On the Predictability of Stock Market Bubbles: Evidence from LPPLS Confidence^{TM} Multi-scale Indicators

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Abstract

We examine the predictive power of market-based indicators over the positive and negative stock market bubbles via an application of the LPPLS Confidence^{TM} Multi-scale Indicators to the $S\&P500$ index. We find that the LPPLS framework is able to successfully capture, ex-ante, some of the prominent bubbles across different time scales, such as the Black Monday, Dot-com, and Subprime Crisis periods. We then show that measures of short selling activity have robust predictive power over negative bubbles across both short and long time horizons, in line with the previous studies suggesting that short sellers have predictive ability over stock price crash risks. Market liquidity, on the other hand, is found to have robust predictive power over both the negative and positive bubbles, while its predictive power is largely limited to short horizons. Short selling and liquidity are thus identified as two important factors contributing to the LPPLS-based bubble indicators. The evidence overall points to the predictability of stock market bubbles using market-based proxies of trading activity and can be used as a guideline to model and monitor the occurrence of bubble conditions in financial markets.

JEL classification: C13, C58, G14
Keywords: Financial bubble indicators, LPPL method, Markov switching, Predictability, Short interest

1. Introduction

Bubble formation in financial markets has always been a topic of great interest, not only from an academic perspective regarding the informational efficiency of markets, but also for practitioners and policy makers who try to mitigate the negative effects of wild price fluctuations and subsequent crashes. Clearly, large sums are at stake when one deals with stock market bubbles. The U.S. market capitalization of firms, as of June 2017, is nearly $25.3 trillion USD (Bloomberg) or 133% of GDP, while the S&P 500 Total Market Capitalization was $21.3 trillion USD on March 31, 2017. Correspondingly, at the end of 2016, $15.9 trillion in corporate equities were held by households and non-profit organizations in the U.S. (Balance Sheet of Households and Nonprofit Organizations (B.101) in Financial Accounts of the United States.). Given the level of penetration of the stock market investment in the U.S., a future crash in the stock market is likely to have widespread negative effects on the U.S. economy (Farmer, 2015; Narayan et al., 2016), as it did in the aftermath of the dotcom bubble in 2000 or of the financial crisis of 2008. Therefore, it is not surprising that a large literature has been devoted to detecting bubbles in the U.S. stock market, especially since the 2008 financial crisis (see for example, Homm and Breitung, 2010; Yan et al., 2012; Brunnermeier and Oehmke 2013; Xiong, 2013; and Balcilar et al., 2016 for detailed reviews in this regard). While significant effort has been spent on explaining how and why bubbles emerge and sustain over long periods (e.g. Kaizoji and Sornette, 2010), a large number of studies have instead focused on developing models to reliably detect bubbles. Consequently, the literature provides various methodologies to detect bubbles that aim to build on the drawback of the existing (earlier) ones (see for example, Brooks and Katsaris, 2005; Anderson et al., 2010; Phillips et al., 2011, 2015; Phillips and Yu, 2011; Shi, 2013; Arora and Shi, 2016 for detailed discussions in this regard).

Against this backdrop and given that the $S\&P500$ is one of the most frequently used stock market indexes to gauge the U.S. stock market performance (Shiller, 2014), representing more than 80% of available market capitalization, the objective of this paper is twofold. First, we present a methodology to detect both positive and negative bubbles [2] for the $S\&P500$ index using the Log-Periodic Power Law Singularity (LPPLS) model (Johansen et al. 1999; 2000, Sornette 2003), not otherwise possible based on the various bubble detection models cited earlier. We then introduce the Multi-scale LPPLS confidence^{TM} indicators to characterise bubbles at different time scales. Second, having constructed indicators that describe positive and negative bubbles, we examine the predictive power of short selling activity and market liquidity over the bubble indicators and provide insight to the predictability of market booms and crashes using market-based indicators. In our predictive tests, we specifically focus on measures of short selling activity and market liquidity as recent studies suggest that short sellers are informed traders who are able to anticipate future aggregate cash flows and that short interest is positively related to stock price crash risk (e.g. Callen and Fang, 2015 and Rapach et al., 2016). We are also interested in the predictive ability of market liquidity over boom and crash indicators as the literature establishes a link between liquidity spikes and market downturns (e.g. Chordia et al., 2001; Pastor and Stambaugh, 2003). To the best of our knowledge, this is the first attempt in examining the predictive ability of market-based indicators over the positive and negative bubbles in the $S\&P500$ index using the LPPLS model. An important finding of this paper is thus to identify some of the factors contributing to the LPPLS-based bubble indicators, namely short selling and liquidity. The LPPLS model, which makes it possible to identify positive and negative bubbles, presents a valuable opening, allowing us to examine the predictability patterns of market booms and crashes separately.

