The Fama French five factor asset pricing model on the JSE

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Abstract

The aim of the research project was to evaluate a number of asset pricing models hinging around the latest research by Gene Fama and Kenneth French who proposed a five-factor asset pricing models using independent variables of: the return of the whole market relative to a risk free investment, value, investment, profitability and size. Previous research had evaluated the model in a number of locations around the world with different results for different regions. Thus, understanding the five factor model in the context of the Johannesburg Stock Exchange (JSE) was a worthwhile academic exercise in addition to being useful to business.

In total, 15 asset pricing models were analysed with combinations of the five factors evaluated. This ranged from the simplest model, the single factor Capital Asset Pricing Model (CAPM), to the full five factor model.

Results show that the five factor model provided the best explanation of share behaviour on the JSE out of all models evaluated. Other findings included: the CAPM does not work well as an explanatory model, more factors in an asset pricing model generally give better results and the results from models with the same number of factors are fairly close together.

Keywords

Capital asset pricing model, Fama French, Johannesburg Stock Exchange, style investing, five factor

Declaration

I declare that this research project is my own work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Gordon Institute of Business Science, University of Pretoria. It has not been submitted before for any degree or examination in any other University. I further declare that I have obtained the necessary authorisation and consent to carry out this research.

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Chapter 1: Introduction to the Research Problem

1.1 Purpose

The purpose of this research was to develop an understanding of how well the latest five factor asset pricing model proposed by (Fama & French, 2015) performs in the context of the Johannesburg Stock Exchange (JSE). In addition to the five-factor model, 14 other models that are the various combinations of the factors including the single factor Capital Asset Pricing Model (CAPM) were evaluated.

1.2 Context of the Study

Asset Pricing Models

Asset pricing models are theoretical models that endeavour to explain why shares or portfolios of shares yield returns in the way that they do. Asset pricing models are used in other asset classes but their most common application is shares on a stock exchange and this is the area of focus for this research topic. Asset pricing models are explanatory models and not predictive models. This means that they do not make any prediction about asset behaviour in the future, they aim to improve the understanding of what is happening in the present.

The seminal work on asset pricing models was done by (Sharpe, 1964) and (Lintner, 1965) who put forward the well-known and widely used Capital Asset Pricing Model (CAPM). The CAPM states that the return on a share or portfolio of shares is dependent on a single independent variable as one can see below.

$$R_{i,t} - R_{f,t} = b_i(R_{m,t} - R_{f,t})$$

R_f is return from a risk free investment, R_m is the return of the market and R_i is the return on the asset of interest (share or portfolio of shares). This model is useful as it provides an indication of the volatility of the asset compared the volatility of the market, if b_i is greater than 1 then the asset is more volatile than the market. Different investors have differing tolerance of volatility and this model helps one to match the asset to the investor based on volatility.

The CAPM was first supplemented by (Ross, 1976) as part of Arbitrage Pricing Theory (APT). The author proposed that it was possible to add additional independent variable terms after the R_m-R_f term, each having its own coefficient.

$$R_{i,t} - R_{f,t} = b_1(R_{m,t} - R_{f,t}) + b_2 \times RP_2 + \ldots + b_n \times RP_n$$

Where RP_n is the risk premium associated with a certain variable. In APT these terms are traditionally macroeconomic variables such as GDP, inflation and the gold price (Investopedia, 2018a). The number of terms is variable and the terms do not have to be macroeconomic in nature, they can be anything that has an effect on the returns of the asset of interest.

The flexibility offered by the generic APT formula has been embraced by various academics to formulate asset pricing models that incorporate stock market and listed company related risk premiums. One of the earlier models proposed was by (Fama & French, 1992), their three factor model added two independent variables to the CAPM. The first was the return premium of small market capitalisation companies when compared to large market capitalisation companies. The second was a return premium of high "value" companies compared to low "value" companies where value is measured by book to market ratio.

Since 1992 a number of different asset pricing models have been proposed with differing terms and numbers of terms. The latest asset pricing model by Gene Fama and Kenneth French contains five terms which includes the terms from their original three factor model plus:

- The return premium of high profitability companies relative to low profitability companies
- The return premium of low investment firms relative to high investment firms

This brief history of asset pricing models is representative of a number of Nobel prizes and a great deal of intellectual effort by some of the world's most intelligent finance academics and practitioners. All of them were aiming to develop our understanding of stock market behaviour. The literature review of chapter 2 goes into detail about some specific findings but overall model accuracy and completeness has improved as time progressed. It is important to note though, that none of the models provide a definitive explanation of behaviour. Asset behaviour is affected by widespread variables: macroeconomic, market behaviour, sentiment in a number of forms, company performance in absolute terms as well as relative terms etc. While the five factor asset pricing model is a much improved model, it is not claimed to be definitive.

An appropriate question to ask at this point would be, “How do asset pricing models link to the real world of investments?” A brief explanation of how the CAPM is used has already been provided but the more complex models have their link in an investment strategy called style investing.

Style Investing

The philosophy of style investing states that it is possible to identify characteristics of companies, their shares, macroeconomic conditions etc. which are predictors of share outperformance for the shares that excel at that characteristic (otherwise known as a style). This is discussed in some detail by (Barberis & Shleifer, 2003). However, the origins are from much earlier in the history of investment strategy development. One of the oldest and most famous strategies is value investing detailed by (Graham & Dodd, 1934), Benjamin Graham was Warren Buffet’s investment mentor.

Value investing finds companies that are “cheap” for their performance or asset value. Typical measures of value are book to market ratio, earnings yield and dividend yield. There are many investment styles and each style has numerous ways to measure it. In total this leads to a wide variety of possible measures, the largest number that was found to be evaluated in one single study was 80 by (Hou, Xue, & Zhang, 2015).

At the time, a major step forward for style investing was the first three factor asset pricing model by (Fama & French, 1992) as it provided a mathematical model which could be tested to see whether a particular style was an out-performing investment strategy. Using the three factor model as an example let us consider what can be gained from it. Apart from the market premium (R_m – R_f) there are also premiums for the performance of small companies over big ones and the performance of high value shares over low value shares.

- The magnitude and sign of each factor (the premium of one characteristic over its opposite) provides an indication of how much outperformance that particular style will give, for example if small market capitalisation companies outperform large ones then the value of this premium gives one an idea of how much better the small market cap style will be.
- The time history of each premium tells one how confident one can be about future performance. A premium which changes sign often may do so again so investing in that style is risky, the vice versa is also true.

- The regression analysis of the share or portfolio of shares based on the asset pricing model gives one an understanding of how exposed that asset is to the styles that are in the model.
- The regression results also tell one how much of the behaviour of the asset is unknown and unexplained. This helps develop an understanding of the unknown risk that is associated with that investment.

In summary, asset pricing models help an investor to identify which styles are currently demonstrating significant outperformance and which styles show a history of doing so. Additionally the influence of each style on an asset's performance can be understood and the level of unknown risk quantified.

A factor, as has already been alluded to, is the difference between the returns on two diametrically opposite investment styles. An example is the difference between the small market capitalisation returns and the large market capitalisation firms. A number of them are discussed in Chapter 2.

1.3 Problem Statement

Numerous studies have analysed asset pricing models in different regions around the world. Some of these studies have also added an analysis that considers the performance of a model formulated on a set of data for a hypothetical "global stock exchange", (Fama & French, 2012) and (Fama & French, 2017) are two examples. The findings all show that the models work differently in different regions and that the model formulated on the "global stock exchange" does not perform as well as the individual regional models. Therefore the results for the five-factor model that have been obtained for North America, Europe, Asia Pacific (Fama & French, 2017), Australia (Chiah, Chai, Zhong, & Li, 2016) and China (Guo, Zhang, Zhang, & Zhang, 2017) are all different. The five-factor asset pricing model by (Fama & French, 2015) has not been tested on the Johannesburg Stock Exchange (JSE) and there is value in understanding how well it explains asset behaviour there.

Evaluating how well a model works in absolute terms does have some value but more value can be derived by comparing it to the performance of the model in other areas. However, possibly the most valuable is to perform a direct comparison with other asset pricing models using the same data and methodology. The decision was taken to do this.

In total 15 models with different factors and varying number of factors were built and analysed with the aim to understand how well each one explains asset behaviour.

1.4 Significance of the Study

Within the context of a business school, research needs to be valuable in a few different ways. It needs to add to the academic body of knowledge, it needs to be useful to business and it needs to be useful within the South African context. Let us consider all three of these questions.

Value for Academia

In academia, two of the earliest pieces of readily available research formulating a theory to explain and predict variations in the capital markets and, by extension, stock exchanges were conducted in London (Jevons, 1866) and New York (Kemmerer, 1911) as per the dates. Both studies aimed to evaluate whether there was annual variation in the markets based on changes in the seasons (Autumn, Winter, Spring and Summer). The age of these studies demonstrate how long ago academia were already wrestling with developing an understanding of market behaviour.

Since the above dates, research aimed at understanding stock exchange behaviour has been occurring continuously and with a substantial amount of work added to the body of knowledge per year. The intellectual challenge of understanding something as dynamic, complex and influenced by so many variables as the stock market is compelling.

Understanding how well the five factor model, along with 14 other asset pricing models, explains variation of shares on the JSE will add to this body of knowledge. In the USA, the five factor model has proven to be better than older and smaller models, a worthwhile academic achievement. If this proves to be the case for the JSE, then it will advance both our understanding of the Johannesburg Stock Exchange and of the Fama French five factor model.

Value for Business

While some academics enjoy accumulating knowledge for the sake of knowledge there are many who are driven to see their research applied, and adding value, in the real world. There can be little doubt that the researchers who have worked on asset pricing models are in the latter group.

(Sharma, 2018) delivered a commentary about stock market behaviour in the New York Times in February 2018. The New York Stock Exchange had just ended a 9-month period of very low volatility with a sudden spike in volatility as well as a general drop in share prices (Wigglesworth & Wells, 2018). It was within this context that the author made two main points. Firstly, that behaviour observed in the past may not be reproduced in the future and secondly that stock markets are both volatile and complex. The unpredictable nature of stock markets means that having an understanding of what drives share behaviour in stock exchanges opens up the opportunity to capitalise on that knowledge to generate investment returns that are above the average. No asset pricing model can claim to provide a perfect understanding of asset behaviour but an investor (either individual or institutional) with an above average understanding has an opportunity to utilise that knowledge to make better investment decisions at a lower level of risk.

With this in mind, it is always worth searching for better ways to select shares.

Value for South Africa

The Fama French models are well recognised and well respected. However, it has already been discussed that a model that works well for one region may not work well in another (Fama & French, 2012). South Africa has the largest capital markets and stock exchange in Africa by a considerable margin. Consequently, understanding how well the five factor model works within the South African context is valuable to both South African business schools and South African businesses.

1.5 Delimitations

A number of delimitations are discussed in detail in chapter 4.

- The study is limited to listed companies on the Johannesburg Stock Exchange
- Only the top 160 listed companies (based on market capitalisation) on the Johannesburg Stock Exchange are studied
- Only information that is publicly available (e.g. data found in annual reports and stock exchange bulletins) were used in the study

1.6 Definition of terms

JSE –	Johannesburg Stock Exchange	CAPM – Capital Asset Pricing Model
APT –	Arbitrage Pricing Theory	NYSE – New York Stock Exchange

HML – The difference in returns between a portfolio of high value shares and a portfolio of low value shares

CMA – The difference in returns between a portfolio of conservative investment shares and a portfolio of aggressive investment shares

RMW – The difference in returns between a portfolio of robust profitability shares and a portfolio of weak profitability shares

SMB – The difference in returns between a portfolio of small market capitalisation shares and a portfolio of big market capitalisation shares

ALSI – The All Share Index on the Johannesburg Stock Exchange

1.7 Assumptions

Within the context of the quantitative analysis performed during the research, the following assumptions were applied. Once again, some of these assumptions are discussed in more detail in chapter 4.

- All companies reported their financials consistently and without any fraud.
- All companies applied the consistency principle when reporting their results. This means that information over a number of years could be compared.
- The Johannesburg Stock Echange reported transactions on their platform accurately.
- The company financial data obtained from IRESS is an accurate reporting of that published by the companies of interest.

Chapter 2: Literature Review

2.1 The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) is attributed to two independent researchers, (Sharpe, 1964) and (Lintner, 1965). William Sharpe was awarded the Nobel prize for this work in 1990 (The Royal Swedish Academy of Sciences, 1990). The CAPM is an explanatory model that states that the return over a set time period in a share or a portfolio of shares is a function of a single independent variable (made from two). The variable is the difference between the return in the whole stock market over that same time period and the return that an investor can make in the lowest risk investment available over that time period. This is known as the risk free rate and is typically assumed to be the 10-year government bond yield.

$$R_{i,t} - R_{f,t} = b_i(R_{m,t} - R_{f,t})$$

Where: $R_{i,t}$ is the return on financial asset i at time t
$R_{f,t}$ is the risk free rate of return at time t (typically the 10-year bond yield)
b_i is the beta value for a financial asset i (a constant)
$R_{m,t}$ is the return on the whole capital market at time t

It is very important to emphasise that the CAPM does not make any predictions about what is going to happen in the future (t+1). It is an explanatory model which aims to *explain* the variation in an asset that is observed at a set point in time (t) given variation in R_m - R_f. This is not only true of the CAPM, the asset pricing models developed by Gene Fama and Kenneth French work in the same way.

2.1.1 Findings in Favour of the CAPM

A study by (Fama & MacBeth, 1973) evaluated the CAPM over a number of sub-periods between 1934 and 1968. This study found that there was a positive relationship between risk (in this case measured by volatility) and the return of the share. This was further confirmed by the same primary author on the NYSE by (Fama & French, 1992). In the time period from 1926 to 1968 the data proved to give a good quality linear relationship between asset return, market return and risk free rate.

Another study which supports the CAPM is (Jagannathan & Wang, 1993) who found that when Beta is allowed to vary over the business cycle the r^2 gave an impressive value of 57%. This result is clearly a success for the CAPM and it implies that doing a regression for Beta that is over too long a period is not ideal for the CAPM. In the South African context the CAPM was evaluated by (Strydom & Charteris, 2013). Their research, which was conducted over sub-periods between 1993 and 2008, found that the CAPM prediction of a positive, linear relationship between Beta and monthly returns was correct provided the appropriate risk free rate is used. This is shown in Figure *i*: Relationship between Beta and monthly returns below for the share asset class. It can be seen that the data points are somewhat haphazardly positioned which implies a low R^2 which does raise some concern.

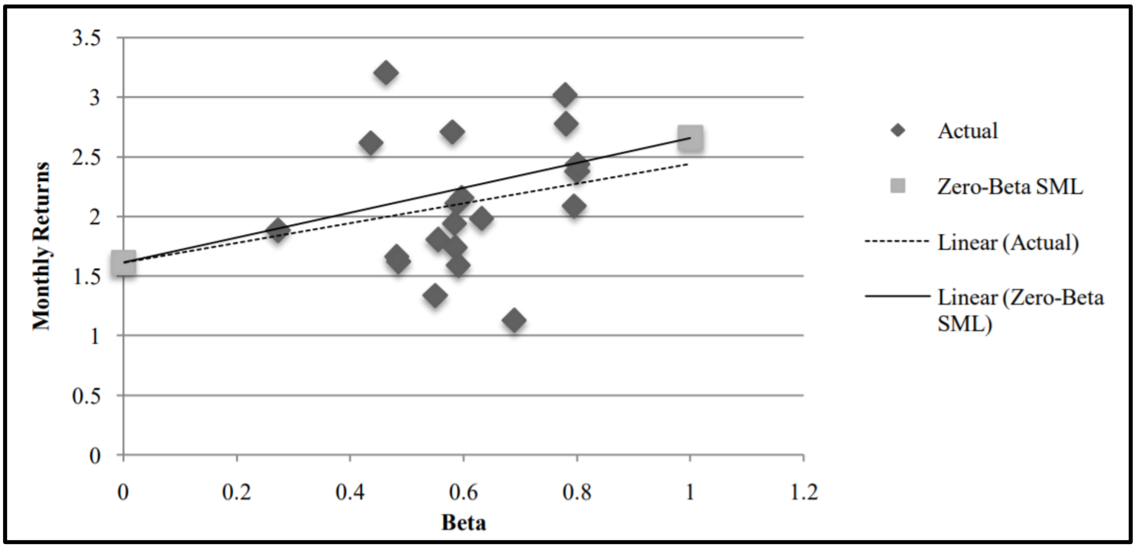

Figure i: Relationship between Beta and monthly returns *(Strydom & Charteris, 2013)*

2.1.2 Findings Contradicting the CAPM

An early study which raised concerns about the CAPM in its traditional form was that of (Jensen, Black, & Scholes, 1972), this study found that the t-statistics for Beta produced by a linear regression over the period from 1931 to 1965 for 10 different portfolios were above 1,85 in just 3 of the 10 portfolios. In a similar study for the period from 1963 to 1988, (Fama & French, 1992) found that the t-statistics were well below the benchmark value of 1,85. A study investigating the British stock exchange was conducted by (Loukeris, 2009). The study was of 39 shares on the London Stock Exchange over the period from 1980 to 1998. The regressions for the CAPM on these shares gave an R^2 value of just 7.3% which means that there is a great deal of unexplained share behaviour.

One of the fundamental principles of the CAPM is that shares which display more volatility should give more returns. A study in North America (Baker, Bradley, & Wurgler, 2011) actually found the opposite, over the period from 1968 to 2008 portfolios formed based on a low value of Beta (which implies low volatility) gave high average returns and low drawdowns. A very similar study was done for the Johannesburg Stock Exchange by (Ward & Muller, 2012) and their finding was exactly the same, that portfolios formed from low Beta stocks significantly outperformed portfolios formed from high Beta stocks as well as outperforming the All Share Index (J203).

Another investigation into the CAPM on the JSE was conducted by (Carter, Muller, & Ward, 2017). This study calculated Beta for a portfolio of shares over a period of 24 months and 60 months. The CAPM was then used to calculate a theoretical return on the portfolio for the subsequent 24 months given the actual risk free rate and the return of the whole market for each specific month. Out of 38 different results (19 sets of data analysed in 2 different ways), there were only 7 sets where the CAPM was not rejected at the 95% confidence level. This demonstrates that overall the CAPM is not a good explanatory model for the behaviour of the Johannesburg Stock Exchange.

Another study, which raises questions about the CAPM, is that of (Frazzini & Pedersen, 2014). They found that a hedge fund strategy that went long on low Beta shares and short on high Beta shares delivered superior returns when adjusted for risk. The implications of this study are that low Beta shares out-perform high Beta shares

2.2 Style Investing

The CAPM implies that the only explanatory variables for an asset's returns are the market's returns and the return from a risk free investment. Therefore, if there are any other variables that provide any explanation as to the returns seen by an asset, then the CAPM must be called into question. The phenomenon known as style investing is exactly this.

2.2.1 Various Investment Styles

Style investing is discussed in (Barberis & Shleifer, 2003). The article explains that when investors group assets together based on a specific common characteristic then they are engaging in style investing. The thesis of these investors is that certain characteristics are predictive of superior or inferior returns. The number of potential investment styles is extremely large. However, the above authors state there is a smaller number that make logical sense and even fewer that have been shown to provide returns that are markedly different from the overall market.

Two very well-known, and contrasting, investment styles known as value and growth which can be identified using a number of different measures. This is detailed *Figure ii: Metrics for value and growth investing* .

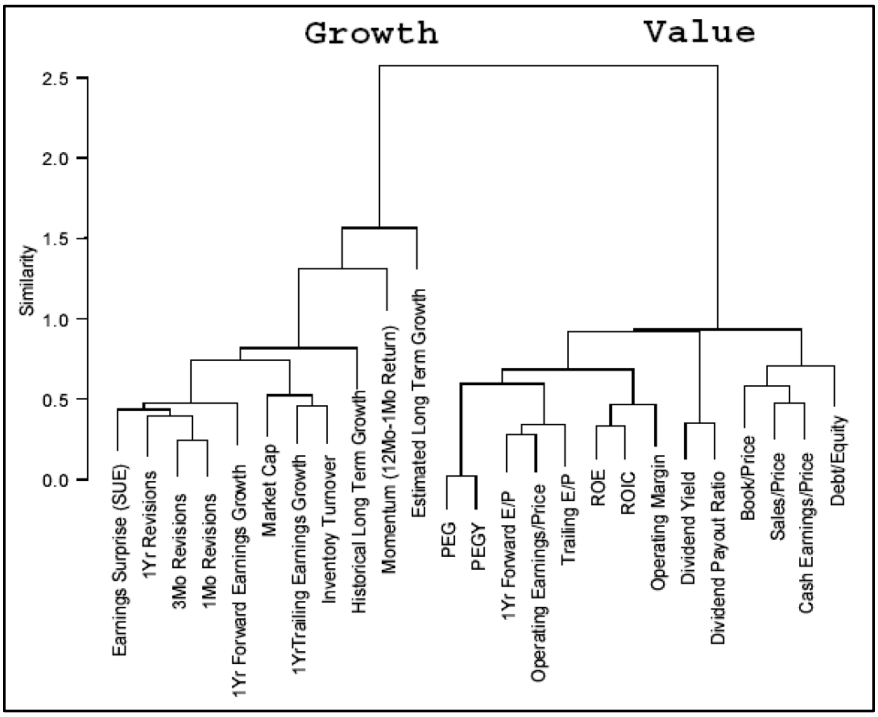

Figure ii: Metrics for value and growth investing (Saville, 2018)

Value Investing

An early reference to value investing is (Graham & Dodd, 1934), the authors advocate searching for assets that are good value for money, this is measured using calculations such as book to market ratio, earnings yield (inverse of PE ratio) and dividend yield (in all cases a high number points to a value stock). A study conducted by (Basu, 1977) found that from 1957 to 1971 low PE ratio shares (high earnings yield) outperformed high PE ratio shares.

Value investing was evaluated on the JSE by (Muller & Ward, 2013) and a graph showing the returns generated by portfolios of dividend yield is shown *Figure iii: Cumulative returns of five dividend yield portfolios* . In this study the JSE all share index was sorted according to dividend yield and divided up into 5 portfolios. The portfolio with the highest dividend yield companies is in “Dividend Yield 1” and the portfolio with the lowest dividend yield companies is called “Dividend Yield 5”. The portfolios were rebalanced every 3 months. One can clearly see that over the period of interest, high dividend yield companies outperform low dividend yield companies by an appreciable margin. There are times when the opposite occurs though and this is shown with the green line where the relative performance between portfolio 1 and 5 is plotted, a negative slope implies portfolio 1 being outperformed by portfolio 5.

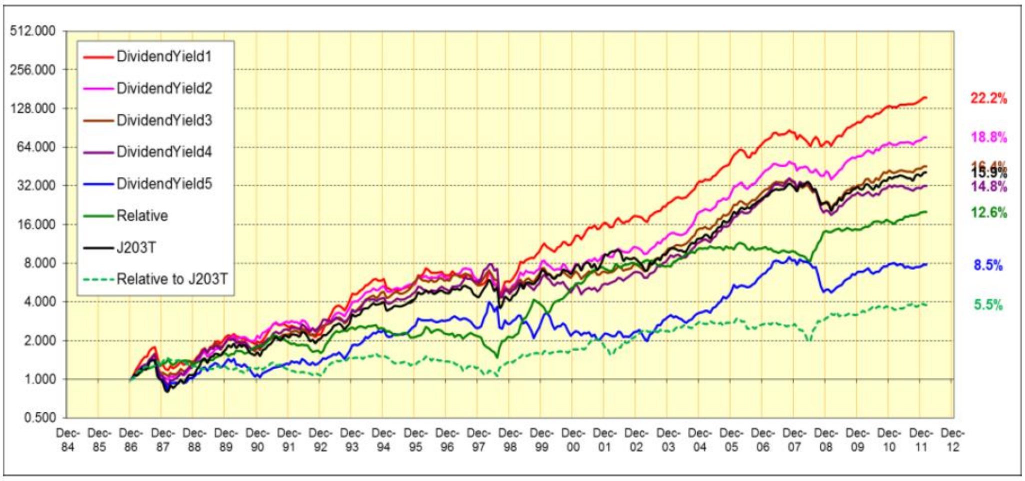

Figure iii: Cumulative returns of five dividend yield portfolios (Muller & Ward, 2013)

As per (Reese, 2015), value investing has under-performed since 2008. This is a long time for an investment style to be out of favour. This change could have come about as a result of a shift in sentiment away from value investing in 2008 which is discussed in some detail in the next section. If this is the case, one could reasonably argue that one day this sentiment could change to favour value investing again.

Growth Investing - Momentum

Growth investing is a philosophy where shares are selected based on metrics which point towards future growth in the company. One of the most well-known is momentum which was evaluated in detail by (Jegadeesh & Titman, 1993). This study which was conducted with data from 1965 to 1989 found that buying past winners (measured over the preceding 6 months) and holding them for 6 months yielded a compound excess return of 12.01%.

This characteristic was also evaluated by (Muller & Ward, 2013) who calculated cumulative results for portfolios of quintiles of highest to lowest momentum. The measure for momentum was the return of the share (or portfolio of shares) of the preceding 12 months excluding the most recent 1 month. One can see in the *Figure iv: Cumulative returns for portfolios of five momentum portfolios* that there is a spread of performance ranging from the highest momentum portfolios to the lowest with the high momentum portfolio outperforming the All Share Index (J203) by a significant margin.

It is interesting to note that the momentum effect is not universal. For example in (Fama & French, 2012) it is discussed that this investment style produces above average returns in many parts of the world but not in Japan.

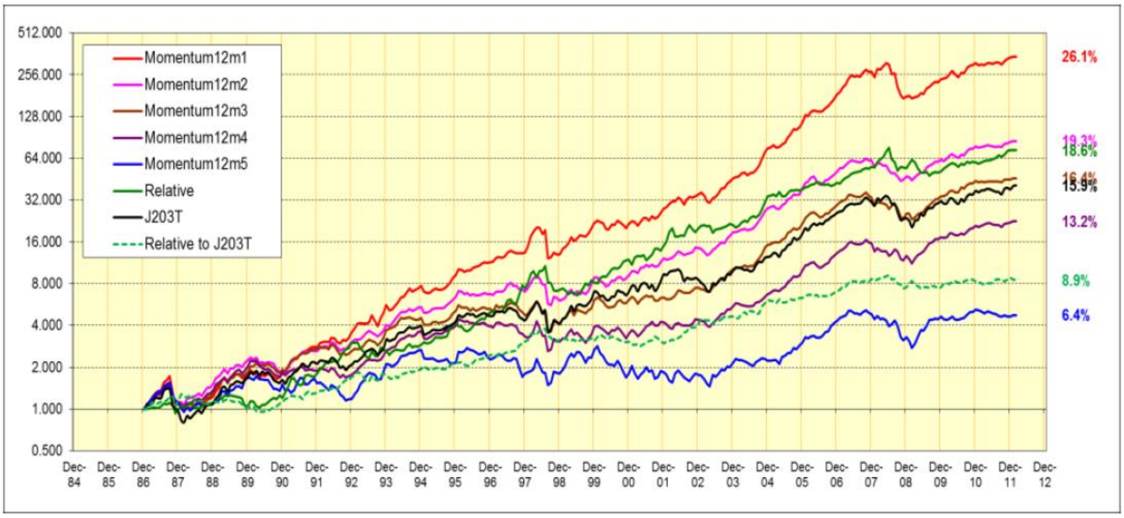

Figure iv: Cumulative returns for portfolios of five momentum portfolios (Muller & Ward, 2013)

Size Effect

Another well-known effect is the size effect. A study by (Banz, 1981) found that portfolios of small market capitalisation shares out-performed portfolios of large market capitalisation shares on the NYSA during the period from 1936 to 1977. This is quite different to the study by (Muller & Ward, 2013) who found that there was very little size effect on the JSE. This is further confirmation of the earlier assertion that style effects vary according to geography.

Other

There are an extremely large number of different potential styles. A few of the many are: industry, board diversity, bid / ask spread, profitability and investment.

2.2.3 Why does Style Investing Work?

Many investment styles make intuitive sense, for a momentum stock one may reasonably expect that, within limits, it will continue to grow. However, some occurrences are not that logical. For example, for many years, value stocks on average outperformed growth stocks, this is the foundation of the investment style advocated by (Graham & Dodd, 1934). This changed dramatically in 2008 at around the same time as the global financial crisis. Since then, value investing strategies have under-performed. A proposal as to why this has happened was put forward by (Reese, 2015). A low PE ratio company (high value) implies that there is some risk of bankruptcy and the events of 2008 have made investors wary of companies going bankrupt.

One theory as to why an investment style can change from giving above average returns to below average returns and vice versa has been put forward by (Froot & Teo, 2008) and it is related to behavioural economics. The study found that a combination of fund inflows and positive returns for a certain style were a predictor of future positive returns. For example if small market capitalisation share is currently doing well and there is an increase in investor interest in small market cap shares, then small market cap shares are likely to give good returns in the future. By contrast, inflows into the opposite style, combined with positive returns in that style, are a predictor of future negative returns for the original style. Therefore, if large market cap is doing well and there is increased investor interest in large market cap shares then small market cap shares would have a tendency to decline. The authors assert that this finding is indicative of sentiment where one style becomes more popular than another. Should this occur, the shares in the styles would simply be subject to the laws of supply and demand with resulting price changes and returns.

2.3 Fama French Asset Pricing Models

The success of style investing raises further doubts about the universal and accurate applicability of the CAPM. However, historical evidence that certain styles demonstrated superior returns than others was not sufficient for academia. A theoretical model that explained this type of asset behaviour was needed.

2.3.1 Fama French 3 Factor Model

An asset pricing model that was recognised as a potential improvement over the CAPM was published by (Fama & French, 1992). Working from previous research into the size effect and value investing on the NYSE, the study confirmed that firm size and firm book to market equity ratio had an influence over the returns that were generated on the stock market. The Fama French 3 Factor model was proposed.

$$R_{i,t} - R_{f,t} = a_i + b_i(R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + e_{i,t}$$

a_i Also known as alpha. It is the return generated even when the market does not change.

s_i A constant. A coefficient that is applicable to an asset (share or portfolio of shares) which gives the extent to which the independent variable SMB influences the return of that asset.

SMB_t Is "small minus big", the return on a portfolio of small stocks (measured by market capitalisation) minus the return on a portfolio of large stocks at time t. Note that that this term was intended to capture any size effect as per (Banz, 1981) in stock exchange behaviour.

h_i A constant. A coefficient that is applicable to an asset (share or portfolio of shares) which gives the extent to which the independent variable HML influences the return of that asset.

HML_t Is "high minus low", the difference between the return on a portfolio of high book-to-market ratio stocks and the return on a portfolio of low book-to-market ratio stocks at time t. This is a term that captures any value – growth effect in stock exchange behaviour.

$e_{i,t}$ Is a correction factor at time t (error).

The significance of this Fama French asset pricing model is that it provides an explanation of how the style effects of value and size will have an influence on an asset's returns.

Value effect – book equity to market equity ratio

Better known by academic literature as simply book to market. This is the most well-known measure of value. In this seminal study by (Fama & French, 1992) they performed regression for Beta, SMB and HML.

- The t statistic for the original CAPM term Beta has fairly low t statistics, the two listed are 0.46 and -1.21 respectively.
- The value term (HML) give t statistics between 4.44 and 5.76, this is very high and indicates that the HML term is capturing a large amount of the variation seen in the dependent variable.

Overall the results showed that the CAPM by itself does not fare well compared to the results for the Fama French 3 factor model.

(Haugen & Baker, 1996) found good t-statistics for portfolios selected based on book to market which implies a strong relationship. The factor HML is used in most models based on the Fama French approach. It can be found in the following studies: (Fama & French, 2015), (Carhart, 1997), (Asness, Moskowitz, & Pedersen, 2013), (Jegadeesh & Titman, 2001). When two opposite investment strategies based on HML were evaluated by

(Lakonishok, Shleifer, & Vishny, 1994), they had much the same findings as Haugen & Baker, that a high book to market ratio generated superior returns. They added to this finding by showing that a low book to market ratio generated inferior returns.

By contrast, (Campbell & Thompson, 2007) found that the book to market strategy gave a t statistic of below 2 for both monthly and annual returns which implies that there is quite a bit of scatter in the data. (Asness, Porter, & Stevens, 2000) investigated book to market equity (price) of firms relative to the industry in which it competed compared to more traditional across market measures, this study found that the within industry method provided a better model. Another variation on value was an investigation conducted by (Asness & Frazzini, 2013) which examined the timing at which a value strategy can be applied and updated, this found that being more pro-active and reacting faster to the data could give superior returns. In the South African context (Muller & Ward, 2013) evaluated price to book and found an inverse relationship, a low price to book (high book to market) gives a high share return which confirms the superior performance of value shares. A similar study by (Plaistowe & Knight, 1986) found that in a declining market, “value” shares declined less than “premium” shares and a similar result was seen by (Fraser & Page, 2000).

Size effect – market capitalisation

The term SMB in the Fama French 3 factor model gives the following results in (Fama & French, 1992) which was conducted with shares data from North American exchanges.

- The results show that the t statistic for the size term (SMB) varies between -1.99 and -3.06

This t statistic value is quite high which shows that one can be fairly confident that it is effective at explaining the variation in the share price.

Size is a popular investment style as well as a popular area for research. There have already been some references relating to the size effect. (Haugen & Baker, 1996) found that market cap was not a significant factor in explaining share price variation. On the other hand (Fama & French, 1992) found that market capitalisation was an important part of their original three factor model to the extent that it is still retained in their latest five factor model (Fama & French, 2015). This was also the case for (Carhart, 1997), (Asness & Frazzini, 2013) and (Jegadeesh & Titman, 2001) who added momentum to the Fama French 3 factor model but retained the size term. It should be noted that these examples did not consider market cap in isolation but rather as part of a larger model. In South Africa market cap was evaluated by (Van Rensburg, 2001) who found that the returns on a size portfolio were seen to be different (lower) than the market at the 95% confidence level. A study on the JSE was conducted by (De Villiers, Lowings, Pettit, & Affleck-Graves, 1986) who found that small market cap firms did not outperform large, in fact the opposite seemed true. The “no size effect” finding was also seen by (Bradfield, Barr, & Affleck-Graves, 1988) and (Page & Palmer, 1992). The most recent investigation into the size effect on the JSE was conducted by (Muller & Ward, 2013) who found no evidence of any size effect in the period from 1984 to 2012.

Fama French 3 factor model in South Africa

The Fama French 3 factor model was evaluated for the JSE by (Basiewicz & Auret, 2010). This study found that for individual shares, “the three factor model can explain the value effect, and goes in the right direction to explain the size effect.” It is clear that there isn’t a high confidence level in the size effect. This is corroborated by (Muller & Ward, 2013), their analysis found no empirical evidence of the size effect.

2.3.2 Style Additions to the Fama French Model Structure

As already discussed, the Fama French 3 factor model is a model that includes widely used, but until then empirically understood, style effects. This was the first time that an asset pricing model had included style effects. The other interesting part of the 3 factor model is that the relatively simple structure of the mathematical model makes changing the number of factors very easy as well as swapping factors. There are a large number of factors that could be used in this type of model structure. For example:

- (Van Rensburg, 2001) evaluated 23 different measures of different investment styles for the South African context
- (Hou, Xue, & Zhang, 2015) evaluated 80 different measures of different styles on the New York Stock Exchange (NYSE) and found that around half were significant (when measured using t statistics)
- (Haugen & Baker, 1996), conducted a detail analysis of 41 different measures and studied the Russel 3000 (an index in the United States of America). Of these 41, 12 were found to regularly have t statistics greater than 2

Having reviewed these papers in combination with a number of other papers the following styles and measures are worth discussing as potential additions to the Fama French 3 factor model.

Interest cover

Operating income over interest charges. One would expect that a low value would bring high risk. This was evaluated by (Haugen & Baker, 1996) but it did not make their 12 styles with t-statistics greater than 2. However, when it was analysed by (Muller & Ward, 2013), they found that an investment strategy based on interest cover yielded excess returns.

Cash flow to debt

A company with a low cash flow to debt ratio could be regarded as higher risk. This ratio was evaluated by (Van Rensburg, 2001) on the JSE who found that a portfolio of shares selected based on shares that had a high cash flow to debt ratio, outperformed the market at a 95% confidence level.

Traded volume / size

Where size is measured by market capitalisation. The study by (Haugen & Baker, 1996) found that it played a role in share price variation. This was also found to give significant results by (Muller & Ward, 2013).

Earning to Price

A high ratio would imply a high "value" share. A great deal of research has been done with this measure. (Haugen & Baker, 1996) found that this was a significant factor in explaining share returns in both the Russell 3000 (USA) and other international markets. This was proposed as a potential factor by (Fama & French, 1992) as a measure of value but was not analysed. A study in North America by (Lakonishok, Shleifer, & Vishny, 1994) found a significant value effect with this measure. An interesting study by (Campbell & Shiller, 2001) evaluated P/E ratio to see if a low value predicted future earnings growth but this was not seen, however it was a good predictor of future share price growth. As part of understanding explanatory abilities, (Campbell & Thompson, 2007) found that when using earning to price for an in-sample regression it gave a t stat of more than 2.5 which indicates a good variable.

On the JSE, earnings to price was investigated by (Van Rensburg, 2001) who found that E/P gave high returns as well as a high t-statistic (3.88). This same study also found that a 2 year lagged P/E ratio gave poor results. An earlier study on the JSE by (Page & Palmer, 1992) found a "significant" E/P effect. The most recent study in the South African context was done by (Muller & Ward, 2013) which gives a very clear indication of the earnings yield effect. This is shown in Figure *v*: Returns for portfolios formed based on earnings yield below where the All Share Index (J203) was divided into quintiles after being sorted in descending order for earnings yield. One can see a fairly clear spread of returns corresponding to the order of the portfolios with portfolio "EarningsYield1" outperforming the J203 by quite a margin.

Figure v: Returns for portfolios formed based on earnings yield *(Muller & Ward, 2013)*

Dividend to Price

Also known as dividend yield, this is another popular measure of value as can be seen by the number of studies. As identified by (Muller & Ward, 2013), it makes sense that high dividend yield companies are well liked by investors which in turn should give good share price appreciation.

This factor was evaluated by (Haugen & Baker, 1996) who found that it did not have a t-statistic greater than 2. On the other hand, (Campbell & Shiller, 2001) found that this factor could be used as a predictor of future share returns. On an annualised basis, dividend to price was found to have t-statistics of around 3 when it was evaluated by (Campbell & Thompson, 2007) which means that it has good explanatory power. On the JSE, this was evaluated by (Van Rensburg, 2001) who found that returns would be well above the market with a greater than 95% confidence level. By contrast, (Bradfield, Barr, & Affleck-Graves, 1988) found that there was no dividend yield effect on the JSE and (Fraser & Page, 2000) found a relatively small effect. In terms of recent studies on the JSE, (Muller & Ward, 2013) produced a very interesting graph which shows that high dividend yield companies consistently outperform low dividend yield companies.

Cash flow to Price

Here a large cash flow and a low price would lead one to expect good share price growth in the future. This measure gives a very good t-statistic in both the USA and in other markets for (Haugen & Baker, 1996). This was also seen by (Lakonishok, Shleifer, &

Vishny, 1994) where shares with high cash flow to price ratios gave better returns than shares with low cash flow to price values. (Muller & Ward, 2013) found that this investment style gave extremely good returns on the JSE. However (Van Rensburg, 2001), found that shares selected based on cash flow to price ratio gave similar returns as returns of the whole market with a low t-statistic.

Profit Margin

This was studied by (Haugen & Baker, 1996) who found that profit margin was not a predictor of superior returns. However, much more recent work by (Hou, Xue, & Zhang, 2015) has used operating profit as a factor in their q-factor model which performed well. Return on capital also gave returns better than the all share index in (Muller & Ward, 2013) for portfolios formed on that basis.

Investment - Asset Growth

When a company shows an increase in assets on the balance sheet, this is a demonstration that it is investing. This factor was included in their successful q factor model from (Hou, Xue, & Zhang, 2015). In South Africa this was examined by (Muller & Ward, 2013) who found that companies who were investing less had higher returns. While this may initially seem counter-intuitive, investment is part of a long-term strategy while returns are measured over a much shorter period. Over short time scales, investors are more impressed by companies that generate profits and dividends than companies that are investing for the long term by increasing their retained earnings.

Momentum (12 – 1 month)

This investment style gave extremely good results for (Haugen & Baker, 1996) for the USA and other international markets. (Asness, Moskowitz, & Pedersen, 2013) and (Asness, Porter, & Stevens, 2000) identified a strong momentum effect with the detailed measure (12 – 1 month). In South Africa (Van Rensburg, 2001) found that 12-month momentum gave the best performance and the highest t-statistics. (Fraser & Page, 2000), also focused on the JSE, found that momentum was a significant factor. (Muller & Ward, 2013) found that a 12-month momentum strategy gave portfolios that outperformed the market by a substantial margin.

When it comes to asset pricing models, (Carhart, 1997) adds 12-month momentum to the Fama French 3 factor model while leaving the other terms unchanged. This exact same model was also evaluated by (Fama & French, 2012) in a number of regions around the world. A similar approach was taken by (Asness & Frazzini, 2013) who added both momentum and a short term reversal in momentum factor to the Fama Franch three factor model.

Business sector

Here one divides a set of shares into sectors based on where the firm performs its primary line of business. Examples would be: construction, resources, industrial, financial, transportation etc. This was studied by (Haugen & Baker, 1996) who found that sector variables did not produce t-statistics greater than 2. (Van Rensburg, 2002) found that a two factor model based on two sectors on the JSE (FINDI and RESI) gave considerably better explanatory power than a model based on the All Share Index. (Muller & Ward, 2013) found that in the time period from 1996 to 2012 there were times where the same two sectors performed very differently and times when they were fairly similar, in this study there was no clear winner in terms of which investment strategy provided out-performance.

2.3.3 Fama-French 5 Factor Model

Model Introduction

The most recent version of asset pricing model proposed by Gene Fama and Kenneth French is the five factor model (Fama & French, 2015). This model has retained all of the factors from their original three factor model and added the factors of profitability and investment. It is interesting to note that momentum, one of the most popular investment styles today, does not feature. This is despite the fact that momentum has been added to asset pricing models in the past as detailed above including (Fama & French, 2012). The five factor model is as follows.

$$R_{i,t} - R_{f,t} = a_i + b_i\left(R_{m,t} - R_{f,t}\right) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{i,t}$$

Previously defined terms are still applicable, new ones are:

r_i A constant. A coefficient that is applicable to an asset (share or portfolio of shares) which gives the extent to which the independent variable RMW influences the return of that asset.

RMW_t Is "robust minus weak", the difference between the returns of portfolios of companies with robust profitability and weak profitability

c_i A constant. A coefficient that is applicable to an asset (share or portfolio of shares) which gives the extent to which the independent variable CMA influences the return of that asset.

