THE DEVELOPMENT OF A MINING METHOD SELECTION MODEL THROUGH A DETAILED ASSESSMENT OF MULTI-CRITERIA DECISION METHODS.

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ABSTRACT
THE DEVELOPMENT OF A MINING METHOD SELECTION MODEL THROUGH A DETAILED ASSESSMENT OF MULTI-CRITERIA DECISION METHODS

In the past decades, attempts were made to build a systematic approach to mining method selection (MMS) (Ooriad et al., 2018). This is because MMS is a complex and irreversible process. Since it can affect the economic potential of a project, the approach must be as thorough, precise, and accurate as possible. The key challenges of the previously established techniques such as the Nicholas and Laubscher method are that, there was a lack of engineering judgement in the process of selecting a mining method. In other instances, not all the parameters required in the mining method selection process were considered; i.e. economics would be the basis of the final decision of a mining method without taking into consideration other factors such as geology (Bogdanovic et al., 2012). While other techniques just considered a few parameters and a limited number of mining methods as alternatives (Nam, 2008). Some techniques were customised procedures for a specific orebody (Nam et al., 2009). Each orebody is unique; therefore, the approach of just adopting the same mining method for similar commodities was not always an effective or realistic approach. Therefore, the existing procedures were found to be inadequate and not applicable for consideration in all MMS processes.

To solve the challenges stated above, an up-to-date approach to MMS is the use of multi-criteria decision-making (MCDM) tools to aid in the process. The MCDM are effective in facilitating a decision-making process; however, the use of MCDM has not gained enough popularity across countries and in the mining industry especially in MMS (Mardani et al., 2015). Their successful implementation in other industries such as in manufacturing companies, water management, quality control, transportation, and product design (Lee et al., 2007) present an opportunity for further exploration in MMS. In this research, these MCDMs were further explored as starting point to solving the challenge faced in MMS.

With the aim of developing a systematic and an unbiased approach that caters for subjective and objective assessment in MMS, this study investigated 10 MCDMs- TOPSIS, TODIM, VIKOR, GRA, PROMETHEE, OCRA, ARAS, COPRAS, SAW, and CP with potential to solve the MMS challenge. The study focused on deriving a model where the MCDMs can be integrated and be successfully used for MMS. Included in the research are factors and mining methods that are necessary MMS. The aim was to use the factors and mining methods as inputs to the developed MMSM.

In the result section, case studies were used to analyse the MCDMs following a descriptive and a statistical analysis (sensitivity analysis, spearman correlation, and Kendall’s coefficient.). PROMETHEE, TOPSIS, and TODIM stood out as methods for use in the selection of mining method in the coal mining industry. From the research findings, it was generally concluded that OCRA, ARAS, CP, SAW, and COPRAS are simplified approaches of the afore-mentioned methods. VIKOR’s rankings were outlying and the conclusion was that it was not a suitable method for MMS. GRA’s conclusion based on the literature view was that there remain many unanswered questions about its mathematical foundations.

The MMSM was developed using the results obtained. In the MMSM, first, the user defines the problem. The approach is of case-based reasoning (CBR); where the user can retrieve, re-use, revise and then retain the information (in the database) for future use. The user can always search within the database for a similar problem to select a MCDM, factors and methods; and this may be one of the future areas of improvement on the developed MMSM because there are a number of factors, MCDMs, and mining methods that the user may need
to go through before getting to the relevant MCDM. One of the recommendations made by the author was that the user must understand the theoretical background of the MCDM before using it in the MMSM. In future studies, algorithms for selection of a suitable MCDM in the MMSM can be developed so that once the problem has been defined and structured; the user may not struggle with knowing which method to use amongst the suggested. Also, an application-based approach may be investigated further.

Key Words: MCDM, MMSM, MMS, factors, mining methods
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<table>
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<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>AHP</td>
<td>Analytic Hierarchy Process</td>
</tr>
<tr>
<td>ARAS</td>
<td>Additive Ratio Assessment</td>
</tr>
<tr>
<td>CBR</td>
<td>Case-Based Reasoning</td>
</tr>
<tr>
<td>COPRAS</td>
<td>Complex Proportional Assessment</td>
</tr>
<tr>
<td>CP</td>
<td>Compromise Programming</td>
</tr>
<tr>
<td>DEA</td>
<td>Data Envelopment Analysis</td>
</tr>
<tr>
<td>ELECTRE</td>
<td>Elimination and Choice Translating Reality</td>
</tr>
<tr>
<td>GDM</td>
<td>Group Decision Making</td>
</tr>
<tr>
<td>GRA</td>
<td>Grey Relational Analysis</td>
</tr>
<tr>
<td>MAUT</td>
<td>Multi Attribute Utility Theory</td>
</tr>
<tr>
<td>MCDA</td>
<td>Multi Criteria Decision Analysis</td>
</tr>
<tr>
<td>MCDM</td>
<td>Multi Criteria Decision Making</td>
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<tr>
<td>MM</td>
<td>Mining Method</td>
</tr>
<tr>
<td>MMS</td>
<td>Mining Method Selection</td>
</tr>
<tr>
<td>MMSM</td>
<td>Mining Method Selection Model</td>
</tr>
<tr>
<td>OCRA</td>
<td>Operational competitiveness rating</td>
</tr>
<tr>
<td>PROMETHEE</td>
<td>Preference Ranking Organisation Method for Enrichment Evaluations</td>
</tr>
<tr>
<td>RQD</td>
<td>Rock Quality Designation</td>
</tr>
<tr>
<td>RMR</td>
<td>Rock Mass Rating</td>
</tr>
<tr>
<td>SAW</td>
<td>Simple Additive Weighting</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>Technique for Order Preference by Similarity to an Ideal Solution</td>
</tr>
<tr>
<td>UBC</td>
<td>University of British Method</td>
</tr>
<tr>
<td>VIKOR</td>
<td>Vlse Kriterijumsk Optimizacija Kompromisno Resenje</td>
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1. CHAPTER ONE: MOTIVATION FOR THE STUDY

The aim of Chapter 1 is to provide a general understanding of a coal mining method selection (MMS) process; how it has evolved over time. Chapter 1 also present a brief review of the models used in selecting the right mining method for a given deposit. Again, the need for an unbiased mining method selection model (MMSM) is emphasised by detailing how the future of coal mining in South Africa- its contribution to GDP, and export markets would need a reliable model for MMS, to preserve and increase the level of confidence for the future of coal mining. The need of a MMSM given the shortcomings of the current methods is presented. Chapter 1 also presents the problem statement, research questions, objectives of the research, how the research was conducted, and the potential it has in the coal mining industry. The organisation of the whole research document is presented as a conclusion of the first chapter. Figure 1 shows a summary of what the first chapter entails.

1.1. INTRODUCTION TO MINING METHOD SELECTION (MMS)

There were attempts to build a systematic approach to mining method selection (MMS) in the past. MMS is a process of selecting an extraction method for a defined deposit (coal or other minerals) (Ooriad et al, 2018). A mining method is selected through proper planning, research, and informed decisions in the presence of experts such as mining engineers and geologists. The process of MMS is one of the most challenging and complex processes. Ooriad et al. (2018) mentions that what makes the process a critical and complex one is that, many factors (such as technical, economical, geological, environmental, and geotechnical) form part of the decision-process. In addition, the nature of each orebody is unique; therefore, the approach of just adopting the same mining method is not always an effective or a realistic approach. Sometimes in the process, the drawbacks that exist are a lack of engineering judgement from experienced experts (Ooriad et al, 2018). While engineering judgement is available for some processes, there are instances where the studies of the MMS process are inadequate or
incomplete; also, the process would not consider all the important aspect to MMS (Ataei & Bitarafan, 2004). There is also no existing specific formula for making decisions on MMS. Despite the challenges presented, it is still of utmost importance to select the right mining method to ensure safety and productivity for the prevailing economic circumstances.

There are four stages in the life of a mine; prospecting, exploration, development and exploitation. In the first two stages, there are tests, examinations, and evaluations to quantify and qualify the possibility of pursuing a mining project. The development process commences thereafter. Development is a means that makes it possible to exploit the orebody deposit. Therefore, the development and exploitation largely depend on the mining method selected for the orebody. Figure 2 shows the four stages of the life of mine. The braces indicate the stages whose dependency is largely on the type of mining method used.

![Figure 2: Stages in a life of a mine (Baloyi, 2018)](image)

The braced ({} stages make the MMS a critical process because the process is irreversible. It is the mining method chosen, which dictates terms on how mining will take place, the type of technology, cycle times, and risks associated with the method and other decisions including personnel to work in the mine (Kant, 2016). Therefore, the process of MMS must be as thorough, precise, and accurate as possible because of its ability to affect the economic potential of a project. (Dehghani et al, 2016). Whichever method chosen, the goal must be clear: to maximise profit, maximise recovery of the deposit, provision of a safe working area for the employees, and adherence to the regulations in place.

In addition, there are different mining methods that form part of the selection process. Input variables/factors that are controllable (e.g. cost) and non-controllable (e.g. geology) are obtained to compare the mining methods; and these factors are studied in detail to understand their influence in the process (Guray et al, 2003). The motivation to study scientifically and technically the used variables is because some deposits and nature of a mining project can only be understood through a detailed analysis of the available data. It must also be understood that every deposit has inherent problems that are unique and must be considered in the MMS process (Ataei & Bitarafan, 2004).
1.2. THE NEED FOR MINING METHOD SELECTION MODELS (MMSM)

To facilitate and manage MMS, there have been extensive studies (As it would be shown in the current study) conducted to find a suitable process of MMS. Different techniques have also been practiced. Sometimes the process of selection would be done by taking into consideration the mining operations within the same area to be developed (As practised in many South African mines). Other times, available decision-making tools, softwares, and logical reasoning from mining experts would be used. Namin et al. (2008) separates these selection methods under three categories: 1. profile and checklist methods, 2. numerical ranking, and 3. decision-making models. All these MMSM have unique drawbacks. The following are some of the unique drawbacks from different MMSM: Some techniques such as Nicholas, Morrison, Hartman and Laubscher methods consider one aspect (such as geology) and neglect the others (Economical, Geotechnical, and Environmental, Hydrological, surface topography, and infrastructure). While others (Boshkov & Wright Method) just consider a few parameters. This makes the existing MMSM inadequate and not applicable for consideration in all MMS processes. (Ooriad et al, 2018)

Research indicates that the ability of a MMSM to identify and address factors that influence a choice of a mining method will determine the success of the process of selection; and most importantly, the success of the implementation and operation of a mine from cradle to grave. Ataei and Bitarafan (2004) further makes an emphasis of factors in the following statement, “Each factor in the method selection can become the principal determining factor, but the obvious predominance of one consideration should not preclude careful evaluations of all parameters.” In addition, in making a mining method choice, it must be noted that there may not be only one feasible method. Therefore, a logical decision must be made (Ooriad et al, 2018).

The following section will summarise some of the MMS approaches; where they have been previously applied and what makes them stand out. Figure 3 is a summary of the outcomes of the previously established MMS techniques, and the need for the research project. All the information presented in the diagram is explained in Section 1.2. Section 1.3 shows the research potential of addressing the need in Section 1.2.
1.2.1. PREVIOUSLY DEVELOPED AND CURRENT BEST PRACTICE MMSM

In the literature, the development of effective MMSM dates to the 1960s. The techno-economic model was used in the late 1960’s and early 1970’s for MMS. The procedure is divided into two phases; preliminary MMS and selection of the most suitable method from amongst a group of methods that are deemed applicable. Its basis is on financial estimations of the financial effects that will result if a specific mining method was selected. Therefore, the mining method whose effects are favourable would be chosen. The method was inadequate because it had its basis on the financial side of implementation and neglected other key parameters (Bogdanovic et al, 2012).

In the early 70s, Boshkov and Wright proposed a qualitative classification for underground mining methods. In this method, geotechnical and geological factors are considered. The selected mining methods, as alternatives, are those which have been applied in similar geological conditions. In addition, up to four methods can be identified as applicable or suitable mining methods (Namin et al, 2009). The limitation in the number of mining method as
alternatives is its drawback (Nicholas, 1993). In 1976, Morrison suggested a classification system that is based on ground conditions only. Laubscher method (method named after his last name) followed in 1981, which was based on the rock mass. His emphasis was on the cavability of the rock mass. The methods in consideration are block caving and stoping methods. Whether one of the methods is used will depend on the rock quality designation (RQD) and the joint characteristics. Laubscher believed that the RQD would determine the ease of cavability (Namin et al, 2009).

The first quantitative approach for underground mining methods was presented in 1981. Nicholas approached the MMS by using a numerical rating. In his quantitative approach, he identified geometry, grade of the orebody and rock mass strength as the most important factors in MMS. In weight assigning, the rock mechanics characteristics are studied. If they make a mining method more suitable, such a method is given a higher point (Guray, 2003). The Nicholas procedure is not currently preferred because of the limitations in the number of parameters used in the process. In addition, on the Nicholas method, the MM alternatives do not leave the decision-makers with enough options. Only 10 MM can be selected; while there are numerous available MM (Namin, 2008).

In 1987, Hartman also developed his own selection method. His qualitative method considers the ground conditions and geometry of the deposit to be the most important parameters to consider (Namin et al, 2009). Appendix 1 presents different flowcharts for these traditional methods.

From the assessed researches, the methods (numerical and checklists) in Figure 4 are considered inadequate to be used in MMS. (Bogdanovic et al, 2012). Their common disadvantage is that they consider a limited number of methods and influencing factors. (Ooriad et al, 2018).

Figure 4: MMS development timeline (Baloyi, 2018)
An up-to-date approach, which has proven to be effective, is the use of multi-criteria decision-making (MCDM) tools to aid in the mining industry and across other sectors to make decisions. Decision-making is considered as a selection process in which the best alternative is chosen. In the process, a problem/opportunity is specified; alternatives and criteria are identified. The criteria are then used to evaluate the alternatives for selection of a preferred alternative (Namin et al, 2009). These techniques are often reliable, and studies have been conducted to increase the level of confidence in applying them to the mining industry (Musungwini, 2016).

Improvements in the MMS process have been seen because of adopting MCDMs to aid in MMS. All factors necessary for a selection of a method can now be considered in the process. MCDMs do not limit the user with a specific number of alternatives to use in the MMS process. However, a general drawback of these recent models is that they are time-consuming because of the calculations to be performed and in understanding the underlying mathematical foundations. They are not widely applied. Lastly, they each have inherent problems that will be presented in Chapter 2 (Namin et al, 2008). The following are some of the MCDM techniques that have gained acceptance within the mining industry and other industries.

Thomas L. Saaty developed the Analytic Hierarchy Process (AHP) in 1980 to aid in incorporating qualitative and quantitative factors where decision-making is concerned. AHP is an easy to use tool and it is accommodative if the number of criteria is large (Velasquez et al, 2013). A hierarchy is developed and at the top part of the structure is the goal of the decision. Through the intermediate levels, the relative importance of the criteria is assessed, and alternatives are compared with respect to each criterion (Bogdanovic et al, 2012).

This is performed through a constructed pair-wise comparison matrix (Saaty, 2008). The AHP has been widely used in the mining industry and other industries. Although it ranks as one of the most used MCDM, it is unsuitable in some areas because of the independence when the considered criteria are rated in isolation; thus, making judgement to be inconsistent (Musungwini, 2016). AHP has successfully assisted in the selection of mining methods such as the Jajarm Bauxite mine, In Iran.

In 1981, Yoon and Hwang proposed the Technique for Order Preferences by Similarity to Ideal Solutions (TOPSIS). Ranking of alternatives is based on the ideal solution and ideal similarity. If the alternative is more similar to the ideal solution, then it is more acceptable (Ooriad et al, 2018) because it uses distances to identify and rank alternatives, it fails to include the correlation of the criterion (Velasquez et al, 2013). However, it is a simple and easy to use method that is applicable in the mining industry; and has been used to select a mining method at Tazerah coal mine (Ooriad et al, 2018).

Incomplete data and problems with a major part of uncertainty can be solved by a mathematical theory, the Grey Analysis Method that was proposed in 1982 by Deng. It was successfully used to select a mining method in Gol-e-Gohar mine, Iran. Its advantage is that it can produce the results of the best and worst alternative of a mining method (Dehghani et al, 2017). The drawback of using GRA is that its mathematical foundation has not been fully proven.

Later, a trend was recognised in the use of Multi Criteria Decision Making Methods (MCDM); it was such that two or more methods would be combined. In some of the previous work, AHP and PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) methods were combined with the aim of selecting a method to mine Coka Marin underground mine in Serbia (Bogdanovic et al, 2012).

The integration of the methods was such the AHP was used for determining the weights of the criteria and to structure the selection process. PROMETHEE on the other side was used to
rank the alternatives in the final selection, and to perform a sensitivity analysis by changing the weights of the criteria. The approach was found to be effective because of the added advantages that PROMETHEE has over other MCDM. (Bogdanovic et al, 2012)

Other MCDM such as, Case-Based Reasoning (CBR), Compromise Programming (CP), ELECTREE (Elimination and choice expressing reality), PROMETHEE, VIKOR, COPRAS, OCRA, and SAW (Simple Additive Weighting) have been developed to be used in and out of the mining industry for different decision-making processes. The success and failure of the MCDMs has been visible over the years as shown in the literature study. However, they are not widely accepted for decision-making; hence the need to study them and assess the possibility of their application in MMS.

1.2.2. WHY IS IT NECESSARY ASSESS AND ANALYSE THE APPLICATION OF MCDM BEFORE THE DEVELOPMENT MMSM?

Over time, any mining enterprise would want to know if the applied mining methods are still as effective as the time they were when initially implemented. In addition, if not, what can be done to correct the initial decisions to maximise production at the prevailing economic circumstance. This would imply that the current coal extraction methods would have to undergo evaluations, audits and assessments to check their effectiveness; and to give proof of which mining method can best suit the current conditions of the deposit. A MMS process can carry out such evaluation. Why is this process important? Additionally, why is it necessary to carry out this research? The following are reasons to quantify the need for the research.

A. Coal Mining’s contribution to the economy.

Mining has helped to shape the economy of South Africa largely. Infrastructure was established, foreign investment attracted, and the employment rate of S.A increased because of the existence of this industry. Even though it is not the highest contributor to the GDP (Mining at 7.3% while finance is at 20%) of the country currently, it still accounts for a major proportion (16%) on foreign direct investments (Chamber of Mines, 2017). Coal accounted for 28% as the largest on a total of almost R460 billion for local and foreign mineral sales in 2017 as shown in Figure 5 (Stats SA, 2017). This means that coal was the largest generator of revenue in the year 2017 in South Africa.
The dependency of South Africa on coal is further validated by the following statistics: 92% of electricity is generated from coal, and over 34% of the liquid fuels used are generated using coal (Fisher et al., 2015). Eskom has 16 running power stations and two more be operational by 2021 (Chamber of mines, 2018). Over 90 000 people have been employed by the coal sector. The fuel import bill would be higher if the country was not richly endowed with hard coal that has been estimated by the Department of Mineral Resource in 2016 to be 30 billion of tons, which represent 3.5% of the world's coal resource. The country is also one of the leading coal export nations and rank as the sixth in the list of exporting nations (Chamber of Mines, 2016). The coal mined is delivered to the Pacific and Atlantic steam coal market. Overall, liquid fuels, basic iron and steel industry, and electricity account for more than 80% of domestic coal demand together (Chamber of Mines, 2018).

Looking into the future, the global demand for coal is expected to rise despite the shift to renewables that is occurring globally. In the December 2017 issue of Pacific Standard Magazine, it is said that the demand for coal in 2022 will increase by 3% or 117 million tons of coal. The forecast is due to the increase demand from India and other countries who depend on coal. The Chamber of Mines coal strategy report of 2018 confirms that India accounts for almost half of the country's total export in terms of volumes. The reasons for the expected growth in demand for India is its dependency on coal for electricity generation. On the other hand, its local supply has not kept up with the growth in demand, hence the need for imports (Chamber of Mines, 2018).

On the contrary, the renewables (wind energy, solar energy, etc.) as alternatives are gaining momentum in most part of the world; such as China, South Korean, Indonesia, Peru, etc. (Steyn et al., 2017). The Department of Energy also intends on reducing the share of coal-generated power in the country's electricity mix from 82% to 31% by 2051. However, until then, coal will continue to play a significant role (Chamber of Mines, 2018). Therefore, the ever-increasing needs of energy, calls for the renewable energy and coal industry to concentrate on innovation, improvements, and new developments of mining methods (for coal).

One such improvement that can be made is to improve the process of selecting the mining methods. Even though it is not yet an issue in the South African coal, the improvement will
help emerging coal companies. This is because if the studies of a discovered deposit do not get it right regardless, mining method selection, the expected production yields may be compromised. This section serves as a reminder that even though the renewable energy is gaining momentum, coal will still play a major role in meeting the energy demands. Therefore, existing, and upcoming mining projects must be as effectively as possible and start by an appropriate MMS.

B. Improve the quality of MMS.
An alternative to the traditional approach of adopting a mining method based on the existing mines within the same region, the MMSM can cater for the uniqueness of each deposit. In this manner, operational and financial difficulties can be eliminated or reduced during the early stages of the mining project.

Owing to its multi-disciplinary nature, the mining industry has been greatly challenged where decision-making is concerned. There are lot of factors to be considered when a decision needs to be made. Based on the literature studies it is believed that by developing a MMSM, the quality of MMS will be improved. The MMSM that follows a multi criteria decision analysis (MCDA) approach will be essential. This is because the approach can successfully facilitate decision-making where many criteria (factors to be considered in MMS) must be considered to arrive at an optimal choice from amongst a collection of alternatives (Balt, 2015). Therefore, in each situation, decision-makers will be able to consider all available alternatives based on a systematic approach (MMSM).

The lack of a quantitative approach in subjective judgement contributed in making the MMS process to be unreliable. Therefore, MMSM as a logical decision-making process will reduce the uncertainty, and present both qualitative and quantitative data before a decision is made. Thus, reliability and dependency on a formal process will be improved.

C. Improve long-term planning of the mine.
When a mining method is selected, the intention must be such that it remains relevant for the life of mine. If adjustments are to be made from one mining method to another, the first mining method must not hinder a successful transition. To ensure the possibilities of a continuous operation until the end of the life of mine, long-term planning is essential. As said by Osanlo et al. (2016), “The risk of an early closure of mine is reduced by a robust mine planning.” Therefore, it is through planning that both the short and the long-term needs of a deposit are catered for. The MMSM will aid in making that possible by providing relevant solutions where the choice of a mining method is concerned.

Fourie et al. (2001) indicated that, “It is the main objective of any mining plan to effectively integrate the activities that are involved in the mining process.” For ease of understanding and use, a well-structured and systematic manner must be followed to approach planning of an operation. Therefore, the MMSM enhances planning by identifying factors that are critical in the specific deposit, and after identification, the factors can be effectively integrated in the planning section. Epstein et al. (2012) also attests that a MMSM will ensure that there is an integration between the chosen mining method and the other downstream processes. Therefore, a well-developed MMSM will also serve as a checklist in the planning phase of a mining project.
D. Diagnose existing problems in the existing mining method.
As extraction process evolves, there may exist a gap between what was initially planned and what is done due to unplanned or unforeseen circumstances. Marle et al. (2012) said in his study of project risk management method selection, “If the risks are not managed pro-actively in a structured approach, then they can result in serious consequences for the project.” Therefore, the MMSM’s ability to evaluate critical factors that poses a greater risk of jeopardising a project is essential. This will result in time and cost savings for any mining project to be undertaken in the future. Again, it will aid in the existing mining extraction systems for the mining companies to assess themselves against the MMSM to confirm and validate their initial decisions of a mining method.

E. Investigate and consider main factors related to MMS.
The MMSM will ensure that any mine planner or decision maker is aware of the main factors that will not just affect the mining method selection process, but the operational life of the mine. For example, the commodity prices are always fluctuating. Their volatile nature affects parameters such as the cut-off grade; that eventually affects the mining process. If the price fall, the low-grade blocks may change to waste and thus reduce the number of blocks to be mined. This in turn affect the long and short-term planning of a mine (Osanloo et al, 2016). Therefore, it is important that the factors affecting the life of a mine be investigated in detail. An MMSM that can evaluate all available factors and prioritise the ones that are critical to the existence of a project will save and preserve the future of coal mining. In addition, will help foresee operational problems to be expected in the future.

Owing to the conflicting, contradictory, and competing nature of factors that leads to a choice of a mining method, it is important that they be studied in detail to understand how they impact on (sensitivity analysis) a given decision. The development of an MMSM in this research will help in making the trade-offs decisions by identifying significant factors, and parameters that are essential when a mining method must be selected. Solving each factor separate is already a complex problem. However, if there is a model that can integrate, eliminate less important factors and considers critical ones, then such a model cannot be ignored or left undeveloped.

F. MMSM are a stepping-stone to novel methods of mining.
As regulations tighten, prices hike up, and mechanisation disrupt the normal way of mining, new mining methods may emerge. To effectively benefit from them, it is important that they be employed at the right deposit. A MMSM that will include these new mining methods as alternatives is essential because the existing approaches have not yet been updated to include the newly emerged methods such as coal gasification as an alternative to coal mining. It will also help the decision makers into considering all-important aspects before making the final decision of the type of a mining method. In addition, through the MMSM, exploring and combining mining methods into new methods is made possible.

G. Increase the level of confidence in using MCDM.
The MMS methods and techniques that have been used in the past are almost similar in their process of executions. Therefore, this research seeks to develop an integrated approach to MMS by evaluating how successful the methods can be when combined to strengthen their advantages while addressing the shortcomings. The research will examine their common applications to assess the correlations that exist. On the same note, Musungwini et al. (2016)
assessed MCDM usage in the mining industry through an analysis of 150 case studies. It was noted that most of the MCDM methods would become inefficient as the number of criteria (factors) increased (especially AHP). In that case, two or more of the MCDM methods would be applied to the same problem to increase the level of confidence. More information on the aforementioned research was explained in the literature study.

Through the literature review it is believed that developing a MMSM is an opportunity to combine and integrate the MCDM that have been used in MMS, together with newly gathered information that will make them effective for application in the mining industry.
1.3. RESEARCH POTENTIAL

Since the expected outcome of the research is a methodology to select coal-mining methods, the intention is to provide a guide to all the coal-mining companies. Table 1 gives a summary of the research potential.

<table>
<thead>
<tr>
<th>Possible beneficiaries from the research project are:</th>
<th>Possible benefits from the study to the beneficiaries:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Existing coal mining companies.</td>
<td>• Improving the MMS decision-making process for the coal mining companies.</td>
</tr>
<tr>
<td>• Upcoming small-scale coal mining companies.</td>
<td>• Provision of a systematic and unbiased approach that caters for subjective and objective analysis in MMS.</td>
</tr>
<tr>
<td>• Existing consulting and research organisation.</td>
<td>• Better insight and understanding of factors that affect coal MMS.</td>
</tr>
<tr>
<td></td>
<td>• Review of the potential yet fully unexplored mining methods such as coal gasification and coal bed methane in South Africa.</td>
</tr>
<tr>
<td></td>
<td>• Increased level of confidence of the MCDM will help South African Mining companies to utilise these models, as application has been limited in the country’s coal mining.</td>
</tr>
</tbody>
</table>

It is the aim of the study to aid in strategic decisions that ought to be made for the future of coal mining for coal mines that would find this study useful. Generally, the author acknowledges the fact that a one-size fits all model cannot be developed in the mining industry due to variations of deposits; however, a general guideline is still a possible solution; especially the MCDM approach to solving MMS because there has been limited mining application. Also, the country that contributed the most to information on MCDMs was Iran. Even though the U.S.A, China, and Australia are some of the largest coal producers in the world, there has not been much from them concerning the MCDMs. It must also be noted that the study will follow a decision-making approach instead of a problem-solving approach. This means that the focus is aimed at the future of coal mining.
1.4. RESEARCH QUESTIONS

The research questions used to formulate the research project are shown in Figure 6:

- What are the available factors in evaluating MMSM?
- How can the importance of factors in mining method selection be ranked?
- What are the available mining methods that can be investigated as alternatives for coal mining?
- What input such as factors necessary in MMS and mining methods can be incorporated in the development?
- What existing techniques/methods have been used?
- How can a MMSM be developed based on the existing MMSM and tools/methods from other industries?
- Are there effective ways of selecting coal-mining methods for a given deposit?
- What existing techniques/methods have been used?
- Are tools/methods from other industries related to method selection relevant?

Figure 6: Flowchart showing research questions (Baloyi, 2018)
1.5. PROBLEM STATEMENT

It is evident that extensive work regarding mining method selection has been done in the past. Decision making models and ranking methods have also been established to assist in building a preferable mining method solution for a selected deposit. MCDA have also been widely applied in the mining industry where decision with a considerable number of criteria and alternatives had to be made. Despite the perceived benefits from the above approaches, a systematic approach towards mining method selection remains a missing link between subjective and objective decision-making. Hence, some mining method selection decisions are based on experience where the outcome of the method adopted is the one similar to the deposit without fully catering for the uniqueness of the deposit in question.

Consequently, the mining industry cannot place much confidence on the previously applied approaches. Most of the decisions cannot be quantified, hence the need of a systematic approach to mining method selection. To address such a need, the author sees it fit to conduct a research that focuses on the development of a mining method selection model.

The project statement states:

*To develop a mining method selection model/methodology through a critical analysis and assessment of the application of MCDA. This is done to address the shortcomings of the existing traditional approaches of MMS.*

The overall aim of the methodology/model is that it must serve as a checklist in conducting a retrospective critique on existing coal mining methods operations.

Again, the decision-making approach must be able to offer a guideline for new coal mining projects on how an optimum mining method can be selected while considering factors that affect MMS.
### 1.6. OBJECTIVES AND METHODOLOGY OF THE STUDY

The objective that were set for the study and the methodologies are shown.

#### Table 2: Objectives and methodology of the study

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Conduct a comprehensive literature review on the existing MMSM and other methods of method selection from other industries.</td>
<td></td>
</tr>
<tr>
<td>a. Investigation of the functionality of each.</td>
<td>QUALITATIVE RESEARCH</td>
</tr>
<tr>
<td>b. Review the application of each to check for correlation amongst methods.</td>
<td></td>
</tr>
<tr>
<td>c. Identify the shortcomings and strength of each method.</td>
<td>• A huge part (1) – (3) of the research will be based on literature; therefore, the study is classified as a desktop study. The following sources will be used for attaining information: Journal databases, Library catalogue, Websites.</td>
</tr>
<tr>
<td>2. Provide summaries of the techniques and methods of coal mining locally and internationally, to be used as input alternatives in the developed MMSM.</td>
<td></td>
</tr>
<tr>
<td>3. Investigate and identify factors that are considered when evaluating a MMSM.</td>
<td></td>
</tr>
<tr>
<td>1. Conduct a descriptive and statistical analysis for the selected MCDMs.</td>
<td></td>
</tr>
<tr>
<td>2. Conduct a sensitivity analysis on each method selection technique.</td>
<td></td>
</tr>
<tr>
<td>3. Determine a procedure of integrating the results of the MCDMs.</td>
<td>QUANTITATIVE APPROACH</td>
</tr>
<tr>
<td></td>
<td>• DATA ANALYSIS: Each method will be evaluated for stability and consistency using existing case studies. The correlation that exists between the methods will determined. An attempt to resolve conflicts amongst the MCDM will be suggested.</td>
</tr>
<tr>
<td>4. Propose a mining method selection model/methodology.</td>
<td>QUALITATIVE + QUANTITATIVE APPROACH</td>
</tr>
<tr>
<td></td>
<td>Necessary information derived from (1) – (5) will be used to develop a MMS decision-making approach.</td>
</tr>
</tbody>
</table>
1.7. ORGANISATION OF THE RESEARCH

Chapter 1 has presented the background of the project, and the objectives as well as the methodology to be used to meet the aim of the research. To meet the objectives set for the research study, a flow chart of the dissertation is presented in Figure 7.

Figure 7: Organisation of the research (Baloyi, 2018)
1.8. LIST OF REFERENCES

Balt, K.D. 2015. A methodology for implementing the analytical hierarchy process to decision-making in mining, *University of Witwatersrand, Masters Engineering*, pp. 9-11


Collings, J. The Sasol Story: A half-century of technological innovation, *Corporate Affairs Department of Sasol Limited* pp. 163


2. LITERATURE REVIEW

2.1. INTRODUCTION

The literature was obtained from different sources; from journal databases to library catalogues. The information collected is presented to form part of the building blocks of the development of an MMSM. The information consists of the previously established techniques, factors as well as underground MM. A summarised version of Chapter 2 is presented in Table 3.

