Efficiency in South African Agriculture: A Two-Stage Fuzzy Approach

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Abstract:

Purpose: The purpose of this paper is to assess the efficiency of agricultural production in South Africa from 1970 to 2014, using an integrated two-stage fuzzy approach.

Design/methodology/approach: Fuzzy technique for order preference by similarity to ideal solution is used to assess the relative efficiency of agriculture in South Africa over the course of the years in the first stage. In the second stage, fuzzy regressions based on different rule-based systems are used to predict the impact of socio-economic and demographic variables on agricultural efficiency. They are compared with the bootstrapped truncated regressions with conditional α levels proposed in Wanke *et al.* (2016a).

Findings: The results show that the fuzzy efficiency estimates ranged from 0.40 to 0.68 implying inefficiency in South African agriculture. The results further reveal that research and development, land quality, health expenditure–population growth ratio have a significant, positive impact on efficiency levels, besides the GINI index. In terms of accuracy, fuzzy regressions outperformed the bootstrapped truncated regressions with conditional α levels proposed in Wanke *et al.* (2015).

Practical implications: Policies to increase social expenditure especially in terms of health and hence productivity should be prioritized. Also policies aimed at conserving the environment and hence the quality of land is needed.

Originality/value: The paper is original and has not been previously published elsewhere.

Keywords: Agriculture; South Africa; Fuzzy TOPSIS; Fuzzy Regression; performance

JEL Classification: C6, D24, Q10, Q28

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1. Introduction

The crucial position of agricultural productivity in the economic and social agenda of developing countries was reiterated by the Malabo Declaration of June 2014, which puts agricultural productivity growth as key to achieving agriculture-led growth and fulfillment of food and nutrition security targets in Africa. To bring hunger in Africa to an end by 2025, the Declaration noted that at least a doubling of agricultural productivity is needed from current levels (Global Strategy to Improve Agricultural and Rural Statistics, 2017). Although the adoption of new technologies have been widely accepted as a means of increasing productivity, output growth is determined by the efficiency with which available technologies are used aside technological innovations (Aye and Mungatana, 2013). In other words, agricultural productivity depends on two components (Global Strategy to Improve Agricultural and Rural Statistics, 2017): the first is the type and quality of the inputs used in the production process and could well be thought of as the production technology. The second component relates to how well these inputs are combined and hence refers to the technical efficiency of the production process. Although agricultural policies have mainly focused on promoting agricultural productivity through technological innovation, there is need to refocus on improving agricultural efficiency given the scarcity of natural resources such as, land and water. Also in the era of pursuing limited environmental footprint of agricultural production (Vlontzos and Pardalos, 2017), the need to rebalance agricultural policies toward improved efficiency cannot be overstressed. A country's agriculture is considered technically efficient if it is producing the maximum potential (frontier) output and inefficient if it is producing below the optimal output given the inputs at its disposal (Ludena, 2012). Achieving high agriculture efficiency has implications for food security and poverty reduction especially for emerging economies. This paper assesses the efficiency of South African agricultural production using the fuzzy technique for order preference by similarity to ideal solution (fuzzy TOPSIS) and fuzzy regressions.

There are a growing number of studies conducted with different methods to assess performance in agriculture. These studies are often grouped into two main approaches, i.e., parametric and non-parametric (Aye and Mungatana, 2011). The most popular parametric method is known as the stochastic frontier approach (SFA), whereas the most popular non-parametric method is data envelopment analysis (DEA) (Tan *et al.*, 2010; Aye and Mungatana, 2011). When put into perspective, however, non-parametric methods are widely used in agricultural efficiency in various countries and regions around the globe (Malana and Malano, 2006; Gomes *et al.*, 2009; Heidari *et al.*, 2012; Aye and Mungatan, 2013; Toma *et al.*, 2015; Iliyasu and Mohamed, 2016; Iliyasu *et al.*, 2016; Vlontzos and Pardalos, 2017; Nsiah and Fayissa, 2017; Gebrehiwot, 2017). Broadly speaking, parametric models allow different types of inferences to be drawn directly from the performance estimates (Kumbhakar *et al.*, 2013). Non-parametric models on the other hand fall short because they need statistical properties for a robust examination of the roots of inefficiency in light of contextual variables. Thus far, bootstrapping – i.e., performance error resampling – is the only statistical tool available to remedy this situation (Bogetoft and Otto, 2010). Studies on agriculture efficiency in South Africa include Piesse *et al.* (1996), van Zyl *et al.* (1995), Pauw *et al.* (2007), Meliko *et al.* (2010), Lovo (2010), Baloyi *et al.* (2012), Dobrowsky (2013), Obi and Kibirige (2014), Kibirige and Obi (2015) and Conradie and Piesse (2015). These studies were conducted mainly at the micro- or meso-level implying that the results may not be generalized for the entire economy. Majority of the studies on agriculture efficiency used DEA in the first stage to obtain efficiency values and OLS or correlation in the second stage to analyze efficiency determinants. A few, however, used the stochastic production frontier, a one-stage approach that simultaneously estimates the production function (with its associated efficiency scores) and the inefficiency effect model for examining determinants of efficiency.

In a traditional DEA model, performance is calculated using *ex post* information (Berger and Humphrey, 1997; Charnes *et al.*, 1978) collected from historical data with respect to inputs and outputs. Battese and Rao (2002) showed that examining performance with DEA presents better discrimination – i.e., efficiency scores that are less biased toward one – if this set of inputs/outputs is considered under a metafrontier that encompasses several years of observation, similarly to what is emulated within the ambit of multi-criteria decision-making models (MCDM). MCDM are also non-parametric by nature because there are no underlying statistical properties whatsoever. Thus far, to the best of our knowledge, MCDMs, such as TOPSIS, have not been used to assess efficiency in agricultural production as is evidenced in the literature review section.

As regards the fundamentals of TOPSIS, this MCDM is based on the concept that the positive ideal alternative has the best level for all criteria considered or for the input/output set, while the negative ideal is the one with the worst values for the input/output set (Wanke, Azad, Barros and Hadi-Vencheh, 2016). Despite its general resemblance to DEA where outputs may be maximized and/or inputs minimized, the determination of the weights of the relative importance of each criteria (namely, benefits and costs, or simply outputs and inputs, respectively) is exogenously defined in TOPSIS, whereas in the case of DEA these weights are endogenously calculated within the ambit of the model (Behzadian *et al.*, 2012). Besides, TOPSIS is computationally simpler because there are virtually no constraints with respect to the number of criteria (inputs/outputs) that can be assessed (Wanke, Azad, Barros and Hadi-Vencheh, 2016).

Although applying these non-parametric methods might be sufficient to determine performance levels, they do not afford details of how contextual variables impact them. To remedy this, several studies proposed two-stage approaches for measuring and explaining performance in different sectors using either DEA or any MCDM as cornerstones (e.g., Wanke, Azad, Barros and Hadi-Vencheh, 2016, Wanke, Barros and Nwaogbe, 2016; Wanke, Barros and Faria, 2015). In the first stage, these methods are used to compute performance levels, while regression models are employed in the second stage to explain their respective drivers (Wanke, Pestana Barros and Chen, 2015). It is important to mention that the underlying uncertainty in performance levels and, therefore, in the input/output set and their relationships with contextual variables, encompasses both randomness and fuzziness. While randomness is related to the intrinsic statistical fluctuation inherent to the data that were collected, fuzziness is related to the underlying vagueness associated to the data collection (Wanke, Barros and Emrouznejad, 2015).

This paper, therefore, fills a literature gap by analyzing and exploring the sources of efficiency in South African agriculture by using a two-stage fuzzy approach, in light of the inherent uncertainty that surrounds the collection of agricultural data in South Africa, scattered over more than four decades. It innovates in this context not only by applying fuzzy TOPSIS to assess efficiency levels but also by adopting different fuzzy regressions to assess the impact of demographic and socio-economic variables on these fuzzy efficiency levels. These results are further compared to those obtained using bootstrapped truncated regressions with conditional α-levels, as proposed in Wanke, Barros and Emrouznejad (2015). This combination of fuzzy and probabilistic approaches also represents a contribution to the emerging literature on possible analytical venues within the ambit of two-dimensional Fuzzy Monte Carlo Analysis (2D FMCA).

In this research, the choice of the techniques used in the two-stage approach adopted here observes the fact that new developments in scientific computing technologies, such as the software R, can be used to support systematic theory testing and development (James *et al.*, 2013). Here, fuzziness implies in adopting the fuzzy version of the TOPSIS technique in the first stage, while in the second stage bootstrapped truncated regressions are employed as proposed by Wanke, Barros and Emrouznejad (2015) to evaluate the sign and significance of contextual variables on performance scores. Fuzzy regressions are further used to boost the predictive power of these significant variables in light of different modeling assumptions. Fuzzy regressions typically do not require any a priori knowledge of a bound on the disturbance and noise and of a bound on the unknown parameters values. They can be used for the robust and adaptive identification of slowly time varying non-linear systems (Kumar *et al.*, 2004).

In summary, the contributions of the present research are the following. First, this research evaluates the evolution of the relative efficiency in South African agricultural production, adding to the scarce literature on this subject. Second, this research uses a fuzzy TOPSIS method to compute efficiency based on triangular fuzzy numbers (TFN) to assess vagueness in agricultural inputs and outputs. As a matter of fact, the agricultural outputs and inputs present different forms of uncertainty within their relationships. For example, credit granting is an input embedded in fuzziness because of the *ex ante* risks associated with farming non-performing loans (World Bank, 2009; Maurer, 2014). On the other hand, the value of the production, which is not a constant number, changes on account of the market value of the commodities. To evaluate agricultural efficiency more realistically and accurately, this study employs the fuzzy TOPSIS model with data specified in bounded forms to capture vagueness and uncertainty to some extent. Third, this research also expands the existing literature due to the use of fuzzy regressions and different rule-based systems to predict and interpret the role of major contextual variables in achieving higher levels of efficiency. Fourth, our analysis covers the period from 1970 to 2014. Finally, our analysis is based on a representative timespan of a given country..

Results presented in this research constitute a contribution to the growing literature on agricultural performance at the country level. These results are consistent with and extend the findings of recently conducted studies as shown in the literature review section, thus adding to the body of knowledge. As regards the findings presented here, although the socio-economic and demographic variables are positively related to agricultural efficiency, it is worth noting that capital accumulation, proxied by the GINI, is also positively related to efficiency levels.

The remainder of the paper is organized as follows: Section 2 presents the contextual setting; Section 3 covers the literature review; and Section 4 presents the methodology. The empirical results are presented and discussed in terms of policy implications in Section 5. Conclusions follow in Section 6.

2. Contextual Setting

Agricultural productivity in South Africa has been fluctuating over the years. The growth is stagnant in some years and in other years, it is either at an increasing rate or at a decreasing rate. For instance, the growth of agricultural productivity was 0.65% per annum prior to 1965, no growth in 1965, and 2.15% between 1965 to 1981, 3.98% between 1981 to 1989, 0.28% between 1989 and 1994 and stagnant around 2008 (DAFF, 2011; Liebenberg and Pardey, 2010). As far back as 1910/11, agriculture contributed about 21% of the total GDP and employed 781,359 people with 54% of these numbers permanently employed in agriculture. Hartzenberg and Stuart (2002) find that the agricultural sector was one of only a few sectors that experienced positive total factor productivity (TFP) growth over all the time periods examined, while Vink (2000) and Thirtle et al. (2000) find evidence of a recovery in agricultural TFP growth during the 1990s. Despite the growth of the agricultural economy in absolute terms from R10.9 billion (US$1.7 billion) in 1910 to R40.9 billion (US$6.4 billion) in 2010 (both measured in 2005 prices), its contribution to GDP has shrunk to as low as 2.5% (Liebenberg, 2013). The number of commercial farmers have dropped from 76,149 in 1918 to 39, 982 by 2007. Looking at the horticultural, field crop and livestock components of South Africa's agriculture, there has been noticeable changes. As at 1918, the contribution of livestock, field crop and horticultural to agriculture GDP was 55%, 33% and 12%

respectively. By 2010, horticulture's contribution had more than doubled to 26.1% while livestock and field crops share dropped to 50.1% and 23.3% respectively (Liebenberg, 2013). The fluctuation in South Africa's agricultural productivity has been attributed to a number of factors including cost – price squeeze experienced by agricultural producers (Vink, 2000), input prices relative to output prices (van Zyl et al., 1993; Kirsten et al, 2003), quick adjustment of farmers to the effects of deregulation (Liebenberg and Pardey, 2010), inflation rates, increased use of mechanization, fertilizer, herbicides, pesticides (DAFF, 2011) and other inputs (Conradie et al., 2009).