CMA_t Is "conservative minus aggressive", the return on a portfolio of companies with high investment minus the return on a portfolio of companies with low investment

Results & Discussion

(Fama & French, 2015) evaluate their model in a number of different ways. The data set is from July 1963 to December 2013 and the sample set was NYSE, AMEX and NASDAQ shares. The following results are important:

- The value of the intercept a_i (commonly known as alpha) was evaluated for three, four and five factor models. A model that perfectly explains asset behaviour will have an alpha of 0. The results showed that having more factors gave a lower value for alpha. However, in general the difference between the full five factor model and a four factor model which does not contain the value term HML is very small (e.g. alpha drops from 0.075 to 0.073 per month for one of the analyses).
- The t-statistics were evaluated in a number of ways but the general trend is as follows.
    - The alpha value has a t-statistic below 2 more often than not
    - The coefficients for HML, RMW and CMA are above 2 more often than not.

Overall, the results indicate that the five factor model is an improvement over the Fama French 3 factor model. The results also indicate that a four factor model which excludes the HML term could be regarded as nearly as good. HML does not contribute meaningfully to the completeness of the model. The authors argue that the return generated by the portfolios in HML is captured in the profitability and investment information factors so that makes it unnecessary. It should also be noted that this result is only data from the USA in the date range 1963 to 2013.

Another example of excluding value (HML) from the 5 factor model is seen in (Hou, Xue, & Zhang, 2015). In this study of USA stock market (NYSE, Amex and Nasdaq) the model produced very good results, better than the 3 factor model by (Fama & French, 1992) and better than the addition of momentum to the 3 factor model as done by (Carhart, 1997).

Exclusion of Momentum

One area of concern regarding the Fama French five factor model is that it excludes momentum. Not only is this an extremely common investment style in asset management, there is also a substantial body of academic work analysing and understanding the momentum effect. An interesting explanation for this is provided by (Hou, Xue, & Zhang, 2015) who argue that a firm's profitability is strongly linked to the momentum that it's share displays on the stock market. Therefore, the profitability factor that is included in the Fama French five factor model as well as the four factor model used by the above authors is analogous to the more common momentum style. A question that has repeatedly come to the mind of the researcher is why have Gene Fama and Kenneth French abandoned momentum in favour of profitability. In (Fama & French, 2015) they argue that profitability is an indication of future dividend payments therefore the price of the share is related to the present value of those future payments. This argument implies that the addition of profitability stays true to the Efficient Markets Hypothesis of (Fama, 1970). By contrast, momentum is regarded as being somewhat related to behavioural economics which is inherently inefficient (Shiller, 2003). Finally it should be noted that in (Fama & French, 2015) the areas for further work specifically stated that they would like to test a six factor model which would add momentum to the five factor model.

2.3.4 Regional Differences

This Fama French 5 factor model was initially evaluated in North America by (Fama & French, 2015). However, it has also been evaluated in a variety of regions around the world.

(Fama & French, 2017) evaluates the application of their five factor model to North America, Europe, Japan and Asia Pacific from 1990 to 2015. Key findings are:

- The intercept of the three, four and five factor models was evaluated in each of the four regions. All regions obtain the lowest values of alpha from the five factor model. However, some of the values are very different (sometimes by a factor of between 2 and 3) in different regions. A different alpha means that there is a difference in the extent to which the model is capturing behaviour.
- The study found that all five factors were needed for North American share behaviour. By contrast, the investment factor CMA was found to be redundant for Europe and Japan.
- A global asset pricing model was evaluated but the results were poor.

Other researchers have also evaluated the Fama French five factor model:

- (Guo, Zhang, Zhang, & Zhang, 2017) evaluated the Fama French Five Factor asset pricing model on the Chinese stock exchange from 1995 to 2015. Evaluation of alpha for the three, four and five factor models showed a significant improvement between the three and four factor model but a much smaller improvement between the four factor and five factor model. The redundant factor was found to be investment (CMA).

Another study by (Chiah, Chai, Zhong, & Li, 2016) evaluated the five factor model for Australian equities in the time period from 1982 to 2013. Findings were:

- The intercept of the models (alpha) were found to be lower with the five factor model than the original three factor model from (Fama & French, 1992).
- The term HML (book to market) was a contributor to the explanatory power of the five factor model. This is different from the findings of (Fama & French, 2015).
- The t statistics for all of the coefficients in the five factor model are above 2 considerably more often than they are below.

One can clearly see from the discussion above that while the model may be useful for a variety of regions, it works differently in each region. Therefore, one cannot take knowledge gained in the application of the model to one region and apply it to another region. Each needs to be analysed individually. In much the same way, it is not possible to formulate a model based on a worldwide mix of assets and then expect that model to be useful and accurate when applied to a specific region like the JSE.

This is further supported by a study that was conducted by (Fama & French, 2012). In this paper the four factor model by (Carhart, 1997) which added momentum to the three factor model was analysed in four regions: North America, Asia, Japan and Asia Pacific. Similar to the global evaluation of the five factor model by (Fama & French, 2017), a combined global model did not work well when applied to specific regions. The most powerful example of a difference is that momentum is an important factor in North America, Europe and Asia Pacific while having little explanatory power in Japan.

Chapter 3: Research Questions and Hypotheses

3.1 Literature Review Summary

It makes sense to summarise the literature review in bullet point format as the starting point to formulating the research questions and hypotheses.

- The CAPM is the traditional model for asset pricing. It was initially well accepted but more recently there is growing concern, supported by a growing body of research, that it is not a good explanatory model of asset behaviour.
- One of the big concerns with the CAPM is that it does not make allowance for style investing, a widespread philosophy founded on the premise that certain characteristics of a listed company are indicative of superior returns in the future. Examples are value, market capitalisation and momentum.
- The Fama French asset pricing models provide an explanatory model that is compatible with style investing. Factors can be swapped, added or subtracted. The first model was a three factor model and this has evolved to a five factor model. This model philosophy has been evaluated by other researchers.
- The Fama French asset pricing models have been evaluated in different regions globally and the results show that regions behave differently.

3.2 Unknowns

The summary above brings to light a number of unknowns that would be useful to understand. The means of achieving this understanding will require some detailed analysis which will be discussed in chapter 4.

- How well does the CAPM provide an explanatory model of asset behaviour on the JSE? Although the CAPM has been evaluated on the JSE by (Ward & Muller, 2012) and (Carter, Muller, & Ward, 2017), it is important to be able to compare CAPM performance with Fama French asset pricing models directly. Thus the same methodology is needed.
- The factors in the Fama French five factor model can be combined in a number of ways to generate a large number of different models ranging from 2 factor models to the 5 factor model. Some of these were done in (Fama & French, 2015). How well does each perform on the JSE? Are there any factors that are very important and are there any factors that are irrelevant?
- How good are the various Fama French models compared to the CAPM on the JSE? This will help identify whether one of the Fama French asset pricing models should be used in preference to the CAPM as an asset pricing model.

3.3 Objectives, Questions, Hypotheses

Combining the literature review and the unknowns raises two research questions and two hypotheses for investigation. It should be noted that each hypothesis (evalution of alpha and R^2 respectively) overlaps both research questions. Additionally should the null hypotheses, $H1b_0$ and $H2b_0$, prove to be incorrect (there is a significant difference), then the approach taken in $H1a_0$ and $H2a_0$ of only selecting a single model from the two factor, three factor etc. would be brought into question.

Research Objectives	Research Questions	Hypothesis 1 – Values for Alpha	Hypothesis 2 – Values for R^2
Understand the application of the CAPM on the JSE and compare to Fama French asset pricing models	**Research Question 1:** How well does the CAPM explain asset returns on the JSE? Performance needs to be evaluated relative to studies of the CAPM on the JSE as well as other stock markets.	**$H1a_0$** – The null hypothesis states that there is no statistically significant difference between the means of the intercepts (alphas) of: the CAPM, one of the two factor asset pricing models, one of the three factor asset pricing models, one of the four factor asset pricing models, the five factor asset pricing model.	**$H2a_0$** – The null hypothesis states that there is no statistically significant difference between the means of the R^2 values of: the CAPM, one of the two factor asset pricing models, one of the three factor asset pricing models, one of the four factor asset pricing models, the five factor asset pricing model.
Understand the application of Fama French asset pricing models on the JSE	**Research Question 2:** How well do the various potential Fama French asset pricing models explain asset returns on the JSE? Performance needs to be evaluated relative to studies of Fama French models on the JSE as well as other stock markets.	**$H1b_0$** – The null hypothesis states that there is no statistically significant difference between the means of the intercepts (alphas) for the groupings of two factor, three factor and four factor Fama French Asset pricing models respectively. Each group of models will be evaluated individually (e.g. the two factor models will only be compared to each other).	**$H2b_0$** – The null hypothesis states that there is no statistically significant difference between the means of the R^2 values for the groupings of two factor, three factor and four factor Fama French Asset pricing models respectively. Each group of models will be evaluated individually (e.g. the two factor models will only be compared to each other).

Chapter 4: Research Methodology

4.1 Choice of Methodology

(Saunders & Lewis, 2012) use an analogy to describe research methodology which is known as the "research onion". In principle one starts at a rather abstract level considering the philosophy behind the work and working through successive layers of the onion which becomes more specific and more detailed until at the end the whole structure of the study, has been determined.

4.1.1 Philosophy

The study analysed data that was available for JSE listed companies. With reference to (Saunders & Lewis, 2012) for the various philosophies. It was a quantitative study and that meant that there were no human perceptions involved. Therefore, it was not interpretivism or critical realism.

Positivism is a very different philosophy, the scientific evaluation of cause and effect but stock market behaviour is not a simple question of cause and effect. This study of asset pricing models gave an extent of cause and effect understanding about stock market behaviour but it did not give absolute cause and effect. Therefore, this project was not positivism. Pragmatism is very much focused on the outcome needed and subsequently evaluating the best way to get the outcome. This approach results in mixed methods and mixed philosophies. Mixed methods was not what was needed for this project so the philosophy was not pragmatism. The approach taken with direct realism is "what you see is what you get". With the Fama French asset pricing models there was a certain amount of this as there was no model that fully described asset returns and one was not expected at the start of the study.

This research was a combination of philosophies. Idealistically, positivism was the ultimate goal where cause and effect were fully understood. However, some direct realism was needed, as there is no positivist model for explaining stock market behaviour.

4.1.2 Approach

The research questions were focused on understanding how well the CAPM and various Fama French asset pricing models including the five factor model (Fama & French, 2015) worked for the JSE and then comparing their relative performance.

In both cases, the approach was taking existing academic theory and aiming to understand it, and its applications, better. Therefore, the approach for this project was deductive.

4.1.3 Methodological Choices

Although there were different sets of analysis needed to extract answers to the research question, the type of quantitative analysis was very much the same from one question to the next. Therefore, the analyses were mono method.

4.1.4 Purpose of the Research Design

The study aimed to develop a better understanding of the factors that contribute to the returns that were observed on the JSE. This is at a deeper level than a simple descriptive study. The work was taking an existing model and applying it in a new environment and in a new way, therefore the purpose was explanatory.

4.1.5 Strategy

The study was going to evaluate how well theoretical models explained behaviour of shares and groups of shares on the JSE. Historic data was used to evaluate whether there was a consistent relationship between variation in the independent variables and variation in the dependent variable. This clearly implied an experimental strategy. The only difference to a traditional experimental strategy is that there would not be any cause and effect proven with 100% certainty, in this case a successful evaluation provided a certain percentage of the variation being explained and the balance being unexplained.

4.1.6 Time Horizon

The data analysis of the historic JSE data to evaluate the asset pricing models was a combination of both longitudinal and cross-sectional.

- The calculation of each Fama French factor was done on a cross-sectional basis for each time step of 1 month.
- The regression analysis to obtain the coefficients for each asset pricing model and associated t-statistics and R^2 values was done using longitudinal data. The detail of how this was done is discussed in more detail in the section on analysis approach.

4.1.7 Techniques and Procedures

The data used in the study was secondary data.

4.2 Population

The population of interest was all companies listed on the Johannesburg Stock Exchange. There were a number of reasons for this selection:

1. They are readily accessible for a would-be investor.
2. Their financials are available for scrutiny and relevant data can be extracted for analysis.
3. Share price and dividends are visible and as a result of the Efficient Markets Hypothesis originated by (Fama, 1970) it is possible to be confident that the share price is generally a good representation of the value that is in the company.

4.3 Unit of Analysis

Before considering the unit of analysis, first consider the unit of observation, this was the resolution at which the data was collected. In this case, the resolution of the data was per company per time step (in this case 1 month). For each company there were a number of measures per time step (e.g. share price, dividend, profitability etc.).

There were two different units of analysis.

- The regression analysis for the CAPM and Fama French models was done at a company level.
- The overall evaluation of CAPM and Fama French models was conducted at two levels. The first was for the top 160 companies on the JSE (very similar to the All Share Index but not exactly the same) and the second was for the top 40 companies on the JSE. The reasoning is discussed in the next section.

4.4 Sampling Size and Method

4.4.1 Sample Size

The type of study lent itself to analysing every single company for which there was data. Analysing one company's data was very similar in duration to analysing a large number of companies' data. However, there were two reasons why the sample size should not have been the full population.

1. (Muller & Ward, 2013) state that at the time of their study there were more than 350 companies on the JSE. However as of 30 April 2018, the All Share Index (ALSI) had 165 companies and covered 99% of the market capitalisation on the JSE (JSE, 2018). The companies outside the All Share Index are so small that they are to a large extent irrelevant and of little interest to bigger investors.

2. In the research done by (Fama & French, 2015), one of the portfolio sorts that they conducted divided their full set of companies into quintiles of size. The quintile of the smallest shares was called "microcap". Their data analysis considered all of the data and then compared that with results excluding the microcap portfolio. The results showed that the model achieved better results when the microcap portfolio was excluded.

As a result of both of these considerations, it was decided that the sample size should be the top 160 shares on the JSE based on market capitalisation. This is very close to the number of companies in the JSE All Share Index (around 165).

A second sample was done with the top 40 shares. Top 40 shares are highly traded, large and subject to a great deal of scrutiny. This makes them likely to be a stronger follower of the efficient markets hypothesis (Fama, 1970) than the smaller shares in the rest of the top 160. Comparing the two would be interesting, whatever the result obtained.

4.4.2 Sample Method

The sample size and qualification criteria for being in the categories defined above were very specific. The author would argue that this was not sampling at all but a census of a certain sub-group of companies within the full population of JSE listed companies.

However, if one wanted to give a sampling classification one would have done the following:

1. Sorted all of the companies on the JSE from largest to smallest (based on market capitalisation).
2. Applied a systematic sampling method but:
    a. Not selected the first one at random, rather selected the top company (in the current environment it would be Naspers)
    b. Instead of sampling 1 in 5 or 1 in 10 sampled 1 in 1
3. Continued the systematic sample until the target number of companies (40 and 160 respectively) was reached.

4.5 Measurement Instrument

This study used secondary data. Therefore, individuals and organisations external to the researcher developed the measurement instrument. It was important that there was confidence that the measurement instrument would give data that was both valid and reliable. Let us consider the definitions as they relate to data collection (Saunders & Lewis, 2012).

- "Validity – the extent to which data collection method or methods accurately measure what they were intended to measure"
- "Reliability – the extent to which data collection methods and analysis procedures will produce consistent findings"

The data for each company was obtained from two sources: firstly, from company financials and secondly from the JSE. Let us consider these two sources of data against the requirements of validity and reliability.

4.5.1 Company Financials

Company financials are populated with data from a company's own internal accounting activities, this is then verified by independent auditors.

Validity

The validity of this data is dependent on the company's internal accounting methods tracking items correctly and the auditors being effective at ensuring that the company's accounting methods are correct. In the 2014-2015 Global Competitiveness Index (Schwab, 2015) South Africa was ranked number 1 in the world for auditing and reporting standards. While there are examples of inaccuracies and errors by both auditors and company accounting departments, the fact that South Africa ranked so high means that it was reasonable to be confident in the validity of the data overall.

Reliability

This is dependent on a company's accounting methods remaining consistent from one year to the next and from one accountant to the next. Consistency is a core principle of accounting (Wüstemann & Wüstemann, 2010). Once again, South Africa's world number 1 ranking for auditing and reporting standards (Schwab, 2015), means that accounting consistency is in all likelihood applied in South African listed firms. Consistency ensures reliability of the data.

4.5.2 Stock Exchange Data

The JSE provides a platform for shares to be traded and publishes the detail about the transactions that have occurred.

Validity & Reliability

The measurement instruments employed by the JSE to report that information are also their internal accounting practices and this is verified by auditors. The JSE was audited by KPMG in 2016 and made a decision, in the interests of regular auditor rotation, to change auditors for 2017 (JSE, 2017). With further reference to the Global Competitiveness Index (Schwab, 2015), South Africa was ranked number 1 in the world for regulation of securities exchanges. Therefore, there is a high likelihood that the JSE data was both valid (accurate) and reliable (repeatable).

4.5.3 Data Cleaning

The author spent time examining data that was extracted from well-recognised commercial sources. Upon examination, it became clear that the data was compromised in a number of ways. Just two examples are:

1. Survivorship bias. Explained in (Investopedia, 2018b) the JSE All Share Index (ALSI) is an example of this. If one looks at the ALSI in the present and then follows those member companies back in time, 20 years ago, there were a large number of companies in the ALSI in 1998 that don't exist now. Therefore, the performance of the failures in the ALSI in history would be missed.
2. Missing data. There was a certain amount of missing data which the database did not have available.

The challenge of cleaning a set of data was significant and would've taken an impractical amount of time for the research project. In addition, if the data was not cleaned correctly, the results would not have added to the academic body of knowledge. As a result, the data set that was used was the one that was originally developed for (Muller & Ward, 2013) and has been maintained to the present day to ensure that it is clean, up to date and available for further research.

The above two issues of data availability and survivorship bias were addressed in this data set. In addition, as per (Muller & Ward, 2013), the following process was applied to ensure clean data.

- The database contained information about all of the companies that were listed on the JSE at each time interval. This made it possible to correct for survivorship bias.
- Spin-offs were manually managed to occur at the end of the current quarter.
- Dividends and scrip dividends were included in share returns.

- Newly listed shares were manually “started” at the beginning of the next quarter. The same principle was followed for delisted shares.
- Name changes were manually tracked through from the old company to the new company.
- The data was examined for data errors (spurious trades at much higher or lower values).

4.6 Data Gathering Process

The data used by (Muller & Ward, 2013) was obtained from two sources. The JSE was accessed for share information and company financial information was obtained from IRESS (used to be INET).

4.7 Analysis Approach

As per (Muller & Ward, 2013), the data is stored in an Access database. Depending on what was required for each calculation, Excel VBA code was written to extract the relevant data from the database, conduct the analysis and then save and display the results. The analysis approach was strongly based on (Fama & French, 2015). This made it possible to be confident in the work that was done as well as review results from other geographical regions which were analysed in a similar way. For ease of reference, the Fama French 5 factor model is repeated:

$$R_{i,t} - R_{f,t} = a_i + b_i\left(R_{m,t} - R_{f,t}\right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{i,t}$$

4.7.1 Fama French Factor Calculation

The data set that was used for the calculation was set up to only work with the largest 160 shares at the point in time of interest.

The factor calculations were done in three different ways in (Fama & French, 2015). All three methods produced very similar end results. As a result, the method that was more technically correct was chosen. This method was the 2 x 2 x 2 x 2 sort and worked as follows for calculating the sub-portfolio SHRC (explained in *Figure vi: Calculation of 2 x 2 x 2 x 2 sub-portfolios and resulting factor calculation*):

1. The 160 shares were ranked in four different ways simultaneously.
   a. From *Smallest* to *Biggest* (measured by market capitalisation) and then split at the median.

b. From *Highest* value to *Lowest* value (measured by book to market ratio) and this was split at the median.
c. From *Robust* profitability to *Weak* profitability (measured by return on assets) and this was split at the median.
d. From *Conservative* investment to *Aggressive* investment (measured by change in total assets from one year to the next) and this was split at the median.

2. The portfolio SHRC was then formulated from all shares that were simultaneously in the: Small market cap section, High value section, Robust profitability section and Conservative investment section.

The above methodology was followed to generate 16 portfolios in total. Thanks to the simultaneous sort used to create the portfolios, it was impossible that a particular share could be in two or more portfolios at the same time. This is an advantage over the other methods that were used by (Fama & French, 2015), three 2 x 3 sorts and three 2 x 2 sorts.

The portfolios were value weighted as per (Fama & French, 2015) and then the return for each portfolio was calculated on a month-to-month basis. *Figure* vi*: Calculation of 2 x 2 x 2 x 2 sub-portfolios and resulting factor calculation* below provides a clear indication of each of the 16 portfolios and how their returns were combined to obtain the final factors by (Fama & French, 2015).

$2 \times 2 \times 2 \times 2$ sorts on *Size*, *B/M*, *OP*, and *Inv*	*Size*: NYSE median	$SMB = (SHRC + SHRA + SHWC + SHWA + SLRC + SLRA + SLWC + SLWA)/8 - (BHRC + BHRA + BHWC + BHWA + BLRC + BLRA + BLWC + BLWA)/8$
	B/M: NYSE median	$HML = (SHRC + SHRA + SHWC + SHWA + BHRC + BHRA + BHWC + BHWA)/8 - (SLRC + SLRA + SLWC + SLWA + BLRC + BLRA + BLWC + BLWA)/8$
	OP: NYSE median	$RMW = (SHRC + SHRA + SLRC + SLRA + BHRC + BHRA + BLRC + BLRA)/8 - (SHWC + SHWA + SLWC + SLWA + BHWC + BHWA + BLWC + BLWA)/8$
	Inv: NYSE median	$CMA = (SHRC + SHWC + SLRC + SLWC + BHRC + BHWC + BLRC + BLWC)/8 - (SHRA + SHWA + SLRA + SLWA + BHRA + BHWA + BLRA + BLWA)/8$

Figure vi: Calculation of 2 x 2 x 2 x 2 sub-portfolios and resulting factor calculation (Fama & French, 2015)

The factors that were finally produced, calculates the difference between the average returns of the one set of portfolios and the average returns of the other set of portfolios as per the Figure vi: Calculation of 2 x 2 x 2 x 2 sub-portfolios and resulting factor calculation .

It should be noted that the measure of profitability was slightly different from that used in (Fama & French, 2015). Their measure was operating profit over book equity, the measure used in this research was return on assets (ROA). Understanding whether this difference in profitability measurement method gives different results could be worth further work.

The (Fama & French, 2015) analysis calculated the shares which belonged in each portfolio for the factor calculations annually. However, in the interests of obtaining a more accurate data set it was decided to do the portfolio rebalancing every 3 months. The analysis was done for a period of 20 years.

For the same time period the CAPM factor Rm – Rf was calculated (the return for the market minus the risk free rate).

4.7.2 Linear Regression for CAPM and Fama French Models

Once the Fama French factors had been calculated the next question was what models were of interest. The simplest model is the CAPM, it only contains one factor which is the difference between the market return and the risk free rate, so this needed inclusion. In (Fama & French, 2015) the researchers did not do the CAPM but analysed 5 models in total: 1 three factor model, 3 four factor models and 1 five factor model. In the interests of completeness, it was decided to do all possible models (with the CAPM factor in every model but the rest being able to be included or not). This resulted in 15 different models.

CAPM	(1 factor, R_m - R_f)
CAPM + HML	(2 factor)
CAPM + SMB	(2 factor)
CAPM + CMA	(2 factor)
CAPM + RMW	(2 factor)
CAPM + HML SMB	(3 factor)
CAPM + HML CMA	(3 factor)
CAPM + HML RMW	(3 factor)
CAPM + SMB CMA	(3 factor)
CAPM + SMB CMA	(3 factor)
CAPM + CMA RMW	(3 factor)
CAPM + HML SMB CMA	(4 factor)
CAPM + HML SMB RMW	(4 factor)
CAPM + SMB CMA RMW	(4 factor)
CAPM + HML SMB CMA RMW	(5 factor)

The linear regression for each of the 15 models for all 160 companies was run every 3 months for the preceding 36 months. It should be noted that (Fama & French, 2015) ran their regressions over 606 months. According to (Jagannathan & Wang, 1993) a shorter time period gives a better chance for the CAPM to give good results so in fairness to the CAPM it was decided to calculate over 36 months. There are positives and negatives for choosing a short or long timeline and investigating this would be valuable further work.

The first set of regressions was run on 31 December 2001 and the last one was run on 30 June 2018. The total number of linear regressions was:

160 shares x 15 asset pricing models x 67 time steps = 160 800

4.7.3 Research Questions 1 & 2: Evaluation of CAPM and Fama French Models

Each linear regression produced three pieces of information for evaluating how good each model was: alpha (the model's intercept), the t-statistics for each term, and R^2 (how much of the variation in the dependant variable was explained by the whole model). The time history of linear regression results was evaluated graphically and with descriptive statistics.

Evaluation of alpha

A well-recognised measure of the accuracy of asset pricing models (Fama & French, 2015) is alpha. This is the intercept of the model when all of the other factors are 0. A non-zero value for alpha implies that there is a consistent movement in the share price which is not explained by the model and this would raise questions about the quality of the model.

The values of alpha from the model were compared with typical values seen in Fama French 5 factor model evaluations done in various regions around the world: North America, Europe, Japan, Asia Pacific, Australia and China by (Fama & French, 2017), (Guo, Zhang, Zhang, & Zhang, 2017) and (Chiah, Chai, Zhong, & Li, 2016). These were good benchmarks upon which it was possible to quantify the accuracy of the relevant asset pricing model on the JSE. Values for alpha were averaged over the cross section for all 160 shares and the top 40 shares.

Evaluation of t statistics

One question that needed to be considered is what value of t statistic would one consider to be statistically significant. One proposal was made by (Harvey, Liu, & Zhu, 2016) who stated that in research topics focused on the cross section of expected for stock exchange behaviour, a factor that was well recognised by the academic community could be regarded as being significant at a t statistic value of 2.0 or above. However, for a proposed new factor the threshold for the t statistic should be raised to 3.0. Values for t statistics were averaged over the cross section for all 160 shares and the top 40 shares.

Evaluation of R^2

The value of R^2 is an overall measure which could be said to be a “combination” of all of the t statistics for that model. As a result, it’s value is in generating an overall feel for the quality of the model rather than the more precise detail that can be obtained from the t-statistics. Based on (Jagannathan & Wang, 1993) it was decided that for asset pricing models an R^2 above 40% was very good while one of less than 20% was not very good. It should be noted that in different studies a value of 40% would be regarded as not being very good but the complexity of a stock market makes a high value of R^2 unrealistic.

4.7.4 Hypotheses 1 & 2: Statistical Comparison of CAPM & Fama French Models

Statistical Evaluation

As per the individual hypotheses, the performance of the models were evaluated in two ways. The first was the value of the intercept alpha (hypothesis H1) and the second was the R^2 value (hypothesis H2). However the methodology was exactly the same. Statistical analysis was conducted in SPSS.

1. For each model at each time step the values for R^2 and alpha were averaged over the top 160 shares and the top 40 shares. Thus, each asset-pricing model had 2 sets of 67 values for alpha and 2 sets of 67 values for R^2.
2. This was plotted graphically which gave an excellent visual understanding of the differences between one model and the next.
3. Homogeneity of variances was tested and this indicated what type of correction was needed for the post-hoc analysis (if this was necessary).
4. An ANOVA single factor test was done, the only factor that was varying was the model being used. This was to ascertain whether there was a difference between one of the means and any of the other means at a 95% confidence level.
5. Once a significant difference had been identified, a post hoc analysis was evaluated.

   i. For equal variances the Bonferroni correction was used.
   ii. For unequal variances, the Games Howell correction was used.
6. The results for each test showed which means had a significant difference and which did not have a significant difference at a 95% confidence level.

$H1a_0$ and $H2a_0$ Approach

For these hypotheses, a limited number of models were analysed. The reasoning behind selection of the model is given.

- The *CAPM*
- A *two factor model*. The work by (Muller & Ward, 2013) found that the size effect was not significant. Therefore, the decision was taken to leave this out of the three factor model by (Fama & French, 1992). The two factor model contains Rm – R and HML.
- The *three factor model* from (Fama & French, 1992). The model contains Rm – Rf, SMB and HML.
- A *four factor* model. When the five-factor model was studied in (Fama & French, 2015), the results indicated that HML was an unnecessary factor as it did not improve the quality of the model. Therefore, it also made sense to evaluate this model. The model contained Rm – Rf, SMB, CMA, RMW. Additionally, this was the model used by (Hou, Xue, & Zhang, 2015).
- The *five-factor model* from (Fama & French, 2015). The model contained Rm – Rf, SMB, HML, CMA & RMW.

$H1b_0$ and $H2b_0$ Approach

To expand on the explanation in chapter 3, the idea was to evaluate whether the results for all of the models that contain the same number of factors are different at a 95% confidence level. The following results were compared:

- The 4 two factor models
- The 6 three factor models
- The 3 four factor models

Chapter 5: Results

5.1 Fama French Factor Calculation

5.1.1 Sub - Portfolio Content

The 2 x 2 x 2 x 2 sort that was employed is the most technically correct as it ensures that one share cannot be in more than one sub-portfolio. The concern with this method is that the shares under consideration are split into 16 different sub-portfolios. If there are not enough shares in each portfolio, the returns generated within that portfolio can be heavily skewed by a single result. The numbers of shares within each individual portfolio is plotted below.

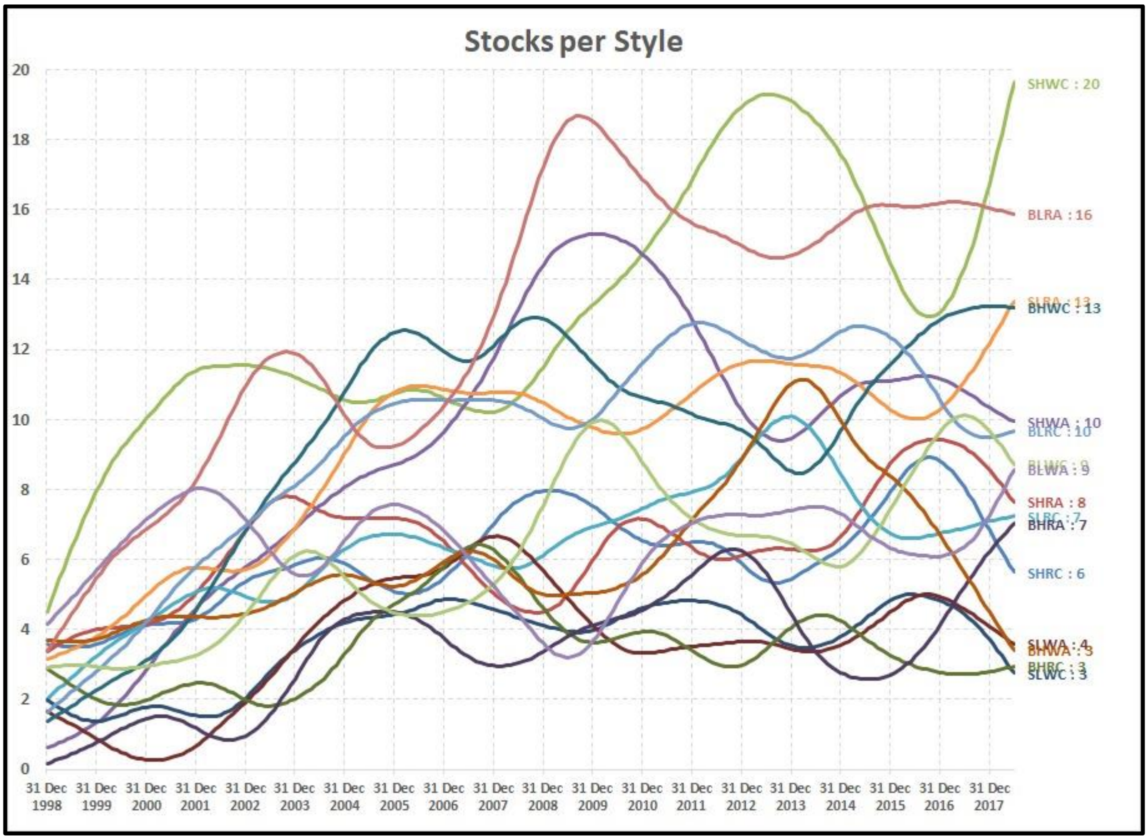

Figure vii: Number of shares in each sub-portfolio

5.1.2 Sub – Portfolio Style Measures

Next, it is necessary to ensure that the sub-portfolios that are going to be combined to give the calculation for each individual factor do indeed represent the right information when they are re-combined. In the interests of brevity, these plots are shown in Appendix 1 for each of the four factors that need to be calculated in that way: SMB, HML, RMW and CMA. The plots show returns for each sub portfolio when measured according to that specific factor's measure.

5.1.3 Sub - Portfolio Returns

The returns that were generated within each sub-portfolio are shown in the Figure *viii*: Sub - portfolio cumulative returns generated over a 20 year period below. It is important to note that these results are cumulative on the graph. Reading a cumulative graph is extremely informative but one needs to take some care with it. This provides one with an initial indication of which styles and combinations of styles will give the best returns. The data that this graph shows is used on a month-to-month return basis to calculate the values for the Fama French factors.

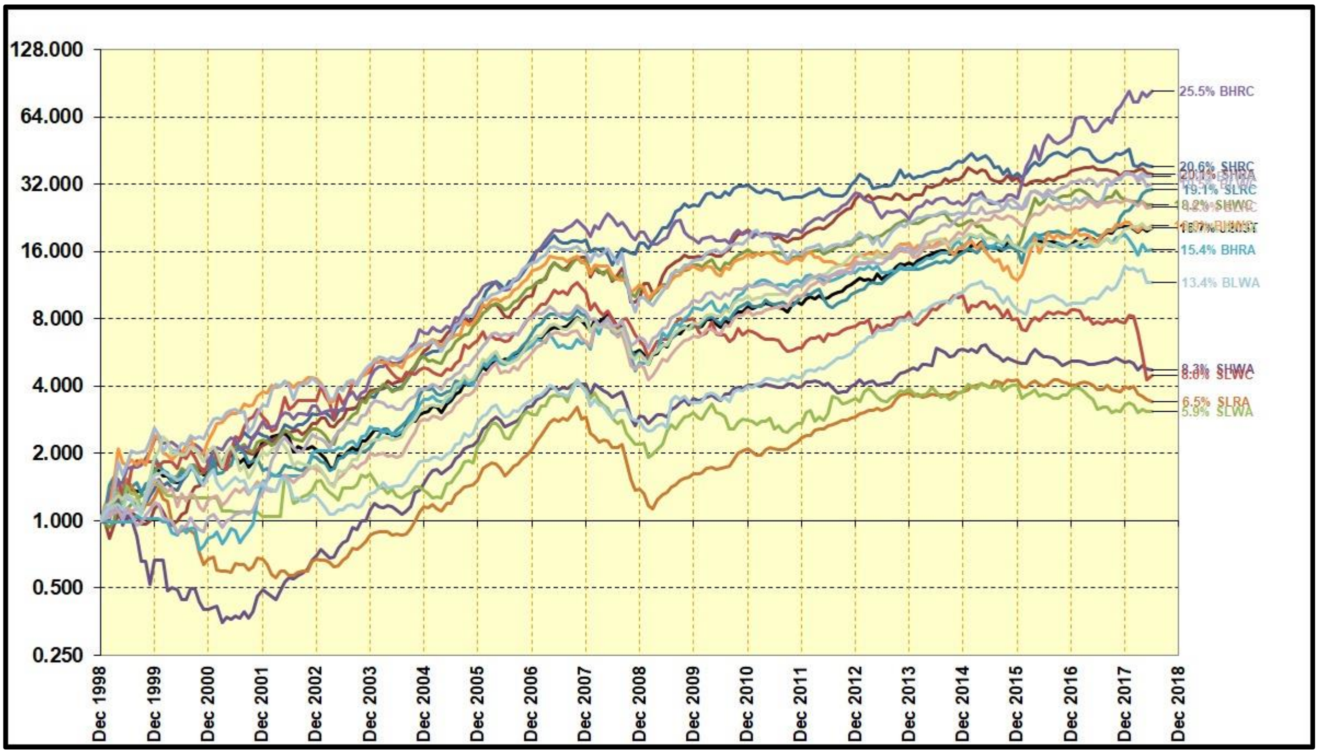

Figure viii: Sub - portfolio cumulative returns generated over a 20 year period

5.1.4 Fama French Factors

Below is a graphical representation of the cumulative returns of each Fama French factor. Once again, one needs to be careful of the fact that this graph is cumulative. For example, when the grey RMW graph possesses a positive gradient, then shares with a robust profitability have performed better than shares with weak profitability. The opposite also applies.

This cumulative graph is a very good way to give a visual representation of what has happened to each Fama French factor over the full period of the analysis. However, when the values for SMB, HML, RMW and CMA are used in linear regressions, the monthly returns are used.

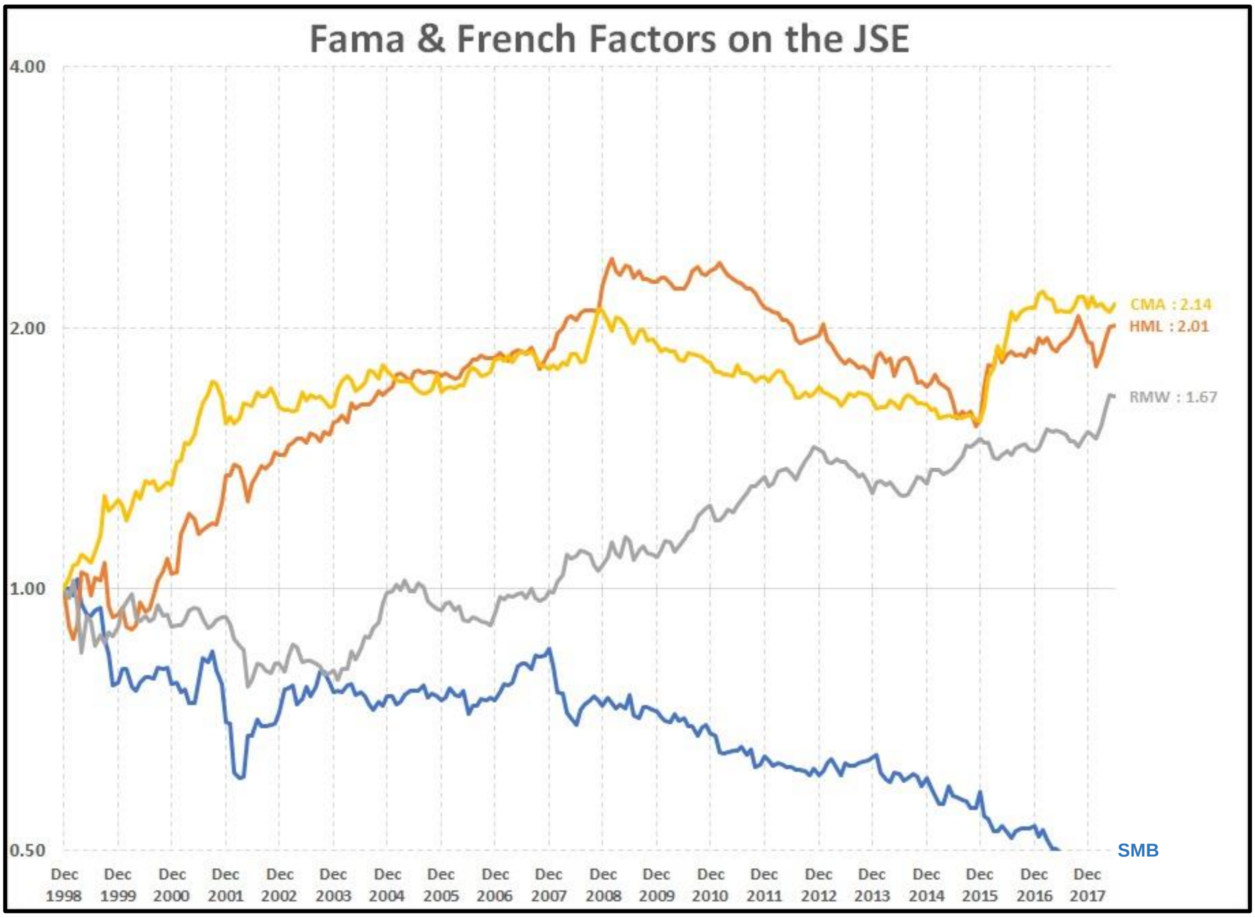

Figure ix: Cumulative returns for each Fama French factor (blue is SMB)

5.2 Research Question 1: CAPM on the JSE

The cross sectional data was averaged in two ways. The first was for the top 160 shares on the JSE and the second top 40 shares on the JSE.

5.2.1 CAPM Alpha

The time history of the average values for alpha when regressed using the CAPM are shown in the Figure *x*: Time history of alpha as per the CAPM below. A second plot for the Top 40 companies is in Appendix 2.

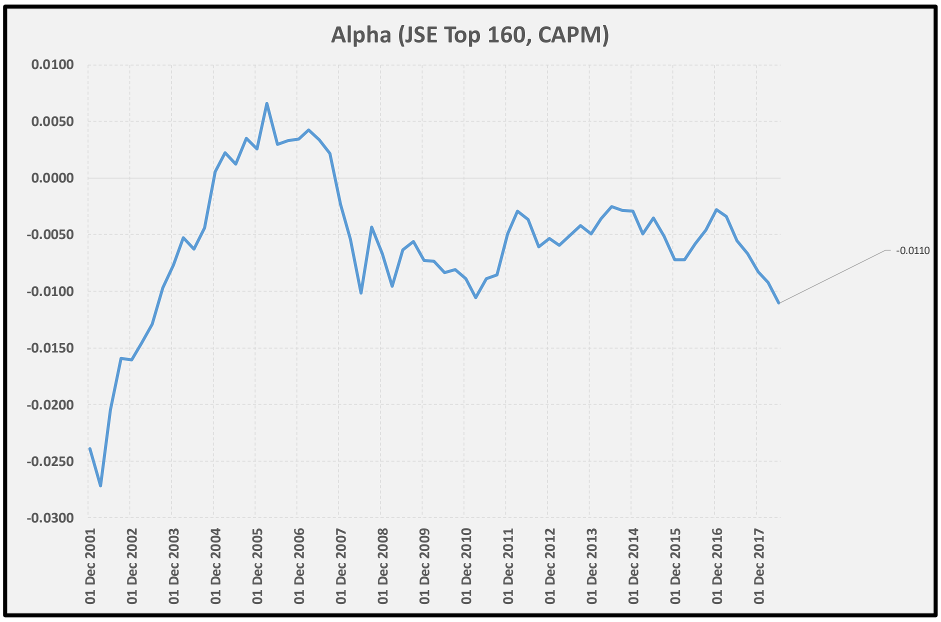

Figure x: Time history of alpha as per the CAPM

The descriptive statistics for alpha over the above period are as per Table *1*: Descriptive statistics for the CAPM Alpha (Top 160) below.