Table 3: Overview of Chapter 2

<table>
<thead>
<tr>
<th>Sub-section</th>
<th>Description</th>
<th>Breakdown</th>
</tr>
</thead>
</table>
| 2.2.        | Existing approaches to Mining method selection, and non-mining method selection | 2.2.1. Mining method selection techniques  
- Boshkov and Wright  
- Morrison, Laubscher  
- Hartman, Nicholas  
- UBC, AHP, TOPSIS  
- TODIM, GRA  
- PROMETHEE, HPV, VIKOR, ELECTREE  
2.2.2. Non-mining method selection techniques  
- OCRA, ARAS  
- COPRAS, SAW, CP |
| 2.3         | Established coal mining methods |  
- Bord and Pillar  
- Pillar extraction  
- Long wall mining  
- Short wall mining  
- Other Caving methods |
| 2.4         | Potential coal mining methods |  
- UCG  
- Coal Bed Methane |
| 2.5         | Factors in mining method selection | Different factors (critical and non-critical) gathered from different studies are presented |
| 2.6         | Significance of the literature review | The section gives a summary of the importance of the literature review |
| 2.7         | List of references | |
2.2. EXISTING APPROACHES TO MMS

The following section present different methods that have been previously used to select mining methods. The first part of the section describes method and classification systems that were adopted in the 1990’s. The latter part introduces MCDM, which may have been used in the mining industry for methods selection. For each method, the functionality, application in mining and other industries, shortcomings and strength are presented.

2.2.1. TECHNIQUES USED IN THE MINING INDUSTRY

A. BOSHKOV & WRIGHT
The method is one of the first qualitative classification developed for the underground mining method selection because of its assumption that surface mining is already eliminated as an option. Boshkov & Wright’s method of 1971 uses factors such as the ore dip, ore strength, ore thickness, strength of the walls (hanging & footwall) and many more. The method provides up to four mining methods as the most suitable to mine a specific deposit. However, the limitations are that it only looks at geological factors. In addition, there is a limited number of mining methods. One of the major disadvantages is that it uses the shortcomings of the mining methods to eliminate them to narrow the selection (Nicholas, 1993). The table for mining method selection using Boshkov & Wright classification system is shown in Table 4.
<table>
<thead>
<tr>
<th>Type of Orebody</th>
<th>Dip</th>
<th>Strength of ore</th>
<th>Strength of walls</th>
<th>Common Application</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thin beds</strong></td>
<td>Flat</td>
<td>Strong</td>
<td>Strong</td>
<td>Open stopes with casual pillars, long wall</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Room-and-pillar, long wall</td>
</tr>
<tr>
<td></td>
<td>Weak or strong</td>
<td>Weak</td>
<td></td>
<td>Open stopes with casual pillars</td>
</tr>
<tr>
<td><strong>Thick beds</strong></td>
<td>Flat</td>
<td>Strong</td>
<td>Strong</td>
<td>Room and pillar</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Top slicing, Sublevel caving.</td>
</tr>
<tr>
<td><strong>Very thick beds</strong></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Same as for &quot;Masses&quot; below</td>
</tr>
<tr>
<td><strong>Very narrow veins</strong></td>
<td>Steep</td>
<td>Strong and weak</td>
<td>Strong and weak</td>
<td>Re-suing</td>
</tr>
<tr>
<td><strong>Narrow veins</strong></td>
<td>Flat</td>
<td>N/A</td>
<td>N/A</td>
<td>Same as for thin beds</td>
</tr>
<tr>
<td>(width up to economic length of stull)</td>
<td>Steep</td>
<td>Strong</td>
<td>Strong</td>
<td>Same as for thin beds</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Open stopes, Shrinkage stopes, Cut-and-fill stopes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Weak</td>
<td>Cut-and-fill stopes, Square-set stopes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Open underhand stopes, Square-set stopes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Strong</td>
<td>Top slicing, Square-set stopes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Weak</td>
<td></td>
</tr>
<tr>
<td><strong>Wide veins</strong></td>
<td>Flat</td>
<td>N/A</td>
<td>N/A</td>
<td>Open underhand stopes, Underground glory hole, Shrinkage stopes, Sublevel stoping,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cut-and-fill stopes, Top slicing, Sublevel caving, Square-set stopes, Combined method</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Strong</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Weak</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Strong</td>
<td></td>
</tr>
</tbody>
</table>

Source: SME Mining Engineering Handbook, 1993
A. MORRISON
The 1976’s classification system of Morrison divides different underground mining methods into 3 groups. 1. Rigid pillar support, 2. Controlled Subsidence, and 3. Caving methods. Then criteria consisting of ore widths, support types and strain energy accumulation is used to select the suitable mining method (Kabwe and Yiming, 2015). The classification figure is shown in Appendix 1, Figure 47. Morrison’s classification systems also select mining methods based on a limited number of geological factors.

B. NICHOLAS METHOD
Nicholas proposed a quantitative selection tool in 1981 that considers rock mechanics characteristics as the most important factors to be considered in MMS of both surface and underground methods. It uses 13 criteria such as orebody characteristics, hanging and footwall, rock strength, fracture spacing and fracture strength. Points are assigned using a numerical ranking from zero (the least preferred) to four (Most preferred). - 49 is used to rule out a method (Guray et al, 2003). The ultimate weight of each mining method is obtained by summing up the scores. Wrong defects in the definition of the weights and a small scoring domain are some of the disadvantages associated with Nicholas method (Azadeh et al, 2010). The support tables for Nicholas method are shown from Table 5 to Table 8.
Table 5: Weighting procedure of the Nicholas MMS technique: Ore geometry attributes (Azadeh et al, 2010)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>General shape</th>
<th>Ore thickness</th>
<th>Ore plunge</th>
<th>Grade distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>T/P</td>
<td>I</td>
<td>N</td>
</tr>
<tr>
<td>Open pit mining</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Block Caving</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>-49</td>
</tr>
<tr>
<td>Sublevel stoping</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sublevel caving</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>-49</td>
</tr>
<tr>
<td>Long wall mining</td>
<td>-49</td>
<td>4</td>
<td>-49</td>
<td>0</td>
</tr>
<tr>
<td>Room and pillar</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Shrinkage stoping</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cut and fill</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Top slicing</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>-49</td>
</tr>
<tr>
<td>Stall stoping</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

M: Massive; T/P: Tabular or Platy; I: Irregular; N: Narrow (<10 m); I: Intermediate (<10-30 m); T: Thick (<30-100 m); VT: Very Thick (<100 m); F: Flat (<20°); I: Intermediate (20-55°); S: steep (>55°); U: Uniform; G: Gradational; E: Erratic

Table 6: Weighting procedure of the Nicholas MMS technique: Ore zone attribute (Azadeh et al, 2010)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Rock substance strength</th>
<th>Fracture Spacing</th>
<th>Fracture strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W</td>
<td>M</td>
<td>S</td>
</tr>
<tr>
<td>Open pit mining</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Block caving</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sublevel stoping</td>
<td>-49</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Sublevel caving</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Long wall mining</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Room and pillar</td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Shrinkage stoping</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Cut and fill</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Top slicing</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Stall stoping</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Rock substance strength- fracture strength: W: weak (<8); M: moderate (8-15); S: strong (>15); fracture spacing: VC: Very close (0-20); C: close (21-40); W: Wide (41-70); VW: Very Wide (71-100).
### Table 7: Weighting procedure of Nicholas MMS technique: Hanging wall attributes (Azadeh et al, 2010)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Criteria</th>
<th>Rock substance strength</th>
<th>Fracture spacing</th>
<th>Fracture strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W</td>
<td>M</td>
<td>S</td>
</tr>
<tr>
<td>Open pit mining</td>
<td></td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Block mining</td>
<td></td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Sublevel stoping</td>
<td>-49</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Sublevel caving</td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Long wall mining</td>
<td></td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Room and pillar</td>
<td></td>
<td>0</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Shrinkage stoping</td>
<td></td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Cut and fill</td>
<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Top slicing</td>
<td></td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Stull stoping</td>
<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table 8: Weighting procedure of the Nicholas MMS technique: Footwall attributes (Azadeh et al, 2010)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Criteria</th>
<th>Rock substance strength</th>
<th>Fracture spacing</th>
<th>Fracture strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W</td>
<td>M</td>
<td>S</td>
</tr>
<tr>
<td>Open pit mining</td>
<td></td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Block caving</td>
<td></td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Sublevel stoping</td>
<td></td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Sublevel caving</td>
<td></td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Long wall mining</td>
<td></td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Room and pillar</td>
<td></td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Shrinkage stoping</td>
<td></td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Cut and fill</td>
<td></td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Top slicing</td>
<td></td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Stull stoping</td>
<td></td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### C. LAUBSCHER METHOD

This 1981 method focuses on selection of mass mining methods (Block caving and stoping methods). The system examines the degree of fracturing, and the Rock Quality Designation (RQD), and from the analysis, block caving or stoping methods may be the suitable method. However, a lack of information prompts the decision maker to rely on guessing which poses a
limitation in applying this method (Kabwe and Yiming, 2015). In 1990, the system was modified to relate the hydraulic radius to rock mass rating. If the area available for undercutting is enough, cavability becomes feasible for a more competent rock when the hydraulic radius is included. (Namin et al, 2009). The classification figures for 1981 and 1990 are shown in Figure 48 and Figure 49 of Appendix 1. Even though Labuscher’s method may seem irrelevant in the coal mining industry because it focuses on mass mining methods, the author believes that it is necessary to include it for two reasons. Firstly, it is a method of selection and it is thus importance for comparison purposes. Secondly, mass mining method may have a potential for future use in the coal mining industry.

A. HARTMAN
The Hartman method of 1987 is developed on a flowchart that is based on the geometry of the deposit, and the ground conditions of the ore zone. It is used for both underground and surface mining; and unlike Boshkov & Wright, the results are more specific to a mining method than offering four options. Some of the assumptions made on Hartman’s method is that the ore strength and rock strength is known, then a method that best suits the ground conditions is chosen (Hartman & Mutmansky, 2002). The one limitation that is visible with this method is that it is neither enough nor complete to decide on a mining method (Kabwe and Yiming, 2015). The flow chart is shown in Error! Reference source not found.

Figure 8: Hartman and Mutmansky selection chart (Alpay and Iphar, 2008)
B. UNIVERSITY OF BRITISH COLUMBIA (UBC) METHOD

UBC was developed in 1995 by Miller-Tait as a modification of the Nicholas method. The scoring domain of the Nicholas method that is between the maximum and minimum was extended (Azadeh et al., 2010). It emphasises the stoping method rather than the mass-mining techniques. This is because it was designed to represent the typical Canadian practice, which is a limitation for use outside Canada (Namin et al., 2009). Additionally, the importance of the criteria was not taken into consideration (Azadeh et al., 2010). The selection process is similar to the Nicholas method procedure. The rankings and characteristics except for grade distribution and plunge are different. The ranking in UBC range from zero to six. Six is given to the characteristic of the most suitable mining method. Additionally, -10 was introduced to the method to strongly discount a method without fully eliminating it. There is also an improvement in the rock mechanics ratings since the internationally recognised rock mass rating is used (Meech et al., 2001). Table 82 in Appendix 1 shows the support chart for the UBC method. A toolkit that utilises the UBC method has been developed and made available in www.edumine.com. It is easy to use given that the user has the required information. However, since the development of UBC was for Canadian practices, the number of mining methods listed are limited. The screen information that appears when the toolkit is launched for use are shown. Figure 9 shows the orebody characteristics criteria. The user can insert the available deposit information, and the orebody cartoon in Figure 11, will show the user how the orebody will look like. The mining methods are then ranked from the best to worst in Figure 10.

![Figure 9: UBC toolkit screenshot 1](image-url)
Mining Method Rankings

(best)

- Open Pit (0)
- Block Caving (0)
- Sublevel Stopping (0)
- Sublevel Caving (0)
- Longwall Mining (0)
- Room and Pillar (0)
- Shrinkage Stopping (0)
- Cut and Fill Stopping (0)
- Top Slicing (0)
- Square Set Stopping (0)

(worst)

Figure 10: UBC toolkit screenshot 3

Figure 11: UBC toolkit screenshot 2
2.2.2. MCDM USED IN THE MINING INDUSTRY

C. ANALYTICAL HIERARCHY PROCESS (AHP)

The development of AHP by Thomas Saaty dates to the 80s. The main purpose of its development was to assist decision makers to make decisions in an organised manner. This is a method that can handle an ill-structured and complicated problem; and still be effective in facilitating the decision-making process (Maletic et al., 2014). AHP’s ability to represent the elements of a problem in a hierarchy form allows the problem to be broken down into smaller constituents’ part; with the objective/goal of the decision-making process on top (Balt, 2015).

In their article, Ataei et al. (2009) described the hierarchy as a dominance structure whereby the elements within the same level can be compared and evaluated against each other. This allows for a relative contribution of each element to the level above them. The following figure is an illustration of a dominance hierarchy formed during the AHP decision-making process.

![Figure 12: Illustration of AHP dominance hierarchy (Balt, 2015)](image)

To simplify how AHP uses a hierarchy: the top level describes the objective of the process. It must be clearly defined what the problem is and why AHP would be an appropriate method for handling the process (Cheng & Li, 2001). The second level will then be evaluated based on the first level. Since criterion (1), (2), and (3) are on the same level, they will be evaluated against each other to check their contribution to the level above, which is the objective. The evaluation is done in pairs; (1) and (2), (1) and (3), finally (2) and (3) will all be compared and their contribution and priorities to the objective will be determined. The last level on the hierarchy consist of alternatives. Each alternative is evaluated according to the criteria from the previous level (Ataei, 2009). The evaluation is subjective in that it relies on the judgements of experts (Velasquez & Hester, 2013). Therefore, users of AHP will express and rate their preferences of the elements in the hierarchy using Saaty’s scale of measurement indicated in Table 9.
<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two activities contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
<td>Judgement favour one over another</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
<td>Judgement strongly favour one over another</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
<td>Strongly favoured and its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Absolute importance</td>
<td>The importance of one over another affirmed on the highest possible order</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values</td>
<td>Used to represent compromise between the priorities listed above</td>
</tr>
</tbody>
</table>

**Source:** Saaty (2012)

A pairwise comparison or judgement matrix is then formed from the second level’s determined relative contribution and priority. When dealing with AHP, it must be noted that the matrix works on a reciprocal and transitivity basis. With reciprocity, if criterion (1) is twice as important as criterion (2), logic dictates that criterion (2) is then half as important as criterion (1). With transitivity, Musungwini and Minnitt (2008) detailed that a relationship is transitive if the relative importance is multiplicative such that, if criterion (2) is said to be twice as important as criterion (1), and criterion (3) is three times as important as criterion (2), then the relationship that exist between criterion (3) and (1) is that (3) is six times as important as criterion (1). It is believed that transitivity yields consistent judgement (Musungwini & Minitt, 2008).

As an additional characteristic to AHP, it can calculate the level of consistency from the pairwise comparison. A consistency ratio (CR) below 10% is acceptable; anything greater is regarded as inconsistent unless there are adequate justifications for its acceptability (Musungwini & Minitt, 2008). In case of inconsistency, the decision-makers should review their expressed preferences in the matrix as well as the objective of the process (Maletic et al, 2014). The following is an equation to calculate the CR:

\[ CR = \frac{CI}{RI} \]  

(1)

Where CI is the consistency index of the matrix in question and calculated as \( \frac{(\lambda_{\text{max}} - n)}{n-1} \) where \( \lambda_{\text{max}} \) is the maximal or principal Eigen value and \( n \) is the size of the matrix. RI is the consistency index of a random matrix size of \( n \). The established RI are shown on the table below. The choice of RI is dependent on the size of the matrix. For example, for a 3 x 3 matrix, RI would be 0.58.

**Table 10: Random Index (RI) For n-th matrix (Cited in Musungwini and Minnit, 2008)**

<table>
<thead>
<tr>
<th>Matrix order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RI</strong></td>
<td>0.00</td>
<td>0.00</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
<td>1.51</td>
<td>1.48</td>
<td>1.56</td>
<td>1.57</td>
<td>1.59</td>
</tr>
</tbody>
</table>

**Source:** (Saaty, 1980)
After performing the consistency test, the global weights of all the criteria are calculated and
alternatives can be ranked (Cheng & Li, 2001). AHP also allows a sensitivity analysis to be
performed to show the effect of altering weights of the criteria on the final ranking of
alternatives (Maletic et al., 2014).

According to Velasquez and Hester (2013), the advantages of AHP are that it is easy to use
and flexible in that its' size can be adjusted to accommodate different decision-making
problems. AHP is also not data-intensive like other MCDA methods. Its ability to handle both
qualitative and quantitative criteria has led to its popularity (Ataei et al., 2008). Data can be
normalised when measured in different scales, and can later be aggregated (Musungwini &
Minnitt, 2008). Moreover, Cheng and Li (2001) believes that AHP is accurate in making
business decisions because of its ability to check the consistency of the expert’s judgement.

Although it has gained power and use across different industries, it has also been criticised
because of how its standard consistency test function (Maletic et al., 2014). In their research,
Maletic et al. (2014) proposed that a quality control approach could be used to conduct the
consistency test. Musungwini and Minnitt (2008) also identified three limitations of this method.
Firstly, calculations can be rendered complex if the number of criteria to be compared
increases. This is illustrated in Table 11.

Table 11: Relationships of criteria with number of pairwise comparison (Cited in Musungwini and Minnit,
2008)

<table>
<thead>
<tr>
<th>Number of criteria</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>n(n−1)</th>
</tr>
</thead>
</table>
| Number of
comparisons       | 0   | 1   | 3   | 6   | 10  | 15  | 21  | \(\frac{n(n−1)}{2}\) |

Source: (Kardi, 2006)

The recommended maximum number of criteria is nine; so, its total comparisons will be 36. If
criteria are greater than nine are, the matrix may be complex and difficult to solve. Secondly,
the final decision (ranking of alternatives) can be affected if the scale of relative importance
was to be increased. Lastly, AHP only works with a positive reciprocal matrix as explained
earlier in the section. Additionally, Hester and Velasquez (2013) indicated that AHP is
susceptible to rank reversal in that, if alternatives are added at the end of the process, the
final; rankings could flip or reverse.

Because of its flexibility and ease of use, AHP has over 1000 doctoral dissertations and 1300
papers released and used by different industries (from the beer industry to transport industry)
for the process decision-making (Balt, 2015). Its use has extended to the mining industry,
particularly in the mining equipment and planning section (Mahase et al., 2016). Ataei et al.
(2008) summarised the application of mining in Table 12.

From the table, it is seen that AHP has been used for selection of an underground mining
method by Alpay and Yavuz (2007) where the number of mining methods alternatives were
five, and the number of criteria with 36 sub-criteria. Bitarafan and Ateai (2004) also used it to
select mining method. Yavuz (2015), used AHP in combination with Yager’ method to select
an underground method. Bogdanovic et al. (2012) used it in combination with PROMETHEE.

The pie chart in Figure 13 shows the frequent use of AHP in combination to other method
where it was used in the mining industry. Because of the existence of AHP, decisions based
on gut feel and intuition can be supported by a structured approach (Mahase et al, 2016). It
must be noted that in this study, AHP will only be used for weight elicitation.
Figure 13: Combination of different methods with AHP in the mining industry (Mahase et al, 2016)

Table 12: Summary of Application of AHP in the mining industry (Ataei et al, 2008)

<table>
<thead>
<tr>
<th>NO</th>
<th>Application areas</th>
<th>No. of attributes (No. of sub-attributes)</th>
<th>No. of alternatives</th>
<th>Proposed by</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Site selection for limestone quarry expansion</td>
<td>4 (13)</td>
<td>3</td>
<td>Kumar Dey, 2008</td>
</tr>
<tr>
<td>2</td>
<td>Optimum support design selection</td>
<td>8</td>
<td>9</td>
<td>Yavuz et al. 2008</td>
</tr>
<tr>
<td>3</td>
<td>Environmental reclamation of an open pit mine</td>
<td>9</td>
<td>4</td>
<td>Bascetin, 2007</td>
</tr>
<tr>
<td>4</td>
<td>Underground mining method selection</td>
<td>6 (36)</td>
<td>5</td>
<td>Alpay &amp; Yavuz, 2007</td>
</tr>
<tr>
<td>5</td>
<td>Rock mass classification on tunnel engineering</td>
<td>11</td>
<td>3</td>
<td>Chen &amp; Liu, 2006</td>
</tr>
<tr>
<td>6</td>
<td>Alumina-cement plant location</td>
<td>5</td>
<td>5</td>
<td>Ataei, 2005</td>
</tr>
<tr>
<td>7</td>
<td>Equipment selection at open pit</td>
<td>4 (10)</td>
<td>4</td>
<td>Bascetin, 2004</td>
</tr>
<tr>
<td>8</td>
<td>Mining method selection</td>
<td>15</td>
<td>7</td>
<td>Bitarafan &amp; Ataei, 2004</td>
</tr>
<tr>
<td>9</td>
<td>Implementation of the AHP with VBA and ArQGIS</td>
<td>4</td>
<td>2</td>
<td>Marinoni, 2004</td>
</tr>
<tr>
<td>10</td>
<td>Drilling waste discharges</td>
<td>3 (5)</td>
<td>8</td>
<td>Sadiq, 2004</td>
</tr>
<tr>
<td>11</td>
<td>Optimal equipment selection in open pit mining</td>
<td>2 (4)</td>
<td>4</td>
<td>Bascetin, 2003</td>
</tr>
<tr>
<td>12</td>
<td>Selection opencast mining equipment</td>
<td>7</td>
<td>5</td>
<td>Samanta et al, 2002</td>
</tr>
<tr>
<td>13</td>
<td>Evaluating the environmental impact of products</td>
<td>5</td>
<td>6</td>
<td>Hertwich, 1997</td>
</tr>
</tbody>
</table>
D. TOPSIS

Hwang and Yoon proposed technique for Order preferences by Similarity to Ideal Solution widely known as TOPSIS in 1981. Amongst the MCDM, TOPSIS is the most straightforward technique (Lee et al., 2007). There is limited subjective input that is needed in TOPSIS expect for weights. The main aim of the method is to ensure that the distance between the best alternative and positive ideal alternative is minimized, while maximising the distance to the negative ideal solution (Olson, 2004).

Unlike AHP, TOPSIS does not have a component to check for the inconsistency of the judgement and expressed preferences. In addition, TOPSIS must rely on other weighting methods such as AHP since it cannot elicit weights. As a result, it can handle large numbers of alternatives (at least 15) and criteria (over 27) (Lee et al., 2007). Therefore, this means that if the weights are not accurate weights, using TOPSIS method may not be viable (Olson, 2004). Like AHP, TOPSIS can also cause rank reversal where the preferences of alternative can change if more criteria are added/removed. However, among many methods, it has the fewest rank reversals (Lee et al., 2007).

On a positive note, TOPSIS can identify the best alternative quicker than many MCDM (Olson, 2004). Its logic is rational and understandable. In addition, the importance of weights can be incorporated into the comparison procedure (Lamata & García-Cascales, 2012). The performance of alternatives and criteria can be visualised on a polyhedron; and the computation process can easily be done using a spreadsheet (Lee et al., 2007).

Over 100 papers have been published where TOPSIS was applied (Zavadskas et al., 2016). Because of its ability to accommodate many alternatives and criteria, it has been applied in various areas such as in manufacturing companies, water management, quality control transportation, and product design. (Lee et al., 2007). TOPSIS has also been used to compare financial performances of companies (Olson, 2004). Additionally, Hester and Vasquez (2013) confirm its use in the supply chain management, logistics, engineering, marketing and environmental management.

Tajvidi et al. (2015) used TOPSIS in selecting an optimum tunnel support system, combining it with methods like SAW. Aghajani and Osanloo (2007) applied the method in combination with AHP when selecting a loading and transportation system for an open pit mine. Ooriad et al. (2018) has used the TOPSIS method to select a suitable mining method for Tazareh Coal Mine (Iran). Amongst 14 mining methods, evaluated against 12 criteria, long wall mining was selected as the most suitable mining method. Table 13 shows more application of the TOPSIS Method.

<table>
<thead>
<tr>
<th>No</th>
<th>Application Areas</th>
<th>No. of attributes</th>
<th>No. of alternatives</th>
<th>Proposed by</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Company financial ratio comparisons</td>
<td>4</td>
<td>7</td>
<td>Deng et al.</td>
</tr>
<tr>
<td>2</td>
<td>Expatriate host country selection</td>
<td>6 (25 sub)</td>
<td>10</td>
<td>Chen &amp; Tzeng</td>
</tr>
<tr>
<td>3</td>
<td>Facility location selection</td>
<td>5</td>
<td>4</td>
<td>Chu</td>
</tr>
<tr>
<td>4</td>
<td>Gear material selection</td>
<td>5</td>
<td>9</td>
<td>Milani et al</td>
</tr>
<tr>
<td>5</td>
<td>High-speed transport system selection</td>
<td>15</td>
<td>3</td>
<td>Janic</td>
</tr>
<tr>
<td>6</td>
<td>Manufacturing plant location analysis</td>
<td>5 (16)</td>
<td>5</td>
<td>Yoon and Hwang</td>
</tr>
</tbody>
</table>
The process of TOPSIS in application is as follows:

Step 1
Develop a matrix like the one shown on the first step. The matrix must have ‘m’ feasible alternatives [A, A₁, A₂…Aₘ] row-wise and ‘n’ evaluation criteria [x₁, x₂, … xₙ] column-wise. Xᵢj represents the performance of an alternative Aᵢ under criterion xⱼ, and Wⱼ is the weight of the criterion such that ∑ₙⱼ=₁ Wⱼ = 1 (Lamata & Garcia-Cascales, 2012).

\[
\begin{pmatrix}
X_{11} & X_{12} & \cdots & X_{1n} \\
X_{21} & X_{22} & \cdots & X_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
X_{m1} & X_{m2} & \cdots & X_{mn}
\end{pmatrix}
\]

Step 2
Normalize the decision matrix using the equation below. Normalization is the process of converting all the scores on the matrix to conform to a norm or standard by removing varying measurements. The normalised values are obtained column-wise and must be positive numbers between zero and one (Shih et al, 2007). It must be noted that the criteria can either be a benefit (increasing effect) or cost criteria (decreasing effect). Their different effects are shown in calculations. The normalised values (rᵢⱼ) will then be calculated using the following formula:

\[
r_{ij} = \frac{X_{ij}}{\sum_{i=1}^{m} X_{ij}^2} \quad i = 1,2, \ldots n, j = 1,2, \ldots n
\]

Step 3
This step shows how important a criterion is over another. The weights can be obtained from AHP or other weighting methods, (Lamata & Garcia-Cascales, 2012). The weighted normalised matrix (aᵢⱼ) is then determined from using the following:

\[
a_{ij} = w_{j}r_{ij} \quad i = 1,2, \ldots m \text{ and } j = 1,2, \ldots n
\]
\[
\begin{pmatrix}
 a_{11} & a_{12} & \cdots & a_{1n} \\
 a_{21} & a_{22} & \cdots & a_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}
\]
Step 4

The positive \((A^+\) and negative \((A^-)\) ideal solution can be determined from the weighted normalised matrix:

\[
A^+ = (a_1^+, a_2^+, a_3^+), i = 1,2, \ldots, n \\
A^- = (a_1^-, a_2^-, a_3^-), i = 1,2, \ldots n
\]

Note that \(A^+\{\max_i a_{ij}, \min_i a_{ij}\} \) if \(j\) is a benefit attribute (larger is better)

and \(A^- = \{\max_i a_{ij}, \min_i a_{ij}\} \) if \(j\) is a cost attribute (Smaller is better)

Step 5

Euclidean distance formula is then used to calculate the distance between the ith-alternative and the ideal alternative as well as the non-ideal alternative solution the following way:

\[
D_i^+ = \sqrt{\sum_{j=1}^{n} (a_{ij} - a_{ij}^+)^2} \quad i = 1,2, \ldots m \tag{3}
\]

\[
D_i^- = \sqrt{\sum_{j=1}^{n} (a_{ij} - a_{ij}^-)^2} \quad i = 1,2, \ldots m \tag{4}
\]

Step 6

The last step involves calculating the relative closeness \((C_i)\) to the ideal solution of each alternative. The higher the value, the greater the preferences on the list of alternatives.

\[
C_i = \frac{D_i^-}{D_i^- + D_i^+}; 0 \leq C_i \leq 1, i = 1,2, \ldots m \tag{5}
\]

For ease of understanding, a graphical representative of the TOPSIS method is presented below. A-E represent alternatives that are a result of the decision made based on criterion C1 and C2. The ideal and anti-ideal are identified. Using the Euclidian distance, the C point is the closest one to the ideal and, while D would be the furthest alternative from the anti-ideal variant (Vavrek et al, 2017).

![Graphical representation of the TOPSIS Method (Vavrek et al, 2017)](image)

Figure 14: Graphical representation of the TOPSIS Method (Vavrek et al, 2017)
E. TODIM

TODIM (Tomada de Decisao Interative Multicriterio in Portuguese) means “Interactive and Multi-Criteria Decision Making” method. It is a distinct method that has its basis on the prospect theory (Zindani et al, 2017). Gomes and Lima to assist in ranking of alternatives where the decision maker has to effectively formulate a decision in the face of risk (Chakraborty, A & Chakraborty S, 2018) founded it in the early 90s.

The prospect theory was derived from a joint research of two Israeli psychologists whose objective of the research was to evaluate the behaviour of human being when a decision that involves risk must be made. From the research, the psychologists observed that when people are faced with a risk, they often opt for a smaller gain to ensure security against the risk instead of running a risk to obtain a greater gain. It was also observed that when humans must make a decision that involves a loss, then people would rather run the risk of a greater loss than to accept a smaller secure risk (Gomes et al, 2011).

A value function of this method is shown in Figure 15 and is similar to the prospect theory gain and loss function. The s-curved function shows a concave curve above the horizontal line that represent the gains. A strong dislike (aversion to risk) is reflected on the concave, while a propensity to risk is represent on the convex side of the graph below the horizontal line (Dehghani et al, 2017). The function is simple constructed from the differences perceived by the decision maker between two alternatives (Gomes et al, 2011).

![Figure 15: Value function of the Todim / prospect theory (Ozturkcan & Sengun, 2016)](image)

The advantage of TODIM is that it is effective in behavioural decision-making. This is because the decision maker’s psychological character is taken into account and can capture loss and again under uncertainty (Huang et al, 2017). The attenuation factor, which can be adjusted, can reflect the risk preference of the decision maker (Yu et al, 2018).

Even professionals without a concrete background of MCDA (Rangel et al, 2009) define the method as a tool that is easy to implement. It can work with both qualitative and quantitative criteria. Other than TODIM the existing MCDA methods look for a solution corresponding to a global measure of a value, while in TODIM, the concept of global measurement is calculated while applying the prospect theory (Chakraborty, A & Chakraborty S, 2018).
TODIM’s application has been limited. Adali et al. (2016) have adopted it as a method of choice to select elective courses for the undergraduate student where the criteria involved the student’s interest and ability. Again, its use has extended to robot selection, material selection, exploration of the importance and performance levels in supply chain practices, and evaluation of real estate properties (Zindani et al. 2017). Chakraborty A and Chakraborty S (2018) applied TODIM in housing project selection. To evaluate broadband internet plans, Rangel et al. (2011) applied TODIM. There has been limited application in the mining industry. However, Dehghani et al. (2017) used the method in a mining method selection process.

