Does real U.K. GDP have a unit root? Evidence from a multi-century perspective

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Abstract

We employ linear and nonlinear unit-root tests to examine the stationarity of five multi-century historical U.K. series of real output compiled by the Bank of England. Three series span 1270 to 2016 and two series span 1700 to 2016. These datasets represent the longest span of historical real output data available and, thus, provide the environment for which unit-root tests are most powerful. A key feature of our test is its simultaneous allowance for two types of nonlinearity: time-dependent (structural breaks) nonlinearity and state-dependent (asymmetric adjustment) nonlinearity. The key finding of the test, contrary to what other more popular nonlinear unit-root tests suggest, provides strong evidence that the main structure of the five series is a stationary process characterized by an asymmetric nonlinear adjustment and a permanent break affecting both the intercept and the trend. A major policy implication of this finding is fiscal and/or monetary stabilization policies have only temporary effects on the output levels of the United Kingdom.

Keywords: Unit root; time-dependence; nonlinearity; state-dependence; Fourier function

I. Introduction

In their seminal paper, Nelson and Plosser (1982) claimed that U.S. real GDP contains a unit root, that is, stochastic trend dominates its movements. This finding has important macroeconomic implications, as it proves inconsistent with the traditional view of the business cycle. Most importantly, it suggests that real factors such as technology shocks play an important role in economic fluctuations, supporting the hypotheses of the real business cycle theory (Christopoulos 2006).

Prior to Nelson and Plosser (1982), the prevailing view argued that real GDP exhibited a stationary process around a deterministic trend (Barro, 1976; Blanchard 1981; Kydland and Prescott 1980). Distinct practical differences exist between trend-stationary and unit-root processes. First, the deterministic trend provides the optimal forecast for a trend-stationary process, while the current value provides the optimal forecast for a unit-root process. Second, a finite zone bounds the MSE of a trend-stationary forecast whereas the MSE of a unit-root forecast grows linearly and, thus, becomes less precise the longer the forecast horizon. Third, the effect of a shock to a trend-stationary process will eventually disappear, or put differently, a trend-stationary process exhibits only a limited memory of its past behaviour, whereas the effect of a shock on a unit-root process does not decay over time, implying a permanent memory.

Almost four decades since Nelson and Plosser (1982), the question of deterministic versus stochastic trend in real GDP remains unresolved. Following Nelson and Plosser (1982), a large body of empirical work failed to reject the hypothesis of unit root for real GNP, leading Stock and Watson (1999) to conclude that the unit-root econometric literature supports the contention of Nelson and Plosser (1982). Empirical studies by Wasserfallen (1986), Perron and Phillips (1987), Campbell and Mankiw (1987), Evans (1989), Papell and Prodan (2004), Ben-David and Papell (1995), Cheung and Chinn (1996), and Murray and Nelson (2000), and so on, reach the conclusion that U.S. real GDP is nonstationary. That is, no evidence exists that the economy self-corrects, in the sense that output never returns to its previous trend. Walton (1988) reaches a similar conclusion for the United Kingdom. Moreover, Kormendi and Meguire (1990), Cogley (1990), Fleissig and Strauss (1999), and Rapach (2002) provide international evidence supporting the null of a unit root in real GDP for OECD economies.

These results, however, are far from conclusive. Other papers since Stock and Watson (1999), using several modifications and extensions, reject the unit-root hypothesis. Ben-David, Lumsdaine, and Papell (2003), Papell and Prodan (2004), Vougas (2007), Beechey and Österholm (2008), Cook (2008), and Shelley and Wallace (2011) find empirical evidence to reject the unit-root hypothesis in real GDP.

The power of the tests lies at the heart of the issue. The power of the standard unit-root tests depends on the specification of the alternative hypothesis. Structural breaks and nonlinearities cause undersized standard unit-root tests, resulting in a reduction of statistical power (Habimana, Månsson, and Sjölander 2018). Perron (1989) first notes that stationary processes with structural breaks are too often mistakenly interpreted as unit-root processes. Perron (1989) suggests that standard unit-root tests such as the standard ADF test probably cannot distinguish the behaviour of a unit-root process from that of a stationary process with structural breaks. Kapetanios, Shin, and Snell (2003), in turn, maintain that the standard unit-root tests suffer from a power problem when applied to data characterized by a nonlinear DGP.

A growing literature, starting with Enders and Granger (1998), relaxes the assumption of linearity implicit in the standard unit-root tests and develops tests that can distinguish linear nonstationary processes from nonlinear stationary processes. These tests examine the unit-root hypothesis against the alternative of a nonlinear stationary process. In this context, the literature analyzes two sources of nonlinearity: state-dependent (regime-wise) nonlinearity (i.e., nonlinearity in the speed of mean reversion) and time-dependent (structural breaks) nonlinearity (i.e., nonlinearity in the deterministic components). Kapetanios, Shin, and Snell (2003) and Sollis (2009) implement state-dependent nonlinear tests. The two tests differ on the dynamics of the speed of adjustment towards equilibrium. Kapetanios, Shin, and Snell (2003) employ the exponential smooth transition autoregressive (ESTAR) model while Sollis (2009) employs the asymmetric exponential smooth transition autoregressive (AESTAR) model. Leybourne, Newbold, and Vougas (1998), Omay (2015), and Çorakcı, Emirmahmutoğlu, and Omay (2017) develop nonlinear structural-break unit-root tests. Leybourne, Newbold, and Vougas (1998) consider a single permanent break; Omay (2015) considers multiple smooth breaks; and Çorakcı, Emirmahmutoğlu, and Omay (2017) consider a single temporary break. Christopoulos and Leon-Ledesma (2010), Omay and Yıldırım (2014), and Omay, Emirmahmutoglu, and Hasanov (2018) implement tests that incorporate both structural break(s) and state-dependent nonlinearity simultaneously. The Omay, Emirmahmutoglu, and Hasanov (2018) test provides the most comprehensive nonlinear unit-root test as it combines the time-dependent nonlinearity of the unit-root test of Leybourne, Newbold, and Vougas (1998) with the state-dependent nonlinearity of the unit-root test of Sollis (2009). Thus, while the Christopoulos and Leon-Ledesma (2010) and Omay and Yıldırım (2014) tests impose a symmetric (ESTAR) nonlinear adjustment, the Omay, Emirmahmutoglu, and Hasanov (2018) test allows for asymmetric (AESTAR) nonlinear adjustment. The Omay, Emirmahmutoglu, and Hasanov (2018) test considers two alternative specifications of the trend function, the logistic transition function, which models a single break, and the Fourier series (Becker, Enders, and Lee 2004, 2006; Enders and Lee 2012; Rodrigues and Taylor 2012), which models multiple breaks. In contrast, the Omay and Yıldırım (2014) test can deal only with a single break, and the Christopoulos and Leon-Ledesma (2010) test can deal only with multiple breaks.

We employ the Omay, Emirmahmutoglu, and Hasanov (2018) test, as well as a battery of other linear and nonlinear tests, to investigate the stationarity properties of five multi-century annual U.K. series of real output compiled by the Bank of England (Thomas and Dimsdale 2017). Three series span 1270 to 2016 and two series span 1700 to 2016. These datasets represent the largest span of historical real output data available and, thus, provide the environment for which unit-root tests are most powerful.

The main results of the Omay, Emirmahmutoglu, and Hasanov (2018) tests are strong and powerful. The tests reject the unit-root hypothesis in each of the five historical U.K. real output series and provides strong evidence that the main structure of the data is stationary with a trend break and an asymmetric nonlinear adjustment. Although we view the findings of the Omay, Emirmahmutoglu, and Hasanov (2018) tests as our main results, for completeness, we also consider several other linear and nonlinear unit-root tests that are popular in the econometric literature. Specifically, we execute a total of ten unit-root tests: two standard linear unit-root tests (Dickey and Fuller 1979; Ng and Perron 2001), two nonlinear state-dependent unit-root tests (Kapetanios, Shin, and Snell 2003; Sollis 2009), three nonlinear time-dependent tests (Leybourne, Newbold, and Vougas 1998; Çorakcı, Emirmahmutoğlu, and Omay 2017; Omay 2015), and three 'hybrid' tests that combine state-dependence and time-dependence tests (Christopoulos and Leon-Ledesma 2010; Omay and Yıldırım 2014; Omay, Emirmahmutoglu, and Hasanov 2018). With the exception of Omay (2015), which employs a fractional Fourier approach, the other nonlinear tests nest in the Omay, Emirmahmutoglu, and Hasanov (2018) tests.[1]

The rest of the paper is organized as follows. Section 2 provides a brief outline of the two versions of Omay, Emirmahmutoglu, and Hasanov (2018) unit-root test. Section 3 presents the findings of the Omay, Emirmahmutoglu, and Hasanov (2018) unit-root tests and the results of the nine other tests. Section 4 conducts a series of linearity tests developed by Luukkonen, Saikkonen, and Terasvirta (1988) and Becker, Enders, and Lee (2004) designed to identify which nonlinear unit-root test is more consistent with the data. Section 5 comments and concludes.

