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Finite element fracture modelling of concrete gravity dams

Q Cai, J M Robberts and B W J van Rensburg

A smeared crack model, based on non-linear fracture mechanics, was developed which allows for either linear or bilinear softening and assumes shear retention dependent on the strain normal to a crack. A mesh objectivity verification study proves that the proposed crack modelling method is mesh objective. The crack model and its computational procedure is verified for a benchmark concrete gravity dam model and an existing concrete gravity dam by comparing the results with those of numerical investigations obtained by other researchers. Furthermore, an existing concrete gravity dam in South Africa is analysed and evaluated with regard to dam safety in terms of the maximum overflow level. A higher imminent failure flood is predicted in the analysis than that obtained by classical strength-based methods. The study proves the usefulness and applicability of the proposed crack model and implementation procedure in predicting crack response and evaluating the safety of concrete gravity dams. A sensitivity study on the material fracture properties and fracture parameters is included for the purpose of investigating the uncertainties often encountered in this type of analysis. The influence of the fracture properties and parameters on the cracking response and the overall structural behaviour is discussed.

INTRODUCTION

As a result of the low tensile resistance of concrete, cracking in concrete dams is a common phenomenon. The accurate prediction and evaluation of crack propagation, and the associated structural response, is important and necessary to ensure dam safety by providing suitable safety margins for crack development in a dam. Concrete gravity dams are generally subjected to both flexure and shear loadings, which induce mixed-mode fracture. This coexistence of crack opening (mode I) and in-plane crack sliding (mode II) influences and complicates prediction of the strain-softening response. (See for example Karihaloo 1995 for a background to fracture mechanics.)

A strain-softening cracking model and a computational procedure for implementing the constitutive model in a finite element (FE) program have been developed by the authors. The objective of this paper is to verify the proposed crack model and to demonstrate the applicability of the crack analysis procedure that was developed in determining the fracture response and safety evaluation of large concrete structures such as gravity dams.

Development of methods for analysing cracking in concrete dams

Using the conventional design methodology, concrete dams are usually designed to have 'no tension' in any part of the dam for normal service loads and to experience only minimum tensile stresses during extreme loading cases. The work of Bažant (1990) reveals that if the size of the dam exceeds a certain limit, the apparently conservative 'no tension' design cannot always be regarded as safe.

The rigid body equilibrium, strength-based criterion was initially adopted where it was assumed that a crack would propagate whenever the principal tensile stress at the crack tip exceeds the specified tensile strength of the concrete. This was the only criterion for determining crack growth in concrete dams before the late 1970s (Saouma *et al* 1990).

The strength-based criterion for crack analysis of concrete dams is based on the assumptions that there is a linear distribution of compressive stresses in the uncracked concrete, and that a crack will propagate horizontally in a plane and extend up to a point where the tensile stress becomes zero. This method of cracking analysis has the following shortcomings:

- The shear stress cannot be taken into account
- Strictly speaking, this theory applies to shallow beams and cannot be applied to concrete dams which are clearly 'deep beams' with base width-to-height ratios of 0,75 to 1
- The stress singularity at the tip of crack cannot be taken into account

Furthermore, in an FE analysis the strength-based criterion can cause the results to be

QINGBO CAI was associated with the Department of Civil Engineering of the University of Pretoria at the time this paper was produced. He is a deputy chief engineer in Directorate: Options Analysis at Department of Water Affairs and Forestry. He obtained his BEng (Civil) from the Southwest Jiao-Tong University in the People's Republic of China, an MSc (Structural Engineering) from the Northern Jiao-Tong University in the People's Republic of China, and a PhD from the University of Pretoria. Over the last 20 years, he has been involved in structural design of various buildings and research on railway prestressed concrete poles and sleepers, and advanced finite element analysis of civil and mechanical structures such as dams and pipes.

Contact details:
759 Lorna Street,
Moreleta Park Ext 10
Pretoria, 0044
T 012-336-8096 (w)
C 082-869-8323
DBE@dwaf.gov.za

JOHN ROBBERTS was associated with the Department of Civil Engineering of the University of Pretoria at the time this paper was produced. John Robberts obtained his BEng (Civil) in 1986 from the University of Pretoria and then joined this institution as lecturer in Structural Engineering. He obtained his MEng (Structural Engineering) from the University of Pretoria, and his PhD from Imperial College London. John also worked at the Atkins bridge office in Epsom (UK) and PBMR (Pretoria). John is currently a director of Nuclear Structural Engineering (Pty) Ltd. He remains involved with the University of Pretoria, as a part-time lecturer, supervising research students and presenting postgraduate courses in Reinforced and Prestressed Concrete.

Contact details:
Nuclear Structural Engineering (Pty) Ltd
Balblair Building, Kildrummy Office Park,
Cnr Witkoppen Road & Umhlanga Road
Paulshof, Johannesburg
T 012-319-7256
C 082-784-8743
john.robberts@nucse.com

BEN VAN RENSBURG is a professor in the in the Department of Civil Engineering at the University of Pretoria where he teaches and does research in structural engineering. He started his career in consulting engineering, worked in a research organization, and subsequently joined the University of Pretoria. He obtained BSc and MSc degrees in Civil Engineering from the University of Pretoria, an MSc (Structural Engineering) from the University of Southampton and a PhD (Civil Engineering) from the University of Pretoria.

Contact details:
Department of Civil Engineering
University of Pretoria
Pretoria, 0002
T 012-420-2439
ben.vanrensburg@up.ac.za

***Table 1** Past investigations into cracking in gravity dams*

Dam	Method	Reference
Fontana gravity dam	LEFM model	Chappell & Ingraffea 1981
Gravity dams	Discrete LEFM model	Ayari 1988
Fontana gravity dam	Mixed-mode LEFM discrete model	Ingraffea 1990
Gravity dams	LEFM model	Linsbauer 1990
Mequinenza gravity dam	2-D & 3-D elastic-fracturing model	Cervera *et al* 1990
Koyna gravity dam	Mixed-mode LEFM discrete model	Gioia *et al* 1992
Lakhwar gravity dam	Discrete LEFM model	Kumar & Nayak 1994
Koyna gravity dam	Smeared NLFM model	Bhattacharjee & Leger 1994
Koyna gravity dam	Rotating smeared NLFM model	Bhattacharjee & Leger 1995
Gravity dam models	LEFM model	Plizzari *et al* 1995
Koyna gravity dam, dam model	Damage mechanics	Ghrib & Tinawi 1995
Gravity dams	LEFM model	Plizzari 1997
Greyrock dam	LEFM & NLFM interface model	Saouma & Morris 1998
Tucurui gravity dam	Discrete NLFM model, damage theory	Araújo & Awruch 1998
Gravity dam models	Cohesive crack model	Barpi & Valente 2001
Gravity dam	Crack-embedded elements model	Horii & Chen 2003
Gravity dam models	Extended fictitious crack model	Shi *et al* 2003

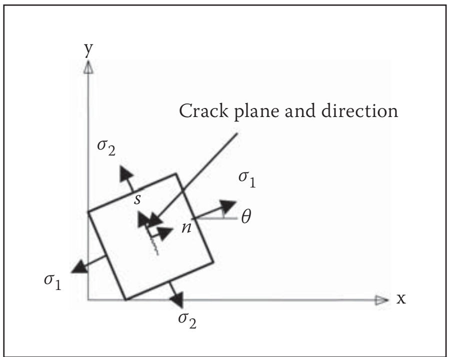

***Figure 1** 2-D Local and global axes*

mesh unobjective, that is, the stresses become progressively larger as the mesh around the crack tip is refined. Because of this, strength-based models are unsuitable for the FE modelling of cracking in concrete structures.

Non-linear FE analysis using material plasticity models such as Drucker-Prager, Mohr-Coulomb (see, for example, Owen & Hinton 1980) and contact simulation of cracking are also often adopted to predict cracking in concrete dams. However, the above non-linear analysis methods can only give a rough idea of where the dams have yielded and of the possible areas of cracking.

Based on energy principles, the fracture mechanics approach is a rational technique for analysing the development and propagation of cracks in concrete structures. The application of fracture mechanics in modelling the cracking process of concrete dams and evaluating dam safety has generated a great deal of interest, as demonstrated in the next section. During the past decades, linear elastic fracture mechanics (LEFM) have been widely used for the analysis of concrete dams, in particular gravity dams. Due to the existence of a fracture process zone (FPZ) at the front of the crack tip (Bažant & Oh 1983), models based on non-linear fracture mechanics (NLFM) should, strictly speaking, be adopted. Currently, NLFM has gained recognition among researchers and has become the main trend for fracture analysis of concrete dams.

Past investigations into static analysis of cracking in concrete gravity dams

Many attempts have been made to model and investigate cracking in concrete gravity dams, using a variety of analysis methods (Cai *et al* 2004 and Cai 2007). Some prior investigations and the methods used are summarised in table 1.

CONSTITUTIVE CRACKING MODEL AND IMPLEMENTATION

The proposed cracking model and the subprogram developed for implementation into a general-purpose FE package (MSC. Marc) have been presented in detail by Cai *et al* (2006) and will only be discussed briefly here. The standard smeared cracking approach may suffer from problems, for instance finite element bias with regard to crack alignment (orientation) and spacing. New developments on improved regularisation methods which include rate-dependent formulations have been put forward (Van Zijl *et al* 2001).

A linear elastic stress-strain relationship in compression and tension, prior to cracking, is assumed. A crack is assumed to occur when the maximum principal stress σ_1 exceeds the concrete tensile strength f_t at a Gauss point. The crack plane is perpendicular to the direction of the maximum principal stress (see figure 1).

