Modelling Long Memory Volatility in the Bitcoin Market: Evidence of Persistence and Structural Breaks

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ABSTRACT

Motivated by the emergence of Bitcoin as a speculative financial investment, the purpose of this paper is to examine the persistence in the level and volatility of Bitcoin price, accounting for the impact of structural breaks. Using parametric and semiparametric techniques, we find strong evidence in favour of a permanency of the shocks and lack of mean reversion in the level series. We also reveal evidence of structural changes in the dynamics of Bitcoin. After accounting for the structural breaks in the level series, evidence of mean reversion is uncovered in some cases. Further analyses show evidence of a long memory in the two measures of volatility (absolute and the squared returns), whereas some cases of short memory are revealed in the squared returns series in particular. Practical implications are discussed on the inefficiency in the Bitcoin market and its importance for Bitcoin users and investors.

Keywords: Bitcoin; Long memory; Structural Breaks

JEL Codes: C22; G1

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1. Introduction

The inception of digital currencies or cryptocurrencies, derived from mathematical cryptography[1], have attracted the attention of the media and economic actors. They represent both a valid form of payment and an alternative to governments-backed currencies. Among many cryptocurrencies in existence such as Litcoin, Ethereum, Ripple, Peer coin, Ripple, and Dogecoin, Bitcoin in particular has emerged and taken an increasingly prominent place in the cryptocurrency markets. As of May 31, 2016, the market capitalization of Bitcoin exceeds 7.00 billion US dollars, constituting more than 90% of the total cryptocurrency capitalisation (https://coinmarketcap.com).

While the literature on the legal aspect, regulation and role of Bitcoin in the payment system is growing (Brito et al., 2014; Trautman et al., 2016), the finance and economics of Bitcoin remain understudied. Rogojanu and Badea (2014) compare Bitcoin to other alternative monetary systems, presenting its pros and cons. Selgin (2105) indicates that Bitcoin has quite similar characteristics to commodity money. Brandvold et al. (2015) and Bouoiyour et al. (2016) explore the contribution of Bitcoin exchanges to price discovery. Ciaian et al. (2016) focus on Bitcoin price formation and market forces of supply/demand in the cryptocurrency markets. Some studies refer to Bitcoin as digital gold (Rogojanu and Badea, 2014; Popper, 2015). While others highlight the diversification benefits of adding Bitcoin to an equity portfolio (Eisl et al., 2015). Dyhrberg (2015) examine the relation between Bitcoin and the UK currency and stock markets.

Independent whether Bitcoin is regarded as a form of payment, an alternative/digital form of a currency, or a financial asset and speculative investment (Yermack, 2013; Bouoiyour and Selmi, 2015; Bouoiyour et al., 2015), the literature on the finance of Bitcoin has to be extended in particular with regards to the analysis of persistence in the

[1] Dwyer (2014) provides a detailed explanation of the principles of Bitcoin.

level and in the volatility of Bitcoin price. The aim of this paper is therefore to contribute to the emerging literature on the economics and finance of Bitcoin through modelling of persistence in the level and in the volatility of Bitcoin.

Such an analysis to whether or not long-term dependence is present in Bitcoin return series could answer a main question of whether or not the Bitcoin market is efficient. This is because the presence of long memory in the high order of Bitcoin series means the statistical dependence between distant observations of a price series are not decreasing very rapidly. Therefore, the presence of persistent dependence between distant Bitcoin observations represents significant evidence against the efficient market hypothesis or random walk model in the Bitcoin market. In this sense, conducting such an analysis is important for Bitcoin users and investors who are both concerned about managing the risk associated with sharp changes in Bitcoin prices. Put differently, evidence of persistence in Bitcoin prices, either with mean reverting behaviour or long memory returns, implies the presence of a predictable component in the dynamics of Bitcoin prices which raises issues regarding tests of efficiency. More practically, users and investors can profit from any inefficiencies in the Bitcoin market to improve the risk-adjusted return (Gil-Alana et al., 2014). Furthermore, the ability of forecasting Bitcoin volatility has important application for hedging (Efimova and Serletis, 2014). Lastly, the recent acceptance of Bitcoin as a commodity and financial product by the US Commodity Futures Trading Commission (CFTC) makes the volatility dynamic of Bitcoin to play a crucial role in any potential derivatives pricing and trading.

However, modelling price volatility and its persistence must account for non-linearity and structural breaks especially with evidence of booms and busts in the Bitcoin market (Cheah and Fry, 2015). Otherwise, if structural breaks such as the Bitcoin price crash of December 2013 have not been addressed properly, the power of the modelling will be very low and the inferences will be invalid (Harris and Sollis, 2005).

The analyses are carried in several phases. First, we focused on the entire sample using parametric and semiparametric approaches. Overall, we found evidence of permanency of shocks and lack of mean reversion. Second, we argued that the presence of long memory can be spurious in the presence of structural breaks. Therefore, we identified several structural breaks in line with Bai and Perron's (2003) method and accordingly investigated the volatility of the subseries. We found evidence of long memory in Bitcoin price volatility. Finally, we focused on the volatility series by looking at the absolute and squared return data and find evidence of a long memory in most of the subseries.

The rest of the paper is divided into three sections. The data and empirical model are presented. They are followed by the empirical results. Finally, concluding remarks are given.