Our findings show that the LPPLS framework, in the implementation presented here, is able to successfully capture, ex-ante, some of the prominent bubbles across different time scales. We observe that some of the great bubbles and subsequent crashes experienced during the Black Monday, Dot-com, and Subprime Crisis periods are successfully captured by the bubble indicators, while the long-term negative bubble indicator diagnoses correctly the transition from a sluggish market to a fast accelerating positive bubble during the mid-90s when the demand for the "New Economy" stocks developed in full force. One can also observe that the medium-term negative bubble indicator also shows a strong signal that the 2008 crisis was ending, providing a precursor to the strong rebound that started in March 2009. Examining the predictability of the negative and positive bubble indicators, our predictive tests reveal several interesting observations. First, we observe that the predictability patterns differ significantly for booms and busts represented by the positive

[2] A positive (resp. negative) bubble is defined as an upward (resp. downward) accelerating price followed by a crash (resp. rally).

and negative bubble indicators, respectively. We find that measures of short selling activity have robust predictive power over negative bubbles, in line with the view that short sellers are able to detect bad news hoarding by managers. The predictive power of short selling activity is robust to alternative measures of short selling employed in our predictive tests and is positively related to the negative bubble indicator, predicting the occurrence of negative bubbles one-month ahead. The predictive power of short selling proxies holds for both the short- and the long-term horizons which is consistent with the recent finding by Callen and Fang (2015) that short interest is positively related to one-year ahead stock price crash risk. On the other hand, our tests show that market liquidity also has robust predictive power over both the negative and positive bubbles in the short-term, suggesting that market liquidity measures can be used to predict the occurrence of both booms and crashes for short horizons. In short, the evidence points to the predictability of both positive and negative stock market bubbles via market-based proxies of trading activity.

The remainder of the paper is organized as follows. Section 2 presents the theory behind the LPPLS model. Section 3 explains the methodology to construct the LPPLS Confidence^{TM} indicator, its application to the $S\&P500$ Index and the Markov Switching (MS) model employed in our predictive tests. Section 4 presents the empirical findings and Section 5 concludes the paper.

2. Construction of the LPPLS Confidence^{TM} Multi-scale Indicators

2.1. The Log-Periodic Power Law Singularity (LPPLS) Model

This section covers the theory behind the LPPLS framework and discusses the application of the method to the $S\&P500$ Index. Following Sornette et al. (1996), a growing body of studies (see for example, Geraskin and Fantazzini, 2011, Sornette et al., 2013 and Zhang et al., 2016 for detailed reviews of this literature) has used critical points with log-periodic corrections, borrowed from statistical physics, to identify bubbles. This methodology (later started to be called Johansen-Ledoit-Sornette (JLS) model or LPPLS analysis) proposes that a bubble can emerge intrinsically out of the natural functioning of the market. Building on the idea of positive feedbacks by imitation, the JLS model proposed by Johansen et al. (1999; 2000) and its extensions includes as a key ingredient the role played by herding behaviour in the formation of bubbles. Considering bubbles as transient phenomena, the existence of positive feedbacks between value investors and noise traders create a super-exponential growth of the price, decorated by deviations around the price growth in the form of oscillations that are approximately periodic in the logarithm of the time to the burst of the bubble and which capture a progressive time contraction of the long-term volatility structure. When imitation reaches a certain threshold, the higher demand for the asset leads to the observable price to increase, bootstrapping on itself, and the market is governed by sentiment rather than some real underlying value (Sornette and Cauwels, 2015). This process is intrinsically unsustainable and the mispricing ends at a critical time, t_c, either smoothly into another regime or abruptly (crash). In short, in this set-up, it is assumed that agents are fully aware of the mispricing but the price continues to rise due to a lack of synchronisation of the arbitragers either due to disagreements about the time of the beginning (Abreu and Brunnermeier, 2003) or of the end (Demos and Sornette, 2017) of the bubble.

In a bubble regime, the observed price trajectory of a given asset decouples from its intrinsic fundamental value (Kindleberger, 1978; Sornette, 2003). For a given fundamental value, the JLS model (Johansen et al., 1999; 2000) assumes that the logarithm of the observable asset price $p(t)$ follows

$$\frac{d(p)}{p} = \mu(t)dt + \sigma(t)dW - kdj \tag{1}$$

where $\mu(t)$ is the expected return, $\sigma(t)$ is the volatility, dW is the infinitesimal increment of a standard Wiener process and dj represents a discontinuous jump such that $j = n$ before and $j = n + 1$ after a crash occurs (where n is an integer). In this specification, the parameter k quantifies the amplitude of a possible crash.

The model considers two types of agents: The first group consists of traders with rational expectations (Blanchard and Watson, 1983), while the second group is characterized by noise traders who tend to exhibit herding behaviour. The model assumes that the collective behaviour of the latter class of traders can destabilize asset prices via correlated trades. Johansen et al. (1999; 2000) propose that their behaviour can be mimicked by writing the *crash hazard rate* under the following form

$$h(t) = \alpha(t_c - t)^{m-1}\big(1 + \beta\, \cos(\omega\, \ln(t_c - t) - \phi'\big) \tag{2}$$

where α, β, ω and t_c are parameters. Eq. (2) suggests that the risk of a crash resulting from herding behaviour is a sum of a power law singularity ($\alpha(t_c - t)^{m-1}$), which is decorated by large scale amplitude oscillations that are periodic in the logarithm of the time to the singularity (or critical time) t_c. In that sense, the power law singularity embodies the positive feedback mechanism associated with the herding behaviour of noise traders. The log-periodic oscillations represent the tension and competition between the two types of agents who tend to create deviations around the faster-than-exponential price growth as the market approaches a finite-time-singularity at t_c. Seyrich and Sornette (2016) have recently presented a model providing a micro-foundation for this singular behavior (2).