TODIM consist of six easy steps to implement:

Step 1
Given that n and m are number of alternatives and criteria respectively, a decision matrix showing the evaluation \( x_{ij} \) of \( i^{th} \) alternative on the criteria is constructed:

\[
\begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1m} \\
  x_{21} & x_{22} & \cdots & x_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{n1} & x_{n2} & \cdots & x_{nm}
\end{bmatrix}
\]

\( i = 1,2,\ldots n; \ j = 1,2,\ldots m \)

Step 2
Like in TOPSIS, the decision matrix is normalised to become dimensionless to allow all the elements to be comparable. To normalise beneficial criteria, the following equation is used:

\[
P_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}}
\]

Non-beneficial (cost) criteria uses the following equation for normalisation:

\[
P_{ij} = \frac{1}{x_{ij}} / \sum_{i=1}^{n} \frac{1}{x_{ij}}
\]

Step 3
This step involves the calculation of weights \( w_j \) for each of the criterion. Shannon entropy or AHP can be used to calculate the relative importance of the criteria. The reference criterion \( C_r \) is then determined by selecting the criterion with the maximum weight \( w_r \), the weight of the reference criterion. The relative weight is calculated as:

\[
W_{cr} = \frac{W_r}{w_r}
\]

When the relative weight is determined, all pairs of differences between performance measurements can be translated to the same dimensions (reference criterion) (Chakraborty, A & Chakraborty S, 2018).
Step 4

The dominance degree of one alternative over another can then be calculated using the following formula:

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \phi_c(A_i, A_j)$$

(9)

The evaluation of the degree of dominance calculated above is performed using the following equation. It is the sum over all the criteria of both relative gains, loss or zeros values (Chakraborty, A & Chakraborty S, 2018).

$$\phi_c(A_i, A_j) = \begin{cases} \frac{w_{cr}(P_{ic} - P_{jc})}{\sum_{c=1}^{m} w_{cr}} & \text{if } (P_{ic} - P_{jc}) > 0 \text{ represent a gain} \\ 0 & \text{if } (P_{ic} - P_{jc}) = 0 \text{ equal contribution} \\ \frac{-1}{\theta} \sqrt{\frac{\sum_{c=1}^{m} w_{cr}(P_{ic} - P_{jc})}{w_{cr}}} & \text{if } (P_{ic} - P_{jc}) < 0 \text{ represent a loss} \end{cases}$$

(10)

Where:

- \(P_{ic}\) and \(P_{jc}\) represent the performances of the alternatives \(A_i\) and \(A_j\) for the criterion, \(c\).
- \(\theta\) is the attenuation factor that is used to represent the scattering of points because of the decision maker’s perception of loss on the adjusted S-curve. It ranges between 0 and 10. The negative quadrant of the prospect theoretical function is affected by this factor and different shapes results from its different values (Zindani, et al., 2017).
- \(\phi_c(A_i, A_j)\) represent the parcel of contribution of criterion \(c\) to function \(\delta(A_i, A_j)\). It is the expression \(\phi_c(A_i, A_j)\) that allows the adjustment of data to the value function of the prospect theory (Chakraborty, A & Chakraborty S, 2018).

Step 5

The overall dominance degree of alternative \(A_i\) \((S_i)\) is determined to rank alternatives. The alternative with a dominating score becomes the best choice. To calculate \(S_i\):

$$S_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)}{\max \sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)}$$

(13)

Step 6

To check the robustness and stability of the final rankings, a sensitivity analysis can be carried out. The elements that can be subjected to changes is the attenuation factor, the choice of the reference criterion, weights and the preference evaluations (Kazancoglu & Burmaoglu, 2013).
F. VIKOR

According to Hayati et al. (2015), VIKOR is a Serbian phrase that means ‘Vlse Kriterijumsk Optimizacija Kompromisno Resenje’, which means “Multi-Criteria Optimization and Compromise Solution”. The method was developed in 1894 to select an alternative as a compromised solution from a list of alternatives in order to make a final decision. According to the method, the closest valid solution to the ideal solution is the compromise solution (Hayati et al, 2015). The VIKOR method ranks alternative according to three scalar quantities ($S_i$, $R_i$ and $Q_i$) which are independently evaluated against the criteria (Caterino et al, 2008).

Unlike AHP, there is no pairwise comparison of the criteria in the VIKOR because criterion can be evaluated independently. In addition, the computations can be less in the face of several criterion. (Hayati et al, 2015). Apart from weight determination, VIKOR only requires the decision maker’s intervention where the coefficient ‘v’ value must be chosen. The TOPSIS and VIKOR have the same approach except that VIKOR allows for weight change through the coefficient ‘v’ (Caterino, et al 2008).

Another added advantage to VIKOR method is that it allows the decision maker to check how far the second-best alternative is from the first. If the method finds that the best alternative in terms of $Q_i$ is the best in terms of the global criteria performance only ($S_i$) or in terms of the performance measurement of single criterion ($R_i$) only, then the first best alternative cannot be considered as the best in isolation, but with other alternatives in a subgroup. Therefore, VIKOR gives satisfaction to acceptability of the final rankings. (Caterino, et al 2008). The method is a useful tool especially where the decision maker is unable to express his/her preferences at the beginning of the process (Thiagarasu & Rengaraj, 2015).

VIKOR has been used in many applications as recorded by Moghassem and Fallahpour (2012). It was used in 2016 by Wang et al. for renewable energy resources selection in China. Kuo et al (2011) to evaluate the quality of airports service used VIKOR again. The successful application of VIKOR has extended too many fields such as manufacturing, material selection, marketing, construction, risk and financial management, supply chain, health-care, performance evaluation and many other areas (Mardani et al, 2015). In the mining industry, there has been limited application of VIKOR. Mahase et al. (2016) identified two areas dealing with mine planning and related studies to have applied this method. Azimi et al used VIKOR in evaluating the strategies of the Iranian mining sector in 2013. To derive the preference order of open pit mines equipment, Bazzazi et al. (2011) applied a modified version of VIKOR. There are over 176 papers published between 2004 and 2015 where VIKOR was applied; either alone or through an integrated approach (Mardani et al, 2015).

VIKOR is performed through the following process:

**Step 1**

Establish a decision matrix like the previous matrix constructed in the other MCDMs.

**Step 2**

Compute the utility (satisfaction-S) and regret (Rejection-R) measures respectively:
The satisfaction ($S$) value indicates the distance of the alternative from the ideal point. So, the $S$-value is obtained for each alternative against a criterion. The sum of all the alternatives against the criteria gives us the $S_j$ option. The $R_j$-value represent the maximum rejection of the alternative because of the distance from the ideal point. Therefore, the highest value ($S_j$) of each option per criterion represent the rejection index ($R_j$) of that alternative (Hayati et al., 2015).

**Step 3**

Calculate the VIKOR index $Q_i$:

$$Q_i = v \left( \frac{S_j - S^*}{S^- - S^*} \right) + (1 - v) \left( \frac{R_j - R^*}{R^- - R^*} \right)$$

Where $S^* = \min_j S_j ; S^- = \max_j S_j ; R^* = \min_j R_j ; R^- = \max_j R_j$. The parameter ‘$v$’ is chosen by the decision maker and must be between zero and one. Common practice is such that a ‘$v$’>0.5 is chosen when the decision maker wants to give more importance to the utility measure. A ‘$v$’< 0.5 is used when the regret measure wants to be given more weight. However, when the two terms are considered equally relevant, then a ‘$v$’ that equals 0.5 should be used (Caterino et al., 2008).

**Step 4**

Rank the order of preferences:

The alternative with the smallest value of $Q$ is determined as the best solution in the VIKOR and ranked as the best alternative (Q minimum) only if the following conditions are met:

**Condition 1**: Acceptable advantage: $Q(A'') - Q(A') \geq DQ$

Where $A'$ and $A''$ are the first and second alternatives respectively with the best rankings in the Q list. $DQ = \frac{1}{m-1}$; $m$ is the number of alternatives.

**Condition 2**: Acceptable stability in decision-making:

In the second condition, $A'$ must be recognised as the best ranked in S and/or R groups.

So, if one of the conditions is not satisfied, the n compromised solutions are proposed as follows:
• Alternatives A’ and A” will be the best option if condition two is not satisfied
• If one of these conditions is not satisfied, then it will not be possible to select the best solution of the set. A subset, of options that are preferable will be defined. In the subset A’ and A” must be included (Caterino et al, 2008).
GREY RELATIONAL ANALYSIS (GRA)

Grey Theory is a mathematical theory that was proposed in 1982 by Deng to solve problems with uncertainty and incomplete information. The grey theory consists of five parts: Grey prediction model, Grey relational analysis (GRA), Grey decision, Grey programming, and Grey control. The area of interest for decision makers is the GRA. This technique treats each alternative as a sequence of data. It then analyses the relational degree between each alternative and the reference sequence (Kuang, 2014).

The reference sequence is the ideal solution that is represented as the best performance when measured against the criteria (Kuang, 2014). The main idea of the GRA is to compare the geometrical similarity between the reference sequence and the data sequences of several alternatives. A higher relational degree means that the sequences (data and reference) are close to each other (Dai et al., 2014). This is illustrated in the reproduced graph from Kuang (2014). In the case of the graph, Alternative 1 would be the best option since it is closer to the reference sequence.

![Figure 16: Grey relational degree illustration (Kuang, 2014)](image)

GRA application has been significant; from agriculture to environment and engineering (in decision-making). Wu (2017) used it for decision making in credit risk analysis. Hasani et al. (2012) determined the optimum process parameters for open-end spinning yarns by applying GRA. Kandasamy and Vinodh (2017) applied GRA in material and end of life strategy selection. GRA has also been used in combination with other MCDA methods such as in evaluating the customer perceptions on in-flight service quality using a fuzzy-grey method by Chen et al. (2010). To optimize multi-response simulation problems, Kuo et al. (2008) used a grey-based Taguchi method. Although it has been successfully applied, GRA has also been criticized for its lack of mathematical foundation (root) to explain its origin, laws and constraints (Lu, 2015).

In the mining industry, Dehghani et al. (2017) used GRA to select a mining method. To assess mine safety, Xu Q and Xi K (2018) used GRA in combination with bow tie. GRA was used to study the coalmines accidents by Shuai and Jin-Long (2008). In 2018, Bao et al. applied GRA in combination to DEA model to evaluating the safety benefits of the mining industry occupational health and safety management systems.
The steps in GRA are as follows:

Step 1
Construction of a matrix:
Like in the previous methods, a matrix of alternatives evaluated against the chosen criteria is drawn.

Step 2
Data pre-processing:
This is a process of transferring data sequences to comparable sequences so that by default, GRA may not yield wrong results. Comparable sequences are normalised sequences ranging between zero and one (Sallehuddin, n.d.). The following three approaches are used:

- If a higher performance is better:
  \[ x^*(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \]  
  \( i = 1,\ldots,m \quad k = 1,\ldots,n \)

- If a lower performance is better:
  \[ x^*(k) = \frac{\max x_i^0(k) - x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \]  
  \( i = 1,\ldots,m \quad k = 1,\ldots,n \)

- If the value to be calculated is closer to a target value \( x^*(k) \) where \( \min x_i^0(k) \leq x^*(k) \leq \max x_i^0(k) \)
  \[ x^*(k) = \frac{1 - |x_i^0(k) - x^*(k)|}{\max\{\max x_i^0(k) - x^*(k), x^*(k) - \min x_i^0(k)\}} \]  
  \( i = 1,\ldots,m \quad k = 1,\ldots,n \)

Where:
- \( i = 1,\ldots,m \)
- \( k = 1,\ldots,n \)
- \( m \) represent the number of criteria.
- \( n \) represent the number of alternatives.
- \( x_i^0(k) \) represent the original sequence.
- \( x^*(k) \) represent the comparability sequence.

Step 3
Derive the reference sequence:
The maximum value \( x_{0j}^\sim \) on the column of each criterion becomes the reference value; collectively, the values make up a reference sequence denoted as:

\[ x_{0j}^\sim = (x_{01}^\sim, x_{02}^\sim, x_{03}^\sim, x_{04}^\sim, \ldots, x_{0m}^\sim) \]

The reference sequence is basically the best performance sequence that any sequence could achieve (Kandasamy & Vinodh, 2017).

Step 4
Generate and calculate the grey relational coefficient:
The measure of similarities between the reference sequence \( x_{0j} \) and the comparability sequence \( x^{*}(k) \) are then calculated to establish the relational coefficient. The following equation is used:

\[
\gamma_{ij}(x_{0j}, x^{*}(k)) = \frac{\Delta min + \delta \Delta max}{\Delta_{0,j} + \delta \Delta max}
\]

(21)

Where:

\[
\Delta min = \min \min |x_{0j} - x^{*}(k)|
\]

\[
\Delta max = \max \max |x_{0j} - x^{*}(k)|
\]

\[
\Delta_{0,j} = |x_{0j} - x^{*}(k)|,
\]

which represent the deviation of the comparability sequence from the reference sequence.

\( \delta \in [0,1] \) is the identification coefficient that is used to adjust the significance of the \( \Delta max \) value. Its role is to make a better distinction between the comparable and the reference sequence by changing the magnitude of the relational coefficient without necessarily changing the rankings. A value of 0.5 is the typically used value because it gives a moderate distinguishing effect as well as stability (Sallehuddin, n.d.).

Step 5

Generation of the Grey Relational Degree (GRD)

The grey relational degree of an alternative with respect to the reference sequence can then be calculated by averaging the grey relational coefficients. This is the level of correlation between the alternatives and the reference sequence. The comparability sequence that is most similar to the reference sequence will have the highest GRD. Therefore, the corresponding alternative of that comparability sequence is the one highly rated (Kandasamy & Vinodh, 2017). The higher the value, the better the ranking of the alternative (Kuang, 2014).

The formula for obtaining the GRD is: 

\[
\gamma(A_0, A_i) = \sum_{j=1}^{m} w_j \gamma(x_{0j}, x^{*}(k))
\]

where the sum of the weights equal one.
H. PROMETHEE

PROMETHEE which stands for Preference Ranking Organisation Method of Enrichment Evaluation was been developed in 1982 by Brans. Since then, PROMETHEE I to VI have been developed to function as outranking methods. Alternatives are compared in pairs with respect to each criterion (Tomic, 2011). A preference function approach is followed in PROMETHEE. A preference function $P_j(a, b)$ for alternatives ‘a’ and ‘b’ depends on the determined difference $[d_j(a, b)]$ of the alternatives for a chosen criterion, $j$. Additionally, it depends on the preference functions that are shown in Table 14. The parameter $q_j$ and $p_j$ represent the indifference (the largest difference when comparing alternatives below which the decision maker considers the alternatives negligible) and preference (smallest difference that justifies strict preference) respectively.

Table 14: generalized preference functions of PROMETHEE. (Lerch et al, 2017)

Using the following steps, PROMETHEE can rank a finite set of alternatives against conflicting criteria. PROMETHEE II and I are used for partial and complete ranking respectively (Iphar & Alpay, 2018).

Step 1

Construct an evaluation matrix where the performance of the alternatives ($i = 1, 2, 3, \ldots m$) can be evaluated using quantitative and qualitative criteria ($j = 1, 2, 3, \ldots n$).
Step 2
Compute the pairwise performance difference between the alternatives for each criterion:

\[ d_j(a, b) = f_j(a) - f_j(b) \] (22)

Step 3
Choose the type of criterion function; indifference, and/or the preference function threshold values for each criterion. The six types of preference functions are used.

In choosing the functions, some of the tips to consider are presented by Kumar and Sultana (2012) as follows:

- The V-shape (Type 3) and the Linear (Type 5) can best work with quantitative criteria (Price, costs, and power).
- Gaussian (Type 6) is hardly used because it is difficult to find its (reversal point) parameter.
- Usual (Type 1) and Level (Type 4) are best for Yes/No (qualitative) criteria scales.
- Level (Type 4) can also be used to differentiate smaller deviations from larger ones.
- The U-shape or Quasi (Type 2) is a special case of the Type 4 preference function and hardly used.

The indifference and preference function are based on the user’s own judgement but must be consistent with the previous studies and a sensitivity analysis can be performed to check for consistency and stability (Kumar & Sultana, 2012).

Step 4
Compute the Multi-criterion preference index:

\[ \pi(a, b) = \sum_{j}^{k} p_j(a, b)w_j \] (23)

This is the overall performance of ‘a’ over ‘b’ for the criteria. \( w_j \) denotes the weight of the \( j^{th} \) criterion. If the calculated value is closer to one, then the greater the preference (Brans & Vincke, 1985).

Step 5
The positive and negative outranking flow of alternative \( a \), in a set of alternatives is then calculated and partially ranks the alternatives (PROMETHEE I):

\[ \phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \pi(a, x) \text{ positive outranking flow} \] (24)

\[ \phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \pi(x, a) \text{ negative outranking flow} \] (25)

The Positive Outranking flow expresses the extent of how an alternative outranks the others. If this value is high, then the alternative is better. (Deshmukh, 2013)
The Negative Outranking flow expresses the extent of how an alternative is outranked (dominated) by the other alternatives. If this value is small, the better the alternative. (Deshmukh, 2013).

Step 6

\[ \phi(a) \] represent the net outranking flow of total ranking (PROMETHEE II). The highest value amongst all the alternatives makes that alternative the most attractive.

\[ \phi(a) = \phi^+(a) - \phi^-(a) \]  

(26)

As a last step, a sensitivity analysis can be performed to verify the stability of the alternative rankings when the weight, indifference and preference functions are altered (Giurca et al, 2014).

Giurca et al (2014) used PROMETHEE in selection of Photovoltaic Panels. PROMETHEE has been successfully applied in strategic planning of natural resources (Kangas et al, 2001). Energy technologies have been previously assessed using PROMETHEE (Oberschmidt et al, 2010). PROMETHEE has also been implemented in the Robotics field (Taillandier & Stinckwich, 2011).

Zooming into application in the mining industry, PROMETHEE was used for a Chromite mine in Turkey to select the most suitable underground ore transport system (Elevli & Demirci, 2004). Bogdanovic et al. (2012) integrated AHP with PROMETHEE for mining method selection. Elevli and Dermici (2004) applied PROMETHEE to select the most suitable underground ore transport. For selecting an underground mining method, Balusa et al, (2018) integrated WPM and PROMETHEE.

A clear advantage the PROMETHEE method has over AHP and other MCDMs is that there is no need to perform a pair-wise comparison when alternatives are removed or added in the evaluation process (Athawale & Chakraborty, 2010). Hyde et al (2003) highlighted some of the limitations of PROMETHEE to include the following: Decision makers finds it difficult to define the preference and indifference thresholds because of limited availability of selection guidelines. The uncertainty of the chosen thresholds is also not fully accounted even though a sensitivity analysis is later performed. The subjective input of the preferences introduces yet uncertainty. Additionally, Hyde et al (2003) further advises the user to note the fact that the six criterion functions introduced do not address the imprecision of the decision matrix constructed from expert judgement. So, difficulties may still be encountered in the process of using PROMETHEE because of these limitations and a considerable amount of uncertainty remains in the ranking process (Hyde et al, 2003).
I. ELECTRE

ELECTRE (Elimination Et Choix Traduisant la REalite) translated to mean: Elimination and Choice Expressing Reality was developed in 1968 by Bernard Roy. Since then, different ELECTRE methods have been developed. ELECTRE I & ELECTRE II are developed for selection problems. ELECTRE TRI is for sorting problems, and ELECTRE II, III, and IV are for ranking problems (Kangas et al, 2001). The method is used for analysing data in a decision matrix to rank a set of alternatives. Like PROMETHEE, the are indexes (concordance and discordance) that are used in the pairwise comparison between alternatives (Yavuz, 2013). The following are steps involved in ELECTRE I:

Step 1
Construction, normalization, and establishing the weighted matrix of the decision matrix.

The step is like the previous ones for the other MCDM methods. The constructed matrix ($R_{ij}$) is normalized using equation (27) for benefit criteria. While non-benefit criteria can be normalised by subtracting equation (27) from 1.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$ where $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$ (27)

Like previously, the normalised matrix is multiplied to the weight to give the weighted matrix.

Step 2
The Ascertainment of concordance and discordance interval sets (Yavuz, 2013).

In a set of alternatives, $A= \{a, b, c,\ldots\}$, the concordance ($C_{ab}$) and discordance ($D_{ab}$) sets are:

$$C_{ab} = \{j | x_{aj} \geq x_{bj}\}$$ (28)

$$D_{ab} = \{j | x_{aj} < x_{bj}\} = J - C_{ab}$$ (29)

Every pair of alternatives either belongs to a concordance or discordance subset. Concordance set consists of all attributes for which 'a' is preferred over 'b'. The complement set, discordance consist of all attributes for which 'b' is preferred over 'a'.

Step 3
Calculate the concordance interval matrix:

$$C_{ab} = \sum_{j \in C_{ab}} w_j$$ (30)

$$C = \begin{bmatrix}
  - & c(1,2) & \cdots & c(1,m) \\
  c(2,1) & - & \cdots & c(2,m) \\
  \vdots & \vdots & - & \vdots \\
  c(m,1) & c(m,2) & \cdots & -
\end{bmatrix}$$
Step 4  
Calculate the discordance interval matrix, which represent the degree of disagreement:

\[ D_{ab} = \frac{\max_{j \in D_{ab}} |x_{aj} - x|}{\max_{j \in I, k \in I} |x_{mj} - x_{nk}|} \]  

(31)

The maximum in the numerator is from the discordance set. The maximum in the denominator is from the overall sets (concordance and discordance).

\[ D = \begin{bmatrix} - & d(1,2) & \cdots & d(1,m) \\ d(2,1) & - & \cdots & - \\ \vdots & \vdots & - & \vdots \\ d(m,1) & d(m,2) & \cdots & - \end{bmatrix} \]

Step 5  
Determine the concordance matrix index:

\[ c^- = \sum_{a=1}^{m} \sum_{b=1}^{m} c(a,b) / m(m-1) \]  

(32)

In addition, the Boolean Matrix (E) is given by:

\[ e(a,b) = \begin{cases} 1 & \text{if } c(a,b) \geq c^- \\ 0 & \text{if } c(a,b) < c^- \end{cases} \]

Step 6  
Determine the discordance index matrix:

\[ d^- = \frac{\sum_{a=1}^{m} \sum_{b=1}^{m}(a,b)}{m(m-1)} \text{ where } m \text{ is the dimension of the matrix} \]  

(33)

The discordance index (F) matrix is given by:

\[ f(a,b) = \begin{cases} 1 & \text{if } d(a,b) \geq d^- \\ 0 & \text{if } d(a,b) < d^- \end{cases} \]

Step 7  
Determine the aggregate dominance matrix, G. It is determined by multiplying corresponding elements of Matrix E and F from step 5 and 6.

Step 8  
Eliminate and rank the alternatives.

In this step, all the columns are checked, and the column with the least amount of number ‘1’s should be chosen as the best (Afshari et al, 2010). If two alternatives have the same amount of number ‘1’, a sensitivity analysis can be performed by changing the concordance and discordance indices since it is an approximate threshold value (Peng et al, 2014). For
better accuracy, Peng et al (2014) suggest that approaches or ways of finding optimal values of the thresholds may be researched further.

One common advantage for many decision-making methods is the ability to handle both qualitative and quantitative criteria. ELECTRE possess such ability. Sometimes ELECTRE fails to sort the alternatives in different ranks, in those cases a hybrid approach may be necessary (Ashfari et al, 2010). A hybrid approach is a process of integrating MCDMs methods to reach a final ranking.

Hobbs and Meier (2000) have used ELECTRE in the civil and environmental engineering. Ashfari et al (2010) used it for personnel selection. Over 540 papers where ELECTRE was applied have been published. The papers represent fields such as energy management, natural resources, environmental management, health, safety, medicine, design, and mechanical engineering. To select optimal technology for surface mining, Stojanovic et al. (2017) applied an integrated AHP-ELECTRE. Bodzióny et al. (2016) used ELECTRE to select surface mining haul trucks.
J. HIERARCHICAL PREFERENCE VOTING SYSTEM (HPVS)

The Preference voting system (PVS) allows voters to select ‘m’ number of candidates from among ‘n’ candidates so that the selected can be ranked from the most to the least preferred. Different ranking places are determined, and each candidate may receive votes from the determined ranking places. To calculate the total score for the candidate, the weighted votes are summed; and the preferred candidate is the one with the highest score (Wang et al, 2007). The formula for calculating the total score is shown below:

\[ Z_i = \sum_{j=1}^{m} v_{ij}w_j \]  \hspace{1cm} (34)

Where \( w_j \) is the importance weight of the j\(^{th}\) \((1, 2, \ldots m)\) ranking place. \( v_{ij} \) represent the vote of the candidate ‘i’ that is being ranked in the j\(^{th}\) place. The table below illustrate the structure of PVS (Nourali et al, 2012).

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Ranking places</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1</td>
<td>... P_j ... P_m</td>
<td></td>
</tr>
<tr>
<td>Weights of the ranking places</td>
<td>W_1 ... W_j ... W_m</td>
<td></td>
</tr>
<tr>
<td>Votes of each candidate</td>
<td>V_{11} ... V_{1j} ... V_{1m}</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Candidate _n</td>
<td>V_{n1} ... V_{nj} ... V_{nm}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z _n = \sum_{j=1}^{m} v_{nj}w_j</td>
</tr>
</tbody>
</table>

The main key to ranking in PVS is to determine the weights associated with the ranking places. Different approaches have been used for determining weights; such as the Broda-Kendall (BK) method. The method has been widely used because of its simplicity. It assigns weights from the highest to the lowest ranking place. The first place is given a weight of ‘m’; the second place is given \( m - 1 \), followed by \( m - 2, \ldots, 2 \). The last place is given a weight of one. Though simple, the production of weights in this method is subjective. As a result, in 1990 Cook and Kress suggested the application of DEA-Data Envelopment Analysis, which reduces subjectivity (Nourali et al, 2012).

In DEA, the candidate can choose his/her own weights such that the overall rating is maximised (Vencheh, 2014). A candidate is said to be DEA efficient when they have the highest score. Because of the freedom to choose weights, often there may be more than one candidate who is DEA efficient. Consequently, Cook and Kress suggested that the gap between weights must be maximised such that only one candidate will be DEA efficient.
suggestion implied that a discrimination intensity function \( \varepsilon \) would need to be determined subjectively; and it may be difficult for the candidate to do so. The suggestion was found to be equivalent to imposing common weights like the BK method (Wang et al, 2007). Therefore, three new models proposed by Wang et al (2007) proved to reduce the difficulty in choosing weights; and all of them lead to a stable full ranking. The models are presented in Table 16.

Table 16: HPV weight models proposed by Wang et al (2007)

<table>
<thead>
<tr>
<th>Model 1: Maximize ( \alpha )</th>
<th>Model 2: Maximize ( \alpha )</th>
<th>Model 3 Maximize:</th>
</tr>
</thead>
</table>
| Subject to: \[
Z_i = \sum_{j=1}^{m} v_{ij}w_j \geq \alpha \quad i = 1, \ldots, n
\]
| Subject to: \[
\alpha \leq Z_i = \sum_{j=1}^{m} v_{ij}w_j \leq 1
\]
| \[
\begin{align*}
w_1 & \geq 2w_2 \geq \cdots mw_m \geq 0 \\
\sum_{j=1}^{m} w_j &= 1
\end{align*}
\]
| \[
\begin{align*}
Z_i &= \sum_{j=1}^{m} v_{ij}w_j \\
w_1 & \geq 2w_2 \geq \cdots mw_m \geq 0 \\
\sum_{j=1}^{n} w_j^2 &= 1
\end{align*}
\]

Model 1 uses a linear DEA model to determine the weights. The model maximises the common lower bound of the total scores. It is suitable for studies with many candidates because it is easy to compute. Model 2 is the same as Model 1 except that the upper bound of the total score equal 1. Model 3 uses nonlinear DEA model. (Nourali et al, 2012).

Nourali et al, (2012) for MMS developed a modified PVS system. It was considered that an MMS problem is hierarchical in nature and could be divided into two objectives. Firstly, a ranking of criteria and determination of their relative weights. Secondly, alternatives are ranked with respect to each criterion. The first and second objectives are illustrated within Table 17 and Table 18.

The relative importance \( (p) \) of each criterion is characterised; from the most (\( IL_1 \)) to the least (\( IL_p \)). Decision makers are then asked to assess each criterion in terms of the importance levels. The number of decision makers assessing each criterion are represented by \( V_{jk} \) appearing within the dotted lines (Nourali et al, 2012). Given the weights associated with each importance level, the total score of the criterion can be obtained using the following formula:

\[
TC_j = \sum_{k=1}^{p} v_{jk}w_j
\]

Table 17: PVS for criteria (Nourali et al, 2012)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Importance Levels</th>
<th>Total Score</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IL_1</td>
<td>( \cdots )</td>
<td>IL_k( \cdots )</td>
</tr>
<tr>
<td>Weights of importance level</td>
<td>( W_1) ( \cdots )</td>
<td>( W_k ) ( \cdots )</td>
<td>( W_p )</td>
</tr>
</tbody>
</table>
The same process is followed for alternatives. Priorities of alternatives are represented in the ranking place from the most to the least and they overall number is represented by \( h \). Let \( v_{ij} \) be the vote of alternative \( i \) being ranked in the \( h \)th ranking place. Then the total score can be calculated as shown in the ultimate score column of Table 18. The highest ultimate score will be the preferred alternative (Nourali et al, 2012).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C_1</th>
<th>\ldots</th>
<th>C_j</th>
<th>\ldots</th>
<th>C_m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights of importance level</td>
<td>W_1</td>
<td>\ldots</td>
<td>W_j</td>
<td>\ldots</td>
<td>W_m</td>
</tr>
<tr>
<td>Ranking places</td>
<td>\ldots</td>
<td>Ranking places</td>
<td>\ldots</td>
<td>Ranking places</td>
<td></td>
</tr>
<tr>
<td>Ultimate Scores</td>
<td>U_{T1}</td>
<td>\ldots</td>
<td>U_{Tj}</td>
<td>\ldots</td>
<td>U_{Tm}</td>
</tr>
<tr>
<td>Weights of each ranking place</td>
<td>w_{11}</td>
<td>\ldots</td>
<td>w_{1h1}</td>
<td>\ldots</td>
<td>w_{1j1}</td>
</tr>
<tr>
<td>Votes of each criterion in each ranking place</td>
<td>A_1</td>
<td>\ldots</td>
<td>V_{11h1}</td>
<td>\ldots</td>
<td>V_{1j1}</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>A_n</td>
<td>\ldots</td>
<td>V_{n11}</td>
<td>\ldots</td>
<td>V_{n1j}</td>
<td>\ldots</td>
</tr>
</tbody>
</table>

Nourali et al (2012) have only used HPV in selection of a mining method. Its application is limited and will not be further tested in this study because of the impossibility of applying it without voters. However, it was mentioned as part of the research so that readers may be aware of its existence.

\[
\sum_{j=1}^{m} v_{ik}w_j
\]

\[
\sum_{j=1}^{m} v_{mk}w_j
\]
2.2.3. TECHNIQUES IN OTHER INDUSTRIES

A. OPERATIONAL COMPETITIVENESS RATING ANALYSIS (OCRA)

Like the other MCDM, OCRA was developed in 1991 by Parkan to calculate the performance of alternatives. It uses an intuitive approach to incorporate the preferences of the decision maker about the relative importance of the criteria (Madic et al, 2015). The decision maker's preferences for the criteria is reflected by the preference ratings of the alternatives (Chatterjee & Chakraborty, 2012).

The advantage of OCRA over some MCDM is that, one can deal with both beneficial and non-beneficial criteria separately without having to lose some information in the process. The method is not a parametric approach and that implies that it is not affected by additional parameters in the process and less steps are required for the whole procedure (Madic et al, 2015). Also, Chakraborty and Chatterjee (2012) add that OCRA can deal with situations where there is a dependency between the relative weights of the criteria and the alternatives.

The following is a procedure for the OCRA method as presented by Madic et al (2015):

Step 1
Construct a decision matrix

Step 2
The preference rating of the decision maker for the non-beneficial criteria for each alternative is determined. The aggregate performance of the alternative with respect to all non-beneficial criteria is calculated using the following formula:

\[
\bar{I}_i = \sum_{k=1}^{q} w_k \times \frac{\text{max}(x_i^k) - x_i^k}{\text{min}(x_i^k)}
\]

(36)

\(q\) represents the number of non-beneficial criteria
\(\bar{I}_i\) represents the measure of the relative performance of the \(i\)-th alternative.
\(x_i^k\) represents the performance score of the \(i\)-th alternative for the \(k\)-th criterion.
\(w_k\) is the weight of the non-beneficial criterion.

If the \(i\)-th alternative is preferred over \(m\)-th alternative with respect to the \(k\)-th criterion, then \(x_i^k < x_m^k\).