2. Data and methodology

2.1 Data

Unlike prior studies, this paper uses two daily series of Bitcoin prices covering two different time periods. The first data series is from Bitstamp, the largest Bitcoin exchange (Brandvold et al., 2015), covering the period from August 19, 2011 to April 29, 2016 (1226 observations). The second data series is the Coindesk Price Index (Cheah and Fry, 2015) from July 18, 2010 to December 15, 2015 (1977 observations). Given the existence of several Bitcoin exchanges around the globe, we choose to analyze the Coindesk Price Index which represents an average of Bitcoin prices across

Bitcoin exchanges. Both data are in US and depicted by data availability. Figures 1 and 2 plot the level of Bitcoin prices from the two series.

Figure 1: The Bitsmap Price Index of Bitcoin

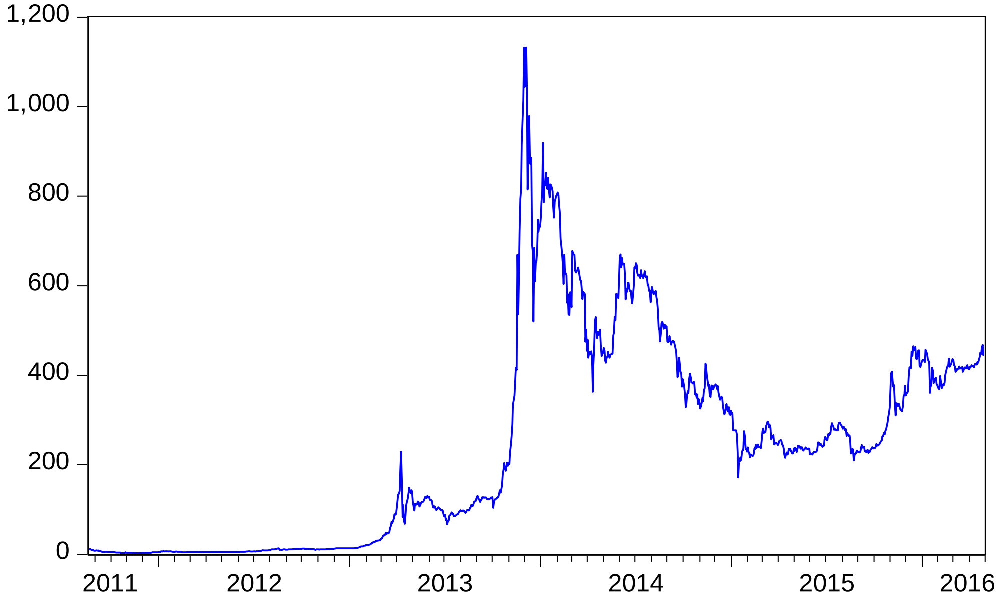
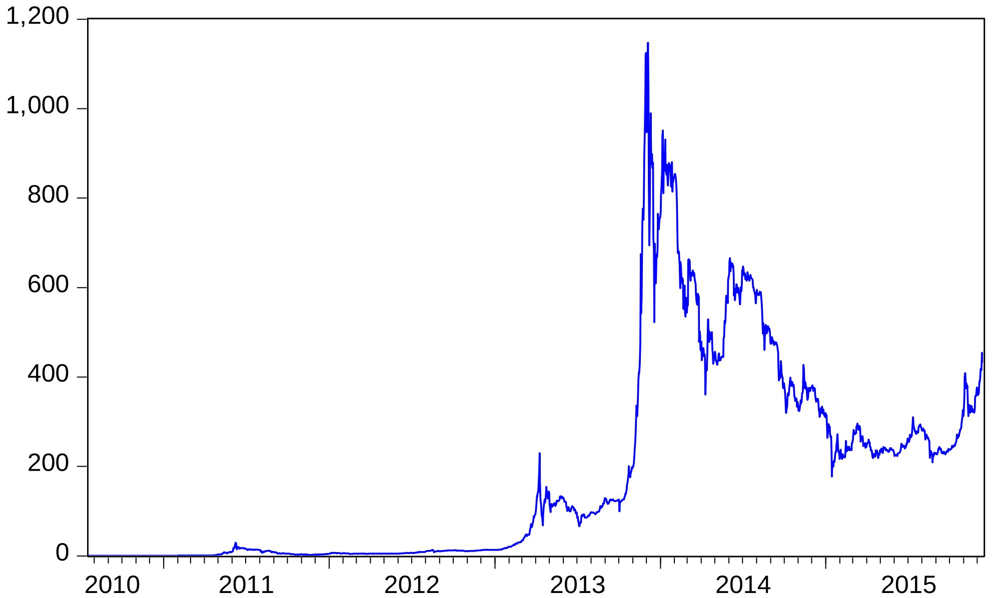


Figure 2: The Coindesk Price Index of Bitcoin



The statistical properties of the series are presented in Table A1 in the Appendix of the paper. Both series are found to be non-normal. We indicate the natural logarithms of the two series considered as LBITS and LCOIN as derived from Bitsmap and Coindesk respectively.

2.2 Methodology

The methodology used in the paper is based on the concept of fractional integration that basically assumes that the number of differences required to render a series to be stationary I(0) is a fractional value.

