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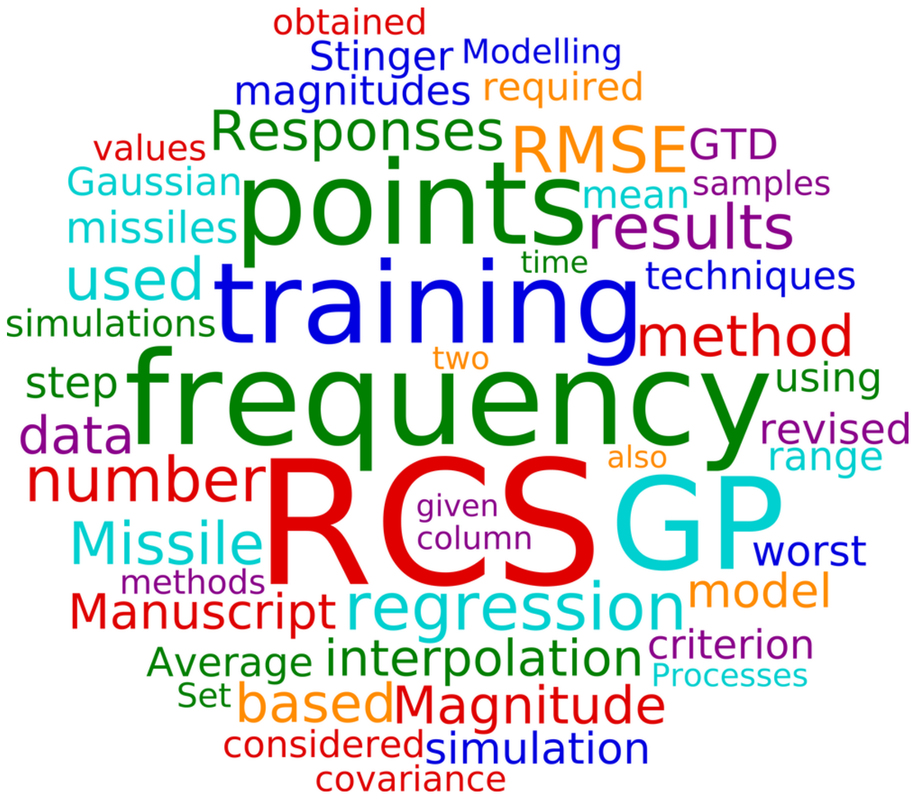

Abbreviations

GP	Gaussian process
GTD	geometric theory of diffraction
JSR	jammer-to-signal ratio
NRF	National Research Foundation of South Africa
RCS	radar cross section
RMSE	root-mean-square error
SE	squared-exponential
SVM	support vector machine
SVR	support vector regression

Efficient Modelling of Missile RCS Magnitude Responses by Gaussian Processes

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***Abstract*—An efficient technique for modelling radar cross section (RCS) magnitude responses versus frequency is presented. The technique is based on Gaussian process (GP) regression and makes it possible to significantly reduce the number of expensive computer simulations required to accurately resolve these responses. Examples of two missiles are used to evaluate the proposed technique. Average predictive normalized root-mean-square errors (RMSEs) of 1.24% and 1.63% were obtained, with the worst RMSE being less than 2.2%. These results were significantly better than results obtained with alternative techniques, including geometric theory of diffraction (GTD)-based modeling and support vector regression.**

***Index Terms*—Gaussian processes, radar cross section (RCS), and modelling.**

I. Introduction

THE radar cross section (RCS) magnitude of a platform is the main factor in the radar-range equation which is not under the control of a radar [1], [2]. As a result, RCS magnitude can be the primary consideration which drives other radar-design decisions [3]. RCS magnitude affects how easily a platform can be detected and influences countermeasure performance via the jammer-to-signal ratio (JSR). The development of stealthy platforms, which primarily aim to reduce RCS magnitude, emphasises the importance of RCS.