Theoretically, agricultural productivity may be improved by either increasing input use especially acreage expansion, improvement in resource use efficiency and or technological change derived from use of new technologies (Aye, 2011). Similar idea was shared by Yang and Xu (2012) who noted that reasons for agricultural productivity decline and sometimes slow growth are increasingly resource scarcity, limited technological innovation, environmental degradation, and insufficient agricultural policy support among others. Where acreage expansion is not possible as it is in many developing countries given increasing population, productivity increase will be left with the option of improved efficiency and technological change. This study focuses on efficiency in agriculture in South Africa. Understanding efficiency therefore has implications for agricultural productivity and hence food prices. This is particularly important for South Africa given the continuing food price rises which has raised concern among consumers and policy makers. The year 2014 started with increase in inflation rate of which energy and food prices were the main contributors. For instance, between December 2013 and January 2014, food prices rose by 1.6% (Statistics South Africa, 2014). Pauw et al. (2007) noted that increased efficiency gains in domestic agricultural sectors is associated with a reduction in food prices, which is an important contributor to the fight against poverty. One of the emphasis in South African agricultural policy is to increase the income of the poorest group by making small-scale agriculture more efficient and internationally competitive, so as stimulate increase in number of small-scale and medium-scale farmers and conserve agricultural natural resources (Meliko et al., 2010; IPTRID, 2000; NDA, 1998). Since South African government emphasises the need for resource use efficiency, a natural step towards this would be to unveil how efficient the agricultural sector is currently and to seek for ways of enhancing it. Hence, the objective of this study and its significance cannot be overemphasized.

3. Literature Review

There is a growing literature on agricultural efficiency. We start with studies outside South Africa. For instance, Vlontzos and Pardalos (2017) assess GHG emissions efficiency and efficiency change of 25 EU countries agriculture sectors using data from 2006 to 2012. Results based on DEA Window analysis and artificial neural networks show that efficiency levels differ significantly across EU countries with less developed countries and countries that depend largely on arable crop production having low efficiency rates. Parichatnon *et al.* (2017) examine the technical efficiency of rice production in four regions of Thailand using a three-stage DEA during the period from 2006 to 2015. They find relatively high level of technical efficiency ranging from 75 to 94 percent with the highest obtained for the northeastern region (94 percent). Also, environmental factors (temperature and rainfall) were found to have significant effect on the production efficiency. Gebrehiwot (2017) analyzed the impact of agricultural extension on farmers' technical efficiencies in Ethiopia using a stochastic production frontier approach. The findings indicate that the average level of technical efficiency is 48 percent and that variables such as gender, the number of crops grown and the number of dependants explained the differences in efficiency levels.

Iliyasu *et al.* (2016) estimate the bias-corrected technical efficiency of different culture systems and species of freshwater aquaculture in Malaysia using bootstrapped DEA. The findings indicate that all technical efficiency scores for all culture systems and species are below the optimal level (i.e. 100 percent). Depending on the culture system and specie, the scores range from 63 to 80 percent. In addition, the results show that farmers' experience, contact with extension workers and household size have a positive and statistically significant impact on technical efficiency. Age of the farmers has a negative and statistically significant impact on technical efficiency. In a related paper, Iliyasu and Mohamed (2016) use DEA in the first stage to estimate technical efficiency and OLS in the second stage to analyze the determinants of efficiency of freshwater pond culture systems in Peninsular Malaysia. Findings show a mean efficiency of 86 percent and that farmers' age, experience, extension training and water management have positive and statistically significant impacts on technical efficiency. Koirala *et al.* (2016) investigate the impact of land ownership on the productivity and technical efficiency of rice farmers in the Philippines using a 2007–2012 Loop Survey and a stochastic frontier function method. The results show that mean technical efficiency is 79 percent, and that educated females and farmers leasing land have higher technical inefficiency. Labajova *et al.* (2016) use multidirectional efficiency analysis (MEA) to calculate technical efficiency indices of each input and output for different pig production types in Sweden and farm efficiency indices by DEA. They further use correlation analyses to identify which of the "farm-specific characteristics"were related to the efficiency indices. Efficiency scores from MEA approach ranges from 85 to 97 percent while those from DEA ranges from 93 to 90 percent. While advisory services, farm location and housing practices were not significant, use of written instructions for feeding and for preventing infectious diseases was associated with higher technical efficiency.

Mekonnen *et al.* (2015) examine how different components of an agricultural innovation system interact to determine levels of technical inefficiency in 85 developing country agriculture using latent class stochastic frontiers and data from 2004 to 2011. The mean technical efficiency score in class 1 countries is 44.1 percent whereas it is 62.7 percent for countries in class 2. Mobile phone subscriptions and the number of scientific and technical journal articles were found to improve technical efficiency of agricultural production in these countries. Aravindakshan *et al.* (2015) use slack-based DEA model to estimate the technical energy input efficiency of wheat farmers under different tillage options in the eastern Indo-Gangetic Plains in South Asia. Results show that the mean efficiency scores ranged from 68 to 91 percent, with conservation tillage farmers being more efficient than traditional ones. Subsequent analysis to examine determinants of energy efficiency using fractional regression model show that conservation tillage training and experience, educational level, credit, split application of NPK fertilizers and distance to input market had positive and significant effect on technical energy efficiency. However, farm size, farmers' reliance on input dealers for crop management advice, negatively affected energy efficiency, distance of farm from main roads and distance to conservation tillage extension advice had negative effect. Using DEA models with production trade-offs between different crops, Atici and Podinovski (2015) analyze the efficiency of agricultural farms in eight regions of Turkey. Results based on both VRS and CRS versions of the DEA model show that efficiency estimates fall between 21 and 98 percent depending on the range of the trade-off. Toma *et al.* (2015) employ DEA to analyze the agriculture efficiency in plain, hill and mountain areas. They find that in majority of the countries, the overall efficiency of agriculture is not reached, with these regions needing to decrease the input levels (especially work hours that are too high compared with productivity) or to increase the output levels (production value) through a better use of fixed capital and higher yields. Using local maximum likelihood (LML) methods, Guesmi *et al.* (2015) assess the technical efficiency of arable crop Kansas farms and compare results with conventional methods. They find that technical efficiency scores derived from the LML approach (90.5 percent) are higher than those of the DEA model under CRS (80.8 percent) and SFA (80.4 percent) and close to DEA–VRS (91.7 percent) ratings.

Ndlovu *et al.* (2014) analyze the productivity and efficiency of maize production under conservation agriculture using panel data from 2008 to 2010 on smallholder farming households across 15 rural districts in Zimbabwe. Using joint stochastic production frontier, the productivity and technical efficiency between conservation agriculture and conventional farming were estimated and compared. The results show that although farmers produce more in conservation agriculture compared with conventional farming, their technical efficiency levels are essentially equal (68 percent) in both technologies. While physical asset and time were positively related to technical efficiency, land was negatively related to efficiency. Ray and Ghose (2014) use DEA to evaluate technical efficiency of individual states over the years 1970–1971 through 2000–2001. Overall output and input efficiency is 79 and 85 percent, respectively, while the Pareto Koopmans efficiency was 70 percent. The second stage regression results show that a number of institutional and demographic factors including education, export orientation, literacy rate, agricultural credit, crop diversification have positive effect on efficiency while Gini ratio of land distribution has negative effect. Ogundari (2014) investigates African agricultural efficiency levels and its drivers over the years based on 442 frontier studies using meta-regression analysis. The results show that the mean efficiency estimates from these studies decrease significantly as year of survey in the primary study increases. Further, education, experience, extension and credit are the major drivers of agricultural efficiency levels in Africa. Baležentis *et al.* (2014) evaluate the efficiency of Lithuanian family farms using data from 2004 to 2009. They use bootstrapped DEA in the first stage and a non-parametric regression in the second stage to assess the impact of selected determinants on efficiency. Results show that average technical efficiency is 50 percent and that production subsidies might be having a negative effect on efficiency.

Diagne *et al.* (2013) investigate rice productivity in the Senegal River Valley using panel data from 2002 to 2006 and a fixed effects simplified translog production function. They obtained technical efficiency scores ranging between 55 and 60 percent. Further, they find that fertilizer, herbicides, bird-chasing efforts, date of sowing and the use of post-harvest technologies such as a thresher-cleaner significantly improved the technical efficiency of rice producers. Bayyurt and Yılmaz (2012) estimate the efficiency of 64 countries for the period 2002–2008 using DEA and subsequently examine the impacts of governance and education on agricultural efficiency using panel data regression. They find that while regulatory quality and type of country (developed or developing) has positive and significant impact of agricultural efficiency, education has a negative effect. Efficiency estimates are however not reported. Hadi-Vencheh and Matin (2011) utilizes an imprecise DEA (IDEA) model to evaluate efficiency of Iranian wheat producer provinces. Out of the 15 provinces studied, 4 were found to be efficient where the remaining 11 were inefficient. This is robust to both irrigation and dry wheat farming. Zhu and Lansink (2010) analyze the impacts of CAP subsidies on technical efficiency of crop farms in Germany, the Netherlands and Sweden using output distance function estimated together with an inefficiency effects model. Their results show that the average technical efficiency from 1995 to 2004 is 64 percent in Germany, 76 percent in the Netherlands and 71 percent in Sweden. They find a negative impact of the share of total subsidies in total farm revenues on technical efficiency in all three countries but the impact of the share of crop subsidies in total subsidies is mixed. Larsén (2010) analyzes the effects of machinery-sharing arrangements on farm efficiency using data for Swedish crop and livestock farms for the time period 2001–2004. Efficiency scores obtained from bootstrap DEA suggest that efficiency is, on average, higher among partnership farms compared to non-partnership farms. Second stage analysis based on Tobit regression shows that partnership arrangements have a positive and statistically significant impact on farm efficiency.

Lio and Hu (2009) use the stochastic frontier production function estimated together with the inefficiency effects model to examine the relationship between six governance indicators and agricultural efficiency for a panel of 118 countries over the 1996, 1998, 2000 and 2002 periods. Their results show that the aggregate agricultural efficiency for all countries over 1996–2002 period is 64.7 percent. For high income countries, the efficiency score is 76.4 percent, upper middle income is 57.7 percent, lower middle income is 64.6 percent and for low income countries is 62.1 percent. Gregg (2009) evaluates the efficiency of Cherokee agriculture using output distance function in the first stage and truncated regression in the second stage. Results show that racial hierarchy was a significant determinant of agricultural efficiency. He also finds a significant inverse-U relationship between efficiency and farm size and significant positive effect of market access, farming experience, age and soil quality. Their results further show that different dimensions of governance have different impacts on agricultural efficiency. Barnes (2008) analyzes the technical efficiency of the cereals, dairy, sheep and beef sub-sectors of Scottish agriculture over the period 1989–2004. Results based on stochastic production frontier indicate that average technical efficiency scores vary between 71 percent for cereals and 82 percent for sheep farms. Also, less favored area' status, farms which either have land or no land within an environmentally sensitive area, and tenure has positive and significant effect on almost all sub-sectors while utilized agricultural area and trend variables have negative effect. Debt ratio has positive and significant effect on dairy and sheep sector's efficiency only.