For the purpose of the present work we define an I(0) process as a covariance stationary process with a spectral density function that is positive and finite at all frequencies in the spectrum. Thus, it includes the white noise model but also all types of stationary Auto-Regressive Moving Average (ARMA) processes. Having said this, we say that a time series y_t is integrated of order d (and denoted as $x_t \sim I(d)$) if it can be represented as:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \ldots, \qquad (1)$$

with $x_t = 0$ for $t \leq 0$, and $d > 0$, where L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is $I(0)$. By allowing d to be fractional, we permit a much richer degree of flexibility in the dynamic specification of the series, not achieved when using the classical approaches based on integer differentiation, i.e., d = 0 and d = 1. Note that the left hand side of equation (1) can be expressed for all real d as:

$$(1 - L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \frac{d(d-1)(d-2)}{6} L^3 + \ldots$$

and thus

$$(1 - L)^d x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \frac{d(d-1)(d-2)}{6} x_{t-3} + \ldots .$$

Accordingly, if d is an integer value, x_t will be a function of a finite number of past observations, while if d is non-integer, x_t depends upon values of the time series far away in the past. Processes with $d > 0$ in (1) display the property of "*long memory*", characterised in this way because the spectral density function of the process is unbounded at its origin. These processes have been widely employed in recent years to describe the dynamics of many economic time series (see, for example, Diebold and Rudebusch, 1989; Sowell, 1992; Baillie, 1996; Gil-Alana and Robinson, 1997; Gil-Alana and Moreno, 2012; etc.).

On the other hand, the estimation of the differencing parameter d is crucial from different perspectives. Thus, for example, it is an indicator of the degree of persistence of the series, since, as it can be seen from the above equation, higher the value of d is, higher is the level of association between the observations. Also, if d is smaller than 0.5, the series is still covariance stationary, unlike what happens with values of $d \geq 0.5$ where the series becomes nonstationary in the sense that the variance of the partial sums increases with d; finally, if $d < 1$, the series is mean reverting with shocks disappearing in the long run unlike what happens with $d \geq 1$ where the shocks persist forever.

In this paper we estimate d using both parametric and semiparametric methods. From a parametric approach, we use the Whittle function in the frequency domain as proposed in Dahlhaus (1989) along with a Lagrange Multiplier (LM) test (Robinson, 1994) that also uses the Whittle function in the frequency domain. The main advantage of the latter approach is that it remains valid even in nonstationary contexts($d \geq 0.5$) and the limit distribution is standard normal and does not change with features of the regressors unlike what happens with other unit root testing methods (Schmidt and Phillips, 1992). Additionally, we use a semiparametric method (Robinson, 1995) that is basically a “local” Whittle function in the frequency domain with the frequencies degenerating to zero.

3. Results

We separate in this section the results for the log-transformed data (and based therefore on the levels) from those based on the volatility proxied by using both the absolute and the squared return series.

3.1 Log-transformed prices

We start this section by estimating d using the parametric approach and considering the following model,

$$y_t = \alpha + \beta t + x_t, \quad (1 - L)^d x_t = u_t, \qquad (2)$$

where y_t is the observed series in natural logarithm, α and β are coefficients referring respectively to the intercept and a linear time trend; L is the lag-operator ($Lx_t = x_{t-1}$); d is the fractional differencing parameter, and u_t is I(0) as defined above, and given the parametric nature of the method employed, we model first u_t as a white noise process, and then using autocorrelation throughout the model of Bloomfield (1973).[2]

Table 1 displays the estimates of d for the three cases of i) no deterministic terms (α and β = 0 a priori), an intercept (α unknown and β = 0 a priori), and an intercept with a linear time trend (α and β unknown), and present the estimates of d along with the 95% confidence band of the non-rejection values of d using Robinson’s (1994) parametric approach.

[2] The model of Bloomfield (1973) is described exclusively in terms of its spectral density function. It produces autocorrelations decaying exponentially as in the AR case, but with a small number of parameters. Moreover, it accomodates extremely well in the context of fractional integration.

Table 1: Estimates of d for the whole simple using a parametric approach

i) White noise disturbances			
Series	No regressors	An intercept	A linear time trend
LBITS	0.978 (0.943, 1.020)	0.994 (0.967, 1.025)	**0.994** **(0.967, 1.025)**
LCOIN	1.036 (1.011, 1.065)	1.033 (1.009, 1.061)	**1.033** **(1.009, 1.060)**
ii) Autocorrelated (Bloomfield) disturbances			
Series	No regressors	Anintercept	A linear time trend
LBITS	0.990 (0.924, 1.047)	**1.070** **(1.013, 1.123)**	1.071 (1.013, 1.123)
LCOIN	1.043 (1.004, 1.084)	1.038 (1.002, 1.079)	**1.039** **(1.002, 1.078)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 2: Estimated coefficients of the selected models in Table 1

i) White noise disturbances			
Series	d	Intercept (t-value)	Time trend (t-value)
LBITS	0.994 (0.967, 1.025)	2.45496 (37.40)	0.00301 (1.68)
LCOIN	1.033 (1.009, 1.060)	-2.40515 (-37.22)	0.00431 (2.32)
ii) Autocorrelated (Bloomfield) disturbances			
Series	d	Intercept (t-value)	Time trend (t-value)
LBITS	1.070 (1.013, 1.123)	2.46210 (37.78)	xxx
LCOIN	1.039 (1.002, 1.078)	-2.40394 (-37.22)	0.00430 (2.22)

Notes: In parenthesis in the second column, the 95% band of non-rejection values of d; in the third and fourth columns they are t-values.

Focusing on the deterministic terms, it can be observed that the time trend is required in the two series for the uncorrelated (white noise) case, but only an intercept for the case of logarithmic Bitcoin price form Bitstamp (LBITS) with autocorrelated (Bloomfield) errors. Table 2 focuses on the selected models according to these deterministic terms and presents the estimated coefficients. We observe that the estimates of d are higher than 1 in most of the cases, Only for LBITS with white noise errors, d is found to be slightly below 1 (0.994), though the unit root null cannot be rejected. Therefore, strong evidence is found in favour of permanency of the shocks and lack of mean reversion.

