Growth volatility and inequality in the U.S.: A wavelet analysis

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Highlights

- Explores relationship between US growth volatility, and income/wealth inequality.
- Considers relationship between output volatility during positive/ negative growth.
- Examines the correlation/causality between two series in time/frequency domains.
- Direction of causality varies across frequencies and time.
- Positive correlation between volatility/inequality across high/low-frequencies.

Abstract

This study applies wavelet coherency analysis to explore the relationship between the U.S. economic growth volatility, and income and wealth inequality measures over the period 1917 to 2015 and 1962 to 2014. We consider the relationship between output volatility during positive and negative growth scenarios. Wavelet analysis simultaneously examines the correlation and causality between two series in both the time and frequency domains. Our findings provide evidence of positive correlation between the volatility and inequality across high (short-run)- and low-frequencies (long-run). The direction of causality varies across frequencies and time. Strong evidence exists that volatilities lead inequality at low-frequencies across income inequality measures from 1917 to 1997. After 1997, however, the direction of causality changes. In the time-domain, the time-varying nature of long-run causalities implies structural changes in the two series. These findings provide a more thorough picture of the relationship between the U.S. growth volatility and inequality measures over time and frequency domains, suggesting important implications for policy makers.

Keywords:
Growth volatility
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1. Introduction

Volatility measures changes over time. The more that a variable fluctuates, the more volatile the variable is. As volatility associates with unpredictability, uncertainty, and, thus, risk, changes in growth volatility can affect macroeconomic

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variables. Volatility can affect growth positively or negatively, although a negative association is the most common. Imbs [1] finds that aggregate volatility slows down economic growth, but more volatile activities grow fast. How would growth volatility affect inequality? At low frequencies, the most likely sources of volatility are macroeconomic and institutional changes (i.e., policy-driven instabilities), whereas at high frequencies, the volatility most likely reflects noise [2]. Thus, Daly [2] recommends calculating volatility over different frequencies as the frequency of data dictates which types of volatility can be seen and, therefore, measured.

This paper examines the short- and long-run relationships between growth volatility and inequality. We consider the relationship between output volatility during positive and negative growth scenarios.

Before researchers found that macroeconomic volatility may actually reduce long-term growth (e.g., [3]), it was generally accepted that the effect of volatility on economic growth and welfare was small and insignificant. Ramey and Ramey [3] examine the relationship between output growth and its volatility. They find an inverse relationship between output volatility and the output growth rate. Thus, policy changes and economic shocks that increase volatility can exert significant long-term negative effects on welfare by reducing economic growth. These findings raise the question of whether volatility also affects other macroeconomic variables.

Over the past three decades, researchers refocused their interest on the causes and consequences of macroeconomic volatility because the recent financial crisis highlighted its costs in terms of increasing inequality. Many of financial crises have been associated with the rapid opening-up of economies to global trade and financial linkages. Jaumotte et al. [4] use panel data of 51 countries over 1981–2003 and report that financial globalization, especially foreign direct investment, associates with an increase in inequality. Gozgor and Ranjan [5] also look at if globalization increases in the distribution of income and show the positive relationship between globalization and inequality and that redistribution is much stronger for OECD countries than for non-OECD countries.[1]

Rather than examining the volatility-growth or the growth-inequality nexuses, the existing literature considers other possible connections between growth volatility and inequality. Hausmann and Gavin [8] directly investigate the relationship between volatility and inequality, finding adverse effects of income volatility on the distribution of income.

How does volatility affect inequality? Theory suggests several channels to explain how growth volatility affects the distribution of income. Volatility can affect the income distribution as individuals possess different levels of risk tolerance and the channels of influence on inequality relate to risk. First, entrepreneurs exhibit higher levels of risk tolerance than salary earners. Also, bearing risk enables entrepreneurs to capture the resulting higher risk premium that contributes to their income and wealth. Caroli and García-Peñalosa [9], focusing on this wage channel, consider an economy where random shocks affect output and, in turn, wages fluctuate. They argue that the share of output captured by entrepreneurs becomes larger the more volatile the output because salaried workers will take a decreased salary to get a constant wage.

Second, Checchi and García-Peñalosa [10], considering the human capital channel, examine the effects of wage volatility on wage differentials between low and high skilled workers. They find that high wage volatility causes a high degree of educational inequality and, as a result, income inequality rises.

Third, volatility makes economic growth less favorable to the poor. Low-income groups do not experience good access to financial and credit markets. These market imperfections can influence occupational outcomes of low-income individuals. Also, they depend more on state grants and social services [11]. The poor receive less diversified sources of income, possess inferior qualifications, and exhibit less mobility than the rich [12–15].

How can we explain the divergence in the patterns of output volatility and income inequality that the data support? Eksi [16] shows that an increase in the time-series variance of micro income shocks lead to increases in both output and income inequality. Moreover, a decrease in the cross-sectional correlation of these shocks across individuals leads to a decrease in output volatility, but to an increase in income inequality. In other words, one variable is an increasing function of the correlation parameter, while the other is a decreasing function of it. Eksi [16] argues that the simultaneity of the changes in output volatility and income inequality during the Great Moderation period is not a coincidence, but reflects the fact that the variables depend on the same parameters of the underlying income microdata.

Many empirical studies find that higher volatility associates with higher income inequality. Hausmann and Gavin [8] find that Latin American countries display higher income inequality and much more volatile economic growth rates. Laursen and Mahajan [14] find that output volatility negatively influences the equality of the income distribution of the bottom 20% income group. With the cross-sectional data of the Gini coefficient and the income share of the top quintile of developing and developed countries, Breen and García-Peñalosa [17] show that higher growth volatility links to higher income inequality.

Numerous empirical studies exist that use panel data. Using a panel data set of 70 countries from 1960 to 2002, Konya and Mouratidis [18] find that volatility affects inequality, but that inequality does not exert a direct effect on volatility. They also find that low growth volatility reduces inequality, whereas high growth volatility leads to more unequal income distribution. In other words, growth volatility reduces inequality in countries with low volatility, while it increases income inequality in

[1] Globalization stimulates global economic growth and enhances social progress, but, it can also raise income inequality and labor-supply competition. Some studies focus on the effect of financial globalization on income inequality, since financial globalization changes substantively where firms and households access capital and financial services. See, for example, Haltiwanger [6], Kose et al. [7], and Gozgor and Ranjan [5] for theoretical and empirical implications of globalization for inequality and redistribution.

countries with high volatility. Calderón and Yeyati [19] use a panel data set of 75 countries over 1970–2005 and also find that output volatility increases income inequality, especially with extremely high volatility, such as macroeconomic crises. They conclude that volatility increases the income share of the highest quintiles at the expense of the middle 40%. Using annual data from the 48 U.S. states over 1945–2004, Huang et al. (2015) find robust results that larger growth volatility positively and significantly associates with higher income inequality. Chauvet et al. [20] also examine the relationship between income volatility and inequality, considering aid and remittances. The authors employ a panel of 142 countries over 1973–2012 and find that volatility increases inequality, where lower income groups are most exposed to the volatility. They also find robust evidence suggesting that aid helps to reduce the negative effects of volatility on the distribution of income.

The effect of output volatility on inequality is well-documented in the literature and most of the studies find that volatility produces an unfavorable effect on the distribution of income. Studies also suggest, however, a possibility of income inequality intensifying macroeconomic volatility. Alesina and Perotti [21] argue that income inequality exerts an indirect effect on macroeconomic volatility via increased political instability. Aghion et al. [22,23] argue that inequality in the form of unequal access to investment opportunities combined with a high level of capital market imperfection may generate persistent credit cycles, resulting in output and investment volatility. Levy [24] uses an AS-AD model and theoretically shows income inequality may influence macro-economic variables by affecting the money multiplier and the trade-off between inflation and output.

One study considers the short- and long-run effects of income volatility on inequality. Bahmani-Oskooee and Motavallizadeh-Ardakani [25] employ linear and nonlinear ARDL approaches on annual U.S. state panel data from 1945 to 2013 and discover short-run asymmetric effects of income volatility on a measure of inequality in many states. The short-run effects translate to long-run asymmetric effects, however, in nineteen states. Only one state, South Dakota, shows long-run symmetric effect wherein increased volatility worsens inequality and decreased volatility improves it. The authors also find that both increased volatility and decreased volatility can create unequalizing effects on income distribution in only Indiana, Michigan and Wyoming and conclude overall that, in the United States, reducing income or output volatility will not help to reduce income inequality.

Given the conclusions in the existing literature, our paper provides three main contributions. First, we extend the existing literature on the effects of income and wealth inequality on output volatility, combining time-series and frequency-domain analyzes. Wavelet analysis allows us to examine the time–frequency historical effects of volatility on U.S. income and wealth inequality. Using wavelet coherency, we can assess the role of income and wealth inequality on growth volatility dynamics at different frequencies and specific moments in time. At the same time, we can indicate the direction of the causality between inequality and volatility at different moments in time. The time- and frequency-varying relationships can provide significant implications for macroeconomic policy makers.

The time-varying relationships indicate that the variables influence each other differently at different points in the business cycle [26]. Frequency-varying relationships reveal short- versus long-term linkages between two variables. In addition, unlike standard tests of Granger causality that require pre-testing for unit roots and cointegration, wavelet analysis provides robust evidence in favor of or against causal relationships between variables under consideration without accounting for issues associated with stationary or non-stationary data and the existence or non-existence of long-run relationships. In other words, we can work with the raw data and do not need to transform the data, which, in turn, often tends to change the definition of the original variables for which we are trying to detect causal relationships.

Second, in contrast to the bulk of the literature that uses output volatility defined as the standard deviation of the rate of output growth, we use the realized volatility calculated by taking the sum over the squared quarterly GNP growth rates. Realized volatility is a nonparametric, ex-post estimate of the return (growth) variation and it provides empirical content to the latent variance variable [27]. Therefore, this approach proves useful for specification testing of the restrictions imposed on volatility by parametric models previously estimated with low-frequency data. Further, realized volatility measures facilitate direct estimation of parametric models.[2]

Finally, we not only examine the aggregate growth volatility but also investigate the volatility related to positive growth (i.e. good volatility) and the volatility connected to negative growth (i.e. bad volatility), which allows deeper examination on the different aspects of volatilities.

Using wavelet analysis, which allows the examination of the relationships between volatility and inequality in different time- and frequency-domains, our empirical results show that the periods and directions of short-term causality between the volatility and inequality vary over time. Volatility mainly leads income inequality measures over the long-term through the early-2000s. As such, our findings generally confirm the theoretical prediction that larger growth volatility worsens the distribution of income.

Our time- and frequency- varying results indicate not only that stabilization policy is required to reduce the level of inequality but also that direct policy, which can reduce the gap of inequality, not providing rent-seeking nor rapid and forced redistribution, can stabilize the volatility. Macroeconomic policy makers can undertake policies that simultaneously reduce volatility and inequality such as keeping the unemployment rate low and adjusting minimum wages. Also, as determinants

2 Please see [27] for detailed discussion on realized volatility.

Table 1
Parameter stability tests in VAR(2) model.

