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Fast Converging Robust Beamforming for Massive MIMO in Heterogeneous Networks

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ABSTRACT The use of massive multiple-input multiple-output (MIMO) base stations in heterogeneous networks (HetNets) offers an increase in throughput without increasing the bandwidth, but with reduced power consumption. In this paper, we investigate the optimization problem of signal-to-interference-plus-noise ratio balancing for the case of imperfect channel state information at the transmitter. We present a fast converging robust beamforming solution for macrocell users in a typical HetNet scenario with massive MIMO at the base station. The proposed method applies the matrix stuffing technique and the alternative direction method of multipliers to give an efficient solution. Simulation results of a single-cell heterogeneous network show that the proposed solution yields performance with modest accuracy, while converging in an efficient manner, compared with optimal solutions achieved by the state-of-the-art modeling languages and interior-point solvers. This is particularly for cases when the number of antennas at the base station increases to large values. This makes the solution method attractive for practical implementation in heterogeneous networks with large-scale antenna arrays at the macrocell base station.

INDEX TERMS Massive MIMO, HetNet, macrocell, beamforming, matrix stuffing, ADMM algorithm.

I. INTRODUCTION

The rapid advancement in wireless communication technologies presents a demand for high data transmission rates. In urban environments, a large proportion of data traffic is generated by highly concentrated groups of users, e.g. in restaurants or at stations and airports. These groups of users are often termed *hotspots* and they comprise low mobility users [1]. In order to cope with the high throughput demand, use of traditional cellular networks, which comprise high-power base stations (BSs), is practically infeasible. The use of small cells has been found to increase the network capacity effectively, while reducing the total transmit power [2]. On the other hand, a large number of small cells diminish network performance and quality of service (QoS) for the user [3]. The small cell radii with BS antennas on and below rooftops cannot support users with high mobility and large area coverage is difficult. This has led to the introduction of the heterogeneous network (HetNet) structure which comprises a macrocell overlaid with small cells [2].

The overlaid cell deployment with frequency reuse of one introduces severe inter-tier interference, especially for cell edge users in the macrocell [4]. This is because the macrocell users at the cell edges are further away from their serving BS and may be in close proximity to a small cell access point (AP). Although the small cell AP transmits with lower power compared to the macrocell BS, it can present severe interference to the macrocell users at the cell edges. Massive multiple-input multiple-output (MIMO) has been applied at the macrocell BS and results have shown that this enhances inter-tier interference mitigation in HetNets. Massive MIMO is a technique where arrays of hundreds of localized or distributed antennas serve many tens of terminals in the same time-frequency resource [5]. Large-scale antenna arrays present a significantly huge improvement in spectral efficiency and energy efficiency in wireless systems over the conventional MIMO used in LTE/4G systems.

Significant performance gains are expected from network topologies where massive MIMO BSs and HetNets coexist, through application of beamforming. Use of massive MIMO at the macrocell BS in a HetNet enables it to concentrate its transmission power on the hotspots it serves, thus providing transmission opportunities to the small cells located in other directions. This is known as spatial blanking, and three interference coordination strategies that use this technique with reduced complexity have been developed [6]. The strategies aim to address the problem of severe inter-tier interference which results from the random geometry of hotspots using the technique of joint spatial division and multiplexing [7].

It was observed that synchronized operation of the time division duplexing (TDD) protocol facilitates channel estimation and enables all the equipment in the network structure to obtain information about the interfering channels with no additional overhead [8]. The coverage probability and spectral efficiency performance were analyzed for both macrocell and small cell tiers in a massive MIMO macrocell that is overlaid with small cells [4]. Hosseini *et al.* [9] consider a reversed TDD (RTDD) protocol, where the two tiers operate in reverse during a transmission period. That is, when the macrocell is in downlink, the small cells will be in uplink and vice versa. This enhances accurate estimation of the interference subspace for fixed BSs of the two tiers. Analysis of performance with RTDD for the case of imperfect CSI at the transmitter has also been carried out [10]. In contrast to [9], Sanguinetti *et al.* [10] make use of a wireless backhaul among the small cells, thus applying RTDD not only between the two tiers, but also among small cell access points.

With a large antenna array at the BS, it was shown that the linear precoding techniques, such as zero-forcing beamforming (ZFBF) and minimum mean square error (MMSE) precoding, can achieve near optimum results in terms of sum rate performance [11], [12]. Thus the less computationally complex linear precoding techniques have been found to be practical to use [13]. The linear precoding schemes, however, cannot account for CSI acquisition errors and other factors that contribute to channel uncertainty. This has led to contributions that propose other efficient robust beamforming techniques for massive MIMO downlink systems [14]. The key tool common to the proposed solutions is the application of robust optimization theory. Robust beamforming is optimization of the beamforming matrix for which the errors in CSI approximation are constrained to lie in an uncertainty set [15]. In other words, it is an uncertainty-based beamforming approach in which the uncertainty model is not stochastic, but rather deterministic and set-based [16].

To solve the problem of maximizing the minimum sum secrecy rate for multi-user MIMO networks with imperfect CSI, an efficient approximation algorithm based on Taylor expansion was developed [17]. Joint robust optimization algorithms for the worst-case weighted sum rate maximization in multicell massive MIMO networks have been presented [18]. The algorithms have different levels of complexity and consider variations in coordination among BSs; however, all the solutions use the semi-definite programming (SDP) approach. A low-complexity robust beamforming method for signal-to-interference-plus-noise ratio (SINR) balancing in multicell massive MIMO networks that is based on iteratively solving second-order cone problems (SOCPs) has been designed [19]. Computational efficiency of the method is achieved by exploiting some properties of the optimization problem's constraints in the robust version of the problem. The complexity of the SDP approach is highly sensitive to the size of the beamforming matrix, and the SDP-based solutions can either incur appreciable computational cost or in certain circumstances the digital resources may not be sufficient to cater for the memory requirements of an SDP-based solution [16]. In addition, the SDP approach can be implemented by a limited choice of solvers, and it is unable to handle various types of uncertainty sets.

Although the RTDD protocol aims to address the problem of imperfect CSI in HetNets with massive MIMO BSs, its backhaul requirement for coordination increases overhead and latency. In this paper we address the problem of maximizing the weighted sum rate for macrocell users in a HetNet scenario. It is worth noting that maximizing the sum rate without increasing the bandwidth can be achieved by increasing the users' SINR. We propose a robust beamforming solution method for the problem of maximizing the minimum achievable SINR of the macrocell users in a HetNet scenario, which is based on iteratively solving SOCPs. The robust scheme accounts for imperfect CSI at the transmitter, and it can be implemented in an uncoordinated network of BSs. We present an efficient method to solve the worst-case SINR balancing problem for macrocell users in HetNets. The two-stage method converges to a near optimal solution much faster compared to the optimal solver. In the first stage, the original non-convex optimization problem is transformed into a standard SOCP using the Smith form reformulation and the matrix stuffing (MS) technique [20], [21]. The resulting SOCP is then solved by an efficient solver which uses the alternative direction method of multipliers (ADMM) [22]. This solution method has been applied to solve optimization problems in dense wireless networks, which are typically used in Cloud-RAN networks [21].