Constitutive relationship during cracking

The *multi-directional crack model* proposed by De Borst and Nauta (1985) and Rots (1988), which is well established, is adopted and has the following main features: a new crack will be initiated whenever the angle between the normal to the crack plane of the last crack and the current principal stress direction exceeds a pre-defined threshold angle *or* the inclined maximum principal stress σ_1 violates the crack onset criterion. To limit the required computing memory and to make the multi-directional crack model more robust, a maximum of six cracks are allowed to form at a Gauss point.

The overall relationship between incremental global stress and strain is as follows:

$$\Delta\sigma = \left\{D^{co} - D^{co}N\left[D^{cr} + N^T D^{co} N\right]^{-1} N^T D^{co}\right\}\Delta\varepsilon \qquad (1)$$

Where

D^{co} = constitutive matrix of uncracked concrete between cracks

$$D^{co} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix}$$

D^{cr} = constitutive matrix of local cracks

$$D^{cr} = \begin{bmatrix} D_1^{cr} & 0 & \cdots \\ 0 & D_2^{cr} & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

$N = \left[N_1\, N_2 \cdots\right]$ is a transformation matrix, combining all the individual crack transformation matrices.

A new feature is added to the adopted crack model in this research. The following mode I local softening modulus for bilinear strain softening was specially developed by the authors (see figure 2):

$$D_{i,bl}^{I} = \frac{\alpha_2 + (1-\alpha_2)\alpha_1^2}{\alpha_2}\left(-\frac{f_t^2 h_c}{2G_f}\right) \qquad (2)$$

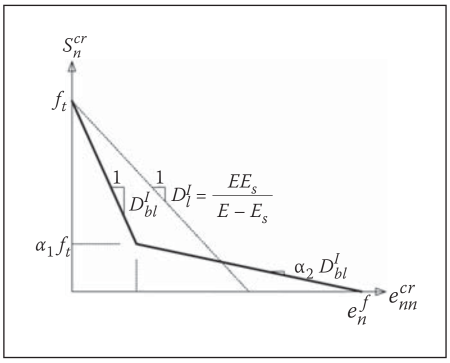

***Figure 2** Linear and bilinear strain-softening cracking models*

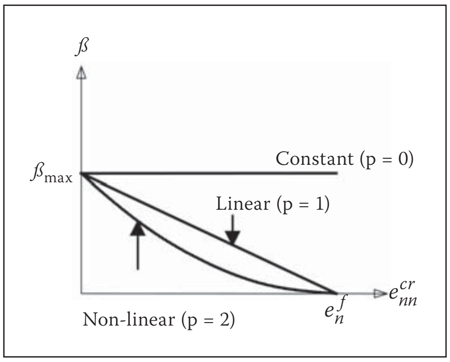

***Figure 3** Relationship between shear retention factor and crack normal strain*

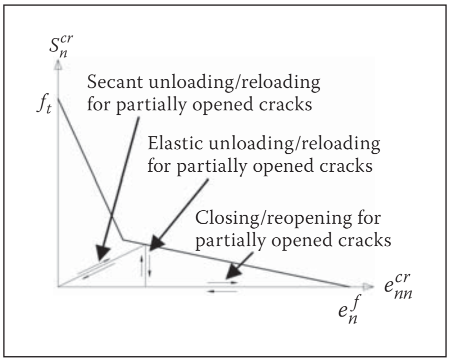

***Figure 4** Unloading/reloading, closing/ reopening crack response*

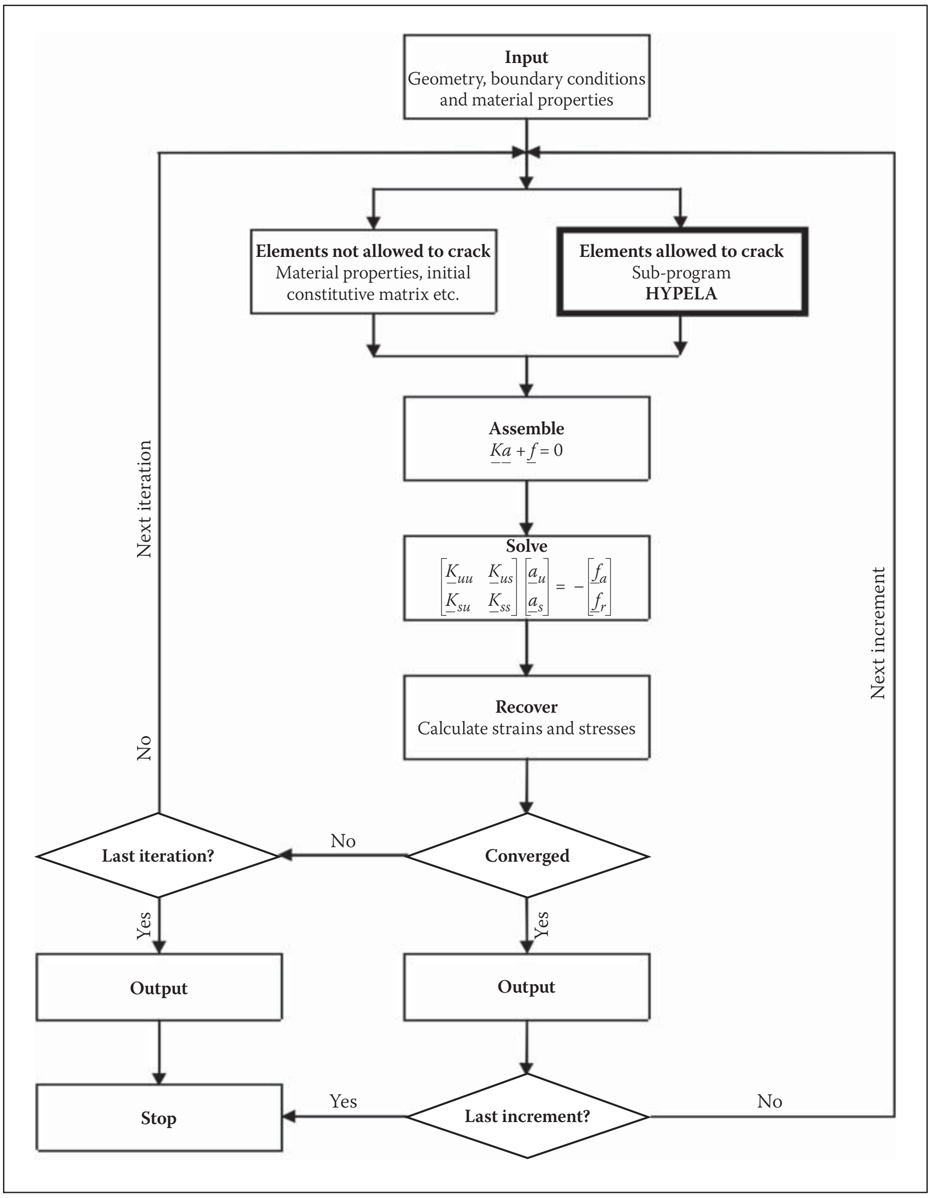

***Figure 5** Flowchart for finite element cracking analysis used in this study*

where α_1 and α_2 are bilinear softening shape parameters. G_f is the fracture energy of the material, f_t is the tensile strength, h_c is the crack band width.

By setting $\alpha_1 = 0$ or $\alpha_2 = 1$, $D_{bl}^I = D_l^I$, the strain softening becomes linear.

In figure 2, S_n^{cr} and e_{nn}^{cr} are respectively the normal stress and normal strain in the local crack, and e_n^f is the ultimate normal crack strain beyond which the tensile stress vanishes.

Shear softening (mode II) and unloading/reloading, closing/ reopening of cracks

An enhanced modification to the shear stiffness of a crack was made in this research. The shear retention factor is defined as a decreasing function of the crack normal strain in equation (3), which is similar to that used by Rots and Blaauwendraad (1989), except that a maximum shear retention factor β_{max} is defined here to limit the maximum shear allowed in a crack.

$$\beta = \beta_{max}\left(1 - \frac{e_{nn}^{cr}}{e_n^f}\right)^p \qquad (3)$$

where e_{nn}^{cr} and e_n^f are as defined previously and p is a constant defining the shear-softening shape. As shown in figure 3, if $p = 0$, $\beta = \beta_{max}$ (constant); if $p = 1$, the shear softening is linear; and if $p = 2$, the shear softening is non-linear.

The unloading/reloading and closing/ reopening strategy applied here is shown in figure 4. A secant unloading approach is adopted which implies that the crack stress-strain relationship follows a path back to the origin upon a strain reduction.

Implementation of the constitutive model in an FE analysis

The flow chart in figure 5 illustrates the general FE organisation and the implementation of the proposed constitutive concrete cracking model. The subroutine HYPELA was specially developed by the authors to implement the proposed constitutive concrete cracking model. The organisation of subroutine HYPELA is presented in figure 6.

VERIFICATION STUDY ON MESH OBJECTIVITY

A centrally loaded notched beam (see figure 7) is used to validate the mesh objectivity of the proposed cracking model. This beam was tested experimentally by Bažant and Pfeiffer (1987) and modelled numerically by Bhattacharjee and Leger (1993) and Cai *et al* (2006).

The model parameters used are listed in table 2.

