate the effect of central bank asset purchases. For this reason the model is kept as parsimonious as possible.

[4] See Poutineau et al. (2015). χ is also referred to as "labour disutility" (Kavli, 2015:149).

where P_t is the aggregate price level. Defining the gross inflation rate as $\pi_t \equiv \frac{P_t}{P_{t-1}}$ (Harrison, 2012:11)[5], equation 4.3 can be expressed as

$$
\begin{aligned}
&C_t + T_t + \frac{M_t}{P_t} + \frac{1}{R_t}\frac{B_t}{P_t} + \frac{1}{R_{L,t}}\frac{B^H_{L,t}}{P_t}(1 + AC_t) \\
&= \frac{W_t}{P_t}N_t + \frac{1}{\pi_t}\left[\frac{M_{t-1}}{P_{t-1}} + \frac{B_{t-1}}{P_{t-1}} + \frac{1}{R_t}\frac{B^H_{L,t-1}}{P_{t-1}}\right]
\end{aligned}
\tag{4.4}
$$

The LHS of equation 4.3 shows that the household allocates their wealth among consumption C_t, payment of a lump-sum tax T_t, real money holdings $\frac{M_t}{P_t}$, and bond purchases. The household holds the nominal value of two types of zero-coupon bonds: short-term (one-period) bonds B_t and long-term bonds $B^H_{L,t}$. The nominal price of these bonds in time t are given by the yields R_t and $R_{L,t}$, respectively.[6] Households therefore have to allocate their discretionary spending between contemporaneous consumption and bond purchases (investment in financial assets). Intuitively, higher expected returns on bonds might encourage households to forgo current consumption in order to invest in bonds which would enable higher future consumption.

The RHS of equation 4.3 represents the household's wealth or total income in time t. Following Andrés et al. (2004), households enter period t with a certain level of real money holdings and a bond portfolio, consisting of maturing one-period bond holdings, purchased in period $t-1$, and net holdings of long-term bonds following transactions[7] in period $t-1$. They earn a nominal wage of W_t during period t. The final two terms in equation 4.3 capture the *ex post* returns on short- and long-term bonds (Harrison, 2012). Total income thus consists of real wage income $\frac{W_t}{P_t}N_t$, real money holdings brought forward from the previous period $\frac{M_{t-1}}{P_t}$ and earnings realised from holding bonds in the previous period. Naturally, the purchasing power of income or wealth brought into period t is eroded by the inflation rate π_t (equation 4.4).

$\frac{B_{t-1}}{P_t}$ and $\frac{B^H_{L,t-1}}{P_t R_t}$ represent real earnings on holdings of short-term bonds B_t and long-term bonds $B^H_{L,t}$ brought forward from the previous period.[8] At time t, the returns (or bond prices or values) R_t and $R_{L,t}$ are known. However, in the presence of a secondary market for bond trading, long-term bonds are subject to price risk before maturity and

[5] Expressed as the log-deviation from the steady state this is equivalent to $\hat{\pi}_t \equiv \hat{p}_t - \hat{p}_{t-1}$.

[6] Bonds are priced at their gross interest rate (Falagiarda, 2014), which is the standard treatment in the literature of zero-coupon bonds. For example, if the face value is \$1,000 (B_t = \$1,000) and the discount rate is 4%, $R_t = 1 + 0.04 = 1.04$ and $\frac{B_t}{R_t} \approx \$961$. The household therefore purchases a one-period bond for \$961 in period t that will pay out \$1,000 in period $t+1$.

[7] Such transactions could include new bond purchases from the government (see section 4.2.3) and/or net sales to the central bank (see section 4.2.4).

[8] Recall that the short-term bond was purchased at a discount in the previous period based on the prevailing interest rate R_{t-1}. It now pays out its nominal value of unity.

therefore $R_{L,t}$ cannot be known at time $t-1$ (Falagiarda, 2014). An agent who buys long-term bonds at time $t-1$ with the view to perhaps sell them in the next period t would be uncertain about the next period's payoffs[9] (or the value of his portfolio). If B_t and $B^H_{L,t}$ are viewed as the household's net holdings of short- and long-term bonds between periods t and $t+1$, that each pay a unit of the consumption good at time $t+j$, it follows from a simple arbitrage argument that in period t long-term bonds represent "identical sure claims to consumption goods at the time of the end of the maturity as newly issued one-period bonds in period t" (Falagiarda, 2014:309). Subsequently long-term bonds are priced at the same rate as one-period bonds, namely the money market rate R_t.[10]

Finally, portfolio adjustment frictions are imposed to allow segmentation in financial markets, which represent "impediments to arbitrage behaviour that would equalise asset returns" (Falagiarda, 2014:309). Households are subsequently assumed to face bond transaction (portfolio adjustment) costs given by

$$AC_t = \left[\frac{\phi_L}{2}\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)^2\right] Y_t \tag{4.5}$$

where κ_L is a parameter describing the steady state ratio of long-term to short-term bond holdings ($\bar{B}^H_L/\bar{B}$) and ϕ_L is a parameter for portfolio adjustment frictions. Households therefore incur a cost whenever their portfolio allocation between short- and long-term bonds deviates from its steady state allocation, which is paid in terms of income Y_t. Adjustment costs are zero in steady state, as the household's portfolio will be allocated optimally.[11] This cost can be intuitively explained by the preferred-habitat theory, where agents could have set preferences for certain bond maturities over others (Kabaca, 2016, Vayanos and Vila, 2009). It would be costly for the household to return to this preferred habitat in the event of a disturbance or shock. Others have interpreted it as reflecting a liquidity risk attached to holding long-term bonds (Andrés et al., 2004), or simply the cost of managing the household's bond portfolio (Falagiarda, 2014).

This adjustment cost framework also allows for balance sheet operations (e.g. LSAPs) by the central bank to directly influence households' spending decisions: Insofar as LSAPs remove long-term bonds from the household's portfolio it will disturb the ratio $B_t/B^H_{L,t}$. The larger the disturbance, the larger to cost to households. This could be

[9]The agent would of course hope that the price he will receive for selling the long-term bond in period t, $R_{L,t}$, is larger than the price he paid for the bond in the previous period, $R_{L,t-1}$.

[10]See also Ljungqvist and Sargent (2012:375–376). In period t, j-term bonds are traded at the price R_t, since R_{t+j} is not known at time t.

[11]In steady state $\kappa_L = \bar{B}^H_L/\bar{B}$ and equation 4.5 thus collapses to zero ($\kappa_L B_t/B^H_{L,t} - 1 = \bar{B}^H_L/\bar{B} \times \bar{B}/\bar{B}^H_L - 1 = 0$).

illustrated by a simple hypothetical example. If we assume the household's 'preferred habitat' is a (steady state) ratio of long-term to short-term bonds of 2:1[12], we have $\kappa_L = \bar{B}_L^H/\bar{B} = 2$. LSAPs by the central bank now removes a significant portion of long-term bonds from the portfolio of households in exchange for cash balances (money holdings). As a direct, immediate result, $B_{L,t}^H$ falls and M_t increases, while B_t remains unchanged. The immediate effect of this central bank action is that the ratio of long-term to short-term bonds $B_{L,t}^H/B_t$ falls to, say, 1:1, and as a result $AC_t = \frac{\phi_L}{2}Y_t > 0$. If the central bank's LSAP programme is even larger and the ratio $B_{L,t}^H/B_t$ thus falls even further, say to 1:2, $AC_t = 9\frac{\phi_L}{2}Y_t > \frac{\phi_L}{2}Y_t > 0$. Clearly, the larger the LSAP programme, the higher the adjustment cost to households. This will be further explored in section 4.2.4 below.

This framework also illustrates how LSAPs would have an expansionary effect on the economy. Following the central bank's intervention, households need to decide how to allocate their new-found liquid reserves. Clearly, there is limited scope for the household to immediately restore its preferred habitat. Immediately allocating these funds to buying short-term bonds will distort the preferred habitat even more, while there is limited availability elsewhere of long-term bonds. The only realistic source of long-term bonds is the government[13], but the contemporaneous supply (and price[14]) of long-term bonds might not be adequate to fully restore the household's preferred habitat. It follows that a portion of this new-found liquidity will be allocated towards additional consumption, ultimately contributing to higher output and inflation. Therefore, the household's return to its preferred habitat is likely to be gradual, rather than immediate.

4.2.1.1 The household's optimisation problem

The household's optimisation problem is derived in detail in Appendix B.1. The key log-linearised first-order conditions are:

Money demand:

$$\hat{m}_t - \hat{p}_t = \frac{1}{\delta}\frac{1}{1-\beta}\left(\sigma\hat{c}_t - \hat{\phi}_t - \beta E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1} - \hat{\phi}_{t+1}]\right) \tag{4.6}$$

[12]The steady state ratio of long- to short-term bonds in the literature ranges from 1.33 (Falagiarda, 2014) to 3 (Harrison, 2012)

[13]In an open economy setting there are of course other options available, such as foreign long-term bonds. The model could also be expanded to include investment in firm capital or other (substitute) financial assets such as equities.

[14]The empirical evidence shows that LSAPs have generally pushed down long-term bond yields (see chapter 3), which is equivalent to an increase in its price (section 3.2.1.1). It will now be more expensive for the household to purchase long-term bonds, which, coupled with the inelastic supply of long-term bonds in the short run, makes it highly unlikely that the household will be able to simply "refill" their portfolio.

Demand for money is an increasing function of contemporaneous consumption, and a decreasing function of expected future consumption and expected inflation.

Labour and wages:

$$\hat{w}_t - \hat{p}_t = \phi \hat{n}_t + \sigma \hat{c}_t - \hat{\phi}_t \tag{4.7}$$

Short-term bonds:

$$E_t\left[\sigma \hat{c}_{t+1} + \hat{\pi}_{t+1} - \hat{\phi}_{t+1}\right] = (\sigma \hat{c}_t + \hat{r}_t - \hat{\phi}_t) + \beta \kappa_L \phi_L \bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) \tag{4.8}$$

Long-term bonds:

$$E_t[\sigma \hat{c}_{t+1} + \hat{\pi}_{t+1} + \hat{r}_{t+1} - \hat{\phi}_{t+1}] = (\sigma \hat{c}_t + \hat{r}_{L,t} - \hat{\phi}_t) + \phi_L \bar{Y}(\hat{b}_t - \hat{b}^H_{L,t}) \tag{4.9}$$

4.2.1.2 The consumption Euler equation

The household's Euler equation can be found by solving for $\hat{c}_t$ in the log-linearised expression for short-term bonds as derived above (equation 4.8), and is given by:

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{1}{\sigma}\beta \kappa_L \phi_L \bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) + \frac{1}{\sigma}E_t\left[\phi_t - \hat{\phi}_{t+1}\right] \tag{4.10}$$

The first two terms are virtually identical to the workhorse model (Galí, 2015:54)[15], which indicates that the household's consumption decision is a function of expected consumption and the expected real interest rate in the next period. In addition to the canonical model, however, the ratio of the household's long- to short-term bond holdings also influences the consumption decision. An increase in this ratio $(\hat{b}^H_{L,t} - \hat{b}_t)$[16] implies that the household now holds relatively more long-term bonds in its portfolio.[17] Intuitively, by foregoing current consumption, households could invest in additional holdings of long-term bonds, which – through the future payoffs of said bond holdings – could generate higher future consumption. Conversely, a fall in this ratio (see the earlier hypothetical example) could imply the removal of long-term bonds from the household's

[15] Notably, the discount factor β from Galí (2015)'s model is absent here. This is the result of the linearisation procedures employed in this paper, where variables are expressed in terms of their log-deviation from steady state (Uhlig, 1999). This approach generally leads to constants (e.g. β) dropping out of the final equation, whereas linearisation by way of Taylor series approximations (employed by Galí (2015)) generally leaves the constants in the final equations. Appendix A provides a more detailed discussion on the approach followed here.

[16] The linearised difference is tantamount to the ratio of the levels of long- to short-term bond holdings $B^H_{L,t}/B_t$.

[17] Note that the short-term bond is a one-period bond, which cannot be traded in the secondary market. The only operations that could influence this ratio is therefore sales or purchases of long-term bonds. In the closed economy, the household can only transact with the central bank.

portfolio in exchange for more liquid assets (e.g. money), which are likely to be used to finance current consumption. $\phi_t > \hat{\phi}_{t+1}$ implies a positive demand shock in period t, and *vice versa*.

4.2.1.3 The term structure

Combining the two log-linearised first-order conditions of the household's short- and long-term bond holdings (equations 4.8 and 4.9) and solving for $\hat{r}_{L,t}$ yields the following expression for the term structure:

$$\hat{r}_{L,t} = \hat{r}_t + E_t[\hat{r}_{t+1}] + \phi_L \bar{Y}(1 + \beta\kappa_L)(\hat{b}^H_{L,t} - \hat{b}_t) \tag{4.11}$$

Theories of the term structure suggests that the long-term interest rate is a function of the expected future path of short-term interest rates and a term premium which captures factors such as investors' preferred habitat and a liquidity premium (Mishkin, 2013). In the absence of portfolio adjustment frictions ($\phi_L = 0$), the liquidity premium disappears and the long-term interest rate is equal to the expected future path of short-term rates. The first two terms of equation 4.11 represent the expected future path of short-term interest rates[18], while the final term is a measure of the household's preferred habitat. Similar to the findings of Harrison (2012) and Falagiarda (2014), the long-term interest rate depends positively on the household's holdings of long-term bonds, and negatively on the household's holdings of short-term bonds, due to the imperfect substitutability between the two asset classes. Therefore, consistent with Tobin's (1969) theory, equation 4.11 suggests that asset purchases by the central bank, by reducing the supply of long-term bonds available to the household, would reduce the long-term interest rate. This is neatly summarised by Falagiarda (2014:314), who states that "to get agents to accept the fact that holding a larger (smaller) fraction of short-term bonds in their portfolio, the spread between the two rates has to decrease (increase)". The implication is therefore also that LSAPs by the central bank should flatten the yield curve by pushing down long-term interest rates.

4.2.2 Firms

The supply block is quite standard, with the firm defined similarly to the model described in Andrés et al. (2004). The consumption good is sold in a monopolistically competitive market, with the firm producing according to the production function

[18]Note that central banks can attempt to influence expectations of the future path of short-term interest rates through explicit forward guidance as the other pillar of unconventional monetary policy. By signaling that the policy rate will remain low, the central bank can anchor long-term interest rates.

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} \tag{4.12}$$

$Y_t(j)$ is firm j's output and $N_t(j)$ represents labour hired from the household. Total labour demand is therefore given by $N_t = \int_0^1 N_t(j)dj$, and the firm's only cost is the wage bill. Z_t is a stochastic technology or productivity shock common to all firms, which evolves over time according to

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z \tag{4.13}$$

Following Galí (2015:58) and Andrés et al. (2004), aggregate output is defined by

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{4.14}$$

Prices are set following Calvo's (1983) staggered pricing framework. That is, in each period t a fraction $1-\theta$ of producers can reset their prices, while the remaining fraction θ keep their prices unchanged[19] (setting their prices equal to P_{t-1}). P_t^* denotes the optimal price set by firms resetting prices in period t. θ^k is the probability that the price set at time t will still hold at time $t+k$, and $\frac{1}{1-\theta}$ represents the average duration of a price. The aggregate price level is therefore given by the weighted average of the prices set by adjusting and non-adjusting firms

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{4.15}$$

Expressing equation 4.15 in terms of inflation, $\pi_t \equiv \frac{P_t}{P_{t-1}}$, yields

$$\pi_t^{1-\varepsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\varepsilon} \tag{4.16}$$

Linearising around the steady state (the firm's optimisation problem is rigourously derived in Appendix B.2) yields an expression for inflation:

$$\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}) \tag{4.17}$$

[19] If all firms can reset prices (i.e. $\theta = 0$) the problem reduces to the case of perfect price flexibility.

Equation 4.17 shows that inflation results from price setting firms reoptimising by choosing a price p_t^* which is different from the average price in the previous period p_{t-1}.[20]

4.2.2.1 Optimal price setting

A firm resetting its price in period t will choose the optimal price P_t^* so as to maximise its discounted stream of future profits while this price remains effective by solving

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left(P_t^* Y_{t+k|t}(j) - TC_{t+k}(Y_{t+k|t}(j))\right)\right] \tag{4.18}$$

subject to the sequence of demand constraints

$$Y_{t+k|t}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \tag{4.19}$$

$\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor, $Y_{t+k|t}(j)$ is output in period $t+k$ for a firm which last reset its price in period t, and $TC_{t+k}(\cdot)$ represents total nominal cost as a function of this output. Firm j's nominal undiscounted profit in period $t+k$ is therefore equal to $P_t^* Y_{t+k|t}(j) - TC_{t+k}(Y_{t+k|t}(j))$. Solving for the optimal price level (see equation B.13) yields

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \left[\theta^k \beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon} Y_{t+k} MC_{t+k|t}^r\right]}{E_t \sum_{k=0}^{\infty} \left[\theta^k \beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon-1} Y_{t+k}\right]} \tag{4.20}$$

In the case of flexible prices ($\theta = 0$), the problem reduces to a one-period problem with $P_t^* = \frac{\varepsilon}{\varepsilon-1} MC_{t|t}$, where $\frac{\varepsilon}{\varepsilon-1}$ represents the "frictionless" or "desired" markup over the (nominal) marginal cost (Galí, 2015:57).

4.2.2.2 Log-linearisation

The key log-linearised equations from the firm's optimisation problem (see Appendix B.2) are given by:

Production function:

$$\hat{y}_t = \hat{z}_t + (1 - \alpha)\hat{n}_t \tag{4.21}$$

[20] Equation 4.17 can be rearranged to show that the price level is determined according to $p_t = \theta p_{t-1} + (1-\theta)p_t^*$ (Galí, 2015). The price level is therefore simply the weighted average of the previous period's prices for the fraction θ of firms who cannot reset and the optimal price set by the fraction $1 - \theta$ of firms who can reset.

Optimal price setting:

$$\hat{p}_t^* = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}^r_{t+k|t} + \hat{p}_{t+k}\right] \tag{4.22}$$

Note that, since $\hat{p}_{t+k} - \hat{p}_{t-1} = \hat{\pi}_{t+k}$, the previous expression is equivalent to

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}^r_{t+k|t} + \hat{\pi}_{t+k}\right]$$

This result is very similar to the optimal price decision in the workhorse New Keynesian model, expressed in Galí (2015) as

$$p_t^* = \mu + (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[mc^r_{t+k} + p_{t+k}\right]$$

Galí (2015:57) defines $\mu = \bar{mc} = \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$. For values of $\bar{mc}$ that are close to one, μ is therefore interpreted as the net markup. Price-setting firms therefore choose a price that "corresponds to their desired markup over a weighted average of the current and expected nominal[21] marginal costs" (Ibid.).

4.2.2.3 Marginal cost and inflation

The marginal product of labour can be derived from the firm's production function (equation 4.12) and is given by

$$\hat{mpn}_t = \hat{z}_t - \alpha\hat{n}_t \tag{4.23}$$

The firm's marginal cost is given by

$$\begin{aligned} \hat{mc}^r_t &= \hat{w}_t - \hat{p}_t - \left(\hat{z}_t - \alpha\left(\frac{\hat{y}_t - \hat{z}_t}{1-\alpha}\right)\right) \\ &= \hat{w}_t - \hat{p}_t - \frac{1}{1-\alpha}(\hat{z}_t - \alpha\hat{y}_t) \end{aligned} \tag{4.24}$$

It follows that inflation can be expressed as (see equation B.29)

[21]The function under the expectations operator is simply the nominal marginal cost at time $t+k$ expressed in logs: $\log MC_{t+k} = \log(MC^r_{t+k}P_{t+k}) = mc^r_{t+k} + p_{t+k}$.

$$\begin{aligned}\frac{\hat{\pi}_t}{1-\theta} &= (1-\theta\beta)[\Theta\hat{mc}^r_t] + \hat{\pi}_t + \theta\beta\frac{E_t[\hat{\pi}_{t+k}]}{1-\theta} \\ \therefore \hat{\pi}_t &= \frac{(1-\theta)(1-\theta\beta)}{\theta}[\Theta\hat{mc}^r_t] + \beta E_t[\hat{\pi}_{t+k}] \\ \therefore \hat{\pi}_t &= \lambda\hat{mc}^r_t + \beta E_t[\hat{\pi}_{t+k}]\end{aligned} \tag{4.25}$$

where $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta$. Combining equations 4.17 and 4.22 and rearranging gives the following equation describing the inflation dynamics (Andrés et al., 2004, Galí, 2015):

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+k}] + \lambda\hat{mc}_t + \hat{a}_t \tag{4.26}$$

where $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta$ (see equation 4.25). We also add a stochastic cost-push shock $\hat{a}_t$, which evolves according to $\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon^a_t$. This can be iterated forward (see Kotzé (2014)) to express current inflation as

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \lambda\hat{mc}_t + \hat{a}_t \tag{4.27}$$

This suggests that inflation is due to the "purposeful price-setting decisions of firms, which adjust prices in light of current and anticipated inflationary conditions" (Kotzé, 2014:55).

4.2.3 Government

The government issues short- and long-term bonds. Short-term (one-period) bonds are sold only to the household, while long-term bonds can be sold to both the household and the central bank. This assumption allows us to mimic central banks that have often purchased long-term securities in order to affect long-term yields. Taxes raised on the household T_t plus new bond issuance finance the government's debt financing costs (repayment of maturing bonds). The government's real budget constraint is therefore given by

$$T_t + \frac{B_t}{P_tR_t} + \frac{B_{L,t}}{P_tR_{L,t}} + \frac{\Delta_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{L,t-1}}{P_tR_t} \tag{4.28}$$

The LHS of equation 4.28 represents the government's income, consisting of taxes and the total value of bond issuance (price × quantity of bonds issued). Δ_t represents the change in the central bank's balance sheet (Falagiarda, 2014, Harrison, 2012) and is discussed in detail in section 4.2.4.2 below. The RHS represents government expenditure, which consists solely of the servicing of outstanding bonds.[22] $B_{L,t} = B^H_{L,t} + B^{CB}_{L,t}$, indicating that the total issuance (supply) of long-term bonds is taken up between households and the central bank.

The government's supply of long-term bonds[23] follows a stochastic AR(1) process of the form

$$\log\left(\frac{B_{L,t}}{\bar{B}_L}\right) = \phi^{BL}\log\left(\frac{B_{L,t-1}}{\bar{B}_L}\right) + \varepsilon_t^{BL} \tag{4.29}$$

where ε_t^{BL} is a stochastic error term with mean of zero and standard deviation of σ^{BL}. Asset purchase shocks are therefore assumed to affect "only the composition of outstanding government liabilities" (Falagiarda, 2014:312), and not the quantity of bonds already in circulation.

The total amount of taxes raised T_t is a function of the government's outstanding liabilities and is expressed as the following passive fiscal rule (Falagiarda, 2014:312):

$$T_t = \bar{T} + \psi_1\left[\frac{B_{t-1}}{P_t} - \frac{\bar{B}}{\bar{P}}\right] + \psi_2\left[\frac{B_{L,t-1}}{R_t P_t} - \frac{\bar{B}_L}{\bar{R}\bar{P}}\right] \tag{4.30}$$

where $\bar{T}$ represents the steady state level of T_t.[24] Taxes therefore react to deviations of government liabilities from its steady state levels, and respond to shortfalls from the previous period. The fiscal rule is passive as long as $\psi_1+\psi_2 > \frac{1}{\beta}-1$ (i.e. $\psi_1+\psi_2 > \bar{R}-1$).

The log-linear forms of the long-term bond supply (equation 4.29) and the tax rule (equation 4.30) are given by (see Appendix B.3):

$$\hat{b}_{L,t} = \phi^{BL}\hat{b}_{L,t-1} + \varepsilon_t^{BL} \tag{4.31}$$

and

$$\hat{t}_t = \frac{1}{\bar{T}\bar{P}}\left[\psi_1\bar{B}(\hat{b}_{t-1} - \hat{p}_t) + \beta\psi_2\bar{B}_L(\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t)\right] \tag{4.32}$$

[22] Equation 4.28 can be rearranged to show that net bond issuance plus the change in the central bank balance sheet finances net transfers to/from households T (Harrison, 2012:9).

[23] This is a simplification of Falagiarda (2014:312)'s specification, where the *real* supply of long-term bonds is considered. This, however, can result in the stock of long-term bonds changing solely in response to a change in the price level, which can unnecessarily distort the household and central bank's balance sheets. We therefore simplify by considering purely the *nominal* long-term bond supply.

[24] In steady state $B_{t-1} = \bar{B}$ and $B_{L,t-1} = \bar{B}_L$, while $P_t = \bar{P}$ and $R_t = \bar{R}$. Therefore equation 4.30 reduces to $T_t = \bar{T}$.

4.2.4 Monetary policy

The central bank plays two roles in the economy. First, it conducts monetary policy through a standard Taylor rule. Second, it trades in long-term government bonds. Ultimately, we wish to consider and compare two possible policy rules: (i) control over the short-term nominal interest rate through a traditional Taylor rule, and (ii) control over the monetary base and long-term bonds via its balance sheet. Under policy (i), the central bank balance sheet is one of the structural equations in the model, where an exogenous shock to central bank asset purchases (equation 4.29) influences the dynamics of the model. Under policy (ii), the central bank has an endogenous "balance sheet rule" which could complement (or even replace) the standard Taylor rule as policy reaction function. Under the latter policy, the central bank could purchase (sell) long-term bonds directly from (to) households, thereby injecting (removing) cash into (from) households' asset portfolio in response to deviations in inflation and output from steady state levels. In the benchmark model, only policy (i) is considered. Policy (ii), central bank asset purchases, is discussed in depth in section 4.4 below.

4.2.4.1 Short-term interest rate

The nominal interest rate is set according to a standard Taylor rule without interest rate smoothing:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi^\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi^y} e^{\varepsilon_t^R} \tag{4.33}$$

where R_t, π_t and Y_t denote the nominal interest rate, inflation rate and output, respectively. ε_t^R is a stochastic error term (monetary policy shock) with mean of zero and standard deviation of σ^R. The parameters $\bar{R}$, $\bar{\pi}$ and $\bar{Y}$ are steady state values for the interest rate, inflation rate and GDP. The central bank responds to deviations in the inflation rate and GDP from their steady state values (the inflation gap and output gap, respectively) in proportion ϕ^π and ϕ^y. Given the nominal interest rate chosen, the central bank adjusts the money supply to ensure equilibrium in the money market. The familiar log-linearised Taylor rule is given by (see Appendix B.4)

$$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t + \varepsilon_t^R \tag{4.34}$$

TABLE 4.1: Balance sheet effects of LSAPs

Non-bank private sector

Assets	Liabilities
− Securities	
+ Deposits	

Central bank

Assets	Liabilities
+ Securities	+ Reserves

Banking sector

Assets	Liabilities
+ Reserves	+ Deposits

4.2.4.2 Central bank balance sheet

The central bank's simplified balance sheet is represented by the money supply M_t on the liabilities side and its holdings of long-term government bonds $B_{L,t}^{CB}$ on the assets side, and is given by

$$M_t = B_{L,t}^{CB}$$

If the central bank purchases long-term government bonds from the non-bank private sector (the household), the non-bank private sector's holdings of these assets falls, while the central bank's holdings increases ($\downarrow B_{L,t}^{H} = \uparrow B_{L,t}^{CB}$, or $\left|\Delta B_{L,t}^{H}\right| = \left|\Delta B_{L,t}^{CB}\right|$), while leaving total long-term bond supply $B_{L,t}$ (the current stock of long-term bonds in circulation) unchanged. The transaction can be financed through the central bank "issuing base money in the form of reserves held by commercial banks" (Bowdler and Radia, 2012:607). The banking sector's balance sheet therefore also expands by these newly created central bank reserves, which are matched against the increased deposits of the non-bank private sector ($\Delta M_t = \Delta B_{L,t}^{CB}$). This is illustrated in Table 4.1:

Absent an explicit banking sector, LSAPs can be modelled as the central bank transferring money balances directly to households (representing the non-bank private sector in Table 4.1) in exchange for some of the households' holdings of long-term government bonds. This is a strong assumption, however, as in reality much of the liquidity injected into the financial system by the Fed during their QE operations ended up being hoarded by banks as excess reserves[25] and did not make its way to households via a commensurate increase in the money supply.[26]

Setting Δ_t to represent the change in the central bank balance sheet, equal to money creation and net asset purchases (Falagiarda, 2014:311), the central bank's nominal balance sheet can be represented by

$$\Delta_t = [M_t - M_{t-1}] - \left[\frac{B^{CB}_{L,t}}{R_{L,t}} - \frac{B^{CB}_{L,t-1}}{R_t}\right] \tag{4.35}$$

where a change in the amount of long-term government bonds held by the central bank is balanced by a commensurate change in the money supply. According to Harrison (2012:11), the level of Δ is dictated by fiscal commitments. Therefore "additional purchases of debt by the central bank, which increase the asset side of its balance sheet, must be financed by an expansion of the liabilities side of the balance sheet via money creation." Rearranging equation 4.35, the central bank's balance sheet can be expressed as a function of net asset purchases.

$$\left[\frac{B^{CB}_{L,t}}{R_{L,t}} - \frac{B^{CB}_{L,t-1}}{R_t}\right] = [M_t - M_{t-1}] - \Delta_t \tag{4.36}$$

Intuitively, asset purchases and a commensurate increase in the money supply would have an expansionary impact on the economy in the short term. By providing households with liquid reserves, immediate consumption spending could be expected to increase, while the increase in the money supply, coupled with potential higher consumption spending, would likely fuel inflation. This disturbance also requires the household to decide how to allocate these reserves: on spending (consumption), saving (short- and long-term bonds), or a combination of these. If asset purchases are fully financed by money creation, Δ_t will be zero and there will be no externality or spillover to the government's budget constraint (equation 4.28). However, if the money supply does not expand in line with net asset purchases, Δ_t will be negative. Central bank asset purchases not fully financed by money creation would therefore mechanically impose a negative externality on the government's income. $\Delta_t < 0$ implies that the government's income is less than its expenditure (see equation 4.28). This could be interpreted as the monetary authority passing the burden of financing their asset purchases to the fiscal authority. In such a case, asset purchases today comes at the cost of future government spending or the need

[25] According to Mishkin (2013:428), during the financial crisis "the huge increase in the monetary base led primarily to a massive rise in excess reserves and bank lending did not increase".

[26] The Fed bought securities mostly from banks and other financial institutions, not directly from the household. The inclusion of a banking sector could perhaps allow this mechanism to be modelled more appropriately, but in the interest of a tractable model the banking sector is dispensed with for now.

to raise additional taxes. Conversely, if $\Delta_t > 0$, asset purchases by the central bank acts as a seignorage-type mechanism, representing an additional source of revenue for the government through an expansion in the money supply.

The central bank's holdings of long-term government bonds are a fraction d_t of the total amount issued, i.e. $B_{L,t}^{CB} = d_t B_{L,t}$. The remaining proportion $(1 - d_t)$ is available to households.[27] The household therefore holds those long-term bonds not held by the central bank. That is,

$$\begin{aligned} B_{L,t}^{H} &= (1 - d_t) B_{L,t} \\ &= B_{L,t} - d_t B_{L,t} \\ &= B_{L,t} - B_{L,t}^{CB} \end{aligned} \tag{4.37}$$

Log-linearising equation 4.37 yields

$$\hat{b}_{L,t}^{H} = \frac{\bar{B}_L}{\bar{B}_L^H} \hat{b}_{L,t} - \frac{\bar{B}_L^{CB}}{\bar{B}_L^H} \hat{b}_{L,t}^{CB} \tag{4.38}$$

Asset purchases by the central bank can be performed by varying the fraction d_t, modelled as the following AR(1) process:

$$\log\left(\frac{d_t}{\bar{\mathcal{D}}}\right) = \phi^{\mathcal{D}} \log\left(\frac{d_{t-1}}{\bar{\mathcal{D}}}\right) + \varepsilon_t^{\mathcal{D}} \tag{4.39}$$

where $\bar{\mathcal{D}}$ is the steady state fraction of long-term bonds held by the central bank $\left(\bar{B}_L^{CB}/\bar{B}_L\right)$ (Falagiarda, 2014), and $\varepsilon_t^{\mathcal{D}}$ is a shock to asset purchases with a mean of zero and standard deviation of $\sigma^{\mathcal{D}}$. If the central bank changes the fraction d_t, it ultimately changes the supply of long-term bonds available to the household, *ceteris paribus*. Based on Tobin (1969:26)'s logic, this will necessarily require that the "rates of return, on this and other assets, must change in a way that induces the public [households] to hold the new supply". Note, however, that the supply of long-term bonds (equation 4.29 or 4.31) is set by the government *independent* of the central bank's choice of d_t. This restriction is deliberately imposed to ensure that the government is not tempted to monetize the debt through over-issuing bonds and expecting that the central bank will simply carry the cost.

The log-linearised version of the central bank's asset purchase equation (4.39) is given by

[27] See Falagiarda (2014:311) and Harrison (2012:11).

$$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \varepsilon_t^{\mathcal{D}} \tag{4.40}$$

Finally, since the fraction d_t represents the central bank's holdings of long-term government bonds relative to the total amount issued, it can be expressed in log-linear terms as

$$\hat{d}_t = \hat{b}_{L,t}^{CB} - \hat{b}_{L,t} \tag{4.41}$$

It is important to note that the central bank's balance sheet does not function as a policy rule in the baseline model specification, since d_t is simply a stochastic process, and not endogenously determined in response to e.g. inflation and output. The purpose of the baseline model is simply to highlight the dynamics and transmission mechanisms following central bank balance sheet operations. The central bank's asset purchases will be converted to a formal balance sheet rule in section 4.4 below.

4.2.5 Consolidating the government sector

The government's budget constraint and the central bank's balance sheet (equations 4.28 and 4.35) are represented (in nominal terms) by:

$$B_{t-1} + \frac{B_{L,t-1}}{R_t} = P_t T_t + \frac{B_t}{R_t} + \frac{B_{L,t}}{R_{L,t}} + \Delta_t \tag{4.42}$$

$$\Delta_t = [M_t - M_{t-1}] - \left[\frac{B_{L,t}^{CB}}{R_{L,t}} - \frac{B_{L,t-1}^{CB}}{R_t} \right] \tag{4.43}$$

The government's debt burden is financed by tax income, new bond issuance and the residual arising from the central bank's balance sheet operations. If money creation exceeds net bond purchases, $\Delta_t > 0$, which can be interpreted as a seignorage type revenue for the government. Substituting the central bank's balance sheet (Δ_t) into the government's budget constraint and simplifying yields

$$P_t T_t = \left[B_{t-1} - \frac{B_t}{R_t} \right] + \left[\frac{B_{L,t-1}}{R_t} - \frac{B_{L,t}}{R_{L,t}} \right] + \left[\frac{B_{L,t}^{CB}}{R_{L,t}} - \frac{B_{L,t-1}^{CB}}{R_t} \right] - [M_t - M_{t-1}] \tag{4.44}$$

This shows that nominal taxes result from the net short-term debt burden, the net long-term debt burden, asset transactions by the central bank, and the change in the money supply. If $\Delta = 0$ (i.e. asset purchases fully financed by money creation) the final two terms drop out and the tax is simply determined by the government's net debt burden. But if $\Delta \neq 0$ central bank asset operations represent either an income or cost to the government. Substituting $B^{CB}_{L,t} = d_t B_{L,t}$ and $B^{CB}_{L,t-1} = d_{t-1} B_{L,t-1}$ (see section 4.2.4.2) in the consolidated budget constraint can be represented by

$$P_t T_t + \frac{B_t}{R_t} + \frac{B_{L,t}}{R_{L,t}} + M_t - M_{t-1} - \frac{d_t B_{L,t}}{R_{L,t}} + \frac{d_{t-1} B_{L,t-1}}{R_t} = B_{t-1} + \frac{B_{L,t-1}}{R_t} \tag{4.45}$$

Log-linearising the above and solving for $\hat{b}_t$ (see Appendix B.5) yields an expression for the short-term bond supply

$$\begin{aligned} \bar{B}\hat{b}_t = {} & \frac{\bar{B}}{\beta}\hat{b}_{t-1} + \bar{B}_L(1-\bar{\mathcal{D}})(\hat{b}_{L,t-1} - \beta\hat{b}_{L,t}) + (\bar{B} - \bar{B}_L(1-\bar{\mathcal{D}}))\hat{r}_t \\ & + \bar{B}_L\beta(1-\bar{\mathcal{D}})\hat{r}_{L,t} - \frac{\bar{M}}{\beta}(\hat{m}_t - \hat{m}_{t-1}) + \bar{\mathcal{D}}\bar{B}_L(\beta\hat{d}_t - \hat{d}_{t-1}) - \bar{T}\bar{P}(\hat{t}_t + \hat{p}_t) \end{aligned} \tag{4.46}$$

Equation 4.46 shows that the supply of short-term bonds is a function of the central bank's asset transactions, the money supply, outstanding short- and long-term debt, short- and long-term interest rates, the tax rate and the price level. It also suggests that asset purchases by the central bank ($\hat{d}_t > \hat{d}_{t-1}$) will, *ceteris paribus*, increase the supply of short-term bonds. This is consistent with the discussion in section 4.2.4.2, where it was suggested that central bank asset purchases not financed by money creation imposes a cost on the government. This cost can clearly be financed through additional issuance of short-term bonds. This also supports the argument that money creation can be interpreted as a seignorage type revenue, which allows the government to fund its expenditure by issuing fewer short-term bonds. Plugging in the calibrated parameters (see section 4.3.2 below) illustrates that short-term bond supply is a negative function of the current long-term bond supply, short-term interest rate, change in the money supply, taxes and the price level. It is a positive function of lagged short- and long-term bond supply, as well as the long-term interest rate and the change in asset purchases.