Step 3
A linear preference rating for the non-beneficial criteria is the determined, so that a zero rating can be assigned to the least preferable alternative. The formula used is:

\[
\bar{\bar{I}}_i = \bar{I}_i - \text{min} (\bar{I}_i)
\]

(37)

Where by, \(\bar{I}_i\) represent the aggregate preference rating for the \(i\)-th alternative with respect to the criteria.
Step 4

The preference rating of the decision maker for the beneficial criteria for each alternative is determined. The aggregate performance of the alternative with respect to all non-beneficial criteria is calculated using the following formula:

$$\bar{O}_i = \sum_{h=1}^{b} w_h \times \frac{x_i^h - \min (x_i^h)}{\min (x_i^h)}, i = 1, 2, \ldots, m$$  \hspace{1cm} (38)

Where:

$b$ represents the number of beneficial criterion and the weight is represented by $w_h$. A higher score implies that the alternative is preferred more.

Step 5

A liner preference rating for beneficial criteria is determined.

$$\bar{\bar{O}}_i = \bar{O}_i - \min(\bar{O}_i)$$  \hspace{1cm} (39)

Step 6

The overall performance ratings of competitive alternative are then computed:

$$P_t = \bar{I}_i + \bar{\bar{O}}_i - \min(\bar{I}_m + \bar{\bar{O}}_m)$$  \hspace{1cm} (40)

The highest overall performance ratings represent the alternative, which is the best choice.

In application, Madic et al. (2015) used the method to select non-conventional machining processes using the OCRA method. Parkan used it in 2002 to measure the operational performance of a public transit company. Chakraborty et al. (2013) applied OCRA with other MCDM to selection location of distribution centres. For Biomass selection, Martinez et al. (2016) used OCRA method with TOPSIS. There has been limited application of the method in the engineering field, especially in the mining industry.
B. ADDITIVE RATIO ASSESSMENT (ARAS)

According to Adali and Isik (2016), ARAS is a method that determines the performance as well as compare alternatives with a chosen ideal alternative. Zavadskas and Turksis developed it in 2010 with an emphasis that a degree of optimality is obtained by determining the ratio of the sum of the weighted normalised values of an alternative to the sum of the values of the weighted normalised of the optimal alternative with respect to criteria (Adali & Isik, 2016). The method is simple and can be performed in Excel. Therefore, the fact that it does not have a complex theoretical background like AHP and the rest makes it favourable to those who wants a simplistic answer (Kocak et al., 2018).

The following is the procedure of the ARAS method as described by Zavadskas and Turksis (2010):

Step 1

The first stage is to develop a decision matrix like in the previous methods; where the \( x_{ij} \) is the performance of the alternative, \( i \) in respect to the criterion, \( j \). 'N' represents the number of alternatives, while 'm' represents the number of criteria.

\[
X = \begin{bmatrix}
     x_{11} & x_{12} & \cdots & x_{1m} \\
     x_{21} & x_{22} & \cdots & x_{2m} \\
     \vdots & \vdots & \ddots & \vdots \\
     x_{n1} & x_{n2} & \cdots & x_{nm}
\end{bmatrix} \quad (i = 1,2,\ldots,n; \ j = 1,2,\ldots,m)
\]

The optimal performance \( x_{0j} \) is determined. It may be given. If not, then for the benefit criteria, the maximum value is assumed as the optimal performance. For the non-beneficial criteria, a minimum value on the column of the criteria is determined as the optimal performance. The equation below shows how the optimal performance for beneficial and non-beneficial criteria can be determined respectively:

\[
x_{0j} = \max_j x_{ij} \\
x_{0j} = \min_j x_{ij}^*
\]

Step 2

In step 2, a normalised decision matrix is constructed, to turn the criteria to dimensionless values. The beneficial criteria are normalised as follows:

\[
\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \tag{41}
\]

The non-beneficial criteria are normalised by applying the 2-stage procedure as follows:

\[
x_{ij} = \frac{1}{x_{ij}^*} \tag{42}
\]

The reciprocal of each criterion is determined and then used in the following equation

\[
\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \tag{43}
\]
Step 3

The weighted normalised matrix is constructed. Note that the weights must be determined as carefully as possible because they influence the solution. Weights must sum up to one.

\[ \sum_{j=1}^{n} w_j = 1 \]

The normalised-weighted values are determined as follows:

\[ x_{ij} = \bar{x}_{ij} \times w_j \]  \hspace{1cm} (44)

Step 4

The optimality function \((S_i)\) is then determined for each alternative as follows:

\[ S_i = \sum_{j=1}^{m} \bar{x}_{ij} \]  \hspace{1cm} (45)

Amongst all alternatives, the highest optimality function becomes the best; and the least becomes the worst. Therefore, the higher the optimality function, there more effective an alternative is.

Step 5

A utility function value \((K_i)\) is then determined. This value determines the relative efficiency of an alternative over the optimal alternative. The value ranges between 0% to 100% (Adali & Isik, 2016). The alternative with the highest utility function is the best choice.

\[ K_i = \frac{S_i}{S_0} \text{ where } S_0 \text{ is the optimality function of the optimal or best alternative.} \]  \hspace{1cm} (46)

The application of ARAS method has been evident over the years. Zavadskas and Turskis (2010) used the method for the first time to evaluate the microclimate in office rooms. Kocak et al (2018) used the method to select a subcontractor in the construction industry. It was used to rank Serbian banks in 2013 by Stanujkic et al. Dahooie et al. (2018) applied ARAS in evaluating oil and gas well drilling projects. Adali and Isik (2016) applied ARAS in an air conditioner selection problem. Nguyen et al (2016) carried a conveyor equipment evaluation out using ARAS and AHP. Like OCRA, there is few or no application in the engineering field, especially mining field.
C. CASE-BASED REASONING (CBR)

The approach to problem solving is often guided by rule-based systems such as the MCDAs, which have been discussed earlier. As successful as these systems have been, their weakness is that all the known information must be encoded into rules, formalised, validated and verified for suitable computing. The process is said to be prone to errors, intentions can be missed because of ill-posed questions, misinterpretations, and misrepresentation. Because of this weakness, Case-Based Reasoning (CBR) was introduced (Wu, 2008 & Becerra-Fernandez et al, 2004).

The starting point of CBR is from the 1982 reminding of Schank who argues that people are automatically reminded of past encounters when faced with a similar situation. He adds that if the encounters are adequately similar, then their solutions are the same (Wu, 2008 & Becerra-Fernandez et al, 2004). CBR works by gathering knowledge and information from previously worked-on cases and store it in memory. The stored cases can be used to derive solutions for new problems. This is done by going back into the information storage (referred to database) and evaluate similar cases’ solutions (Ziba, 2015).

Therefore, the solution of the most suitable case can be adapted to be the solution of the new problem. If necessary, adjustments, and transformations can be made to the solution to perfectly fit it in for the new problem. CBR systems also have instruments, which helps in implementing the algorithms, which can compare cases, as well as adapting solutions. Therefore, the underlying bases of the method is the presumption that similar cases have similar solutions. (Ziba, 2015)

The application of CBR can be summarised as an operation cycle in Figure 17.

![Figure 17: A cycle of CBR (Ziba, 2015)](image)

The advantage CBR has in application is that it has a database of previous information; consequently, its solution-finding capabilities are high. There is also continuous learning where CBR is adopted because new cases are always being added into the database and
accessible for solving future cases (Ziba, 2015). Application of CBR leads to avoiding personal influence of individual experts when decisions must be made. Experience is not generalised into rules with the use of CBR (Wu, 2008 & Becerra-Fernandez et al, 2004).

Some of the setbacks of the CBR system are that it may be difficult to calculate (compute) the similarity distance between the new case (actual) and the old case (desired). However, there are methods such as Euclidean distance, Jaccard Method, and Tversky method that can be used to calculate the similarity degree (Mulyana et al, 2015). Some cases are poorly documented, and that means inaccurate information may rollover the new cases. In addition, if there is too little diversity in the many cases stored in the database, then significant gap coverage may result. Sometimes solutions may conflict with each other (Wu, 2008 & Becerra-Fernandez et al, 2004).

CBR has gain much popularity especially in the financial sector. Zima (2015) has used it in performing a cost estimation of a construction project. The application has extended vastly to the health field. Mulyana et al (2015) used the method with input text processing to diagnose mood disorders. Drilling engineering has successfully applied case based reasoning and different applications in petroleum engineering is present by Shokouhi et al (2012). In Korea, it has been used as a decision-making support system to select a method for a construction project (Yoon et al, 2016). Bjuren (2013) has used it to predict energy usage. There has not been much recorded use in the engineering field except where CBR was used as a Decision support method. Figure 18 shows in a broad perspective where CBR can be applied.

![Figure 18: Application of CBR (Gabel T, 2010)](image)

University of Stirling (2015) identified some indicators that can be used to judge if CBR is suitable for use in the given problem. These indicators are:

- Existing records of previously solved cases.
- There is often referral to historical cases when solving new cases.
- Rather than referring to general rules or policies, human experts tend to talk about the problem domain in terms of examples.
- A problem that is not well defined and difficult to understand.
- Experience is seen as valuable as textbook knowledge.

In applying CBR, the similarity degree of the cases can be calculated using different methods like the ones shown below. It has been suggested that the Modified-Tversky method outperforms the Jaccard and the Tversky method in terms of consistency (Mulyana et al, 2015).
Let \( \text{Sim} (X, Y) \) represent similarity between two cases with a finite no of criteria, the following formulas can then be used:

**Jaccard Method:**

\[
\text{Sim}(X,Y) = \frac{|X \cap Y|}{|X \cup Y|} = \frac{\#\text{SAME}(X,Y)}{\#\text{SAME}(X,Y) + \#\text{DIFFER}(X,Y)}
\]

Where \( \text{SAME} \) represent the number of features that are similar in the cases and \( \text{DIFFER} \) refers to the features that differ.

**Tversky Method:**

\[
\text{Sim}(X,Y) = \frac{\alpha |X \cap Y|}{\alpha |X \cap Y| + \beta |X \emptyset Y|} = \frac{\alpha (\#\text{SAME}(X,Y))}{\alpha (\#\text{SAME}(X,Y)) + \beta (\#\text{DIFFER}(X,Y))}
\]

Where: \(|X \emptyset Y| = |X \cup Y| - |X \cap Y| \) and the value \( \alpha \) and \( \beta \) are formulated by experts parallel to their respective significance of value (Mulyana et al, 2015)

And the Modified Tversky Method with an addition of weight is calculated as follows:

\[
\text{Sim}(X,Y) = \frac{2|X \cap Y|}{|X| + |Y|} = \frac{2\alpha (\#\text{SAME}(X,Y))}{2\alpha (\#\text{SAME}(X,Y)) + \beta (\#\text{DIFFER}(X,Y))}
\]

The method described above cannot be used as an MCDM; however, in the development of the MMSM, it can aid as storage for the previously conducted studies that relate to mining method.
D. COMPLEX PROPORTIONAL ASSESSMENT (COPRAS)

The development of COPRAS by Zavadskas and other researchers dates to 1996. It is a fast-developed method to deal with real problems. The method can be performed without difficulty even when the attribute and alternatives are large; and it can handle both qualitative and quantitative criteria (Mousavi-Nasabi & Sotoudeh-Anvari, 2017). However, COPRAS is less stable when a sensitivity analysis is performed and gives different rankings when there are changes in the weights (Podvezko, 2011).

It has been applied in a supplier selection problem because of its simplicity and advantage of plugging values onto EXCEL for a faster implementation (Madic et al., 2014). Chatterjee and Charaborty (2014) used it for a manufacturing firm to select the most appropriate flexible manufacturing system. Assessment of road design has been done by COPRAS (Zavadskas et al., 2007). In combination with the fuzzy AHP, Das et al (2012) used COPRAS to measure the relative performance of Indian technical institution. It has also successfully been applied in the construction as well as property management (Petkovic et al., 2015).

The procedure of COPRAS is best presented by Madic et al. (2014) in the following way:

Step 1

Construct a matrix, X, where the $x_{ith}$ is the performance of the alternative, $i$ in respect to the criterion, $j$.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \quad (i = 1, 2, \ldots, n; \ j = 1, 2, \ldots, m)$$

Step 2

Normalize the constructed matrix, R.

$$R = r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad \quad (47)$$

Step 3

Determine the weighted normalised matrix, D; by multiplying the elements of the matrix with the weights. The sum of the new elements must always equal the weight of the criterion.

$$D = y_{ij} = r_{ij} \times w_j \quad \quad (48)$$

$$\sum_{i=1}^{m} y_{ij} = w_j$$

Step 4

Calculate the sum of the benefit and non-beneficial criteria for each alternative:

$$S_{+i} = \sum_{j=1}^{n} y_{+ij} \ for \ beneficial \ criteria; \ S_{-i} = \sum_{j=1}^{n} y_{-ij} \ for \ non-beneficial \ criteria$$
Step 5

Establish the relative significance value of the alternatives that shows the priority of an alternative.

\[ Q_i = S_{+i} + \frac{S_{-\min} \cdot \sum_{i=1}^{m} S_{-i}}{S_{-i} \cdot \sum_{i=1}^{m} \left( \frac{S_{-\min}}{S_{-i}} \right)}, \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,m, \] (49)

Where \( S_{-\min} \) is the minimum value of \( S_{-i} \).

Step 6

Calculate the quantitative utility for each alternative:

\[ U_i = \frac{Q_i}{Q_{\max}} \cdot 100\%, \quad (50) \]

Where \( Q_{\max} \) is the maximum relative significance value.

The utility values thus range from 0% to 100%. A higher value implies that the alternative is ranked higher.
E. SIMPLE ADDITIVE WEIGHTING (SAW)

Hwang and Yoon suggested the SAW method in 1981. It is also called the weighted linear combination method (Tajvidi et al, 2015). It is described as one of the most straight-forward MCDM methods. Its application is usually for benchmarking; to evaluate results from other techniques (Mousavi-Nasabi & Sotoudeh-Anvari, 2017).

The advantages of the method is that it is intuitive to decision makers, and there is no need for any complex compute program as the computations are easy. However, the drawbacks are that the criteria should be a maximizing criterion before any calculation; this means that minimizing criteria must be turned to maximizing, and that leads to the method not reflecting real situation problems. As a result, the results obtained may not be logical. Despite the drawbacks, the application of SAW ranges from water management, to business and financial management (Velasquez & Hester, 2013). Afshari et al. (2010) applied the method in personnel selection problems. Setyani and Saputra to determine flood-prone area at Semarang City used the SAW method in 2016. There has been limited use of SAW in the mining industry.

Tajvidi et al (2015) present the procedure for ranking alternatives using SAW as follows:

Step 1

Construct a matrix as previously explained using other methods such as AHP.

Step 2

Normalization of the matrix is performed using positive and negative linear method respectively for the criterion:

\[ r_{ij} = \frac{x_{ij}}{\max_i \{x_{ij}\}} \quad i = 1,\ldots,m \text{ and } j = 1,\ldots,n \]  

\[ r_{ij} = \frac{\min_i \{x_{ij}\}}{x_{ij}} \]  

Step 3

Determine the weight of each criterion, \( w_j \) : AHP or any other determining weights method can be used to determine the weights (relative importance) of the criterion.

Step 4

Calculate the evaluation score: The alternatives’ evaluation \( A_i \), score as explained by Afshari et al (2010), is calculated by multiplying the normalised value with the weight of the criterion. It is then followed by summing the products of all criteria for each alternative. The following formula is used:

\[ A_i = \sum_{j=1}^{m} w_j r_{ij} \]  

Step 5

Rank the alternatives: The alternative with the highest evaluation score is the suitable choice.
F. COMPROMISE PROGRAMMING (CP)

According to Park et al. (2015), Zeleny proposed CP in the 70s for identification of an alternative that is closest to the ideal solution based on the distance measure $L_p$. Poff et al (2010) applied CP to evaluate forest management approaches. Park et al (2015) emphasised that CP is effective in solving environmental problems. It has also been used in the field of water resource management. However, when compared with other methods, it is significantly less used (Stanujkic et al, 2013). The method has proven to be robust and sensitive to the weight and the ‘p’ value chosen by the decision maker. It is therefore advisable to perform a sensitivity analysis to check for stability in the answers obtained (Poff et al, 2010).

The procedure of the CP is as follows as presented by Kumar (N.D):

**Step 1:**
Construct a decision matrix of alternative ‘a’ with respect to the j-th criterion with a value of $F_j(a)$.

**Step 2:**
Determine the weight of each criterion, $w_j$

**Step 3:**
Specify the parameter ‘p’.

The parameter $p$ reflects the decision maker’s choice (weight) on how he/she compensate for the deviations. It governs the distance between the ideal point and the solution. This non-dominated parameter takes on the values between one and any largest deviations such that $1 \leq p \leq \infty$ (Park et al, 2015). When $p = 1$, all the deviations from the ideal value are taken in direct proportion to their magnitude. When $p = \infty$, the largest deviation from the ideal is considered to have the greatest weight (Kumar, N.D).

**Step 4:**
Compute the distance metric of the alternative $L_p$, which is used to estimate the degree of closeness between points in multi-dimensional space (Li et al, 2013).

$$L_p(a) = \left( \sum_{j=1}^{j=J} w_j^p \left[ \left( \frac{M_j - F_j(a)}{M_j - m_j} \right)^p \right] \right)^{\frac{1}{p}}$$

Where $L_p(a)$ represent the distance metric of the alternative ‘a’. $J$ represent the total number of criteria. $M_j$ represent the best (maximum) value in the criterion set, and $m_j$ represent the worst (minimum) value

**Step 5:**
Rank alternatives based on the distance metric of the alternative. The best alternative will have the lowest $L_p$. Which implies that it is closest to the ideal solution.
2.3. EXISTING COAL MINING METHODS

In the following section, different underground coal mining methods are discussed for a better understanding of these operations.

2.3.1. BORD-AND-PILLAR MINING

Bord-and-pillar, known as ‘room and pillar’ mining is developed by extracting coal in a series of narrow roads. It is a checker-board method. These roadways are separated by a block of coal that is parallel to them. At the right angle to the roadways, the second set of roadways are established and connected to the first ones. Square pillars, whose size depends on the depth and width of the roadways are formed between these roadways (Raghavan et al., 2014). At times, depillaring or pillar extraction may be performed. This is a process where pillars are mined/taken out and the roof is allowed to collapse on its own into an area known as the goaf (Nayak & Dalai, 2010).

The process of mining in Bord-and-Pillar layout was previously performed by the conventional method of drilling and blasting. However, the introduction of Continuous Miners (CM) has improved the conventional way of mining. The coal is extracted using a continuous miner (CM) whose components, such as the rotating drum with sharp picks, allows it to cut and load the coal simultaneously. The extracted coal is then loaded onto shuttle cars and/or conveyor belts; it will then be transported to surface. A roof bolter is used to install support to the mining face. Figure 19, illustrates the workings of bord-and-pillar operations.

![Figure 19: Typical Bord-and-Pillar layout (Harraz, 2014)](image)

According to Nayak and Dalai (2010), bord-and-pillar, mining is suitable for flat or gently dipping deposits whose seam thickness is greater than 1.5m. Raghavan et al. (2014) argues that the method has been successful in thinner seams (1.2) and thicker seams (up to 4.8m).
Bord-and-Pillar is performed at depths less than 300m because greater depths may induce pillar crushing. However, in India depth of 600m has been reached because of a higher coal thickness (Raghavan et al. 2014). The seam type must be such that there are free or minimum stone or dirt bands are not gassy, and have a strong roof and floor which can stand for longer periods even after the development is completed (Nayak & Dalai, 2010).

The following are the advantages and disadvantage of bord-and-pillar mining as quoted from Wagner (1980) and Singh (2004):

- Bord-and-pillar provides flexible operating procedures. Therefore, if need be, the initial planning can be modified if geological disturbances are encountered.
- The capital investments are low.
- The method is not sensitive to geological disturbances, which implies that it can deal well with variability in geology unlike methods such as Long wall mining.
- The integrity of the roof strata is maintained.
- The sequence of extraction is visible.
- Since the pillars are left behind as support, there are cost reductions for supports.

The following are drawbacks:

- An increase in depth requires large pillars for support.
- As a result, the extraction percentage reduces as depth increases.
- It is difficult to control ventilation in the system because of numerous connections.
- Subsidence can sometimes be experienced.
- There is a higher risk of spontaneous combustion.

The method has been widely used in the United States of America (USA). In the 1970’s, 50% of the production was accounted for by bord and pillar. Brazilian underground coal mines employ bord-and-pillar for extraction. The wide use of the method is because it can be used in hard rock (limestone, dolomite, metals such as lead, copper, zinc and gold) and in soft rock such as coal, potash and salt. The recovery can reach 85% (Harraz, 2015). Most of the South African underground coal mines also employs bord-and-pillar because of the advantages mentioned above.

2.3.2. PILLAR EXTRACTION

As previously, mention, pillars left behind during bord-and-pillar can be mined to increase the ratio of extraction, especially as the depth increases. There must be an increased safety factor since a support (pillar) is being extracted in these high extraction methods. The overlying strata is allowed to come in (goaf) and that may result in negative impacts such as surface subsidence and damage to the underground water structure leading to inflows of water into the workings. It is therefore necessary that in planning such aspects should be taken into consideration (University of Pretoria, 2018). It must also be noted that total pillar recovery’s application has been curtailed in most countries due to increased risk (Spearing, 2019).

The following are some of the pillar extraction methods that exist:

A. ANGULAR CUT PILLAR EXTRACTION
In angular cut pillar extraction, diagonal cuts are made through the pillar. The focus was on maximum recovery of the pillar. That has changed. The current method ensures that critical intersections are protected by leaving a rib (1m) or snook along the diagonal after three cuts have been made. This is done to protect the CM during the cutting process. It is applicable to depth of up to 30m and pillars with a safety factor of 1.6. A good production is achieved (University of Pretoria, 2018). The figure below illustrates the angled cut method sequence. Pillars are extracted from left to right in a straight line.

![Figure 20: Left: Angled cut pillar EXTRACTION SEQUENCE and ventilation layout. Right: cutting sequence in angled pillar extraction (University of Pretoria, 2018)](image)

**B. SPLIT AND FENDER METHOD**

It is the commonly practiced method in the South African mines. The pillar in a pre-developed panel is split into two or three ribs (fenders) and are then extracted in a similar manner to rib pillar mining. Approximately pillar sizes of 18m are favourable; and an overall extraction that varies from 66% to 80%. The main driver behind the pillar design is that the geometry must be able to allow the splitting of the pillar into fenders of 6m. The safety factor must be above 1.6 (University of Pretoria, 2018). Pillars are then extracted from left to right as depicted in

![Figure 21: Left: Extraction and ventilation layout in Split and fender method. Right: cutting sequence of split and fender method](image)
C. NEVID METHOD OF EXTRACTION

The Nevid method was developed to overcome most of problems encountered when extracting pillars in horizontal stressed zones. A flexible method can be applied in all circumstances. The top middle pillars are split as shown in the figure below. However, they are not fully mined to prevent goafing. Extraction of 60% to 64% is achieved through the Nevid method (University of Pretoria, 2018)

![Nevid Method Diagram](image)

Figure 22: The Nevid method sequence (University of Pretoria, 2018)

2.3.3. LONG WALL MINING

Long wall mining involves the removal of coal in large blocks using a mechanized shearer. The panels can be as wide as 5km in length. The shearer used for coal extraction is mounted on rails, which serve as a guide in moving the shearer back and forth along the coal face. Once the coal is cut, the Armoured Face Conveyor (AFC) transport the coal to the adjoining road that the coal may be conveyed to surface. Hydraulic shields are used to support the roof above the working area. The shields advances as mining proceeds, and that causes the roof behind the shields to collapse since it is not supported (Ralston et al, 2017).
Long wall mining is characterized by high productivity (Vardhan & Kumar, 2017). 95% of recovery rate is expected in such operations (Scott et al, 2010). Higher productivity and recovery rate are because of its ability to operate at greater depths of greater than 300m. It is also less labour intensive unlike room and pillar mining. Because of the connection between the equipment working in a long wall, it is almost a continuous process (Scott et al, 2010).

Long wall mining can either retreat or advance depending on the mine planning. In the advance method, development or the opening of the passageways and panel extraction take place simultaneously. However, the development moves slightly ahead of the face. On retreat mining, which is the mostly preferred by Americans (Holman, 1999), developments are driven all the way to the end; and extraction process starts at the far end and move towards the main entries. (Kgweetsi, 2016)

Longwall is unsuitable where the geology is irregular. In irregular and harsh working conditions, the machinery used can be subjected to shock and vibrations, which may lead to either unplanned maintenance or total replacement of the parts. This may be time-consuming and financially unviable. Another drawback is that once there is a breakage in the machinery, the whole production is stopped. Automation of the mining process in long wall aims to address some of the shortcomings of the method (Vardhan & Kumar, 2017). In addition, there is substantial subsidence with this method. As coal extraction progresses, subsidence begins. It is dependent on time, depth of mining, thickness of the coal bed and the strength of the overlying rock (Kgweetsi, 2016). Long wall mining is capital intensive.

Long wall mining method is one of the two methods that are used to mine underground coal; and the most common underground method in Australia and China (Scott et al, 2010). It is best suited for almost all geological conditions, especially thinner seams of less than 1.8m thickness. Greater depth, gassy seams, and seams prone to spontaneous heating can be mined using this without difficulty (Raghavan et al, 2014).
2.3.4. SHORT WALL MINING

When the depths of a coal seam are such that bord-and-pillar cannot be employed, and there are geological features limiting the use of long wall mining, short wall mining is introduced to overcome the shortcomings of the two methods. A CM, shuttle cars, and hydraulic supports are used to extract pillars during the initial development to form panels. A wall of 90m would be economically optimum to perform short wall mining (Kushwaha & Banerjee, 2005). In addition, coal face heights of short wall mining typically range between 4.5m to 6.0m (Yu-de et al, 2008). Short wall mining was developed to overcome the disadvantages of long-wall and bord-and-pillar mining. A typical layout of a short wall panel is shown in Figure 25.

![Figure 24: Short Wall Mining Trueman (1984)](image)

The advantages of short wall mining method in use are the following as presented by Fauconnier (1982).

- Hydraulic support shields personnel from fall of grounds.
- Ventilation systems makes the environment conducive for working.
- Less strenuous work and labour in a panel.
• Higher productivity and recovery rate (approximately 85%).

The limitations as a result of little consideration given to the overall strata control factors are presented below:

• There are abutment stresses build-up that result in continuous miners being less efficient. Consequently, the supports would tilt towards the previously extracted panels and a loss of support is observed. This is a major consideration hence it has only limited application and mainly in South Africa (Spearing, 2019)

2.4. POTENTIAL COAL MINING METHODS

The following methods have not been assessed in any of the existing MMSM. However, it is necessary to take them into consideration because they have the potential to grow in the coal mining industry. The methods are underground coal gasification, and coal bed methane.

2.4.1. UNDERGROUND COAL GASIFICATION (UCG)

UCG is one of the most effective method in converting coal to useful forms of energy. The process of conversion is much cleaner than in normal combustion processes, which releases hazardous pollutants. In coal gasification, solid coal is converted to combustible gases through scientific reactions without the need to mine it (Dzimba, 2011).

The process of UCG is in several steps. Firstly, series of boreholes are drilled into the coal seam in situ. These bore holes are connected underground by either directional drilling or fracturing (Dzimba, 2011). Coal is then ignited by injecting highly pressured oxidants such as air, steam or oxygen. The combustion of coal and the oxidant forms carbon dioxide and heat. The heat produced is used in the subsequent reaction in which the carbon dioxide reacts with steam to form hydrogen gas, carbon monoxide, carbon dioxide and methane (Sajjad & Rasul, 2014).

Gases produced (syngas) are collected through the production wells. It is then distributed through pipes to its destination for various applications (Dzimba, 2011). Syngas can be used directly as fuel, to power gas turbines for electricity generation, chemical feedstock to produce liquid fuels (South African coal road map, 2011). So, it is evident that UCG is influenced by factors such as temperature, type and composition of coal, coal seam thickness, water incoming rate to the gasification chamber, pressure and the length and section of the gasification channel (Sajjad & Rasul, 2014) Water in the coal seam flows in the gasification cavity so that it can be utilized for the gasification process (Shafirovich et al, 2008).
The advantages associated with UCG as highlighted by Sajjad & Rasul (2014), are that: UCG increases efficiency in electricity generation by using waste heat from the product gas. There are fewer emissions of gases such as SO₂, NOₓ, and CO₂. UCG is applicable in coal seams that would even be deemed sub-economic, un-mineable and unsafe. Personnel only work from surface to drill wells and this means the UCG working environment is safe. There are low capital investment costs unlike in conventional coal mining. The gasification cavity can be used for carbon dioxide sequestration (Shafirovich et al., 2008).

However, in deeper coal seams, the costs of drilling, operating pressure, injection of oxidants may increase. While on shallow depths, surface subsidence may occur because of UCG (Sajjad & Rasul, 2014). There may be difficulties experienced in linking the wells if the coal seam thickness is insufficient. Potential loss of control over UCG could result in uncontrolled underground coal fires. Fortunately, most of the drawbacks can be resolved such as obtaining reliable geological data and using specialized linkage technologies for wells. Also, ground water pollution may be the result if the coal seams are located closer to the underground water table (Shafirovich et al, 2008).
2.4.2. COAL BED METHANE (CBM)

Coal bed methane is a natural gas formed during coalification where it remains trapped under pressure in the coal seam and the host rock. It is composed of methane, carbon dioxide, elemental nitrogen, and heavier carbon compounds such as ethane, traces of propane, and butanes. Coal rank and coal seam depth determines the amount of methane trapped. The higher the coal rank, the higher the amount of methane. At greater depth, coal seams contain a higher amount of methane than a shallower seam of the same rank. When coal seams are injected with a high-pressure water, foam, and sand mix, the coal around the borehole fractures. The water and gas will then flow to the surface when fractures are kept opened by the sand mix (Xaba and Jeffrey, 2002).

Like UCCG, it provides an opportunity to exploit coal resources that were previously referred to as uneconomic and unmineable. Fortunately, with CBM, there is no surface subsidence. The methane produced is of good quality and can be fed directly to the gas distribution network owing to its low carbon dioxide content. Gases like methane can be reduced prior to mining activities (Xaba and Jeffrey, 2002). There are low emissions of carbon dioxide, no ash or toxins. It offers profit opportunities in unmineable coal where it is not possible to mine with high-volume mining equipment in thinner seams with poor quality of coal and difficult mining conditions. Since there is a growing demand for natural gas, CB is also growing to be more attractive as a fuel (Thakur and Steve, 2014).

Factors that have been hindering the development of CBM are geotechnical in nature such as low coal permeability, variation, and low-quality gas. Economics and institutional barriers are a hindrance to the development. (Xaba and Jeffrey, 2002). Knowledge of the origin of coalbed has become a prerequisite for successful CBM exploration (Thakur and Steve, 2014).

It is applied in Belgium, Australia, USA and China. South Africa has the potential to use CBM; if extracted, methane would prevent the future import needs for natural gas (Xaba and Jeffrey, 2002). The strong economic potential of CBM renders it viable for exploration. It can be used in electricity generation or co-fired with coal at power plants to reduce SOx and NOx. CBM can also be used in turbines or fuel cells for power generation (Thakur and Steve, 2014).
2.5. FACTORS CONSIDERED IN MMS

A number of research studies have been carried out to list factors that are important in the selection of a mining method. In his research, Jianpu (2011) stated that the main influencing factors on safe coal mining and eventually coal mining method selection are the coal seam thickness, stability, and the structure of the coal seam, its physical properties, place of occurrence, variation of the coal seams as well as the coal seam roof and floor. Thick coal seams that are structurally simple have large reserves and may be easy to mine. He further argued that thin coal seams that changes abruptly and if the structure is complicated, might pose mining difficulties. All these properties also assist in selecting mining machinery. It is further stated that the occurrence of a coal seam is of importance to the design, construction, and production of the shaft (Jianpu, 2011).

In the study conducted by Jeffrey (2002), geotechnical factors that had an impact on secondary extraction of two of the South African coalfields were identified and ranked. He further went on to indicate factors that were identified to have a major impact on the selection of a mining method (for secondary extraction). These are: the size of the remaining reserves, surface infrastructure, period since primary extraction, the thickness and lithology of the overburden, depth below surface, extractable thickness, multi-seam extraction, coal strength, geological features such as dykes & sills, primary mining method, equipment, safety factor and mining history (Jeffrey, 2002).