RCS varies rapidly with frequency [1], [2], so large numbers of simulations are required to determine the RCS magnitude of a target. As a result, only the RCS magnitude averaged over frequency is specified in many cases, with [3] only providing seven RCS values from 35 MHz to 9 GHz for the missile type considered here, despite the significant variations with frequency shown in simulations below. Most targets of interest are large in terms of wavelength, meaning that RCS simulations are extremely time and memory intensive, even when high-frequency techniques are used [2].

In this letter, the use of Gaussian process (GP) regression [4], [5] to reduce the number of points at which the RCS magnitude must be determined via simulation is demonstrated. A GP model employing a composite covariance function intended for modelling quasi-periodic responses [4], [6] is shown to accurately predict RCS magnitudes despite the complicated frequency responses of the targets. GP regression is shown to significantly outperform the generic, machine-learning, and physics-based interpolation techniques considered.

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II. Modelling with Gaussian Processes

The roots of GP regression lie in Bayesian statistics; the main implementation steps are summarized below, following [4].

Suppose a training set $\{(x_i, g_i)|i = 1, \ldots, n\}$ of a limited number of noise-free observations of the latent function is available (in the present work, the scalar inputs x_i are frequency values, and the scalar outputs g_i are the corresponding RCS magnitude values; the latent function is the full RCS response). It is desired to predict values of the latent function for new (i.e. test) inputs $\{x_{*,j}|j = 1, \ldots, n_*\}$ (i.e. frequency values not included among the training inputs). First, a jointly Gaussian prior distribution is constructed over the n training outputs (contained in vector $\mathbf{g}$) and the n_* unknown test outputs ($\mathbf{g}_*$):

$$\begin{bmatrix} \mathbf{g} \\ \mathbf{g}_* \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} K(\mathbf{x},\mathbf{x}) & K(\mathbf{x},\mathbf{x}_*) \\ K(\mathbf{x}_*,\mathbf{x}) & K(\mathbf{x}_*,\mathbf{x}_*) \end{bmatrix}\right). \quad (1)$$

In (1), $\mathbf{x}$ and $\mathbf{x}_*$ are vectors containing all training and test inputs respectively; $K(\mathbf{x},\mathbf{x}_*)$ is an $n \times n_*$ matrix containing the covariances between all pairs of training and test outputs (other sub-matrices $K(\cdot)$ are similarly defined); and $N(\mathbf{u}, V)$ denotes a multivariate normal distribution with mean vector $\mathbf{u}$ and covariance matrix V.

The posterior distribution, given by $\mathbf{g}_*|\mathbf{x}_*, \mathbf{x}, \mathbf{g} \sim N(\mathbf{m}, \Sigma)$ [4], is obtained by conditioning the test outputs on the known training outputs $\mathbf{g}$. The posterior mean $\mathbf{m}$ is given by

$$\mathbf{m} = K(\mathbf{x}_*,\mathbf{x}) K(\mathbf{x},\mathbf{x})^{-1} \mathbf{g} \quad (2)$$

which contains the most probable RCS magnitudes associated with the test frequencies in $\mathbf{x}_*$.

Elements of the covariance matrices in (1) and (2) are given by the covariance function. For example, the squared-exponential (SE) covariance function is a standard covariance function previously used for antenna modelling [5]: $k_{SE}(x, x') = \sigma_f^2 \exp(-0.5r^2)$, where σ_f^2 governs the variance of the process, and $r^2 = (x - x')^2/\tau^2$ where τ is a positive length-scale parameter [4].

Training in GP regression entails determining the hyperparameters of the covariance function (e.g. σ_f^2 and τ above) that

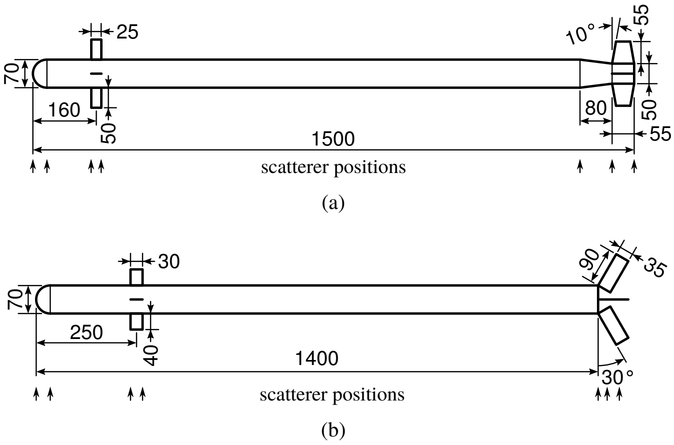


Fig. 1. The dimensions of the (a) Stinger and (b) Strela-2 missile models in mm with the positions of the main scatterers indicated.