A	Realized volatility equation		Gini equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	54.28	<0.01	11.97	0.108	55.33	<0.01
Mean-F	24.88	<0.01	5.70	0.066	20.04	<0.01
Exp-F	23.12	<0.01	3.79	0.078	23.89	<0.01
B	Realized volatility equation		Top 10% equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	65.16	<0.01	16.48	0.018	39.09	<0.01
Mean-F	23.21	<0.01	5.33	0.086	24.99	<0.01
Exp-F	28.32	<0.01	4.90	0.029	16.44	<0.01
C	Realized volatility equation		Top 1% equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	58.13	<0.01	15.67	0.026	44.18	<0.01
Mean-F	23.45	<0.01	5.58	0.072	23.04	<0.01
Exp-F	24.83	<0.01	4.96	0.027	18.30	<0.01
D	Realized volatility equation		p90p100 equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	143.83	<0.01	515.18	<0.01	25.26	<0.01
Mean-F	13.95	<0.01	38.18	<0.01	14.45	<0.01
Exp-F	68.25	1.000	253.93	1.000	9.81	<0.01
A	Positive volatility equation		Gini equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	32.12	<0.01	12.10	0.103	45.42	<0.01
Mean-F	11.32	<0.01	4.77	0.127	17.06	<0.01
Exp-F	12.09	<0.01	3.33	0.119	18.73	<0.01
B	Positive volatility equation		Top 10% equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	34.41	<0.01	19.30	0.005	39.93	<0.01
Mean-F	14.35	<0.01	5.05	0.105	21.30	<0.01
Exp-F	13.13	<0.01	5.81	0.013	16.41	<0.01
C	Positive volatility equation		Top 1% equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	31.84	<0.01	27.21	<0.01	40.79	<0.01
Mean-F	14.11	<0.01	7.79	0.014	19.63	<0.01
Exp-F	12.33	<0.01	9.55	<0.01	16.86	<0.01
D	Positive volatility equation		p90p100 equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	46.32	<0.01	975.95	<0.01	27.27	<0.01
Mean-F	11.80	<0.01	46.55	<0.01	13.30	<0.01
Exp-F	19.50	<0.01	484.31	1.000	10.46	<0.01
A	Negative volatility equation		Gini equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	28.00	<0.01	15.44	0.027	42.54	<0.01
Mean-F	13.15	<0.01	5.58	0.072	16.56	<0.01
Exp-F	9.86	<0.01	4.22	0.053	17.02	<0.01
B	Negative volatility equation		Top 10% equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	24.73	<0.01	79.73	<0.01	32.09	<0.01
Mean-F	11.70	<0.01	15.85	<0.01	15.11	0.002
Exp-F	8.22	0.002	35.60	1.000	12.22	<0.01
C	Negative volatility equation		Top 1% equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	26.91	<0.01	72.37	<0.01	27.29	0.003
Mean-F	12.85	<0.01	16.41	<0.01	13.70	0.005
Exp-F	9.27	0.01	32.03	0.435	9.60	0.005

(*continued on next page*)

Table 1 (*continued*).

D	Negative volatility equation		p90p100 equation		VAR(2) system	
	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value	Statistics	Bootstrap *p*-value
Sup-F	N/A	N/A	N/A	N/A	83.75	<0.01
Mean-F	N/A	N/A	N/A	N/A	13.17	<0.01
Exp-F	N/A	N/A	N/A	N/A	38.21	<0.01

Note: The parameter stability tests exhibit non-standard asymptotic distributions. Using the parametric bootstrap procedure, Andrews [28] and Andrews and Ploberger [29] report the critical values and *p* values for the non-standard asymptotic distributions of these tests. We obtain the critical values and *p* values using asymptotic distribution constructed by means of Monte Carlo simulations using 10,000 samples generated from a VAR model with constant parameters. Besides, according to Andrews [28], 15-percent trimming from both ends of the sample is required for the Sup-F, Mean-F and Exp-F. Hence, we apply the tests to the fraction of the sample in (0.15, 0.85).

Table 2
Parameter stability tests in long-run relationship FM-OLS.

Realized volatility	Gini		Top 10%		Top 1%		p90p100	
	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value
Lc	2.05	<0.01	7.77	<0.01	7.43	<0.01	1.87	<0.01
Positive volatility	Gini		Top 10%		Top 1%		p90p100	
	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value
Lc	1.02	0.013	5.35	<0.01	4.98	<0.01	2.04	<0.01
Negative volatility	Gini		Top 10%		Top 1%		p90p100	
	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value	Stats	Bootstrap *p*-value
Lc	1.28	<0.01	3.68	<0.01	3.67	<0.01	0.55	0.126

Note: We apply the Lc test proposed by Nyblom [30] and Hansen [31] to investigate the long-run parameter stability with the long-run relationship estimated using the Fully Modified ordinary least squares (FM-OLS) estimator of Phillips and Hansen [32]. We calculate *p*-value using 10,000 bootstrap repetitions.

Table 3
Wavelet phase difference (Volatility, logarithm of Atkinson Index).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1958	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05
	1959–2015	$(0, \frac{\pi}{2})$, In-phase	+	Atkin05 → Volatility
Low frequency	1917–1997	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05
	1998–2015	$(0, \frac{\pi}{2})$, In-phase	+	Atkin05 → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1964	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05
	1965–2003	$(0, \frac{\pi}{2})$, In-phase	+	Atkin05 → Volatility
	2004–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05
Low frequency	1917–1998	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05
	1999–2015	$(0, \frac{\pi}{2})$, In-phase	+	Atkin05 → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1951	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05
	1952–2015	$(0, \frac{\pi}{2})$, In-phase	+	Atkin05 → Volatility
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Atkin05

of volatility differ between long- and short-term volatility, the policy implications differ according to which volatility is targeted for stabilization.

The rest of the paper is organized as follows. Sections 2 and 3 present the methodology and the data, respectively. Sections 4 and 5 present the empirical results. Section 6 concludes the paper.

Table 4
Wavelet phase difference (Volatility, logarithm of Gini coefficient).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1960	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini coefficient
	1961–1983	$(0, \frac{\pi}{2})$, In-phase	+	Gini → Volatility
	1984–1985	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
	1986–1987	$(0, \frac{\pi}{2})$, In-phase	+	Gini → Volatility
	1988–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1978	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
	1979–1987	$(0, \frac{\pi}{2})$, In-phase	+	Gini → Volatility
	1988–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1946	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
	1947–1976	$(0, \frac{\pi}{2})$, In-phase	+	Gini → Volatility
	1977–1993	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
	1994–2015	$(0, \frac{\pi}{2})$, In-phase	+	Gini → Volatility
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini

Table 5
Wavelet phase difference (Volatility, logarithm of the Relative Mean Deviation).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1960	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
	1961–2015	$(0, \frac{\pi}{2})$, In-phase	+	Rmeandev → Volatility
Low frequency	1917–2012	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
	2013–2015	$(0, \frac{\pi}{2})$, In-phase	+	Rmeandev → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1968	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
	1969–1989	$(0, \frac{\pi}{2})$, In-phase	+	Rmeandev → Volatility
	1990–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
Low frequency	1917–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
	2015	$(0, \frac{\pi}{2})$, In-phase	+	Rmeandev → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1945	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
	1946–1979	$(0, \frac{\pi}{2})$, In-phase	+	Rmeandev → Volatility
	1980–1990	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev
	1991–2015	$(0, \frac{\pi}{2})$, In-phase	+	Rmeandev → Volatility
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Rmeandev

2. Methodology: Wavelet coherency and phase difference

Wavelet analysis can extract time- and frequency-localized information not only from stationary series but also from non-stationary and locally stationary series as well as series with structural changes [33]. Economic processes emerge as outcomes of the actions of numerous agents at different frequencies, which implies that a macroeconomic time series incorporates information that operates at different time domains. Wavelet analysis separates the time series into several

Table 6
Wavelet phase difference (Volatility, logarithm of Theil Index).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1954	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
	1955–1988	$(0, \frac{\pi}{2})$, In-phase	$+$	Theil → Volatility
	1989–2015	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
Low frequency	1917–2012	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
	2013–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Theil → Volatility
Good/$(+)$ Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1961	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
	1962–1986	$(0, \frac{\pi}{2})$, In-phase	$+$	Theil → Volatility
	1987–2015	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
Low frequency	1917–2007	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
	2008–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Theil → Volatility
Bad/$(-)$ Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1951	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
	1952–1978	$(0, \frac{\pi}{2})$, In-phase	$+$	Theil → Volatility
	1979–1992	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil
	1993–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Theil → Volatility
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Theil

Table 7
Wavelet phase difference (Volatility, logarithm of Top 10%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility
Low frequency	1917–2008	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Top 10%
	2009–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility
Good/$(+)$ Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1931	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility
	1932–1963	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Top 10%
	1964–2006	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility
	2007–2015	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Top 10%
Low frequency	1917–2007	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Top 10%
	2008–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility
Bad/$(-)$ Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility
Low frequency	1917–2005	$(\frac{-\pi}{2}, 0)$, In-phase	$+$	Volatility → Top 10%
	2006–2015	$(0, \frac{\pi}{2})$, In-phase	$+$	Top 10% → Volatility

sub-series, which may associate with a particular time domain and which narrows the focus to provide fruitful insights on economic phenomena [34,35].

2.1. Continuous wavelet transform

There are two kinds of wavelet transforms exist: discrete wavelet transforms (DWT) and continuous wavelet transforms (CWT). The DWT reduces noise and compresses data whereas the CWT extracts features and detects data self-similarities [36,37].

Table 8
Wavelet phase difference (Volatility, logarithm of Top 5%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1918	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	1919	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
	1920–1921	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	1922–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
Low frequency	1917–2003	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	2004–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1926	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
	1927–1959	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	1960–2009	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
	2010–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
Low frequency	1917–2004	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	2005–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1927	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	1928–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility
Low frequency	1917–2000	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 5%
	2001–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 5% → Volatility

Table 9
Wavelet phase difference (Volatility, logarithm of Top 1%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
Low frequency	1917–2001	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
	2002–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2012	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
	2013–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
Low frequency	1917–2001	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
	2002–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1940	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
	1941–1960	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
	1961–1970	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
	1971–1972	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
	1973	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
	1974–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility
Low frequency	1917–2002	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
	2003–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 1% → Volatility

The continuous wavelet transforms, with respect to the wavelet ψ, is a function

$$W_x(s, \tau) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s}\right) dt,$$

where $*$ denoted complex conjugation. The parameter s is scaling factor that controls the length of the wavelet and τ is a location parameter that indicates where the wavelet is centered. Scaling a wavelet simply means stretching it (if $|s| > 1$), or compressing it (if $|s| < 1$).

Table 10
Wavelet phase difference (Volatility, logarithm of Top 0.5%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility
Low frequency	1917–2004	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.5%
	2005–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2014	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility
	2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.5%
Low frequency	1917–2004	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.5%
	2005–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1943	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.5%
	1944–1957	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility
	1958–1964	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.5%
	1965–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility
Low frequency	1917–2004	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.5%
	2005–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.5% → Volatility

Table 11
Wavelet phase difference (Volatility, logarithm of Top 0.1%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1938	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	1939–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
Low frequency	1917–2004	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	2005–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
Low frequency	1917–2004	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	2005–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1946	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	1947–1952	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
	1953–1954	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	1955	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
	1956–1957	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	1958	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
	1959–1972	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	1973–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility
Low frequency	1917–2007	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.1%
	2008–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.1% → Volatility

If the wavelet function $\psi(t)$ is complex,[3] the wavelet transformW_x will also be complex. The transform can then be divided into the real part ($\Re W_x$) and imaginary part (IW_x), or amplitude, $|W_x|$, and phase, $tan^{-1}(\frac{IW_x}{\Re W_x})$. The phase of a given

[3] The wavelet transform is a method to decompose an input signal into wavelets via "mother wavelet" function. In this study, a morlet wavelet – a complex valued wavelet with optimal joint time–frequency concentration – is used as "mother wavelet" as it brings in information on the amplitude and phase which both are essential to study synchronism between different time-series. See Goupillaud et al. [38] and Aguiar-Conraria et al. [39] for detailed information of the mother wavelet and Morlet wavelet.