II. The Omay, Emirmahmutoglu, and Hasanov (2018) unit-root tests

The Omay, Emirmahmutoglu, and Hasanov (2018) tests are the newest and most comprehensive nonlinear unit-root tests. We offer the second application of the tests. Omay, Emirmahmutoglu, and Hasanov (2018) first applied the procedure to test the purchasing power parity (PPP) hypothesis using both the trade-weighted real effective exchange rate (REER) (Bahmani-Oskooee, Kutan, and Zhou 2007) and bilateral real exchange rates. The key findings of the tests suggest that the PPP holds in the majority of the countries in the sample, which details the importance of employing highly complex models in the analysis and tests of aggregate data. Omay, Emirmahmutoglu, and Hasanov (2018) utilize the following equation for modelling the deterministic and stochastic components of an observed time series y_t:

$$y_t = \phi(t) + u_t, \tag{1}$$

where $\phi(t)$ is the nonlinear deterministic trend function and u_t is the stochastic deviation from the trend. Omay, Emirmahmutoglu, and Hasanov (2018) consider two specifications of $\phi(t)$. The first specification combines the time-dependent (time-varying) nonlinearity of Leybourne, Newbold, and Vougas (1998) and the state-dependent (regime-wise) nonlinearity of the AESTAR model of Sollis (2009) and models one permanent structural break. The break is modelled with the logistic transition function following Leybourne, Newbold, and Vougas (1998) under three alternative models:

$$\text{Model A} : y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t, \tag{2a}$$

$$\text{Model B} : y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t, \tag{2b}$$

$$\text{Model C} : y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 S_t(\gamma, \tau)t + \varepsilon_t, \tag{2c}$$

where $t = 1,2, \ldots, T$, ε_t is a zero mean process, and $S_t(\gamma, \tau)$ is the logistic transition function with a sample size of T. That is,

$$S_t(\gamma, \tau) = [1 + exp\{-\gamma(t - \tau T)\}]^{-1}. \tag{3}$$

In this framework, a smooth transition process between different regimes governs structural change as in Leybourne, Newbold, and Vougas

[1]Other researchers independently propose slightly different versions of the Omay et al. (2014, 2018) tests. Two, in particular, warrant mention. First, Chen and Xie (2015) develop the Leybourne et al. (1998) version of the AESTAR test and examine current account sustainability. Second, Ranjbar et al. (2018) develop the Fourier version of the AESTAR test and reexamine real interest rate parity for 12 OECD countries. An important difference between our paper and these two other papers relates to the critical-value problem. That is, Omay, et al. (2018) obtain convergent critical values (see Tables 1 and 2, Omay et al. 2018), whereas Chen and Xie (2015) and Ranjbar et al. (2018) obtain divergent critical values (see Table 1, Chen and Xie, 2015, and Table 1, Ranjbar et al. 2018). Unlike Chen and Xie (2015) and Ranjbar et al. (2018), we apply the Simplex method to compute critical values for the Leybourne et al. (1998) type of detrending. Omay and Emirmahmutoğlu (2017) show that the Genetic and Simplex methods are the appropriate optimizing algorithms for the Leybourne et al. (1998) type of unit-root testing.

(1998), rather than an instantaneous structural break as in Lumsdaine and Papell (1997) and Lee and Strazicich (2003). This reflects the now prevailing view in the current literature that the cyclical behaviour of real GDP is best represented by a nonlinear model rather than a linear model with structural breaks (Beechey and Österholm 2008). That is, real GDP movements between peaks and troughs occur gradually and not instantaneously. The model in Equations. (1)-(3) captures a regime-switching model with two regimes associated with the extreme values of the transition function $S_t(\gamma,\tau) = 0$ and $S_t(\gamma,\tau) = 1$, where the transition from one regime to the other occurs gradually. $S_t(\gamma,\tau)$ is a continuous function, and the parameters γ and τ determine the smoothness or speed of transition and location between the two regimes, respectively. Since the value of $S_t(\gamma,\tau)$ depends on the value of the parameter γ, the transition between the two regimes occurs slowly for small values of γ whereas the transition between the regimes becomes almost instantaneous at time $t = \tau T$ for large values of γ. When $\gamma = 0$, then $S_t(\gamma,\tau) = 0.5$ for all values of t. Therefore, in Model A, y_t is stationary around a mean that changes from α_1 to $\alpha_1 + \alpha_2$.Model B allows for a fixed slope term β_1, whereas the intercept term changes from α_1 to $\alpha_1 + \alpha_2$. Model C allows, in addition to the similar changes in the intercept, the slope changes from β_1 to $\beta_1 + \beta_2$ at the same time. See, for further details, Leybourne, Newbold, and Vougas (1998).

The second specification of the test utilizes the Fourier series (Enders and Lee 2012; Omay 2015) to model multiple smooth breaks:

$$\phi(t) = \alpha_0 + \delta t + \sum_{k=1}^{n} a_k \sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{n} b_k \cos\left(\frac{2\pi kt}{T}\right) + u_t, \quad (4)$$

where $n \leq T/2$ represents the number of frequencies, k is the selected frequency in the approximation process, and a_i and b_i are the measurements for the amplitude and displacement of the sinusoidal components of the function. As stated in Omay, Emirmahmutoglu, and Hasanov (2018), the Fourier series with an appropriate lag order in most cases can approximate any function with unknown numbers of breaks of unknown forms. Under the assumption of $a_i = b_i = 0$ for all i, the Fourier function becomes a linear model without a structural break. As a result, rejecting the null of $a_i = b_i = 0$ implies a structural break in the series. If Equation (4) allows for a structural break, the minimum frequency component must equal at least one.

To model the stochastic component, Omay, Emirmahmutoglu, and Hasanov (2018) utilize the asymmetric exponential smooth transition autoregressive (AESTAR) model of Sollis (2009), which captures the nonlinear asymmetric adjustment process towards equilibrium. The AESTAR model considers both a logistic function and an exponential function as follows:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + (1 - F_t(\theta_2, u_{t-1}))\rho_2\} u_{t-1} + \epsilon_t, \quad (5)$$

$$G_t(\theta_1, u_{t-1}) = 1 - \exp\left(-\theta_1\left(u_{t-1}^2\right)\right), \quad \theta_1 > 0 \ and \quad (6)$$

$$F_t(\theta_2, u_{t-1}) = [1 + \exp(-\theta_2(u_{t-1}))]^{-1}, \quad \theta_2 > 0 \quad (7)$$

where $\epsilon_t \sim iid(0, \sigma^2)$. $F_t(\theta_2, u_{t-1})$ is the logistic transition function for two regimes, determined by the positive and negative deviations from the equilibrium of u_t (i.e., the sign of disequilibrium). $G_t(\theta_1, u_{t-1})$ is the U-shaped symmetric exponential transition function, defined over the range from 0 to 1, determined by the small and large deviations from the equilibrium in absolute terms.

The AESTAR function implies a globally stationary process, which requires $\theta_1 > 0$, $\rho_1 < 0$, and $\rho_2 < 0$ as stated in Sollis (2009). If $\rho_1 \neq \rho_2$, the adjustment process captures not only sign but also size adjustment to the equilibrium. On the other hand, if $\rho_1 = \rho_2$, the adjustment to the equilibrium becomes a symmetric exponential smooth transition autoregressive (ESTAR) process.

We can test the null hypothesis of a linear unit root against the alternative hypothesis of a globally stationary AESTAR process. The hypotheses are as follows:

$$H_0 : \theta_1 = 0, \quad (8)$$

$$H_1 : \theta_1 > 0. \quad (9)$$

Failure to reject the null provides evidence of a unit root. Testing the null hypothesis proves problematic, since ρ_1, ρ_2, and θ_2 are unidentified nuisance parameters under the null. To overcome this problem, Sollis (2009) applies a first-order Taylor expansion and derives the following auxiliary equation:

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \omega_t. \quad (10)$$

Under Equation (10), the null hypothesis in Equation (8) becomes $H_0 : \varphi_1 = \varphi_2 = 0$. Equation (5) assumes a serially uncorrelated error term. To allow for serial correlation, we augment the regression equation as follows:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + (1 - F_t(\theta_2, u_{t-1}))\rho_2\} u_{t-1} + \sum\nolimits_{j=1}^{p} \delta_j \Delta u_{t-j} + \epsilon_t, \quad (11)$$

where $\epsilon_t \sim iid(0, \sigma^2)$. Therefore, we use the following auxiliary regression to test the null hypothesis $H_0 : \varphi_1 = \varphi_2 = 0$:

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \sum\nolimits_{j=1}^{p} \delta_j \Delta u_{t-j} + \vartheta_t \quad (12)$$

The testing procedure in Omay, Emirmahmutoglu, and Hasanov (2018) consists of two steps (see, also, Kapetanios, Shin, and Snell 2003; Leybourne, Newbold, and Vougas 1998; Sollis 2009). First, Omay, Emirmahmutoglu, and Hasanov (2018) estimate (a) the Fourier model (by OLS) for the frequency k over the range $1 \le k \le k_{max}$ and obtain the optimal k that minimizes RSS through a grid search over the interval $1 \le k \le k_{max}$ or (b) the logistic model (by NLS). Second, using the residuals from (a) or (b), Omay, Emirmahmutoglu, and Hasanov (2018) estimate Equation (12) (by OLS), and test the null hypothesis $H_0 : \varphi_1 = \varphi_2 = 0$, using a conventional F-test. The F-test statistic is denoted as F_{LBAE} (logistic transition function version), or as F_{FSAE} (Fourier function version). Omay, Emirmahmutoglu, and Hasanov (2018) obtain the critical values of F_{LBAE} and F_{FSAE} via stochastic simulation and show that the tests possess satisfactory size and power small-sample properties.