minimise the negative log marginal likelihood. In the general GP regression formulation, observations are assumed to be noisy, i.e. of the form $y(x) = g(x) + \eta(x)$, with $g(x)$ being the latent function and $\eta(x)$ representing zero-mean Gaussian noise with variance σ_n^2. The negative log likelihood is then given by [4]

$$\log\left[p\left(\mathbf{y}|\mathbf{x}\right)\right] = -0.5\mathbf{y}^T\left(K+\sigma_n^2 I\right)^{-1}\mathbf{y} - 0.5\log\left|K+\sigma_n^2 I\right| - 0.5n\log\left(2\pi\right) \quad (3)$$

where $\mathbf{y}$ is a vector containing the n training outputs; K contains the covariances between all possible pairs of training outputs calculated using the covariance function; $|\cdot|$ denotes the determinant of a matrix; and I is the identity matrix. In the present work, σ_n was initialized to a very small value ($\approx 10^{-6}$) to reflect the assumption of noise-free RCS observations, so after training, RCS predictions at training frequencies coincided with the training values.

Standard covariance functions such as the SE function are not sufficiently expressive to model the complex RCS magnitude responses of Section III [6]. Instead, a composite covariance function of the form $k(x,x') = k_{SE}(x,x') \times k_{PER}(x,x')$ [4] can be used, with k_{SE} as given above, and $k_{PER}(x,x')$ being a periodic covariance function used to model one-dimensional functions that comprise exact repetitions of a basic function [4]:

$$k_{PER}(x,x') = \exp\left[\left(-2\sin^2\left[\pi\left|x-x'\right|/\lambda\right]\right)/\theta^2\right]. \quad (4)$$

In (4), the hyperparameter λ determines the intervals between repetitions, and θ is a length-scale parameter.

III. Verification Examples

Missile RCS magnitude responses relevant to a real-world application were considered [3]. Fig. 1 shows models of the FIM-92 Stinger and 9K32 Strela-2 (SA-7 Grail) derived from information freely available on the internet. Frequencies from 5 to 15 GHz were considered in 10-MHz steps for a total of 1 001 data points per missile.

FEKO release 2017.1 [7] was used to simulate the two targets on axis from the front (i.e. left to right in Fig. 1) as these missiles will fly straight towards the most vulnerable targets (helicopters and aircraft during take-off and landing). Both electrical and magnetic symmetry were used to reduce simulation run times and memory requirements. Run times were further reduced by separately meshing the targets at each simulation frequency, with the maximum triangle edge length being specified as a fifth of a wavelength. High-frequency techniques cannot be used here because the main scatterers (see Fig. 1) are smaller than five wavelengths [2], even at the highest frequency.

The small size of the targets allows full-wave simulations to be performed at high frequencies. Despite the small target sizes, a grand total of 301 hours of simulation time and a maximum of 16.8 GB of memory were required to simulate both missiles on a computer with a six-core 2.30-GHz processor.

Twenty sets of training data were compiled for each missile to verify that results are not sensitive to the specific training-data configuration. For each set, the interval of 5 to 15 GHz was divided into 164 equal sub-intervals, with one frequency point being uniformly randomly selected from each subinterval. If absent, the extreme values (5 and 15 GHz) were added for a maximum of 166 points. This method of selection ensured that points were not clumped together, but spread over the entire frequency range as shown in Fig. 2. The average simulation time for each set of training data was 25 hours, so the total simulation time (on average) for both missiles' training data was 50 hours.