In a mining method selection study conducted by Balusa and Singam (2017), it was indicated that factors that affect underground selection of mining method can be grouped as: physical parameters, mechanical parameters, economical parameters and technical parameters. The parameters that were used in the selection process were the dip of the deposit, its shape, its thickness, the Rock Mass Rating (RMR) of the ore, footwall and hanging wall, technology, orebody depth, ore uniformity, dilution, production, and recovery (Balusa and Singam, 2017). In comparing two most used underground mining method within Australia, Scott et al. (2009) reviewed capital cost, productivity, recovery, versatility, and mine safety as necessary in mining method selection.

Namin et al. (2009) confirmed the practical application of decision-making techniques in selecting mining methods in Iran. It was indicated that underground factors that affect the mining method selection are deposit geometry (size, shape, depth), geology, hydrology conditions (mineralogy, petrology, uniformity, alteration and weathering), geotechnical properties (elasticity, state of stress, competency and physical properties), economic considerations (mining costs, rate of production, reserves tonnage, life of mine, productivity), and technological factors (mine recovery, dilution, flexibility, selectivity, concentration of workings, capital, labour, and mechanization). Lastly, environmental factors such as ground control surface subsidence and atmospheric control.

Williams (2005) indicated in a presentation that the following physical features are key factors in selecting coal underground mining methods: coal seam thickness, depth of the overburden and its characteristics, reserve configuration, geologic features, surface features, previous mining activities, obstacles, regulation, mobility, flexibility and lastly, capital commitment.

Ooriad et al. (2018) stated that engineers and geologists must work together to identify factors that affect the mining method selection. The factors that are of major impact as stated are: physical and mechanical characteristics of the deposit such as the ground conditions of the
ore seam, the hanging and foot wall, thickness of the seam, general shape, dip, plunge and depth below the surface, how grade is distributed and the quality of resources. Capital costs, the operating costs, mineable ore tonnages and mineral value were stated as major economic factors. Technical (mine recovery, flexibility, machinery, and mining rate) and productivity (annual productivity, equipment, and environmental conditions) factors also play a role. The same factors were used in selecting a Tazerah underground coal mine. (Ooria et al, 2018), bauxite mine (Mohsen et al, 2009); underground coal mine by Yavuz (2015), and a Columbian coal mine (Gelvez and Aldana, 2014).

Bashari et al. (2013) stated that for the Angouran Zinc-Lead mine located in Iran that the factors considered must fully cover the problem at hand. The main characteristics are the orebody thickness, dip, shape, and depth, RSS (ore, hanging and footwall), RMR (ore, hanging and footwall). The same criteria were chosen and defined in detail for a successful selection of an underground fluorine mine in Iran (Javanshirgiv and Safari, 2017). An iron mine using fuzzy dominance method by Bitafaran and Ataei (2004) used the same criteria. Karadogan et al. (2008) also used the criteria mentioned previously, but in addition, the subsidence effect, nearness of the residential area, and existence of methane were used as criteria.

Bogdanovic et al. (2012) classified the most important factors to consider for underground mining method selection into three main groups: geological, technical, and economic. It was further stated that MMS is a difficult process because of relationship that exist between these factors. While one mining method may perform well in the geological factors rating, it may not be justified as the best from the financial point of view. Hence the need of effective MMSM that can be used by decision makers. In Gol-e-gohar iron mine in 2015, Deghani et al (2017) used the geological factors previously mentioned to select a mining method using the Todim and Grey analysis. 32 factors were used in selecting a mining method for a coal mine in Iran. Some of the factors that have not been mentioned above are the climate of the area, ability to mechanise and occupational considerations (Nourali et al, 2012).

From the results above, important factors were derived. They will be shown in table format in the result section and was used in the developed MMSM.
2.6. SIGNIFICANCE OF THE LITERATURE REVIEW

Chapter 2 is divided into three sections. The first section focused on studying the approaches available for method selection. Different approaches and techniques that have been used in the mining industry across different commodities have been reviewed to understand their functionality and how they will be of aid in developing a coal MMSM. The study was not limited to the mining methods selection techniques only, but has expanded into other industries to understand how decisions of method selection are handled given the complex nature presented in such procedures.

The methods identified in this section can be evaluated according to their ability of handling complex, conflicting and contradicting information. This is done so that the methods are compared in terms of functionality, application, shortcomings and strength. The results arising from the comparison was used to assist the author in developing a MMSM without compromising the strength of each method.

A MMSM must be developed that can deal with complex data to assist the user in making a mining method selection decision. The overall focus will be on how to integrate and maximise the strength the techniques have in order to overcome their individual disadvantages.

The second section introduced the mining methods that can be used as input alternatives to the MMSM. The significance of this section was to introduce the number of alternative coal mining methods that have recently been developed. Previously developed MMS techniques have not been updated to include these recent methods. Also, to understand the requirements and functionality of each mining method so that the background information can form part of the MMSM database.

The last section focused on the factors/parameters that are considered in selecting a coal mining method. The aim of studying the factors was that they could be ranked, based on their importance, and can be used as input criteria in the MMSM to qualify an optimal mining method amongst alternatives in the decision-making process.
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3. RESULT & RESULT ANALYSIS

3.1. INTRODUCTION

Chapter 3 will summarise the findings from studying the method selection techniques, factors, and mining methods. It is the aim of this chapter to class approaches and techniques to methods selection, to test the techniques against each other in different case studies, to check for consistency in decision making, and stability of the final rankings of alternatives. Two analysis approaches will be carried out; statistical and descriptive. A summary of factors that have been found consistently used in the previous MMSM will be presented so that they can be used as inputs to the proposed MMSM.

The information from chapter 3 will be used to develop the proposed MMSM in chapter 4. Its capability and applicability will be detailed. The process of getting to the model and all the calculations will be shown. Chapter 5, 6, and 7 will conclude the study, recommend, and suggest to the interested parties in the coal industry what can be done to further refine the study.

The results section will be divided into two section; a descriptive and statistical approach to comparing the MCDM. Factors and coal mining methods will be presented as conclusions of this chapter.

3.2. CASE STUDY EVALUATION

The following two case studies will be used to evaluate the MCDM methods that were introduced in the previous section. Saaty & Ergu introduced a set of criteria that will be used to evaluate the MCDMs. The Saaty & Ergu list of criteria was used because it highlights major criteria that can be used for comparison. The set is shown in Table 19. In addition to the criteria, the MCDMs’ extent of application will be evaluated as well.

Table 19: Criteria to evaluate MCDMs (Zavadaskas et al, 2016)

<table>
<thead>
<tr>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Simplicity of execution</td>
</tr>
<tr>
<td>2. Logical mathematical procedure</td>
</tr>
<tr>
<td>3. Input parameters</td>
</tr>
<tr>
<td>4. Synthesis of judgements with merging function</td>
</tr>
<tr>
<td>5. Rank of tangibles</td>
</tr>
<tr>
<td>6. Generalizability to ranking intangibles</td>
</tr>
<tr>
<td>7. Rank preservation and reversal</td>
</tr>
<tr>
<td>8. Sensitivity analysis</td>
</tr>
<tr>
<td>9. Conflict resolution</td>
</tr>
<tr>
<td>10. Trustworthiness and validity of the approach</td>
</tr>
</tbody>
</table>
3.2.1. CASE STUDY 1: INTEGRATED APPROACH TO MMS

This case study illustrates the process of selecting a mining method through an integrated approach. A fuzzy AHP and TOPSIS were applied to facilitate the decision-making process.

The case study uses information from: Shariat S, Yazdani-chamzini A, Bashari B.P, 2013, Mining method selection using an integrated model, International research journal of applied and Basic Sciences, 6(2): 199-214. The mine that was investigated is located in the Wester Zanjn province in Iran. It forms part of the major producers of zinc. The orebody is located within a metamorphic basement plunging east ward at 10-20. It is 600m long in the N-S line and 200-400m across.

In this case, a fuzzy AHP determined the weights of the criteria (shown in Table 20), and the criteria were used to rate and rank the importance of the mining method alternatives in the TOPSIS model. Amongst the alternatives, cut and fill method ranked the highest and was confirmed through a sensitivity analysis. The selection process was performed for an Angouran Zn-Pb mine in Iran.

The significance of the selected case study was that it illustrated the decision process of MMS by using TOPSIS as one of the investigated MCDM in the literature of this current study. Even though it is not a coal mining example, it better illustrated the use of MCDMs in decision making, especially where a mining method had to be selected.

The mine started as an open pit. However, as depth increased (to 2880m), it was required that a mining method be suggested for continued operations. The criteria used for this specific problem and the alternatives are summarised in Table 20.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weights</th>
<th>Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1: Orebody thickness</td>
<td>0.005</td>
<td>A1: Block Caving</td>
</tr>
<tr>
<td>C2: Orebody dip</td>
<td>0.244</td>
<td>A2: Sublevel Stoping</td>
</tr>
<tr>
<td>C3: Orebody shape</td>
<td>0.048</td>
<td>A3: Sublevel Caving</td>
</tr>
<tr>
<td>C4: Grade distribution</td>
<td>0.051</td>
<td>A4: Cut &amp; Fill</td>
</tr>
<tr>
<td>C5: Orebody depth</td>
<td>0.147</td>
<td>A5: Top Slicing</td>
</tr>
<tr>
<td>C6: Orebody RSS</td>
<td>0.092</td>
<td>A6: Square Set Stoping</td>
</tr>
<tr>
<td>C7: Footwall RSS</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>C8: Hanging wall RSS</td>
<td>0.096</td>
<td></td>
</tr>
<tr>
<td>C9: Orebody RMR</td>
<td>0.074</td>
<td></td>
</tr>
<tr>
<td>C10: Footwall RMR</td>
<td>0.134</td>
<td></td>
</tr>
<tr>
<td>C11: Hanging wall RMR</td>
<td>0.013</td>
<td></td>
</tr>
</tbody>
</table>

To illustrate how TOPSIS was applied, the information from the selected case study will be shown in seven steps that TOPSIS comprises of.

**Step 1: Development of a Matrix**

Fifteen decision makers were involved in evaluating alternative mining methods based on the given criteria (Table 20) for the selected case study 1. All the criteria are benefit criteria and the higher the score, the better the performance of the mining method (Shariati et al, 2013). For example, A2 (sub-level caving), was given a rating of four based on the judgement of the decision makers on the mine information provided. The rating is higher amongst the alternatives in the criterion 4. The rating means that the grade distribution (C4) performed...
better than the other criteria for alternative (A2). How the TOPSIS method work is explained in the literature review. Combined ratings of the 15 decision makers are shown below.

Table 21: Performance ratings of alternatives from case study 1 (Shariati et al, 2013)

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>A3</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>A5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Step 2: Normalisation of the Matrix

Equation (2) was used to normalise the scores. As initially explained, the scores in Table 21 are not necessarily similar in terms of measurements; therefore, normalisation converts all the scores to conform to the standard for ease and fair comparison. Table 22 shows the normalised values.

Table 22: Normalised performance rating (Baloyi, 2018)

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,442</td>
<td>0,378</td>
<td>0,667</td>
<td>0,338</td>
<td>0,359</td>
<td>0,000</td>
<td>0,000</td>
<td>0,229</td>
<td>0,338</td>
<td>0,433</td>
<td>0,577</td>
</tr>
<tr>
<td>A2</td>
<td>0,590</td>
<td>0,189</td>
<td>0,500</td>
<td>0,676</td>
<td>0,539</td>
<td>0,743</td>
<td>0,857</td>
<td>0,688</td>
<td>0,676</td>
<td>0,433</td>
<td>0,385</td>
</tr>
<tr>
<td>A3</td>
<td>0,590</td>
<td>0,189</td>
<td>0,500</td>
<td>0,338</td>
<td>0,539</td>
<td>0,371</td>
<td>0,171</td>
<td>0,459</td>
<td>0,507</td>
<td>0,433</td>
<td>0,577</td>
</tr>
<tr>
<td>A4</td>
<td>0,147</td>
<td>0,567</td>
<td>0,167</td>
<td>0,507</td>
<td>0,359</td>
<td>0,557</td>
<td>0,343</td>
<td>0,459</td>
<td>0,338</td>
<td>0,577</td>
<td>0,385</td>
</tr>
<tr>
<td>A5</td>
<td>0,295</td>
<td>0,378</td>
<td>0,167</td>
<td>0,169</td>
<td>0,359</td>
<td>0,000</td>
<td>0,343</td>
<td>0,229</td>
<td>0,169</td>
<td>0,289</td>
<td>0,192</td>
</tr>
<tr>
<td>A6</td>
<td>0,000</td>
<td>0,567</td>
<td>0,000</td>
<td>0,169</td>
<td>0,180</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,169</td>
<td>0,144</td>
<td>0,000</td>
</tr>
</tbody>
</table>

Step 3: Weighted normalised matrix

The weights obtained by the Fuzzy AHP for each of the 11 criteria, are now applied on the normalised matrix by multiplying each normalised value with the corresponding weight. The results are shown on Table 23:

Table 23: Weighted Normalised Matrix

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0,055</td>
<td>0,244</td>
<td>0,048</td>
<td>0,051</td>
<td>0,147</td>
<td>0,092</td>
<td>0,048</td>
<td>0,095</td>
<td>0,074</td>
<td>0,134</td>
<td>0,013</td>
</tr>
<tr>
<td>A1</td>
<td>0,024</td>
<td>0,092</td>
<td>0,032</td>
<td>0,017</td>
<td>0,053</td>
<td>0,000</td>
<td>0,000</td>
<td>0,022</td>
<td>0,025</td>
<td>0,058</td>
<td>0,008</td>
</tr>
<tr>
<td>A2</td>
<td>0,032</td>
<td>0,046</td>
<td>0,024</td>
<td>0,034</td>
<td>0,079</td>
<td>0,068</td>
<td>0,041</td>
<td>0,065</td>
<td>0,050</td>
<td>0,058</td>
<td>0,005</td>
</tr>
<tr>
<td>A3</td>
<td>0,032</td>
<td>0,046</td>
<td>0,024</td>
<td>0,017</td>
<td>0,079</td>
<td>0,034</td>
<td>0,008</td>
<td>0,044</td>
<td>0,038</td>
<td>0,459</td>
<td>0,008</td>
</tr>
<tr>
<td>A4</td>
<td>0,008</td>
<td>0,138</td>
<td>0,008</td>
<td>0,026</td>
<td>0,053</td>
<td>0,051</td>
<td>0,016</td>
<td>0,044</td>
<td>0,025</td>
<td>0,077</td>
<td>0,005</td>
</tr>
<tr>
<td>A5</td>
<td>0,016</td>
<td>0,092</td>
<td>0,008</td>
<td>0,009</td>
<td>0,053</td>
<td>0,000</td>
<td>0,016</td>
<td>0,022</td>
<td>0,013</td>
<td>0,039</td>
<td>0,003</td>
</tr>
<tr>
<td>A6</td>
<td>0,000</td>
<td>0,138</td>
<td>0,000</td>
<td>0,009</td>
<td>0,026</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,013</td>
<td>0,019</td>
<td>0,000</td>
</tr>
</tbody>
</table>
Step 4: Determine the Positive and negative ideal solution.

In this step, the ideal solutions are derived from each column. It must be noted that all the criteria are benefit criteria; which means that a higher score is a better performance and will make up the positive ideal. The negative ideal implies that a lower performance is a better performance. The derived scores are shown for each criterion in Table 24.

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>0.032</td>
<td>0.138</td>
<td>0.032</td>
<td>0.034</td>
<td>0.079</td>
<td>0.068</td>
<td>0.041</td>
<td>0.065</td>
<td>0.050</td>
<td>0.077</td>
</tr>
<tr>
<td>-</td>
<td>0.000</td>
<td>0.046</td>
<td>0.000</td>
<td>0.009</td>
<td>0.026</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.013</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Step 5: Distance calculation from the ideal solution

The distance of each alternative from the ideal solution is calculated. This separation measure indicates by how much the alternatives are further from the ideal solutions. Table 25 shows the obtained distance measures from the positive and negative ideal solutions. The equations used are (3) and (4):

<table>
<thead>
<tr>
<th></th>
<th>(D_i^+)</th>
<th>(D_i^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.1116</td>
<td>0.0818</td>
</tr>
<tr>
<td>A2</td>
<td>0.0946</td>
<td>0.1366</td>
</tr>
<tr>
<td>A3</td>
<td>0.1101</td>
<td>0.0990</td>
</tr>
<tr>
<td>A4</td>
<td>0.0628</td>
<td>0.1341</td>
</tr>
<tr>
<td>A5</td>
<td>0.1202</td>
<td>0.0654</td>
</tr>
<tr>
<td>A6</td>
<td>0.1449</td>
<td>0.0922</td>
</tr>
</tbody>
</table>

Step 6: Calculation of the relative closeness

The alternative with the highest relative closeness is rated as the preferred alternative. Equation (5) was used to obtain the relative closeness. A4 (Cut and Fill Method) was chosen as the most suitable to exploit the given deposit. While the least preferred option is A5.

<table>
<thead>
<tr>
<th></th>
<th>(C_i)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.4231</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0.5908</td>
<td>2</td>
</tr>
<tr>
<td>A3</td>
<td>0.4735</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>0.6809</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>0.3524</td>
<td>6</td>
</tr>
<tr>
<td>A6</td>
<td>0.3889</td>
<td>5</td>
</tr>
</tbody>
</table>

A sensitivity analysis was conducted to check the robustness of the results. The parameters that were chosen for the analysis were the weights of the criteria. The weights were increased...
by 20% and it was concluded that the results were not sensitive to the change because A4 still emerged as the most suitable.

To compare the performances of the MCDM, the author saw it fit to use the case study for the other methods with the following aim in mind: to check for consistency in the above results. Also, how further apart are the results. To illustrate how the other methods are applied, in this case study, all the steps of the other methods will be shown. In case study 2, only the results of the ranking will be shown.

**TODIM RANKING CASE STUDY 1**

**Step 1: Development of a Matrix**

The matrix is already developed and shown in Table 21.

**Step 2: Normalisation of the Matrix**

The evaluations in the matrix are evaluated using equation (6) for beneficial and (7) for non-beneficial criteria. In the above case study, it was noted that all criteria are beneficial; therefore, equation (6) is applicable. The results of the normalised values are shown.

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A1</strong></td>
<td>0.214</td>
<td>0.167</td>
<td>0.333</td>
<td>0.154</td>
<td>0.154</td>
<td>0.000</td>
<td>0.000</td>
<td>0.111</td>
<td>0.154</td>
<td>0.188</td>
<td>0.273</td>
</tr>
<tr>
<td><strong>A2</strong></td>
<td>0.286</td>
<td>0.083</td>
<td>0.250</td>
<td>0.308</td>
<td>0.231</td>
<td>0.444</td>
<td>0.500</td>
<td>0.333</td>
<td>0.308</td>
<td>0.188</td>
<td>0.182</td>
</tr>
<tr>
<td><strong>A3</strong></td>
<td>0.286</td>
<td>0.083</td>
<td>0.250</td>
<td>0.154</td>
<td>0.231</td>
<td>0.222</td>
<td>0.100</td>
<td>0.222</td>
<td>0.231</td>
<td>0.188</td>
<td>0.273</td>
</tr>
<tr>
<td><strong>A4</strong></td>
<td>0.071</td>
<td>0.250</td>
<td>0.083</td>
<td>0.231</td>
<td>0.154</td>
<td>0.333</td>
<td>0.200</td>
<td>0.222</td>
<td>0.154</td>
<td>0.250</td>
<td>0.182</td>
</tr>
<tr>
<td><strong>A5</strong></td>
<td>0.143</td>
<td>0.167</td>
<td>0.083</td>
<td>0.077</td>
<td>0.154</td>
<td>0.000</td>
<td>0.200</td>
<td>0.111</td>
<td>0.077</td>
<td>0.125</td>
<td>0.091</td>
</tr>
<tr>
<td><strong>A6</strong></td>
<td>0.000</td>
<td>0.250</td>
<td>0.000</td>
<td>0.077</td>
<td>0.077</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.077</td>
<td>0.063</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Step 3: Calculation of weights**

Once weights are determined, the reference criterion, which is the criterion with the highest weight rating, is determined. The relative weight can then be calculated using equation (8).

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>W_r = 0.244</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W_c</td>
<td>0.055</td>
<td>0.244</td>
<td>0.048</td>
<td>0.051</td>
<td>0.147</td>
<td>0.092</td>
<td>0.048</td>
<td>0.095</td>
<td>0.074</td>
<td>0.134</td>
<td>0.013</td>
</tr>
<tr>
<td>W_c'r</td>
<td>0.225</td>
<td>1.000</td>
<td>0.197</td>
<td>0.209</td>
<td>0.602</td>
<td>0.377</td>
<td>0.197</td>
<td>0.389</td>
<td>0.303</td>
<td>0.549</td>
<td>0.053</td>
</tr>
</tbody>
</table>

**Step 4: Dominance degree calculation**

Equation (9) to (12) are then used in calculating the dominance degree. The degree is calculated to determine the extent of which one alternative dominates the other; and it is
calculated for every alternative. The following tables from Table 29 to Table 34 shows the results of dominance.

### Table 29: The dominance of A1 over other alternatives for case study 1

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-A1</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>A1-A2</td>
<td>-1.140</td>
<td>0.143</td>
<td>0.063</td>
<td>-1.738</td>
<td>-0.724</td>
<td>-2.199</td>
<td>-3.229</td>
<td>-1.530</td>
<td>-1.443</td>
<td>0.000</td>
<td>0.034</td>
</tr>
<tr>
<td>A1-A3</td>
<td>-1.140</td>
<td>0.143</td>
<td>0.063</td>
<td>0.000</td>
<td>-0.724</td>
<td>-1.555</td>
<td>-1.444</td>
<td>-1.082</td>
<td>-1.020</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>A1-A4</td>
<td>0.143</td>
<td>-0.585</td>
<td>0.109</td>
<td>-1.229</td>
<td>0.000</td>
<td>-1.904</td>
<td>-2.042</td>
<td>-1.082</td>
<td>0.000</td>
<td>-0.683</td>
<td>0.034</td>
</tr>
<tr>
<td>A1-A5</td>
<td>0.063</td>
<td>0.000</td>
<td>0.109</td>
<td>0.063</td>
<td>0.000</td>
<td>0.000</td>
<td>-2.042</td>
<td>0.000</td>
<td>0.075</td>
<td>0.091</td>
<td>0.049</td>
</tr>
<tr>
<td>A1-A6</td>
<td>0.109</td>
<td>-0.585</td>
<td>0.126</td>
<td>0.063</td>
<td>0.106</td>
<td>0.000</td>
<td>0.000</td>
<td>0.103</td>
<td>0.075</td>
<td>0.129</td>
<td>0.060</td>
</tr>
</tbody>
</table>

### Table 30: The dominance of A2 over other alternatives for case study 1

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2-A1</td>
<td>0.063</td>
<td>-0.585</td>
<td>-1.318</td>
<td>0.089</td>
<td>0.106</td>
<td>0.202</td>
<td>0.155</td>
<td>0.145</td>
<td>0.107</td>
<td>0.000</td>
<td>-2.646</td>
</tr>
<tr>
<td>A2-A2</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.089</td>
<td>0.000</td>
<td>0.143</td>
<td>0.138</td>
<td>0.103</td>
<td>0.075</td>
<td>0.000</td>
<td>-2.646</td>
</tr>
<tr>
<td>A2-A3</td>
<td>0.109</td>
<td>-0.827</td>
<td>0.089</td>
<td>0.063</td>
<td>0.106</td>
<td>0.101</td>
<td>0.120</td>
<td>0.103</td>
<td>0.107</td>
<td>-0.683</td>
<td>0.000</td>
</tr>
<tr>
<td>A2-A4</td>
<td>0.089</td>
<td>-0.585</td>
<td>0.089</td>
<td>0.108</td>
<td>0.106</td>
<td>0.202</td>
<td>0.120</td>
<td>0.145</td>
<td>0.131</td>
<td>0.091</td>
<td>0.034</td>
</tr>
<tr>
<td>A2-A5</td>
<td>0.125</td>
<td>-0.827</td>
<td>0.109</td>
<td>0.108</td>
<td>0.150</td>
<td>0.202</td>
<td>0.155</td>
<td>0.178</td>
<td>0.131</td>
<td>0.129</td>
<td>0.049</td>
</tr>
<tr>
<td>A2-A6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

### Table 31: The dominance of A3 over other alternatives for case study 1

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3-A1</td>
<td>0.063</td>
<td>-0.585</td>
<td>-1.318</td>
<td>0.000</td>
<td>0.106</td>
<td>0.143</td>
<td>0.069</td>
<td>0.103</td>
<td>0.075</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>A3-A2</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-1.738</td>
<td>0.000</td>
<td>-1.555</td>
<td>-2.888</td>
<td>-1.082</td>
<td>-1.020</td>
<td>0.000</td>
<td>0.034</td>
</tr>
<tr>
<td>A3-A3</td>
<td>0.109</td>
<td>-0.827</td>
<td>0.089</td>
<td>-1.229</td>
<td>0.106</td>
<td>-1.100</td>
<td>-1.444</td>
<td>0.000</td>
<td>0.075</td>
<td>-0.683</td>
<td>0.034</td>
</tr>
<tr>
<td>A3-A4</td>
<td>0.089</td>
<td>-0.585</td>
<td>0.089</td>
<td>0.063</td>
<td>0.106</td>
<td>0.143</td>
<td>-1.444</td>
<td>0.103</td>
<td>0.107</td>
<td>0.091</td>
<td>0.049</td>
</tr>
<tr>
<td>A3-A5</td>
<td>0.125</td>
<td>-0.827</td>
<td>0.109</td>
<td>0.063</td>
<td>0.150</td>
<td>0.143</td>
<td>0.069</td>
<td>0.145</td>
<td>0.107</td>
<td>0.129</td>
<td>0.060</td>
</tr>
<tr>
<td>A3-A6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

### Table 32: The dominance of A4 over other alternatives for case study 1

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A4-A1</td>
<td>-1.612</td>
<td>0.143</td>
<td>-2.283</td>
<td>0.063</td>
<td>0.000</td>
<td>0.175</td>
<td>0.098</td>
<td>0.103</td>
<td>0.000</td>
<td>0.091</td>
<td>-2.646</td>
</tr>
<tr>
<td>A4-A2</td>
<td>-1.975</td>
<td>0.202</td>
<td>-1.864</td>
<td>-1.229</td>
<td>-0.724</td>
<td>-1.100</td>
<td>-2.501</td>
<td>-1.082</td>
<td>-1.443</td>
<td>0.091</td>
<td>0.000</td>
</tr>
<tr>
<td>A4-A3</td>
<td>-1.975</td>
<td>0.202</td>
<td>-1.864</td>
<td>0.101</td>
<td>-1.738</td>
<td>0.101</td>
<td>0.069</td>
<td>0.000</td>
<td>-1.020</td>
<td>0.091</td>
<td>-2.646</td>
</tr>
<tr>
<td>A4-A4</td>
<td>-1.140</td>
<td>0.143</td>
<td>0.000</td>
<td>0.089</td>
<td>0.000</td>
<td>0.175</td>
<td>0.000</td>
<td>0.103</td>
<td>0.075</td>
<td>0.129</td>
<td>0.034</td>
</tr>
<tr>
<td>A4-A5</td>
<td>0.063</td>
<td>0.000</td>
<td>0.063</td>
<td>0.089</td>
<td>0.106</td>
<td>0.175</td>
<td>0.098</td>
<td>0.145</td>
<td>0.075</td>
<td>0.158</td>
<td>0.049</td>
</tr>
<tr>
<td>A4-A6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

106
Step 5: Calculation of the overall dominance degree

The overall dominance degree is calculated using equation (13) and the alternatives are ranked based on the scores. The alternative with the highest score dominates the other alternatives. The results are shown below:

Table 35: TODIM rankings for Case study one

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.708</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>0.867</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>0.730</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>0.412</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to the result obtained from using TODIM, A2, which is Sub-level Stoping, dominates the rest of the alternatives. A4, which was proven as the best in TOPSIS (original data) and after the sensitivity analysis, is dominated by two more methods in TODIM. Even though the ranking of the above two methods differ, a sensitivity analysis is yet to be carried out for TODIM to validate its results.

The following method to be checked is VIKOR.

VIKOR Case study 1

Step 1: establishment of the matrix
Step 2: determine the best and worst criterion

In Step 2, the best criterion is represented by the maximum criterion, and the worst is represented by the minimum criterion as shown in Error! Reference source not found.

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,0000</td>
<td>3,0000</td>
<td>4,0000</td>
<td>3,0000</td>
<td>4,0000</td>
<td>5,0000</td>
<td>3,0000</td>
<td>4,0000</td>
<td>4,0000</td>
<td>3,0000</td>
<td></td>
</tr>
<tr>
<td>0,0000</td>
<td>1,0000</td>
<td>0,0000</td>
<td>1,0000</td>
<td>1,0000</td>
<td>0,0000</td>
<td>0,0000</td>
<td>0,0000</td>
<td>1,0000</td>
<td>1,0000</td>
<td>0,0000</td>
</tr>
</tbody>
</table>

Step 3: computation of the utility and regret measures:

The utility (S) values are shown in Error! Reference source not found., and they indicate the distance of the alternative from an ideal point. While the regret measure (R) is the maximum distance amongst the utility values from the ideal point. Equation (15) and (16) were used to perform the calculations:

<table>
<thead>
<tr>
<th></th>
<th>S_J</th>
<th>R_J</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,541</td>
<td>0,122</td>
</tr>
<tr>
<td>A2</td>
<td>0,305</td>
<td>0,244</td>
</tr>
<tr>
<td>A3</td>
<td>0,475</td>
<td>0,244</td>
</tr>
<tr>
<td>A4</td>
<td>0,305</td>
<td>0,074</td>
</tr>
<tr>
<td>A5</td>
<td>0,666</td>
<td>0,122</td>
</tr>
<tr>
<td>A6</td>
<td>0,757</td>
<td>0,147</td>
</tr>
</tbody>
</table>

Step 5: calculation of the VIKOR index $Q_i$

The VIKOR index is calculated for each alternative for the final ranking. The alternative with the smallest index will be determined as the best. However, it must be able to meet the following conditions:

**Condition 1**: Acceptable advantage: $Q(A'') - Q(A') \geq DQ$

Where $A'$ and $A''$ are the first and second alternatives respectively with the best rankings in the $Q$ list. $DQ = \frac{1}{m-1}$; $m$ is the number of alternatives.

**Condition 2**: Acceptable stability in decision-making:

In the second condition, $A'$ must be recognised as the best ranked in $S$ and/or $R$ groups.

In this case study, VIKOR Index were calculated and are shown on Error! Reference source not found.. The coefficient ‘v’ of 0, 5 was chosen.
Table 38: Vikor indeces

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Q.1</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,403</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>0,500</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>0,689</td>
<td>5</td>
</tr>
<tr>
<td>A4</td>
<td>0,000</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>0,542</td>
<td>4</td>
</tr>
<tr>
<td>A6</td>
<td>0,716</td>
<td>6</td>
</tr>
</tbody>
</table>

A4 has the lowest index, and therefore it is the best option. A4 must therefore meet the above-stated conditions to be qualified as the best option.

For condition 1:

\[ Q(A'') - Q(A') \geq DQ \]
\[ Q(A1) - Q(A4) \geq \frac{1}{6 - 1} \]
\[ 0,403 - 0,000 \geq 0,2 \]

For condition 2:

\[ S(A4) = 0,305 \text{ and } R(A4) = 0,074. \]

Therefore, all conditions are met. From the calculated index, A4 (Cut and Fill method) is confirmed to be the best by the VIKOR method. This confirms the TOPSIS results. However, whether the results are robust or will change if any of the parameters were changed will be confirmed by sensitivity analysis.

The following method to be applied in the case study is Grey Relational Analysis

**GRA Case Study 1**

**Step 1: Construction of a Matrix**

Like in the previous methods, a matrix is constructed and shown in Table 21.

**Step 2: Normalisation of a Matrix**

The evaluation scores are normalised using equation (18) - (20). Error! Reference source not found. below shows the normalised values. It must be noted that each row is referred to a comparability sequence.