III. Empirical results

The dataset contains five multi-century annual U.K. real output series that the Bank of England recently compiled in the database *A Millennium of Macroeconomic Data* maintained at https://www.bankofengland.co.uk/statistics/research-datasets. Three series span 1270 to 2016, and two series span 1700 to 2016. We use Version 3.1 of the database, updated to 2016. For detailed information about the historical sources of the data, see Thomas and Dimsdale (2017). The five data series are defined as follows:

Series 1: Real U.K. GDP at market prices (1700-2016), geographically-consistent estimate based on post-1922 borders, millions of British pounds, chained volume measure, 2013 prices;

Series 2: Real U.K. GDP at factor cost (1700-2016), geographically-consistent estimate based on post-1922 borders, millions of British pounds, chained volume measure, 2013 prices;

Series 3: Real GDP of England at market prices (1270-2016), millions of British pounds, chained volume measure, 2013 prices;

Series 4: Real GDP of England at factor cost (1270-2016), millions of British pounds, chained volume measure, 2013 prices; and

Series 5: Composite estimate of English and (geographically-consistent) U.K. real GDP at factor cost (1270-2016), 2013=100.

As a preliminary step, we first apply two linear unit-root tests: the standard Augmented Dickey-Fuller (ADF 1979) test (constant, and constant and linear trend) and the four versions of the Ng and Perron (2001) test (constant and linear trend). The Ng and Perron (2001) procedure yields substantial power gains over the standard unit-root test. Significant modifications of existing unit-root tests improve their power and size. The MZa and MZt tests modify the Phillips (1987) and Phillips and Perron (1988) Za and Zt tests, respectively; the MSB test relates to the Bhargava (1986) R1 test; and the MPT test modifies the Elliott, Rothenberg, and Stock (1996) point-optimal test. Tables 1 and 2 report the results of applying these tests. We choose the proper lag length by the SIC criterion from a maximum of 12 lags. We cannot reject the null hypothesis of a unit root for any of the five series. As mentioned above, linear unit-root

Table 1. ADF unit-root test results.

Output Series	Constant	Constant and trend
Series 1	7.791	5.348
Series 2	7.293	4.676
Series 3	11.426	10.927
Series 4	12.197	11.713
Series 5	11.740	11.231
Test critical values:		
1%	−3.438	−3.420
5%	−2.865	−2.910
10%	−2.568	−2.620

*denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 2. Ng and Perron (2001) unit-root test results.

Output Series	MZa	MZt	MSB	MPT
Series 1	2.891	1.957	0.677	136.294
Series 2	2.943	2.061	0.700	145.605
Series 3	−5.623	−1.099	0.195	15.221
Series 4	−6.181	−1.177	0.190	14.597
Series 5	−6.275	−1.196	0.191	14.498
Test critical values:				
1%	−23.800	−3.420	0.143	4.030
5%	−17.300	−2.910	0.168	5.480
10%	−14.200	−2.620	0.185	6.670

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

Table 3. Kapetanios, Shin, and Snell (2003) unit-root test results.

Output Series	Constant	Constant and trend
Series 1	−0.900	−1.513
Series 2	−0.747	−1.379
Series 3	−1.184	−1.758
Series 4	−0.764	−1.306
Series 5	−5.868***	−8.628***
Test critical values:		
1%	−3.480	−3.970
5%	−2.930	−3.400
10%	−2.660	−3.130

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

Table 4. Sollis (2009) unit-root test results.

Output series	Constant	Constant and trend
Series 1	1.638	2.412
Series 2	8.292***	2.140
Series 3	29.473***	20.859***
Series 4	25.062***	15.647***
Series 5	17.421***	50.774***
Test critical values:		
1%	6.883	8.531
5%	4.954	6.463
10%	4.157	5.460

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 5. Leybourne, Newbold, and Vougas (1998) unit-root test results.

Output Series	Model A	Model B	Model C
Series 1	−3.023	−3.447	−3.447
Series 2	−3.186	−5.882***	−5.882***
Series 3	−6.699***	−0.579	−7.126***
Series 4	−7.320***	−0.543	−7.697***
Series 5	−7.130***	−0.451	−7.468***
Test critical values:			
1%	−4.882	−5.479	−5.560
5%	−4.232	−4.771	−5.011
10%	−3.909	−4.427	−4.697

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

tests can suffer from power problems in the presence of nonlinearities in the data leading to a bias towards the non-rejection of the null hypothesis.

We next consider the results of two tests that allow for state-dependent nonlinearity. That is, the tests allow for symmetric nonlinearity (Kapetanios, Shin, and Snell 2003) and asymmetric nonlinearity (Sollis 2009), but ignore the possibility of structural breaks. Tables 3 and 4 tabulate the results, respectively, of the Kapetanios, Shin, and Snell (2003) unit-root tests of a symmetric ESTAR model and of the Sollis tests of an asymmetric ESTAR (AESTAR) model. The Kapetanios, Shin, and Snell (2003) test rejects the null of a unit root in only one series (Series 5), for the constant, and constant and trend cases. Conversely, the Sollis (2009) unit-root test rejects the null of unit root in four series (Series 2, 3, 4, and 5) in the constant case, and in three series (Series 3, 4, and 5) in the constant and trend case. This provides some evidence that asymmetric adjustment proves an important characteristic of state-dependent nonlinearity.

Then, we consider three time-dependent unit-root tests designed to consider structural breaks. Table 5 presents the Leybourne, Newbold, and Vougas (1998) unit-root test results. The test results show that allowing for a permanent break causes a more frequent rejection of the null hypothesis than the Omay (2015) and Çorakcı, Emirmahmutoğlu, and Omay (2017) tests presented in Tables 6 and 7. The Omay (2015) test models multiple smooth structural breaks using the fractional version of the Fourier function, while the Çorakcı, Emirmahmutoğlu, and Omay (2017) test models temporary structural breaks. The Omay (2015) and Çorakcı, Emirmahmutoğlu, and Omay (2017) tests do not reject the null in any of the output series. In contrast, Model C of the Leybourne, Newbold, and Vougas (1998) unit-root test rejects the unit-root hypothesis in four of the five series (Series 2, 3, 4, and 5). Weaker evidence of rejection appears for Model A, which rejects the null in three of the five series (Series 3, 4, and 5), while Model

Table 6. Omay (2015) unit-root test results.

Output Series	Constant	Constant and trend
Series 1	1.238	−1.912
Series 2	1.236	−1.945
Series 3	11.005	6.943
Series 4	11.365	7.985
Series 5	10.957	7.661
Test critical values:		
1%	−4.31	−4.94
5%	−3.67	−4.35
10%	−3.33	−4.05

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

Table 7. Çorakcı, Emirmahmutoğlu, and Omay (2017) unit-root test results.

Output Series	Model A	Model B	Model C
Series 1	−1.006	−1.649	−1.687
Series 2	−0.932	−1.481	−2.056
Series 3	−0.164	−0.787	−0.876
Series 4	−0.010	−0.008	−0.698
Series 5	−1.504	−2.997	−1.954
Test critical values:			
1%	−5.017	−5.544	−5.797
5%	−4.374	−4.900	−5.166
10%	−4.051	−4.572	−4.844

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

Table 8. Christopoulos and Leon-Ledesma (2010) unit-root test results.

Output Series	Constant	Constant and trend
Series 1	3.560	−0.442
Series 2	3.712	−0.359
Series 3	6.693	4.045
Series 4	7.188	4.634
Series 5	7.113	6.322
Test critical values:		
1%	−4.41	−4.44
5%	−3.86	−3.86
10%	−3.54	−3.57

* denotes 10% significance level; ** denotes 5% significance level; *** denotes 1% significance level.

Table 9. Omay and Yıldırım (2014) unit-root test results.