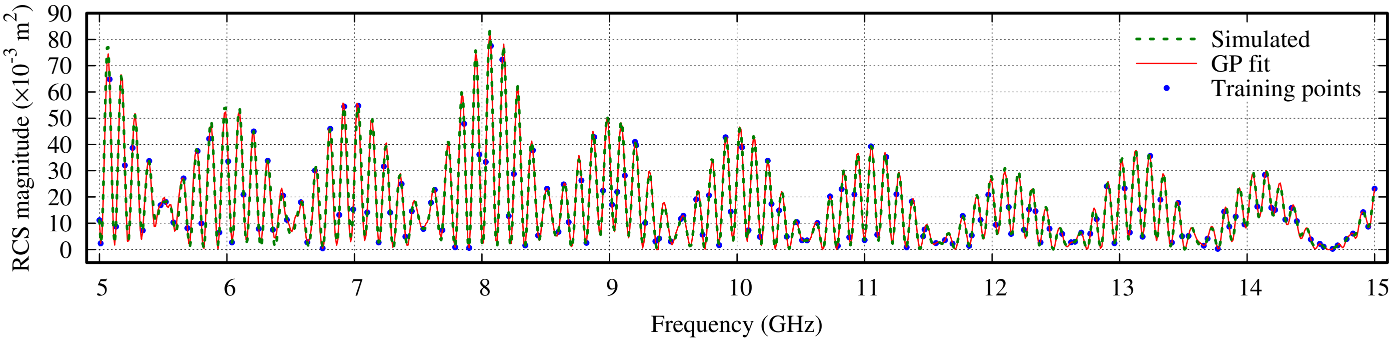
The missiles are approximately 1.5 m long, and only the magnitude of the RCS is considered. A frequency step of 50 MHz or less is thus necessary to ensure that the nose and tail of a missile can be resolved [2]. Stated differently, the fastest variations in the data have a period of approximately 100 MHz, so the Shannon-Nyquist criterion requires at least two samples over this interval [8]. This estimate ignores higher-order effects, so a smaller frequency spacing is actually required. The use of 164 to 166 frequency points is thus based on a frequency spacing which is 20% larger than required by the Shannon-Nyquist criterion.

Training consisted of numerical optimization of (3). The problem is multi-modal, so it was necessary to consider multiple sets of random hyperparameter starting values to ensure that good results were obtained (300 sets were used). The results for the set that gave the lowest negative log-likelihood are reported. This automatic procedure took 6 to 13 minutes per GP model, so this time was negligible compared to the simulation time.

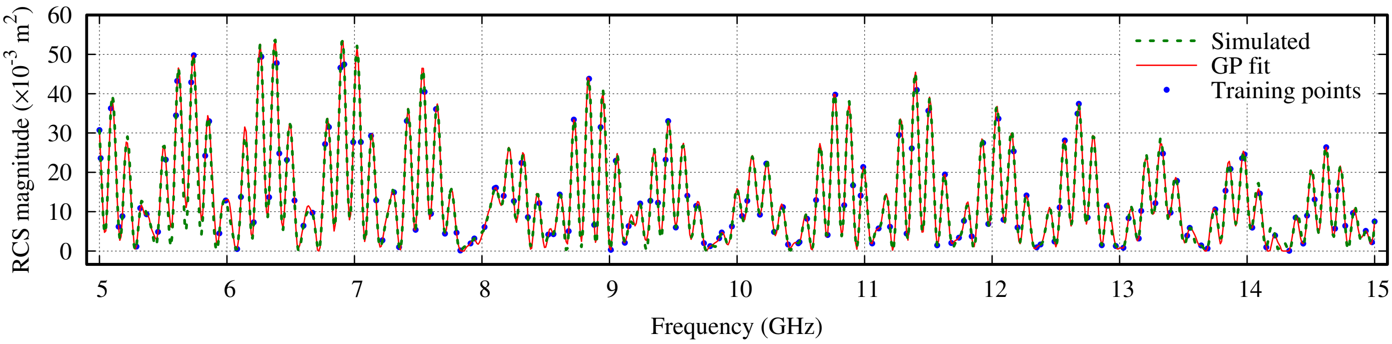
The root-mean-square errors (RMSEs) for each model were computed from

$$\mathrm{RMSE} = \frac{\sqrt{\sum_{l=1}^{n_*}\left[\mathrm{RCS}_p\left(l\right) - \mathrm{RCS}_s\left(l\right)\right]^2}}{\max\limits_{l}\ \mathrm{RCS}_s\left(l\right)} \quad (5)$$

where $\mathrm{RCS}_p(l)$ and $\mathrm{RCS}_s(l)$ are the predicted and simulated linear RCS magnitudes at frequency index l respectively. GP regression ensures that the errors at the training points are negligible, so all results reported below consider the RMSEs