The mesh objectivity verification analyses carried out by Bhattacharjee and Leger (1993) and Cai *et al* (2006) have the following limitations:

***Table 2** Model parameters (mesh objectivity verification)*

Constitutive parameters and dimensions of the FE model			
Young's modulus E (MPa)	27 413	Tensile strength f_t (MPa)	2,89
Poisson's ratio ν	0,18	Fracture energy G_f (N/m)	40,29
Thickness of beam (mm)	38,10	Depth of beam d (mm)	304,80

***Table 3** Model parameters (Case 1)*

Constitutive concrete parameters		Constitutive rock parameters	
Young's modulus E (MPa)	24 000	Young's modulus E (MPa)	41 000
Poisson's ratio ν	0,15	Poisson's ratio ν	0,1
Tensile strength f_t (MPa)	1,5	Mass density (kg/m^3)	0
Fracture energy G_f (N/m)	150		
Mass density (kg/m^3)	2 400		

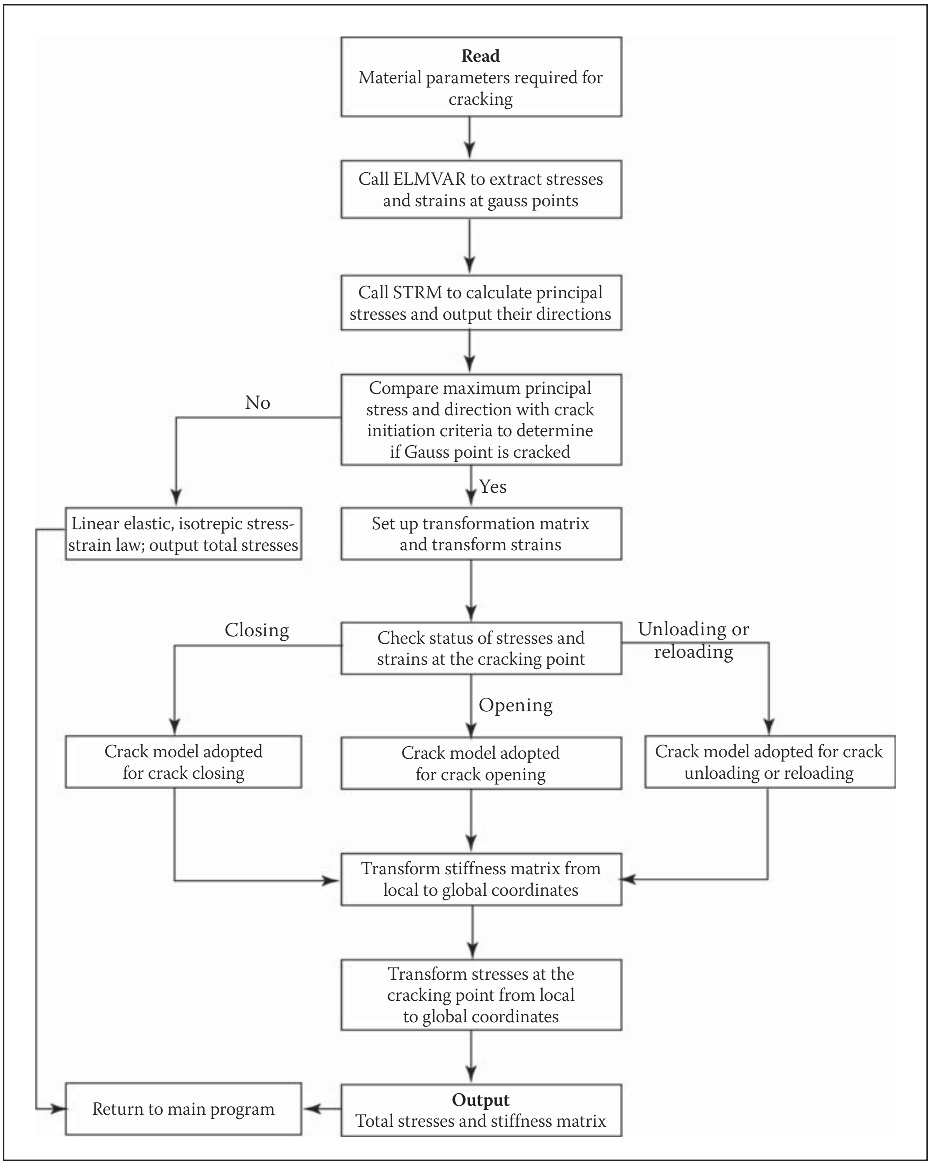

***Figure 6** Flowchart for subroutine HYPELA*

- The width of the notch in the three meshes is not fixed but varies with the element size used in the mesh
- The loadings in the three mesh models are not applied at the same distance to the centreline of the models, but vary with the element size used

To eliminate the above limitations, three FE models with 6, 12 and 24 elements through the depth of the beam were created, in which the position of the loadings and the width of the notches are kept the same in order to achieve the aim of this mesh objectivity verification (see figures 8 to 10).

Comparison of the results from this research with the experimental results is shown in figure 11. It can be seen that for the crack analysis method and procedures developed, convergence to a unique global response appears to be found with mesh refinement. Different meshes only result in a maximum discrepancy of approximately 7 % in the result of the $\frac{P_u}{P_0}$ ratio. The load P_0 required to cause crack-tip tensile stress equal to the tensile strength f_t is determined using elastic bending theory and the peak loading resistance P_u is obtained from the analyses. The difference in results between the strain-softening model and the experimental findings, as explained by Bhattacharjee and Leger (1993), stems from the fact that the constitutive model parameters had to be assumed since they were not available from the experimental results.

CASE STUDIES

Case study 1: A concrete gravity dam adopted by NW-IALAD

The internet network for the Integrity Assessment of Large Concrete Dams (Network IALAD 2005) was established for collaboration amongst researchers from across Europe. The objective of IALAD Task 2.4 was the systematic comparison of existing finite/boundary element methods, based on fracture/damage mechanics, for the fracture analysis of selected benchmark concrete dams.

The benchmark concrete gravity dam model selected for case study 1 is shown in figure 12. The dam has a height of 80 m, with a crest width of 5 m and a base width of 60 m. The boundary to the rock foundation was set at 120 m from each edge of the dam wall and 80 m deep below the base of the dam, fixing all degrees of freedom at the boundary (see figure 13). A perfect bond between the concrete wall and the rock foundation is assumed.

The loads applied to the model were the self-weight of concrete and a horizontal hydrostatic pressure, with the water level in the dam gradually increasing to the crest level (80 m) and then continuing to overflow to the maximum water level. Only the concrete wall is allowed to crack and no cracking is allowed in the rock.

The constitutive model parameters used in the analysis are given in table 3.