Output Series	Model A	Model B	Model C
Series 1	−4.152**	−3.865	−5.142***
Series 2	−4.506***	−3.701	−5.478***
Series 3	−5.051**	−2.716	−5.174***
Series 4	−5.027***	−2.332	−5.120***
Series 5	−23.873***	−1.540	−26.669***
Test critical values:			
1%	−4.443	−4.777	−5.041
5%	−3.821	−4.202	−4.411
10%	−3.509	−3.889	−4.090

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

B cannot reject the unit-root hypothesis in four of the five series (Series 1, 3, 4, and 5). Given the historical length of the data, and the fact that all five output series, as expected, display a positive trend, we find more appropriate to emphasize Model C, which allows for breaks in both the intercept and the slope of the trend. The findings from Model C provide some evidence that the structure of the historical series does not include a single temporary break or multiple smooth breaks.

Finally, we report in Table 8–11 the results of the three 'hybrid' tests, which consider simultaneously the two types of nonlinearity: time dependence (structural breaks) and state dependence (transitional adjustment). Table 8 reports the Christopoulos and Leon-Ledesma (2010) test results. The test statistics for the constant, and constant and trend cannot reject the null of a unit root for all five series. This confirms that a model with multiple smooth breaks and symmetric adjustment does not adequately support the hypothesis of stationarity. The critical values for the constant only test statistic come from Christopoulos and Leon-Ledesma (Table 2). The critical values for the constant and trend test statistic are not available from Christopoulos and Leon-Ledesma (2010) and are obtained by our Monte Carlo simulation for T=500 and $k = 1$.[2]

Table 9 reports the Omay and Yıldırım (2014) test results. The test employs the Leybourne, Newbold, and Vougas (1998) structure but assumes symmetric adjustment (ESTAR). The test statistics for Model A reject the null of a unit root at the 1-per cent level in Series 1, 2, 4, 5 and at the 5-per cent level in Series 3. The results of model B, on the other hand, indicate failure to reject the null for all series. The test statistics for Model C, in contrast, reject the null of unit root at the 1-per cent level in all five series. The results from Model C, which for the reasons mentioned above we find more suitable, provide strong evidence that favours the presence of both a single permanent break and nonlinear symmetric adjustment for the five historical real output series.

Tables 10 and 11 report the results of the two versions of the Omay, Emirmahmutoglu, and Hasanov (2018) unit-root tests, namely the logistic function version and the Fourier function version. Table 10 reports the F_{LBAE} of the logistic transition function for Models A, B, and C. The test results of

[2]We also computed the critical values for T = 2500. We report them for additional information. They are, respectively, −4.42, −3.86, and −3.56 for the 1%, 5%, and 10% significance level, respectively.

Table 10. Omay, Emirmahmutoglu, and Hasanov (2018) unit-root test results (logistic trend function version).

Output Series	Model A	Model B	Model C
Series 1	10.720**	9.377*	18.759***
Series 2	12.230***	9.651**	22.811***
Series 3	25.248***	52.249***	28.732***
Series 4	30.734***	9.756**	34.318***
Series 5	386.214***	42.540***	433.129***
Test critical values:			
1%	10.756	12.681	13.621
5%	8.110	9.642	10.617
10%	7.101	8.339	9.209

*denotes 10% significance level; **denotes 5% significance level; ***denotes 1% significance level.

Table 11. Omay, Emirmahmutoglu, and Hasanov (2018) unit-root test results (Fourier trend function version).

Output Series	Constant	Constant and trend
Series 1	21.346***	4.867
Series 2	21.805***	4.726
Series 3	73.806***	40.620***
Series 4	70.126***	37.119***
Series 5	47.809***	35.947***
Test critical values:		
1%	8.68	10.61
5%	6.36	7.93
10%	5.31	6.75

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

single break version of the test are quite similar to those in Model A and Model C of Omay and Yıldırım (2014), confirming the relevance of the single permanent structural break over the multiple smooth breaks. We reiterate again that we deem Model C the most appropriate. Table 11 reports the F_{FSAE} test statistic for the constant, and constant and trend. We find that the logistic version of the Omay, Emirmahmutoglu, and Hasanov (2018) unit-root test is also preferred to the Fourier version, which allows for the possibility of multiple smooth breaks, since the logistic version rejects the null in all five series compared to the rejection of only three series in the Fourier function case.

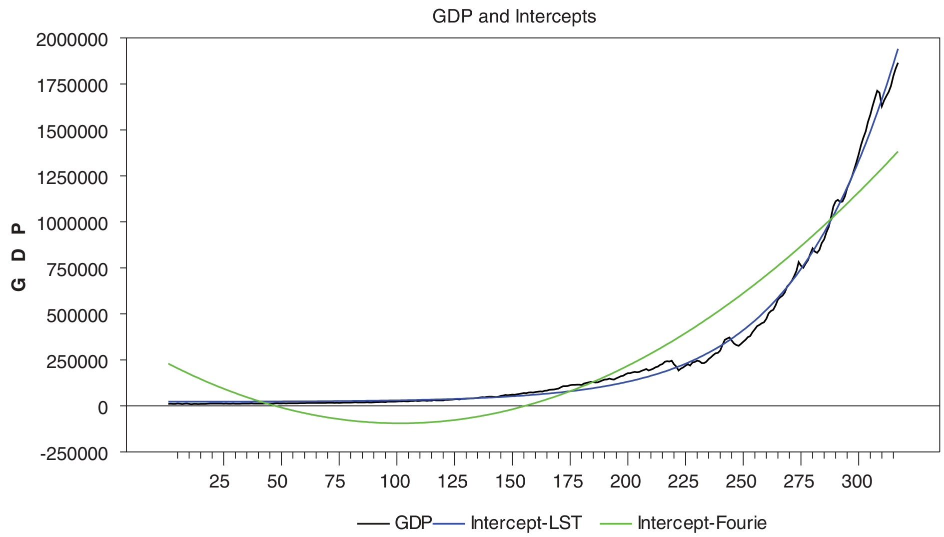
Figures 1–5 present in panels (a) and (b) the historical GDP series along with estimated nonlinear trend functions and the corresponding detrended data. Visual inspection of the series reveals the importance of taking account of structural breaks when analysing these historical series.

IV. Identification tests of the stochastic structure of the data

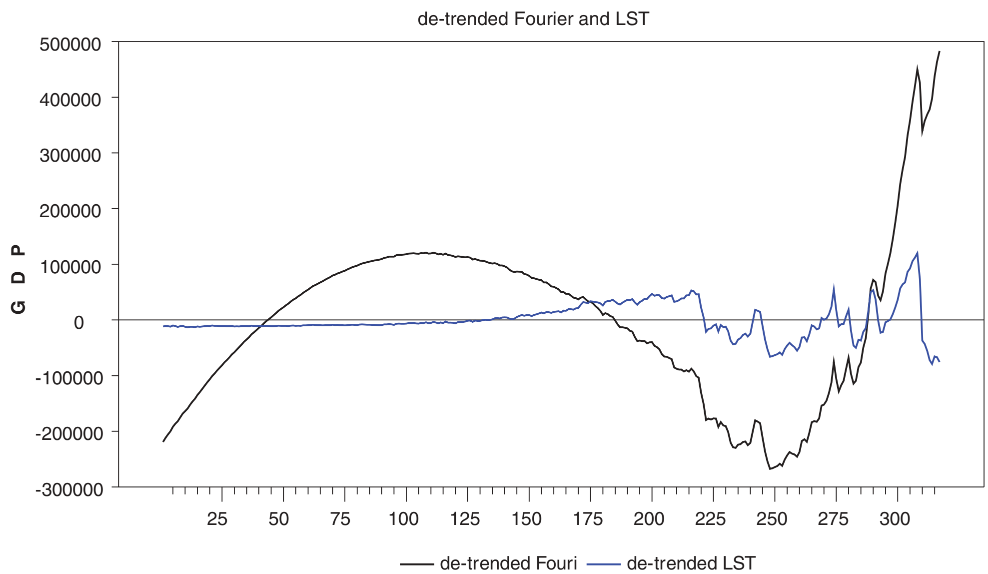
In the previous section, we have determined the integration order of the five series of U.K. historical real output by using various state-dependent and time-dependent nonlinear tests. Three tests, in particular, namely Leybourne, Newbold, and Vougas (1998), Omay and Yıldırım (2014), and Omay, Emirmahmutoglu, and Hasanov (2018) identify the data as stationary processes with different nonlinear structures. Still, the identification of the stochastic features of the data is scant, since in all tests the null hypothesis is one of unit-root linearity. Unit-root tests, however, do not identify which test is most consistent with the data. To do that, we conduct in this section a sequence of auxiliary identification tests. Linearity testing is an integral part of nonlinear modelling.