The no-arbitrage condition imposes that the excess return $\mu(t)$ during a bubble phase is proportional to the crash hazard rate given by Eq. (2). Indeed, setting $E[dp] = 0$, and assuming that no-crash has yet occurred ($dj = 0$), we get $\mu = kh(t)$ since $E[dj] = h(t)dt$ by definition of $h(t)$. By integration, we obtain the expected trajectory of the price logarithm during a bubbly trajectory, conditional on the crash not yet happening, as

$$E[\ln p(t)] = A + B|t_c - t|^m + C|t_c - t|^m \cos\big(\omega\, \ln|t_c - t| - \phi\big) \tag{3}$$

where $B = -k\alpha/m$ and $C = -k\alpha\beta/\sqrt{m^2 + \omega^2}$. Note that the formula extends the price dynamics beyond t_c by replacing $t_c - t$ by $|t_c - t|$, which corresponds to assuming symmetric behavior of the average of the log-price around the singularity at t_c. This assumption is made for the sake of simplicity and provides a convenient extension to minimize the biases that occur in calibration exercises when imposing $t < t_c$.

Bubble regimes are in general characterized by $0 < m < 1$ and $B < 0$. The first condition $m < 1$ suggests that a singularity exists (i.e. the momentum of the expected log-price diverges at t_c for $m < 1$), while $m > 0$ ensures that the price remains finite at the critical time t_c. The second condition $B < 0$ ensures that the price is indeed growing super-exponentially towards t_c (for $0 < m < 1$),

2.2. Estimation and Calibration of the Model

Filimonov and Sornette (2013) rewrite Eq. (3) by expanding the term $C\cos[.]$ to replace the two parameters C and ϕ by two linear parameters $C_1 = C\cos\phi$ and $C_2 = C\sin\phi$. This representation reduces the complexity of the calibration of the LPPLS model by reducing the number of nonlinear parameters from 4 (m, ω, t_c, ϕ) to 3 (m, ω, t_c), while augmenting the set of linear parameters to 4 (A, B, C_1, C_2).

We use the formulation in terms of the four linear parameters A, B, C_1, C_2 and three nonlinear parameter m, ω, t_c (Filimonov and Sornette, 2013) so that the log-price given by Eq. (3) can be written as

$$fLPPL(\phi, t) = A + B(f) + C_1(g) + C_2(h) \tag{4}$$

where $\phi = \{A, B, C_1, C_2, m, \omega, t_c\}$ is a (1×7) vector of parameters we want to determine and

$$f \equiv (tc - t)^m \tag{5}$$
$$g \equiv (t_c - t)^m \cos(\omega\ \ln(t_c - t)) \tag{6}$$
$$h \equiv (t_c - t)^m \sin(\omega\ \ln(tc - t)). \tag{7}$$

Fitting Eq. (4) to the log-price time-series amounts to search for the parameter set ϕ^* that yields the smallest *N-dimensional* distance between realization and theory. Mathematically, using the L^2 norm, we form the following sum of squares of residuals

$$F(t_c, m, \omega, A, B, C_1, C_2) = \sum_{i=1}^{N}\Big[\ln[P(t_i)] - A - B(f_i) - C_1(g_i) - C_2(h_i)\Big]^2 \tag{8}$$

for $i = 1, \ldots, N$. We proceed in two steps. First, slaving the linear parameters $\{A, B, C_1, C_2\}$ to the remaining nonlinear parameters $\phi = \{t_c, m, \omega\}$ yields the cost function $\chi^2(\phi)$ as

$$\chi^2(\phi) := F_1(t_c, m, \omega) = \min_{\{A,B,C_1,C_2\}} F(t_c, m, \omega, A, B, C_1, C_2) = F(t_c, m, \omega, \widehat{A}, \widehat{B}, \widehat{C}_1, \widehat{C}_2) \tag{9}$$

where the hat symbol $\widehat{}$ indicates the estimated parameters. This is obtained by solving the optimization problem

$$\{\widehat{A}, \widehat{B}, \widehat{C}_1, \widehat{C}_2\} = arg \min_{\{A,B,C_1,C_2\}} F(t_c, m, \omega.A, B, C_1, C_2) \tag{10}$$

which can be computed analytically by solving the following matrix equations

$$\begin{bmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum f_i^2 & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum g_i^2 & \sum g_i h_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h_i^2 \end{bmatrix} \begin{bmatrix} \widehat{A} \\ \widehat{B} \\ \widehat{C}_1 \\ \widehat{C}_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i f_i \\ \sum y_i g_i \\ \sum y_i h_i \end{bmatrix} \tag{11}$$

In the second step, we solve the nonlinear optimization problem involving the remaining nonlinear parameters m, ω, t_c expressed as

$$\{\widehat{t}_c, \widehat{m}, \widehat{\omega}\} = arg \min_{\{t_c,m,\omega\}} F_1(t_c, m, \omega). \tag{12}$$

The model is calibrated on the data using the Ordinary Least Squares method, providing estimations of all parameters t_c, ω, m, A, B, C_1, C_2 in a given time window of analysis. For each fixed data point t_2 (corresponding to a fictitious "present" up to which the data is recorded), we fit the price time series in shrinking windows (t_1, t_2) of length $dt := t_2 - t_1$ decreasing from 750 trading days to 30 trading days. We shift the start date t_1 in steps of 5 trading days, thus giving us 142 windows to analyze for each t_2. In order to minimize calibration problems and address the sloppiness of the model in Eq. (3) with respect to some of its parameters (in particular t_c), we use a number of filters to condition the solutions as summarized in Table 1.[3] These filters derive from the empirical evidence gathered in investigations of previous bubbles (Zhou and Sornette, 2003a,b; Zhang et al., 2015; Jiang et al., 2010; Sornette et al., 2015). It must be noted that only those calibrations that meet the conditions given in Table (1) are considered valid and the others are discarded. However, previous calibrations of the JLS model have further shown the value of additional constraints imposed on the nonlinear parameters in order to remove spurious calibrations, i.e. false positive identification of bubbles (Sornette and Johansen, 2001; Geraskin and Fantazzini, 2011; Bree et al., 2013; Sornette et al., 2013; Demos and Sornette, 2017).

