

# The stock-bond nexus and investors' behavior in mature and emerging markets: Evidence from long-term historical data

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## Abstract

**Purpose** – Portfolio construction and diversification is a prominent challenge for investors. It reflects market agents' behavior and response to market conditions. This paper aims to investigate the stock-bond nexus in the case of two emerging and two mature markets, India, South Africa, the UK and the USA, using long-term historical monthly data.

**Design/methodology/approach** – To address the issue at hand, copula quantile-on-quantile regression (C-QQR) is used to model the correlation structure. Although this technique is driven by copula-based quantile regression model, it retains more flexibility and delivers more robust and accurate estimates.

**Findings** – Results suggest that there is substantial heterogeneity in the bond-stock returns correlation across the countries under study point to different investors' behavior in the four markets examined. Additionally, the findings reported herein suggest that using C-QQR in portfolio management can enable the formation of tailored response strategies, adapted to the needs and preferences of investors and traders.

**Originality/value** – To the best of the authors' knowledge, no previous study has addressed in a comparative setting the stock-bond nexus for the four countries used here using long-term historical data that cover the periods 1920:08-2017:02, 1910:01-2017:02, 1933:01-2017:02 and 1791:09-2017:02 for India, South Africa, the UK and the USA, respectively.

**Keywords** Copula, Quantile regression, Stock-bond nexus

**Paper type** Research paper

## 1. Introduction

The nexus between the stock and bond markets is of particular interest for both portfolio managers and policymakers as it has important repercussions for asset allocation and risk management strategies. Both reflect and encapsulate investors' behavior and responses to

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changing market conditions as well as their forward looking perspectives or the economy (Dajman, 2012; Dajman, 2015; Geoffrey, 2017; Aslanidis and Christiansen, 2012, 2014; Dunis *et al.*, 2013). Hence, not surprisingly, the determinants and the dynamics that govern the relationship between the two markets have been a popular theme in the relevant literature (Baur and Lucey, 2009; Connolly *et al.*, 2007; Andersson *et al.*, 2008; Li and Zou, 2008; Baele *et al.*, 2010; Chui and Yang, 2012).

A large proportion of the literature on the stock-bond association focuses on developed markets. Relatively less attention has been given to emerging markets or to comparative studies that include both developed, mature markets and emerging ones. Such comparative studies can shed more light in how market agents' behavior as reflect in the stock-bond correlation differs between mature and emerging markets. This is a theme taken up by this study. It examines the correlation between the two markets in the cases of India and South Africa, two emerging markets, and the USA and the UK, two of the more mature and largest markets globally. Furthermore, the majority of previous studies investigate the dynamic correlation between mean returns of stock and bond markets. In this paper, we opt to focus on quantiles to probe deeper into how this can change when returns are more located into the tails. To this effect, we examine the entire dependence structure of the quantile of stock return and the quantile of bond return, thereby extending the copula quantile regression (C-QR) to the copula quantile-quantile regression (C-QQR), given that the framework now enables a regressor to be itself a conditional quantile. Beyond the central tendency of correlation (i.e. the average level), the C-QQR allows capturing the dynamic correlations ranging from non-extreme events (when returns are centrally located) to extreme events (when returns are more located into the tails). Finally, we use longest possible spans of historical data for the four countries under consideration, i.e. the monthly periods of 1920:08-2017:02, 1910:01-2017:02, 1933:01-2017:02 and 1791:09-2017:02 for India, South Africa, the UK and the USA, respectively, and thus track the historical correlation between the bond and stock markets of these countries as they evolve. A brief overview of the literature and the reported findings is contained in Section 2, while the methodology used is briefly presented in Section 3. The data used are shown in Section 4 where the findings are also reported and discussed. Section 5 concludes the paper.

## 2. Literature review: a bird's eye view

As Wu *et al.* (2017) point out, modeling correlations and linkages between financial asset returns are important for international investors and for policymakers. Early studies investigating the stock-bond correlation mainly focused on the time invariant relationship between the two markets. The papers of Keim and Stambaugh (1986); Schwert (1989); Shiller and Beltratti (1992); Campbell and Ammer (1993) and Fleming *et al.* (1998) can be cited among the first seminal works that probed into the nexus between the two markets and the factors that affect it. For instance, Shiller and Beltratti (1992) report a positive correlation between stock returns and bond yields, attributed to the common discount rate effect. Campbell and Ammer (1993) report that real interest rate shocks affect positively this relationship while, on the other hand, an inflation shock has not a clear directional effect. In subsequent studies, a time-varying correlation between bond and stock markets is assumed. In the relevant literature, examining the main determinants of the underlying dynamic intertemporal relation between the stocks and bond assets, real interest rates, inflation, the unemployment rate and the economic growth rate are cited as key macroeconomic variables that exert an influence. It has been frequently argued that macroeconomic stability is the main cause for the positive relationship between stock and bond returns that is occasionally observed. Macroeconomic stability is determined by common macroeconomic factors such

as inflation expectations or expected economic growth (Asgharian *et al.*, 2015, 2016; Christiansen, 2010; Imanen, 2003; Connolly *et al.*, 2005; Dimic *et al.*, 2016; Dacjman, 2012; Kim *et al.*, 2006; Skintzi, 2017).

Imanen (2003), using US data reports evidence showing that at high levels of inflation, changes in the discount rates dominate the changes in cash flow expectations, thereby inducing a positive stock-bond return correlation. In a similar vein, Li (2004) shows that greater concerns for future inflation tend to result in stronger co-movements between stocks and bonds. According to Andersson *et al.* (2008), the stock and bond prices move in the same direction during periods of high inflation expectations, while periods of a negative stock-bond return correlation coincide with the lowest levels of inflation expectations. Moreover, the stock-bond returns correlation is unaffected by economic growth expectations.

As shown by a number of studies, the relationship between the two markets can differ between the short and long run time horizons. Over longer periods, the aforementioned mentioned macroeconomic factors affect positively this relationship. However, over the short run there can be a negative relationship between bonds and stocks that is mainly attributed to the high level of stock market volatility that leads to a decoupling between these asset prices providing evidence for the well-known flight-to-quality phenomenon (Andersson *et al.*, 2008). In other words, in times of stock market turbulence, investors become more risk-averse and adjust their portfolios from risky assets such as stocks to safer assets such as long-term government bonds (Chang and Hsueh, 2013; Durand *et al.*, 2010; Yang *et al.*, 2009, 2010; Baur and Lucey, 2009; Capiello *et al.*, 2006; Gulko, 2002; Thomadakis, 2012).

Bond inclusion to stock market portfolios in times of stock market turbulence can enhance portfolio returns stability (Gulko, 2002). Connolly *et al.* (2005) stress that cross-market hedging and flight-to-quality are present with increased stock market volatility. Dacjman (2012), focusing on Eurozone countries, reports evidence pointing to significant flight-to-quality effects during financial crises prior to 2010. However, during the European Sovereign Debt Crisis, the flight-to-quality effect is only observed in the case of Germany.

Perhaps worth mentioning here is that exogenous events such as for instance natural or anthropogenic catastrophes, social unrest, political upheavals, terrorism and other violent events such as conflict and war, can also exert an impact on the stock-bond correlation over the short run (Schneider and Troeger, 2006; Apergis *et al.*, 2017; Guidolin and La Ferrara, 2010; Wisniewski, 2016). These largely unanticipated events have the potential to generate uncertainty, adversely influence risk perceptions and exert a negative effect on investors' sentiment and their concomitant assessment of markets. As a result, markets' volatility and portfolio allocation decisions between stocks and bonds are influenced by such exogenous events (Brune *et al.*, 2015; Aslam and Kang, 2015; Kaplanski and Levy, 2010; Kollias *et al.*, 2013). In a similar vein, Gupta *et al.* (2017) using a recently published news implied volatility index of Manela and Moreira (2017) and its main components, examine how the nexus between the two markets is affected by news and the concomitant uncertainty they potentially cause. Evidence for a flight-to-quality effect is found since bond returns are positively influenced by the increased uncertainty concerning security markets news. However, bond returns are negatively influenced from uncertainty induced by government policy news. Correlation between the two markets is found to increase significantly over specific types of uncertainty news for both the UK and the USA. However, in case of the UK uncertainty news associated with conflict, trigger diversification benefits implied by the appearance of a negative correlation between stock and bond market.