The rest of the paper is organized as follows: Section II presents the system model for both perfect and imperfect CSI acquisition at the transmitter. In Section III, the proposed SOCP-based solution is presented. Section IV discusses the simulation results, and a conclusion is given in Section V.

II. SYSTEM MODEL

Consider a typical single cell of a HetNet shown in Fig. 1. The macrocell BS consists of M transmit antennas transmitting to K single-antenna macrocell user equipment (MUE). The macrocell is overlaid with S small cells, which contain a single-antenna BS serving a few small-cell user equipment (SUE), each equipped with a single antenna. Typically $M \gg K$ and the system is also referred to as multi-user MIMO. The K users and the small cells are served in the same time-frequency resource and we consider the channel to be Rayleigh fading.

The signal received at the k^{th} MUE is modelled as

$$y_{\text{MUE},k} = \sqrt{P_b}\mathbf{h}_k^T\mathbf{s} + z_{MUE,k}, \tag{1}$$

where P_b is the transmit power of the macrocell BS, $\mathbf{h}_k$ is the channel vector between the kth MUE and the macrocell BS, $\mathbf{s} = \mathbf{W}\mathbf{x}$ is the transmitted vector with $\mathbf{x} \in \mathbb{R}^K$ containing

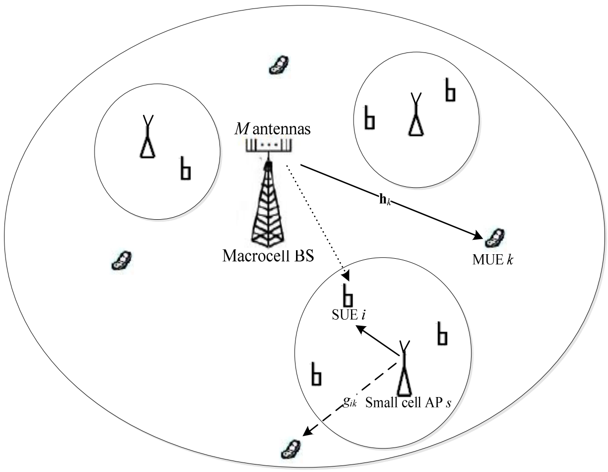

FIGURE 1. Typical single cell of a HetNet with massive MIMO at the macrocell BS.

the data symbols for the K users, and $z_{\mathrm{MUE,k}}$ represents interference plus noise. Let $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]$ be the $M \times K$ channel matrix, and $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_K]$ be an $M \times K$ precoding matrix. The interference and noise terms are defined as

$$z_{\mathrm{MUE},k} = \Sigma_{i \in S}\, \breve{g}_{ik} x_{\mathrm{SCA,i}} + n_{\mathrm{MUE},k}, \tag{2}$$

where $n_{\mathrm{MUE},k}$ is the additive white Gaussian noise with variance σ^2, $S = \{1, 2, \ldots S\}$ is the set of small-cell BSs, $\breve{g}_{ik}$ represents the channel from the k^{th} MUE to small cell BS i, and $x_{\mathrm{SCA,i}}$ is the data symbol transmitted to the i^{th} SUE from its associated BS.

A. MODELLING FOR PERFECT CSI

In the absence of CSI acquisition errors, the WSRMax optimization problem is given by

$$\begin{aligned} &\text{maximize } \Sigma_{k=1}^{K} \alpha_k \log_2(1+\gamma_k), \\ &\text{subject to } \Sigma_{k \in B_u} \|\mathbf{w}_k\|_2^2 \le P_b, \end{aligned} \tag{3}$$

where P_b is the transmit power of the corresponding BS, α_k is a positive weighting factor for user k, B_u is the set of all macrocell users served by the BS, and $\mathbf{w}_k$ is the beamforming vector for the k^{th} user. γ_k is the SINR of the k^{th} macrocell user, which is given by

$$\gamma_k = \frac{P_b \left|\mathbf{h}_k^T \mathbf{w}_k\right|^2}{\sigma^2 + P_b \Sigma_{i \ne k} \left|\mathbf{h}_i^T \mathbf{w}_i\right|^2 + P_{\mathrm{SCA}} \Sigma_{i=1}^{S} \left|\breve{g}_{ik}\right|^2}, \tag{4}$$

where P_{SCA} is the transmit power of the small cell BS.

By setting a threshold for the minimum achievable rate for each user, r, problem (3) can be solved by a bi-section method and the resulting optimization problem is given as

$$\begin{aligned} &\text{minimize } \|\mathbf{w}\|_2 \\ &\text{subject to } \alpha_k \log_2(1+\gamma_k) \ge r, \quad \forall k \\ &\qquad \Sigma_{k \in B_u} \|\mathbf{w}_k\|_2^2 \le P_b, \end{aligned} \tag{5}$$

where $\mathbf{w}$ represents the vectorized precoding matrix. This problem can be reformulated as an SOCP optimization problem. The solution method is based on determining the upper bound for the users' sum rate using the uplink-downlink duality theory [23], and then applying the max-min fairness optimization algorithm to solve the SOCP problem [24]. For the case of perfect CSI acquisition at the transmitter, the rate upper bound was determined by using the capacity equation from [25] and applying Jensen's inequality after noting that the log function is concave [26]. The resulting capacity limit is given by

$$C_k \le \log_2\left(\det\left[\mathbf{I}_k + \frac{1}{\sigma^2} E\left(\mathbf{h}_k^T \mathbf{h}_k\right)\right]\right), \quad \forall k, \tag{6}$$

where $\mathbf{I}_k$ is a $K \times K$ identity matrix and $E[(\mathbf{h}_k^T \mathbf{h}_k)]$ is the autocorrelation matrix of the channel vector $\mathbf{h}_k$.

It was shown that this solution method closely reaches the performance of the optimal branch-and-bound (BRB) method as the number of antennas at the macrocell BS is increased to more than a hundred [27]. This ADMM-based method achieves this near-optimal performance while converging much faster than the interior-point methods because the structure of the transformed problem can be solved by parallel closed forms.

B. IMPERFECT CSI MODEL

It is practically impossible to achieve perfect CSI at the transmitter, and use of TDD cannot guarantee error-free CSI acquisition at the BS especially for users with high mobility. The premise of the TDD protocol is that the channel is assumed to remain constant during the channel coherence time. This can be valid for cases where the user remains in the same vicinity relative to the BS during the channel coherence interval. However, for future generation networks that aim to support very high mobility users, this assumption is not accurate. Therefore, the focus of this work is to investigate and optimize the performance of the massive MIMO system for macrocell users in a HetNet scenario, in the presence of channel estimation errors. We model the channel vectors to account for uncertainty in the CSI acquisition, and they take the form

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \tag{7}$$

where $\hat{\mathbf{h}}_k$ is the estimated channel vector for the kth user, and ek is the corresponding downlink channel error vector. The channel estimation error vectors are assumed to be bounded and to lie in an uncertainty set U, defined as

$$U_k = \{\mathbf{e}_k : \|\mathbf{e}_k\| \le \rho_k, \ \forall k\}, \tag{8}$$

where $\|\cdot\|$ is an appropriate absolute norm described by the parameter ρ_k, which is chosen based on the desired channel uncertainty model [28].