Table 1: Descriptive statistics for the CAPM Alpha (Top 160)

Model	Mean	Median	Standard Dev.
CAPM	-0.0058	-0.0054	0.0062

Table 2: Descriptive statistics for the CAPM Alpha (Top 40)

Model	Mean	Median	Standard Dev.
CAPM	-0.0067	-0.0064	0.0033

The associated histogram for the top 160 shares is shown below while the histogram for the top 40 is in appendix 2.

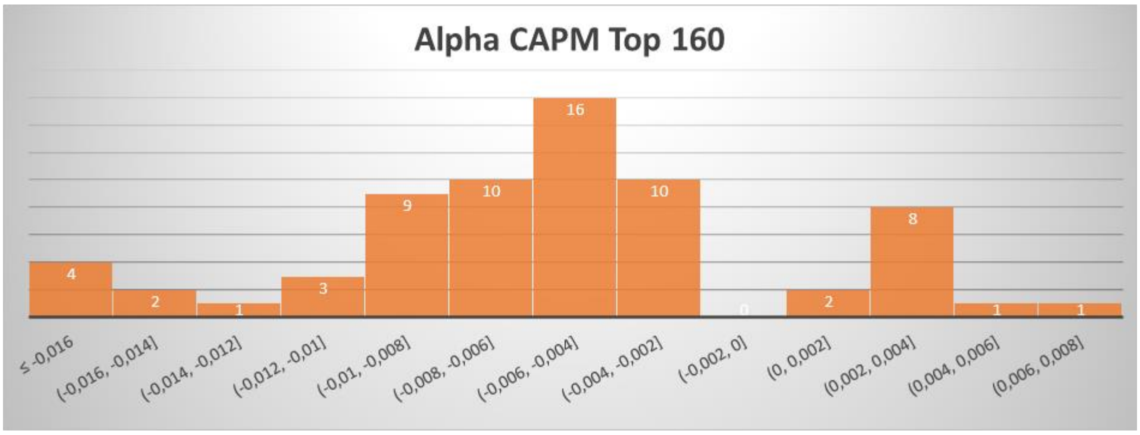

Figure xi: Histogram of Alpha values (Top 160)

5.2.2 CAPM T-statistics

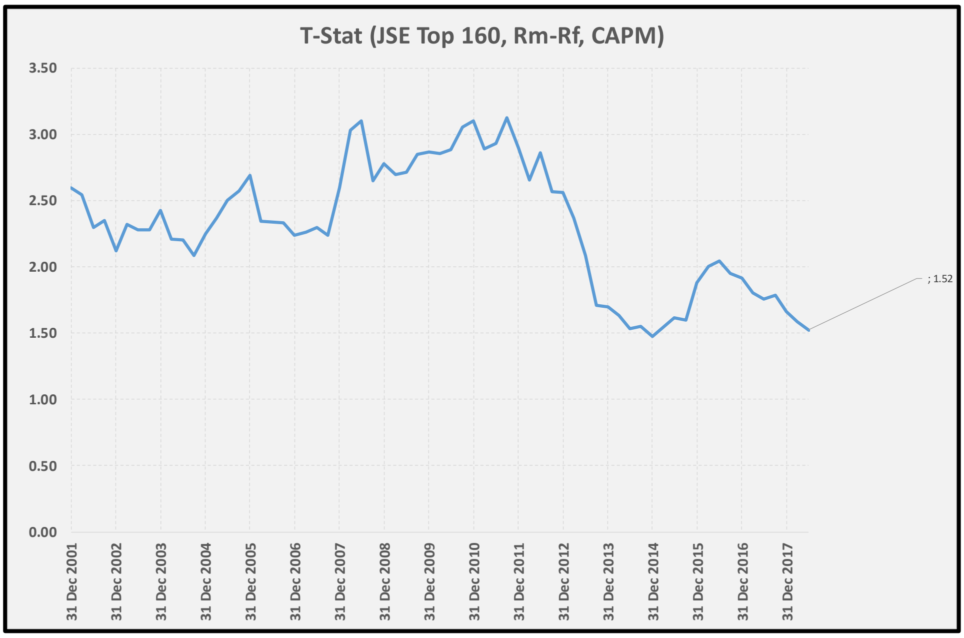

Figure xii: T-statistic of Rm-Rf coefficient (beta) for the CAPM (Top 160)

The above Figure *xii*: T-statistic of Rm-Rf coefficient (beta) for the CAPM (Top 160) tracks the t-statistic of Beta as it varies through time. It has been calculated according to the methodology detailed in chapter 4 (previous 36 months of data). The descriptive statistics for this is as follows.

Table 3: T-statistics for the CAPM Beta (Top 160)

Model	Mean	Median	Standard Dev.
CAPM	2.3074	2.3200	0.4707

Table 4: T-statistics for the CAPM Beta (Top 40)

Model	Mean	Median	Standard Dev.
CAPM	3.6124	3.7467	0.6103

The associated histogram is as follows. The histogram for the top 40 shares can be found in Appendix 2:

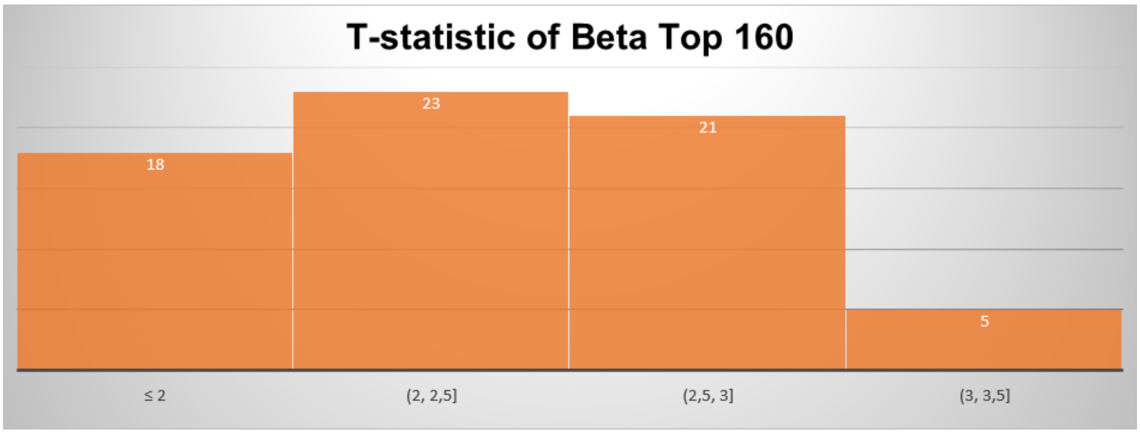

Figure xiii: Histogram of t-stat values for Beta (Top 160)

5.2.3 CAPM R^2

The time history of R^2 is plotted below. Once again, the values are based on a linear regression performed over the previous 36 months. The graph for the top 40 shares is shown in appendix 2.

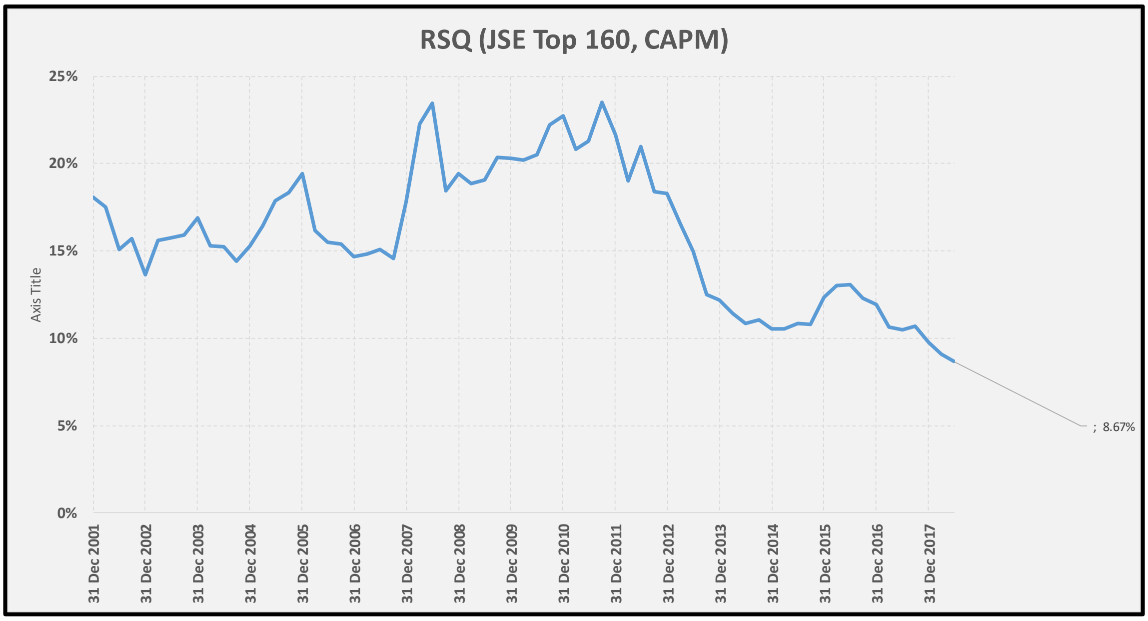

Figure xiv: R^2 for the CAPM (Top 160)

The descriptive statistics for the time history of R^2 are as follows:

Table 5: R^2 for the CAPM Beta (Top 160)

Model	Mean	Median	Standard Dev.
CAPM	0.1592	0.1558	0.0391

Table 6: R^2 for the CAPM Beta (Top 40)

Model	Mean	Median	Standard Dev.
CAPM	0.2752	0.2912	0.0593

Below is the histogram for the R^2 value of the top 160 shares. The graph for the top 40 can be found in appendix 2.

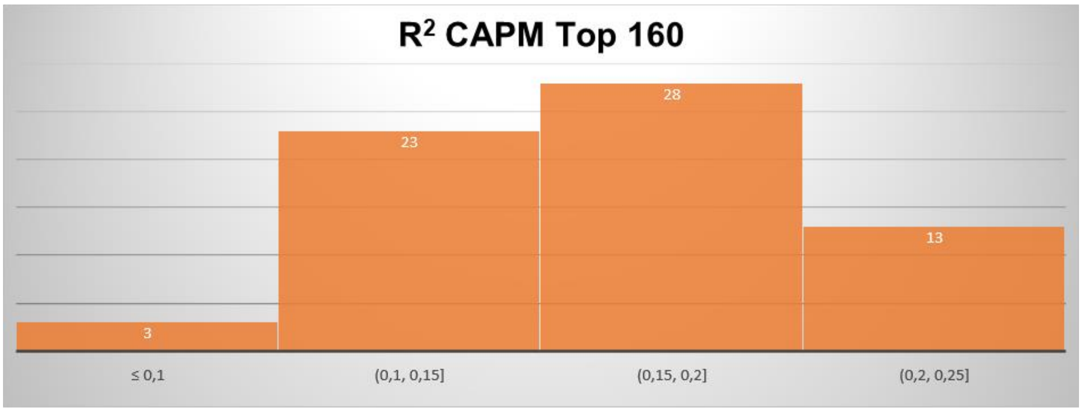

Figure xv: Histogram of R^2 values for CAPM (Top 160)

5.3 Research Question 2: Fama French on the JSE

5.3.1 Fama French Alpha

The time history of the alpha values for the top 160 shares is in the Figure *xvi*: Time history of alpha as per the various FF models below. The graph for the top 40 shares is in appendix 3.

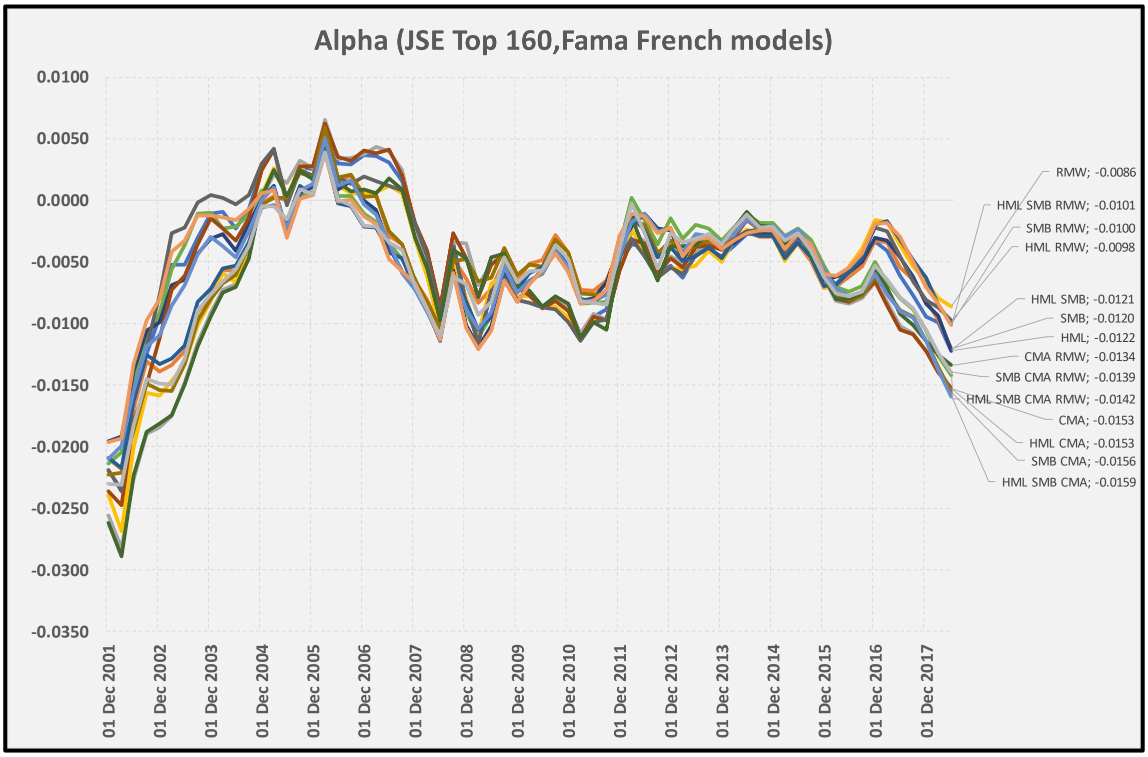

Figure xvi: Time history of alpha as per the various FF models

The descriptive statistics are as follows:

Table 7: Alpha for the Fama French models (Top 160)

Model	Mean	Median	Standard Dev.
CAPM + HML	-0.0050	-0.0045	0.0055
CAPM + SMB	-0.0055	-0.0049	0.0049
CAPM + CMA	-0.0063	-0.0054	0.0069
CAPM + RMW	-0.0060	-0.0054	0.0058
CAPM + HML SMB	-0.0052	-0.0043	0.0044
CAPM + HML CMA	-0.0053	-0.0047	0.0061
CAPM + HML RMW	-0.0047	-0.0045	0.0053
CAPM + SMB CMA	-0.0059	-0.0049	0.0055
CAPM + SMB RMW	-0.0058	-0.0049	0.0046
CAPM + CMA RMW	-0.0065	-0.0055	0.0066
CAPM + HML SMB CMA	-0.0057	-0.0054	0.0049
CAPM + HML SMB RMW	-0.0049	-0.0040	0.0043
CAPM + SMB CMA RMW	-0.0062	-0.0056	0.0052
CAPM + HML SMB CMA RMW	-0.0052	-0.0041	0.0048

Table 8: Alpha for the Fama French models (Top 40)

Model	Mean	Median	Standard Dev.
CAPM + HML	-0.0061	-0.0060	0.0029
CAPM + SMB	-0.0069	-0.0073	0.0032
CAPM + CMA	-0.0072	-0.0075	0.0035
CAPM + RMW	-0.0067	-0.0071	0.0033
CAPM + HML SMB	-0.0061	-0.0063	0.0029
CAPM + HML CMA	-0.0063	-0.0069	0.0032
CAPM + HML RMW	-0.0058	-0.0054	0.0028
CAPM + SMB CMA	-0.0072	-0.0071	0.0036
CAPM + SMB RMW	-0.0070	-0.0069	0.0029
CAPM + CMA RMW	-0.0073	-0.0078	0.0032
CAPM + HML SMB CMA	-0.0064	-0.0060	0.0035
CAPM + HML SMB RMW	-0.0057	-0.0057	0.0027
CAPM + SMB CMA RMW	-0.0073	-0.0073	0.0033
CAPM + HML SMB CMA RMW	-0.0059	-0.0059	0.0034

The histograms for all of these models for the two portolios (top 160 and top 40) can be found in appendix 3.

5.3.2 Fama French T-Statistics

There are t-statistics available for each of the five factors in the Fama French models but not all of the models use all of the factors. The first t-statistic is for the R_m-R_f coefficient. The graph for the top 40 can be seen in appendix 3.

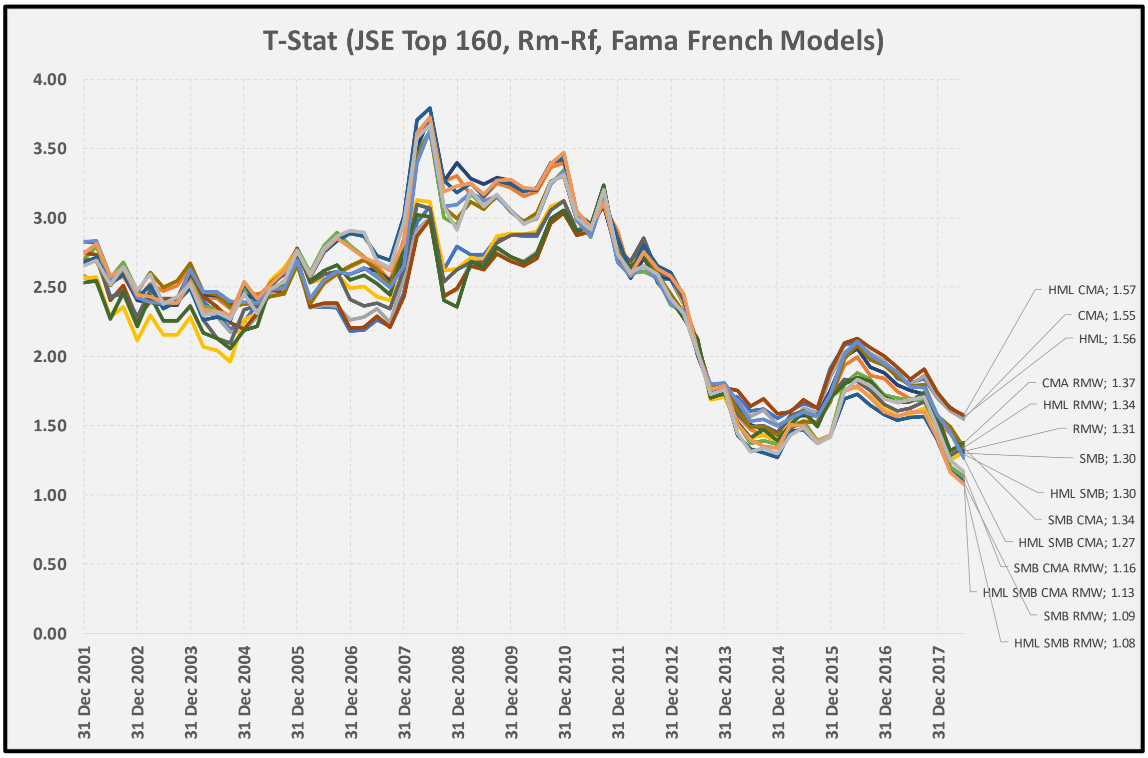

Figure xvii: Time history of t-statistic for the Rm-Rf coefficient for Fama French models (Top 160)

Table 9: T-Statistics for the Fama French models' Rm-Rf coefficient (top 160)

Model	Mean	Median	Standard Dev.
CAPM + HML	2.3414	2.3807	0.4554
CAPM + SMB	2.4337	2.5280	0.6172
CAPM + CMA	2.3002	2.3485	0.4332
CAPM + RMW	2.2766	2.3850	0.5448
CAPM + HML SMB	2.4471	2.5453	0.6096
CAPM + HML CMA	2.3206	2.3867	0.4133
CAPM + HML RMW	2.3062	2.4120	0.5203
CAPM + SMB CMA	2.4172	2.5133	0.5655
CAPM + SMB RMW	2.4085	2.5281	0.6978
CAPM + CMA RMW	2.2725	2.4073	0.5012
CAPM + HML SMB CMA	2.4165	2.4833	0.5561
CAPM + HML SMB RMW	2.4132	2.5778	0.6765
CAPM + SMB CMA RMW	2.3884	2.5214	0.6417
CAPM + HML SMB CMA RMW	2.3837	2.5098	0.6187

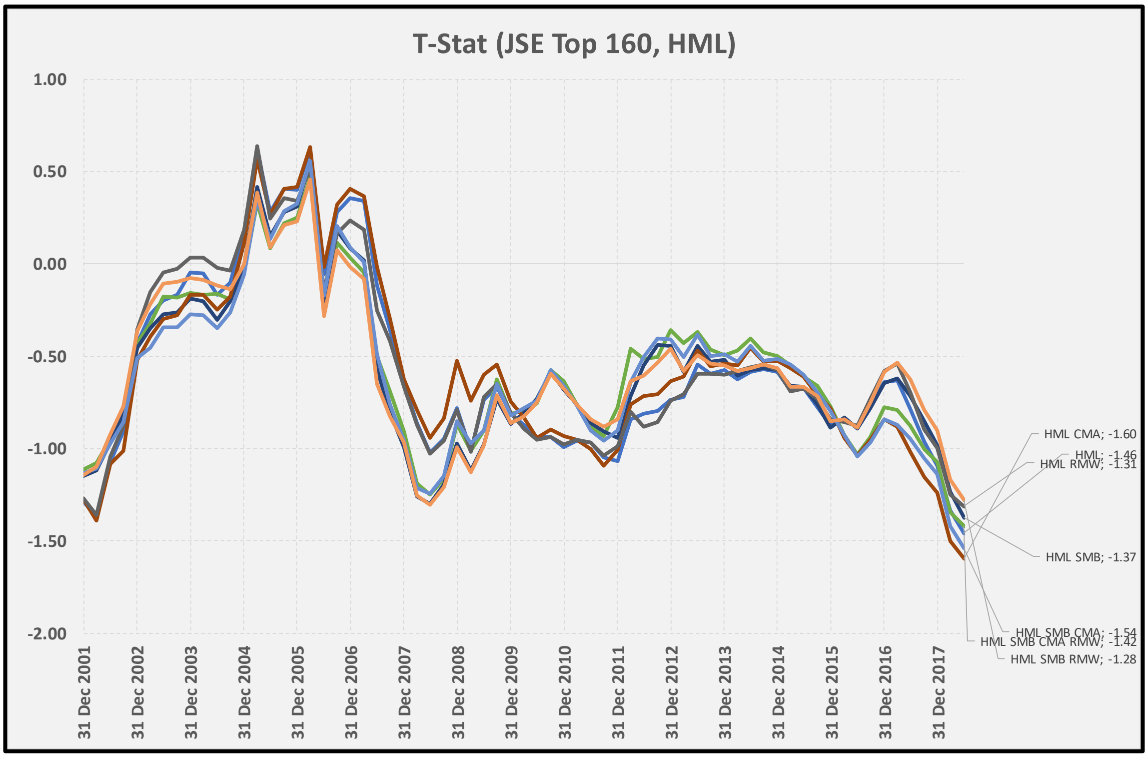

Figure xviii: Time history of t-statistic for the HML coefficient for Fama French models (Top 160)

Table 10: T-Statistics for the Fama French models' HML coefficient (top 160)

Model	Mean	Median	Standard Dev.
CAPM + HML	-0.5786	-0.7190	0.4947
CAPM + HML SMB	-0.5968	-0.6607	0.4273
CAPM + HML CMA	-0.5777	-0.6231	0.5024
CAPM + HML RMW	-0.5667	-0.6933	0.4758
CAPM + HML SMB CMA	-0.6069	-0.6414	0.4351
CAPM + HML SMB RMW	-0.5827	-0.6364	0.4134
CAPM + HML SMB CMA RMW	-0.5791	-0.6094	0.4189

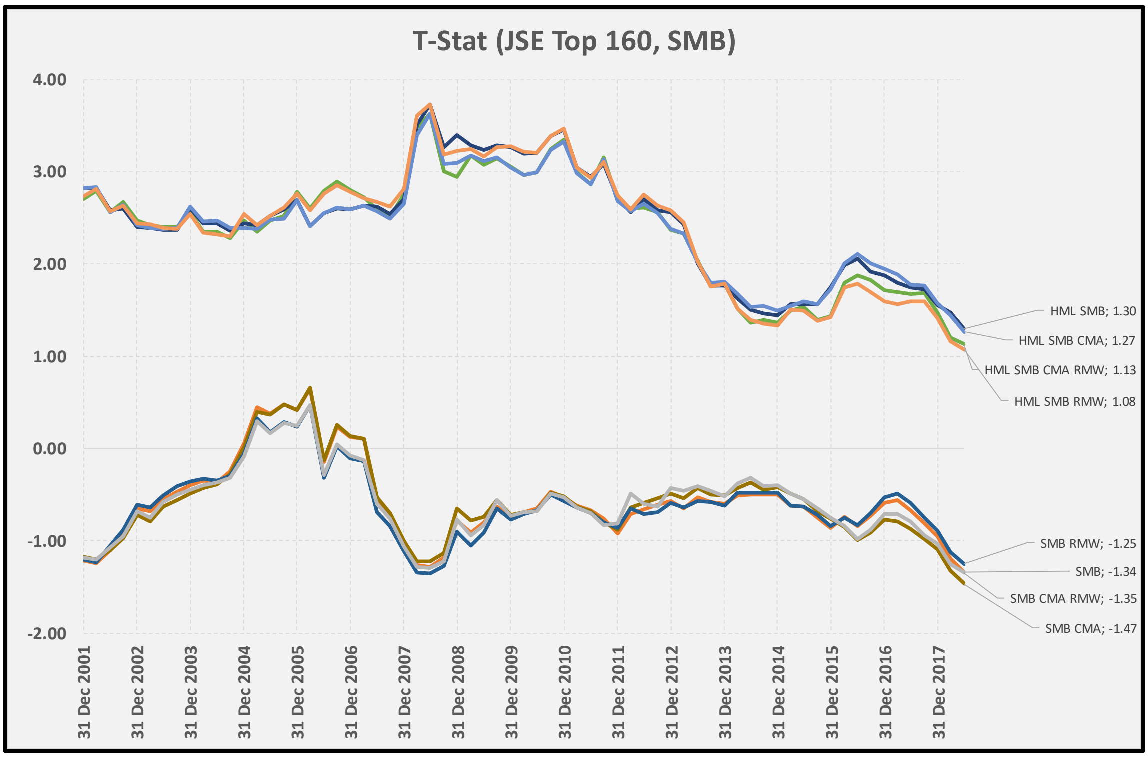

Figure xix: Time history of t-statistic for the SMB coefficient for Fama French models (Top 160)

Table 11: T-Statistics for the Fama French models' SMB coefficient (top 160)

Model	Mean	Median	Standard Dev.
CAPM + SMB	-0.2838	-0.2735	0.2174
CAPM + HML SMB	3.2836	3.4574	0.6138
CAPM + SMB CMA	-0.2990	-0.2992	0.2405
CAPM + SMB RMW	-0.3066	-0.2751	0.2047
CAPM + HML SMB CMA	3.2482	3.3711	0.5679
CAPM + HML SMB RMW	3.1988	3.4006	0.7035
CAPM + SMB CMA RMW	-0.3148	-0.3046	0.2223
CAPM + HML SMB CMA RMW	3.1619	3.3260	0.6448

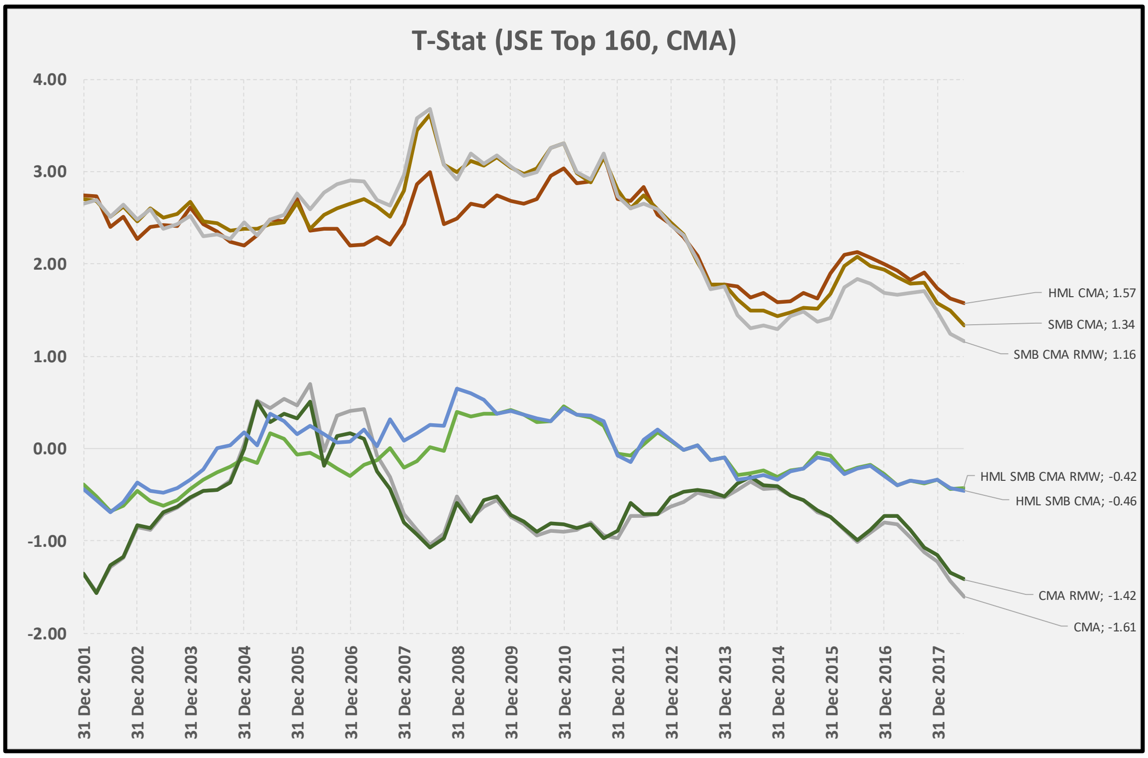

Figure xx: Time history of t-statistic for the CMA coefficient for Fama French models (Top 160)

Table 12: T-Statistics for the Fama French models' CMA coefficient (top 160)

Model	Mean	Median	Standard Dev.
CAPM + CMA	-0.6045	-0.7083	0.5105
CAPM + HML CMA	2.3206	2.3867	0.4133
CAPM + SMB CMA	2.4172	2.5133	0.5655
CAPM + CMA RMW	-0.6108	-0.6938	0.4489
CAPM + HML SMB CMA	-0.0287	0.0050	0.3220
CAPM + SMB CMA RMW	2.3884	2.5214	0.6417
CAPM + HML SMB CMA RMW	-0.1146	-0.1333	0.2878

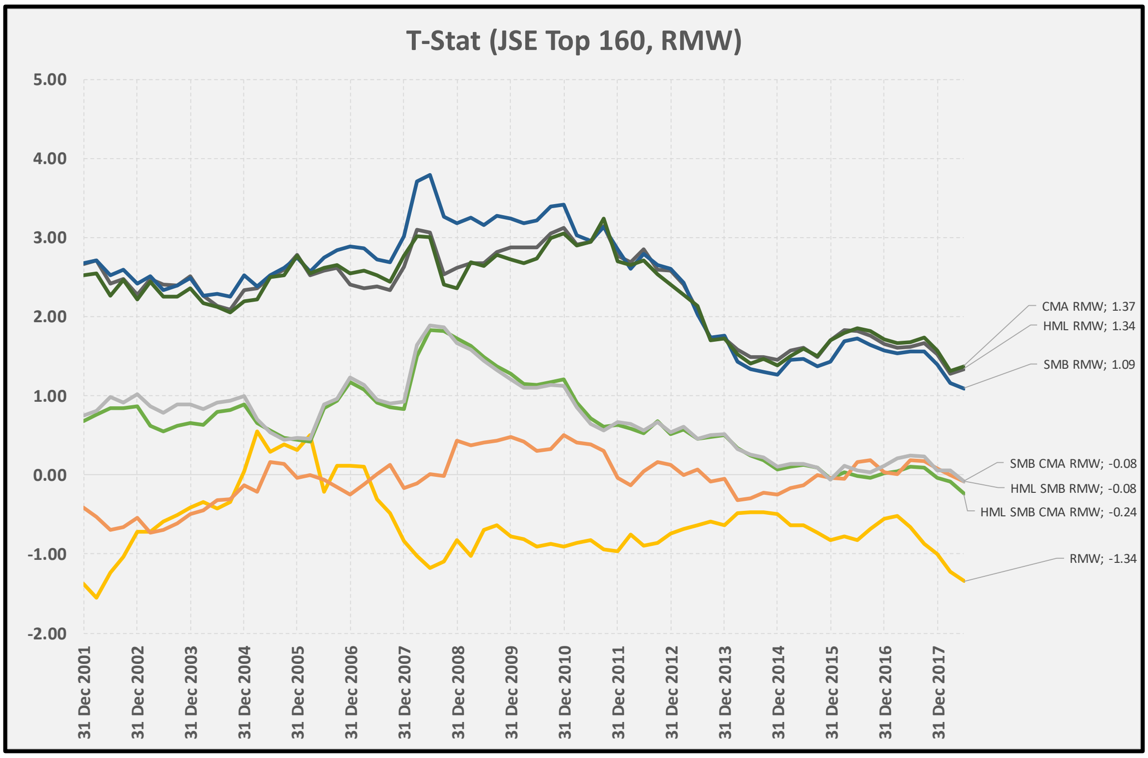

Figure xxi: Time history of t-statistic for the RMW coefficient for Fama French models (Top 160)

Table 13: T-Statistics for the Fama French models' RMW coefficient (top 160)

Model	Mean	Median	Standard Dev.
CAPM + RMW	-0.62694	-0.69299	0.441481
CAPM + HML RMW	2.306151	2.411955	0.520271
CAPM + SMB RMW	2.408497	2.528142	0.697817
CAPM + CMA RMW	2.27249	2.407288	0.501165
CAPM + HML SMB RMW	-0.04666	-0.03259	0.308393
CAPM + SMB CMA RMW	0.714297	0.709081	0.475247
CAPM + HML SMB CMA RMW	0.660856	0.631845	0.49315

5.3.3 Fama French R^2

The t-statistics give an understanding of how well each individual independent variable in the model performs, therefore the values are important for explaining *how* the model works and identifying how it can be improved. However, the most important question for the model is how well it works as a whole at explaining variation in share price. This is quantified with the R^2 values.

There is value in considering the R^2 results for both the top 160 shares on the JSE as well as the top 40 in the main results.

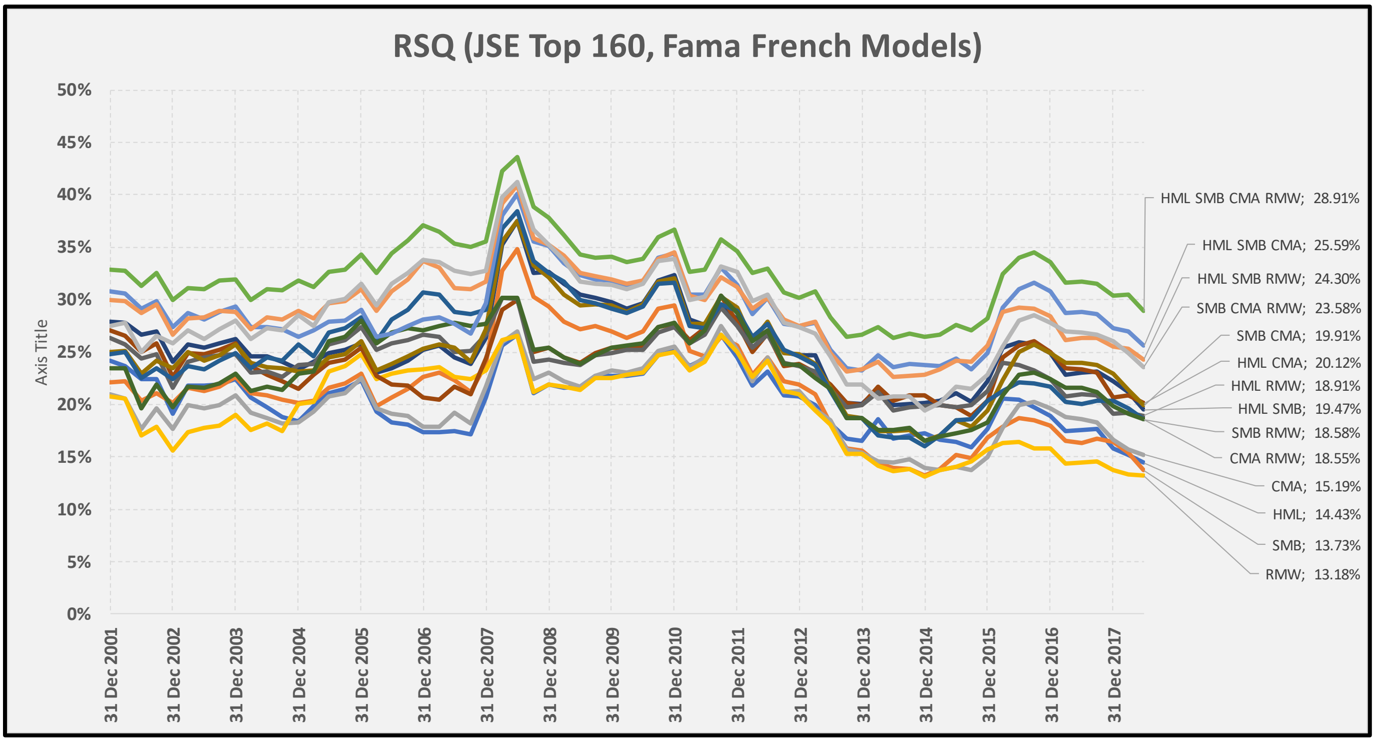

Figure xxii: Time history of R^2 for the various Fama French models (Top 160)

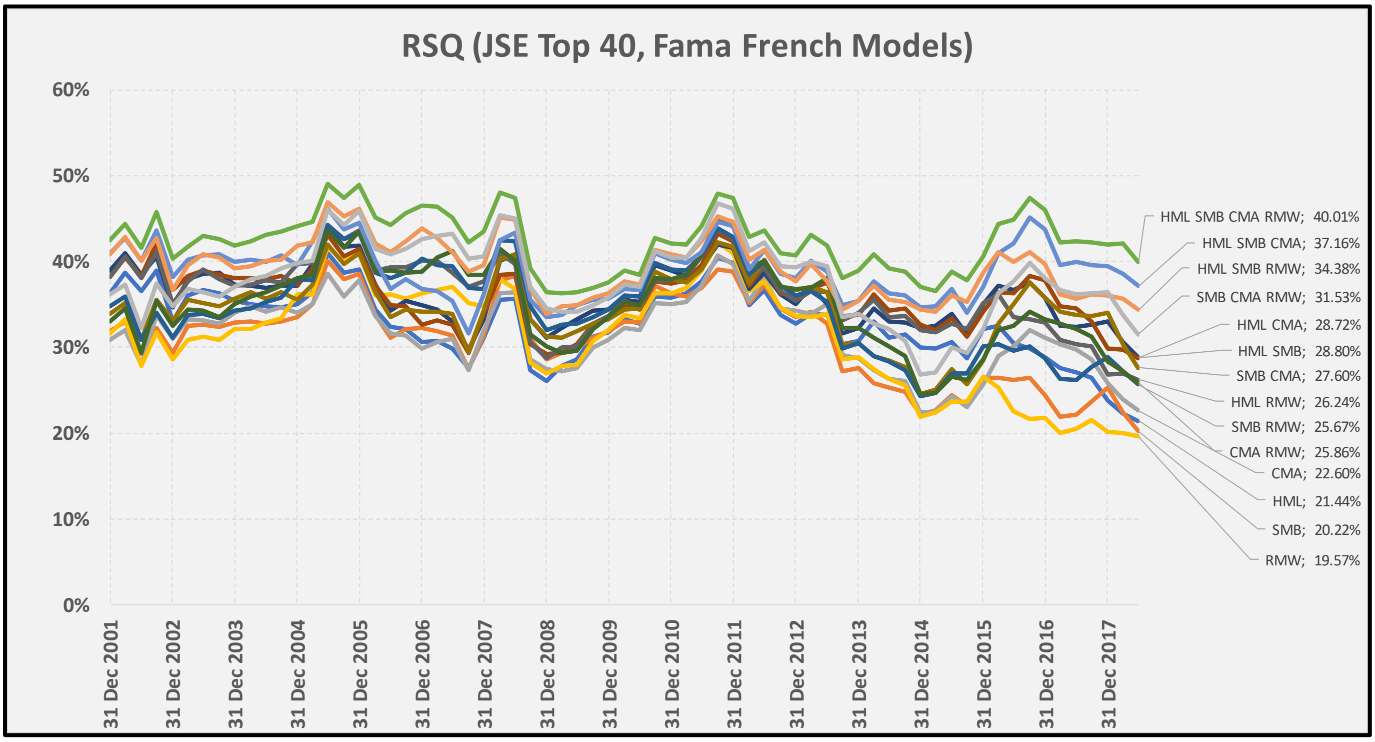

Figure xxiii: Time history of R^2 for the various Fama French models (Top 40)

The next area of interest is the descriptive stats of the R^2 values for each separate Fama French model.