For South Africa, there are also some studies that have analyzed agriculture efficiency. For instance, van Zyl *et al.* (1995) use DEA to estimate efficiency of farms in the former home land in South Africa. Their findings show that for KaNgwane, the mean level of total efficiency is 35.8 percent, Lebowa (42.7 percent) and Venda (47.6 percent). Using ordinary regression, they find an inverse relationship between farm size and efficiency in the commercial farming areas for the range of farms analyzed. Piesse *et al.* (1996) use DEA and 1990–1991 maize production data for small-holders in the Northern Transvaal homelands of KaNgwane, Lebowa and Venda to assess efficiency. They show that mean overall efficiency score is 35.8 percent in KaNgwane, 42.7 percent in Lebowa and 47.6 percent in Venda. Pauw *et al.* (2007) rather looked at the effect of agricultural efficiency gain on welfare in South Africa using a computable general equilibrium model. The simulation results show that technological advances in agriculture leads to welfare gains from declining prices albeit a negative impact on agricultural employment.

Meliko *et al.* (2010) determine the efficiency of the small-scale irrigation sector of Limpopo province for the production 2006/2007 year using policy analysis matrix. Their results show that all 12 production systems studied were profitable under market condition with existing policies and all except dry land maize had comparative advantage suggesting efficiency in the systems. Lovo (2010) examines liquidity constraints and household technical efficiency using a sample of farm households in the KwaZulu Natal province of South Africa. Results based on DEA revealed the presence of large inefficiencies with Vhembe having the highest efficiency of 46 percent while Zululand had the lowest (26 percent). The overall efficiency for all nine districts is 36 percent. OLS and 2SLS regression results show that access to liquidity, income diversification and district-level employment rate have a positive effect on household technical efficiency. Baloyi *et al.* (2012) investigate the determinants of efficiency among 120 small-scale maize producers in GaMothiba, Limpopo province in South Africa. Using Cobb-Douglas, their results show that there is significant positive relationship between farm size and fertilizer with technical efficiency while cost of tractor hours (the proxy for capital) had negative effect. Using both DEA and SPF models, Dobrowsky (2013) assessed technical and allocative efficiency in the organizational form of agriculture in South Africa. The results show that mean technical efficiency score is 96 percent under the VRS DEA and 80 percent under the CRS DEA while it is 86 percent under stochastic frontier analysis.

Obi and Kibirige (2014) examine the relationship between farmers' goals and efficiency in small-scale maize production in Eastern Cape province of South Africa. On average, smallholder farmers were technically inefficient in maize production with a score of about 44 percent. The perceived farmers' goal found to have a positive and significant impact on technical efficiency was farm status, while farmers' goal related to business (profit maximization) had a negative relationship with technical efficiency. Farm and farmers' characteristics found to be significantly related to technical efficiency included household size, years spent in school, access to training on agronomy, crop incomes and government social grants. Kibirige and Obi (2015) estimate the allocative and technical efficiencies of smallholder farmers in Eastern Cape province of South Africa using Cobb-Douglas production function and stochastic frontier analysis. The findings indicate smallholder farmers were, however, technically efficient at approximately 98.8 percent and determinants of this efficiency based on OLS regression include household size, farming experience, use of agro-chemicals, off-farm income, and gross margins earned from maize, and household commercialization level. Conradie and Piesse (2015) employ DEA to benchmark extensive sheep operations in Laingsburg in the Central Karoo, South Africa, with data from the 2012 production season. The top third producers' overall efficiency score was 99 percent while the bottom third was 34.6 percent. Correlation analysis shows that the overall efficiency was correlated with stocking density, flock size, unit production cost and profitability, cumulative family experience of farming and the use of family labor. Nsiah and Fayissa (2017) estimate the trends in the agricultural sector production efficiency using data for 49 African countries from 1995 to 2012. Employing Malmquist Total Factor Productivity Index and dynamic GMM, they find that 12 African countries in including South Africa were on the efficiency frontier each year of the analysis while 17 countries were never efficient in any of the period under consideration. Their results further show that agriculture aid, capital infrastructure for the agriculture industry, sanitation and good governance are the main determinants of agriculture efficiency and its growth. Clearly, the foregoing shows that none of these studies have employed fuzzy TOPSIS in analyzing efficiency in agriculture. Hence, we contribute in this regard.

4. Methodology

This section presents the major methodological steps adopted in this research. After presenting in Section 4.1 the data collected in terms of inputs, outputs, and contextual – socio-economic and demographic - variables, the two stage fuzzy approach is explained in detail. Section 4.2 presents a preamble on Triangular Fuzzy Numbers and Triangular Fuzzy Matrices, which is deemed necessary to follow the subsequent sections. Section 4.3 depicts the Fuzzy TOPSIS method used in the first stage, while Section 4.4 sets out the fundamentals and origins of fuzzy regression. In section 4.5, the bootstrapped truncated regression with conditional α-levels, proposed by Wanke et al. (2016a), is reviewed in light of the emerging literature on 2-Dimensional Fuzzy Monte Carlo Analysis. At last, section 4.6 addresses a number of different possible rule-based systems embedded within the ambit of fuzzy regressions, as discussed in Riza et al. (2015).

4.1. The data

The data on South African agriculture were obtained from different sources such as FAOSTAT and World Bank Development Indicators and encompassed the period from 1970 to 2014. The inputs and the outputs considered observed not only those commonly found in the literature review but also the availability of data. The input variables included the land or cultivable area (in 1000 Ha); the fertilizer or the consumption of nutrients (in tonnes of nutrients); the labor or the employment in agriculture (in 1000 persons); and the capital stock of equipment used in agriculture (in million Rands, 2005 prices). Output variables included a desirable one, the total agricultural production in value (in million Rands, 2005 prices) and an undesirable one, the CO_2 emissions (in CO_2 equivalent Gigagrams). Their descriptive statistics are presented in Table 1. Within the ambit of the Fuzzy TOPSIS model depicted in Section 4.3, the inputs and the undesirable output were considered with a negative sign, and the desirable output, with a positive one.

In addition, a number of socio-economic and demographic variables were collected to explain differences in the efficiency levels. These variables are also presented in Table I and are related to the: agriculture research spending (R&D), measured as a percentage of GDP; land quality, measured as a percentage of cultivable area that is dedicated to permanent crops; health expenditure, measured as a percentage of GDP; education level, measured as the primary gross enrollment ratio of both sexes (percent); annual GDP growth (percent); annual population growth (percent); net inflows of foreign direct investment (FDI), measured as a percentage of GDP; and the GINI inequality index, as estimated by the World Bank.

Based on the literature (Mekonnen *et al.*, 2015; Gregg, 2009; Obi and Kibirige, 2014; Ray and Ghose, 2014; Toma *et al.*, 2015; Zhu and Lansink, 2010; Kinkingninhoun-Mêdagbé *et al.*, 2010; Quiroga *et al.*, 2014; Ostry *et al.*, 2014; *The Economist*, 2014), our a priori expectations on the contextual variables are: R&D (+), land quality (+), health expenditure (+), education (+), FDI (+), GDP growth (+), GINI(±) and population growth (±).

Table 1. Descriptive statistics for the inputs, outputs and the contextual variables

Variables		Min	Max	Mean	SD	CV
Input	Land	12446	14197	13172	531.355	0.040
	Fertilizer	347260	*24378253*	5682522	6963239	1.225
	Labour	613.7	*11964.2*	4362.2	3498.714	0.802
	Capital	162.2	14832.9	4139.1	4257.258	1.028
Output	Production	18869	46456	32511	7837.56	0.241
	CO_2	6760	35679	28010	4092.073	0.146
Socio-economic and demographic variables	R&D	1.970	9507.796	1418.654	2446.129	1.724
	Land Quality	0.200	0.643	0.355	0.073	0.206
	Education	75.41	117.34	99.53	13.18	0.132
	Health Expenditure	7.275	8.933	8.049	0.518	0.064
	Population	1.267	2.597	2.035	0.426	0.209
	GDP Growth	-2.137	6.621	2.598	2.258	0. 869
	FDI	-0.866	5.983	0.834	1.242	1.489
	GINI	57.77	75.96	64.68	4.403	0.068

4.2. Background on TFNs and TFMs

TFNs (triangular fuzzy numbers) are commonly found in Fuzzy TOPSIS uses (Izadikhah, 2009). On the other hand, it is well known that the matrix formulation of a mathematical formula provides additional ease of handling for a specific problem (Shyamal and Pal, 2007). This being the case, Shyamal and Pal (2007) introduced and derived the properties of TFMs (triangular fuzzy matrices) in a seminal work. On the other hand, fuzzy matrices were introduced for the first time by Thomason (1977), who discussed the convergence of powers of fuzzy matrices. In this line, several authors have presented a number of results on the convergence of the power sequence of fuzzy matrices (Hashimoto, 1983; Kandel, 1996; Kolodziejczyk, 1988). Broadly speaking, a Triangular Fuzzy Number "a" may be represented by $(a-\alpha, a, a+\beta)$, where α and β represents, respectively, the spreads to the left and to the right from a. Alternatively, fuzzy numbers of these kind can be expressed such as $\langle a, \alpha, \beta \rangle$. If M is a TFN expressed as $M = \langle a, \alpha, \beta \rangle$ its membership function is given by:

$$\mu_M(x) = \begin{cases} 0 & for\ x \leq m - \alpha \\ 1 - \frac{m-x}{\alpha} & for\ m - \alpha < x < m \\ 1 & for\ x = m \\ 1 - \frac{x-m}{\beta} & for\ m < x < m + \beta \\ 0 & for\ x \geq m + \ \beta \end{cases} \qquad (1)$$

The membership function equals 1 when *x* reaches the mean value, *m*. Besides, considering α and β to be, respectively, the spreads to the left and to the right of the TFN M, it is possible to affirm that this number is symmetrical around the mean if both spreads assume the same value, that is, if $\alpha = \ \beta$.

Several researches have attempted to define the arithmetic operations of TFNs over the course of time. They were pioneered by Dubois and Prade (1980), who introduced the definitions of their arithmetic operations. Consider $M = \langle m, \alpha, \beta \rangle$ and $N = \langle n, \gamma, \delta \rangle$ to be two Triangular Fuzzy Numbers. Their arithmetic operations are defined as it follows (Shyamal and Pal, 2007):

Addition: $M + N = \ \langle m + n, \alpha + \gamma, \beta + \delta \rangle$.

Scalar multiplication: If λ is scalar, $\lambda M = \ \langle \lambda m, \lambda\alpha, \lambda\beta \rangle$ when $\lambda \geq 0$.
Otherwise $\lambda M = \ \langle \lambda m, -\lambda\beta, -\lambda\alpha \rangle$ when $\lambda \leq 0$. Particularlly, $-M = \ \langle -m, \beta, \alpha \rangle$

Subtraction: $M - N = \ \langle m, \alpha, \beta \rangle - \ \langle n, \gamma, \delta \rangle = \ \langle m - n, \alpha + \delta, \beta + \gamma \rangle$.
Given two triangular Fuzzy Numbers, M and N, their addition, subtraction, and scalar multiplication, i.e., $M \ + N, M - N$ and λM are all TFNs.

Multiplication: One may show that the membership function shape of $M.N$ is not necessarily triangular. However, if the spreads of M and N are sufficiently small compared to their mean values m and n, then this shape follows a trianglular form. A robust decision rule is given next (Shyamal *and Pal*, 2007):

(a) When $M \geq 0$ and $N \geq 0$ ($M \geq 0$, if $m \geq 0$)
$M.N = \ \langle m, \alpha, \beta \rangle.\langle n, \gamma, \delta \rangle \ \simeq \ \langle mn, m\gamma + n\alpha, m\delta + n\beta \rangle$.

(b) When $M \leq 0$ and $N \geq 0$
$M.N = \ \langle m, \alpha, \beta \rangle.\langle n, \gamma, \delta \rangle \ \simeq \ \langle mn, n\alpha - m\delta, n\beta - m\gamma \rangle$

(c) When $M \leq 0$ and $N \leq 0$
$M.N = \langle m, \alpha, \beta \rangle.\langle n, \gamma, \delta \rangle \ \simeq \ \langle mn, -n\beta - m\gamma, -n\alpha - m\gamma \rangle$

When spreads are not small compared with mean values, a better approximation is given next:

$\langle m, \alpha, \beta \rangle.\langle n, \gamma, \delta \rangle \ \simeq \ \langle mn, m\gamma + n\alpha - \alpha\gamma, m\delta + n\beta + \ \beta\delta \rangle$ for $\mathrm{M} > 0, N > 0$.

