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Feasibility Study and Strategic Location Modelling of a centralised distribution centre for Conways

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Abstract

Facility location modelling has proven to be a unique and powerful tool in operations research. Location models are widely used to solve complex engineering problems across various industries. For Conways, a facility model is invaluable in determining the optimal location for their new centralised distribution warehouse. This project aims to develop a facility location model to identify possible locations, given the locations of the five warehouses, the new centre will stock. A new distribution centre will assist Conways in consolidating these five warehouses and order their products through one account. A single account, rather than five separate accounts, will allow them to make better use of the quantity discounts offered by Wispeco. Since only one account will handle the procurement, the procurement problem was also investigated to identify the best combination of purchases at the lowest possible cost. A centralised centre will ultimately provide the opportunity to improve Conways' procurement system.

This report provides the problem investigation and an in-depth literature review of the procurement problem as well as the facility location model. The mathematical models used to solve both problems are discussed, along with the results. The first model identified the best combination purchases in order to obtain the lowest cost. This combination will save Conways R 766 002,00 annually. The second model determined that the optimal location for a distribution centre is in the Pretoria region. The Tannery Industrial Park is the preferred location.

Declaration of Originality

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Declaration

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Chapter 1

Introduction

1.1 Background

Wispeco Aluminium is the largest aluminium extruder in Africa, with three different factories located in Johannesburg, Vereeniging and Cape Town. They manufacture aluminium profiles for industrial applications, such as mill finish extrusions, powder coated extrusions and anodised extrusions, as well as architectural applications. Wispeco's architectural profiles are referred to as Crealco fenestration systems. Crealco fenestration systems refer to all aluminium finishes of the openings of a building and include windows, doors and storefronts. These systems are available directly from Wispeco or from one of Wispeco's Appointed Stockists [33].

A stockist is a company who purchases bulk aluminium profiles and components required to assemble different fenestration systems. Hereafter the stockist sells the profiles and components to fabricators to install in buildings and renovation projects. Wispeco Appointed Stockists are Wispeco established companies with aligned ethical values to sell Crealco fenestration systems to fabricators [4].

Conways is one of Wispeco's Appointed Stockists and is the largest and leading aluminium profile stockist in Southern Africa. Conways' branches are located in Pretoria, Randburg, Edenvale, Limpopo, Nelspruit, Durban, Port Elizabeth, and Cape Town [3].

Conways strive to deliver the architectural aluminium market with innovation and service with passion. One of their goals is to become the lowest cost stockist in South Africa. They aim to streamline their procurement system in the Gauteng region (Pretoria, Randburg, Edenvale, Limpopo, and Nelspruit) before implementing this system nationwide. In order to improve their procurement system, they first have to decrease their procurement costs by making better use of the quantity discounts offered by Wispeco.

Quantity discounts offered by Wispeco are divided into three groups: M, S and T. The first group, M, is the minimum amount available for purchase, 250 kg, with either a 0% or 4% discount, depending on the product. The S-group refers to the purchase of 500 kg, where a discount of 4% or 8% is offered. The final group, T, is the purchase of 1 ton with a discounted price of 8%. At the moment, Conways is only able to purchase group M, and only occasionally group S. Each warehouse individually receives stock from the Wispeco production factory in Alrode, preventing them from placing larger orders.

Figure 1.1 shows the distribution of Conways warehouses around the Wispeco factory, adapted from Google Maps.

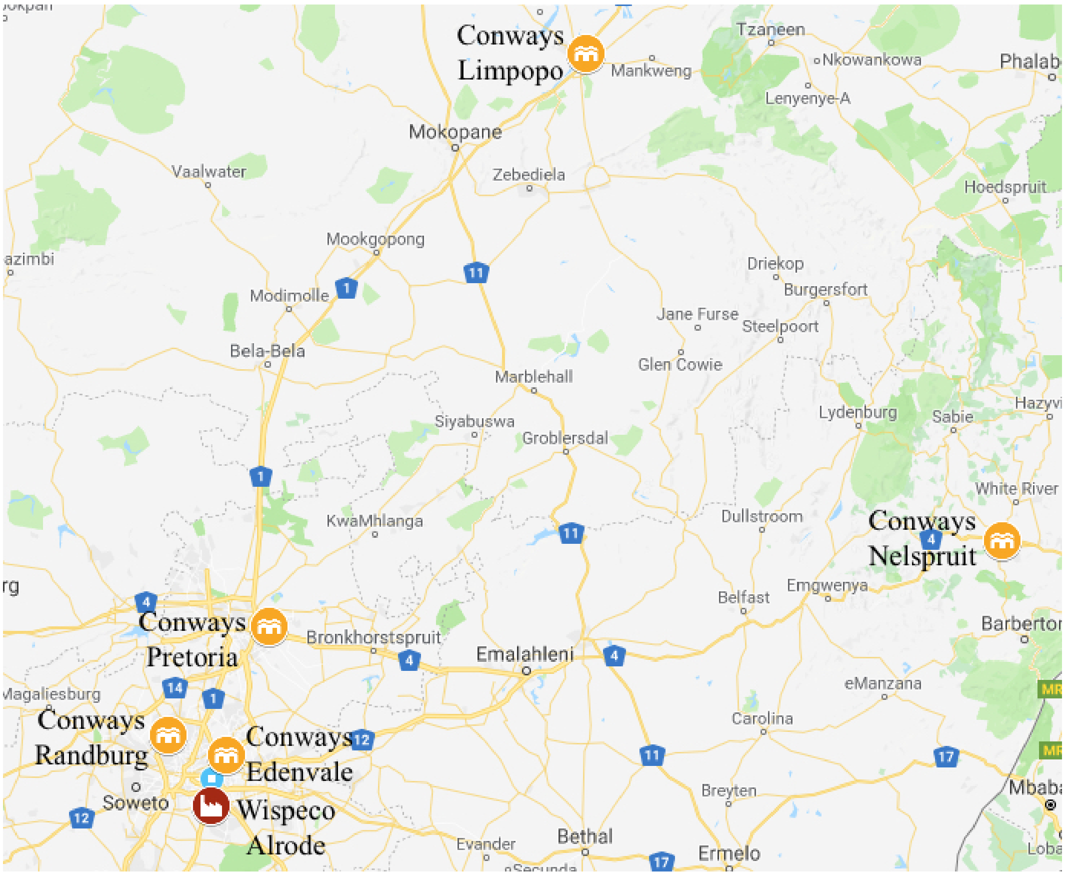

Figure 1.1: Distribution of Conways warehouses relative to the Wispeco plant.

In order to consolidate these five warehouses, a centralised distribution centre is required. A centralised distribution centre will increase the available storage space and will create a connecting point between the supply, Wispeco, and demand, the five warehouses. Conways can use this warehouse to store the inventory of their bulk purchases and distribute products to the other warehouses as needed.

1.2 Problem Investigation

1.2.1 Problem Statement

For this project, a feasibility study was performed to investigate the advantages of stocking the five warehouses through one centralised distribution centre. Through the use of quantity discounts, different combinations of bulk purchases were explored to identify the lowest cost option. Hereafter the optimal location and size of the distribution centre were determined through the use of strategic location modelling.

1.2.2 Project Aim

The aim of this project was to determine the costs Conways will save by making use of quantity discounts as well as determining the optimal location for a centralised distribution centre. To achieve these aims, the project was divided into two phases, each with its own aim.

Phase one focused on the procurement problem, specifically making better use of quantity discounts. The aim of phase one was to determine the possible cost savings should Conways use the quantity discounts. The current purchases of each warehouse were investigated to identify the items that are purchased for more than one warehouse. The expected cost savings were determined by calculating the difference in cost should the overlapping purchases be ordered together, rather than separately. This calculation indicated an expected cost saving of one million rand per year. An algorithm was required to obtain the best combination of purchases to achieve the lowest cost.

Phase two aimed to obtain an optimal location for the distribution centre which will be used to consolidate the five warehouses. An algorithm based on the discrete facility location model theory was required.

1.2.3 As-is Analysis

Problem investigation, more specifically as-is analysis was needed to investigate the current problem and process fully. This section discusses the problem in more detail than the problem statement.

Conways consists of eight warehouses across South Africa, however, this project only considers the Northern five: Limpopo, Nelspruit, Pretoria, Randburg, and Edenvale.

At the moment, two Conways employees are situated at Wispeco Alrode and manage their orders from the head office. Each warehouse has its own account at Wispeco and their orders are determined and placed independently.

The independence of orders reduces the administration required to separate the orders for each warehouse, but it also divides the orders of the same products into smaller groups. Dividing the orders eliminates the opportunity of larger quantity discounts.

Since each warehouse's account is handled separately, the products ordered are only divided into the MST group within that account. Figure 1.2 indicates the percentage of products under each category of MST for each warehouse.