Imperfect CSI for robust optimization can be modelled either statistically or by using the worst case design. The worst case design guarantees a certain system performance for channels sufficiently close to the estimated values. On the other hand, statistically modelled imperfect CSI guarantees a certain system performance based on averages obtained over the number of channel realizations [29]. Selection of the modelling design depends on the source of CSI errors at the BS. There are two sources of uncertainty in CSI acquisition, namely estimation error and quantization error [28]. Massive MIMO systems rely on the law of large numbers where the RF chains are built with low-cost components in order to achieve economic production, hence it can be assumed that a few bits are reserved for CSI quantization. Considering that current and next generation channel estimation methods provide accurate estimates, it is highly probable that the dominant source of uncertainty in our model is the quantization errors. Taking into account that the quantization errors are bounded, it can be justified to adopt the worst case design model for imperfect CSI, which is based on the bounded channel error model [19].

In this paper we assume that the channel error statistics are known and we use a new capacity upper bound which accounts for the imperfect CSI. Considering the channel errors for each user k to be i.i.d. Gaussian entries, with zero mean and variance $\sigma_{e,k}^2$, which is customary for conventional channel estimation schemes [26], the new capacity upper bound is determined as

$$C_k \leq \log_2\left(\det\left[\mathbf{I}_k + \frac{1}{\sigma^2}E\left[\mathbf{h}_k^T\mathbf{h}_k + \sigma_{e,k}^2\mathbf{I}\right]\right]\right), \quad \forall k. \quad (9)$$

From equations (6) and (9), it can be seen that the channel errors reduce the effective capacity upper bound. This reduction in the capacity upper bound negatively affects the solution that is given by applying the max-min fairness algorithm. The same applies to other solution methods that make use of the rate upper bound, such as the ADMM algorithm.

III. FAST CONVERGING ROBUST BEAMFORMING FOR IMPERFECT CSI

The robust counterpart of any optimization problem is usually either intractable or more complex to solve. The resulting robust optimization problem becomes intractable because the set of constraints becomes infinite. "Commonly employed approximation schemes usually increase the complexity of the original problem by one degree; that is, a linear program becomes an SOCP, and an SOCP transforms to an SDP problem" [19]. Common solution methods are either SDP-based or SOCP-based. When dealing with the robust counterpart of any optimization problem, it is either difficult to represent in tractable form or once a tractable form is obtained, the complexity of the problem becomes much greater than the original problem. The significance of increased complexity when designing uncertainty immune beamforming matrices is further enhanced when some of the parameters involved in the system take up very large values as in the case of massive MIMO systems.

There are two main approaches to solving general convex optimization problems. The first approach makes use of a parser/solver, where the original problem is canonicalized to obtain an SOCP. The SOCP is then solved to provide a solution of the original problem. Examples of parser/solvers that use such an approach are CVX [6], YALMIP [30], and SeDumi [31]. These commonly used optimal solvers are not computationally efficient for problems with large dimensions, e.g. with massive antenna arrays at the BS. This is because the transformations required in order to deploy the algorithms onto embedded systems are time-consuming. The second approach makes use of a parser/generator combination, where the problem is analyzed in advance. The generator then generates a custom SOCP solver, and the parameters of the problem are mapped to the custom solver to give a solution. An example of a parser/generator which uses this approach is CVXGEN [32]. Although this approach reduces the transformation overhead of the aforementioned approach, the code generation step is also time-consuming and this makes it unsuitable for rapid prototyping [20]. Also, CVXGEN and similar frameworks, such as FORCES [33] and ACADO [34], are limited to solving quadratic programs.

In this paper, we follow the parser/generator approach of [20], where a problem family is canonicalized, and lightweight code for mapping the parameters into an SOCP is generated. The canonicalization is done using the Smith form reformulation [35] and the matrix stuffing technique is used for the mapping. We then adopt the ADMM to solve the resulting SOCP-based optimization problem as done in [22]. It has been shown that use of the matrix stuffing technique and the ADMM algorithm provides an efficient near-optimum solution of the weighted sum rate maximization (WSRMax) problem for macrocell users in a HetNet scenario [27]. The flexibility of the solution allows for easy scaling of the network structure for very large antenna arrays at the BS without much increase in computational overhead.

A. SOCP-BASED ROBUST OPTIMIZATION

We propose a robust beamforming design that is based on iteratively solving SOCPs. By not specifying any particular norm for the uncertainty set of the channel error vectors, the SOCP-based scheme is capable of handling a wide variety of uncertainties. We obtain a tractable robust formulation which incorporates uncertainty with second order cone constraints.

In order to obtain the robust counterpart of the WSRMax problem in (5), it is convenient to linearize the objective function and make it data independent [28]. To do so, we introduce a new variable t,which is minimized as the objective. The resultant robust formulation of (5) is written as

$$\begin{aligned} &\text{minimize } t \\ &\text{subject to } \|\mathbf{w}\|_2 \leq t \\ &\qquad\qquad \alpha_k \log_2(1+\gamma_k) \geq r, \quad \forall k \\ &\qquad\qquad \Sigma_{k\in B_u} \|\mathbf{w}_k\|_2^2 \leq P_b. \end{aligned} \quad (10)$$

In order to solve the robust beamforming optimization problem efficiently, with acceptable definitive optimal accuracy, we present a solution that uses the matrix stuffing technique and the ADMM algorithm. Such a solution converges to a near-optimal solution in a much shorter time because the structure of the transformed problem can be solved by parallel closed forms. The solution is based on a two-stage technique. The first stage initially transforms the optimization problem of (5) into an ADMM-compliant form, and in the second stage, the operator splitting method [22] is used by the ADMM solver to obtain a solution for the ADMM-compliant problem.

B. SOCP PROBLEM FORMULATION

The ADMM-compliant form of an optimization problem is the standard form of an SOCP, which is represented as

$$\begin{aligned} \mathscr{P}_{\text{ADMM}} : &\text{minimize } \mathbf{c}^T\mathbf{x} \\ &\text{subject to } \mathbf{Ax} + \mu = \mathbf{b} \\ &\qquad (\mathrm{x}, \mu) \in \mathbb{R}^n \times V, \end{aligned} \tag{11}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the optimization variable, $\mu \in \mathbb{R}^m$ denotes the slack variable, $V = \{0\}^r \times S^{m_1} \times \cdots \times S^{m_q}$ with S^p as the standard second-order cone of dimension $p \cdot \mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$, and $\mathbf{c} \in \mathbb{R}^n$.