On the other hand, a Triangular Fuzzy Matrix (TFM) of order $m \ \times n$ can be given as $\mathrm{A} = (\mathrm{a}_{\mathrm{ij}})_{\mathrm{m} \times \mathrm{n}}$, where $\mathrm{a}_{\mathrm{ij}} = \ \langle \mathrm{m}_{\mathrm{ij}}, \alpha_{\mathrm{ij}}, \beta_{\mathrm{ij}} \rangle$ is the ijth elements of A, m_{ij} is the mean value of a_{ij} and $\alpha_{\mathrm{ij}}, \beta_{\mathrm{ij}}$ are, respectively, the left and right spreads of a_{ij}. Likewise classical matrix algebra, let us consider the following operations involving TFMs, given that $\mathrm{A} = (\mathrm{a}_{\mathrm{ij}})$ and $\mathrm{A} = (\mathrm{b}_{\mathrm{ij}})$ are two TFMs of same order. In such cases, the following relationships are observed (Shyamal and Pal, 2007).

(i) $A + B = (a_{ij} + b_{ij})$

(ii) $A - B = (a_{ij} - b_{ij})$,

(iii) For $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{n\times p}$, C= $A.B = (c_{ij})_{m\times p}$

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \,.\, b_{kj} \ , i = 1,2,\dots, m \text{ and } j = 1,2,\dots,p$$

(iv) $A' = (a_{ji})$ (the transpose of A)

(v) $k.A = (ka_{ij})$, where k is a scalar.

4.3. Fuzzy TOPSIS

The TOPSIS method (Technique for Order Preference by Similarity Ideal Solution) suggested by Hwang and Yoon (1981), belongs to the group of pattern linear ordering methods of multidimensional objects. Broadly speaking, the ordering of objects from the best one to the worst one considering an assumed synthetic measure, which is not subjected to a direct measurement, belongs to the task of linear ordering (Dudek and Jefmanski, 2015). A characteristic feature of TOPSIS is a way to evaluate a synthetic criterion's values, which takes into consideration the distance of an evaluated object from a positive-ideal solution as well as from a negative-ideal solution. Barros and Wanke (2015) and Wanke et al. (2015a, 2015b) are examples of applications of the TOPSIS method in efficiency measurement problems.

The fuzzy TOPSIS method was proposed by Chen (2000). An example of applying this method can be found, among others, in the studies of: Chang and Tseng (2008), Uyun and Riadi (2011), Madi and Tap (2011), Yayla *et al.* (2012), Jannatifar *et al.* (2012), Erdoğan *et al.* (2013), Ataei (2013) and Kia *et al.* (2014). The major difference between the fuzzy TOPSIS method and the original one is that fuzzy numbers are used in the computation of the rankings and the performance scores of the observations. In this research, TFN are used to capture vagueness in agricultural inputs and outputs. As previously discussed, a TFN may be represented by (l, m, u) where l, m and u denote, respectively, the minimal, the mean and the maximal value of a given variable. A TFN may be symmetrical around the mean or not. Broadly speaking, TFNs are the most common and intuitive form for representing vagueness since they allow inputs and outputs to be measured simultaneously in terms of their mean, minimal and maximal values. TFNs constitute a simple but effective way for assessing vagueness (Wanke, Azad, Barros and Hadi-Vencheh, 2016).

Several assumptions were made about the nature of output and input data in South African agriculture over the course of time to capture the vagueness in the collection. First, variations in inputs and outputs were considered to be linear. Second, inputs and outputs were all represented by means of TFNs. Third, the minimum inputs and outputs during the researched timespan were considered as the lower values of the TFNs. Likewise, the maximum values of inputs and outputs were considered as the upper values of the TFNs, while the mean values of inputs/outputs were considered as the middle values of the respective TFNs.

Let us assume that a certain set of objects $A = \{Ai \mid i = 1,..., n\}$ and a set of criteria $C = \{Cj \mid j = 1,...,m\}$, where $\tilde{X} = \left\{\tilde{x}_{ij}\middle| i = 1, \dots, n, j = 1, \dots, m\right\}$ stand for a set of fuzzy evaluation criterion and $\widetilde{W} = \left\{\widetilde{w}_j \middle| \, j = 1, \dots, m\right\}$ a set of fuzzy weights. The linear ordering of objects with the application of the fuzzy TOPSIS method with the above outlined assumptions requires the accomplishment of the following steps (Chen, 2000):

Step 1. Calculation of normalized fuzzy evaluation criteria:

$$\tilde{Z}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{x}_{ij}^2}} \quad , \qquad i = 1, \dots, n; \;\; j = 1, \dots, m. \tag{2}$$

Step 2. Calculation of weighted normalized fuzzy evaluation criteria:

$$\tilde{v}_{ij} = \widetilde{w}_j \tilde{z}_{ij} \tag{3}$$

Step 3. Appointing positive-ideal solution A^+ and negative-ideal solution A^- development:

$$\tilde{A}^+ = \{\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_m^+\} = \{(\max_i \tilde{v}_{ij} \mid \, j \in \, J_1), (\min_i \tilde{v}_{ij} \mid j \in \, J_2) \;\mid\; i = 1, \dots, n\} \tag{4}$$

$$\tilde{A}^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_m^-\} = \{(\min_i \tilde{v}_{ij} \mid \, j \in \, J_1), (\max_i \tilde{v}_{ij} \mid j \in \, J_2) \;\mid\; i = 1, \dots, n\} \tag{5}$$

where J_1 and J_2 are respectively the benefit criterion and the cost criterion.

Step 4. Calculation for each object of a distance from positive-ideal solution d_i^+ and negative-ideal solution d_i^- (in the original work it is a Euclidean distance).

Step 5. Calculation of a synthetic measure:

$$CC_i^+ = \frac{d_i^-}{d_i^+ + d_i^-} \quad , \qquad i = (1, \dots, n). \tag{6}$$

Measure values (6) are normalized in an interval $< 0;1 >$. The smaller the distance of an object from a positive-ideal solution and the bigger from a negative-ideal solution, the closer the value of a synthetic measure is to cohesion.

Step 6. Establishing the objects' ranking. The best object has the biggest value of a synthetic measure.

In summary. the fuzzy TOPSIS method assumes in the first step of procedure, the normalization of fuzzy numbers according to the formula of linear scale transformation. According to the second step of the fuzzy TOPSIS method's procedure, the weights of particular criteria can be expressed in the form of TFN. In this research, for the sake of simplicity, the same system of weights has been assumed for all variables, therefore the parameters' values of fuzzy numbers representing weights are the same and come to 1. Besides, R codes provided in Dudek and Jefmanski (2015) were used in this research for computation of the fuzzy TOPSIS scores.

4.4. Fuzzy Regression

Fuzzy regression was introduced by Tanaka et al. (1982) to model situations in which the practitioner cannot accurately measure the dependent variable. As long as traditional statistical regression models can only fit crisp data, fuzzy linear regression models can be used to fit both fuzzy and crisp data (Chang and Ayyub, 2001). For such data, fuzzy set theory provides a means for modelling linguistic variables utilizing membership functions. In contrast to the traditional statistical regression models which are based on probability theory, fuzzy regression is based simultaneously on possibility theory (Dubois and Prade, 1988) and fuzzy set theory (Zadeh, 1965; 1978).

Since the introduction of fuzzy linear regression, the literature on the subject has grown rapidly (Pasha et al., 2007). Traditionally, there are two approaches in fuzzy regression analysis: the linear programming-based method (Hojati et al., 2005; Nasrabadi and Nasrabadi, 2004; Peters, 1994; Sakawa, 1992) and the fuzzy least squares method (Chang et al, 1996; Dubois and Prade, 1980; Modarres et al., 1995, Savic and Pedrycz, 1991). The first method is based on minimizing fuzziness as an optimal criterion. Its major advantage is simplicity in programming and computation. The second method uses a fuzzy least-squares approach to minimize the errors between the observed and predicted values.

In statistical regression analysis, the errors derived from the adjustment of a regression model into the observed data are assumed to be observational errors caused by a random variable following some statistical distribution (e.g., normal, with constant variance and zero mean). However, fuzzy regression analysis views these errors as the underlying uncertainty or fuzziness that exists within the model structure, as proposed by Tanaka et al. (1982, 1988, 1989). This being the case, according to Chang and Ayyub (2001), statistical regressions are meant for handling random errors determined from crisp estimated and observed data. These errors are different in nature from fuzziness or uncertainty. On the other hand, fuzzy regression analyses are meant to model observed fuzzy data.

As one would expect, when fuzzy data approach their crisp state in fuzzy regression (e.g. $\alpha = 1$), the results should approach those obtained from the statistical regression analysis (Chang and Ayyub, 2001). This property, however, still does not exist in actual fuzzy regression models. The basic reason is that fuzzy regression takes the fuzziness assumption as a substitute for the randomness assumption in statistical analysis. In other words, fuzziness is treated as a replacement to randomness, rather than being modeled in a complementary fashion to the underlying randomness. Chang and Ayyub (2001) called this aspect as the "limiting behavior" of fuzzy regression methods. This behavior has unfortunately segregated the used of fuzzy regression from the well-received ordinary least-squares regression. For the same reason, the use of fuzzy regression methods has drawn some criticism from statisticians, for example, Redden and Woodall (1994). Two distinct approaches that try to overcome such limitations are discussed next: the bootstrapped Truncated Regression with conditional α-levels proposed in Wanke et al. (2016a) the application of different fuzzy rule-based systems within the ambit of fuzzy regressions (Riza et al., 2015).

4.5. Bootstrapped Truncated Regression with conditional α-levels

Before proceeding, and for the sake of the readability of this section, it is worth mentioning to readers the intrinsic characteristics of the α-level approach and its operationalization into different α values when assessing vagueness during the measurement process of a variable. The fuzzy α-level analysis (also known as α-cut analysis) is widely used in assessing uncertainty or vagueness in the measurement of a variable. Uncertain variables can be treated as fuzzy numbers such as the TFN previously discussed. They can be manipulated by specially designed operators, in our case, the different levels of α, by assigning a given value ranging between 0 and 1 (say, 0, 0.1, …). In fact, the α-level is the degree of sensitivity of a given variable to vagueness. At some point, as the information value diminishes (lower values of α, thus implying higher values of fuzziness or vagueness), one no longer wants to be "bothered" by the data. In many systems, due to the inherent limitations of the mechanisms of observation, the information becomes suspect below a certain level of reliability. When α is equal to one, it is possible to say there is no fuzziness (vagueness) and there is full reliability (apart, of course, from random intrinsic effects).

Wanke et al. (2016a) departed from the approach of Simar and Wilson (2007) and proposed conditional bootstrapped truncated regression to analyze the crisp values derived from fuzzy efficiency models – where scores range between 0 and 1 - based on α-levels. The following conditional modeling was proposed:

$$\theta_j \mid \alpha = k + Z_j\delta + F_j\gamma + \varepsilon_j,\ j = 1,....,n \tag{7}$$

The modeling can be understood as the first-order approximation of the unknown true relationship. In eq. (7), α is a real value bounded between 0 and 1 and represents the α-level of the membership function for the efficiency score, k is the constant term, ε_j is statistical noise, F_j is vector of dummy variables that represent the fixed effects for the type of the fuzzy models used, whenever different models are used, and Z_j is a vector of the contextual variables for observation j that is expected to be related to the observation's efficiency score, θ_j, taken as a crisp value.

Specifically, noting that the distribution of ε_j is restricted by the condition $\varepsilon_j \geq 1 - k - Z_j\delta - F_j\gamma$ (since both sides of (7) are bounded by unit), the steps proposed in Simar and Wilson (2007) were followed in Wanke et al. (2015a, 2015b), and it was assumed that this distribution is truncated normal with zero mean (before truncation), unknown variance, and (left) truncation point determined by this very condition. Furthermore, replacing the true but unobserved regressand in (6), θ_j, by the respective fuzzy efficiency estimate, $\bar{\theta}_j$, the conditional econometric model formally becomes:

$$\bar{\theta}_j \mid \alpha \approx k + Z_j\delta + F_j\gamma + \varepsilon_j,\ j = 1,....,n, \tag{8}$$

where

$$\varepsilon_j \sim N(0, \sigma_\varepsilon^2) \text{, so that } \varepsilon_j \geq 1 - k - Z_j\delta - F_j\gamma,\ j = 1,...,n, \tag{9}$$

which is evaluated via maximal likelihood estimation as regards $(\delta, \sigma_\varepsilon^2)$ obtained from the data. It should be noted that in this research only one type of fuzzy TOPSIS model was used, thus implying the discard of vector F_j. Besides, the respective computations for the parametric bootstrap for this conditional regression were carried out with R codes developed by Wanke et al. (2015a, 2015b).