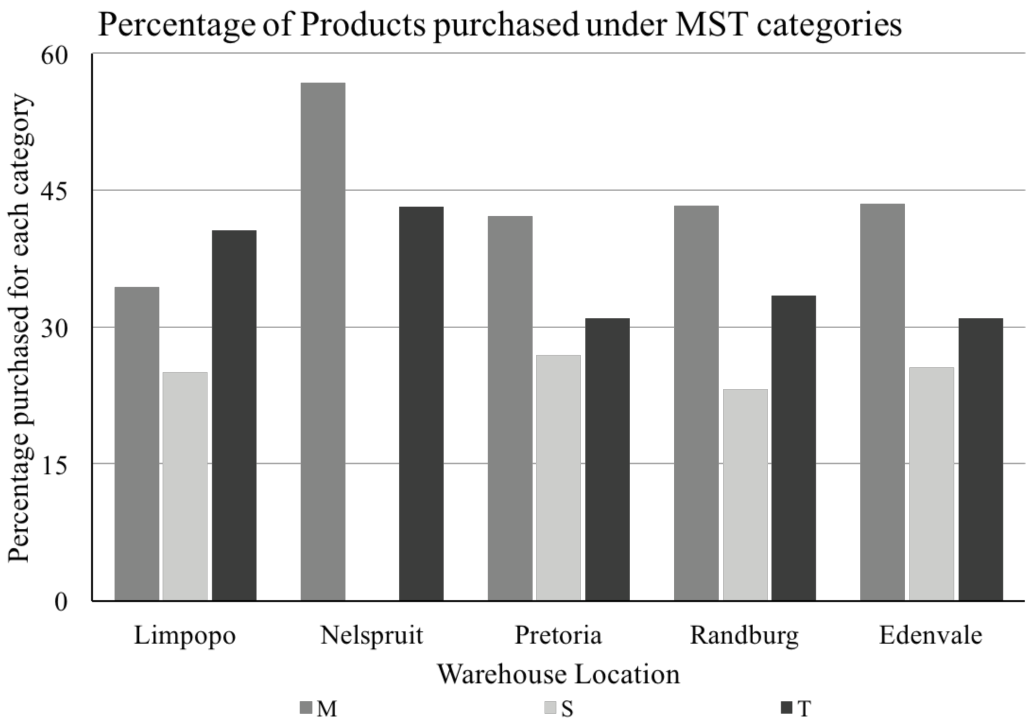

Figure 1.2: Percentage of products purchased under each category of MST.

Figure 1.2 clearly shows that the majority of products are purchased under M, the lowest discount option. Should Conways purchase all their products through one account, more products will shift to either S or T, decreasing the overall procurement cost.

The MST discounts are only applied to the mill finished products. Mill finished refers to the state of the aluminium when it exits the extrusion mill. No other mechanical or chemical finishing was performed. Each product is then anodised and powder-coated as required.

The transportation of the products is handled by a third party, employed by Wispeco. The transportation manager assured that the project will not influence the transportation costs or salaries of the drivers. A constant price is given for transport, no matter the route or distance. The drivers receive a fixed salary, which will not be influenced by the distance they travel.

1.3 Project Approach

The final deliverables of this project were possible cost savings should Conways use the discount policy and an optimal location for a distribution centre. In order to complete this project successfully, the remainder of this section explains the methodology.

1.3.1 Problem Investigation

The problem statement in Section 1.2.1 merely defined the problem. In order to understand the problem fully, a further as-is analysis was required. The current procurement system of Conways was investigated to verify whether the problem could be solved through the use of Industrial Engineering techniques. This included interviews with management and data collection to understand the current process as well as to develop initial solution ideas. As-is analyses were conducted in Section 1.2.3.

1.3.2 Investigation of best practices

After the problem investigation, an in-depth literature study was conducted to identify the best practices for solving these problems. A comprehensive insight into the theory of the problem and applicable Industrial Engineering techniques are essential. Hereafter, case studies of similar problems were considered to determine the best possible methods and solution approaches for this project. An in-depth literature review of both the procurement problem and facility location modelling are conducted in Chapter 2.

1.3.3 Development of alternative solutions

Mathematical models were developed for both the procurement problem and facility location modelling. All the relevant data and information was gathered to be used as inputs. In order to solve both models, algorithms were required. The model formulations consisted of an objective function, variables, and constraints. The model formulations are described in Chapters 3 and 4.

1.3.4 Solution verification and validation

The developed solutions were verified using sensitivity analysis. The final model results were presented to Conways and compared to the initial aims of the project for validation. The models were verified by changing certain inputs to ensure that those inputs were taken into account. Possible improvements were identified and recommendations were made.

The remainder of the report is structured as follows: An in-depth literature study of the best practices is conducted in Chapter 2. Chapter 3 and 4 provides the model formulation and results for both the procurement problem and the facility location model respectively. The report is concluded in Chapter 5.

Chapter 2

Literature Review

In this section, the literature of procurement problems and facility location modelling is examined. The literature provides extensive insight into the theory of the problems as well as existing algorithms which were used to solve similar problems. The most applicable methods are studied for this project.

2.1 Total Quantity Discount Policy

Procurement problems, specifically those involving discount policies, have been investigated since 1994. Katz *et al.* [28] and Sadrian and Yoon [31] identified a discount policy wherein a supplier offers a percentage discount based on the total cost of all the purchased goods. Later Crama *et al.* [34] investigated a similar procurement problem where the discount is offered as a function of the cost rather than a percentage. Goossens [12] defined another policy, where the number of products ordered, determines the volume interval in which the buyer falls and this interval determines the discount. This policy is referred to as the Total Quantity Discount policy (TQD). This policy further assumes that the product prices apply to all the units purchased, which is referred to as the all-unit discount policy and was introduced by Munson and Rosenblatt [27]. Wispeco's discount policy is identified as a TQD policy.

Goossens [12] established three variants of the TQD policy. The first variant allows the buyer to impose higher and lower limits on the number of purchased goods. The second recognises that the buyer is willing to purchase more products than needed in order to reach the lowest cost. Lastly, Goossens [12] considers that the buyer may want to limit the number of suppliers to purchase from. The latter is not applicable in this case, since the buyer, Conways, focusses on their purchases from only one seller, Wispeco.

The TQD policy can be mathematically formulated and solved using the min-cost flow based branch-and-bound, linear programming based branch-and-bound or the branch-and-cut algorithm.

The purpose of this formulation is to identify the best combination of purchases to obtain the lowest cost.

The following parameters are required:

$$\boldsymbol{G} = \text{the set of } k \text{ items} \tag{2.1}$$
$$\boldsymbol{S} = \text{the set of } i \text{ suppliers} \tag{2.2}$$
$$d_k \triangleq \text{the number of items } k \in \boldsymbol{G} \text{ to be procured} \tag{2.3}$$
$$\tag{2.4}$$

For each supplier $i \in \boldsymbol{S}$, associate a volume interval $Z_i = \{0,1,...,\max i\}$ indexed by j.

For each supplier $i \in \boldsymbol{S}$ and interval $j \in Z_i$, define the minimum, l_{ij}, and the maximum, u_{ij}, number of items required from supplier i to be in interval j.

For each supplier $i \in \boldsymbol{S}$, for each interval $j \in Z_i$, and each item $k \in \boldsymbol{G}$, let c_{ijk} be the price of one item k purchased from supplier i in its j-th interval.

In order to satisfy these parameters, the following assumptions are made:

$$[l_{ij}, u_{ij}) \cap [l_{ij'}) = \varnothing \qquad \forall i \in \boldsymbol{S}, j \neq j' \in Z_i \tag{2.5}$$
$$c_{ijk} \geq c_{i,j+1,k} \qquad \forall i \in \boldsymbol{S}, j \in Z_i \setminus \{max_i\}, k \in \boldsymbol{G} \tag{2.6}$$
$$c_{ijk} \geq 0, l_{ij} \geq 0, u_{ij} \geq 0, d_k \geq 0 \qquad \forall i \in \boldsymbol{S}, j \in Z_i, k \in \boldsymbol{G} \tag{2.7}$$

The first assumption (2.5) ensures that a supplier's interval should not overlap, and (2.6) expresses that the prices of items remain constant between intervals. The last assumption (2.7) indicates that all prices, quantities and demands are nonnegative.