Table 12
Wavelet phase difference (Volatility, logarithm of Top 0.01%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1943	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.01%
	1944–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.01% → Volatility
Low frequency	1917–2008	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.01%
	2009–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.01% → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1929	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.01%
	1930–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.01% → Volatility
Low frequency	1917–2005	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.01%
	2006–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.01% → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1974	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.01%
	1975–2015	$(0, \frac{\pi}{2})$, In-phase	+	Top 0.01% → Volatility
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 0.01%

Table 13
Wavelet phase difference (Volatility, Net personal wealth held by p90p100).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1975	$(0, \frac{\pi}{2})$, In-phase	+	p90p100 → Volatility
	1976–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
Low frequency	1962–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1976	$(0, \frac{\pi}{2})$, In-phase	+	p90p100 → Volatility
	1977–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
Low frequency	1962–2001	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
	2002–2014	$(0, \frac{\pi}{2})$, In-phase	+	p90p100 → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1985	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
	1986–2014	$(0, \frac{\pi}{2})$, In-phase	+	p90p100 → Volatility
Low frequency	1962–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100

time series $x(t)$ is parameterized in radians, ranging from $-\pi$ to π. In order to separate the phase and amplitude information of a time series, it is important to make use of complex wavelets.

2.2. Wavelet coherency and phase difference

Hudgins et al. [40] and Torrence and Compo [41] develop methodologies of the cross-wavelet power, the cross-wavelet coherency, and the phase difference. Wavelet analysis closely links to Fourier analysis; but, it possesses certain advantages. Wavelet analysis conserves information in both time and frequency domains by conducting the estimation of spectral characteristics of a time series as a function of time [39]. Also, wavelet analysis applies to non-stationary or locally stationary series [33]. Wavelet coherency involves a three-dimensional analysis, which counts the time and frequency elements at the same time as well as the strength of the correlation between the time-series elements [37]. Thus, we can observe both the time- and frequency-variations of the correlation between two series in a time–frequency domain. When the frequency components exhibit non-stationarity, the traditional approach may miss such frequency components. Wavelet analysis provides the time- and frequency-localized information with structural breaks. Thus, we can avoid the need to assume stationarity [42].

Table 14
Wavelet phase difference (Volatility, Net personal wealth held by p50p90).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1978	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
	1979–2014	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility
Low frequency	1962–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1692–1978	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
	1979–2014	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility
Low frequency	1962–1998	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility
	1999–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1964	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
	1965–1967	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility
	1968–1972	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
	1973–1979	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility
	1980–1981	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
	1982–1983	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility
	1984–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p50p90
Low frequency	1962–2014	$(0, \frac{\pi}{2})$, In-phase	+	p50p90 → Volatility

Table 15
Wavelet phase difference (Volatility, Net personal wealth held by p0p50).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–2006	$(0, \frac{\pi}{2})$, In-phase	+	p0p50 → Volatility
	2007–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p0p50
Low frequency	1962–2014	$(0, \frac{\pi}{2})$, In-phase	+	p0p50 → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–2014	$(0, \frac{\pi}{2})$, In-phase	+	p0p50 → Volatility
Low frequency	1962–1989	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p0p50
	1990–2014	$(0, \frac{\pi}{2})$, In-phase	+	p0p50 → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1988	$(0, \frac{\pi}{2})$, In-phase	+	p0p50 → Volatility
	1989–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p0p50
Low frequency	1962–2014	$(0, \frac{\pi}{2})$, In-phase	+	p0p50 → Volatility

As a result, wavelet coherency delivers a better measure of the co-movement between variables, U.S. income and wealth inequality and output volatility, in comparison to conventional causality and correlation analysis. Following the approach of Li et al. [26], we estimate wavelet coherency by using the cross-wavelet and auto-wavelet power spectrums as follow:

$$R^2_{xy}(\tau, s) = \frac{|S(s^{-1}W_{xy}(\tau, s))|^2}{S(s^{-1}|W_x(\tau, s)|^2)S(s^{-1}|W_y(\tau, s)|^2)},$$

where complex argument $\arg W_{xy}(\tau, s)$ is the local relative phase between x_t and y_t, $|W_x(\tau, s)|^2$ represents the wavelet power, $\arg W_x(\tau, s)$ is local phase, and S represents a smoothing operator.[4] The ratio of the cross-wavelet spectrum to the

4 Without smoothing, the squared wavelet coherency is always equal to 1 at any frequency and time. Torrence and Compo [41] show that smoothing in time or frequency increases the degrees of freedom of each point and increases the confidence of the wavelet spectrum.

Table 16
Wavelet phase difference (Volatility, Net personal wealth held by p99p100).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–2005	$(0, \frac{\pi}{2})$, In-phase	+	p99p100 → Volatility
	2006–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p99p100
Low frequency	1962–2014	$(0, \frac{\pi}{2})$, In-phase	+	p99p100 → Volatility
Good/(+) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1988	$(0, \frac{\pi}{2})$, In-phase	+	p99p100 → Volatility
	1989–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p99p100
Low frequency	1962–2000	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p99p100
	2001–2014	$(0, \frac{\pi}{2})$, In-phase	+	p99p100 → Volatility
Bad/(−) Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–2014	$(0, \frac{\pi}{2})$, In-phase	+	p99p100 → Volatility
Low frequency	1962–2014	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p99p100

Table 17
Results of Granger causality in different frequencies.

	Frequencies decomposed by the MODWT						Granger causality	
Frequency	Short term		Medium term		Long term		Whole sample period	
Null hypothesis	*F*-Stat.	Prob.	*F*-Stat.	Prob.	*F*-Stat.	Prob.	*F*-Stat.	Prob.
BadRV does not Granger Cause Gini	1.493		9.242	***	8.258	***	2.258	
BadRV does not Granger Cause Top10	0.737		2.630	*	2.210		2.581	*
BadRV does not Granger Cause p90p100	1.369		2.964	*	21.689	***	0.238	
GoodRV does not Granger Cause Gini	0.890		3.039	*	1.064		1.620	
GoodRV does not Granger Cause Top10	2.730	*	1.040		2.330		5.097	***
GoodRV does not Granger Cause p90p100	0.074		13.758	***	26.868	***	1.960	
RV does not Granger Cause Gini	4.130	**	9.626	***	0.539		3.296	**
RV does not Granger Cause Top10	0.846		0.170		0.416		2.308	
RV does not Granger Cause p90p100	0.296		8.695	***	45.455	***	2.463	*
Gini does not Granger Cause GoodRV	9.670	***	0.311		12.314	***	3.808	**
Gini does not Granger Cause BadRV	0.521		1.369		20.946	***	0.307	
Gini does not Granger Cause RV	9.725	***	1.219		15.749	***	0.885	
Top10 does not Granger Cause GoodRV	0.384		2.976	*	11.050	***	1.327	
Top10 does not Granger Cause BadRV	11.650	***	6.532	***	23.970	***	0.529	
Top10 does not Granger Cause RV	8.215	***	3.329	**	16.996	***	0.473	
p90p100 does not Granger Cause GoodRV	1.846		3.808	**	4.212	**	0.010	
p90p100 does not Granger Cause BadRV	0.528		2.841	*	1.488		0.093	
p90p100 does not Granger Cause RV	3.135	*	6.765	***	5.508	***	0.040	

Note:
***Indicate significance at the 0.01 levels.
**Indicate significance at the 0.05 levels.
*Indicate significance at the 0.1 levels.
We use the MODWT based on the Daubechies and decompose our data up to level 8.

product of the spectrum of each series equals the local correlation of the two series. This formula gives a quantity between 0 and 1 in a time–frequency window. Zero coherency indicates that no co-movement occurs between the volatility, and the income and wealth inequality measures, while the highest coherency implies the strongest co-movement between the two series. On the wavelet coherency plots, red and blue colors correspond to strong and weak co-movements, respectively.

As the wavelet coherency is squared, we cannot easily distinguish between positive and negative co-movements. Rather, we use the phase difference to provide information on positive and negative co-movements as well as the lead–lag relationships between the two series.[5] Bloomfield et al. [43] characterize the phase difference relationship between $x(t)$

[5] The term phase means the position in the pseudo-cycle of the series as a function of frequency.

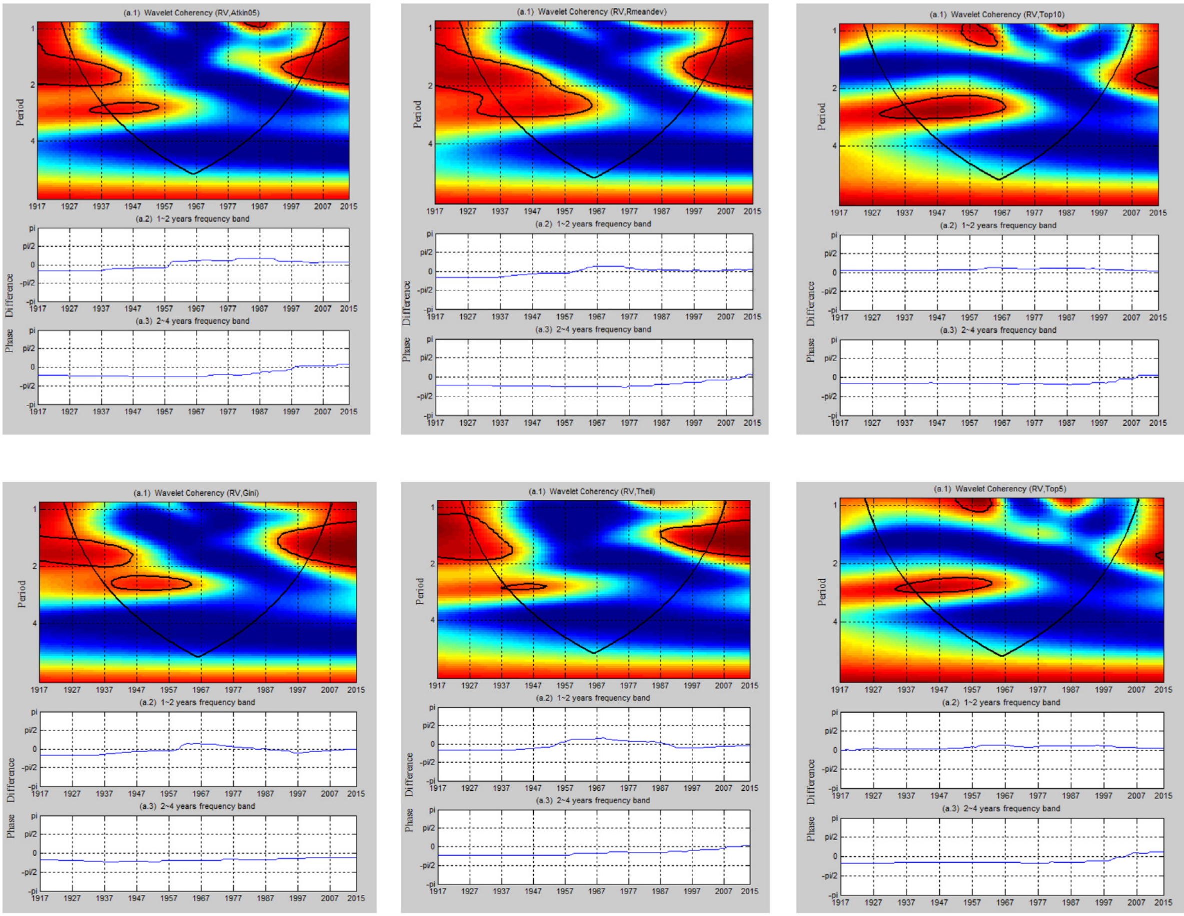

Fig. 1. Causal relationship between Aggregate Output Volatility and Income Inequality measures. **Note:** Wavelet Coherency between the aggregate output volatility and income inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period over the period 1917–2015.

and $y(t)$ such that:

$$\phi_{xy} = tan^{-1}\left(\frac{\Im\left\{S\left(s^{-1}W_{xy}\left(\tau,s\right)\right)\right\}}{\Re\left\{S\left(s^{-1}W_{xy}\left(\tau,s\right)\right)\right\}}\right),\ with\ \phi_{xy} \in [-\Pi, \Pi],$$

where $\Im$ is the imaginary part of the smoothed cross-wavelet transform and $\Re$ represents the real part of the smoothed cross-wavelet transform.