(a)



(b)

Fig. 2. The worst GP results for the (a) Stinger and (b) Strela-2 compared to the simulated (true) RCS magnitude.

of only the test points (the subsets of the 1 001 available points remaining after the training points were removed).

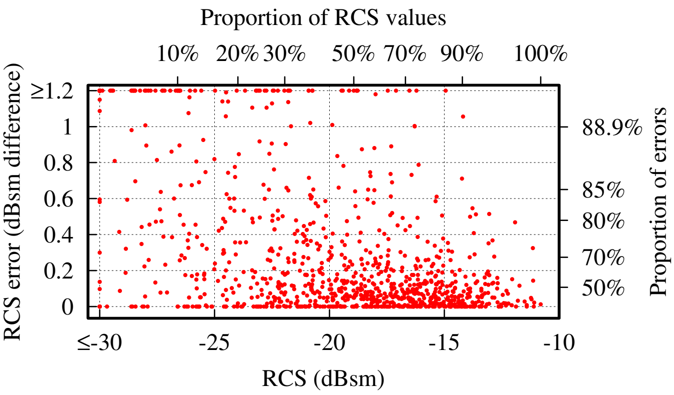
The simulated responses, shown for each missile in Fig. 2, are challenging to model as they contain quasi-periodicities at a minimum of two levels: a rapidly-varying oscillation and a slowly-varying envelope which has no clear pattern. Fig. 2 also shows the GP fit for each missile, specifically the result with the worst RMSE for the 20 training-data sets. While the predicted RCS magnitudes have slight errors at some frequencies, the detailed structure of both responses is correctly predicted. GP regression is thus shown to accurately model the variation of the RCS magnitudes with respect to frequency for the missile models considered.

The accuracy of GP regression is explored further in Fig. 3, where the RCS and error magnitudes are compared. The results with the worst RMSEs from the 20 training data sets are again reported to highlight how well the GP approach performs in all cases. The right scales of Fig. 3 show that the error is less than 1 dB for 88.9% and 86.5% of the 1 001 frequency points for the Stinger and Strela-2 models respectively. Fig. 3 also shows that larger RCS magnitudes tend to have smaller errors than smaller RCS magnitudes. Less than 1.0% and 2.6% of the largest 50% of the 1 001 RCS magnitudes exceed a 1-dB error in the two cases, and fewer than 1.2% and 2.1% of the points which are within 10 dB of the maximum RCS magnitude for each missile have an error of greater than 1 dB. The mean is most strongly influenced by large values, so errors in the larger RCS magnitudes will have a greater effect on the average RCS magnitudes used for system design.

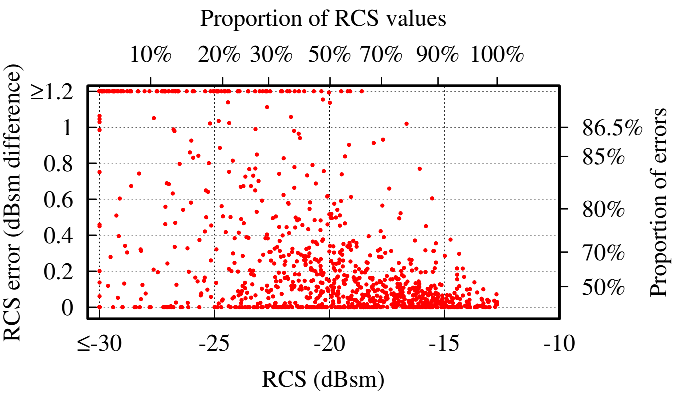
The RMSEs obtained using GP regression and three other modelling techniques are compared in Table I for the same 20 sets of training points. Even the best results for cubic-spline interpolation, a generic interpolation technique, were poor (RMSEs of 18.8% and 17.5% for each missile respectively). Support vector regression (SVR), a kernel-based machine learning method often used for microwave modelling (e.g. [9]), yielded best results when the epsilon-insensitive band was set to 2% of the maximum RCS magnitude in the training set (the penalty parameter and the scaling parameter of the Gaussian kernel were optimized using a grid search with cross-validation). While the SVR RMSEs improved on the cubic-spline results, they still were poor, and depended strongly on the training point configurations. Of the three alternative techniques, a physics-based interpolation technique based on geometric theory of diffraction (GTD) [10] performed best. The associated scatterers were initially positioned as shown in Fig. 1, after which the model parameters were numerically