More recently, a number of studies using wavelet analysis investigate the stock bond correlation in European and Emerging markets. For instance, Kim and In (2005) study the G7 group of countries and argue that the relationship depends on the time scale, with a

negative relation in most economies both over the short and long horizons. On the other hand, [Dajcman \(2015\)](#) examining ten Eurozone countries yield positive wavelet correlations between the two markets, with Portugal being the only exception. [Asgharian et al. \(2015\)](#) report negative long-run correlations when the economy is weak, providing evidence of the flight-to-quality phenomenon. [Aloui et al. \(2015\)](#) conduct wavelet-based analysis in the Gulf Corporation Council countries and yield similar results. [Dimic et al. \(2016\)](#) apply the wavelet approach to investigate the stock-bond return correlation patterns in emerging markets. Their results reveal sustainable negative correlations over the short-run, once again pointing to a flight-to-quality and mostly positive comovements over the long run. According to [Ferrer et al. \(2016\)](#), the UK equities display the strongest linkage with the bond market followed by Germany, France, The Netherlands and Spain while much weaker interdependence between bond yields and stock returns is reported for Portugal, Ireland and Greece. [Bayraci et al. \(2017\)](#) report evidence of positive co-movements in the G7 bond-stock markets, which vary over time and across investment horizons. The higher co-movement is found to be more concentrated in the lower frequency bands. The correlations are highly volatile and significantly increase across different time scales during the episodes of equity market turbulence. This result can mainly be attributed to flights from stocks to safer bond investments due to significant changes in investor sentiment and risk aversion at times of market stress. In the following section, the data and methodology are presented.

### **3. Methodology: an outline**

It is recognized that the historical data of time series are the result of complex economic processes which include policy shifts, structural changes, sudden shocks, political tensions and other changes and developments that can exert a significant and traceable effect. The combined influence of these various events are the root of distributional characteristics of financial and macroeconomic time series such as asymmetry, nonlinearity, heavy-tailness and extreme values ([Bouoiyour and Selmi, 2017](#); [Sukcharoen and Leatham, 2016](#)). Among many asset classes, the association of the asset returns has been documented to deviate from the perceived correlation especially over turbulent times. In this context, the major contribution of the present paper is to effectively capture dynamic dependencies in data. Traditional studies consider the interdependences among financial markets as effects of the conditional means of the return of one market onto the conditional means of chronologically succeeding returns of another market. The average can vary greatly depending on the sample used. Standard linear regression techniques display the average dependence between a set of regressors and the dependent time series. This provides only a partial view of the relationship studied, as we might be interested in depicting the relation at different points in the conditional distribution of dependent variable. The estimate of the mean is in this case compromised. Although commonly applied regressions focus on the mean, deviations from the regression line can greatly affect the fit of the ordinary least squares (OLS). Median estimators and more general quantile estimators are generally less impacted by outlying observations in the response variable conditional on the covariates ([Koenker and Bassett, 1978](#); [Koenker, 2005](#)). Further, it is important to recognize that covariates can have an influence on the dispersion of the response variable as well as its location (i.e. heteroskedasticity). When this occurs, quantile regression unlike the OLS or the mean regression provides a more flexibility of covariate effects. The existing literature is quite rich in tools to predict conditional quantiles. The most frequently employed technique is the linear quantile regression ([Koenker and Bassett, 1978](#)) which can be viewed as the extension of the OLS estimation used to predict the conditional mean. These simple linear methods have been refined to control for nonlinear effects via additive models ([Koenker, 2005](#)),

semiparametric quantile regression (Noh *et al.*, 2015) and nonparametric quantile regression (Li *et al.*, 2013). This study uses a new econometric tool to model the entire dependence structure of two assets (bonds and stocks) by using their return information to characterize the environments of the respective financial markets. Although this technique is based on the quantile regression paradigm, it departs from the conventional framework, as the exogenous variable may be itself a quantile[1]. This enables us to document the detailed correlation structure across return quantiles. Inspired by Sim (2015), a copula approach will be used to examine rigorously the joint dependence of quantiles. When the correlation dynamics are of interest, the copula approach becomes highly useful as the copula parameter gives accurate outcomes regarding the correlation structure. Sim (2015) evaluated the dependence between the quantile of Australian return and the quantile of USA return, thereby extending the C-QR developed by Bouyè and Salmon (2009) to the copula quantile-quantile regression (C-QQR). Exploiting the C-QQR, we analyze from a new perspective the correlation dynamics across bond return and stock returns for India, South Africa, the UK and the USA. The copula-QQR model complements a rich body of existing methodologies for investigating the correlation structure and the interdependence among financial markets. The earliest empirical research on stock markets relationships have concentrated on the analysis of short-term benefits of international portfolio diversification (Levy and Sarnat, 1970; Solnik, 1974). Other works have analyzed the interdependence between stock markets via the covariance of excess returns (Phylaktis, 1999). Another strand of literature has assessed the linkages across stock markets using bivariate (Taylor and Tonks, 1989) or multivariate cointegration methodology (Kasa, 1992). One can also point to the linear quantile regression that estimates the average dependence as well as the upper and lower tails (Koenker and Bassett, 1978); the regime-switching model to capture the jumps in correlation between normal and extreme states (Guidolin and Timmermann, 2005); extreme value theory to model the dependence between the tails and the extreme realizations of the distributions of returns (Ang and Chen, 2002; Poon *et al.*, 2004; Heffernan and Tawn, 2004); and a copula approach to examine the correlation structure with an explicit measure of tail dependence (Patton, 2006; Bouyè and Salmon, 2009; Sukcharoen and Leatham, 2016; Tang *et al.*, 2014; Wu *et al.*, 2017).

Operationally, we specify a model aimed at providing a flexible way of modeling correlation corresponding to a spectrum of non-extreme to extreme events, where the quantile of one variable is dependent on the quantile of another variable. Let the superscript  $STR$  and  $BdR$  denote the quantiles of stock return and bond return, respectively; for each country under study, we postulate a model for the  $\theta$ -quantile of  $BdR$  as a function of history and  $STR$  denoted as:

$$BdR\_India_t = \beta^\theta STR\_India_t + \alpha^\theta BdR\_India_{t-1} + \varepsilon_t^\theta \quad (1)$$

$$BdR\_SouthAfrica_t = \beta^\theta STR\_SouthAfrica_t + \alpha^\theta BdR\_SouthAfrica_{t-1} + \xi_t^\theta \quad (2)$$

$$BdR\_UK_t = \beta^\theta STR\_UK_t + \alpha^\theta BdR\_UK_{t-1} + \zeta_t^\theta \quad (3)$$

$$BdR\_US_t = \beta^\theta STR\_US_t + \alpha^\theta BdR\_US_{t-1} + \mathfrak{s}_t^\theta \quad (4)$$

where  $\varepsilon_t^\theta$ ,  $\xi_t^\theta$ ,  $\zeta_t^\theta$ ,  $\varsigma_t^\theta$  is an error term that has a zero  $\theta$ -quantile. To analyze the correlation between  $\theta$ -quantile of bond return and the  $\tau$ -quantile of stock return, we linearize the function  $\beta^\theta(\cdot)$  by taking a first order Taylor expansion of  $\beta^\theta(\cdot)$  around  $STR^\tau$ , which yields:

$$\beta^\theta(STR_{India_t}) \approx \beta^\theta(STR_{India}^\tau) + \beta^{\theta'}(STR_{India}^\tau)(STR_{India_t} - STR_{India}^\tau) \quad (5)$$

$$\begin{aligned} \beta^\theta(STR_{SouthAfrica_t}) &\approx \beta^\theta(STR_{SouthAfrica}^\tau) \\ &\quad + \beta^{\theta'}(STR_{SouthAfrica}^\tau) \\ &\quad (STR_{SouthAfrica_t} - STR_{SouthAfrica}^\tau) \end{aligned} \quad (6)$$

$$\beta^\theta(STR_{UK_t}) \approx \beta^\theta(STR_{UK}^\tau) + \beta^{\theta'}(STR_{UK}^\tau)(STR_{UK_t} - STR_{UK}^\tau) \quad (7)$$

$$\beta^\theta(STR_{US_t}) \approx \beta^\theta(STR_{US}^\tau) + \beta^{\theta'}(STR_{US}^\tau)(STR_{US_t} - STR_{US}^\tau) \quad (8)$$