Table 14: Descriptive statistics for R^2 of all Fama French models (top 160)

Model	Mean	Median	Standard Dev.
CAPM + HML	20.27%	20.58%	2.92%
CAPM + SMB	21.41%	21.31%	4.84%
CAPM + CMA	19.80%	19.61%	3.38%
CAPM + RMW	19.54%	20.28%	4.00%
CAPM + HML SMB	25.56%	25.16%	3.84%
CAPM + HML CMA	23.80%	24.02%	2.62%
CAPM + HML RMW	23.91%	24.11%	2.73%
CAPM + SMB CMA	24.91%	24.68%	4.37%
CAPM + SMB RMW	25.07%	24.82%	4.98%
CAPM + CMA RMW	23.37%	23.13%	3.64%
CAPM + HML SMB CMA	28.86%	28.58%	3.45%
CAPM + HML SMB RMW	29.15%	28.98%	3.77%
CAPM + SMB CMA RMW	28.48%	27.80%	4.57%
CAPM + HML SMB CMA RMW	32.44%	32.50%	3.53%

Table 15: Descriptive statistics for R^2 of all Fama French models (top 40)

Model	Mean	Median	Standard Dev.
CAPM + HML	32.71%	32.81%	4.22%
CAPM + SMB	30.82%	31.91%	4.98%
CAPM + CMA	31.39%	31.66%	4.30%
CAPM + RMW	31.11%	32.09%	6.15%
CAPM + HML SMB	35.96%	35.59%	3.34%
CAPM + HML CMA	35.85%	36.32%	3.56%
CAPM + HML RMW	36.10%	37.08%	4.20%
CAPM + SMB CMA	34.34%	34.37%	4.08%
CAPM + SMB RMW	34.43%	34.73%	5.24%
CAPM + CMA RMW	34.87%	34.85%	4.94%
CAPM + HML SMB CMA	39.02%	39.55%	3.20%
CAPM + HML SMB RMW	39.35%	39.71%	3.34%
CAPM + SMB CMA RMW	37.83%	37.37%	4.54%
CAPM + HML SMB CMA RMW	42.39%	42.35%	3.32%

While viewing all of the histograms is both time consuming and unlikely to add much value it does make sense to examine some sample histograms. Two models were selected for both the top 160 shares and the top 40 shares:

- The original three factor Fama French model (CAPM + HML, SMB)
- The latest five factor Fama French model (CAPM + HML, SMB, CMA, RMW)

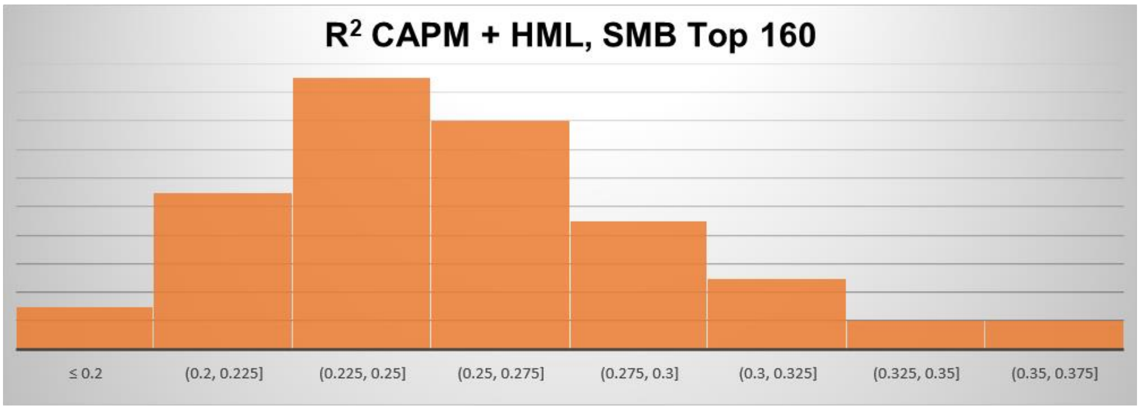

Figure xxiv: Histogram for R^2 values of Fama French 3 factor model (Top 160 shares)

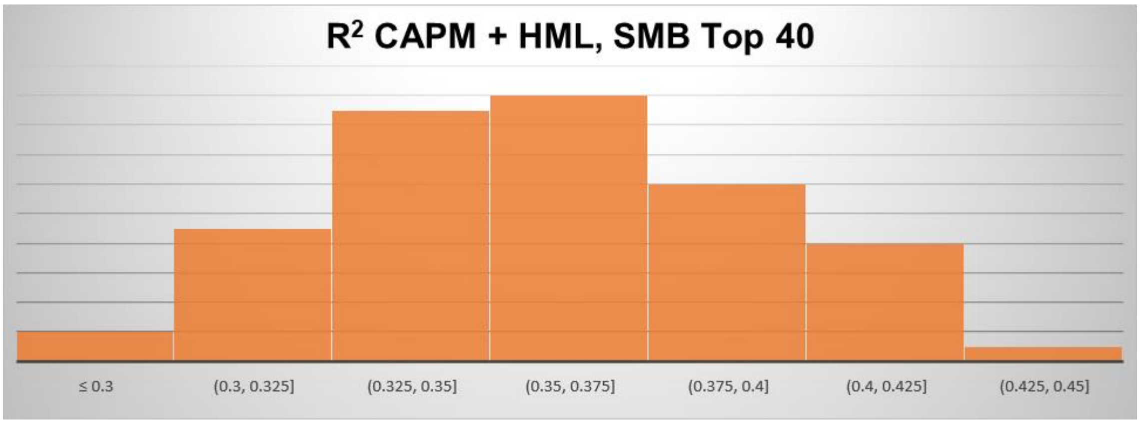

Figure xxv: Histogram for R^2 values of Fama French 3 factor model (Top 40 shares)

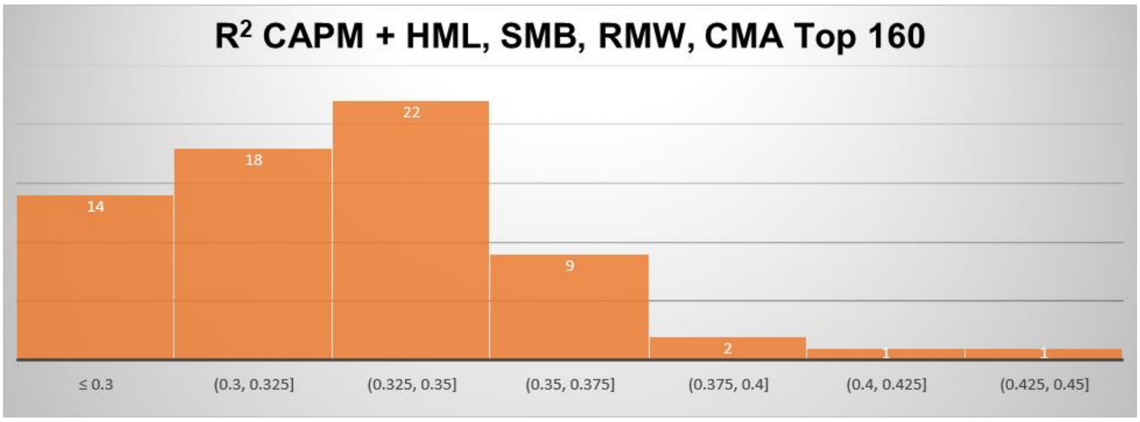

Figure xxvi: Histogram for R^2 values of Fama French 5 factor model (Top 160 shares)

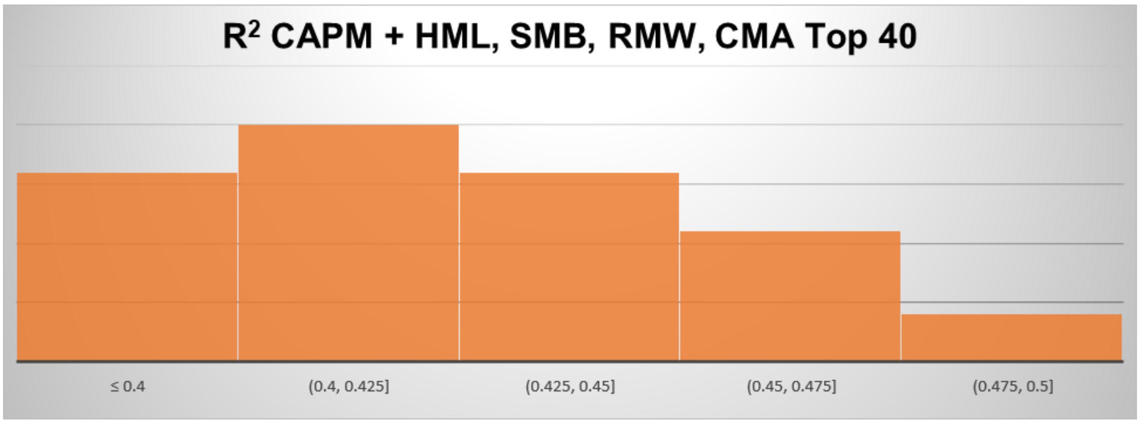

Figure xxvii: Histogram for R^2 values of Fama French 5 factor model (Top 160 shares)

5.4 Hypothesis 1: Evaluation of Alpha

5.4.1 H1a$_0$: Alpha Comparison between CAPM and Selected Fama French

The first step is to evaluate the 5 models graphically for alpha. The graphs for the top 40 shares are shown in appendix 4.

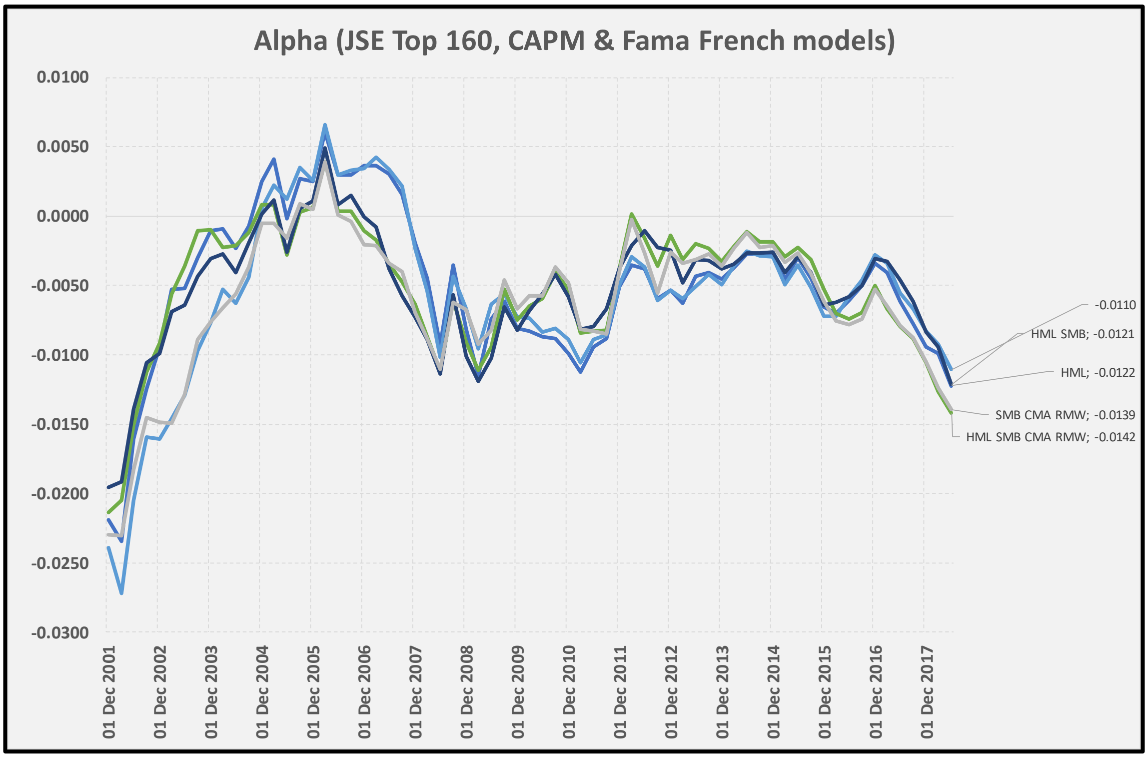

Figure xxviii: Time history of alpha for the CAPM and selected Fama models (Top 160)

The next aspect of hypothesis is to determine whether there is a difference between the various means under consideration at a 95% confidence level. The detailed results from the analyses are shown in appendix 4. The final results are as follows.

Top 160

Table 16: Mean of alpha for CAPM and Selected Fama French models (Top 160)

Model	Mean
CAPM	-0.005783
CAPM + HML	-0.005021
CAPM + HML SMB	-0.005229
CAPM + SMB CMA RMW	-0.006217
CAPM + HML SMB CMA RMQ	-0.005242

The results for the one-way ANOVA shown in appendix 4 provide a definitive answer that there is no significant difference at the 95% confidence level between any of the mean values in Table *16*: Mean of alpha for CAPM and Selected Fama French models (Top 160) for the top 160 shares. As a result, no post-hoc analysis was necessary.

Top 40

Table 17: Mean of Alpha for CAPM and Selected Fama French models (Top 40)

Model	Mean
CAPM	-0.006665
CAPM + HML	-0.006126
CAPM + HML SMB	-0.006059
CAPM + SMB CMA RMW	-0.007295
CAPM + HML SMB CMA RMQ	-0.005943

The results for the one-way ANOVA shown in appendix 4 state that there is no significant difference at the 95% confidence level between any of the mean values in Table *17*: Mean of Alpha for CAPM and Selected Fama French models (Top 40) for the top 40 shares. As a result, no post-hoc analysis was necessary.

5.4.2 $H1a_0$: Alpha Comparison between groupings of Fama French

For this section, it is worth reviewing the results for each grouping of model individually. Supplementary results and information about $H1a_0$ can be found in appendix 5.

Two Factor Models

Table 18: Alpha values for the two factor models (Top 160)

Model	Mean
CAPM + HML	-0.0050
CAPM + SMB	-0.0055
CAPM + CMA	-0.0063
CAPM + RMW	-0.0060

The analysis showed that there is no significant difference between the means shown in Table *18*: Alpha values for the two factor models (Top 160).

Table 19: Alpha values for the two factor models (Top 40)

Model	Mean
CAPM + HML	-0.0061
CAPM + SMB	-0.0069
CAPM + CMA	-0.0072
CAPM + RMW	-0.0067

The analysis showed that there is no significant difference between the means for all two factor models in the above *Table 19: Alpha values for the two factor models (Top 40)*

Three Factor Models

Table 20: Alpha values for the three factor models (Top 160)

Model	Mean
CAPM + HML SMB	-0.0052
CAPM + HML CMA	-0.0053
CAPM + HML RMW	-0.0047
CAPM + SMB CMA	-0.0059
CAPM + SMB RMW	-0.0058
CAPM + CMA RMW	-0.0065

The calculations showed that there is no significant difference between the means shown in Table *20*: Alpha values for the three factor models (Top 160).

Table 21: Alpha values for the three factor models (Top 40)

Model	Mean
CAPM + HML SMB	-0.0061
CAPM + HML CMA	-0.0063
CAPM + HML RMW	-0.0058
CAPM + SMB CMA	-0.0072
CAPM + SMB RMW	-0.0070
CAPM + CMA RMW	-0.0073

The analysis in appendix 5 shows that there is no significant difference between the means shown in *Table 21: Alpha values for the three factor models (Top 40)*.

Four Factor Models

Table 22: Alpha values for the four factor models (Top 160)

Model	Mean
CAPM + HML SMB CMA	-0.0057
CAPM + HML SMB RMW	-0.0049
CAPM + SMB CMA RMW	-0.0062

The calculations showed that there is no significant difference between the means shown in Table *22*: Alpha values for the four factor models (Top 160).

Table 23: Alpha values for the four factor models (Top 40)

Model	Mean
CAPM + HML SMB CMA	-0.0064
CAPM + HML SMB RMW	-0.0057
CAPM + SMB CMA RMW	-0.0073

The results of the analysis for these models is shown in appendix 5. The following means have a significant difference with a 95% confidence level:

- CAPM + HML SMB RMW and CAPM + SMB CMA RMW (with the latter being larger)

5.5 Hypothesis 2: Evaluation of R^2

5.5.1 $H2a_0$: R^2 Comparison between CAPM and Selected Fama French

The first step is to evaluate the 5 models graphically for R^2. The graphs for the top 40 shares are shown in appendix 6.

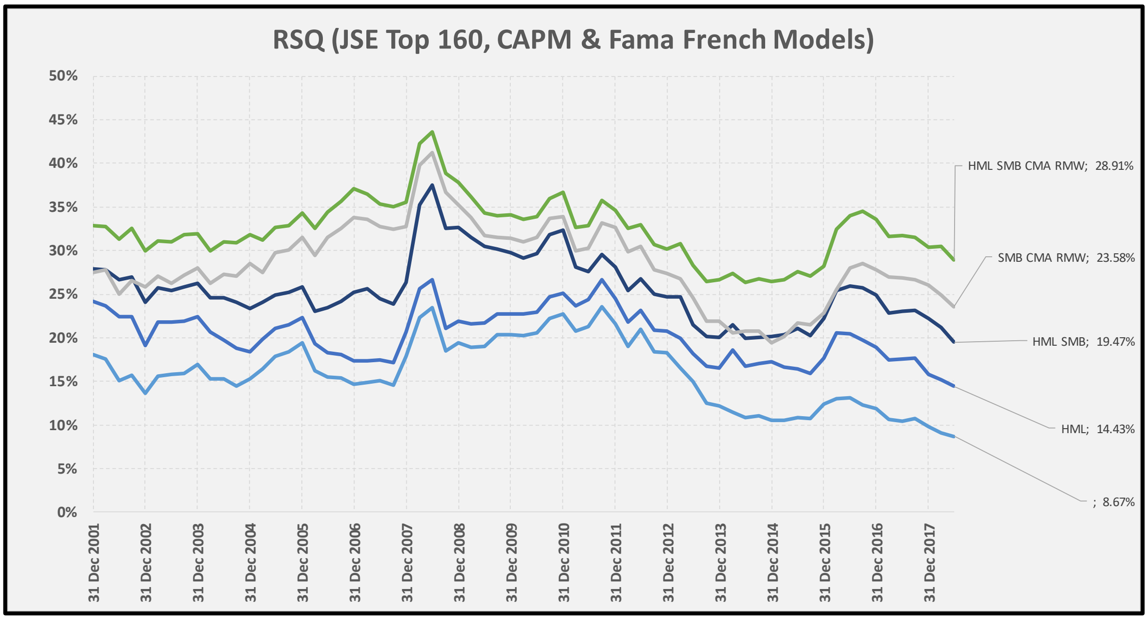

Figure xxix: Time history of R^2 for the CAPM and selected Fama models (Top 160)

Next, it is necessary to determine whether there is a difference between the various means under consideration at a 95% confidence level. The detailed results from the analyses are shown in appendix 6. The final results are as follows.

Top 160

Table 24: Mean of R^2 for CAPM and Selected Fama French models (Top 160)

Model	Mean
CAPM	15.92%
CAPM + HML	20.27%
CAPM + HML SMB	25.56%
CAPM + SMB CMA RMW	28.48%
CAPM + HML SMB CMA RMQ	32.44%

Appendix 6 provides the results from the SPSS analyses. The results are as follows. The mean value of the R^2 values showed a significant difference to a 95% confidence level in the following models for the top 160 shares

- CAPM and CAPM + HML
- CAPM and CAPM + HML SMB
- CAPM and CAPM + SMB CMA RMW
- CAPM and CAPM + HML SMB CMA RMW
- CAPM + HML and CAPM + HML SMB
- CAPM + HML and CAPM + SMB CMA RMW
- CAPM + HML and CAPM + HML SMB CMA RMW
- CAPM + HML SMB and CAPM + SMB CMA RMW
- CAPM + HML SMB and CAPM + HML SMB CMA RMW
- CAPM + SMB CMA RMW and CAPM + HML SMB CMA RMW

The mean value of the R^2 values showed no significant difference to a 95% confidence level in any of the comparisons for the top 160 shares.

Top 40

Table 25: Mean of R2 for CAPM and Selected Fama French models (Top 40)

Model	Mean
CAPM	27.52%
CAPM + HML	32.71%
CAPM + HML SMB	35.96%
CAPM + SMB CMA RMW	37.83%
CAPM + HML SMB CMA RMQ	42.39%

The mean value of the R^2 values showed a significant difference to a 95% confidence level in the following models for the top 40 shares:

- CAPM and CAPM + HML
- CAPM and CAPM + HML SMB
- CAPM and CAPM + SMB CMA RMW

- CAPM and CAPM + HML SMB CMA RMW
- CAPM + HML and CAPM + HML SMB
- CAPM + HML and CAPM + SMB CMA RMW
- CAPM + HML and CAPM + HML SMB CMA RMW
- CAPM + HML SMB and CAPM + HML SMB CMA RMW
- CAPM + SMB CMA RMW and CAPM + HML SMB CMA RMW

The mean value of the R^2 values did not show a significant difference to a 95% confidence level in the following comparisons for the top 40 shares:

- CAPM + HML SMB and CAPM + SMB CMA RMW

5.5.2 $H2b_0$: R^2 Comparison between groupings of Fama French

For this section, it is worth reviewing the results for each grouping of model individually. SPSS analysis results for $H2b_0$ can be found in appendix 7.

Two Factor Models

Table 26: R^2 values for the two factor models (Top 160)

Model	Mean
CAPM + HML	20.27%
CAPM + SMB	21.41%
CAPM + CMA	19.80%
CAPM + RMW	19.54%

The analysis showed that there is no significant difference between the means shown in Table *26*: R^2 values for the two factor models (Top 160).

Table 27: R^2 values for the two factor models (Top 40)

Model	Mean
CAPM + HML	32.71%
CAPM + SMB	30.82%
CAPM + CMA	31.39%
CAPM + RMW	31.11%

The values for the means of R^2 for the top 40 companies is shown in Table *27*: R^2 values for the two factor models (Top 40). The analysis shows that there is no significant difference between the means.

Three Factor Models

Table 28: R^2 values for the three factor models (Top 160)

Model	Mean
CAPM + HML SMB	25.56%
CAPM + HML CMA	23.80%
CAPM + HML RMW	23.91%
CAPM + SMB CMA	24.91%
CAPM + SMB RMW	25.07%
CAPM + CMA RMW	23.37%

The calculations showed that there is a significant difference between the following means shown in Table *28*: R^2 values for the three factor models (Top 160):

- CAPM + HML CMA & CAPM + HML SMB (with the latter being larger)
- CAPM + CMA RMW & CAPM + HML SMB (with the latter being larger)

Table 29: R^2 values for the three factor models (Top 40)

Model	Mean
CAPM + HML SMB	35.96%
CAPM + HML CMA	35.85%
CAPM + HML RMW	36.10%
CAPM + SMB CMA	34.34%
CAPM + SMB RMW	34.43%
CAPM + CMA RMW	34.87%

The values for the means of R^2 for the top 40 companies are shown in *Table 29: R2 values for the three factor models (Top 40)*. The analysis shows that there is no significant difference between the means.

Four Factor Models

Table 30: R^2 values for the four factor models (Top 160)

Model	Mean
CAPM + HML SMB CMA	39.02%
CAPM + HML SMB RMW	39.35%
CAPM + SMB CMA RMW	37.83%

The calculations showed that there is no significant difference between the means shown in Table *30*: R^2 values for the four factor models (Top 160).

Table 31: R^2 values for the four factor models (Top 40)

Model	Mean
CAPM + HML SMB CMA	39.02%
CAPM + HML SMB RMW	39.35%
CAPM + SMB CMA RMW	37.83%

The values for the means of R^2 for the top 40 companies are shown in

Table *31*: R^2 values for the four factor models (Top 40). The analysis shows that there is no significant difference between the means.

Chapter 6: Discussion of Results

6.1 Fama French Factor Calculation

It is very important that the Fama French factors are correctly calculated as this gives the model the best chance of being a realistic reflection of behaviour on the JSE.

6.1.1 Shares per Portfolio

There was some concern that the 2 x 2 x 2 x 2 sorts would result in too many simultaneous portfolios with too few shares in them. This is shown in Figure *vii*: Number of shares in each sub-portfolio. The results show that from around 2005 there is a somewhat even split of shares in each sub-portfolio. There are some that have quite a few more shares than the others, SHWC (Small, High value, Weak profitability, Conservative investment) is consistently higher than average.

There are 4 sub-portfolios which have numbers of shares which one could argue are lower than one would like them to be (SLWA, BHWA, BHRC and SLWC end with 3 or 4 shares in each portfolio). Out of these four sub-portfolios, three of them are consistently low in numbers of shares. When one only has 3 shares in a sub-portfolio then an extra good return or extra bad return for one of those shares will significantly influence the overall return for that portfolio. However, it should be remembered that each sub-portfolio has its returns combined with 7 other sub-portfolios as per *Figure vi: Calculation of 2 x 2 x 2 x 2 sub-portfolios and resulting factor calculation* from (Fama & French, 2015) and this will still "hide" a single share's abnormal results in the factor values. It was concluded that provided there were only a limited number of sub-portfolios with a small number of shares, it is not a major concern.

Another item that is noticeable is that near the beginning of the data set in Figure *vii*: Number of shares in each sub-portfolio there is a low number of shares in all sub-portfolios. There is a concern that there may be survivorship bias. The calculations to generate each sub-portfolio were set up with the express goal of eliminating survivorship bias but for some reason it may have been unsuccessful. Unfortunately, time constraints have made it impossible to investigate this further. However one can see in the figure that the number of shares in each portfolio increases substantially and very quickly soon after. Therefore, even if there is survivorship, it is believed that there are enough shares to provide data for the analyses. Additionally, this is a comparative study for models run on the same set of data so even if there is survivorship bias, the comparisons are still valid.

6.1.2 Sub-portfolio Style Measures

The results for this are shown from Figure *xxxvi*: Sub-portfolios used in SMB calculation ranked by market cap to Figure *xxxix*: Sub-portfolios used in CMA calculation ranked by asset growth and form a good check for the sub-portfolio formation. Each of the four figures plots each of the four styles that are used in the Fama French five-factor model. Each portfolio's value for that specific style characteristic is calculated and then plotted over time.

As an example, consider Figure *xxxvi*: Sub-portfolios used in SMB calculation ranked by market cap. One can clearly see that the market cap of the 8 sub-portfolios with a name starting with "B" score higher than the 8 sub-portfolios with a name starting with "S". This trend can be seen in all of the figures where measuring the sub-portfolios by a certain style measure separates out the sub-portfolios according to the letter naming that they have been given. These figures give one confidence that the calculations to create the sub-portfolios are working in the way that they should be.

6.1.3 Sub-portfolio Returns

The cumulative returns generated by each individual sub-portfolio (16 of them) are shown in Figure *viii*: Sub - portfolio cumulative returns generated over a 20 year period. This figure is by no means a clear indicator of what the factors will look like but they are useful nonetheless. For example, when one looks at the returns it is possible to see that on average the sub-portfolios with the letter "C" in the name (Conservative investment) appear to be outperforming those with "A" in the name (Aggressive investment). Even before calculating and plotting the far clearer Fama French factors, it is possible to see in this a confirmation of the work by (Muller & Ward, 2013) in the same context (JSE).

Another example in the same figure is the returns of the shares with Robust profitability vs the returns of the shares with Weak profitability. Here one can see that generally speaking the "R" shares tend to out-perform the "W" shares. This is a particular investment style that makes a great deal of sense. As argued by (Fama & French, 2015), a share with a high net present value of future profitability and dividends should generate a premium price today.

One final item in Figure *viii*: Sub - portfolio cumulative returns generated over a 20 year period which should be considered is J203, the all share index. This shows that the J203 sits roughly in the middle. When one is considering all of the companies on the All Share

Index, it makes sense that there should be an even spread above and below the J203 and goes some way to alleviating the concerns of survivorship bias highlighted in 6.1.1. The J203 is a good investment but ideally one would want to find shares that have a good chance of out-performing the market. This can be done by finding shares, or styles of shares, that *consistently* out-perform or under-perform the market.

When it comes to investment strategies, consider the returns generated by SHRC. Over the full 20 year period, it has results considerably better than the market. Yet, since 2013 this sub-portfolio has had a gradient which is less than J203 (under-performed). In an ideal world, if one had been following an SHRC investment strategy, this should have been abandoned in 2013 and something else would have proven more effective since then. Interestingly enough, BHRC could have proven to be singularly effective at this time.

6.1.4 Fama French Factors

Style investing is not just about finding an investment style that is successful at a specific point in time. It is more important that a style remains successful over a long period of time. If an investor were given a choice between high returns for an unknown duration followed by negative returns for an unknown duration OR lower returns forever, they would prefer the predictability of lower returns forever.

The graphs in Figure *ix*: Cumulative returns for each Fama French factor (blue is SMB) show the performance and persistence of four different investment styles over the past 20 years.

Size: A somewhat surprising result. The graph shows a consistent negative gradient over the 20 years under consideration. This implies that large market cap companies outperform small market cap companies on the JSE. As already mentioned the exact opposite was seen by (Banz, 1981) and (Fama & French, 1992) who found that small companies tend to outperform large companies. There were numerous studies on the JSE who found no size effect at all: (Bradfield, Barr, & Affleck-Graves, 1988), (Page & Palmer, 1992) and (Muller & Ward, 2013). There is only one reference to the negative size effect (big outperforming small) which is (De Villiers, Lowings, Pettit, & Affleck-Graves, 1986).

Value: This is shown by HML in the figure. The graph shape until 2 years ago is a very close reflection of the various pieces of evidence available about

Value investing. It was clearly a powerful investment style up until 2008. (Lakonishok, Shleifer, & Vishny, 1994) had the same findings as (Haugen & Baker, 1996), that there were good t-statistics for a superior return of a value portfolio. (Plaistowe & Knight, 1986) and (Fraser & Page, 2000) found that value shares were a useful haven in a declining market. However since 2008, value investing has not performed well, as per (Reese, 2015). Interestingly enough, the positive gradient in the last 2 years indicates that the value effect may be returning to the JSE. Nonetheless, the negative persistence of value investing in the recent past is not confidence inspiring for this philosophy.

Profitability: The graph shows that since 2004 the profitability investment style has given good returns and maintained reasonable persistence. This is consistent with (Hou, Xue, & Zhang, 2015) and (Muller & Ward, 2013).

Investment: The graph for the investment Fama French factor shows a negative gradient from 2008 to 2015 (Aggressive investment outperforming conservative investment) while it is positive the rest of the time period. This is somewhat different from (Muller & Ward, 2013) who found that capital accumulation tended to be a negative indicator of share performance. There is some discrepancy.

It is very important to note that a factor which changes gradient from positive to negative or vice versa highlights a difficult time for asset managers and an even worse time for those hedge fund managers who have chosen that factor as their strategy. However, for a Fama French asset-pricing model, it may not have an effect on the accuracy of the model. Theoretically speaking, if the returns on an asset changed from positive to negative at the same time as the factor changed sign then the asset pricing model would reflect the change without any change in the model’s explanatory ability. It should be possible to see whether this is the case in real-life by examining graphs of the t-statistics and R2 and see what happens to them around the same time as the sign change in a factor.

The above paragraph highlights an important difference between asset pricing models and investment strategy selection. Someone working on investment strategy would do well to spend a lot of time with factor graphs to understand how well a specific investment style is performing and how often it changes sign. By contrast, an academic will spend more time with the regression results to see how well the model explains behaviour and can accommodate factor sign changes.

6.2 Research Question 1: CAPM on the JSE

The methodology which (Carter, Muller, & Ward, 2017) used to evaluate the CAPM on the JSE was a very logical and reasoned approach.

1. Given variation in the market and the risk free rate, calculate the co-efficients (Alpha and Beta).
2. Using alpha and beta determined above, calculate what the out of sample returns on a specific asset should be given actual variation in R_m – R_f.
3. If the CAPM explains exactly what is happening, then if one plotted a scatter graph of actual return with the return explained by Fama French, there would be a 45 degree line (a 1:1 relationship so their values would be the same).

This is an elegant way to evaluate how good the CAPM is. However, it is very different to the methodology conducted by (Fama & French, 2015) when they evaluated the more complex five factor asset pricing model. To enable this Fama-French model analysis to have some kind of comparative value, it makes sense to use a similar methodology to that used by Gene Fama and Kenneth French. As a result, the evaluation of the CAPM was done in the same way.

There are other studies that can be referenced for this type of methodology. For example, the detailed study by (Fama & French, 1992) provides t-statistics for Beta on the New York Stock Exchange.

- From 1941 to 1965 the t-statistic was 1.82
- From 1966 to 1990 the t-statistic was 0.06

The study found the interesting situation where the CAPM used to be a good model but appears to have “lost” its explanatory ability. A possible reason for this is that in that period, investors sub-consciously invested as though the stock markets behaved in the way that the CAPM says it does. If everyone believed in the idea that more volatile shares should give better returns then people would buy those shares looking for returns. By the laws of supply and demand, this would drive up the price of those shares and thus give high returns for them. Over the period 1966 to 1990, maybe investors became more imaginative, applying more complex strategies in the hopes of generating superior returns. When enough investors were ignoring the traditional CAPM theory, it no longer proved to be a good explanatory model. This is speculation and cannot be verified in this study. Nonetheless, the data does indicate a change over time.

The study of the CAPM by (Jagannathan & Wang, 1993) obtained a value of 57% for R^2. This result was dependant on the regression making allowance for variations in the business cycle. It is believed that this study has allowed for this by regressing over the previous 36 months, well inside the timescale of a typical business cycle. Another study, which calculated R^2 values, is that of (Loukeris, 2009) who obtained a value of 7.3% on the London Stock Exchange.

6.2.1 CAPM Alpha

One can see in Figure *x*: Time history of alpha as per the CAPM that the value is consistently negative. The value indicates a considerably less accurate model near the beginning of the time period but this improves. It should be noted that the results are absolute numbers while other studies typically quote percentage values. For the purposes of this discussion the percentage values will be quoted.

The descriptive stats have relatively similar numbers for mean and median. However, the standard deviation for the Top 40 is only half that of the Top 160. This leads one to speculate that maybe the Top 40 shares are a lot more predictable, which causes the CAPM, and its associated alpha, to also be stable.

- The mean values for alpha of 0.58% per month (Top 160) and 0.67% per month (Top 40).
- (Cooper, Gutierrez Jr, & Hameed, 2004) obtained values of 1.12% per month (for 36 months of UP markets), 0.01% per month (for 36 months of DOWN markets and -2.22% per month (for UP and DOWN market periods). This analysis was conducted on a combination of NYSE and AMEX shares.
- One can see that the numbers in this analysis are fairly typical of the values seen in CAPM calculations, not significantly different in either direction. However, it is not possible to extract any conclusions from this data.

The histogram Figure *xi*: Histogram of Alpha values (Top 160) shows a shape that is not a perfect normal distribution but is nonetheless reasonably close.

6.2.2 CAPM T-Statistics

The t-statistic plot in Figure *xii*: T-statistic of Rm-Rf coefficient (beta) for the CAPM (Top 160) shows the information needed without much need for a comprehensive set of descriptive statistics. It is clear that the t-statistic for Beta is greater than 2 for a majority of the time. As stated in the methodology section, a t-statistic that is greater than 2 for a well-recognised factor will work well. There can be little doubt that the term containing Market Return - Risk Free Rate is well recognised so 2 is a good benchmark.

The descriptive stats for the t-statistic give the following conclusion:

- The mean of the t-statistics for the top 160 shares is 2.3074 while the mean for the top 40 shares is 3.6124.
- The CAPM value for Beta t-statistic is at the 95% confidence level for both the Top 40 and Top 160 shares.
- The model works better for the Top 40 shares.
- The study by (Fama & French, 1992) found that the t-statistic from 1941 to 1965 the t-statistic was 1.82 while from 1966 to 1990 the t-statistic was 0.06.
- The difference between the results obtained in this study and (Fama & French, 1992) is rather concerning. There may be regional differences but this is still rather large.

The histogram Figure *xiii*: Histogram of t-stat values for Beta (Top 160) shows a shape that is not a perfect normal distribution but is nonetheless reasonably close.

6.2.3 CAPM R^2

While the t-statistics provide information on each individual dependant variable in a multiple linear regression, the final measure is how good the model as a whole is at explaining variation in the dependant variable. The graph in Figure *xiv*: R^2 for the CAPM (Top 160) shows a relatively stable value for R^2, it could be argued that there is a downward trend but there isn't enough historic information to be able to be sure of this at any confidence level.

The descriptive statistics provide some other interesting viewpoints.

- The mean of the CAPM R^2 for the top 160 shares is at 15.92%. This is considerably lower than the 57% seen by (Jagannathan & Wang, 1993) but quite a bit higher than the 7.3% seen by (Loukeris, 2009).
- The mean of the CAPM R^2 for the top 40 shares is at 27.52%. Once again this is a lot lower than the above-mentioned value.
- The standard deviation is 3.91% (Top 160) and 5.93% (Top 40) respectively. This indicates that over the 20-year period of interest, the new CAPM model is relatively consistent in terms of its explanatory ability.

The large difference between this study's R^2 values and the two very different values is a concern. However the 57% appears to be the exception rather than the rule. Studies such as (Carter, Muller, & Ward, 2017) and (Baker, Bradley, & Wurgler, 2011), while not providing actual R^2 values, provide enough information for one to be confident that the R^2 values would be very low. The benchmark values of R^2 that are needed in order to meet certain criteria were proposed in Chapter 4, below 20% is poor and above 40% is reasonably good. Based on these criteria, it is clear that the CAPM is only just good enough in the Top 40 data set while it is poor for the Top 160 shares.

The histogram plot for the R^2 in Figure *xv*: Histogram of R^2 values for CAPM (Top 160) supports the relatively low standard deviation value seen in the descriptive stats. As for the other histogram shapes in this study, it not the ideal normal distribution but it is reasonably close.

6.3 Research Question 2: Fama French on the JSE

The Fama French five factor model has four factors which are additional to the traditional CAPM factor of $R_m – R_f$. The decision was made to evaluate all possible combinations of these factors, from two-factor models all the way to the full five-factor model. This provides 14 different models to evaluate. While it is not practical to go into detail in all of these models, a quick review of the descriptive statistics and linear regression results can provide a great deal of information.

6.3.1 Fama French Alpha

Below are the values for alpha that were obtained by (Fama & French, 2017) on the NYSE for a few different asset pricing models containing differing combinations of the five-factor model factors. The Figure *xxx*: Selected results for alpha for a few different asset pricing models is a selection of typical results rather than the full set which is rather large. It should be noted that the factor SMB was included in all of the models detailed, therefore the model titled HML is in fact a three-factor model (Rm – Rf, SMB and HML).

	2 × 3 Factors				2 × 2 Factors				2 × 2 × 2 × 2 Factors			
	GRS	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$	*GRS*	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$	*GRS*	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$
Panel A: 25 Size-B/M portfolios												
HML	3.62	0.102	0.54	0.38	3.54	0.101	0.53	0.36	3.40	0.096	0.51	0.36
HML RMW	3.13	0.095	0.50	0.24	3.11	0.096	0.51	0.26	3.29	0.089	0.47	0.24
HML CMA	3.52	0.101	0.53	0.39	3.46	0.100	0.53	0.37	3.18	0.096	0.51	0.35
RMW CMA	2.84	0.100	0.53	0.22	2.78	0.093	0.49	0.19	2.78	0.087	0.46	0.13
HML RMW CMA	2.84	0.094	0.50	0.23	2.80	0.093	0.49	0.23	2.82	0.088	0.46	0.18
Panel B: 25 Size-OP portfolios												
HML	2.31	0.108	0.68	0.51	2.31	0.109	0.68	0.51	1.91	0.089	0.56	0.37
RMW	1.71	0.067	0.42	0.12	1.82	0.078	0.49	0.16	1.73	0.059	0.37	0.05
HML RMW	1.64	0.062	0.39	0.16	1.74	0.058	0.36	0.03	1.62	0.064	0.40	0.06
HML CMA	3.02	0.137	0.86	0.90	2.85	0.135	0.85	0.86	2.06	0.102	0.64	0.49
RMW CMA	1.87	0.075	0.47	0.12	1.67	0.066	0.42	0.05	1.61	0.068	0.43	0.05
HML RMW CMA	1.87	0.073	0.46	0.12	1.73	0.066	0.42	0.06	1.60	0.069	0.43	0.07
Panel C: 25 Size-Inv portfolios												
HML	4.56	0.112	0.64	0.57	4.40	0.107	0.61	0.53	4.32	0.100	0.57	0.56
CMA	4.03	0.105	0.60	0.47	4.05	0.106	0.61	0.47	4.23	0.123	0.70	0.62
HML RMW	4.40	0.106	0.61	0.57	4.26	0.103	0.59	0.52	4.45	0.116	0.66	0.66
HML CMA	4.00	0.099	0.57	0.43	3.97	0.098	0.56	0.41	3.70	0.084	0.48	0.35
RMW CMA	3.33	0.085	0.49	0.29	3.28	0.082	0.47	0.26	3.50	0.082	0.47	0.27
HML RMW CMA	3.32	0.085	0.49	0.29	3.27	0.082	0.47	0.27	3.59	0.082	0.47	0.28

Figure xxx: Selected results for alpha for a few different asset pricing models *(Fama & French, 2015)*

The results obtained above indicate that adding factors has reduced the value of alpha and the authors use this as an indication that the model is capturing more information on what is influencing asset fluctuation. The assumption being that a full model will never have unexplained share price changes (non-zero alpha). This makes a great deal of sense.

This methodology is also used in (Guo, Zhang, Zhang, & Zhang, 2017) who evaluated the model on the Chinese stock market. The results are shown below and reflect a similar trend, that the models containing more factors have lower alpha.

Market factors	p(GRS)	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{As^2(a_i)}{A(a_i^2)}$	p(GRS)	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{As^2(a_i)}{A(a_i^2)}$	p(GRS)	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{As^2(a_i)}{A(a_i^2)}$
	25 Size-B/M portfolios				*25 Size-ROE portfolios*				*25 Size-InvA portfolios*			
Three-factor	0.148	0.250	0.50	0.73	0.000	0.458	1.15	0.24	0.007	0.311	0.87	0.40
Four-factor	0.230	0.210	0.42	1.07	0.003	0.271	0.68	0.63	0.186	0.210	0.59	1.10
Five-factor	0.192	0.213	0.43	1.04	0.003	0.273	0.68	0.62	0.200	0.197	0.55	1.14
	25 Size-InvB portfolios				*32 Size-B/M-ROE portfolios*				*32 Size-B/M-InvA portfolios*			
Three-factor	0.059	0.318	0.95	0.51	0.039	0.388	0.78	0.50	0.160	0.292	0.61	0.75
Four-factor	0.575	0.172	0.51	1.64	0.466	0.237	0.48	1.10	0.717	0.201	0.42	1.45
Five-factor	0.612	0.168	0.50	1.66	0.469	0.244	0.49	1.03	0.695	0.198	0.42	1.40
	32 Size-B/M-InvB portfolios				*32 Size-ROE-InvA portfolios*				*32 Size-ROE-InvB portfolios*			
Three-factor	0.135	0.304	0.67	0.84	0.002	0.449	1.06	0.38	0.000	0.461	1.04	0.42
Four-factor	0.792	0.177	0.39	1.94	0.151	0.236	0.56	1.04	0.052	0.298	0.67	0.97
Five-factor	0.800	0.179	0.39	1.80	0.171	0.233	0.55	1.00	0.060	0.285	0.64	1.02

Figure xxxi: Values for Alpha for three, four and five factor Fama French models *(Guo, Zhang, Zhang, & Zhang, 2017)*

Unfortunately, the results of alpha for the 14 asset pricing models considered on the JSE show something very different. The graph Figure *xvi*: Time history of alpha as per the various FF models shows clearly that the time histories of the various values for alpha are very similar. This is reflected in the descriptive statistics for alpha in both the top 40 and the top 160 shares. The mean and standard deviation reflect marginal differences between the various models and there is no trend, not even a slight one, as the models increase in complexity from the two-factor models to the final five-factor model.