As all five series are I(0), we can use the linearity test of Luukkonen, Saikkonen, and Terasvirta (1988) and the *Trig*-test of Becker, Enders, and Lee (2004) for identification of alternative nonlinearities such as state-dependent nonlinearity and time-dependent nonlinearity. We conduct the identification of the true structure of the data with a sequential procedure, which consists of five stages as follows:

(1) Apply the unit-root tests. If the results indicate that the data are stationary I(0), move to Step 2. If the unit-root null is not rejected, then accept the nonstationary hypothesis and stop.
(2) Apply the linearity test of Luukkonen, Saikkonen, and Terasvirta (1988) to determine whether the stationary data are state-dependent or time-dependent (time-varying).
(3) If the stationary data are state-dependent, determine whether the state-dependent nonlinearity is of the ESTAR or AESTAR type.
(4) If the stationary data are time-dependent, determine whether the nonlinear trend is logistic (LSTR), exponential (ESTR), Fourier with integer frequency (IFFF), or Fourier with fractional frequency (FFFF).
(5) For the last step, check whether residual nonlinearity (remaining nonlinearity after de-trending the nonlinear trend) exists after applying Step 4. For this purpose, we use the Becker, Enders, and Lee (2004) Trig-test and Luukkonen, Saikkonen, and Terasvirta (1988) test.



(a)



(b)

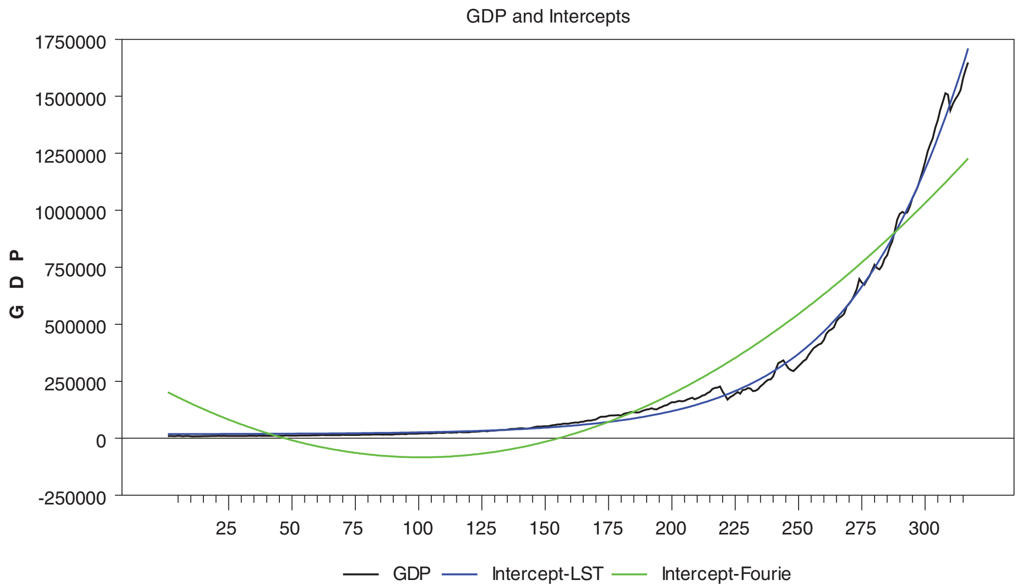
Figure 1. Series 1: Real U.K. GDP at market prices (1700–2016), geographically consistent estimate based on post-1922 borders; millions of British pounds, chained volume measure, 2013 prices.

Linearity tests against smooth transition autoregressive (STAR) models assume that the time series under investigation is stationary. Since we find that the assumption of stationarity is generally satisfied in the Leybourne, Newbold, and Vougas (1998), Omay and Yıldırım (2014), and Omay, Emirmahmutoglu, and Hasanov (2018) tests, we proceed to stage 2 and apply the first linearity test to determine whether the series exhibit state- or time-dependent nonlinearity.

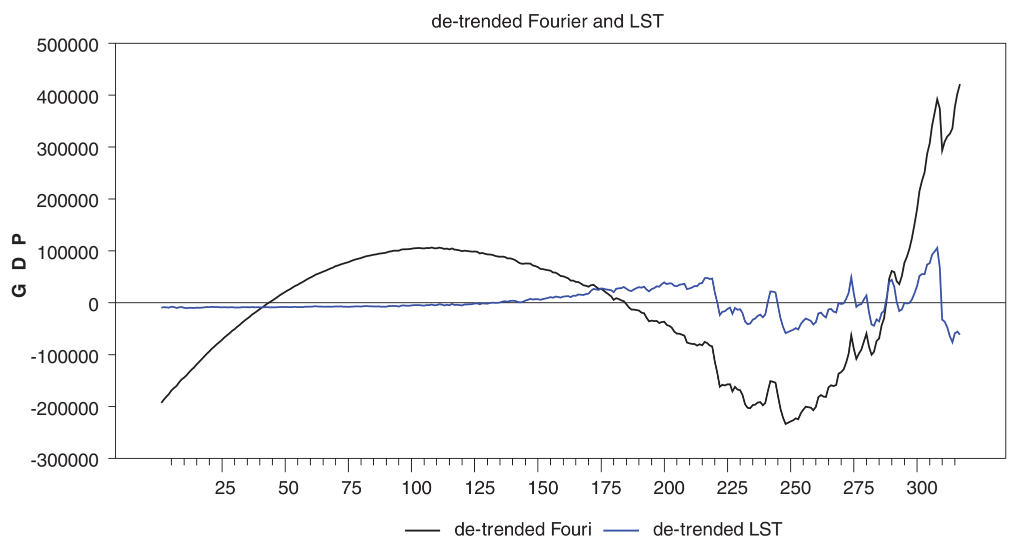
We apply the linearity tests of Luukkonen, Saikkonen, and Terasvirta (1988) and consider the class of two-regime STAR processes represented as

$$y_t = \phi_1 x_{1t} + \phi_2 x_{2t} F(s_t; \gamma, \tau) + u_t \qquad (13)$$

for some transition function $F(s_t; \gamma, c)$, where S_t is the transition variable, $\gamma > 0$ is the transition scale, and τ is the threshold. Luukkonen, Saikkonen, and Terasvirta (1988), and the vast majority of empirical research since then, consider the logistic and exponential transition functions. Within this framework, the tests of linearity are complicated by the presence of an unidentified nuisance



(a)



(b)

Figure 2. Series 2: Real U.K. GDP at factor cost (1700–2016), geographically consistent estimate based on post-1922 borders; millions of British pounds, chained volume measure, 2013 prices.

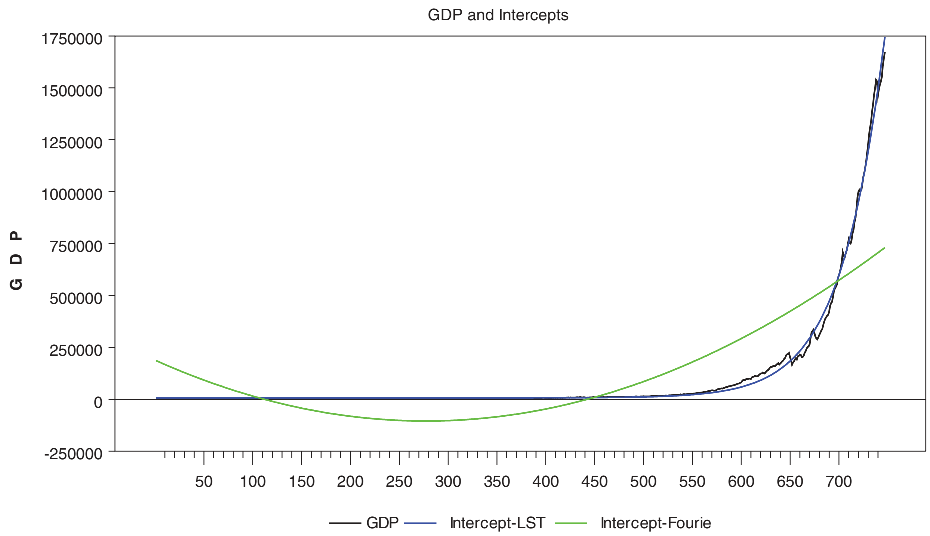
parameter under the null hypothesis, since we can test the hypothesis of linearity in two ways: as a test that the parameters in the two regimes equal each other, H_0: $\phi_1 = \phi_2$ and as a test that the scale parameter γ equals zero, H'_0: $\gamma = 0$. To overcome this problem, Luukkonen, Saikkonen, and Terasvirta (1988) replace the transition function with the appropriate Taylor approximation.