4.2.6 Closing the model

4.2.6.1 Goods market

The goods market will clear when the economy is at equilibrium, i.e. if the resource constraint holds. Following Falagiarda (2014:313), but ignoring government spending G, the economy's resource constraint is specified by

$$\begin{aligned} Y_t &= C_t + \frac{B^H_{L,t}}{P_t R_{L,t}} AC^L_t \\ \therefore Y_t &= C_t + \frac{B_{L,t}}{P_t R_{L,t}} \left[\frac{\phi_L}{2} \left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1 \right)^2 \right] Y_t \end{aligned} \tag{4.47}$$

The log-linearised resource constraint (Appendix B.5) reduces to

$$\hat{y}_t = \hat{c}_t \tag{4.48}$$

Aggregate consumption C_t is determined by the household's Euler equation (4.10). Substituting the resource constraint 4.48 into the Euler equation yields the dynamic IS curve

$$\hat{y}_t = E_t\left[\hat{y}_{t+1}\right] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{\beta \kappa_L \phi_L \bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) + E_t\left[\phi_t - \hat{\phi}_{t+1}\right] \tag{4.49}$$

4.2.6.2 The Phillips curve

The New Keynesian Phillips curve is given by (Appendix B.2.5):

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t[\hat{\pi}_{t+1}] + \hat{a}_t \tag{4.50}$$

where

$\kappa = \lambda \left(\sigma + \frac{\phi + \alpha}{1 - \alpha} \right)$.

4.2.7 Final model equations

The model contains the following endogenous variables, each of which are described by a linear equation:

Table 4.2: Endogenous variables

Variable	Description
y	Output / aggregate demand
c	Consumption
m	Money demand
n	Labour
w	Nominal wages
p	Price level
b	Short-term bonds (held by households)
b_L^H	Long-term bonds (held by households)
b_L^{CB}	Long-term bonds (held by central bank)
x	Fraction of long-term bonds held by central bank
b_L	Supply of long-term bonds
a	Technology
π	Inflation
t	Taxes
r	Short-term interest rate
r_L	Long-term interest rate
ys	Yield spread or term premium $(r_L - r)$

The model contains five possible shocks in the form of demand (preference) ε_t^d, supply ε_t^a, monetary policy ε_t^R, asset purchase $\varepsilon_t^{\mathcal{D}}$ and long-term bond supply ε_t^{BL} shocks.

Table 4.3: Exogenous variables (shocks)

Variable	Description
ε_t^d	Demand (preference) shock
ε_t^a	Supply (cost push) shock
ε_t^R	Monetary policy shock
ε_t^{BL}	LT bond supply shock
$\varepsilon_t^{\mathcal{D}}$	Asset purchase shock

Finally, the linear model equations are given by the following:

(1) **IS curve:** $\hat{y}_t = E_t\left[\hat{y}_{t+1}\right] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}])$
$$- \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) + E_t\left[\phi_t - \hat{\phi}_{t+1}\right] + \varepsilon^d_t$$

(2) **Taylor rule:** $\hat{r}_t = \phi^\pi\hat{\pi}_t + \phi^y\hat{y}_t + \varepsilon^R_t$

(3) **NK Phillips curve:** $\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t[\hat{\pi}_{t+1}] + \hat{a}_t$

(4) **Term structure:** $\hat{r}_{L,t} = \hat{r}_t + E_t[\hat{r}_{t+1}] + \phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t)(\beta\kappa_L + 1)$

(5) **LT bond supply:** $\hat{b}_{L,t} = \phi^{BL}\hat{b}_{L,t-1} + \varepsilon^{BL}_t$

(6) **Tax rule:** $\hat{t}_t = \frac{1}{\bar{T}\bar{P}}\left[\psi_1\bar{B}(\hat{b}_{t-1} - \hat{p}_t) + \beta\psi_2\bar{B}_L(\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t)\right]$

(7) **CB LT bond holdings:** $\hat{b}^{CB}_{L,t} = \hat{d}_t + \hat{b}_{L,t}$

(8) **LSAPs:** $\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \varepsilon^{\mathcal{D}}_t$

(9) **HH LT bond holdings:** $\hat{b}^H_{L,t} = \frac{\bar{B}_L}{\bar{B}^H_L}\hat{b}_{L,t} - \frac{\bar{B}^{CB}_L}{\bar{B}^H_L}\hat{b}^{CB}_{L,t}$

(10) **ST bonds:** $\hat{b}_t = \frac{1}{\beta}\hat{b}_{t-1} + \frac{\bar{B}_L}{\bar{B}}(1-\bar{\mathcal{D}})(\hat{b}_{L,t-1} - \beta\hat{b}_{L,t})$
$$+ (1 - \frac{\bar{B}_L}{\bar{B}}(1-\bar{\mathcal{D}}))\hat{r}_t + \frac{\bar{B}_L}{\bar{B}}\beta(1-\bar{\mathcal{D}})\hat{r}_{L,t}$$
$$- \frac{\bar{M}}{\bar{B}}\beta(\hat{m}_t - \hat{m}_{t-1}) + \bar{\mathcal{D}}\frac{\bar{B}_L}{\bar{B}}(\beta\hat{d}_t - \hat{d}_{t-1}) - \frac{\bar{T}\bar{P}}{\bar{B}}(\hat{t}_t + \hat{p}_t)$$

(11) **Production function:** $\hat{y}_t = \hat{z}_t + (1-\alpha)\hat{n}_t$

(12) **Cost push shock:** $\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon^a_t$

(13) **Wages:** $\hat{w}_t - \hat{p}_t = \phi\hat{n}_t + \sigma\hat{c}_t$

(14) **Resource constraint:** $\hat{y}_t = \hat{c}_t$

(15) **Money demand:** $\hat{m}_t - \hat{p}_t = \frac{1}{\delta}\frac{1}{1-\beta}\left(\sigma\hat{c}_t - \hat{\phi}_t - \beta E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1} - \hat{\phi}_{t+1}]\right)$

(16) **Price level:** $\hat{p}_t = \hat{\pi}_t + \hat{p}_{t-1}$

The dynamic IS curve (1) is given by 4.49 and is the result of substituting the economy's resource constraint (4.48) into the household's Euler equation 4.10. The Taylor rule (2) represents the central bank's response to inflation and output, and is given by 4.34. The New Keynesian Phillips curve (3) is standard, and links inflation to the output gap and expected future inflation (4.50). It also contains a cost-push shock $\hat{a}_t$.

The term structure (4) is derived by combining the household's first-order conditions for short- and long-term bonds (4.8 and 4.9), and solving for the long-term interest rate.

The real supply of long-term bonds (5) evolves according to a simple AR(1) process (4.31). Taxes (6) finance net bond issuance (4.32). The central bank's holdings of long-term bonds (7) are dictated by the fraction d_t, which represents the fraction of the total long-term bond supply held by the central bank. Varying this fraction (8) represents net asset purchases or sales by the central bank (4.40).

The household's holdings of long-term bonds (9) is the residual of the central bank's holdings of long-term bonds out of its total supply (4.38). The supply of short-term bonds (10) is the result of the government closing its budget constraint (equation 4.46) and equals the household's holdings of short-term bonds.

The firm's production function (11) is given by 4.21 and indicates that production is determined by technology and the amount of labour employed. The cost push shock (12) evolves according to a simple AR(1) process (4.13). The real wage (13) is a function of labour supply and consumption, and is the result of the household's optimisation problem (4.7).

The economy wide resource constraint (14) is determined by 4.48. The money demand equation (15) is the result of the household's optimisation problem (4.6). Finally, since inflation is simply the difference between the current and previous price level, the price level (16) can be expressed as the sum of the price level from the previous period and current inflation.

4.3 Results: Baseline model

4.3.1 Static solution

The flow of the model is as follows[28]: The goverment supplies long-term bonds, which are distributed between the central bank and the household. The central bank chooses its long-term bond holdings (by varying the fraction d_t), with the residual taken up by the household. The government raises taxes, based on its outstanding debt burden from the previous period, and then sets the supply of short-term bonds to meet its budget constraint. This supply is taken up by the household, whose bond portfolio is now realised, who finally decides – based on their net wealth and income after taxes and investment activities – on its optimal levels of consumption and money holdings aimed at maximising lifetime consumption.

[28]While all these activities theoretically take place at the same time t, it is useful to illustrate the sequence of decisions in this manner.

In steady state, the household's portfolio thus consists of long- and short-term bonds, which together generate future returns, represented by the interest rates on these bonds. These returns are subsequently used to finance future spending, investment and money holdings. In each period, the government supplies a quantity of new long-term bonds, which are taken up by the central bank and the household at the steady state ratio. There are thus no asset transactions between the central bank and the household in the steady state. The government pays interest and face value on maturing bonds, which are partially financed by the issuance of new bonds. The fiscal shortfall (where bond financing costs exceed the income from new bond issuance) is funded by a lump-sum tax on the household.

The equilibrium can be disturbed by the following shocks: (i) a demand (or preference) shock, which enters the IS curve; (ii) a supply shock, which enters the Phillips curve as an exogenous increase in inflation; (iii) a monetary supply shock, which increases the policy rate through the Taylor rule; (iv) a bond supply shock, which changes the quantity of long-term bonds available to households and the central bank; and (v) an asset purchase shock, which changes the fraction of the long-term bond supply taken up by the central bank. This study is primarily interested in the asset purchase shock (v). Since the supply of long-term bonds is essentially fixed in the short run, a positive asset purchase shock, in the form of higher long-term bond purchases by the central bank, will necessarily mean that the central bank purchases these assets from the household. Such a transaction will change the household's holdings of real money balances and its portfolio mix, disturbing the preferred habitat, and pass through to consumption decisions and ultimately aggregate output and the price level. Furthermore, the change in relative supplies of and demand for these assets will affect its yields and prices, thereby changing the composition and cost of outstanding government debt.

4.3.2 Calibration

The model's standard parameters are described in Table 4.4, while the key parameters specifically related to the central bank's asset purchases are described in Table 4.5 below:

TABLE 4.4: Calibration of standard parameters

Parameter	**Value**	**Description**
Preferences and technology		
β	0.994	Discount rate
σ	2	CRRA
ϕ_L	0.01	Portfolio adjustment frictions
δ	7	Elasticity of money demand
ϕ	1	Elasticity of labour supply
χ	8	Labour disutility
α	0.36	Capital share of production
ρ_z	0.95	AR(1) technology coefficient
ρ_a	0.5	Persistence of cost push shock
θ	0.75	Calvo sticky price parameter
ε	6	CES
κ	0.556	Slope of the Phillips curve
Fiscal and monetary policy		
ψ_1	0.3	Fiscal response to short-term debt
ψ_2	0.3	Fiscal response to long-term debt
ϕ^{π}	1.5	Monetary policy response to inflation
ϕ^{y}	0.5	Monetary policy response to output

TABLE 4.5: Calibration of key parameters and steady states

Parameter	**Value**	**Description**
$\bar{B} + \bar{B}_L$	0.496	Total debt to GDP
$\bar{B}$	0.188	ST debt to total debt
$\bar{B}_L$	0.308	LT debt to total debt
$\bar{B}_L^H$	0.250	LT debt held by households
$\bar{B}_L^{CB}$	0.058	LT debt held by central bank
$\phi^{\mathcal{D}}$	0.83	Persistence of asset purchases
ϕ^{BL}	0.75	AR(1) bond supply coefficient

The standard parameters are calibrated in line with the general approach in the literature. The majority of the calibrations, and all of the key calibrated values, were obtained from Falagiarda (2014)'s calibration for the US economy. The Calvo probability of firms not being able to change price $\theta = 0.75$ is taken from Harrison (2012:17), and is guided by the assumption that “firms change prices on average once a year”. The CES (constant elasticity of consumption) parameter $\varepsilon = 6$ is taken from Galí (2015).[29] The Taylor rule parameters $\phi^{\pi} = 1.5$ and $\phi^{y} = 0.5$ are widely used. $\chi = 8$ is taken from Kavli (2015).

Normalising output to 1 (Falagiarda, 2014:315) enables us to solve for the other steady state variables, either through our steady state equations or calculating relevant ratios based on real world data. A collection of all the steady state equations consists of

$$
\begin{aligned}
&P_t = P_{t-1} = P_{t+1} = \bar{P} \;\Rightarrow\; \pi_t = \pi_{t-1} = \pi_{t+1} = \bar{\pi} = 1 \\
&(\bar{M}/\bar{P})^{-\delta} = \bar{C}^{-\sigma}(1-\beta) \;;\; \bar{N}^{\phi} = \chi^{-1}(\bar{W}/\bar{P})\bar{C}^{-\sigma} \\
&\kappa_L = \bar{B}_L^H/\bar{B} \;;\; \bar{R} = \frac{1}{\beta} \;;\; \bar{R}_L = \frac{1}{\beta^2} \\
&\bar{Y} = \bar{A}\bar{N}^{1-\alpha} \;;\; \bar{Y} = \bar{C} \\
&M\bar{C}^r = \frac{\varepsilon - 1}{\varepsilon} \;;\; M\bar{P}N = \bar{A}(1-\alpha)\bar{N}^{-\alpha}
\end{aligned}
$$

Combining these equations with the benchmark calibration of the other parameters (Tables 4.4 and 4.5), the model's steady state parameters can be derived (Table 4.6):

TABLE 4.6: Steady state values

Parameter	**Value**	**Description**
$\bar{Y}$	1	Output (normalised to 1)
$\bar{C}$	1	$\bar{C} = \bar{Y}$ from steady state
$\bar{T}$	0.1972	Ratio of taxes-output
$\bar{N}$	0.3/1.3	Ratio of market to non-market activities
$\bar{R}^{\dagger}$	1.006036	Gross money market rate
$\bar{R}_L{}^{\dagger}$	1.012109	Long-term bond rate
$\kappa_L{}^{\ddagger}$	1.329787	SS ratio of LT to ST bonds
$\bar{\mathcal{D}}^{\star}$	0.188312	SS fraction of LT bonds held by the CB

† Calculated from $\bar{R} = 1/\beta$ and $\bar{R}_L = 1/\beta^2$

‡ Calculated as $\kappa_L = \bar{B}_L^H/\bar{B}$

* Calculated as $\bar{\mathcal{D}} = \bar{B}_L^{CB}/\bar{B}_L$

[29] Harrison (2012:17) uses a CES of 5.

From the steady state equation we have $\bar{Y} = \bar{C} = 1$. The values for $\bar{T}$, $\bar{N}$ and $\bar{X}$ were obtained from Falagiarda (2014). Finally, the price level is also normalised to 1 ($\bar{P} = 1$).[30]

The model will likely be highly sensitive to the calibration of κ_L (the household's 'preferred habitat', see discussion in section 4.2.1). The higher (lower) κ_L, i.e. the more long-term bonds relative to short-term bonds in the household's steady state bond portfolio, the more (less) sensitive the household would be to an asset purchase shock. There is some divergence in the literature on this variable, given that different data and samples are used: Falagiarda (2014) assigns $\kappa_L \approx 1.33$, while Harrison (2012) uses $\kappa_L = 3$. This will be analysed in more depth in Section 4.5 below.

[30] This follows Falagiarda (2014)'s normalising the price level to 1 ($\Pi = \beta\bar{R} = 1$). Even though this yields an unrealistically high simulated steady state money supply, the fact that the model is specified in terms of log-deviations from the steady state implies that the level of the steady states are of no consequence to the dynamics.

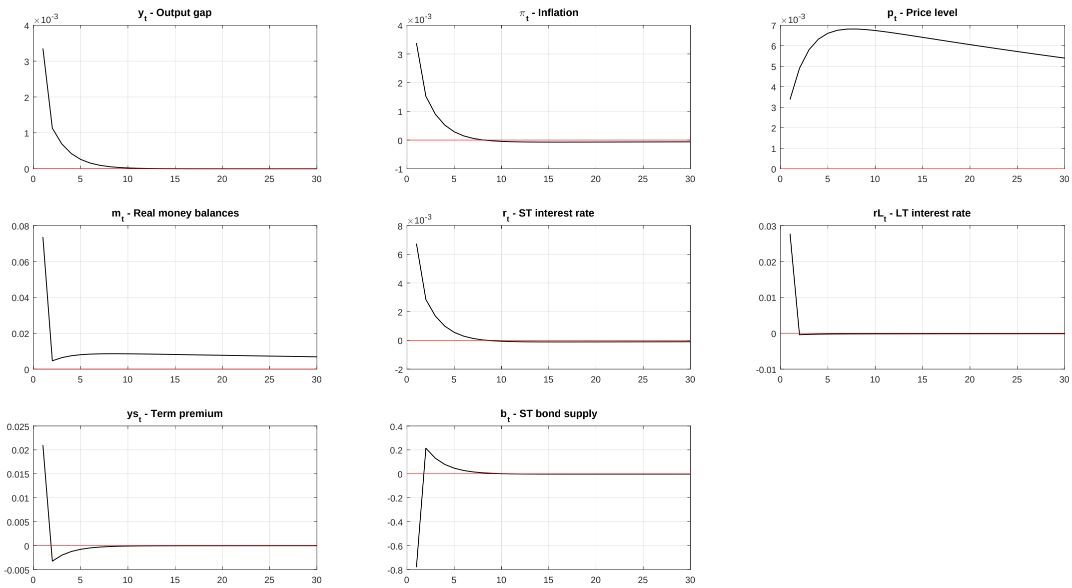
4.3.3 Impulse response functions

The baseline model estimates the impact of various shocks when the monetary policy response function is exclusively represented by the Taylor interest rate rule. All impulse response functions are shown as percentage deviations from the steady state, while the horizontal axis represents quarters. All shocks are briefly discussed here to illustrate the model's transmission mechanisms. However, the majority of the discussion to follow will be devoted to the asset purchase shock as the primary focus of this study, and its interaction with the Taylor rule.

4.3.3.1 Demand (preference) shock

A positive demand shock through the IS curve, illustrated in figure 4.1, has the immediate effect of increasing output, inflation and the price level. In response, the central bank increases the short-term interest rate through the Taylor rule in an attempt to cool down the economy. The demand for real money balances also increases. There is no effect on the supply or relative holdings of long-term bonds. The increase in the short-term interest rate implies that it is now more expensive for the government to issue short-term bonds, while the increase in the money supply (matching the increase in money demand) represents seignorage income to the government. As a result, the supply of short-term bonds falls (but this is short-lived). Since the household's short-term bond holdings now fall, but there is no trade in long-term bonds (therefore no change in long-term bond holdings), the term premium increases (equation 4.11). The long-term interest rate consequently increases by more than the short-term interest rate, reflected in a steeper yield curve.

FIGURE 4.1: Impulse response functions – Demand shock

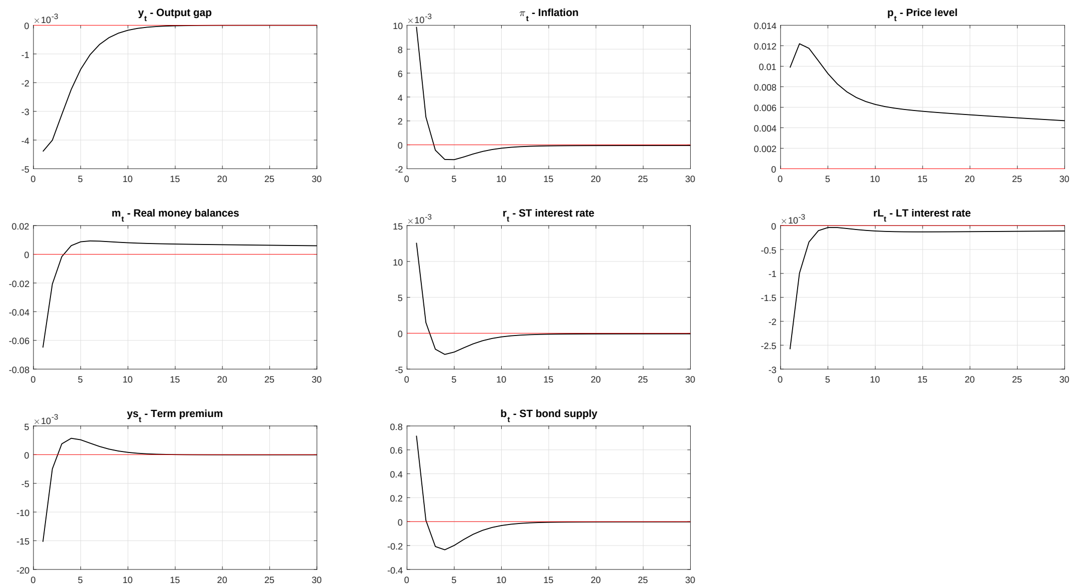


4.3.3.2 Supply shock

A negative supply (cost-push) shock, which enters through the Phillips curve (equation 4.50), increases inflation and the price level while reducing output. Through the Taylor rule, the short-term interest rate is increased to combat the inflationary pressure.[31] Money demand falls in line with output, while there is no effect on the supply or relative holdings of long-term bonds. In contrast to the demand shock, the short-term bond supply *increases* following a supply shock. The fall in the money supply implies a drain on government income (see equation 4.28 and the discussion in section 4.2.4.2), which would suggest the issue of additional short-term bonds to make up the shortfall. This dominates the effect of higher financing costs, due to the increase in the short-term interest rate, and the net effect is an increase in the short-term bond supply. The long-term interest rate only gradually responds to the supply shock. The initial increase in the short-term interest rate is almost entirely countered by a fall in the term premium due to the increase in short-term bond supply. Thereafter, the term premium increases as the short-term bond supply falls. The larger initial increase in the short-term relative to the long-term interest rate flattens the yield curve.

[31]The standard approach in the New-Keynesian literature is to specify a larger weight on inflation than the weight on output in the Taylor rule. In such a setting the central bank would rather apply contractionary policy to combat inflation than apply expansionary policy to counter the fall in output. This is due to the consensus that inflation, if left unchecked, would permanently harm output in the longer term.

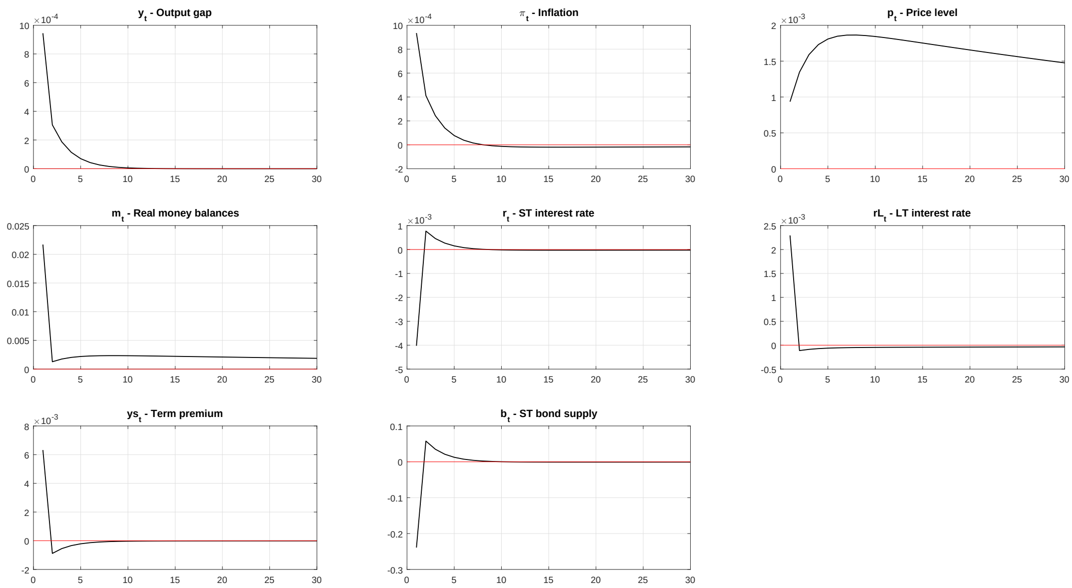
FIGURE 4.2: Impulse response functions – Supply shock



4.3.3.3 Monetary policy shock

The response to a monetary policy shock (an increase in the short-term interest rate via the Taylor rule, equation 4.34) is consistent with the standard New-Keynesian model. Figure 4.3 illustrates the impact of a decrease in the short-term interest rate. Output, money demand, inflation and the price level increase. The short-term interest rate is subsequently increased by the central bank in response to this expansion. Again, there is no effect on the supply or relative holdings of long-term bonds. The contemporaneous effect of the increase in the money supply is to decrease the supply of short-term bonds. However, this effect is virtually immediately countered by the subsequent increase in the short-term interest rate. Similar to the effect of a supply shock, the short-term bond supply falls, which increases the term premium; this effect is, however, not persistent. The initial reduction in the short-term bond supply, coupled with the increase in the short-term relative to the long-term interest rate, contribute to an immediate steepening of the yield curve.

Figure 4.3: Impulse response functions – Monetary policy shock

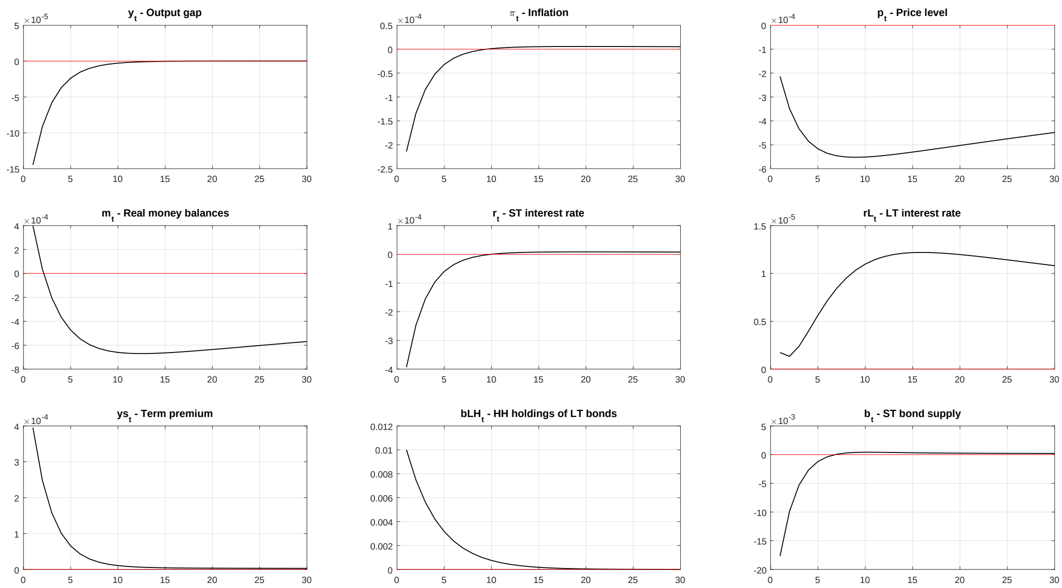


4.3.3.4 Bond supply shock

An increase in the supply of long-term bonds by the government (equation 4.31) has the immediate effect of lowering consumption and output (figure 4.4). Both the central bank and the household now takes up additional long-term bonds, thereby lowering the household's current ability to consume. This pushes down inflation and the price level, which leads the central bank to respond by lowering the short-term interest rate through the Taylor rule. Money demand falls. The excess supply of long-term bonds causes a fall in its price, which is captured by a higher term premium and an increase in the long-term interest rate. This leads to a steeper yield curve.[32] The short-term bond supply falls as a result of the increased long-term bond supply, which leads to an adjustment in the household's asset portfolio.

[32]This prediction is consistent with the empirical literature on the yield curve. Long-term government borrowing, by way of issuing long-term bonds, can be expected to increase the productive capacity of the economy, which would increase the slope of the yield curve. See e.g. Mishkin (2013).

FIGURE 4.4: Impulse response functions – Long-term bond supply shock



4.3.3.5 Asset purchase shock

The effect of an increase in the amount of long-term bonds purchased by the central bank (by increasing the fraction d_t from equation 4.41) is illustrated in figure 4.5. The magnitude of the asset purchase shock is set equal to 1, implying an increase of 100% in the amount of long-term bonds held by the central bank. This results in a fall in the amount of long-term bonds held by the household by 23.2%, which is in line with the empirical evidence and virtually identical to the result in Falagiarda (2014).[33] The equivalent response – despite the substantial differences in constructing this model relative to the Falagiarda (2014) model – is highly encouraging, as it suggests that this is a quite robust framework within which central bank balance sheet operations could be analysed. The liquidity created by virtue of the household now holding fewer long-term bonds – having been partially relieved thereof by the central bank – can now be allocated toward additional consumption, money holdings[34] and investment in short-term bonds. Output and inflation duly increases, which is in turn countered by the central bank increasing the short-term interest rate through the Taylor rule.

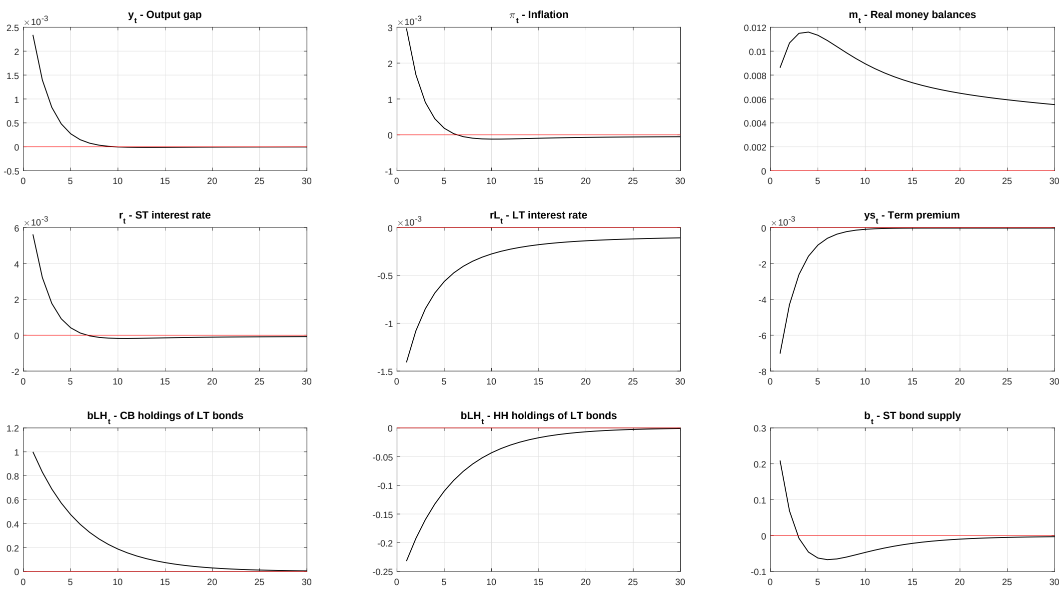
[33] Falagiarda (2014:317) finds that doubling the central bank's holdings of long-term bonds "reduces the amount of long-term bonds at the disposal of households by around 23%". In effect, this transaction is really nothing more than a reallocation of long-term bonds from the household to the central bank in line with the steady state ratio of these holdings.

[34] Several empirical studies, such as those discussed in Chapters 2 and 3 above, found that the money supply or monetary base increased as a result of central bank asset purchases.

The increased demand for long-term bonds by the central bank drives up its price, which is illustrated by a fall in the long-term interest rate. The lower interest rate on long-term bonds lowers the government's future debt burden, which allows the government to gradually issue fewer short-term bonds to finance this debt. The higher short-term interest rate makes issuing short-term bonds somewhat more expensive, which will also contribute to the decrease in the supply of short-term bonds. However, given the inertia present in the short-term bond supply (equation 4.46), this is necessarily a much slower adjustment. The initial gradual fall in short-term bond supply, mirroring the gradual fall in the short-term interest rate, is consistent with Harrison (2012:3)'s suggestion that "reductions [increases] in the short-term nominal interest rate reduce [increase] the relative supply of short-term bonds".

Finally, the yield curve flattens as short rates increase and long rates fall. Specifically, the short-term interest rate increases by 19 basis points while the long-term interest rate falls by 19 basis points, implying a net fall in the term premium of 38 basis points. Falagiarda (2014) simulates a ZLB by imposing a highly restrictive smoothing parameter in the Taylor rule, thus preventing the short-term interest rate from responding to the expansionary effect of asset purchases. His results include a 47 basis point fall in the term premium, which is solely due to a commensurate fall in the long-term interest rate. In this model the short-term interest rate response counteracts much of the potential dynamics in the long rate visible in Falagiarda (2014) and other models where the short-term interest rate is not allowed to respond.

FIGURE 4.5: Impulse response functions – Asset purchase shock



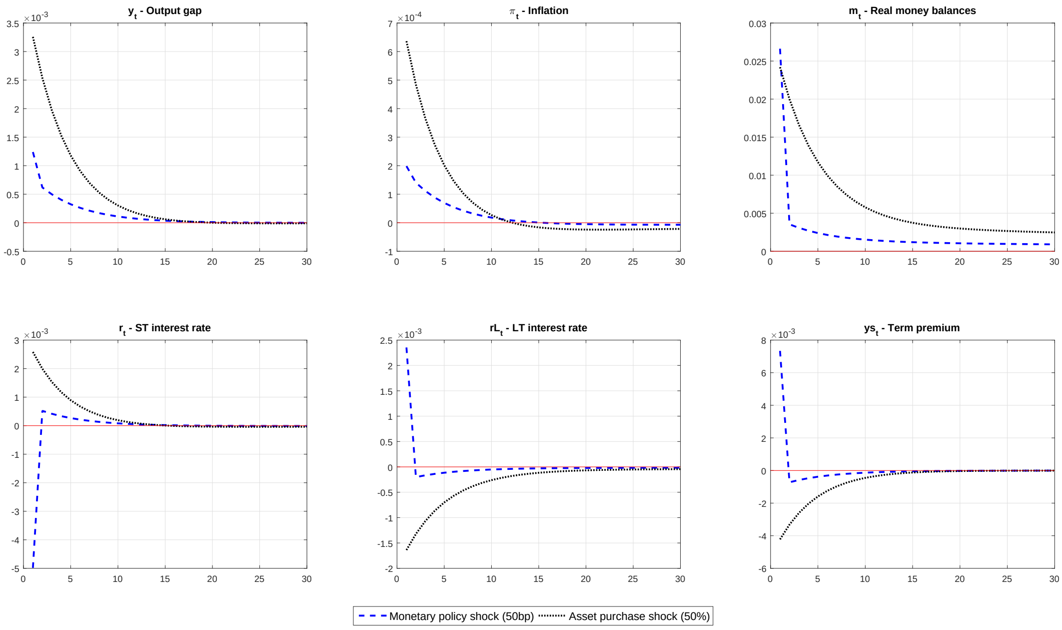
Clearly, asset purchases (sales) increases (decreases) both output and inflation. In that sense, an asset purchase shock has a similar effect as a demand shock. In both these cases, the central bank responds by increasing (decreasing) the short-term interest rate in order to counteract the expansionary (contractionary) effect on the economy. Furthermore, in this model balance sheet operations in itself are capable of stimulating the economy (i.e. without requiring a reduction in short rates), which further supports the view that balance sheet policy alone could be an effective substitute for expansionary interest rate policy for an economy stuck at the ZLB. In addition, balance sheet policy could potentially be a useful policy tool even in "normal" (i.e. non-ZLB) times. This will be discussed in detail in section 4.4 below. The effect on the term premium from this model is more comparable to Krishnamurthy and Vissing-Jorgensen (2011a) and Chen et al. (2012), who estimate, respectively, a fall in 33 and 30 basis points in the term premium as a result of QE2 in the USA.

4.3.4 Two conflicting policy levers?

The interaction between two separate monetary policy tools (the short-term interest rate and balance sheet operations as illustrated in figure 4.5 above) are of particular interest. The way the model is set up here (enabling an immediate adjustment in the short-term interest rate through the Taylor rule), implies that the central bank is essentially pulling two policy levers in seemingly opposing directions at the same time. On the one hand, LSAPs expand the economy, while the mechanical response is to tighten the short-term interest rate to counter this expansion. Expansionary balance sheet operations might therefore, to some degree, be countered by contractionary interest rate policy, and *vice versa*. Balance sheet operations are clearly effective at stimulating the economy, while the long-term effect on inflation is potentially favourable (inflation eventually falls below its steady state path, albeit marginally). Conversely, expansionary balance sheet policy could potentially be utilised to soften the undesirable impact of contractionary interest rate policy. For example, if the central bank applies contractionary interest rate policy in response to a negative supply shock (figure 4.2), the negative impact on output might to some degree be ameliorated by expansionary LSAPs. However, expansionary LSAPs are in turn associated with a further increase in the short-term interest rate (figure 4.5), which would require the central bank to walk a fine tightrope in implementing the optimal policy response mix.

In reality, during much of the Fed's QE programmes the US economy and financial markets were so fragile that short rates hardly ever responded to the expansionary effect of LSAPs. Effectively, the Fed only had one policy lever available[35], which trivialises the question of interaction during the crisis. However, exploring this interaction for non-crisis times when both policy levers are available might be valuable. The transmission of these two policy levers is neatly captured in figure 4.6, by comparing figure 4.5 with an expansionary Taylor rule shock where only one policy lever is utilised (figure 4.3).

FIGURE 4.6: Expansionary monetary policy shock vs. asset purchase shock



A 50% increase[36] in central bank asset purchases has a larger effect (more than double) on output and inflation than a 50 basis point decrease in the short-term interest rate. Under the balance sheet shock, output and inflation return to their steady states within 23 and 14 quarters, respectively. Under the interest rate shock, output and inflation return to their steady states within 29 and 16 periods, respectively. The interest rate shock is therefore somewhat more persistent relative to both output and inflation. The impact on short- and long-term interest rates are initially the opposite of one another, but this effect is short-lived. After two quarters both rates start trending together. The same applies to the term premium. This interaction is explored in more detail in section 4.4, where a formal balance sheet rule will be constructed.

[35]The Fed's objective during its various QE programmes was ultimately to stimulate the economy and increase inflation. Therefore, they really had no incentive to counter their expansionary balance sheet policies with contractionary interest rate policy. The Taylor rule was effectively "switched off" for all intents and purposes.

[36]For the purpose of illustration, the asset purchase shock is reduced to 50% from 100% in figure 4.5.

4.4 Extending the baseline model: Balance sheet rules

The simulations on the baseline model demonstrated the model's transmission mechanisms. These findings are broadly consistent with the literature and empirical results, which suggests that the baseline model could be utilised to, in the spirit of Cúrdia and Woodford (2011:54), analyse " 'unconventional' dimensions of policy alongside traditional interest-rate policy." From the results and discussion of Figure 4.5 above, the following can be summarised:

1. LSAPs have an expansionary effect on the economy.
2. LSAPs reduce the long-term interest rate. This, coupled with an increase in the short-term interest rate (Taylor rule response), flattens the yield curve.
3. LSAPs expand the money supply.