This result is something of a concern, the use of alpha as an indicator of the completeness of a model is common to asset pricing model studies and the results of this study do not demonstrate this in any way. One aspect of the analysis, which could be causing this, is the duration for each linear regression (36 months). By contrast, (Fama & French, 2015) used 606 months. While 36 months seems like it has the potential to be too short, the 606 months seems too long. The duration over which the regression is conducted is something that could be investigated further.

6.3.2 Fama French T-Statistics

The t-statistics can help one understand how well each individual factor fits with the data that is being analysed. Let us first consider the t-statistics results from (Fama & French, 2015).

Below is *Figure xxxii: T-statistics for the Fama French five-factor asset pricing model* (Fama & French, 2017) which gives the results for 25 value weighted Size – Book to Market portfolios. Panel B is the panel of interest as it covers the five-factor model. The right hand column are the values for the t-statistics for alpha, the coefficient of HMLO (orthogonal version of HML), the coefficient of RMW and the coefficient of CMA. One can clearly see that the averages of the t-statistics of h, r and c are well above the benchmark value of 2 (ideally 3). The one t-statistic that does not give such good results is the t-statistic of a (alpha), the average is below 2.

It is unclear why the authors chose to exclude the t-statistics for R_m – R_f and SMB. However, based on previous commentary in the article, it is likely that the authors assumed these factors were “non negotiable” inclusions.

$$R(t)-R_F(t)=a+b[R_M(t)-R_F(t)]+sSMB(t)+hHMLO(t)+rRMW(t)+cCMA(t)+e(t).$$

$B/M \rightarrow$	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor intercepts: R_M-R_F, SMB, and HML										
			a					*t(a)*		
Small	−0.49	0.00	0.02	0.16	0.14	−5.18	0.07	0.40	2.88	2.37
2	−0.17	−0.04	0.12	0.07	−0.02	−2.75	−0.80	2.24	1.40	−0.38
3	−0.06	0.06	0.02	0.06	0.12	−0.98	0.92	0.33	0.96	1.66
4	0.14	−0.10	−0.04	0.07	−0.08	2.24	−1.46	−0.55	1.05	−0.94
Big	0.17	0.02	−0.07	−0.11	−0.18	3.53	0.40	−0.95	−1.86	−1.92
Panel B: Five-factor coefficients: R_M-R_F, SMB, HMLO, RMW, and CMA										
			a					*t(a)*		
Small	−0.29	0.11	0.01	0.12	0.12	−3.31	1.61	0.17	2.12	1.99
2	−0.11	−0.10	0.05	−0.00	−0.04	−1.73	−1.88	0.95	−0.04	−0.64
3	0.02	−0.01	−0.07	−0.02	0.05	0.40	−0.10	−1.06	−0.25	0.60
4	0.18	−0.23	−0.13	0.05	−0.09	2.73	−3.29	−1.81	0.73	−1.09
Big	0.12	−0.11	−0.10	−0.15	−0.09	2.50	−1.82	−1.39	−2.33	−0.93
			h					*t(h)*		
Small	−0.43	−0.14	0.10	0.27	0.52	−10.11	−4.38	3.90	10.12	17.55
2	−0.46	−0.01	0.29	0.43	0.69	−15.22	−0.45	11.77	16.78	24.44
3	−0.43	0.12	0.37	0.52	0.67	−14.70	3.71	12.28	17.07	18.75
4	−0.46	0.09	0.38	0.52	0.80	−15.18	2.76	11.03	15.88	20.26
Big	−0.31	0.03	0.26	0.62	0.85	−14.12	1.09	7.54	21.05	18.74
			r					*t(r)*		
Small	−0.58	−0.34	0.01	0.11	0.12	−13.26	−10.56	0.31	3.89	3.95
2	−0.21	0.13	0.27	0.26	0.21	−6.75	4.89	10.35	9.86	7.04
3	−0.21	0.22	0.33	0.28	0.33	−6.99	6.77	10.36	8.98	8.88
4	−0.19	0.27	0.28	0.14	0.25	−6.06	7.75	7.99	4.16	6.14
Big	0.13	0.25	0.07	0.23	0.02	5.64	8.79	2.07	7.62	0.49
			c					*t(c)*		
Small	−0.57	−0.12	0.19	0.39	0.62	−12.27	−3.46	6.59	13.15	19.10
2	−0.59	0.06	0.31	0.55	0.72	−17.76	1.94	11.27	19.39	22.92
3	−0.67	0.13	0.42	0.64	0.78	−20.59	3.64	12.52	18.97	19.62
4	−0.51	0.31	0.51	0.60	0.79	−15.11	8.33	13.35	16.41	18.03
Big	−0.39	0.26	0.41	0.66	0.73	−16.08	8.38	10.80	19.88	14.54

Figure xxxii: T-statistics for the Fama French five-factor asset pricing model *(Fama & French, 2015)*

	Low	2	3	4	High	Low	2	3	4	High
Panel A: the three-factor model										
			a_i					$t(a_i)$		
Small	−0.14	0.19	0.21	0.55	0.69	−0.83	0.80	1.02	2.11**	2.62***
2	−0.51	0.40	−0.01	0.08	0.52	−3.00***	2.23**	−0.07	0.38	2.64***
3	−0.48	−0.18	0.03	0.35	−0.01	−1.88*	−0.88	0.17	1.96**	−0.04
4	−0.28	−0.35	0.26	−0.01	0.34	−1.16	−2.34**	1.83*	−0.04	2.05**
Large	−0.40	−0.13	−0.06	0.14	0.36	−0.95	−0.80	−0.47	1.09	1.88*
			Adjusted R^2							
Small	0.85	0.42	0.41	0.38	0.46					
2	0.78	0.58	0.52	0.37	0.50					
3	0.64	0.55	0.57	0.58	0.63					
4	0.60	0.72	0.72	0.63	0.64					
Large	0.30	0.73	0.79	0.79	0.60					
Panel B: the five-factor model										
			a_i					$t(a_i)$		
Small	−0.07	0.04	0.07	0.22	0.13	−0.42	0.16	0.36	0.86	0.54
2	−0.30	0.25	−0.32	−0.18	0.06	−1.75*	1.37	−1.69*	−0.84	0.36
3	0.02	−0.29	−0.15	0.20	−0.16	0.09	−1.41	−0.84	1.11	−0.85
4	0.12	−0.37	0.16	−0.20	0.34	0.50	−2.36**	1.06	−1.24	1.92*
Large	0.02	0.08	−0.09	−0.02	0.10	0.06	0.48	−0.69	−0.15	0.54
			b_i					$t(b_i)$		
Small	1.01	0.81	0.66	0.86	0.95	24.99**	13.37**	13.16**	13.63**	15.59**
2	0.95	0.86	0.93	0.72	0.92	22.32**	19.48**	20.23**	13.37**	21.37**
3	1.00	0.99	0.90	0.85	1.07	16.37**	19.42**	20.42**	19.21**	23.54**
4	0.93	1.01	1.01	0.98	0.92	16.00**	26.44**	27.78**	24.50**	21.48**
Large	0.90	1.03	1.05	1.09	0.93	8.71**	26.40**	32.67**	34.90**	19.66**
			h_i					$t(h_i)$		
Small	−0.22	0.17	0.17	0.14	−0.07	−4.46**	2.21*	2.72**	1.76	−0.93
2	−0.24	0.10	0.12	0.14	0.07	−4.63**	1.90	2.12*	2.04*	1.23
3	−0.38	0.17	0.14	−0.04	−0.03	−5.03**	2.64**	2.55**	−0.71	−0.45
4	−0.29	0.05	0.02	0.04	−0.02	−4.01**	1.10	0.48	0.74	−0.43

Figure xxxiii: T-statistics for the five-factor model on the Australian stock exchange (Chiah, Chai, Zhong, & Li, 2016)

The above *Figure xxxiii: T-statistics for the five-factor model on the Australian stock exchange* (Chiah, Chai, Zhong, & Li, 2016) gives the t-statistics for the five-factor model in Australia. Here the values for b (the coefficient for Rm – Rf) and h (the coefficient for HML) show that they are both statistically significant when used in this model.

Let us compare these results with that values obtained from this study from section 5.3.2.

T-statistics for Rm – Rf coefficient

The results for this coefficient are in *Figure xvii: Time history of t-statistic for the Rm-Rf coefficient for Fama French models (Top 160)* and *Table 9: T-Statistics for the Fama French models' Rm-Rf coefficient (top 160)*. It is clear that in all cases the use of Rm – Rf is an important part of the model. This vindicates the Fama and French position that inclusion of Rm – Rf is non-negotiable.

T-statistics for HML coefficient

In all cases, one can see that the t-statistic is not statistically significant to the 95% confidence level. This is in *Figure xviii: Time history of t-statistic for the HML coefficient for Fama French models (Top 160)* and Table 10: T-Statistics for the Fama French models' HML coefficient (top 160). It is interesting to see that this result is rather different from the figure above which both show that the t-statistic for the HML coefficient is significant.

T-statistics for SMB coefficient

Unfortunately, there was no success in looking for other sources of t-statistics for this coefficient. This is a very interesting result for this data analysis. The results are given in *Figure xix: Time history of t-statistic for the SMB coefficient for Fama French models (Top 160)* and *Table 11: T-Statistics for the Fama French models' SMB coefficient (top 160)*

The coefficient is statistically significant (t-statistic > 2) in the following models: CAPM + HML SMB, CAPM + HML SMB CMA, CAPM + HML SMB RMW, CAPM + HML SMB CMA RMW

The coefficient is not statistically significant (t-statistic < 2) in the following models: CAPM + SMB, CAPM + SMB CMA, CAPM + SMB RMW, CAPM + SMB CMA RMW

The difference between these sets of results is interesting. One can see that the models that have a statistically significant coefficient for SMB only occur when SMB is together with HML in a model (the Fama French three-factor model plus associated extensions). The implication from this is that, even though HML does not have a good t-statistic, in order to obtain the best models these two factors should be used in conjunction.

T-statistics for CMA coefficient

The results in *Figure xx: Time history of t-statistic for the CMA coefficient for Fama French models (Top 160)* show a similar situation to SMB.

The coefficient is statistically significant (t-statistic > 2) in the following models: CAPM + HML CMA, CAPM + SMB CMA, CAPM + HML SMB CMA RMW

The coefficient is not statistically significant (t-statistic < 2) in the following models: CAPM + CMA, CAPM + CMA RMW, CAPM + HML SMB CMA, CAPM + HML SMB CMA RMW

Unfortunately, there is no clear trend for when CMA is statistically significant or not. Additionally, one can see in *Figure xxxii: T-statistics for the Fama French five-factor asset pricing model* that the coefficient has statistical significance in most of the portfolios that were evaluated in the North American context.

T-statistics for RMW coefficient

The results in *Figure xxi: Time history of t-statistic for the RMW coefficient for Fama French models (Top 160)*Figure *xx*: Time history of t-statistic for the CMA coefficient for Fama French models (Top 160) show a similar situation to CMA.

The coefficient is statistically significant (t-statistic > 2) in the following models: CAPM + HML RMW, CAPM + SMB RMW, CAPM + CMA RMW

The coefficient is not statistically significant (t-statistic < 2) in the following models: CAPM + RMW, CAPM + HML SMB RMW, CAPM + SMB CMA RMW, CAPM + HML SMB CMA RMW

This result shows that the coefficient for RMW is only statistically significant in the asset pricing models with fewer factors. The implication is that RMW is possibly overlapping with other factors and is therefore unnecessary for four or five factor models. However, before drawing any conclusions regarding this, it needs to be evaluated in the context of the R^2 values as that is the ultimate measure of how “good” the model is.

One can see in *Figure xxxii: T-statistics for the Fama French five-factor asset pricing model* that the coefficient has statistical significance in most of the portfolios that were evaluated by (Fama & French, 2015) so there a difference between the two studies. Once again it would be interesting to evaluate whether the time scale over which the regression is run has an effect on the results.

6.3.3 Fama French R^2

The value for R^2 could be regarded as analogous to the “bottom line” of a set of financial statements. It is the overall measure of how well the model explains variation in the dependent variable. A value of 100% means that the model is an exact reflection of reality and a value of 0% means that the model has no reflection of reality.

While some research topics could be expected to have a fairly high value for the model to have credence, models explaining stock market behaviour are subject to a wide range of influencing factors.

- Behavioural economics as described by (Mullainathan & Thaler, 2000) states that an extent of stock market behaviour is driven by human psychology and sentiment.
- The efficient markets hypothesis (Fama, 1970) states the exact opposite, that the stock market is a reflection of rational investors who are all equally informed about the company performance and what shares they should be buying or selling.
- Macroeconomic and geopolitical factors can affect an organisation and its share price significantly even though it has nothing to do with how well the organisation is being managed (or not).

While the Fama French models aim to be able to explain these phenomena in a rational and efficient way, it is clear that there are many other factors that can have an influence. For these reasons, it is asserted that it would be unfair to set a high qualification criteria for inclusion.

The available literature also provides some information about other investigations where the R^2 was evaluated for the Fama French five-factor model. Once such example is (Chiah, Chai, Zhong, & Li, 2016) and the relevant results are in *Figure xxxiv: Adjusted*

R2 statistics for portfolios formed on bivariate sorts of market capitalisation and value (Chiah, Chai, Zhong, & Li, 2016).

Table 6 (*continued*)

	Low	2	3	4	High
Large	−0.02	0.25	0.12	0.07	−0.46
			s_i		
Small	1.47	0.71	0.76	1.00	1.41
2	0.88	0.70	0.68	0.78	1.01
3	0.34	0.58	0.50	0.66	0.65
4	0.05	0.21	0.24	0.21	0.20
Large	−0.32	−0.20	−0.05	−0.16	−0.34
			p_i		
Small	−0.25	0.18	0.31	0.51	0.69
2	−0.23	0.27	0.43	0.37	0.65
3	−0.52	0.30	0.24	0.27	0.38
4	−0.6	0.00	0.14	0.25	0.03
Large	−0.68	−0.38	0.06	0.21	0.30
			l_i		
Small	0.18	0.06	−0.14	−0.03	0.18
2	−0.10	−0.06	0.03	0.04	0.04
3	−0.30	−0.16	0.04	−0.06	−0.20
4	0.00	0.03	0.02	0.06	−0.03
Large	0.07	0.09	−0.02	0.04	0.10
			Adjusted R^2		
Small	0.86	0.42	0.45	0.43	0.55
2	0.79	0.60	0.57	0.41	0.63
3	0.69	0.58	0.58	0.60	0.68
4	0.65	0.72	0.73	0.65	0.64
Large	0.35	0.76	0.79	0.80	0.62

Figure xxxiv: Adjusted R^2 statistics for portfolios formed on bivariate sorts of market capitalisation and value *(Chiah, Chai, Zhong, & Li, 2016)*

Table 1
Empirical properties of the q-factors

Panel A: Descriptive statistics

	Mean	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	R^2
r_{ME}	0.31	0.23	0.17				0.06
	(2.12)	(1.62)	(4.33)				
		0.04	0.02	0.99	0.17		0.93
		(1.09)	(1.59)	(57.37)	(7.05)		
		0.01	0.02	0.99	0.19	0.03	0.94
		(0.15)	(2.40)	(61.51)	(7.34)	(2.16)	
$r_{I/A}$	0.45	0.52	−0.15				0.13
	(4.95)	(5.93)	(−5.58)				
		0.33	−0.06	−0.02	0.39		0.50
		(4.85)	(−3.66)	(−0.81)	(11.98)		
		0.28	−0.05	−0.02	0.41	0.05	0.52
		(3.85)	(−3.24)	(−0.87)	(11.94)	(1.97)	
r_{ROE}	0.58	0.63	−0.11				0.04
	(4.81)	(5.62)	(−2.38)				
		0.77	−0.09	−0.33	−0.20		0.20
		(6.94)	(−2.08)	(−5.75)	(−2.38)		
		0.50	−0.03	−0.33	−0.10	0.28	0.40
		(4.75)	(−0.98)	(−4.38)	(−1.48)	(6.27)	

Figure xxxv: R^2 for various asset-pricing models *(Hou, Xue, & Zhang, 2015)*

The above *Figure xxxv: R2 for various asset-pricing models* (Hou, Xue, & Zhang, 2015) contains the results for R^2 for a number of different asset pricing models. The first is the CAPM, the second is the Fama French three-factor model and the third adds momentum (UMD) to the three factor model, this four factor model was originally proposed by

(Carhart, 1997). One can see that as more terms are added to the model, the R^2 gets higher so the model is capturing more of the variation.

The analysis results are in Figure *xxii*: Time history of R^2 for the various Fama French models (Top 160) and Figure *xxiii*: Time history of R^2 for the various Fama French models (Top 40). Firstly, it is important to note that as more terms are added to the model, the higher the R^2 value. This is the same trend as what was seen by (Hou, Xue, & Zhang, 2015) even though the data was obtained in different ways from different sources.

The values in this study range from 16% to 28% (depending on the model) for the JSE top 160 while it ranges from 32% to 39% for the top 40. In both cases this is somewhat lower than the data recorded for the five-factor model in (Chiah, Chai, Zhong, & Li, 2016) which ranges from 35% to 80% and also higher than the work by (Hou, Xue, & Zhang, 2015) with results between 6% to 94% depending on the model. The histograms for R^2 demonstrate that the value has a reasonably good spread of data with the shape approaching a normal distribution. Overall, however, it is asserted that the trends seen in other research is also seen in this evaluation on the JSE, that more factors give better R^2 and that the values are similar. One can also see that the values are well above 20% (the benchmark between poor and acceptable) and 40% (the benchmark between acceptable and very good).

6.4 Hypothesis 1: Evaluation of Alpha

The main aim for both Hypothesis 1 and Hypothesis 2 is to reach a conclusion as to which asset pricing model is better and have some confidence in the result. As already mentioned there are two ways that the overall performance of a model is measured. The first is the value of alpha and the second is the value of R^2.

Figure *xxviii*: Time history of alpha for the CAPM and selected Fama models (Top 160) plots the values for alpha of each of the selected models. The lines for the various models overlap substantially and the lines also cross each other. Based on a visual evaluation, one would state that there is no discernible difference between the various models. However this needs to be checked statistically.

6.4.1 $H1a_0$: Alpha Comparison between CAPM and Selected Fama French

The detail of how the difference of means calculation was conducted can be found in section 4.7.4. The results for the alpha evaluation, Table *16*: Mean of alpha for CAPM and Selected Fama French models (Top 160) and Table *17*: Mean of Alpha for CAPM and Selected Fama French models (Top 40) show that the means of the various alphas are not different to the 95% confidence level. Supplementary information is in appendix 4.

The use of alpha to evaluate the quality of a model is used extensively in academic literature and it also makes a great deal of intuitive sense. A better model should have a better (lower) alpha (Fama & French, 2015). The results for this study do not demonstrate this in any way and this is a concern.

The ANOVA shows that there is not a statistically significant difference. One could argue that the duration over which the analysis was done (36 months) influences the result, this would be worth investigating further. This result is true for both the top 160 shares as well as the top 40 shares.

***H1a$_0$* Statement**

Based on the above discussion we fail to reject the null hypothesis.

6.4.2 H1b$_0$: Alpha Comparison between groupings of Fama French

As has already been discussed the values obtained for alpha show very little difference from one model to the next and this is different from other studies. This needs to considered statistically though.

Two Factor Models

The values for alpha are all relatively similar for the two factor models. This is confirmed by the ANOVA which stated that there was no significant difference between any of the results in both the top 160 and the top 40 shares. It is interesting to note that the selected Fama French two factor model (CAPM + HML) displays the lowest value for alpha (low alpha implies the best result) but it is not statistically significant so it could be a coincidence.

Three Factor Models

The three factor models show relatively similar values for alpha. In both the top 160 and the top 40 shares the differences are not statistically significant.

While there is not statistical significance, it is interesting that out of the values closest to 0, the Fama French three factor model (CAPM + HML SMB) has the second closest (Fama & French, 1992). This is not anything more than interesting though.

Four Factor Models

The results for the difference of mean tests for the four factor models show that there is no significant difference between any of the values for alpha in any of the models for the top 160 shares.

In the top 40 shares the selected four factor model (CAPM + SMB CMA RMW) obtained from (Hou, Xue, & Zhang, 2015) has the largest magnitude of alpha as well as being statistically significantly different from the model with the smallest value of alpha (CAPM + HML SMB RMW). This is somewhat concerning, but it should be kept in mind that the results for alpha are not typical of other studies. As a result, one should view conclusions with some care.

Predominantly, the Fama French models that are grouped by their numbers of factors have means of Alpha that are very similar. There is only one difference of means that is statistically significant at the 95% confidence level. Additionally, $H1a_0$ demonstrates that models with differing numbers of factors are predominantly different to a statistically significant degree.

$H1b_0$ Statement

Based on the above discussion we reject the null hypothesis and accept the alternative hypothesis.

It should be noted though, that out of all of the comparisons that were conducted, there was only one difference in means that was statistically significant.

6.5 Hypothesis 2: Evaluation of R^2

The rationale for the selection of the limited number of asset pricing models is explained in Chapter 5. The decision to limit the number is somewhat justified when one reviews the results for the graphs for Figure *xxii*: Time history of R^2 for the various Fama French models (Top 160) and Figure *xxiii*: Time history of R^2 for the various Fama French models (Top 40). Here one can see that the results for R^2 are predominantly grouped according to how many factors they have, with the larger number of factors giving higher values for

R^2. This suggests that for a given model with a given number of factors of interest, one could select almost any of the models available. This is evaluated in the b_0 hypotheses.

6.5.1 H2a$_0$: R^2 Comparison between CAPM and Selected Fama French

The difference of means analysis produced interesting results when analysing R^2. For the top 160 shares there is a significant difference between the means of all the models at a 95% confidence level. For the top 40 shares there is a significant difference between all but one model when measured at a 95% confidence level. The exception are the means of R^2 for the models CAPM + HML SMB and CAPM + SMB CMA RMW.

The fact that there are significant differences between so many of the models allows us to reach some effective conclusions.

- The CAPM is not a good as any of the 14 other Fama French asset pricing models that were evaluated.
- The model, which proved to be the best explanatory model for asset behaviour on the JSE, was the five-factor asset pricing model. This was true for both the Top 160 shares on the JSE and the Top 40 shares.
- The results for the Top 40 shares are considerably better (higher R^2) than the result for the Top 160. One potential explanation is that the Top 40 shares behave in a way that is closer to the efficient market hypothesis of (Fama, 1970). This would make them better at following a rational asset pricing model than shares which are thinly traded, more subject to market sentiment and less scrutinised (the Top 160).
- The mean R^2 value of the five-factor asset is 42.4% for the Top 40 shares. Viewed in isolation it could be argued that it is not good enough. However when compared to other models available, it is considerably better and it should be viewed in that light. Stock exchange behaviour is subject to many different variables that influence behaviour. Thus, an R^2 value greater than 70% is unlikely.

H2a$_0$ Statement

Based on the above discussion we reject the null hypothesis and accept the alternative hypothesis. There are statistically significant differences between the means of the R^2 values of the CAPM the four selected Fama French asset pricing models.

It should additionally be noted that all but one of the means are statistically significantly different to all of the others.

6.5.2 H2b^0: R^2 Comparison between groupings of Fama French

In general, the values for R^2 show a larger spread than what was seen for the results for alpha. Thus, it is felt that more credence should be given to R^2 than alpha. Let us consider the statistical results.

Two factor models

For both the top 160 and the top 40 shares on the JSE there is no statistically significant difference between the means. Thus, one could argue that any of the two factor models would have worked as well any of the others.

The two factor model that was selected from the literature was CAPM + HML. While not statistically significant, it is interesting that it has the highest R^2 value for the top 40 shares and the second largest for the top 160 shares. A larger data set may or may not provide some statistical support that CAPM + HML is one of the better two factor models and could form the basis for further work.

Three Factor Models

For the top 160 shares the Fama French 3 factor model (CAPM + HML SMB) from (Fama & French, 1992) has the highest value of R^2 and the value is larger in a statistically significant way to the following three factor models: CAPM + HML CMA and CAPM + CMA RMW. This indicates that the prior research and this research corroborate each other, that the traditional three factor Fama French model is possibly the best available three factor model. Once again, out of the six models there are some statistically insignificant differences that could potentially be proven or disproven with more data.

For the top 40 shares CAPM + HML SMB was the second highest value of R^2 but only just lower than CAPM + HML RMW. All differences between the six models are not statistically different. The results can be viewed in Table *29*: R^2 values for the three factor models (Top 40). However there are no statistically significant differences.

Four Factor Models

There are no statistically significant differences between any of the values of R^2 for both the top 160 and top 40 shares. The values can be seen in Table *30*: R^2 values for the four factor models (Top 160) and Table *31*: R^2 values for the four factor models (Top 40).

For all of these groupings of two factor, three factor and four factor models there are very small differences between models with the same number of factors. At the same time $H2a_0$ demonstrates that models with differing numbers of factors are predominantly different to a statistically significant degree.

$H2b_0$ Statement

Based on the above discussion we reject the null hypothesis and accept the alternative hypothesis. There are statistically significant differences between the means of the R^2 values for the various groupings of Fama French asset pricing models.

It should be additionally be noted that statistically significant differences are in the minority. Out of all of the comparisons that were conducted there were only two differences that were statistically significantly different.

Chapter 7: Conclusions

7.1 Principal Findings

The aim of this research was to evaluate 15 different asset-pricing models.

Factors in the asset pricing model	Detail of model and factors
Rm-Rf	The CAPM by (Sharpe, 1964) and (Lintner, 1965) where Rm-Rf is the difference between the return of the market and the return of a risk-free investment
Rm-Rf & HML	A two-factor model where HML is the difference between the return on a portfolio of high value shares and the return on a portfolio of low value shares (value is measured by book to market ratio)
Rm-Rf & SMB	A two-factor model where SMB is the difference between the return on a portfolio of small company shares and the return on a portfolio of large company shares (size is measured by market capitalisation)
Rm-Rf & CMA	A two-factor model where CMA is the difference between the return on a portfolio of conservative investment company shares and the return on a portfolio of aggressive investment company shares (investment is measured by asset growth)
Rm-Rf & RMW	A two-factor model where RMW is the difference between the return on a portfolio of companies with robust profitability and the return on a portfolio of companies with weak profitability (profitability is measured by return on assets)
Rm-Rf, HML & SMB	The three-factor model originally proposed by (Fama & French, 1992)
Rm-Rf, HML & CMA	A three-factor model
Rm-Rf, HML & RMW	A three-factor model
Rm-Rf, SMB & CMA	A three-factor model
Rm-Rf, SMB & RMW	A three-factor model
Rm-Rf, CMA & RMW	A three-factor model
Rm-Rf, HML, SMB & CMA	A four-factor model
Rm-Rf, HML, SMB & RMW	A four-factor model
Rm-Rf, SMB, CMA & RMW	The four-factor model evaluated by (Hou, Xue, & Zhang, 2015)
Rm-Rf, HML, SMB, CMA & RMW	The five-factor model proposed by (Fama & French, 2017)

The evaluation was done in three ways:

1. The CAPM was evaluated to see how well it explained share behaviour. The performance was compared to previous studies for alpha, t-statistics and R^2 as well as standard benchmarks

2. The 14 other models which contained all possible combinations of the other factors in the Fama French five-factor asset pricing model were evaluated to see how well they explained asset returns. The performance was compared to previous studies for alpha, t-statistics and R^2 as well as standard benchmarks
3. Five models' were selected for a more in-depth evaluation, highlighted in grey above. Their results were compared to each other to verify whether there was a significant difference between their results. This analysis was conducted with the values for alpha as well as the values for R^2.
4. The groupings of two factor were evaluated statistically on both alpha and R^2 to see whether all two factor models performed similarly or differently. This was also done for groupings of three factor and four factor models.

The above analyses were conducted for the top 160 companies and the top 40 companies on the JSE when sorted by market capitalisation.

The findings can be summarised as follows:

CAPM

The CAPM is not a good explanatory model for share behaviour on the JSE. This confirms the contemporary consensus about the model as well as work conducted by (Carter, Muller, & Ward, 2017). Work in other regions of the world has similar findings: (Fama & French, 1992) and (Loukeris, 2009).

Alpha

A common measure of asset pricing model ability is the value of alpha. Typically the smaller the value of alpha the better the model is. This study gave something very different, with values or alpha being very similar for all 15 models. This was confirmed statistically by the ANOVA difference of means test, which found that there was no significant difference in the means for five selected asset pricing models.

Fama French variations

The 14 models that considered all possible combinations of the four additional factors used in the Fama French asset pricing model showed a consistent picture with regard to their R^2 values. One can see in the results in Table *14*: Descriptive statistics for R^2 of all Fama French models (top 160) and Table *15*: Descriptive statistics for R^2 of all Fama French models (top 40) that in general the more factors there are in the model the higher the R^2 value is.

This leads to the conclusion that the five factor asset pricing model as proposed by (Fama & French, 2015) is the best available. This result is at odds with the findings of the above authors. Their evaluation showed that excluding the value term, HML, yielded very similar results to the five-factor model. This does not mean that the results are incorrect, studies have already shown that the performance of the model varies from one region to the next, it is simply an interesting difference. The R^2 values for the five factor model are considerably better than the other models and compare favourably with other evaluations of R^2. While some statisticians may be disappointed with the mean values of 32.43% for the top 160 and 42.39% for the top 40 models respectively, a model that can deal with the extremely unpredictable nature of stock markets and give those results can be said to be rather impressive. Especially when it is compared to the R^2 values of other asset pricing models, the CAPM in particular.

The results also show that all asset pricing models explain more variation in share prices for the top 40 companies than the top 160 companies. A proposed explanation is that the top 40 shares are more heavily traded and subject to a great deal more scrutiny and analysis. Therefore, they are more consistent with the efficient markets hypothesis (Fama, 1970) so a mathematical model has a better chance of describing their behaviour. The smaller shares, which are included in the top 160, are subject to less rational considerations, which makes it more difficult for a model to explain their behaviour.

Results for R^2 Evaluation of Selected Fama French

In the top 160 analysis, the difference of means for the five selected models confirmed that the means for R^2 for the various models were different at a 95% confidence level. In fact, in some cases it was considerably more than 95%. This is shown is section 5.5.1. This serves as a confirmation of the commentary above that the five factor model is indeed the best asset pricing model of the 15 that were evaluated.

In the top 40 analysis there was a difference of means at the 95% confidence level for all of the models except the three factor model of (Fama & French, 1992) and the four factor model of (Hou, Xue, & Zhang, 2015). However, the main conclusion that the five factor asset pricing model is the best out of all the ones evaluated is also confirmed for the top 40 companies.

Groupings of Fama French Models

All two factor models were evaluated against each other and the same was done for the three and four factors groups of models. There were there were a few instances of differences at the 95% confidence level but there were far more where it could be said that there were no statistically significant differences. The rather surprising conclusion is that the performance of an asset pricing model is largely dependent on the number of factors that it contains and not necessarily on which factors that constitute it. It should be noted though, that this finding is only for asset pricing models formed from the factors that make up the five factor model, it could not be expanded to cover all feasible factors.

7.2 Implications for Management

It is very important to understand what can be done with the increased knowledge that has been accumulated with this study. The five factor asset pricing model demonstrates the ability to explain share price fluctuation on the JSE to a level of completeness which is not possible with the other models. Therefore, it gives a better understanding of what is driving variation in share prices.

Understanding how stock exchanges behave is an academic challenge of no small measure. A significant amount of time and effort has gone into developing an understanding with many very different theories proposed. For example, the efficient markets hypothesis of (Fama, 1970) is almost diametrically opposed to the ideas of behavioural economics of (Mullainathan & Thaler, 2000). It is clear that a comprehensive and accurate model and understanding of the stock market does not exist. As a result, research that contributes to developing a better understanding has value. This specific item of research has identified that on the JSE, the five factor model explains 32% to 42% of share price fluctuation. This leaves 58% to 68% still to be understood and explained. While this may seem like a long way to go, consider that the traditional asset pricing model (CAPM) leaves 72% to 84% unexplained. Progress has been made.

With respect to how this can guide investment strategies, some thoughts are as follows:

- Style investing selects shares to invest in based on certain characteristics such as value, profitability, momentum. The five factor model provides four factors that are worth considering as they all contribute to the overall value of R^2 for the JSE environment. These are investment (CMA), profitability (RMW), size (SMB) and value (HML).
- Each factor is divided into two different investment styles:

- CMA is divided between high investment companies and low investment companies
- RMW is divided between high profitability companies and low profitability companies
- SMB is divided between small market cap companies and large market cap companies
- HML is divided between high value companies and low value companies

- The plots of the various factors such as the one shown Figure *ix*: Cumulative returns for each Fama French factor (blue is SMB) show:
  - Of the two styles making up each factor, which style is out-performing the other
  - The time history of each factor provides an indication of how stable the out-performance is. Consistent out-performance gives some confidence that this will continue for an appreciable time. Outperformance which “flips” regularly is the opposite and one should be careful of investing in that style.
- The value of R2 for asset pricing models helps one understand how much of the returns are understood and how much of the returns are not understood. The unknown portion contributes to an understanding of the risk involved in a certain strategy and helps an investor balance this against their risk.

7.3 Limitations of the Research

No research analysis is perfect and neither is this one. The following concerns have been identified through the duration of the research and write-up.

- The multiple linear regressions that were conducted calculated the coefficients for use in the various models were conducted on a continuous basis (every 3 months) for the previous 36 months. This duration is very different to the one that was used by (Fama & French, 2015) who conducted their analysis over 606 months. Regressing over such a long period means that the model assumes that the coefficient remains constant for the full period. This may not be true or appropriate and regressing over the shorter period could give a more accurate model. On the other hand, there is concern that in this analysis the number of data points per regression (36 per variable) may be too few.
- The sorts that relate to SMB split the portfolios at the midpoint of rank of market capitalisation. This means that there are 80 shares in the “large” portfolio and 80 shares in the “small” portfolio. Considering the high concentration of very large

market cap shares on the JSE it could be argued that some of the shares that are inside the top 80 shares based on market cap should be classified as small, for example the 80th share (AFE – 14 619 market cap) is 1.1% of the largest share (BTI – 1 329 956 market cap). It may make more sense for the split between large and small to be at a lower number of shares, possibly around the 40th largest company on the JSE.

- A large number of analyses which evaluate asset pricing models use a Gibbons, Ross, Shanken (GRS) methodology first used by (Gibbons, Ross, & Shanken, 1989). The GRS statistic evaluates whether the model is a complete description of the portfolio price fluctuations. The GRS analysis in (Fama & French, 2015) found that the models analysed did not provide a complete description, as a result it was assumed that this would also be the case for this research. Additionally, this research worked with each individual share and not portfolios. While the values for R^2 point to the fact that the models are also incomplete models, it may have been worthwhile to evaluate this question with the more accepted method (GRS).
- The data set used in the analysis may have survivorship bias in it. This could have affected the results. However, it is felt that this is only a big concern for the first 2 to 3 years of the data set.

7.4 Further work

The limitations listed in section 7.3 above have already identified some suggestions for further work. In addition to those, the following opportunities for further work have been identified and there may be value in researching these specific questions in some detail.

- (Fama & French, 2015) state that they would like to evaluate a 6 factor model which adds the well-known investment style momentum. This would be a very interesting analysis but unfortunately may prove to be impractical on the relatively small JSE. It is highly likely that the portfolios would end up too small. However it may make sense to evaluate a different five factor model on the JSE. For example, adding in Resources / Non resources and Momentum while removing SMB which was brought into question by (Muller & Ward, 2013) and HML which was found to not play a big part by (Fama & French, 2015). This is the biggest opportunity for further work and development of understanding.
- The 2 x 2 x 2 x 2 x 2 is the purest means of evaluating the factors as it prevents a share appearing more than one sub-portfolio. However, there is some concern

that there are not enough shares in each sub-portfolio. One of the other sorts could be used, for example combinations of 2 x 2 sorts and 2 x 3 sorts.

- The plot of HML in Figure *ix*: Cumulative returns for each Fama French factor (blue is SMB) shows a positive and flat gradient since December 2015. This is after a negative gradient between 2008 and 2015. This may indicate a return of the value effect. Over it may be worth checking this at a later date when more data has been accumulated.
- The fact that alpha is very similar for all models is rather strange as the magnitude of alpha being an indication of model ability is a well-recognised phenomenon.
- The t-statistics for the five factor model are below 2 for four of them (Rm-Rf is above). This is rather strange as the R^2 for the five factor model has the highest value. This is worth investigating further to improve understanding this.