Table 39: GRA's normalised matrix

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,75</td>
<td>0,50</td>
<td>1,00</td>
<td>0,33</td>
<td>0,50</td>
<td>0,00</td>
<td>0,00</td>
<td>0,33</td>
<td>0,33</td>
<td>0,67</td>
<td>1,00</td>
</tr>
<tr>
<td>A2</td>
<td>1,00</td>
<td>0,00</td>
<td>0,75</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>0,67</td>
<td>0,67</td>
</tr>
<tr>
<td>A3</td>
<td>1,00</td>
<td>0,00</td>
<td>0,75</td>
<td>0,33</td>
<td>1,00</td>
<td>0,50</td>
<td>0,20</td>
<td>0,67</td>
<td>0,67</td>
<td>0,67</td>
<td>1,00</td>
</tr>
<tr>
<td>A4</td>
<td>0,25</td>
<td>1,00</td>
<td>0,25</td>
<td>0,67</td>
<td>0,50</td>
<td>0,75</td>
<td>0,40</td>
<td>0,67</td>
<td>0,33</td>
<td>1,00</td>
<td>0,67</td>
</tr>
<tr>
<td>A5</td>
<td>0,50</td>
<td>0,50</td>
<td>0,25</td>
<td>0,00</td>
<td>0,50</td>
<td>0,00</td>
<td>0,40</td>
<td>0,33</td>
<td>0,00</td>
<td>0,33</td>
<td>0,33</td>
</tr>
</tbody>
</table>
Step 3: Derivation of the Reference sequence

The reference sequence is determined by looking for the maximum value in each column. The following are the results for the above-normalised values:

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref C</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
</tr>
</tbody>
</table>

Step 4: Generation and calculation of the grey relational coefficient:

The relational coefficient measures the similarities between the reference sequence and the comparability sequences. A value of zero, 5 is used as the identification coefficient in the equation (21) that is used to calculate the Grey relational coefficient shown in Error! Reference source not found..

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,667</td>
<td>0,500</td>
<td>1,000</td>
<td>0,429</td>
<td>0,500</td>
<td>0,333</td>
<td>0,333</td>
<td>0,429</td>
<td>0,429</td>
<td>0,600</td>
<td>1,000</td>
</tr>
<tr>
<td>A2</td>
<td>1,000</td>
<td>0,333</td>
<td>0,667</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,600</td>
<td>0,600</td>
</tr>
<tr>
<td>A3</td>
<td>1,000</td>
<td>0,333</td>
<td>0,667</td>
<td>0,429</td>
<td>1,000</td>
<td>0,500</td>
<td>0,385</td>
<td>0,600</td>
<td>0,600</td>
<td>0,600</td>
<td>1,000</td>
</tr>
<tr>
<td>A4</td>
<td>0,400</td>
<td>1,000</td>
<td>0,400</td>
<td>0,600</td>
<td>0,500</td>
<td>0,667</td>
<td>0,455</td>
<td>0,600</td>
<td>0,429</td>
<td>1,000</td>
<td>0,600</td>
</tr>
<tr>
<td>A5</td>
<td>0,500</td>
<td>0,500</td>
<td>0,400</td>
<td>0,333</td>
<td>0,500</td>
<td>0,333</td>
<td>0,455</td>
<td>0,429</td>
<td>0,333</td>
<td>0,429</td>
<td>0,429</td>
</tr>
<tr>
<td>A6</td>
<td>0,333</td>
<td>1,000</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
<td>0,333</td>
</tr>
</tbody>
</table>

Step 6: Generation of the GRD

The grey relational coefficients are then averaged to obtain the grey relational degree. This degree shows the similarity of the comparability and reference sequence. The higher the value of the GRD, the better the ranking. The calculated GRD are shown in Table 42. A2 (sub-level stoping) is the highly ranked; then A4 comes second. A5 is the least preferred alternative. The results will further be validated by a sensitivity analysis.

<table>
<thead>
<tr>
<th></th>
<th>GRD</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,515</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0,764</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>0,596</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>0,703</td>
<td>2</td>
</tr>
</tbody>
</table>
The following methods to be evaluated on Case study 1 are PROMETHEE I and II which will give us a partial and complete ranking respectively. A Visual PROMETHEE software was used to perform the rankings and the results are shown in the following procedure.

**PROMETHEE Case Study 1**

**Step 1: construction of a matrix**

The matrix table has already been constructed and is shown in Table 21.

**Step 2: Pairwise performance difference**

The pairwise performance was calculated within the Visual PROMETHEE software. If manual calculations were carried out, Equation (22) would have been used.

**Step 3: Choosing criteria function**

The criteria function chosen for all criterion is the Level (type 4) function. The choice was motivated by previous studies such as the integrated approach into mining method selection by Bogdanovic et al. (2012). The choice of the criteria function will influence whether the indifference (q) and preference (p) functions are to be chosen. These functions are based on the evaluator’s judgment, as long as they are consistent with previous studies, then they can be accepted. A sensitivity analysis can still be performed to check the robustness of the results (Kumar & Sultana, 2012).

In the case of the case study 1: q=1, and p=2.

**Step 4: Multi-criterion preference index**

Step 4 was calculated within the Visual PROMETHEE software. Manual calculations would have been carried out using equation (23).

**Step 5: Calculation of the positive and negative outranking**

The outranking flows were calculated within the software and are depicted by Figure 29. These outranking flows partially ranks the alternatives. A2 is a better alternative amongst the 6, since a higher positive value expresses the extent of how an alternative outranks the others. What the exact values are is shown by the PROMETHEE network in Figure 30. The negative
outranking shows that A4 is a better alternative because its value is smaller, and it expresses the extent of how an alternative is outranked by the others. In this case, the others outrank A6.

Figure 29: PROMETHEE I partial ranking of alternatives for Case Study 1

Figure 30: PROMETHEE’s diamond network ranking of alternatives for Case Study 1

Step 6: determination of the net outranking flow
The total ranking is performed and A2 was found as the alternative that outranks the others. A4 in this case came second. A sensitivity analysis will be performed to validate these results. The total rankings are shown in Figure 31.

Figure 31: PROMETHEE II final ranking of alternatives for Case Study 1

**ELECTREE case study 1**

ELECTREE is also one of the MCDM under investigation. Its steps are as follows:

**Step 1: Construct, normalise, and establish a weighted matrix**

The matrix referred to is in Table 21.

The weighted normalised matrix is like that of TOPSIS in Table 23

**Step 2: Determine the concordance and discordance sets**

In step 2, the concordance and discordance sets are determined according to equation (28) and (29). It must be noted that either ever pair of alternatives belongs to a concordance subset where alternative ‘a’ is preferred over ‘b’ or a complement set (discordance). This is illustrated in the following tables from Table 43 to Table 48.

<table>
<thead>
<tr>
<th>A1</th>
<th>C=1</th>
<th>C=2</th>
<th>C=3</th>
<th>C=4</th>
<th>C=5</th>
<th>C=6</th>
<th>C=7</th>
<th>C=8</th>
<th>C=9</th>
<th>C=10</th>
<th>C=11</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (1,2)</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>D (1,2)</td>
</tr>
<tr>
<td>C (1,3)</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>D (1,3)</td>
</tr>
<tr>
<td>C (1,4)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>D (1,4)</td>
</tr>
<tr>
<td>C (1,5)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (1,5)</td>
</tr>
<tr>
<td>C (1,6)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (1,6)</td>
</tr>
</tbody>
</table>
### Table 44: A2's concordance and discordance set for case study 1

<table>
<thead>
<tr>
<th>A2</th>
<th>C=1</th>
<th>C=2</th>
<th>C=3</th>
<th>C=4</th>
<th>C=5</th>
<th>C=6</th>
<th>C=7</th>
<th>C=8</th>
<th>C=9</th>
<th>C=10</th>
<th>C=11</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (2,1)</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>D (2,1)</td>
</tr>
<tr>
<td>C (2,2)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (2,2)</td>
</tr>
<tr>
<td>C (2,3)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>D (2,3)</td>
</tr>
<tr>
<td>C (2,4)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>D (2,4)</td>
</tr>
<tr>
<td>C (2,5)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (2,5)</td>
</tr>
<tr>
<td>C (2,6)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (2,6)</td>
</tr>
</tbody>
</table>

### Table 45: A3's concordance and discordance set for case study 1

<table>
<thead>
<tr>
<th>A3</th>
<th>C=1</th>
<th>C=2</th>
<th>C=3</th>
<th>C=4</th>
<th>C=5</th>
<th>C=6</th>
<th>C=7</th>
<th>C=8</th>
<th>C=9</th>
<th>C=10</th>
<th>C=11</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (3,1)</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (3,1)</td>
</tr>
<tr>
<td>C (3,2)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>D (3,2)</td>
</tr>
<tr>
<td>C (3,3)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (3,3)</td>
</tr>
<tr>
<td>C (3,4)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>D (3,4)</td>
</tr>
<tr>
<td>C (3,5)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (3,5)</td>
</tr>
<tr>
<td>C (3,6)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (3,6)</td>
</tr>
</tbody>
</table>

### Table 46: A4's concordance and discordance set for case study 1

<table>
<thead>
<tr>
<th>A4</th>
<th>C=1</th>
<th>C=2</th>
<th>C=3</th>
<th>C=4</th>
<th>C=5</th>
<th>C=6</th>
<th>C=7</th>
<th>C=8</th>
<th>C=9</th>
<th>C=10</th>
<th>C=11</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (4,1)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>D (4,1)</td>
</tr>
<tr>
<td>C (4,2)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>D (4,2)</td>
</tr>
<tr>
<td>C (4,3)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>D (4,3)</td>
</tr>
<tr>
<td>C (4,4)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>D (4,4)</td>
</tr>
<tr>
<td>C (4,5)</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (4,5)</td>
</tr>
<tr>
<td>C (4,6)</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (4,6)</td>
</tr>
</tbody>
</table>

### Table 47: A5's concordance and discordance set for case study 1

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<th>C=2</th>
<th>C=3</th>
<th>C=4</th>
<th>C=5</th>
<th>C=6</th>
<th>C=7</th>
<th>C=8</th>
<th>C=9</th>
<th>C=10</th>
<th>C=11</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (5,1)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td></td>
<td>-</td>
<td>D (5,1)</td>
</tr>
<tr>
<td>C (5,2)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
<td>D (5,2)</td>
</tr>
<tr>
<td>C (5,3)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
<td>D (5,3)</td>
</tr>
<tr>
<td>C (5,4)</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
<td>D (5,4)</td>
</tr>
<tr>
<td>C (5,5)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (5,5)</td>
</tr>
</tbody>
</table>

114
### Table 48: A6’s concordance and discordance set for case study 1

<table>
<thead>
<tr>
<th>A6</th>
<th>C=1</th>
<th>C=2</th>
<th>C=3</th>
<th>C=4</th>
<th>C=5</th>
<th>C=6</th>
<th>C=7</th>
<th>C=8</th>
<th>C=9</th>
<th>C=10</th>
<th>C=11</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (6,1)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>D (6,1)</td>
</tr>
<tr>
<td>C (6,2)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>D (6,2)</td>
</tr>
<tr>
<td>C (6,3)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>D (6,3)</td>
</tr>
<tr>
<td>C (6,4)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>D (6,4)</td>
</tr>
<tr>
<td>C (6,5)</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>D (6,5)</td>
</tr>
<tr>
<td>C (6,6)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>D (6,6)</td>
</tr>
</tbody>
</table>

### Step 3: Concordance interval Matrix

For every concordance set occurring in a similar row, all the weights where ‘a’ is preferred over ‘b’ are summed using equation (30). The summations are shown as follows:

#### Table 49: Concordance interval matrix for case study 1

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,00</td>
<td>0,439</td>
<td>0,490</td>
<td>0,337</td>
<td>0,953</td>
<td>0,757</td>
</tr>
<tr>
<td>A2</td>
<td>0,696</td>
<td>0,000</td>
<td>0,988</td>
<td>0,623</td>
<td>0,757</td>
<td>1,001</td>
</tr>
<tr>
<td>A3</td>
<td>0,709</td>
<td>0,641</td>
<td>0,000</td>
<td>0,432</td>
<td>0,709</td>
<td>0,757</td>
</tr>
<tr>
<td>A4</td>
<td>0,885</td>
<td>0,391</td>
<td>0,664</td>
<td>0,000</td>
<td>0,946</td>
<td>1,001</td>
</tr>
<tr>
<td>A5</td>
<td>0,626</td>
<td>0,244</td>
<td>0,292</td>
<td>0,298</td>
<td>0,000</td>
<td>0,757</td>
</tr>
<tr>
<td>A6</td>
<td>0,384</td>
<td>0,244</td>
<td>0,244</td>
<td>0,244</td>
<td>0,461</td>
<td>0,000</td>
</tr>
</tbody>
</table>

### Step 4: Discordance interval matrix

The discordance matrix is formed from the discordance set using equation (31), and the results are represented as follows:

#### Table 50: Discordance interval matrix for case study 1

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,00</td>
<td>1,00</td>
<td>0,74</td>
<td>1,00</td>
<td>0,69</td>
<td>1,00</td>
</tr>
<tr>
<td>A2</td>
<td>0,67</td>
<td>0,00</td>
<td>0,07</td>
<td>1,00</td>
<td>0,67</td>
<td>1,00</td>
</tr>
<tr>
<td>A3</td>
<td>1,00</td>
<td>1,00</td>
<td>0,00</td>
<td>1,00</td>
<td>1,00</td>
<td>1,00</td>
</tr>
<tr>
<td>A4</td>
<td>0,47</td>
<td>0,29</td>
<td>0,29</td>
<td>0,00</td>
<td>0,16</td>
<td>0,00</td>
</tr>
<tr>
<td>A5</td>
<td>1,00</td>
<td>1,00</td>
<td>0,74</td>
<td>1,00</td>
<td>0,00</td>
<td>1,00</td>
</tr>
<tr>
<td>A6</td>
<td>0,84</td>
<td>0,74</td>
<td>0,57</td>
<td>1,00</td>
<td>0,57</td>
<td>0,00</td>
</tr>
</tbody>
</table>
Step 5: Determination of the concordance matrix index

Using Equation (32), the concordance index was calculated to be 0.599. In addition, the Boolean Matrix is shown in Table 51.

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 6: Determination of the discordance matrix index

Equation (33) was used to determine the discordance index (0.750) and its matrix is shown in Table 52.

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A5</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A6</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

STEP 7: AGGREGATE THE DOMINANCE MATRIX

The final matrix is obtained by multiplying the matrix in Step 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>A6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
From the Aggregate dominance matrix, all the columns are checked. The column with the least amount of ‘1’, its alternative will be chosen as the best. From the Matrix, A3 only has zeroes, therefore is the best alternative. A2, A4, and A5 have the same number of ‘1’. In this case, a sensitivity analysis can be carried out by changing the indexes in Step 5 and Step 6 since those values are threshold values. A1 follows and lastly, the least preferred method is A6. A sensitivity analysis will be performed since ELECTRE failed to sort the alternatives in different ranks.

The following methods have not been used before in the MMS process. However, they are included for comparison purposes. They are OCRA, ARAS, CORPAS, CP and SAW. Their processes of reaching a final ranking will be detailed in the following pages.

**OCRA Case study 1**

OCRA is carried out in 6 straightforward steps:

**Step 1: Construction of a matrix**

A matrix used is in Table 21.

**Step 2/3: Preference rating for non-beneficial criteria**

Since in the given case study, there are non-beneficial criterion, then step 2 and 3 are not applicable. However, for their calculations, Equation (36) and (37) would have been used.

**Step 4: Preference rating for beneficial**

Preferential ratings for each alternative with respect to the criteria are calculated using equation (38). The results are shown in Table 54.

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1,595</td>
<td>0,244</td>
<td>1,872</td>
<td>0,051</td>
<td>0,147</td>
<td>0,000</td>
<td>0,000</td>
<td>0,855</td>
<td>0,074</td>
<td>0,268</td>
</tr>
<tr>
<td>A2</td>
<td>2,145</td>
<td>0,000</td>
<td>1,392</td>
<td>0,153</td>
<td>0,294</td>
<td>3,588</td>
<td>2,352</td>
<td>2,755</td>
<td>0,222</td>
<td>0,268</td>
</tr>
<tr>
<td>A3</td>
<td>2,145</td>
<td>0,000</td>
<td>1,392</td>
<td>0,051</td>
<td>0,294</td>
<td>1,748</td>
<td>0,432</td>
<td>1,805</td>
<td>0,148</td>
<td>0,268</td>
</tr>
<tr>
<td>A4</td>
<td>0,495</td>
<td>0,488</td>
<td>0,432</td>
<td>0,102</td>
<td>0,147</td>
<td>2,668</td>
<td>0,912</td>
<td>1,805</td>
<td>0,074</td>
<td>0,402</td>
</tr>
<tr>
<td>A5</td>
<td>1,045</td>
<td>0,244</td>
<td>0,432</td>
<td>0,000</td>
<td>0,147</td>
<td>0,000</td>
<td>0,912</td>
<td>0,855</td>
<td>0,000</td>
<td>0,134</td>
</tr>
<tr>
<td>A6</td>
<td>0,000</td>
<td>0,488</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
</tr>
</tbody>
</table>

**Step 5: Linear preference rating for beneficial**

Linear preference ratings are determined according to equation (39) and they are shown in Table 55.

<table>
<thead>
<tr>
<th></th>
<th>linear P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>5,483</td>
</tr>
<tr>
<td>A2</td>
<td>13,416</td>
</tr>
<tr>
<td>A3</td>
<td>8,66</td>
</tr>
<tr>
<td>A4</td>
<td>7,698</td>
</tr>
<tr>
<td>A5</td>
<td>3,886</td>
</tr>
<tr>
<td>A6</td>
<td>0,488</td>
</tr>
</tbody>
</table>
**STEP 6: OVERALL PERFORMANCE RATING**

The overall ratings of the alternatives are determined using equation (40) and are shown below. A2 is the alternative with the highest overall performance; followed by A3. The worst performing alternative is A6. A sensitivity analysis will be carried out to check the robustness of the answer if weights and other possible parameters were altered.

**Table 56: OCRA Final ranking for case study 1**

<table>
<thead>
<tr>
<th>Rank</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

**ARAS case study 1**

Like OCRA, Aras is straightforward with only 5 steps:

**Step 1: Develop a Decision Matrix**

A developed Matrix is shown in Table 21.

**Step 2: Normalisation of the Matrix**

The matrix in Table 57, was normalised using equation (41-43).

**Table 57: ARAS' normalised matrix for case study 1**

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,214</td>
<td>0,167</td>
<td>0,333</td>
<td>0,154</td>
<td>0,154</td>
<td>0,000</td>
<td>0,000</td>
<td>0,111</td>
<td>0,154</td>
<td>0,188</td>
</tr>
<tr>
<td>A2</td>
<td>0,286</td>
<td>0,083</td>
<td>0,250</td>
<td>0,308</td>
<td>0,231</td>
<td>0,444</td>
<td>0,500</td>
<td>0,333</td>
<td>0,308</td>
<td>0,188</td>
</tr>
<tr>
<td>A3</td>
<td>0,286</td>
<td>0,083</td>
<td>0,250</td>
<td>0,154</td>
<td>0,231</td>
<td>0,222</td>
<td>0,100</td>
<td>0,222</td>
<td>0,231</td>
<td>0,188</td>
</tr>
<tr>
<td>A4</td>
<td>0,071</td>
<td>0,250</td>
<td>0,083</td>
<td>0,231</td>
<td>0,154</td>
<td>0,333</td>
<td>0,200</td>
<td>0,222</td>
<td>0,154</td>
<td>0,250</td>
</tr>
<tr>
<td>A5</td>
<td>0,143</td>
<td>0,167</td>
<td>0,083</td>
<td>0,077</td>
<td>0,154</td>
<td>0,000</td>
<td>0,200</td>
<td>0,111</td>
<td>0,077</td>
<td>0,125</td>
</tr>
<tr>
<td>A6</td>
<td>0,000</td>
<td>0,250</td>
<td>0,000</td>
<td>0,077</td>
<td>0,077</td>
<td>0,000</td>
<td>0,000</td>
<td>0,000</td>
<td>0,077</td>
<td>0,063</td>
</tr>
</tbody>
</table>

**Step 3: Weighted Normalised Matrix**

Like in the previous MCDMs, a weighted normalised matrix was constructed by multiplying the weight with each score from step 3. The results are shown in Table 58.

**Table 58: ARAS weighted normalised matrix**

<table>
<thead>
<tr>
<th>w</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,012</td>
<td>0,041</td>
<td>0,016</td>
<td>0,008</td>
<td>0,023</td>
<td>0,000</td>
<td>0,000</td>
<td>0,011</td>
<td>0,011</td>
<td>0,025</td>
<td>0,004</td>
</tr>
<tr>
<td>A2</td>
<td>0,016</td>
<td>0,020</td>
<td>0,012</td>
<td>0,016</td>
<td>0,034</td>
<td>0,041</td>
<td>0,024</td>
<td>0,032</td>
<td>0,023</td>
<td>0,025</td>
<td>0,002</td>
</tr>
</tbody>
</table>
Step 4: Determine an optimality function (OF)

The higher the optimality function, the more effective an alternative is. The calculated OFs are shown in Table 59.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>OF</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.150</td>
</tr>
<tr>
<td>A2</td>
<td>0.244</td>
</tr>
<tr>
<td>A3</td>
<td>0.182</td>
</tr>
<tr>
<td>A4</td>
<td>0.212</td>
</tr>
<tr>
<td>A5</td>
<td>0.123</td>
</tr>
<tr>
<td>A6</td>
<td>0.090</td>
</tr>
</tbody>
</table>

Step 5: Determine and rank utility function values

The alternatives are then ranked through the utility function value. This value determined the relative efficiency of an alternative over the optimal alternative. The results are shown below. A2 is rated as the best choice, followed by A4. The worst choice is A6. The graph in Figure 36, depicts the results.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Utility Function Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>61%</td>
</tr>
<tr>
<td>A2</td>
<td>100%</td>
</tr>
<tr>
<td>A3</td>
<td>74%</td>
</tr>
<tr>
<td>A4</td>
<td>87%</td>
</tr>
<tr>
<td>A5</td>
<td>50%</td>
</tr>
<tr>
<td>A6</td>
<td>37%</td>
</tr>
</tbody>
</table>

COPRAS case study 1

CORPAS’ procedure is similar to that of ARAS with a few differences towards the final steps.

Step 1: Construct a matrix

See Table 21

Step 2: Normalise the constructed matrix

See Table 57

Step 3: Weighted normalise matrix

See Table 58
Step 4-5-6: sum of beneficial and non-beneficial, determine the relative significance value (Q), and calculation of quantitative utility (U).

Once the beneficial criteria are summed up, a relative significance value of the alternative that shows the priority of the alternatives can then be calculated using (49). From the calculations, a quantitative utility value can then be calculated using equation (50). A higher value implies a higher ranking. In the rankings shown in Table 61 and Figure 34, A2 emerged as the winning option. While A6 was ranked the worst amongst the alternatives. The results will be further validated by a sensitivity analysis.

<table>
<thead>
<tr>
<th></th>
<th>Sum</th>
<th>Q</th>
<th>U</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.150</td>
<td>0.150</td>
<td>61%</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0.244</td>
<td>0.244</td>
<td>100%</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>0.182</td>
<td>0.182</td>
<td>74%</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>0.212</td>
<td>0.212</td>
<td>87%</td>
<td>2</td>
</tr>
<tr>
<td>A5</td>
<td>0.123</td>
<td>0.123</td>
<td>50%</td>
<td>5</td>
</tr>
<tr>
<td>A6</td>
<td>0.090</td>
<td>0.090</td>
<td>37%</td>
<td>6</td>
</tr>
</tbody>
</table>

**Figure 34: COPRAS final ranking for case study 1**

**CP case study 1**

The following is a procedure used in CP, a method that determines alternatives based on a distance measure.

**Step 1: establish a matrix**

See Table 21.

**Step 2: determine the weight of each criterion**

The weights have been determined in the original case study and they are shown in the TOPSIS procedure.

**Step 3: specify parameter, p**

The parameter ‘p’ reflects the decision maker’s choice and how deviations are compensated. It ranges between 1 and infinity. The choice in this case is that of 1, which implies that deviations from the ideal value are taken in direct proportion to their magnitude.

\[ p=1 \]
Step 4: computation of the distance metric

The distance metric is then computed using equation (54). The measure will then be used to estimate how close an alternative is to the ideal solution. They are shown in Table 62.

<table>
<thead>
<tr>
<th>P=1</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.014</td>
<td>0.122</td>
<td>0.000</td>
<td>0.034</td>
<td>0.074</td>
<td>0.092</td>
<td>0.048</td>
<td>0.063</td>
<td>0.049</td>
<td>0.045</td>
<td>0.000</td>
</tr>
<tr>
<td>A2</td>
<td>0.000</td>
<td>0.244</td>
<td>0.012</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.045</td>
<td>0.004</td>
</tr>
<tr>
<td>A3</td>
<td>0.000</td>
<td>0.244</td>
<td>0.012</td>
<td>0.034</td>
<td>0.000</td>
<td>0.046</td>
<td>0.038</td>
<td>0.032</td>
<td>0.025</td>
<td>0.045</td>
<td>0.000</td>
</tr>
<tr>
<td>A4</td>
<td>0.041</td>
<td>0.000</td>
<td>0.036</td>
<td>0.017</td>
<td>0.074</td>
<td>0.023</td>
<td>0.029</td>
<td>0.032</td>
<td>0.049</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>A5</td>
<td>0.028</td>
<td>0.122</td>
<td>0.036</td>
<td>0.051</td>
<td>0.074</td>
<td>0.092</td>
<td>0.029</td>
<td>0.063</td>
<td>0.074</td>
<td>0.089</td>
<td>0.009</td>
</tr>
<tr>
<td>A6</td>
<td>0.055</td>
<td>0.000</td>
<td>0.048</td>
<td>0.051</td>
<td>0.147</td>
<td>0.092</td>
<td>0.048</td>
<td>0.095</td>
<td>0.074</td>
<td>0.134</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Step 5: Ranking of alternatives

The alternatives are then ranked based on the calculations performed in Step 4. The best alternative will have the lowest distance metric, and that implies that it is closest to the ideal solution. A4 appears slightly lower than A2, but they are both closest to the ideal solution compared to the rest of the alternatives, with A6 being the furthest. The results are depicted in Table 63 and Figure 35.

<table>
<thead>
<tr>
<th></th>
<th>L_P</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.541</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0.3050</td>
<td>2</td>
</tr>
<tr>
<td>A3</td>
<td>0.475</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>0.3049</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>0.666</td>
<td>5</td>
</tr>
<tr>
<td>A6</td>
<td>0.757</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 35 CP final alternatives ranking for case study 1:
SAW Case study 1

The last MCDM to be compared is SAW. The following procedure is followed for this method.

Step 1: Construction of a matrix

See Table 21.

Step 2: Normalisation of a matrix

The SAW matrix is normalised using equation (51) and (52). The results are as follows.

Table 64: SAW’s normalised matrix for case study 1

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,750</td>
<td>0,667</td>
<td>1,000</td>
<td>0,500</td>
<td>0,667</td>
<td>0,000</td>
<td>0,000</td>
<td>0,333</td>
<td>0,500</td>
<td>0,750</td>
<td>1,000</td>
</tr>
<tr>
<td>A2</td>
<td>1,000</td>
<td>0,333</td>
<td>0,750</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,667</td>
</tr>
<tr>
<td>A3</td>
<td>1,000</td>
<td>0,333</td>
<td>0,750</td>
<td>0,500</td>
<td>1,000</td>
<td>0,500</td>
<td>0,200</td>
<td>0,667</td>
<td>0,750</td>
<td>0,750</td>
<td>1,000</td>
</tr>
<tr>
<td>A4</td>
<td>0,250</td>
<td>1,000</td>
<td>0,250</td>
<td>0,750</td>
<td>0,667</td>
<td>0,750</td>
<td>0,400</td>
<td>0,667</td>
<td>0,500</td>
<td>1,000</td>
<td>0,667</td>
</tr>
<tr>
<td>A5</td>
<td>0,500</td>
<td>0,667</td>
<td>0,250</td>
<td>0,250</td>
<td>0,667</td>
<td>0,000</td>
<td>0,400</td>
<td>0,333</td>
<td>0,250</td>
<td>0,500</td>
<td>0,333</td>
</tr>
<tr>
<td>A6</td>
<td>0,000</td>
<td>1,000</td>
<td>0,000</td>
<td>0,250</td>
<td>0,333</td>
<td>0,000</td>
<td>0,000</td>
<td>0,250</td>
<td>0,250</td>
<td>0,000</td>
<td>0,000</td>
</tr>
</tbody>
</table>

Step 3: Weight determination

Weights were determined in the original case study method, which in this case is the TOPSIS method.

Step 4: Calculations of evaluations score

The evaluation scores are simply calculated by multiplying each score with the weight. Then, all these new values are summed up for each alternative and will give the final score of the alternative. This is shown by Table 65.

Table 65: SAW’s evaluation score for case study 1

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,041</td>
<td>0,163</td>
<td>0,048</td>
<td>0,026</td>
<td>0,098</td>
<td>0,000</td>
<td>0,000</td>
<td>0,032</td>
<td>0,037</td>
<td>0,101</td>
<td>0,013</td>
</tr>
<tr>
<td>A2</td>
<td>0,055</td>
<td>0,081</td>
<td>0,036</td>
<td>0,051</td>
<td>0,147</td>
<td>0,092</td>
<td>0,048</td>
<td>0,095</td>
<td>0,074</td>
<td>0,101</td>
<td>0,009</td>
</tr>
<tr>
<td>A3</td>
<td>0,055</td>
<td>0,081</td>
<td>0,036</td>
<td>0,026</td>
<td>0,147</td>
<td>0,046</td>
<td>0,010</td>
<td>0,063</td>
<td>0,056</td>
<td>0,101</td>
<td>0,013</td>
</tr>
<tr>
<td>A4</td>
<td>0,014</td>
<td>0,244</td>
<td>0,012</td>
<td>0,038</td>
<td>0,098</td>
<td>0,069</td>
<td>0,019</td>
<td>0,063</td>
<td>0,037</td>
<td>0,134</td>
<td>0,009</td>
</tr>
<tr>
<td>A5</td>
<td>0,028</td>
<td>0,163</td>
<td>0,012</td>
<td>0,013</td>
<td>0,098</td>
<td>0,000</td>
<td>0,019</td>
<td>0,032</td>
<td>0,019</td>
<td>0,067</td>
<td>0,004</td>
</tr>
<tr>
<td>A6</td>
<td>0,000</td>
<td>0,244</td>
<td>0,000</td>
<td>0,013</td>
<td>0,049</td>
<td>0,000</td>
<td>0,000</td>
<td>0,019</td>
<td>0,034</td>
<td>0,000</td>
<td>0,000</td>
</tr>
</tbody>
</table>

Step 5 Ranking of alternatives

The alternative with the highest score is the most suitable. In this case, A2 appears to be the highest followed by A4. A6 with a score of 0,358 is the least preferred option.
Table 66: SAW’s final ranking of alternatives

<table>
<thead>
<tr>
<th></th>
<th>SUM</th>
<th>RANK</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0,558</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>0,789</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>0,633</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
<td>0,737</td>
<td>2</td>
</tr>
<tr>
<td>A5</td>
<td>0,454</td>
<td>5</td>
</tr>
<tr>
<td>A6</td>
<td>0,358</td>
<td>6</td>
</tr>
</tbody>
</table>

Conclusion of Case Study 1

The combined results of all the 10 MCDM yield the following frequency of ratings. The table shows how many times each alternative appeared in a rank. A2 from the combined MCDM results was rated as the best option because it emerged as the most suitable method in 7 out of the 10 rating methods; while A6 emerged as the worst option in 8 out 10 ratings.

Table 67: Ranking frequencies of MCDM in case study 1

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Ranks→</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A2</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A3</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A4</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>A6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

A2 > A4 > A3 > A1 > A5 > A6

The following is a table that sorts the MCDM’s according to the results of their ratings. Group 1 shows the MCDM with the same rankings of alternatives and so on. This information will help the author in checking for consistency when the other case studies are assessed.

Table 68: Grouping of MCDM case study 1

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
<th>Group 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROMETHEE, CORPAS, ARAS, SAW</td>
<td>TODIM, OCRA</td>
<td>TOPSIS</td>
<td>CP</td>
<td>GRA</td>
<td>VIKOR</td>
</tr>
</tbody>
</table>

A sensitivity analysis was conducted for each method and the results are shown in section 3.2.5.