The bootstrapped truncated regression with conditional α-levels lends a contribution to the emerging literature on combined probabilistic-fuzzy approaches, where randomness and uncertainty have their useful properties jointly considered (Arunaj et al., 2013). More specifically, 2-Dimentional Fuzzy Monte Carlo Analysis (2D FMCA) uses a combination of probability and possibility theory to include probabilistic and imprecise information in the same analytical model. In this research, a specific application of the 2D FMCA approach is developed to assess the efficiency levels and their determinants in the South African agriculture. More precisely, the approach used here starts off from the Fuzzy TOPSIS models – where production inputs and outputs are treated as triangular fuzzy numbers (as in Puri and Yadav, 2013; Wanke et al., 2016a) with a 20% offset from their central values – and culminates with the proposed conditional bootstrapped truncated regression. They are performed each time for a given α-level (say 0; 0.1; 0.2; ... ; 1, as in Hsiao et al., 2011). Readers should be aware that the α-level values within this set are primarily used in the Fuzzy TOPSIS so as to determine crisp values for the input and the output bounds, thus allowing the computation of their respective efficiency levels.

4.6. Rule-based systems in fuzzy regression

Fuzzy set theory was proposed by Zadeh (1965), as an extension of the classical set theory to model sets whose elements have degrees of membership. So, instead of just having two values: member or non-member, fuzzy sets allow for degrees of set membership, defined by a value between zero and one. A degree of one means that an object is a member of the set, a value of zero means it is not a member, and a value somewhere in-between shows a partial degree of membership. During the last forty years, scientific research has been growing steadily and the available literature is vast (Riza et al., 2015). A lot of monographs provide comprehensive explanations about fuzzy theory and its techniques, for example in Klir and Yuan (1995); Pedrycz and Gomide (1998). One of the most fruitful developments of fuzzy set theory are fuzzy-rule based systems (FRBSs).

FRBSs are an extension of classical rule-based systems (also known as production systems or expert systems). Basically, they are expressed in the form "IF A THEN B" where A and B are fuzzy sets. A and B are called the antecedent and consequent parts of the rule, respectively. During the modeling of an FRBS, there are two important steps that need to be conducted: structure identification and parameter estimation (Riza et al., 2015). Nowadays, there exists a wide variety of algorithms to generate fuzzy IF-THEN rules automatically from numerical data, covering both steps. Approaches that have been used in the past are, e.g., heuristic procedures, neuro-fuzzy techniques, clustering methods, genetic algorithms, squares methods, etc. With respect to the structure of the rule, there exist two basic FRBS models: the Mamdani and Takagi-Sugeno-Kang (TSK) models. Other variants have been proposed in order to improve the accuracy and to handle specific problems. Their drawback is that they usually have higher complexity and are less interpretable. For example, the disjunctive normal form (DNF) fuzzy rule type has been used in González, et al. (1993).

Constructing an FRBS means defining all of its components, especially the database and rulebase of the knowledge base. Basically, there are two different strategies to build FRBSs, depending on the information available (Wang, 1994). The first strategy is to get information from human experts. The second strategy is to obtain FRBSs by extracting knowledge from data by using learning methods (Riza et al., 2015). Generally the learning process involves two steps: structure identification and parameter estimation (Sugeno and Yasukawa 1993; Pedrycz 1996). In the structure identification step, we determine a rulebase corresponding to pairs of input and output variables, and optimize the structure and number of the rules. Then, the parameters of the membership function are optimized in the parameter estimation step. The processing steps can be performed sequentially or simultaneously. Learning methods are usually classified into five groups (Riza et al., 2015): space partition; genetic algorithms; clustering; neural networks; and gradient descent A FRBS can be used just like other regression models and their corresponding packages in R and technique principles are duly described in Table 2.

Table 2. Learning methods and implementation details in R (built upon Riza et al., 2015).

Learning method	FRBS model	Model acronym in R	Acronym meaning	Description of the technique
Fuzzy neural Networks	TSK	ANFIS	Adaptive-network-based fuzzy inference system	The systems in this group are commonly also called neuro-fuzzy systems or fuzzy neural networks (FNN; Buckley and Hayashi 1994) since they combine artificial neural networks (ANN) with FRBSs. An FRBS is laid upon the structure of an ANN and the learning algorithm of the latter is used to adapt the FRBS parameters, usually the membership function parameters. There exist many variants of methods based on FNNs, such as the adaptive network-based fuzzy inference system ("ANFIS") and the hybrid neural fuzzy inference system ("HYFIS"). **Adaptive-network-based fuzzy inference system ("ANFIS").** This method was proposed by Jang (1993). It considers a TSK FRBS model which is built out of a five-layered network architecture. The "ANFIS" learning algorithm consists of two processes, the forward and the backward stage. The forward stage goes through the five layers as follows: **Layer 1:** The fuzzification process which transforms crisp into linguistic values using the Gaussian function as the shape of the membership function. **Layer 2:** The inference stage using the t-norm operator (the AND operator). **Layer 3:** Calculating the ratio of the strengths of the rules. **Layer 4:** Calculating the parameters for the consequent parts. **Layer 5:** Calculating the overall output as the sum of all incoming signals. The backward stage is a process to estimate the database which consists of the parameters of the membership functions in the antecedent part and the coefficients of the linear equations in the consequent part. Since this method uses the Gaussian function as membership function, we optimize two parameters of this function: mean and variance. In this step, the least squares method is used to perform the parameter learning. For the prediction phase, the method performs normal fuzzy reasoning of the TSK model.
	MAMDANI	HYFIS	Hybrid neural fuzzy inference system	**Hybrid neural fuzzy inference system ("HYFIS").** This learning procedure was proposed by Kim and Kasabov (1999). It uses the Mamdani model as its rule structure. There are two phases in this method for learning, namely the knowledge acquisition module and the structure and parameter learning. The knowledge acquisition module uses the techniques of Wang and Mendel. The learning of structure and parameters is a supervised learning method using gradient descent-based learning algorithms. The function generates a model that consists of a rule database and parameters of the membership functions. "HYFIS" uses the Gaussian function as the membership function. So, there are two parameters which are optimized: mean and variance of the Gaussian function for both antecedent and consequent parts. Predictions can be performed by the standard Mamdani procedure.
Clustering	CLUSTERING	SBC	Subtractive clustering	**Subtractive clustering ("SBC").** This method is proposed by Chiu (1996). For generating the rules in the learning phase, the "SBC" method is used to obtain the cluster centers. It is an extension of Yager and Filev's mountain method (Yager and Filev 1994). It considers each data point as a potential cluster center by determining the potential of a data point as a function of its distances to all the other data points. A data point has a high potential value if that data point has many nearby neighbors. The highest potential is chosen as the cluster center and then the potential of each data point is updated. The process of determining new clusters and updating potentials repeats until the remaining potential of all data points falls below some fraction of the potential of the first cluster center. After getting all the cluster centers from "SBC", the cluster centers are optimized by fuzzy c-means.

	CLUSTERING	DENFIS	Dynamic evolving neural fuzzy inference system	**Dynamic evolving neural fuzzy inference system ("DENFIS").** This method is proposed by Kasabov and Song (2002). There are several steps in this method that are to determine the cluster centers using the evolving clustering method (ECM), to partition the input space and to find optimal parameters for the consequent part of the TSK model, using a least squares estimator. The ECM algorithm is a distance-based clustering method which is determined by a threshold value, Dthr. This parameter influences how many clusters are created. In the beginning of the clustering process, the first instance from the training data is chosen to be a cluster center, and the determining radius is set to zero. Afterwards, using the next instance, cluster centers and radius are changed based on certain mechanisms of ECM. All of the cluster centers are then obtained after evaluating all the training data. The next step is to update the parameters on the consequent part with the assumption that the antecedent part that we got from ECM is fixed. Actually, ECM can perform well as an online clustering method, but here it is used in an offline mode.
Gradient descent	TSK	FIR.DM	Fuzzy inference rules by descent method	**Fuzzy inference rules with descent method ("FIR.DM").** This method is proposed by Nomura, Hayashi, and Wakami (1992). "FIR.DM" uses simplified fuzzy reasoning where the consequent part is a real number (a particular case within the TSK model), while the membership function on the antecedent part is expressed by an isosceles triangle. So, in the learning phase, "FIR.DM" updates three parameters which are center and width of the triangle and a real number on the consequent part using a descent method.
	TSK	FS.HGD	FRBS using heuristics and gradient descent method	**FRBS using heuristics and the gradient descent method ("FS.HGD").** This method is proposed by Ishibuchi et al. (1994). It uses fuzzy rules with non-fuzzy singletons (i.e., real numbers) in the consequent parts. The techniques of space partitioning are implemented to generate the antecedent part, while the initial consequent part of each rule is determined by the weighted mean value of the given training data. Then, the gradient descent method updates the value of the consequent part. Furthermore, the heuristic value given by the user affects the value of weight of each data point.
Genetic fuzzy systems	APPROXIMATE	GFS.FR.MOGUL	Genetic fuzzy for fuzzy rule learning based on the MOGUL methodology	**Genetic fuzzy systems for fuzzy rule learning based on the MOGUL methodology** ("GFS.FR.MOGUL"). This method is proposed by Herrera et al. (1998). It uses a genetic algorithm to determine the structure of the fuzzy rules and the parameters of the membership functions simultaneously. To achieve this, it uses the approximative approach as mentioned in Section 2.2. Each fuzzy rule is modeled as a chromosome, which consists of the parameter values of the membership function. So, every rule has its own membership function values. A population contains many such generated chromosomes, based on the iterative rule learning approach (IRL). IRL means that the best chromosomes will be generated one by one according to the fitness value and covering factor. The method carries out the following steps: **Step 1:** Genetic generation process involving the following steps: Create an initial population, evaluate individual fitness, perform genetic operators, obtain the best rule and collect it, and repeat this process until the stopping criterion has been met. **Step 2:** Tuning process: Repetitively adjust the best individual until the stopping criterion is met. **Step 3:** Obtain an FRBS model as the output.

Space partition	MAMDANI	WM	Wang and Mendel’s technique	**Wang and Mendel’s technique ("WM").** It was proposed by Wang and Mendel (1992) using the Mamdani model. For the learning process, there are four stages as follows: **Step 1**: Divide equally the input and output spaces of the given numerical data into fuzzy regions as the database. In this case, fuzzy regions refer to intervals for the linguistic terms. Therefore, the length of the fuzzy regions is related to the number of linguistic terms. For example, let us assume a concept of temperature between 1 and 5. Then, we define the linguistic terms “cold”, “neutral”, and “hot”, and we define the length of fuzzy regions as 2. This now gives us the fuzzy regions as intervals [1, 3], [2, 4], [3, 5], respectively, and we can construct triangular membership functions. E.g., in the first case, we have the corner points a = 1, b = 2, and c = 3 where b is a middle point whose degree of the membership function equals one. **Step 2**: Generate fuzzy IF-THEN rules covering the training data, using the database from Step 1. First, we calculate degrees of the membership function for all values in the training data. For each instance and each variable, a linguistic value is determined as the linguistic term whose membership function is maximal in this case. Then, we repeat the process for all instances in the training data to construct fuzzy rules covering the training data. **Step 3:** Determine a degree for each rule. Degrees of each rule are determined by aggregating degrees of membership functions in the antecedent and consequent parts. In this case, we are using the product aggregation operators. **Step 4**: Obtain a final rulebase after deleting redundant rules. Considering the degrees of rules, we can delete a redundant rule with a lower degree.