TABLE I
RMSEs OF TEST DATA

Model		Best	Median	Worst	Mean
Stinger	GP	0.88%	1.25%	1.68%	1.24%
	Spline	18.8%	19.8%	22.2%	19.9%
	SVM	12.5%	13.6%	15.5%	13.7%
	GTD	4.36%	4.82%	12.6%	5.46%
Strela-2	GP	1.32%	1.54%	2.11%	1.63%
	Spline	17.5%	19.0%	23.2%	19.2%
	SVM	12.4%	14.2%	16.4%	14.2%
	GTD	7.48%	14.8%	16.6%	12.8%



(a)



(b)

Fig. 3. The relationship between the error and simulated (true) RCS magnitudes for the worst (a) Stinger and (b) Strela-2 GP models.

optimised to minimise the RMSE. The results obtained by the GTD-based model depend strongly on the training-point configurations, and even the best results obtained are notably poorer than the worst GP results.

IV. Conclusion

Many applications require accurate RCS magnitude information, but the complexity of the responses means that simulations at large numbers of frequencies are required. GP regression has been shown to accurately model the RCS magnitudes of missiles using a frequency step which is 20% larger than required by a first-order estimate of the Shannon-Nyquist rate.

The GP-regression models are extremely accurate and consistent despite the number of simulated frequency points being below the Shannon-Nyquist limit. Significantly, even the worst GP results substantially improved upon the best results obtained by other techniques.

References

[1] F. Neri, *Introduction to Electronic Defense Systems*, 2nd ed. Rayleigh, NC: SciTech, 2006.

[2] E. Knott, J. Schaeffer, and M. Tulley, *Radar Cross Section*, 2nd ed. Rayleigh, NC: SciTech, 2004.

[3] A. A. Black, "Pulse Doppler for missile approach warning," *J. Electron. Defence (JED)*, vol. 14, no. 8, pp. 44–52, Aug. 1991.

[4] C. E. Rasmussen and C. K. I. Williams, *Gaussian processes for machine learning*. Cambridge, MA: MIT Press, 2006.

[5] J. P. Jacobs and S. Koziel, "Two-stage framework for efficient Gaussian process modeling of antenna input characteristics," *IEEE Trans. Antennas Propag.*, vol. 62, no. 2, pp. 706–713, Feb. 2014.

[6] J. P. Jacobs and D. I. L. de Villiers, "Gaussian process modeling of aperture efficiency ripple in reflector antennas," in *Loughborough Antennas Propag. Conf. (LAPC)*, Loughborough, UK, 2-3 Nov. 2015, pp. 1–4.

[7] *FEKO User Manual*, Altair Hyperworks, 2017, release 2017.1.

[8] B. P. Lathi and Z. Ding, *Modern digital and analog commuication systems*, 4th ed. New York, NY: Oxford University Press, 2010.

[9] G. Angiulli, M. Cacciola, and M. Versaci, "Microwave devices and antennas modelling by support vector regression machines," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1589–1592, Apr. 2007.

[10] L. C. Potter, D.-M. Chiang, R. Carriere, and M. J. Gerry, "A GTD-based parametric model for radar scattering," *IEEE Trans. Antennas Propag.*, vol. 43, no. 10, pp. 1058–1067, Oct. 1995.