We can redefine  $\beta^\theta(STR^\tau)$  and  $\beta^{\theta'}(STR^\tau)$ , respectively, as  $\beta_0(\theta, \tau)$  and  $\beta_1(\theta, \tau)$ . Thereafter, the equations (5) to (8) are denoted as:

$$\beta^\theta(STR_{India_t}) \approx \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR_{India_t} - STR_{India}^\tau) \quad (9)$$

$$\begin{aligned} \beta^\theta(STR_{SouthAfrica_t}) &\approx \beta_0(\theta, \tau) \\ &\quad + \beta_1(\theta, \tau)(STR_{SouthAfrica_t} - STR_{SouthAfrica}^\tau) \end{aligned} \quad (10)$$

$$\beta^\theta(STR_{UK_t}) \approx \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR_{UK_t} - STR_{UK}^\tau) \quad (11)$$

$$\beta^\theta(STR_{US_t}) \approx \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR_{US_t} - STR_{US}^\tau) \quad (12)$$

Then, we substitute the equations (1) to (4) into equations (9) to (12), respectively, to obtain:

$$\begin{aligned} BdR_{India_t} &= \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR_{India_t} - STR_{India}^\tau) \\ &\quad + \alpha(\theta)BdR_{India_{t-1}} + \varepsilon_t^\theta \end{aligned} \quad (13)$$

$$\begin{aligned} BdR_{SouthAfrica_t} &= \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR_{SouthAfrica_t} - STR_{SouthAfrica}^\tau) \\ &\quad + \alpha(\theta)BdR_{SouthAfrica_{t-1}} + \varepsilon_t^\theta \end{aligned} \quad (14)$$

$$BdR\_UK_t = \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR\_UK_t - STR\_India_\tau) + \alpha(\theta)BdR\_UK_{t-1} + \varepsilon_t^\theta \quad (15)$$

$$BdR\_US_t = \beta_0(\theta, \tau) + \beta_1(\theta, \tau)(STR\_US_t - STR\_US_\tau) + \alpha(\theta)BdR\_US_{t-1} + \varepsilon_t^\theta \quad (16)$$

The copula is then applied to provide a rich source of potential nonlinear dynamics depict intertemporal association depending on the behavior of bond and stock markets, and also allows us to distinguish the investigated dependence from the specification of the marginal (stationary) distribution of the response. The copula function is used to express the dependences between bond returns and stock returns. In bivariate modeling, the theory of copula states that for every joint distribution  $F$  of  $X_1$  (dependent variable: bond return) and  $X_2$  (independent variable: stock return) having marginal distributions  $FX_1$  and  $FX_2$ , there exists a unique copula function  $h$  with copula parameter  $\rho$  satisfying  $F(STR, BdR) = h(F(BdR), F(STR); \rho)$ , so that the copula combines the marginal distributions to yield the joint distribution of the covariates. Hypothesizing the dependence function  $h$ , the quantile dependence equation (Bouyé and Salmon, 2009; Sim, 2015) is denoted as:

$$BdR_t = h(STR_t; \rho(ut, vt))$$

where  $\rho$  is the correlation coefficient between  $BdR$  and  $STR$ ;  $u_t$  is the i.i.d. innovation in  $BdR_t$  with distribution function  $F(\cdot)$ ,  $u_t$  and  $v_t$  are independent.

Having accurate information into the dynamic dependencies between these two assets can help investors to construct an optimal portfolio. Some concerns like the existence of asymmetric dependence structure or the time-varying dependence may help to appropriately measuring and assessing risks. In addition, the benefits of international diversification of assets can be largely affected by the asymmetric dependence structures. Therefore, appropriately determining this dependence is of great importance in portfolio and risk management. The existing literature suggests that a time-invariant copula is irrelevant since it does not allow the parameters in a copula function to change over time (Patton, 2006; Harvey, 2010; Busetti and Harvey, 2011). To avoid possible econometric pitfalls, this study considers different copula functions with symmetric and asymmetric tail behavior, even if we control for time-varying dependence (see Appendix 1 for more details). In terms of goodness of fit, the Student's  $t$  copula seems more suitable than the rest of copula functions, as it gives the lowest AIC and BIC[2].

## 4. Findings and discussion

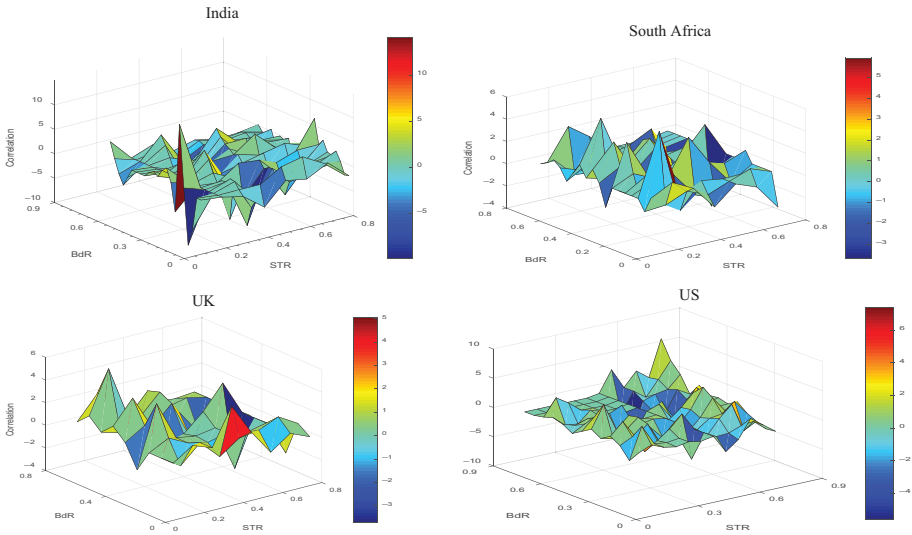
### 4.1 Data

The financial data set used in our empirical estimations, consists of monthly data on Indian, South African, American and British bond and stock returns. Perhaps a novelty of the study and the results presented herein is the fact that it simultaneously addresses four markets with notable differences. The first two are widely considered two of the most important emerging markets globally. Both India and South Africa are members of the BRICS group of countries that are forecasted to increasingly affect global economic affairs. Hence, given their growth prospects and increasing importance, their stock and bond markets are increasingly attracting the attention of market agents and investment funds. The latter two, i.e. the US and the UK markets, are two of the largest and most important economies worldwide with large and mature bond and stock markets that greatly influence the global economy in

general and capital and money markets in particular. The four markets present a rich database extending back to 1920:08, 1910:01, 1933:01, and 1791:09 for India, South Africa, UK and US respectively. The series time-length offers the opportunity for greater insights into the long-term association between the two markets and hence for more robust inferences. In particular, the Indian, South African, US and UK stock log returns are calculated from the, Bombay Stock Exchange, Johannesburg All Share, FTSE All Share, and S&P500 indices respectively, with returns being computed as the first-differences of the natural logs of these indices. The corresponding bond log returns for the countries are extracted from the 10-year government bond total return indices, with data for stocks and bond prices being recovered from the Global Financial Database. While, the starting dates for each country varies based on data availability, the end point for the four countries is 2017:02. The data has been plotted in [Appendix 2](#) and summarized in [Appendix 3](#) of the paper. The returns are non-normal as indicated by the Jarque–Bera test, thus justifying an analysis of correlation based on extreme dependence.