3. Positive and Negative Bubbles and LPPLS Confidence™ indicators

3.1. Capturing Positive and Negative Bubbles

As mentioned earlier, the methodology presented in this paper not only permits one to decouple the analysis of bubbles into different time-scales, but also allows one to focus on positive or negative bubbles separately. In the case of positive bubbles, the asset price grows super-exponentially towards t_c and ends with a change of regime (in general a crash), whereas negative bubbles are the exact $y \to -y$ mirror of positive bubbles with respect to the horizontal axis and exhibit an accelerating price drop ending with a change of regime, in general a potential "negative" crash, i.e. a substantial price appreciation (i.e. price rebound). This feature is captured by the LPPLS model through parameter B with the estimated parameter $\hat{B} < 0$ indicating a positive bubble and $\hat{B} > 0$ indicating a negative bubble. In both positive and negative bubbles, the critical time t_c denotes the time at which the bubble ends.

[3]For further information about the sloppiness of the LPPLS model, we refer the reader to Demos and Sornette (2017) and Filimonov et al. (2017).

3.2. Definition of LPPLS Confidence^{TM} indicators

The LPPLS Confidence$^{\mathrm{TM}}$ indicator was introduced by Sornette et al. (2015) and used in details by Zhang et al. (2016b). It is also one of the key indicators powering the Financial Crisis Observatory[4] at ETH Zurich. It is defined as the fraction of fitting windows whose calibrations meet the filtering condition depicted in Table (1). It thus measures the sensitivity of the observed bubble pattern to the 142 time windows of duration from 30 to 750 trading days. A large value indicates that the LPPLS pattern is found at most scales and is thus more reliable. If the value is close to one, the pattern is practically insensitive to the choice of the window size $dt := t_2 - t_1$. A small value of the indicator signals a possible fragility since it is presented in a few fitting windows.

3.3. Multi-scale Indicators

In order to incorporate bubbles of different scales into the analysis, we introduce the Multi-scale LPPLS Confidence$^{\mathrm{TM}}$ Indicator which is constructed as follows:

- Short-term bubble: The short-term bubble indicator at time t_2 is a number $\in [0, 1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30 : 90]$ business days for this t_2. As an example, if a fit is qualified at a given window i (i.e. the filtering conditions are met) then we set its index to $Q_i = 1$. If that is not the case, $Q_i = 0$. For a total of 13 fits $((90 - 30)/5 + 1)$, the short-term indicator is simply the average over these 13 windows of their index: $Short_{ind} = \frac{1}{13}\sum_{i=1}^{13} Q_i$.
- Medium-term bubble: The medium-term bubble indicator at time t_2 is a number $\in [0, 1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [90 : 300]$ business days for this t_2. For a total of 43 fits $((300 - 90)/5 + 1)$, using the same definition of the index Q_i for each of these 43 time windows, the medium-term bubble indicator is simply $Medium_{ind} = \frac{1}{43}\sum_{i=1}^{43} Q_i$.
- Long-term bubble: The long-term bubble indicator at time t_2 is a number $\in [0, 1]$ which denotes the fraction of qualified fits for estimation windows of length $\in [300 : 745]$ business days for this t_2. For a total of 90 fits $((745 - 300)/5 + 1)$, using the same definition of the index Q_i for each of these 90 time windows, the long-term bubble indicator is simply $Long_{ind} = \frac{1}{90}\sum_{i=1}^{90} Q_i$.

3.4. Smoothed LPPLS Confidence^{TM} Multi-scale Indicators

The above defined short-term / medium term/ long-term bubble indicators exhibit significant statistical fluctuations. For the purpose of facilitating the visual interpretation of these indicators, we perform an exponential smoothing of these LPPLS confidence indicators via $AR(1)$ moving averages as follows

$$
\begin{aligned}
CLPPLS_{Short}(t) &= \alpha_{short}\, CLPPLS_{short}(t-1) + (1-\alpha_{short}) Short_{ind}(t) \ , && (13)\\
CLPPLS_{Medium}(t) &= \alpha_{medium}\, CLPPLS_{medium}(t-1) + (1-\alpha_{medium}) Medium_{ind}(t) \ , && (14)\\
CLPPLS_{Long}(t) &= \alpha_{long}\, CLPPLS_{long}(t-1) + (1-\alpha_{long}) Long_{ind}(t) \ . && (15)
\end{aligned}
$$

where $\alpha_{short} = 0.980, \alpha_{medium} = 0.995$ and $\alpha_{long} = 0.998$ corresponding respectively to time scales of 50, 200 and 500 days that are in synchrony with the respective time scales of the short-term / medium term/ long-term bubble indicators. In other words, given the fact that the short-term bubble indicator is constructed by using time windows of size $\in [30 : 90]$ business days, we perform a smoothing exponential averaging over the last 50 days for each t_2. Similarly, given the fact that the medium-term bubble indicator is constructed by using time windows of size $\in [90 : 300]$ business days, we perform a smoothing exponential averaging over the last 200 days for each t_2. Lastly, given the fact that the long-term bubble indicator is constructed by using time windows of size $\in [300 : 745]$ business days, we perform a smoothing exponential averaging over the last 500 days for each t_2.

The time series of these three smoothed bubble indicators, both for positive and negative bubbles, are shown in Fig. (1) for the period $t \in [Jan.1973 : Dec.2014]$ and for the financial time series obtained by taking the ratio of the $S\&P500$ Index divided by the capital weighted dividends of the constituting firms.