All of the above, with the exception of the short-term interest rate effect, are consistent with the empirical observations around the Fed's LSAPs (see chapters 2 and 3). Short-term interest rates, however, did not rise in response to the expansionary effect of LSAPs, presumably because the US economy and financial markets were still very fragile for quite some time after the height of the crisis. The way the Taylor rule is specified here, however, allows the short-term interest rate to immediately react to changes in economic conditions following asset purchases.[37] This specification is deliberate, as it allows balance sheet policy to be considered alongside interest rate policy during "normal" times, and we do not wish to simulate a ZLB.

4.4.1 Asset purchases as monetary policy instrument

The central bank's balance sheet, as discussed in section 4.2.4.2 above, functions in conjunction with the Taylor rule. However, central bank asset purchases have thus far been modelled purely as an exogenous process (see, for example, Figure 4.5), and only produces dynamics if a shock is introduced. In this sense, therefore, the central bank balance sheet does not function as an instrument of monetary policy and asset purchases are simply a stochastic process, with its persistence dependent on the AR(1) parameter. For the central bank balance sheet to qualify as an *instrument* an explicit rule is required, where net asset purchases are endogenised to respond to developments in

[37] An alternative specification can be seen in Falagiarda (2014:321), who includes a large interest rate smoothing parameter in his Taylor rule specification which gives rise to extreme inertia in the short-term interest rate. This specification is highly effective at shutting down a short-term interest rate response, simulating a ZLB environment.

other macroeconomic variables. This problem has two dimensions: (i) to which macroeconomic variable(s) should the balance sheet react?, and (ii) what are the appropriate parameters within which such a rule would be feasible? One also has to balance such a rule with the existing Taylor rule, either by modifying the Taylor rule to find a joint parameter space within which both rules can function in a complementary manner, or concentrate each rule on a separate target.[38] In the latter scenario, the Taylor rule could e.g. be applied to stabilise inflation, while the balance sheet rule is applied to stabilise output.

Given the relatively larger weight on inflation than on output in the standard formulation of the Taylor rule ($\phi^{\pi} = 1.5$ to $\phi^{y} = 0.5$), the short-term interest rate (instrument) mainly attempts to control the inflation rate. This is most evident in the case of an adverse supply shock (causing a fall in output and an increase in inflation). The Taylor rule predicts a mechanical tightening of the short-term interest rate in response to a supply shock, which, while combatting the higher inflation, further suppresses output. Given the empirical evidence that "successful disinflations entail a period of output contraction" (Ascari and Ropele, 2013:77), the Taylor rule therefore in reality represents an instrument which predominantly works on one target: the desired inflation rate. (Admittedly, there is an extensive literature on the adverse effect of inflation on output in the long run. By aggressively targeting inflation in the short run the central bank also implicitly targets output in the medium to long run, which contributes to the eventual stabilisation of output.) If, however, a second instrument can be introduced which could target output in a complementary manner, the central bank might be able to achieve more desired levels of both inflation and output (e.g. by reducing the sacrifice ratio). This approach has its roots in the Tinbergen principle (Klein, 2004), where an equal number of instruments (in this case, the short-term interest rate and the central bank balance sheet) should be applied to the number of desired targets (inflation and output).[39]

Balance sheet operations could therefore in this framework be utilised as a monetary policy reaction to boost a flagging economy (net asset purchases), or potentially cool down an overheating economy (net asset sales). A balance sheet rule could therefore be constructed in the same vein as a Taylor rule which responds to deviations of inflation and output from its steady state values in the following manner:

[38] For example, Ascari and Ropele (2013:80) do not include the output gap in their Taylor rule, thus allowing the central bank's interest rate rule to focus on inflation only.

[39] The Tinbergen principle technically applies to *independent* targets. While inflation and output are obviously not entirely independent, this does not detract from the fact that the availability of an additional instrument could improve the central bank's optimal policy mix.

$$d_t = f\left(\frac{\pi_t}{\overline{\pi}}, \frac{Y_t}{\overline{Y}}\right) \tag{4.51}$$

Furthermore, if the balance sheet rule could be combined with the Taylor rule, the central bank could simultaneously set both B_L^{CB} and R_t (i.e. controlling both ends of the yield curve, the short-term interest rate directly and the long-term interest rate indirectly), allowing the key policy variables to be endogenously determined.

The simplest formulation of a balance sheet rule could be to suggest that asset purchases or sales could augment the short-term interest rate in the Taylor rule. That is, instead of responding to deviations in output and inflation by setting the short-term interest rate, the central bank responds only to inflation through the interest rate, while responding to output by way of direct asset market intervention. While the Taylor rule's optimal parameters ϕ^π and ϕ^y are largely agreed upon, these parameters (say γ^π and γ^y) are unknown for a balance sheet rule. An important focus of this section is therefore to establish the appropriate parameter space within which such a balance sheet rule could potentially be applied.

4.4.2 Evaluating alternative monetary policy rules

According to Woodford (2003:381), there is "a fair amount of consensus in the academic literature that a desirable monetary policy is one that achieves a low expected value of a discounted loss-function, where the losses each period are a weighted average of terms quadratic in the deviation of inflation from a target rate and in some measure of output relative to potential." He subsequently proposes the following loss function[40] for a basic Calvo-pricing New-Keynesian model (Woodford, 2003:400) :

$$L_t = \pi_t^2 + \lambda(y_t - y^*)^2 \tag{4.52}$$

where the relative weight on output gap variability is given by λ. If $\lambda = 1$, the central bank is neutral on its preferences. It is assumed here that we have an inflation targeting central bank which attaches a higher weight to inflation than output, and therefore we set $\lambda = 0.5$. Finally, since the model is constructed here in terms of log-deviations from steady state, the loss function takes the form

$$L_t = \hat{\pi}_t^2 + \lambda \hat{y}_t^2 \tag{4.53}$$

[40] This form of the loss function is widely used.

This simple loss function enables us to calculate the value of L_t for any given monetary policy rule or set of rules, and distinguish across parameters or targets, to ultimately determine the "best" (or least costly) monetary policy framework for a given model economy.

4.4.3 Instruments and targets: Towards a balance sheet rule

4.4.3.1 Balance sheet rule to stabilise output and/or inflation

Balance sheet policy could potentially be applied instead of, or in conjunction with, the Taylor rule. In the latter case, the central bank would simultaneously have control over both the short and long ends of the yield curve. Asset purchases were shown to have an expansionary effect on the economy, by increasing both output and inflation, while asset sales would have the opposite effect. It follows then that balance sheet policy should be applied counter-cyclically, e.g. the central bank would respond to a positive demand shock (see figure 4.1 above) by selling long-term bonds to the household to cool down the economy. A balance sheet rule can therefore be constructed in the form

$$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \gamma^{\pi} \hat{\pi}_t + \gamma^{y} \hat{y}_t + \varepsilon_t^{\mathcal{D}} \tag{4.54}$$

where the counter-cyclical nature of the response dictates that $\gamma^{\pi} < 0$ and $\gamma^{y} < 0$. This equation would then replace equation 4.40 in the model specification, and could be used in conjunction with the Taylor rule

$$\hat{r}_t = \phi^{\pi} \hat{\pi}_t + \phi^{y} \hat{y}_t + \varepsilon_t^{R} \tag{4.55}$$

as a suite of policy tools available to the central bank.

Table 4.7 below illustrates three potential monetary policy responses, with various combinations of the four monetary policy parameters ϕ^{π}, ϕ^{y}, γ^{π} and γ^{y}. The first rule is (i) the standard Taylor rule response, where central bank asset purchases are present as a purely exogenous process (i.e. identical to the baseline model constructed and discussed above). The second rule is a (ii) dual policy rule, where the short-term interest rate responds to inflation only, while central bank asset purchases respond to output only. Finally, a hybrid rule (iii) is proposed, where the short-term interest rate and the central bank balance sheet are combined into one response. It should be noted that the aim is not to change the Taylor rule, but rather to find a parameter space for the balance sheet rule to complement existing interest rate policy; therefore the Taylor rule parameters remain unchanged.

Table 4.7: Comparing policy rules

	ϕ^{π}	ϕ^{y}	γ^{π}	γ^{y}	Functional form (no shocks)	
(i) Taylor rule	1.5	0.5	0	0	$\hat{r}_t = \phi^{\pi}\hat{\pi}_t + \phi^{y}\hat{y}_t$	$\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1}$
(ii) Dual (targeted) rule	1.5	0	0	-10	$\hat{r}_t = \phi^{\pi}\hat{\pi}_t$	$\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \gamma^{y}\hat{y}_t$
(iii) Hybrid rule	1.5	0.5	-1.5	-10	$\hat{r}_t = \phi^{\pi}\hat{\pi}_t + \phi^{y}\hat{y}_t$	$\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \gamma^{\pi}\hat{\pi}_t + \gamma^{y}\hat{y}_t$

The persistence paramater $\phi^{\mathcal{D}}$ ensures some degree of stability in the balance sheet rule. However, the degree of persistence in asset purchases could play an important role in its efficiency. The sensitivity of the model relative to changes in this parameter, as well as some other parameters, are rigorously explored in section 4.5 below. For now we set $\phi^{\mathcal{D}} = 0.83$ as in the baseline model above.

Figures 4.7 to 4.10 below illustrate the response of these three distinct monetary policy rules to various shocks. The green line represents the standard Taylor rule response, and serves as a useful benchmark against which alternative policy rules can be evaluated. The dotted black line represents a dual policy rule, where the short-term interest rate responds to inflation only and the central bank balance sheet responds to output only. Finally, the dashed blue line represents a hybrid policy rule, where both the short-term interest rate and the balance sheet are allowed to respond to inflation and output. The various policy rules are evaluated against the loss function 4.53, based on simple parameter spaces, with the results reported in Table 4.8.

Table 4.8: Policy rules and loss function

Rule	ϕ^{π}	ϕ^{y}	γ^{π}	γ^{y}	ε_t^{d} $(\times 10^{-5})$	ε_t^{a} $(\times 10^{-3})$	ε_t^{R} $(\times 10^{-8})$	ε_t^{BL} $(\times 10^{-7})$
(i)	1.5	0.5	-	-	3.667	1.012	2.814	1.510
(ii)	1.5	-	-	-10	6.171	1.183	4.714	2.726
(iii)	1.5	0.5	-1.5	-10	2.902	0.977	2.234	1.141

Figure 4.7: Optimal policy rules: Demand shock

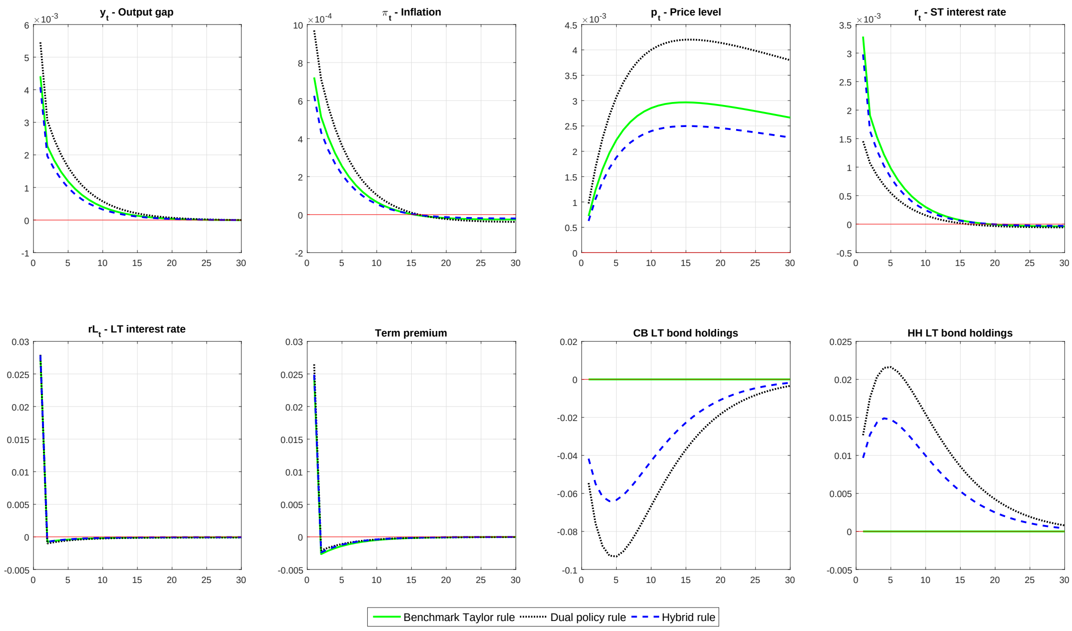


Figure 4.8: Optimal policy rules: Supply shock

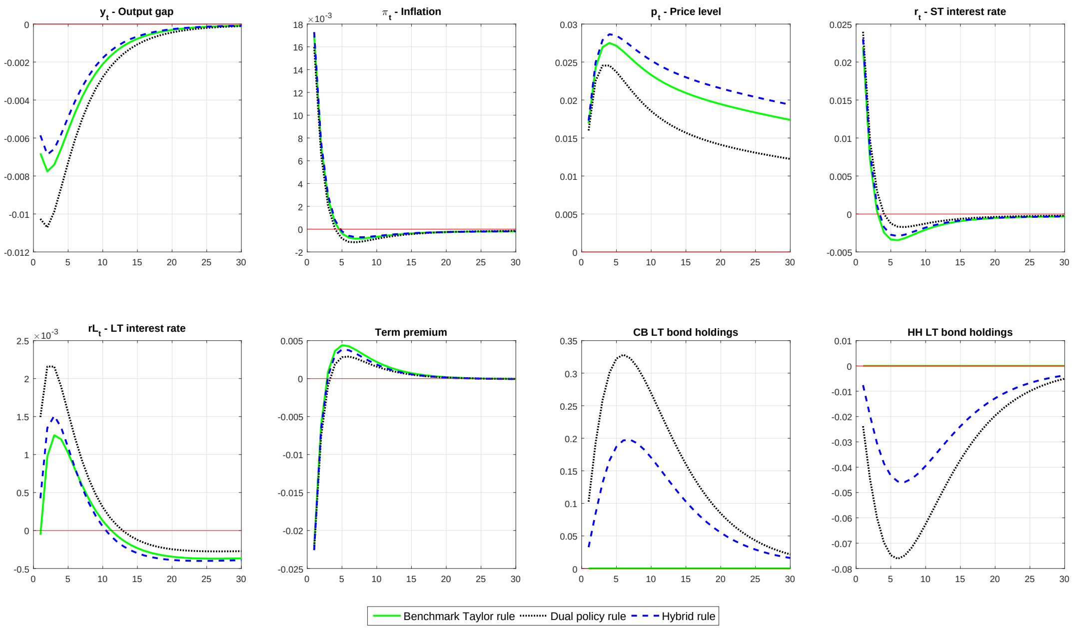


FIGURE 4.9: Optimal policy rules: Monetary policy shock

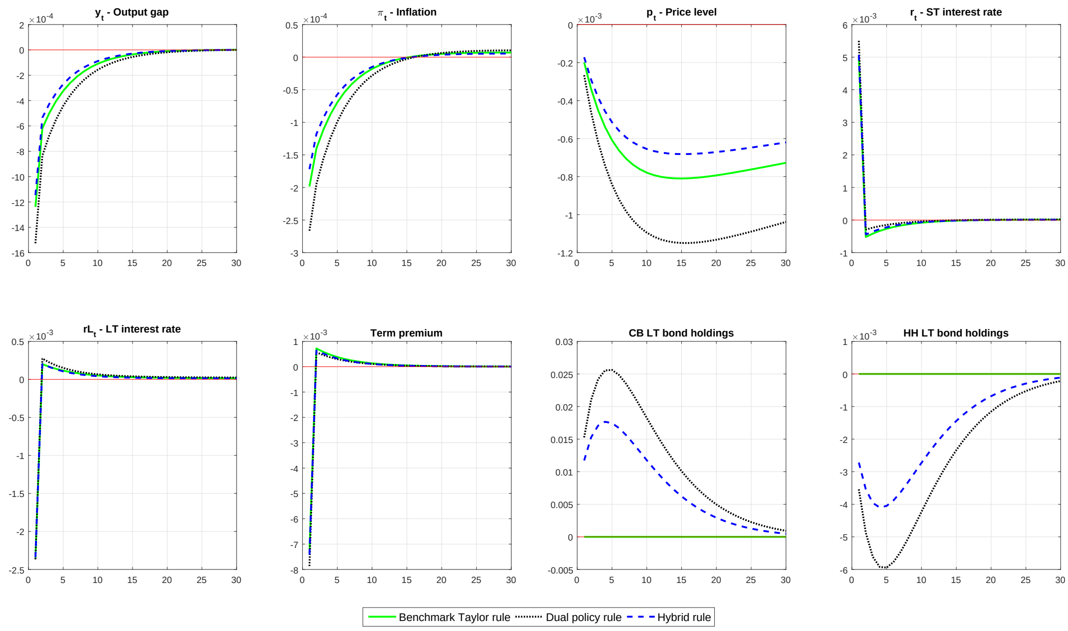
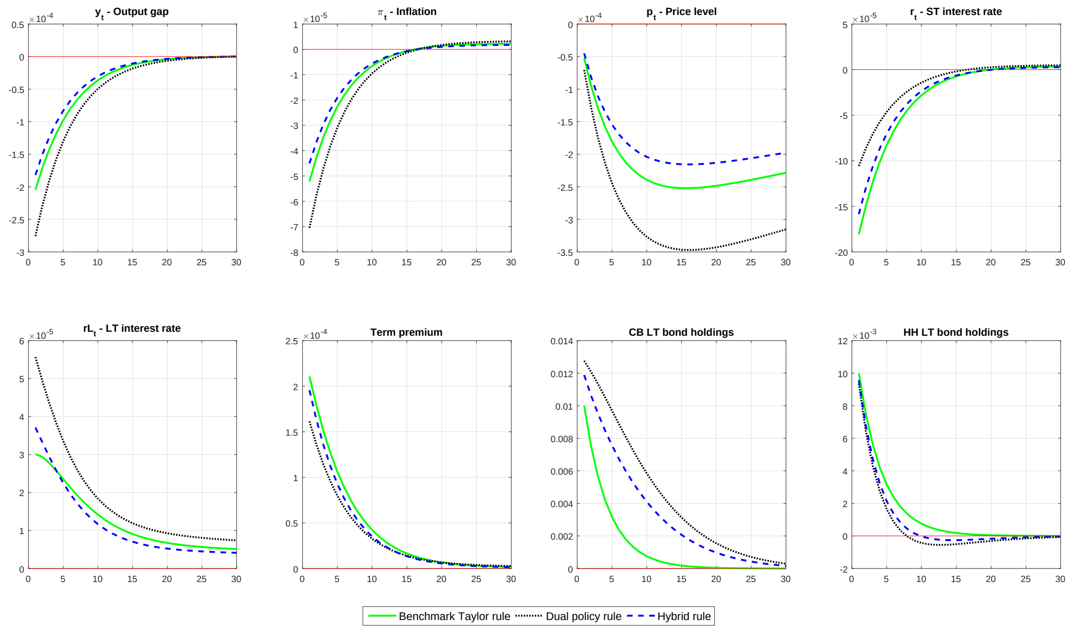


FIGURE 4.10: Optimal policy rules: LT bond supply shock



Based on the results from Table 4.8, the dual rule clearly performs worse than the Taylor rule, in that it produces a higher welfare loss under each shock. On the other hand, the hybrid rule is superior to the Taylor rule. These results are substantiated by Figures 4.7 to 4.10. The volatility of both output and inflation are demonstrated to be smallest under the hybrid rule, irrespective the type of shock. The variability in output and inflation is therefore minimised under the hybrid rule, with limited impact on the adjustment paths of other variables of interest. This would suggest that the central bank could utilise both instruments in order to achieve its target, but at a somewhat smaller cost to the economy than under only an interest rate rule.

4.5 Sensitivity analyses

4.5.1 Weight on output in the loss function

In the loss function (4.53), it was assumed that the central bank places a higher weight on inflation than on output, with $\lambda = 0.5$. However, central banks around the world might have different preferences based on individual political economy considerations or local economic conditions. For example, an emerging economy might place a higher weight on output stabilisation (e.g. $\lambda = 1$), whereas an economy fighting persistently high inflation could place a higher weight on inflation stabilisation (e.g. $\lambda = 0.25$). Tables 4.9 and 4.10 below accordingly recalculates the loss function based on these two alternative weights on output, while keeping the benchmark calibration unchanged.

TABLE 4.9: Policy rules and loss function: $\lambda = 1$

Rule	ϕ^{π}	ϕ^{y}	γ^{π}	γ^{y}	ε_t^d $(\times 10^{-5})$	ε_t^a $(\times 10^{-3})$	ε_t^R $(\times 10^{-8})$	ε_t^{BL} $(\times 10^{-7})$
(i)	1.5	0.5	-	-	3.541	0.657	2.720	1.428
(ii)	1.5	-	-	-10	5.930	0.872	4.534	2.574
(iii)	1.5	0.5	-1.5	-10	2.812	0.604	2.167	1.081

TABLE 4.10: Policy rules and loss function: $\lambda = 0.25$

Rule	ϕ^{π}	ϕ^{y}	γ^{π}	γ^{y}	ε_t^d $(\times 10^{-5})$	ε_t^a $(\times 10^{-3})$	ε_t^R $(\times 10^{-8})$	ε_t^{BL} $(\times 10^{-7})$
(i)	1.5	0.5	-	-	3.919	1.722	3.003	1.674
(ii)	1.5	-	-	-10	6.654	1.806	5.076	3.029
(iii)	1.5	0.5	-1.5	-10	3.083	1.722	2.370	1.260

Similar to Table 4.8, we consistently find the lowest value of the loss function under the hybrid policy rule (iii), irrespective the nature of the shock and the relative weights on inflation and output. Again, the dual rule (ii) performs poorly. The only possible exception is the supply shock, which shows an identical value for the loss function (1.722) under the Taylor and hybrid rules for an inflation-focused central bank (Table 4.10). However, this merely suggests that the hybrid rule is equal to the Taylor rule under this parameterisation. If one drives the weight on inflation even higher ($\lambda = 0.1$), the value of the loss function for a supply shock under the hybrid rule becomes even larger (3.957×10^{-3}), while under the Taylor rule it becomes 3.852×10^{-3}. Based on these values, the Taylor rule is therefore preferable to the hybrid rule in this particular scenario. However, in general, we can conclude that the hybrid rule robustly outperforms the Taylor rule across all shocks, unless the weight on inflation is very large.

4.5.2 Persistence of asset purchases

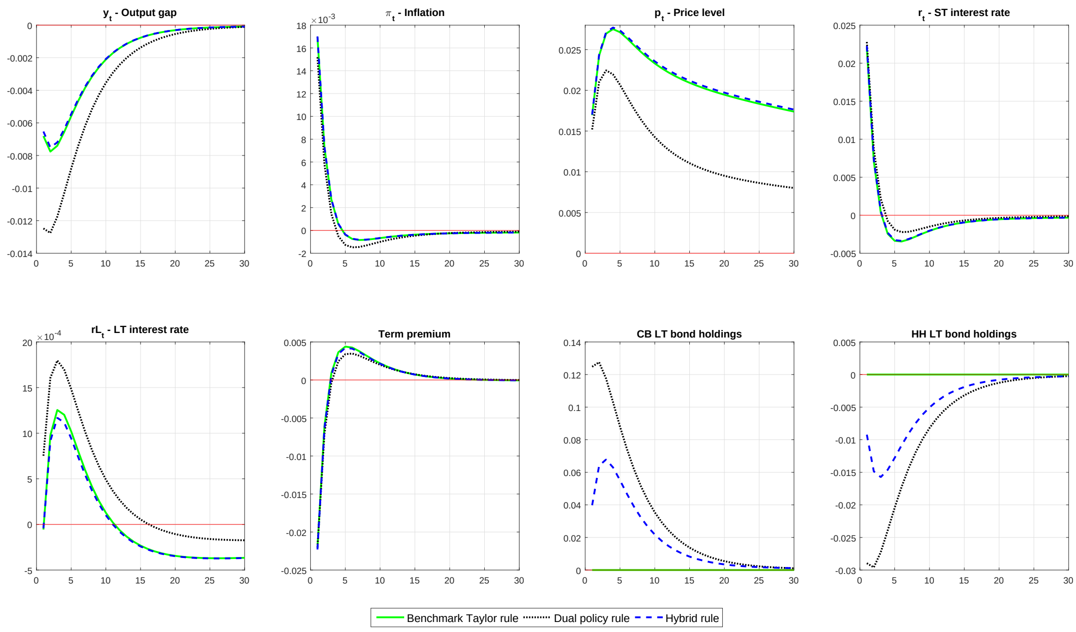
In the calibration of the benchmark model, it is assumed that the central bank follows a medium-term exit strategy following asset purchases (Falagiarda, 2014). That is, the assets accumulated during the central bank's LSAPs are gradually sold over the following five–six years (illustrated by the duration of the return of the central bank's long-term bond holdings to steady state of just under 24 quarters according to figure 4.5). A decrease in the parameter $\phi^{\mathcal{D}}$ represents a lower persistence in asset purchases (equation 4.40), which is tantamount to a faster exit strategy. Conversely, a slower exit strategy is represented by a higher parameter $\phi^{\mathcal{D}}$. A more gradual approach (higher $\phi^{\mathcal{D}}$) accurately reflects the Fed's QE3 programme, where they committed to small, continuous, monthly transactions. On the other hand, the early QE programmes (QE1 and QE2) saw extremely large and blunt "once-off" transactions ($\phi^{\mathcal{D}} \to 0$).

If we set $\phi^{\mathcal{D}} = 0$, the Taylor and hybrid rules become virtually indistinguishable.[41] That is, under normal economic conditions, there is nothing to gain by following a hybrid balance sheet rule with no persistence rather than a pure Taylor rule. For example, Figure 4.11 illustrates the response to a supply shock where there is no persistence in asset purchases, which shows the overlap between the Taylor and hybrid rules. Furthermore, the loss function delivers identical results under the Taylor rule and almost identical results under the hybrid rule. It is therefore clear that *persistent* central bank asset market interventions are required for the balance sheet rule to improve upon the conventional Taylor rule.

Of interest, however, is the fact that the QE3 period saw relatively high persistence of asset purchases by the central bank, yet the impact on yields were comparatively small (see the discussion in Section 3.3.3.1). It may be the case that the efficacy of central bank asset purchases is lower in non-crisis times (QE3), and that it was easier for the Fed to move the market during the QE1 period. Nonetheless, the result that balance sheet policies can complement its conventional counterparts does not ride on the absolute efficiency of such policies. However, it would play a part in determining the practical feasibility of implementing such policies.

[41] The exception of course is that the Taylor rule does not induce any changes to long-term bond holdings.

FIGURE 4.11: Optimal policy rules: Supply shock ($\phi^{\mathcal{D}} = 0$)



4.5.3 Portfolio adjustment costs

The portfolio adjustment cost parameter ϕ_L is what gives the portfolio balance channel its traction. If $\phi_L = 0$ the financial frictions fall away, and the model effectively collapses to the canonical model with no role for relative bond holdings and central bank balance sheet policy. It is therefore imperative that there exists a portfolio adjustment cost, however small and trivial but non-zero. It was suggested earlier that this cost could represent the household's preference for a certain maturity structure, the liquidity risk attached to holding long-term bonds, or the cost of managing its bond portfolio (see Section 4.2.1). However, this parameter may feasibly be larger in economies with less-developed financial and bond markets. Long-term bonds could be inherently more risky, or it could be more difficult to manage one's portfolio in the absence of deep or broad financial markets. However, increasing this parameter to, say, $\phi_L = 0.05$ does not qualitatively impact the model. The economy's response to a monetary policy or LSAP shock remains almost identical. However, given less-developed financial markets, the balance sheet response to a demand or supply shock are necessarily larger. Because of the higher portfolio adjustment cost, it is now more costly and thus less effective for the central bank to implement balance sheet policies. As a result, the magnitude of the intervention is required to be larger to achieve the same outcomes.

4.5.4 Steady state portfolio holdings

The ratio of short- to long-term bonds in the household's asset portfolio is calculated here from recent US data, and is approximately equal to $\kappa_L \approx 1.33$ (Falagiarda, 2014). On the other hand, in his similar analysis, Harrison (2012) uses $\kappa_L = 3$, also for the US economy but based on a different data set. However, these different parameters do not significantly change the model's response to a demand shock or monetary policy shock. There are, however, subtle differences in the model's response to a supply shock and long-term bond supply shock. Figures 4.12 and 4.13 illustrate the response to these two shocks under the calibration of $\kappa_L = 3$.

FIGURE 4.12: Optimal policy rules: Supply shock ($\kappa_L = 3$)

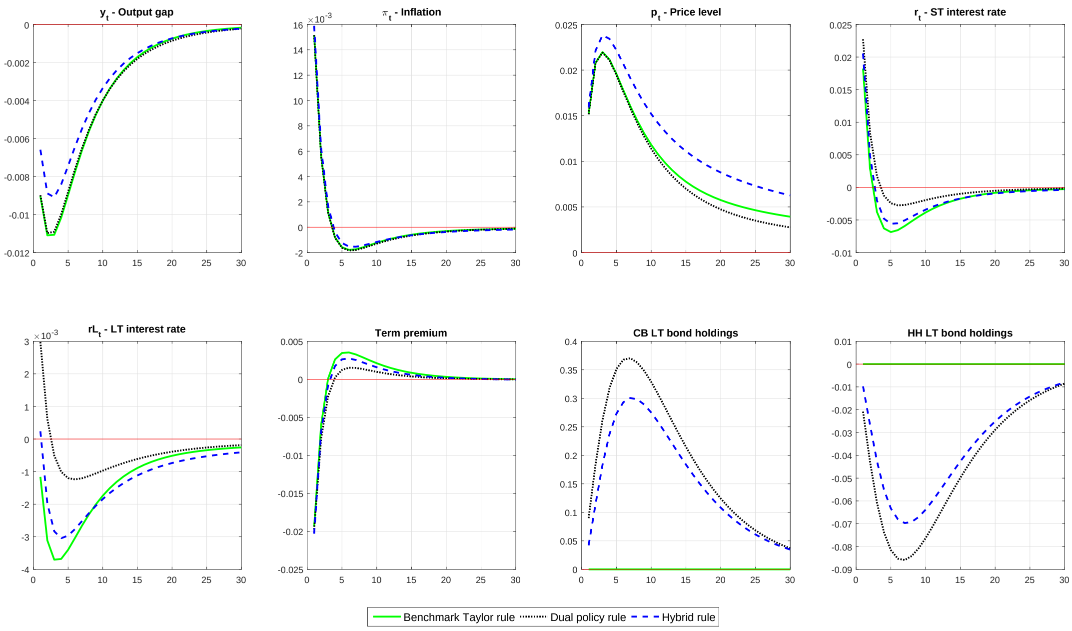
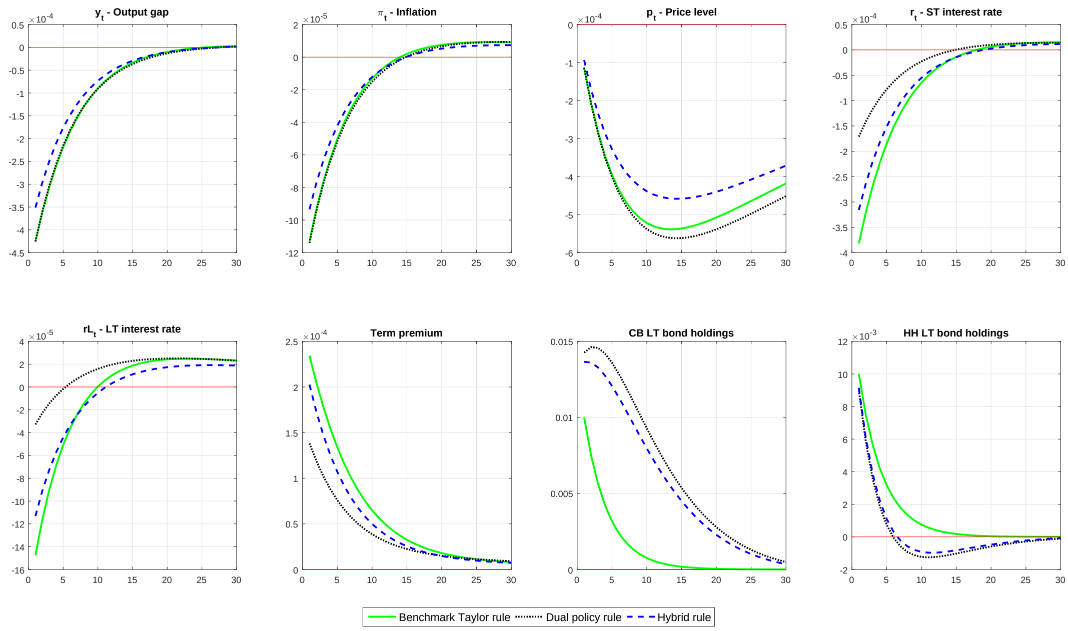


FIGURE 4.13: Optimal policy rules: Long-term bond supply shock ($\kappa_L = 3$)



Notably, under both shocks the reaction of the long-term interest rate is almost the symmetric opposite to the benchmark calibration (see Figures 4.8 and 4.10). These effects are very small in magnitude (the result of a relatively small shock applied to the model), yet the discrepancy is worth some attention. While the movements in the short-term interest rate and term premium are comparable, the size of the short-term interest rate response is substantially different. Under all three policy rules, the short-term interest rate response is somewhat sharper in the face of a supply shock. Specifically, the short-term interest rate increases by roughly 2 basis points more under each policy rule. The change in the long-term interest rate is negligble. On the other hand, in the event of a long-term bond supply shock, the short-term interest rate does not fall as much under a smaller κ_L. The long-term interest rate falls under $\kappa_L = 3$, while it increases under $\kappa_L = 1.33$. This implies that the smaller κ_L, the smaller the Taylor rule response to a long-term bond supply shock, while the reverse holds in the event of a supply shock.

Under both these calibrations the household holds more long-term than short-term bonds in its steady state portfolio. However, under $\kappa_L = 3$ the relative weight of long- to short-term bonds is somewhat larger than under $\kappa_L = 1.3$. If the household holds relatively more long-term bonds, a change in its holdings of long-term bonds (due to e.g. central bank asset purchases or a long-term bond supply shock) will have a proportionally larger effect. Under these conditions balance sheet rules become more powerful, and we see a smaller role for the interest rate rule. Conversely, relatively smaller holdings of long-term to short-term bonds would reduce the effect of changes in the household's long-term bond holdings. This reduces the efficacy of balance sheet rules and therefore requires a larger role for interest rate policy - hence the stronger Taylor rule response under these conditions.

4.6 Conclusion

This chapter constructed a DSGE model in order to simulate alternative monetary policy rules. The model introduces financial frictions in the form of the household's balance sheet and an imperfect adjustment to deviations from the household's steady state holdings of long- to short-term bonds, as well as an explicit central bank balance sheet, to an otherwise standard DSGE model.

First, various shocks are simulated. Of particular interest is the reaction of macroeconomic variables in response to a central bank asset purchase shock, to demonstrate how the model could be used to gauge the impact of LSAPs and QE policies on markets and the economy. The findings are consistent with related studies from the literature, suggesting that this is a robust framework within which these types of asset market interventions could be analysed. Subsequently the asset purchase shock was converted into a balance sheet rule, by stipulating that the central bank's asset purchases are no longer a simple exogenous shock process, but an endogenous response to developments in output and inflation. Under these conditions, the balance sheet rule shares many properties with the conventional Taylor rule.

Various parameter spaces and combinations of the Taylor and balance sheet rules were considered. A loss function was constructed, and the social welfare cost of each policy mix in response to various exogenous shocks were estimated. These results, in conjunction with impulse response functions for the same shocks, show that a hybrid policy rule, where the central bank responds to deviations in output and inflation by utilising both the short-term interest rate and its balance sheet, is superior to a conventional Taylor rule where only the short-term interest rate is utilised. This supports the notion that central bank balance sheet operations could complement the Taylor rule, even under normal conditions. That is, balance sheet interventions need not be limited to crisis times only.

The model's sensitivity to a number of key parameters were considered. It was found that the higher the weight on inflation in the central bank's loss function, the stronger the performance of the Taylor rule. Unless the weight on inflation is very large, however, the hybrid rule outperforms the Taylor rule. Importantly, it is the *persistence* of asset purchases/sales which gives the balance sheet rule its traction. If asset market interventions are a once-off event, the impact on the economy is qualitatively similar to the standard Taylor rule. Portfolio adjustment costs are necessary to ensure the imperfect substitutability between short- and long-term bonds in the household's balance sheet. Higher adjustment costs reduce the efficiency of the central bank's balance sheet interventions. This can be supported by the notion that, under less-developed capital markets, the central bank's interventions need to be larger to achieve the same outcomes. Finally, the composition of households' balance sheets is also important. If households hold relatively more long-term bonds, central bank balance sheet interventions will have a proportionally larger impact, and the balance sheet rule becomes even stronger.

Chapter 5

Monetary policy rules and international capital flows

5.1 Introduction

In Chapters 2 to 4 the impact of QE policies on the domestic economy was considered. However, the massive liquidity boost from such programmes inevitably spilled over to other economies. Such spillovers influenced asset markets and asset prices – and subsequently macroeconomic variables such as exchange rates, inflation and economic growth – in many foreign economies. The objective of this chapter is therefore to address the international spillovers of these QE policies from the perspective of a small open economy. The theoretical model developed in Chapter 4 is opened up to allow for a foreign sector. The domestic household now has access to foreign consumption goods, while foreign agents are able to trade in domestic financial assets and domestic consumption goods. Specifically, the domestic central bank's monetary policy responses to disruptions in local asset markets as a result of large foreign capital flows is simulated as an example of an application of this framework. The balance sheet rule developed in Chapter 4 is extended and proposed as a possible additional policy instrument. There would be a neat symmetry if the disruptions in local asset markets, as a result of unconventional policies employed elsewhere, could be managed by employing the exact same instruments domestically.[1]

[1]This is not to say that balance sheet policy is the "best" answer to international capital flows. A substantial literature exists which tests the efficacy of capital controls and other policy measures. However, in keeping with the theme of this study, which compares conventional (interest rate) to unconventional (balance sheet) policy, capital controls and other policy options are not considered here.