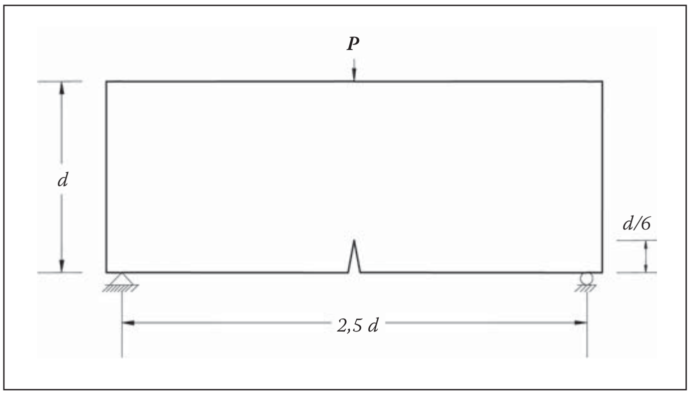

***Figure 7** Geometric configuration and boundary conditions of the beam for the mesh objectivity verification*

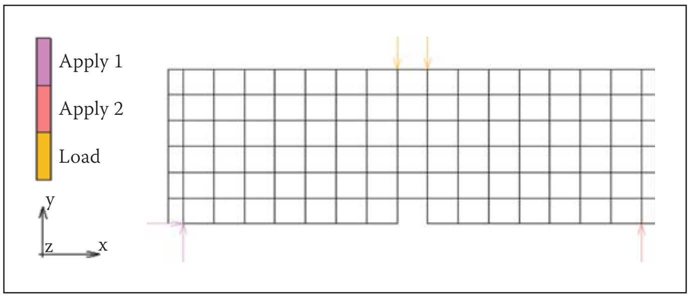

***Figure 8** Coarse model – 6 elements in depth*

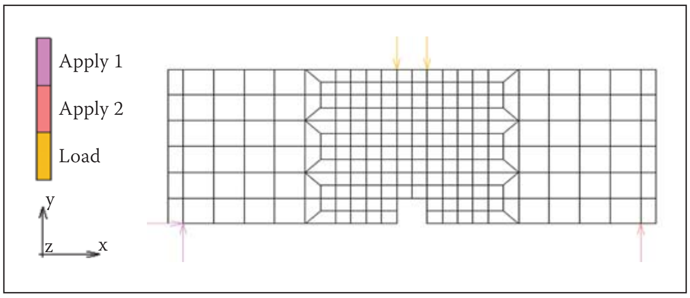

***Figure 9** Medium model – 12 elements in depth*

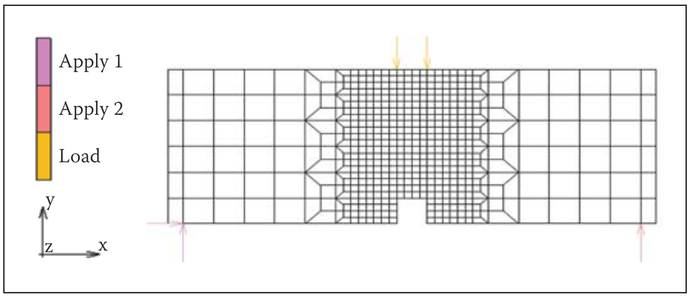

***Figure 10** Fine model – 24 elements in depth*

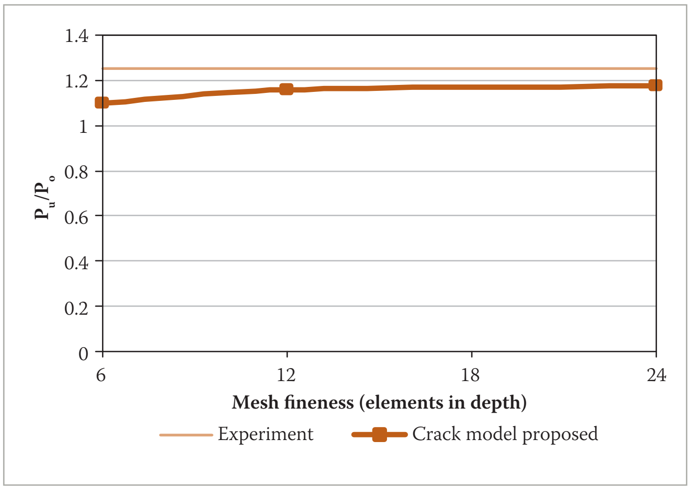

***Figure 11** Comparison of mesh objectivity*

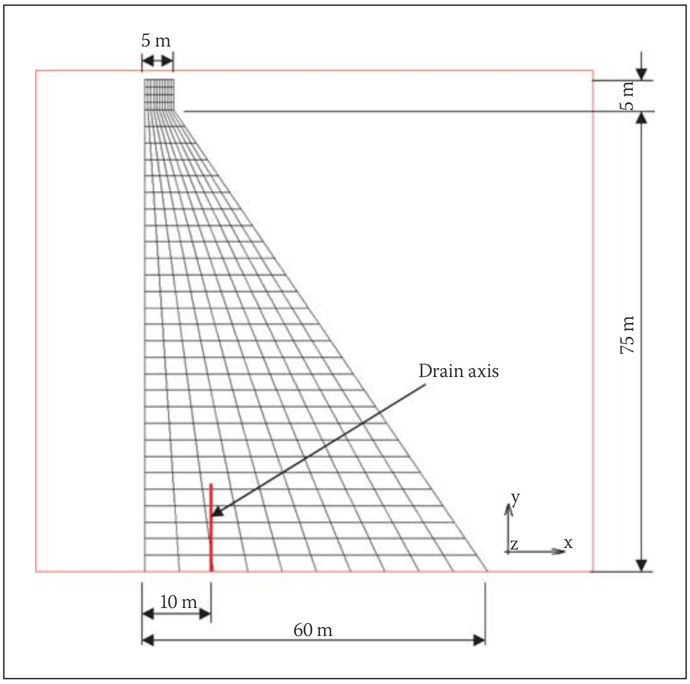

***Figure 12** Geometric configuration of concrete dam (Case 1)*

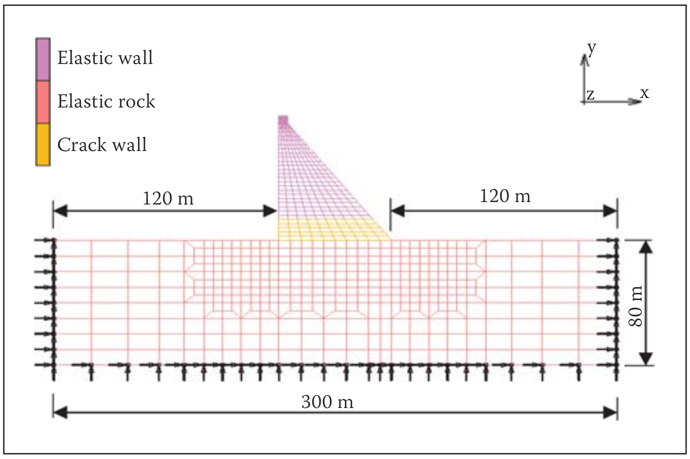

***Figure 13** Finite element model (Case 1)*

The above model was also analysed by other researchers (Jefferson *et al* 2005) using the FE programs LUSAS and DIANA. The same model parameters and loadings were assumed, except for the maximum hydrostatic overflow loading, which was set to 100 m and 90 m respectively for LUSAS and DIANA. A concrete model was used in LUSAS. The model uses a local Coulomb yield function to simulate directional fracture and isotropic compressive behaviour. The concrete was modelled with the total strain based rotating crack model in the DIANA analysis. The results from LUSAS and DIANA are presented for illustrative purposes.

The linear and bilinear softening models proposed in this research were verified by analysing the fracture response of the model dam. Four-noded quadrilateral plane strain elements were used and a modified Newton-Raphson solution technique was adopted for the non-linear equations (Owen & Hinton 1980).

The equivalent total strain resulting from bilinear softening in this research was plotted together with the predicted LUSAS crack planes (figure 14, red lines) as reported by Jefferson (2003). In this smeared cracking approach, cracking is indicated by high strains and the equivalent total strain in MSC.Marc gives an indication of how and where the crack grows. The strain plots show good agreement with the