1) SMITH FORM REFORMULATION

In order to transform the optimization problem in (5) into the standard ADMM-compliant form, we apply the Smith form reformulation, where a new variable is introduced for each sub-expression of the objective function and all the constraints. The resulting problem is convex if all expressions used are affine, and thus all equality constraints are relaxed in order to make them affine.

By introducing new variables, t_0 and $\mathbf{t}_1$, for the sub-expressions in the objective function, min $\|\mathbf{w}\|_2$, the representative Smith form is given as

$$\begin{aligned} &\text{minimize } t_0 \\ &\text{subject to } (t_0, \mathbf{t}_1) \in \mathbb{Q}^{N+1} \\ &\qquad \mathbf{t}_1 = \mathbf{w} \in \mathbb{R}^N, \end{aligned} \tag{12}$$

where $N = KM$.

For the per-antenna power constraint $\|\mathbf{w}_m\|_2 \leq \sqrt{P_m}$, the introduction of new variables, v_0 and $\mathbf{v}_1$, results in the representative formulation

$$\begin{aligned} (v_0^m, \mathbf{v}_1^m) &\in \mathbb{Q}^{K+1} \\ v_0^m &= \sqrt{P_m} \in \mathbb{R} \\ \mathbf{v}_1^m &= \mathbf{w}_m \in \mathbb{R}^K. \end{aligned} \tag{13}$$

With the aim to make the QoS constraint convex, i.e. for all sub-expressions to be affine, let $\theta_k = 2^{\gamma/\alpha_k} - 1$, and we have the expression

$$\frac{P_b \left|\mathbf{h}_k^T \mathbf{w}_k\right|^2}{\sigma^2 + P_b \Sigma_{i \neq k} \left|\mathbf{h}_i^T \mathbf{w}_i\right|^2 + P_{\text{SCA}} \Sigma_{i=1}^S \left|\check{g}_{ik}\right|^2} \geq \theta_k, \tag{14}$$

which is equivalent to

$$\|\mathbf{C}_k \mathbf{w} + \mathbf{q}_k\|_2 \leq \beta_k \mathbf{r}_k^T \mathbf{w}, \tag{15}$$

where $\mathbf{q}_k = [\mathbf{0}_K^T, \sigma_k]^T \in \mathbb{R}^{K+1}$, $\beta_k = \sqrt{1 + 1/\theta_k} \in \mathbb{R}$, $\mathbf{r}_k = [\mathbf{0}_{(K-1)M}^T, \mathbf{h}_k^T, \mathbf{0}_{(K-k)M}^T]^T \in \mathbb{R}^N$, and $\mathbf{C}_k$ is given by

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{h}_k^T & & \\ & \ddots & \\ & & \mathbf{h}_k^T \\ \hline & \mathbf{0}_N^T & \end{bmatrix} \in \mathbb{R}^{(K+1) \times N}. \tag{16}$$

By introducing new variables, y_0^k, $\mathbf{y}_1^k$, $\mathbf{y}_2^k$ and $\mathbf{y}_3^k$, the reformulated Smith form representation of (15) is given as

$$\begin{aligned} (y_0^k, \mathbf{y}_1^k) &\in \mathbb{Q}^{K+1} \\ y_0^k &= \beta_k \mathbf{r}_k^T \mathbf{w} \in \mathbb{R} \\ \mathbf{y}_1^k &= \mathbf{y}_2^k + \mathbf{y}_3^k \in \mathbb{R}^{K+1} \\ \mathbf{y}_2^k &= \mathbf{C}_k \mathbf{w} \in \mathbb{R}^{K+1} \\ \mathbf{y}_2^k &= \mathbf{q}_k \in \mathbb{R}^{K+1}. \end{aligned} \tag{17}$$

The resultant relaxed Smith form reformulation of the original problem in (5) is represented as

$$\begin{aligned} &\text{minimize } t_0 \\ &\text{subject to } \mathcal{G}_0, \ \mathcal{G}_1(m), \ \mathcal{G}_2(k), \quad \forall k, m, \end{aligned} \tag{18}$$

where $\mathcal{G}_0$ is the relaxed Smith form representation of the objective function, which is given by

$$\mathcal{G}_0 : \left\{ \begin{array}{l} (t_0, \mathbf{t}_1) \in \mathbb{Q}^{N+1} \\ \mathbf{t}_1 = \mathbf{w} \in \mathbb{R}^N \end{array} \right\}. \tag{19}$$

$\mathcal{G}_1(m)$ is the relaxed Smith form representation of the power constraint given as

$$\mathcal{G}_1(m) : \left\{ \begin{array}{l} \left(v_0^m, \mathbf{v}_1^m\right) \in \mathbb{Q}^{K+1} \\ v_0^m = \sqrt{P_b} \in \mathbb{R} \\ \mathbf{v}_1^m = \mathbf{w}_m \in \mathbb{R}^K \end{array} \right\}, \tag{20}$$

and $\mathcal{G}_2(k)$ represents the relaxed Smith form reformulation for the QoS constraint of MUE$_k$, given as

$$\mathcal{G}_2(k) : \left\{ \begin{array}{l} \left(y_0^k, \mathbf{y}_1^k\right) \in \mathbb{Q}^{K+1} \\ y_0^k = \beta_k \mathbf{r}_k^T \mathbf{w} \in \mathbb{R} \\ \mathbf{y}_1^k = \mathbf{y}_2^k + \mathbf{y}_3^k \in \mathbb{R}^{K+1} \\ \mathbf{y}_2^k = \mathbf{C}_k \mathbf{w} \in \mathbb{R}^{K+1} \\ \mathbf{y}_3^k = \mathbf{q}_k \in \mathbb{R}^{K+1} \end{array} \right\}. \tag{21}$$

The standard SOCP representation of the reformulated problem is represented by the vector of variables $\mathbf{x}$, which includes the original variables and the new variables, and the problem data, which are defined in $\mathbf{A}$, $\mathbf{b}$, and $\mathbf{c}$. The optimization variables are given by

$$\mathbf{x} = \left[t_0; v_0^1; \ldots; v_0^M; y_0^1; \ldots; y_0^K; \mathbf{w}\right] \in \mathbb{R}^n, \tag{22}$$

and the vector of coefficients is consequently given as $\mathbf{c} = [1; \mathbf{0}_n - 1]$. The SOCP problem's structure is characterized by the cone dimensions and the representation of the problem data. The cone dimensions of the reformulated problem, n, m, and V, are given by

$$n = 1 + M + K + N, \tag{23}$$
$$m = (M + K) + (M + 1) + M(K + 1) + K(K + 2), \tag{24}$$
$$V = (\mathbb{Q}^1)^{M+K} \times \mathbb{Q}^{N+1} \times (\mathbb{Q}^{K+1})^L \times (\mathbb{Q}^{K+2})^K, \tag{25}$$