Furthermore, two additional variables are added:

$$x_{ijk} \triangleq \text{the number of item k to be purchased from supplier } i \text{ in interval } j \tag{2.8}$$

$$y_{ij} = \begin{cases} 1, & \text{if supplier } i \text{ and interval } j \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \tag{2.9}$$

Given that the purpose of the TQD policy is to determine the combination of purchases in order to obtain the lowest cost, the objective function is:

$$\min Z = \sum_{i \in \boldsymbol{S}} \sum_{j \in Z_i} \sum_{k \in \boldsymbol{G}} c_{ijk} x_{ijk} \tag{2.10}$$

subject to

$$\sum_{i \in \boldsymbol{S}} \sum_{j \in Z_i} x_{ijk} = d_k \qquad \forall k \in \boldsymbol{G} \tag{2.11}$$

$$\sum_{j \in Z_i} y_{ij} \leq 1 \qquad \forall i \in \boldsymbol{S} \tag{2.12}$$

$$\sum_{k \in \boldsymbol{G}} x_{ijk} - y_{ij} l_{ij} \geq 0 \qquad \forall i \in \boldsymbol{S}, j \in Z_j \tag{2.13}$$

$$\sum_{k \in \boldsymbol{G}} x_{ijk} - y_{ij} u_{ij} \leq 0 \qquad \forall i \in \boldsymbol{S}, j \in Z_i \tag{2.14}$$

$$x_{ijk} \geq 0 \qquad \forall i \in \boldsymbol{S}, j \in Z_i, k \in \boldsymbol{G} \tag{2.15}$$

$$y_{ij} \in \{0, 1\} \qquad \forall i \in \boldsymbol{S}, j \in Z_i \tag{2.16}$$

The objective function (2.10) aims to minimise the total price of the number of items k ordered from supplier i in interval j. Constraint (2.11) ensures that the demand is met and constraint (2.12) verifies that only one interval per supplier is selected. Constraints (2.13) and (2.14) guarantee that the number of items chosen lies within the given bounds. Constraint (2.15) indicates the nonnegativity of all values and (2.16) defines the binary decision variable.

For the first variant, the following constraint can be added to account for the upper and/or lower limit set by the buyer:

$$q_{i,k} \leq \sum_{j \in Z_i} x_{ijk} \leq Q_{i,k} \tag{2.17}$$

With $q_{i,k}$ the lowest number of items k to be purchased from supplier i and $Q_{i,k}$ the highest number.
In order to allow the buyer to purchase more products than required, as in the second variant, the following constraint can be added:

$$\sum_{i \in S} \sum_{j \in Z_i} x_{ijk} \geq d_k \qquad \forall k \in \boldsymbol{G} \tag{2.18}$$

This constraint (2.18) replaces constraint (2.11) to allow the buyer to purchase more than the demand.

Goossens [12] used this mathematical formulation and used the three different types of algorithms to test their performance. The computational results proved that all three algorithms obtained an exact solution, however, the min-cost flow-based algorithm is most accurate for a small number of suppliers and when applying upper and/or lower bounds.

It is evident from the literature that Goossens' mathematical formulation can be adapted to determine the best combination of purchases from Wispeco to obtain the lowest price. The constraints pertaining to the different suppliers can be altered to allow for only one supplier.

2.2 Facility Location Modelling

Operations research is a unique and powerful approach to decision making. Operations research can be defined as the study of how to form and apply mathematical models and analytical methods to assist in decision making [17].

One of the branches of operations research is facility location modelling [25]. Facility location modelling has been used to solve numerous complex problems [1]. The theory supports decision making with regards to identifying an optimal location for a facility, serving a specified function. Application of facility location modelling includes the location of schools, ambulance dispatch points and a supermarket within a community [14].

Location theory was introduced by Alfred Weber [10], who modelled the problem of locating a single facility to minimise the distance travelled between the facility and demand. Later, Harold Hotelling [16] studied the location of two facilities. His model strategically positioned two vendors close to the customers to maximise their market share. Hereafter, Walter Isard [18] combined economics and location theory. Daskin [6] classified location models into four categories as depicted in Figure 2.1.

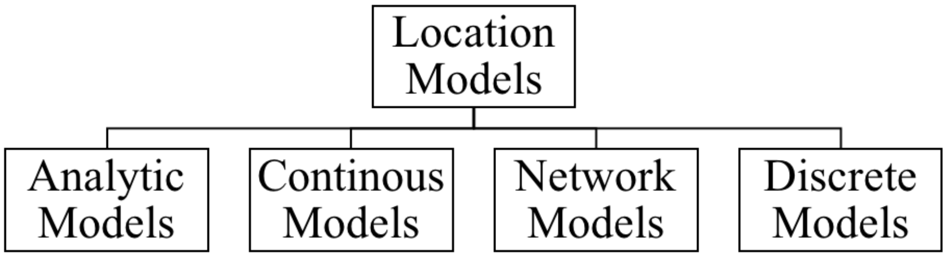

Figure 2.1: Taxonomy of location models [6].

The simplest location model is the analytical model, which assumes that the demand spans over a sector and that a facility location can be found within that sector. This model can easily be solved using calculus or other basic techniques. Continuous models, rather than analytical models, assume that the demand is located at discrete points or nodes and the facility can only be located at one of these points [29].

Network models assume that the demand consists of a framework of points and links and a facility can be located anywhere on the network [19]. The final model is the discrete location model, which assumes that demand arises on points and a facility can be located at a predetermined set of locations [7].

The demand for this project refers to the five warehouses, which are located at specific nodes, and possible locations for the distribution centre will be predetermined. With these parameters, the applicable method is discrete modelling.

Discrete models are divided into three categories: Covering-based, median-based, and other models [13]. The classification of discrete location models is shown in Figure 2.2.

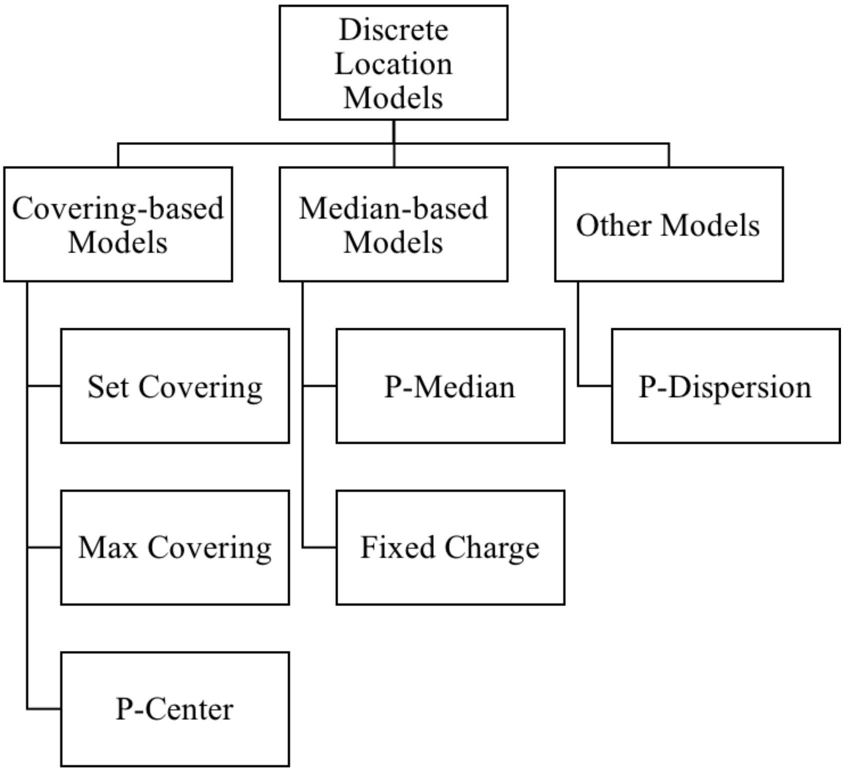

Figure 2.2: Classification of discrete location models [6].

Covering-based models, also referred to as maximum service distance problems, assume that there is some critical covering metric, such as distance or time, within which the demand needs to be met [2]. This model's objective is to maximise the coverage within the vicinity of the facility [22]. Median-based models, also known as the average distance models, are concerned with optimising the distance a customer has to travel to the facility.

This model minimises the average distance between the customer (demand point) and the facility [5]. Each demand node can be given a weight to influence the distance [15]. Lastly, some models do not fall under these two models. An example is the p-dispersion model, which is only concerned with the distance between facilities. The model's objective is to maximise the minimum distance between facilities [20].

Given that all five Conways warehouses have to be stocked through one centralised distribution centre, covering models are not appropriate for this project. Median-based models are suitable since the model aims to locate one facility on the demand network [9]. More specifically, the P-median model is investigated, since the model obtains the location of a fixed number of P facilities in order to minimize the average weighted distance of the network [24].