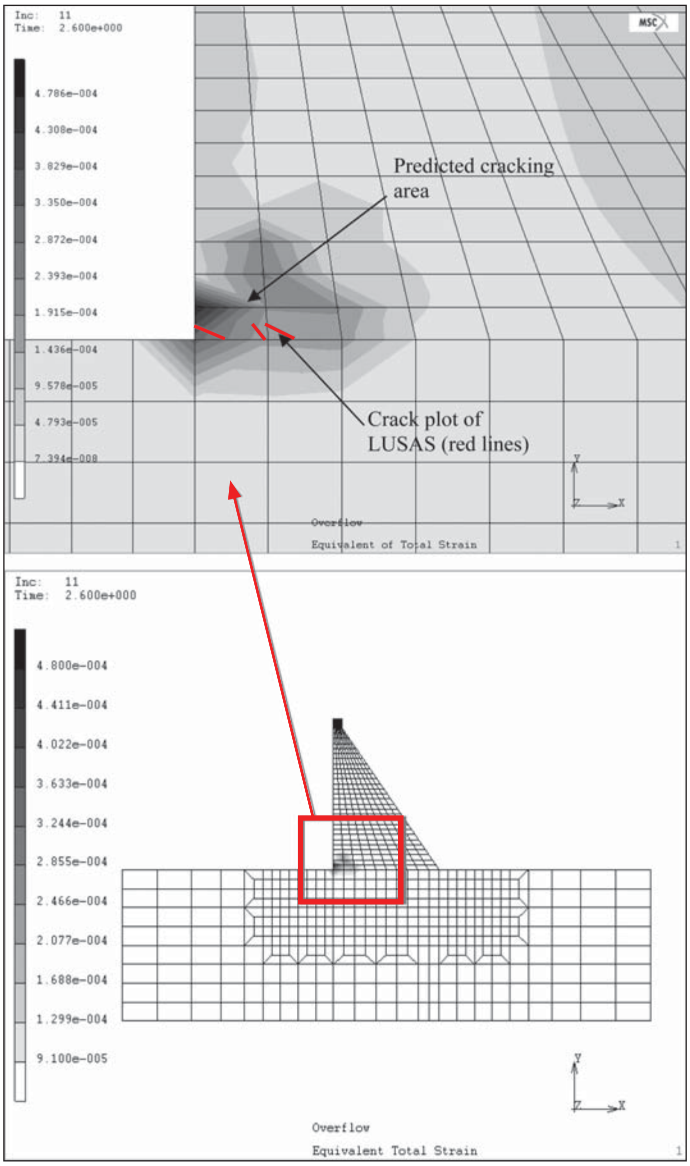

***Figure 14** Crack plots (Case 1)*

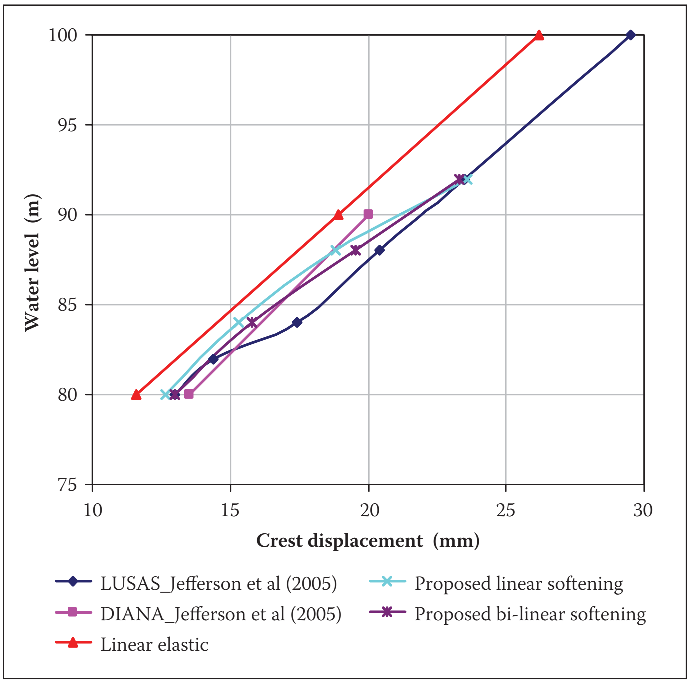

***Figure 15** Water level (overflow) vs. crest displacement (Case 1)*

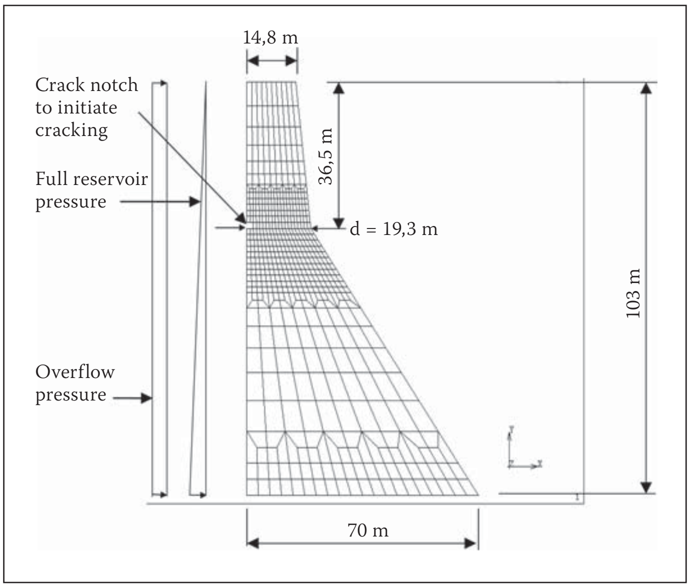

***Figure 16** Finite element model of Koyna Dam and applied loads (Case 2)*

***Table 4** Model parameters (Case 2)*

Dimensions of the model (m)		**Constitutive parameters**	
Dam height	103	Young's modulus E (MPa)	25 000
Crest width	14,8	Poisson's ratio ν	0,2
Bottom width	70	Tensile strength f_t (MPa)	1,0
Width of dam at the level of initial notch d	19,3	Fracture energy G_f (N/m)	100 or 200
Depth of initial notch	0,1d	Mass density (kg/m^3)	2 450

predicted cracks, although in this analysis the crack extends slightly further and also over a wider area (refer to figure 14).

The relationship between the water level (overflow above 80 m) and the crest displacement, as obtained from this research, is compared with the results from LUSAS and DIANA (Jefferson *et al* 2005) in figure 15. The LUSAS results clearly show a change in overall stiffness after cracking, while the DIANA results do not exhibit this behaviour. In the results from this research, the linear softening shows less deformation than the bilinear softening, indicating that the bilinear softening simulates the loss in stiffness caused by fracture more accurately than the linear softening. Nevertheless, both the linear and bilinear softening models exhibit a strain-softening behaviour that is in good agreement with the LUSAS and DIANA results. In this research the analysis was terminated at a water level of approximately 92 m. This should not be regarded as the failure water level since no effort was made to increase the accuracy at failure by refining the mesh or adjusting the convergence tolerance.

Case study 2: Koyna Dam

Gioia *et al* (1992) analysed the Koyna Dam (subjected to reservoir overflow loading) using a plasticity-based model and LEFM. In the study of Gioia *et al* (1992), three positions of a pre-set crack were studied and it was found that a crack located on the upstream side, as shown in figure 16, is the most critical position. This pre-set notch was created for the 'seeding' of crack propagation. Severe damage actually occurred at this location when an earthquake struck the Koyna Dam in 1967. All the past investigations (Gioia *et al* 1992; Bhattacharjee & Leger 1994; Ghrib & Tinawi 1995) have adopted the same 'seeding' position in order to have a meaningful comparison. Bhattacharjee and Leger (1994) and Ghrib and Tinawi (1995) analysed this dam using a smeared NLFM with a linear softening

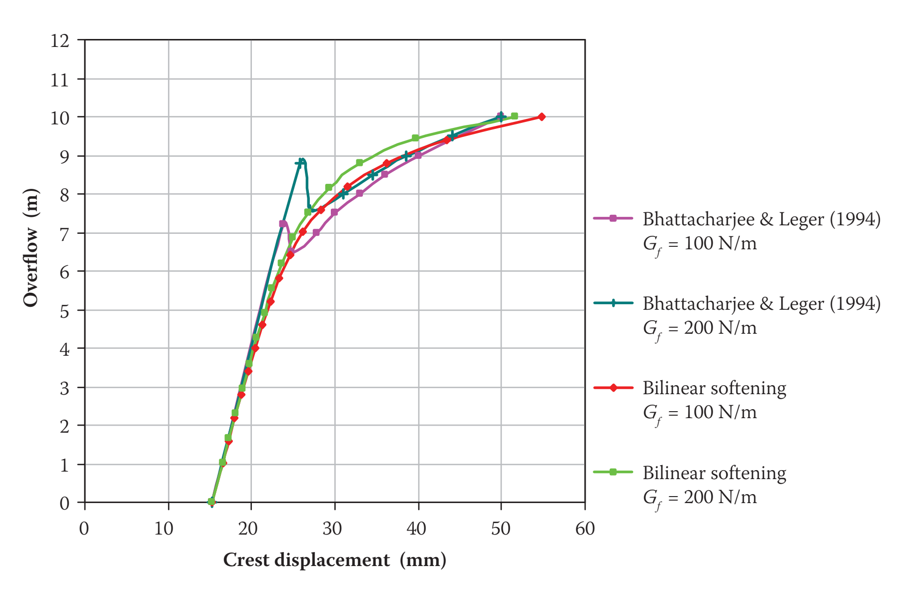

***Figure 17** Influence of fracture energy G_f on predicted structural response for bilinear softening models (Case 2)*

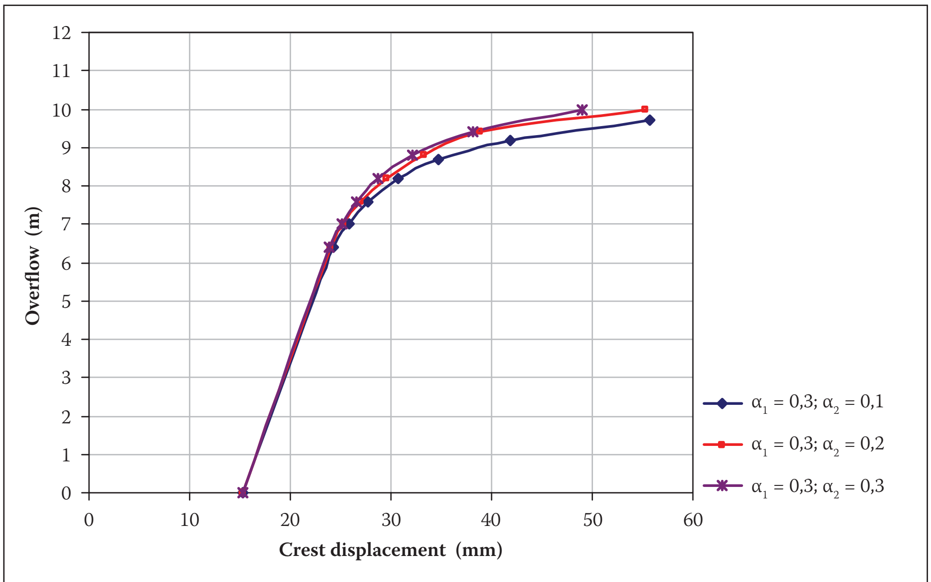

***Figure 18** Influence of bilinear softening parameters α_1=0,3; α_2=0,1, 0,2 and 0,3 respectively on predicted structural response (Case 2)*

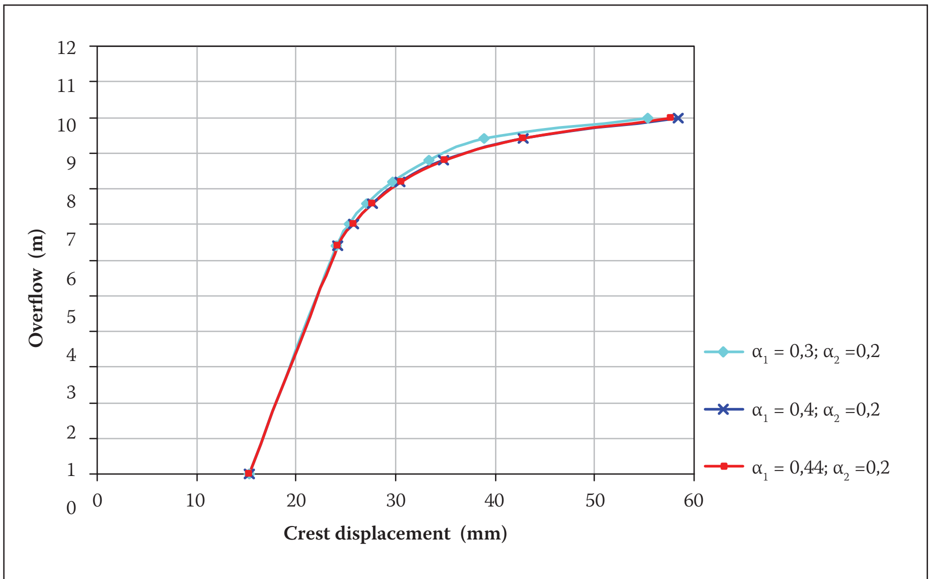

***Figure 19** Influence of bilinear softening parameters α_1=0,3, 0,4 and 0,44; α_2=0,2 respectively on predicted structural response (Case 2)*

and a damage mechanics approach respectively. In this study, the geometric configuration of the FE model is the same as that adopted by Bhattacharjee and Leger (1994) and Ghrib and Tinawi (1995), as shown in figure 16.

To be consistent with the Bhattacharjee and Leger (1994) model, and to allow comparison of the results, a four-noded, full integration, plane stress element is adopted. The dam structure should be analysed with plane strain elements as done in the next case study 3. The reason of using plane stress approach is to compare the results with the past investigations where plane stress elements were also adopted. The model is subjected to gravity loads together with a hydrostatic pressure at full-reservoir level and an overflow loading. Water pressures inside the cracks are not considered in this study. The overflow-crest displacement relationship and the crack profile agree closely with the results obtained by Ghrib and Tinawi (1995) and Bhattacharjee and Leger (1994).