where V is the Cartesian product of $2(M + K) + 1$ closed convex cones, and $\mathbf{A} \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^{m\times 1}$ are given as

$$\mathbf{A} = \left[\begin{array}{ccccccc|c} 1 & & & & & & & \\ & \ddots & & & & & & \\ & & 1 & & & & & \\ \hline & & & 1 & & & & -\beta_1 \mathbf{r}_1^T \\ & & & & \ddots & & & \vdots \\ & & & & & & 1 & -\beta_k \mathbf{r}_k^T \\ \hline -1 & & & & & & & -\mathbf{I}_N \\ \hline & -1 & & & & & & -\mathbf{I}_K^m \\ \hline & & \vdots & & & & & \vdots \\ \hline & & -1 & & & & & -\mathbf{I}_K^M \\ \hline & & & -1 & & & & -\mathbf{C}_1 \\ \hline & & \vdots & & & & & \vdots \\ \hline & & & & & & -1 & -\mathbf{C}_K \end{array}\right], \tag{26}$$

where the number of columns of $\mathbf{A}$ is equal to the total number of variables, i.e. the original variables plus the new variables introduced by the reformulation. The rows of $\mathbf{A}$ are composed of the atoms of the reformulation, where each atom is a block that is underlined.

$$\mathbf{b} = \left[\begin{array}{c} \sqrt{P_1} \\ \vdots \\ \sqrt{P_M} \\ \hline 0 \\ \vdots \\ 0 \\ \hline \mathbf{0}_N \\ \hline 0 \\ \mathbf{0}_K \\ \hline 0 \\ \mathbf{q}_1 \\ \hline \vdots \\ \hline 0 \\ \mathbf{q}_K \end{array}\right]. \tag{27}$$

2) MATRIX STUFFING

The canonicalization procedure carried out through the Smith form reformulation simply creates the SOCP structure of the original problem. For a given set of the HetNet's parameters, i.e. the number of BS antennas, MUEs, and SUEs, the ADMM-compliant problem's structure is fixed. Thus the structures of $\mathbf{A}$, $\mathbf{b}$, $\mathbf{c}$ and the description of V can be generated and stored offline, since the number of BS antennas, MUEs, and SUEs can be assumed to remain constant for a long period.

The problem data are then copied to the corresponding data in $\mathscr{P}_{\text{ADMM}}$ for any specific network realization. This comprises the maximum per-antenna transmit power of the macrocell BS, i.e., $\sqrt{\text{P}_m}$'s in $\mathbf{b}$, the per-user SINR thresholds γ_k, i.e., β_k's in $\mathbf{A}$, and the channel realizations $\mathbf{h}_k$'s, i.e., $\mathbf{r}_k$'s and $\mathbf{C}_k$'s in $\mathbf{A}$. This is the so called matrix stuffing technique, which provides significantly faster transformation compared to the modelling framework of CVX.

C. ADMM-BASED SOLUTION

To solve the formulated standard SOCP problem, we apply a first-order method for solving very large cone programs. The method solves the homogeneous self-embedding of a primal-dual pair of the optimization problem by using an operator splitting method known as the ADMM algorithm. This approach scales favorably to convex conic problems with very large dimensions, and it is well suited to distributed antenna setup since it can be applied in parallel across multiple processors [24]. This first-order method can provide reliable, modestly accurate solutions in a relatively efficient manner because of significantly faster convergence compared to the interior-point methods, at the cost of lower accuracy.

1) HOMOGENEOUS SELF-DUAL EMBEDDING

The premise of homogeneous self-dual embbeding is to encode the primal and dual pair of an optimization problem into a single feasible problem. This entails finding a feasible (non-zero) point in the intersection of a convex set and a subspace. A non-zero solution of the original pair is taken as the optimal solution, otherwise a certificate of infeasibility is generated that proves that either the primal or dual is infeasible.

The primal and dual pair, $\mathscr{P}_{\text{ADMM}}$ and $\mathscr{D}_{\text{ADMM}}$, of the original problem is given as

$$\begin{aligned} \mathscr{P}_{\text{ADMM}}:\ & \text{minimize } \mathbf{c}^T\mathbf{x} \\ & \text{subject to } \quad \mathbf{A}\mathbf{x} + \mu = \mathbf{b} \\ & \qquad\qquad\quad (\mathbf{x}, \mu) \in \mathbb{R}^n \times V \\ \mathscr{D}_{\text{ADMM}}:\ & \text{minimize } -\mathbf{b}^T\boldsymbol{\eta} \\ & \text{subject to } \quad -\mathbf{A}^T\boldsymbol{\eta} + \boldsymbol{\lambda} = \mathbf{c} \\ & \qquad\qquad\quad (\boldsymbol{\eta}, \boldsymbol{\lambda}) \in \{0\}^n \times V^*, \end{aligned} \tag{28}$$

where $\boldsymbol{\eta} \in \mathbb{C}^m$ is the dual's optimization variable, $\boldsymbol{\lambda} \in \mathbb{C}^n$ being the dual slack variable. V^* is the dual cone of the non-empty closed convex cone V and $\{0\}^n$ is the dual cone of $\mathbb{R}^n$.

In order to solve the problem optimally, the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient, and they require strong duality, i.e. when

$$\mathbf{A}\mathbf{x} + \boldsymbol{\mu} = \mathbf{b}, \tag{29}$$
$$\mathbf{A}^T\boldsymbol{\eta} + \mathbf{c} = \boldsymbol{\lambda}, \tag{30}$$
$$\mathbf{c}^T\mathbf{x} + \mathbf{b}^T\boldsymbol{\eta} = \mathbf{0}, \tag{31}$$

with $(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\eta}) \in \mathbb{R}^n \times V \times \{0\}^n \times V^*$ satisfying the KKT conditions and being primal-dual optimal. The primal-dual pair of (28) is converted into a single problem by embedding the KKT conditions into a single system of equations and including the optimal points that the primal and dual must jointly satisfy. This embedding gives

$$\begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{A}^{\mathrm{T}} \\ -\mathbf{A} & \mathbf{0} \\ \mathbf{c}^{\mathrm{T}} & \mathbf{b}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\eta} \end{bmatrix} + \begin{bmatrix} \mathbf{c} \\ \mathbf{b} \\ \mathbf{0} \end{bmatrix}, \quad (\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\lambda}, \boldsymbol{\eta}) \in \mathbb{R}^n \times V \times \{0\}^n \times V^*. \tag{32}$$

In this case, if equation (28) is primal or dual infeasible, then equation (32) has no solution. Homogeneous self-dual embedding addresses this shortcoming by introducing two new non-negative variables τ and κ, which encode the different possible outcomes of the solution [36]. The homogeneous self-dual embedded system of equations is then given as

$$\begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \kappa \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{A}^T & \mathbf{c} \\ -\mathbf{A} & 0 & \mathbf{b} \\ -\mathbf{c}^T & -\mathbf{b}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\eta} \\ \tau \end{bmatrix}. \tag{33}$$