The parametrization of the respective models in the **frbs** R package (Riza et al., 2015) is given in Table 3.

Table 3. Parameters for the FRBS.

Methods	Parameters
ANFIS	num.labels = 5, max.iter = 300, step.size = 0.01, type.mf = 3
HYFIS	num.labels = 5, max.iter = 200, step.size = 0.01
SBC	r.a = 0.3, eps.high = 0.5, eps.low = 0.15
DENFIS	Dthr = 0.15, max.iter = 5000, step.size = 0.01, d = 2 "
FIR.DM	num.labels = 5, max.iter = 1000, step.size = 0.01
FS.HGD	num.labels = 5, max.iter = 100, step.size = 0.01, alpha.heuristic = 1
GFS.FR.MOGUL	persen_cross = 0.9, max.iter = 300, max.gen = 200, max.tune = 500, persen_mutant = 0.3, epsilon = 0.95
WM	num.labels = 15, type.mf = 3, type.defuz = 1, type.tnorm = 1, type.snorm = 1

5. Results and Discussion

The distributions of the efficiencies scores calculated using Fuzzy TOPSIS for South African agriculture from 1970 to 2014, using a meta-frontier (O'Donnell et al., 2007) and the set of inputs and outputs generated based on TFN and the α-level approach, are given in Fig. 1. In general terms, the fuzzy estimates mostly ranged from to 0.40 to 0.68 and appear to be increasing again, after a period of decay (1970-1980) and stagnation (1980-2000). This finding may not be surprising given that agricultural productivity trend in South Africa has equally been fluctuating and one of the reasons for this was attributed to changing resource use (DAFF, 2011; Liebenberg, 2013; Kirsten et al., 2003)

It is interesting to note that, in a quite analogous way to what happen with bootstrapped estimates in frontier methods, fuzzy TOPSIS efficiencies are higher when there is no fuzziness at all (α-level = 1, represented by the solid bold line in Fig. 1 on the left). On the other hand, fuzzy TOPSIS efficiencies systematically decay with the value of α-level (thus representing increased fuzziness). Their minimal values are obtained when α-level = 0 and fuzziness is maximal. The dashed fine line in Fig. 1 on the left represents this. Under bootstrapping, the newer efficiency estimates computed statistically tend to be lower than the original ones.

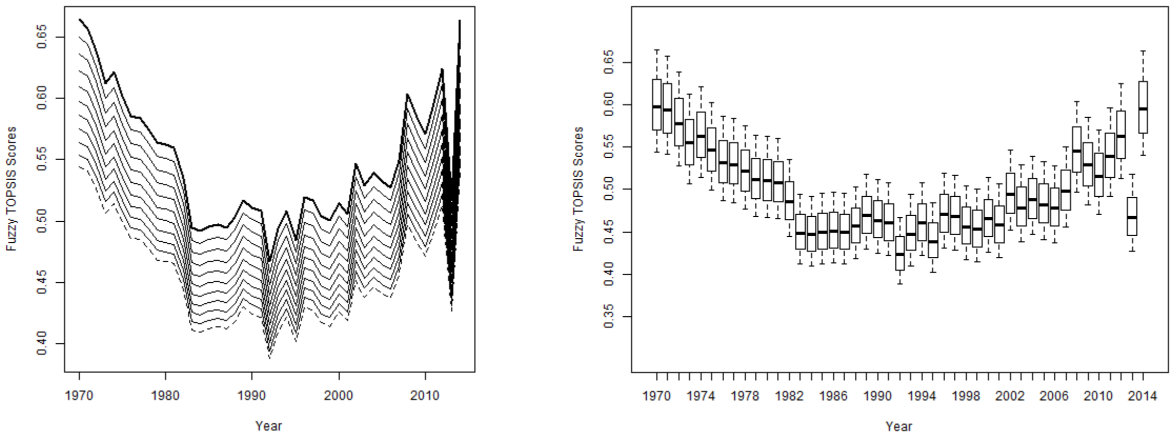


Fig. 1. Fuzzy estimates of South African agricultural efficiency levels.

Results for the conditional bootstrapped truncated regression performed on different α-levels reflect the impact of socio-economic and demographic variables on agricultural efficiency in South Africa under different levels of fuzziness. They are presented in Fig. 2. From a quick inspection, several conclusions can be drawn. A solid line marks the zero in each graph, thus indicating whether a contextual variable is significant or not for a given value of α. Several contextual variables are not statistically significant, regardless of the uncertainty level in inputs and outputs: education, FDI, and GDP growth.

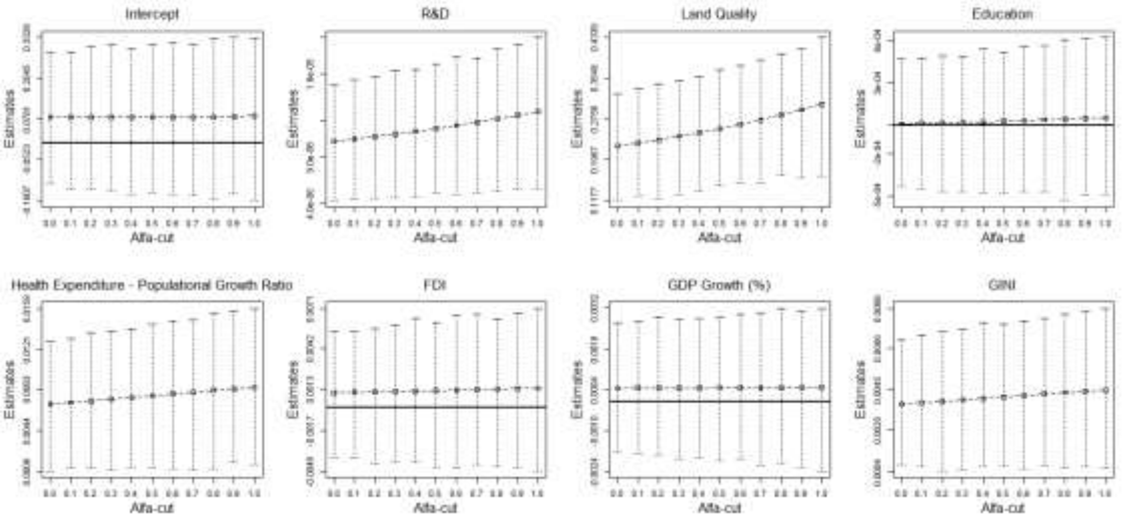


Fig. 2. Coefficient estimates for the conditional bootstrapped truncated regression.

However, the reminder contextual variables are significant regardless of the α-values: R&D expenditure, land quality, health expenditure–population growth ratio and GINI index. They all present a positive impact on agricultural efficiency levels in South Africa, thus suggesting that, although land quality may be a prerequisite for higher productivity in agricultural systems, the impact of R&D expenditure on genetic modified seeds, new fertilizers, equipment/machinery for grain harvest and storage and other production methods cannot be neglected. Besides, there is an interesting counterbalance of social welfare and capital accumulation in agricultural productivity in South Africa. Although efficiency tends to increase with a higher health expenditure–population growth ratio, higher GINI levels, which reflects income inequality and capital accumulation to some extent also, exerts a positive effect on efficiency. These results suggest that agricultural efficiency in South Africa is driven not only by wealth accumulation, but also by the dissemination of social welfare. Generally speaking, this would imply a macro-economic trade-off to be managed by policy-makers.

It is also interesting to note that the sign of the significant impact did not depend upon the fuzziness level that the input and output data are subjected to. This suggests that uncertainty and randomness do not interact in the input/output level. This lack of ambiguity -which is not so often found in fuzzy systems applied to efficiency measurement (Wanke et al. 2016b and Wanke et al., 2016a) - opens the room for researchers and practitioners to investigate further the actual sources of efficiency using different rule-based systems within the ambit of fuzzy regressions. The inherent statistical limitations of fuzzy regressions previously discussed in Section 4.4 should be, of course, observed.

Bootstrapping was also used to build confidence intervals for the log-likelihood measurements for the conditional regression models considering different α-values. These results are presented in Fig. 3. Although one cannot claim that these log-likelihoods are significantly different, since their error bars overlap, it is interesting to note how the regression's performance is affected by the extreme values of uncertainty in the measurement of inputs and outputs. The best fit to the data was verified under $\alpha = 0$, that is, when fuzziness was maximal. A similar behavior was found in Wanke et al. (2015a, 2015b) and Wanke et al. (2016a, 2016b) when applying this bootstrapped regression to different decision-making contexts. This also suggest the use of different rule-based systems to assess the problem of predicting efficiency levels in South African agriculture.