Table 4 presents the data used in the FE model and analysis.

The present analysis is aimed at determining the fracture response of a full gravity dam subjected to general gravity loads and hydrostatic pressures, and to investigate the sensitivity of the following parameters:

- Fracture energy, G_f
- Bilinear softening shape parameters (α_1 and α_2)
- Threshold angle, ϕ
- Maximum shear retention factor β_{max}

The influence of the fracture energy G_f on the predicted structural response is shown in figure 17. When G_f is increased from 100 to 200 N/m, the initial peak crack resistance of the structure is also increased. After cracking, the initial stiffer response associated with the higher fracture energy (G_f = 200 N/m) gradually reduces to approach the response of the lower fracture energy (G_f = 100 N/m), eventually yielding a similar ultimate response for the two values of G_f.

The influence of the bilinear softening shape parameters, α_1 and α_2, on the predicted structural response is shown in figures 18 and 19. The fracture parameters used in these analyses are G_f = 100 N/m, threshold angle = 30° and maximum shear retention factor β_{max} = 0,1. In figure 18, where α_1 is fixed at 0,3 while α_2 is increased from 0,1 to 0,3, the structural responses are similar, with a slight increase in stiffness as α_2 increases.

In theory, when α_2 increases, the first softening modulus (absolute value) will decrease, while the second softening modulus (absolute value) will increase. This

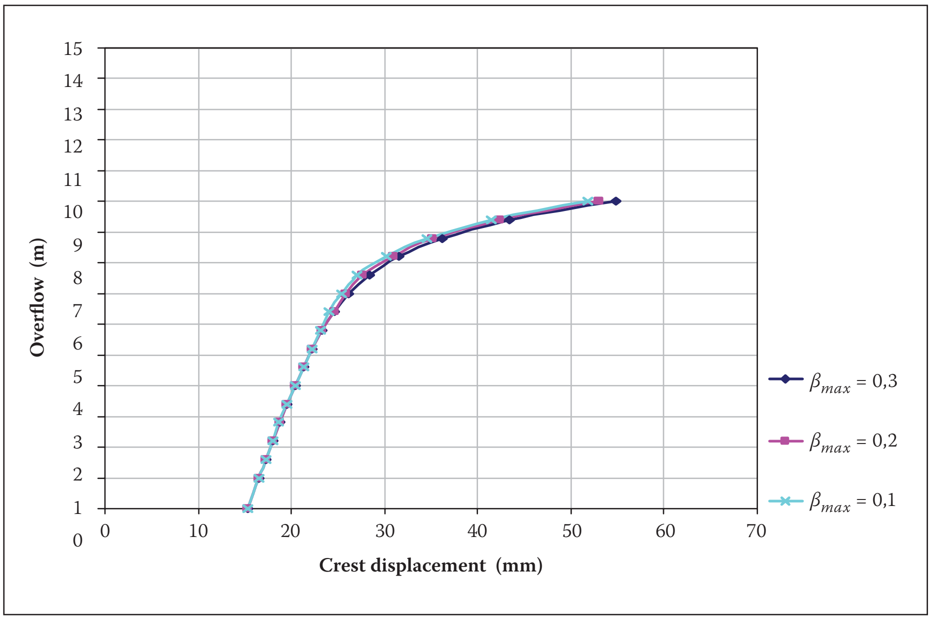

***Figure 20** Influence of maximum shear retention factor β_{max} on predicted structural response (Case 2)*

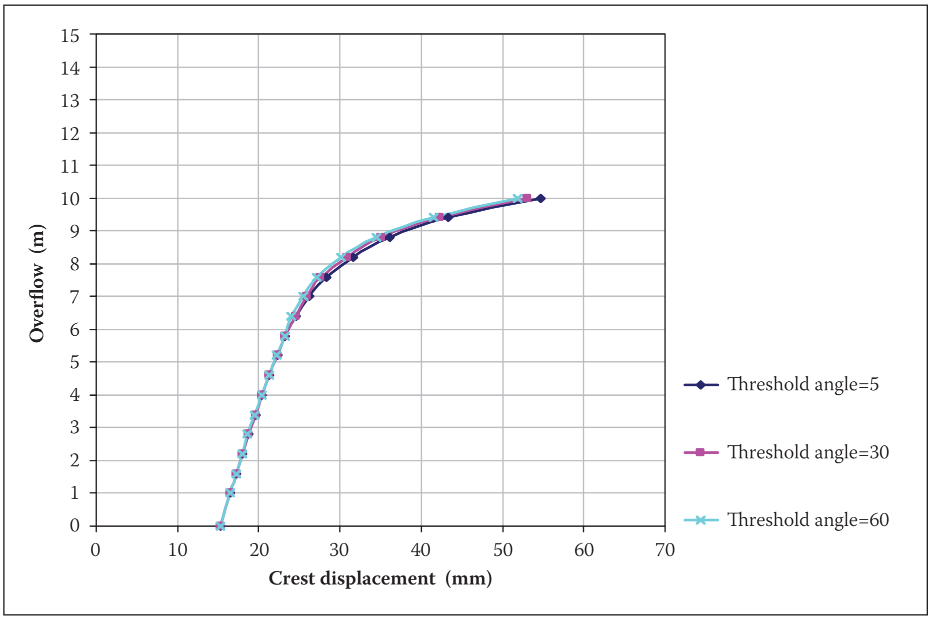

***Figure 21** Influence of threshold angle on predicted structural response (Case 2)*

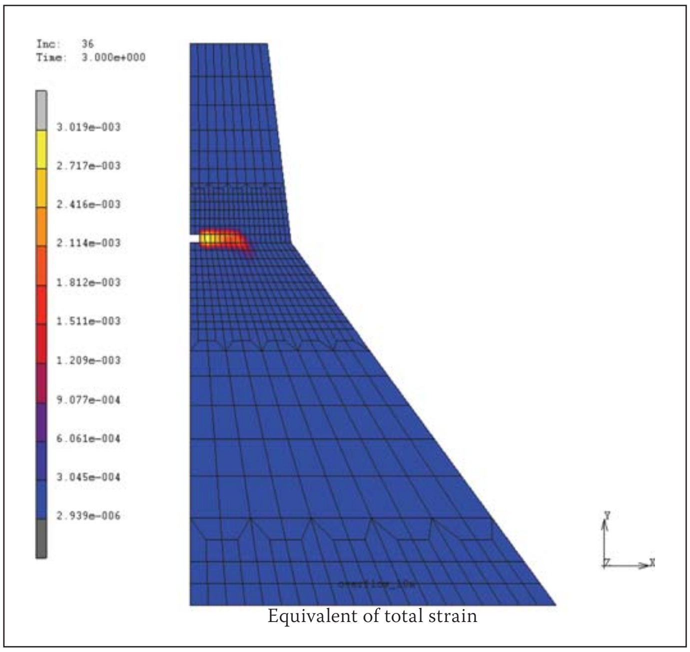

***Figure 22** Crack profile (Case 2)*

implies that the first softening modulus plays a more important (dominant) role when the structure starts to crack. The correspondence of the smaller first softening modulus to the greater α_2 value means that the localised softening provides a stiffer structural response. Gradually, the second softening modulus starts to influence the structural response, leading to a similar ultimate response with different values of α_2.

When α_2 is set equal to 0,2 while α_1 increases from 0,3 to 0,44, the predicted structural responses are similar, indicating that α_1 has a minor influence on the structural response (see figure 19).

Similarly, the maximum shear retention factor β_{max} and the threshold angle for the crack onset criterion have a minor influence on the predicted structural response, as shown in figures 20 and 21 respectively.

The crack profiles predicted by introducing the different constitutive fracture parameters (such as G_f, α_1, α_2, etc), are in close agreement with the crack profiles predicted by Bhattacharjee and Leger (1994). Initially, the crack profile extends horizontally before gradually bending downwards due to the existence of compressive stresses on the downstream side. A typical crack profile is shown in figure 22.

From the above sensitivity study it is observed that the localised fracturing as influenced by the constitutive fracture parameters (G_f, threshold angle, β_{max}, α_1 and α_2) does not have a significant role on the 'overall'structural response (such as crest displacement). The fracture parameters would however have a much greater influence on the 'local" fracturing behaviours such as the crack propagation path as shown in the case study 3.