The linearity test obtained from the first-order Taylor approximation results in the following auxiliary regression[3]

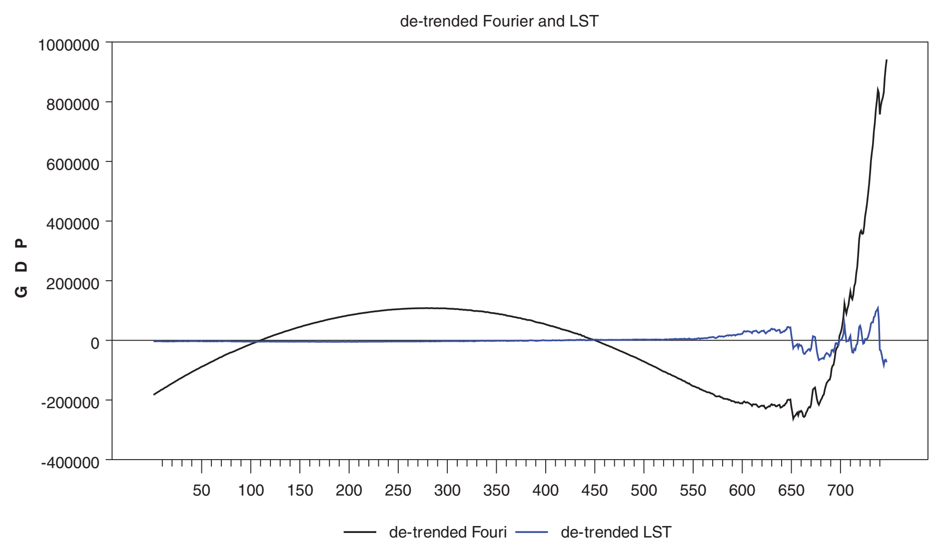
$$y_t = \beta_{0,0} + \beta_0' x_t + \beta_{1,0} s_t + \beta_1' x_t s_t + e_t \qquad (14)$$

where the regression parameters $\beta_{0,0}$, β'_0, and β'_1 depend on the parameters ϕ_1, ϕ_2, γ and τ and e_t is the disturbance term, which comprises the original shocks, u_t, as well as the error term arising from the Taylor approximation. We can express the null hypothesis of linearity as $H''_0 : \beta_{1,0} = \beta'_1 = 0$, that is, the parameters associated with the auxiliary regressors equal zero. We test this null hypothesis by a standard variable

[3]For further details, see Luukkonen, Saikkonen, and Terasvirta (1988).



(a)



(b)

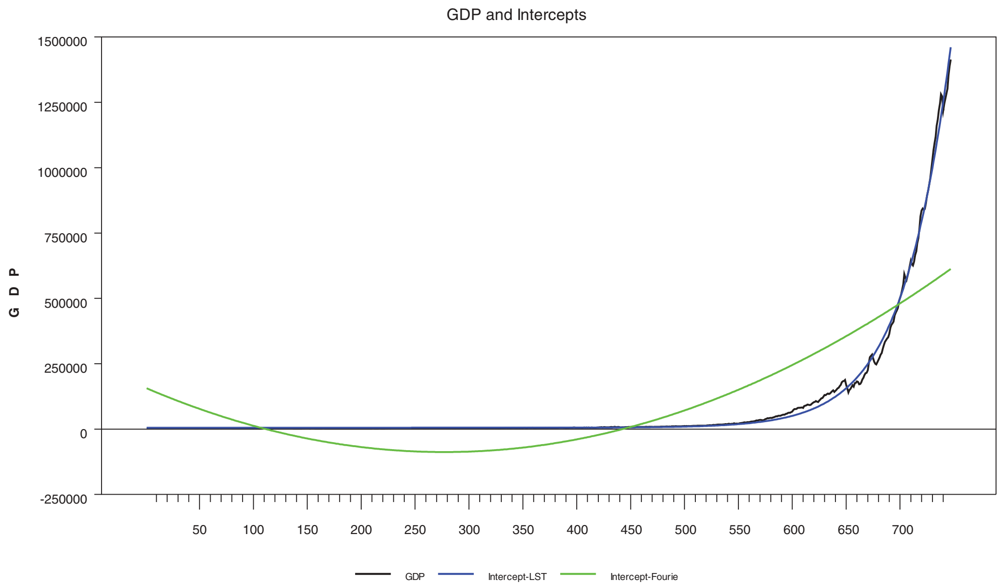
Figure 3. Series 3: Real GDP of England at market prices (1270–2016), millions of British pounds, chained volume measure, 2013 prices.

addition test in a straightforward manner. The test statistic, denoted by LM1, is asymptotically distributed as χ^2 with p+1 degrees of freedom, where p is the dimension of the vector x_t.[4] As noted by Luukkonen, Saikkonen, and Terasvirta (1988), the LM1 test statistic has no power in situations where only the intercept differs across regimes. Luukkonen, Saikkonen, and Terasvirta (1988) suggest in this case the replacement of the transition function $F(s_t;\gamma,c)$ with a third-order Taylor approximation. This results in the following auxiliary model:

$$y_t = \beta_{0,0} + \beta_0^{'}x_t + \beta_{1,0}s_t + \beta_1^{'}x_ts_t + \beta_{2,0}s_t^2 + \beta_2^{'}x_ts_t^2 + \beta_{3,0}s_t^3 + \beta_3^{'}x_ts_t^3 + e_t. \quad (15)$$

The null hypothesis now corresponds to $H_0^{''}:\beta_i^{'}=0,\ i=1,2,3$, which we can again test by a standard LM-type test. Under the null hypothesis of linearity, the test statistic, denoted by LM3, has an asymptotic χ^2 distribution with 3(p +1) degrees of freedom. Since only the parameters corresponding to s_t^2 and s_t^3 are functions of ϕ_1 and ϕ_2, we can obtain

[4]In Equation (13), we assume that the transition variable s_t is not one of the elements in x_t. If this is not the case, we drop the term $\beta_{1,0}s_t$ from the auxiliary regression.



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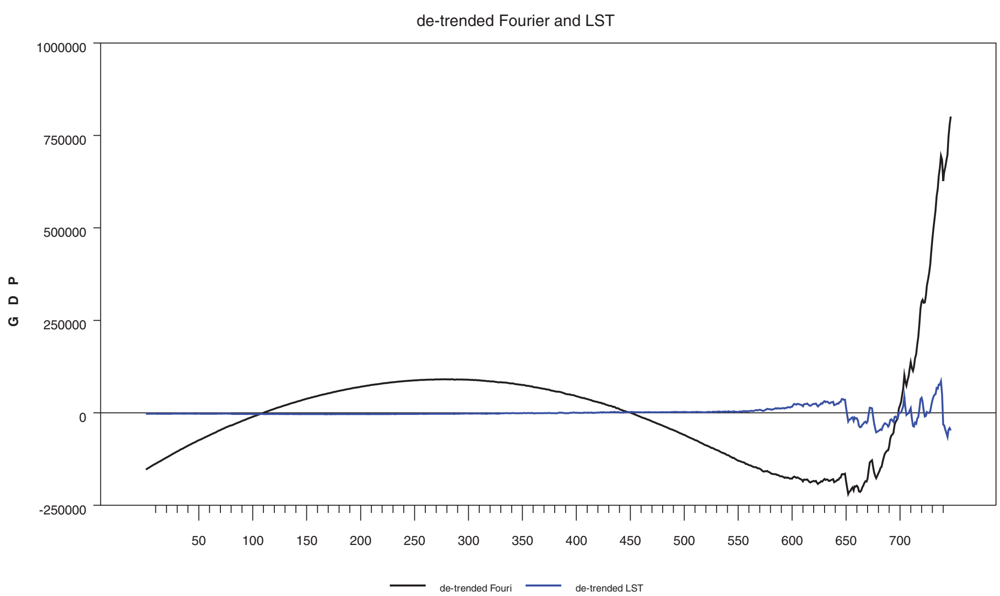

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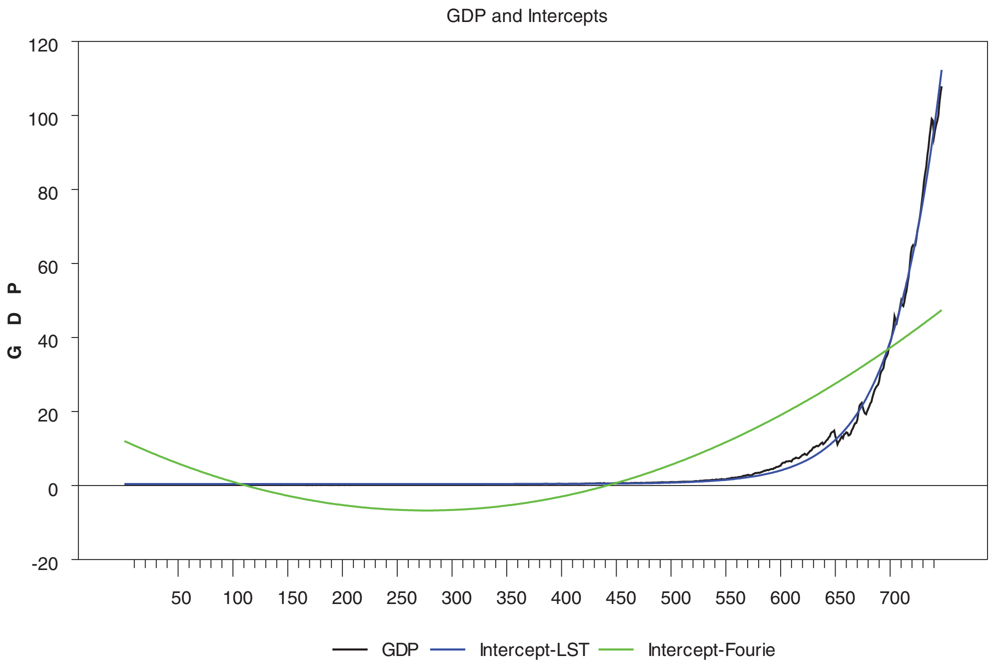
Figure 4. Series 4: Real GDP of England at factor cost (1270–2016), millions of British pounds, chained volume measure, 2013 prices.