This chapter is structured as follows: Section 5.2 provides a brief overview of the international spillover effects of US QE. In Section 5.3 the DSGE model developed in Section 4.2 is extended to incorporate the foreign sector. The final model equations and calibration is presented in Section 5.4, and the results are reported in Section 5.5. The sensitivity of the results to changes in the balance sheet rule parameters are considered in Section 5.6. Section 5.7 concludes.

5.2 QE and international spillovers

As discussed in Chapter 3 above, the Fed's large-scale asset purchases (LSAPs) induced changes in relative characteristics (such as risk or liquidity considerations) among asset classes. Such relative adjustments between asset classes, coupled with the removal from the market of substantial amounts of long-term fixed-income securities, led investors to adjust the composition of their asset portfolios by substituting among different asset classes. This then changed the relative demand for various assets, which caused changes in asset prices.

Much attention in the literature has been paid to domestic US asset markets, such as bond markets (both Treasuries and corporate), equities and real estate, and there exists a burgeoning literature on the impact of the Fed's unconventional monetary policy on asset markets. The most prominent framework is to assess the effects of LSAPs on bond market yields and subsequent spillovers to other asset prices such as equities. Influential papers in this category include Bauer and Rudebusch (2014), D'Amico and King (2013), D'Amico, English, Lopez-Salido, and Nelson (2012), Krishnamurthy and Vissing-Jorgensen (2011b) and Gagnon, Raskin, Remache, and Sack (2011b).[2] A common approach in this framework is that of utilising event studies in order to isolate price and yield effects on various asset markets in a narrow window around official statements pertaining to LSAPs by the Federal Reserve Open Market Committee (FOMC). Developments in bond markets are theorised to subsequently influence other asset markets through various channels. Insofar as these assets are deemed appropriate substitutes for one another – evaluated by the degree of similarity between asset classes in terms of exposure to various types of risk, liquidity considerations, balance sheet effects etc. – any changes in the price/yield of one asset class is expected to lead to price and yield changes in other asset classes. For example, LSAPs generally had the effect of lowering yields on longer-term US Treasuries. This has been ascribed to both a signaling channel,

[2] See Bhattarai and Neely (2016) for a recent comprehensive summary of this literature. Joyce, McLaren, and Young (2012) and Breedon, Chadha, and Waters (2012) investigate the question from the UK perspective. Fawley and Neely (2013) discuss different approaches to QE among the Fed, Bank of England, European Central Bank and the Bank of Japan.

where the Fed's LSAPs signal their intention of keeping short rates lower for longer, as well as a portfolio balance channel which broadly lowers the term premium component of long rates. These channels represent changes in the relative characteristics of US Treasuries relative to other domestic asset classes, and had the effect of boosting other asset prices such as equities and corporate bonds.

However, investors are not limited to domestic assets only in rebalancing their portfolios. International assets, such as foreign sovereign debt, Eurobonds or foreign stock markets, were also impacted by the Fed's LSAPs. While the majority of research has been focused on domestic asset markets, there is a growing literature on LSAPs' international spillover effects. Neely (2015:102) argues that "unconventional policies could affect international asset prices through the signaling and PB [portfolio balance] channels". The signaling channel suggests that "investors interpret asset purchase announcements as implying a lower path for future short-term interest rates" (Bauer and Neely, 2014:24), leading to a fall in the expectations component of long rates. The portfolio balance channel suggests that LSAPs could "affect prices of imperfectly substitutable assets" (Ibid.) by reducing the term premium in both US long-term yields and international substitutes. These are also the two dominant channels identified in the literature on domestic spillover effects (see Chapter 3), and could theoretically be applied to all asset classes, domestic or international, to the extent that these assets could be deemed substitutes for one another. Indeed, portfolio balance effects are argued to be "consistent with the degree of substitutability across international bonds" (Bauer and Neely, 2014:24), whereas "signaling effects tend to be large for countries with strong yield responses to conventional U.S. monetary policy surprises" (Ibid.). Chen, Filardo, He, and Zhu (2016:64) argue that the Fed's "QE policies helped to stabilise global financial markets and prevented an even further collapse in the global economic activity". However, there have also been concerns that these policies may have created excessive global liquidity, leading to massive capital flows to emerging market economies (EMEs). This surge in capital flows "is widely blamed for appreciation pressures on EME currencies, a build-up of financial imbalances and asset price bubbles in EMEs, high credit growth and the threat of an over-heating of the domestic economies" (Fratzscher, Lo Duca, and Straub, 2013:4). Moreover, the Fed's QE announcements during 2008–2009 "substantially reduced international long-term bond yields and the spot value of the dollar" (Neely, 2015:101).

The empirical evidence suggests that the Fed's QE interventions led to a reduction in foreign long-term bond yields, an exchange rate appreciation vs. the US dollar and an increase in capital flows to these countries, and sometimes a stock market boom. However, the responses differ substantially among countries, as well as to different QE programmes. For example, Fratzscher et al. (2013) show that QE1 is associated with a portfolio rebalancing out of emerging economies into US bonds and equities, while these flows are reversed under QE2. QE1 boosted bond and equity prices globally, and especially in the US, and led to an appreciation in the US dollar. Conversely, the impact of QE2 was much larger on global asset prices, and saw a depreciation in the dollar. The event study approach is also used, where the response of international financial variables are evaluated in a narrow (1- or 2-day) window around the Fed's QE announcements. This follows much of the empirical work on the effects of QE on domestic asset markets. A critical assumption of the event study framework is that "markets are efficient in the sense that all the effects on yields occur when market participants update their expectations and not when actual purchases take place" (Gagnon et al., 2011b:16). Therefore, portfolio decisions – and subsequently current asset prices – at the time of the announcements are driven by changes in expectations of future asset prices. Fratzscher et al. (2013), however, argue that the impact of Fed announcements have been dwarfed by the actual operations.[3] This implies that investors did not react only to the announcements, as Gagnon et al. (2011b) would suggest, but also responded to the actual operations in a somewhat sluggish way.

The consensus in recent literature is that the global financial crisis of 2007–2010 and subsequent unconventional monetary policy in the form of quantitative easing (QE), notably practiced by the US Fed and the Bank of England and to a lesser extent the European Central Bank and the Bank of Japan, have contributed to recent volatility in international capital flows.[4] During the early part of the crisis global risk aversion was extremely high, leading capital to flow out of EMEs and into traditional safe havens. Conversely, the subsequent massive injections of liquidity through the various QE programmes contributed to a glut of liquidity in developed economies, of which a significant portion flowed to EMEs. The fact that short-term interest rates were effectively stuck at the zero lower bound (ZLB) in much of the developed world, while EME interest rates were generally a few percentage points higher, has most likely exacerbated this "search for yield".

Small open economies (SOEs), especially EMEs, are exceedingly vulnerable to large and volatile international capital flows. According to Kitano and Takaku (2017:2), "volatile capital flows amplify boom-bust cycles and destabilise emerging market economies".

[3] Virtually all of the FOMC's announcements pertaining to QE did not involve contemporaneous asset purchases. The LSAPs were implemented by the Federal Reserve Bank of New York, and these operations occurred well after the official announcements. Furthermore, several of the later QE announcements indicated small, continuous, purchases of securities, in contrast to the large, bulk, purchases of e.g. QE1.

[4] See, among others, Tillmann (2016), Alpanda and Kabaca (2015), Bhattarai, Chatterjee, and Park (2015), Bowman, Londono, and Sapriza (2012), Neely (2015), Bauer and Neely (2014), Fratzscher et al. (2013) and Kim and Yang (2011).

While capital inflows are generally desirable from an EME perspective[5], it can often be a two-edged sword. Large capital inflows could lead the domestic currency to appreciate drastically, thereby making exports less competitive, while leading to higher inflation and output volatility. Davis and Presno (2014:2) argue that “surges in capital inflows” could lead to the “danger of overheating in many emerging markets”, while Rey (2013) suggests that such capital flows could potentially lead to asset price bubbles, excessive credit creation and domestic financial instability. Conversely, capital outflows lead to a depreciation of the currency, thereby contributing to higher domestic inflation, “imported” through the purchase of foreign goods. Furthermore, if capital inflows are suddenly reversed the negative economic effects could be substantial.[6] The precarious position of EMEs is neatly framed by Fratzscher et al. (2017:1), who suggest that “QE policies have created excessive global liquidity and caused an acceleration of capital flows to EMEs”, leading to “appreciation pressures on EME currencies and a build-up of financial imbalances in EMEs” (Ibid.).

Broadly, there thus exists a healthy tension between capital inflows and capital outflows, provided that volatility and shocks to capital flows can be managed. The situation is, however, complicated by the extremities on both sides of the scale. On the one hand, sharp capital inflows could lead to an overheating economy and potential asset price bubbles, while sharp capital outflows or sudden stops (what Tillmann (2016:137) terms “fierce reversal of capital flows back into mature economies”) are also highly undesirable. Kim and Yang (2011:294) warn that the rapid asset price appreciation and the recent surge in capital inflows witnessed in Asian economies are “cause for pre-emptive policy responses to capital inflows from abroad”, since “they might lead to financial instability and adverse consequences to the real economy”. This raises the question of what the appropriate policy response of an EME central bank should be to such sharp capital flows. The puzzle is, however, that central banks are often caught between competing objectives when faced with such developments. Capital inflows appreciates the domestic currency, making exports less competitive, while at the same time putting upward pressure on domestic asset prices. The latter could potentially fuel asset price bubbles and even lead to costly boom-bust cycles, while at the same time challenging the central bank's management of domestic inflation. Capital outflows, on the other hand, depreciates the domestic currency, again imparting inflationary pressures, as well as eroding asset prices, even though it might boost exports by making domestic goods cheaper to foreigners.

[5]This includes benefits and positive externalities such as risk sharing and the development of financial markets, while a capital account surplus enables a current account deficit, contributing to higher domestic spending and economic activity in general.

[6]See, for example, Calvo (1998) and the literature on “sudden stops” inspired by Dornbusch, Goldfajn, Valdés, Edwards, and Bruno (1995).

The more volatile the flows, of course, the harder the situation is to manage. Theoretically, *conventional* monetary policy, referring to the use of the short-term interest rate as tool, could combat undesirable or volatile capital flows by adjusting the interest rate. Sharp capital inflows could to some degree be sterilised by lowering the policy interest rate, which should lead to a fall in the foreign demand for domestic financial assets – and subsequently its prices – through a smaller interest rate differential with the rest of the world, as well as a weakening of the exchange rate. However, lower domestic interest rates would increase domestic liquidity (it is now cheaper to borrow locally); such extra liquidity could again inflate asset prices and overheat the economy. On the other hand, if interest rate policy is used in an attempt to attract foreign capital in response to a capital outflow, by way of increasing the interest rate in order to make domestic financial assets more attractive, it could further slow down an already-contracting economy. Therefore, using interest rate policy to combat one dimension of the problem (e.g. asset price stability), could force the central bank to compromise on another goal (e.g. inflation).

To this end, the remainder of this chapter is concerned with establishing the most appropriate monetary policy response to international capital flows. Specifically, this chapter aims to identify a monetary policy framework within which the central bank utilises its balance sheet to manage international capital flows. The Fed's QE policies is but one recent example of capital flow volatility; however, the results could apply to any cause of international capital flows, be it global risk aversion, currency crises etc. There is, however, a neat symmetry in domestically utilising unconventional monetary policies to manage international capital flows which were – in the first place – largely caused by unconventional monetary policies abroad.

5.3 Open-economy DSGE model

The model constructed here is a small open-economy (SOE) extension of the model derived in the previous chapter. It combines the canonical open-economy model developed by Galí and Monacelli (2005) (extensively described in Galí (2015)) with the central bank balance sheet and households' portfolios and portfolio frictions introduced in the closed-economy model above. It is assumed that the *home* economy is a small emerging economy (where domestic factors do not influence the rest of the world), which is restricted to being simply a "recipient of net capital inflows" (Devereux and Sutherland, 2009:181). The *foreign* economy is advanced. Following Galí (2015:223), we introduce the exchange rate, net exports, the terms of trade, and international financial markets, with the aim of ultimately assessing "the implications of alternative monetary policy rules in an open economy" with financial frictions. We are interested in determining whether any of the results found in chapter 4 above would still hold in the open-economy case, and whether the existence of exchange rate or capital flow dynamics would influence the optimal monetary policy response. Moreover, the efficiency of the balance-sheet rules derived earlier can now be tested in the open-economy setting. We further assume complete international financial markets, and the law of one price (Galí, 2015), while maintaining staggered price setting by domestic firms (sticky prices) à la Calvo (1983).

The home household can now purchase both domestic and foreign consumption goods, modeled as a "single world good" (Devereux and Sutherland, 2009:184) produced both at home and abroad. There are no transaction costs in the international goods market. The home household saves for future consumption by purchasing domestic financial assets. As in the closed-economy model, there is no financial sector or intermediary.

A foreign investor, who trades in domestic financial assets, is introduced. The only domestic asset the foreign investor can invest in is domestic long-term government bonds. The purpose of this restriction is to mimic large capital flows out of e.g. the US economy in search of substitute investment goods for US long-term and fixed-income securities. Because the Fed's LSAPs removed significant amounts of such assets from US financial markets, investors were forced to look elsewhere – including abroad – for similar products. Higher demand for domestic long-term bonds would increase its price, effectively increasing the household's wealth or income, and enable higher domestic consumption. Furthermore, the transaction would remove long-term bonds from the household's portfolio in exchange for liquid reserves (cash or money holdings). A substantial portion of this will be spent on consumption of foreign-produced goods, which will balance the capital account surplus (net capital inflows from foreign investors) with the current account deficit (net imports of consumption goods). This transmission mechanism also applies to the case of capital outflows. Foreign investors can liquidate their holdings of domestic securities by selling it back to the home household. Excess supply leads to a drop in the price of long-term bonds, eroding the home household's wealth. The home household also now has less liquidity available to spend on consumption.

Financial frictions enter the model through the balance sheet of the home household, where the household holds a combination of imperfectly substitutable domestic short- and long-term government bonds. This specification of the home household's bond portfolio therefore allows capital flows to disrupt the domestic economy, which is similar to Davis and Presno (2014:4)'s framework where "constraints [on the household's balance sheet] provide a means through which cycles of capital flows into and out of the small open economy cause financial instability".[7] The externality created by the financial frictions imposed here (the influence of asset prices and the relative holdings of bonds on the household's consumption decisions) gives a potential role for monetary policy to attempt to counter or sterilise the effect of capital flows on domestic asset prices.

5.3.1 Households

The domestic household is modelled similarly to chapter 4. We, however, simplify by ignoring the preference shock ϕ_t.[8] The household maximises utility according to

$$\max_{C_t, \frac{M_t}{P_t}, N_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{1}{1-\delta} \left(\frac{M_t}{P_t} \right)^{1-\delta} - \chi \frac{N_t^{1+\phi}}{1+\phi} \right] \tag{5.1}$$

subject to the nominal budget constraint

$$\begin{aligned} P_t C_t + P_t T_t + M_t + \frac{B_t}{R_t} + \frac{B_{L,t}^H}{R_{L,t}} (1 + AC_t^L) \\ = W_t N_t + M_{t-1} + B_{t-1} + \frac{B_{L,t-1}^H}{R_t} \end{aligned} \tag{5.2}$$

Total consumption now consists of consumption of both home and foreign goods, with C_t representing a composite consumption index given by

$$C_t \equiv \left((1-\upsilon)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + \upsilon^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{5.3}$$

where $C_{H,t}$ and $C_{F,t}$ represent consumption of home and foreign goods, respectively (Galí, 2015:225). $\upsilon > 0$ is a measure of openness and $\eta > 0$ measures the degree of substitutability between foreign and domestic goods. It is easy to see that in the extreme case of $\upsilon \to 0$ (i.e. a closed economy), total consumption consists of domestic consumption only. Finally, it can be shown that the cost of total consumption is conveniently given by $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ (Galí, 2015:226-227).

Two types of securities are traded in domestic financial markets: short-term (one-period) and long-term bonds, both issued by the home government. New issues of long-term

[7] The constraints on the household's balance sheet are modelled here as a preference for a certain portfolio mix, or 'preferred habitat', while Davis and Presno (2014) relates the constraints to collateral and borrowing costs.

[8] The effect of a preference (demand) shock in this framework was exhaustively discussed in the previous chapter. Here, the main concern is capital flow shocks, and, as such, other shocks are not revisited.

bonds can be purchased by home households, the home central bank and foreign households (investors) in the primary market, while there exists a secondary market for trading these instruments. The home household alone is allowed to purchase short-term bonds. The home household therefore holds a portfolio of domestic short- and long-term government bonds (B_t and $B^H_{L,t}$, respectively), which generates returns to be used mainly for future consumption. Deviations in this portfolio from the household's "preferred habitat"[9] imposes an adjustment cost, which is paid in terms of income Y_t, and is given by

$$AC^L_t = \left[\frac{\phi_L}{2}\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)^2\right] Y_t \tag{5.4}$$

5.3.1.1 Capital flows and the household's balance sheet

The home and foreign economies are linked by the markets for international goods and domestic long-term government bonds. Foreign investors can purchase securities from home households (capital inflows), which is mirrored by a current account deficit in the goods market, and *vice versa*. When the foreign investor's demand for domestic bonds increases, "the small open economy experiences a capital inflow" (Liu and Spiegel, 2015:11). The balance of payments then dictates that the (real value of the) current account balance (ca_t) must equal net foreign capital outflows:[10]

$$\begin{aligned}
\frac{CA_t}{P_t} &= -\frac{1}{P_t}\left(B^F_t - B^F_{t-1}\right) \\
&= -\left(\frac{B^F_t}{P_t} - \frac{B^F_{t-1}}{P_t}\right) \\
&= -\left(\frac{B^F_t}{P_t} - \frac{B^F_{t-1}}{\pi_t P_{t-1}}\right) \\
\therefore ca_t &= -\left(b^F_t - \frac{b^F_{t-1}}{\pi_t}\right)
\end{aligned} \tag{5.5}$$

Equation 5.5 confirms the intuition that the current account is simply the mirror of the capital account. If net (real) capital flows are positive ($b^F_t > \frac{b^F_{t-1}}{\pi_t}$) the current account will be negative, which suggests that the home household is able to finance additional consumption by increasing imports of goods and running a negative trade balance.

[9] See the discussion in section 4.2.1 above.

[10] Contrary to Liu and Spiegel (2015) we restrict agents in the home economy from holding foreign assets. Therefore the only source of net capital flows is gross flows originating in the foreign economy.

Extending trade in domestic long-term government bonds to foreign investors allows international capital inflows (outflows) to enter the model as an increase (decrease) in the demand for domestic long-term bonds on the secondary market, which mimics the search for substitute investments following the Fed's QE programmes.[11] The total amount of domestic long-term bonds issued by the home government can now be held between the home household, home central bank and foreign investor. That is,

$$B_{L,t} = B^H_{L,t} + B^{CB}_{L,t} + B^F_{L,t} \tag{5.6}$$

where $B^H_{L,t} = h_t B_{L,t}$, $B^{CB}_{L,t} = d_t B_{L,t}$ and $B^F_{L,t} = f_t B_{L,t}$ represent, respectively, home household, central bank and foreign holdings of domestic long-term bonds at time t. New bonds issued by the government are taken up by agents in these same ratios. The home household's holdings of long-term bonds can be expressed as the fraction of total bonds in circulation not held by the home central bank or foreign investor (or the residual after transactions by the central bank and foreign investors):

$$B^H_{L,t} = (1 - d_t - f_t)B_{L,t} = h_t B_{L,t} \tag{5.7}$$

where the home household and central bank's holdings of long-term government bonds are given by the fractions h_t and d_t of the total amount issued, and f_t denotes the fraction held by foreigners. Therefore, both a varying in the fraction d_t, through the central bank's balance sheet rule or asset purchase programmes, and the fraction f_t, approximating a change in foreign demand for domestic long-term bonds, would introduce a wedge in the household's optimisation problem by disturbing his preferred portfolio mix. An exogenous shock to f_t increases foreign demand for domestic long-term bonds. The foreign investor can only purchase such securities from the home household, which pushes up the price and lowers the yield on domestic long-term bonds. The home household's additional liquidity as a result of these transactions can now be allocated towards additional consumption, real money holdings or further investment. The home household is, however, not able to invest in foreign securities[12], which requires that the additional liquidity be allocated domestically. Of course, if $f_t = 0$ the household's bond holdings evolve identically to the the closed economy specification.

[11] These programmes removed significant amounts of long-term securities from the US market. Investors were forced to look elsewhere, including foreign markets, for securities or investments that could serve as substitutes or near-substitutes.

[12] Intuitively, the fact that foreign investors demand domestic financial assets implies that substitute foreign securities were not available in the first place. See also Kavli (2015) and Liu and Spiegel (2015), where a complete restriction is imposed on agents in the home (emerging) economy from investing in the foreign (advanced) economy.

The resulting liquidity that flows into the home economy enters through the capital account of the balance of payments. Given that the home household is restricted from holding foreign assets, the capital account position is therefore equal to the foreign holdings of domestic assets. The home economy therefore has a negative net foreign asset (NFA) position[13], while $f_t > 0$ represents a positive NFA position from the foreign economy perspective. A surplus on the capital account enables the home economy to finance a deficit on the current account, which implies that "the capital flow in the financial market will always be balanced by a current account deficit in the goods market" (Kavli, 2015:132). Capital inflows (outflows) are therefore associated with higher (lower) imports – equivalent to smaller (larger) net exports – and therefore higher (lower) aggregate domestic consumption.

The fraction of long-term bonds held by the household (see equation 5.7) can be represented as follows:

$$\frac{B^H_{L,t}}{B_{L,t}} = h_t = 1 - d_t - f_t \tag{5.8}$$

We assume that the foreign investor transacts only with the home household, so that $\Delta f_t = -\Delta h_t$ (assuming no change in d_t). That is, any transaction that changes the fraction of total domestic long-term bonds held by the foreign sector must have the home household as counterparty. For example, a hypothetical increase in f_t from 30% to 35% (i.e. 5 p.p.) must be "financed" by a commensurate fall in h_t from e.g. 50% to 45%. Indeed, the first difference of equation 5.8 reduces to

$$\Delta h_t = -\Delta d_t - \Delta f_t \tag{5.9}$$

Any change in the fraction of long-term bonds held by the home central bank or foreign investor would therefore influence the fraction held by the home household.[14]

Finally, the steady state stock of long-term bonds is distributed according to

$$\begin{aligned} \bar{B}_L &= \bar{B}^H_L + \bar{B}^{CB}_L + \bar{B}^F_L \\ &= \left(\frac{\bar{B}^H_L}{\bar{B}_L} + \frac{\bar{B}^{CB}_L}{\bar{B}_L} + \frac{\bar{B}^F_L}{\bar{B}_L} \right) \bar{B}_L \\ &= (\bar{\mathcal{H}} + \bar{\mathcal{D}} + \bar{\mathcal{F}}) \bar{B}_L \end{aligned} \tag{5.10}$$

[13] A country's NFA position is the value of foreign assets and securities that country owns MINUS the value of domestic assets owned by foreigners.

[14] The fraction f_t thus plays an identical role to the fraction d_t, as discussed in the particular case of the closed economy above, where an increase in either removes long-term bonds from the home household's balance sheet.

where $\bar{\mathcal{H}}$, $\bar{\mathcal{D}}$ and $\bar{\mathcal{F}}$ represent the steady state fractions of long-term bonds held by the home household, home central bank and foreign investor, respectively. Clearly, $\bar{\mathcal{H}} + \bar{\mathcal{D}} + \bar{\mathcal{F}} = 1$. As will be discussed below, $\bar{\mathcal{F}} > 0$, i.e. a negative NFA from the home economy perspective, is compatible with balanced trade in the steady state (see section 5.3.6).

5.3.1.2 Household's optimisation problem

The household's optimisation problem is therefore essentially identical to the closed-economy problem. The only differences to the open-economy problem are (i) the composition of the consumption basket (equation 5.3) and the role of international considerations[15] in determining the aggregate price level P_t and CPI inflation $\hat{\pi}_t$, and (ii) the potential disturbance to the household's preferred habitat as a result of foreign asset purchases or sales. However, since we have a homogenous global consumption good, the household is indifferent to consuming foreign or domestically produced goods. In addition, foreign asset purchases enter the household's optimisation problem as a pure exogenous shock, similar to the central bank asset purchase shock modelled in section 4 above. We therefore find the key log-linearised first-order conditions broadly similar to the closed-economy model:

Money demand:

$$\hat{m}_t - \hat{p}_t = \frac{1}{\delta}\frac{1}{1-\beta}\left(\sigma\hat{c}_t - \beta E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1}]\right) \tag{5.11}$$

Labour and wages:

$$\hat{w}_t - \hat{p}_t = \phi\hat{n}_t + \sigma\hat{c}_t \tag{5.12}$$

Short-term bonds:

$$E_t\left[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1}\right] = (\sigma\hat{c}_t + \hat{r}_t) + \Psi_1(\hat{b}^H_{L,t} - \hat{b}_t) \tag{5.13}$$

Long-term bonds:

$$E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1} + \hat{r}_{t+1}] = (\sigma\hat{c}_t + \hat{r}_{L,t}) + \phi_L\bar{Y}(\hat{b}_t - \hat{b}^H_{L,t}) \tag{5.14}$$

[15] This includes foreign prices, the exchange rate and the terms of trade. These variables are, however, all interdependent (see Appendix C.1).

The Euler equation:

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{\Psi_1}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) \tag{5.15}$$

The term structure:

$$\hat{r}_{L,t} = \hat{r}_t + E_t[\hat{r}_{t+1}] + \Psi_2(\hat{b}^H_{L,t} - \hat{b}_t) \tag{5.16}$$

$\Psi_1 = \beta\kappa_L\phi_L\bar{Y}$ and $\Psi_2 = \phi_L\bar{Y}(1+\beta\kappa_L)$ are convolutions of the steady state parameters. Foreign developments can therefore enter the model in a limited number of ways:

1. Inflationary or price level pressure from abroad, which influences the terms of trade and therefore the aggregate (CPI) price level;
2. Foreign purchases/sales of domestic securities which disturb the ratio $\hat{b}^H_{L,t} - \hat{b}_t$ in the household's portfolio. For example, foreign purchases of long-term bonds from the home household would lower $\hat{b}^H_{L,t}$, which increases consumption through the Euler equation and lowers the long-term interest rate through the term structure.

5.3.2 Monetary policy

The home central bank has access to two monetary policy instruments: the short-term interest rate and the central bank balance sheet. The central bank can use either, or a combination, of the two rules to respond to a capital flow shock. As was discussed in section 5.3.1.1 above, capital inflows will have an expansionary effect on the domestic economy. Through the household's balance sheet, consumption, output and inflation will increase, while the long-term interest rate would fall as a result of increased demand for long-term securities. In response, the central bank could (i) increase the short-term interest rate through a Taylor rule (the standard policy response in much of the New-Keynesian literature) and/or (ii) *sell* domestic long-term government bonds back to the home household in an attempt to sterilise the effect of capital inflows on the household's balance sheet.[16] The Taylor rule targets the short end of the yield curve and therefore only indirectly the long end, while the balance sheet rule targets long rates directly. The latter is due to the liquidity premium term in equation 5.16, which suggests a positive relationship between the long-term interest rate and the ratio of long- to short-term bonds in the household's portfolio. Sales of long-term bonds to the household

[16] The latter option will, of course, only be feasible if the central bank holds a sufficient stock of domestic long-term government bonds. There is thus a clear upper bound to the central bank's ability to intervene. This bears similarities to the central bank's ability under a fixed exchange rate regime to intervene in foreign exchange markets by selling foreign reserves to counter exchange rate movements.

would therefore push long-term interest rates back up, countering to some degree the expansionary effect of capital inflows.

5.3.2.1 Taylor rule

Identical to the closed-economy description above, the nominal interest rate is set following a standard Taylor rule

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi^\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi^y} e^{\varepsilon_t^R} \tag{5.17}$$

which can be log-linearised as

$$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t + \varepsilon_t^R \tag{5.18}$$

The central bank therefore responds to *aggregate* (CPI) inflation and domestic output. The difficulty facing the central bank, however, is that there could exist undesirable externalities in its decision of the appropriate interest rate response. Higher foreign demand for domestic financial assets increases domestic asset prices and lowers yields. This has an expansionary effect on the domestic economy. If the central bank attempts to counter the expansionary effect of increasing asset prices by lowering the policy rate in order to temper the desirability of domestic assets, it could further overheat the economy through domestic liquidity considerations. Conversely, tightening the policy rate would rein in domestic consumption and inflation, but it could attract further capital inflows due to higher yields on domestic financial assets[17], in which case the cycle is likely to repeat itself. Furthermore, the central bank only holds a finite stock of domestic long-term government bonds. If the capital inflows are so large or so persistent that it dwarfs the central bank's holdings, the central bank may very quickly find its balance sheet eroded and vulnerable.

5.3.2.2 Central bank balance sheet

The open-economy central bank balance sheet is identical to the closed-economy version (equation 4.35):

[17] While the open-economy literature often assumes that the small open economy is too small to influence developments in the rest of the world, the evidence on the impact of domestic developments on capital flows cannot be ignored. One could therefore set up foreign capital flows to respond to interest rate differentials.

$$\left[\frac{B_{L,t}^{CB}}{R_{L,t}} - \frac{B_{L,t-1}^{CB}}{R_t}\right] + \Delta_t = [M_t - M_{t-1}] \tag{5.19}$$

Δ_t again represents the change in the central bank balance sheet, equal to money creation and net asset purchases (Falagiarda, 2014:311). Similar to the closed-economy model, central bank asset purchases not fully financed by money creation ($\Delta_t < 0$) would impose a negative externality on the government's income, whereas money creation in excess of net asset purchases ($\Delta_t > 0$) represents a seignorage-type income for the government (see the government's budget constraint, equation 5.25 below).

The central bank's holdings of long-term government bonds are a fraction d_t of the total amount issued, i.e. $B_{L,t}^{CB} = d_t B_{L,t}$. Asset purchases (sales) can be performed by varying the fraction d_t, and is expressed as the following stochastic AR(1) process

$$\log\left(\frac{d_t}{\bar{\mathcal{D}}}\right) = \phi^{\mathcal{D}} \log\left(\frac{d_{t-1}}{\bar{\mathcal{D}}}\right) + \varepsilon_t^{\mathcal{D}} \tag{5.20}$$

where $\bar{\mathcal{D}}$ is the steady state fraction of long-term bonds held by the central bank $\left(\bar{B}_L^{CB}/\bar{B}_L\right)$ and $\varepsilon_t^{\mathcal{D}}$ is a shock to asset purchases with a mean of zero and standard deviation of $\sigma^{\mathcal{D}}$. The log-linearised version of the central bank's asset purchase equation (5.20) is given by

$$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \varepsilon_t^{\mathcal{D}} \tag{5.21}$$

Finally, the fraction $d_t = B_{L,t}^{CB}/B_{L,t}$ can also be expressed in log-linear terms as

$$\hat{d}_t = \hat{b}_{L,t}^{CB} - \hat{b}_{L,t} \tag{5.22}$$

5.3.2.3 Balance sheet rules

In the previous section, central bank asset purchases were modelled simply as a stochastic process, which could be used to illustrate the model's transmission mechanism in response to LSAP or QE-type programmes. To construct a rule, equation 5.21 would need to endogenised by expressing $\hat{d}_t$ as a response to certain variables, similar to the approach in Chapter 4. These could potentially include inflation, the output gap, asset prices (the long-term interest rate), or foreign asset purchases.

Under a balance sheet rule, direct intervention in asset markets could have the effect of stabilising the household's balance sheet, and therefore domestic economic activity, without having to resort to changes in the policy rate. This might prevent, or at least limit, the undesirable side-effects from interest rate policy as discussed above. The central bank could sell (purchase) long-term bonds directly to (from) the home household, thereby removing (injecting) cash from (into) the household's asset portfolio in response to capital inflows (outflows).[18] In this way, the central bank could to varying degrees (depending on the parameters of the policy rule) sterilise the effect of capital flows on domestic economic conditions. Therefore, net asset purchases could respond to changes in foreign holdings of domestic securities so that $\hat{d}_t = f(\hat{f}_t, \cdot)$ or $\hat{b}^{CB}_{L,t} = f(\hat{b}^F_{L,t}, \cdot)$. The central bank will indirectly respond to foreign asset transactions in order to directly influence the home household's balance sheet so as to minimise local economic disruption due to the portfolio balance channel. The balance sheet rule can be therefore be expressed in the form

$$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \gamma^f \hat{f}_t + \varepsilon^{\mathcal{D}}_t \tag{5.23}$$

where $-1 < \gamma^f < 0$.[19] Asset purchases are now driven by two factors: (i) the standard "day-to-day" transactions[20], driven by the stochastic process (the parameter $\phi^{\mathcal{D}}$), and (ii) the response to an exogenous capital flow shock. Absent a foreign demand shock, $\hat{f}_t = 0$ and equation 5.23 collapses to 5.21.

The simplest balance sheet rule would be for the central bank to fully and immediately sterilise net portfolio flows. In this way, the central bank can sell securities to absorb the cash that would otherwise circulate through the domestic economy. If foreign investors purchase domestic bonds from the household, the central bank could counter this by immediately selling the identical value back to the household. This would theoretically keep the household's balance sheet unchanged, and effectively shut down the portfolio balance channel. It is easy to verify from equation 5.9 that an increase in f_t requires an equivalent decrease in d_t for h_t to remain unchanged. Such a rule would therefore be required to achieve $\hat{d}_t = -\hat{f}_t$. We also need to account for the inertia from the AR(1) term and the parameter $\phi^{\mathcal{D}}$, which would suggest that the central bank will only gradually arrive at the point where $\Delta d_t = -\Delta f_t$. In the presence of inertia and smoothing behaviour, it is therefore impossible that the adjustment to the household's portfolio would be full and immediate. If the central bank responds too sharply and persistently (i.e. $\gamma^f \to -1$ and $\phi^{\mathcal{D}} \to 1$), there is a danger of excessive intervention and

[18] Such a rule could potentially also be extended to other shocks and disturbances.

[19] It may of course be possible for $\gamma^f < -1$, where the central bank responds more than one-to-one to foreign asset purchases. However, to protect the central bank from running out of resources we impose this restriction.

[20] This could be likened to regular open-market operations, where the central bank manages market liquidity by buying and selling (mostly short-term) securities.

thus overshooting the target. However, if we simply set $\phi^{\mathcal{D}} = 0$ and $\gamma^f = -1$, equation 5.23 would imply that $\hat{d}_t = -\hat{f}_t$. Even then, however, it would not necessarily follow that the net effect on the household's balance sheet is zero, mainly due to continued purchases of new issues of long-and short-term bonds. Furthermore, one of the results from Chapter 4 was that asset purchases need to be persistent to ensure it's efficiency as policy instrument. For this reason we cannot have $\phi^{\mathcal{D}} = 0$.

However, the balance sheet rule in the closed-economy model was assumed to react to output and inflation, in some combination with the Taylor rule (see equation 4.54). Moreover, because the central bank's ultimate goal is to manage inflation and output, we cannot simply assume that directly targeting capital flows would necessarily be the optimal policy.[21] Therefore, to ensure consistency between the models, the following functional form is proposed for the open-economy balance sheet rule:

$$\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \gamma^{\pi}\hat{\pi}_t + \gamma^y\hat{y}_t + \gamma^f\hat{f}_t + \varepsilon_t^{\mathcal{D}} \tag{5.24}$$

Since there are no clear guidelines in the literature, various sets of calibrations will be considered below. For example, setting $\gamma^f = 0$, the balance sheet rule responds only to output and inflation and is identical to the rule in Chapter 4. On the other hand, setting $\gamma^{\pi} = \gamma^y = 0$ implies that the balance sheet rule responds solely to capital flows (equation 5.23). Finally, the interaction between the persistence parameter $\phi^{\mathcal{D}}$ and the weight on capital flows γ^f should be carefully considered. Since γ^{π} and γ^y are both negative, an additional contractionary parameter ($\gamma^f < 0$) poses the risk of an overly-contractionary policy response, especially in combination with a high persistence parameter.

5.3.3 Government

The domestic government issues short- and long-term bonds, while raising a tax on the home household. Similar to the closed-economy model, short-term (one-period) bonds are sold only to the home household, while long-term bonds can be sold to the home household and central bank, as well as foreign investors. The government's budget constraint is identical to the closed-economy specification and is given by

$$T_tP_t + \frac{B_t}{R_t} + \frac{B_{L,t}}{R_{L,t}} + \Delta_t = B_{t-1} + \frac{B_{L,t-1}}{R_t} \tag{5.25}$$

[21] This can be compared to the literature on whether a central bank should be targeting asset prices (see for example Bernanke and Gertler (1999), Posen (2006), Roubini (2006)).

where Δ_t again represents the change in the central bank's balance sheet (see equation 5.19). Similar to the closed-economy model, the government raises a tax on the home household. The supply of long-term bonds evolves according to a stochastic process, while the supply of short-term bonds ensures that the government's budget constraint binds. The linearised tax rule, long-term bond supply and short-term bond supply are unchanged from the closed-economy specification and given below.