#### 4.2 Copula quantile-on-quantile regression results

In this section, the findings from estimating the C-QQR model are reported ([Figure 1](#)). The C-QQR is able to model correlation in a flexible way. To evaluate the dependence between bearish markets,  $\theta$  (bond return) and  $\tau$ (stock return) may be set to 0.1, 0.2, and 0.3 as this captures the association between the 10th, 20th and 30th percentiles of bond and stock



**Notes:** This graph depicts the estimates of the slope coefficient,  $\beta_1(\theta, \tau)$ , which is placed on the z-axis against the quantiles of the  $BdR(\theta)$  on the y-axis and the quantiles of  $STR(\tau)$  on the x-axis. The colors in the color bar measure the degree of the co-movement between two assets under investigation ( $BdR$  and  $STR$ ). The red color corresponds to positive and growing values of the slope coefficient, while the dark blue color corresponds to negative and strong values of the slope coefficient. The light blue and the light green colors correspond to weak or insignificant values of the slope coefficient

**Figure 1.**

The correlation between bond return and stock return by country



returns when markets are bear. Pertaining to normal state, one possibility is to set  $\theta$  and  $\tau$  to 0.4, 0.5 and 0.6 and look at the focal relationship across the median returns. For correlation among markets during bull regimes,  $\theta$  and  $\tau$  can be set to 0.7, 0.8 and 0.9.

We, first, find that correlation between stock return and bond return for the four countries under study is asymmetric. It is sometimes positive and sometimes negative depending to whether the markets are functioning around the bear, normal or bull states. Generally speaking, a positive association means that the prices of the two assets move in the same direction. However, a negative correlation indicates that the two assets move in opposite directions in price action. In general, investors and traders seek to include negatively correlated assets to deal with excessive volatility and risks for the overall portfolio. More interestingly, our findings reveal that investors and traders face a series of probability distributions for future returns and are uncertain as to which of these will apply. For India, for example, we find that the correlation is positive between the 80th and 90th stock returns (bull state) and the 50th and 60th bond returns (normal mode), while it is negative between the 70th and 80th bond returns (bullish market) and the 10th and 20th stock returns (bearish market). The dependence seems stronger around the bottom quantile levels (bear regimes). For the case of South Africa, we notice that the dependence between the 20th and 30th stock returns (when investors in stock market are pessimistic) and the 70th and 90th bond returns (when investors in bond market are optimistic) is positive, and negative among the 70th and 80th equity returns (bull state) and the 10th and 20th bonds returns (bear state). Different outcomes are found for UK case; the interdependence between share returns and bond returns seems negative when the bond market is normal (50th) or bull (80th) and the stock market is normal (50th), while it appears positive under bear bond market (40th) and normal stock market (50th). With reference to US case, a negative association is found between BdR whatever its state (bear, normal or bull regime, i.e. lower, middle and higher quantile levels) and STR in bear or normal mode (bottom and middle quantiles), while a positive connection is shown between bond and stock markets in bullish states (upper quantiles).

Table I summarizes the significant (negative and positive) correlations between the quantiles of the  $STR(\tau)$  and the quantiles of  $BdR(\theta)$  to better clarify the C-QQR results reported in Figure 1, and in turn, to provide more accurate information to investors by giving them the opportunity to choose the adequate time to enter a particular market.

#### 4.3 Test of the effectiveness of copula quantile-on-quantile regression

The data generation processes of the asset market returns may be nonlinear, non-stationary and heavy-tailed. In addition, the marginal distributions may be asymmetric, leptokurtic and show conditional heteroscedasticity (Cholette *et al.*, 2009). Some works use the dynamic

**Table I.** Summary of the C-QQR results: the dependence structure between the quantiles of  $STR(\tau)$  and the quantiles of the  $BdR(\theta)$

	India	South Africa	The UK	The USA
Positive correlation	$(\tau = 0.8; \theta = 0.5)$	$(\tau = 0.2; \theta = 0.7)$	$(\tau = 0.5; \theta = 0.5)$	$(\tau = 0.9; \theta = 0.9)$
	$(\tau = 0.8; \theta = 0.6)$	$(\tau = 0.2; \theta = 0.9)$	$(\tau = 0.5; \theta = 0.8)$	
	$(\tau = 0.9; \theta = 0.5)$	$(\tau = 0.3; \theta = 0.7)$		
	$(\tau = 0.9; \theta = 0.6)$	$(\tau = 0.3; \theta = 0.9)$		
Negative correlation	$(\tau = 0.7; \theta = 0.1)$	$(\tau = 0.7; \theta = 0.1)$	$(\tau = 0.5; \theta = 0.4)$	$(\tau = 0.3; \theta = 0.5)$
	$(\tau = 0.7; \theta = 0.2)$	$(\tau = 0.7; \theta = 0.2)$		$(\tau = 0.3; \theta = 0.8)$
	$(\tau = 0.8; \theta = 0.1)$	$(\tau = 0.8; \theta = 0.1)$		$(\tau = 0.6; \theta = 0.3)$
	$(\tau = 0.8; \theta = 0.2)$	$(\tau = 0.8; \theta = 0.2)$		$(\tau = 0.6; \theta = 0.6)$ $(\tau = 0.6; \theta = 0.9)$

conditional correlation (DCC)-GARCH models (Engle, 2002) or Hamiltonian regime-switching models (Ang and Chen, 2002) in modeling the joint dynamics of returns. These papers, nevertheless, fail to detect the asymmetric tail dependence across markets since they assume that innovations follow a normal or student-t distribution, which is a symmetric distribution (Garcia and Tsafak, 2009). The use of copula approaches has been and continues to be a common practice in financial literature when it comes to dealing with asymmetric dependence structure. In this study, the dynamic C-QQR correlation is benchmarked against the correlation estimated from the DCC-GARCH framework of Engle (2002). Despite our awareness about the contrast between the two modeling techniques, this comparison serves to underscore the efficacy of the dynamic C-QQR correlation when examining the asymmetric correlations between stock and bond returns under different bond market states and various stock market circumstances.

Even though the C-QQR and DCC methods are dissimilar (the C-QQR method belongs to a class of quantile regression and the DCC econometric tool belongs to a class of GARCH models), the C-QQR correlation results bear a sharp resemblance to the DCC outcomes (Figure 2). We note also that for the four countries under study, the C-QQR tends to lead the DCC, and this is clearly noticeable by comparing the peaks and troughs.

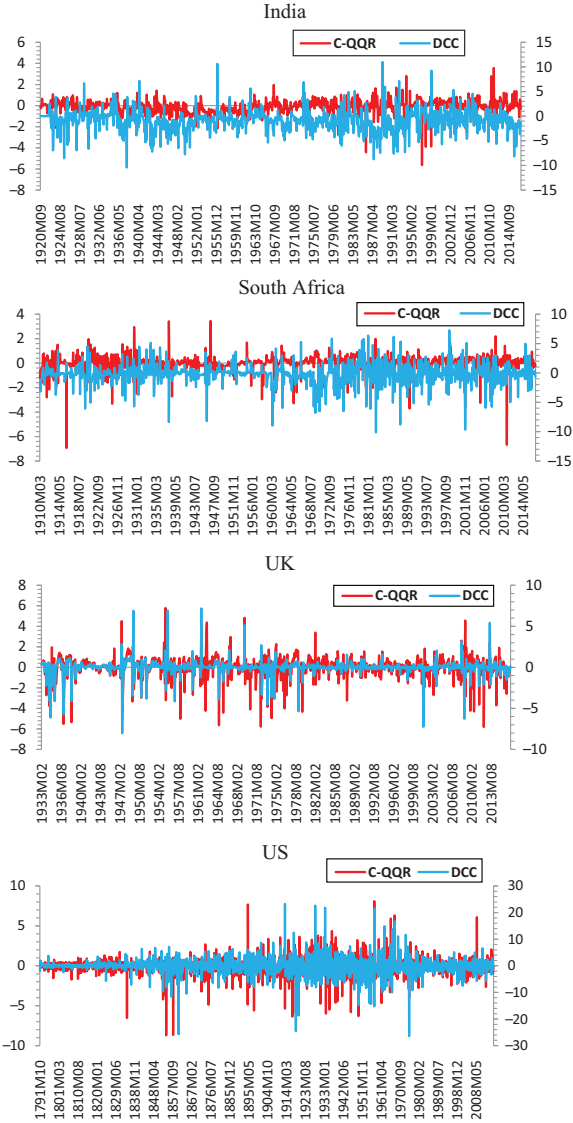
To test the effectiveness of C-QQR, different Granger causality tests involving the C-QQR correlation and the DCC are pursued. First, we use a standard linear Granger causality test. Though many researchers have utilized the linear Granger causality test to investigate the causality between data variables, this test can work effectively only when the “true” causal link is linear, but loses a lot of power if this is no longer the case. To avoid the limitations standard Granger causality test, we use a nonlinear causality test based on a Taylor expansion (Peguin-Feissolle and Terasvirta, 1999), assuming that that  $x_t$  does not cause  $y_t$  if the past values of  $x_t$  does not contain any information about  $y_t$  that is already contained in the past values of  $y_t$  itself.