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Appendix 1: Fama French Factor Results

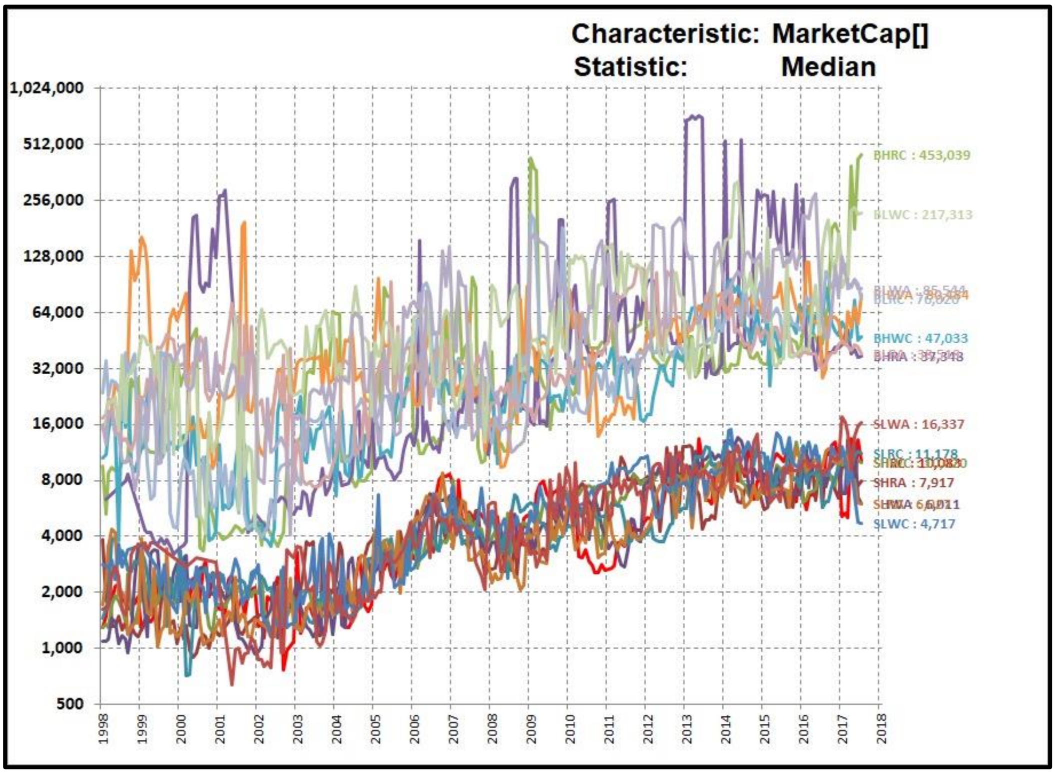

Figure xxxvi: Sub-portfolios used in SMB calculation ranked by market capitalisation

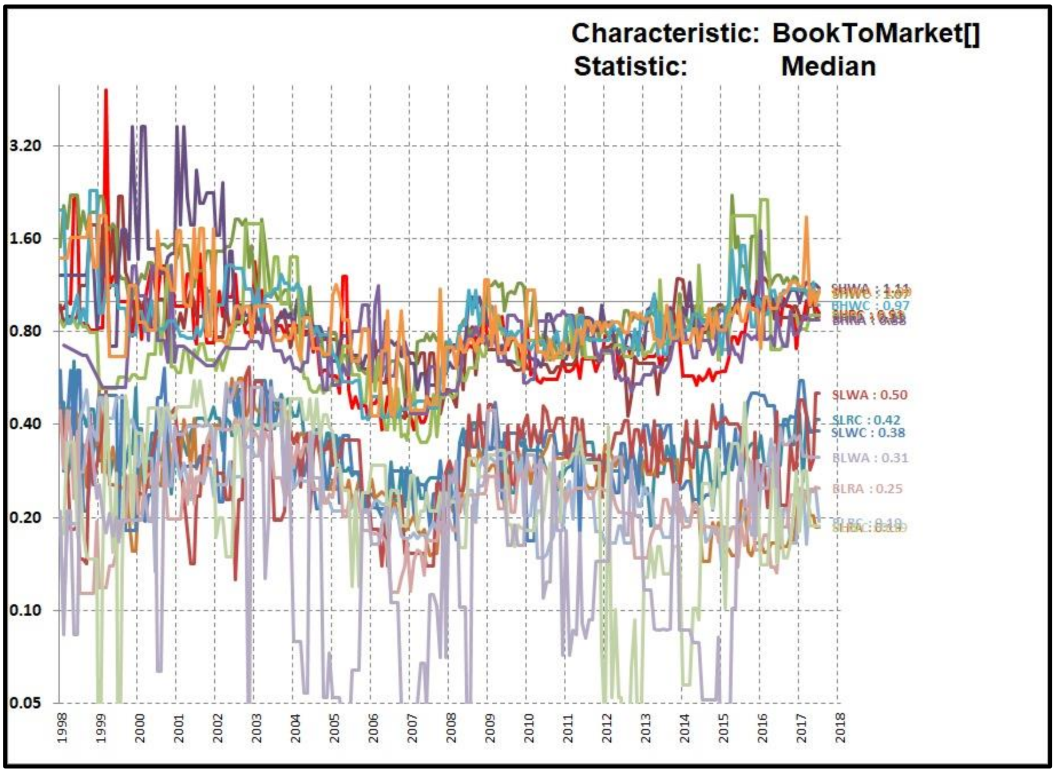

Figure xxxvii: Sub-portfolios used in HML calculation ranked by book to market ratio

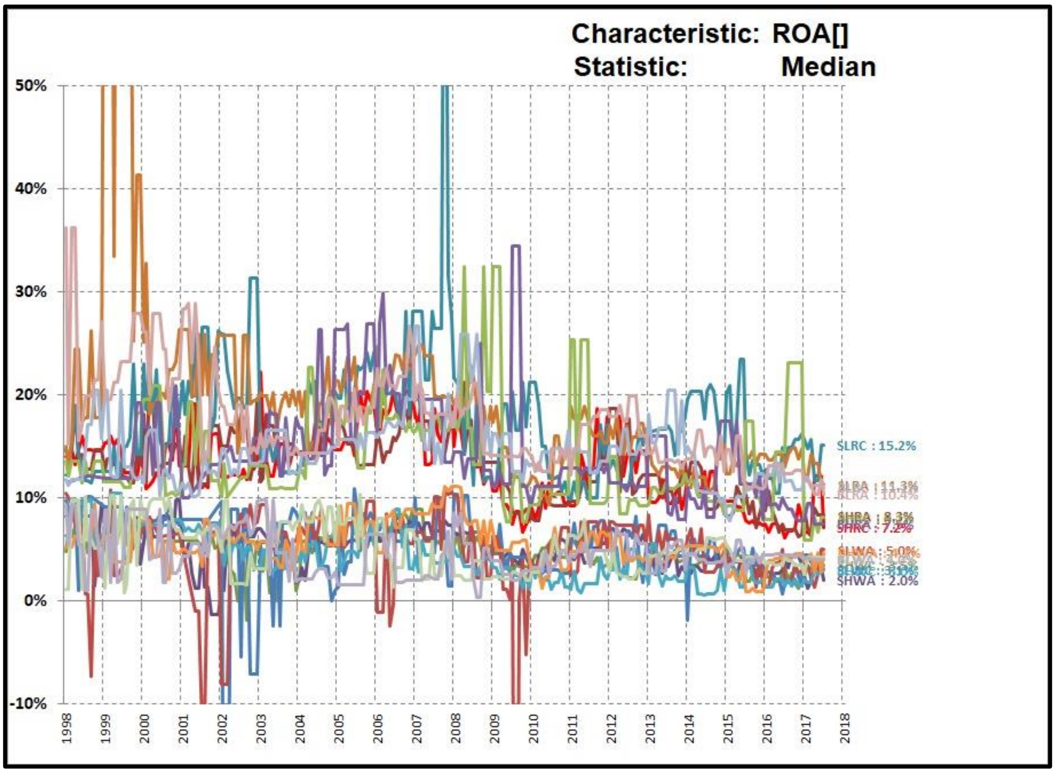

Figure xxxviii: Sub-portfolios used in RMW calculation ranked by return on assets (ROA)

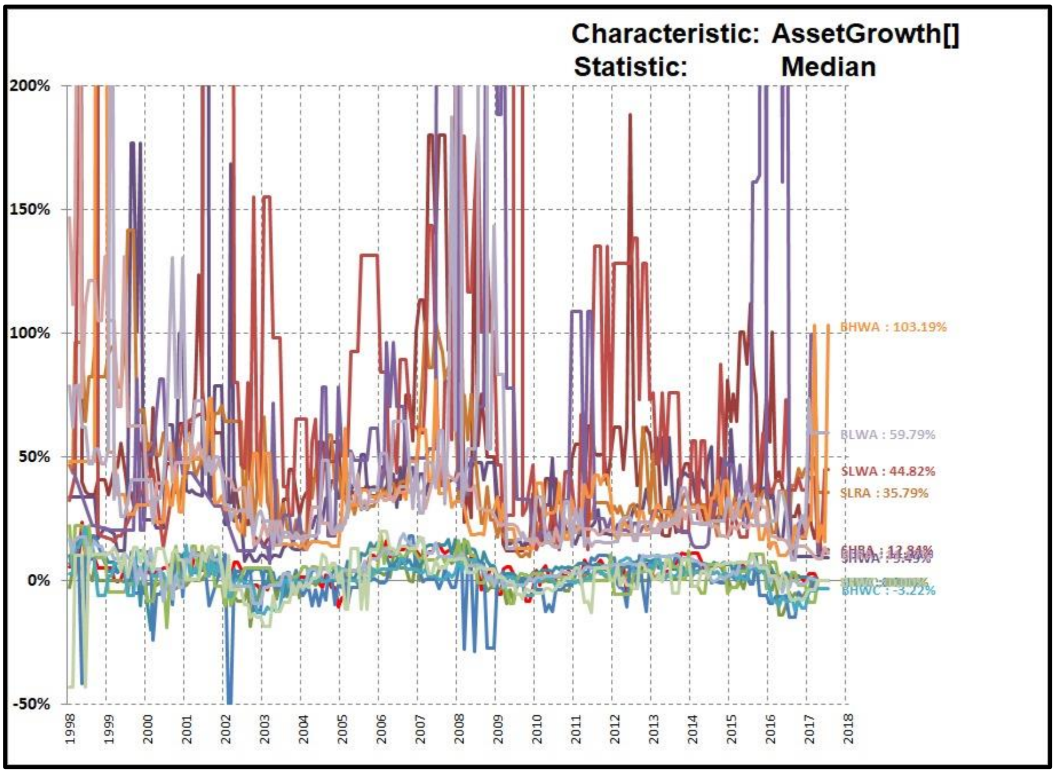

Figure xxxix: Sub-portfolios used in CMA calculation ranked by asset growth

Appendix 2: RQ1 CAPM Evaluation

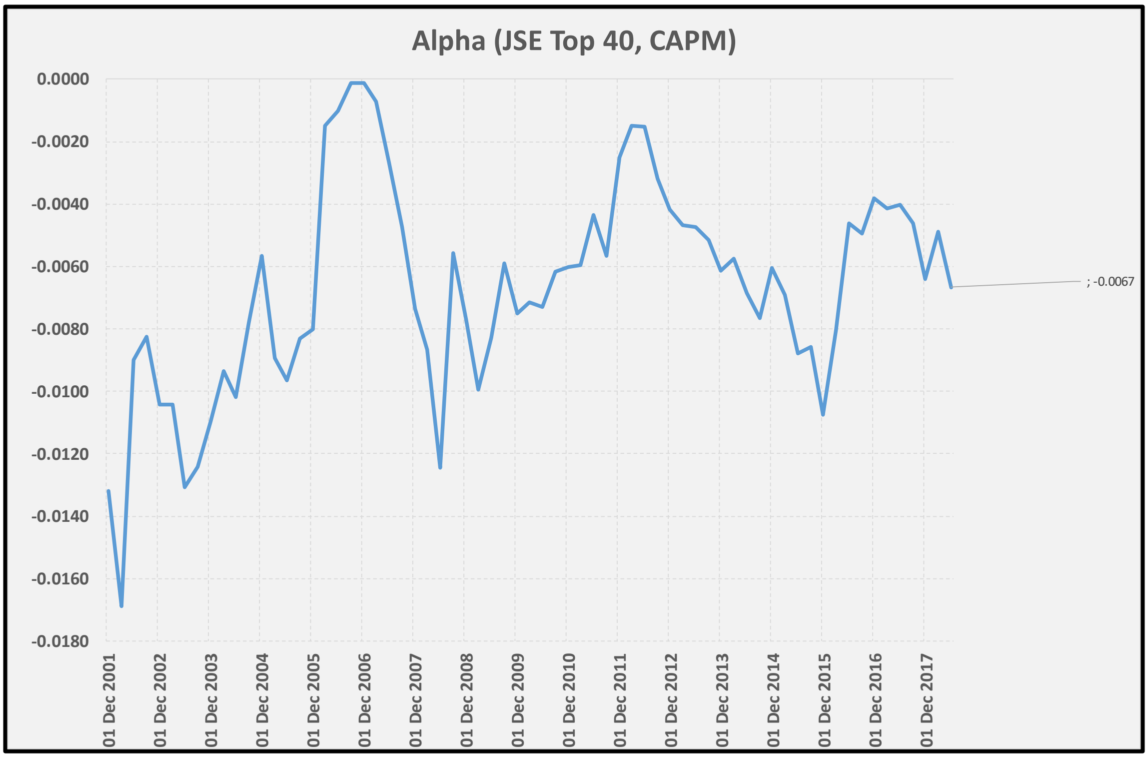

Figure xl: Alpha as regressed by the CAPM for Top 40

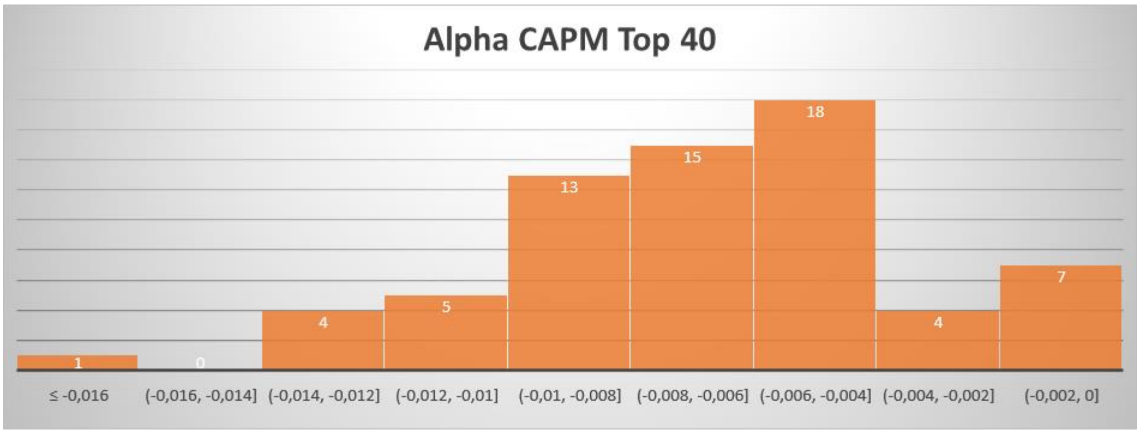

Figure xli: Histogram of Alpha values (Top 40)

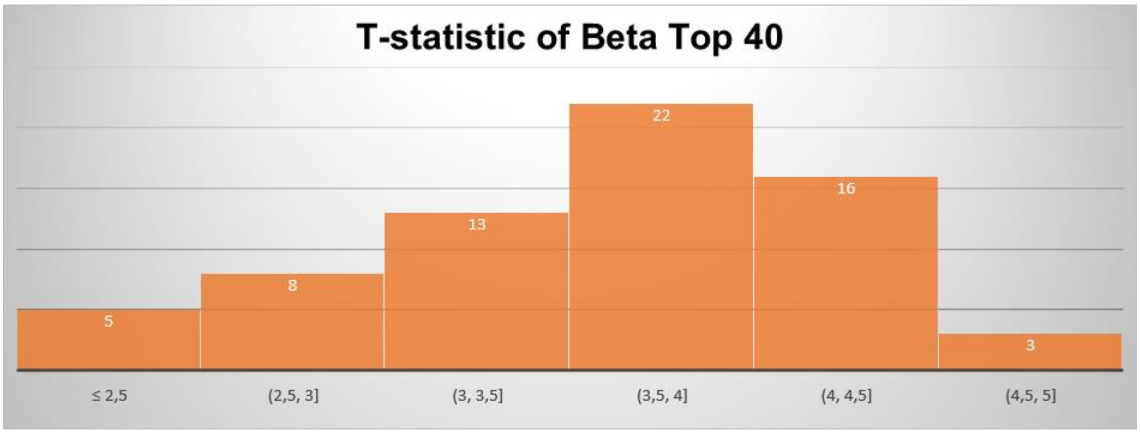

Figure xlii: Histogram of t-stat values for Beta (Top 40)

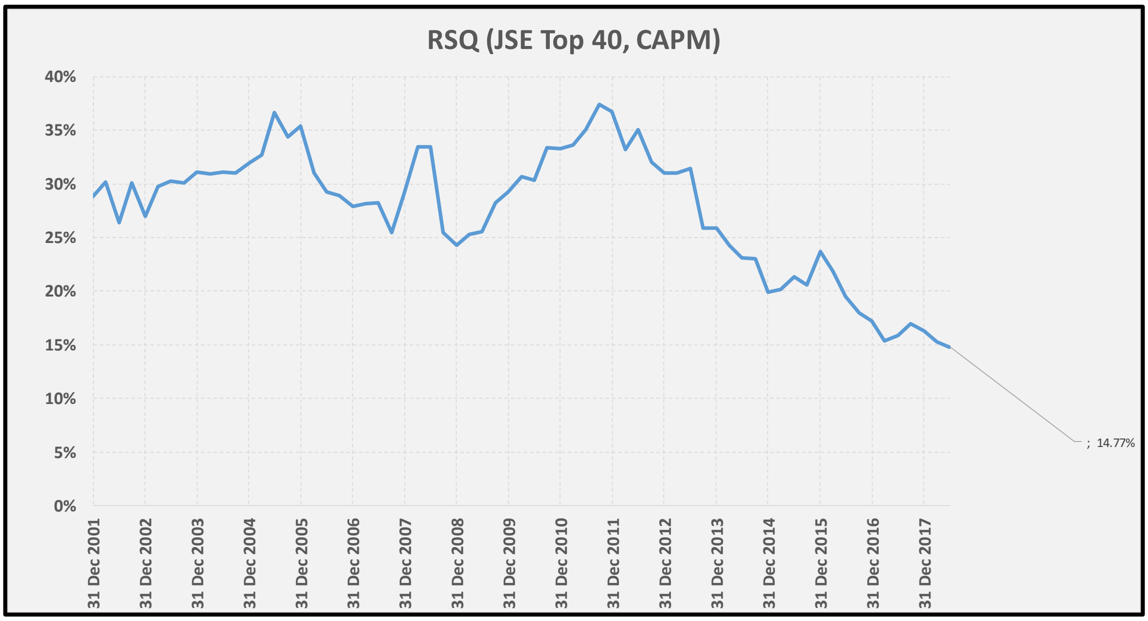

Figure xliii: R^2 for the CAPM (Top 40)

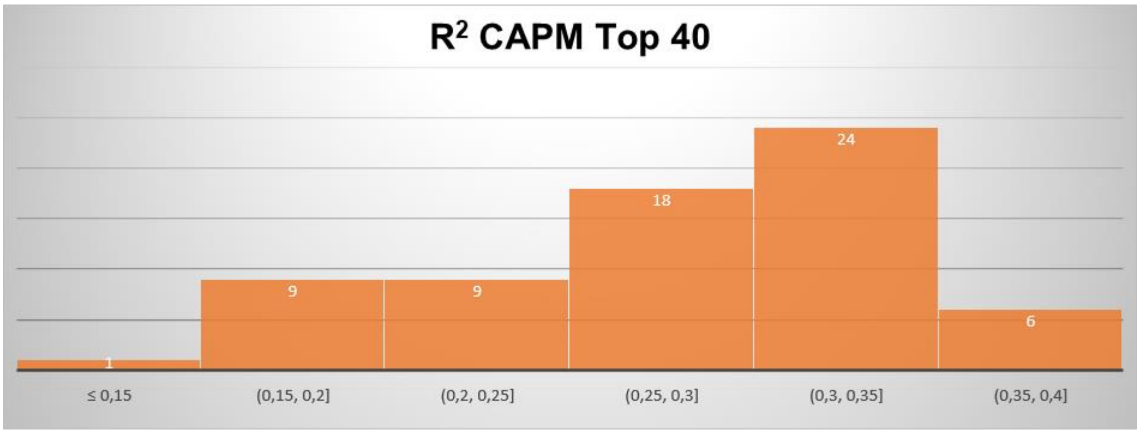

Figure xliv: Histogram of R^2 values for CAPM (Top 40)

Appendix 3: RQ2 Fama French Models Evaluation

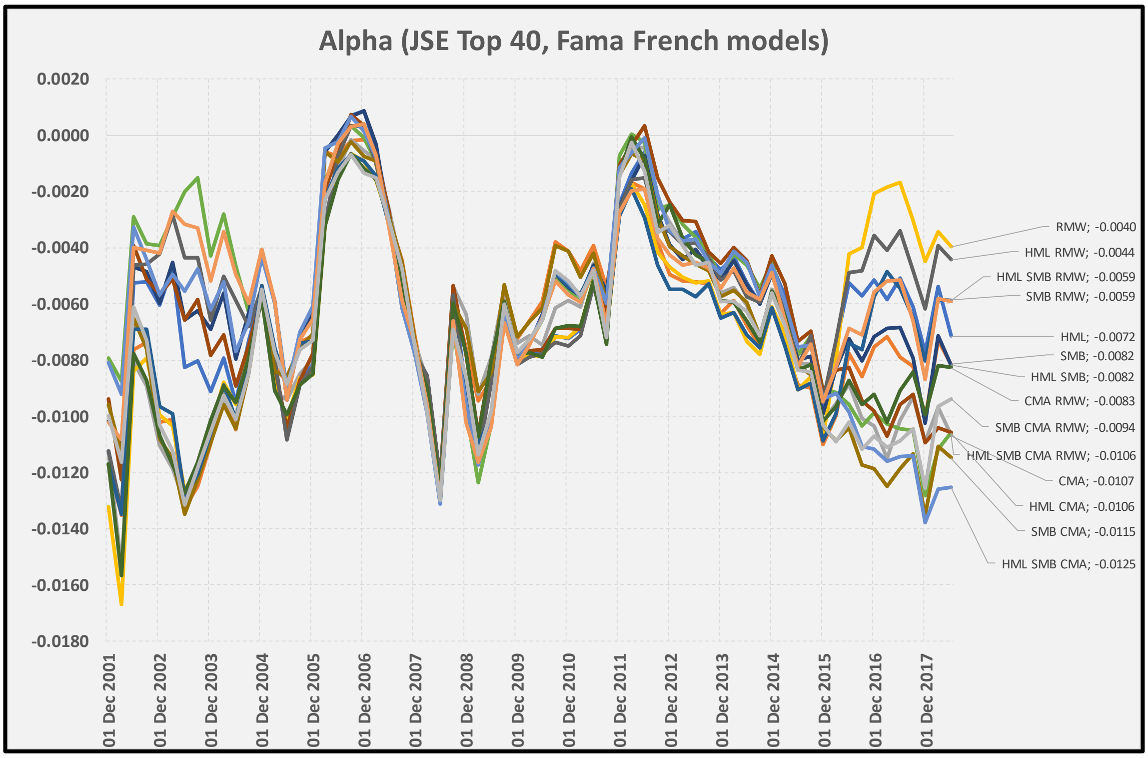

Figure xlv: Time history of alpha as per the various FF models (Top 40)

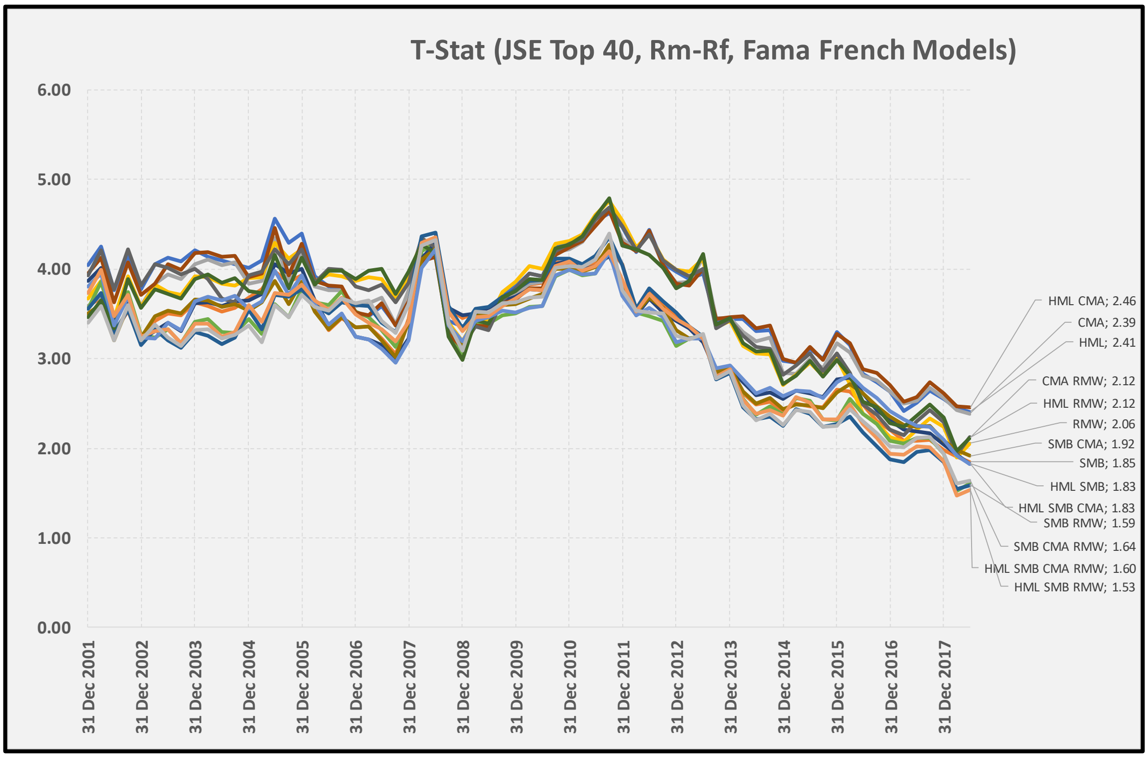

Figure xlvi: Time history of t-statistic for the Rm-Rf coefficient for Fama French models (Top 40)

Table 32: T-Statistics for the Fama French models' Rm-Rf coefficient (top 40)

Model	Mean	Median	Standard Dev.
CAPM + HML	3.6747	3.8024	0.5926
CAPM + SMB	3.2607	3.4804	0.6495
CAPM + CMA	3.5891	3.7124	0.5644
CAPM + RMW	3.5624	3.8387	0.7086
CAPM + HML SMB	3.2836	3.4574	0.6138
CAPM + HML CMA	3.6390	3.7828	0.5520
CAPM + HML RMW	3.5909	3.8296	0.6789
CAPM + SMB CMA	3.2417	3.4618	0.5902
CAPM + SMB RMW	3.1884	3.3575	0.7393
CAPM + CMA RMW	3.5378	3.7494	0.6522
CAPM + HML SMB CMA	3.2482	3.3711	0.5679
CAPM + HML SMB RMW	3.1988	3.4006	0.7035
CAPM + SMB CMA RMW	3.1604	3.3267	0.6772
CAPM + HML SMB CMA RMW	3.1619	3.3260	0.6448

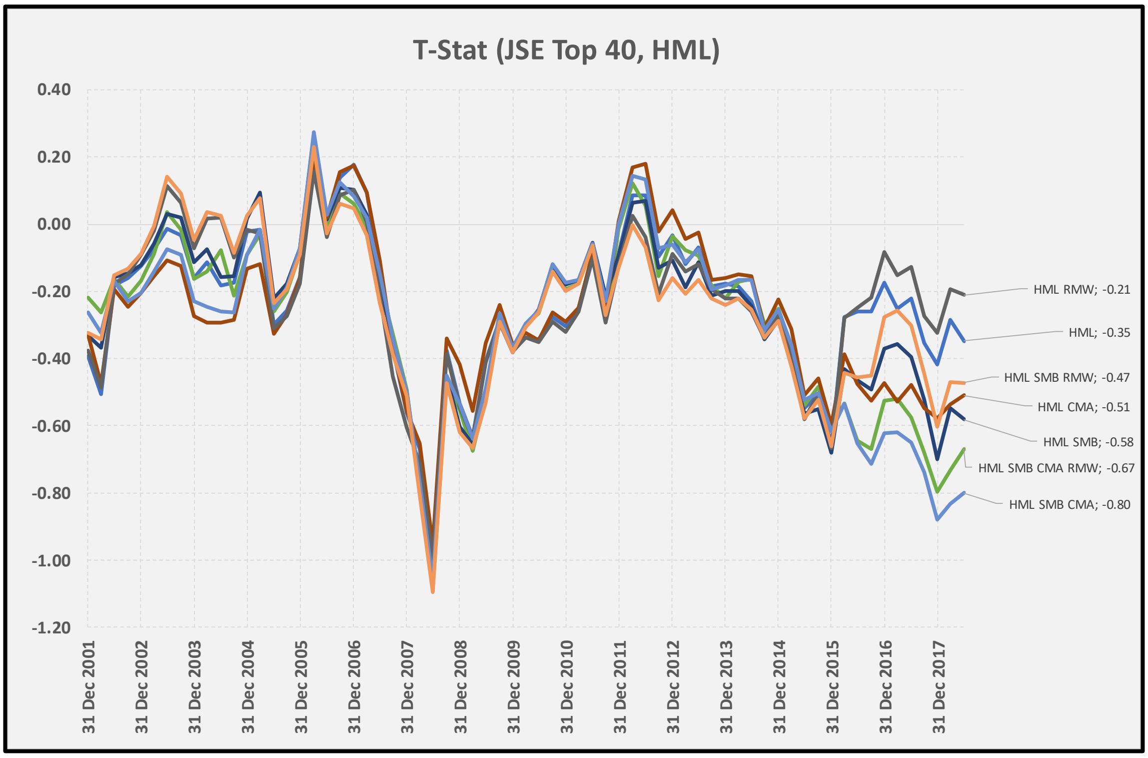

Figure xlvii: Time history of t-statistic for the HML coefficient for Fama French models (Top 40)

Table 33: T-Statistics for the Fama French models' HML coefficient (top 40)

Model	Mean	Median	Standard Dev.
CAPM + HML	-0.5786	-0.7190	0.4947
CAPM + HML SMB	-0.5968	-0.6607	0.4273
CAPM + HML CMA	-0.5777	-0.6231	0.5024
CAPM + HML RMW	-0.5667	-0.6933	0.4758
CAPM + HML SMB CMA	-0.6069	-0.6414	0.4351
CAPM + HML SMB RMW	-0.5827	-0.6364	0.4134
CAPM + HML SMB CMA RMW	-0.5791	-0.6094	0.4189

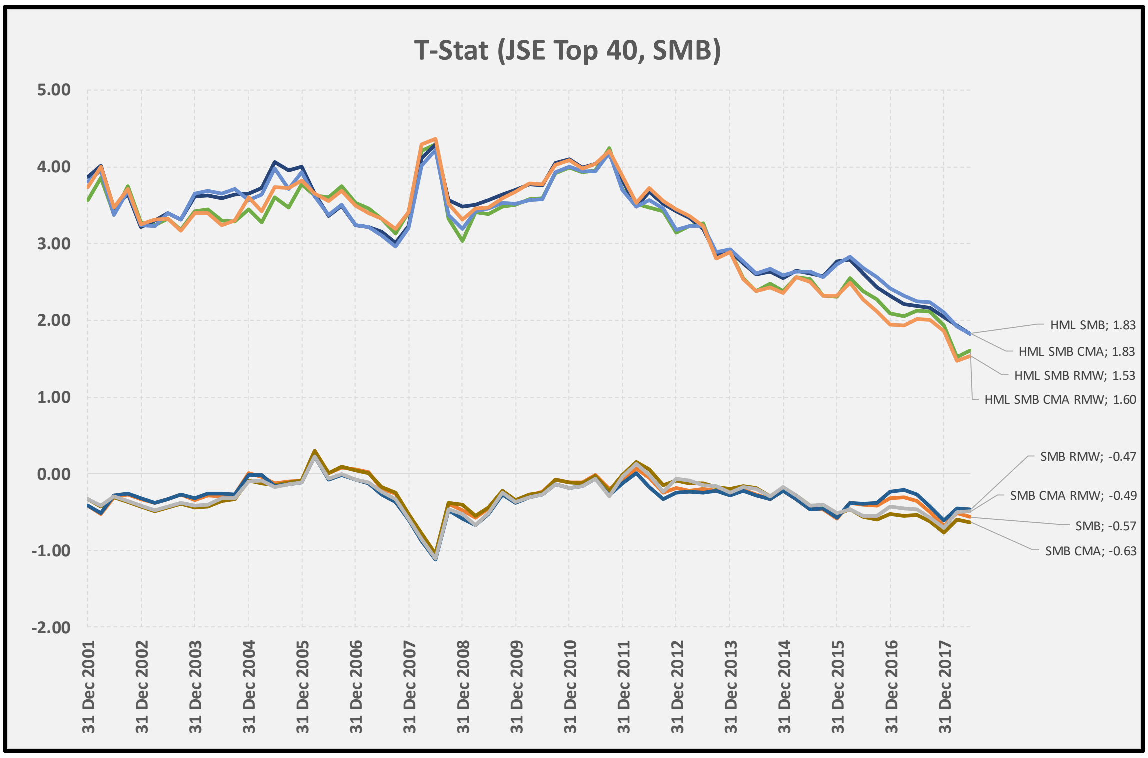

Figure xlviii: Time history of t-statistic for the SMB coefficient for Fama French models (Top 40)

Table 34: T-Statistics for the Fama French models' SMB coefficient (top 40)

Model	Mean	Median	Standard Dev.
CAPM + SMB	-0.5804	-0.6335	0.4367
CAPM + HML SMB	2.4471	2.5453	0.6096
CAPM + SMB CMA	-0.5831	-0.6344	0.4448
CAPM + SMB RMW	-0.6122	-0.6392	0.3912
CAPM + HML SMB CMA	2.4165	2.4833	0.5561
CAPM + HML SMB RMW	2.4132	2.5778	0.6765
CAPM + SMB CMA RMW	-0.6034	-0.6252	0.3963

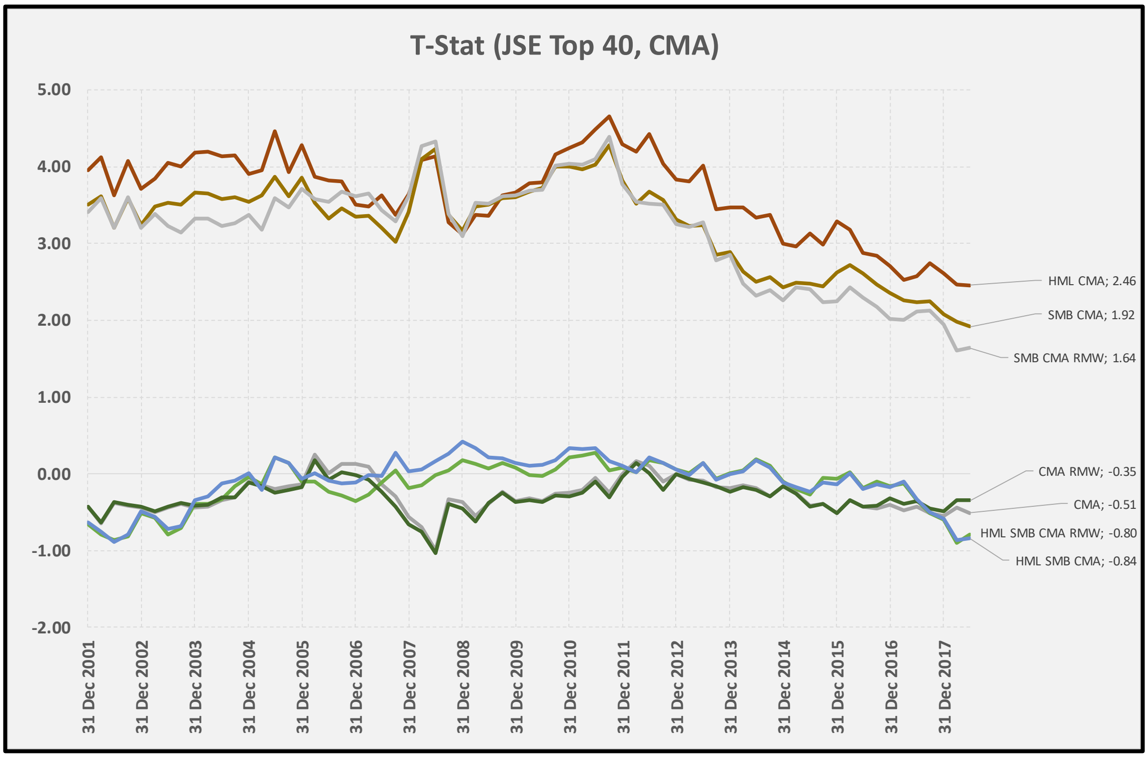

Figure xlix: Time history of t-statistic for the CMA coefficient for Fama French models (Top 40)

Table 35: T-Statistics for the Fama French models' CMA coefficient (top 40)

Model	Mean	Median	Standard Dev.
CAPM + CMA	-0.2864	-0.3231	0.2217
CAPM + HML CMA	3.6390	3.7828	0.5520
CAPM + SMB CMA	3.2417	3.4618	0.5902
CAPM + CMA RMW	-0.3038	-0.3217	0.2011
CAPM + HML SMB CMA	-0.0999	-0.0167	0.3266
CAPM + SMB CMA RMW	3.1604	3.3267	0.6772
CAPM + HML SMB CMA RMW	-0.1588	-0.0968	0.3050

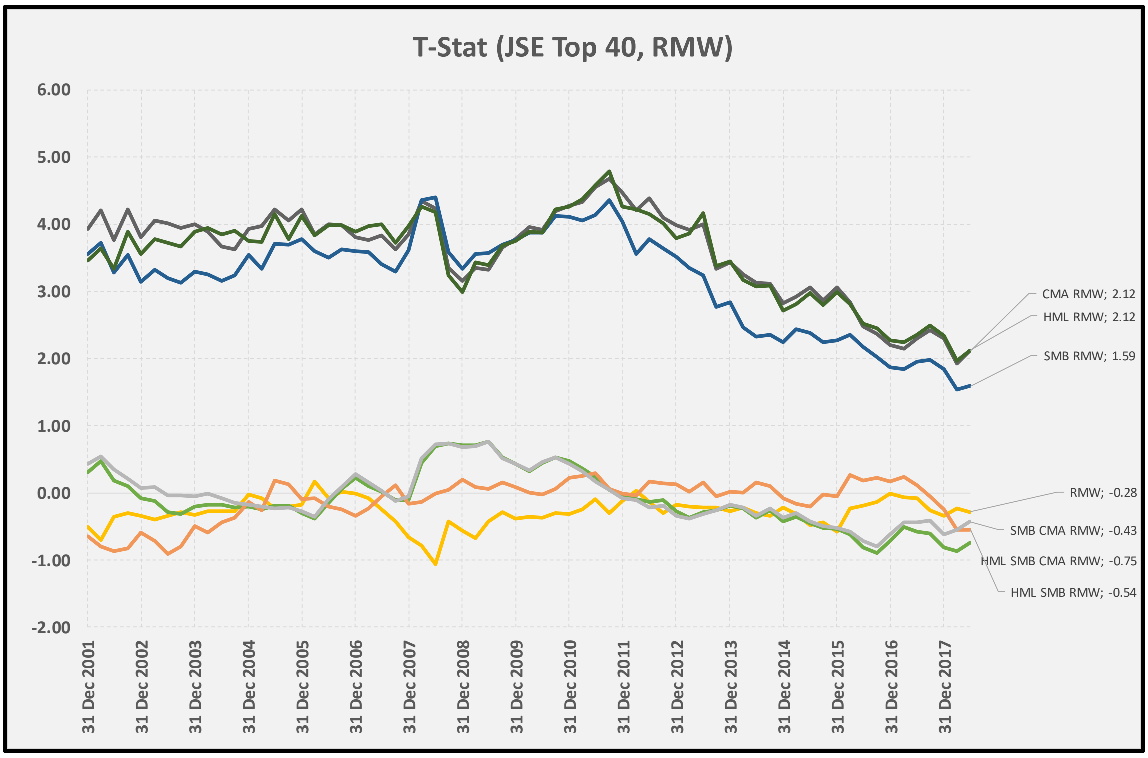

Figure I: Time history of t-statistic for the RMW coefficient for Fama French models (Top 40)

Table 36: T-Statistics for the Fama French models' RMW coefficient (top 40)

Model	Mean	Median	Standard Dev.
CAPM + RMW	-0.2874	-0.2669	0.2005
CAPM + HML RMW	3.5909	3.8296	0.6789
CAPM + SMB RMW	3.1884	3.3575	0.7393
CAPM + CMA RMW	3.5378	3.7494	0.6522
CAPM + HML SMB RMW	-0.1197	-0.0252	0.3157
CAPM + SMB CMA RMW	-0.0455	-0.1024	0.3924
CAPM + HML SMB CMA RMW	-0.0980	-0.1771	0.4207

Appendix 4: $H1a_0$ Alpha for CAPM & Selected Fama French

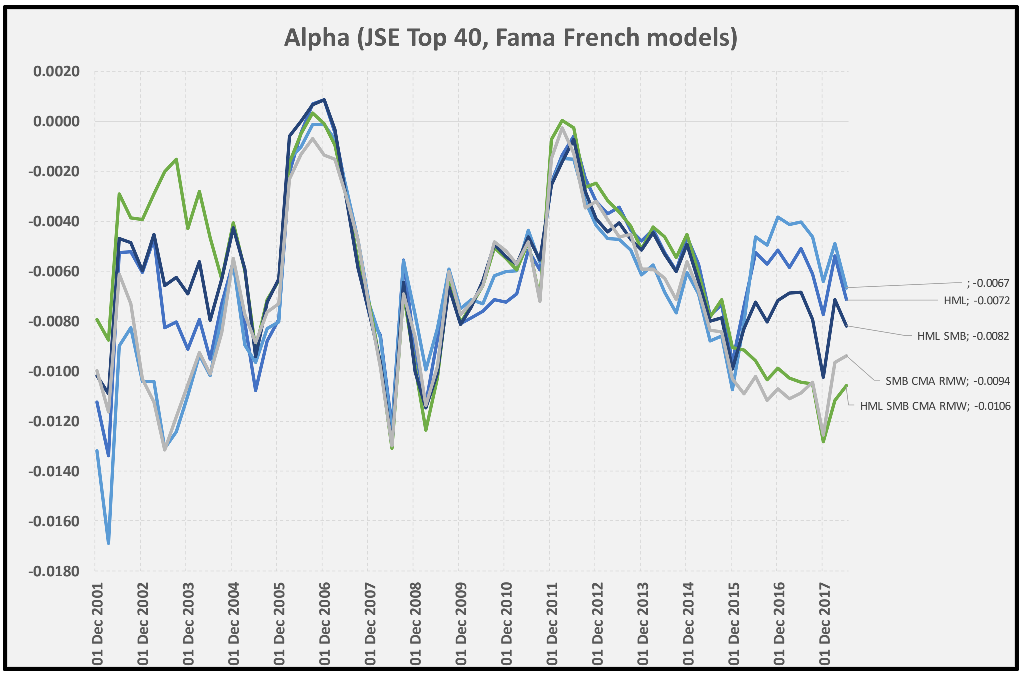

Figure li: Time history of alpha for the CAPM and selected Fama models (Top 160)

Difference of Means for Alpha (Top 160)

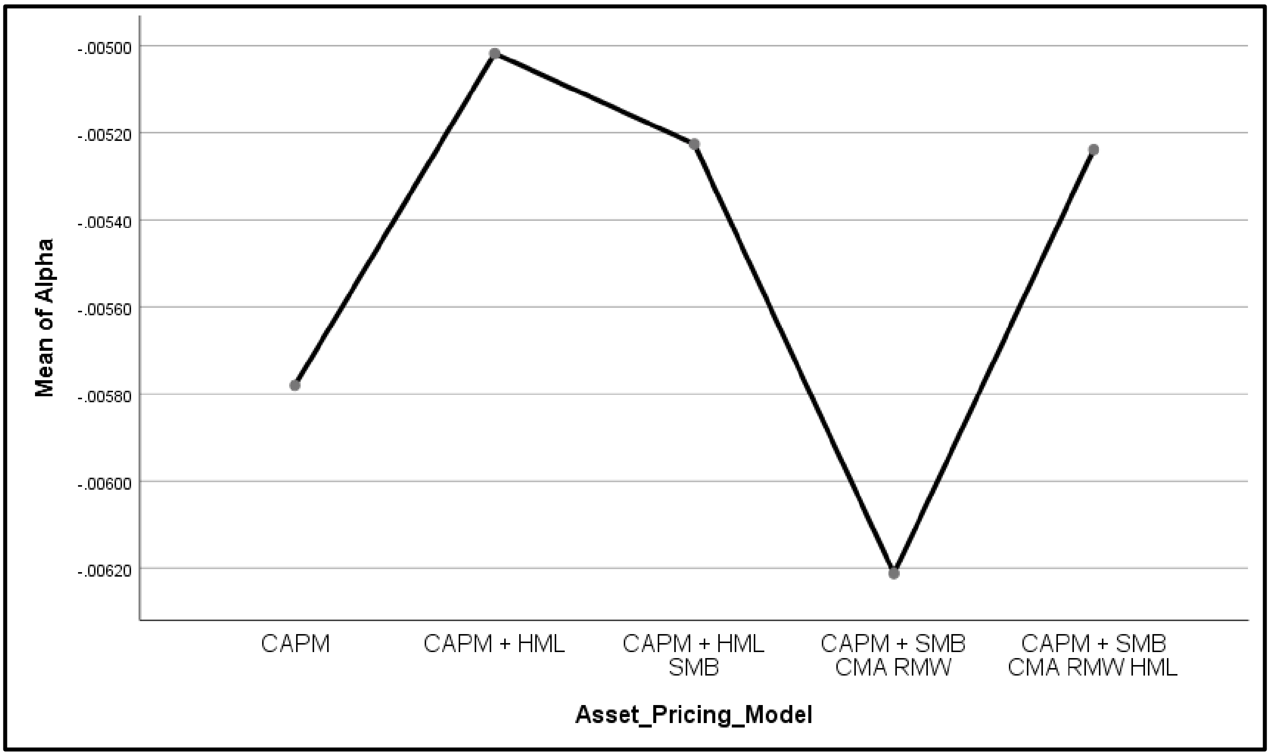

Figure lii: Plot of means of Alpha for CAPM & Selected Fama French (Top 160)

Table 37: Descriptive statistics for means of Alpha CAPM & Selected Fama French (Top 160)

Descriptives								
Alpha								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM	67	-.0057800	.00621468	.00075924	-.0072959	-.0042641	-.02720	.00655
CAPM + HML	67	-.0050179	.00557482	.00068107	-.0063777	-.0036581	-.02344	.00607
CAPM + HML SMB	67	-.0052261	.00442939	.00054114	-.0063065	-.0041457	-.01958	.00491
CAPM + SMB CMA RMW	67	-.0062122	.00522082	.00063782	-.0074857	-.0049388	-.02306	.00389
CAPM + SMB CMA RMW HML	67	-.0052387	.00483429	.00059060	-.0064178	-.0040595	-.02138	.00388
Total	335	-.0054950	.00527704	.00028832	-.0060621	-.0049278	-.02720	.00655

Table 38: Homogeneity of variances for means of Alpha CAPM & selected Fama French (Top 160)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	.715	4	330	.582
	Based on Median	.729	4	330	.573
	Based on Median and with adjusted df	.729	4	307.046	.573
	Based on trimmed mean	.719	4	330	.579

Table 39: ANOVA results for means of Alpha CAPM & selected Fama French (Top 160)

ANOVA					
Alpha					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	4	.000	.575	.681
Within Groups	.009	330	.000		
Total	.009	334			

Difference of Means for Alpha (Top 40)