Tax rule:

$$\hat{t}_t = \frac{1}{\bar{T}\bar{P}}\left[\psi_1 \bar{B}(\hat{b}_{t-1} - \hat{p}_t) + \beta\psi_2 \bar{B}_L(\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t)\right] \tag{5.26}$$

Long-term bond supply:

$$\hat{b}_{L,t} = \phi^{BL}\hat{b}_{L,t-1} + \varepsilon_t^{BL} \tag{5.27}$$

Short-term bond supply:

$$\begin{aligned}
\therefore \bar{B}\hat{b}_t &= \frac{\bar{B}}{\beta}\hat{b}_{t-1} + \bar{B}_L(1-\bar{\mathcal{D}})(\hat{b}_{L,t-1} - \beta\hat{b}_{L,t}) + (\bar{B} - \bar{B}_L(1-\bar{\mathcal{D}}))\hat{r}_t \\
&\quad + \bar{B}_L\beta(1-\bar{\mathcal{D}})\hat{r}_{L,t} - \frac{\bar{M}}{\beta}(\hat{m}_t - \hat{m}_{t-1}) + \bar{\mathcal{D}}\bar{B}_L(\beta\hat{d}_t - \hat{d}_{t-1}) - \bar{T}\bar{P}(\hat{t}_t + \hat{p}_t) \\
\therefore \hat{b}_t &= \frac{1}{\beta}\hat{b}_{t-1} + \frac{\bar{B}_L}{\bar{B}}(1-\bar{\mathcal{D}})(\hat{b}_{L,t-1} - \beta\hat{b}_{L,t}) + (1 - \frac{\bar{B}_L}{\bar{B}}(1-\bar{\mathcal{D}}))\hat{r}_t \\
&\quad + \frac{\bar{B}_L}{\bar{B}}\beta(1-\bar{\mathcal{D}})\hat{r}_{L,t} - \frac{\bar{M}}{\bar{B}}\beta(\hat{m}_t - \hat{m}_{t-1}) + \bar{\mathcal{D}}\frac{\bar{B}_L}{\bar{B}}(\beta\hat{d}_t - \hat{d}_{t-1}) - \frac{\bar{T}\bar{P}}{\bar{B}}(\hat{t}_t + \hat{p}_t)
\end{aligned} \tag{5.28}$$

5.3.4 Firms

Similar to the closed-economy setting, the representative home firm j produces a homogenous good according to the production function[22]

$$Y_t(j) = Z_t N_t(j) \tag{5.29}$$

Z_t is a stochastic productivity shock, with $\hat{z}_t \equiv \log Z_t$, which evolves over time according to

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z \tag{5.30}$$

[22]Following Galí and Monacelli (2005:715) we simplify by setting $\alpha = 0$ (i.e. no capital share in production).

Total labour demand is again given by $N_t = \int_0^1 N_t(j)dj$, while aggregate domestic output is defined by

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{5.31}$$

The optimal price setting decision of the domestic firm is identical to the expression derived earlier in the closed-economy setting (Galí, 2015:233), and is given by[23]

$$\bar{p}_{H,t} = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}^r_{t+k|t} + \hat{p}_{H,t+k} \right] \tag{5.32}$$

As $\theta \to 0$ (the flexible price limit), the home firms' optimal price setting rule becomes $\bar{p}_{H,t} = \hat{mc}^r_t + \hat{p}_{H,t}$ (i.e. a fixed markup over the domestic price level).

5.3.5 Exports

Following Galí (2015:234), the home economy's exports in a two-economy model are given by

$$\begin{aligned} X_t &= \upsilon \left(\frac{P_{H,t}}{\mathcal{E} P^\star_t} \right)^{-\eta} Y^\star_t \\ &= \upsilon \mathcal{S}^\eta_t Y^\star_t \end{aligned} \tag{5.33}$$

where $\mathcal{S}_t = \frac{\mathcal{E}_t P^\star_t}{P_{H,t}}$ represents the terms of trade (see Appendix C.1.1). This result holds "under the assumption that the preferences of households in the rest of the world are identical to those of domestic households" and the fact that global goods market clearing requires that $C^\star_t = Y^\star_t$ (Galí, 2015:234). The home economy's exports are therefore a function of trade openness, the terms of trade (i.e. the ratio of the price of foreign to home goods), the degree of substitutability between home and foreign goods, and world output (equivalent to aggregate world demand). The terms of trade, in turn, is a positive function of the nominal exchange rate ($\mathcal{E}_t$) and the world price level, and a negative function of the domestic price level. Intuitively, a weaker domestic currency (captured by an increase in $\mathcal{E}_t$) and higher world prices would increase foreign demand for home goods, while higher home production prices would lower foreign demand for home goods.

[23] Note the change in notation from the closed-economy setting, where $\bar{p}_{H,t}$ now denotes newly set home prices instead of $\hat{p}^*_t$. This is to avoid confusion with the asterisk * used to describe world economy variables.

In the symmetric steady state, where $\bar{\mathcal{S}} = 1$, steady state exports is given by $\bar{X} = \upsilon \bar{Y}^\star$. Log-linearising equation 5.33 around this steady state yields

$$\begin{aligned} X_t &= \upsilon \mathcal{S}_t^\eta Y_t^\star \\ \therefore \bar{X} e^{\hat{x}_t} &= \upsilon (\bar{\mathcal{S}} e^{\hat{s}_t})^\eta \bar{Y}^\star e^{\hat{y}_t^\star} \\ &= \upsilon \bar{\mathcal{S}}^\eta \bar{Y}^\star e^{\eta \hat{s}_t + \hat{y}_t^\star} \\ \therefore \bar{X}(1 + \hat{x}_t) &\approx \upsilon \bar{Y}^\star (1 + \eta \hat{s}_t + \hat{y}_t^\star) \\ \therefore \hat{x}_t &= \eta \hat{s}_t + \hat{y}_t^\star \end{aligned} \tag{5.34}$$

Furthermore, since in steady state $\bar{C}_F = \upsilon \bar{C}$ (C.1.2) and $\bar{C} = \bar{Y}^\star$ (from the risk sharing condition, C.1.5) it follows that $\bar{C}_F = \upsilon \bar{C} = \upsilon \bar{Y}^\star = \bar{X}$ (i.e. imports = exports). Therefore, “trade is balanced at the steady state” (Galí, 2015:234).

5.3.6 The current account and trade balance

The components of the *current account* are the trade balance, or net exports of goods and services (exports - imports) PLUS net factor income, or income from abroad (NY) PLUS net current/cash transfers (NCT). $CA = (X - M) + NY + NCT$. In our simple model, a current account deficit means that the economy is consuming more than it is producing. The *capital account* consists of foreign direct investment (FDI) PLUS portfolio investment PLUS other investment PLUS reserve account. A current account deficit can be financed by a capital account surplus, either through foreign debt, FDI or portfolio investment from abroad, or the economy running down its foreign reserves.

Given the components of the current account as defined above, it is possible to posit a steady state current account deficit in our model even under the assumption of balanced trade ($\bar{X} - \bar{M} = 0$). If there is a steady state stock of home bonds held by foreigners, these bonds will periodically pay interest to the foreign bond owners and will be counted as an outflow, i.e. $\overline{NY} < 0$. While a change in ownership of these assets (e.g. the foreign investor sells some of his holdings to the home household) will be recorded in the capital account, the net income derived from these assets is always recorded in the current account. It is easy to see that in our model, where the only internationally traded asset is domestic long-term bonds and home agents are not allowed to own foreign assets, *net factor income will always be negative*. This in turn allows us to include in the steady state a quantity of domestic securities owned by foreigners ($\bar{B}_L^F$ and $\mathcal{F}$ from section 5.3.1.1), which enables us to construct a tractable model and intuitive set of dynamic equations. Furthermore, the simple structure of international asset markets used here (only one security, complete markets and perfect risk sharing) keeps the model computationally convenient by obviating the need for higher-order estimations of the equilibrium portfolios or net foreign asset (NFA) positions.[24]

Net exports, in terms of domestic output and expressed as a fraction of steady state output (Galí, 2015:236), is given by

$$nx_t \equiv \left(\frac{1}{\bar{Y}}\right)\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right) \tag{5.35}$$

Steady state net exports can therefore be expressed as

$$\overline{nx} = \frac{1}{\bar{Y}}\left(\bar{Y} - \frac{\bar{P}}{\bar{P}_H}\bar{C}\right) = \frac{\bar{Y} - \bar{C}}{\bar{Y}} \tag{5.36}$$

Equation 5.36 shows that steady state net exports is equal to the fraction of steady state output not consumed domestically. Under the assumption that "trade is balanced in the steady state" (Galí, 2015:236), net exports is equal to zero and it follows from equation 5.36 that $\overline{nx} = 0 \Rightarrow \bar{Y} = \bar{C}$. Log-linearising equation 5.35 around this steady state, noting that $\hat{p}_t - \hat{p}_{H,t} = \upsilon\hat{s}_t$ (equation C.5), yields

$$\begin{aligned}
nx_t &= \frac{1}{\bar{Y}}\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right) \\
\therefore nx_t &= \frac{1}{\bar{Y}}\left(\bar{Y}e^{\hat{y}_t} - \frac{\bar{P}e^{\hat{p}_t}}{\bar{P}_H e^{\hat{p}_{H,t}}}\bar{C}e^{\hat{c}_t}\right) \\
&= \frac{1}{\bar{Y}}\left(\bar{Y}e^{\hat{y}_t} - \bar{C}e^{\hat{p}_t - \hat{p}_{H,t} + \hat{c}_t}\right) \\
&= e^{\hat{y}_t} - e^{\hat{p}_t - \hat{p}_{H,t} + \hat{c}_t} \\
\therefore nx_t &\approx (1 + \hat{y}_t) - (1 + \upsilon\hat{s}_t + \hat{c}_t) \\
\therefore nx_t &= \hat{y}_t - \hat{c}_t - \upsilon\hat{s}_t
\end{aligned} \tag{5.37}$$

5.3.7 Bond market clearing

The long-term bond market clears when the home household, home central bank and foreign investor demand all issued bonds. The total holdings of long-term bonds is therefore distributed between these three agents (see equation 5.7), and can be expressed in log-linearised form as

[24]See, for example, Coeurdacier and Gourinchas (2016), Devereux and Sutherland (2009, 2010, 2011), Rabitsch, Stepanchuk, and Tsyrennikov (2015) and Schmitt-Grohé and Uribe (2003).

$$B_{L,t} = B^H_{L,t} + B^{CB}_{L,t} + B^F_{L,t}$$
$$\therefore \bar{B}_L e^{\hat{b}_{L,t}} = \bar{B}^H_L e^{\hat{b}^H_{L,t}} + \bar{B}^{CB}_L e^{\hat{b}^{CB}_{L,t}} + \bar{B}^F_L e^{\hat{b}^F_{L,t}}$$
$$\therefore \bar{B}_L(1+\hat{b}_{L,t}) \approx \bar{B}^H_L(1+\hat{b}^H_{L,t}) + \bar{B}^{CB}_L(1+\hat{b}^{CB}_{L,t}) + \bar{B}^F_L(1+\hat{b}^F_{L,t})$$

From steady state we know that $\bar{B}_L = \bar{B}^H_L + \bar{B}^{CB}_L + \bar{B}^F_L$. The constants therefore drop out, and we can divide by $\bar{B}_L$ to find

$$\begin{aligned} \hat{b}_{L,t} &= \frac{\bar{B}^H_L}{\bar{B}_L}\hat{b}^H_{L,t} + \frac{\bar{B}^{CB}_L}{\bar{B}_L}\hat{b}^{CB}_{L,t} + \frac{\bar{B}^F_L}{\bar{B}_L}\hat{b}^F_{L,t} \\ &= \bar{\mathcal{H}}\hat{b}^H_{L,t} + \bar{\mathcal{D}}\hat{b}^{CB}_{L,t} + \bar{\mathcal{F}}\hat{b}^F_{L,t} \end{aligned} \tag{5.38}$$

where $\bar{\mathcal{H}}$, $\bar{\mathcal{D}}$ and $\bar{\mathcal{F}}$ represent the steady state shares of long-term bonds held by the home household, home central bank and foreign investor, respectively.

Similar to the central bank's asset holdings, foreign asset holdings evolve according to the stochastic process

$$\log\left(\frac{f_t}{\bar{\mathcal{F}}}\right) = \phi^{\mathcal{F}} \log\left(\frac{f_{t-1}}{\bar{\mathcal{F}}}\right) + \varepsilon^{\mathcal{F}}_t \tag{5.39}$$

where $\bar{\mathcal{F}}$ is the steady state fraction of long-term bonds held by foreign investors $\left(\bar{B}^F_L/\bar{B}_L\right)$ and $\varepsilon^{\mathcal{F}}_t$ is a shock to foreign asset demand with a mean of zero and standard deviation of $\sigma^{\mathcal{F}}$. The log-linearised version of equation 5.39 is given by

$$\hat{f}_t = \phi^{\mathcal{F}} \hat{f}_{t-1} + \varepsilon^{\mathcal{F}}_t \tag{5.40}$$

Finally, since the fraction $f_t = B^F_{L,t}/B_{L,t}$ represents the foreign investor's holdings of long-term government bonds relative to the total amount issued, it can be expressed in log-linear terms as

$$\hat{f}_t = \hat{b}^F_{L,t} - \hat{b}_{L,t} \tag{5.41}$$

5.3.8 Goods market clearing

The domestic goods market will clear when local production is fully consumed. That is, all locally-produced goods are either consumed by the home household or exported.

Following Galí (2015:234) and adding our portfolio adjustment friction, the resource constraint therefore becomes

$$Y_t = C_{H,t} + X_t + \frac{B^H_{L,t}}{R_{L,t}} AC^L_t \tag{5.42}$$

Substituting in the expressions we know for $C_{H,t}$ and X_t[25], the resource constraint can be expressed as

$$Y_t = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \upsilon \mathcal{S}^\eta_t Y^\star_t + \frac{b^H_{L,t}}{R_{L,t}} AC^L_t \tag{5.43}$$

In the symmetric steady state, where $\bar{\mathcal{S}} = 1$ and $\bar{P} = \bar{P}_H$ (see Appendix C.1), the resource constraint becomes[26]

$$\bar{Y} = (1-\upsilon)\bar{C} + \upsilon \bar{Y}^\star \tag{5.44}$$

Under the assumption of balanced trade we have $\bar{Y} = \bar{C}$. It then follows from equation 5.44 that $\bar{Y}^\star = \bar{Y} = \bar{C}$. Utilising this, $\bar{P} = \bar{P}_H$ (equation C.1) and $\hat{p}_t - \hat{p}_{H,t} = \upsilon \hat{s}_t$ (equation C.5), we can log-linearise the resource constraint around the symmetric steady state:[27]

$$\begin{aligned}
Y_t &= (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \upsilon \mathcal{S}^\eta_t Y^\star_t + \frac{b^H_{L,t}}{R_{L,t}} AC^L_t \\
\therefore \bar{Y} e^{\hat{y}_t} &= (1-\upsilon)\left(\frac{\bar{P}_H e^{\hat{p}_{H,t}}}{\bar{P} e^{\hat{p}_t}}\right)^{-\eta} \bar{C} e^{\hat{c}_t} + \upsilon \left(\bar{\mathcal{S}} e^{\hat{s}_t}\right)^\eta \bar{Y}^\star e^{\hat{y}^\star_t} + 0 \\
&= (1-\upsilon)\left(e^{\hat{p}_{H,t} - \hat{p}_t}\right)^{-\eta} \bar{C} e^{\hat{c}_t} + \upsilon \bar{Y}^\star e^{\eta \hat{s}_t + \hat{y}^\star_t} \\
\therefore \bar{Y}(1 + \hat{y}_t) &\approx (1-\upsilon)\bar{C}(1 - \eta(\hat{p}_{H,t} - \hat{p}_t) + \hat{c}_t) + \upsilon \bar{Y}^\star (1 + \eta \hat{s}_t + \hat{y}^\star_t) \\
\therefore \bar{Y} \hat{y}_t &= (1-\upsilon)\bar{C}(\eta \upsilon \hat{s}_t + \hat{c}_t) + \upsilon \bar{Y}^\star (\eta \hat{s}_t + \hat{y}^\star_t) \\
\therefore \hat{y}_t &= (1-\upsilon)(\eta \upsilon \hat{s}_t + \hat{c}_t) + \upsilon(\eta \hat{s}_t + \hat{y}^\star_t) \\
&= (1-\upsilon)\hat{c}_t + \upsilon \eta (2-\upsilon)\hat{s}_t + \upsilon \hat{y}^\star_t
\end{aligned} \tag{5.45}$$

Combining equation 5.45 (the linearised resource constraint) with equation C.14 (the risk sharing condition linking domestic consumption with world output and the terms of trade) yields the following expression for the terms of trade:

[25] $C_{H,t} = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$ (Appendix C.1.2) and $X_t = \upsilon \mathcal{S}^\eta_t Y^\star_t$ (equation 5.33).

[26] Recall that in steady state AC^L_t collapses to zero (see the discussion of equation 4.5).

[27] In equation B.35 it was shown that log-linearising the final term (adjustment cost) around the steady state equals zero. We therefore skip explicitly showing this again.

$$
\begin{aligned}
\hat{y}_t &= (1-\upsilon)\left(\hat{y}_t^\star + \frac{1-\upsilon}{\sigma}\hat{s}_t\right) + \upsilon\eta(2-\upsilon)\hat{s}_t + \upsilon\hat{y}_t^\star \\
&= \hat{y}_t^\star(1-\upsilon+\upsilon) + \hat{s}_t\left[\frac{(1-\upsilon)^2}{\sigma} + \upsilon\eta(2-\upsilon)\right] \\
&= \hat{y}_t^\star + \hat{s}_t\left[\frac{(1-\upsilon)^2 + \sigma\upsilon\eta(2-\upsilon)}{\sigma}\right] \\
\therefore \hat{s}_t &= \sigma_\upsilon(\hat{y}_t - \hat{y}_t^\star)
\end{aligned}
\tag{5.46}
$$

where $\sigma_\upsilon = \frac{\sigma}{(1-\upsilon)^2+\sigma\upsilon\eta(2-\upsilon)} > 0$. Following Galí (2015:235), this can be simplified to $\sigma_\upsilon \equiv \sigma\Phi > 0$, with $\Phi \equiv \frac{1}{1+\upsilon(\varpi-1)} > 0$ and $\varpi \equiv \sigma\eta + (1-\upsilon)(\sigma\eta - 1)$.

Finally, combining equation 5.37 (net exports) with equations 5.45 (the resource constraint) and C.14 (the risk sharing condition) allows net exports to be expressed as a function of the terms of trade (Galí, 2015:236):

$$
\hat{nx}_t = \upsilon\left(\frac{\varpi}{\sigma} - 1\right)\hat{s}_t \tag{5.47}
$$

5.3.9 Dynamic IS curve

The open-economy consumption Euler equation is identical to the closed-economy specification (equation 5.15).[28] However, domestic inflation in the open economy is now a function of both the rate of change in domestic goods prices and the change in the terms of trade ($\hat{\pi}_t = \hat{\pi}_{H,t} + \upsilon\Delta\hat{s}_t$, see equation C.6), and not simply the former as in the closed-economy scenario. Therefore, the expanded open-economy Euler equation is given by

$$
\begin{aligned}
\hat{c}_t &= E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) \\
&= E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1} + \upsilon\Delta\hat{s}_{t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) \\
&= E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) + \frac{\upsilon}{\sigma}E_t[\Delta\hat{s}_{t+1}] - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)
\end{aligned}
\tag{5.48}
$$

The difference between the closed- and open-economy Euler equations is therefore the addition of one term which captures the terms of trade and trade openness, which both influence aggregate domestic inflation, and therefore the home household's consumption decision.

[28]See Galí (2015:228) or Kotzé (2014:71).

The dynamic IS curve can now be derived by combining the Euler equation (5.48) with the resource constraint (5.45) and the terms of trade, expressed as a function of domestic and foreign output (5.46), and is given by (detailed derivation in C.1.6)

$$\hat{y}_t = E_t\hat{y}_{t+1} + \Omega_1\left[E_t\Delta\hat{y}^\star_{t+1}\right] - \Omega_2(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \Omega_2\beta\kappa_L\phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) \tag{5.49}$$

where $\Omega_1 = \left(\frac{-\upsilon-\Omega}{1+\Omega}\right) > 0$ and $\Omega_2 = \frac{1}{\sigma}\left(\frac{1-\upsilon}{1+\Omega}\right) > 0$ are convolutions of the parameters. $\Omega = \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma} - \upsilon\eta(2-\upsilon)\sigma_\upsilon < 0$, $(1+\Omega) > 0$, $(1-\upsilon) > 0$ and $(-\upsilon-\Omega) > 0$. It is easy to verify that for $\upsilon \to 0$ we will have $\Omega \to 0$, and thus $\Omega_1 \to 0$ and $\Omega_2 \to \frac{1}{\sigma}$. Therefore, if the degree of openness is set equal to zero, the IS curve collapses to the familiar closed-economy representation (identical to equation 4.49). Applying some algebra on the Ω_i coefficients it can be shown that $\Omega_1 = \upsilon(\varpi - 1)$ and $\Omega_2 = \frac{1}{\sigma_\upsilon}$. This yields the equivalent IS curve with the more familiar open-economy coefficients (Galí, 2015:235)

$$\begin{aligned}\hat{y}_t = E_t\hat{y}_{t+1} + \upsilon(\varpi-1)\left[E_t\Delta\hat{y}^\star_{t+1}\right] - \frac{1}{\sigma_\upsilon}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}])\\ - \frac{1}{\sigma_\upsilon}\Psi_1(\hat{b}^H_{L,t} - \hat{b}_t)\end{aligned} \tag{5.50}$$

Domestic output is therefore a positive function of expected future output and world demand, and a negative function of the real interest rate and the ratio of long- to short-term bonds held by the household.

5.3.10 Inflation dynamics and the Phillips curve

Following Galí and Monacelli (2005:717) domestic inflation can be expressed in terms of real marginal cost as[29]

$$\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \lambda\hat{mc}^r_t \tag{5.51}$$

where $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$. From the production function, the marginal product of labour is equal to $MPN_t = \frac{\partial Y_t(j)}{\partial N_t(j)} = Z_t$[30], while the log-linearised aggregate production function can be expressed as

$$\hat{y}_t = \hat{z}_t + \hat{n}_t \tag{5.52}$$

[29]This equation is analogous to the closed-economy version. See section B.2.5 for the derivation.
[30]In log-linear terms this becomes $\hat{mpn}_t = \hat{z}_t$.

Real marginal costs, expressed in home prices, are common across domestic firms (Galí and Monacelli, 2005:715) and is given by

$$\begin{aligned} \hat{mc}_t^r &= \hat{w}_t - \hat{p}_{H,t} - \hat{mpn}_t \\ &= \hat{w}_t - \hat{p}_{H,t} - \hat{z}_t \end{aligned} \tag{5.53}$$

Given the assumption that domestic conditions do not affect world output, it can be shown that domestic real marginal costs are related to the output gap according to[31]

$$\begin{aligned} \hat{mc}_t^r &= (\hat{w}_t - \hat{p}_{H,t}) - \hat{z}_t \\ &= (\hat{w}_t - \hat{p}_t) + (\hat{p}_t - \hat{p}_{H,t}) - \hat{z}_t \\ &= (\sigma\hat{c}_t + \phi\hat{n}_t) + \upsilon\hat{s}_t - \hat{z}_t \end{aligned}$$

From equations C.14 and 5.52 we have $\hat{c}_t = \hat{y}_t^\star + \frac{1-\upsilon}{\sigma}\hat{s}_t$ and $\hat{n}_t = \hat{y}_t - \hat{z}_t$. Substituting these into the previous expression and simplifying yields

$$\hat{mc}_t^r = \sigma\hat{y}_t^\star + \phi\hat{y}_t + \hat{s}_t - (1+\phi)\hat{z}_t$$

Real domestic marginal cost is therefore a positive function of the terms of trade and world output. The domestic real wage is influenced by both these variables, "through the wealth effect on labour supply resulting from their impact on domestic consumption" (Galí and Monacelli, 2005:718), while "changes in the terms of trade have a direct effect on the product wage, for any given real wage" (Ibid.). Finally, substituting in equation 5.46 for $\hat{s}_t$ allows real marginal cost to be expressed in terms of world output, domestic output and local technology:

$$\hat{mc}_t^r = (\sigma_\upsilon + \phi)\hat{y}_t + (\sigma - \sigma_\upsilon)\hat{y}_t^\star - (1+\phi)\hat{z}_t \tag{5.54}$$

For $\upsilon \to 0$ we will have $\sigma_\upsilon \to \sigma$, and real marginal cost will be identical to the closed-economy specification. Following Galí and Monacelli (2005:718), the output gap and real marginal cost is related according to

$$\hat{mc}_t^r = (\sigma_\upsilon + \phi)\hat{y}_t \tag{5.55}$$

which can be combined with equation 5.51 to obtain the open-economy specification of the New-Keynesian Phillips curve:

[31] This utilises equations 5.12 and C.5.

$$\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \kappa_\upsilon \hat{y}_t \tag{5.56}$$

where $\kappa_\upsilon \equiv \lambda(\sigma_\upsilon + \phi)$.[32] It can be verified that for $\upsilon \to 0$ the slope coefficient will be analogous to the closed-economy specification.

5.4 Final model equations and calibration

5.4.1 Final model equations

The linear model equations are given by the following:

[32] For $\alpha \neq 0$ this becomes $\kappa_\upsilon \equiv \lambda(\sigma_\upsilon + \frac{\phi+\alpha}{1-\alpha})$ (Galí, 2015:238).

(1) **IS curve (5.50):** $\hat{y}_t = E_t[\hat{y}_{t+1}] + \upsilon(\varpi - 1)\left[E_t \Delta \hat{y}^\star_{t+1}\right] - \frac{1}{\sigma_\upsilon}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{1}{\sigma_\upsilon}\beta\kappa_L\phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t)$

(2) **Taylor rule (5.18):** $\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t + \varepsilon^R_t$

(3) **NK Phillips curve (5.56):** $\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \kappa_\upsilon \hat{y}_t$

(4) **Term structure (5.16):** $\hat{r}_{L,t} = \hat{r}_t + E_t[\hat{r}_{t+1}] + \phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t)(\beta\kappa_L + 1)$

(5) **LT bond supply (5.27):** $\hat{b}_{L,t} = \phi^{BL}\hat{b}_{L,t-1} + \varepsilon^{BL}_t$

(6) **Tax rule (5.26):** $\hat{t}_t = \frac{1}{\bar{T}\bar{P}}\left[\psi_1\bar{B}(\hat{b}_{t-1} - \hat{p}_t) + \beta\psi_2\bar{B}_L(\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t)\right]$

(7) **CB LT bonds (5.22):** $\hat{b}^{CB}_{L,t} = \hat{d}_t + \hat{b}_{L,t}$

(8a) **LSAPs (5.21):** $\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \varepsilon^{\mathcal{D}}_t$

(8b) **Balance sheet rule (5.24):** $\hat{d}_t = \phi^{\mathcal{D}}\hat{d}_{t-1} + \gamma^\pi\hat{\pi}_t + \gamma^y\hat{y}_t + \gamma^f\hat{f}_t + \varepsilon^{\mathcal{D}}_t$

(9) **HH LT bonds (5.38):** $\bar{\mathcal{H}}\hat{b}^H_{L,t} = \hat{b}_{L,t} - \bar{\mathcal{D}}\hat{b}^{CB}_{L,t} - \bar{\mathcal{F}}\hat{b}^F_{L,t}$

(10) **ST bonds (5.28):**

$$\hat{b}_t = \frac{1}{\beta}\hat{b}_{t-1} + \frac{\bar{B}_L}{\bar{B}}(1 - \bar{\mathcal{D}})(\hat{b}_{L,t-1} - \beta\hat{b}_{L,t}) + (1 - \frac{\bar{B}_L}{\bar{B}}(1 - \bar{\mathcal{D}}))\hat{r}_t + \frac{\bar{B}_L}{\bar{B}}\beta(1 - \bar{\mathcal{D}})\hat{r}_{L,t} - \frac{\bar{M}}{\bar{B}}\beta(\hat{m}_t - \hat{m}_{t-1}) + \bar{\mathcal{D}}\frac{\bar{B}_L}{\bar{B}}(\beta\hat{d}_t - \hat{d}_{t-1}) - \frac{\bar{T}\bar{P}}{\bar{B}}(\hat{t}_t + \hat{p}_t)$$

(11) **Production function (5.52):** $\hat{y}_t = \hat{z}_t + \hat{n}_t$

(12) **Technology (5.30):** $\hat{z}_t = \rho_z\hat{z}_{t-1} + \varepsilon^z_t$

(13) **Wages (5.12):** $\hat{w}_t - \hat{p}_t = \phi\hat{n}_t + \sigma\hat{c}_t$

(14) **Resource constraint (5.45):** $\hat{y}_t = (1 - \upsilon)\hat{c}_t + \upsilon\eta(2 - \upsilon)\hat{s}_t + \upsilon\hat{y}^\star_t$

(15) **Money demand (5.11):** $\hat{m}_t - \hat{p}_t = \frac{1}{\delta}\frac{1}{1 - \beta}(\sigma\hat{c}_t - \beta E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1}])$

(16) **CPI price level (C.5):** $\hat{p}_t = \hat{p}_{H,t} + \upsilon \hat{s}_t$

(17) **CPI inflation (C.6):** $\hat{\pi}_t = \hat{\pi}_{H,t} + \upsilon \Delta \hat{s}_t$

(18) **Home price level (from 5.56):** $\hat{p}_{H,t} = \hat{\pi}_{H,t} + \hat{p}_{H,t-1}$

(19) **World output (C.16):** $\hat{y}^\star_t = \phi^{y\star} \hat{y}^\star_{t-1} + \varepsilon^{y\star}_t$

(20) **Terms of trade (5.46):** $\hat{s}_t = \sigma_\upsilon (\hat{y}_t - \hat{y}^\star_t)$

(21) **Nominal ER (from C.7):** $\hat{e}_t = \hat{s}_t - (\hat{p}^\star_t - \hat{p}_{H,t})$

(22) **Real ER (C.9):** $\hat{q}_t = (1 - \upsilon) \hat{s}_t$

(23) **Foreign bond purchases (5.40):** $\hat{f}_t = \phi^{\mathcal{F}} \hat{f}_{t-1} + \varepsilon^{\mathcal{F}}_t$

(24) **Foreign bond holdings (5.41):** $\hat{f}_t = \hat{b}^F_{L,t} - \hat{b}_{L,t}$

(25) **World prices (C.17):** $\hat{p}^\star_t = \phi^{p\star} \hat{p}^\star_{t-1} + \varepsilon^{p\star}_t$

World variables (output/demand and prices) are assumed to be exogenously determined according to an AR(1) process.

5.4.2 Calibration

The calibration is virtually identical to the closed-economy calibration described in section 4.3.2 above. An additional set of parameters, specific to the open-economy extension, is described in Table 5.3 below. The latter category of parameters is adopted from Galí and Monacelli (2005). Output is normalised to 1 (i.e. $\bar{Y} = 1$). All other steady state variables are calibrated according to data and expressed in relation to this.

Table 5.1: Calibration of standard parameters

Parameter	Value	Description
Preferences and technology		
β	0.994	Discount rate
σ	2	CRRA
ϕ_L	0.01	Portfolio adjustment frictions
δ	7	Elasticity of money demand
ϕ	1	Elasticity of labour supply
χ	8	Labour disutility
α	0	Capital share of production
ρ_z	0.95	AR(1) technology coefficient
ρ_a	0.5	Persistence of cost push shock
θ	0.75	Calvo sticky price parameter
ε	6	CES
κ_υ †	0.284	Slope of the Phillips curve
Fiscal and monetary policy		
ψ_1	0.3	Fiscal response to short-term debt
ψ_2	0.3	Fiscal response to long-term debt
ϕ^π	1.5	Monetary policy response to inflation
ϕ^y	0.5	Monetary policy response to output

† Calculated as $\kappa_\upsilon \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}(\sigma_\upsilon + \frac{\phi+\alpha}{1-\alpha})$, $\alpha = 0$.

Table 5.2: Calibration of key parameters and steady states

Parameter	Value	Description
$\bar{B} + \bar{B}_L$	0.496	Total debt to GDP
$\bar{B}$	0.188	ST debt on total debt
$\bar{B}_L$	0.308	LT debt on total debt
$\bar{B}_L^H$	0.190	LT debt held by households
$\bar{B}_L^{CB}$	0.062	LT debt held by central bank
$\bar{\mathcal{H}}$	0.5	Fraction of LT debt held by HH
$\bar{\mathcal{D}}$	0.2	Fraction of LT debt held by CB
$\phi^{\mathcal{D}}$	0.83	Persistence of CB asset purchases
ϕ^{BL}	0.75	AR(1) bond supply coefficient

For illustrative purposes, the calibration of the debt instruments somewhat arbitrarily assumes that long-term debt is held 30% by foreign investors, 20% by the home central bank and 50% by the home household.[33] The ratio of short- to long-term to GDP is unchanged from the closed-economy calibration. The persistence of foreign asset purchases is somewhat arbitrarily set to 0.7. Again, because this is unexplored territory, there are no benchmark values to consider from the literature. The calibration of η and υ follow Galí (2015).

TABLE 5.3: Calibration of open-economy parameters

Parameter	**Value**	**Description**
η	1	Elasticity of substitution between home and foreign goods
υ	0.4	Index of openness
$\bar{B}_L^F$	0.118	LT debt held by FH
$\bar{\mathcal{F}}$	0.3	Fraction of LT debt held by FH
$\bar{\mathcal{F}}$	0.3	Fraction of LT debt held by FH
$\phi^{y^\star}$		Persistence of world demand shock
$\phi^{p^\star}$		Persistence of world price shock

5.5 Impulse response functions

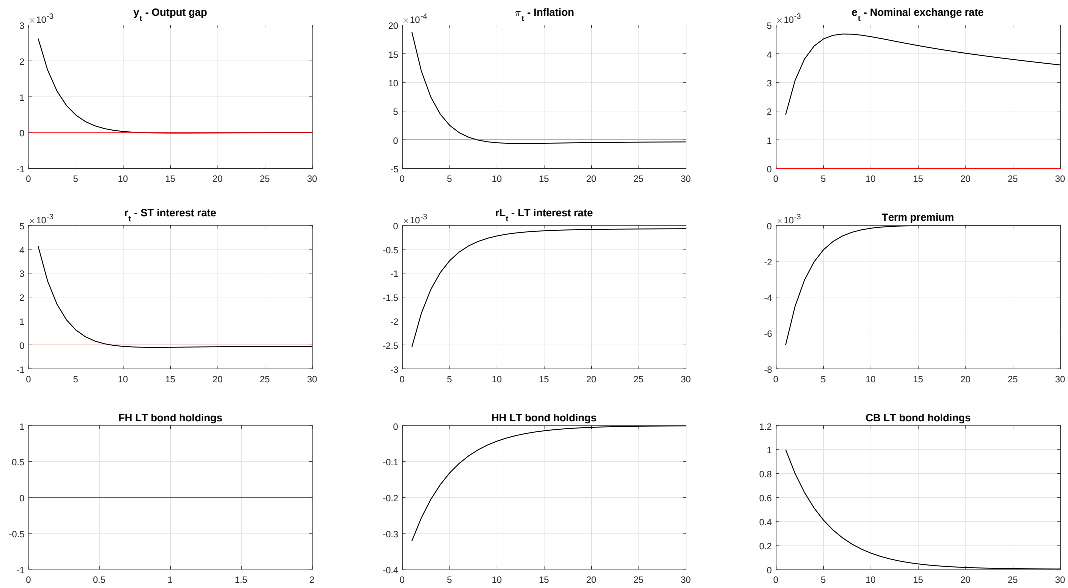
5.5.1 Domestic LSAPs

Figure 5.1 demonstrates the adjustment of the home economy following a domestic asset purchase shock. This illustrates how a typical QE-type intervention would influence the home economy. The central bank doubles its holdings of domestic long-term government bonds (a deliberately large shock), which removes a large portion of these securities from the household's balance sheet. The excess liquidity increases home demand and consumption, and subsequently domestic inflation. Higher home demand leads to an increased demand for foreign goods, which leads to a depreciation in the nominal exchange rate.[34] The exchange rate gradually returns to equilibrium as excess home demand subsides. The increased demand for long-term securities leads to a fall in long-term interest rates. This, coupled with the increase in the short-term interest rate (the central bank's Taylor rule response), leads to a fall in the term premium. There is, of course, no change to foreign holdings of domestic assets, as this is a purely domestic shock.

[33] In reality, this final portion is held predominantly by domestic institutional investors, e.g. pension funds and insurers. We allocate these to the home household in the absence of a financial sector.

[34] This follows from the higher demand for foreign currency to purchase foreign goods, which increases the "price" of foreign currency and thus implies a depreciation in the home currency.

FIGURE 5.1: Impulse response functions – LSAP shock



5.5.2 Capital inflows and balance sheet rules

Similar to the approach followed in Section 4.4.3.1, various combinations of monetary policy parameters are considered in order to evaluate different policy frameworks. As was alluded to earlier, central bank balance sheet operations could be applied to target inflation and output only, or capital flows only, or some combination. Table 5.4 proposes five possible policy "stances" or "mixes", each with a distinct parameter space and aimed at different targets. We retain the (i) Taylor and (ii) hybrid rules from the closed-economy estimation, as they were shown to be strong and robust policy rules. They are then contrasted against three alternative policy frameworks: (iii) A capital flow rule, which leaves the Taylor rule to stabilise inflation only, and the balance sheet rule to stabilise capital flows; (iv) a Taylor/capital flow rule, which combines the conventional Taylor rule (stabilising both inflation and output) with a balance sheet rule to stabilise capital flows; and (v) a composite rule, which extends the hybrid rule to also target capital flows.

As was alluded to earlier, the combination of negative γ^{π} and γ^{y} with an additional contractionary parameter ($\gamma^{f} < 0$) poses the risk of an overly-contractionary policy response, especially in combination with a high persistence parameter. We therefore start out with a small response to capital flows ($\gamma^{f} = -0.1$), while maintaining the persistence parameter at $\phi^{\mathcal{D}} = 0.83$. Other calibrations will be considered in the model's sensitivity analyses (Section 5.6).

Table 5.4: Comparing policy rules: Open economy

	ϕ^π	ϕ^y	γ^π	γ^y	γ^f	Functional form (no shocks)	
(i) Taylor rule	1.5	0.5	0	0	0	$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t$	$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1}$
(ii) Hybrid rule	1.5	0.5	-1.5	-10	0	$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t$	$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \gamma^\pi \hat{\pi}_t + \gamma^y \hat{y}_t$
(iii) Capital flow rule	1.5	0	0	0	-0.1	$\hat{r}_t = \phi^\pi \hat{\pi}_t$	$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \gamma^f \hat{f}_t$
(iv) Taylor/capital flow rule	1.5	0.5	0	0	-0.1	$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t$	$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \gamma^f \hat{f}_t$
(v) Composite rule	1.5	0.5	-1.5	-10	-0.1	$\hat{r}_t = \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t$	$\hat{d}_t = \phi^{\mathcal{D}} \hat{d}_{t-1} + \gamma^\pi \hat{\pi}_t + \gamma^y \hat{y}_t + \gamma^f \hat{f}_t$

The various policy stances are evaluated against the loss function (equation 4.53) for the various shocks, and the results are reported in Table 5.5 below.