$$y_t = f^*(y_{t-1}, \dots, y_{t-q}, x_{t-1}, \dots, x_{t-n}, \theta^*) + \varepsilon_t \quad (18)$$

where  $\theta^*$  is a parameter vector and  $\varepsilon_t \sim \text{mid}(0, \sigma^2)$ ; The functional form of  $f^*$  is unknown but we assume that is adequately represents the causal relationship between  $x_t$  and  $y_t$  By imposing, for the two tests, 10 lags on each time series, we show:

- (1) Using the linear Granger causality test, the  $p$ -value of the  $F$ -test for the null hypothesis that “the DCC does not Granger-cause the C-QQR correlation” is 0.5742 for Indian case, 0.3869 for South Africa, 0.4768 for UK and 0.1897 for US. The  $p$ -values for the null of “the C-QQR correlation does not Granger-cause the DCC” are 0.0049 for India, 3.68e-05 for South Africa, 4.198e-06 for UK and 0.0006 for US.
- (2) By carrying out a nonlinear Granger causality test, the null hypothesis that the DCC does not Granger-cause the C-QQR correlation is not rejected for all the countries studied ( $p$ -values are equal to 0.3425, 0.4976, 0.3855 and 0.5188 for India, South Africa, UK and US, respectively), while there exists an evidence that the C-QQR correlation significantly Granger-causes the DCC for India ( $p$ -value = 5.124e-05), South Africa ( $p$ -value = 4.193e-05), UK ( $p$ -value = 7.028e-06) and US ( $p$ -value = 5.631e-05), emphasizing the tendency of the C-QQR correlation to lead the DCC.

In short, our results displayed in Figure 2 indicate that the upturn and the downturn are likely to be shown earlier from the C-QQR correlation. This highlights the efficacy of the C-QQR method to capture the salient features in the correlation dynamics. For portfolio



**Note:** The C-QQR and DCC correlations are plotted for the bond return and stock return for the cases of India, South Africa, UK and US. The red line is associated with the 60-month moving average C-QQR correlation. The blue line is associated with the DCC, which is based on Engle's (2002) Dynamic Conditional Correlation model assuming that the returns series follow a GARCH(1,1) process

**Figure 2.** C-QQR vs DCC

managers, the peaks and troughs in correlation warrant a more defensive investment strategy.

## 5. Concluding remarks

Stocks and bonds are the two major classes of assets traded in national and international capital markets. Due to their different risk-return characteristics, they constitute the foundations for the vast majority of investment portfolios. Not surprisingly, therefore, a steadily expanding number of studies have addressed the stock-bond nexus focusing on its determinants and its dynamics. As invariably stressed in many studies, the association between the two markets is important not only for portfolio managers but also for policymakers and monetary policy decision-making institutions in particular. Within this body of literature, this paper set out to investigate the correlation between the two markets in the cases of four different countries: India, South Africa, USA and the UK. The former are two of the most important and bigger emerging markets while the latter two countries are two of the biggest mature markets with the largest capitalization globally. To probe into the issue at hand, we used a Copula quantile-on-quantile regression (C-QQR) to assess how the quantile of bond return react to the quantile of stock return.

As one would intuitively expect, the results were not uniform. Indeed, they point to significant heterogeneity in the bond-stock returns correlation in the four countries examined herein. Perhaps a significant finding that was chiseled out of the estimations was that the use of C-QQR in portfolio management can help the construction of tailored response strategies that are adapted to the needs of particular investors and traders. Nowadays, asset markets are more complex, fast-moving and unpredictable than ever before; nevertheless, a market participant' objectives are still generally the same: the agents want portfolios that will rise in value and lessen the risk of losses. When thinking about portfolio diversification, investors instinctively focus on the sign and the strength of correlation. The results reported herein offer fresh insights on when the association between bond return and stock return is negative, and when it is positive (also when it is weak and when it is strong), conditional on the different bond and stock market conditions (i.e. bear, normal or bull regimes). Specifically, the C-QQR approach employed herein indicates that:

- For India, a positive (negative) correlation between bond returns and stock returns under bull bond market state and bull (bear) stock market.
- For South Africa, a positive (negative) correlation under bear (bull) stock market and bull (bear) bond market conditions.
- For UK, a positive (negative) dependence when the bond market is normal or bullish (bearish) and the stock market is mildly normal or bearish (normal or bullish).
- For US, a positive (negative) correlation under bull (normal) bond market scenario and bull (bear or normal) stock market regime.

These findings suggest that it is favorable for investors in India and South Africa with a long position in the bull bond market and a short position in the bear stock market to obtain extreme profits. The results also indicate that an investor or a trader with a long position in the stock market, and a short position in the bond market faces the minimum systemic risk. However, it is more likely for market participants in US with long positions in bullish bond and stock markets circumstances. In a nutshell the slope coefficients derived through the C-QQR can be viewed as true systemic risks derived from various extreme-return states which seem very prominent for efficaciously determining the value-at-risk, and may in turn have relevant implications for risk management and international asset pricing. Additionally,

these findings may be useful for portfolio construction and diversification, as keeping up with markets' behavior give investors the opportunity to choose the time to enter a particular market.

## Notes

1. Even though the quantile regression focuses on modeling the conditional quantile of the endogenous variable, the Copula-QQR approach captures the dependence between the distributions of bond and stock returns. Thus, it uncovers two nuance features in the bond return and stock return dependence.
2. Akaike (AIC) and the Bayes (BIC) information criteria and the log likelihood (LL) are used to assess the performance of the copula models summarized in [Appendix 1](#). These results are available upon request.

## References

- Aloui, C., Hammoudeh, S. and Hamida, H.B. (2015), "Co-movement between sharia stocks and sukuk in the GCC markets: a time-frequency analysis", *Journal of International Financial Markets, Institutions and Money*, Vol. 34, pp. 69-79.
- Andersson, M., Krylova, E. and Vahamaa, S. (2008), "Why does the correlation between stock and bond returns vary over time?", *Applied Financial Economics*, Vol. 18 No. 2, pp. 139-151.
- Ang, A. and Chen, J. (2002), "Asymmetric correlations of equity portfolios", *Journal of Financial Economics*, Vol. 63 No. 3, pp. 443-494.
- Apergis, N., Bonato, M., Gupta, R. and Kyei, C. (2017), "Does geopolitical risks predict stock returns and volatility of leading defense companies? Evidence from a nonparametric approach", *Defence and Peace Economics*, doi: [org/10.1080/10242694.2017.1292097](https://doi.org/10.1080/10242694.2017.1292097).
- Asgharian, H., Christiansen, C. and Hou, A.J. (2015), "Effects of macroeconomic uncertainty on the stock and bond markets", *Finance Research Letters*, Vol. 13, pp. 10-16.
- Asgharian, H., Christiansen, C. and Hou, A.J. (2016), "Macro-finance determinants of the long-run stock-bond correlation: the DCC-MIDAS specification", *Journal of Financial Econometrics*, Vol. 14 No. 3, pp. 617-642.
- Aslam, F. and Kang, H.-G. (2015), "How different terrorist attacks affect stock markets", *Defence and Peace Economics*, Vol. 26 No. 6, pp. 634-648.
- Aslanidis, N. and Christiansen, C. (2014), "Quantiles of the realized stock-bond correlation and links to the macroeconomy", *Journal of Empirical Finance*, Vol. 28, pp. 321-331.
- Aslanidis, N. and Christiansen, C. (2012), "Smooth transition patterns in the realized stock-bond correlation", *Journal of Empirical Finance*, Vol. 19 No. 4, pp. 454-464.
- Baele, L., Bekaert, G. and Inghelbrecht, K. (2010), "The determinants of stock and bond return comovements", *Review of Financial Studies*, Vol. 23 No. 6, pp. 2374-2428.
- Baur, D. and Lucey, B. (2009), "Flights and contagion. An empirical analysis of stock-bond correlations", *Journal of Financial Stability*, Vol. 5 No. 4, pp. 339-352.
- Bayraci, S., Demiralay, S. and Gencer, H.G. (2017), "Stock-bond co-movements and flight-to-quality in G7 countries: a time-frequency analysis", *Bulletin of Economic Research*, doi: [10.1111/boer.12118](https://doi.org/10.1111/boer.12118).
- Bouoiyour, J. and Selmi, R. (2017), "The dependence between US and China stock return quantiles: new insights via quantile-on-quantile approach", Working paper, CATT, University of Pau.
- Bouyè, E. and Salmon, M. (2009), "Dynamic copula quantile regressions and tail area dynamic dependence in forex markets", *European Journal of Finance*, Vol. 15 Nos 7/8, pp. 721-750.