***Table 5** Material properties of concrete and rock (Case 3)*

Concrete wall		Rock foundation	
Young's modulus E (MPa)	28 000	Young's modulus E (MPa)	30 000
Poisson's ratio ν	0,2	Poisson's ratio ν	0,22
Tensile strength f_t (MPa)	1,5	Tensile strength f_t (MPa)	2,5
Mass density (kg/m^3)	2 455	Mass density (kg/m^3)	0
Cohesion (MPa)	2,41	Cohesion (MPa)	1 ~ 10
Frictional angle	55º	Frictional angle	39º
Coefficient of thermal expansion	10^{-5}/ºC		

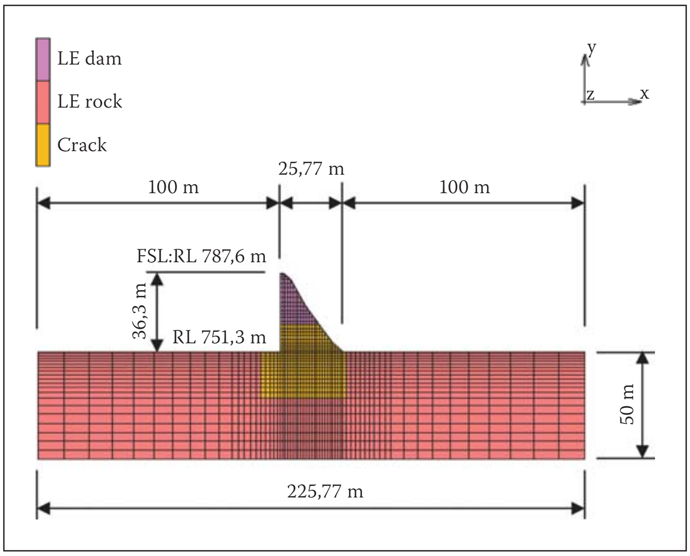

***Figure 23** Finite element model (Case 3)*

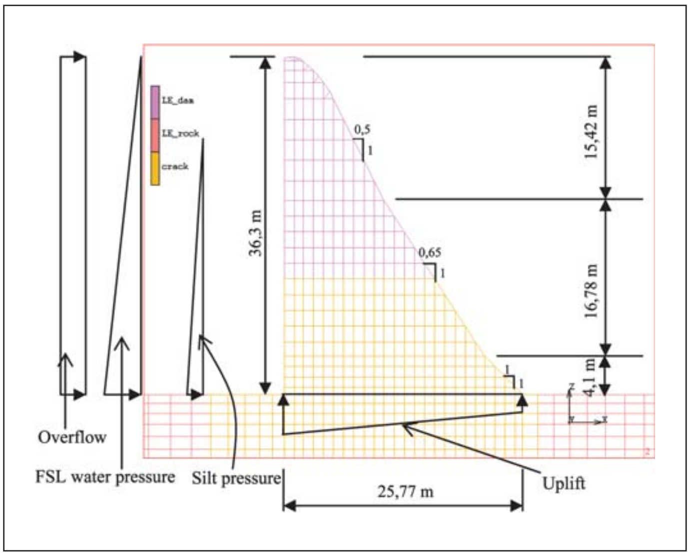

***Figure 24** Finite element model and external loadings applied (Case 3)*

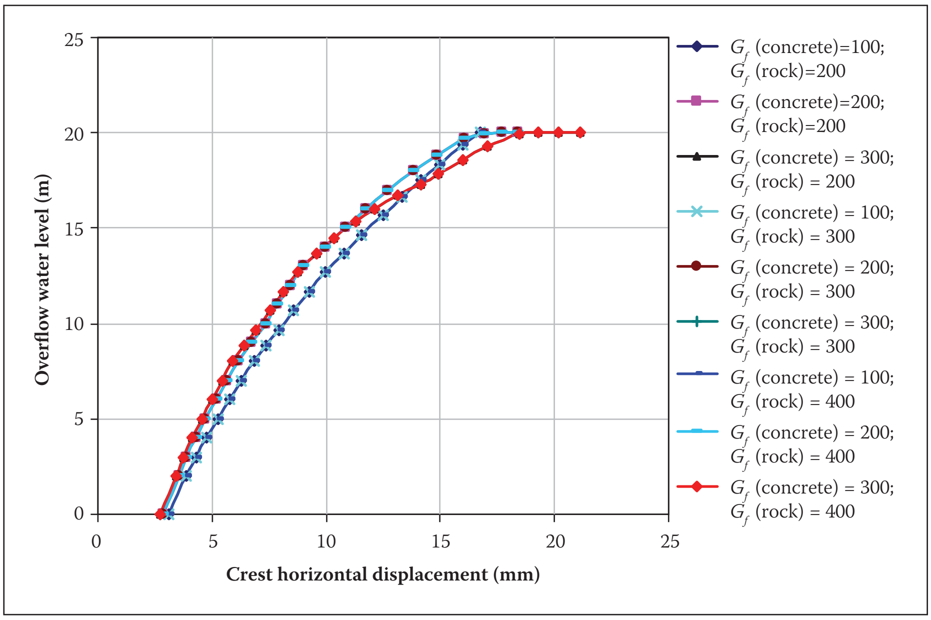

***Figure 25** Crest horizontal displacement vs overflow (Case 3)*

Case study 3: An old existing gravity dam in South Africa

Cracking in concrete gravity dams endangers their safety and it therefore needs to be accurately simulated and analysed. For the third case study, the authors applied the crack analysis method developed to one of South Africa's existing gravity dams, firstly to predict crack propagation and secondly, to evaluate the safety of the dam during crack development.

The dam is a 33 m high concrete gravity dam completed in 1925. In the FE model shown in figures 23 and 24 it was conservatively assumed that the average critical level of the concrete/rock interface, over the central part of the dam, was 5,7 m below the riverbed level (reduced level (RL) of interface = 751,30 m) (Seddon *et al* 1998). This level was extracted from the site 'progress of construction' drawing and is confirmed by borehole logs in Schall's report (1988). First-order, full integration, plane strain elements with bilinear strain softening were used in the analysis. Both the horizontal and vertical translation degrees of freedom were fixed at all nodes along the outer edges of the foundation, excluding the nodes at the top face supporting the base of the dam.

The dam is loaded by self-weight, hydrostatic pressure at full supply level (FSL), silt pressure, overflow up to 20 m, uplift pressure and a seasonal temperature drop in the dam wall.

The concrete material properties for the dam were determined from tests on drilled cores (Van der Spuy 1992) and are summarised in table 5.

Samples of the rock foundation were obtained by drilling five vertical holes through the dam wall and into the rock. Visual inspection and laboratory testing of the rock samples indicated that the bedrock is sound dolerite of excellent quality (Schall 1988). The material properties of the rock are also presented in table 5.