Farahani [9] discusses the classic model for the basic P-median problem. In order to formulate the model, the following parameters are required:

$$\boldsymbol{S} = \text{the set of demand nodes } i \tag{2.19}$$

$$\boldsymbol{G} = \text{the set of candidate facility locations } j \tag{2.20}$$

$$\boldsymbol{P} = \text{the number of facilities to be established} \tag{2.21}$$

$$h_i \triangleq \text{the demand of node } i \tag{2.22}$$

$$n \triangleq \text{number of nodes} \tag{2.23}$$

$$d_{ij} \triangleq \text{the distance between the demand node } i \text{ and the facility location } j. \tag{2.24}$$

$$X_{ij} = \begin{cases} 1, & \text{if the demand of node } i \text{ is covered by the facility at } j \\ 0, & \text{otherwise} \end{cases} \tag{2.25}$$

$$Y_j = \begin{cases} 1, & \text{if facility is located at node } j \\ 0, & \text{otherwise} \end{cases} \tag{2.26}$$

This model was proposed by ReVelle and Swain [30]. The aim is to minimise the total average distance, therefore the objective function is:

$$\min Z = \sum_{i \in \boldsymbol{S}} \sum_{j \in \boldsymbol{G}} h_i d_{ij} X_{ij} \tag{2.27}$$

subject to:

$$\sum_{j \in \boldsymbol{G}} X_{ij} = 1 \qquad \forall i \in \boldsymbol{S} \tag{2.28}$$

$$\sum_{j \in \boldsymbol{G}} Y_j = P \tag{2.29}$$

$$X_{ij} \leq Y_j \qquad \forall i \in \boldsymbol{S}, j \in \boldsymbol{G} \tag{2.30}$$

$$X_{ij}, Y_j \in \{0, 1\} \qquad \forall i \in \boldsymbol{S}, j \in \boldsymbol{G} \tag{2.31}$$

The first constraint (2.28) states that each demand node should only be serviced by one facility. The second constraint (2.29) ensures that the chosen number of facilities is equal to the required number (P) of facilities. Constraint 3 (equation 2.30) states the demand node i can be serviced by facility node j. Lastly, (2.31) confirms the binary variables.

In order to solve the P-median problem, three possible algorithm types may be used: Exact method, Heuristic, and metaheuristic algorithms. The exact algorithm finds the optimal solution along with proof of its optimality, if one exists. A heuristic algorithm, however, finds a solution close to optimal. Metaheuristic algorithms are approaches to the design of a heuristic algorithm. Most location problems are solved using heuristic methods [26].

Heuristics are divided into three groups: Constructive (CH), Local Search (LS), and those based on Mathematical Programming (MP). Each group consists of different subgroups. CH heuristics typically starts with a set of open facilities and facilities are either added or removed to reach the number p [8, 21]. LS heuristics are divided into the alternate and interchange method. The alternate method requires an initial solution set L and p facilities are located within this set. The demand is assigned to the closest facility and the p-median problem is solved for each facility. Hereafter, the process is iterated using new locations [23]. The interchange method starts with p facilities, which are then moved, one by one, to open locations in order to reduce the total cost. The iterations are stopped once a minimum total cost has been reached [32].

MP formulation consists of three subgroups, Dynamic Programming (DP), Lagrangian heuristics (LH), and Aggregation (AG). DP starts with an open set L and locations are iteratively added. The best solutions, q are stored for each iteration. The process stops once p locations are reached. LH is based on the equations (2.28 - 2.31), with a multiplier u_i added to the objective function. The Lagrangian model is solved with different values for u_i [11]. AG is the process of reducing the number of demand points in order to reduce the computational time or for confidential reasons [26].

Mladenović [26] concluded that, even though heuristic methods are usually used to solve the p-median problem, there are advantages to using metaheuristics. Exact methods and classic heuristics are successful in solving relatively small problems, whereas metaheuristics' solutions are more accurate on a larger scale. Given that there are only five demand nodes for this project, an exact method, such as the branch and bound method, will be sufficient and accurate.

Table 2.1 compares the different solution methods using four criteria. Each criteria is assigned a weight from one to four, depending on the importance to this project. Every method is given a rating from one to three, with three being the best and one the worst, for each criteria. The weighted total is calculated by multiplying each score with the respective weight and summing the scores for each method.

Table 2.1: Evaluation of different solution methods.

		Exact Method	Heuristics	Metaheuristics
Criteria	Assigned Weight	*Proposed Software LINGO*	*Proposed Software Python*	*Proposed Software MATLAB*
Best Applicable	4	3	2	1
Computational Complexity	3	3	2	1
Efficiency	2	3	1	2
Execution Time	1	2	1	3
Weighted Total		**29**	**17**	**14**

Chapter 3 discusses the TQD Policy Model including the formulation, results, validation, verification, and recommendations. Chapter 4 describes these same topics, but for the Facility Location Model.

Chapter 3

Total Quantity Discount Policy

The mathematical model regarding the Total Quantity Discount Policy is discussed in this chapter. The method developed by Goossens [12] was used to formulate and solve the procurement problem of Conways. This chapter also includes the model verification and validation.

3.1 Model Inputs

As discussed in the literature review in Chapter 2, the proposed model was used to determine the best combination of purchases in order to obtain the lowest cost. Given that only one supplier was used in this project, the model inputs were few and therefore linear programming was suitable to find a solution.

The model was formulated based on the available procurement data from Conways and limited to the model's capability to describe the procurement process. The following assumptions were made:

- The demand for the products remain constant
- The prices of the products remain the same
- The discounts offered will not change
- No new products will be added

The monthly demand of the five warehouses was combined to obtain a total demand per product. Since the TQD Policy offered by Wispeco was given in kilograms, only the products purchased in kilograms were taken into account. The demand for every product was updated every two to three months, but the purchases based on weight, mainly remained the same.

Wispeco divides all their products into three segments, each segment with a different discount policy. The applicable percentage is then subtracted from the product's base price. The segments along with the respective discount policies are shown in Table 3.1.

Table 3.1: Discount policy of each segment.

Segment	M (250kg)	S (500kg)	T (1000kg)
A	-0%	-8%	-8%
B	-4%	-4%	-8%
C	-0%	-4%	-8%

3.2 Mathematical Model

The mathematical formulation of Goossens [12] was adapted and formulated as follows:

Let:
$\boldsymbol{G}$ = the set of products k such that

$$\boldsymbol{G} = \begin{cases} 1 = & \text{Product code 11163} \\ 2 = & \text{Product code 11165} \\ 3 = & \text{Product code 11193} \\ ... & \\ 83 = & \text{Product code 70963} \end{cases} \tag{3.1}$$

$\boldsymbol{Z}$= the set of volume interval j such that

$$\boldsymbol{Z} = \begin{cases} 1 = & \text{M (250 - 499 kg)} \\ 2 = & \text{S (500 - 999 kg)} \\ 3 = & \text{T (1000kg or more)} \end{cases} \tag{3.2}$$

$$d_k \triangleq \text{the demand of product } k, \forall\ k \in \boldsymbol{G} \tag{3.3}$$
$$l_j \triangleq \text{the minimum weight required to be in interval } j, \forall\ j \in \boldsymbol{Z} \tag{3.4}$$
$$u_j \triangleq \text{the maximum weight required to be in interval } j, \forall\ j \in \boldsymbol{Z} \tag{3.5}$$
$$c_{jk} \triangleq \text{the price of product } k \text{ in interval } j, \forall\ k \in \boldsymbol{G}, j \in \boldsymbol{Z} \tag{3.6}$$
$$x_{jk} \triangleq \text{the total weight of product } k \text{ to be purchased in interval } j, \tag{3.7}$$
$$\forall\ k \in \boldsymbol{G}, j \in \boldsymbol{Z} \tag{3.8}$$
$$y_{jk} = \begin{cases} 1, & \text{if interval } j \text{ is selected for product } k \\ 0, & \text{otherwise} \end{cases} \tag{3.9}$$

With the objective function:

$$\min Z = \sum_{j \in \boldsymbol{Z}} \sum_{k \in \boldsymbol{G}} c_{jk} x_{jk} \tag{3.10}$$

subject to

$$\sum_{j \in \boldsymbol{Z}} x_{jk} \geq d_k \qquad \forall k \in \boldsymbol{G} \tag{3.11}$$

$$x_{jk} - y_{jk} l_j \geq 0 \qquad \forall j \in \boldsymbol{Z}, k \in \boldsymbol{G} \tag{3.12}$$

$$x_{jk} - y_{jk} u_j \leq 0 \qquad \forall j \in \boldsymbol{Z}, k \in \boldsymbol{G} \tag{3.13}$$

$$\sum_{j \in \boldsymbol{Z}} y_{jk} \leq 1 \qquad \forall k \in \boldsymbol{Z} \tag{3.14}$$

$$x_{jk} \geq 0 \qquad \forall j \in \boldsymbol{Z}, k \in \boldsymbol{G} \tag{3.15}$$

$$y_{jk} \in \{0, 1\} \qquad \forall j \in \boldsymbol{Z}, k \in \boldsymbol{G} \tag{3.16}$$

The objective function (3.10) minimises the total cost of products purchased. Constraint (3.11) creates the possibility to procure more products than the demand. Constraints (3.12) and (3.13) ensures that the weight of products lies within the interval's limits. Constraint (3.14) limits the model to purchase products from only one interval per product. Constraint (3.15) indicates the nonnegativity of the number of goods to be purchased. Since Conways allows the purchase of more products than needed to obtain the lowest price, no upper limit is set.

3.3 Model Results

The Optimisation Modeling Software, LINGO, was used to program and solve the model. A linear program was developed to include the sets, variables, objective function, and constraints as defined in Section 3.1. The model was solved to obtain the total minimum procurement cost, along with a list of how much to purchase per product per interval. The total minimum monthly cost is R 2 606 830. The detailed list of purchases is shown in Table 3.2.

Table 3.2: Purchase quantities, in kilograms, of each product.