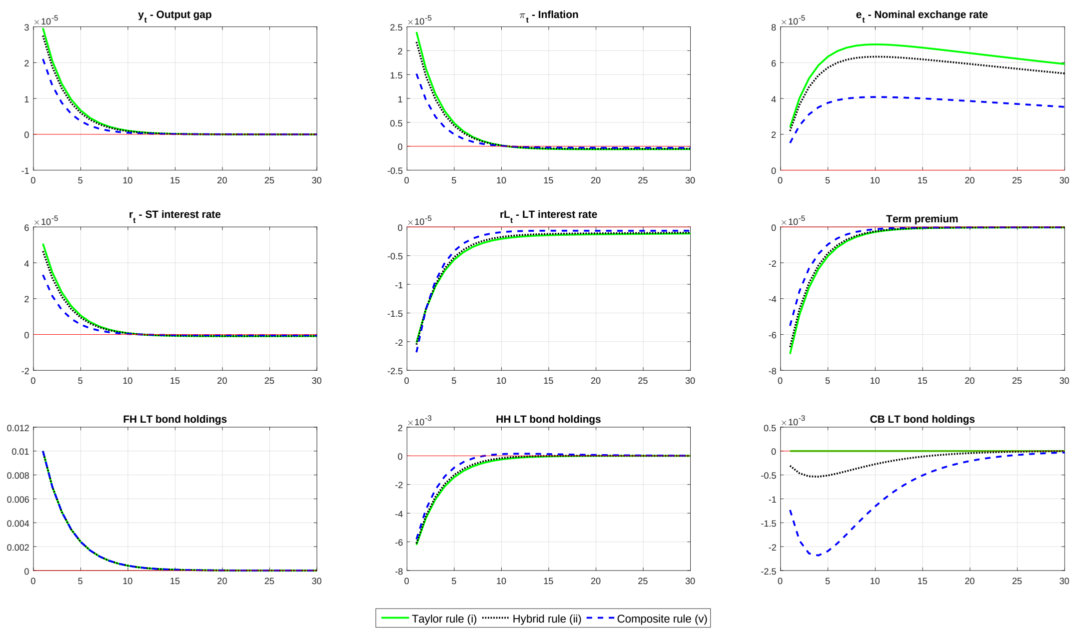
Table 5.5: Policy rules and loss function: Open economy

Rule	ϕ^π	ϕ^y	γ^π	γ^y	γ^f	ε_t^d $(\times 10^{-5})$	ε_t^a $(\times 10^{-4})$	ε_t^R $(\times 10^{-6})$	ε_t^{BL} $(\times 10^{-7})$	ε_t^f $(\times 10^{-9})$
(i)	1.5	0.5	-	-	-	3.763	4.767	2.972	1.220	3.817
(ii)	1.5	0.5	-1.5	-10	-	3.198	4.949	2.531	1.013	3.178
(iii)	1.5	-	-	-	-0.1	6.147	4.451	4.842	2.056	3.120
(iv)	1.5	0.5	-	-	-0.1	3.763	4.767	2.972	1.220	1.857
(v)	1.5	0.5	-1.5	-10	-0.1	3.198	4.949	2.531	1.013	1.552

The main result from Table 5.5 is the distinction between internal and external shocks. There is no difference between the performance of the Taylor rule (i) and the Taylor/-capital flow rule (iv), just as there is virtually no difference between the hybrid rule (ii) and the composite rule (v) for the four *internal* shocks (demand, supply, interest rate and long-term bond supply). This is hardly surprising, however, since under domestic shocks $\hat{f}_t = 0$ and therefore the role of γ^f is trivial. Furthermore, under these four shocks the hybrid rule (ii) still outperforms the Taylor rule, consistent with the findings from Chapter 4, while the capital flow rule (iii) performs significantly poorer. The exception is the cost-push shock, where the hybrid rule is outperformed by both the Taylor rule as well as the capital flow rule. However, in the event of an *external* (capital flow) shock, any policy mix that does target capital flows (i.e. $\gamma^f \neq 0$) improves upon policy stances that do not target capital flows. This even applies to the capital flow rule (iii), which performs very poorly against internal shocks but is superior to rules (i) and (ii) under an external shock. This suggests that capital flow shocks might be managed most effectively by directly targeting such flows with the central bank's balance sheet, in combination with the conventional Taylor rule. Moreover, internal shocks could be better managed by utilising both interest rate and balance sheet policies, instead of just interest rate policy, given the superior performance of the hybrid rule (ii) over the pure Taylor rule (i).

Figure 5.2 below illustrates the responses of policy rules (i), (ii) and (v) to a capital inflow shock. The composite rule (v) is the only policy mix of the three that directly targets capital inflows. This rule was shown to realise the lowest value of the loss function (Table 5.5), and from Figure 5.2 it is clear why this is the case. The deviation in both output and inflation from its steady state levels are noticably smaller under the composite rule than any other policy stance. Volatility in the nominal exchange rate, both the short- and long-term interest rates and the term premium are smaller. It is, however, notable that the net effect of capital inflows is not an exchange rate appreciation, as might have been expected, but an exchange rate *depreciation*. Clearly, the appreciating pressure originating from foreign demand for local assets (and thus local currency) is dominated by the depreciation pressure resulting from the home household's demand for foreign consumption goods (and thus foreign currency). The disruption to the household's portfolio, illustrated by the smaller response of the household's long-term bond holdings under the composite rule, is also somewhat less pronounced. The larger central bank balance sheet intervention thus contributes to alleviating disruptions elsewhere, and reduces the costs associated with the central bank's policy intervention.

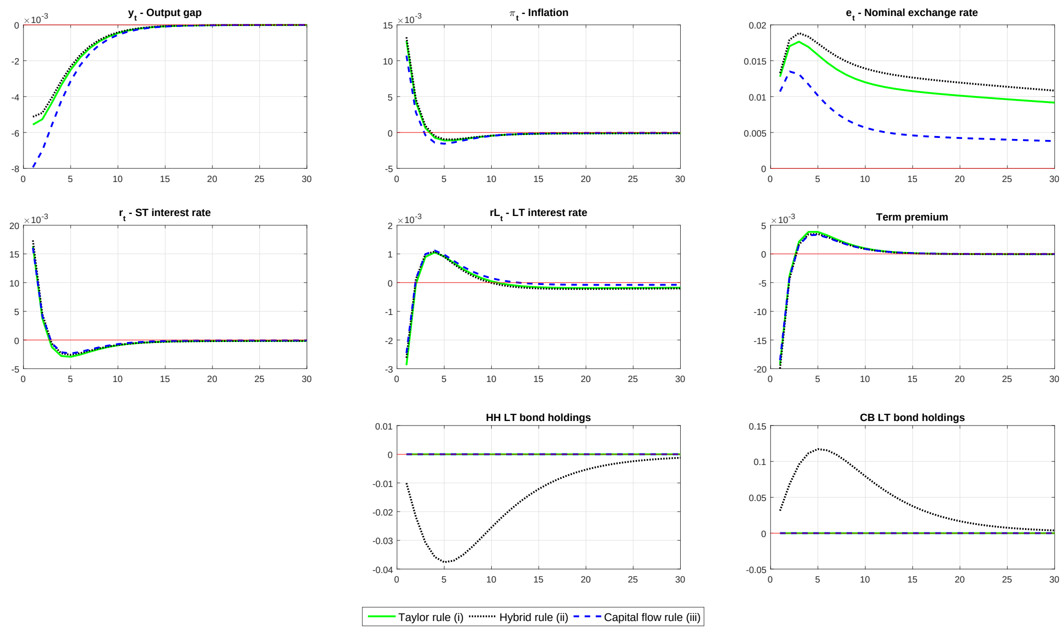
Figure 5.2: Optimal open-economy policy rules: Capital inflows



5.5.3 Supply shock

As a point of comparison, Figure 5.3 below illustrates the response of these policy mixes to a supply shock, replacing the composite rule (v) with the capital flow rule (iii) which was shown to be superior in Table 5.5. There is, of course, no change in foreign holdings of domestic assets, therefore the bottom-left panel (illustrating changes in foreign holdings of domestic long-term bonds) is simply removed, while the Taylor rule response would not affect holdings of long-term bonds.

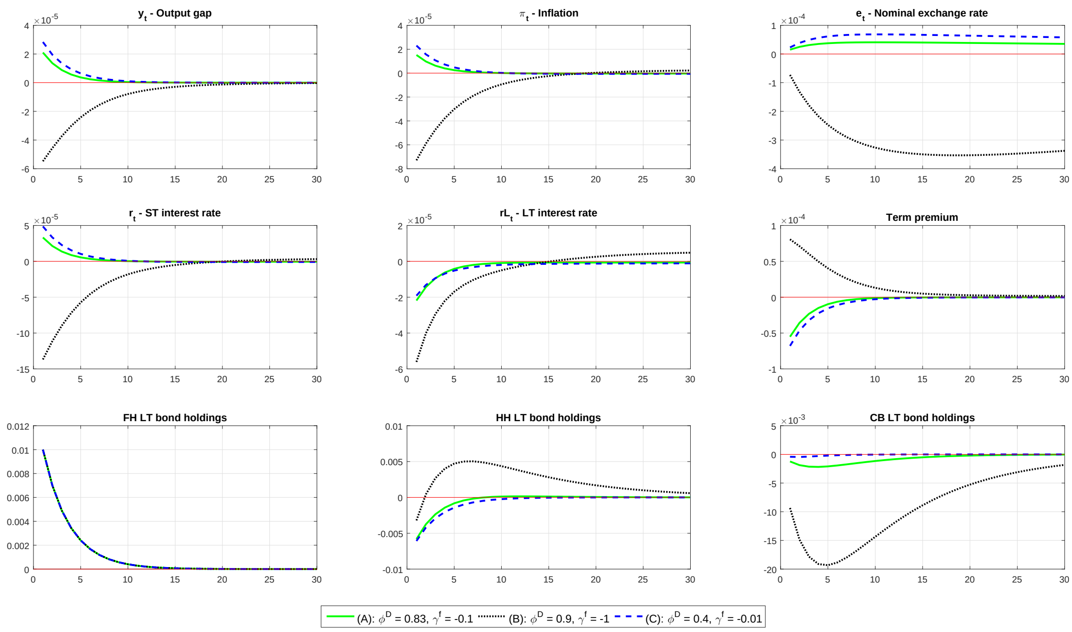
FIGURE 5.3: Optimal open-economy policy rules: Supply shock



5.6 Sensitivity analyses

Figure 5.2 illustrated the response of three policy mixes to a foreign capital flow shock, under the assumption that central bank asset transactions are persistent ($\phi^{\mathcal{D}} = 0.83$) and that the central bank's weight on these flows are relatively low ($\gamma^f = -0.1$). It was shown that a composite rule, in which the central bank balance sheet directly responds to capital inflows, is superior to rules which do not explicitly respond to capital flows. However, how would this composite rule change under different calibrations of $\phi^{\mathcal{D}}$ and γ^f? It was suggested earlier that there might exist a tradeoff between these two parameters, in that a higher persistence should accommodate a smaller response to capital flows (and *vice versa*), to prevent an overly-contractionary policy response. Figure 5.4 contrasts (A) the benchmark composite rule against (B) a composite rule with both high persistence ($\phi^{\mathcal{D}} = 0.9$) and a strong capital flow response ($\gamma^f = -1$) and (C) a composite rule with low persistence ($\phi^{\mathcal{D}} = 0.4$) and a weak capital flow response ($\gamma^f = -0.01$).

FIGURE 5.4: Composite rules: Parameter sensitivity



Clearly, the combination of high persistence and a strong capital flow response (B) is too strong, and causes the economy to contract. The central bank now sells *too many* bonds to the household, draining cash and actually *increasing* the household's holdings of long-term bonds, which leads to a contraction in consumption spending and economic activity. Since capital inflows are generally expected to have an expansionary impact on domestic output, consumption and inflation, such a strong policy response is undesirable.

On the other hand, a weak policy response (C) is somewhat less effective at reining in the expansionary impact of capital inflows. This final result is also supported by the findings in Chapter 4, where it was argued that it is the *persistence* of central bank asset market interventions which makes the policy effective.

5.7 Conclusion

In this chapter, the theoretical closed-economy model developed previously was extended to an open-economy model. The main aim of this chapter was therefore to evaluate the balance sheet rule, as proposed in Chapter 4, in an open-economy setting. The open-economy model introduced a foreign investor, who holds an asset portfolio which consists in part of domestic long-term bonds. The foreign investor serves as a mechanism through which international capital flows could enter the model: Capital inflows (outflows) can be modeled as an increase (decrease) in foreign demand for home financial assets. To mimic the international search for yield and substitute investment goods following the Fed's LSAP programmes in the US, the foreign investor's interaction with the home economy is restricted to trading in domestic long-term government bonds only. Capital inflows sees the foreign investor purchasing domestic long-term bonds from the home household. This has an expansionary effect on the home economy, as the home household now holds larger cash balances and a smaller long-term bond portfolio, which translates to higher consumption spending and inflationary pressure.

Various shocks (both internal and external) were simulated under various combinations of monetary policy mixes. These policy mixes simply varied the parameters of the Taylor and balance sheet rules in an attempt to find an optimal policy mix. Based on the central bank's loss function the superior policy mix was determined as the parameter space with the lowest welfare cost. Under three of the four internal shocks (demand, monetary policy and asset purchase shocks) the hybrid rule, which allows both the short-term interest rate as well as the central bank balance sheet to respond to the shock, was found to be superior to the traditional Taylor rule. This result is consistent with the results from Chapter 4. However, in response to a supply shock, the Taylor rule is preferred to the hybrid rule. Moreover, a lower social cost, captured by the loss function, is realised under a Taylor rule which responds to inflation only than under a traditional Taylor which responds to both inflation and output. The international capital flow shock, the only external shock, is best managed by a composite rule, under which the Taylor and balance sheet rules exist side-by-side. Under this policy mix the balance sheet explicitly responds to inflation, output and capital flows. These results suggest that the central bank's balance sheet could potentially be used to complement the Taylor rule, leading to a softer disruption of the domestic economy in response to a number of different shocks.

Finally, consistent with the results from Chapter 4, the open-economy model is also sensitive to the persistence of the central bank's asset market interventions. If the persistence parameter is too low, the balance sheet rule loses its power. Conversely, an overly-aggressive response to capital flows, coupled with a high persistence parameter, poses the danger of the central bank intervention overshooting its target, leading to unintended consequences. Admittedly, these parameters tested here are somewhat arbitrarily selected due to the highly limited literature. They should therefore be interpreted as "first guesses" towards a more robust empirical estimation and framework. However, the intuitive application and theoretical foundation of this model and its transmission mechanisms could be construed as a strong framework upon which future studies could be built.

Chapter 6

Conclusion

This thesis evaluated the impact of unconventional monetary policy on asset markets and economic activity. Unconventional monetary policy was defined as "the central bank [using] its balance sheet to affect asset prices and financial conditions beyond the short-term interest rate" (Borio and Disyatat, 2009:25).

In Part I, the impact of the central bank utilising its balance sheet to directly intervene in long-term bond markets was considered both from a theoretical and empirical perspective.

The theoretical overview (Chapter 2) involved identifying various channels through which unconventional policy could influence the yields and price of a wide spectrum of longer-term bonds and fixed-income securities. The two notable channels, according to the literature, is a *signalling* and *portfolio balance* channel. Through the signalling channel central bank asset purchases "convey a signal that monetary policy is likely to be easier going forward, which reduces investors' expectations of the path of the federal funds rate and thereby has a broad impact on asset prices" (Krishnamurthy and Vissing-Jorgensen, 2013:59). The portfolio balance channel works through changes in relative asset holdings, which "will induce portfolio rebalancing and movements in asset prices" (Bowdler and Radia, 2012:609). Several other (smaller) transmission channels were further identified, the majority of which could be interpreted as special cases of the portfolio balance channel. The consensus that emerges is that central bank asset purchases work mainly through the term premium component of long-term interest rates. Through the purchasing of long-term fixed-income securities, thereby changing relative asset holdings of market participants, the central bank induces a fall in the term premium, which results in a fall in long-term interest rates.

These theories were substantiated by the empirical analyses conducted in Chapter 3. The behaviour of asset markets, in particular bond market yields and stock prices, throughout the Fed's various programmes were investigated. Long-term trends were identified, coinciding with various QE programmes. Furthermore, asset markets' response to individual FOMC QE announcements were measured using an event study approach. The empirical results generally support the theoretical considerations from Chapter 2, as well as the findings from the mainstream literature.

Part II consists of two DSGE models: A closed-economy (Chapter 4) and open-economy (Chapter 5) setting. The overarching objective of these two models is to consider the application of unconventional monetary policy, by way of central bank asset market intervention through its balance sheet, in conjunction with the conventional Taylor (short-term interest rate) rule. The main result is that unconventional monetary policy can complement conventional policy, provided the balance sheet rule and its parameters are appropriately configured.

In Chapter 4 a first closed-economy model was constructed to simulate the effect of central bank balance sheet policies on the economy. The goal was to obtain a tractable and intuitive framework which reflects the theoretical and empirical considerations of Chapters 2 and 3. This model yielded results consistent with comparable studies in the literature. The next step was to convert the central bank's asset purchase equation, which up to here was simply modelled as a stochastic process subject to exogenous shocks, to a balance sheet *rule*. To this end, the central bank's (net) asset purchase decision was endogenised to respond to output and inflation in a countercyclical manner. Given the uniqueness of this approach there does not exists a literature upon which to base the balance sheet rule or its parameters. Consequently the rule is constructed and parameters chosen somewhat arbitrarily, and should be interpreted as a first step towards refining such a rule. Nonetheless, the simulation results were promising. Various combinations of the Taylor and balance sheet rules (the policy mix) were considered and evaluated based on a simple loss function. It was determined that a combination of the Taylor and balance sheet rules yielded a lower welfare loss than a pure (conventional) Taylor rule. The interpretation of this result is therefore that there is indeed scope for unconventional monetary policy to be pursued in conjunction with conventional policy in conventional times.

Chapter 5 considers one possible application of the framework derived in the previous chapter. Here the economy is open up to allow for international capital flows, and a similar comparison between the Taylor and balance sheet rules are conducted. The novelty of this model is that international capital flows are modelled as foreign demand for domestic long-term government bonds. As the foreign investor's demand for domestic securities increase, capital flows into the home economy by way of the foreign investor purchasing domestic securities from the home household. The home household's additional cash holdings then filter through the home economy through the same channels as derived previously. The balance sheet rule now plays the role of sterilising, to some degree, these capital inflows in order to manage its expansionary effect on domestic inflation and output. Similar to Chapter 4, various policy mixes are compared based on each mix's loss function. Again, the combination of the Taylor and balance sheet rules are shown to be a superior policy framework in response to the majority of shocks, including international capital flows.

This thesis demonstrated that unconventional monetary policy need not be limited to crisis times, or when short-term interest interest rate policy is ineffective due to the ZLB. In fact, it was found that unconventional policy can complement conventional policy. Admittedly, the two DSGE models are highly theoretical constructions. While the majority of calibration and parameters were based on actual data, a number of key parameters were somewhat arbitrarily chosen. This does, however, provide a solid and rigorous foundation upon which any number of similar studies could be built.

Appendix A

Notation and technical approach

According to Devereux and Sutherland (2011:338), "the usual method of analysis in DSGE models is to take a linear approximation around a nonstochastic steady state". Therefore, our equations are log-linearised in terms of variables' log (percentage) deviations from their steady state values, following the approach of Uhlig (1999). In the steady state we define any variable $X_t = X_{t+1} = \bar{X}$. Variable X's deviation from its steady state level is therefore defined as $\hat{x}_t = \log(X_t) - \log(\bar{X})$. The rule $X_t = \bar{X}e^{x_t} \approx \bar{X}(1+\hat{x}_t)$ (Uhlig, 1999:5) will be applied throughout.

The derivation of the model equations makes use of the following notational conventions: Variables in levels are denoted by capital letters, real variables are denoted by small letters (e.g. $m_t = M_t/P_t$, and log deviations of variables are denoted by small hatted letters (e.g. $\hat{x}_t = \log(X_t) - \log(\bar{X})$).

Appendix B

Derivation of closed-economy equations

B.1 Solving the household's optimisation problem

B.1.1 First-order conditions

To derive the optimality conditions we set up the Lagrangian $\mathcal{L}$ from the household's utility function (equation 4.1) and real budget constraint (equation 4.3), and utilising the fact that the price level can be expressed as $P_{t+1} \equiv \pi_{t+1} P_t$:

$$\mathcal{L}_{C_t, \frac{M_t}{P_t}, N_t, B_t, B_{L,t}^H} = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \phi_t \left[\frac{1}{1-\sigma} C_t^{1-\sigma} + \frac{1}{1-\delta} \frac{M_t}{P_t}^{1-\delta} - \frac{\chi}{1+\phi} N_t^{1+\phi} \right] \right.$$
$$\left. -\lambda_t \left[C_t + T_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + \frac{B_{L,t}^H}{P_t R_{L,t}}(1 + AC_t) - \frac{W_t}{P_t} N_t - \frac{M_{t-1}}{P_t} - \frac{B_{t-1}}{P_t} - \frac{B_{L,t-1}^H}{P_t R_t} \right] \right\}$$

This yields the following first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow 0 = C_t + T_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + \frac{B^H_{L,t}}{P_t R_{L,t}}(1 + AC_t)$$
$$- \frac{W_t}{P_t}N_t - \frac{M_{t-1}}{P_t} - \frac{B_{t-1}}{P_t} - \frac{B^H_{L,t-1}}{P_t R_t}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow 0 = \phi_t C_t^{-\sigma} - \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = 0 \Rightarrow 0 = \beta\phi_{t+1} C_{t+1}^{-\sigma} - \lambda_{t+1}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \frac{M_t}{P_t}} = 0 &\Rightarrow 0 = \left(\frac{M_t}{P_t}\right)^{-\delta} - \lambda_t + \frac{\partial}{\partial \frac{M_t}{P_t}}\left\{E_t\left[\lambda_{t+1}\frac{M_t}{P_{t+1}}\right]\right\} \\ &\Rightarrow 0 = \left(\frac{M_t}{P_t}\right)^{-\delta} - \lambda_t + E_t\left[\lambda_{t+1}\frac{\partial}{\partial \frac{M_t}{P_t}}\left\{\frac{M_t}{P_t}\frac{1}{\pi_{t+1}}\right\}\right] \\ &\Rightarrow 0 = \left(\frac{M_t}{P_t}\right)^{-\delta} - \lambda_t + E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right]\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow 0 = -\phi_t \chi N_t^{\phi} + \lambda_t \frac{W_t}{P_t}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = 0 \Rightarrow 0 = -\frac{\lambda_t}{P_t R_t} - \frac{\partial}{\partial B_t}\left\{\lambda_t \frac{B^H_{L,t}}{P_t R_{L,t}}(1 + AC_t)\right\} + \frac{\partial}{\partial B_t}\left\{E_t\left[\lambda_{t+1}\frac{B_t}{P_{t+1}}\right]\right\}$$

Seperately solving for the two partial derivatives yields:

$$\begin{aligned}\frac{\partial}{\partial B_t}\left\{\lambda_t \frac{B^H_{L,t}}{P_t R_{L,t}}(1 + AC_t)\right\} &= \frac{\lambda_t B^H_{L,t}}{P_t R_{L,t}}\frac{\partial}{\partial B_t}\left\{1 + \frac{\phi_L Y_t}{2}\left(\kappa_L \frac{B_t/P_t}{B^H_{L,t}/P_t} - 1\right)^2\right\} \\ &= \frac{\lambda_t B^H_{L,t}}{P_t R_{L,t}}\left\{0 + \phi_L Y_t\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)\left(\frac{\kappa_L}{B^H_{L,t}}\right)\right\} \\ &= \frac{\lambda_t \kappa_L \phi_L Y_t\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)}{P_t R_{L,t}}\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial B_t}\left\{E_t\left[\lambda_{t+1}\frac{B_t}{P_{t+1}}\right]\right\} &= \frac{\partial}{\partial B_t}\left\{E_t\left[\lambda_{t+1}\frac{B_t}{P_t\pi_{t+1}}\right]\right\} \\
&= \frac{\partial}{\partial B_t}\left\{E_t\left[\frac{\lambda_{t+1}}{P_t\pi_{t+1}}\right]B_t\right\} \\
&= \frac{1}{P_t}E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right]
\end{aligned}$$

$$\therefore \frac{\partial \mathcal{L}}{\partial B_t} = 0 \Rightarrow 0 = -\frac{\lambda_t}{R_t} - \frac{\lambda_t \kappa_L \phi_L Y_t\left(\kappa_L \frac{B_t}{B^H_{L,t}} - 1\right)}{R_{L,t}} + E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right]$$

$$\frac{\partial \mathcal{L}}{\partial B^H_{L,t}} = 0 \Rightarrow 0 = \frac{\partial}{\partial B^H_{L,t}}\left\{\left(-\lambda_t\frac{B^H_{L,t}}{P_tR_{L,t}}\right)(1+AC_t)\right\} + \frac{\partial}{\partial B^H_{L,t}}\left\{E_t\left[\lambda_{t+1}\frac{B^H_{L,t}}{P_{t+1}R_{t+1}}\right]\right\}$$

Seperately solving for the two partial derivatives yields:

$$\begin{aligned}
&\frac{\partial}{\partial B^H_{L,t}}\left\{\left(-\lambda_t\frac{B^H_{L,t}}{P_tR_{L,t}}\right)(1+AC_t)\right\} \\
&\quad= (1+AC_t)\left(\frac{-\lambda_t}{P_tR_{L,t}}\right) + \left(-\frac{\lambda_t B^H_{L,t}}{P_tR_{L,t}}\right)\frac{\partial}{\partial B^H_{L,t}}\{1+AC_t\} \\
&\quad= -\frac{\lambda_t}{P_tR_{L,t}}(1+AC_t) - \frac{\lambda_t B^H_{L,t}}{P_tR_{L,t}}\frac{\partial}{\partial B^H_{L,t}}\left\{\frac{\phi_L Y_t}{2}\left(\kappa_L\frac{B_t/P_t}{B^H_{L,t}/P_t} - 1\right)^2\right\} \\
&\quad= -\frac{\lambda_t}{P_tR_{L,t}}(1+AC_t) - \frac{\lambda_t B^H_{L,t}}{P_tR_{L,t}}\left\{\phi_L Y_t\left(\kappa_L\frac{B_t}{B^H_{L,t}} - 1\right)\left(-\frac{\kappa_L B_t}{\left(B^H_{L,t}\right)^2}\right)\right\} \\
&\quad= -\frac{\lambda_t}{P_tR_{L,t}}(1+AC_t) + \frac{\lambda_t B^H_{L,t}\phi_L Y_t\left(\kappa_L\frac{B_t}{B^H_{L,t}} - 1\right)\kappa_L B_t}{P_tR_{L,t}\left(B^H_{L,t}\right)^2} \\
&\quad= -\frac{\lambda_t}{P_tR_{L,t}}(1+AC_t) + \frac{\lambda_t\phi_L Y_t B_t\kappa_L\left(\kappa_L\frac{B_t}{B^H_{L,t}} - 1\right)}{P_tR_{L,t}b^H_{L,t}}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial B_{L,t}^{H}}\left\{E_t\left[\lambda_{t+1}\frac{B_{L,t}^{H}}{P_{t+1}R_{t+1}}\right]\right\} &= \frac{\partial}{\partial B_{L,t}^{H}}\left\{E_t\left[\lambda_{t+1}\frac{B_{L,t}^{H}}{P_t\pi_{t+1}R_{t+1}}\right]\right\} \\
&= \frac{1}{P_t}E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}R_{t+1}}\right]
\end{aligned}
$$

$$
\begin{aligned}
\therefore \frac{\partial \mathcal{L}}{\partial B_{L,t}^{H}} = 0 \Rightarrow 0 = &-\frac{\lambda_t}{R_{L,t}} - \frac{\lambda_t\phi_L Y_t\left(\kappa_L\frac{B_t}{B_{L,t}^{H}}-1\right)^2}{2R_{L,t}} + \frac{\lambda_t\phi_L Y_t B_t\kappa_L\left(\kappa_L\frac{B_t}{B_{L,t}^{H}}-1\right)}{R_{L,t}B_{L,t}^{H}} \\
&+ E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}R_{t+1}}\right]
\end{aligned}
$$

The first-order conditions can be combined to find the household's optimal decision making rules (see equations B.1–B.5 from Appendix B.1).

From $\frac{\partial \mathcal{L}}{\partial C_t}$ and $\frac{\partial \mathcal{L}}{\partial C_{t+1}}$:

$$
\begin{aligned}
\lambda_t &= \phi_t C_t^{-\sigma} \\
\lambda_{t+1} &= \beta\phi_{t+1}C_{t+1}^{-\sigma}
\end{aligned}
\tag{B.1}
$$

Substituting these into the remaining first-order conditions and rearranging yields a number of optimality conditions:

From $\frac{\partial \mathcal{L}}{\partial \frac{M_t}{P_t}}$ and equation B.1 we can derive the **money demand function**:

$$
\left(\frac{M_t}{P_t}\right)^{-\delta} = \phi_t C_t^{-\sigma} - \beta E_t\left[\frac{\phi_{t+1}C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]
\tag{B.2}
$$

From $\frac{\partial \mathcal{L}}{\partial N_t}$ and equation B.1 we can derive the consumer's **labour supply equation**:

$$
\begin{aligned}
\frac{W_t}{P_t} &= \frac{1}{\lambda_t}\chi N_t^{\phi} = \chi\frac{1}{\phi_t C_t^{-\sigma}}N_t^{\phi} \\
\therefore N_t^{\phi} &= \frac{1}{\chi}\phi_t C_t^{-\sigma}\frac{W_t}{P_t}
\end{aligned}
\tag{B.3}
$$

From $\frac{\partial \mathcal{L}}{\partial B_t}$:

$$
\beta E_t\left[\frac{\phi_{t+1}C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] = \frac{\phi_t C_t^{-\sigma}}{R_t} + \frac{\phi_t C_t^{-\sigma}\kappa_L\phi_L Y_t\left(\kappa_L\frac{B_t}{B_{L,t}^{H}}-1\right)}{R_{L,t}}
\tag{B.4}
$$

From $\frac{\partial \mathcal{L}}{\partial B_t^H}$:

$$\beta E_t\left[\frac{\phi_{t+1}C_{t+1}^{-\sigma}}{\pi_{t+1}R_{t+1}}\right]=\frac{\phi_t C_t^{-\sigma}}{R_{L,t}}+\frac{\phi_t C_t^{-\sigma}\phi_L Y_t\left(\kappa_L\frac{B_t}{B_{L,t}^H}-1\right)^2}{2R_{L,t}}-\frac{\phi_t C_t^{-\sigma}\phi_L Y_t B_t\kappa_L\left(\kappa_L\frac{B_t}{B_{L,t}^H}-1\right)}{R_{L,t}B_{L,t}^H} \tag{B.5}$$

B.1.1.1 Steady state

In steady state we note that $\pi_t=\pi_{t+1}=\bar{\pi}=1$ and $\phi_t=\phi_{t+1}=\bar{\phi}=1$. The steady state versions of equations B.2 and B.3 are therefore given by

$$\text{B.2: } \left(\frac{\bar{M}}{\bar{P}}\right)^{-\delta}=\bar{\phi}\bar{C}^{-\sigma}-\beta E_t\left[\frac{\bar{\phi}\bar{C}^{-\sigma}}{\bar{\pi}}\right]=\bar{C}^{-\sigma}-\beta\bar{C}^{-\sigma}=\bar{C}^{-\sigma}(1-\beta)$$

$$\text{B.3: } \bar{N}^{\phi}=\chi^{-1}\frac{\bar{W}}{\bar{P}}\bar{\phi}\bar{C}^{-\sigma}=\chi^{-1}\frac{\bar{W}}{\bar{P}}\bar{C}^{-\sigma}$$

Finally, in steady state we have $\kappa_L=\bar{B}_L^H/\bar{B}=B_{L,t}^H/B_t$, implying that $(\kappa_L\bar{B}/\bar{B}_L^H-1)$ and subsequently AC_t collapse to zero. This yields the steady state versions of equations B.4 and B.5:

$$\begin{aligned}\text{B.4: } \beta E_t\left[\frac{\bar{\phi}\bar{C}^{-\sigma}}{\bar{\pi}}\right]&=\frac{\bar{\phi}\bar{C}^{-\sigma}}{\bar{R}}\\ \therefore \beta&=\frac{1}{\bar{R}}\\ \text{or } \bar{R}&=\frac{1}{\beta}\end{aligned}$$

$$\begin{aligned}\text{B.5: } \beta E_t\left[\frac{\bar{\phi}\bar{C}^{-\sigma}}{\bar{\pi}\bar{R}}\right]&=\frac{\bar{\phi}\bar{C}^{-\sigma}}{\bar{R}_L}\\ \therefore \frac{\beta}{\bar{R}}&=\frac{1}{\bar{R}_L}\\ \therefore \bar{R}_L=\frac{1}{\beta}\bar{R}&=\frac{1}{\beta^2}\end{aligned}$$

These equations give us steady state expressions for real money demand ($\frac{\bar{M}}{\bar{P}}$), labour supply ($\bar{N}$), and short- and long-term interest rates ($\bar{R}$ and $\bar{R}_L$). The latter two equations also indicate that the steady state values of the short- and long-term interest rates are linked by the discount rate β. If $\beta=0.99$, which is a standard calibration in the literature, $\bar{R}\approx 1.01$ and $\bar{R}_L\approx 1.02$.

B.1.1.2 Log-linearisation

Log-linearising[1] equations B.1–B.5 yields the following:

Money demand:

$$\begin{aligned}
\left(\frac{M_t}{P_t}\right)^{-\delta} &= \phi_t C_t^{-\sigma} - \beta E_t\left[\frac{\phi_{t+1}C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] \\
\therefore \left(\frac{\bar{M}e^{\hat{m}_t}}{\bar{P}e^{\hat{p}_t}}\right)^{-\delta} &= \bar{\phi}e^{\hat{\phi}_t}(\bar{C}e^{\hat{c}_t})^{-\sigma} - \beta E_t\left[\frac{\hat{\phi}e^{\hat{\phi}_{t+1}}(\bar{C}e^{\hat{c}_{t+1}})^{-\sigma}}{(\bar{\pi}e^{\hat{\pi}_{t+1}})}\right] \\
\therefore \left(\frac{\bar{M}}{\bar{P}}\right)^{-\delta} e^{-\delta(\hat{m}_t-\hat{p}_t)} &= (\bar{C}^{-\sigma})(e^{\hat{\phi}_t-\sigma\hat{c}_t}) - \beta(\bar{C}^{-\sigma})E_t\left[\frac{e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}}}{e^{\hat{\pi}_{t+1}}}\right] \\
&= \bar{C}^{-\sigma}\left(e^{\hat{\phi}_t-\sigma\hat{c}_t} - \beta E_t\left[e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}}\right]\right)
\end{aligned}$$

From steady state we can derive that $\left(\frac{\bar{M}}{\bar{P}}\right)^{-\delta}/\bar{C}^{-\sigma} = 1-\beta$. Therefore

$$\begin{aligned}
(1-\beta)e^{-\delta(\hat{m}_t-\hat{p}_t)} &= e^{\hat{\phi}_t-\sigma\hat{c}_t} - \beta E_t\left[e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}}\right] \\
\therefore (1-\beta)(1-\delta(\hat{m}_t-\hat{p}_t)) &\approx (1+\hat{\phi}_t-\sigma\hat{c}_t) - \beta E_t[1+\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}] \\
&\approx 1+\hat{\phi}_t-\sigma\hat{c}_t-\beta+\beta E_t[\sigma\hat{c}_{t+1}+\hat{\pi}_{t+1}-\hat{\phi}_{t+1}] \\
&\approx (1-\beta)+(\hat{\phi}_t-\sigma\hat{c}_t)+\beta E_t[\sigma\hat{c}_{t+1}+\hat{\pi}_{t+1}-\hat{\phi}_{t+1}] \\
\therefore 1-\delta(\hat{m}_t-\hat{p}_t) &\approx 1+\frac{\beta}{1-\beta}E_t[\sigma\hat{c}_{t+1}+\hat{\pi}_{t+1}-\hat{\phi}_{t+1}] - \frac{1}{1-\beta}(\sigma\hat{c}_t-\hat{\phi}_t)
\end{aligned}$$

$$\therefore \hat{m}_t-\hat{p}_t = \frac{1}{\delta}\frac{1}{1-\beta}\left(\sigma\hat{c}_t-\hat{\phi}_t-\beta E_t[\sigma\hat{c}_{t+1}+\hat{\pi}_{t+1}-\hat{\phi}_{t+1}]\right) \tag{B.6}$$

Demand for money is therefore an increasing function of contemporaneous consumption, and a decreasing function of expected future consumption and expected inflation.