The system in equation (33) is homogeneous because if $(\mathbf{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\lambda}^*, \boldsymbol{\eta}^*)$ is a solution to the embedded problem, then $(a\mathbf{x}^*, a\boldsymbol{\mu}^*, a\boldsymbol{\lambda}^*, a\boldsymbol{\eta}^*)$ is also a solution for any $a \geq 0$. The embedded problem is also self-dual, and the proof is shown in [22]. To simplify the representation of the embedded problem, let

$$\mathbf{r} = \begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\mu} \\ \kappa \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 0 & \mathbf{A}^T & \mathbf{c} \\ -\mathbf{A} & 0 & \mathbf{b} \\ -\mathbf{c}^T & -\mathbf{b}^T & 0 \end{bmatrix}, \ \mathbf{p} = \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\eta} \\ \tau \end{bmatrix},$$

where $(\mathbf{p}, \mathbf{r}) \in \mathbb{R}^{n+m+1}$, $\mathbf{Q} \in \mathbb{R}^{(n+m+1)\times(n+m+1)}$. The embedded problem is then given as

$$\begin{aligned} &\text{find } (\mathbf{p}, \mathbf{r}) \\ &\text{subject to } \ \mathbf{r} = \mathbf{Q}\mathbf{p} \\ &\qquad\qquad (\mathbf{p}, \mathbf{r}) \in \mathbb{C}\times\mathbb{C}^* \end{aligned} \tag{34}$$

where $\mathbb{C} = \mathbb{R}^n \times V^* \times \mathbb{R}^+$ is a cone with dual cone $\mathbb{C}^* = \{0\}^n \times V \times \mathbb{R}^+$.

2) ADMM ALGORITHM

The ADMM algorithm is a first-order method for solving optimization problems, which is based on the operator splitting method [37]. The method is well-suited for large-scale convex optimization problems. The algorithm has been adopted to solve optimization problems in cloud computing environments because it can handle complex problems fairly well, while it is scalable enough to process data with large parameters. We propose to adopt this algorithm to solve the homogeneous self-dual embedded problem for robust beamforming optimization for macrocell users in a typical HetNet.

The basic operator splitting method solves convex problems of the form

$$\begin{aligned} &\text{minimize } [f(y) + g(z)] \\ &\text{subject to } Ay + Bz = c, \end{aligned} \tag{35}$$

where $A \in \mathbb{R}^{p\times n}$, $B \in \mathbb{R}^{p\times m}$, and the variable of the original problem $x \in \mathbb{R}^n$, is split into two parts; that is $y \in \mathbb{R}^n$ and $z \in \mathbb{R}^n$ in this case, provided the objective function of the original problem is separable across the splitting. It is assumed that f and g are convex, and they may be nonsmooth or may take on infinite values to encode implicit constraints.

The iterations of the ADMM algorithm consist of the steps

$$y^{k+1} = \text{argmin } L_\rho(y, z^k, d^k), \tag{36}$$
$$z^{k+1} = \text{argmin } L_\rho(y^{k+1}, z, d^k), \tag{37}$$
$$d^{k+1} = d^k + \delta(Ay^{k+1} + Bz^{k+1} - c), \tag{38}$$

where d is the dual variable, and $\delta > 0$ is a penalty multiplier of the augmented Lagrangian, L_ρ. The initial points z^0 and d^0 are arbitrary, but are usually taken to be zero. The algorithm comprises a y-minimization step, a z-minimization step, and a step to update the dual variable. This is basically similar to the method of multipliers, which takes the form

$$(y^{k+1}, z^{k+1}) = \text{argmin } L_\rho(y, z^k, d^k), \tag{39}$$
$$d^{k+1} = d^k + \delta(Ay^{k+1} + Bz^{k+1} - c). \tag{40}$$

Unlike this method of multipliers where the augmented Lagrangian for the two variables is jointly minimized as in (39), in the ADMM the variables are updated in an alternating manner, as given in (36) and (37). Hence the term *alternating direction* in ADMM.

In order to apply the operator splitting method, the embedded problem in (34) is transformed into the form

$$\begin{aligned} &\text{minimize } I_{\mathbb{C}\times\mathbb{C}^*}(p, r) + I_{Q\tilde{x}}(\tilde{p}, \tilde{r}) \\ &\text{subject to } (p, r) = (\tilde{p}, \tilde{r}), \end{aligned} \tag{41}$$

where I_S denotes the indicator function of the set S.Directly applying the ADMM to the self-dual embedding in (41) yields the following algorithm:

$$\begin{aligned} (\tilde{p}^{k+1}, \tilde{r}^{k+1}) &= \prod\nolimits_{Qp=r} (p^k + \lambda^k, r^k + \mu^k) \\ p^{k+1} &= \prod\nolimits_{\mathbb{C}} (\tilde{p}^{k+1} - \lambda^k) \\ r^{k+1} &= \prod\nolimits_{\mathbb{C}}^{*} (\tilde{r}^{k+1} - \mu^k) \\ \lambda^{k+1} &= \lambda^k - \tilde{p}^{k+1} + p^{k+1} \\ \mu^{k+1} &= \mu^k - \tilde{r}^{k+1} + r^{k+1}, \end{aligned} \tag{42}$$

where $\Pi_S(x)$ denotes the Euclidean projection of x onto the set S, and λ and μ are dual variables for the equality constraints on p and r, respectively.

IV. NUMERICAL RESULTS

We evaluate the performance of the proposed solution method for the SINR balancing problem in terms of the average worst case SINR for all the MUEs. For all simulation setups, we consider a single cell of a typical HetNet, where the macrocell is overlaid with four small cells ($S = 4$), each serving one user within 10 m. Our focus is on evaluating the performance of the macrocell users, which are uniformly distributed within the macrocell with a radius of 60 m. We assume the channels to exhibit small-scale Rayleigh fading, and we model them similar to Case 1 for heterogeneous deployments in the 3GPP LTE standard [38].

We compare the performance of the solution method, which uses the MS technique and the ADMM algorithm to solve the convex optimization problem, with an optimal solution that uses CVX and the BRB algorithm. We also compare the performance of the optimization methods with ZFBF and MMSE precoding, which are prominent linear precoding techniques. To implement the ADMM, we use the splitting conic solver (SCS) [22] toolbox in Matlab, whereas the solution method that uses the BRB algorithm is implemented with the SeDuMi solver. We consider the efficiency of the proposed solution in terms of SINR maximization accuracy and average time taken to converge to a solution compared to the optimal solution.