3.2.2. CASE STUDY 2 AHP AND PROMETHEE APPROACH TO MMS
The second case study is an example of the use of two MCDM: AHP and PROMETHEE. Like in the previous case study, AHP was used for determining the weights of the criteria. PROMETHEE was used for ranking purposes. This approach was performed for ‘Coka Marin’ underground mine in Serbia. Using this case study, gave the author an opportunity to differentiate the performances of the MCDM methods where non-beneficial (such as cost) and beneficial (such as production rate) criteria were used. In conclusion, the methods are grouped according to their similarities, and a final ranking that resolve the conflict is given.

Five possible mining methods and eleven criteria were used in evaluating the MMS process; and are shown in Table 69.

Table 69: Criteria and Alternatives of Case study 2

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>• C1: Thickness</td>
<td>• A1: Room and pillar</td>
</tr>
<tr>
<td>• C2: Dip</td>
<td>• A2: Room and pillar with fill</td>
</tr>
<tr>
<td>• C3: Rock Substance strength of ore</td>
<td>• A3: Shrinkage</td>
</tr>
<tr>
<td>• C4: Crack system of ore</td>
<td>• A4: Cut and fill</td>
</tr>
<tr>
<td>• C5: Shape</td>
<td>• A5: Sublevel caving</td>
</tr>
<tr>
<td>• C6: Coefficient of development</td>
<td></td>
</tr>
<tr>
<td>• C7: Ore excavation efficiency</td>
<td></td>
</tr>
<tr>
<td>• C8: Ore dilution</td>
<td></td>
</tr>
<tr>
<td>• C9: Excavation costs</td>
<td></td>
</tr>
<tr>
<td>• C10: Work safety</td>
<td></td>
</tr>
<tr>
<td>• C11: Terrain surface preservation</td>
<td></td>
</tr>
</tbody>
</table>

The mine under consideration is located in Serbia with total ore reserves of 1 160 000 tons. The commodities extracted are Copper, Lead, Zinc, Gold and Silver. The physical properties of the ‘Coka Marin’ underground mine are shown in Table 70.

Table 70: Physical properties of ‘Coka Marin’ Deposity (Bogdanovi et al, 2012)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Ore</th>
<th>Footwall</th>
<th>Hanging wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volumetric mass</td>
<td>t/m³</td>
<td>4.07</td>
<td>2.68</td>
<td>2.55</td>
</tr>
<tr>
<td>Porosity</td>
<td>%</td>
<td>3.1</td>
<td>4.57</td>
<td>3.09</td>
</tr>
<tr>
<td>Uniaxial compressive strength</td>
<td>MPa</td>
<td>64.7</td>
<td>42.1</td>
<td>56.1</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>MPa</td>
<td>9.1</td>
<td>5.2</td>
<td>7.3</td>
</tr>
<tr>
<td>Inner frictional angle</td>
<td>Degree</td>
<td>34</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Module of elasticity</td>
<td>GPa</td>
<td>37.4858</td>
<td>21.1705</td>
<td>25.5683</td>
</tr>
<tr>
<td>Module of deformation</td>
<td>GPa</td>
<td>36.207</td>
<td>19.9077</td>
<td>23.847</td>
</tr>
<tr>
<td>Poisson coefficient</td>
<td></td>
<td>0.189</td>
<td>0.22</td>
<td>0.214</td>
</tr>
<tr>
<td>Cohesion</td>
<td>MPa</td>
<td>17.5</td>
<td>11.6</td>
<td>11.2</td>
</tr>
<tr>
<td>Thickness</td>
<td>M</td>
<td>40-50m</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dip</td>
<td>Degree</td>
<td>65-75</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Type</td>
<td></td>
<td>Massive</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Depth</td>
<td>M</td>
<td>20-270</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

After calculations were performed for each MCDMs, the following results were obtained. The tables show rankings of different alternatives based on the MCDM used. Since the initial study used PROMETHEE, the calculations were repeated to confirm the results of the case study. The initial results of case study 2 were $A_5 > A_3 > A_1 > A_4 > A_2$. The following table’s shows results of different case MCDMs applied in case study 2 problem.
Table 71: MCDMs results for case study 2

<table>
<thead>
<tr>
<th></th>
<th>TOPSIS Ranking CS 2</th>
<th>TODIM Ranking CS 2</th>
<th>VIKOR Ranking CS 2</th>
<th>GRA Ranking CS 2</th>
<th>OCRA Ranking CS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
<td>0.05</td>
</tr>
<tr>
<td>A2</td>
<td>0.2</td>
<td>1</td>
<td>1</td>
<td>0.6</td>
<td>0.1</td>
</tr>
<tr>
<td>A3</td>
<td>0.4</td>
<td>0.8</td>
<td>0.8</td>
<td>0.4</td>
<td>0.15</td>
</tr>
<tr>
<td>A4</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>A5</td>
<td>0.8</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.25</td>
</tr>
</tbody>
</table>

- TOPSIS: Technique for Order Preference by Similarity to Ideal Solution
- TODIM: Technique for Order Preference by Similarity to Ideal Solution
- VIKOR: Vector Iterative KOHonen Rule
- GRA: Generalized TOPSIS
- OCRA: Ordered Categorical Ranking
From the rankings depicted by the graphs above, the following conclusions can be extracted. The frequency of occurrences in each ranking are shown by Table 72. In addition, Table 73 shows the groupings of MCDM in terms of similarities in how they ranked the alternatives. It is visible that the groupings are not the same as the groupings of case study 1. There is a level of inconsistency that is detected so far. However, the results will be confirmed by a sensitivity analysis. For this case study, the combined rankings of the MCDMs have been obtained as $A_5 > A_3 > A_1 > A_4 > A_2$; where 8 out of 10 methods rated $A_5$ as the best, and 8 out 10 methods rated $A_2$ as the worst method. The results confirm the initial obtained results of case study 2 since the rankings did not change.

Table 72: Frequency of rankings for case study 2

<table>
<thead>
<tr>
<th>Ranks→</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>A2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>A3</td>
<td>2</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A4</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>A5</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 73: Groups of MCDMs for case study 2

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>• PROMETHEE</td>
<td>• TOPSIS</td>
<td>• OCRA</td>
<td>• CP</td>
<td>• GRA</td>
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<td>• CORPAS</td>
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<td>• VIKOR</td>
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<td>• TOPSIS</td>
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In the following section, method will be analysed based on descriptions and will be rated on a scale of Low, Medium and High.

3.2.3. DESCRIPTIVE ANALYSIS OF THE MCDM
Saaty and Ergu described a set of criteria that can be used to evaluate different MCDMs in order to answer the question: “When is a Decision-Making method trustworthy?” The following set of criteria are evaluated against the MCDMs described in the literature review.

A. SIMPLICITY OF EXECUTION
In simplicity of execution, Saaty and Ergu (2015) suggested that the user of the MCDM must be able to perform the decision-making process without the need of an expert to supervise the process. The mathematics and underlying logic of the procedure must not hinder a successful decision-making process. A scale to rate the ease of use for the methods is given as, low: if the method’s logic is complicated and not suitable to be used by non-expert; medium if there is much effort that goes into learning it; high if it can be implemented in almost all studies and can be easily understood by layman.

The author has assessed the methods as follows:

- **TOPSIS**
  From the current research experience with the application of TOPSIS, the method is user friendly and one experiences a high simplicity of use. This claim is backed up by the extent of application of the TOPSIS. It was been previously mentioned in this current research that over 100 papers have been published where TOPSIS is applied. The following are some of the comments from the previous users of TOPSIS.

  Pavic and Novoselac (2013) attest to the simplicity of TOPSIS in their publication; that the method has a simple mathematical model and is a practical method since the user can rely on computer support for computations. Yavuz (2012) confirmed the ease of use of TOPSIS compared to the other MCDM methods when he used it for wheel loader selection in a coalmine. The computational process of TOPSIS is said to be straightforward by Garcia-Cascales & Lamata (2012).

- **TODIM**
From the current research experience, the author found that the method is simple to use once the user understands the procedure. However, there can be computational mistakes in the process because of the effort that goes onto its implementation. In terms of the pre-defined ratings of simplicity of execution, TODIM is rated medium. It was previously mentioned in the literature review that Rangel et al (2009) said the method is easy to be implemented even by users who are not professionals.

- **VIKOR**
  According to the current research, VIKOR is simple to use and is rated highly. However, the need for input of some other parameters such as the ‘v’ parameter may make the process tedious.

- **GRA**
  Wu (2002) mentioned some of the advantages of GRA. Amongst them was the simplicity in computations and the straightforwardness of the method. In this current research. GRA was used, and the author found the method easy to implement. The rating GRA is given in this study is a high rating.

- **PROMETHEE**
  The simplicity of PROMETHEE in application and conception has led to its widespread use and a fast-growth (Balali et al, 2014). However, the difficulty of PROMETHEE shows up when the preference function has to be chosen; and may be difficult when the decision maker has no experience of using this MCDM. Therefore, the simplicity is given a medium rating.

- **ELECTRE**
  ELECTRE is a complex method and difficult for non-experts. Balali et al (2014) attest that ELECTRE does suffer from sophisticated mathematical formulation. A low rating is given to ELECTRE.

- **OCRA, ARAS, COPRAS, SAW, and CP.**
  The above methods are simple to execute in decision-making and will therefore be given a high rating of simplicity.

### B. LOGICAL, MATHEMATICAL PROCEDURE

This criterion simply means that a method must have a mathematical representation, logical reasoning behind the theory and justification. A low rating is given to a method with just a simple mathematical logical procedure; a medium rating is given to a method that uses reference sequences or relative difference to rank alternatives; and high rating for methods using pairwise comparison technique to determine the dominance of one criterion over another.

- **TOPSIS**
  The mathematical approach of TOPSIS is well structured and uses relative difference of the distances for ranking. Therefore, it is given a medium rating.

- **TODIM**
  TODIM uses pairwise comparison to determine the dominance of one criterion over another. In addition, from its equations, it can eliminate inconsistencies that do arise from the pairwise comparison technique. Therefore, it is given a high rating.

- **VIKOR**
There are no pairwise comparisons in the VIKOR method and criterion can be evaluated independently. A relative difference is thus used at the final ranking of the alternatives; and VIKOR is rated medium in this case.

- **GRA**
  According to Lu (2015), the theory behind GRA does not have any solid foundation in mathematics. The assumption made in GRA is that the data is exponential; however, there are no further explanations on why such a claim is made. This makes it difficult to know the interpretations if the data is not exponential. Some other challenges faced and could be the reason for limited application of GRA are the quality of English as well as the writing style, the limitation in the theory application and limitation if the audience (readers). A low rating is given.

- **PROMETHEE**
  It does not consider discordance but does use the pairwise comparison to determine the dominance degree of one alternative over the other. PROMETHEE therefore receives a high rating.

- **ELECTRE**
  It does consider discordance and does use the pairwise comparison to determine the dominance degree of one alternative over the other. A high rating is therefore given to ELECTRE.

- **OCRA, COPRAS, SAW, ARAS**
  These methods are given a low rating because they just use a simple mathematical equation to show and justify their procedure

- **CP**
  CP’s procedure uses a relative difference and is therefore given a medium rating.

### C. INPUT PARAMETERS

A method must be justified in at least three ways; in its procedures, consequence of the procedures and approaches. If there are input parameters, there must be justifiable theory behind. A low rating is given to methods without any justifications of the parameters; Medium rating if it involves parameters in some part, and high if it involves complete and logical reasoning to input parameters.

- **TOPSIS**
  The only input on the TOPSIS method are the weights given by the decision maker to each criterion. This means that subjectivity is further reduced on this method and therefore it is given a high rating.

- **TODIM**
  In TODIM, the input parameters are the weights of the criterion and the attenuation factor. This factor can be adjusted between 1 and 10. 1 is usually used because it signifies that the losses would contribute with their real values. The reasoning for both the parameters is logical; therefore, a high rating is given to TODIM.

- **VIKOR**
  A decision maker intervenes in the VIKOR process to determine the weights of the criteria and to choose the value of the coefficient ‘v’, which should be between 0 and 1. This parameter gives the importance of weight of the measures. A v equal 0.5 is usually chosen so that both the utility and regret measures are given equal weight. A value less than 0.5 gives more weight to the regret measure; while a value greater than 0.5 places more importance to
the utility measure. The parameter is logical and therefore VIKOR is given a high rating.

- GRA
  There is a lack of axiomatic foundation for GRA. In addition, it was noted that there is missing proof of reliability of the method by theoretical research. Clarification needs to be done or it may hinder adoption in many industries for application. Thus, a low rating is given

- PROMETHEE
  There are thresholds as input of the decision maker’s preference. It was suggested that the use of the thresholds must be based on previous studies for guidance. That justifies the use of preferences in PROMETHEE. In addition, one of the studies that formed part of the research suggested two ways of setting the thresholds; one is to set the indifference to zero and preference threshold as the maximum evaluation between the alternatives. The other approach suggested by the author was to set the indifference as the minimum and preference as the maximum alternative. PROMETHEE is given a medium rating.

- ELECTRE
  In the procedure of ELECTRE, there are threshold (c and d) that the decision process depends on. The values of these thresholds depend on the Decision maker. It is believed that these values have an influence on the final ranking, and the fact that it is not ensured that using a higher c and a lower d will lead to small number of non-dominated solutions (Caterino et al, 2008), it is not suggested that ELECTRE be used for decision making process for MMS.

- OCRA, ARAS, COPRAS and SAW
  The above listed methods do not have any other input parameter except the weights. Medium ratings are given to these methods.

- CP
  The weights of the criteria and the p-parameters are the input of the decision maker in the CP process. The p-parameter shows how the decision maker compensate for the deviations in the process of decision-making. Medium rating is given to CP

D. SYNTHESIS OF JUDGEMENTS WITH MERGING FUNCTIONS

In this criterion, the judgements from different experts are synthesised. To obtain an overall rank, the evaluations must be synthesised. If a method synthesizes the evaluations by averaging weights, it is rated low. A method will be rated medium if a simple weighted method is used. A high rating is given if there is a rigorous merging function with reasonable weights used.

- TOPSIS
  In the aggregation process, TOPSIS uses an equation that considers the distance from the positive ideal and from the negative ideal. TOPSIS does not consider the relative importance of this distance between the alternatives. A high rating is given to TOPSIS.

- TODIM
  TODIM measures the dominance degree of each alternative by calculating the partial and overall dominance of each alternative. From the dominance degree, rankings can be made. It follows a rigorous procedure and can be highly rated.
• VIKOR
  In the aggregation process of VIKOR, a L\textsubscript{p}-metric, which is a distance function, is calculated. It represents group regret that an idea cannot be chosen. L\textsubscript{1} is represented by S-group as the sum of all the individual regrets. While L\textsubscript{\infty} represent the R-group. That is the maximum regret that an alternative could have (Tseng & Opricovic, 2007). Q aggregates the S- and R-group with the ‘v’ parameter. The method is therefore rated highly because of the rigorous merging process.

• GRA
  The magnitude of correlation between alternatives and the reference sequence is calculated using the grey relational degree. A high rating is given to GRA.

• PROMETHEE
  A net preference flow is introduced as an aggregating utility function, and the equations used are shown the previous mentioned steps for PROMETHEE. Research found that the foundation of net flow if PROMETHEE and the S-Group of VIKOR have the same foundation (Tseng & Opricovic, 2007); and their results are similar if PROMETHEE uses its Linear (Type 5) function. A high rating is given. A high rating is given

• ELECTRE
  The output of the ELECTRE process is a set of concordance of alternatives, which indicates how one alternative dominates the other. The ranking is partial in ELECTRE because some alternatives remains incomparable. Medium rating is given

• OCRA, ARAS, COPRAS, CP, SAW
  Medium ratings are given to the methods above because while others average weights in their process, simple weighted methods are used.

E. RANKING OF TANGIBLES
  Alternatives are ranked either higher, lower or equal to the other alternatives they are competing with on the evaluation of the tangible criteria. If a method does not involve ranking, it is ranked low. If it uses ordinal scale, it uses medium and high if cardinal scale is used to rank alternatives.

  • All the methods can deal with both quantitate and qualitative data and uses cardinal scale; therefore, they are rated highly in this criterion.

F. GENERALIZATION TO RANKING OF INTANGIBLES
  Intangible criteria are often part of a decision problem; and they need to be quantified. If a method is applicable to both tangibles and intangibles, and asses the intangibles by using pairwise comparison technique, it is then rated high. If it transforms intangibles into cardinal numbers by using interval. Ratio/absolute scale, it is rated medium; and if it just assigns arbitrary ordinary numbers to quantify the intangibles, it is rated low.

  • PROMETHEE AND ELECTRE
    Alternatives are evaluated on a pairwise comparison technique. The deviations between two evaluations of alternatives are considered. They are therefore given a high rating.
TOPSIS, TODIM, VIKOR, GRA, CORPAS, ARAS, CP, SAW, OCRA: Medium rating since they use a cardinal absolute scale, but not a pairwise comparison technique.

G. RANK PRESERVATION AND REVERSAL

MCDMs’ one of the significant drawbacks is due to the phenomenon called: rank reversal. This phenomenon explains the change of alternatives ranking if one or more alternatives are added or removed from a decision problem. Sometimes the best alternative can become the worst alternative, especially where the rank reversal totally inverts the ordering. A method which does not deal with rank reversal at all is rated low; one which basically deals with it is rated medium, and one which implements ways in its procedure for interpreting reasons for rank preservation and reversal is rated highly.

- TOPSIS
  The above-mentioned phenomenon has made the validity of TOPSIS debatable. Because rank reversal would clearly mean that a better decision/alternative depends on the number of alternatives. Fortunately, Garcia-cascales & Lamata (2012) identified two points that causes rank reversal in TOPSIS. Namely; the ideal solutions and the normalisation process. In their research, they modified the above-mentioned points and rank reversal was dealt with. Because there have been previous attempts to deal with rank reservation and preservation, TOPSIS would therefore be rated medium because the solution has not yet been widely accepted and when applied, it gives different rankings compared to the original TOPSIS.

- TODIM, ARAS, CORPAS, GRA, VIKOR
  TODIM is also mentioned as one of the methods that do suffer from rank reversal, however, a solution for it is its normalisation procedure. Therefore, it is rated medium.

- PROMETHEE
  There has been limited studies concerning rank reversal for PROMETHEE. The first people to address it were De Keyser & Peeters in 1996. It was only in 2013 that Veryl and De Smet investigated the probability of rank reversal in PROMETHEE I and II. It was shown that these two classes of PROMETHEE do suffer from rank reversal. However, in 2016, Brans & De Smet showed that the removal or additions of alternatives does not lead to rank reversal in PROMETHEE. Therefore, it was tested in the current studies and found that it is stable. So, a high rating is given to PROMETHEE concerning rank reversal.

- ELECTRE
  In ELECTRE, the rank reversal is caused by the pairwise comparison. It is also noted that rank reversal probability of occurrences increases as the number of alternatives are increased. Also, under equal weights for criteria, there is more rank reversal. Therefore, the method is rated low because there is no proven method to deal with rank reversal in ELECTRE.

- SAW, OCRA
  Because of its Normalisation procedure, SAW and OCRA suffers less from rank reversal. A medium rating is given.

- CP
  CP suffers from rank reversals and has been proven in this study as sensitivity analysis was carried out. Therefore, a low rating is given.
H. SENSITIVITY ANALYSIS
A method is rated low if it only assesses a single parameter; medium if it works on two to three parameters; and high if it can assess more parameters.

- TOPSIS
  A medium rating is given to TOPSIS because it can assess the weights of the criterion and the evaluations of each criterion against the alternatives.

- TODIM
  A high rating for TODIM is given because it can assess the following parameters for sensitivity: attenuation factor, criteria weights, the choice of the reference criterion, and the performance evaluations of the alternatives.

- VIKOR
  The v-parameter and weights can be changed in VIKOR. Therefore, a medium rating is given to the method.

- GRA
  Only two parameters can be varied in this method; weights of the criterion and the identification coefficient. Therefore, a medium rating is given to GRA.

- PROMETHEE
  In PROMETHEE, preference, indifference, preference functions, and the weights of the criterion can be changed to assess the influences of each. Therefore, a high rating is given to PROMETHEE.

- ELECTRE
  ELECTRE assesses 3 parameters; concordance and discordance index, as well as the weights. It is therefore given a high rating.

- OCRA, ARAS, COPRAS and SAW
  These methods do not have special input parameters except the weights of the criterion. Since they only assess one parameter, they are given a low rating.

- CP
  Only two parameters can be varied; the ‘p’ value and the weights. Therefore, CP is given a medium rating.

I. APPLICABILITY TO CONFLICT RESOLUTION
A method must be able to resolve the conflict that exist within the criteria of making a decision. There must be fair trade-offs in the process; such as normalisation to find the best solution where conflict is concerned. A low rating is given to methods which use a simple mathematical compensation technique; medium rating for methods using analytical methods, and high rating for methods providing an understandable, acceptable, practical and flexible way of resolving the conflicts in criteria.

- TOPSIS
  TOPSIS process of normalising uses a vector normalisation. It must be noted that the normalised value could be different for different evaluation unit of a criterion (Tseng & Opricovic, 2004). For example, if a problem with two alternatives is evaluated against 3 criteria and the evaluations are 3, 4, and 5 for A1 and 2, 3, 9 for A2: the normalised values of 3 will be different. Therefore, TOPSIS is given a medium rating.

- TODIM
  A high rating is given to the normalisation procedure of TODIM.
• **VIKOR**
  In the normalisation procedure of VIKOR, a linear transformation is used; and it does not depend on the unit of the criterion, or whether it is a minimum or maximum criterion. The normalisation procedure is aggregated in calculation the utility and regret measures.

• **PROMETHEE**
  Conflict resolution is resolved in the aggregation process. A high rating is given.

• **GRA, ELECTRE, OCRA, ARAS, COPRAS, CP, SAW**
  They all can resolve conflict that exist in the criteria and they are all given a high rating in this case.

**J. TRUSTWORTHINESS AND VALIDITY OF THE APPROACH**

The quality of a method and what makes it trustworthy must be considered. Questions to be asked are: can the method yield the choices that accurately reflect the values of the user? If a method has been widely applied, it provides a platform to be trusted and can be rated high. Medium ranking is for methods which have limited application, and low rating is for methods which have not been applied in the field of question.

• **TOPSIS**
  TOPSIS has proven itself and has provided it's on platform for future applications in almost all industries. The number of papers that have been published where TOPSIS was applied are over 100. In different journals such as expert systems with applications, applied soft computing, knowledge-based systems, information sciences, and many more. Therefore, in terms of trustworthiness and validity of the approach, TOPSIS is highly rated.

• **TODIM**
  Limited application of TODIM in mining method selection. Medium rating is given.

• **VIKOR**
  There is limited application of VIKOR in mining method selection. Medium rating is given.

• **GRA**
  GRA has enjoyed wide application in agriculture, environment, and marketing industry. However, there is limited application in the mining method selection industry. Medium rating is given.

• **ELECTRE**
  The disadvantages of ELECTRE makes it unsuitable for use in the mining industry because in the ranking process, it often does not lead to one solution. It is therefore suitable for decision problems that have few alternatives and less criteria. Low rating is given.

• **PROMETHEE**
  A high rating is given to PROMETHEE like TOPSIS because there have been numerous applications in the mining method selection industry.

• **OCRA, ARAS, COPRAS, CP, SAW**
  The methods have not been applied in mining method selection before and therefore, they are given low rating since there is no proof of application and the level of confidence is low.
3.2.4. RANK AND ELIMINATE LESS PREFERRED MMSM

The last section of 3.2 will summarise the finds based on the descriptive analysis performed in 3.2.3

Firstly, the descriptive analysis results based on the ratings from Section 3.2.3 are shown. As rated H- represent High, M-Medium and L-Low. From the results presented, the author has low confidence in ELECTRE, SAW, COPRAS, ARAS, OCRA, and CP. The methods that stand out as a result of the descriptive analysis are TOPSIS, and PROMETHEE. VIKOR, GRA, and TODIM’s confidence is neither low nor high; and will therefore be assessed based on the final decision of the author considering the other analysis performed. It must be noted that ELECTREE was eliminated in the first stages because of its inability to rank results and therefore could not be analysed with the other methods except in descriptive analysis since it is based on literature and the author’s experience in application of the methods.

Table 74: Descriptive analysis rating results

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<th>TODIM</th>
<th>TODIM</th>
<th>VIKOR</th>
<th>GRA</th>
<th>PROMETH</th>
<th>ELECTRE</th>
<th>OCRA</th>
<th>ARAS</th>
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<tr>
<td>A. Simplicity of execution</td>
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<td>B. Logical, mathematical procedure</td>
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<td>C. Input parameters</td>
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<td>D. Synthesis of judgement with merging functions</td>
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<td>E. Ranking of tangibles</td>
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<td>F. Generalizability to ranking intangibles</td>
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<td>G. Rank preservation and reversal</td>
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<td>H. Sensitivity analysis</td>
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<td>I. Applicability to conflict resolution</td>
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<td>J. Trustworthiness and validity of the approach</td>
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3.2.5. STATISTICAL ANALYSIS

A. SENSITIVITY ANALYSIS

Sensitivity analysis is of paramount importance for the MCDMs because of their nature of input parameters that are subjective. The ability to test for the robustness, and uncertainty is relevant where group decision making is concerned. The results from the sensitivity analysis help in increasing confidence, and credibility of the results. Also, the overall risk associated with the decision-making process is thus reduced. It was found by Triantaphyllou (2000) that the most sensitive criterion in decision problem is the one with the highest weight if weight changes are measured in relative terms (%). To relate the rest of the criteria to match the changes of the critical criterion weight, the equation taken from Leoneti (2016) was used. However, the author modified the critical criterion percentage from just considering 10% to considering any percentage for a good stability check and to ensure that the sum of the final weights would still equal 1.

\[ w_{n}^* = \frac{w_{n}(1 - w_{i}^*)}{(1 - w_{i})} \]

Where

- \( w_{i} \) represent the original weighting of the critical criterion.
- \( w_{i}^* \) represent the original weighting of the critical criterion plus the % change
- \( w_{n} \) represent the original weight of criterion n
- \( w_{n}^* \) recalculated weighting for criterion.

Case study 1 results will be used for performing a sensitivity analysis. The first method to be evaluated was TOPSIS. Only the weights of the method were modified between -50% and 50% changes. The results obtained are depicted Figure 36.
The radar graph shows the changes in rankings when weights are adjusted. It is observed that A1’s ranking did not change throughout the adjustments; while A5 changed between rank 5 and 6. A6 recorded the most changes by moving around all the ranks except for rank 1.

The changes became stable as the weights were being reduced. At +10% and -10%, there were no changes in rankings; however, as the weights were further reduced by 20% to 50%, a stable ranking was observed and has been taken as the final rank for TOPSIS.

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<th>original ranking</th>
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TODIM allows for changes in weights of the criterion, the attenuation factor which ranges between 1 and 10, reference criterion, and the performance evaluation. The weights of the criteria were changed as per the formula presented above. Adjustments of the weights were made from -50% to +50% and there were no changes in the rankings of the alternatives. The reference criterion was also changed. Initially, the highest weighted criterion was chosen as the reference criterion. In sensitivity analysis, the lowest criterion was checked, and it did not lead to changes in the rankings. Since for this study, the performance evaluations of the alternatives against the criteria will not be analysed because the author wants to maintain the original evaluations, the last parameter to be checked was the attenuation factor. The factor
was ranged between 1 and 10, and even though there were changes in the final values, they were too minimal to cause changes in the rankings. The following table shows the initial rankings, and the rankings after the performing sensitivity analysis.

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VIKOR’s input parameters were also check for stability in their rankings. The $v$-parameter ranges from 0 to 1. For the initial rankings, a value of 0.5 was used. The values were varied between 0 and 1 and a stable ranking could not be obtained. The rankings were similar between 0,0 and 0,3. At 0,4 and 0,5 the rankings were different. Between 0,6 and 0,8 the rankings were similar again and changed at 0,9 but remained constant to 1. The results indicated that the rank depends on the ‘$v$’ that is used, and one cannot depend on the rankings of VIKOR to base the final decision. The results of the ranks are shown Figure 37.

![Figure 37: VIKOR's sensitivity analysis with varying 'v' parameters](image)

The weight variations were the input parameters that were checked as well. Initially, the ‘$v$’ parameter was kept constant as the weights were varied. However, a stable ranking could not be obtained. The results of variation of weights were $v=0.5$ are shown in Appendix 2 in Figure 50. The ‘$v$’ parameter was changed to 1 since the ranks using $v=1$ showed similarity with the ranks of other MCDM. The results from $v=1$, were found to be stable. The instability was considered negligible and the final rankings of VIKOR were determined from $v=1$ with weight variations Figure 38.
The final rankings have been chosen as shown in Error! Reference source not found..

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>2</td>
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<td></td>
<td></td>
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<tr>
<td>3</td>
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<td>1</td>
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<tr>
<td>6</td>
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</tr>
</tbody>
</table>

GRA's rankings were then checked against weight variations as well the grey coefficients. Firstly, the weights were varied between -50% and +50%. A6 was found to be the most unstable as the weights were changed. It moved from rank 6 at -50% change to rank 3 at +50% change. Variation of weights resulted in a lot of instability, but only outside the -10% to 10% change. The results of the sensitivity analysis for varying weights are shown Figure 39.
The Grey coefficient was varied between 0.1 and 1. The changes were minimal and were only between A4 and A6 as shown in Figure 40. The overall changes when the grey coefficients and weights were varied were negligible. Therefore, the results of GRA are stable and were not changed as shown in Error! Reference source not found.

Figure 39: GRA sensitivity analysis for varying weights

Figure 40: GRA sensitivity analysis for varying coefficients
The Stability of OCRA’s ranking was assessed and there were no changes even when the weights were varied from -50% to 50%. The rankings remained the same. In the case of OCRA, a stability check was done on equal weights of the criterion since no other input parameter could be varied. The rankings changed. However, they will not be considered as the final rankings of OCRA since it was stable in weight variations. The results are shown below:

<table>
<thead>
<tr>
<th>initial ranking</th>
<th>Rankings after sensitivity analysis</th>
<th>equal weights ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

ARAS stability check was also on the variation of weights. The rankings remained stable; and the equal weight criterion rating were checked. The only change observed was a swap between A3 and A4. The rest of the rankings remained stable.

<table>
<thead>
<tr>
<th>Initial rankings</th>
<th>Rankings after sensitivity analysis</th>
<th>Equal weights rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

In CORPAS stability check, the input parameter were the weights variations. The sensitivity results of weight variation between -50% and 50% were all the same but different from the initial rankings because of a swap of A3 and A4. The equal weight results are similar to initial rankings. However, the rankings after the sensitivity analysis will be used.
CP method was checked. The input parameter 'v' was checked at p=1, p=2, and p=10, 20, 100 (which represent infinity). The rankings only became stable at p=10 as shown in Figure 41.

Figure 41: CP sensitivity analysis for varying 'p' parameters

Varying weights were checked at the different p's (1, 2, and 10). P=2 and p=10 did not provide stable rankings and can be seen in appendix 2. The results shown in Figure 42 are those of varying weights where p=1. At p=1, the weights were not similar. As the weights were reduced, the ratings were similar but different from the initial rankings. As the weights were being increased, the ratings remained similar to the original ratings. Both sides of the +/-10% are shown Error! Reference source not found..

Figure 42: CP sensitivity analysis for varying weights

The rankings accepted as the final rankings were the initial rankings shown in Error! Reference source not found..
SAW method’s stability was checked based on weight variations between -50% and 50%. The changes were minimal such that the initial rankings were accepted as the final ranking of alternatives.

![SAW Sensitivity Analysis](image)

**Figure 43: SAW sensitivity analysis**

PROMETHEE was the last method to be checked for its stability. The VISUAL PROMETHEE was used and the results are shown Figure 44: PROMETHEE sensitivity analysis. The input parameters that were varied were the preference and indifference thresholds as explained in the descriptive analysis of PROMETHEE. On weights, the equal weights, -10%, and +10% weight scenarios were checked. The overall ratings indicate that PROMETHEE is stable even after parameter changes. Though the values changed and there was a swap between A4 and A3, the changes were not enough to effect changes of the ranks.
To conclude on the sensitivity analysis, the new rankings from the sensitivity analysis of all the 10 MCDMs are shown in Table 75. The initial ratings of the following methods have changed: TOPSIS and VIKOR. TODIM, GRA, PROMETHE, OCRA, ARAS, SAW and CP’s overall rankings were stable and therefore did not change. However, out of the 10 methods, 6 methods have the same rankings unlike before sensitivity. It can therefore be concluded that the ratings for case study 1 should have been A2>A4>A3>A1>A5>A6. This confirms the frequency of ratings of case study 1.