- Brune, A., Hens, T., Rieger, O. and Wang, M. (2015), "The war puzzle: contradictory effects of international conflicts on stock markets", *International Review of Economics*, Vol. 62 No. 1, pp. 1-21.
- Campbell, J. and Ammer, J. (1993), "What moves the stock and bond markets? A variance decomposition for long-term asset returns", *Journal of Finance*, Vol. 48 No. 1, pp. 3-37.
- Cappiello, L., Engle, R. and Sheppard, K. (2006), "Asymmetric dynamics in the correlations of global equity and bond returns", *Journal of Financial Econometrics*, Vol. 4 No. 4, pp. 537-572.
- Chang, C.L. and Hsueh, P.L. (2013), "An investigation of the flight-to-quality effect: evidence from Asia-Pacific countries", *Emerging Markets Finance and Trade*, Vol. 49 No. 4, pp. 53-69.
- Christiansen, C. (2010), "Decomposing European bond and equity volatility", *International Journal of Finance and Economics*, Vol. 15 No. 2, pp. 105-122.
- Chui, C.M. and Yang, J. (2012), "Extreme correlation of stock and bond futures markets: international evidence" *The Financial Review*, Vol. 47 No. 3, pp. 565-587.
- Connolly, R., Stivers, C. and Sun, L. (2005), "Stock market uncertainty and the stock-bond return relation", *Journal of Financial and Quantitative Analysis*, Vol. 40 No. 1, pp. 161-194.
- Connolly, R., Stivers, C. and Sun, L. (2007), "Commonality in the time-variation of stock-stock and stock-bond return comovements", *Journal of Financial Markets*, Vol. 40, pp. 192-218.
- Dajcman, S. (2012), "Comovement between stock and bond markets and the flight-to-quality during financial market turmoil – a case of the Eurozone countries most affected by sovereign debt crisis of 2010–2011", *Applied Economics Letters*, Vol. 19, pp. 1655-1662.
- Dajcman, S. (2015), "An empirical investigation of the nexus between sovereign bond yields and stock market returns – a multiscale approach", *Engineering Economics*, Vol. 26, pp. 108-117.
- Dimic, N., Kiviahho, J., Pijak, V. and Äijö, J. (2016), "Impact of financial market uncertainty and macroeconomic factors on stock-bond correlation in emerging markets", *Research in International Business and Finance*, Vol. 36, pp. 41-51.
- Dunis, C., Sermpinis, G. and Karampelias, M.F. (2013), "Stock market linkages among new EMU members and the euro area: implications for financial integration and portfolio diversification", *Studies in Economics and Finance*, Vol. 30 No. 4, pp. 370-388.
- Durand, R.B., Junker, M. and Szimayer, A. (2010), "The flight-to-quality effect: a copula-based analysis", *Accounting and Finance*, Vol. 50 No. 2, pp. 281-299.
- Engle, R.F. (2002), "Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models", *Journal of Business and Economic Statistics*, Vol. 20 No. 3, pp. 339-350.
- Ferrer, R., Bolos, V.J. and Benitez, R. (2016), "Interest rate changes and stock returns: a European multi-country study with wavelets", *International Review of Economics and Finance*, Vol. 44, pp. 1-12.
- Garcia, R. and Tsafak, G. (2009), *Dependence Structure and Extreme Comovements in International Equity and Bond Markets*, CIRANO Scientific series, Montreal, Quebec.
- Geoffrey, L. (2017), "The impact of global financial market uncertainty on the risk-return relation in the stock markets of G7 countries", *Studies in Economics and Finance*, Vol. 34 No. 1, pp. 2-23.
- Guidolin, M. and La Ferrara, E. (2010), "The economic effects of violent conflict: evidence from asset market reactions", *Journal of Peace Research*, Vol. 47 No. 6, pp. 671-684.
- Guidolin, M. and Timmermann, A. (2005), "Economic implications of bull and bear regimes in UK stock and bond returns. Economic", *Journal*, Vol. 115, pp. 111-141.
- Gulko, L. (2002), "Decoupling", *The Journal of Portfolio Management*, Vol. 28 No. 3, pp. 59-66.
- Gupta, R., Kollias, C., Papadamou, S. and Wohar, M. (2017), "News implied volatility and the stock-bond nexus: evidence from historical data for the USA and the UK markets", WP No. 201730, University of Pretoria.

- Harvey, A.C. (2010), "Tracking a changing copula", *Journal of Empirical Finance*, Vol. 17 No. 3, pp. 485-500.
- Heffernan, J.E. and Tawn, J.A. (2004), "A conditional approach for multivariate extreme values", *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, Vol. 66 No. 3, pp. 497-546.
- Ilmanen, A. (2003), "Stock-bond correlations", *The Journal of Fixed Income*, Vol. 13 No. 2, pp. 55-66.
- Kaplanski, G. and Levy, H. (2010), "Sentiment and stock prices: the case of aviation disasters", *Journal of Financial Economics*, Vol. 95 No. 2, pp. 174-201.
- Kasa, K. (1992), "Common stochastic trends in international stock markets", *Journal of Monetary Economics*, Vol. 29 No. 1, pp. 95-124.
- Keim, D.B. and Stambaugh, R.F. (1986), "Predicting returns in the stock and bond markets", *Journal of Financial Economics*, Vol. 17 No. 2, pp. 357-390.
- Kim, S. and In, F. (2005), "The relationship between stock returns and inflation: new evidence from wavelet analysis", *Journal of Empirical Finance*, Vol. 12 No. 3, pp. 435-444.
- Kim, S., Moshirian, F. and Wu, E. (2006), "Evolution of international stock and bond market integration: Influence of the European monetary union", *Journal of Banking and Finance*, Vol. 30 No. 5, pp. 1507-1534.
- Koenker, R. (2005), *Quantile Regressions*, Cambridge University Press, Cambridge.
- Koenker, R. and Bassett, G. (1978), "Regression quantiles", *Econometrica*, Vol. 46 No. 1, pp. 33-50.
- Kollias, C., Papadamou, S. and Arvanitis, V. (2013), "Does terrorism affect the stock-bond covariance? Evidence from European countries", *Southern Economic Journal*, Vol. 79 No. 4, pp. 832-848.
- Levy, H. and Sarnat, M. (1970), "International diversification of investment portfolios", *American Economic Review*, Vol. 60 No. 4, pp. 668-675.
- Li, L. (2004), "Macroeconomic factors and the correlation of stock and bond returns", *Proceeding of the American Finance Association Meeting*, Vol. 2004.
- Li, X. and Zou, L. (2008), "How do policy and information shocks impact co-movements of China's T-bond and stock markets?", *Journal of Banking and Finance*, Vol. 32 No. 3, pp. 347-359.
- Manela, A. and Moreira, A. (2017), "News implied volatility and disaster concerns", *Journal of Financial Economics*, Vol. 123 No. 1, pp. 137-162.
- Patton, A.J. (2006), "Modelling asymmetric exchange rate dependence", *International Economic Review*, Vol. 47 No. 2, pp. 527-555.
- Peguin-Feissolle, A. and Terasvirta, T. (1999), "A general framework for testing the Granger non-causality hypothesis", Stockholm School of Economics Working Paper Series in Economics and Finance n° 343.
- Phylaktis, K. (1999), "Capital market integration in the Pacific Basin region: an impulse response analysis", *Journal of International Money and Finance*, Vol. 18 No. 2, pp. 267-287.
- Poon, S.H., Rockinger, M. and Tawn, J. (2004), "Extreme value dependence in financial markets: diagnostics, models and financial implications", *Review of Financial Studies*, Vol. 17 No. 2, pp. 581-610.
- Schneider, G. and Troeger, V. (2006), "War and the world economy, stock market reactions to international conflicts", *Journal of Conflict Resolution*, Vol. 50 No. 5, pp. 623-645.
- Schwert, G.W. (1989), "Why does stock market volatility change over time?", *The Journal of Finance*, Vol. 44 No. 5, pp. 1115-1153.
- Shiller, R.J. and Beltratti, A.E. (1992), "Stock prices and bond yields: can their comovements be explained in terms of present value models?", *Journal of Monetary Economics*, Vol. 30 No. 1, pp. 25-46.
- Sim, N. (2015), "The dependence of quantile returns: a new approach to modeling correlations", Working paper, University of Adelaide.
- Skintzi, V. (2017), "Determinants of stock-bond market comovement in the Eurozone under model uncertainty", MPRA Paper No. 78278.