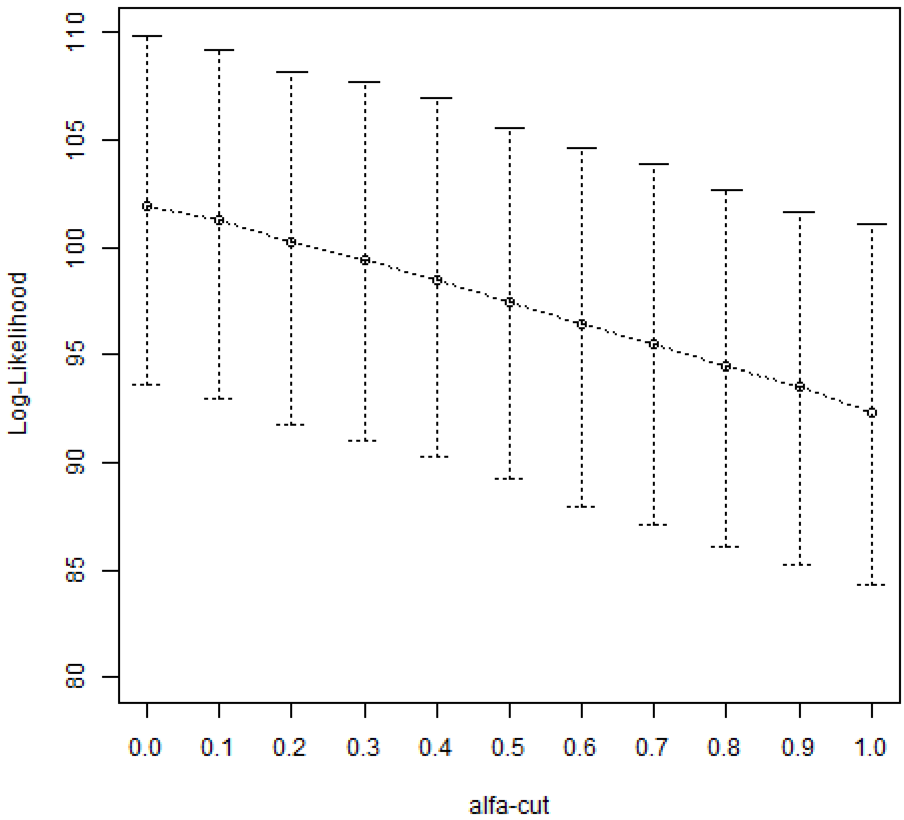
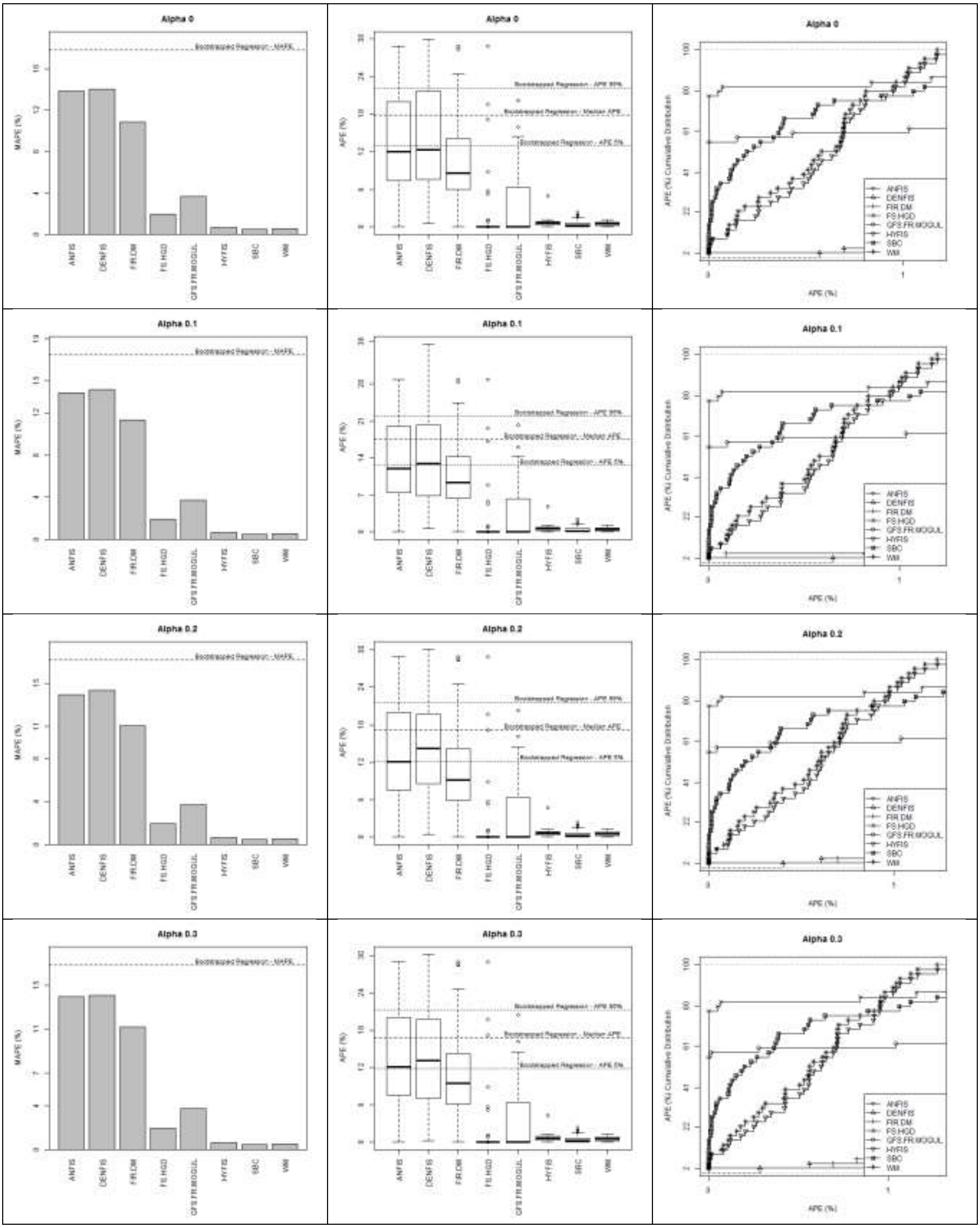
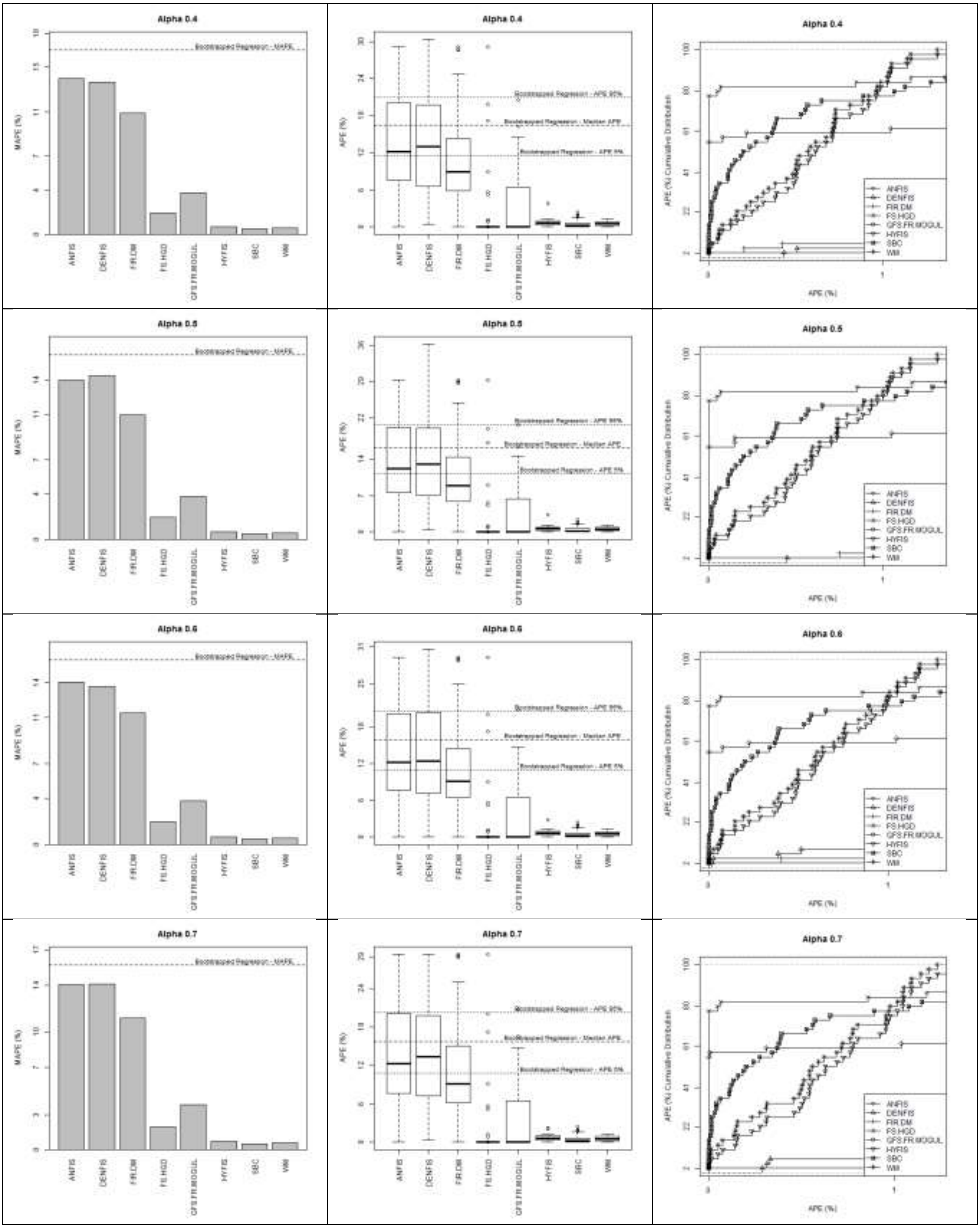


Fig. 3. Log-likelihood confidence intervals for the conditional bootstrap truncated regressions.

Therefore, the results of the rule-base systems methods presented in Tables 2 and 3 and considered for comparison within the ambit of fuzzy regressions are now discussed. To compare the results, we computed the APE – Average Percent Error. Two panels with the full set of results are shown in Figs. 4 and 5. One is organized by alpha-levels; the other, by methods. It can be seen that the eight FRBS methods tested outperformed bootstrapped truncated regression results in terms of MAPE, with the exception of DENFIS method for α-levels 0.8 and 0.9. With respect to the distribution of the APEs, the average errors were substantially smaller under HYFIS, SBC, FS.HGD, GFS.FR.MOGUL, and WM models, and comparable in central tendency and distributional characteristics to those obtained via bootstrap under ANFIS, DENFIS, and FIR.DM. This suggests that outperformance in terms of APE and MAPE can be achieved in several FRBS models, regardless of the underlying learning method. The absence of interaction between fuzziness and randomness, as detected in the bootstrapped regression, altogether with the fact that the best likelihood model is the one obtained for highest input/output fuzziness, may help in explaining why several FRBS methods performed superiorly best. Further research, however, is deemed necessary to confirm this conjecture under circumstances where there are interactions between randomness and fuzziness.


Alpha 0
Alpha 0.1
Alpha 0.2
Alpha 0.3
MAPE (%)
APE (%)
APE (%) Cumulative Distribution
ANFIS
DENFIS
FIR.DM
FS.HGD
GFS.FR.MOGUL
HYFIS
SBC
WM


Alpha 0.4
Alpha 0.5
Alpha 0.6
Alpha 0.7
MAPE (%)
APE (%)
APE (%) Cumulative Distribution
Bootstrapped Regression - MAPE
Bootstrapped Regression - APE 95%
Bootstrapped Regression - Median APE
Bootstrapped Regression - APE 5%
ANFIS
DENFIS
FIR.DM
FS.HGD
GFS.FR.MOGUL
HYFIS
SBC
WM