Product Code	M	S	T	Product Code	M	S	T
11163	250	0	0	52695	0	500	0
11165	250	0	0	53033	0	750	0
11193	250	0	0	53098	0	0	1000
11493	250	0	0	53101	0	0	1000
11517	0	0	1000	53151	0	500	0
18079	0	0	1000	53194	0	0	1250
18501	250	0	0	53365	0	750	0
20423	0	750	0	53406	0	500	0
23224	250	0	0	53407	250	0	0
24494	250	0	0	53566	0	750	0
24498	250	0	0	53888	250	0	0
25022	250	0	0	53955	250	0	0
25029	250	0	0	54329	0	500	0
26150	250	0	0	54348	0	0	1500
26154	0	0	1000	54651	0	750	0
26282	0	0	1500	54720	0	0	1250
28475	250	0	0	54858	0	750	0
28643	0	750	0	54964	0	750	0
30607	0	750	0	54965	0	750	0
31533	0	500	0	54966	0	500	0
31535	0	500	0	54986	0	500	0
31537	0	500	0	54987	250	0	0
31538	0	500	0	55111	0	500	0
31539	0	0	1000	55112	0	500	0
31590	0	0	1250	55113	0	500	0
31789	0	500	0	55209	250	0	0
32347	0	0	1250	55363	0	500	0
33921	0	750	0	55365	0	500	0
34853	0	500	0	55433	0	500	0
35807	250	0	0	55434	0	500	0
42758	250	0	0	55456	0	0	1000
44083	0	750	0	55501	250	0	0
44085	0	750	0	55824	0	500	0
44086	0	0	1250	55891	0	500	0
44087	0	500	0	56205	0	0	1500
44089	0	0	1500	56206	0	0	1250
44090	0	750	0	56543	250	0	0
44091	0	0	1250	56607	250	0	0
44092	0	750	0	56616	250	0	0
44093	0	0	1250	70519	0	0	1500
44094	0	0	1000	70963	0	0	1000
52538	0	0	1000				

3.4 Model Validation and Verification

The new procurement list was proposed and accepted by Conways. This new policy will save R 63 833,50 per month, which will lead to an annual saving of R 766 002,00. Conways confirmed that these results are valid, given that the estimated cost savings was one million rand per year. The first aim of this project was to determine possible cost savings should Conways use the quantity discounts. These results indicate that the first phase of the project aim has been successfully completed.

In order to verify the results of the model, sensitivity analyses were conducted. Different model inputs were changed to test the sensitivity of the model outputs.

3.4.1 Demand Input

The total demand for each product was increased by 250kg and 500kg. The model outputs were recorded after every change. Figure 3.1 indicates how many products shifted to the next volume interval, as the demand increased.

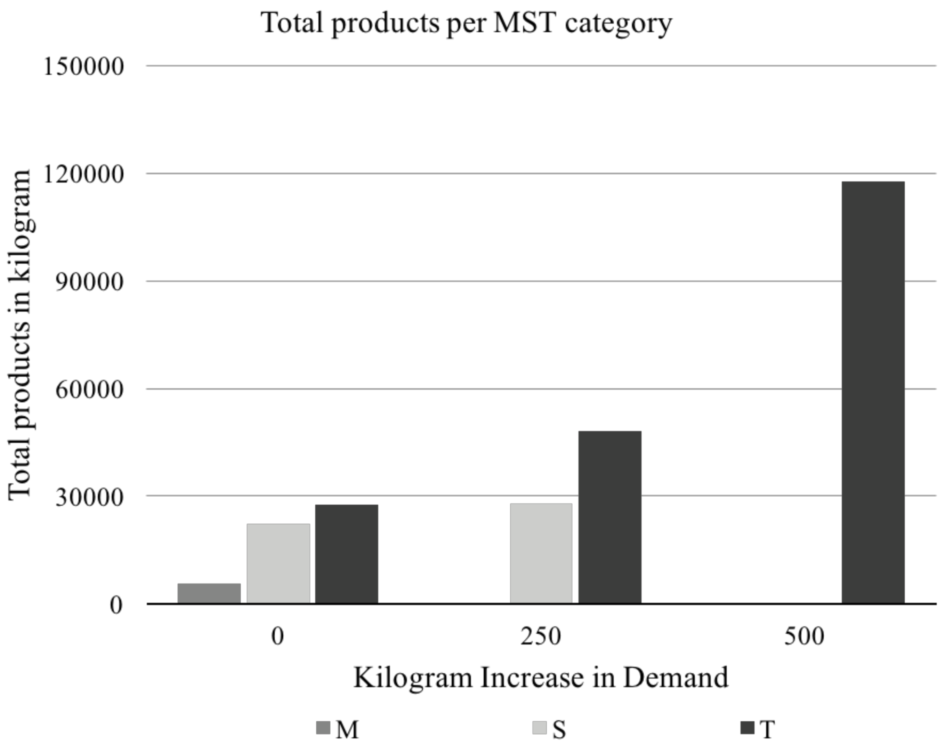

Figure 3.1: Total products to purchase per MST category.

The total demand was then decreased by 250kg and 500kg. The model outputs were recorded and the results are shown in Figure 3.2.

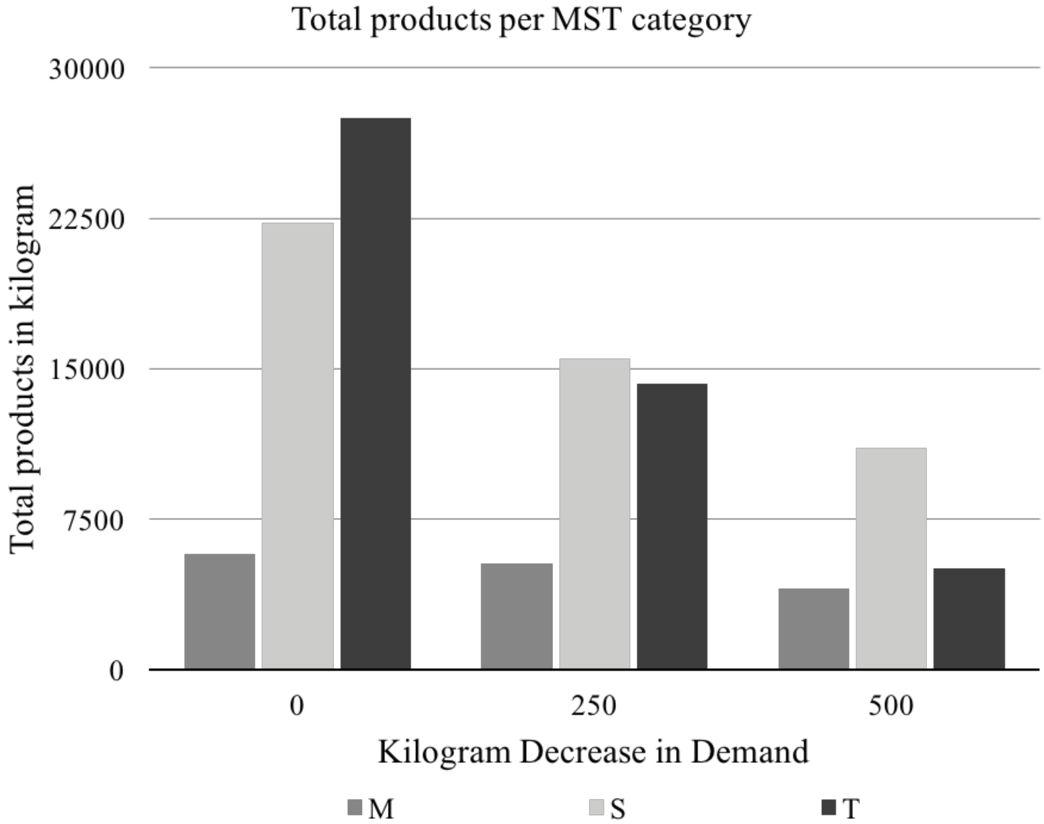

Figure 3.2: Total products to purchase per MST category.

The above graphs clearly show that as the demand increased, the number of products to purchase in the higher volume intervals increased, while the number in the lower intervals decreased. These results indicate that the model outputs are very sensitive to changes in the demand of products.

3.4.2 Category Boundaries Input

The upper and lower limit of each category of M, S and T were changed and the model was executed. The changes in the boundaries are shown in Table 3.3.

Table 3.3: Changes in upper and lower limits.

	M		S		T	
	Lower	Upper	Lower	Upper	Lower	Upper
Original	250	499	500	999	1 000	-
Change 1	250	749	750	1 249	1 250	-
Change 2	0	250	251	1 000	1 001	-
Change 3	0	250	251	749	750	-

The number of products to purchase under each category was recorded after every change. These results are available in Figure 3.3.