- Sklar, A. (1959), *Fonctions de Répartition à n Dimensions et Leurs Marges*, Publications de l'Institut Statistique de l'Université de Paris, Paris, France, 8, 229-231.
- Sukcharoen, K. and Leatham, D. (2016), "Dependence and extreme correlation among US industry sectors", *Studies in Economics and Finance*, Vol. 33 No. 1, pp. 26-49.
- Tang, J.C., Sriboonchitta, S., Ramos, V. and Wong, W.K. (2014), "Modelling dependence between tourism demand and exchange rate using copula-based GARCH model", *Current Issues in Method and Practice*, Vol. 19 No. 9, pp. 1-19.
- Thomadakis, A. (2012), *Contagion or Flight-to-Quality Phenomena in Stock and Bond Returns DP 06/12 Department of Economics*, University of Surrey, Guildford, Surrey.
- Wisniewski, T.P. (2016), "Is there a link between politics and stock returns? A literature survey", *International Review of Financial Analysis*, Vol. 47, pp. 15-23.
- Wu, W., Lau, M.C.K. and Vigne, S.A. (2017), "Modelling asymmetric conditional dependence between Shanghai and Hong Kong stock markets", *Research in International Business and Finance*, Vol. 42, pp. 1137-1149.
- Yang, J., Zhou, Y. and Wang, Z. (2009), "The stock-bond correlation and macroeconomic conditions: one and a half centuries of evidence", *Journal of Banking and Finance*, Vol. 33 No. 4, pp. 670-680.
- Yang, J., Zhou, Y. and Wang, Z. (2010), "Conditional co-skewness in stock and bond markets: time series evidence", *Management Science*, Vol. 56 No. 11, pp. 2031-2049.

### Further reading

- Engle, R.F. (2009), *Anticipating Correlations: A New Paradigm for Risk Management*, Princeton University Press, Princeton, NJ.
- Koenker, R. and Xiao, Z. (2002), "Inference on the quantile regression process", *Econometrica*, Vol. 81, pp. 1583-1612.
- Koenker, R. and Xiao, Z. (2006), "Quantile autoregression", *Journal of the American Statistical Association*, Vol. 101 No. 475, pp. 980-990.
- Ohmi, H. and Okimoto, T. (2016), "Trends in stock-bond correlations", *Applied Economics*, Vol. 48 No. 6, pp. 536-552.
- Shik Lee, H. (2004), "International transmission of stock market movements: a wavelet analysis", *Applied Economics Letters*, Vol. 11 No. 3, pp. 197-201.

### Appendix 1. An overview on the different copula functions

This paper attempts to assess, from a new perspective, the dependence structure between the stock returns ( $X_1$ ) and the bonds returns ( $X_2$ ). For this purpose, a copula is used to assess the bivariate joint distribution function,  $F_{X_1X_2}(x_1, x_2)$ . For the two random variables, based on Sklar's theorem (Sklar, 1959), the joint distribution of two random variables can be defined in terms of a copula after transforming marginal distributions into uniform distributions. In this way, the joint distribution of the two variables can be depicted through a copula function  $C$  denoted as:

$$F_{X_1X_2}(x_1, x_2) = C(u, v), \quad (a1)$$

where  $u = F_{X_1}(x_1)$  and  $v = F_{X_2}(x_2)$ .

The joint probability density of the variables  $X_1$  and  $X_2$  is then derived via the copula density,  $(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v}$ , expressed as following:



$$f_{X_1 X_2}(x_1, x_2) = c(u, v) f_{X_1}(x_1) f_{X_2}(x_2), \quad (\text{a2})$$

where  $f_{X_1}(x_1)$  and  $f_{X_2}(x_2)$  are, respectively, the marginal densities of the random variables  $X_1$  and  $X_2$ .

Patton (2006) by extending the Skalar's theorem demonstrates the lower (left) and upper (right) tail dependence of two random variables for the copula to be as follows:

$$\tau^L = \lim_{u \rightarrow 0} \Pr(F_{X_1}(x_1) \leq u \mid F_{X_2}(x_1) \leq u) = \lim_{u \rightarrow 0} \frac{C(u, u)}{u} \quad (\text{a3})$$

$$\tau^U = \lim_{\alpha \rightarrow 1} \Pr(F_{X_1}(x_1) > u \mid F_{X_2}(x_1) > u) = \lim_{u \rightarrow 1} \frac{1 - 2u - C(u, u)}{1 - u}, \quad (\text{a4})$$

where  $\tau^L$  and  $\tau^U \in [0, 1]$ .

Throughout the study, we consider different copula functions with symmetric and asymmetric tail behavior, even if we control for time-varying dependence. We first consider elliptical Gaussian and Student-t copulas which are usual choices for the market dependence structure. These functions are denoted as:

$$C^{Gaussian}(u_t, v_t; \rho) = \Phi\left(\Phi^{-1}(u_t)\Phi^{-1}(v_t)\right) \quad (\text{a5})$$

$$C^{Student-t}(u_t, v_t; \rho, v) = T_v\left(t_v^{-1}(u_t), t_v^{-1}(v_t)\right) \quad (\text{a6})$$

where  $\Phi$  is the bivariate standard normal CDF with correlation  $\rho$  ( $-1 < \rho < 1$ ),  $\Phi^{-1}(u_t)$  and  $\Phi^{-1}(v_t)$  are standard normal quantile functions,  $T$  is the bivariate Student-t CDF with the degree-of-freedom parameter  $v$  and the correlation  $\rho$  ( $-1 < \rho < 1$ ) and  $t_v^{-1}(u_t)$  are the quantile functions of the univariate Student-t distributions. Both copulas display symmetric dependence, even though the Gaussian has zero tail dependence and the Student-t displays tail dependence given by:  $\lambda^U = \lambda^L = 2t_{v+1}\left(-\sqrt{v+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right) > 0$ .

The study also accounts for copulas with symmetric tail dependence, namely, the Plackett and the Frank copulas, expressed, respectively, as:

$$C^{Plackett}(u_t, v_t; \pi) = \frac{1}{2(\pi - 1)}$$

$$\left(1 + (\pi - 1)(u_t + v_t) - \sqrt{(1 + (\pi - 1)(u_t + v_t))^2 - 4\pi(\pi - 1)u_t v_t}\right) \quad (\text{a7})$$

$$C^{Frank}(u_t, v_t; \lambda) = \frac{-1}{\lambda} \log \frac{(1 - e^{-\lambda}) - (1 - e^{-\lambda u_t})(1 - e^{-\lambda v_t})}{(1 - e^{-\lambda})} \quad (\text{a8})$$

where  $\pi \in [0, \infty] \setminus \{1\}$  and  $\lambda \in (-\infty, \infty) \setminus \{0\}$ . It must be pointed out at this stage that the two above copulas exhibit tail independence.