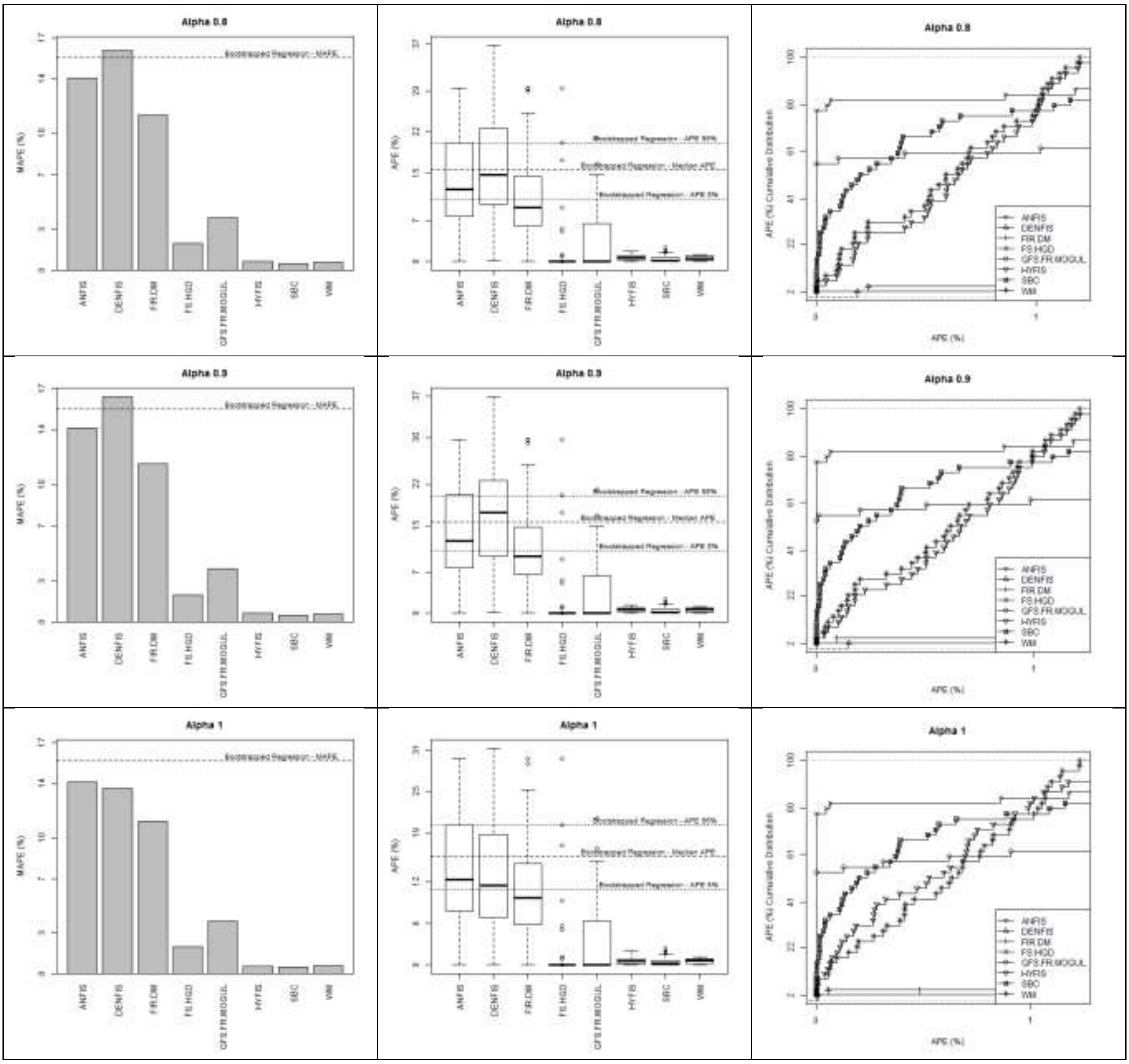
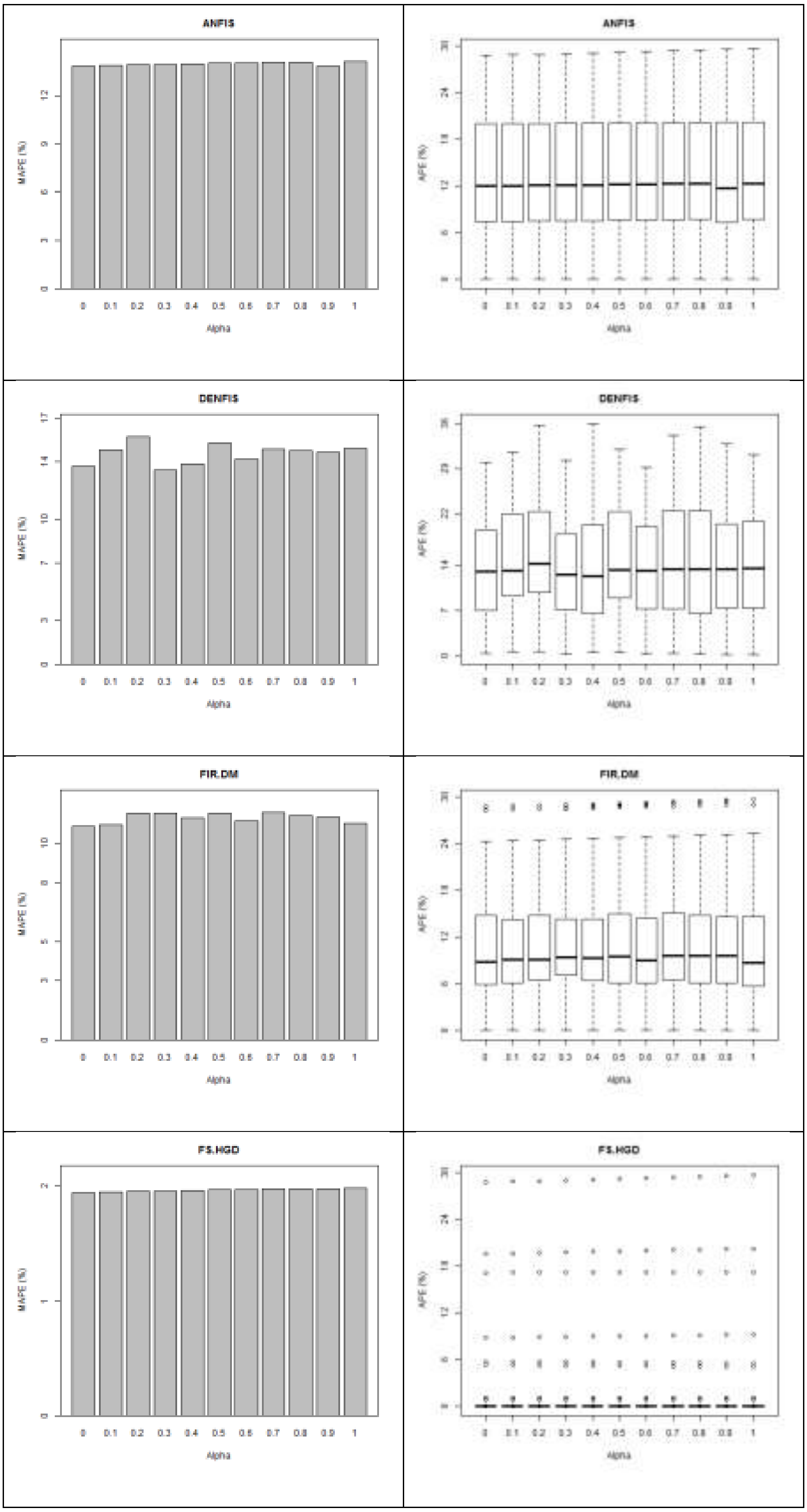


Fig. 4. FRBS regression results grouped by α-levels.

On the other hand, Figure 5 illustrates the impact of the fuzziness level on the errors achieved by each FRBS model. It illustrates that the different α-levels do not impact much on the distributional aspect of the APE and its central tendency (median), although average errors (MAPE) tend to slightly increase with increasing fuzziness (α-level = 0) only under the HYFIS approach. Under the remainder approaches, either the APE increases or remains stagnant with lower fuzziness levels. This suggests that FRBS generally works better under higher fuzziness environments. The methodological implications of these findings may be related to the learning power of neural networks, the underlying assumption within the HYFIS model, where the linguistic terms used to connect efficiency with contextual variables – i.e. higher land quality imply higher agricultural efficiency – prevail to the detriment of other rules such as space partition or clustering. The possibility of deriving a discourse on how things happen rather than fitting parameters for space partition or cluster membership helps not only theory consolidation, but also in establishing a common basis of comparison for quantitative unreliable data obtained from different sources scattered over the course of time, although qualitatively comparable in meaning.


ANFIS
MAPE (%)
APE (%)
Alpha
DENFIS
FIR.DM
FS.HGD
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

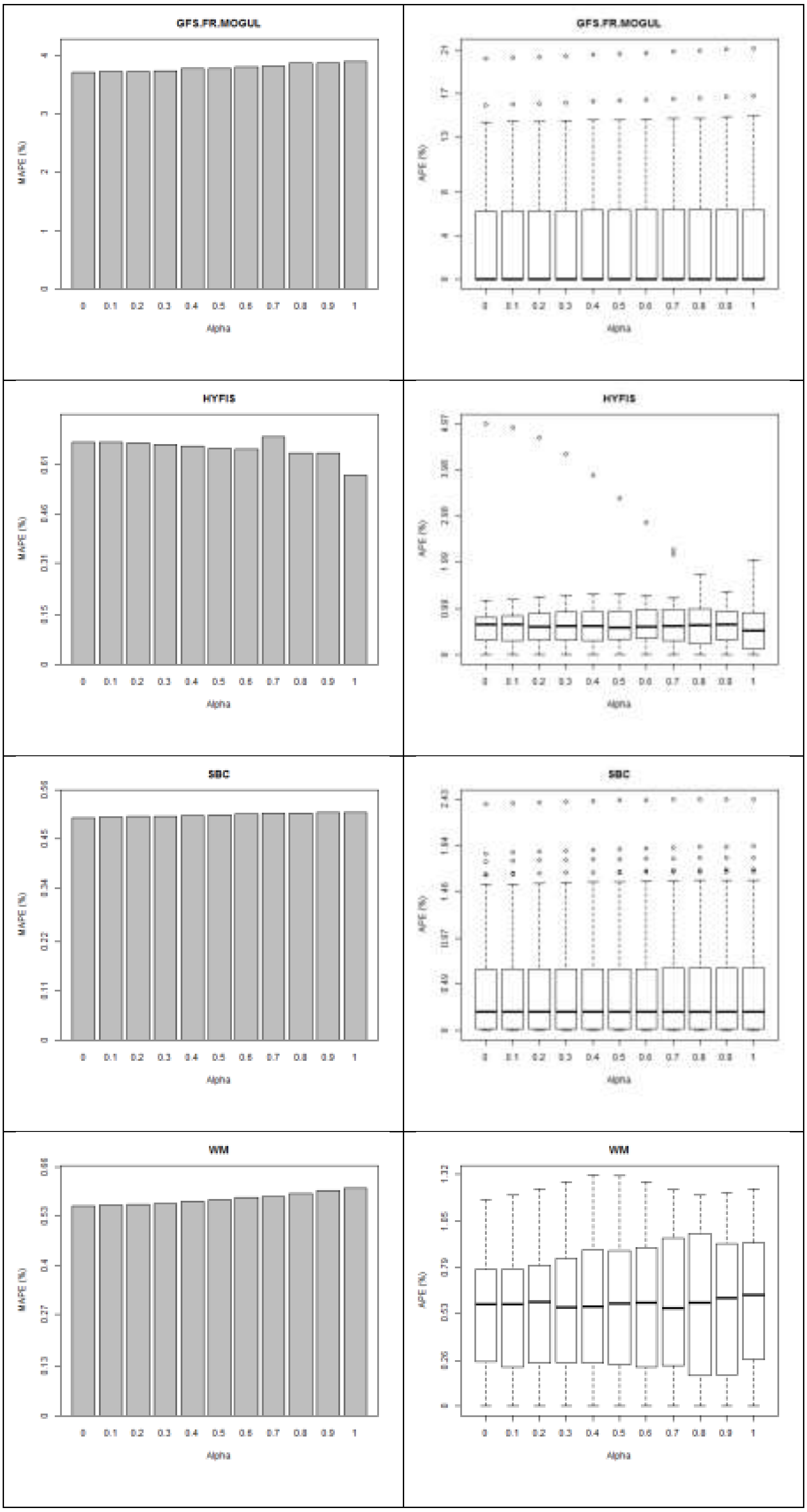


Fig. 5. FRBS regression results grouped by models.

The ruling implications for decision-makers based on this practical application in South African agriculture are related to very nature of the fuzziness and randomness of the problem under analysis. Considering that such interactions may exist, and can be detected in sign reversal for different alpha levels, one should consider using bootstrapped conditional regressions and FRBS regressions in a complimentary fashion. The first one should be used in detecting significant contextual variables; the other one, for prediction purposes. On the other hand, putting the case for the South African agriculture into perspective and considering that there was no interaction found between randomness and fuzziness embedded within the collected data, decision makers should adopt a combination approach, where different forecasts are weighted and combined into a final number.

6. Conclusion

This paper presents an analysis of South African agricultural efficiency using Fuzzy TOPSIS and fuzzy based regressions in a two-stage approach. Fuzzy TOPSIS enables different sources of uncertainty and vagueness to be handled while computing the efficiency scores. Based on the results of the fuzzy analysis performed in the second stage, it is possible to explain the causes of inefficiency in terms of socio-economic and demographic variables. Agricultural efficiency in South Africa appears to be explained by the countervailing forces of capital accumulation and social-welfare, building upon land quality and R&D expenditure. These findings have implications for policy in South Africa. Policies to increase social expenditure especially in terms of health is necessary. This will enhance the productive capacity of the farm families. More health care facilities in both the urban and rural areas with adequate and qualified personnel needs to be provided and at affordable costs. Also there is need to put up policies that will conserve the environment and hence the quality of land. This can be done side by side with increased investment in R&D to enable the farming units access to genetically modified seeds, new fertilizers, equipment/machinery for grain harvest and storage, and other production methods that can improve efficiency and subsequently productivity yet leave the environment sustainable.

As regards the two-stage fuzzy approach developed here, the technical contributions of this paper are built upon two pillars. In the first stage, fuzzy logic was employed to assess the relative differences and rankings obtained from TOPSIS. These scores have proved to be useful in accounting for the impact of higher levels of vagueness, over the course of time. In the second stage, different fuzzy regression models were employed in a competitive fashion. This allowed the determination of the discourse structure of the most relevant contextual variables in terms of explaining agricultural efficiency levels in South Africa.

The methodology employed in this research also constitute an advance in the field of agricultural efficiency measurement using fuzzy approaches, building upon previous studies in the area, besides using a comprehensive data set of countries over several decades. First of all, it adds to the scant previous studies conducted at the macro-economic level that have recognized the importance of using fuzzy logic to handle inherent uncertainty in input/output measurement. Second, this study for the first time used fuzzy regressions and a comprehensive set of different inference systems to map the relationships between efficiency scores and their major socio-economic and demographic drivers. Rather than focusing on the statistical properties of randomness, a different venue of uncertainty, this paper aimed at the predictive power of fuzzy regressions and their capability of unveiling hidden non-linear relationships covered up with vagueness.

The implications for policy-makers are related to a number of measures that can assure that agricultural efficiency can be multi-dimensional and be explained beyond the technological choices. Indeed, those choices may be determined by social and environmental factors and political decisions. Thus, a transition to a more efficient agricultural sector in South Africa will depend on how we address certain socio-political factors, such as pursuing a more health care, while managing social inequality and capital accumulation.

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