[4] http://tasmania.ethz.ch/pubfco/fco.html

3.5. Predictive Tests

As mentioned earlier, a large value for a confidence indicator suggests that the LPPLS pattern is found over several time windows and is thus more reliable whereas a small value for the indicator signals a possible fragility since it is present in only a few fitting windows. Taking into account the specification of the bubble indicator in which greater values indicate the presence of a bubble, we utilize a regime switching model that incorporates market states representing bubble and non-bubble regimes. Therefore, having computed the positive and negative bubble indicators for short and long time horizons, we examine the predictive ability of short selling and liquidity-based indicators by estimating a Markov Switching predictive model specified as

$$Ind(t) = \gamma_{0,S_t} + \gamma_{1,S_t} X_{t-1} + \epsilon_t, \tag{16}$$

where $Ind(t)$ is either $Short_{ind}(t)$, $Medium_{ind}(t)$, or $Long_{ind}(t)$, S_t is a discrete regime variable taking values in $(0, 1)$, following a two-state Markov process and ϵ_t is the error term. X_{t-1} is a vector of the predictors measured at the end of month $t-1$. As explained in the next section, the predictive model is applied to alternative proxies for short selling activity in order to check the robustness of the findings.

Here, we stress that we use the non-smoothed indicators $Short_{ind}(t)$, $Medium_{ind}(t)$ and $Long_{ind}(t)$ as the dependent variables in (16), and not the smoothed ones $CLPPLS(t)$. Using the later would lead to spurious regressions and inaccurate p-values due to their build-in correlation structure.

4. Data and Empirical Findings

4.1. Data

The dataset used to construct the LPPLS Confidence^{TM} Indicators includes monthly price-to-dividend (P/D) ratios for the $S\&P500$ Index over the period January 1973 through December 2014. As mentioned earlier, we focus on the predictive power of short selling and market liquidity measures over the confidence indicators representing positive and negative bubbles in the index. For this purpose, we examine various alternative proxies for each market-based predictor. Short selling activity is measured via two proxies. The first is the short interest index (SII) of Rapach et al. (2016) as an aggregate measure of short interest, constructed using firm-level short interest data. Rapach et al. (2016) argue that short sellers are informed traders and show that short interest is arguably the strongest predictor of aggregate stock returns, both in- and out-of-sample. The data is available on David Rapach's website. The second proxy for short selling activity is the short interest ratio (SIR) by Callen and Fang (2015), defined as the total number of shares sold short divided by total shares outstanding from the last month of the fiscal year. Callen and Fang (2015) show that this ratio is positively related to one-year ahead stock price crash risk. Following Callen and Fang (2015), we calculate this ratio using short interest data from Compustat.

Evidence that associates high stock market volume with periods of high market volatility has already been well-established in the literature (e.g. Karpoff, 1987; Gallant et al. 1992; Jones et al. 1994). Therefore, we use liquidity as a control variable in our predictive tests in order to check the robustness of the predictive ability of short selling measures. Following a number of studies including Amihud (2002) and Avramov et al. (2006), we use the stock market turnover (TURNO) as a proxy for market liquidity. We compute monthly turnover values as the number of shares traded divided by shares outstanding for all NYSE and AMEX firms from the CRSP files. Following Campbell et al. (1993), we detrend the monthly log turnover series by subtracting a one-year backward moving average of log turnover, yielding a triangular moving average of turnover growth rates.

4.2. Empirical Findings

Figure (1) presents the estimated positive and negative multi-scale LPPLS confidence^{TM} bubble indicators for the $S\&P500$ index divided by dividends. The short, medium and long-term bubble indicators are depicted in different colors and the log price-to-dividend ratio for the $S\&P500$ index is represented as the black solid line. Note that a large value for the indicator indicates that the LPPLS pattern is found for many windows in the corresponding scale range (of short-term, medium-term and long-term) and is thus more reliable. Looking at Figure (1), we observe remarkable "*spikes*" in the smoothed indicators at the eve of regime changes. For example, the long-term indicator successfully captures, ex-ante, all the great bubbles and subsequent crashes suffered by the $S\&P500$ index (Black-Monday - 1987, Dot-com - 2000 and Subprime - 2008) when using a threshold $\geq 50\%$. Similarly, the negative long-term indicator remarkably shows the start of a positive bubble at the beginning of 1995 where its value reaches ≈ 1. The exponential damping structure after each peak is due to the AR(1) smoothing explained in section 3.4.

It is also interesting to notice the number of small bubbles (green shaded region on the upper panel) permeating the bubbly period that stretches from 1994 to the burst of the dot-com bubble in 2000. Note also that throughout this period, the positive long-term indicator is ever increasing as well as the medium-term indicator, thus suggesting the maturation of the bubble towards instability across several distinct time-scales. Overall, these results support our claim that the LPPLS framework is a flexible tool for detecting bubbles across different time-scales.

Having constructed the series of positive and negative bubble indicators, we next examine their predictability using the regime-switching specification in Eq.(16). Table 2 reports the estimates for the Markov Switching model for the short-term bubble indicator, $Short_{ind}$.[5] Panels A and B report the findings when short selling activity is measured by the short interest index of Rapach et al. (2016) and the short interest ratio of Callen and Fang (2015), respectively. The two-state specification identifies two distinct regimes corresponding to bubble and non-bubble market states for each indicator series. Examining the findings for the negative bubble indicators, we see that short sellers indeed have significant predictive power over market crashes, consistent across both measures of short selling activity in Panels A and B. The model yields positive and highly significant estimates for both short selling measures, suggesting that higher level of short selling activity predicts the occurrence of negative bubbles in the short term. As expected, none of the short selling proxies have predictive significance in the case of the non-bubble regimes. Consistent with the positive coefficients observed for the negative bubble indicators, we see that the short interest ratio, defined as the total number of shares sold short divided by total shares outstanding, has predictive power over the positive bubble indicator with a negative coefficient, suggesting that higher short selling activity predicts lower occurrence for a positive bubble.