Table 3: Semiparametric estimates of d for LBITS data

Bandwidth (m)	d	95% Lower I(1) Band	95% Upper I(1) Band
20	**1.182**	0.816	1.184
30	1.204	0.849	1.150
31	1.205	0.852	1.147
32	1.221	0.854	1.145
33	1.235	0.856	1.143
34	1.255	0.858	1.141
35~ $(T)^{0.5}$	1.244	0.860	1.139
36	1.230	0.862	1.137
37	1.213	0.864	1.135
38	1.229	0.866	1.133
39	1.225	0.868	1.131
40	1.234	0.869	1.130
50	1.193	0.883	1.116

Notes: In bold, evidence of unit roots (d = 1) at the 5% level.

Table 4: Semiparametric estimates of d for LCOIN data

Bandwidth (m)	d	95% Lower I(1) Band	95% Upper I(1) Band
30	1.217	0.849	1.150
40	1.311	0.869	1.130
41	1.315	0.871	1.128
42	1.298	0.873	1.127
43	1.315	0.874	1.125
44 ~ $(T)^{0.5}$	1.313	0.876	1.123
45	1.332	0.877	1.122
46	1.315	0.878	1.121
47	1.309	0.880	1.119
48	1.291	0.881	1.118
49	1.299	0.882	1.117
50	1.280	0.883	1.116
60	1.218	0.893	1.106

Tables 3 and 4 displays the estimates of d for the two series using a semiparametric method of Robinson (1995) for a selected group of bandwidth numbers (m).[3] Using other most updated approaches also based on this technique (e.g., Phillips and Shimotsu, 2004; Abadir et al., 2007) produced essentially the same results.[4] Looking at the results in these two tables we obtain the same conclusions as with the parametric tests: evidence of d > 1 in all cases, especially for the logarithmic price index of Bitcoin from Coindesk (LCOIN).

Table 5: Break dates using Bai and Perron's (2003) methodology

Series	N. of breaks	Break dates
LBITS	4	6/15/2012; 3/04/2013; 11/14/2013; 1/13/2015
LCOIN	5	5/10/2011; 3/01/2012; 1/26/2013; 11/18/2013; 1/11/2015

On the other hand, many authors argue that long memory can be a spurious phenomenon caused by the existence of structural breaks not taking into account (Diebold and Inoue, 2001; Granger and Hyung, 2004). Because of this, we use Bai and Perron's (2003) methodology of detecting the potential presence of multiple structural breaks. In this regard, we regress the log-level of the two series under consideration on a constant and a trend to capture breaks in its mean and trend. Then we use the powerful UDmax and WDmax tests of 1 to M globally determined breaks proposed by Bai and Perron (2003), with a proposed 15% trimming of end-points, maximum of 5 breaks, and allowing for error distributions to differ across the breaks. The results have been reported in Table 5, which shows four breaks in case of the LBITS series and five for LCOIN, so we have five subsamples in LBITS and six in case of the LCOIN series.

[3] The choice of the bandwidth number (m) clearly shows the trade-off between bias and variance: the asymptotic variance is decreasing with m while the bias is growing with m.

[4] These methods require additionally more user-chosen parameters.

Table 6: Estimates of d for the subsamples with WHITE NOISE errors

i) Series: LBITS			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.954 (0.867, 1.068)	**0.882 (0.822, 0.968)**	0.882 (0.822, 0.968)
2nd sub-sample	1.013 (0.925, 1.126)	1.123 (1.034, 1.231)	**1.124 (1.033, 1.232)**
3rd sub-sample	1.022 (0.944, 1.132)	**1.044 (0.947, 1.162)**	1.044 (0.945, 1.162)
4rd sub-sample	1.023 (0.956, 1.106)	**0.881 (0.828, 0.964)**	0.882 (0.817, 0.959)
5rd sub-sample	1.028 (0.957, 1.108)	**0.980 (0.905, 1.177)**	0.981 (0.910, 1.176)
ii) Series: LCOIN			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	1.023 (0.958, 1.115)	0.988 (0.927, 1.069)	**0.988 (0.926, 1.071)**
2nd sub-sample	1.016 (0.956, 1.094)	**1.025 (0.944, 1.126)**	1.025 (0.944, 1.124)
3rd sub-sample	1.009 (0.937, 1.090)	0.879 (0.817, 0.948)	**0.879 (0.816, 0.949)**
4rd sub-sample	1.113 (1.032, 1.212)	1.189 (1.090, 1.302)	**1.185 (1.084, 1.299)**
5rd sub-sample	1.005 (0.945, 1.089)	**0.945 (0.888, 1.022)**	0.945 (0.887, 1.021)
6rd sub-sample	0.978 (0.902, 1.056)	**0.948 (0.864, 1.036)**	0.937 (0.857, 1.032)