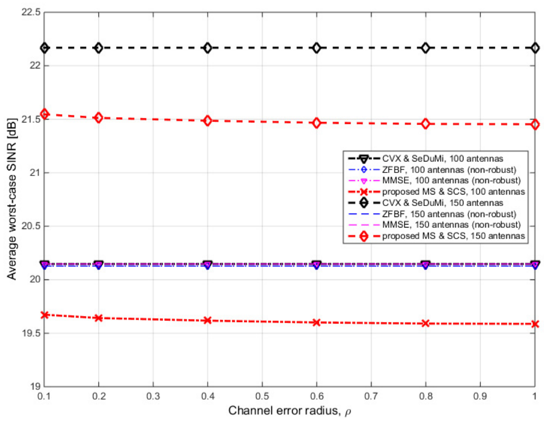

FIGURE 2. **Average worst-case SINR performance against error uncertainty set radius, ρ, bounded by l_2-norm for number of macrocell users, $K = 20$, and BS power, $P_b = 1$dB.**

A. SIMULATION RESULTS FOR PERFECT CSI

Fig. 2 shows plots of the average worst case SINR against the radius of the channel errors' uncertainty set, ρ. The figure depicts the accuracy and robustness performance of the proposed solution, which uses MS and SCS, by comparing it to the aforementioned methods. The worst case SINR was averaged over 10 channel realizations with the error uncertainty bounded by the l_2-norm, for $K = 20$ MUEs, and the transmit power for the BS set as 1 dB. It can be seen from the result of Fig. 2 that for the case of $M = 100$ antennas at the BS, the solution method that uses the MS technique and the SCS solver is outperformed by the optimal method, which uses SeDuMi and CVX, and the solutions that use ZFBF and MMSE.

From Fig. 2 it can be observed that the worst-case SINR performance of all the solution methods, in terms of the average worst case SINR value, increases as M increases. For the case of $M = 150$ antennas, the performance of the MS and SCS method improves to be fairly close to the performance of the optimal method which uses CVX and SeDuMi. It is also worth noting that the MS and SCS method outperforms the ZFBF and MMSE solutions in this case of $M = 150$, unlike in the case of $M = 100$. As M was increased to 200 and beyond, the simulations showed this trend of the performance of the MS and SCS method closely approaching that of CVX and SeDuMi, while the gap between these two optimization solutions and the two linear precoding methods increased.

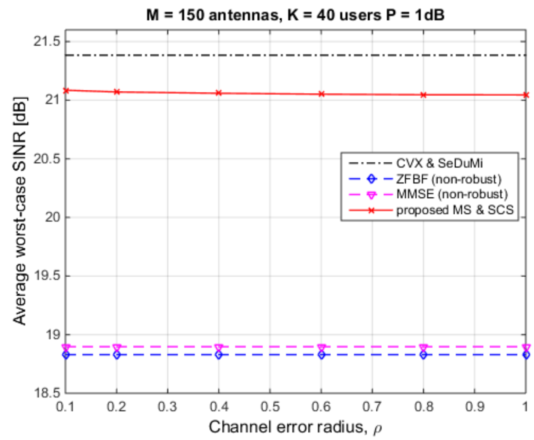

FIGURE 3. **Average worst-case SINR performance against error uncertainty set radius, ρ, bounded by l_2-norm for number of BS antennas, $M = 150$, number of macrocell users, $K = 40$, and BS power, $P_b = 1$dB.**

Fig. 3 shows the effect of increasing the number of users while the number of BS antennas remains 150 as in the case of Fig. 2. A comparison of the result of Fig. 2 and the the result of Fig. 3 shows that the average worst-case SINR performance of all the beamforming methods diminishes. The trend, however, remains the same with the MS and SCS method significantly outperforming the ZFBF and MMSE solutions, but being outperformed by the CVX and SeDuMi method.

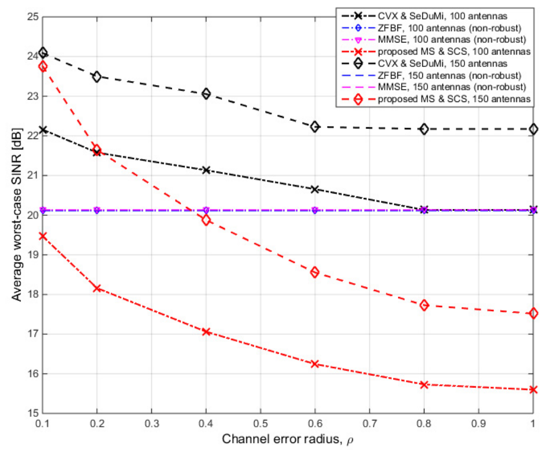

FIGURE 4. **Average worst-case SINR performance against error uncertainty set radius, ρ, bounded by l_∞-norm for number of macrocell users, $K = 20$, and BS transmit power, $P_b = 1$dB.**

B. SIMULATION RESULTS FOR IMPERFECT CSI

Fig. 4 shows the result of evaluating the performance of the proposed solution method in terms of robustness to the channel error uncertainty. In this case, the channel error vectors are bounded within a multi-dimensional box of size ρ. That is, $\|\mathbf{e}_k\|_\infty \leq \rho$, and in this case the channel error uncertainty is said to be bounded by the l_∞-norm. It is observed that the average worst case SINR performance for both the MS and SCS method, and the CVX and SeDuMi method is lower than that for the case when the error uncertainty set radius is bounded by the l_2-norm, for the same value of ρ. This is because the l_∞-norm defines a smaller feasible set than the l_2-norm for the same ρ [19], which leads to the degradation in performance. This also explains the reduced robustness of the two optimization methods for this case of l_∞-norm, which is observed from the greater range of worst-case SINR variation with ρ.

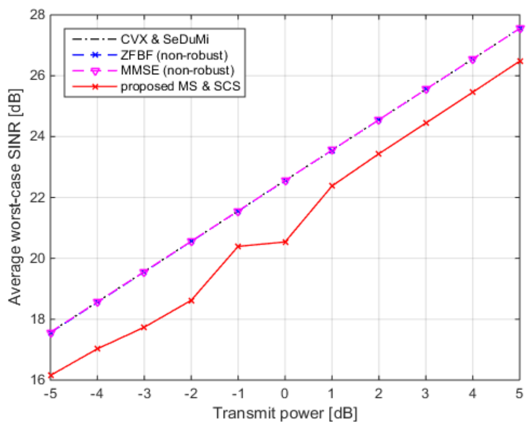

FIGURE 5. Average worst-case SINR performance against BS transmit power for number of BS antennas, $M = 150$, number of macrocell users, $K = 20$, and error uncertainty set radius, $\rho = 0.8$, bounded by l_2-norm.

We also evaluated how the average worst-case SINR scales with the BS transmit power, P_b. The performance of the proposed solution method was compared with that of the optimal method together with the other two non-robust methods, as shown in Fig. 5. From Fig. 5, it can be seen that the SINR performance of the MS and SCS solution method increases linearly, similar to that of the ZFBF, MMSE and the CVX and SeDuMi methods, although with reduced accuracy. The SINR performance of the MS and SCS method does not show a perfectly linear variation with BS transmit power. This is due to the limited number of channel realizations that were carried out for the simulation.

The accuracy of the MS and SCS solution method improves significantly as the number of BS antennas, M, is increased, to achieve near optimum performance of the other beamforming methods. This was observed when M was increased to 200 and beyond, where the gap between the performance of the proposed method and the other beamforming methods reduced as M increased.