a parsimonious version of the LM3 statistic, denoted as $LM3^E$, by augmenting the auxiliary model (13) only with regressors s_t^2 and s_t^3. That is,

$$y_t = \beta_{0,0} + \beta_0^{'} x_t + \beta_1^{'} x_t s_t + \beta_{2,0} s_t^2 + \beta_{3,0} s_t^3 + e_t \quad (16)$$

The resultant statistic is the $LM3^E$ statistic. As transition variable, s_t, we select y_{t-1}. This is consistent with the structure of the unit-root tests. Table 12 displays the $LM3^E$ version of the test for the five real output series. The results indicate that time-dependent nonlinearity (structural breaks) is the dominant structure of the data in levels, although the test also supports the state-dependent nonlinearity for series 4 and 5 as well. Thus, this outcome can explain why the Kapetanios, Shin, and Snell (2003) test finds ESTAR stationarity for series 5 and the Sollis (2009) test produces AESTAR stationarity results for series 4 and 5.



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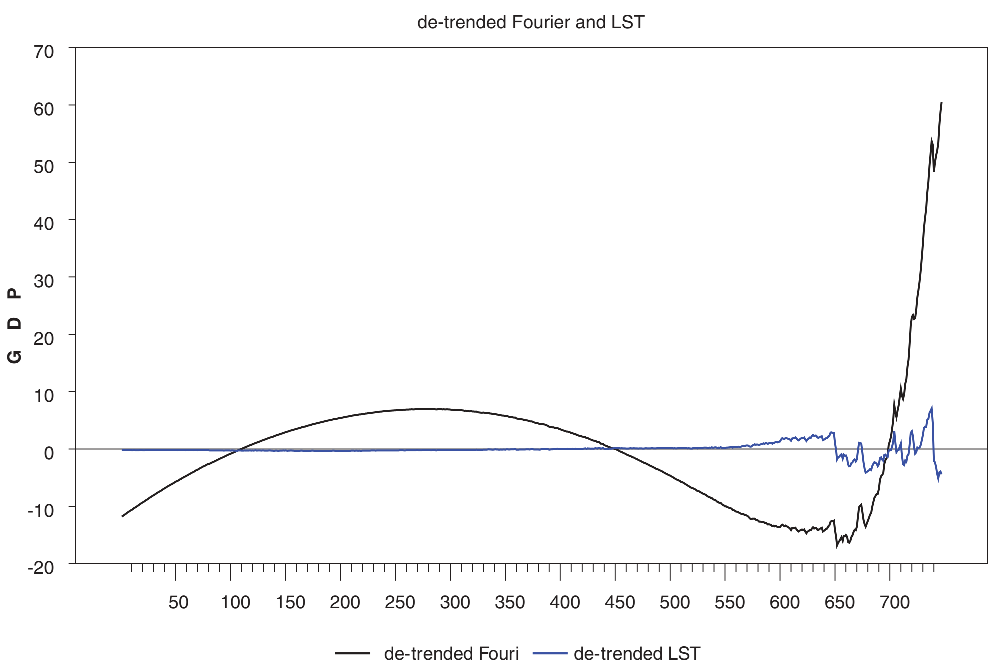

(b)

Figure 5. Series 5: Composite Estimate of English and (geographically consistent) U.K. real GDP at factor cost (1270–2016), 2013 = 100.

In stage 3, we consider the choice between the ESTAR and LSTAR models and the ESTAR and AESTAR models. To choose between the ESTAR and LSTAR models, Terasvirta (1994) suggests using a decision rule based on a sequence of tests in Equation (15). Specifically, Terasvirta (1994) proposes to test the following set of null hypotheses:

(i) $H_{03} : \beta_4 = 0.$
(ii) $H_{02} : \beta_2 = 0|\beta_4 = 0.$
(iii) $H_{03} : \beta_1 = 0|\beta_4 = \beta_2 = 0.$

Table 12. Linearity tests (level data).

Output Series	State-Dependent Nonlinearity $s_t = y_{t-1}$ F-test	Significance	Time-Varying Nonlinearity $s_t = t$ F-test	Significance	Result
Series 1	1.178	0.315	8.927	0.000	TV
Series 2	1.208	0.299	8.617	0.000	TV
Series 3	1.185	0.311	8.726	0.000	TV
Series 4	2.410	0.027	6.649	0.000	SD-TV
Series 5	2.624	0.016	6.669	0.000	SD-TV

Note: TV indicates time-varying or time-dependent nonlinearity, SD indicates state-dependent nonlinearity.

These hypotheses can be tested by ordinary F tests, denoted as F3, F2, and F1, respectively. The decision rule is as follows: If the p-value corresponding to F2 is the smallest, then the ESTAR model is selected, while in all other cases, the LSTAR model is selected.

For the choice between the ESTAR and AESTAR models, we use the following extended auxiliary regression:

$$y_t = \beta_{0,0} + \beta_0' x_t + \beta_{1,0} s_t + \beta_{1,}' x_t s_t + \beta_{2,0} s_t^2 + \beta_{2,}' x_t s_t^2 + \beta_{3,0} s_t^3 + \beta_{3,}' x_t s_t^3 + \beta_{4,0} s_t^4 + \beta_{4,}' x_t s_t^4 + e_t \quad (17)$$

together with the following set of null hypotheses:

(i) $H_{03} : \beta_3 = 0.$
(ii) $H_{02} : \beta_2 = 0 | \beta_3 = 0.$
(iii) $H_{03} : \beta_1 = 0 | \beta_2 = \beta_3 = 0.$

These hypotheses are again tested by ordinary F tests, denoted as F4, F2, and F1, respectively. The decision rule is as follows: If the p-value corresponding to F2 is the smallest, then the ESTAR model is selected, while in all other cases, the AESTAR model is selected. Since we found in stage 2, however, that the dominant structure is time-dependent nonlinearity, we do not apply these tests and proceed to stage 4.

In this stage, we use the Becker, Enders, and Lee (2004) *Trig*-test for determining which nonlinear trend is consistent or more appropriate with the data. The test is valid under the null of stationarity. Becker, Enders, and Lee (2004) test the null hypothesis of linearity against the alternative of a nonlinear trend with given frequency k. Enders and Lee (2012) use the following F-statistic:,

$$F(k) = \frac{(SSR_0 - SSR_1(k))/2}{SSR_1(k)/(T-q)}$$

where $SSR_1(k)$ denotes the sum of squared residuals (SSR) when Fourier transforms enter into the equation, q is the number of regressors, and SSR_0 denotes the SSR from the regression without the trigonometric terms. By analogy, we can use the following F-statistic for the smooth transition type of nonlinear trends:

$$F(k) = \frac{(SSR_0 - SSR_1(\gamma, \tau))/2}{SSR_1(\gamma, \tau)/(T-q)}$$

where $SSR_1(\gamma, \tau)$ denotes the sum of squared residuals (SSR) when smooth transition functions enter into the equation.

We present the results of the *Trig*-test in Table 13 with the choice of the best fitting trend in bold type. The *Trig*-test results suggest that the best fitting nonlinear trend is Model C of Leybourne, Newbold, and Vougas (1998). This is consistent with the unit-root test results. Therefore, we can safely claim that all real output series are characterized by breaks that are prevalent in the intercept and trend. Thus, the Luukkonen, Saikkonen, and Terasvirta (1988) linearity test and the *Trig*-test of Becker, Enders, and Lee (2004) both support Model C of Omay, Emirmahmutoglu, and Hasanov (2018) and Omay and Yıldırım (2014) tests. Consequently, in stage 5, we concentrate on which of the three tests is more consistent with the data under investigation.

That is, in this stage, we check which of Model C of Leybourne, Newbold, and Vougas (1998), Model C of Omay, Emirmahmutoglu, and Hasanov (2018), or Model C of Omay and Yıldırım (2014) is more consistent with the data. All three tests successfully detect stationarity among the five real output series. Recall that Leybourne, Newbold, and Vougas (1998), Omay and Yıldırım (2014), and Omay, Emirmahmutoglu, and Hasanov (2018) are two-step tests. The first step removes the structural

Table 13. Selection of the best fitting trend function by using F-test.