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 7: Estimates of d for the subsamples with AUTOCORRELATED errors

i) Series: LBITS			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.897 (0.756, 1.105)	**0.922** **(0.839, 1.052)**	0.930 (0.841, 1.051)
2nd sub-sample	1.054 (0.903, 1.246)	1.063 (0.954, 1.233)	**1.062** **(0.952, 1.222)**
3rd sub-sample	1.034 (0.905, 1.225)	0.978 (0.808, 1.209)	**0.978** **(0.832, 1.208)**
4rd sub-sample	0.967 (0.856, 1.104)	**1.041** **(0.908, 1.221)**	1.042 (0.910, 1.221)
5rd sub-sample	0.973 (0.887, 1.122)	0.971 (0.882, 1.133)	**0.980** **(0.877, 1.134)**
ii) Series: LCOIN			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.992 (0.890, 1.153)	0.991 (0.892, 1.117)	**0.992** **(0.888, 1.122)**
2nd sub-sample	1.035 (0.932, 1.156)	**0.900** **(0.819, 1.006)**	0.900 (0.819, 1.005)
3rd sub-sample	1.011 (0.898, 1.152)	0.978 (0.876, 1.123)	**0.978** **(0.865, 1.123)**
4rd sub-sample	1.059 (0.949, 1.212)	1.112 (0.993, 1.324)	**1.108** **(1.001, 1.317)**
5rd sub-sample	1.011 (0.912, 1.132)	**0.872** **(0.784, 1.002)**	0.883 (0.769, 1.002)
6rd sub-sample	0.978 (0.867, 1.090)	0.826 (0.733, 0.967)	**0.822** **(0.729, 0.955)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 8: Estimated coefficients from models in Table 6 (white noise)

i) Series: LBITS			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.882 (0.822, 0.968)	2.43476 (28.95)	---
2nd sub-sample	1.124 (1.033, 1.232)	1.85486 (57.66)	0.00979 (2.32)
3rd sub-sample	1.044 (0.947, 1.162)	3.70272 (39.57)	---
4rd sub-sample	0.881 (0.828, 0.964)	6.08838 (94.49)	---
5rd sub-sample	0.980 (0.905, 1.177)	5,15055 (145,32)	---
ii) Series: LCOIN			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.988 (0.926, 1.071)	-2.42648 (-27.29)	0.01272 (2.73)
2nd sub-sample	1.025 (0.944, 1.126)	1.75788 (19.57)	---
3rd sub-sample	0.879 (0.816, 0.949)	1.52958 (45.04)	0.00399 (4.23)
4rd sub-sample	1.185 (1.084, 1.299)	2.85804 (44.70)	0.01571 (1.72)
5rd sub-sample	0.945 (0.888, 1.022)	6.30871 (123.18)	---
6rd sub-sample	0.948 (0.864, 1.036)	5.57167 (156.75)	---

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 9: Estimated coefficients from the models in Table 7

i) Series: LBITS			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.922 (0.839, 1.052)	2.45271 (28.88)	---
2nd sub-sample	1.062 (0.952, 1.222)	1.85249 (57.27)	0.00959 (3.03)
3rd sub-sample	0.978 (0.832, 1.208)	3.69738 (39.45)	0.01237 (2.06)
4rd sub-sample	1.041 (0.908, 1.221)	6.00385 (93.30)	---
5rd sub-sample	0.980 (0.877, 1.134)	5.14729 (145.02)	0.00284 (1.64)
ii) Series: LCOIN			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.992 (0.888, 1.122)	-2.42349 (-27.25)	0.01271 (2.59)
2nd sub-sample	0.900 (0.819, 1.006)	1.77646 (20.06)	---
3rd sub-sample	0.978 (0.865, 1.123)	1.52540 (44.48)	0.00408 (2.54)
4rd sub-sample	1.108 (1.001, 1.317)	2.86173 (44.26)	0.01396 (2.18)
5rd sub-sample	0.872 (0.784, 1.002)	6.33130 (124.69)	---
6rd sub-sample	0.822 (0.729, 0.955)	5.53702 (159.89)	0.00148 (2.01)

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Tables 6 and 7 report the estimates of d using the parametric method for each subsample with white noise and autocorrelated errors respectively, while Tables 8 and 9 display the estimated coefficients for each case. We observe across these tables some cases of mean reversion here, particularly for LBITS in the first and fourth subsamples, and for LCOIN in the third one. However, under autocorrelated errors (Tables 7 and 9), mean reversion only takes place for LCOIN during the last subsample.

3.2 Volatility series

As earlier mentioned, we approximate the volatility by using two standard measures, the absolute and the squared returns, and we conduct the same type of analysis as in Section 3.1 for the corresponding series. Absolute returns were employed among others by Ding et al. (1993), Granger and Ding (1996), Bollerslev and Wright (2000), Gil-Alana (2005), Cavalcante and Assaf (2004), Sibbertsen (2004) and Cotter (2005), whereas squared returns were used in Lobato and Savin (1998), Gil-Alana (2003), Cavalcante and Assaf (2004) and Cotter (2005).

Table 10: Estimates of d based on the absolute and squared returns

i) White noisedisturbances			
Series	No regressors	An intercept	A linear time trend
Absolutertns. LBITS	0.281 (0.252, 0.313)	0.269 (0.2400.302)	**0.281** **(0.252, 0.313)**
Absolutertns. LCOIN	0.302 (0.2820.330)	0.293 (0.2680.321)	**0.290** **(0.264 0.319)**
Squaredrtns. LBITS	0.234 (0.2000.272)	**0.231** **(0.197 0.269)**	0.228 (0.1930.267)
Squaredrtns. LCOIN	0.222 (0.1940.256)	0.215 (0.1850.242)	**0.209** **(0.181 0.237)**
ii) Autocorrelated (Bloomfield) disturbances			
Series	No regressors	An intercept	A linear time trend
Absolutertns. LBITS	0.372 (0.321, 0.446)	**0.354** **(0.301, 0.430)**	0.357 (0.297, 0.422)
Absolutertns. LCOIN	0.362 (0.3140.417)	0.334 (0.2830.372)	**0.334** **(0.279 0.386)**
Squaredrtns. LBITS	0.301 (0.2380.400)	**0.298** **(0.240 0.388)**	0.297 (0.2330.386)
Squaredrtns. LCOIN	0.252 (0.2140.316)	0.233 (0.2040.283)	**0.228** **(0.186 0.281)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 10 displays the estimates for the four series (that is, absolute and squared returns of LBITS and LCOIN) again for the three cases of no regressors, an intercept, and an intercept with a linear time trend. As expected evidence of long memory (d > 0) is found here in all cases, with the estimated values of d ranging between 0.2 and 0.4 in all cases.