Given the fact that the dependence between the variables of interest may behave dissimilarly over diverse market conditions (bear, normal or bull regime), this study implements copula functions with asymmetric tail dependence structures. The Gumbel copula reflects upper tail dependence, while its rotation reflects lower tail dependence, expressed, respectively, as:

$$\mathcal{C}^{Gumbel}(u_t, v_t; \delta) = \exp\left(-\left((-\log u_t)^\delta + (-\log v_t)^\delta\right)^{1/\delta}\right) \quad (\text{a9})$$

$$\mathcal{C}^{Rotated\_Gumbel}(u_t, v_t; \delta) = u_t + v_t - 1 + \mathcal{C}^{Gumbel}(1 - u_t, 1 - v_t; \delta) \quad (\text{a10})$$

where  $\delta \in (1, \infty)$ . The upper and lower tail dependence structures of the Gumbel copula are  $\lambda_U = 2 - 2^{1/\delta}$  and  $\lambda_L = 0$ , respectively, while the opposite holds for the rotated Gumbel.

Moreover, we utilize the symmetrized Joe-Clayton (SJC) copula since it takes into account both lower and upper tail dependence, accounting for possible asymmetry.

$$\begin{aligned} \mathcal{C}^{SJC}(u_t, v_t; \lambda_U^{SJC}, \lambda_L^{SJC}) &= 0.5\left(\mathcal{C}^{JC}(u_t, v_t; \lambda_U^{JC}, \lambda_L^{JC})\right) \\ &+ \mathcal{C}^{JC}(1 - u_t, 1 - v_t; \lambda_U^{JC}, \lambda_L^{JC}) + u_t + v_t - 1 \end{aligned} \quad (\text{a11})$$

where

$$\begin{aligned} \mathcal{C}^{JC}(u_t, v_t; \lambda_U^{JC}, \lambda_L^{JC}) &= 1 - \left(1 - \left\{[1 - (1 - u_t)^\kappa]^{-\gamma}\right.\right. \\ &\quad \left.\left.+ [1 - (1 - v_t)^\kappa]^{-\gamma} - 1\right\}^{-1/\gamma}\right)^{1/\kappa}, \\ \kappa &= 1/\log_2(2 - \lambda_U^{JC}) \text{ and } \gamma = -1/\log_2(\lambda_L^{JC}). \end{aligned}$$

For this copula function, the tail dependence coefficients are themselves the parameters of the copula. So, if  $\lambda_U^{SJC} = \lambda_L^{SJC}$ , then the market structure is symmetric, otherwise it is asymmetric.

Then, we account for possible time-varying dependence structure. For the Gaussian and Student-t copulas, we are able to describe the dynamics of the linear dependence parameter as evolving flexibly over time based on a new econometric tool proposed by [Patton \(2006\)](#), expressed as follows:

$$\rho_t = \Lambda\left(\Psi_0 + \Psi_1\rho_{t-1} + \Psi_2\frac{1}{10}\sum_{j=1}^{10}\Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})\right) \quad (\text{a.12})$$

where  $\Lambda$  denotes the logistic transformation  $\Lambda(x) = (1 - e^{-x})(1 + e^{-x})^{-1}$  that is employed to keep  $\rho_t$  within  $(-1, 1)$ . For the Student-t copula,  $\Phi^{-1}(x)$  is substituted by  $t_{\nu}^{-1}(x)$ . For the conditional Gumbel copula and its rotation, the evolution of  $\delta$  is specified based on ARMA (1,10) process, that can be denoted as:

$$\delta_t = \Psi_0 + \Psi_1 \delta_{t-1} + \Psi_2 \frac{1}{10} \sum_{j=1}^{10} \left| u_{t-j} - v_{t-j} \right| \quad (\text{a13})$$

For the SJC copula, the variation of upper and lower tail dependencies over time can be given by an ARMA(1,10) process as follows:

$$\begin{aligned} \tau_{U,t}^{SJC} &= \Lambda \left( \Psi_0^U + \Psi_1^U \tau_{U,t-1}^{SJC} + \Psi_2^U \frac{1}{10} \sum_{j=1}^{10} \left| u_{t-j} - v_{t-j} \right| \right) \\ \tau_{L,t}^{SJC} &= \Lambda \left( \Psi_0^L + \Psi_1^L \tau_{L,t-1}^{SJC} + \Psi_2^L \frac{1}{10} \sum_{j=1}^{10} \left| u_{t-j} - v_{t-j} \right| \right) \end{aligned} \quad (\text{a14})$$

## Appendix 2

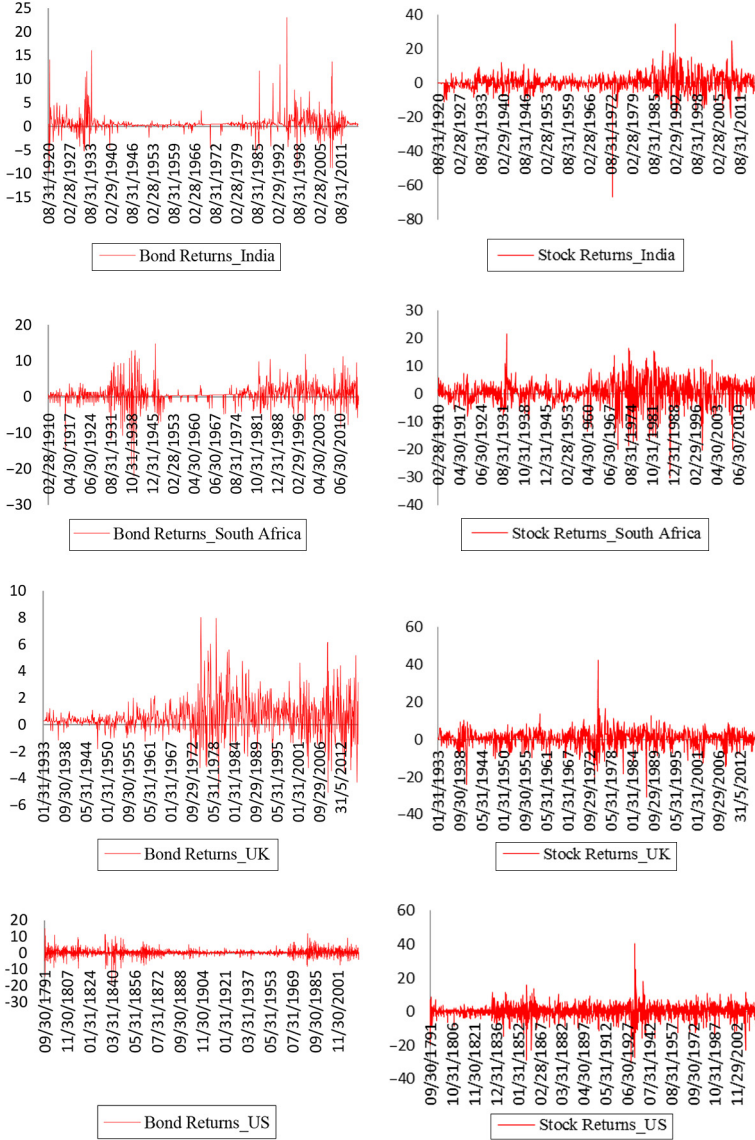


Figure A1. C-QQR vs DCC

### Appendix 3

**Table AI.** Summary statistics

Statistic	Returns							
	India		South Africa		The UK		The USA	
	Bond	Stock	Bond	Stock	Bond	Stock	Bond	Stock
Mean	0.5169	0.4192	0.6247	0.6143	0.5585	0.5095	0.4067	0.2498
Median	0.4158	0.2061	0.4365	0.7082	0.3844	0.9327	0.3869	0.2567
Maximum	23.0191	34.6992	14.7838	21.6409	8.0191	42.3197	14.9968	40.7459
Minimum	-9.8806	-66.8341	-22.0266	-30.2285	-5.1090	-30.9241	-18.7497	-30.7528
SD	1.8842	5.1751	2.6496	4.4871	1.3468	4.8357	2.0099	3.8396
Skewness	3.0893	-1.5102	-0.6234	-0.6896	0.8048	-0.1614	-0.3775	-0.5859
Kurtosis	36.1139	30.1890	14.3993	7.3702	7.3820	11.8057	14.9635	14.7762
Jarque-Bera	54796.67	36139.77	7040.676	1124.419	917.1279	3267.503	16201.56	15790.98
<i>p</i> -value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	1159	1159	1285	1285	1010	1010	2706	2706

**Notes:** SD symbolizes the standard deviation; *p*-value corresponds to the null of normality based on the Jarque-Bera test