A phase difference of zero reveals that the two underlying series move together, while a phase difference of $\pi(-\pi)$ indicates that the two series move in the opposite directions. If $\phi_{xy} \in (0, \pi/2)$, then the series move in phase (positively co-move) with $y(t)$ leading $x(t)$. If $\phi_{xy} \in (\pi/2, \pi)$, then the series move out of phase (negatively co-move) with $x(t)$ leading $y(t)$. If $\phi_{xy} \in (-\pi, -\pi/2)$, then the series move out of phase with $y(t)$ leading $x(t)$. Finally, if $\phi_{xy} \in (-\pi/2, 0)$, then the series move in phase with $x(t)$ leading $y(t)$. Also, the phase difference indicates causality between $x(t)$ and $y(t)$ in both the time and frequency domains. Overall, wavelet analysis enables a deeper understanding than the conventional causality test, which assumes that a single causal link holds for the whole sample period as well as at each frequency [36,44]. For instance, in wavelet analysis, if $x(t)$ leads $y(t)$, then a causal relationship runs from $x(t)$ to $y(t)$ at a particular time and frequency [26].

3. Data

The U.S. economy experienced several episodes of high and low growth volatility, such as low volatility of output from the mid-1980s up to 2008 (called the Great Moderation), and increased growth volatility characterizing the late 1960s and

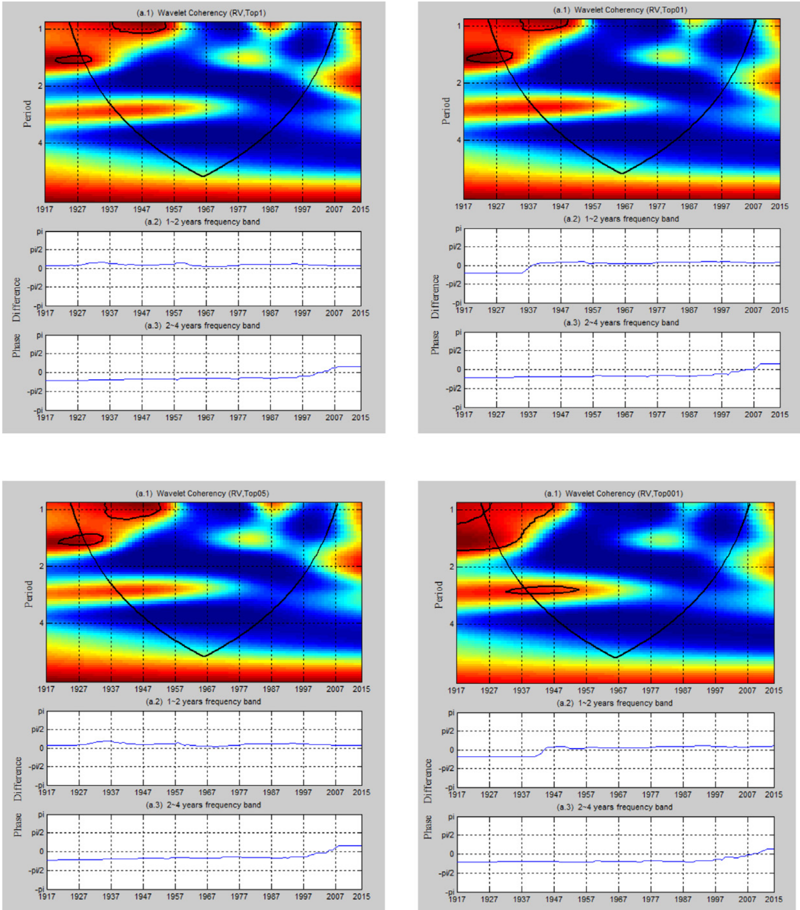

Fig. 1. (*continued*).

1970s (called the Great Inflation) and from 1929 to the start of World War II (Great Depression). In addition, movements in inequality conform to certain periods of time, including 1945 to 1979 (called the Great Compression) and 1980 to the present (called the Great Divergence). Our analysis provides clarification on the causality between income and wealth inequality and growth volatility, at different frequencies and at different moments in time. We use data with an annual frequency covering 1917 to 2015 for volatility and income inequality and 1962 to 2014 for volatility and wealth inequality. Data for the quarterly real GNP over 1917Q1 to 2015Q2 come from Omay et al. [45][6] and from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis from 2015Q3 to 2015Q4. Using quarterly GNP data, we calculate the annual realized volatility by taking the sum of quarterly squared growth rates. In our analysis, we not only use output volatility but we also categorize it into positive/good and negative/bad volatilities. We first create dummy variable, 1 for positive quarterly growth rate of output and 0 otherwise, and multiply the growth rate with the dummy variable. We do the same as above for the cases of negative quarterly growth rates. Then we take sum of the squared positive or negative quarterly growth rates of output over a specific year to obtain a measure of good or bad realized volatility respectively. Income inequality measures — Atkinson Index, Gini Coefficient, the Relative Mean Deviation, Theil's entropy Index, Top 10%, Top 5%, Top 1%, Top 0.5%,

[6] The authors explain how they compute the unique dataset, which is the longest possible data on U.S. output available at a quarterly frequency (i.e., the most relevant frequency at which to measure output globally). First, the observations covering the period 1875:Q1-1946:Q4 used by Omay et al. [45] (and in our case 1917:Q1-1946:Q4) come from National Bureau of Economic Research (NBER), available for download at: http://www.nber.org/data/abc/, with the actual sources being the tables of quarterly data corresponding to Appendix B of Gordon (1986). As Omay et al. [45] point out, this is the only existing source for the pre-1947 quarterly data on U.S. GNP and the GNP deflator with National Income and Product Account (NIPA) quarterly data series non-existent before 1947. Second, Omay et al. [45] use data from 1947:Q1-2015:Q2 from the FRED database. Note that the dataset compiled by Gordon (1986) runs through 1983:Q4 with 1972 as the base year of the GNP deflator. Given that nominal GNP and the GNP deflator data based on the NIPA are available from 1947:Q1, Omay et al. [45] decided to use, for those variables, the FRED database, rather than the Gordon (1986) one, which, in any case, only runs through 1983:Q4. Omay et al. [45] update the base year of the GNP deflator for the period 1875:Q1-1946:Q4 from 1972 to 2009 to correspond to the base year of the GNP deflator based on the NIPA. Thus, the real GNP is ultimately in constant 2009 prices.

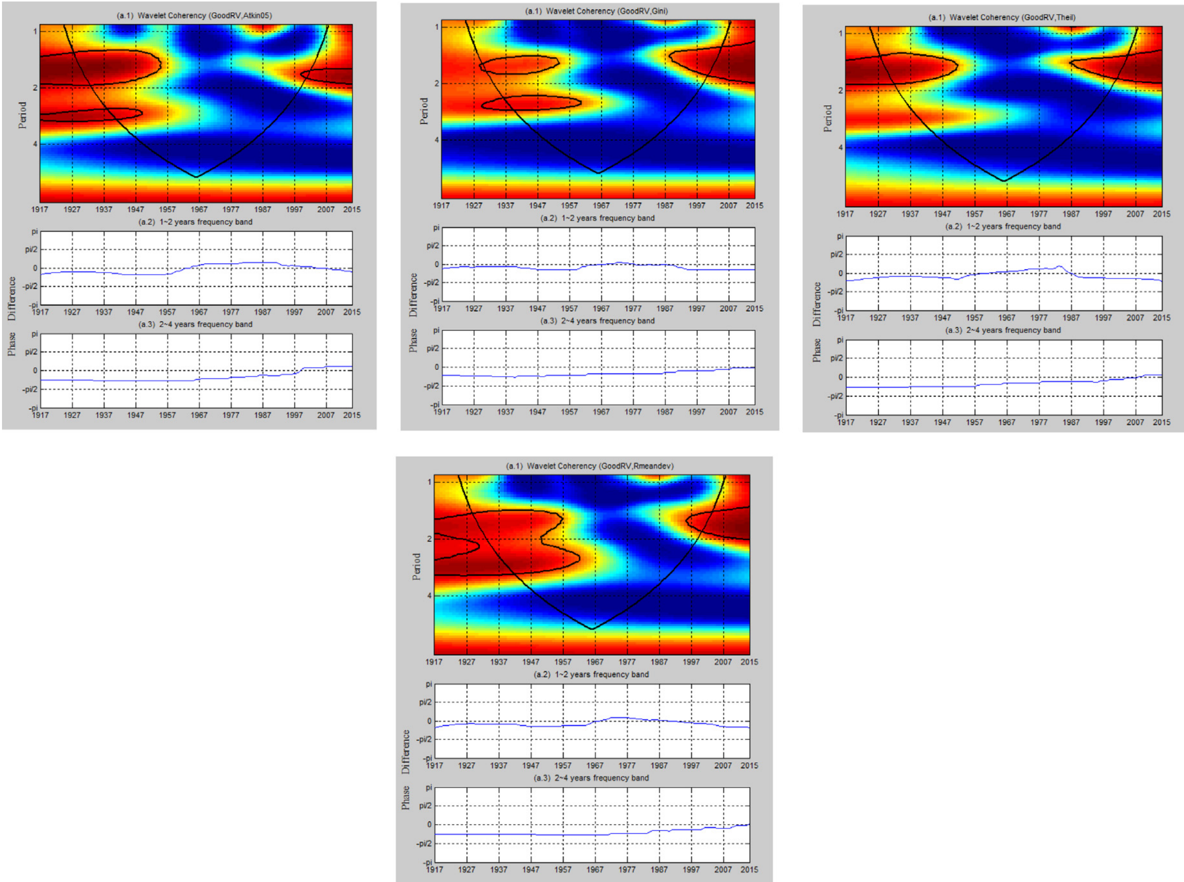

Fig. 2. Causal relationship between Positive Output Volatility and Income Inequality measures. **Note:** Wavelet Coherency between the positive output volatility and income inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period over the period 1917–2015.