3.2.1 Absolute return series

Focusing now on the same breaks as in the previous cases, the results for the two cases of uncorrelated and autocorrelated errors are displayed in Tables 11 and 12, and their corresponding estimates for the selected models are reported in Tables 13 and 14.

Table 11: Estimates of d for the subsamples with WHITE NOISE errors

i) Series: Absolute returns of LBITS			
Sub-Series	No regressors	Anintercept	A linear time trend
1st sub-sample	0.135 (0.071, 0.213)	0.111 (0.065, 0.189)	**0.038** **(-0.044, 0.119)**
2nd sub-sample	0.246 (0.146, 0.364)	**0.209** **(0.123, 0.322)**	0.209 (0.117, 0.322)
3rd sub-sample	0.468 (0.395, 0.568)	**0.457** **(0.368, 0.552)**	0.446 (0.354, 0.551)
4rd sub-sample	0.445 (0.352, 0.557)	0.379 (0.299, 0.478)	**0.404** **(0.303, 0.508)**
5rd sub-sample	0.142 (0.087, 0.228)	0.125 (0.072, 0.198)	**0.130** **(0.072, 0.202)**
ii) Series: Absolute returns of LCOIN			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.344 (0.261, 0.443)	**0.317** **(0.242, 0.407)**	0.317 (0.242, 0.407)
2nd sub-sample	0.297 (0.222, 0.384)	0.266 (0.186, 0.355)	**0.256** **(0.179, 0.355)**
3rd sub-sample	0.244 (0.178, 0.329)	**0.230** **(0.166, 0.324)**	0.232 (0.164, 0.323)
4rd sub-sample	0.436 (0.354, 0.526)	**0.406** **(0.337, 0.497)**	0.412 (0.335, 0.503)
5rd sub-sample	0.290 (0.247, 0.364)	0.254 (0.202, 0.312)	**0.234** **(0.189, 0.309)**
6rd sub-sample	0.325 (0.248, 0.422)	0.289 (0.215, 0.365)	**0.314** **(0.237, 0.435)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 12: Estimates of d for the subsamples with AUTOCORRELATED errors

i) Series: Absolute retunrs of LBITS			
Sub-Series	No regressors	Anintercept	A linear time trend
1st sub-sample	0.280 (0.180, 0.413)	0.225 (0.143, 0.341)	**0.117** **(-0.015, 0.275)**
2nd sub-sample	0.334 (0.162, 0.590)	**0.254** **(0.090, 0.466)**	0.254 (0.015, 0.465)
3rd sub-sample	0.648 (0.466, 0.835)	**0.608** **(0.392, 0.815)**	0.598 (0.371, 0.818)
4rd sub-sample	0.437 (0.289, 0.622)	0.317 (0.205, 0.489)	**0.355** **(0.203, 0.614)**
5rd sub-sample	0.267 (0.159, 0.418)	0.218 (0.112, 0.346)	**0.241** **(0.133, 0.505)**
ii) Series: Absolute returns of LCOIN			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.516 (0.336, 0.823)	**0.443** **(0.309, 0.660)**	0.443 (0.309, 0.660)
2nd sub-sample	0.288 (0.125, 0.454)	0.226 (0.082, 0.373)	**0.218** **(0.055, 0.390)**
3rd sub-sample	0.254 (0.133, 0.424)	**0.237** **(0.122, 0.408)**	0.237 (0.122, 0.409)
4rd sub-sample	0.440 (0.317, 0.607)	**0.384** **(0.267, 0.532)**	0.400 (0.267, 0.549)
5rd sub-sample	0.343 (0.279, 0.443)	0.272 (0.200, 0.378)	**0.262** **(0.177, 0.350)**
6rd sub-sample	0.217 (0.139, 0.332)	0.177 (0.115, 0.291)	**0.240** **(0.123, 0.497)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 13: Estimated coefficients from models in Table 11 (white noise)

i) Series: Absolute returns of LBITS			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.038 (-0.044, 0.119)	0.08199 (7.74)	-0.000312 (-3.70)
2nd sub-sample	0.209 (0.123, 0.322)	0.02463 (5.06)	---
3rd sub-sample	0.457 (0.368, 0.552)	0.05872 (1.92)	---
4rd sub-sample	0.404 (0.303, 0.508)	0.14571 (5.78)	-0.000402 (-2.63)
5rd sub-sample	0.130 (0.072, 0.202)	0.03138 (6.12)	-0.000046 (-1.83)
ii) Series: Absolute returns of LCOIN			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.317 (0.242, 0.407)	0.06932 (3.58)	---
2nd sub-sample	0.256 (0.179, 0.355)	0.08750 (4.25)	-0.000189 (-1.65)
3rd sub-sample	0.230 (0.166, 0.324)	0.01823 (3.45)	---
4rd sub-sample	0.406 (0.337, 0.497)	0.04503 (2.34)	---
5rd sub-sample	0.234 (0.189, 0.309)	0.05954 (6.09)	-0.000113 (-2.92)
6rd sub-sample	0.314 (0.237, 0.435)	0.05243 (5.33)	-0.000114 (-2.30)