Similarly, examining the estimated coefficients for turnover, we see that market liquidity also commands significant predictive ability over both the negative and positive bubble indicators. The significant predictive power observed for turnover is consistent with the finding by Nneji (2015) that market liquidity has a prevalent effect on stock bubbles and that liquidity shocks provide warning signals of impending bubble collapses. Interestingly however, the highly significant and positive estimates observed for turnover indicate that high market turnover can serve as a predictor of bubble occurrence in either direction, i.e. a booming or collapsing market condition. Shin (2006) also highlights the connection between available liquidity and rising asset prices. His argument stresses that strong balance sheets induce banks to increase their lending which, In turn, raises asset prices, leading to stronger balance sheets and so forth.

The findings for the medium- and long-term bubble indicators reported in Tables 3 and 4 further confirm the predictive power of short selling proxies over dropping markets (i.e. negative bubbles) across both the short and long horizons. We observe highly significant and positive coefficient estimates for both short selling proxies in the models for the negative bubble indicator, suggesting that short selling activity predicts greater occurrences of negatively trending markets over both short and long time scales. This finding is not inconsistent with Callen and Fang (2015) who document that short interest is positively related to one-year ahead stock price crash risk. To that end, our results confirm short sellers' predictive ability over developing market loss risks which are successfully captured by our implementation of the LPPLS framework presented in this study.

Interestingly, however, while market turnover retains its predictive ability in the medium term, we observe that the sign of the estimated coefficients for turnover in Table 3 flips to negative, suggesting that high turnover predicts lower occurrence of bubbles (in either direction) in the medium term. Similarly, in the case of the long term bubble indicator reported in Table 4, turnover loses its significance for the negative bubble indicator. These observations suggest that market liquidity has only a transient and relatively short-term

[5] The model is estimated using the non-smoothed bubble indicators explained in section 3.3 as the smoothed indicators may lead to spurious regressions and inaccurate p-values, as already mentioned.

impact on prices, which is detected for the short-term indicator, but not for the longer time scales of the bubble indicators, which are themselves more robust to detect the overall bubble sentiment at long-time scales.

Overall, our findings suggest that market-based indicators can indeed be utilized to predict the occurrence of market booms and collapsing market regimes, implied by the significant predictive ability observed for short selling proxies for negative bubble regimes across both the short and long horizons. On the other hand, market liquidity is found to predict the occurrence of both decreasing and booming market conditions while its predictive power is limited to shorter time horizons. These findings are encouraging news for market regulators as the results show that short selling proxies can be used to model and monitor negative bubble market conditions, while market liquidity can be used to supplement forecasting models for both boom and bust market conditions.

5. Conclusion

This paper has examined the predictability of stock market booms and crashes via an application of the LPPLS Confidence^{TM} Multi-scale Indicators to the $S\&P500$ index. First, we presented a methodology to detect positive and negative bubbles for the $S\&P500$ index using the Log-Periodic Power Law Singularity (LPPLS) model (Johansen et al. 1999; 2000, Sornette 2003), something not possible by other bubble detection models. Next, we provided insight to the predictability of market booms and crashes using market-based indicators by examining the predictive power of short selling activity and market liquidity over the constructed bubble indicators. To the best of our knowledge, this is the first attempt in examining the predictive ability of market-based indicators over the positive and negative bubbles in the $S\&P500$ index using the LPPLS model.

Our findings suggest that the LPPLS framework is able to successfully capture, ex-ante, some of the prominent bubbles across different time scales. We show that some of the great bubbles and subsequent crashes experienced during the Black Monday, Dot-com, and Subprime Crisis periods are successfully captured by the constructed bubble indicators. Our predictive tests indicate that measures of short selling activity have robust predictive power over negative bubbles, in line with the previous studies that short sellers have predictive ability over stock price crash risks. The predictive ability of short selling activity is robust to alternative measures of short selling as well as to short and long time horizons, consistent with the recent finding by Callen and Fang (2015) that short interest is positively related to one-year ahead stock price crash risk. On the other hand, our tests show that market liquidity has robust predictive power over both the negative and positive bubbles, however in the short-term, suggesting that market liquidity measures can be used to predict the occurrence of both booms and collapses for short horizons. We have thus identified short selling and liquidity as two important factors contributing to the LPPLS-based bubble indicators. The evidence overall points to the predictability of both positive and negative stock market bubbles via market-based proxies of trading activity and can be used as a guideline to model and monitor bubble conditions in stock markets.

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Figure 1: Positive (upper panel) and negative (lower panel) multi-scale LPPLS Confidence bubble indicator. The black continuous line denotes the logarithm of the monthly Price over Dividend (P/D) time-series for the $S\&P500$ Index from January 1973 to December 2014. The short, medium and long-term bubble indicators are depicted in green, magenta and red respectively. We refer to Sec. (3) for the construction of the indexes.