Labour and wages:

[1] See Appendix A for a discussion on the linearisation techniques employed here.

$$
\begin{aligned}
N_t^{\phi} &= \chi^{-1}\frac{W_t}{P_t}\phi_t C_t^{-\sigma} \\
\therefore (\bar{N}e^{\hat{n}_t})^{\phi} &= \chi^{-1}\frac{\bar{W}e^{\hat{w}_t}}{\bar{P}e^{\hat{p}_t}}\hat{\phi}e^{\hat{\phi}_t}(\bar{C}e^{\hat{c}_t})^{-\sigma} \\
\therefore \left[\bar{N}^{\phi}\right]e^{\phi\hat{n}_t} &= \left[\chi^{-1}\frac{\bar{W}}{\bar{P}}\bar{C}^{-\sigma}\right]e^{\hat{w}_t-\hat{p}_t+\hat{\phi}_t-\sigma\hat{c}_t}
\end{aligned}
$$

We know that in steady state $\bar{N}^{\phi} = \chi^{-1}\frac{\bar{W}}{\bar{P}}\bar{C}^{-\sigma}$.

$$
\begin{aligned}
&\therefore 1 + \phi\hat{n}_t \approx 1 + \hat{w}_t - \hat{p}_t + \hat{\phi}_t - \sigma\hat{c}_t \\
&\therefore \hat{w}_t - \hat{p}_t \approx \phi\hat{n}_t + \sigma\hat{c}_t - \hat{\phi}_t
\end{aligned}
\tag{B.7}
$$

Short-term bonds:

$$
\begin{aligned}
\beta E_t\left[\frac{\phi_{t+1}C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] &= \frac{\phi_t C_t^{-\sigma}}{R_t} + \frac{\phi_t C_t^{-\sigma}\kappa_L\phi_L Y_t\left(\kappa_L\frac{B_t}{B_{L,t}^H} - 1\right)}{R_{L,t}} \\
\therefore \beta E_t\left[\frac{\bar{\phi}e^{\hat{\phi}_{t+1}}(\bar{C}e^{\hat{c}_{t+1}})^{-\sigma}}{\bar{\pi}e^{\hat{\pi}_{t+1}}}\right] &= \frac{(\bar{\phi}e^{\hat{\phi}_t}\bar{C}e^{\hat{c}_t})^{-\sigma}}{\bar{R}e^{\hat{r}_t}} + \frac{\bar{\phi}e^{\hat{\phi}_t}(\bar{C}e^{\hat{c}_t})^{-\sigma}\kappa_L\phi_L(\bar{Y}e^{\hat{y}_t})\left(\kappa_L\frac{\bar{B}e^{\hat{b}_t}}{\bar{B}_L^H e^{\hat{b}_{L,t}^H}} - 1\right)}{\bar{R}_L e^{\hat{r}_{L,t}}} \\
\therefore \beta\bar{C}^{-\sigma}E_t\left[\frac{e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}}}{e^{\hat{\pi}_{t+1}}}\right] &= \frac{\bar{C}^{-\sigma}}{\bar{R}}\frac{e^{\hat{\phi}_t-\sigma\hat{c}_t}}{e^{\hat{r}_t}} + \frac{\bar{C}^{-\sigma}\kappa_L\phi_L\bar{Y}}{\bar{R}_L}\frac{e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t}}{e^{\hat{r}_{L,t}}}\left(\kappa_L\frac{\bar{B}}{\bar{B}_L^H}\frac{e^{\hat{b}_t}}{e^{\hat{b}_{L,t}^H}} - 1\right)
\end{aligned}
$$

Dividing through by $\bar{C}^{-\sigma}$ and noting that in steady state we have $\kappa_L = \bar{B}_L^H/\bar{B}$, as well as the observation that $1/\bar{R} = \beta$ and $1/\bar{R}_L = \beta^2$ (from the steady state of equations B.3 and B.4), the above equation can be simplified to

$$
\begin{aligned}
\beta E_t\left[e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}}\right] &= \beta e^{\hat{\phi}_t-\sigma\hat{c}_t-\hat{r}_t} + \beta^2\kappa_L\phi_L\bar{Y}(e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}})(e^{\hat{b}_t-\hat{b}_{L,t}^H} - 1) \\
\therefore \beta E_t\left[e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}}\right] &= \beta e^{\hat{\phi}_t-\sigma\hat{c}_t-\hat{r}_t} + \beta^2\kappa_L\phi_L\bar{Y}(e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+\hat{b}_t-\hat{b}_{L,t}^H} - e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}}) \\
\therefore E_t\left[1 + \hat{\phi}_{t+1} - \sigma\hat{c}_{t+1} - \hat{\pi}_{t+1}\right] &\approx (1 + \hat{\phi}_t - \sigma\hat{c}_t - \hat{r}_t) + \beta\kappa_L\phi_L\bar{Y}(1 + \hat{\phi}_t - \sigma\hat{c}_t + \hat{y}_t - \hat{r}_{L,t} \\
&\quad + \hat{b}_t - \hat{b}_{L,t}^H - (1 + \hat{\phi}_t - \sigma\hat{c}_t + \hat{y}_t - \hat{r}_{L,t}))
\end{aligned}
$$

$$
\therefore E_t\left[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1} - \hat{\phi}_{t+1}\right] = (\sigma\hat{c}_t + \hat{r}_t - \hat{\phi}_t) + \beta\kappa_L\phi_L\bar{Y}(\hat{b}_{L,t}^H - \hat{b}_t) \tag{B.8}
$$

Long-term bonds:

$$
\begin{aligned}
&\beta E_t\left[\frac{\phi_{t+1}C_{t+1}^{-\sigma}}{\pi_{t+1}R_{t+1}}\right]\\
&\quad=\frac{\phi_t C_t^{-\sigma}}{R_{L,t}}+\frac{\phi_t C_t^{-\sigma}\phi_L Y_t\left(\kappa_L\frac{B_t}{B_{L,t}^H}-1\right)^2}{2R_{L,t}}-\frac{\phi_t C_t^{-\sigma}\phi_L Y_t B_t\kappa_L\left(\kappa_L\frac{B_t}{B_{L,t}^H}-1\right)}{R_{L,t}B_{L,t}^H}\\
&\therefore \beta E_t\left[\frac{\bar\phi e^{\hat\phi_{t+1}}(\bar C e^{\hat c_{t+1}})^{-\sigma}}{(\bar\pi e^{\hat\pi_{t+1}})(\bar R e^{\hat r_{t+1}})}\right]\\
&\quad=\frac{\bar\phi e^{\hat\phi_t}(\bar C e^{\hat c_t})^{-\sigma}}{\bar R_L e^{\hat r_{L,t}}}+\frac{\bar\phi e^{\hat\phi_t}(\bar C e^{\hat c_t})^{-\sigma}\phi_L\bar Y e^{\hat y_t}}{2\bar R_L e^{\hat r_{L,t}}}\left(\kappa_L\frac{\bar B e^{\hat b_t}}{\bar B_L^H e^{\hat b_{L,t}^H}}-1\right)^2\\
&\quad-\frac{\bar\phi e^{\hat\phi_t}(\bar C e^{\hat c_t})^{-\sigma}\phi_L\bar Y e^{\hat y_t}\bar B e^{\hat b_t}\kappa_L}{\bar R_L e^{\hat r_{L,t}}\bar b_L^H e^{\hat b_{L,t}^H}}\left(\kappa_L\frac{\bar B e^{\hat b_t}}{\bar B_L^H e^{\hat b_{L,t}^H}}-1\right)\\
&\therefore \frac{\beta}{\bar R}\bar C^{-\sigma}E_t[e^{\hat\phi_{t+1}-\sigma\hat c_{t+1}-\hat\pi_{t+1}-\hat r_{t+1}}]\\
&\quad=\frac{\bar C^{-\sigma}}{\bar R_L}e^{\hat\phi_t-\sigma\hat c_t-\hat r_{L,t}}+\frac{\bar C^{-\sigma}\phi_L\bar Y}{2\bar R_L}e^{\hat\phi_t-\sigma\hat c_t+\hat y_t-\hat r_{L,t}}\left(\kappa_L\frac{\bar B}{\bar B_L^H}e^{\hat b_t-\hat b_{L,t}^H}-1\right)^2\\
&\quad-\frac{\bar C^{-\sigma}\phi_L\bar Y\bar b\kappa_L}{\bar R_L\bar b_L^H}e^{\hat\phi_t-\sigma\hat c_t+\hat y_t+\hat b_t-\hat r_{L,t}-\hat b_{L,t}^H}\left(\kappa_L\frac{\bar B}{\bar B_L^H}e^{\hat b_t-\hat b_{L,t}^H}-1\right)
\end{aligned}
$$

Noting that we can divide through by the constant $\bar C^{-\sigma}$, that $\frac{\bar B}{\bar B_L^H}\kappa_L=\frac{1}{\kappa_L}\kappa_L=1$ and that $\frac{1}{\bar R}=\beta$ and $\frac{1}{\bar R_L}=\beta^2$, the expression simplifies to

$$
\begin{aligned}
\beta^2E_t[e^{\hat\phi_{t+1}-\sigma\hat c_{t+1}-\hat\pi_{t+1}-\hat r_{t+1}}]&=\beta^2e^{\hat\phi_t-\sigma\hat c_t-\hat r_{L,t}}+\frac{\beta^2\phi_L\bar Y}{2}e^{\hat\phi_t-\sigma\hat c_t+\hat y_t-\hat r_{L,t}}(e^{\hat b_t-\hat b_{L,t}^H}-1)^2\\
&-\beta^2\phi_L\bar Y e^{\hat\phi_t-\sigma\hat c_t+\hat y_t+\hat b_t-\hat r_{L,t}-\hat b_{L,t}^H}(e^{\hat b_t-\hat b_{L,t}^H}-1)\\
\therefore E_t[e^{\hat\phi_{t+1}-\sigma\hat c_{t+1}-\hat\pi_{t+1}-\hat r_{t+1}}]&=e^{\hat\phi_t-\sigma\hat c_t-\hat r_{L,t}}+\frac{\phi_L\bar Y}{2}\left[e^{\hat\phi_t-\sigma\hat c_t+\hat y_t-\hat r_{L,t}}(e^{\hat b_t-\hat b_{L,t}^H}-1)^2\right]\\
&-\phi_L\bar Y\left[e^{\hat\phi_t-\sigma\hat c_t+\hat y_t+\hat b_t-\hat r_{L,t}-\hat b_{L,t}^H}(e^{\hat b_t-\hat b_{L,t}^H}-1)\right]\\
\therefore E_t[e^{\hat\phi_{t+1}-\sigma\hat c_{t+1}-\hat\pi_{t+1}-\hat r_{t+1}}]&=e^{\hat\phi_t-\sigma\hat c_t-\hat r_{L,t}}+\frac{\phi_L\bar Y}{2}\left[e^{\hat\phi_t-\sigma\hat c_t+\hat y_t-\hat r_{L,t}}(e^{\hat b_t-\hat b_{L,t}^H}-1)^2\right]\\
&-\phi_L\bar Y\left[e^{\hat\phi_t-\sigma\hat c_t+\hat y_t+\hat b_t-\hat r_{L,t}-\hat b_{L,t}^H+\hat b_t-\hat b_{L,t}^H}-e^{\hat\phi_t-\sigma\hat c_t+\hat y_t+\hat b_t-\hat r_{L,t}-\hat b_{L,t}^H}\right]
\end{aligned}
$$

Multiplying out the square root in the middle term yields

$$
\begin{aligned}
e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}}(e^{\hat{b}_t-\hat{b}^H_{L,t}}-1)^2 &= e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}}(e^{2\hat{b}_t-2\hat{b}^H_{L,t}}-2e^{\hat{b}_t-\hat{b}^H_{L,t}}+1)\\
&= e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+2\hat{b}_t-2\hat{b}^H_{L,t}}-2e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+\hat{b}_t-\hat{b}^H_{L,t}}+e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}}
\end{aligned}
$$

Substituting back into the above and simplifying yields

$$
\begin{aligned}
E_t[e^{\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}-\hat{r}_{t+1}}] &= e^{\hat{\phi}_t-\sigma\hat{c}_t-\hat{r}_{L,t}}\\
&+\frac{\phi_L\bar{Y}}{2}\Big[e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+2\hat{b}_t-2\hat{b}^H_{L,t}}-2e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+\hat{b}_t-\hat{b}^H_{L,t}}\\
&+e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}}\Big]\\
&-\phi_L\bar{Y}\Big[e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t+\hat{b}_t-\hat{r}_{L,t}-\hat{b}^H_{L,t}+\hat{b}_t-\hat{b}^H_{L,t}}-e^{\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t+\hat{b}_t-\hat{r}_{L,t}-\hat{b}^H_{L,t}}\Big]\\
\therefore E_t[1+\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}-\hat{r}_{t+1}] &\approx (1+\hat{\phi}_t-\sigma\hat{c}_t-\hat{r}_{L,t})\\
&+\frac{\phi_L\bar{Y}}{2}\Big[(1+\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+2\hat{b}_t-2\hat{b}^H_{L,t})\\
&-2(1+\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t}+\hat{b}_t-\hat{b}^H_{L,t})+\\
&(1+\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t-\hat{r}_{L,t})\Big]\\
&-\phi_L\bar{Y}\Big[(1+\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t+\hat{b}_t-\hat{r}_{L,t}-\hat{b}^H_{L,t}+\hat{b}_t-\hat{b}^H_{L,t})\\
&-(1+\hat{\phi}_t-\sigma\hat{c}_t+\hat{y}_t+\hat{b}_t-\hat{r}_{L,t}-\hat{b}^H_{L,t})\Big]\\
\therefore E_t[1+\hat{\phi}_{t+1}-\sigma\hat{c}_{t+1}-\hat{\pi}_{t+1}-\hat{r}_{t+1}] &\approx (1+\hat{\phi}_t-\sigma\hat{c}_t-\hat{r}_{L,t})+\frac{\phi_L\bar{Y}}{2}(0)-\phi_L\bar{Y}(\hat{b}_t-\hat{b}^H_{L,t})
\end{aligned}
$$

$$
\therefore E_t[\sigma\hat{c}_{t+1}+\hat{\pi}_{t+1}+\hat{r}_{t+1}-\hat{\phi}_{t+1}] = (\sigma\hat{c}_t+\hat{r}_{L,t}-\hat{\phi}_t)+\phi_L\bar{Y}(\hat{b}_t-\hat{b}^H_{L,t}) \tag{B.9}
$$

B.1.2 The Euler equation and the term structure

The household's Euler equation can be found by solving for $\hat{c}_t$ in the log-linearised expression for short-term bonds as derived above (equation 4.8):

$$
\begin{aligned}
E_t\Big[\sigma\hat{c}_{t+1}+\hat{\pi}_{t+1}-\hat{\phi}_{t+1}\Big] &= (\sigma\hat{c}_t+\hat{r}_t-\phi_t)+\beta\kappa_L\phi_L\bar{Y}(\hat{b}^H_{L,t}-\hat{b}_t)+E_t\Big[\hat{\phi}_{t+1}-\phi_t\Big]\\
\therefore \hat{c}_t = E_t[\hat{c}_{t+1}]+\frac{1}{\sigma}E_t[\hat{\pi}_{t+1}]&-\frac{1}{\sigma}\hat{r}_t-\frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t}-\hat{b}_t)+\frac{1}{\sigma}E_t\Big[\hat{\phi}_{t+1}-\phi_t\Big]
\end{aligned}
$$

$$\therefore\ \hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) + \frac{1}{\sigma}E_t\left[\hat{\phi}_{t+1} - \phi_t\right] \quad \text{(B.10)}$$

The term structure can be derived by substituting the household's Euler equation into the log-linearised first-order condition for long-term bonds as derived above:

$$\begin{aligned}
E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1} + \hat{r}_{t+1} - \hat{\phi}_{t+1}] &= \sigma\hat{c}_t + \hat{r}_{L,t} - \phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) \\
\therefore E_t[\sigma\hat{c}_{t+1} + \hat{\pi}_{t+1} + \hat{r}_{t+1}] &= \sigma E_t[\hat{c}_{t+1}] - (\hat{r}_t - E_t[\hat{\pi}_{t+1}]) \\
&- \beta\kappa_L\phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) + \hat{r}_{L,t} - \phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) \\
\therefore \hat{r}_{L,t} &= \hat{r}_t + E_t[\hat{r}_{t+1}] + \phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t)(\beta\kappa_L + 1)
\end{aligned}$$

B.2 Solving the firm's optimisation problem

B.2.1 Aggregate price dynamics

Linearising around the steady state (where $\pi_t = \bar{\pi} = 1$ and $P^*_t = P_t = P_{t-1} = \bar{P}$) yields

$$\begin{aligned}
\pi_t^{1-\varepsilon} &= \theta + (1-\theta)\left(\frac{P^*_t}{P_{t-1}}\right)^{1-\varepsilon} \\
\therefore \left(\bar{\pi}e^{\hat{\pi}_t}\right)^{1-\varepsilon} &= \theta + (1-\theta)\left(\frac{\bar{P}e^{\hat{p}^*_t}}{\bar{P}e^{\hat{p}_{t-1}}}\right)^{(1-\varepsilon)} \\
\therefore 1 + (1-\varepsilon)\hat{\pi}_t &\approx \theta + (1-\theta)(1 + (1-\varepsilon)(\hat{p}^*_t - \hat{p}_{t-1})) \\
\therefore \hat{\pi}_t &\approx \frac{1}{1-\varepsilon} + (1-\theta)(\hat{p}^*_t - \hat{p}_{t-1})
\end{aligned}$$

Around the steady state the constant $\frac{1}{1-\varepsilon}$ is small enough to be safely ignored.

B.2.2 Optimal price setting

A firm resetting its price in period t will choose the optimal price P^*_t so as to maximise its discounted stream of future profits while this price remains effective by solving

$$\max_{P_t^*} \quad \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left(P_t^* Y_{t+k|t}(j) - TC_{t+k}(Y_{t+k|t}(j)) \right) \right] \tag{B.11}$$

subject to the sequence of demand constraints

$$Y_{t+k|t}(j) = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \tag{B.12}$$

$\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor, $Y_{t+k|t}(j)$ is output in period $t+k$ for a firm which last reset its price in period t, and $TC_{t+k}(\cdot)$ represents total nominal cost as a function of this output. Firm j's nominal undiscounted profit in period $t+k$ is therefore equal to $P_t^* Y_{t+k|t}(j) - TC_{t+k}(Y_{t+k|t}(j))$. Inserting the demand constraint into the profit function allows the problem to be rewritten as an unconstrained Lagrangian

$$\begin{aligned}
\mathcal{L} &= \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left(P_t^* \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} - TC_{t+k|t} \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right) \right) \right] \\
&= \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left(P_t^{*1-\varepsilon} P_{t+k}^{\varepsilon} Y_{t+k} - TC_{t+k|t} \left(P_t^{*-\varepsilon} P_{t+k}^{\varepsilon} Y_{t+k} \right) \right) \right]
\end{aligned}$$

This yields the first-order condition:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_t^*} = 0 = &\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left((1-\varepsilon) P_t^{*-\varepsilon} \left(\frac{1}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right. \right. \\
&\left. \left. - MC_{t+k|t} \left(-\varepsilon \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon-1} \frac{1}{P_{t+k}} Y_{t+k} \right) \right) \right] \\
= &\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left((1-\varepsilon) \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right. \right. \\
&\left. \left. + \varepsilon MC_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon-1} \left(\frac{1}{P_{t+k}} \right) Y_{t+k} \right) \right]
\end{aligned}$$

where $MC_{t+k|t}$ represents the nominal marginal cost in $t+k$ for a firm which last reset its price in t. Substituting back the demand constraint (equation B.12) yields

$$
\begin{aligned}
0 &= \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \left((1-\varepsilon) Y_{t+k|t}(j) \right.\right. \\
&\quad \left.\left. + \varepsilon MC_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} \right)^{-1} \left(\frac{1}{P_{t+k}} \right) Y_{t+k|t}(j) \right) \right] \\
&= \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} Y_{t+k|t}(j) \left((1-\varepsilon) + \varepsilon MC_{t+k|t} \left(\frac{1}{P_t^*} \right) \right) \right] \\
&= \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} Y_{t+k|t}(j) \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k|t} \right) \right]
\end{aligned}
$$

Multiplying this out yields

$$
\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} Y_{t+k|t}(j) P_t^* \right] = \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} Y_{t+k|t}(j) MC_{t+k|t} \right]
$$

Replacing the stochastic discount factor $\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$ and demand constraint $Y_{t+k|t}(j) = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$ we can solve for P_t^*:

$$
\begin{aligned}
&\sum_{k=0}^{\infty} \theta^k E_t \left[\left(\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \right) \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right) P_t^* \right] \\
&= \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \theta^k E_t \left[\left(\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \right) \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right) MC_{t+k|t} \right] \\
\therefore &\sum_{k=0}^{\infty} \theta^k E_t \left[\beta^k C_{t+k}^{-\sigma} C_t^{\sigma} P_t P_{t+k}^{\varepsilon - 1} P_t^{*1-\varepsilon} Y_{t+k} \right] \\
&= \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \theta^k E_t \left[\beta^k C_{t+k}^{-\sigma} C_t^{\sigma} P_t P_{t+k}^{\varepsilon - 1} P_t^{*-\varepsilon} Y_{t+k} MC_{t+k|t} \right] \\
\therefore &\, P_t^{*1-\varepsilon} \sum_{k=0}^{\infty} \theta^k E_t \left[\beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon-1} Y_{t+k} \right] = P_t^{*-\varepsilon} \frac{\varepsilon}{1-\varepsilon} \sum_{k=0}^{\infty} \theta^k E_t \left[\beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon} Y_{t+k} \frac{MC_{t+k|t}}{P_{t+k}} \right] \\
\therefore &\, P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \left[\theta^k \beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon} Y_{t+k} MC_{t+k|t}^r \right]}{E_t \sum_{k=0}^{\infty} \left[\theta^k \beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon - 1} Y_{t+k} \right]}
\end{aligned}
\tag{B.13}
$$

B.2.3 Steady state

In the steady state with zero inflation we have $\bar{\pi} = \frac{P_t^*}{P_{t-1}} = \frac{P_t^*}{P_t} = \frac{P_t^*}{P_{t+k}} = 1$, $Y_{t+k|t} = \bar{Y}$ and $\Lambda_{t,t+k} = \beta^k$. Real marginal cost can be derived from the frictionless markup and is given by $MC^r_{t+k|t} = \bar{MC}^r_{t|t} = \frac{MC_{t|t}}{P_t} = \frac{MC_{t|t}}{P_t^*} = \frac{\varepsilon-1}{\varepsilon} = \bar{MC}^r$. Finally, the steady state aggregate production function (see equation 4.12) is given by

$$\bar{Y} = \bar{Z}\bar{N}^{1-\alpha} \tag{B.14}$$

B.2.4 Log-linearisation

Defining the optimal price in terms of inflation[2] by dividing equation B.13 through by P_{t-1} yields

$$\frac{P_t^*}{P_{t-1}} E_t \sum_{k=0}^{\infty} \left[\theta^k \beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon-1} Y_{t+k}\right] = \frac{\varepsilon}{\varepsilon-1} E_t \sum_{k=0}^{\infty} \left[\theta^k \beta^k C_{t+k}^{-\sigma} P_{t+k}^{\varepsilon} Y_{t+k} MC^r_{t+k|t}\right] \frac{1}{P_{t-1}}$$

Applying Uhlig's (1999) rule ($X_t = \bar{X}e^{\hat{x}_t} \approx \bar{X}(1+\hat{x}_t)$) separately to the LHS and RHS, while noting that $p_t^* - p_{t-1} = \pi_t^{*3}$, yields

LHS:

$$\begin{aligned}
&\frac{\bar{P}e^{\hat{p}_t^*}}{\bar{P}e^{\hat{p}_{t-1}}} E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[(\bar{C}e^{\hat{c}_{t+k}})^{-\sigma} (\bar{P}e^{\hat{p}_{t+k}})^{\varepsilon-1} (\bar{Y}e^{\hat{y}_{t+k}})\right] \\
&= \left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right) E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[e^{\hat{p}_t^* - \hat{p}_{t-1} - \sigma\hat{c}_{t+k} + (\varepsilon-1)\hat{p}_{t+k} + \hat{y}_{t+k}}\right] \\
&\approx \left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right) E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[1 + \hat{p}_t^* - \hat{p}_{t-1} - \sigma\hat{c}_{t+k} + (\varepsilon-1)\hat{p}_{t+k} + \hat{y}_{t+k}\right] \\
&\approx \left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right) E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[1 + \hat{\pi}_t^* - \hat{\sigma} c_{t+k} + (\varepsilon-1)\hat{p}_{t+k} + \hat{y}_{t+k}\right]
\end{aligned}$$

RHS:

[2] $\pi_t^* = \frac{P_t^*}{P_{t-1}}$

[3] $\log \pi_t^* = \log\left(\frac{P_t^*}{P_{t-1}}\right) = \log P_t^* - \log P_{t-1} = p_t^* - p_{t-1} = \pi_t^*$

$$
\begin{aligned}
&\frac{\varepsilon}{\varepsilon-1}\frac{1}{\bar{P}e^{\hat{p}_{t-1}}}E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[(\bar{C}e^{\hat{c}_{t+k}})^{-\sigma}(\bar{P}e^{\hat{p}_{t+k}})^{\varepsilon}(\bar{Y}e^{\hat{y}_{t+k}})(\bar{MC}^r e^{\hat{mc}^r_{t+k|t}})\right]\\
&=\frac{\varepsilon}{\varepsilon-1}\left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\bar{MC}^r\right)E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[e^{-\hat{p}_{t-1}-\sigma\hat{c}_{t+k}+\varepsilon\hat{p}_{t+k}+\hat{y}_{t+k}+\hat{mc}^r_{t+k|t}}\right]\\
&\approx\frac{\varepsilon}{\varepsilon-1}\bar{MC}^r\left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right)E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[1-\hat{p}_{t-1}-\sigma\hat{c}_{t+k}+\varepsilon\hat{p}_{t+k}+\hat{y}_{t+k}+\hat{mc}^r_{t+k|t}\right]\\
&\approx\left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right)E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[1-\hat{p}_{t-1}-\sigma\hat{c}_{t+k}+\varepsilon\hat{p}_{t+k}+\hat{y}_{t+k}+\hat{mc}^r_{t+k|t}\right]
\end{aligned}
$$

Equating the LHS and RHS and simplifying yields

$$
\begin{aligned}
&\left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right)E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[1+\hat{\pi}^*_t-\hat{\sigma}c_{t+k}+(\varepsilon-1)\hat{p}_{t+k}+\hat{y}_{t+k}\right]\\
&=\left(\bar{C}^{-\sigma}\bar{P}^{\varepsilon-1}\bar{Y}\right)E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[1-\hat{p}_{t-1}-\sigma\hat{c}_{t+k}+\varepsilon\hat{p}_{t+k}+\hat{y}_{t+k}+\hat{mc}^r_{t+k|t}\right]\\
&\therefore E_t\sum_{k=0}^{\infty}\theta^k\beta^k[\pi^*_t]=E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[\hat{mc}^r_{t+k|t}+\hat{p}_{t+k}-\hat{p}_{t-1}\right]\\
&\therefore E_t\sum_{k=0}^{\infty}\theta^k\beta^k[\hat{p}^*_t-\hat{p}_{t-1}]=E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[\hat{mc}^r_{t+k|t}+\hat{p}_{t+k}-\hat{p}_{t-1}\right]
\end{aligned}
$$

From the sum to infinity of a convergent geometric progression we know that $\sum_{k=1}^{\infty}a^k=\frac{a}{1-a}$ for $|a|<1$. Therefore $\sum_{k=0}^{\infty}a^k=a^0+\frac{a}{1-a}=1+\frac{a}{1-a}=\frac{1-a+a}{1-a}=\frac{1}{1-a}$, and $\sum_{k=0}^{\infty}(\theta\beta)^k=\frac{1}{1-\theta\beta}$ [4] and we have

$$
\begin{aligned}
\frac{\hat{p}^*_t-\hat{p}_{t-1}}{1-\theta\beta}&=E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[\hat{mc}^r_{t+k|t}+\hat{p}_{t+k}-\hat{p}_{t-1}\right]\\
\therefore\hat{p}^*_t-\hat{p}_{t-1}&=(1-\theta\beta)E_t\sum_{k=0}^{\infty}\theta^k\beta^k\left[\hat{mc}^r_{t+k|t}+\hat{p}_{t+k}-\hat{p}_{t-1}\right]
\end{aligned}
$$

The two k-invariant terms drop out on both sides, leaving us with an expression for the optimal price

[4] Since both $0<\theta<1$ and $0<\beta<1$ it follows that $0<\theta\beta<1$.

$$\hat{p}_t^* = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}^r_{t+k|t} + \hat{p}_{t+k} \right]$$

Note that, since $\hat{p}_{t+k} - \hat{p}_{t-1} = \hat{\pi}_{t+k}$, the previous expression is equivalent to

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}^r_{t+k|t} + \hat{\pi}_{t+k} \right]$$

Finally, the log-linearised aggregate production function can be easily derived as follows:

$$\begin{aligned} Y_t &= Z_t N_t^{1-\alpha} \\ (\bar{Y} e^{\hat{y}_t}) &= (\bar{Z} e^{\hat{z}_t})(\bar{N} e^{\hat{n}_t})^{1-\alpha} \\ \therefore \; \bar{Y}(1+\hat{y}_t) &\approx \bar{Z}\bar{N}^{1-\alpha}(1+\hat{z}_t + (1-\alpha)\hat{n}_t) \end{aligned}$$

We know that in steady state $\bar{Y} = \bar{Z}\bar{N}^{1-\alpha}$. Therefore the production function becomes

$$\hat{y}_t = \hat{z}_t + (1-\alpha)\hat{n}_t \tag{B.15}$$

B.2.5 Marginal costs and inflation

The marginal product of labour can be derived from the firm's production function (equation 4.12):

$$MPN_t = \frac{\partial Y_t(j)}{\partial N_t(j)} = Z_t(1-\alpha)N_t(j)^{-\alpha} \tag{B.16}$$

Log-linearising the above, noting that in steady state $M\bar{P}N = \bar{Z}(1-\alpha)\bar{N}^{-\alpha}$, yields

$$\begin{aligned} M\bar{P}N e^{m\hat{p}n_t} &= \bar{Z} e^{\hat{a}_t}(1-\alpha)(\bar{N} e^{\hat{n}_t})^{-\alpha} \\ [M\bar{P}N](1+m\hat{p}n_t) &\approx [\bar{Z}(1-\alpha)\bar{N}^{-\alpha}](1+\hat{z}_t - \alpha\hat{n}_t) \\ m\hat{p}n_t &= \hat{z}_t - \alpha\hat{n}_t \end{aligned} \tag{B.17}$$

The firm's total cost depends on wages and the level of employment, and is given by

$$W_tN_t = W_t \int_0^1 \left(\frac{Y_t(j)}{Z_t}\right)^{\frac{1}{1-\alpha}} dj = W_t \left(\frac{1}{Z_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 Y_t(j)^{\frac{1}{1-\alpha}} dj \tag{B.18}$$

The (nominal) marginal cost of producing one additional unit of output can be calculated as follows (Kotzé, 2014:52):

$$MC_t = \frac{\partial W_tN_t}{\partial Y_t(j)} = W_t \left(Z_t(1-\alpha)N_t(j)^{-\alpha}\right)^{-1} \tag{B.19}$$

Real marginal cost is determined by dividing through by the price level:

$$MC_t^r = \frac{W_t}{P_t} \left(Z_t(1-\alpha)N_t(j)^{-\alpha}\right)^{-1} \tag{B.20}$$

Substituting in the marginal product of labour (equation B.17) we arrive at

$$MC_t^r = \frac{W_t}{P_t} MPN_t^{-1} \tag{B.21}$$

Log-linearising, again noting that in steady state $\bar{MC}^r = \frac{\bar{W}}{\bar{P}} M\bar{P}N^{-1}$, yields

$$\begin{aligned} \bar{MC}e^{\hat{mc}_t^r} &= \frac{\bar{W}e^{\hat{w}_t}}{\bar{P}e^{\hat{p}_t}}(M\bar{P}Ne^{\hat{mpn}_t})^{-1} \\ \therefore [\bar{MC}]e^{\hat{mc}_t^r} &= \left[\frac{\bar{W}}{\bar{P}}M\bar{P}N^{-1}\right](e^{\hat{w}_t-\hat{p}_t-\hat{mpn}_t}) \\ \therefore 1+\hat{mc}_t^r &\approx 1+\hat{w}_t-\hat{p}_t-\hat{mpn}_t \\ \therefore \hat{mc}_t^r &= \hat{w}_t-\hat{p}_t-\hat{mpn}_t \\ &= \hat{w}_t-\hat{p}_t-(\hat{z}_t-\alpha\hat{n}_t) \end{aligned} \tag{B.22}$$

with the final step substituting in the marginal product of labour derived earlier (equation B.17). Rearranging the production function $\hat{y}_t = \hat{z}_t + (1-\alpha)\hat{n}_t$ and substituting the labour market condition $\hat{n}_t = \frac{\hat{y}_t-\hat{z}_t}{1-\alpha}$ we have

$$\begin{aligned} \hat{mc}_t^r &= \hat{w}_t-\hat{p}_t-\left(\hat{z}_t-\alpha\left(\frac{\hat{y}_t-\hat{z}_t}{1-\alpha}\right)\right) \\ &= \hat{w}_t-\hat{p}_t-\frac{1}{1-\alpha}(\hat{z}_t-\alpha\hat{y}_t) \end{aligned} \tag{B.23}$$

Defining real marginal cost in $t+k$, given the costs in t, as

$$\begin{aligned} \hat{mc}^r_{t+k|t} &= \hat{w}_{t+k} - \hat{p}_{t+k} - \hat{mpn}_{t+k|t} \\ &= \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{z}_{t+k} - \alpha \hat{y}_{t+k|t}) \end{aligned} \tag{B.24}$$

Substracting equation B.23, evaluated at time $t+k$, from equation B.24 yields

$$\begin{aligned} \hat{mc}^r_{t+k|t} - \hat{mc}^r_{t+k} &= \left[\hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{z}_{t+k} - \alpha \hat{y}_{t+k|t})\right] \\ &- \left[\hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{z}_{t+k} - \alpha \hat{y}_{t+k})\right] \\ &= \frac{\alpha}{1-\alpha}(\hat{y}_{t+k|t} - \hat{y}_{t+k}) \end{aligned} \tag{B.25}$$

Making use of the log-linearised demand schedule[5] $\hat{y}_{t+k|t} = -\varepsilon(\hat{p}^*_t - \hat{p}_{t+k}) + \hat{y}_{t+k}$ we have

$$\begin{aligned} \hat{mc}^r_{t+k|t} - \hat{mc}^r_{t+k} &= \frac{\alpha}{1-\alpha}\left[-\varepsilon(\hat{p}^*_t - \hat{p}_{t+k}) + \hat{y}_{t+k} - \hat{y}_{t+k}\right] \\ \therefore \hat{mc}^r_{t+k|t} &= \hat{mc}^r_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(\hat{p}^*_t - \hat{p}_{t+k}) \end{aligned} \tag{B.26}$$

Substituting this into the firm's optimal price setting equation[6] (Kotzé, 2014:54) yields

$$\begin{aligned} \hat{p}^*_t - \hat{p}_{t-1} &= (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}^r_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(\hat{p}^*_t - \hat{p}_{t+k}) + \hat{p}_{t+k} - \hat{p}_{t-1}\right] \\ &= (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\Theta \hat{mc}^r_{t+k} + \hat{p}_{t+k} - \hat{p}_{t-1}\right] \end{aligned} \tag{B.27}$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$. Following Kotzé (2014), this equation can be expressed as

[5] See equation B.12.
[6] See equation 4.22.

$$\begin{aligned}\hat{p}_t^* - \hat{p}_{t-1} = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\Theta \hat{mc}_{t+k}^r\right] + \\ (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{p}_{t+k} - \hat{p}_{t-1}\right] \\ = (1-\theta\beta)E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\Theta \hat{mc}_{t+k}^r\right] + E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{\pi}_{t+k}\right] \\ = (1-\theta\beta)[\Theta \hat{mc}_t^r] + \hat{\pi}_t + \theta\beta[\hat{p}_{t+1}^* - \hat{p}_t]\end{aligned} \tag{B.28}$$

From equation 4.17 we know that $\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$. It follows that inflation can be expressed as (Kotzé, 2014)

$$\begin{aligned}\frac{\hat{\pi}_t}{1-\theta} = (1-\theta\beta)[\Theta \hat{mc}_t^r] + \hat{\pi}_t + \theta\beta\frac{E_t[\hat{\pi}_{t+k}]}{1-\theta} \\ \therefore \hat{\pi}_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}[\Theta \hat{mc}_t^r] + \beta E_t[\hat{\pi}_{t+k}] \\ \therefore \hat{\pi}_t = \lambda \hat{mc}_t^r + \beta E_t[\hat{\pi}_{t+k}]\end{aligned} \tag{B.29}$$

where $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta$. This expression suggests that "the level of current inflation is positively dependent on marginal costs" and therefore that "inflation would be largely due to the purposeful price-setting decisions of firms, which adjust prices in light of current and anticipated inflationary conditions" (Kotzé, 2014:55). It follows that the New-Keynesian Phillips curve can be expressed as a function of the output gap and expected inflation (Kotzé, 2014:57) as

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} \tag{B.30}$$

where $\kappa = \lambda\left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)$.