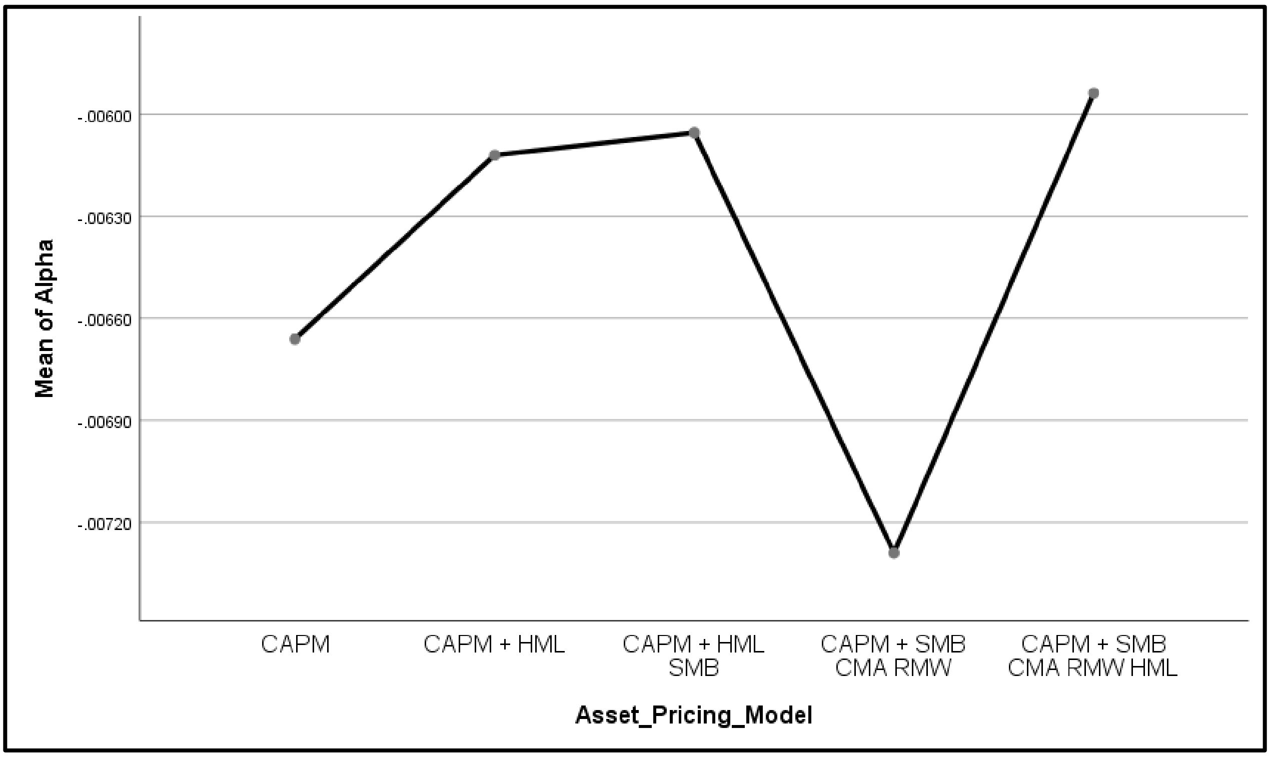

Figure liii: Plot of means of Alpha for CAPM & Selected Fama French (Top 40)

Table 40: Descriptive statistics for means of R^2 CAPM & Selected Fama French (Top 40)

Descriptives

Alpha

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
CAPM	67	-.0066607	.00335601	.00041000	-.0074793	-.0058422	-.01689	-.00012
CAPM + HML	67	-.0061204	.00297005	.00036285	-.0068449	-.0053960	-.01337	.00085
CAPM + HML SMB	67	-.0060546	.00289940	.00035422	-.0067618	-.0053474	-.01295	.00087
CAPM + SMB CMA RMW	67	-.0072896	.00334783	.00040900	-.0081062	-.0064730	-.01315	-.00026
CAPM + SMB CMA RMW HML	67	-.0059381	.00344932	.00042140	-.0067794	-.0050967	-.01309	.00035
Total	335	-.0064127	.00323269	.00017662	-.0067601	-.0060653	-.01689	.00087

Table 41: Homogeneity of variances for means of Alpha CAPM & selected Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	1.536	4	330	.191
	Based on Median	1.555	4	330	.186
	Based on Median and with adjusted df	1.555	4	327.642	.186
	Based on trimmed mean	1.541	4	330	.190

Table 42: ANOVA Results for means of Alpha CAPM & selected Fama French (Top 40)

ANOVA					
Alpha					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	4	.000	2.060	.086
Within Groups	.003	330	.000		
Total	.003	334			

Appendix 5: $H1b_0$ Alpha for Groupings of Fama French

Difference of Means for Alpha – Two Factor (Top 160)

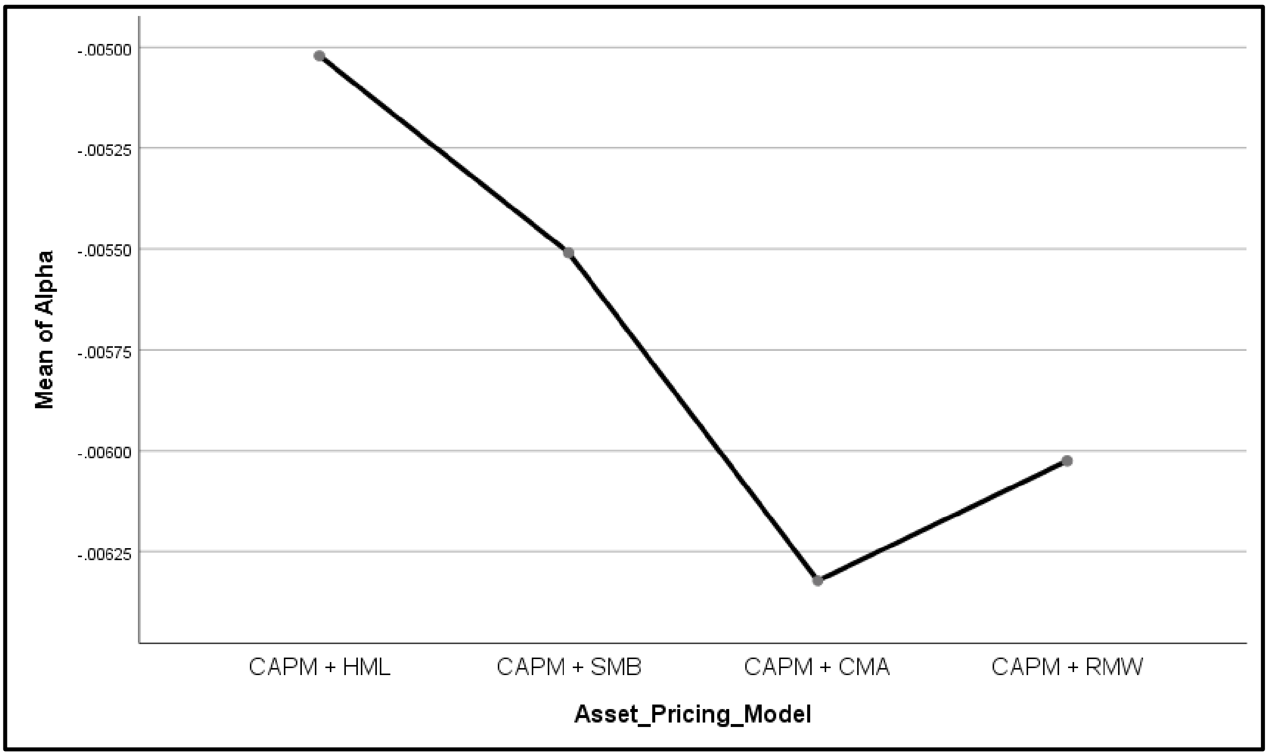

Figure liv: Plot of means of Alpha for two factor Fama French (Top 160)

Table 43: Descriptive statistics for means of Alpha of two factor Fama French (Top 160)

Descriptives

Alpha

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML	67	-.0050210	.00557766	.00068142	-.0063815	-.0036605	-.02345	.00607
CAPM + SMB	67	-.0055093	.00497101	.00060731	-.0067218	-.0042967	-.02178	.00591
CAPM + CMA	67	-.0063221	.00698641	.00085353	-.0080262	-.0046180	-.02822	.00656
CAPM + RMW	67	-.0060255	.00585116	.00071483	-.0074527	-.0045983	-.02686	.00461
Total	268	-.0057195	.00588004	.00035918	-.0064267	-.0050123	-.02822	.00656

Table 44: Homogeneity of variances for means of Alpha of two factor Fama French (Top 160)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	1.911	3	264	.128
	Based on Median	1.895	3	264	.131
	Based on Median and with adjusted df	1.895	3	251.445	.131
	Based on trimmed mean	1.915	3	264	.128

Table 45: ANOVA results for means of Alpha of two factor Fama French (Top 160)

ANOVA

Alpha

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	3	.000	.636	.592
Within Groups	.009	264	.000		
Total	.009	267			

Difference of Means for Alpha – Three Factor (Top 160)

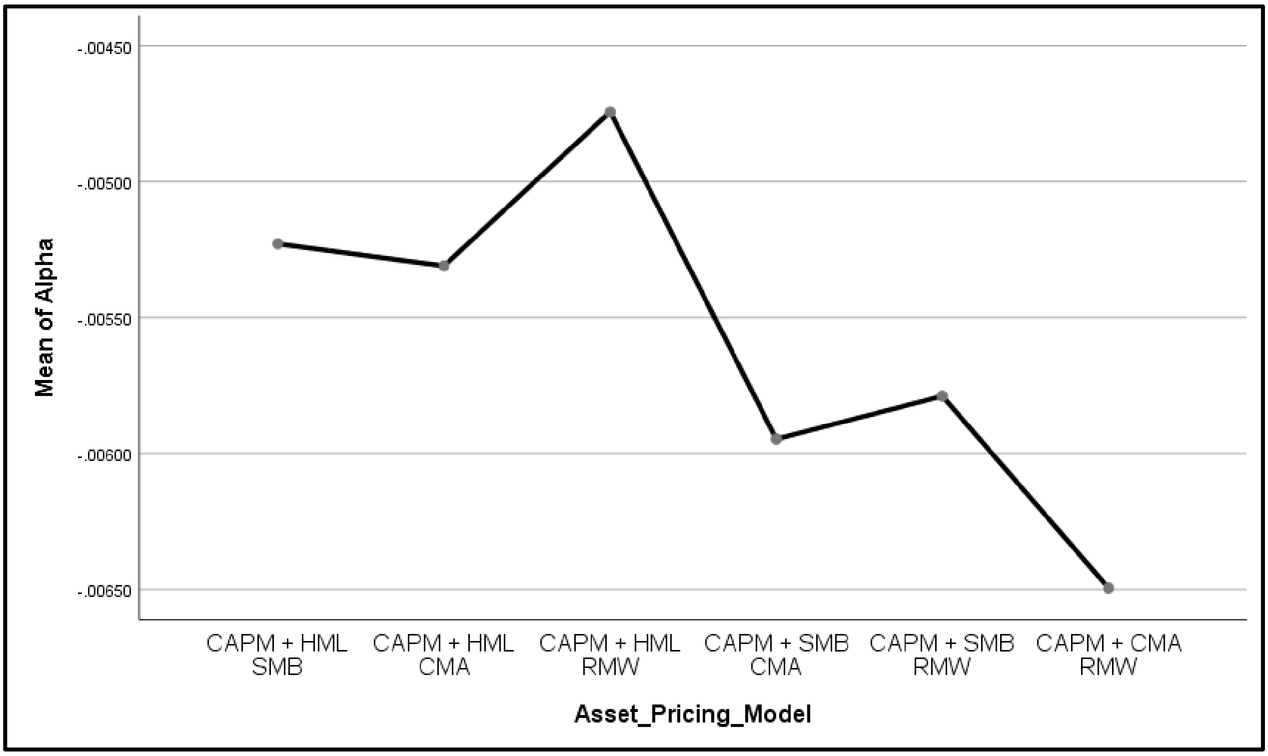

Figure lv: Plot of means of Alpha for three factor Fama French (Top 160)

Table 46: Descriptive statistics for means of Alpha of three factor Fama French (Top 160)

Descriptives

Alpha

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML SMB	67	-.0052290	.00443096	.00054133	-.0063098	-.0041482	-.01958	.00491
CAPM + HML CMA	67	-.0053099	.00614307	.00075049	-.0068083	-.0038114	-.02476	.00621
CAPM + HML RMW	67	-.0047439	.00538892	.00065836	-.0060583	-.0034294	-.02362	.00477
CAPM + SMB CMA	67	-.0059464	.00555769	.00067898	-.0073020	-.0045908	-.02230	.00591
CAPM + SMB RMW	67	-.0057888	.00461197	.00056344	-.0069138	-.0046639	-.02176	.00392
CAPM + CMA RMW	67	-.0064943	.00661275	.00080788	-.0081073	-.0048814	-.02891	.00459
Total	402	-.0055854	.00550664	.00027465	-.0061253	-.0050454	-.02891	.00621

Table 47: Homogeneity of variances for means of Alpha of thee factor Fama French (Top 160)

Test of Homogeneity of Variances

		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	2.036	5	396	.073
	Based on Median	1.955	5	396	.084
	Based on Median and with adjusted df	1.955	5	371.060	.085
	Based on trimmed mean	2.015	5	396	.076

Table 48: ANOVA results for means of Alpha of three factor Fama French (Top 160)

ANOVA

Alpha

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	5	.000	.842	.521
Within Groups	.012	396	.000		
Total	.012	401			

Difference of Means for Alpha – Four Factor (Top 160)

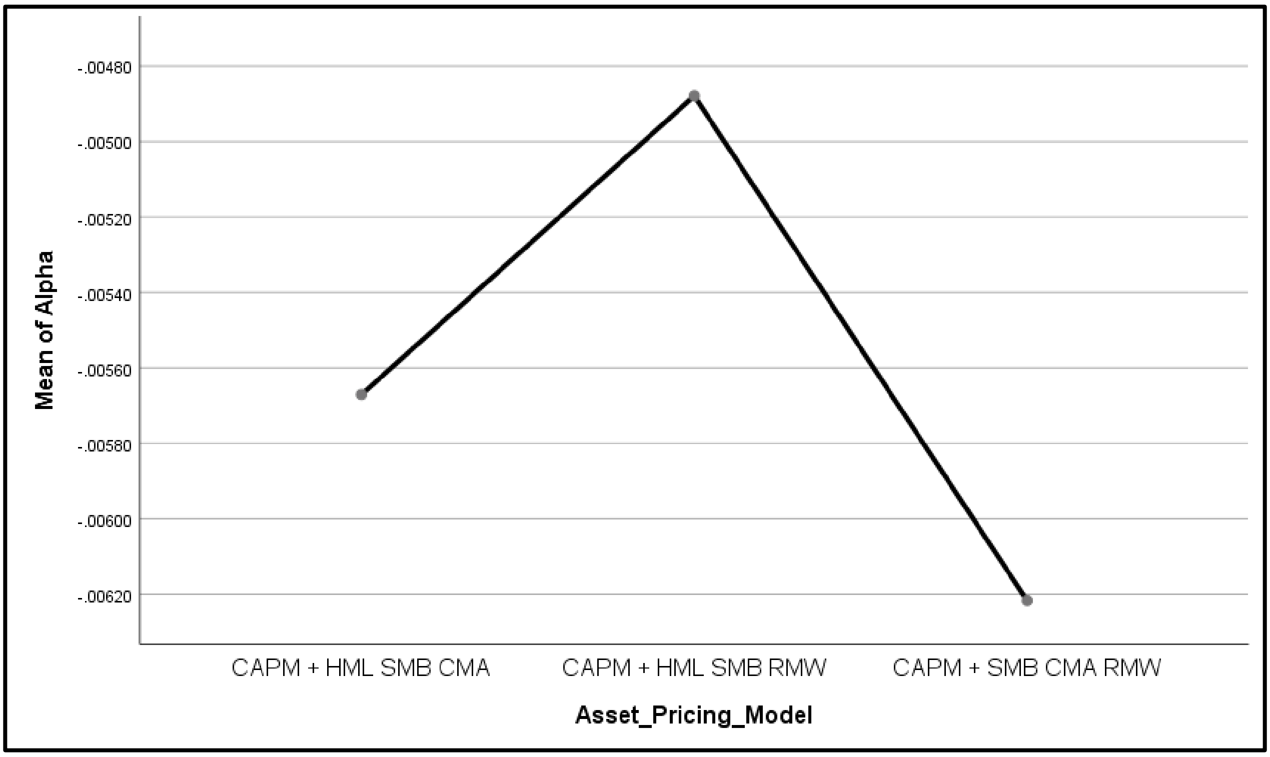

Figure lvi: Plot of means of Alpha for four factor Fama French (Top 160)

Table 49: Descriptive statistics for means of Alpha of four factor Fama French (Top 160)

Descriptives

Alpha

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
CAPM + HML SMB CMA	67	-.0056709	.00491561	.00060054	-.0068699	-.0044719	-.02096	.00505
CAPM + HML SMB RMW	67	-.0048791	.00432511	.00052840	-.0059341	-.0038241	-.01964	.00378
CAPM + SMB CMA RMW	67	-.0062173	.00522138	.00063789	-.0074909	-.0049437	-.02306	.00389
Total	201	-.0055891	.00484222	.00034154	-.0062626	-.0049156	-.02306	.00505

Table 50: Homogeneity of variances for means of Alpha of four factor Fama French (Top 160)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	.680	2	198	.508
	Based on Median	.751	2	198	.473
	Based on Median and with adjusted df	.751	2	193.974	.473
	Based on trimmed mean	.711	2	198	.492

Table 51: ANOVA results for means of Alpha of four factor Fama French (Top 160)

ANOVA					
Alpha					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	2	.000	1.297	.276
Within Groups	.005	198	.000		
Total	.005	200			

Difference of Means for Alpha – Two Factor (Top 40)

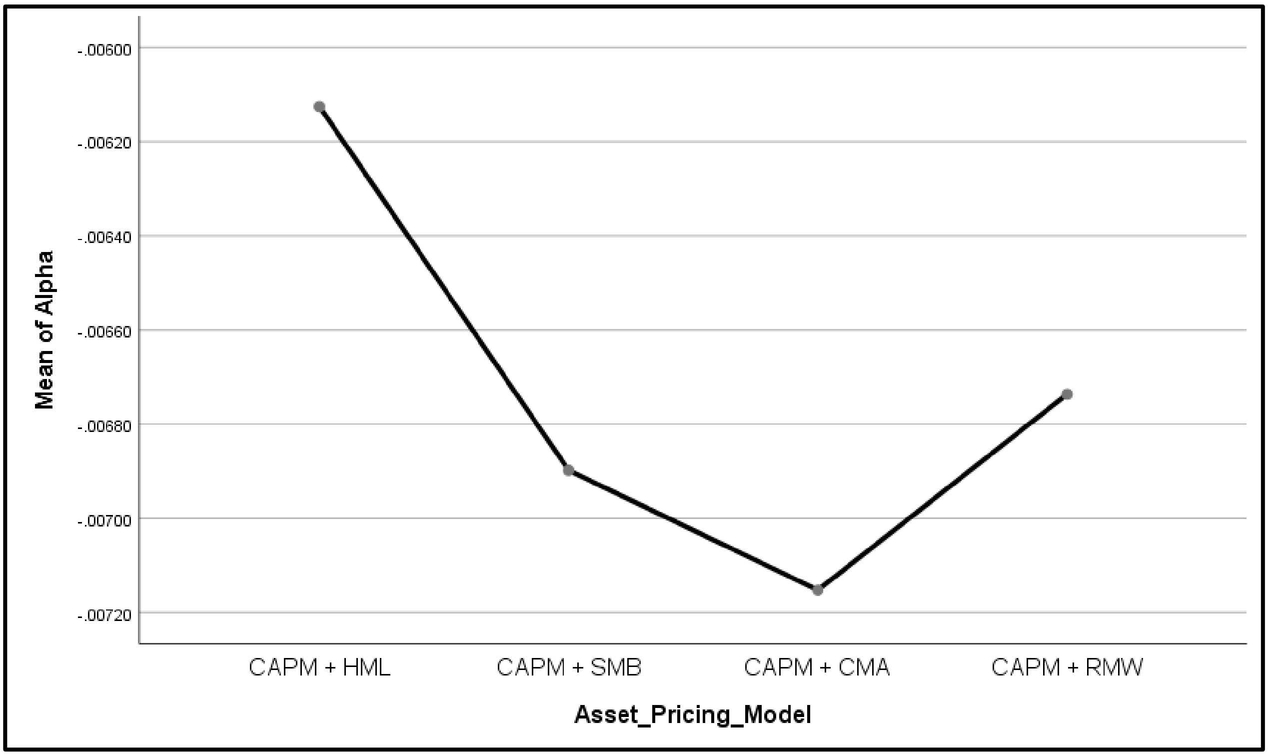

Figure lvii: Plot of means of Alpha for two factor Fama French (Top 40)

Table 52: Descriptive statistics for means of Alpha of two factor Fama French (Top 40)

Descriptives								
Alpha								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML	67	-.0061258	.00297115	.00036298	-.0068505	-.0054011	-.01338	.00086
CAPM + SMB	67	-.0068981	.00318017	.00038852	-.0076738	-.0061224	-.01351	-.00017
CAPM + CMA	67	-.0071524	.00349700	.00042723	-.0080054	-.0062994	-.01537	-.00015
CAPM + RMW	67	-.0067367	.00334985	.00040925	-.0075538	-.0059196	-.01668	-.00065
Total	268	-.0067282	.00325919	.00019909	-.0071202	-.0063363	-.01668	.00086

Table 53: Homogeneity of variances for means of Alpha of two factor Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	1.014	3	264	.387
	Based on Median	.931	3	264	.426
	Based on Median and with adjusted df	.931	3	262.681	.426
	Based on trimmed mean	1.014	3	264	.387

Table 54: ANOVA results for means of Alpha of two factor Fama French (Top 40)

ANOVA					
Alpha					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	3	.000	1.205	.308
Within Groups	.003	264	.000		
Total	.003	267			

Difference of Means for Alpha – Three Factor (Top 40)

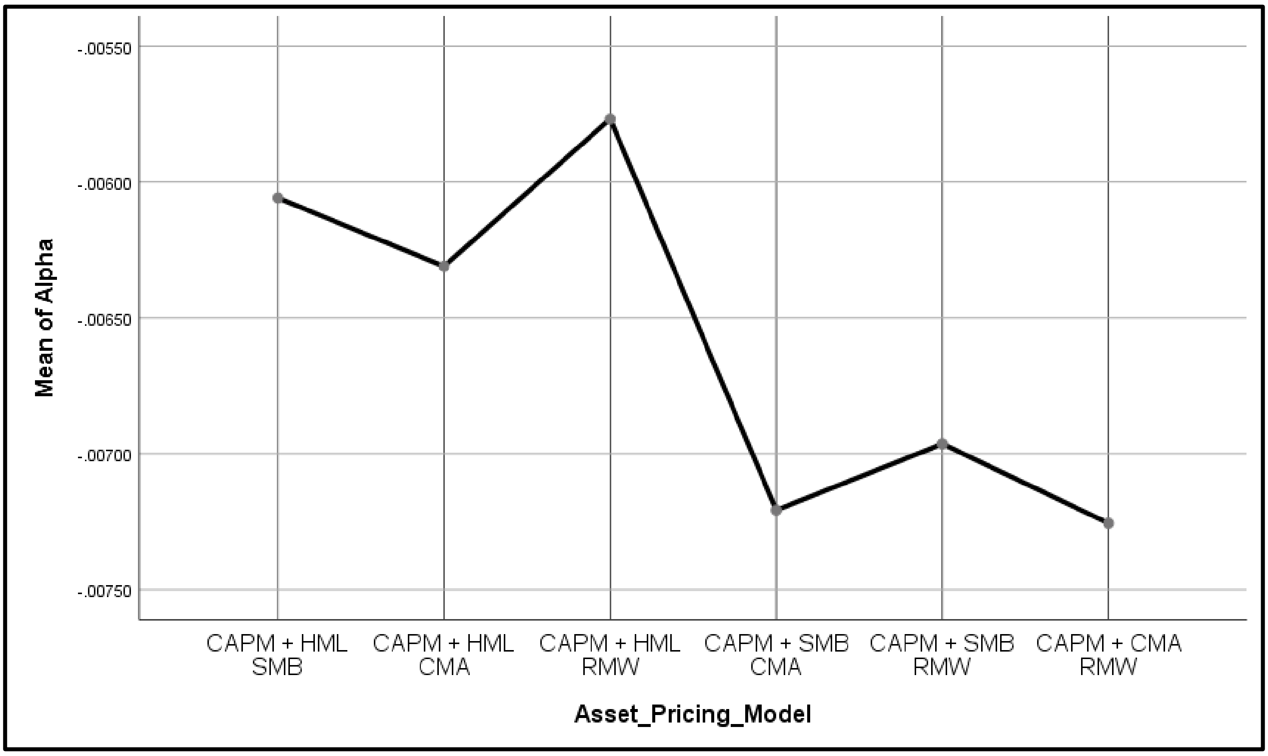

Figure lviii: Plot of means of Alpha for three factor Fama French (Top 40)

Table 55: Descriptive statistics for means of Alpha of three factor Fama French (Top 40)

Descriptives

Alpha

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML SMB	67	-.0060587	.00290099	.00035441	-.0067663	-.0053510	-.01296	.00087
CAPM + HML CMA	67	-.0063103	.00321657	.00039297	-.0070949	-.0055257	-.01225	.00073
CAPM + HML RMW	67	-.0057682	.00285242	.00034848	-.0064640	-.0050725	-.01337	.00034
CAPM + SMB CMA	67	-.0072072	.00364013	.00044471	-.0080951	-.0063193	-.01351	-.00021
CAPM + SMB RMW	67	-.0069634	.00292607	.00035748	-.0076772	-.0062497	-.01349	-.00068
CAPM + CMA RMW	67	-.0072557	.00325346	.00039747	-.0080493	-.0064621	-.01566	-.00006
Total	402	-.0065939	.00317702	.00015846	-.0069054	-.0062824	-.01566	.00087

Table 56: Homogeneity of variances for means of Alpha of three factor Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	1.859	5	396	.101
	Based on Median	1.707	5	396	.132
	Based on Median and with adjusted df	1.707	5	389.656	.132
	Based on trimmed mean	1.860	5	396	.100

Table 57: ANOVA results for means of Alpha of three factor Fama French (Top 40)

ANOVA Alpha	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	5	.000	2.711	.020
Within Groups	.004	396	.000		
Total	.004	401			

Table 58: Post hoc analysis for means of Alpha three factor Fama French (Top 40)

Multiple Comparisons

Dependent Variable: Alpha

	(I) Asset_Pricing _Model	(J) Asset_Pricing_Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Bonferroni	CAPM + HML SMB	CAPM + HML CMA	.00025164	.00054314	1.000	-.0013523	.0018556
		CAPM + HML RMW	-.00029045	.00054314	1.000	-.0018944	.0013135
		CAPM + SMB CMA	.00114851	.00054314	.526	-.0004555	.0027525
		CAPM + SMB RMW	.00090478	.00054314	1.000	-.0006992	.0025087
		CAPM + CMA RMW	.00119701	.00054314	.422	-.0004069	.0028010
	CAPM + HML CMA	CAPM + HML SMB	-.00025164	.00054314	1.000	-.0018556	.0013523
		CAPM + HML RMW	-.00054209	.00054314	1.000	-.0021461	.0010619
		CAPM + SMB CMA	.00089687	.00054314	1.000	-.0007071	.0025008
		CAPM + SMB RMW	.00065313	.00054314	1.000	-.0009508	.0022571
		CAPM + CMA RMW	.00094537	.00054314	1.000	-.0006586	.0025493
	CAPM + HML RMW	CAPM + HML SMB	.00029045	.00054314	1.000	-.0013135	.0018944
		CAPM + HML CMA	.00054209	.00054314	1.000	-.0010619	.0021461
		CAPM + SMB CMA	.00143896	.00054314	.126	-.0001650	.0030429
		CAPM + SMB RMW	.00119522	.00054314	.425	-.0004087	.0027992
		CAPM + CMA RMW	.00148746	.00054314	.097	-.0001165	.0030914
	CAPM + SMB CMA	CAPM + HML SMB	-.00114851	.00054314	.526	-.0027525	.0004555
		CAPM + HML CMA	-.00089687	.00054314	1.000	-.0025008	.0007071
		CAPM + HML RMW	-.00143896	.00054314	.126	-.0030429	.0001650
		CAPM + SMB RMW	-.00024373	.00054314	1.000	-.0018477	.0013602
		CAPM + CMA RMW	.00004851	.00054314	1.000	-.0015555	.0016525
	CAPM + SMB RMW	CAPM + HML SMB	-.00090478	.00054314	1.000	-.0025087	.0006992
		CAPM + HML CMA	-.00065313	.00054314	1.000	-.0022571	.0009508
		CAPM + HML RMW	-.00119522	.00054314	.425	-.0027992	.0004087
		CAPM + SMB CMA	.00024373	.00054314	1.000	-.0013602	.0018477
		CAPM + CMA RMW	.00029224	.00054314	1.000	-.0013117	.0018962
	CAPM + CMA RMW	CAPM + HML SMB	-.00119701	.00054314	.422	-.0028010	.0004069
		CAPM + HML CMA	-.00094537	.00054314	1.000	-.0025493	.0006586
		CAPM + HML RMW	-.00148746	.00054314	.097	-.0030914	.0001165
		CAPM + SMB CMA	-.00004851	.00054314	1.000	-.0016525	.0015555
		CAPM + SMB RMW	-.00029224	.00054314	1.000	-.0018962	.0013117

Difference of Means for Alpha – Four Factor (Top 40)

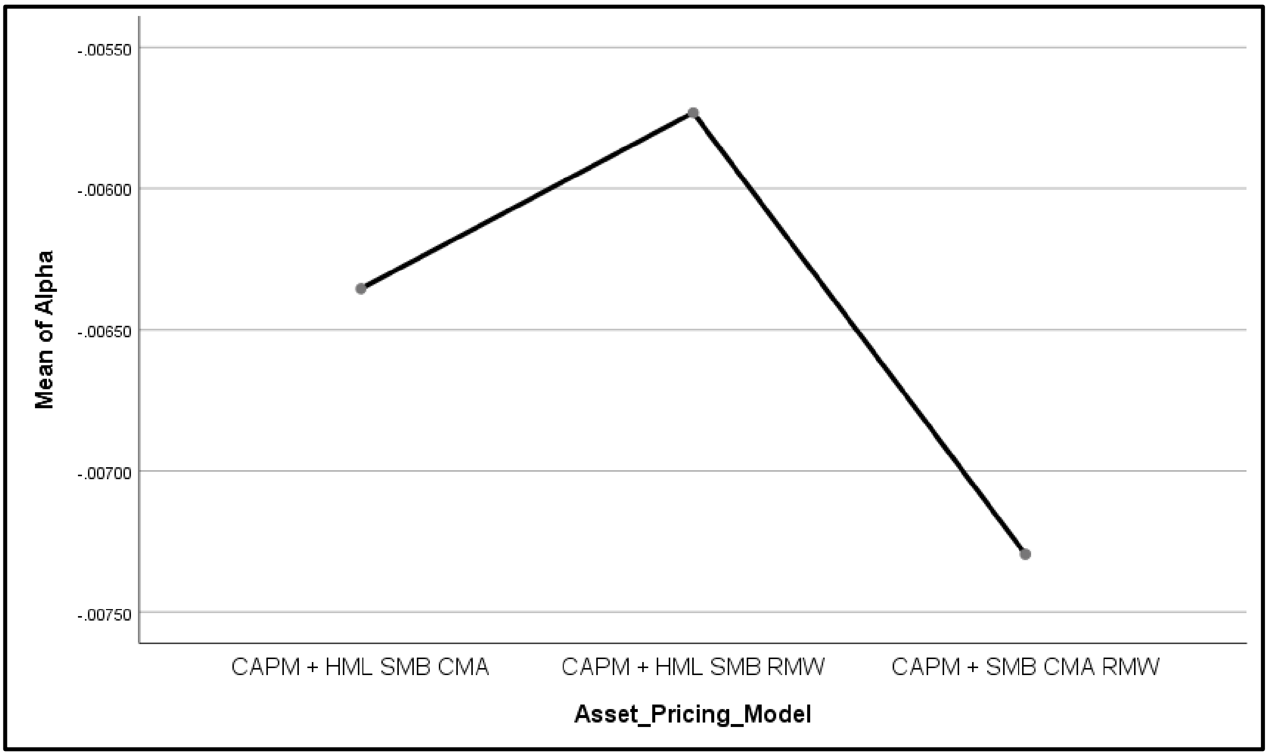

Figure lix: Plot of means of Alpha for four factor Fama French (Top 40)

Table 59: Descriptive statistics for means of Alpha of four factor Fama French (Top 40)

Descriptives

Alpha

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML SMB CMA	67	-.0063549	.00352772	.00043098	-.0072154	-.0054944	-.01379	.00067
CAPM + HML SMB RMW	67	-.0057319	.00276987	.00033839	-.0064076	-.0050563	-.01295	.00041
CAPM + SMB CMA RMW	67	-.0072954	.00334761	.00040898	-.0081119	-.0064788	-.01316	-.00026
Total	201	-.0064607	.00327900	.00023128	-.0069168	-.0060047	-.01379	.00067

Table 60: Homogeneity of variances for means of Alpha of four factor Fama French (Top 40)

Test of Homogeneity of Variances

		Levene Statistic	df1	df2	Sig.
Alpha	Based on Mean	2.666	2	198	.072
	Based on Median	2.509	2	198	.084
	Based on Median and with adjusted df	2.509	2	191.660	.084
	Based on trimmed mean	2.663	2	198	.072

Table 61: ANOVA results for means of Alpha of four factor Fama French (Top 40)

ANOVA

Alpha

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	2	.000	3.975	.020
Within Groups	.002	198	.000		
Total	.002	200			

Table 62: Post hoc analysis for means of Alpha four factor Fama French (Top 40)

Multiple Comparisons

Dependent Variable: Alpha

	(I) Asset_Pricing_Model	(J) Asset_Pricing_Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Bonferroni	CAPM + HML SMB CMA	CAPM + HML SMB RMW	-.00062299	.00055828	.797	-.0019709	.0007250
		CAPM + SMB CMA RMW	.00094045	.00055828	.281	-.0004075	.0022884
	CAPM + HML SMB RMW	CAPM + HML SMB CMA	.00062299	.00055828	.797	-.0007250	.0019709
		CAPM + SMB CMA RMW	.00156343*	.00055828	.017	.0002155	.0029114
	CAPM + SMB CMA RMW	CAPM + HML SMB CMA	-.00094045	.00055828	.281	-.0022884	.0004075
		CAPM + HML SMB RMW	-.00156343*	.00055828	.017	-.0029114	-.0002155

*. The mean difference is significant at the 0.05 level.

Appendix 6: H2a$_0$ R^2 for CAPM & Selected Fama French

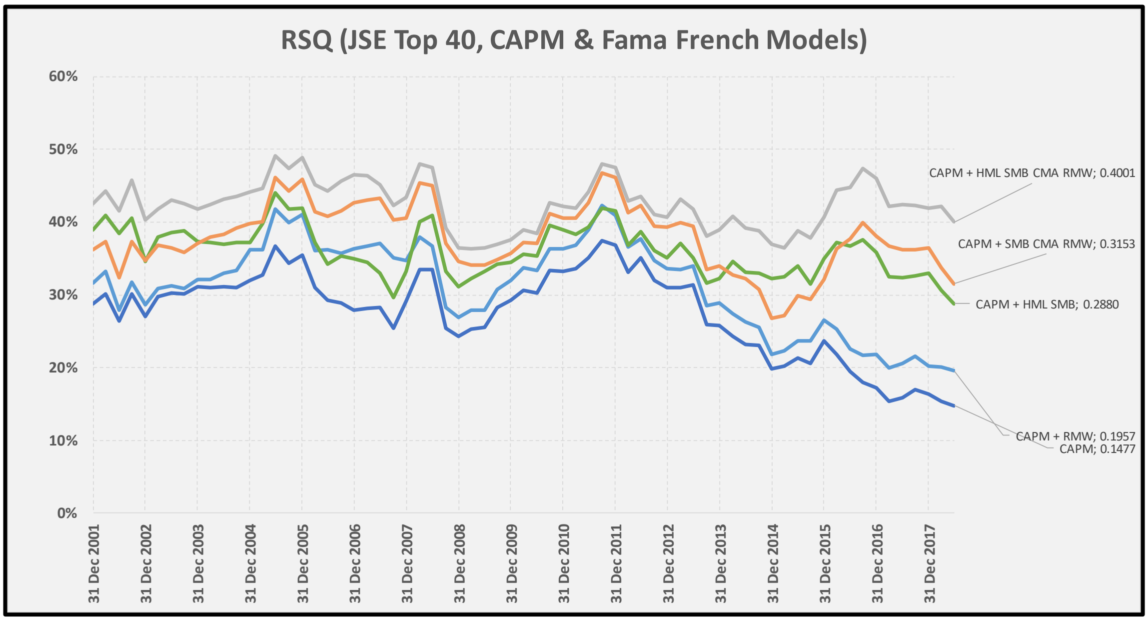

Figure lx: Time history of R^2 for the CAPM and selected Fama models (Top 40)

Difference of Means for R^2 (Top 160)

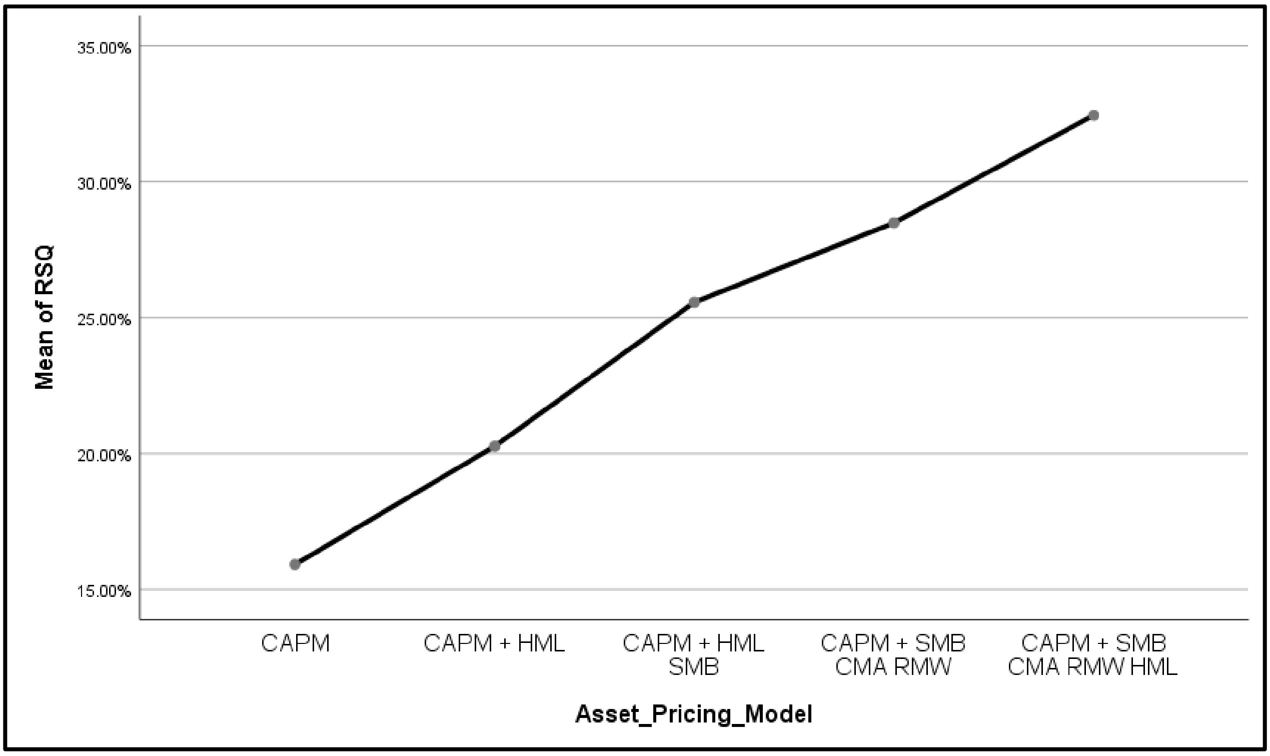

Figure lxi: Plot of means of R^2 for CAPM & Selected Fama French (Top 160)

Table 63: Descriptive statistics for means of R^2 CAPM & Selected Fama French (Top 160)

Descriptives

RSQ

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean: Lower Bound	95% Confidence Interval for Mean: Upper Bound	Minimum	Maximum
CAPM	67	15.9216%	3.93943%	0.48128%	14.9607%	16.8825%	8.67%	23.50%
CAPM + HML	67	20.2670%	2.94200%	0.35942%	19.5494%	20.9846%	14.43%	26.66%
CAPM + HML SMB	67	25.5601%	3.86826%	0.47258%	24.6166%	26.5037%	19.47%	37.48%
CAPM + SMB CMA RMW	67	28.4842%	4.60581%	0.56269%	27.3608%	29.6077%	19.40%	41.23%
CAPM + SMB CMA RMW HML	67	32.4366%	3.55244%	0.43400%	31.5701%	33.3031%	26.37%	43.64%
Total	335	24.5339%	6.98712%	0.38175%	23.7830%	25.2848%	8.67%	43.64%

Table 64: Homogeneity of variances for means of R2 CAPM & selected Fama French (Top 160)

Test of Homogeneity of Variances

		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	2.723	4	330	.030
	Based on Median	2.448	4	330	.046
	Based on Median and with adjusted df	2.448	4	293.621	.046
	Based on trimmed mean	2.646	4	330	.033

Table 65: ANOVA results for means of R^2 CAPM & selected Fama French (Top 160)

ANOVA

RSQ

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	11489.723	4	2872.431	196.819	.000
Within Groups	4816.107	330	14.594		
Total	16305.831	334			

Table 66: Post hoc analysis for means of R^2 CAPM & selected Fama French (Top 160)

Multiple Comparisons							
Dependent Variable: RSQ							
	(I) Asset_Pricing_Model	(J) Asset_Pricing_Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Games-Howell	CAPM	CAPM + HML	-4.34541%*	0.60068%	.000	-6.0087%	-2.6822%
		CAPM + HML SMB	-9.63853%*	0.67451%	.000	-11.5041%	-7.7729%
		CAPM + SMB CMA RMW	-12.56260%*	0.74044%	.000	-14.6112%	-10.5140%
		CAPM + SMB CMA RMW HML	-16.51498%*	0.64806%	.000	-18.3077%	-14.7223%
	CAPM + HML	CAPM	4.34541%*	0.60068%	.000	2.6822%	6.0087%
		CAPM + HML SMB	-5.29312%*	0.59373%	.000	-6.9369%	-3.6493%
		CAPM + SMB CMA RMW	-8.21720%*	0.66769%	.000	-10.0685%	-6.3659%
		CAPM + SMB CMA RMW HML	-12.16957%*	0.56351%	.000	-13.7289%	-10.6102%
	CAPM + HML SMB	CAPM	9.63853%*	0.67451%	.000	7.7729%	11.5041%
		CAPM + HML	5.29312%*	0.59373%	.000	3.6493%	6.9369%
		CAPM + SMB CMA RMW	-2.92407%*	0.73482%	.001	-4.9573%	-0.8908%
		CAPM + SMB CMA RMW HML	-6.87644%*	0.64163%	.000	-8.6513%	-5.1016%
	CAPM + SMB CMA RMW	CAPM	12.56260%*	0.74044%	.000	10.5140%	14.6112%
		CAPM + HML	8.21720%*	0.66769%	.000	6.3659%	10.0685%
		CAPM + HML SMB	2.92407%*	0.73482%	.001	0.8908%	4.9573%
		CAPM + SMB CMA RMW HML	-3.95237%*	0.71062%	.000	-5.9196%	-1.9852%
	CAPM + SMB CMA RMW HML	CAPM	16.51498%*	0.64806%	.000	14.7223%	18.3077%
		CAPM + HML	12.16957%*	0.56351%	.000	10.6102%	13.7289%
		CAPM + HML SMB	6.87644%*	0.64163%	.000	5.1016%	8.6513%
		CAPM + SMB CMA RMW	3.95237%*	0.71062%	.000	1.9852%	5.9196%
*. The mean difference is significant at the 0.05 level.							