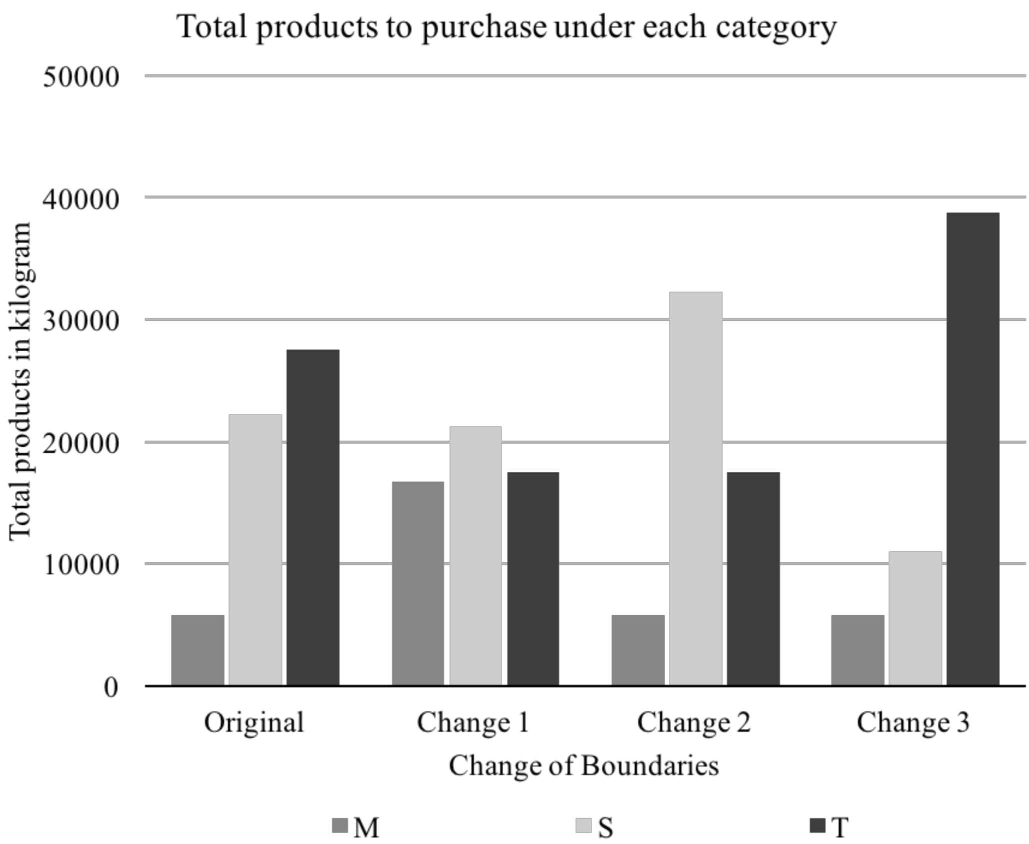

Figure 3.3: Total products to purchase per MST category.

The above figures indicates that as the upper and lower limits of every category changes, the number of products per category change. These results prove that the model is particularly sensitive to changes in the category boundaries.

3.4.3 Product Price Input

The price of every product under each category is calculated using a base price. The applicable percentage discount is subtracted from the given base price. The base price of every product was increased by different percentages to evaluate the effect on the model outputs. The total minimum monthly cost was recorded after every price change. These results are shown in Figure 3.4.

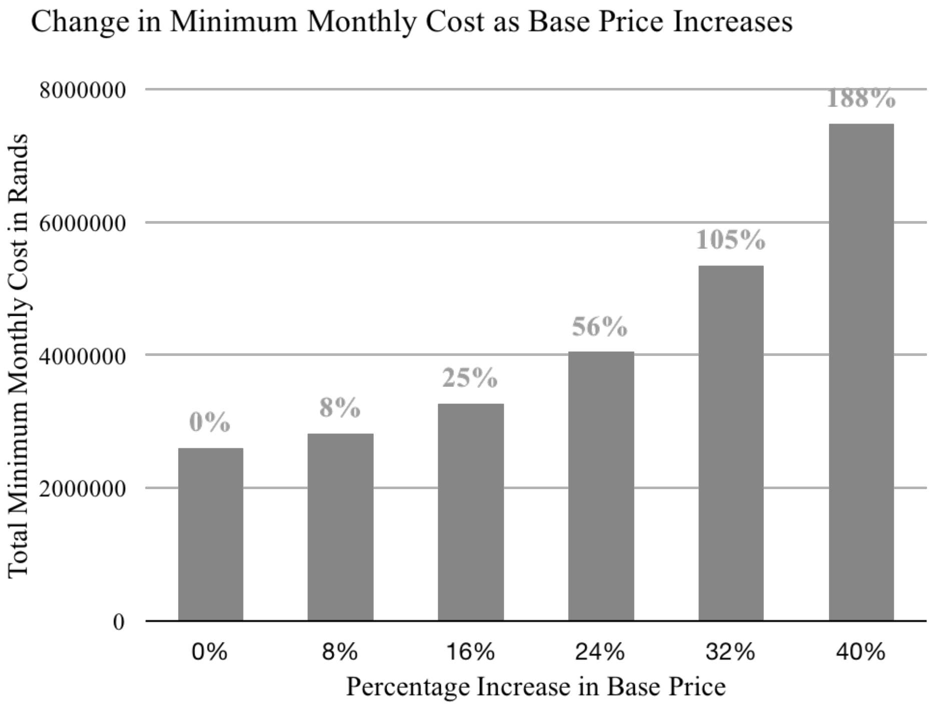

Figure 3.4: Change in Minimum Monthly Cost as the Base Price is Increased.

Figure 3.4 indicates that the minimum monthly cost increases as the base price increases. These results show that the model is highly sensitive to changes in the prices of the products.

3.5 Bi-Monthly Procurement System

Given that the demand for each product is only updated every two to three months, the possibility of purchasing bi-monthly was investigated. This procurement approach will increase the initial procurement costs, but may decrease the total annual costs.

The model was adapted to purchase bi-monthly by changing the demand to twice the amount. This change resulted in a bi-monthly total cost of R5 137 895,00 and an annual cost of R30 827 370,00. The additional annual savings of purchasing every second month is therefore R454 590,00.

3.6 Recommendations

Following the above analyses, it is recommended that Conways should combine the different warehouses' accounts and purchase the total demand under one account and according to the new policy presented in Table 3.2.

Furthermore, if Conways has the necessary cash flow and storage, it is recommended to purchase bi-monthly. This will allow Conways to purchase more products under the higher discount category, which leads to an annual saving of R 1 220 592,00.

Chapter 4

Facility Location Modeling

This chapter discusses the required inputs, the mathematical model, and the methods used to solve the facility location model. The p-median based model, as proposed by ReVelle and Swain [30], was used to formulate the location model. As discussed in the literature review in Chapter 2, an exact method, such as linear programming, is suitable to solve the location model. The solution verification and validation are also considered in this chapter.

4.1 Model Inputs

The model formulated is merely a simulation of reality and therefore has restrictions. The following assumptions were made:

- The demand of each node is constant
- The chosen candidate facility locations are available

The demand nodes refer to the five different warehouses of Conways as shown in Figure 1.1, Chapter 1. Each warehouse has a monthly demand, which is updated every two to three months. Conways provided the previous five sets of data. These data sets refer to the monthly demand as updated every other month, of which the average demand is used for this model. The demand, in units, for each warehouse is shown in Figure 4.1.

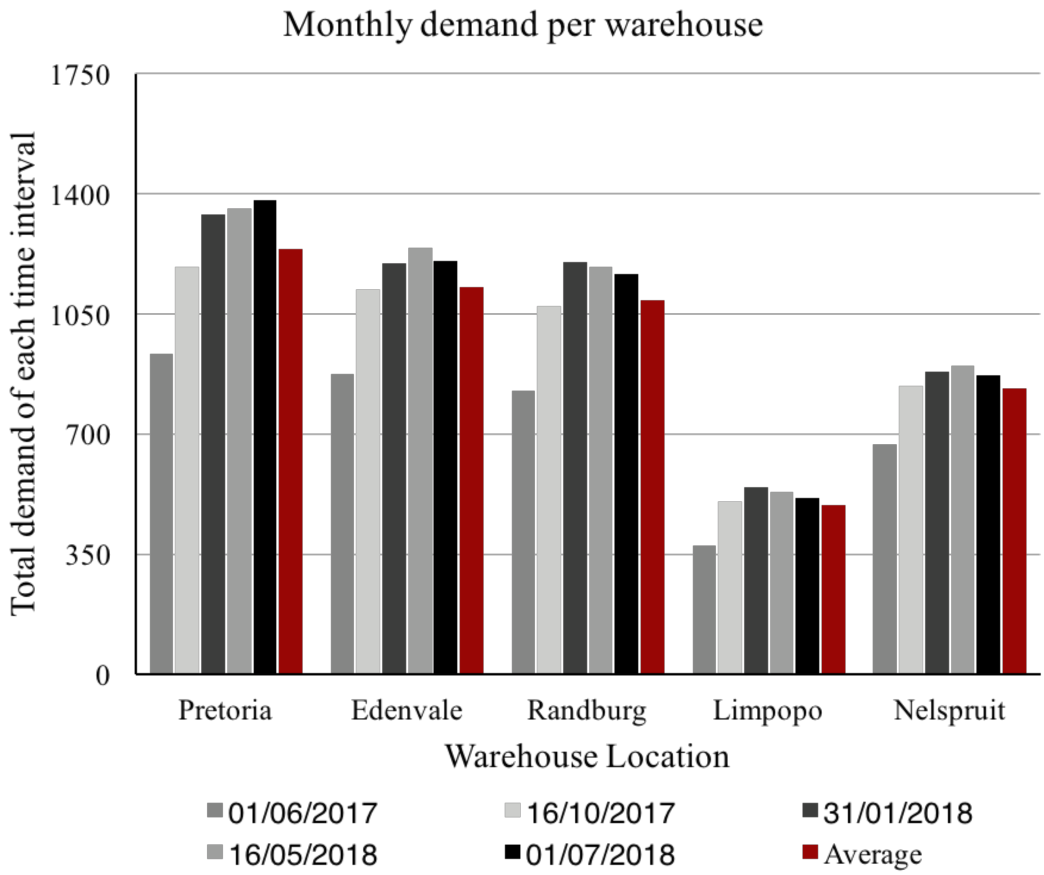

Figure 4.1: Total monthly demand per warehouse.