B.3 Government

B.3.1 Tax rule

$$
\begin{aligned}
T_t &= \bar{T} + \psi_1 \left[\frac{B_{t-1}}{P_t} - \frac{\bar{B}}{\bar{P}} \right] + \psi_2 \left[\frac{B_{L,t-1}}{R_t P_t} - \frac{\bar{B}_L}{\bar{R}\bar{P}} \right] \\
\therefore \bar{T} e^{\hat{t}_t} &= \bar{T} + \psi_1 \left[\frac{\bar{B} e^{\hat{b}_{t-1}}}{\bar{P} e^{\hat{p}_t}} - \frac{\bar{B}}{\bar{P}} \right] + \psi_2 \left[\frac{\bar{B}_L e^{\hat{b}_{L,t-1}}}{\bar{R} e^{\hat{r}_t} \bar{P} e^{\hat{p}_t}} - \frac{\bar{B}_L}{\bar{R}\bar{P}} \right] \\
&= \bar{T} + \psi_1 \frac{\bar{B}}{\bar{P}} (e^{\hat{b}_{t-1} - \hat{p}_t} - 1) + \psi_2 \frac{\bar{B}_L}{\bar{R}\bar{P}} (e^{\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t} - 1) \\
\therefore \bar{T}(1 + \hat{t}_t) &\approx \bar{T} + \psi_1 \frac{\bar{B}}{\bar{P}} (1 + \hat{b}_{t-1} - \hat{p}_t - 1) + \psi_2 \frac{\bar{B}_L}{\bar{R}\bar{P}} (1 + \hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t - 1) \\
\therefore \bar{T} + \bar{T}\hat{t}_t &\approx \bar{T} + \psi_1 \frac{\bar{B}}{\bar{P}} (\hat{b}_{t-1} - \hat{p}_t) + \psi_2 \frac{\bar{B}_L}{\bar{R}\bar{P}} (\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t) \\
\therefore \bar{T}\hat{t}_t &\approx \psi_1 \frac{\bar{B}}{\bar{P}} (\hat{b}_{t-1} - \hat{p}_t) + \psi_2 \frac{\bar{B}_L}{\bar{R}\bar{P}} (\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t) \\
\therefore \hat{t}_t &\approx \psi_1 \frac{1}{\bar{T}} \frac{\bar{B}}{\bar{P}} (\hat{b}_{t-1} - \hat{p}_t) + \psi_2 \beta \frac{1}{\bar{T}} \frac{\bar{B}_L}{\bar{P}} (\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t) \\
\therefore \hat{t}_t &\approx \frac{1}{\bar{T}\bar{P}} \left[\psi_1 \bar{B} (\hat{b}_{t-1} - \hat{p}_t) + \beta \psi_2 \bar{B}_L (\hat{b}_{L,t-1} - \hat{r}_t - \hat{p}_t) \right]
\end{aligned}
$$

B.3.2 Long-term bond supply

$$
\begin{aligned}
\log \left(\frac{B_{L,t}}{\bar{B}_L} \right) &= \phi^{BL} \log \left(\frac{B_{L,t-1}}{\bar{B}_L} \right) + \varepsilon_t^{BL} \\
\therefore \log \left(\frac{\bar{B}_L e^{\hat{b}_{L,t}}}{\bar{B}_L} \right) &= \phi^{BL} \log \left(\frac{\bar{B}_L e^{\hat{b}_{L,t-1}}}{\bar{B}_L} \right) + \varepsilon_t^{BL} \\
\therefore \log e^{\hat{b}_{L,t}} &= \phi^{BL} \log e^{\hat{b}_{L,t-1}} + \varepsilon_t^{BL} \\
\therefore \hat{b}_{L,t} &= \phi^{BL} \hat{b}_{L,t-1} + \varepsilon_t^{BL}
\end{aligned}
$$

B.4 Central bank rules

B.4.1 Taylor rule

Log-linearising the Taylor rule (equation 4.33) is straightforward:

$$\begin{aligned}
\frac{R_t}{\bar{R}} &= \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi^\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi^y} e^{\varepsilon_t^R} \\
\frac{\bar{R}e^{\hat{r}_t}}{\bar{R}} &= \left(\frac{\bar{\pi}e^{\hat{\pi}_t}}{\bar{\pi}}\right)^{\phi^\pi} \left(\frac{\bar{Y}e^{\hat{y}_t}}{\bar{Y}}\right)^{\phi^y} e^{\varepsilon_t^R} \\
\therefore \log e^{\hat{r}_t} &= \phi^\pi \log e^{\hat{\pi}_t} + \phi^y \log e^{\hat{y}_t} + \log e^{\varepsilon_t^R} \\
\therefore \hat{r}_t &= \phi^\pi \hat{\pi}_t + \phi^y \hat{y}_t + \varepsilon_t^R
\end{aligned} \tag{B.31}$$

B.4.2 Central bank asset purchases

The household's holdings of long-term government bonds (4.37) can be log-linearised as follows:

$$\begin{aligned}
B_{L,t}^H &= B_{L,t} - B_{L,t}^{CB} \\
\therefore \bar{B}_L^H e^{\hat{b}_{L,t}^H} &= \bar{B}_L e^{\hat{b}_{L,t}} - \bar{B}_L^{CB} e^{\hat{b}_{L,t}^{CB}} \\
\therefore \bar{B}_L^H (1 + \hat{b}_{L,t}^H) &\approx \bar{B}_L (1 + \hat{b}_{L,t}) - \bar{B}_L^{CB} (1 + \hat{b}_{L,t}^{CB}) \\
\therefore \bar{B}_L^H \hat{b}_{L,t}^H &\approx \bar{B}_L \hat{b}_{L,t} - \bar{B}_L^{CB} \hat{b}_{L,t}^{CB}
\end{aligned}$$

The central bank's asset purchase equation (4.39) can be log-linearised as follows:

$$\begin{aligned}
\log\left(\frac{x_t}{\bar{X}}\right) &= \phi^x \log\left(\frac{x_{t-1}}{\bar{X}}\right) + \varepsilon_t^x \\
\therefore \log x_t - \log \bar{X} &= \phi^x \left[\log x_{t-1} - \log \bar{X}\right] + \varepsilon_t^x \\
\therefore \log\left(\bar{X}e^{\hat{x}_t}\right) - \log \bar{X} &= \phi^x \left[\log\left(\bar{X}e^{\hat{x}_{t-1}}\right) - \log \bar{X}\right] + \varepsilon_t^x \\
\therefore \log \bar{X} + \log e^{\hat{x}_t} - \log \bar{X} &= \phi^x \left[\log \bar{X} + \log\left(e^{\hat{x}_{t-1}}\right) - \log \bar{X}\right] + \varepsilon_t^x \\
\therefore \hat{x}_t &= \phi^x \hat{x}_{t-1} + \varepsilon_t^x
\end{aligned} \tag{B.32}$$

The fraction x_t represents the central bank's holdings of long-term government bonds relative to the total amount issued. It can be expressed in log-linear terms – noting that in steady state $\bar{X} = \bar{B}_L^{CB}/\bar{B}_L$ – as

$$
\begin{aligned}
x_t &= \frac{B_{L,t}^{CB}}{B_{L,t}} \\
\therefore \bar{X}e^{\hat{x}_t} &= \frac{\bar{B}_L^{CB}e^{\hat{b}_{L,t}^{CB}}}{\bar{B}_L e^{\hat{b}_{L,t}}} \\
\therefore \bar{X}(1+\hat{x}_t) &\approx \frac{\bar{B}_L^{CB}}{\bar{B}_L}(1+\hat{b}_{L,t}^{CB}-\hat{b}_{L,t})
\end{aligned}
$$

B.5 Market clearing

B.5.1 Goods market clearing

The economy's resource constraint is given by

$$
\begin{aligned}
Y_t &= C_t + \frac{b_{L,t}^H}{R_{L,t}}(AC_t^L) \\
\therefore Y_t &= C_t + \frac{b_{L,t}}{R_{L,t}}\left[\frac{\phi_L}{2}\left(\kappa_L \frac{B_t}{B_{L,t}^H}-1\right)^2\right]Y_t
\end{aligned}
$$

Noting that $B_t = b_t P_t$ and $B_{L,t}^H = b_{L,t}^H P_t$[7], the resource constraint can be expressed as

$$
Y_t = C_t + \frac{b_{L,t}}{R_{L,t}}\left[\frac{\phi_L}{2}\left(\kappa_L \frac{b_t}{b_{L,t}^H}-1\right)^2\right]Y_t \tag{B.33}
$$

In steady state we know that AC_t^L collapses to zero.[8] Therefore the economy-wide steady state resource constraint is given by

$$
\bar{Y} = \bar{C} \tag{B.34}
$$

Applying Uhlig's (1999) rule to the resource constraint (equation B.33) yields

[7]We know that $b_t = \frac{B_t}{P_t}$ and $b_{L,t}^H = \frac{B_{L,t}^H}{P_t}$.

[8]In steady state we have $\kappa_L = \frac{\bar{B}_L^H}{\bar{B}} = \frac{\bar{b}_L^H \bar{P}}{\bar{b}_t \bar{P}} = \frac{\bar{b}_L^H}{\bar{b}_t} = \frac{b_{L,t}^H}{b_t}$.

$$
\begin{aligned}
\bar{Y}e^{\hat{y}_t} &= \bar{C}e^{\hat{c}_t} + \frac{\bar{b}_L e^{\hat{b}_{L,t}}}{\bar{R}e^{\hat{r}_{L,t}}}\frac{\phi_L}{2}\left(\kappa_L \frac{\bar{b}e^{\hat{b}_t}}{\bar{b}_L^H e^{\hat{b}_{L,t}}} - 1\right)^2 \bar{Y}e^{\hat{y}_t} \\
\therefore \bar{Y}e^{\hat{y}_t} &= \bar{C}e^{\hat{c}_t} + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}\left(\frac{e^{\hat{b}_{L,t}}}{e^{\hat{r}_{L,t}}}\right)\left(\frac{e^{\hat{b}_t}}{e^{\hat{b}_{L,t}}} - 1\right)^2 e^{\hat{y}_t} \\
\therefore \bar{Y}e^{\hat{y}_t} &= \bar{C}e^{\hat{c}_t} + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}\left(e^{\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t}\right)\left(e^{\hat{b}_t-\hat{b}_{L,t}} - 1\right)^2 \\
\therefore \bar{Y}e^{\hat{y}_t} &= \bar{C}e^{\hat{c}_t} + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}\left(e^{\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t}\right)\left(e^{2\hat{b}_t-2\hat{b}_{L,t}} - 2e^{\hat{b}_t-\hat{b}_{L,t}} + 1\right) \\
\therefore \bar{Y}e^{\hat{y}_t} &= \bar{C}e^{\hat{c}_t} + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}\left(e^{\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t+2\hat{b}_t-2\hat{b}_{L,t}} - 2e^{\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t+\hat{b}_t-\hat{b}_{L,t}} + e^{\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t}\right) \\
\therefore \bar{Y}e^{\hat{y}_t} &= \bar{C}e^{\hat{c}_t} + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}\left(e^{-\hat{r}_{L,t}+\hat{y}_t+2\hat{b}_t-\hat{b}_{L,t}} - 2e^{-\hat{r}_{L,t}+\hat{y}_t+\hat{b}_t} + e^{\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t}\right) \\
\therefore \bar{Y}(1+\hat{y}_t) &\approx \bar{C}(1+\hat{c}_t) + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}\Big((1-\hat{r}_{L,t}+\hat{y}_t+2\hat{b}_t-\hat{b}_{L,t}) \\
&\quad -2(1-\hat{r}_{L,t}+\hat{y}_t+\hat{b}_t) + (1+\hat{b}_{L,t}-\hat{r}_{L,t}+\hat{y}_t)\Big) \\
\therefore \bar{Y}(1+\hat{y}_t) &\approx \bar{C}(1+\hat{c}_t) + \frac{\phi_L}{2}\frac{\bar{Y}\bar{b}_L}{\bar{R}}(0) \\
\therefore \bar{Y} + \bar{Y}\hat{y}_t &\approx \bar{C} + \bar{C}\hat{c}_t
\end{aligned}
\tag{B.35}
$$

From steady state we know that $\bar{Y} = \bar{C}$. Therefore the resource constraint reduces to

$$
\begin{aligned}
\bar{Y}\hat{y}_t &= \bar{C}\hat{c}_t \\
\therefore \hat{y}_t &= \frac{\bar{C}}{\bar{Y}}\hat{c}_t = \hat{c}_t
\end{aligned}
\tag{B.36}
$$

B.5.2 Dynamic IS curve

Combining the household's Euler equation (4.10) with the goods market clearing condition (equation 4.48) yields

$$
\hat{y}_t = E_t[\hat{y}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}_{L,t}^H - \hat{b}_t) + E_t\left[\phi_t - \hat{\phi}_{t+1}\right] \tag{B.37}
$$

B.5.3 Labour market clearing

Aggregate labour supply is given by

$$N_t = \int_0^1 N_t(j)dj \tag{B.38}$$

From the individual firm's production function $Y_t(j) = Z_t N_t(j)^{1-\alpha}$ we can solve for $N_t(j)$. Substituting this into the above yields the broad employment index (Kotzé, 2014)

$$N_t = \int_0^1 \left(\frac{Y_t(j)}{Z_t}\right)^{\frac{1}{1-\alpha}} dj \tag{B.39}$$

Following Kotzé (2014:51), substituting in the demand constraint $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$ yields

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{Y_t(j)}{Z_t}\right)^{\frac{1}{1-\alpha}} dj \\ &= \left(\frac{Y_t}{Z_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} dj \end{aligned} \tag{B.40}$$

Log-linearising the above around the steady state yields

$$(1-\alpha)\hat{n}_t = \hat{y}_t - \hat{z}_t + d_t \tag{B.41}$$

where $d_t = (1-\alpha)\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} dj$. In a zero inflation steady state $d_t \approx 0$ (Kotzé, 2014:52). Rearranging the above then yields

$$\hat{y}_t = \hat{z}_t + (1-\alpha)\hat{n}_t \tag{B.42}$$

which is equal to the log-linearised aggregate production function (4.21).

B.5.4 Short-term bond supply

Log-linearising the consolidated government budget constraint, recalling the steady state relationships $\frac{1}{R} = \beta$ and $\frac{1}{R_L} = \beta^2$, yields an expression for the short-term bond supply

$$
\begin{aligned}
&\bar{P}e^{\hat{p}_t}\bar{T}e^{\hat{t}_t} + \frac{\bar{B}e^{\hat{b}_t}}{\bar{R}e^{\hat{r}_t}} + \frac{\bar{B}_L e^{\hat{b}_{L,t}}}{\bar{R}_L e^{\hat{r}_{L,t}}} + \bar{M}e^{\hat{m}_t} - \bar{M}e^{\hat{m}_{t-1}} - \frac{\bar{X}e^{\hat{x}_t}\bar{B}_l e^{\hat{b}_{L,t}}}{\bar{R}_L e^{\hat{r}_{L,t}}} + \frac{\bar{X}e^{\hat{x}_{t-1}}\bar{B}_L e^{\hat{b}_{L,t-1}}}{\bar{R}e^{\hat{r}_t}} \\
&= \bar{B}e^{\hat{b}_{t-1}} + \frac{\bar{B}_L e^{\hat{b}_{L,t-1}}}{\bar{R}e^{\hat{r}_t}} \\
&\therefore \bar{P}\bar{T}e^{\hat{p}_t+\hat{t}_t} + \frac{\bar{B}}{\bar{R}}e^{\hat{b}_t-\hat{r}_t} + \frac{\bar{B}_L}{\bar{R}_L}e^{\hat{b}_{L,t}-\hat{r}_{L,t}} + \bar{M}(e^{\hat{m}_t} - e^{\hat{m}_{t-1}}) - \frac{\bar{X}\bar{B}_L}{\bar{R}_L}e^{\hat{x}_t+\hat{b}_{L,t}-\hat{r}_{L,t}} \\
&+ \frac{\bar{X}\bar{B}_L}{\bar{R}}e^{\hat{x}_{t-1}+\hat{b}_{L,t-1}-\hat{r}_t} = \bar{B}e^{\hat{b}_{t-1}} + \frac{\bar{B}_L}{\bar{R}}e^{\hat{b}_{L,t-1}-\hat{r}_t} \\
&\therefore \bar{B}e^{\hat{b}_{t-1}} + \bar{B}_L\beta e^{\hat{b}_{L,t-1}-\hat{r}_t} = \bar{P}\bar{T}e^{\hat{p}_t+\hat{t}_t} + \bar{B}\beta e^{\hat{b}_t-\hat{r}_t} + \bar{B}_L\beta^2 e^{\hat{b}_{L,t}-\hat{r}_{L,t}} + \bar{M}(e^{\hat{m}_t} - e^{\hat{m}_{t-1}}) \\
&- \bar{X}\bar{B}_L\beta^2 e^{\hat{x}_t+\hat{b}_{L,t}-\hat{r}_{L,t}} + \bar{X}\bar{B}_L\beta e^{\hat{x}_{t-1}+\hat{b}_{L,t-1}-\hat{r}_t} \\
&\therefore \bar{B}(1+\hat{b}_{t-1}) + \bar{B}_L\beta(1+\hat{b}_{L,t-1}-\hat{r}_t) \approx \bar{P}\bar{T}(1+\hat{p}_t+\hat{t}_t) + \bar{B}\beta(1+\hat{b}_t-\hat{r}_t) \\
&+ \bar{B}_L\beta^2(1+\hat{b}_{L,t}-\hat{r}_{L,t}) + \bar{M}(1+\hat{m}_t-1-\hat{m}_{t-1}) \\
&- \bar{X}\bar{B}_L\beta^2(1+\hat{x}_t+\hat{b}_{L,t}-\hat{r}_{L,t}) + \bar{X}\bar{B}_L\beta(1+\hat{x}_{t-1}+\hat{b}_{L,t-1}-\hat{r}_t)
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ \bar{B}\hat{b}_t &\approx \frac{\bar{B}}{\beta}\hat{b}_{t-1} + (\bar{B}_L - \bar{X}\bar{B}_L)\hat{b}_{L,t-1} + (\bar{X}\bar{B}_L\beta - \bar{B}_L\beta)\hat{b}_{L,t} + (\bar{B} + \bar{X}\bar{B}_L - \bar{B}_L)\hat{r}_t \\
&+ (\bar{B}_L\beta - \bar{X}\bar{B}_L\beta)\hat{r}_{L,t} - \frac{\bar{M}}{\beta}\hat{m}_t + \frac{\bar{M}}{\beta}\hat{m}_{t-1} + \bar{X}\bar{B}_L\beta\hat{x}_t - \bar{X}\bar{B}_L\hat{x}_{t-1} - \bar{T}\bar{P}\hat{t}_t - \bar{T}\bar{P}\hat{p}_t \\
\therefore \bar{B}\hat{b}_t &\approx \frac{\bar{B}}{\beta}\hat{b}_{t-1} + \bar{B}_L(1-\bar{X})(\hat{b}_{L,t-1} - \beta\hat{b}_{L,t}) + (\bar{B} - \bar{B}_L(1-\bar{X}))\hat{r}_t \\
&+ \bar{B}_L\beta(1-\bar{X})\hat{r}_{L,t} - \frac{\bar{M}}{\beta}(\hat{m}_t - \hat{m}_{t-1}) + \bar{X}\bar{B}_L(\beta\hat{x}_t - \hat{x}_{t-1}) - \bar{T}\bar{P}(\hat{t}_t + \hat{p}_t)
\end{aligned}
$$

Appendix C

Derivation of open-economy equations

C.1 Open-economy identities

The standard open-economy identities, as described in Galí (2015:229–232) and insofar relevant to our model, are briefly discussed below.

C.1.1 Terms of trade

The terms of trade is defined as the ratio of the price of foreign goods ($P_{F,t}$) to home goods ($P_{H,t}$)[1] in terms of domestic currency:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} \tag{C.1}$$

In steady state, this becomes $\bar{\mathcal{S}} = \frac{\bar{P}_F}{\bar{P}_H}$. Equation C.1 can be log-linearised around the steady state as follows:

[1]Steinbach et al (2009:231) defines the terms of trade as the "relative price of imports to domestically produced goods".

$$\begin{aligned} \mathcal{S}_t &= \frac{P_{F,t}}{P_{H,t}} \\ \therefore\ \bar{\mathcal{S}}e^{\hat{s}_t} &= \frac{\bar{P}_F}{\bar{P}_H}e^{\hat{p}_{F,t}-\hat{p}_{H,t}} \\ \therefore\ \bar{\mathcal{S}}(1+\hat{s}_t) &\approx \frac{\bar{P}_F}{\bar{P}_H}(1+\hat{p}_{F,t}-\hat{p}_{H,t}) \\ \therefore\ \hat{s}_t &\approx \hat{p}_{F,t}-\hat{p}_{H,t} \end{aligned} \tag{C.2}$$

An increase in $\hat{s}_t$ represents a depreciation in the terms of trade, since foreign goods are now relatively more expensive than domestic goods.

C.1.2 Home price level

According to Galí (2015:226), the "optimal allocation of expenditures between domestic and imported goods" can be given by

$$C_{H,t} = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \ ;\ C_{F,t} = \upsilon\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{C.3}$$

where the domestic price level (CPI) is given by

$$P_t^{1-\eta} = (1-\upsilon)P_{H,t}^{1-\eta} + \upsilon P_{F,t}^{1-\eta} \tag{C.4}$$

In steady state, this becomes $\bar{P}^{1-\eta} = (1-\upsilon)\bar{P}_H^{1-\eta} + \upsilon\bar{P}_F^{1-\eta}$. Log-linearising equation C.4 around a symmetric steady state with $\bar{\mathcal{S}} = 1$[2] yields

$$\begin{aligned} P_t^{1-\eta} &= (1-\upsilon)P_{H,t}^{1-\eta} + \upsilon P_{F,t}^{1-\eta} \\ (\bar{P}e^{\hat{p}_t})^{1-\eta} &= (1-\upsilon)(\bar{P}_He^{\hat{p}_{H,t}})^{1-\eta} + \upsilon(\bar{P}_Fe^{\hat{p}_{F,t}})^{1-\eta} \\ \bar{P}^{1-\eta}(1+(1-\eta)\hat{p}_t) &\approx (1-\upsilon)\bar{P}_H^{1-\eta}(1+(1-\eta)\hat{p}_{H,t}) \\ &\quad + \upsilon\bar{P}_F^{1-\eta}(1+(1-\eta)\hat{p}_{F,t}) \end{aligned}$$

The steady state and constants drop out and the expression simplifies to

$$\hat{p}_t = (1-\upsilon)\hat{p}_{H,t} + \upsilon\hat{p}_{F,t}$$

Finally, subsituting in the log-linearised terms of trade (equation C.2) yields an expression for the domestic CPI price level

[2]From equation C.1 this implies that $\bar{P} = \bar{P}_H = \bar{P}_F$. See Galí (2015:228).

$$\begin{aligned}\hat{p}_t &= (1-\upsilon)\hat{p}_{H,t} + \upsilon\hat{p}_{F,t} \\ &= \hat{p}_{H,t} - \upsilon\hat{p}_{H,t} + \upsilon\hat{s}_t + \upsilon\hat{p}_{H,t} \\ &= \hat{p}_{H,t} + \upsilon\hat{s}_t\end{aligned} \tag{C.5}$$

Domestic inflation is defined as $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ (Galí, 2015:229). Utilising equation C.5, it follows that CPI (aggregate) inflation can be expressed as a function of domestic inflation and the terms of trade (see also Kotzé (2014:72)):

$$\begin{aligned}\hat{\pi}_t &= \hat{p}_t - \hat{p}_{t-1} \\ &= (\hat{p}_{H,t} + \upsilon\hat{s}_t) - (\hat{p}_{H,t-1} + \upsilon\hat{s}_{t-1}) \\ &= \hat{p}_{H,t} - \hat{p}_{H,t-1} + \upsilon(\hat{s}_t - \hat{s}_{t-1}) \\ &= \hat{\pi}_{H,t} + \upsilon\Delta\hat{s}_t\end{aligned} \tag{C.6}$$

C.1.3 Law of one price

The assumption of the law of one price dictates that if a good has a price P_t in the domestic market, its price in the foreign market will be equal to this price times the nominal exchange rate. That is, $P_{F,t} = \mathcal{E}_t P_t^\star$ (Galí, 2015:229), where $\mathcal{E}_t$ is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. An increase in $\mathcal{E}_t$ therefore represents a depreciation in the domestic currency (foreign currency is now more expensive in terms of domestic currency). $P_t^\star$ is the price of foreign goods expressed in foreign currency.[3] Plugging this into the definition of the terms of trade (equation C.1) yields $\mathcal{S}_t = \frac{\mathcal{E}_t P_t^\star}{P_{H,t}}$, which can be log-linearised[4] as follows:

$$\begin{aligned}\bar{\mathcal{S}}e^{\hat{s}_t} &= \frac{\bar{\mathcal{E}}e^{\hat{e}_t}\bar{P}^\star e^{\hat{p}_t^\star}}{\bar{P}_H e^{\hat{p}_{H,t}}} \\ \therefore\ \bar{\mathcal{S}}(1+\hat{s}_t) &\approx \frac{\bar{\mathcal{E}}\bar{P}^\star}{\bar{P}_H}(1+\hat{e}_t+\hat{p}_t^\star-\hat{p}_{H,t}) \\ \therefore\ \hat{s}_t &= \hat{e}_t + \hat{p}_t^\star - \hat{p}_{H,t}\end{aligned} \tag{C.7}$$

The terms of trade can therefore be expressed in terms of the (nominal) exchange rate and the difference between world and home price levels.

[3] $P_t^\star$ can also be interpreted as a world price index, since the "size of the small open economy is assumed to be negligible relative to the rest of the world" (Galí, 2015:229).

[4] Note that in steady state $\bar{\mathcal{S}} = \frac{\bar{\mathcal{E}}\bar{P}^\star}{\bar{P}_H}$.

C.1.4 Real exchange rate and UIP

The real exchange rate is defined as the ratio of world to domestic CPI, expressed in domestic currency (Galí, 2015:230), and is given by

$$\mathcal{Q}_t \equiv \frac{P_{F,t}}{P_t} \tag{C.8}$$

Log-linearising equation C.8, utilising the results from equations C.2 and C.5 and noting that in steady state $\bar{\mathcal{Q}} = \frac{\bar{P}_F}{\bar{P}_t}$, yields

$$\begin{aligned}
\bar{\mathcal{Q}}e^{\hat{q}_t} &= \frac{\bar{P}_F e^{\hat{p}_{F,t}}}{\bar{P}e^{\hat{p}_t}} \\
\therefore\ \ \bar{\mathcal{Q}}(1+\hat{q}_t) &\approx \frac{\bar{P}_F}{\bar{P}}(1+\hat{p}_{F,t}-\hat{p}_t) \\
\therefore\ \ \hat{q}_t &= \hat{p}_{F,t}-\hat{p}_t \\
&= \hat{s}_t + \hat{p}_{H,t} - \hat{p}_t \\
&= \hat{s}_t + \hat{p}_{H,t} - \hat{p}_{H,t} - \upsilon\hat{s}_t \\
&= (1-\upsilon)\hat{s}_t
\end{aligned} \tag{C.9}$$

Finally, the uncovered interest parity (UIP) condition can be expressed as

$$\hat{r}_t = \hat{r}_t^{\star} + E_t[\Delta\hat{e}_{t+1}] \tag{C.10}$$

(Galí, 2015:232), which could then be used to "relate the interest rate differential to the terms of trade or real exchange rate" (Kotzé, 2014:76).

C.1.5 International risk sharing

The FOC from which the Euler equation (5.15) is derived is a simplified version of equation the household's intertemporal optimality condition (see Appendix B.1) , and is given by

$$\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] = \frac{1}{R_t} + \frac{\kappa_L\phi_L Y_t\left(\kappa_L\frac{B_t}{B^H_{L,t}}-1\right)}{R_{L,t}} \tag{C.11}$$

Following Galí (2015:230), we assume a "complete set of state-contingent securities traded internationally" (i.e. complete markets for international securities). This implies that a condition analogous to equation C.11 must also hold for foreign households. That is

$$\beta E_t\left[\left(\frac{C^\star_{t+1}}{C^\star_t}\right)^{-\sigma}\left(\frac{P^\star_t}{P^\star_{t+1}}\right)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)\right]=\frac{1}{R_t}+\frac{\kappa_L\phi_L Y_t\left(\kappa_L\frac{B_t}{B^H_{L,t}}-1\right)}{R_{L,t}} \tag{C.12}$$

where the presence of the exchange rate terms reflects the fact that the securities' payoffs are "expressed in the currency of the small open economy" (Galí, 2015:230). Combining the law of one price ($P_{F,t}=\mathcal{E}_t P^\star_t$) with the real exchange rate ($\mathcal{Q}_t\equiv\frac{P_{F,t}}{P_t}$, eq. C.8) gives $\mathcal{Q}_t=\frac{\mathcal{E}_t P^\star_t}{P_t}$. This result, combined with equations C.11 and C.12 yields

$$\begin{aligned}
\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right]&=\beta E_t\left[\left(\frac{C^\star_{t+1}}{C^\star_t}\right)^{-\sigma}\left(\frac{P^\star_t}{P^\star_{t+1}}\right)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)\right]\\
\therefore E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right]&=E_t\left[\left(\frac{C^\star_{t+1}}{C^\star_t}\right)^{-\sigma}\left(\frac{P^\star_t}{P^\star_{t+1}}\right)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)\right]\\
\therefore E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\right]&=E_t\left[\left(\frac{C^\star_t}{C^\star_{t+1}}\right)^{\sigma}\left(\frac{\mathcal{E}_t P^\star_t}{P_t}\right)\left(\frac{P_{t+1}}{\mathcal{E}_{t+1}P^\star_{t+1}}\right)\right]\\
\therefore C_t^\sigma&=E_t\left[\left(\frac{C_{t+1}}{C^\star_{t+1}}\right)^{\sigma}(C^\star_t)^\sigma\left(\frac{\mathcal{E}_t P^\star_t}{P_t}\Big/\frac{\mathcal{E}_{t+1}P^\star_{t+1}}{P_{t+1}}\right)\right]\\
&=E_t\left[\left(\frac{C_{t+1}}{C^\star_{t+1}}\right)^{\sigma}(C^\star_t)^\sigma\left(\frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}}\right)\right]\\
\therefore C_t&=\vartheta C^\star_t\mathcal{Q}_t^{\frac{1}{\sigma}}
\end{aligned} \tag{C.13}$$

where $\vartheta=\frac{C_{t+1}}{C^\star_{t+1}\mathcal{Q}_{t+1}^{\frac{1}{\sigma}}}$ is a "constant which will generally depend on initial conditions regarding relative net asset positions" (Galí, 2015:231).[5] Assuming symmetric initial conditions it follows that $\vartheta=1$. Moreover, since we have assumed an infinitesimally small home economy, it follows that $C^\star_t=Y^\star_t$, where $Y^\star_t$ represents world output. Log-linearising this condition around the steady state where $\bar{C}=\vartheta\bar{Y}^\star\bar{\mathcal{Q}}^{\frac{1}{\sigma}}=\bar{Y}^\star$[6], and noting that $\hat{q}_t=(1-\upsilon)\hat{s}_t$, yields the risk sharing condition

[5] See also Devereux and Sutherland (2009:187) for the theoretical considerations regarding the steady state net foreign asset position.

[6] In steady state, $\bar{\mathcal{Q}}=1$ (Appendix C.1.4).

$$
\begin{aligned}
C_t &= \vartheta Y_t^{\star} \mathcal{Q}_t^{\frac{1}{\sigma}} \\
\therefore \bar{C} e^{\hat{c}_t} &= \vartheta \left(\bar{Y}^{\star} e^{\hat{y}_t^{\star}} \right) \left(\bar{\mathcal{Q}} e^{\hat{q}_t} \right)^{\frac{1}{\sigma}} \\
\therefore e^{\hat{c}_t} &= e^{\hat{y}_t^{\star} + \frac{1}{\sigma} \hat{q}_t} \\
\therefore 1 + \hat{c}_t &\approx 1 + \hat{y}_t^{\star} + \frac{1}{\sigma} \hat{q}_t \\
\therefore \hat{c}_t &= \hat{y}_t^{\star} + \frac{1 - \upsilon}{\sigma} \hat{s}_t
\end{aligned}
\tag{C.14}
$$

C.1.6 Dynamic IS curve

Combining the Euler equation (5.48) with the resource constraint (5.45) and the terms of trade, expressed as a function of domestic and foreign output (5.46), yields the following

$$
\begin{aligned}
&\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) + \frac{\upsilon}{\sigma}E_t[\Delta\hat{s}_{t+1}] - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)\\
&\therefore \frac{1}{1-\upsilon}\left(\hat{y}_t - \upsilon\eta(2-\upsilon)\hat{s}_t - \upsilon\hat{y}^\star_t\right)\\
&\quad = E_t\left[\frac{1}{1-\upsilon}\left(\hat{y}_{t+1} - \upsilon\eta(2-\upsilon)\hat{s}_{t+1} - \upsilon\hat{y}^\star_{t+1}\right)\right] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}])\\
&\quad + \frac{\upsilon}{\sigma}E_t[\sigma_\upsilon(\hat{y}_{t+1} - \hat{y}^\star_{t+1}) - \sigma_\upsilon(\hat{y}_t - \hat{y}^\star_t)] - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)\\
&\therefore \frac{1}{1-\upsilon}\left(\hat{y}_t - \upsilon\eta(2-\upsilon)\sigma_\upsilon(\hat{y}_t - \hat{y}^\star_t) - \upsilon\hat{y}^\star_t\right)\\
&\quad = E_t\left[\frac{1}{1-\upsilon}\left(\hat{y}_{t+1} - \upsilon\eta(2-\upsilon)\sigma_\upsilon(\hat{y}_{t+1} - \hat{y}^\star_{t+1}) - \upsilon\hat{y}^\star_{t+1}\right)\right] - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}])\\
&\quad + \frac{\upsilon}{\sigma}E_t[\sigma_\upsilon(\hat{y}_{t+1} - \hat{y}^\star_{t+1}) - \sigma_\upsilon(\hat{y}_t - \hat{y}^\star_t)] - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)\\
&\therefore \frac{1}{1-\upsilon}\left[\hat{y}_t\left(1 - \upsilon\eta(2-\upsilon)\sigma_\upsilon\right) + \hat{y}^\star_t\left(\upsilon\eta(2-\upsilon)\sigma_\upsilon - \upsilon\right)\right]\\
&\quad = \frac{1}{1-\upsilon}E_t\left[\hat{y}_{t+1}\left(1 - \upsilon\eta(2-\upsilon)\sigma_\upsilon\right) + \hat{y}^\star_{t+1}\left(\upsilon\eta(2-\upsilon)\sigma_\upsilon - \upsilon\right)\right]\\
&\quad - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)\\
&\therefore \frac{1}{1-\upsilon}\left[\hat{y}_t\left(1 - \upsilon\eta(2-\upsilon)\sigma_\upsilon + \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma}\right) + \hat{y}^\star_t\left(\upsilon\eta(2-\upsilon)\sigma_\upsilon - \upsilon - \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma}\right)\right]\\
&\quad = \frac{1}{1-\upsilon}E_t\left[\hat{y}_{t+1}\left(1 - \upsilon\eta(2-\upsilon)\sigma_\upsilon + \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma}\right)\right.\\
&\quad \left.+\hat{y}^\star_{t+1}\left(\upsilon\eta(2-\upsilon)\sigma_\upsilon - \upsilon - \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma}\right)\right]\\
&\quad - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)
\end{aligned} \tag{C.15}
$$

Noticing the common terms in each coefficient ($\upsilon\eta(2-\upsilon)\sigma_\upsilon$ and $\frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma}$), we can express the above as

$$
\begin{aligned}
&\frac{1}{1-\upsilon}\left[\hat{y}_t\left(1+\Omega\right) + \hat{y}^\star_t\left(-\upsilon-\Omega\right)\right] = \frac{1}{1-\upsilon}E_t\left[\hat{y}_{t+1}\left(1+\Omega\right) + \hat{y}^\star_{t+1}\left(-\upsilon-\Omega\right)\right]\\
&- \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t)
\end{aligned}
$$

where $\Omega = \frac{\upsilon(1-\upsilon)\sigma_\upsilon}{\sigma} - \upsilon\eta(2-\upsilon)\sigma_\upsilon < 0$. From the calibration (see section 5.4.2 below) it can be shown that $1+\Omega > 0$, $(1-\upsilon) > 0$ and $(-\upsilon-\Omega) > 0$.

$$\begin{aligned}
\therefore \left(\frac{1+\Omega}{1-\upsilon}\right)\hat{y}_t &= \left(\frac{1+\Omega}{1-\upsilon}\right)E_t\hat{y}_{t+1} + \left(\frac{-\upsilon-\Omega}{1-\upsilon}\right)\left[E_t\hat{y}^{\star}_{t+1} - \hat{y}^{\star}_t\right] \\
&\quad - \frac{1}{\sigma}(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}(\hat{b}^H_{L,t} - \hat{b}_t) \\
\therefore \hat{y}_t &= E_t\hat{y}_{t+1} + \left(\frac{-\upsilon-\Omega}{1-\upsilon}\right)\Big/\left(\frac{1+\Omega}{1-\upsilon}\right)\left[E_t\hat{y}^{\star}_{t+1} - \hat{y}^{\star}_t\right] \\
&\quad - \frac{1}{\sigma}\Big/\left(\frac{1+\Omega}{1-\upsilon}\right)(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{\beta\kappa_L\phi_L\bar{Y}}{\sigma}\Big/\left(\frac{1+\Omega}{1-\upsilon}\right)(\hat{b}^H_{L,t} - \hat{b}_t) \\
&= E_t\hat{y}_{t+1} + \left(\frac{-\upsilon-\Omega}{1+\Omega}\right)\left[E_t\Delta\hat{y}^{\star}_{t+1}\right] \\
&\quad - \frac{1}{\sigma}\left(\frac{1-\upsilon}{1+\Omega}\right)(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \frac{1}{\sigma}\left(\frac{1-\upsilon}{1+\Omega}\right)\beta\kappa_L\phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t) \\
&= E_t\hat{y}_{t+1} + \Omega_1\left[E_t\Delta\hat{y}^{\star}_{t+1}\right] - \Omega_2(\hat{r}_t - E_t[\hat{\pi}_{H,t+1}]) - \Omega_2\beta\kappa_L\phi_L\bar{Y}(\hat{b}^H_{L,t} - \hat{b}_t)
\end{aligned}$$

where $\Omega_1 = \left(\frac{-\upsilon-\Omega}{1+\Omega}\right)$ and $\Omega_2 = \frac{1}{\sigma}\left(\frac{1-\upsilon}{1+\Omega}\right)$ are convolutions of the parameters.

C.1.7 World variables

Finally, following Galí (2015:230), we assume that world demand and the world price level are "taken as exogenous to the small open economy". That implies that these two variables evolve according to the following stochastic processes:

$$\hat{y}^{\star}_t = \phi^{y\star}\hat{y}^{\star}_{t-1} + \varepsilon^{y\star}_t \tag{C.16}$$

$$\hat{p}^{\star}_t = \phi^{p\star}\hat{p}^{\star}_{t-1} + \varepsilon^{p\star}_t \tag{C.17}$$

Appendix D

Federal Reserve Open-Market Committee (FOMC) Policy Statements

Table D.1: URLs for Federal Reserve Policy Statements involving QE actions

Date	**Programme**	**URL**
11/25/2008	QE1	http://www.federalreserve.gov/newsevents/press/monetary/20081125b.htm
3/18/2009	QE1	http://www.federalreserve.gov/newsevents/press/monetary/20090318a.htm
8/10/2010	QE1	http://www.federalreserve.gov/newsevents/press/monetary/20100810a.htm
3/11/2010	QE2	http://www.federalreserve.gov/newsevents/press/monetary/20101103a.htm
9/21/2011	MEP	http://www.federalreserve.gov/newsevents/press/monetary/20110921a.htm
6/20/2012	MEP	http://www.federalreserve.gov/newsevents/press/monetary/20120620a.htm
9/13/2012	QE3	http://www.federalreserve.gov/newsevents/press/monetary/20120913a.htm
12/12/2012	QE3	http://www.federalreserve.gov/newsevents/press/monetary/20121212a.htm
12/18/2013	QE3	http://www.federalreserve.gov/newsevents/press/monetary/20131218a.htm
10/29/2014	Exit	http://www.federalreserve.gov/newsevents/press/monetary/20141029a.htm

Source: Adapted and updated from Fawley and Neely (2013).

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