Top 0.1%, and Top 0.01%[7] - come from the online data segment of Professor Mark W. Frank's website.[8] Wealth inequality measures — Top 10% net personal wealth (p90p100), Middle 40% (p50p90), Bottom 50% (p0p50), and Top 1% (p99p100) - come from World wealth and income database (WID) with data range from 1962 to 2014.[9] We employ the frequency cycles in the analysis. The first cycle (1-2-years cycle) associates with the short-run, or with high-frequency bands. The second cycle (2-4-years cycle) associates with the long, or with low frequency bands.[10]

4. Preliminary analysis

To examine the short- and long-run stability of the coefficients of the VAR model formed by inequality measures — Gini coefficient the Top 10% of net personal wealth as well as the Top 10% and Top 1% income shares,[11] – and volatility measures – aggregate as well as positive and negative volatility, we apply the Lc tests of Nyblom [30] and Hansen [47], which test the null hypothesis of constant parameters against the alternative hypothesis of parameters that follow a random-walk process [48]. We also apply the tests for stability of the short-run parameters, using the three different test statistics: Sup-F, Ave-F, and

7 Top income shares serve as useful proxies for inequality across the income distribution [46].

8 See http://www.shsu.edu/eco_mwf/inequality.html. Professor Frank constructed the dataset based on the Internal Revenue Service (IRS) information, which has a limitation of omission of some individual earning less than a threshold level of gross income. For this reason, we focus more on top income shares as primary indicators of inequality measures.

9 The data is available for download from: http://wid.world/.

10 We focus on two frequency bands: 1–2 and 2-4 years, as volatility and inequality the most coherent regions are between the 1–4 years band.

11 Out of fourteen income and wealth inequality measures, we use four inequality measures for the preliminary analysis.

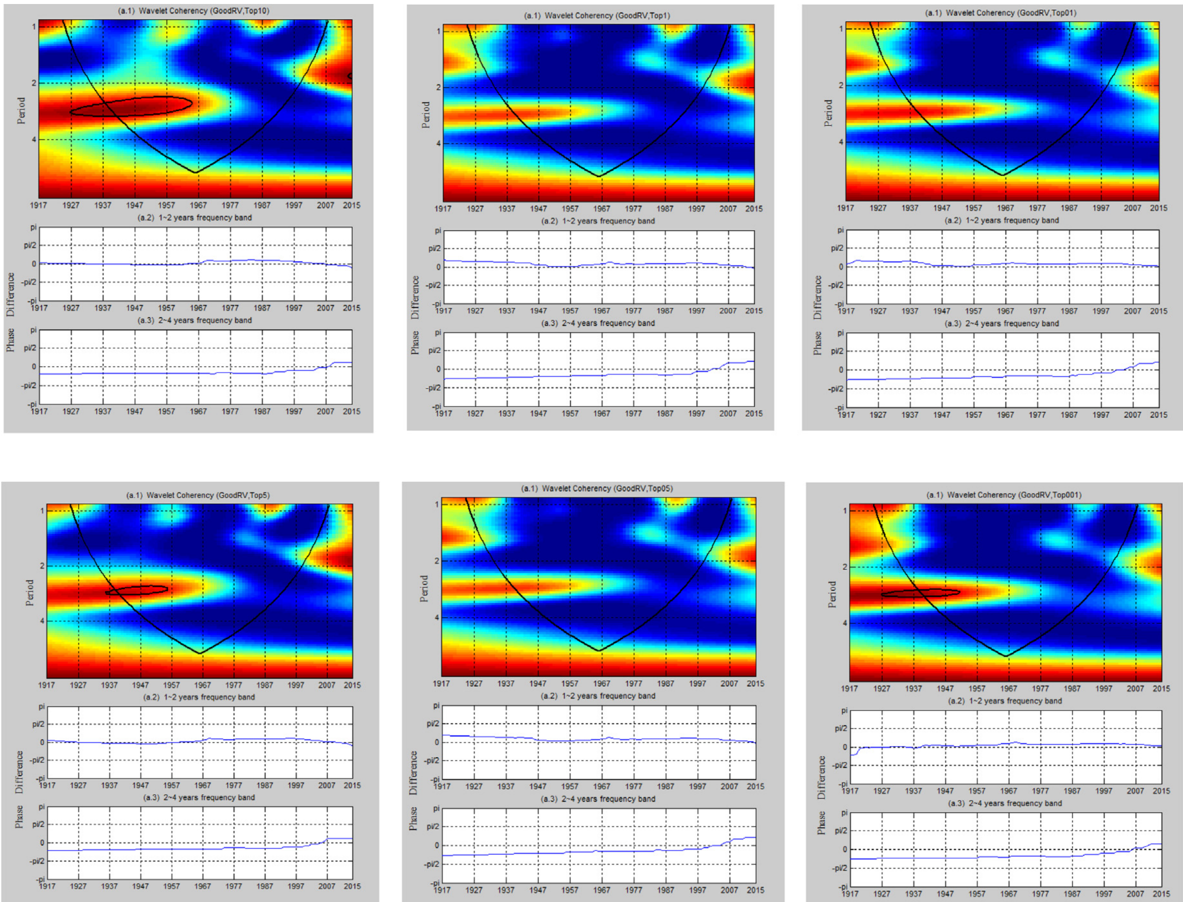

Fig. 2. (*continued*).

Exp-F. According to Andrews [28] and Andrews and Ploberger [29], the F-statistics test[12] the null hypothesis of no structural break against the alternative hypothesis of a single shift of unknown change point.

Tables 1 and 2, report the results of the parameter stability tests for the volatility measures and the inequality measures. Andrews and Ploberger [29] suggest that the use of the Sup-F, Mean-F, and Exp-F tests, which test the same null hypothesis, but differ in the alternative hypotheses, depends on the purpose of the test. The Sup-F statistic tests parameter constancy against a one-time sharp shift in parameters, so that the alternative hypothesis for the Sup-F test is an immediate shift in the regime. If the system shifts gradually, however, then the Mean-F and Exp-F statistics, which assume that parameters follow a martingale process, are suitable. Both statistics test the global constancy of the parameters, implying that the Mean-F and Exp-F tests appropriately investigate whether the underlying relationships among the variables stays stable over time. Table 1 shows that the Sup-F, Mean-F, and Exp-F tests reject the null hypothesis of parameter constancy, implying parameter non-constancy in the volatility equations, whereas the results report significant evidence of parameter non-constancy in the inequality equations, but not in the null of overall stability of the VAR (2) model.

Investigating the causal relationship between the variables, using short-run parameters of the differenced or cointegrated VAR, can lead to meaningless results with biased inference and inaccurate forecasts. Moreover, Granger causality tests will show sensitivity to changes in the sample period. Overall, the parameter stability tests show that the cointegrated VAR model possesses unstable short- and long-run parameters, suggesting the existence of structural changes.

To check for the robustness of long-run stability of the parameters, we also apply Lc tests to the FM-OLS estimates, where Table 2 reports the results. The FM-OLS estimator for negative volatility and Top 10% net personal wealth and for positive volatility and the Gini, the Nyblom–Hansen Lc test cannot reject the null hypothesis of long-term parameter stability at the 10- and 5-percent levels, respectively. For the rest FM-OLS estimators, the Nyblom–Hansen Lc test rejects the null hypothesis

12 Contrary to the Lc test, the F-tests require trimming from the ends of the sample. The *p* values and critical values for all stability tests come from parametric bootstrapping, which avoids the use of asymptotic distribution.

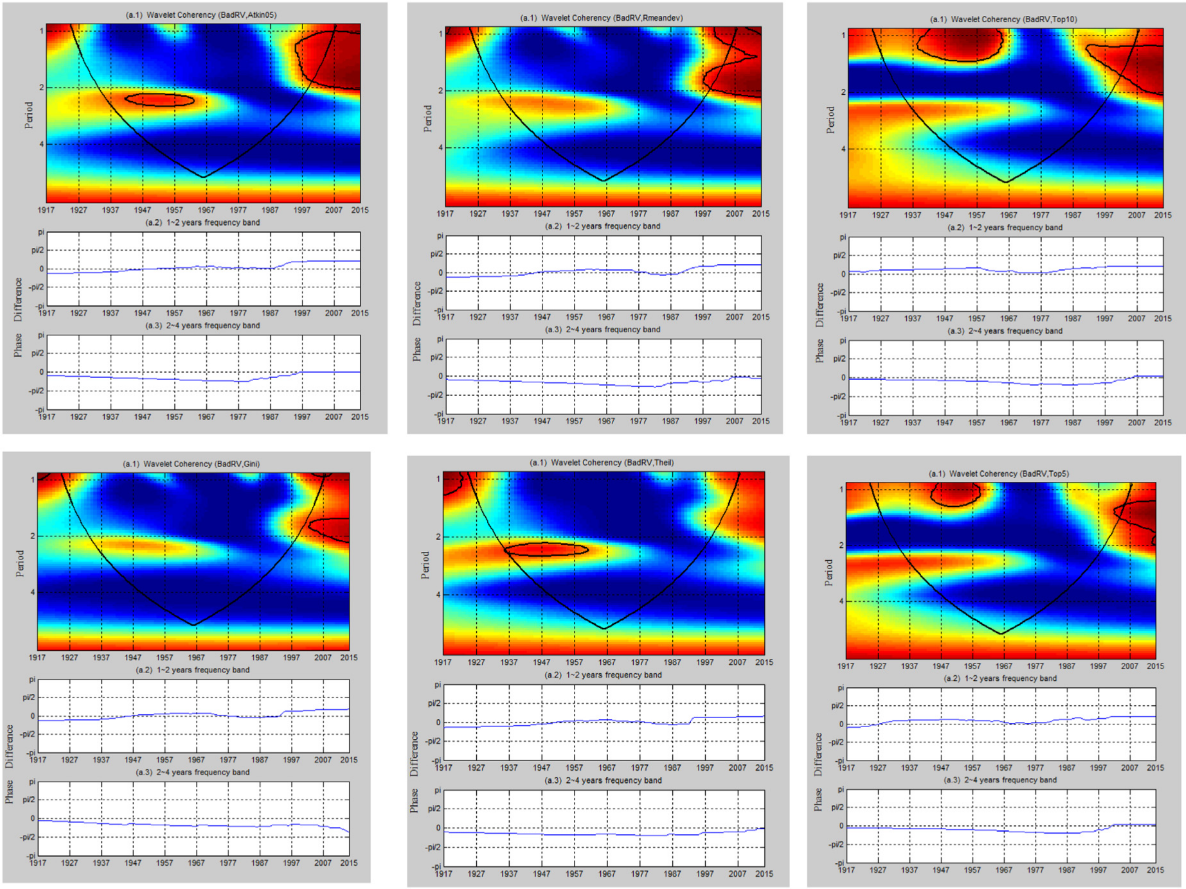

Fig. 3. Causal relationship between Negative Output Volatility and Income Inequality measures. **Note:** Wavelet Coherency between the negative output volatility and income inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period over the period 1917–2015.

at the 1-percent level. Thus, we observe both short- and long-run instability, motivating wavelet coherency analysis. (See Tables 3–16)

5. Main analysis

We simultaneously look at the correlation and the causal relationship between (i) income and wealth inequality, and growth volatility (ii) income and wealth inequality, and positive volatility, and (iii) income and wealth inequality, and negative volatility.

The results of wavelet coherency indicate correlation between two variables. The wavelet coherency between volatility[13] and the various income inequality measures show statistically significant high coherency across high- and low-frequencies in Fig. 1. Across the high- and low-frequency bands, at least two significant islands exist of high coherency between output volatility and the income inequality measures. With the wealth inequality measures in Fig. 4, we observe the consistent strong positive correlation between growth volatility and inequality measures at the 2–4 years frequency. Only weak correlation appears with wealth inequality measures across the 1–2 year frequency.