The candidate facility locations selected are widely distributed in order to ensure sufficient options for the model to choose from. Each candidate was selected to be in either an industrial area or a business park. Figure 4.2 indicates the candidate locations relative to the warehouses.

Figure 4.2: Distribution of candidate locations relative to the warehouses.

The last input variable required is the distance between each warehouse and each candidate location. Google Maps was used to obtain the driving distances in kilometres. Table 4.1 shows the distances between each warehouse and candidate facility location.

Table 4.1: Distance between each warehouse and each candidate location.

Warehouse	Candidates											
	1	2	3	4	5	6	7	8	9	10	11	12
Pretoria	78	67	50	39	6	112	99	153	211	308	279	269
Edenvale	18	14	31	37	67	156	126	184	255	331	303	313
Randburg	42	45	20	32	70	160	157	213	258	362	334	317
Limpopo	328	317	299	288	274	175	291	165	64	304	174	6
Nelspruit	343	321	350	339	302	413	212	229	357	3	152	301

4.2 Mathematical Model

The model, as discussed by Farahani [9] as well as ReVelle and Swain [30], was adapted and formulated.

Since only one facility is required for the distribution centre, p is set to one.

Then let:

$\boldsymbol{S}$ = the set of demand nodes i such that

$$\boldsymbol{S} = \begin{cases} 1 = & \text{Conways warehouse in Pretoria} \\ 2 = & \text{Conways warehouse in Edenvale} \\ 3 = & \text{Conways warehouse in Randburg} \\ 4 = & \text{Conways warehouse in Limpopo} \\ 5 = & \text{Conways warehouse in Nelspruit} \end{cases} \tag{4.1}$$

$\boldsymbol{G}$ = the set of candidate facility locations j such that

$$\boldsymbol{G} = \begin{cases} 1 = & \text{Droste Business Park, Johannesburg} \\ 2 = & \text{N12 Idustrial Park, Boksburg} \\ 3 = & \text{Kyalami Business Park, Midrand} \\ 4 = & \text{N1 Industrial Park, Pretoria} \\ 5 = & \text{N4 Gateway Industrial Park, Pretoria} \\ 6 = & \text{R101 Business Park, Bela-Bela} \\ 7 = & \text{Marelden Industrial Park, Emalahleni} \\ 8 = & \text{Bank Street, Groblersdal} \\ 9 = & \text{Old Industrial Area, Mokopane} \\ 10 = & \text{Riverside Industrial Park, Nelspruit} \\ 11 = & \text{Steelpoort Industrial Park, Randburg} \\ 12 = & \text{Platinum Park, Limpopo} \end{cases} \tag{4.2}$$

$$h_i \triangleq \text{the average demand of node } i, \ \forall\, i \in \boldsymbol{S} \tag{4.3}$$

$$d_{ij} \triangleq \text{the distance between the demand node } i \text{ and the candidate} \tag{4.4}$$

$$\text{facility location } j, \ \forall\, i \in \boldsymbol{S}, \ \forall\, j \in \boldsymbol{G} \tag{4.5}$$

$$X_{ij} = \begin{cases} 1, & \text{if the demand of node } i \text{ is covered by the facility at } j \\ 0, & \text{otherwise} \end{cases} \tag{4.6}$$

$$Y_j = \begin{cases} 1, & \text{if facility is located at node } j \\ 0, & \text{otherwise} \end{cases} \tag{4.7}$$

With the objective function:

$$\min Z = \sum_{i \in \boldsymbol{S}} \sum_{j \in \boldsymbol{G}} h_i d_{ij} X_{ij} \tag{4.8}$$

Subject to:

$$\sum_{j \in G} X_{ij} = 1 \qquad \forall i \in \boldsymbol{S} \tag{4.9}$$

$$\sum_{j \in G} Y_j = 1 \tag{4.10}$$

$$X_{ij} \leq Y_j \qquad \forall i \in \boldsymbol{S}, j \in \boldsymbol{G} \tag{4.11}$$

$$X_{ij}, Y_j \in \{0, 1\} \qquad \forall i \in \boldsymbol{S}, j \in \boldsymbol{G} \tag{4.12}$$

The objective function (4.8) minimises the total weighted average distance. Constraint (4.9) ensures that each demand node is only serviced by one facility and constraint (4.10) ensures that only one facility is chosen. Constraint (4.11) states that node i can be serviced by facility j. Constraint (4.12) defines the binary variables.

4.3 Model Results

The same Optimisation Modeling Software, LINGO, was used to program and solve the model. A linear program was developed to include the sets, variables, objective function, and constraints as defined in Section 4.2. The program was then executed to obtain an exact solution. After execution, a global optimal solution was found. The minimum total average distance is 515 103 km, with the optimal location at Candidate 4, N1 Industrial Park, Pretoria. The location of the selected candidate is indicated in Figure 4.3.

Figure 4.3: Location of the optimal Candidate.

4.4 Model Validation and Verification

The facility location model was presented to Conways and they were satisfied with the formulation thereof as well as the final optimal location. The results were also compared to the project aim. The second aim of this project was to obtain an optimal location for the distribution centre. The formulated model obtained an optimal location for the new distribution centre relative to the five warehouses. This model completed the second phase successfully.

The model was verified by means of a sensitivity analysis. The total monthly demand input of each warehouse was increased by different percentages. The model was executed after every increase and the minimum average distance was recorded. Figure 4.4 indicates these results. The percentage change in output was also added.

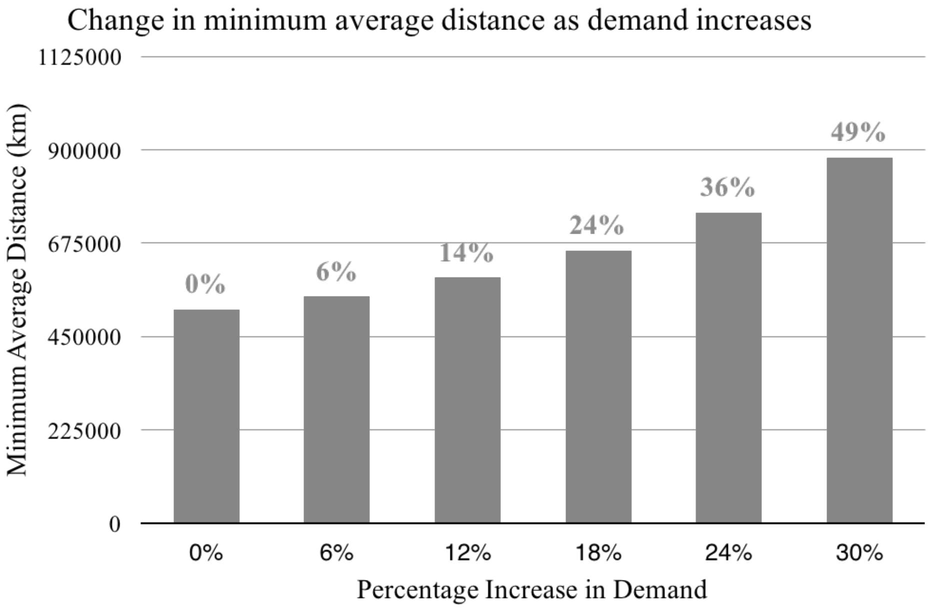

Figure 4.4: Change in minimum total average distance if demand increases.

The total monthly demand was then decreased and the model outputs were recorded. These results are shown in Figure 4.5, which includes the percentage change in output as well.