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 14: Estimated coefficients from the models in Table 12 (autocorrelation)

i) Series: Absolute returns of LBITS			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.117 (-0.015, 0.275)	0.07912 (5.40)	-0.000293 (-2.57)
2nd sub-sample	0.254 (0.090, 0.466)	0.02611 (2.81)	---
3rd sub-sample	0.608 (0.392, 0.815)	0.05455 (1.10)	---
4rd sub-sample	0.355 (0.203, 0.614)	0.12483 (5.69)	-0.000342 (-2.72)
5rd sub-sample	0.241 (0.133, 0.505)	0.03760 (4.73	-0.000070 (-1.78)
ii) Series: Absolute returns of LCOIN			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.443 (0.309, 0.660)	0.08973 (2.24)	---
2nd sub-sample	0.218 (0.055, 0.390)	0.08607 (4.84)	-0.000180 (-1.85)
3rd sub-sample	0.237 (0.122, 0.408)	0.02066 (2.52)	---
4rd sub-sample	0.384 (0.267, 0.532)	0.03708 (1.60)	---
5rd sub-sample	0.262 (0.177, 0.350)	0.06103 (5.49)	-0.000116 (-2.63)
6rd sub-sample	0.240 (0.123, 0.497)	0.04266 (5.58)	-0.000084 (-2.22)

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Starting with the case of uncorrelated errors, evidence of long memory is found here in all cases except for the first subsample in LBITS where the null of d = 0 cannot be rejected. The same evidence is obtained with autocorrelated (Bloomfield) disturbances, and long memory is found in all subsamples except for the first one with LBITS.

3.2.2 Squared return series

The results for the case of the squared returns are displayed across Tables 15 – 18. Starting again with white noise errors, evidence of long memory is obtained in all cases except for the first subsamples in the case of the LBITS data. In these two cases we cannot reject the null of I(0) or short memory behaviour.

Table 15: Estimates of d for the subsamples with WHITE NOISE errors

i) Series: Squared returns of LBITS			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.080 (0.013, 0.165)	0.075 (0.012, 0.156)	**0.025** **(-0.065, 0.117)**
2nd sub-sample	0.082 (0.015, 0.203)	**0.076** **(-0.022, 0.196)**	0.066 (-0.035, 0.195)
3rd sub-sample	0.347 (0.255, 0.445)	**0.344** **(0.254, 0.435)**	0.338 (0.234, 0.423)
4rd sub-sample	0.389 (0.288, 0.507)	0.355 (0.267, 0.511)	**0.678** **(0.326, 0.862)**
5rd sub-sample	0.077 (0.028, 0.157)	0.070 (0.024, 0.144)	**0.093** **(0.028, 0.165)**
ii) Series: Squared returns of LCOIN			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.236 (0.149, 0.346)	**0.211** **(0.136, 0.300)**	0.207 (0.135, 0.301)
2nd sub-sample	0.201 (0.124, 0.280)	0.184 (0.117, 0.283)	**0.166** **(0.089, 0.277)**
3rd sub-sample	0.244 (0.155, 0.353)	**0.245** **(0.155, 0.355)**	0.246 (0.155, 0.352)
4rd sub-sample	0.396 (0.309, 0.506)	**0.387** **(0.294, 0.497)**	0.390 (0.303, 0.503)
5rd sub-sample	0.309 (0.231, 0.380)	0.279 (0.212, 0.357)	**0.265** **(0.190, 0.343)**
6rd sub-sample	0.426 (0.302, 0.581)	0.390 (0.284, 0.572)	**0.524** **(0.384, 0.676)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 16: Estimates of d for the subsamples with AUTOCORRELATED errors

i) Series: Squared retunrs of LBITS			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.165 (0.067, 0.312)	0.166 (0.055, 0.295)	**0.073 (-0.069, 0.239)**
2nd sub-sample	0.065 (-0.111, 0.283)	**0.048 (-0.091, 0.266)**	0.048 (-0.121, 0.266)
3rd sub-sample	0.508 (0.295, 0.829)	**0.508 (0.272, 0.829)**	0.477 (0.225, 0.826)
4rd sub-sample	0.179 (0.054, 0.355)	0.158 (0.040, 0.279)	**0.178 (0.022, 0.299)**
5rd sub-sample	0.163 (0.051, 0.326)	0.153 (0.041, 0.286)	**0.266 (0.089, 0.846)**
ii) Series: Squared returns of LCOIN			
Sub-Series	No regressors	An intercept	A linear time trend
1st sub-sample	0.576 (0.034, 1.180)	**0.444 (0.264, 1.214)**	0.480 (0.264, 1.233)
2nd sub-sample	0.164 (0.014, 0.314)	0.126 (0.007, 0.288)	**0.065 (-0.080, 0.267)**
3rd sub-sample	0.089 (-0.034, 0.258)	**0.086 (-0.034, 0.259)**	0.087 (-0.035, 0.259)
4rd sub-sample	0.252 (0.117, 0.466)	**0.236 (0.106, 0.424)**	0.257 (0.104, 0.466)
5rd sub-sample	0.212 (0.140, 0.354)	0.182 (0.134, 0.280)	**0.154 (0.090, 0.273)**
6rd sub-sample	0.018 (-0.073, 0.132)	0.016 (-0.078, 0.135)	**0.044 (-0.056, 0.222)**