13 As we cannot observe our major independent variable, growth volatility, we must derive an operational measure of volatility to consider the effect of volatility on inequality. In our paper, we utilize realized volatility, as it is a model-free estimate, which was introduced in the literature by Taylor and Xu (1997) and Andersen and Bollerslev (1998). To generate alternative proxies for growth volatility based on parametric approaches, we estimate a GARCH (1,1) model and collect the fitted conditional variances as parametric proxies of growth volatility. Please see the appendix A.1 and B.1 to B.4 for the results of wavelet coherency and phase difference between growth volatility and inequality.

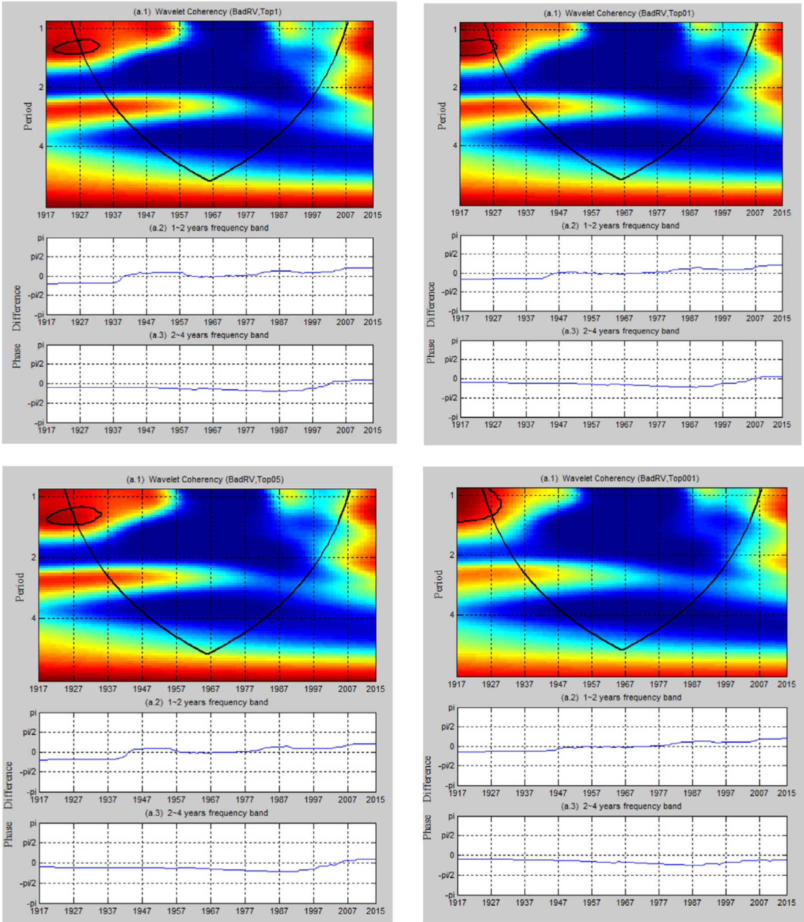

Fig. 3. (*continued*).

The coherency results of positive volatility and income inequality measures also show statistically significant high coherency islands over the short- and long-term in Fig. 2. Especially from 1917 to the 1960s, all income inequality measure indicate strong co-movement across low-frequency. Only weak correlation appears with top income shares across high-frequency band from 1935 to 1997 and with wealth inequality measures across low frequencies in Fig. 5. Compared to the aggregate output volatility, positive volatility shows less strong co-movement with top income shares across high-frequency.

The results of negative volatility show statistically significant high coherency across 1–2 year frequency band for all inequality measures in Fig. 3. Across the 2–4 years frequency band, we observe a significant island from 1935–1961 and 1942–1963, which relates to World War II. Sign of strong correlation appears with the Top 1%, Top 0.5%, Top 0.1% and Top 0.01% of income inequality and with wealth inequality measures across high-frequency bands in Fig. 6. Fig. 3 also shows stronger correlations between the negative volatility and inequality over the short-term than positive volatility. That is, negative volatility exerts a bigger effect on inequality than positive volatility over the short-term.

Our empirical evidence shows that volatility and inequality relate positively, which a number of studies show. This positive relationship appears in Hausmann and Gavin [8], Breen and García-Peñalosa [17], Laursen and Mahajan [14], and Gordon [49].

The phase differences of Figs. 1 to 6 indicate the causality between two series (see Fig. 7 for compiled results). Across the 2–4 year frequency band in Fig. 7, for all three volatility measures, volatility leads the income inequality measures. The change of direction of causality from volatility leads to inequality leads in the early 2000s probably indicates a structural break.

At low frequency, volatility leads the wealth inequality measures Top 10% (p90p100) and Middle 40% (p50p90) in 1962–2014, whereas Bottom 50% (p0p50) and Top 1% (p99p100) lead volatility. Negative volatility leads Top 10% and Top 1%, whereas Middle 40% and Bottom 50% lead negative volatility in 1962–2014. Positive volatility leads Top 10% and Top 1% through the early 2000s and the direction of causality changes after that. Positive volatility also leads Bottom 50% through the late 1980s and the direction of causality changes after that. Middle 40% leads positive volatility from 1962 through the

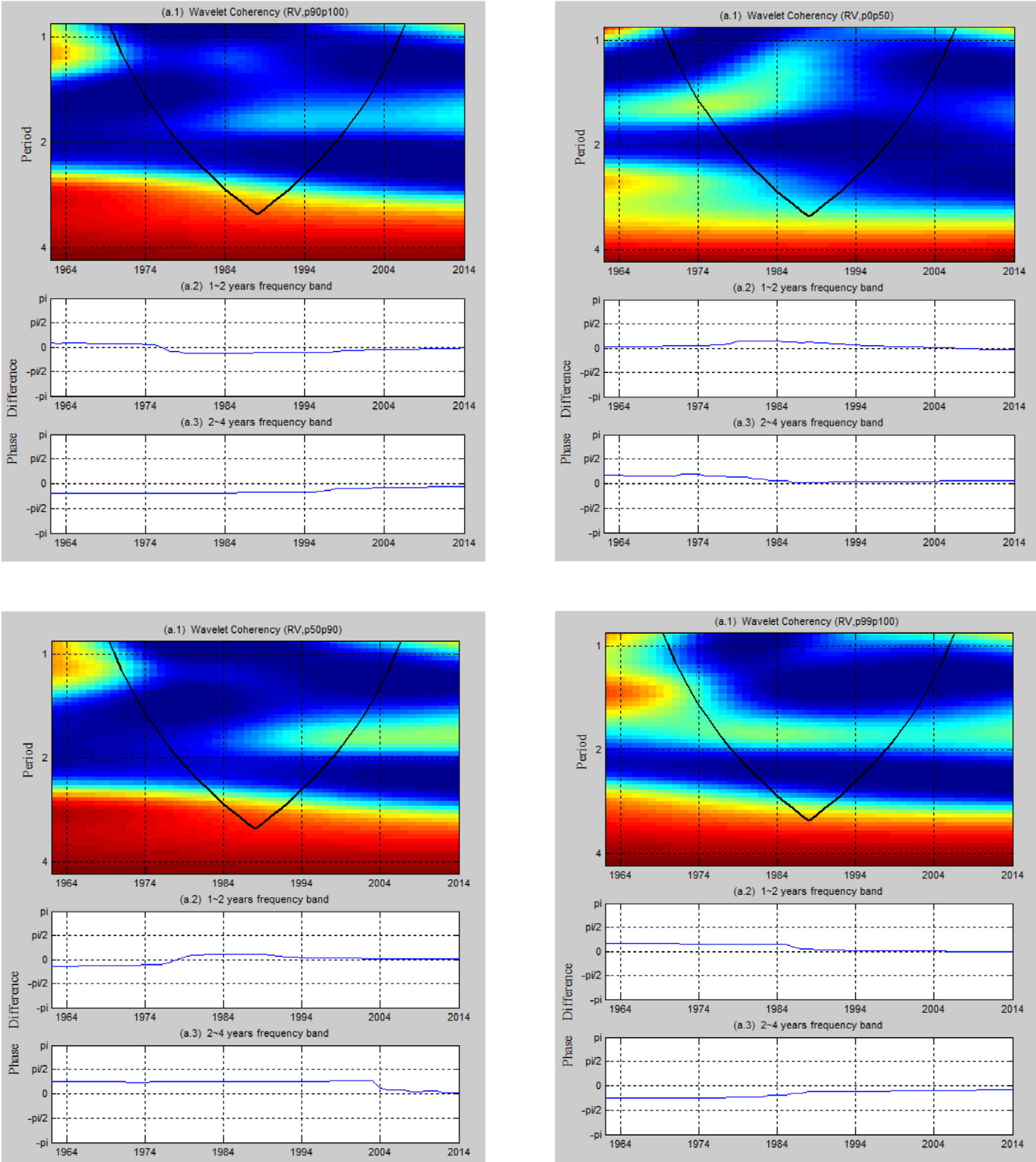

Fig. 4. Causal relationship between Aggregate Output Volatility and Wealth Inequality measures. **Note:** Wavelet Coherency between the aggregate output volatility and wealth inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period over the period 1962–2014.

late 1990s and causality changes after that. For Top 10% at low frequency, aggregate and negative volatility lead wealth inequality. Bottom 50% leads aggregate and negative volatility from 1962 to 2014.

Compared to long-term causality, more movement occurs in changes of direction of causality in the short-term. Volatility leads the Atkinson Index and the Relative Mean Deviation from 1917 to the late 1950s, while the Atkinson Index and the Relative Mean Deviation lead volatility after that. Volatility also leads the Gini coefficient and the Theil index from 1917 to the late 1950s and from the late 1980s to 2014, while the Gini coefficient and the Theil index lead volatility from 1961 to the late 1980s. The Top income shares, however, lead volatility, except in 1917–1921, when volatility leads Top 5%, in