Table 75: Aggregated Final sensitivity analysis.

<table>
<thead>
<tr>
<th></th>
<th>TOPSIS</th>
<th>TODIM</th>
<th>VIKOR</th>
<th>GRA</th>
<th>PROMETHEE</th>
<th>OCRA</th>
<th>ARAS</th>
<th>CORPAS</th>
<th>SAW</th>
<th>CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>A3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A4</td>
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<td>3</td>
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<td>2</td>
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<td>3</td>
<td>2</td>
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<td>2</td>
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<tr>
<td>A5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<td>5</td>
<td>5</td>
</tr>
<tr>
<td>A6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>6</td>
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<td>6</td>
<td>6</td>
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</tr>
</tbody>
</table>

B. SPEARMAN CORRELATION

The 10 MCDMs that have been investigated and detailed in the previous pages will now be compared based on their final performances that led to the rankings. The spearman correlation coefficient (rho) will be used in this case. The coefficient helps to determine the strength of the relationships between the MCDMs. In other words, it is used to measure the similarities between two sets of rankings. The value obtained from the correlation ranges between -1 and
+1. If the value is large and closer to +1, it then indicates a good agreement between the MCDMs. The formula below is used to calculate this coefficient:

$$r_s = 1 - \frac{6 \sum d_i^2}{n^2 - n}$$  \hspace{1cm} (49)

Where:

- $d_i$ represent the difference between MCDM ranks
- $n$ represent the sample size (in this case; the alternatives)

For case study 1, the spearman correlation coefficients were calculated for the 10 MCDMs and are shown in Table 76. According to the observations, the coefficients range between 0.371 and 1.000. TODIM is similar to OCRA. PROMETHEE is similar to ARAS, CORPAS, and SAW. And according to the Table 83 of the characteristics of co-efficient R in Appendix 2, the methods have a very strong relationship. The results of TOPSIS compared to GRA, and CP are satisfactory. However, VIKOR shows to be an outlier and its similarity to the rest of the methods is low and depicted by the black lined-graph in Figure 45: Spearman correlation MCDM comparisons.

### Table 76: Spearman’s correlation results

<table>
<thead>
<tr>
<th>Method</th>
<th>TOPSIS</th>
<th>TODIM</th>
<th>VIKOR</th>
<th>GRA</th>
<th>PROMETHEE</th>
<th>OCRA</th>
<th>ARAS</th>
<th>CORPAS</th>
<th>CP</th>
<th>SAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOPSIS</td>
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<td>0,771</td>
<td>0,600</td>
<td>0,943</td>
<td>0,886</td>
<td>0,771</td>
<td>0,886</td>
<td>0,886</td>
<td>0,943</td>
<td>0,886</td>
</tr>
<tr>
<td>TODIM</td>
<td>1,000</td>
<td>0,371</td>
<td>0,886</td>
<td>0,943</td>
<td>1,000</td>
<td>0,943</td>
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<td>0,829</td>
<td>0,943</td>
<td>0,943</td>
</tr>
<tr>
<td>VIKOR</td>
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<td>0,486</td>
<td>0,600</td>
<td>0,371</td>
<td>0,600</td>
<td>0,600</td>
<td>0,600</td>
<td>0,714</td>
<td>0,600</td>
<td>0,600</td>
</tr>
<tr>
<td>GRA</td>
<td>1,000</td>
<td>0,943</td>
<td>0,886</td>
<td>0,943</td>
<td>0,943</td>
<td>0,943</td>
<td>0,943</td>
<td>0,943</td>
<td>0,943</td>
<td>0,943</td>
</tr>
<tr>
<td>PROMETHEE</td>
<td>1,000</td>
<td>0,943</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>0,943</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>OCRA</td>
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<td>0,943</td>
<td>0,829</td>
<td>0,943</td>
<td>0,943</td>
<td>0,829</td>
<td>0,943</td>
<td>0,943</td>
<td>0,943</td>
</tr>
<tr>
<td>ARAS</td>
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<td>0,943</td>
<td>1,000</td>
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<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
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</tr>
<tr>
<td>CORPAS</td>
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<td>1,000</td>
<td>1,000</td>
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<tr>
<td>SAW</td>
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<td></td>
<td></td>
<td></td>
<td>1,000</td>
</tr>
</tbody>
</table>

145
C. KENDALL’S COEFFICIENT

To check for the overall similarity of the rankings, the Kendall’s coefficient is calculated for the overall rankings using the formula below. This value ranges between 0 and 1; with 0 indicating that there is no agreement and 1 shows the agreement between the MCDMs. Table 77 shows the summary of the calculations. The coefficient was calculated as 0.866 which suggest that there is almost a perfect agreement between all the considered MCDMs. According to the Kendall’s coefficient of concordance interpretation shown in Appendix 2, Table 84, and a coefficient great than 0.7 shows a strong agreement.

\[ W = \frac{12 \sum d_i^2}{m^2 \times n(n^2 - 1)} \]

Where:

- \( m \) is the number of judges/rate(rs); in this case, the MCDMs.
- \( n \) id the number of alternatives
- \( d \) shows the differences of the ranks

| Table 77: Kendall’s coefficient for the 10 MCDMs |
|-----------------|-----------------|
| \( W \)          | 0.866           |
| chi-square       | 43.31           |
| Degrees of freedom | 5              |
| p-value          | 0.00            |

A Null Hypothesis: \( H_0 \) means that there is no statistically significant degree of agreement amongst the MCDM; while \( H_1 \) means that there is a statistically significant degree of agreement between the MCDM. The P-value that indicates the level of significance has been
calculated to be 0.00 in Table 77. The hypothesis says if \( p < 0.05 \), the null hypothesis is rejected, and the alternative hypothesis is accepted.

**D. AGREEMENT ON THE TOP 3 RANKS**

In this section, the test based on the agreement of the top 3 ranked alternatives is performed. (1, 2, 3) means the first three ranks match. (1, 2, #) means the first two ranks match, and (#, #, #) means there is no match. In the rankings of MCDMs for case study 1, the table shows that the following sets match in their first 3 ranks: TOPSIS and CP; TODIM and OCRA; GRA, PROMETHEE, ARAS, COPRAS and SAW. VIKOR results in the maximum number of mismatches because its first three ranked alternatives do not match with any of the other MCDMs.

**Table 78: Agreement on Top 3 ranks**

<table>
<thead>
<tr>
<th></th>
<th>TOPSIS</th>
<th>TODIM</th>
<th>VIKOR</th>
<th>GRA</th>
<th>PROMETHEE</th>
<th>OCRA</th>
<th>ARAS</th>
<th>CORPAS</th>
<th>CP</th>
<th>SAW</th>
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<tbody>
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<td>#,#,#</td>
<td>1,#,3</td>
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<td>#,#,#</td>
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<td>1,2,3</td>
<td>1,#,3</td>
<td>1,2,3</td>
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</tr>
<tr>
<td>ARAS</td>
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<td>1,2,3</td>
<td>1,2,3</td>
<td>1,2,3</td>
<td>1,#,3</td>
<td>1,2,3</td>
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</tr>
<tr>
<td>CORPAS</td>
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<td>1,2,3</td>
<td>1,2,3</td>
<td>1,2,3</td>
<td>1,#,3</td>
<td>1,2,3</td>
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</tr>
<tr>
<td>CP</td>
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<td>1,2,#</td>
<td>1,2,3</td>
<td>1,2,3</td>
<td>1,2,3</td>
<td>1,#,3</td>
<td>1,2,3</td>
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<td></td>
</tr>
<tr>
<td>SAW</td>
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<td>1,2,#</td>
<td>1,2,3</td>
<td>1,2,3</td>
<td>1,2,3</td>
<td>1,#,3</td>
<td>1,2,3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**E. RANKS MATCHING PERCENTAGE**

The test in this section refers to the number of ranks matched (1-6) expressed as the percentage of the number of alternatives. The only methods with 100% matches are between TODIM and OCRA; PROMETHEE, ARAS, COPRAS, and SAW. Even though TOPSIS and CP match in the first three ranks, overall, the match is only 67%. VIKOR and GRA’s rankings do not have 100% matches with any of the MCDMs. The highest match percentage for VIKOR is 33% with CP.
Table 79: Ranks matching percentage

<table>
<thead>
<tr>
<th></th>
<th>TOPSIS</th>
<th>TODIM</th>
<th>VIKOR</th>
<th>GRA</th>
<th>PROMETHEE</th>
<th>OCRA</th>
<th>ARAS</th>
<th>COPRAS</th>
<th>CP</th>
<th>SAW</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOPSIS</td>
<td>100%</td>
<td>17%</td>
<td>17%</td>
<td>67%</td>
<td>33%</td>
<td>17%</td>
<td>33%</td>
<td>33%</td>
<td>67%</td>
<td>33%</td>
</tr>
<tr>
<td>TODIM</td>
<td>100%</td>
<td>17%</td>
<td>33%</td>
<td>67%</td>
<td>100%</td>
<td>67%</td>
<td>67%</td>
<td>67%</td>
<td>67%</td>
<td>67%</td>
</tr>
<tr>
<td>VIKOR</td>
<td>100%</td>
<td>0%</td>
<td>17%</td>
<td>17%</td>
<td>17%</td>
<td>17%</td>
<td>33%</td>
<td>17%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRA</td>
<td>100%</td>
<td>67%</td>
<td>33%</td>
<td>67%</td>
<td>67%</td>
<td>33%</td>
<td>33%</td>
<td>67%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PROMETHEE</td>
<td>100%</td>
<td>67%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>67%</td>
<td>67%</td>
<td>67%</td>
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</tr>
<tr>
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<td>ARAS</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
<td>67%</td>
<td>100%</td>
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<td></td>
</tr>
<tr>
<td>CORPAS</td>
<td></td>
<td></td>
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<td>CP</td>
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<td>SAW</td>
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<td>100%</td>
</tr>
</tbody>
</table>

3.2.6. RESOLVING CONFLICTING MCDMs

A. GROUP DECISION MAKING

On the previous pages, different MCDMs resulted in different rankings of alternatives. This is because the decision-process of each method is different. Another way of resolving conflicting results from the MCDMs is to use group decision making (GDM). In GDM, individual interests are reduced and integrated to form a group preference (Banarjee & Ghosh, 2013). Two rules are used; additive ranking rule and multiplicative ranking rule. In additive ranking rule, the rankings are summed up and an average of the rankings is obtained as the final rank. In multiplicative rankings, a product of the rankings from the MCDMs is raised to the power of 1/MCDMs. The following are the results of the group decision making for case study 1.

Table 80: Group decision making

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Additive ranking</th>
<th>Multiplicative ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>3,800</td>
<td>3,732</td>
</tr>
<tr>
<td>A2</td>
<td>1,400</td>
<td>1,282</td>
</tr>
<tr>
<td>A3</td>
<td>3,000</td>
<td>2,911</td>
</tr>
<tr>
<td>A4</td>
<td>1,900</td>
<td>1,761</td>
</tr>
<tr>
<td>A5</td>
<td>5,100</td>
<td>5,071</td>
</tr>
<tr>
<td>A6</td>
<td>5,800</td>
<td>5,785</td>
</tr>
</tbody>
</table>

The results obtained from either additive or multiplicative confirms the results of the sensitivity analysis. The results agree with 6 out of the 10 studied MCDMs. Therefore, a conclusion of which method is less preferred will be made in the following pages.

3.3. FACTORS THAT CAN BE USED IN MMSM

The factors that affect mining method selection were also part of the study. Different researchers’ results were analysed and below are factors that can be used for selection of coal mining method. Table 81 shows the factors identified in different researches.
It is observed that the most selected factors common in all the studies are ore thickness, ore depth, deposit dip, and orebody shape. The least used are life of mine, place of occurrence of the coal seam, nearness to the residential area and the existence of methane. These table can be used should future researches need to select a coal mining method. Also, the to-be developed MMSM will use these factors as input criteria.
Table 81: Factors that can be used in MMS

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<tbody>
<tr>
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<td>✓</td>
<td>✓</td>
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<td>Foot wall</td>
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<td>Ore plunge</td>
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<td>Nearness to residential area</td>
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<tr>
<td>Existence of methane</td>
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<td>✓</td>
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</table>
3.4. SELECTION OF COAL MINING METHODS
Different mining methods that have been used in the coal industry have been identified and can be used as part of the data base in the MMSM. The methods are:

- Bord-and-Pillar
- Pillar extraction
- Long wall
- Short wall
- Underground coal gasification
- Coal bed methane.

These methods will be used as input in the developed coal mining method selection model in the next chapter. For more information on what the method can do, their advantages and disadvantages are mentioned in the Literature Survey in 2.3 and 2.4.

3.5. SIGNIFICANCE OF THE RESULTS
The result presented above are important in the development of the mining method selection model. The MCDMs were studied and analysed. This will help in choosing which MCDMs can be used in the MMSM. Also, the results provide a good starting point for future research about the existing MCDMs. Factors and mining methods were also presented. In Chapter 4, a Mining Method Selection Model will be presented based on the information from chapter 3 and the literature review. The functionality, as well as the advantages and disadvantages of the model will be presented.
3.6. LIST OF REFERENCES


Yavuz Mahmut. 2012. Application of the TOPSIS Method to solve some decision-making problems in mining operations, Journal of underground resources, pp. 22-34
4. PROPOSED MMSM PROCEDURE

4.1. INTRODUCTION TO THE PROPOSED MMSM

Using the results obtained, and the literature reviewed, the author developed the MMSM. This chapter introduces the MMSM. The testing of the MCDM was not necessary since each MCDM was tested in the result section and the functionality of each have been explained. The result section analysed the MCDMs using different analysis methods, and the MCDM methods that emerged as the best were then used to develop the MMSM.

Case study 1 and 2 were used to test the functionality of the MCDMs. Initially, there were variations in the rankings. The results were used for the statistical analysis in section 3.2.5.

A sensitivity analysis was performed. The results from the sensitivity analysis helped in increasing confidence, and credibility of the results. Also, the overall risk associated with the decision-making process is thus reduced. It was found by Triantaphyllou (2000) that the most sensitive criterion in decision problem is the one with the highest weight if weight changes are measured in relative terms (%). To relate the rest of the criteria to match the changes of the critical criterion weight, the equation taken from Leoneti (2016) was used. However, the author modified the critical criterion percentage from just considering 10% to considering any percentage for a good stability check and to ensure that the sum of the final weights would still equal 1.

From the sensitivity analysis (3.2.5-A), all 6 of the methods except TODIM, GRA, CP, and OCRA agreed on the rankings. Initially, the groupings of the MCDMs in case study 1 results also showed that VIKOR and TOPSIS’ ratings were different, and they were grouped differently. However, after the sensitivity analysis, their ratings changed and agreed with the 4 methods to make up 6 agreeing methods. It was then concluded that 6 out of 10 methods are fit to be used in the MCDM. The results and application of these methods can only be validated by an actual mining problem as this is theoretical approach.

From the Spearman correlation test in section 3.2.5-B, 1 (VIKOR) of the six methods did not correlate with the rest of the methods; and ranked between low and moderate in terms of its agreement with the rest of the methods. TOPSIS did not have a 100% correlation with the remaining 4 (PROMETHEE, ARAS, COPRAS, and SAW) methods but showed a very high correlation which represent a strong relationship and will therefore not be eliminated. In the first three ranks test (3.2.5-D) and the ranks percentage match test (3.2.5-E), the four remaining method were still in agreement. However, CP and TOPSIS had similar first three ranks. GRA agreed with the four methods. VIKOR still remained an outlier. According to the ranking % match, the 4 methods agreed and had 100% match. GRA, TODIM, OCRA and CP had a 67% agreement. TOPSIS and VIKOR’s percentage agreement with the 4 were low at 33% and 17% respectively.

Saaty and Ergu described a set of criteria that can be used to evaluate different MCDMs in order to answer the question: “When is a Decision-Making method trustworthy?” In this study, a section of descriptive analysis (3.2.3) used the criteria to compare the 10 MCDMs. The MCDMs were checked on simplicity of execution and rated based on the author’s experience. The logical and mathematical procedure used in each method was assessed. Some methods were found to be simpler than others. While some had no mathematical foundation to base their existence. How each method synthesises and merge judgements to produce final ranking was also analysed. MCDMs have a tendency of not preserving rankings when new information
is added or removed from the systems. So, a criterion: rank preservation and reversal was used to rate how each method deals with the phenomenon of ranking reversal.

From the descriptive analysis, the author has high confidence in PROMETHEE, TOPSIS, and TODIM. The author does not recommend the rest of the methods. The following conclusion can be made. CP is a simplified approach between VIKOR and TOPSIS. OCRA is a simplified version of TODIM. COPRAS, ARAS, and SAW are simplified versions of PROMETHEE. GRA’s lack of a mathematical foundation, explanation, and the proven fact that it does not match with any MCDM’s rankings makes it impossible to be included in the model.

Therefore, the methods to be included in the model are TOPSIS, PROMETHEE, and TODIM. This does not mean these methods do not have shortcomings, however, they are less risky to use. It must be noted that their shortcomings will form part of future studies. For example, in PROMETHEE, there are thresholds that must be used as inputs, and they form part of the user’s preferences/choice. That makes the method subjective and difficult in that an inexperienced user may not know what or how to choose the thresholds. To deal with such a short coming, a solution has been introduced in section 3.2.3 (C). With TODIM, the author’s experience in using the method is that the method is prone to errors because of the complexity in computations. On the other side, TOPSIS has rank reversal problems. So, all these methods still have shortcomings even though they are suggested for use.

4.2. DEVELOPMENT PROCESS OF THE MMSM

The model has been developed and is below. Each step of the model is broken down below:

1. Form a decision-making team and define the problem.

The first thing the users of the model must do is to define the problem at hand. The users must define the mine under investigation, its geological and any other information that will be critical when selecting a mining method. An ill-structured problem may prove difficult to solve. When the problem has been sufficiently defined, the users must identify the decision goal. It is recommended that a neutral third party facilitates the decision process. Key players such as geologists, mine planners, engineers, and other relevant parties must be brought together for the decision to be made.

2. Search in MMS Database for a similar problem.

The approach is of case-based reasoning (CBR); where the user can retrieve, re-use, revise and then retain the information for future use. The user can always search within the database for a similar problem before selecting a MCDM. The author recommends PROMETHEE, TOPSIS, and TODIM to be used. However, depending on the nature of the problem, any other MCDM can be used.
The reason the author suggests the CBR approach on the developed MMSM is because CBR offers a platform for continuous learning as each solved problem is added to the database. Its solution-finding capabilities are high because the user can always find a similar problem within the database and that saves time.

1. Form a decision-making team and define the problem.
2. Search in MMS Database for a similar problem
3. Select MCDM method
4. Select suitable factors as criteria
5. Identify suitable alternative mining methods
6. Calculate and assign weights to the criteria
7. Evaluate alternatives against the criteria in MCDM
8. Perform a sensitivity analysis
9. Select the preferred alternative

4.1. MM and 5.1. Factors Database
2.1. Previous solved MMS cases Database
Problems that are difficult to solve can always be compared to similar ones in the database to reach a solution. A CBR process is illustrated and what each step entail is shown in Figure 46.

![Figure 46: Illustration of the MMSM Database](Image)

### 3. Select MCDM method

After the problems in the data base have been searched and compared to the defined problem, a MCDM must be selected. It is recommended that TODIM, PROMETHHE, or TOPSIS be used for the MMS problems. The background principles of selecting an alternative when using MCDMs are similar. The processes of the 3 recommended MCDM are shown below. See section 2.2.2 for more details. A Matrix is constructed based on the preferences of the decision makers. Weights are determined, and the normalised matrix is calculated. The user must take note of the type of criteria when selecting the criterion in step 4 of the developed model. The correct formulas must be used for the type (benefit or cost criteria) of criteria. An ideal solution for TOPSIS, relative criterion for TODIM, and criterion function, preference, and indifference parameter are identified as explained in the literature. Then the preferences are calculated. The higher the value, the more an alternative is preferred.

<table>
<thead>
<tr>
<th>Method</th>
<th>TOPSIS</th>
<th>TODIM</th>
<th>PROMETHEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Construct a Matrix</td>
<td>Construct a Matrix</td>
<td>Construct a Matrix</td>
</tr>
<tr>
<td>Step 2</td>
<td>Normalise a Matrix</td>
<td>Normalise a Matrix</td>
<td>Compute pairwise performances</td>
</tr>
<tr>
<td>Step 3</td>
<td>Determine the weighted Normalised Matrix</td>
<td>Determine the weighted Normalised Matrix</td>
<td>Choose a criterion function, indifference, and preference</td>
</tr>
<tr>
<td>Step 4</td>
<td>Identify an ideal solution</td>
<td>Identify a relative criterion</td>
<td>Determine the multi-criteria preference index</td>
</tr>
<tr>
<td>Step 5</td>
<td>Calculate the Euclidian distance</td>
<td>Determine the dominance degree of each alternative</td>
<td>Calculate the outranking flows</td>
</tr>
</tbody>
</table>
From the factors and mining method database, the user can then select suitable criteria and alternatives respectively. The CBR approach is still utilised for both mining method and factors data bases. Even though factors are many and different, the discussed factors can be grouped under these categories:

- Geological and hydrological factors
- Geotechnical factors
- Environmental factors
- Economic factors
- Technological factors
- Spatial characteristics of the investigated deposit.

So, as more factors are added into the decision-process, they can be stored under the aforementioned categories for ease of searching in the future. More categories can be added should the need arise. The same applies for the mining methods. This MMSM does not limit the user to the described mining methods only, and that is an added advantage compared to the traditional techniques of MMS.

To perform the evaluation process, weights must be assigned to the criteria. In the literature review, AHP was introduced as one of the MCDMs. However, AHP was used for weight assigning in this study. In AHP, the decision makers construct a pairwise comparison matrix, and find the relative priorities of the criteria. The calculation of weights is a subjective process; fortunately, AHP allows for a consistency ratio to be calculated for accuracy. Should the weight-assigning process be found to be inconsistent, the decision-makers need to evaluate their priority ratings and make necessary changes.

In step 7, the alternatives are then evaluated against the criteria. Rankings will be derived from the evaluations. The are two routes after obtaining the rankings; the user can perform a statistical analysis or take the rankings as the final decision. In the statistical analysis, three
tests are performed; Spearman correlation (7.1.a), agreement of the first 3 ranks (7.2.a), and rank match percentages (7.3.a).

The spearman correlation determines the strength of the relationships of the MCDMs by calculating the similarity between two set of rankings. The test is based on the agreement of the top 3 ranked alternatives. (1, 2, 3) means the first three ranks match; (1, 2, #) means the first two ranks match, and (#, #, #) means that there is no match. The last test refers to the number of ranks matched expressed as % of the no. of alternatives. This route is applicable if more than 1 MCDM were used to obtain the rankings.

A Kendall coefficient is then calculated in 7.5a to check for the agreement in the MCDMs. If the coefficient equal 1, then the process is ended, and the final rankings will then be derived based on all the tests. If the coefficient is less than 1, then a conflict resolution can be applied. In conflict resolution, additive and multiplicative rankings are determined, and the rankings are obtained. A sensitivity analysis is then performed, then the final decision is taken.

Alternatively, after the evaluations of the alternative against the criteria in step 7, a sensitivity analysis can be performed directly without doing a statistical analysis. In sensitivity analysis, weights are re-assigned based on the agreement of the decision-makers. Other scenarios based on the controllable variables within each MCDM can be created in the process to confirm the results. For example; in TODIM, other than the weights, the attenuation factor and the choice of the reference criterion can be adjusted.

After observing the effects of the changes on the final rankings through a sensitivity analysis, the process comes to an end. In step 9, a decision is reached. A preferred alternative will be taken as the mining method to be used in the specific mine. The users can always confirm the final rankings with experiential knowledge.
Some of the advantages of the developed MMSM

- The MMSM allows the user freedom to choose MCDM; so the user is not limited to a single method with its shortcomings.
- The user can easily compare the results after using multiple MCDMs.
- The information used can be stored into the database for future use.
- There is no limitation on the number of criteria and alternatives that can be used as inputs in the system.
- The procedure provides a good platform for future developments into an app-based format or software
- The MMSM can be used even for other commodities outside coal mining.

The disadvantage of the MMSM:

- Users need to understand the theoretical background of the MCDMs before making a choice on which one to use for the decision-process. However, in future studies the author intends to develop an application-based procedure so that the functionality of each MCDM may be built in and the user will just insert the evaluation performance of alternatives against criteria to obtain the final rankings.
5. CONCLUSIONS

The aim of project was to develop a mining method selection model through a detailed assessment of MCDAs. This is because attempts to build a systematic approach to mining method selection have been made in the past. However, there has been limitation from the traditional approaches presented. Therefore, objectives were set to achieve the aim of the project. The main aim was to study in detail the MCDMs that were previously used in decision-making in and outside the mining industry.

Ten MCDMs- TOPSIS, TODIM, VIKOR, GRA, PROMETHEE, OCRA, ARAS, COPRAS, SAW, and CP were studied in detail; their application, functionality, advantages and disadvantages. ELECTRE and HPV were also introduced. However, they could not be studied in detail and are not recommended. ELECTRE fails to sort the alternatives ratings in ranks. While HPV cannot be implemented in the absence of voters. AHP was introduced as well. However, in this study, it was only used for weight elicitation since the introduced MCDM cannot assign weights.

In the results section, the MCDMs were analysed following a descriptive and a statistical analysis. In the descriptive analysis, a set of criteria was introduced and used to evaluate the MCDMs. In the statistical analysis, tools such as sensitivity analysis, spearman correlation, and Kendall's coefficient were used. Two ways (additive, and multiplicative) of resolving conflict in the ranks were introduced and the final ranking of the combined MCDM was obtained. After such a comprehensive analysis, it was found that PROMETHEE, TOPSIS, and TODIM stand out and can be successfully used in the selection of mining method in the coal mining industry. The other methods (OCRA, ARAS, CP, SAW, and COPRAS) have been found to be simplified approaches of the aforementioned methods. VIKOR’s rankings were outlying and it was concluded that it is not a suitable method for MMS. GRA’s conclusion based on the literature view is that there is no founded mathematical explanation behind its existence because there remain may unanswered questions about its foundations.

Factors such as ore thickness, depth, ore plunge, and mining methods such as bord-and-pillar, long wall, gasification, and coal bed methane were presented, were also studied and presented as part of the literature review. They formed part of the database of the CBR in the developed MMSM.

The potential this study has is that it will be a provision of a systematic approach that caters for subjective and objective analysis in MMS. Also, it will result in an increased level of confidence of the MCDM so that South African Mining companies can utilise these models as application has been limited in the country’s coal mining industry.

The last section of the project presented a MMSM procedure of choosing a mining method. The approach has been simplified and can be implemented by any user given that the background information presented in this research is understood.
6. RECOMMENDATIONS

- The user must understand the discussed MCDMs and must acknowledge that the model developed is a simplified approach and can only be useful if there is an understanding of the theoretical background behind the MCDMs.
- Fit-for-purpose criteria and alternatives may be added in the database for the specific problem being investigated, should the factors and methods in the results section be insufficient.
- For effective and reliable results, at least 2 of the MCDMs can be used in the MMSM to observe and record any variations in the final ranks.
- In MMSM, a problem or an objective must be defined appropriately before the MMSM is used to avoid inconsistency in the final rankings.

7. SUGGESTIONS FOR FURTHER WORK

- A limitation in the study is that only AHP was used to elicit weights. This means that a room for other methods with capabilities to elicit and calculate weights is left. Therefore, a future study could be to investigate other weight elicitation methods and their influence on the final ranks.
- One of the limitations in the study is that some of the articles, and journals reviewed were a translation from other languages to English. Therefore, in future, more articles from other languages can also be reviewed for more information on MCDMs.
- In future studies, algorithms for selection of a suitable MCDM in the MMSM can be developed so that once the problem has been defined and structured, the user may not struggle with knowing which method to use amongst the suggested.
- Since all the MCDMs have their unique strengths and shortcomings, it is suggested that a group-decision making approach be further refined.
- A sensitivity analysis approach may need to be refined or specifically developed for the MMSM.
- To develop an application-based procedure so that the functionality of each MCDM may be built in and the user will just insert the evaluation performance of alternatives against criteria to obtain the final rankings.
APPENDICES
APPENDIX 1

The classification and support systems for the traditional approaches to mining method selection are presented in this appendix.

Figure 47: Support chart for MMS tool by Morrison (Peskens, 2013)
Figure 48: support graph developed by Laubscher in 1981. Source: Peskens (2013)

Figure 49: Support Graph developed by Laubscher in 1990. Source: Peskens (2013)
Table 82: Support Table for UBC MMS by Miller-Tait of 1995 (Peskens, 2013)

<table>
<thead>
<tr>
<th>General shape</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Equi-dimensional</td>
<td>All dimensions are of the same order</td>
</tr>
<tr>
<td>Platy-tabular</td>
<td>Two dimensions are larger than the thickness</td>
</tr>
<tr>
<td>Irregular</td>
<td>Dimension vary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orebody thickness</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrow</td>
<td>&lt;10m</td>
</tr>
<tr>
<td>Intermediate</td>
<td>10 – 30m</td>
</tr>
<tr>
<td>Thick</td>
<td>30 – 100m</td>
</tr>
<tr>
<td>Very Thick</td>
<td>&gt;100m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deposit plunge</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>&lt;20°</td>
</tr>
<tr>
<td>Intermediate</td>
<td>20° – 55°</td>
</tr>
<tr>
<td>Steep</td>
<td>&gt;55°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade distribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>Grade is equal to the mean grade throughout the orebody</td>
</tr>
<tr>
<td>Gradational</td>
<td>Grade has a zonal characteristic which gradually changes from zone to zone</td>
</tr>
<tr>
<td>Erratic</td>
<td>Grade can change quickly over distance without any pattern</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Deposit depth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow</td>
<td>&lt; 100m</td>
</tr>
<tr>
<td>Intermediate</td>
<td>100- 600m</td>
</tr>
<tr>
<td>Deep</td>
<td>&gt;600m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RMR</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Very weak</td>
<td>0 – 20</td>
</tr>
<tr>
<td>Weak</td>
<td>20 – 40</td>
</tr>
<tr>
<td>Medium</td>
<td>40 – 60</td>
</tr>
<tr>
<td>Strong</td>
<td>60 – 80</td>
</tr>
<tr>
<td>Very strong</td>
<td>80 – 100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rock Substance Strength (RSS)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Weak</td>
<td>&lt;5</td>
</tr>
<tr>
<td>Weak</td>
<td>5 – 10</td>
</tr>
<tr>
<td>Medium</td>
<td>10 – 15</td>
</tr>
<tr>
<td>Strong</td>
<td>&gt;15</td>
</tr>
</tbody>
</table>
APPENDIX 2
In this appendix, tables and figures that support the results section. Table 83 shows the characteristics of the spearman correlation. Table 84 shows the characteristics of the Kendall’s coefficient. Figure 51-53 supports the sensitivity analysis in the results section.

Table 83: Characteristics of Co-efficient R (Banerjee & Ghosh, 2013)

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Nature of correlation</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9-1.0</td>
<td>Very High</td>
<td>Very strong relationship</td>
</tr>
<tr>
<td>0.7-0.9</td>
<td>High</td>
<td>Marked relationship</td>
</tr>
<tr>
<td>0.4-0.7</td>
<td>Moderate</td>
<td>Substantial relationship</td>
</tr>
<tr>
<td>0.2-0.4</td>
<td>Low</td>
<td>Definite relationship</td>
</tr>
<tr>
<td>&lt;0.2</td>
<td>Slight</td>
<td>Small relationship</td>
</tr>
</tbody>
</table>

Table 84: Characteristics of Co-efficient W

<table>
<thead>
<tr>
<th>W</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;=0.3</td>
<td>Weak agreement</td>
</tr>
<tr>
<td>0.3&lt;W&lt;=0.5</td>
<td>Moderate agreement</td>
</tr>
<tr>
<td>0.5&lt;W&lt;=0.7</td>
<td>Good agreement</td>
</tr>
<tr>
<td>W&gt;0.7</td>
<td>Strong agreement</td>
</tr>
</tbody>
</table>

Figure 50: Sensitivity analysis of VIKOR @ v=0, 5 with varying weights
Figure 51: CP sensitivity analysis for varying weights at p=2

Figure 52: CP sensitivity analysis for varying weights at p=10