Difference of Means for R^2 (Top 40)

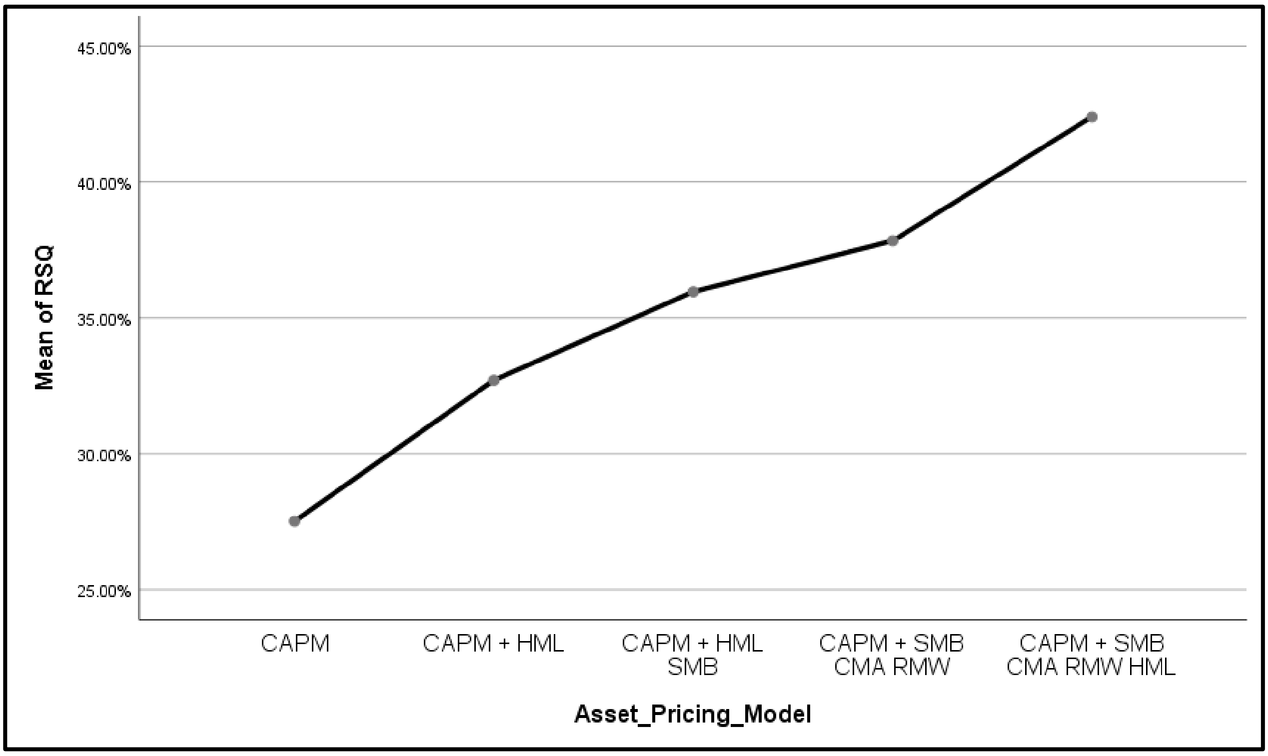

Figure lxii: Plot of means of Alpha for CAPM & Selected Fama French (Top 40)

Table 67: Descriptive statistics for means of R^2 CAPM & Selected Fama French (Top 40)

Descriptives

RSQ

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM	67	27.5186%	5.97174%	0.72956%	26.0620%	28.9752%	14.77%	37.41%
CAPM + HML	67	32.7051%	4.25277%	0.51956%	31.6678%	33.7424%	21.44%	40.97%
CAPM + HML SMB	67	35.9615%	3.36752%	0.41141%	35.1401%	36.7829%	28.80%	44.01%
CAPM + SMB CMA RMW	67	37.8349%	4.57617%	0.55907%	36.7187%	38.9511%	26.82%	46.72%
CAPM + SMB CMA RMW HML	67	42.3945%	3.34969%	0.40923%	41.5775%	43.2116%	36.30%	49.06%
Total	335	35.2829%	6.64733%	0.36318%	34.5685%	35.9973%	14.77%	49.06%

Table 68: Homogeneity of variances for means of R^2 CAPM & selected Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	8.409	4	330	.000
	Based on Median	6.242	4	330	.000
	Based on Median and with adjusted df	6.242	4	247.104	.000
	Based on trimmed mean	8.084	4	330	.000

Table 69: ANOVA results for means of R^2 CAPM & selected Fama French (Top 40)

ANOVA					
RSQ					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	8340.002	4	2085.000	107.198	.000
Within Groups	6418.477	330	19.450		
Total	14758.478	334			

Table 70: Post hoc analysis for means of R^2 CAPM & selected Fama French (Top 40)

Multiple Comparisons

Dependent Variable: RSQ

	(I) Asset_Pricing_Model	(J) Asset_Pricing_Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Games-Howell	CAPM	CAPM + HML	-5.18653%*	0.89566%	.000	-7.6675%	-2.7056%
		CAPM + HML SMB	-8.44292%*	0.83757%	.000	-10.7681%	-6.1177%
		CAPM + SMB CMA RMW	-10.31632%*	0.91914%	.000	-12.8609%	-7.7717%
		CAPM + SMB CMA RMW HML	-14.87591%*	0.83650%	.000	-17.1983%	-12.5535%
	CAPM + HML	CAPM	5.18653%*	0.89566%	.000	2.7056%	7.6675%
		CAPM + HML SMB	-3.25639%*	0.66272%	.000	-5.0907%	-1.4221%
		CAPM + SMB CMA RMW	-5.12979%*	0.76322%	.000	-7.2409%	-3.0187%
		CAPM + SMB CMA RMW HML	-9.68939%*	0.66137%	.000	-11.5200%	-7.8587%
	CAPM + HML SMB	CAPM	8.44292%*	0.83757%	.000	6.1177%	10.7681%
		CAPM + HML	3.25639%*	0.66272%	.000	1.4221%	5.0907%
		CAPM + SMB CMA RMW	-1.87340%	0.69413%	.060	-3.7956%	0.0488%
		CAPM + SMB CMA RMW HML	-6.43300%*	0.58028%	.000	-8.0380%	-4.8280%
	CAPM + SMB CMA RMW	CAPM	10.31632%*	0.91914%	.000	7.7717%	12.8609%
		CAPM + HML	5.12979%*	0.76322%	.000	3.0187%	7.2409%
		CAPM + HML SMB	1.87340%	0.69413%	.060	-0.0488%	3.7956%
		CAPM + SMB CMA RMW HML	-4.55960%*	0.69284%	.000	-6.4783%	-2.6409%
	CAPM + SMB CMA RMW HML	CAPM	14.87591%*	0.83650%	.000	12.5535%	17.1983%
		CAPM + HML	9.68939%*	0.66137%	.000	7.8587%	11.5200%
		CAPM + HML SMB	6.43300%*	0.58028%	.000	4.8280%	8.0380%
		CAPM + SMB CMA RMW	4.55960%*	0.69284%	.000	2.6409%	6.4783%

*. The mean difference is significant at the 0.05 level.

Appendix 7: H2b$_0$ R^2 for Groupings of Fama French

Difference of Means for R^2 – Two Factor (Top 160)

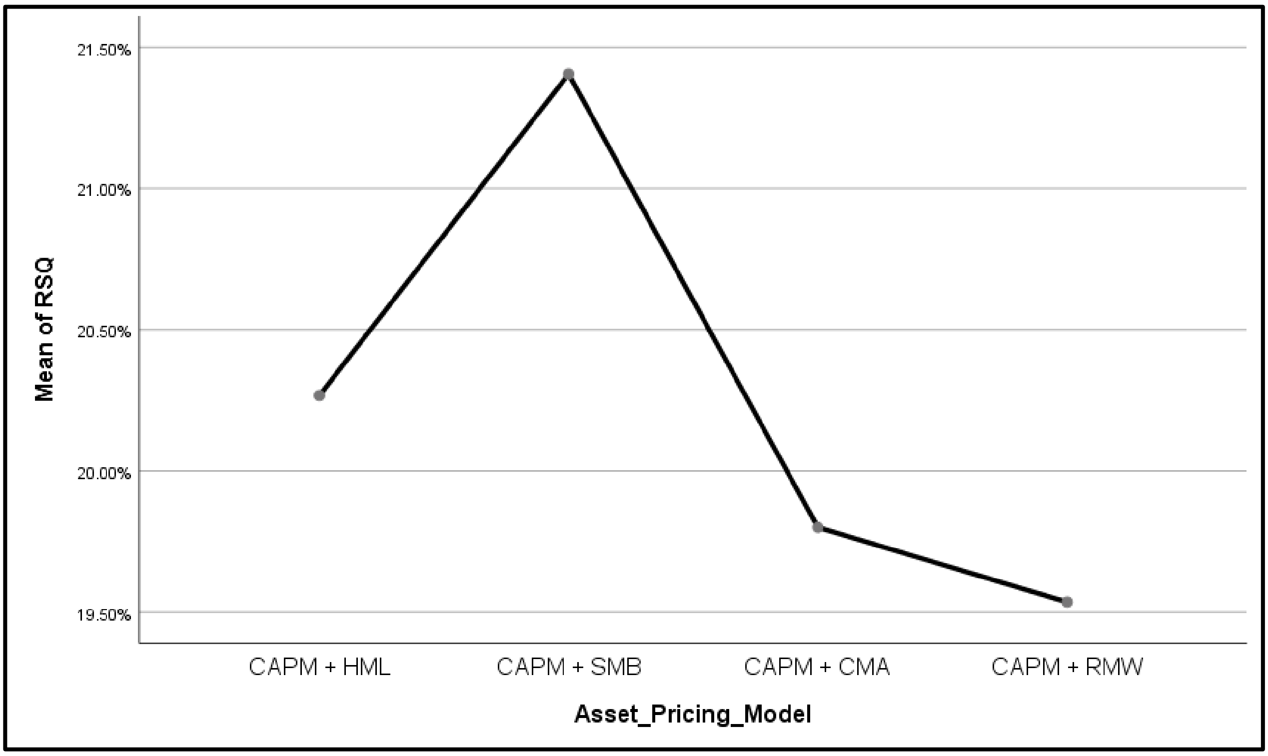

Figure lxiii: Plot of means of R^2 for two factor Fama French (Top 160)

Table 71: Descriptive statistics for means of R^2 of two factor Fama French (Top 160)

Descriptives								
RSQ								
					95% Confidence Interval for Mean			
	N	Mean	Std. Deviation	Std. Error	Lower Bound	Upper Bound	Minimum	Maximum
CAPM + HML	67	20.2670%	2.94200%	0.35942%	19.5494%	20.9846%	14.43%	26.66%
CAPM + SMB	67	21.4054%	4.88144%	0.59636%	20.2147%	22.5961%	13.26%	34.84%
CAPM + CMA	67	19.8001%	3.40142%	0.41555%	18.9704%	20.6298%	13.72%	27.44%
CAPM + RMW	67	19.5358%	4.03062%	0.49242%	18.5527%	20.5190%	13.07%	26.61%
Total	268	20.2521%	3.92674%	0.23986%	19.7798%	20.7243%	13.07%	34.84%

Table 72: Homogeneity of variances for means of R^2 of two factor Fama French (Top 160)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	5.507	3	264	.001
	Based on Median	5.223	3	264	.002
	Based on Median and with adjusted df	5.223	3	205.735	.002
	Based on trimmed mean	5.502	3	264	.001

Table 73: ANOVA results for means of R^2 of two factor Fama French (Top 160)

ANOVA RSQ					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	137.191	3	45.730	3.034	.030
Within Groups	3979.759	264	15.075		
Total	4116.950	267			

Table 74: Post hoc analysis for means of R^2 two factor Fama French (Top 160)

Multiple Comparisons Dependent Variable: RSQ							
	(I) Asset_Pricing_ Model	(J) Asset_Pricing_ Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Games-Howell	CAPM + HML	CAPM + SMB	-1.13836%	0.69630%	.364	-2.9553%	0.6785%
		CAPM + CMA	0.46692%	0.54942%	.830	-0.9631%	1.8969%
		CAPM + RMW	0.73119%	0.60964%	.628	-0.8570%	2.3194%
	CAPM + SMB	CAPM + HML	1.13836%	0.69630%	.364	-0.6785%	2.9553%
		CAPM + CMA	1.60528%	0.72686%	.127	-0.2890%	3.4995%
		CAPM + RMW	1.86955%	0.77339%	.079	-0.1438%	3.8829%
	CAPM + CMA	CAPM + HML	-0.46692%	0.54942%	.830	-1.8969%	0.9631%
		CAPM + SMB	-1.60528%	0.72686%	.127	-3.4995%	0.2890%
		CAPM + RMW	0.26427%	0.64433%	.977	-1.4129%	1.9415%
	CAPM + RMW	CAPM + HML	-0.73119%	0.60964%	.628	-2.3194%	0.8570%
		CAPM + SMB	-1.86955%	0.77339%	.079	-3.8829%	0.1438%
		CAPM + CMA	-0.26427%	0.64433%	.977	-1.9415%	1.4129%

Difference of Means for R^2 – Three Factor (Top 160)

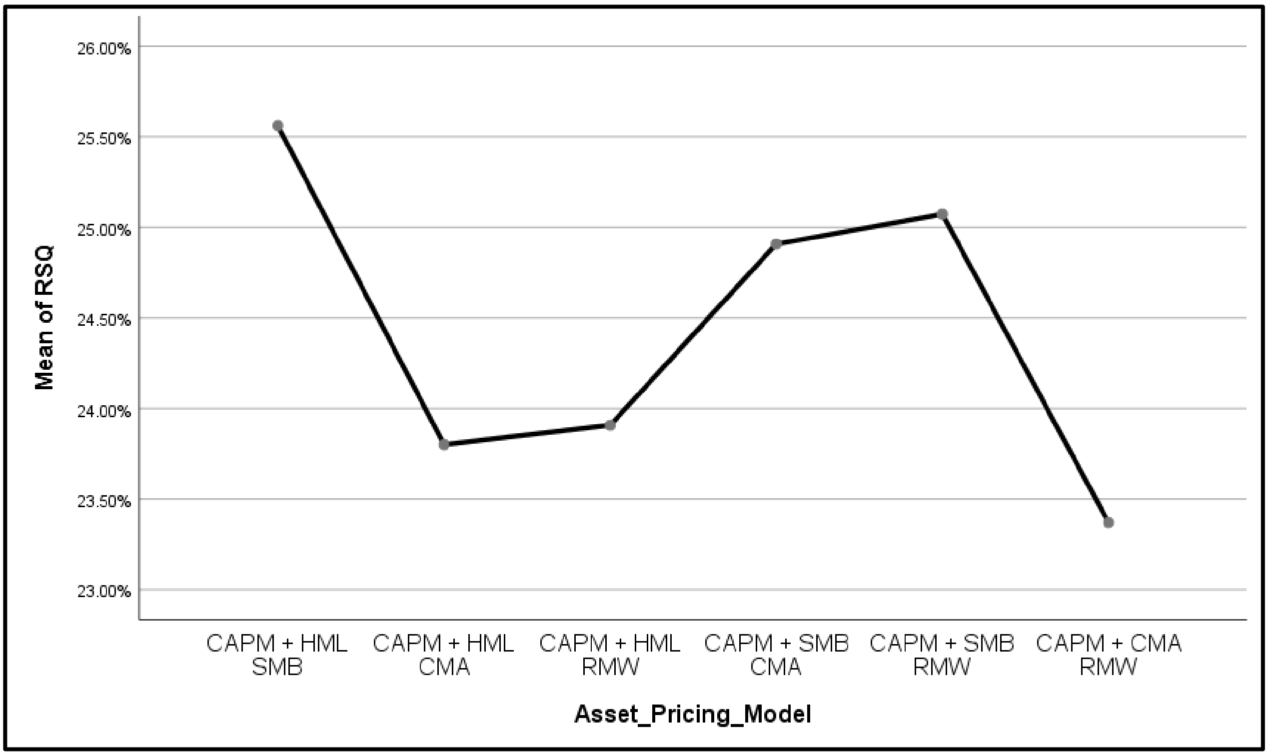

Figure lxiv: Plot of means of R^2 for three factor Fama French (Top 160)

Table 75: Descriptive statistics for means of R^2 of three factor Fama French (Top 160)

Descriptives								
RSQ								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML SMB	67	25.5601%	3.86826%	0.47258%	24.6166%	26.5037%	19.47%	37.48%
CAPM + HML CMA	67	23.8011%	2.63681%	0.32214%	23.1579%	24.4443%	18.95%	30.36%
CAPM + HML RMW	67	23.9074%	2.75538%	0.33662%	23.2353%	24.5795%	18.91%	30.21%
CAPM + SMB CMA	67	24.9078%	4.40511%	0.53817%	23.8333%	25.9823%	16.56%	37.51%
CAPM + SMB RMW	67	25.0717%	5.01424%	0.61259%	23.8486%	26.2947%	15.96%	38.47%
CAPM + CMA RMW	67	23.3699%	3.66393%	0.44762%	22.4762%	24.2636%	16.54%	30.38%
Total	402	24.4363%	3.87531%	0.19328%	24.0564%	24.8163%	15.96%	38.47%

Table 76: Homogeneity of variances for means of R^2 of three factor Fama French (Top 160)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	6.450	5	396	.000
	Based on Median	6.278	5	396	.000
	Based on Median and with adjusted df	6.278	5	321.002	.000
	Based on trimmed mean	6.299	5	396	.000

Table 77: ANOVA results for means of R^2 of three factor Fama French (Top 160)

ANOVA					
RSQ					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	248.539	5	49.708	3.409	.005
Within Groups	5773.704	396	14.580		
Total	6022.243	401			

Table 78: Post hoc analysis for means of R^2 three factor Fama French (Top 160)

Multiple Comparisons

Dependent Variable: RSQ

	(I) Asset_Pricing_Model	(J) Asset_Pricing_Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Games-Howell	CAPM + HML SMB	CAPM + HML CMA	1.75906%*	0.57193%	.031	0.1017%	3.4164%
		CAPM + HML RMW	1.65277%	0.58022%	.057	-0.0279%	3.3334%
		CAPM + SMB CMA	0.65231%	0.71621%	.943	-1.4195%	2.7241%
		CAPM + SMB RMW	0.48847%	0.77369%	.988	-1.7512%	2.7281%
		CAPM + CMA RMW	2.19024%*	0.65092%	.013	0.3077%	4.0728%
	CAPM + HML CMA	CAPM + HML SMB	-1.75906%*	0.57193%	.031	-3.4164%	-0.1017%
		CAPM + HML RMW	-0.10629%	0.46593%	1.000	-1.4538%	1.2412%
		CAPM + SMB CMA	-1.10674%	0.62722%	.493	-2.9267%	0.7132%
		CAPM + SMB RMW	-1.27059%	0.69212%	.448	-3.2817%	0.7406%
		CAPM + CMA RMW	0.43119%	0.55149%	.970	-1.1661%	2.0285%
	CAPM + HML RMW	CAPM + HML SMB	-1.65277%	0.58022%	.057	-3.3334%	0.0279%
		CAPM + HML CMA	0.10629%	0.46593%	1.000	-1.2412%	1.4538%
		CAPM + SMB CMA	-1.00045%	0.63478%	.616	-2.8414%	0.8405%
		CAPM + SMB RMW	-1.16430%	0.69898%	.557	-3.1944%	0.8658%
		CAPM + CMA RMW	0.53748%	0.56007%	.930	-1.0841%	2.1591%
	CAPM + SMB CMA	CAPM + HML SMB	-0.65231%	0.71621%	.943	-2.7241%	1.4195%
		CAPM + HML CMA	1.10674%	0.62722%	.493	-0.7132%	2.9267%
		CAPM + HML RMW	1.00045%	0.63478%	.616	-0.8405%	2.8414%
		CAPM + SMB RMW	-0.16384%	0.81541%	1.000	-2.5226%	2.1949%
		CAPM + CMA RMW	1.53793%	0.69999%	.246	-0.4875%	3.5633%
	CAPM + SMB RMW	CAPM + HML SMB	-0.48847%	0.77369%	.988	-2.7281%	1.7512%
		CAPM + HML CMA	1.27059%	0.69212%	.448	-0.7406%	3.2817%
		CAPM + HML RMW	1.16430%	0.69898%	.557	-0.8658%	3.1944%
		CAPM + SMB CMA	0.16384%	0.81541%	1.000	-2.1949%	2.5226%
		CAPM + CMA RMW	1.70178%	0.75870%	.226	-0.4954%	3.8990%
	CAPM + CMA RMW	CAPM + HML SMB	-2.19024%*	0.65092%	.013	-4.0728%	-0.3077%
		CAPM + HML CMA	-0.43119%	0.55149%	.970	-2.0285%	1.1661%
		CAPM + HML RMW	-0.53748%	0.56007%	.930	-2.1591%	1.0841%
		CAPM + SMB CMA	-1.53793%	0.69999%	.246	-3.5633%	0.4875%
		CAPM + SMB RMW	-1.70178%	0.75870%	.226	-3.8990%	0.4954%

*. The mean difference is significant at the 0.05 level.

Difference of Means for R^2 – Four Factor (Top 160)

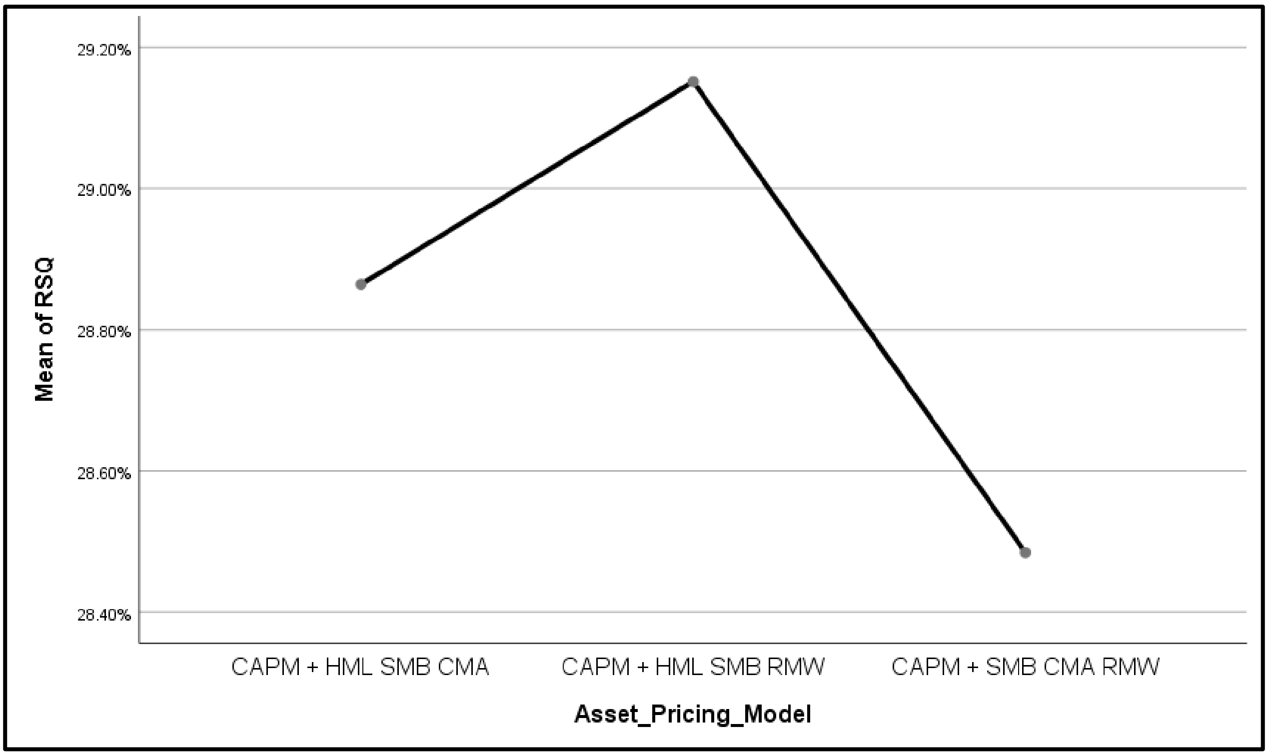

Figure lxv: Plot of means of R^2 for four factor Fama French (Top 160)

Table 79: Descriptive statistics for means of R^2 of four factor Fama French (Top 160)

Descriptives

RSQ

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
CAPM + HML SMB CMA	67	28.8642%	3.47857%	0.42497%	28.0157%	29.7127%	23.27%	40.14%
CAPM + HML SMB RMW	67	29.1512%	3.79798%	0.46400%	28.2248%	30.0776%	22.64%	40.91%
CAPM + SMB CMA RMW	67	28.4842%	4.60581%	0.56269%	27.3608%	29.6077%	19.40%	41.23%
Total	201	28.8332%	3.97854%	0.28062%	28.2798%	29.3866%	19.40%	41.23%

Table 80: Homogeneity of variances for means of R^2 of four factor Fama French (Top 160)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	2.983	2	198	.053
	Based on Median	2.700	2	198	.070
	Based on Median and with adjusted df	2.700	2	189.896	.070
	Based on trimmed mean	2.990	2	198	.053

Table 81: ANOVA results for means of R^2 of four factor Fama French (Top 160)

ANOVA RSQ					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	15.001	2	7.501	.471	.625
Within Groups	3150.747	198	15.913		
Total	3165.748	200			

Difference of Means for R^2 – Two Factor (Top 40)

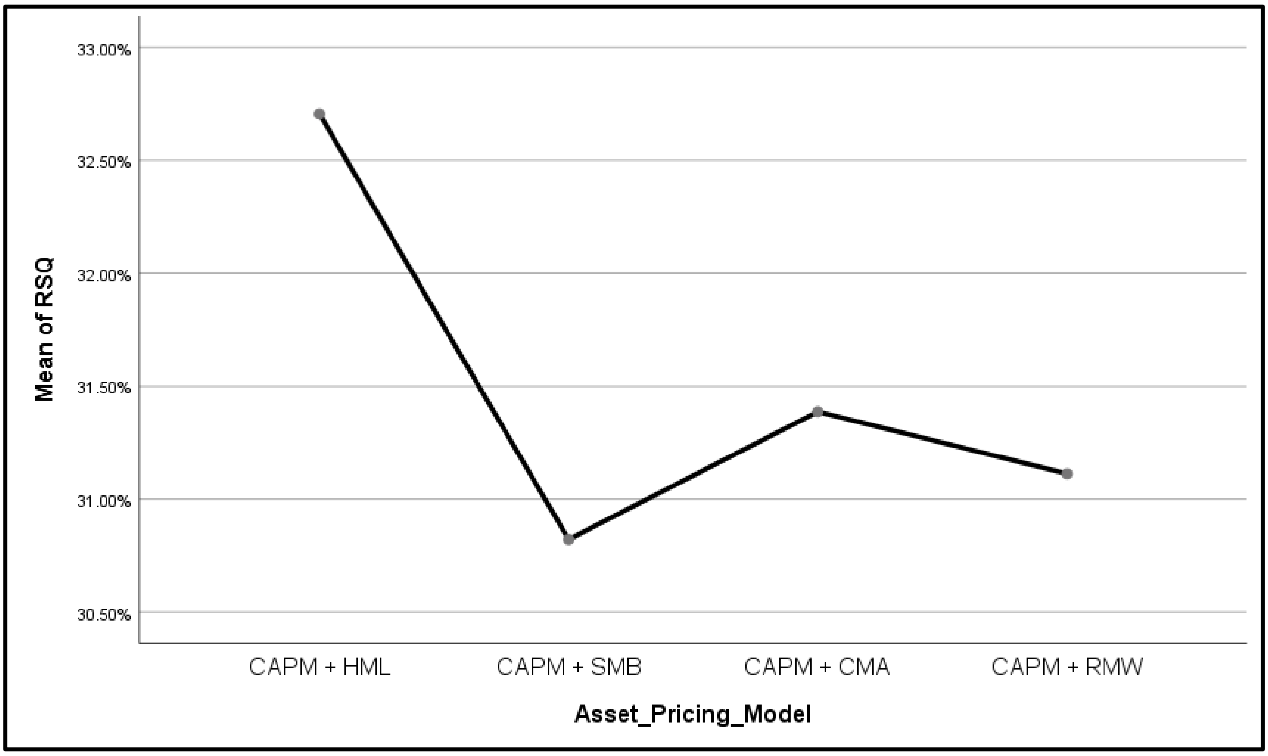

Figure lxvi: Plot of means of R^2 for two factor Fama French (Top 40)

Table 82: Descriptive statistics for means of R^2 of two factor Fama French (Top 40)

Descriptives								
RSQ								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML	67	32.7051%	4.25277%	0.51956%	31.6678%	33.7424%	21.44%	40.97%
CAPM + SMB	67	30.8216%	5.01487%	0.61266%	29.5984%	32.0448%	20.22%	40.03%
CAPM + CMA	67	31.3869%	4.33615%	0.52974%	30.3292%	32.4445%	22.42%	40.67%
CAPM + RMW	67	31.1124%	6.19936%	0.75737%	29.6002%	32.6245%	19.57%	42.23%
Total	268	31.5065%	5.03547%	0.30759%	30.9009%	32.1121%	19.57%	42.23%

Table 83: Homogeneity of variances for means of R^2 of two factor Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	5.827	3	264	.001
	Based on Median	4.685	3	264	.003
	Based on Median and with adjusted df	4.685	3	241.755	.003
	Based on trimmed mean	5.759	3	264	.001

Table 84: ANOVA results for means of R^2 of two factor Fama French (Top 40)

ANOVA					
RSQ					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	139.053	3	46.351	1.845	.139
Within Groups	6630.976	264	25.117		
Total	6770.029	267			

Difference of Means for R^2 – Three Factor (Top 40)

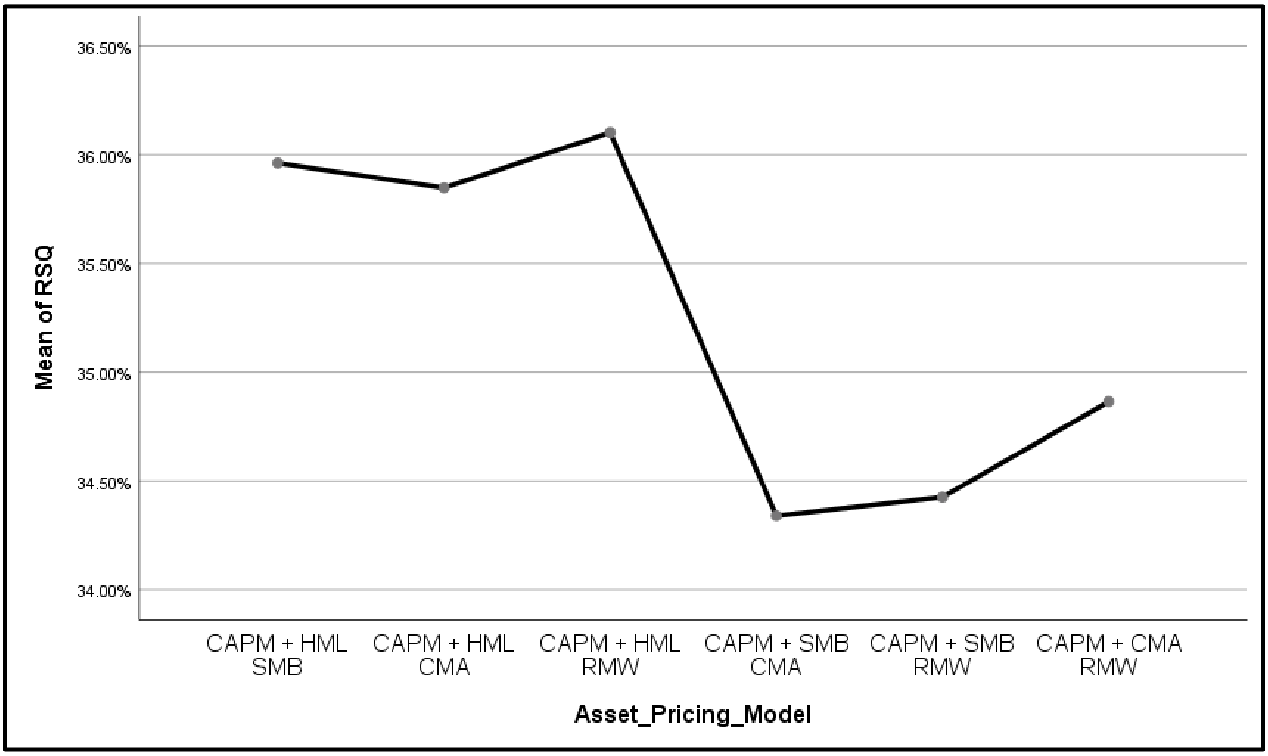

Figure lxvii: Plot of means of R^2 for three factor Fama French (Top 40)

Table 85: Descriptive statistics for means of R^2 of three factor Fama French (Top 40)

Descriptives

RSQ

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML SMB	67	35.9615%	3.36752%	0.41141%	35.1401%	36.7829%	28.80%	44.01%
CAPM + HML CMA	67	35.8475%	3.58530%	0.43801%	34.9729%	36.7220%	28.72%	43.26%
CAPM + HML RMW	67	36.1014%	4.22801%	0.51653%	35.0701%	37.1326%	26.24%	44.27%
CAPM + SMB CMA	67	34.3404%	4.10669%	0.50171%	33.3387%	35.3421%	24.54%	42.27%
CAPM + SMB RMW	67	34.4262%	5.27874%	0.64490%	33.1386%	35.7138%	24.28%	44.23%
CAPM + CMA RMW	67	34.8650%	4.97541%	0.60784%	33.6515%	36.0786%	24.54%	45.12%
Total	402	35.2570%	4.34769%	0.21684%	34.8307%	35.6833%	24.28%	45.12%

Table 86: Homogeneity of variances for means of R^2 of three factor Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	4.540	5	396	.000
	Based on Median	4.364	5	396	.001
	Based on Median and with adjusted df	4.364	5	365.763	.001
	Based on trimmed mean	4.530	5	396	.000

Table 87: ANOVA results for means of R^2 of three factor Fama French (Top 40)

ANOVA					
RSQ					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	217.210	5	43.442	2.337	.041
Within Groups	7362.652	396	18.593		
Total	7579.862	401			

Table 88: Post hoc analysis for means of R^2 three factor Fama French (Top 40)

Multiple Comparisons

Dependent Variable: RSQ

	(I) Asset_Pricing_Model	(J) Asset_Pricing_Model	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Games-Howell	CAPM + HML SMB	CAPM + HML CMA	0.11403%	0.60093%	1.000	-1.6240%	1.8520%
		CAPM + HML RMW	-0.13985%	0.66035%	1.000	-2.0510%	1.7713%
		CAPM + SMB CMA	1.62113%	0.64882%	.132	-0.2563%	3.4986%
		CAPM + SMB RMW	1.53529%	0.76495%	.345	-0.6828%	3.7534%
		CAPM + CMA RMW	1.09646%	0.73398%	.669	-1.0306%	3.2235%
	CAPM + HML CMA	CAPM + HML SMB	-0.11403%	0.60093%	1.000	-1.8520%	1.6240%
		CAPM + HML RMW	-0.25388%	0.67725%	.999	-2.2133%	1.7055%
		CAPM + SMB CMA	1.50710%	0.66601%	.217	-0.4195%	3.4337%
		CAPM + SMB RMW	1.42125%	0.77959%	.455	-0.8379%	3.6804%
		CAPM + CMA RMW	0.98242%	0.74922%	.778	-1.1875%	3.1524%
	CAPM + HML RMW	CAPM + HML SMB	0.13985%	0.66035%	1.000	-1.7713%	2.0510%
		CAPM + HML CMA	0.25388%	0.67725%	.999	-1.7055%	2.2133%
		CAPM + SMB CMA	1.76098%	0.72009%	.148	-0.3216%	3.8435%
		CAPM + SMB RMW	1.67513%	0.82626%	.333	-0.7161%	4.0664%
		CAPM + CMA RMW	1.23630%	0.79767%	.633	-1.0715%	3.5441%
	CAPM + SMB CMA	CAPM + HML SMB	-1.62113%	0.64882%	.132	-3.4986%	0.2563%
		CAPM + HML CMA	-1.50710%	0.66601%	.217	-3.4337%	0.4195%
		CAPM + HML RMW	-1.76098%	0.72009%	.148	-3.8435%	0.3216%
		CAPM + SMB RMW	-0.08585%	0.81708%	1.000	-2.4510%	2.2793%
		CAPM + CMA RMW	-0.52468%	0.78815%	.985	-2.8053%	1.7559%
	CAPM + SMB RMW	CAPM + HML SMB	-1.53529%	0.76495%	.345	-3.7534%	0.6828%
		CAPM + HML CMA	-1.42125%	0.77959%	.455	-3.6804%	0.8379%
		CAPM + HML RMW	-1.67513%	0.82626%	.333	-4.0664%	0.7161%
		CAPM + SMB CMA	0.08585%	0.81708%	1.000	-2.2793%	2.4510%
		CAPM + CMA RMW	-0.43883%	0.88621%	.996	-3.0019%	2.1242%
	CAPM + CMA RMW	CAPM + HML SMB	-1.09646%	0.73398%	.669	-3.2235%	1.0306%
		CAPM + HML CMA	-0.98242%	0.74922%	.778	-3.1524%	1.1875%
		CAPM + HML RMW	-1.23630%	0.79767%	.633	-3.5441%	1.0715%
		CAPM + SMB CMA	0.52468%	0.78815%	.985	-1.7559%	2.8053%
		CAPM + SMB RMW	0.43883%	0.88621%	.996	-2.1242%	3.0019%

Difference of Means for R^2 – Four Factor (Top 40)

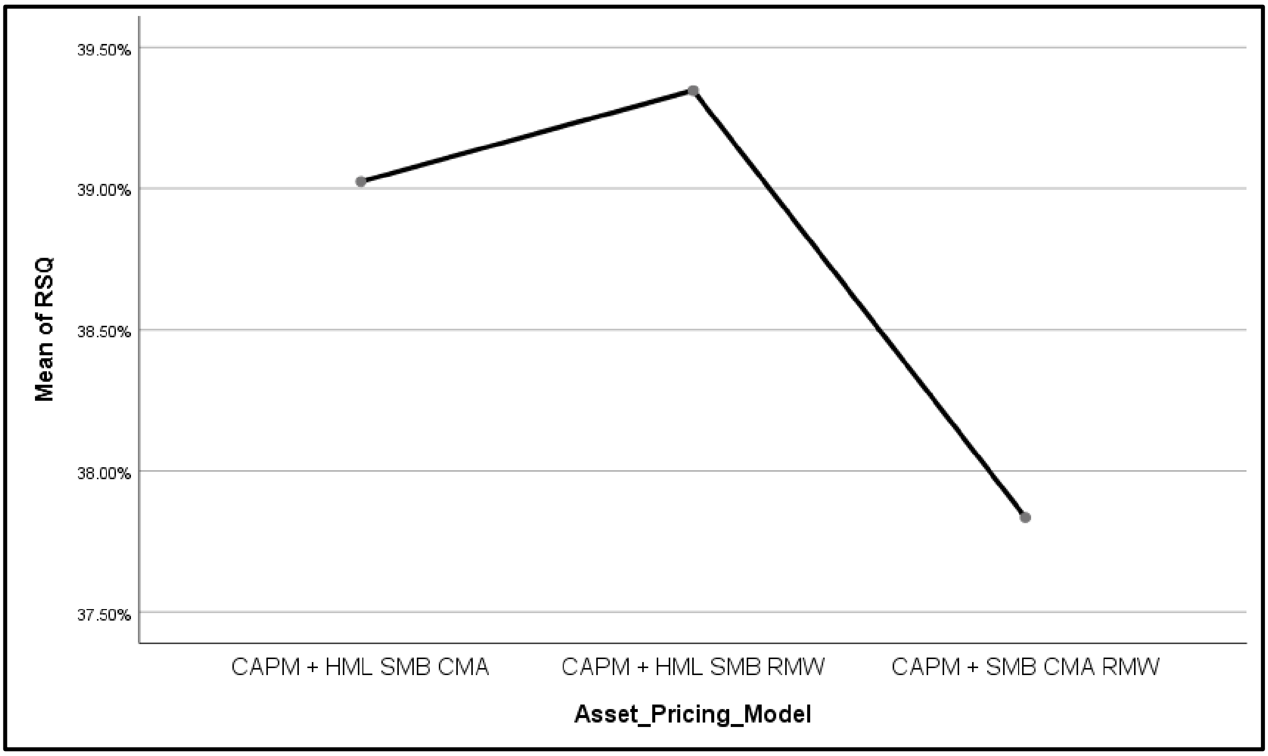

Figure lxviii: Plot of means of R^2 for four factor Fama French (Top 40)

Table 89: Descriptive statistics for means of R^2 of two factor Fama French (Top 40)

Descriptives

RSQ

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
CAPM + HML SMB CMA	67	39.0242%	3.22905%	0.39449%	38.2366%	39.8119%	31.62%	46.17%
CAPM + HML SMB RMW	67	39.3466%	3.36456%	0.41105%	38.5259%	40.1673%	33.91%	46.91%
CAPM + SMB CMA RMW	67	37.8349%	4.57617%	0.55907%	36.7187%	38.9511%	26.82%	46.72%
Total	201	38.7352%	3.80945%	0.26870%	38.2054%	39.2651%	26.82%	46.91%

Table 90: Homogeneity of variances for means of R^2 of four factor Fama French (Top 40)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
RSQ	Based on Mean	4.049	2	198	.019
	Based on Median	3.888	2	198	.022
	Based on Median and with adjusted df	3.888	2	171.836	.022
	Based on trimmed mean	4.065	2	198	.019

Table 91: ANOVA results for means of R^2 of four factor Fama French (Top 40)

ANOVA

RSQ

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	84.948	2	42.474	2.985	.053
Within Groups	2817.429	198	14.229		
Total	2902.378	200			