C. ALGORITHM CONVERGENCE EFFICIENCY

Fig. 6 is a result of an investigation of how the average worst-case SINR scales with the number of antennas at the BS. For this simulation, a varying number of antennas were serving $K = 20$ macrocell users, with the transmit power, P_b set

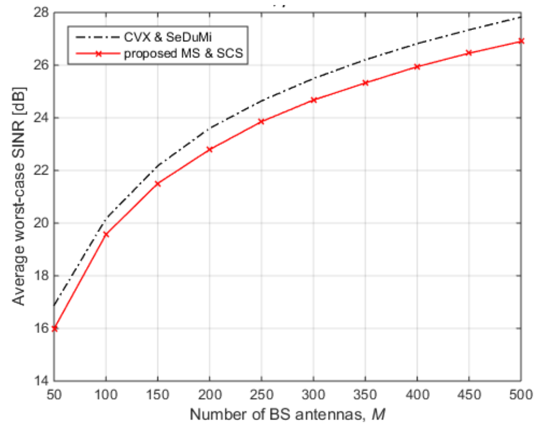

FIGURE 6. Average worst-case SINR performance for varying number of BS antennas, with error uncertainty set radius, $\rho = 0.4$, bounded by l_2-norm for number of macrocell users, $K = 20$, and BS power, $P_b = 1$dB.

to be 1dB, and the uncertainty set radius, ρ, being 0.4 for channel errors bounded by the l_2-norm. It can be seen that the MS and SCS method is outperformed by the ZFBF, MMSE, and the CVX and SeDuMi methods for the number of BS antennas considered, although it closely follows their average worst-case SINR. Analysis of the efficiency of the proposed method, with comparison to the optimal method which uses CVX and SeDuMi, was carried out and the results are given in Table 1.

TABLE 1. Comparison of SINR performance and convergence efficiency for the two optimization solutions for various BS antenna cases.

No. of BS Antennas	SINR [dB]		Convergence time [s]		Local growth order	
	CVX + BRB	MS + SCS	CVX + BRB	MS + SCS	CVX + BRB	MS + SCS
50	16.8934	15.9472	27.3516	1.4780		
100	20.1081	19.6042	38.6138	2.0304	0.4974	0.4581
150	22.2393	21.5002	47.7635	4.5101	0.5244	1.9683
200	23.5986	22.8178	58.5188	8.7485	0.7059	2.3030
250	24.6431	23.8083	71.6505	14.4775	0.9072	2.2573
300	25.5000	24.6085	77.3890	20.8652	0.4225	2.0046
350	26.2029	25.3361	88.5346	28.5974	0.8728	2.0449
400	26.7980	25.8784	104.080	36.0553	1.2114	1.7354
450	27.3167	26.4306	119.968	44.3111	1.2061	1.7505
500	27.8233	26.9173	137.292	52.5670	1.2802	1.6215

Table 1 shows the results of run-time analysis of the two optimization solutions, i.e. the CVX and SeDuMi method, and the MS and SCS method. The table gives a comparison of the maximum achievable worst-case SINR and the time taken to converge to a solution for various numbers of antennas at the BS. The convergence time includes both the time taken to transform the original problem and the time taken by the solver to reach a solution. For this comparison, the number of MUEs, K, was set to be 20 as the number of antennas at the BS was varied. The simulations were executed on a

64-bit Intel CORE i5 desktop computer with 8 GB RAM. The bisection procedure for both solution methods was set to terminate when the difference between the objective values of two bisection steps, $\epsilon \leq 0.001$. The maximum number of iterations was set to be 2500 for both optimal solvers. It is seen from Table 1 that the MS and SCS method converges more than twice faster, with the average worst case SINR less than 1 dB in all cases, compared to the solution that uses CVX and SeDuMi.

Since algorithms are platform-independent and machine-independent, we also analyzed the run-time efficiency of the two methods by considering the local growth order of the solutions, in addition to the convergence time. For run times t_1 and t_2, with their corresponding input parameters n_1 and n_2, the local order of growth, o, is given by

$$o = \frac{\log(t_2/t_1)}{\log(n_2/n_1)} \tag{43}$$

The local growth order gives an indication of how the run-time of an algorithm increases as the input parameter increases. In this case, the input parameter is the number of antennas at the BS, and the local growth orders of the two optimization solutions for the varying number of BS antenna cases are also given in Table 1. It should be noted that owing to the computation of the local growth order as given in equation (43), the first row of the local growth order fields do not contain any values.

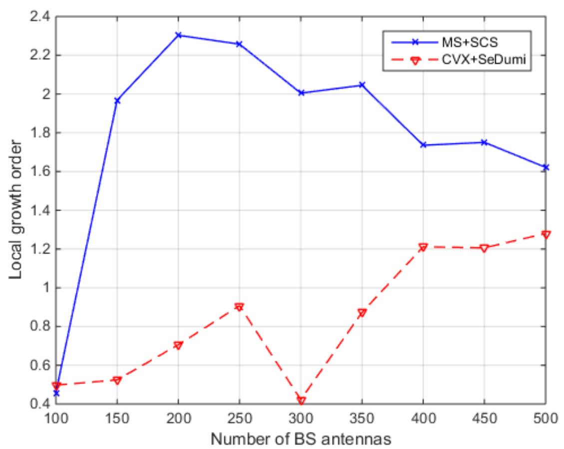

FIGURE 7. Variation of local growth order with increase in number of BS antennas for the two optimization algorithms.

To obtain a clearer picture of the trends in the local growth orders of the two solution methods, the results in Table 1 are illustrated in Fig. 7. The figure shows that the growth orders of both solution algorithms initially show an increasing trend as the number of BS antennas, M, increases. Although the local growth order of the MS and SCS method initially shows a higher rate of increase, it begins to show a decreasing trend as M increases beyond 250. The CVX and SeDuMi method, on the other hand, continues to show an increasing trend for the local growth rate as M increases.

V. CONCLUSION

A fast converging robust beamforming solution for the SINR balancing problem, that accounts for CSI acquisition errors at the transmitter for macrocell users in a HetNet configuration, is proposed and presented. This solution is based on applying the matrix stuffing technique and the ADMM algorithm to transform and solve the original optimization problem. Simulation results show that this method gives average worst-case SINR performance with accuracy averaging 97% for all cases considered compared to the optimal method which uses CVX and the BRB algorithm, while converging to a solution more than twice faster than the interior-point method. In addition, the growth order of the ADMM algorithm's solution generally decreases with an increasing number of antennas at the macrocell BS. This makes the proposed beamforming method attractive for practical application in HetNets with large antenna arrays at the macrocell BS in scenarios with channel uncertainty.

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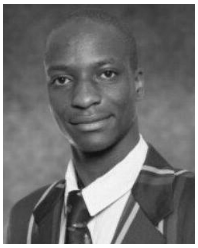

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