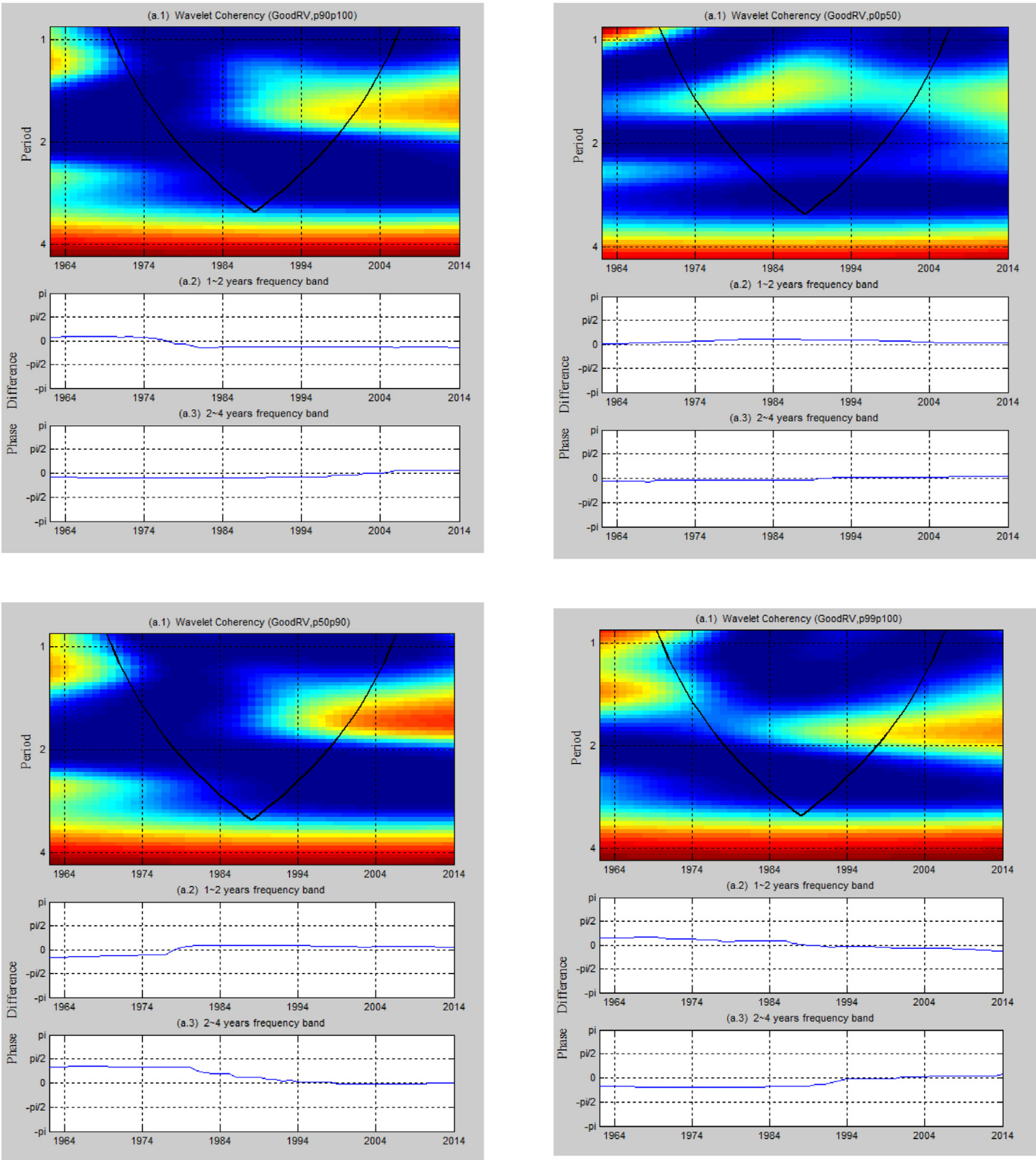

Fig. 5. Causal relationship between Positive Output Volatility and Wealth Inequality measures. **Note:** Wavelet Coherency between the positive output volatility and wealth inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period over the period 1962–2014.

1917–1938, when volatility leads Top 0.1%, and in 1917–1943, when volatility leads Top 0.01%. For high frequencies, the Top 0.1% leads positive volatility and Top 10% leads negative volatility from 1917 to 2015. The direction of causality of the wealth inequality measures Top 10% (p90p100) and Middle 40% (p50p90) changes in the mid and late 1970s. For Bottom 50% (p0p50) and Top 1% (p99p100), the direction of causality changes in the mid-2000s. The 1970s saw two oil price spikes,

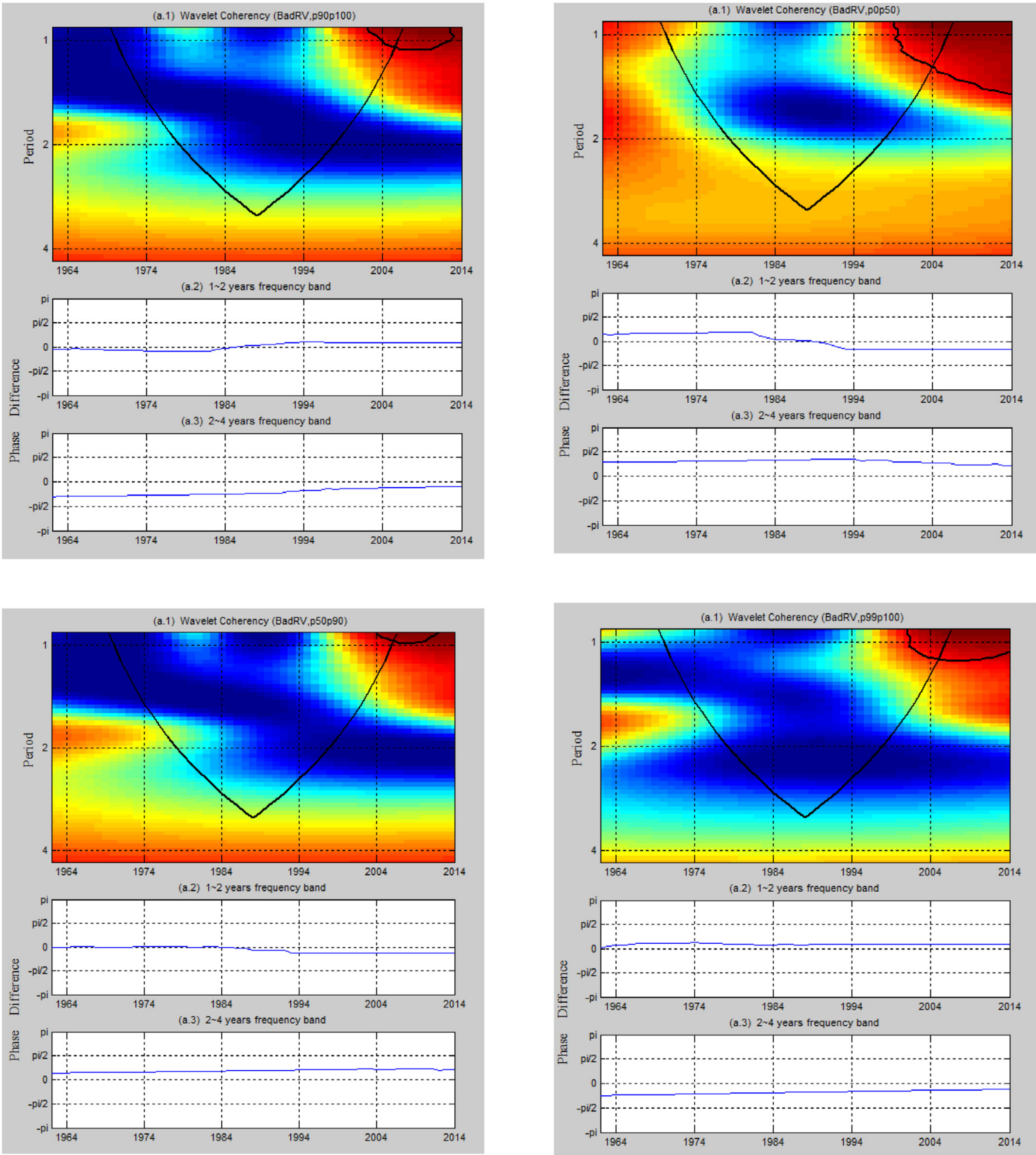

Fig. 6. Causal relationship between Negative Output Volatility and Wealth Inequality measures. **Note:** Wavelet Coherency between the negative output volatility and wealth inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period over the period 1962–2014.

as OPEC began affecting prices. Also, the Vietnam War covered the 1967–1972 period, where, in turn, productivity growth slowed.

Similar to the causality with aggregate growth volatility, the direction of causality of the wealth inequality measures Top 10% and Middle 40% changes in the mid and late 1970s for positive volatility. The Top 10% leads positive volatility from 1917 to 1976, while positive volatility leads Top 10% from 1977 to 2014. In contrast, Middle 40% leads positive volatility from 1979

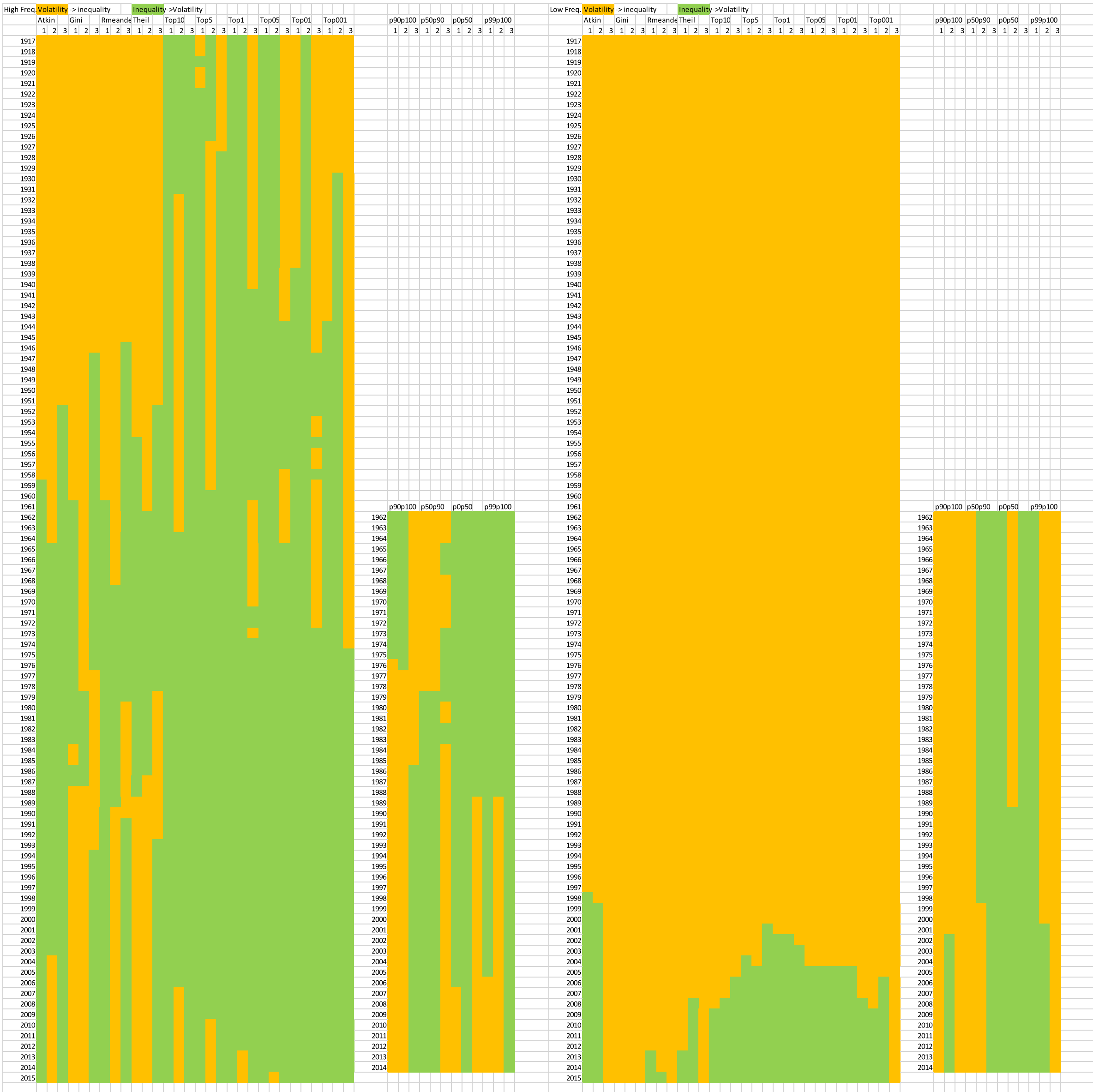

Fig. 7. Short and Long Run Causality. **Note:** First two figures from the left indicate the short run causality relationship between volatility and inequality. 1, 2 and 3 indicate aggregate volatility, positive volatility and negative volatility. Orange color indicates that the volatility leads and Green color indicates that inequality leads. Third and fourth figures from the left show the long run causality. *Y*-axis indicates the year. .

to 2014, while positive volatility leads Middle 40% from 1962 to 1978. Top 1% leads positive volatility from 1917 to 1988 and positive volatility takes lead from 1989, whereas Bottom 50% leads negative volatility in 1962–2014.