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 17: Estimated coefficients from models in Table 15 (white noise)

i) Series: Squared returns of LBITS			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.025 (-0.065, 0.117)	0.01648 (4.16)	-0.000083 (-2.64)
2nd sub-sample	0.076 (-0.022, 0.196)	0.00125 (3.34)	---
3rd sub-sample	0.344 (0.254, 0.435)	0.00962 (0.87)	---
4rd sub-sample	0.678 (0.326, 0.862)	0.14069 (10.32)	-0.000350 (-2.17)
5rd sub-sample	0.093 (0.028, 0.165)	0.00224 (3.75)	-0.000005 (-1.74)
ii) Series: Squared returns of LCOIN			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.211 (0.136, 0.300)	0.01067 (2.87)	---
2nd sub-sample	0.166 (0.089, 0.277)	0.01579 (3.24)	-0.000050 (-1.85)
3rd sub-sample	0.245 (0.155, 0.355)	0.00112 (0.72)	---
4rd sub-sample	0.387 (0.294, 0.497)	0.00495 (1.04)	---
5rd sub-sample	0.265 (0.190, 0.343)	0.00772 (3.51)	-0.000020 (-2.37)
6rd sub-sample	0.524 (0.384, 0.676)	0.01682 (6.20)	-0.000048 (-2.68)

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Table 18: Estimated coefficients from models in Table 16 (autocorrelation)

i) Series: Squared returns of LBITS			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.073 (-0.069, 0.239)	0.01606 (3.30)	-0.000080 (-2.10)
2nd sub-sample	0.048 (-0.091, 0.266)	0.00146 (2.22)	---
3rd sub-sample	0.508 (0.272, 0.829)	0.01173 (0.54)	---
4rd sub-sample	0.178 (0.022, 0.299)	0.01599 (4.37)	-0.000058 (-2.93)
5rd sub-sample	0.266 (0.089, 0.846)	0.00412 (3.48)	-0.000012 (-2.08)

ii) Series: Squared returns of LCOIN			
Sub-Series	d	Intercept (t-value)	Time trend (t-value)
1st sub-sample	0.444 (0.264, 1.214)	0.01867 (1.28)	---
2nd sub-sample	0.065 (-0.080, 0.267)	0.01552 (4.77)	-0.000049 (-2.67)
3rd sub-sample	0.086 (-0.034, 0.259)	0.00134 (1.08)	---
4rd sub-sample	0.236 (0.106, 0.424)	0.00344 (0.90)	---
5rd sub-sample	0.154 (0.090, 0.273)	0.00701 (5.01)	-0.000019 (-3.46)
6rd sub-sample	0.044 (-0.056, 0.222)	0.00251 (5.01)	-0.000006 (-2.69)

Notes: In bold the selected models according to the deterministic terms. In parenthesis the 95% band of non-rejection values of d.

Allowing for autocorrelated disturbances, we see in Table 18 that there are some more cases of short memory (i.e. d= 0). They are those corresponding to the first two subsamples in LBITS, and contrary to previous tables, to the second, the third and the sixth subsamples in LCOIN.

4. Conclusion and implications

Along with the rapid emergence of Bitcoin as a financial asset, there has been a growing interest among Bitcoin users, investors, and policy-makers in understanding the issue of persistence in Bitcoin prices, especially because the Bitcoin market exhibited periods of boom and bust coupled with extreme market volatility. Such a behaviour implies non-linearity dynamics in the price of Bitcoin.

While limited studies have focused on the economics and finance of Bitcoin, no previous studies have examined whether or not the Bitcoin market is efficient. This paper addresses this literature gap and thus participates to the advancement of knowledge and debate on this newly emerged crypto-currency. In particular, it used two Bitcoin indices and employed long range dependence techniques based on fractional integration in order to focus on the persistence and volatility of Bitcoin accounting for non-linearity dynamics in the price of Bitcoin.

The empirical results are summarized as follows. First, we found strong evidence supporting the permanency of shocks and lack of mean reversion in the log-transformed prices in both using parametric and semi-semiparametric methods. Second, we identified at least four structural breaks in each of the data series, one of which corresponds to the price crash of December 2013, and accounted for their importance in describing the dynamics of the Bitcoin market. Specifically, we observed cases of mean reversion in some subsamples in both data series. Third, we used two standard measures of volatility, the absolute and the squared returns, and found evidence of long memory in almost all subsamples in the absolute returns series. While evidence of long memory was also reported in the squared return series, the results differed between the two volatility measures as evidence of short memory was more pronounced in the squared return series.

Our empirical findings imply the importance of accounting for the long memory property in an empirical analysis that consider the finance of Bitcoin such as optimal hedging estimation, risk portfolio management, and potential option valuation. The findings also offer market participants and analysts an interesting opportunity to get benefits from the inefficiencies in the Bitcoin market. As such, they can potentially improve the risk-adjusted performance of their portfolios by using long memory based frameworks.

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APPENDIX:

Table A1: Summary Statistics:

Statistic	BITCOIN (Bitstamp)	BITCOIN (Coindesk)
Mean	241.5266	181.4523
Median	227.92	76.8900
Maximum	1132.01	1147.2500
Minimum	2.24	0.0500
Std. Dev.	232.6023	230.2312
Skewness	0.865282	1.3644
Kurtosis	3.195702	4.2699
Jarque-Bera	154.9434	746.2470
Probability	0.0000	0.0000
Observations	1226	1977

Notes: Std. Dev.: Standard Deviation; Probability corresponds to the rejection of the hypothesis of normality associated with Jarque-Bera test.