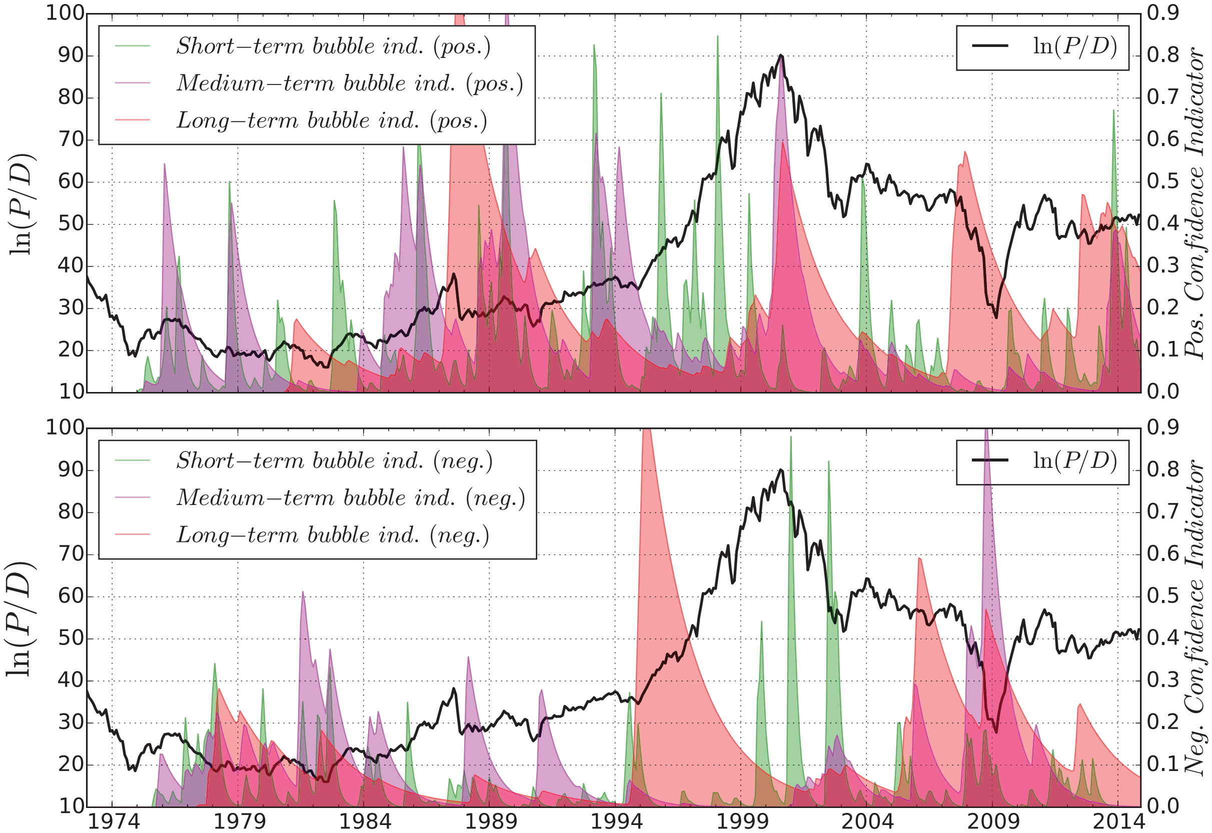

Table 1: Search space and filter conditions for the qualification of valid LPPLS fits. Within the JLS framework, the condition that the crash hazard rate $h(t)$ is non-negative by definition translates into a value of the Damping parameter $\frac{m|B|}{\omega|C|}$ larger than or equal to 1.

Item	Notation	Search space	Filtering condition 1	Filtering condition 2
3 nonlinear parameters	m	[0, 2]	[0.01, 1.2]	[0.01, 0.99]
	ω	[1, 50]	[6, 13]	[6, 13]
	t_c	$[t_2 - 0.2dt,$ $t_2 + 0.2dt]$	$[t_2 - 0.05dt,$ $t_2 + 0.1dt]$	$[t_2 - 0.05dt,$ $t_2 + 0.1dt]$
Number of oscillations	$\frac{\omega}{2} \ln \lvert \frac{t_c - t_1}{t_2 - t_1} \rvert$	—	$[2.5, +\infty)$	$[2.5, +\infty)$
Damping	$\frac{m\lvert B\rvert}{\omega\lvert C\rvert}$	—	$[0.8, +\infty)$	$[1, +\infty)$
Relative error	$\frac{p_t - \widehat{p}_t}{\widehat{p}_t}$	—	[0, 0.05]	[0, 0.2]

Table 2: The predictive ability of short interest and turnover on the *short–term* bubble indicator, $Short_{ind}(t)$.

Panel A: Short selling activity measured by the short interest index of Rapach et al. (2016)				
	Negative Bubble		**Positive Bubble**	
	Regime 1 (bubble regime)			
Constant	0.2988***	(0.0122)	0.4502***	(0.0131)
Short interest index, SII	0.0870***	(0.0122)	0.0167	(0.0127)
Turnover	0.5402*	(0.2317)	2.1897***	(0.2292)
	Regime 2 (non-bubble regime)			
Constant	0.0082***	(0.0012)	0.0099***	(0.0026)
Short interest index, SII	0.0007	(0.0012)	-0.001	(0.0029)
Turnover	-0.0167	(0.0167)	0.0838**	(0.0365)
AIC	-4.211		-2.593	
log L	1067.960		661.050	
Panel B: Short selling activity measured by the short interest ratio of Callen and Fang (2015)				
	Negative Bubble		**Positive Bubble**	
	Regime 1 (bubble regime)			
Constant	0.1235***	(0.0241)	0.5046***	(0.0199)
Short interest ratio, SIR	9.9473***	(1.7216)	-2.0706***	(0.5316)
Turnover	0.5620***	(0.2010)	2.0469***	(0.2273)
	Regime 2 (non-bubble regime)			
Constant	0.00650***	(0.0018)	0.0137***	(0.0037)
Short interest ratio, SIR	0.071153	(0.0478)	-0.129462	(0.0966)
Turnover	-0.016927	(0.0180)	0.0815**	(0.0356)
AIC	-4.136		-2.616	
log L	1042.885		662.960	