Top 1%, Top 0.5%, Top 0.1%, and Top 0.01% income shares mostly lead positive volatility in our data range. Top 10% and Top 5% show similar patterns and directions of causality. Positive volatility leads the Relative Mean Deviation, and the Theil index in 1917 through the 1960s and in the late 1980s through 2015, while the two measures of inequality lead positive volatility in the rest of period. Positive volatility leads the Gini coefficient from 1917 to 2015 except from 1979 to 1987. Also, positive volatility leads the Atkinson index from 1917 to 1964 and from 2004 to 2015.

With negative volatility at high frequencies, the results show that all the inequality measures lead negative volatility from 1994 to 2015, whereas negative volatility leads all the inequality measures except Top 10% and Top 5% from 1917 to 1940. In the 1940s, the direction of causality changes from negative volatility leads to inequality leads, which relates to wage compression during the 1940s. Negative volatility leads Top 0.01% in 1917–1974 and Top 0.01% leads negative volatility from

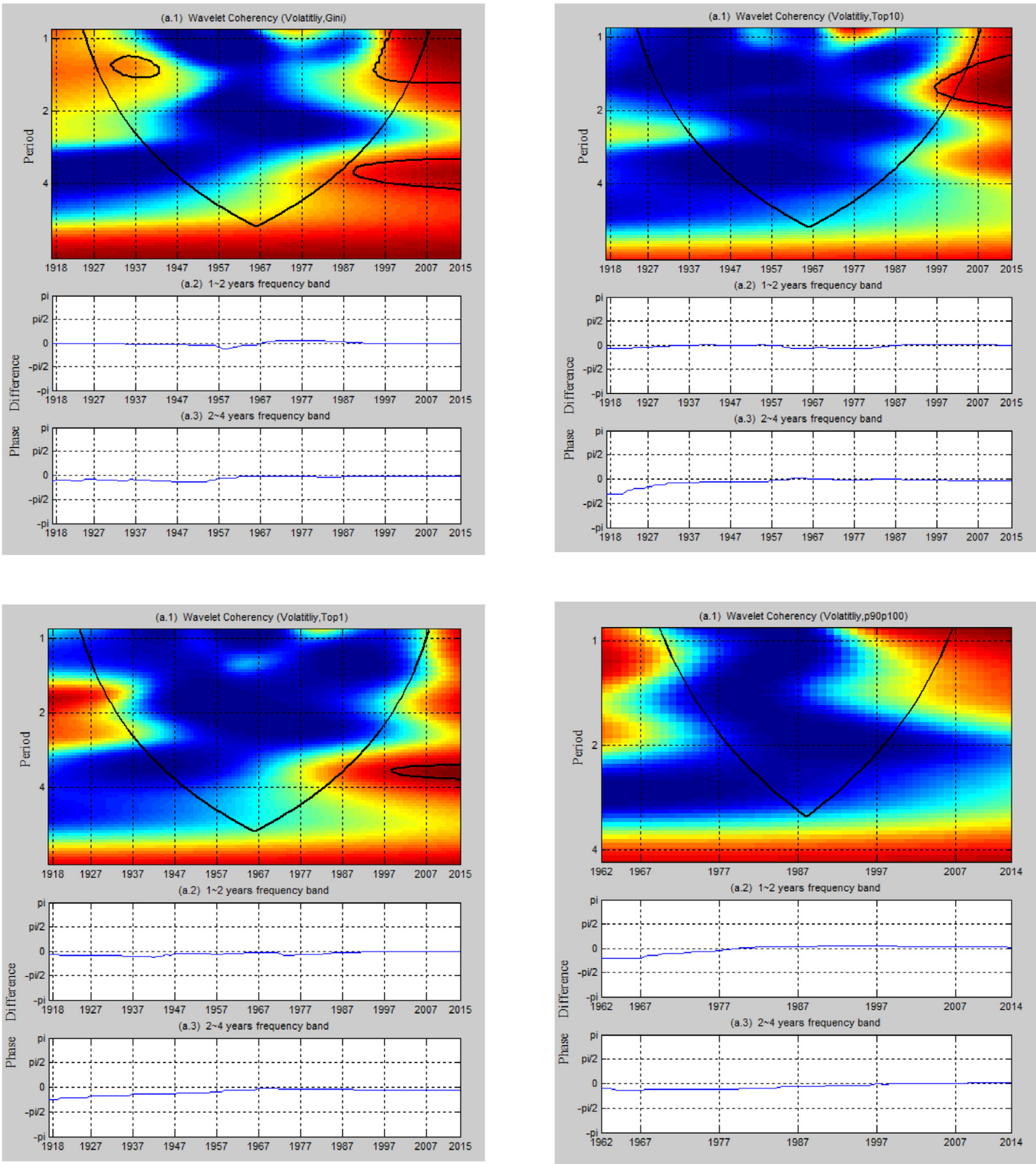

Fig. A.1. Causal relationship between Output volatility and Inequality measures. **Note:** GARCH (1,1) model is fitted to quarterly GNP to generate the variances. Wavelet Coherency between the output volatility and inequality measures. The black contour indicates a 5% significance level. The significance levels are based on 10,000 draws from Monte Carlo simulations estimated on an ARMA(1,1) null of no statistical significance. The line around figure is the cone-of-influence (COI) earmarking the areas affected by the edge effects or phase. The color code for power ranges from blue (low power) to red (high power). The *y*-axis refers to the frequencies (measured in years); the *x*-axis refers to the time period.

1975. For wealth inequality, Top 1% (p99p100) leads negative volatility from 1962 through 2014. The direction of causality of Top 10% and Bottom 50% changes mid and late 1980s.

We observe that the directions of causality vary and the changes of direction mostly coincide with the business cycle (NBER). This probably relates to business cycle movements that associate with large permanent effects on the long-run level of output ([50,51]).

We also decompose the time-series into high-, medium- and low-frequency using a Maximal Overlap Discrete Wavelet Transform (MODWT) and employ Granger causality to see in which frequencies the underlying driver lies in.[14] Table 17 reports the results of Granger causality tests in the different frequency domain. Top 10% income share and Top 10% net personal wealth (p90p100) are observed to Granger cause volatility in medium- and long term. Gini coefficient does not Granger cause volatilities in medium term. Negative volatility does not cause inequality in short term and over all volatility and positive volatility does not cause income inequality in long term. We find one stable causality holding for the whole sample period which Top 10% income share causes positive negative and overall volatility, however, in general, the causality findings exhibit substantial time- and frequency-dependence.

The phase difference results in Fig. 7 show that volatility, including positive and negative volatilities, mostly leads income inequality until the 2000s across low frequencies and changes direction from volatility leads to income inequality leads from the 2000s onward. In contrast to the short term, long-term causality patterns and directions are robust to different measures of income inequality. Across high frequencies, the income share inequality measures lead volatilities, but directions of causality vary across frequencies and evolve with time. If we restrict our analysis to classical time series, we cannot find any information about differences across frequencies.

6. Conclusion

Policy makers attempt to reduce inequality through economic growth, fiscal policy, monetary policy, aid programs, and so on. The relationship between inequality and the various policy instruments receives much discussion and analysis in the existing literature. As numerous variables affect each other simultaneously or at different points of time, rendering net causality and correlation results difficult to document. This paper investigates the causal relationships between U.S. income and wealth inequality measures, and output volatility. We use wavelet analysis, which allows the causal relationship between the two series to vary over time and frequency. Wavelet analysis is robust to lag length [42], stationarity [33], model specification [52], and cointegration as wavelet analysis allows time- and frequency-varying approach. Furthermore, it permits to measure local co-movement between two time series in the time–frequency domain and discover the lead–lag relationship between two time series. We use annual time-series data from 1917 to 2015 for volatility and income inequality and 1962 to 2014 for volatility and wealth inequality, which cover numerous economic expansions and contraction.

Our results show that the periods and directions of short-term causality vary over time. Volatility mainly leads income inequality measures over the long-run through the early-2000s. At high frequencies, causality changes direction — from volatility leading to inequality leading. Our results also show that higher positive and negative volatility leads to increases in inequality. This implies that economic growth does not trickle down to the bottom income group as they experience more fluctuations in output growth. In addition, we find that volatility not only matters for inequality but also inequality matters for volatility, especially in more recent years.

A positive correlation always exists between volatility and inequality. This finding proves consistent with the view that larger growth volatility strongly associates with higher income/wealth inequality. Hausmann and Gavin [8], Breen and García-Peñalosa [17], Laursen and Mahajan [14], and Calderón and Yeyati [19] as well as the theoretical conjectures of Caroli and García-Peñalosa [9] and Checchi and García-Peñalosa [10] support this finding. In addition,this correlation also proves consistent with higher inequality contributing to higher growth volatility, as in Alesina and Perotti [21] and Aghion et al. [22,23].

As our long-term results show, changes in the direction of causality from volatility leads to income inequality leads coincide with the end of the Great moderation era. This could imply that a threshold level of inequality exists below which macroeconomic variables influence inequality and above which they do not. This can be part of future research. Policy makers can use direct policy, such as enlarging the tax bracket for low-income households, raising taxes on high-income households, or increasing state aid programs, to reduce inequality, which can also moderate volatility. Our findings also imply that stabilization policies can affect income inequality. Thus, stabilization policy can provide an important instrument to reduce income inequality. This finding corresponds with studies[15] that find a significant effect from aid programs and/or remittances on inequality via stabilizing effects on volatility.

To fully understand the effects of volatility on inequality, we need a detailed examination of all possible channels, as different mechanisms may require different policy implications. We leave this issue for future study.

Appendix A

See Fig. A.1.

Appendix B

See Tables B.1–B.4.

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14 Testing causality in frequency domain collapses the time dimension into a single point in time and information is lost on the time variation in causality.

15 See [53–55] for the related study .

Table B.1
Wavelet phase difference (Volatility, logarithm of Gini coefficient).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1967	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini coefficient
	1968–1991	$(0, \frac{\pi}{2})$, In-phase	+	Gini → Volatility
	1992–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Gini

Table B.2
Wavelet phase difference (Volatility, logarithm of Top 10%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–1939	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%
	1940–1943	$(0, \frac{\pi}{2})$, In-phase	+	Top 10% → Volatility
	1944–1953	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%
	1954–1955	$(0, \frac{\pi}{2})$, In-phase	+	Top 10% → Volatility
	1956–1988	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%
	1989–2011	$(0, \frac{\pi}{2})$, In-phase	+	Top 10% → Volatility
	2012–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%
Low frequency	1917–1961	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%
	1962–1969	$(0, \frac{\pi}{2})$, In-phase	+	Top 10% → Volatility
	1970–1981	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%
	1982–1986	$(0, \frac{\pi}{2})$, In-phase	+	Top 10% → Volatility
	1987–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 10%

Table B.3
Wavelet phase difference (Volatility, logarithm of Top 1%).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%
Low frequency	1917–2015	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → Top 1%

Table B.4
Wavelet phase difference (Volatility, Net personal wealth held by p90p100).

Volatility				
	Period	Phase	Sign of co-movement	Causality
High frequency	1962–1979	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
	1980–2014	$(0, \frac{\pi}{2})$, In-phase	+	p90p100 → Volatility
Low frequency	1962–2003	$(\frac{-\pi}{2}, 0)$, In-phase	+	Volatility → p90p100
	2004–2014	$(0, \frac{\pi}{2})$, In-phase	+	p90p100 → Volatility

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