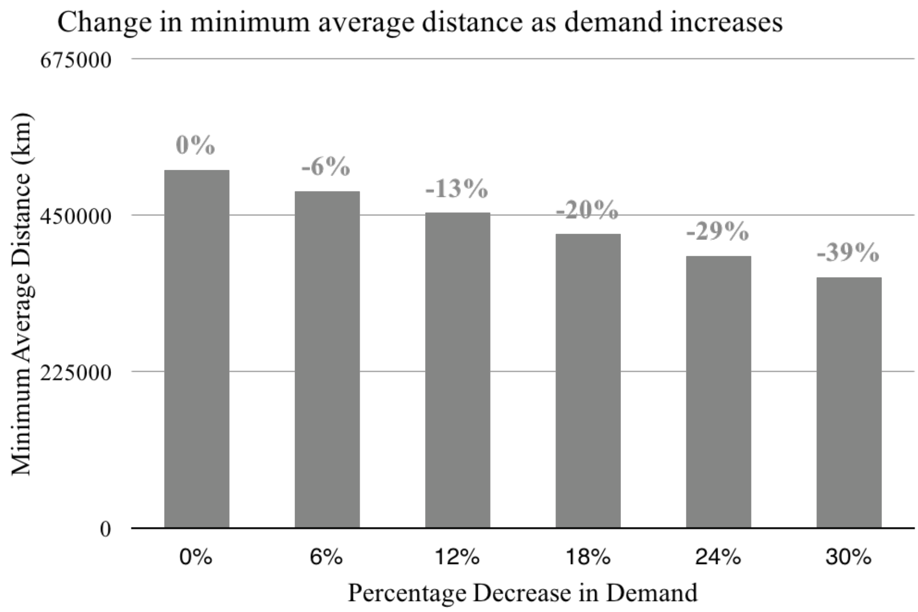

Figure 4.5: Change in minimum total average distance if demand decreases.

From the above figures, it can be seen that the minimum average distance increases as the demand of each warehouse was increased, and decreases as the demand decreased. The model output was not extremely sensitive for the change in demand input, up until the 12% increase or decrease mark. The model was more sensitive for any increase or decrease above 12%.

4.5 Pretoria Centered Location Model

The global optimal solution selected by the model, Candidate 4, was removed from the candidate locations and the model was executed. The new optimal solution chosen was Candidate 5, N4 Gateway Industrial Park, Pretoria. Given that both preferred locations are situated in Pretoria, it is probable that Pretoria is the optimal city for a new distribution network.

Further investigation into different locations in Pretoria was conducted. Seven new candidate locations were chosen to be in Industrial Parks in the Pretoria region. These locations are shown in Figure 4.6.

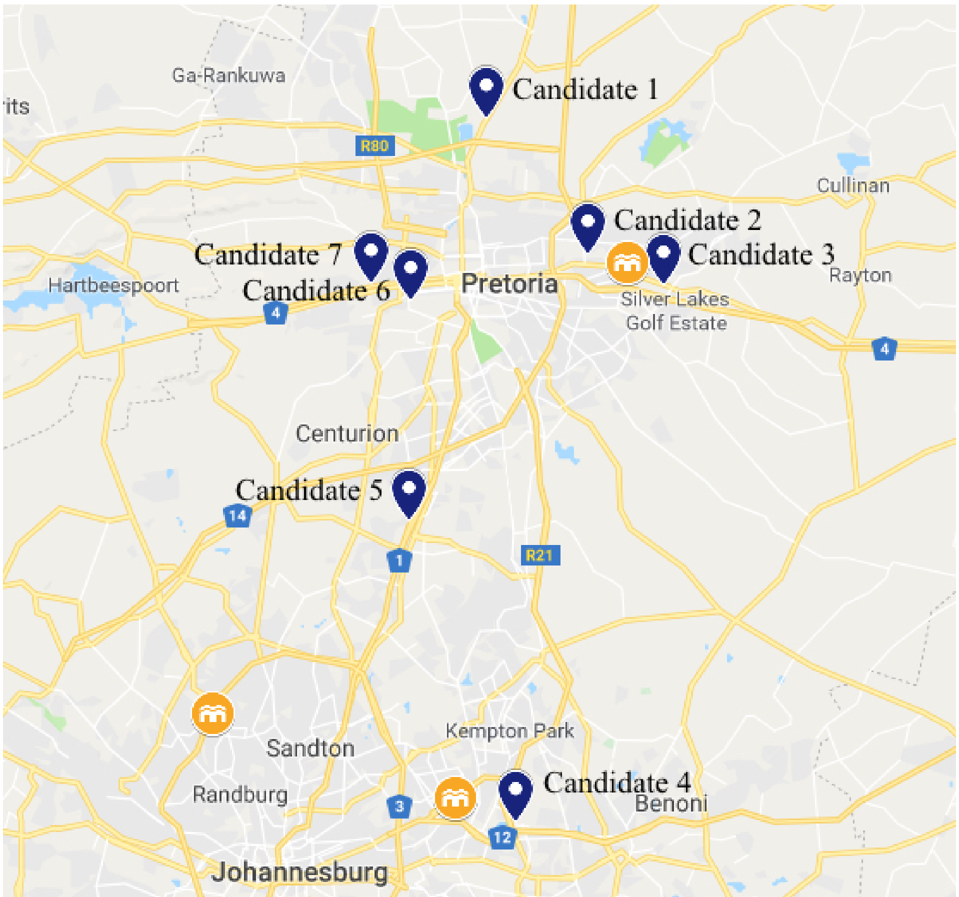

Figure 4.6: Locations of the new Candidates.

The same mathematical model was used as defined in Section 4.2. The set of candidate facility locations $\boldsymbol{G}$ was modified to reflect the new candidates.

$$\boldsymbol{G} = \begin{cases} 1 = & \text{Wonderboom Industrial Park} \\ 2 = & \text{Tannery Industrial Park} \\ 3 = & \text{N4 Gateway Industrial Park} \\ 4 = & \text{Anchor Industrial Park} \\ 5 = & \text{N1 Industrial Park} \\ 6 = & \text{50 Delfos Road, Industrial Park} \\ 7 = & \text{Industrial Park, Pretoria East} \end{cases} \tag{4.13}$$

The distance variable, d_{ij}, was also changed to include the new distances between each warehouse and new candidate. The new distances are indicated in Table 4.2.

Table 4.2: Distance between each warehouse and each new candidate location.

Warehouse	Candidates						
	1	2	3	4	5	6	7
Pretoria	31	5	5	44	32	23	27
Edenvale	74	61	67	12	38	58	64
Randburg	78	64	70	44	32	53	59
Limpopo	253	261	272	312	289	287	280
Nelspruit	334	313	305	324	341	326	330

4.5.1 Results

The changed model was executed and a new global optimal solution was found. The optimal location is Candidate 2, Tannery Industrial Park, with a total minimum weighted distance of 502 560 km. This weighted distance is 12 543 km less than the previous optimal location, the previous Candidate 4, situated at the N1 Industrial Park.

The new optimal location is therefore situated at Tannery Industrial Park, Pretoria.

Chapter 5

Conclusion

This project was the initial phase of Conways' goal to improve their procurement system. Currently, they are not using the TQD offered by Wispeco to their advantage. Conways was interested in knowing the possible cost savings of using the TQD. In order to do so, the five warehouses should combine their accounts. One account would allow Conways to order a larger quantity of products at once and therefore obtain greater discounts. The best combination of purchases was identified through mathematical programming. This formulation indicated the number of products Conways should purchase to achieve the lowest cost.

The new procurement system will allow Conways to save R 766 002,00 per year. Should Conways change this new system to purchase bi-monthly, they will save R 1 220 592,00 annually.

An additional warehouse is required to create a connection point between the bulk purchases from Wispeco and the five warehouses of Conways. An optimal location for the new distribution centre was required to reduce the total average distance. A facility location model was formulated to identify the best locations. The results will assist Conways in the decision making of, if and where the new facility should be located. The recommended location is Tannery Industrial Park, Pretoria.

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Final Year Project Mentorship Form
2018

Introduction

An industry mentor is the key contact person within a company for a final year project student. The mentor should be the person that could provide the best guidance on the project to the student and is most likely to gain from the success of the project.

The project mentor has the following important responsibilities:

1. To select a suitable student/candidate to conduct the project.
2. To confirm his/her role as project mentor, duly authorised by the company by signing this **Project Mentor Form**. Multiple mentors can be appointed, but is not advised.
3. To ensure that the **Project Definition** adequately describes the project.
4. To review and approve the **Project Proposal**, ensuring that it clearly defines the problem to be investigated by the student and that the project aim, scope, deliverables and approach is acceptable.
5. To review and approve all subsequent project reports, particularly the **Final Project Report** at the end of the second semester, thereby ensuring that information is accurate and the solution addresses the problems and/or design requirements of the defined project.
6. Ensure that sensitive confidential information or intellectual property of the company is not disclosed in the document and/or that the necessary arrangements are made with the Department regarding the handling of the reports.

Project Mentor Details

Company:	**Conways - Aluminium Profile Stockists**
Project Description:	**Feasibility study and strategic location modelling of a centralised distribution centre for the Conways stockist group**
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