Note: This table reports the estimates for the Markov Switching model specified in Eq.(16). Market liquidity is measured by stock market turnover, computed as the number of shares traded divided by shares outstanding for all NYSE and AMEX firms from the CRSP files. Following Campbell et al. (1993), we detrend the log turnover series by subtracting a one-year backward moving average of log turnover. Panels A and B report the findings for when short selling activity is measured by the short interest index of Rapach et al. (2016) and the short interest ratio of Callen and Fang (2015), respectively. The numbers in parentheses are the standard errors. ***, **, and * represent significance at 1, 5, and 10 percent, respectively.

Table 3: The predictive ability of short interest and turnover on the *medium–term* bubble indicator, $Medium_{ind}(t)$.

Panel A: Short selling activity measured by the short interest index of Rapach et al. (2016)				
	Negative Bubble		**Positive Bubble**	
Regime 1 (bubble regime)				
Constant	0.1613***	(0.0100)	0.571***	(0.0164)
Short interest index, SII	0.0421***	(0.0065)	-0.0303*	(0.0171)
Turnover	-0.5756***	(0.0806)	-2.6188***	(0.2745)
Regime 2 (non-bubble regime)				
Constant	0.0046***	(0.0009)	0.0101***	(0.0018)
Short interest index, SII	0.0008	(0.0009)	-0.0021	(0.0018)
Turnover	-0.0239*	(0.0126)	-0.0123	(0.0291)
AIC	-4.840		-3.389	
log L	1226.355		861.355	
Panel B: Short selling activity measured by the short interest ratio of Callen and Fang (2015)				
	Negative Bubble		**Positive Bubble**	
Regime 1 (bubble regime)				
Constant	0.107***	(0.0099)	0.6521***	(0.0184)
Short interest ratio, SIR	1.9061***	(0.1914)	-5.8897***	(0.4582)
Turnover	-0.7668***	(0.1086)	-2.3249***	(0.1473)
Regime 2 (non-bubble regime)				
Constant	0.0036***	(0.0012)	0.0108***	(0.0026)
Short interest ratio, SIR	0.0242	(0.0304)	-0.0576	(0.0687)
Turnover	-0.0202*	(0.0116)	-0.0194	(0.0255)
AIC	-4.958		-3.473	
log L	1248.589		877.131	

Note: This table reports the estimates for the Markov Switching model specified in Eq.(16). Market liquidity is measured by stock market turnover, computed as the number of shares traded divided by shares outstanding for all NYSE and AMEX firms from the CRSP files. Following Campbell et al. (1993), we detrend the log turnover series by subtracting a one-year backward moving average of log turnover. Panels A and B report the findings for when short selling activity is measured by the short interest index of Rapach et al. (2016) and the short interest ratio of Callen and Fang (2015), respectively. The numbers in parentheses are the standard errors. ***, **, and * represent significance at 1, 5, and 10 percent, respectively.

Table 4: The predictive ability of short interest and turnover on the *long–term* bubble indicator, $Long_{ind}(t)$.

Panel A: Short selling activity measured by the short interest index of Rapach et al. (2016)				
	Negative Bubble		**Positive Bubble**	
	Regime 1 (bubble regime)			
Constant	0.5143***	(0.0128)	0.5297***	(0.0111)
Short interest index, SII	0.0379***	(0.0128)	0.0152	(0.0085)
Turnover	-0.3156	(0.2553)	-1.4654***	(0.1753)
	Regime 2 (non-bubble regime)			
Constant	0.0069***	(0.0020)	0.0092	(0.0020)
Short interest index, SII	0.0014	(0.0019)	-0.002	(0.0019)
Turnover	-0.0217	(0.0275)	0.0074	(0.0228)
AIC	-3.186		-3.217	
log L	810.157		818.126	
Panel B: Short selling activity measured by the short interest ratio of Callen and Fang (2015)				
	Negative Bubble		**Positive Bubble**	
	Regime 1 (bubble regime)			
Constant	0.4347***	(0.0203)	0.5447***	(0.0158)
Short interest ratio, SIR	2.516***	(0.4817)	-0.3782	(0.3294)
Turnover	0.2515	(0.2661)	-1.3063***	(0.1675)
	Regime 2 (non-bubble regime)			
Constant	0.0039	(0.0030)	0.0062**	(0.0029)
Short interest ratio, SIR	0.0908	(0.0768)	0.1091	(0.0770)
Turnover	-0.0139	(0.0269)	0.0063	(0.0282)
AIC	-3.202		-3.208	
log L	809.589		811.064	

Note: This table reports the estimates for the Markov Switching model specified in Eq.(16). Market liquidity is measured by stock market turnover, computed as the number of shares traded divided by shares outstanding for all NYSE and AMEX firms from the CRSP files. Following Campbell et al. (1993), we detrend the log turnover series by subtracting a one-year backward moving average of log turnover. Panels A and B report the findings for when short selling activity is measured by the short interest index of Rapach et al. (2016) and the short interest ratio of Callen and Fang (2015), respectively. The numbers in parentheses are the standard errors. ***, **, and * represent significance at 1, 5, and 10 percent, respectively.