The fracture parameters used for all analyses in this case study are: bilinear

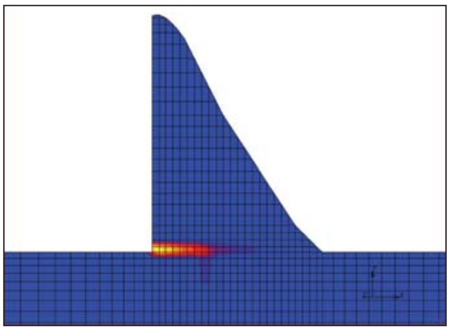

***Figure 26** Crack profile for G_f^c = 100 N/m and G_f^r = 400 N/m (Case 3)*

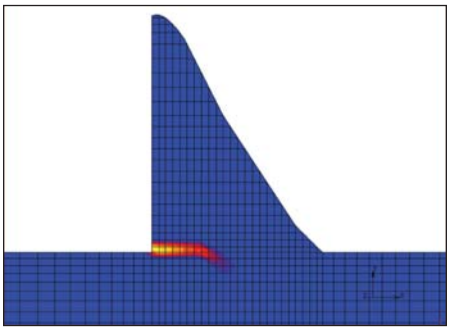

***Figure 27** Crack profile for G_f^c = 200 N/m and G_f^r = 400 N/m (Case 3)*

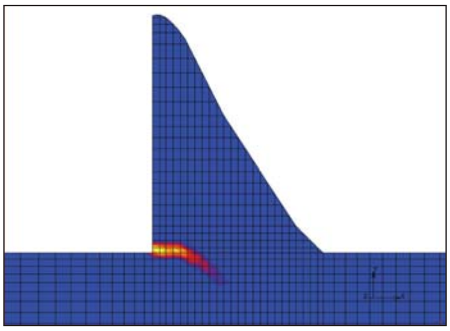

***Figure 28** Crack profile for G_f^c = 300 N/m and G_f^r = 400 N/m (Case 3)*

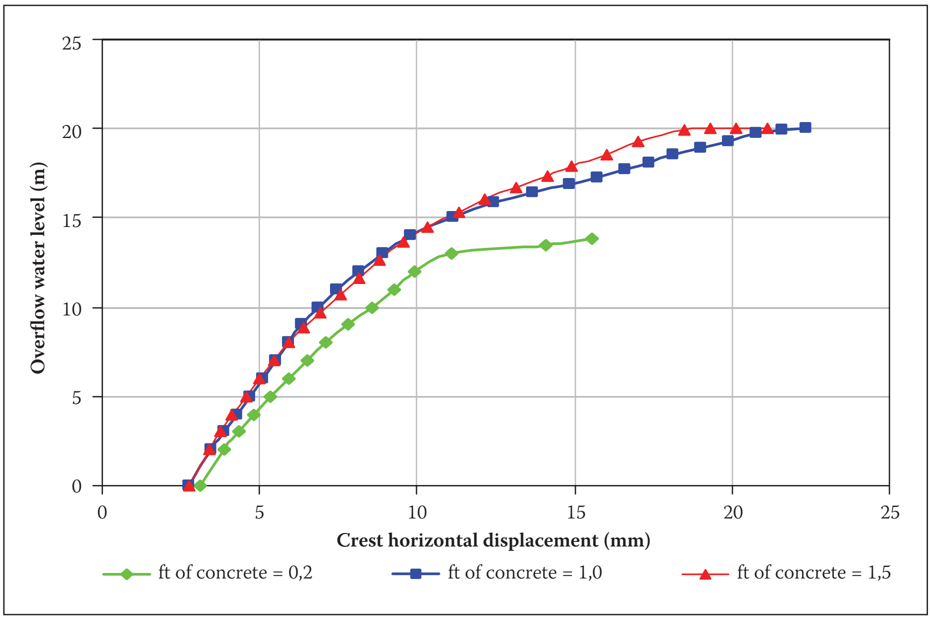

***Figure 29** Crest horizontal displacement vs overflow (Case 3)*

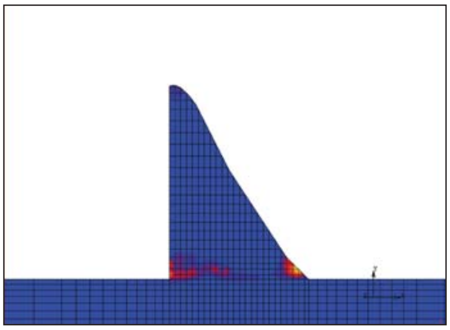

***Figure 30** Crack profile for f_t^c = 0,002 MPa and f_t^r = 2,5 MPa (Case 3)*

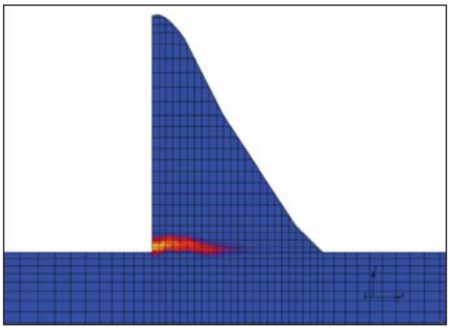

***Figure 31** Crack profile for f_t^c = 0,2 MPa and f_t^r = 2,5 MPa (Case 3)*

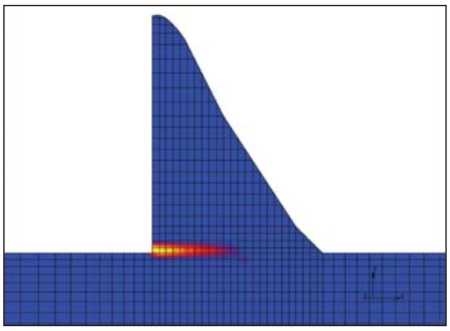

***Figure 32** Crack profile for f_t^c = 1,0 MPa and f_t^r = 2,5 MPa (Case 3)*

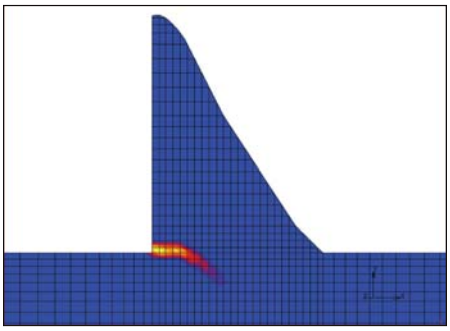

***Figure 33** Crack profile for f_t^c = 1,5 MPa and f_t^r = 2,5 MPa (Case 3)*

shape parameters α_1 = 0,4 and α_2 = 0,05; and threshold angle ϕ = 30º.

Parametric study on the fracture energy of concrete and rock

A sensitivity study on the concrete fracture energy, for G_f^c ranging from 100 to 300 N/m and the rock fracture energy G_f^r ranging from 200 to 400 N/m, was carried out. A maximum shear retention factor β_{max} = 0,1 and tensile strengths for the concrete and rock of f_t^c = 1,5 and f_t^r = 2,5 MPa respectively, were selected. The different combinations of the fracture energy of concrete and rock based on the above ranges were used in the crack analysis of this dam.

The results of crest horizontal displacement versus overflow water level are shown in figure 25. The fracture energy of the rock G_f^r appears to have little influence on the crack response of the dam. The structural behaviours of the same fracture energy of concrete G_f^c with different fracture energies of rock G_f^r are nearly identical. At low overflow water level, the lower fracture energy of concrete G_f^c (100 N/m) results in a higher crest deformation. As the overflow water level increases to a higher level (approximately 17 m overflow), the crest deformation for the higher fracture energy of concrete G_f^c (300 N/m) becomes larger and increases at a higher rate. The fracture energy of concrete G_f^c = 300 N/m and rock G_f^r= 400 N/m would cause the highest deformation in the dam.

It appears that the fracture energy of concrete and rock in general do not have a significant influence on the overall dam deformation. Nevertheless, the fracture energy of concrete G_f^c has a significant influence on the crack propagation paths in the dam structure, as shown in figures 26 to 28. As the fracture energy of concrete G_f^c increases, the crack tends to deviate more from the initial horizontal direction along the concrete/rock interface and extends further into the rock foundation.

Parametric study on the tensile strength of concrete

Testing of concrete cores taken from the dam concrete showed that the concrete has a tensile strength of 1,5 MPa (Van der Spuy 1992). The sensitivity of the crack response of the dam to the tensile strength of the concrete is investigated by fixing the tensile strength of the rock f_t^r at 2,5 MPa, while increasing the tensile strength of the concrete f_t^c from 0,002 to 1,5 MPa. Fracture energies of G_f^c= 300 N/m and G_f^r= 400 N/m, and a maximum shear retention factor β_{max} = 0,1 were assumed.

For f_t^c = 0,002 MPa, representing no tensile strength at the concrete/rock interface (Seddon *et al* 1998), the dam would crack through and fail even before water reached the full supply level (FSL). Increasing f_t^c results in a smaller crest displacement with the crack response significantly influenced by the tensile strength of the concrete, as shown in figure 29.

As f_t^c increases, the crack tends to deviate from the concrete/rock interface and extend into the rock foundation, as shown in figures 30 to 33.

Comparison study and safety evaluation of the dam

A linear elastic analysis was also carried out and the results are presented in figure 34. Sensitivity studies on other fracture parameters, such as the bilinear shape parameters α_1 and α_2, the crack onset threshold angle ϕ and the maximum shear retention factor β_{max}, were also carried out, but due to the space limitation, these responses are not presented here. For the fracture analysis of the dam, the crest displacement increases rapidly at the overflow water level of approximately 17 m above FSL. It therefore appears that the dam can

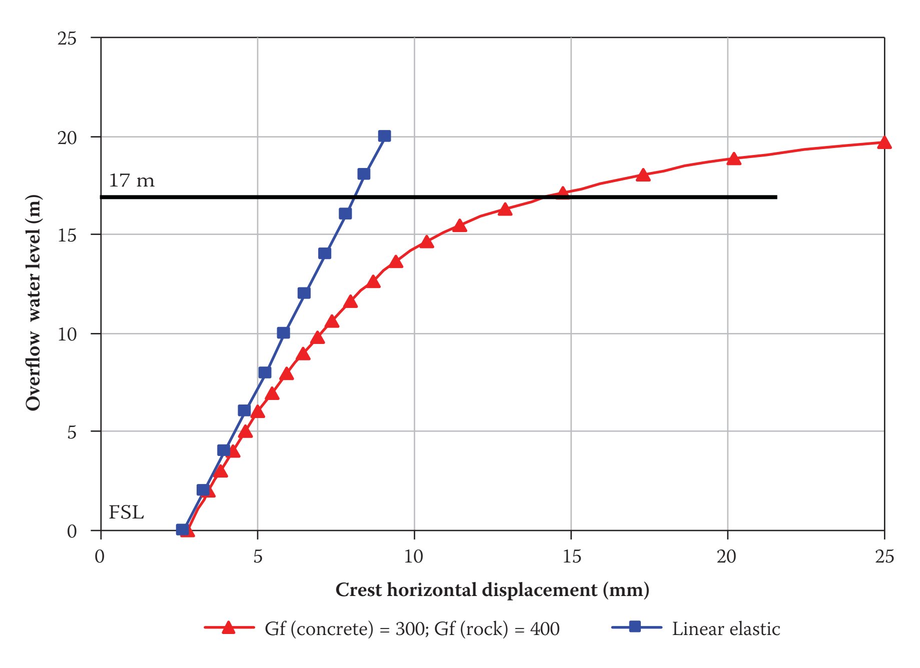

***Figure 34** Crest horizontal displacement vs overflow (Case 3)*

be regarded as unsafe when the overflow water level reaches approximately 17 m which is higher than the failure flood level obtained by the classical strength-based methods.

CONCLUSIONS

A smeared crack model, based on non-linear fracture mechanics, which allows for either linear or bilinear softening and assumes a shear retention dependent on the strain normal to a crack, has been presented. A mesh objectivity verification study has been carried out and it was shown that the proposed crack modelling method is mesh objective. Three case studies (a gravity dam benchmark model and two existing gravity dams) have been undertaken, which have indicated the usefulness and applicability of the proposed cracking constitutive model and implementation procedure in predicting the crack response of concrete gravity dams and evaluating the safety of a dam against cracking. A higher imminent failure flood level is predicted for the existing gravity dam in South Africa (case study 3) in this smeared fracture analysis than that obtained by the classical strength-based methods. In the smeared crack approach, crack orientation may subject to element type or alignment.

To cover uncertainties regarding the material fracture properties and the fracture parameters of the concrete, a sensitivity study of their influence on the fracture response of Koyna Dam (case study 2) and parametric analyses for an appropriate structural evaluation concerning the safety of the old gravity dam in South Africa (case study 3) have been undertaken. The influence of the fracture parameters on the cracking response of the dams can be summarised as follows:

- In general, the localised fracturing, which is affected by material fracture properties and fracture parameters such as the fracture energy G_f, the bilinear softening shape parameters α_1/α_2, the tensile strength of concrete f_t, the maximum shear retention factor β_{max} and the threshold angle ϕ, etc, does not significantly influence the 'overall' structural displacement behaviour.
- Nevertheless, the above fracture properties and parameters would have a considerable influence on the path of crack propagation along the interface of the concrete wall and the rock foundation, as in case study 3.

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