Smooth Transition trends

Output Series	A	γ	τ	B	γ	τ	C	γ	τ
Series 1 LSTR	64151.6	0.024	0.124	889.141	0.002	0.387	**69966.06**	0.026	0.0661
Series 2 LSTR	72588.5	0.024	0.124	988.977	0.002	0.394	**77395.99**	0.024	0.1291
Series 3 LSTR	**40493.4**	0.020	0.380	738.938	0.001	0.366	516.028	0.006	0.9554
Series 4 LSTR	14143.134	0.025	0.939	1331.931	0.993	0.332	**19231.932**	0.036	0.998
Series 5 LSTR	14947.579	0.025	0.925	1430.391	0.995	0.321	**19449.298**	0.035	0.918
Series 1 ESTR	34.444	2.1E-07	0.451	1025.101	0.000	0.896	58576.69	0.000	0.641
Series 2 ESTR	91.648	2.1E-07	0.404	1045.669	0.000	0.892	952.204	0.000	0.457
Series 3 ESTR	96.162	1.6E-07	0.403	1154.020	0.000	0.939	3111.42	0.000	0.510
Series 4 ESTR	158.981	0.000	0.262	633.401	0.000	0.992	15621.332	0.000	0.529
Series 5 ESTR	132.497	0.000	0.244	719.193	0.000	0.993	11144.209	0.000	0.627

Fourier trends

Output Series Frequency	Intercept	k	Intercept and trend	k
Series 1 Integer	111.741	1	448.361	1
Series 2 Integer	112.490	1	453.563	1
Series 3 Integer	114.008	1	459.518	1
Series 4 Integer	110.414	1	544.879	1
Series 5 Integer	112.674	1	563.911	1
Series 1 Fractional	298.760	0.1	1101.983	0.1
Series 2 Fractional	301.552	0.1	1119.738	0.1
Series 3 Fractional	305.859	0.1	1137.801	0.1
Series 4 Fractional	406.288	0.1	2425.552	0.1
Series 5 Fractional	421.181	0.1	2583.837	0.1

Table 14. Linearity test of the residuals of Model C (Leybourne, Newbold, and Vougas 1998).

	State-Dependent Nonlinearity $s_t = y_{t-1}$		Time-Dependent Nonlinearity $s_t = t$		Result
Output Series	F-test	Significance	F-test	Significance	
Series 1	3.142	0.004	1.613	0.140	SD
Series 2	0.611	0.721	8.140	0.000	TV
Series 3	3.240	0.003	1.457	0.190	SD
Series 4	0.420	0.865	7.192	0.000	TV
Series 5	1.201	0.305	6.838	0.000	TV

Note: TV indicates time-varying or time-dependent nonlinearity; SD indicates state-dependent nonlinearity.

break and the second step uses the linear ADF test (Leybourne, Newbold, and Vougas 1998), the nonlinear Kapetanios, Shin, and Snell (2003) test (Omay and Yıldırım 2014), or the nonlinear Sollis (2009) test (Omay, Emirmahmutoglu, and Hasanov 2018). Since in the first step, Omay and Yıldırım (2014) and Omay, Emirmahmutoglu, and Hasanov (2018) use the same Model C of Leybourne, Newbold, and Vougas (1998), we apply the Luukkonen, Saikkonen, and Terasvirta (1988) linearity test to the residuals obtained from Model C of the Leybourne, Newbold, and Vougas (1998) model. The results appear in Table 14. The test can be viewed as a linearity test of the residuals (or remaining nonlinearity after de-trending the series with nonlinear trend) and also this nonlinear trend can be classified as nonlinear attractor which the asymmetric mean adjustment occurs around this nonlinear attractor; hence, we are determining this nonlinearity in the residual terms.

As Table 14 shows, the Luukkonen, Saikkonen, and Terasvirta (1988) test results on the residuals of the Leybourne, Newbold, and Vougas (1998) unit-root test do not support linearity. That is, the Model C of Leybourne, Newbold, and Vougas (1998) obtained residuals are not linear. All the test values indicate that a nonlinear structure persists in the residuals. This allows us to exclude the Leybourne, Newbold, and Vougas (1998) as consistent with the data[5] The results for series 1 and 3 support state-dependent nonlinearity, while the results for series 2, 4 and 5 support time-dependent (time-varying) nonlinearity. The tests that comprise the time-dependent structure of series 2, 4, and 5 are Model

[5]As Kapetanios, Shin, and Snell (2003) indicate, the ADF test is still powerful in some form of ESTAR nonlinearity. Therefore, the test results that are obtained as stationarity may be found for this reason..

C of Omay, Emirmahmutoglu, and Hasanov (2018) and Model C of Omay and Yıldırım (2014).

Enders and Lee (2012) observe that the Fourier function can imitate the ESTAR structure. It follows that the Fourier structure can also imitate the Kapetanios, Shin, and Snell (2003) and Sollis (2009) models. This, in our view, can explain the time-varying nonlinearity results. The test results and the unit-root test results are super consistent and imply that the structure underlying the U.K. GDP data conform to either Model C of Omay, Emirmahmutoglu, and Hasanov (2018) or Model C of Omay and Yıldırım (2014). Model C of Omay, Emirmahmutoglu, and Hasanov (2018) employs the AESTAR model, while Model C of Omay and Yıldırım (2014) employs the ESTAR model. Since the AESTAR model nests the ESTAR model, we can safely conclude that the true structure of the U.K. real output series is characterized by nonlinear AESTAR stationarity around a break, which occurs in the intercept and trend. These, in turn, are the main features of the Omay, Emirmahmutoglu, and Hasanov (2018) unit-root test.

V. Conclusions

We employ the nonlinear unit-root test recently developed by Omay, Emirmahmutoglu, and Hasanov (2018), as well as a battery of linear and nonlinear tests, to examine the stationarity of five multi-century historical U.K. series of real output compiled by the Bank of England (Thomas and Dimsdale 2017). Three series span 1270 to 2016 and two series span 1700 to 2016. These datasets represent the longest span of historical real output data available and, thus, provide the environment for which unit-root tests are most powerful.

Linear unit-root tests, such as the ADF and the Ng and Perron (2001) tests, systematically fail to reject the unit root in all five historical real output series.

Nonlinear unit-root tests exhibit mixed success. Time-dependent tests, such as Leybourne, Newbold, and Vougas (1998), which impose on the structure of the data a single break, reject the unit-root hypothesis in four of the five real output series (Series 2, 3, 4, and 5). Oddly, state-dependent tests, such as Sollis (2009), which imposes asymmetric adjustment, reject the null of unit root also in four of the five series (Series 2, 3, 4, and 5). This shows that time-dependent nonlinearity in the form of a single structural break, and state-dependent nonlinearity in the form of asymmetric adjustment can imitate each other. That is, a break in trend and intercept can also be modelled by an AESTAR type nonlinearity. In contrast, time-dependent tests such as Omay (2015) and Çorakcı, Emirmahmutoğlu, and Omay (2017), which impose multiple smooth breaks and one temporary break, respectively, consistently fail to reject the unit-root hypothesis. The results of the Christopoulos and Leon-Ledesma (2010) tests also fail to reject the unit-root hypothesis, confirming that the structure of the series is not inclusive of multiple smooth structural breaks. State-dependent tests with symmetric adjustment, such as Kapetanios, Shin, and Snell (2003), also fail to reject the null of a unit root. Thus, the above-mentioned tests are capable, on their own, of delivering some bits and pieces of empirical information about the structure of the five historical series. Applied to the first series, however, none of these tests provides evidence of stationarity.

In contrast, the key findings of the Omay, Emirmahmutoglu, and Hasanov (2018) unit-root test, Model C, provides strong evidence that the main structure of all the five series is stationary with a break in the intercept and the trend and an asymmetric nonlinear adjustment. This finding is highly significant from the perspective of current macroeconomic debate because it refutes, for the historical U.K. series at least, the most stylized fact that real output follows a non-stationary process. This result is highly at odds with the much more popular nonlinear tests that consider only one facet of the nonlinear process, such as the Kapetanios, Shin, and Snell (2003) unit-root test that allows for state-dependent nonlinearity, but ignores structural breaks, or the Christopoulos and Leon-Ledesma (2010) unit-root test that allows for multiple smooth breaks but ignore asymmetric adjustments.

Finally, since nonlinear unit-root tests identify nonlinearity under the alternative but not under the null, which remains one of linearity, we pay attention to the issue of model identification by means of a sequence of linearity tests. We infer from this sequence that Model C of Omay, Emirmahmutoglu, and Hasanov (2018) is consistent with the time-dependence and AESTAR structure of the data. Therefore, by using this 5-step identification procedure, we introduce an empirical strategy to the researcher for better identification of their data.

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